TECHNICAL BULLETIN No. 691 OCTOBER 1939

THE GLUED LAMINATED WOODEN ARCH

By

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UNITED STATES DEPARTMENT OF AGRICULTURE, WASHINGTON, D. C.

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By T. R. C. WILSON [2]

Senior engineer, Forest Products Laboratory,[3] Forest Service

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[3] Maintained by the U. S. Department of Agriculture at Madison, Wis., in cooperation with the University of Wisconsin.

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INTRODUCTION

For many centuries the arch has been an important element in the structural and artistic design of buildings and bridges. Structurally it is a means of supporting loads over an opening between supports. As distinguished from the truss or the simple beam that may serve this same purpose, an arch carrying vertical loads exerts horizontal thrust as well as vertical pressure on its supports. An arch may consist either of a solid rib or ring or of a framework similar to a truss.

The arch of masonry blocks was one of the earliest forms but lacking strength in tension it had to be thick and heavy in order to avoid tensile stress. Later, metals and more recently, reinforced concrete, materials with tensile and bending as well as compressive strength, permitted more efficient, less massive, and more graceful construction.

Wood has good resistance to tension and compression and to the bending to which arch ribs may be subjected, but its use in arches has been retarded by the difficulty of making the required shapes without incurring certain disadvantages. Ribs of the necessary size and curvature cannot be made by bending a single piece of timber. They can be formed only by assembling pieces sawed to shape or by superimposing bent layers or laminae. Both methods have been practiced but the arches have lacked efficiency: the first type because the assembly is weakened by the necessary joints and by nonparallelism of wood fibers to the axis; the second because available mechanical connections such as nails, screws, bolts, or dowels permit sliding of one lamina on another and thus do not cause the assembly of laminations to act as a unit.

Efficient laminated wooden arch ribs and other members are now available through the use of glues of proven durability to bond laminae together as a unit so that resistance to sliding or shear is as great between the layers as within the wood itself.

PURPOSE AND SCOPE

The purpose of this bulletin is to discuss recent research on the strength of glued laminated construction as conducted at the Forest Products Laboratory and to present the resulting recommendations for specifications and design stresses, together with other topics and information useful to engineers, architects, and builders. Much of the information applies to other structural members, such as straight, cambered, or curved beams and curved chords for trusses, as well as to arches. Framework arches [4] are not considered.

Glued laminated structural members as discussed herein include:

1. Arches and other curved members formed by bending boards, or laminations, to the required curvature and gluing them together. Bending and gluing are done in one operation. Thickness of laminations is so adjusted to the curvature that steaming or other softening treatment as used in bending wood for other purposes is not required.

2. Straight members for use as beams made by gluing boards or laminations together with their width horizontal, that is, parallel to the neutral surface.

[4] Development during recent years of improved means for joining wooden parts increases the efficiency of timber in framed arches, trusses, and towers. These are discussed and data for their use presented under Modern Connectors in the Wood Handbook (*30*).[5]

[5] Italic numbers in parentheses refer to Literature Cited, p. 121.

ADVANTAGES AND DISADVANTAGES OF GLUED LAMINATED CONSTRUCTION

Among the advantages and possibilities that may be cited for glued laminated construction of structural members are the following:

Arches to span large unobstructed areas with superior architectural effect are made available. An arch rib, furthermore, being in a single large cross section, involves no intricate framing and has better fire resistance than an equivalent truss composed of smaller pieces.

Material of the sizes used in laminated construction can be dried in a short time. Hence, thoroughly seasoned members that will be subject to only a minimum of warping, twisting, and shrinkage after installation can be provided quickly.

Members can be built up to larger cross section and greater length than are otherwise readily available or than can be shipped conveniently over long distances.

Members can be built up from material that is too small to be structurally useful otherwise.

Laminations can be positioned in accordance with their strength characteristics as determined by species, density, and defects, thus gaining the advantage of material of high strength at points where such material is needed.

Beams can be cambered to overcome the undesirable appearance resulting from obvious sagging. Members may be tapered in depth for more graceful appearance and to save material that in a member of constant cross section contributes but little to strength and stiffness.

In some types of trusses, curved chords continuous through several panels can be substituted for panel-length chord parts with resultant simplification of joints.

Glued laminated construction has a record of successful use in Europe during a third of a century. It is reported by Swiss engineers to be highly resistant to chemical deterioration and is, consequently, widely used where metal structures or metal in connector-built wooden structures would be subject to corrosion.

Among the disadvantages and limitations of glued laminated construction may be cited the following:

In some parts of Europe, glued laminated members are said to be less economical than other forms of construction and other materials. Continued popularity and current use in other parts indicate that any increased cost is offset by superior appearance and other desirable qualities. American experience is that at least some types of structures cost less with glued laminated arches as roof supports than with alternative constructions because the required headroom can be provided with lesser height of side walls and with less total space under the roof. The character of the glues that are now applicable does not permit the use of glued laminated wooden members under all conditions of exposure. Further developments in glues, together with the use of wood treated to increase its durability and fire resistance, may be expected to extend the field of use of such members.

Glued laminated construction is not applicable to structures that must be completed within 2 or 3 weeks after the material is cut from the log.

The efficiency of glued laminated construction, depending as it does upon proper gluing, requires skilled workmanship and careful manipulation during assembly.

USES OF GLUED LAMINATED CONSTRUCTION

The most extensive use of glued laminated construction has been in Germany, Sweden, and Switzerland, with lesser development in Denmark and Norway and with some reported use in Austria, Czechoslovakia, France, and Italy. Glued laminated construction has only recently been introduced in the United States. American applications are illustrated in figures 1 to 9 and in numerous figures presented later.

European uses of glued laminated arches include erection forms for stone and concrete arch bridges; and roof supports in airplane hangars

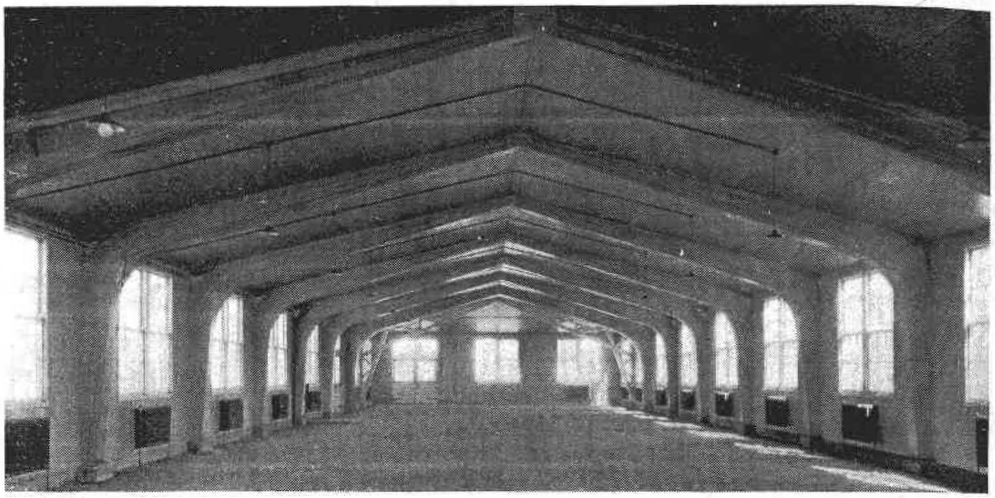

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FIGURE 1.—Service building at Forest Products Laboratory, 46 feet wide by 160 feet long, with nine arch spans spaced 16 feet. The five central spans (D-type) are glued laminated arches of rectangular cross section and constant width tapering from a maximum depth at the knee, near the junction of wall and roof, to lesser depths at the foundation and the roof peak. Next to each end span is a wooden arch of double I-section (C-type) composed of plywood webs, similarly tapering in depth, and glued laminated flanges of constant cross section. End spans are a trussed type framed with modern wood connectors. For drawings and dimensions of arches see figure 38.

(*23*), auditoriums (*22*), bandstands, bathhouses, chemical and other factories, churches, concourses of important railway stations, exhibit halls, engine houses (*2*, *4*, *22*), garages, growing houses for plants, gymnasiums, planetariums, residences, riding academies (*25*), schools, stock barns (*25*), streetcar barns (*23*), tennis halls (fig. 10), theaters, and warehouses; and in addition a few foot and vehicle bridges. European applications of glued laminated construction include also, particularly in Sweden and Switzerland, a large number of railway station platform roof supports (*7*, *23*) involving a variety of members, such as curved braces (fig. 11, *A*), false arches (fig. 11, *B*), curved beams (fig. 11, *C*) and arches (fig. 11, *D*).

The principal conclusions from inspections made by the author on glued laminated construction in Europe are presented together with other data on European developments and practice on pages 87–94.

Definite information on the first uses of glue in structural members in the United States is lacking but apparently no extensive development occurred prior to the installation of glued laminated arches in the service building (fig. 1) at the Forest Products Laboratory in 1935 (*27*). The company that built these arches has subsequently made glued laminated arches and other members for nearly a hundred buildings distributed in a number of States. Included are churches, community halls, garages, and gymnasiums. The same company, together with another specializing in farm structures, has supplied

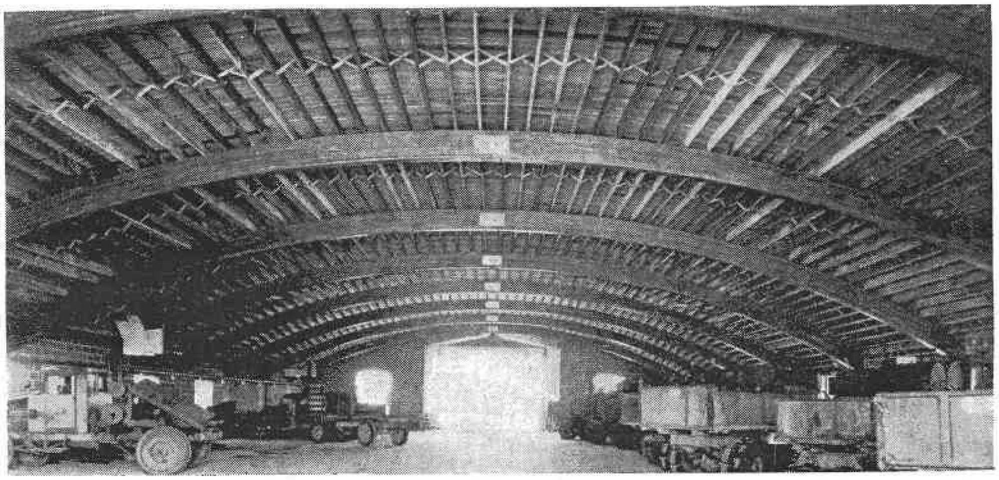

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FIGURE 2.—Garage at Altoona, Wis. Glued laminated arches on buttressed walls. Span, 80 feet; spacing of arches, 16 feet.

glued laminated rafters for gothic-style dairy barns and other farm buildings in the upper Mississippi Valley.

The use of glued laminated arches and other members in American structures was suggested by the favorable record of glued construction

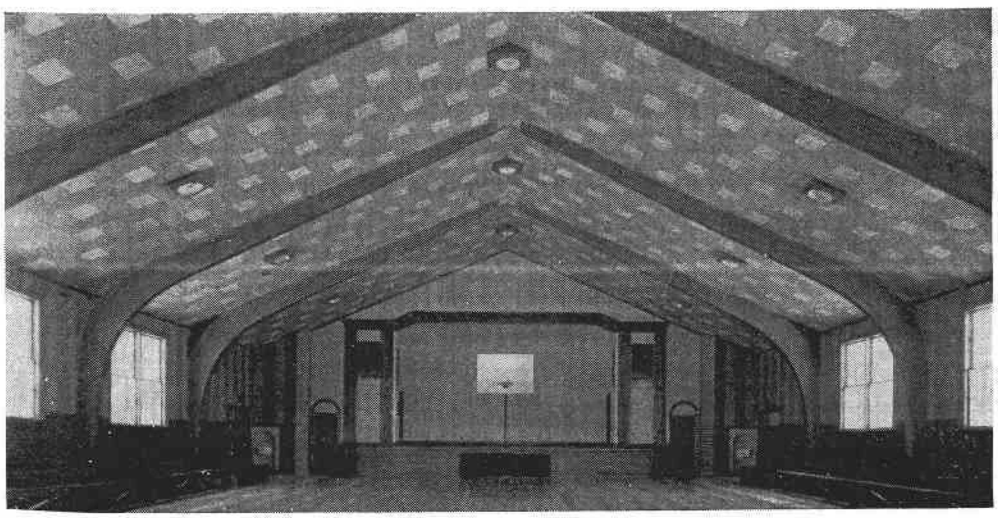

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FIGURE 3.—Gymnasium, at Goodman, Wis. Span, 58 feet; rise, 25 feet; spacing of glued laminated arches, 16 feet.

in Europe. Such use would extend the applicability of wood as a construction material by providing new types of wooden members with possibilities of economy through the use of at least some low-grade, narrow-width, and short-length material.

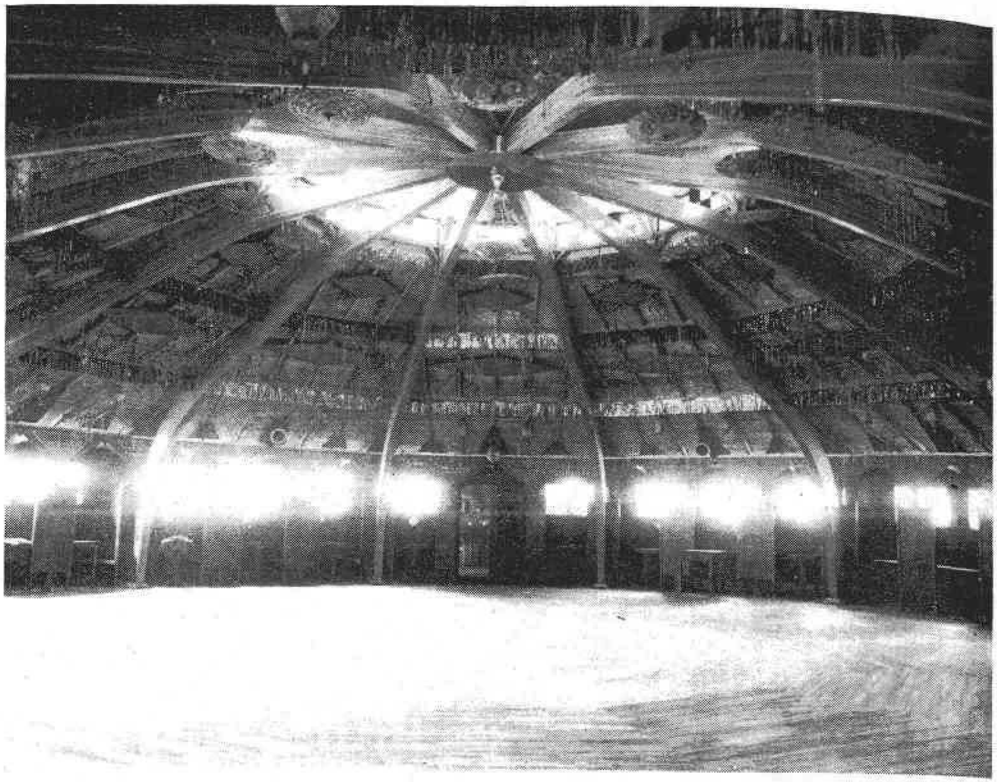

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FIGURE 4.—Bayshore Night Club, Marinette, Wis. Diameter of building, 57⅔ feet; spacing of glued laminated arches around periphery, 15 feet. Arch ribs 5½ inches wide, 12 inches deep at base, 16 inches at knee, and 8 inches at crown.

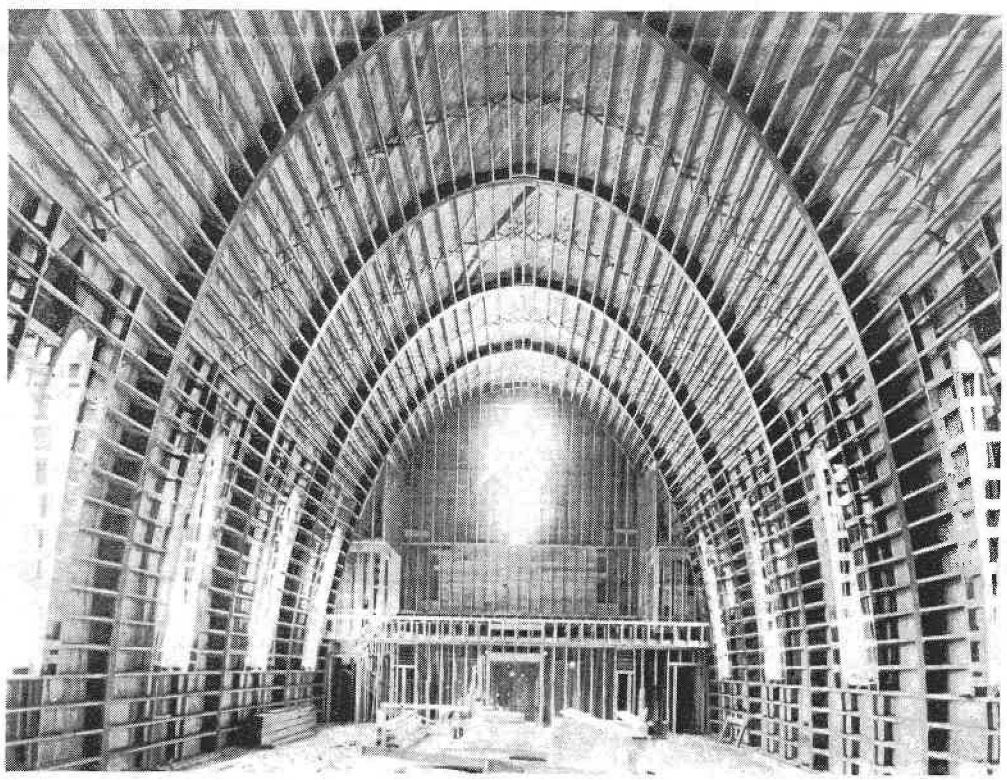

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FIGURE 5.—St. Austin's Catholic Church at Minneapolis, Minn. Span, 42¾ feet; rise, 40 feet; spacing of glued laminated arches, 14 feet.

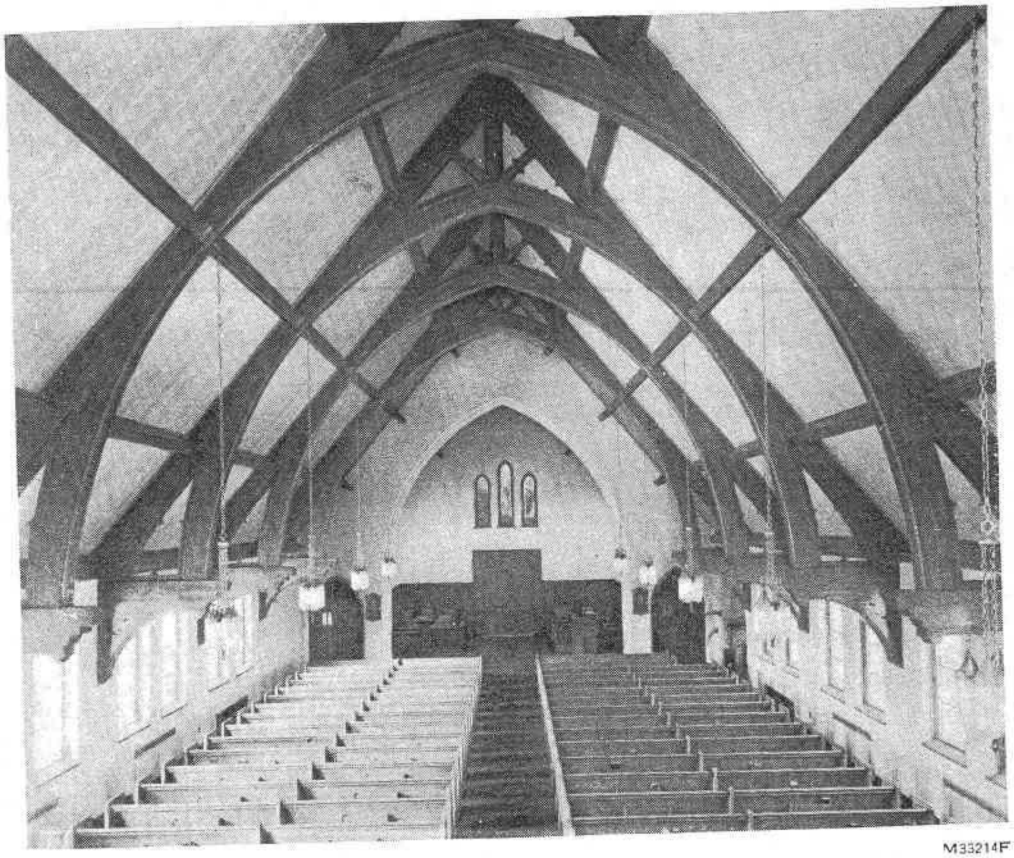

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FIGURE 6.—St. Peter's Reformed Church at Kiel, Wis. Span, 42 feet; spacing of glued laminated arches, 12 feet.

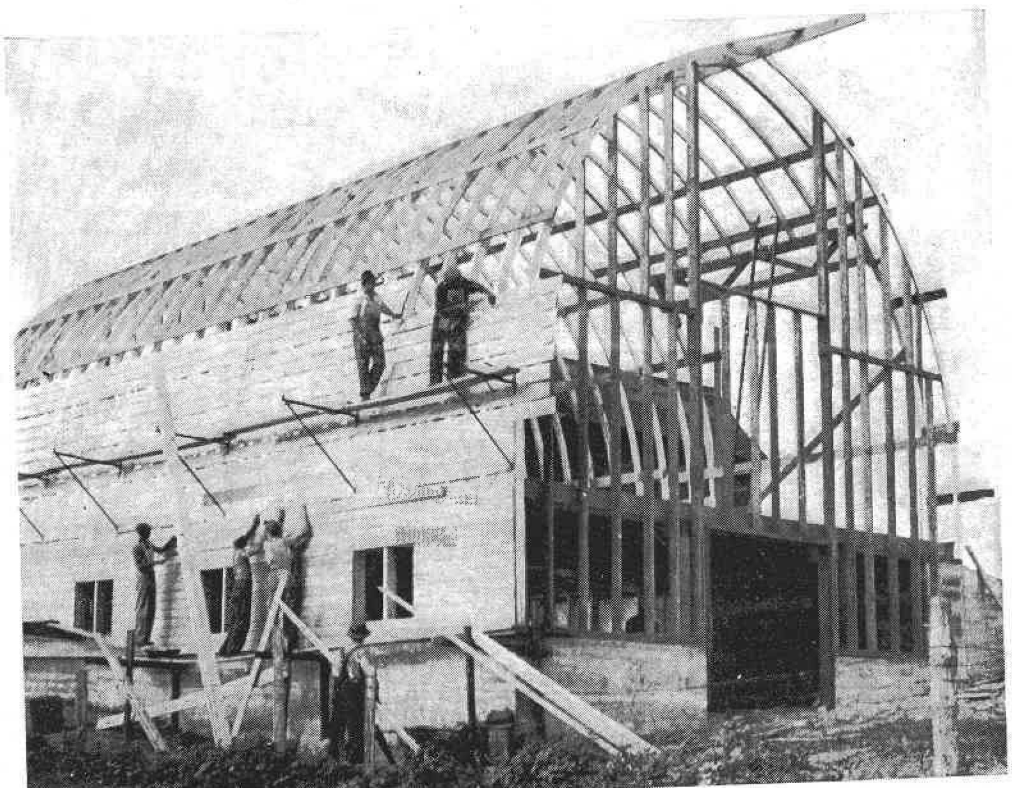

FIGURE 7.—Barn near Verona, Wis., with glued laminated rafters (1⅝ by 5½ inches in cross section) continuous from foundation to roof peak. Spacing of rafters, 2 feet.

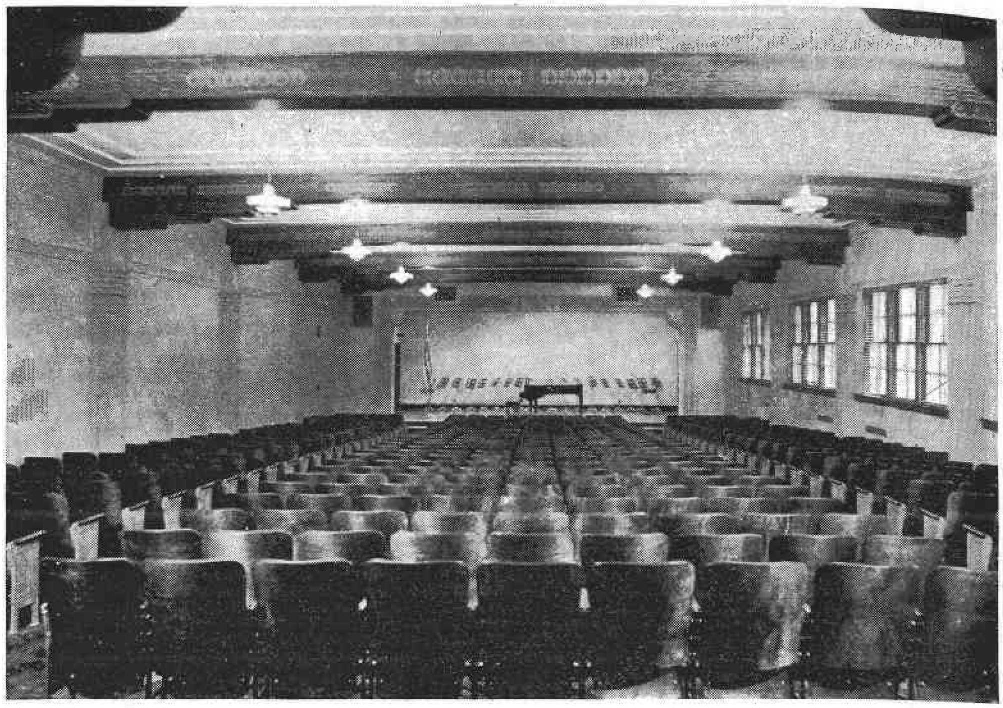

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FIGURE 8.—Music room at Henry Mitchell Junior High School, Racine, Wis. Ceiling (roof) carried by glued laminated beams. Span, 40 feet; width of beams, 15 inches; depth of beams, 20 inches.

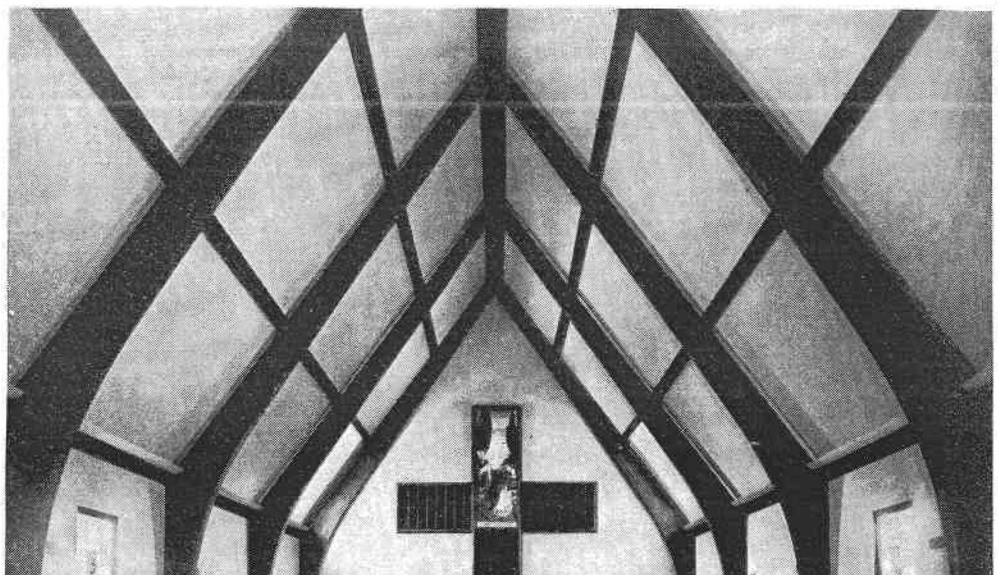

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FIGURE 9.—United Brethren Church at Seattle, Wash. Width, 32½ feet; height of glued laminated arches, 36⅔ feet.

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FIGURE 10.—Tennis hall at Falkenburg, Sweden. Roof arches of glued laminated construction; span about 150 feet.

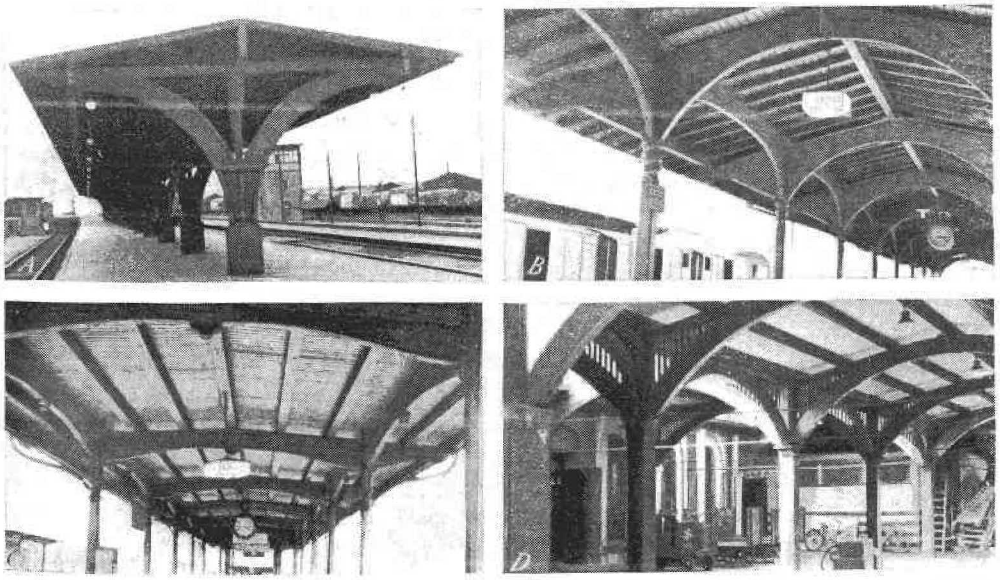

FIGURE 11.—Examples of glued laminated construction in railway station platform roof supports: *A*, Curved braces, Trondhjem, Norway; *B*, false arches, Landquart, Switzerland; *C*, curved beams, Gossau, Switzerland; *D*, arches, Gothenburg, Sweden.

TESTS OF THE STRENGTH OF GLUED LAMINATED CONSTRUCTION

Although European experience is useful as a guide, particularly with respect to the permanence of glued laminated construction, it does not provide test data applicable to United States conditions and woods. Most available European test data have related to straight laminated members rather than to curved ones and only incomplete information about the character of the material and other factors that may have important effects is included (*6, 11, 14*). Aside from those reported herein, tests recently made at the University of Illinois on two small glued laminated arches by Oliver (*21*) are the only American tests on this type of construction that are known to the author.

The dryness essential for good gluing precludes steaming or soaking laminations to make them flexible. Consequently, bending to the curvature sometimes required induces stress of considerable magnitude. The effect of this stress on the strength of the resulting members has not previously been investigated other than theoretically. It has been a moot question (*24, 26*) with European engineers but has usually been disregarded in designing. The effect of knots in laminae has not been given adequate study. Laminae of the full length of the member have generally been required, which is inconsistent with conditions in the United States where lumber must be shipped long distances.

The principal object of the tests described herein was to afford information that of itself or in conjunction with other available data could be used to determine (1) the effect on glued-up curved members of the stress produced in bending laminations to the required curvature, (2) acceptable methods of joining pieces end to end to form laminations of greater length than are conveniently available as single pieces, (3) the extent to which material with knots and other defects can be used, and (4) stresses for use in design.

Two kinds of test were made. For preliminary study of the effect of initial stresses, laminated curved members were subjected to transverse load. Other objects were served by tests that subjected curved members to end thrust in the same way as an arch rib is stressed in service.

TRANSVERSE TESTS ON CURVED AND STRAIGHT MEMBERS

The effect of initial stress was studied in tests on southern yellow pine (*Pinus palustris, p. taeda,* or *p. echinata*), Douglas fir (*Pseudotsuga taxifolia*), oak (*Quercus* sp.), and Sitka spruce (*Picea sitchensis*). The thickness of laminations was varied. Assemblies were made to circular curves, the radii of which ranged from 80 to 320 times the thickness of a lamination. Only material free from defects was used. Laminae prepared in pairs consisted of two pieces from end-to-end or side-by-side positions in a flat-sawn board. One lamina of a pair was used in a curved assembly, and the other in the same position in an assembly of straight pieces. The straight and curved assemblies were glued at the same time and after a period of seasoning were tested under transverse loading. Half the curved members were tested with the convex side up as indicated by figure 12 and the other half in the reverse position. Subsequent to the transverse tests, specimens cut from uninjured portions were tested for strength in compression parallel to grain.

Data from the transverse tests are shown in figures 13 to 15. Figures 13 and 14, in which each plotted point represents the strength value for a single curved specimen expressed as its ratio to the same strength value of the matched straight specimen, are each for two species as indicated. Average ratios for Sitka spruce, southern yellow pine, and Douglas fir are shown in figure 15. The dotted curves shown in these figures represent the formula later suggested for relating design values to curvature. Results of compression tests are summarized in table 1.

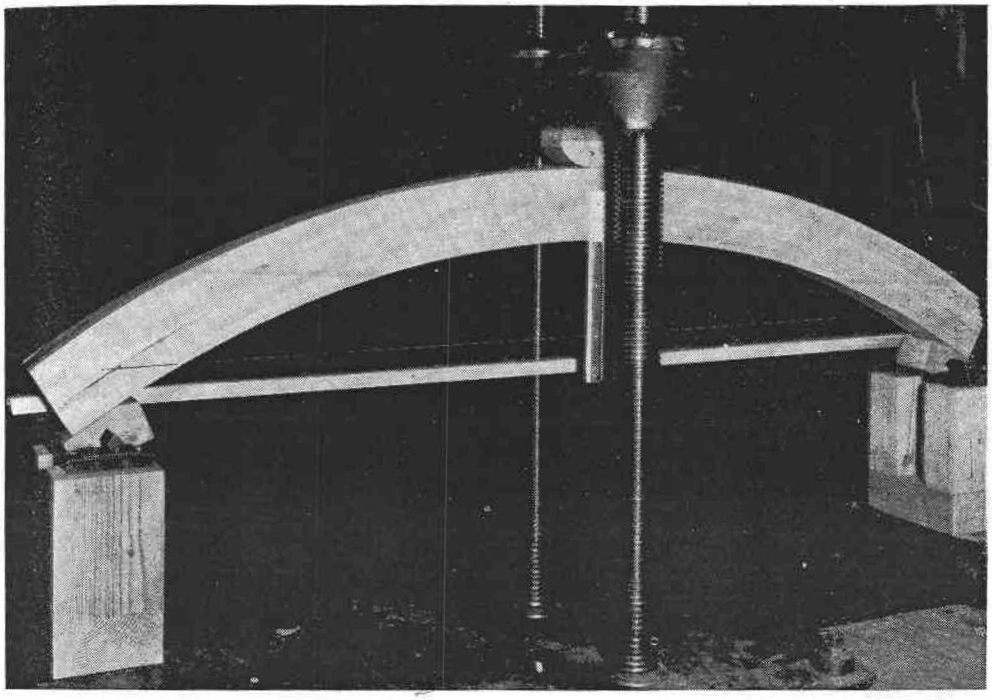

FIGURE 12.—Curved member in testing machine in position for transverse test.

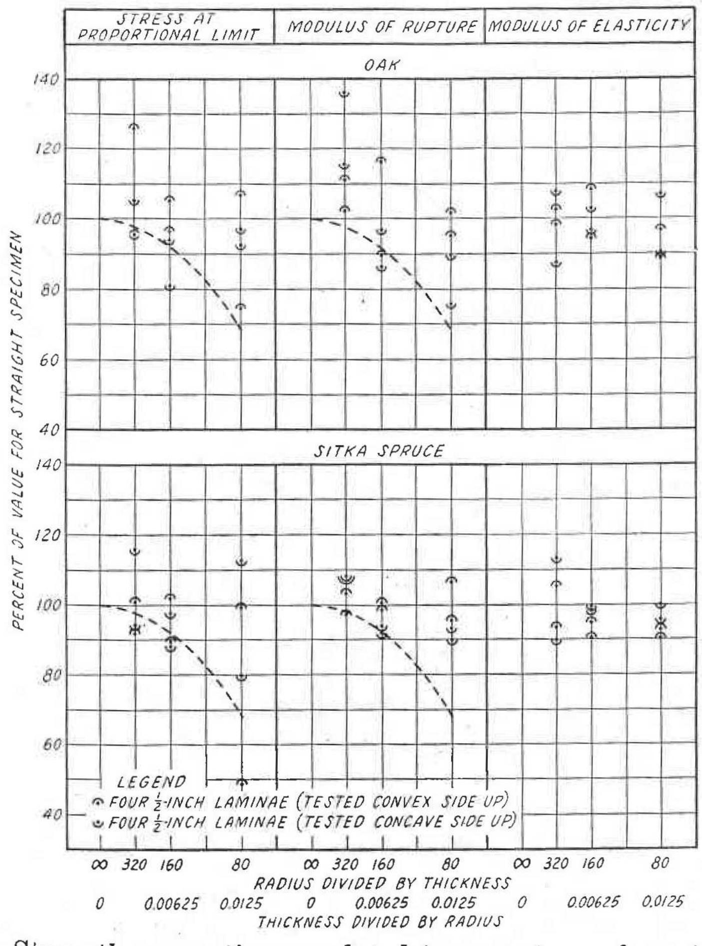

FIGURE 13.—Strength properties as related to curvature—from transverse tests on oak and Sitka spruce. The dotted curves represent the formula used for relating design stresses to curvature.

TABLE 1.—*Ratio of maximum crushing strength (compression parallel to grain) of curved to matched straight members; by species, thickness of laminae, and ratio of radius of curvature to thickness of laminae*

Species and thickness of laminae (inches)	Ratio of curvature=320			Ratio of curvature=174		
	Average	Maximum	Minimum	Average	Maximum	Minimum
Southern yellow pine:						
½	112.4	120.5	95.3			
¾	97.6	99.6	93.2			
1⅜ [1]				107.4	111.9	102.8
Douglas fir:						
½	98.7	104.2	92.2			
¾	100.5	105.5	91.5			
1⅜ [1]				99.5	100.0	99.0
Sitka spruce:						
½	101.8	104.3	96.2			
Oak:						
½	100.4	104.5	97.4			
All species and thicknesses	101.9	120.5	91.5	103.4	111.9	99.0
	Ratio of curvature=160			Ratio of curvature=80		
Southern yellow pine:						
½	101.4	116.4	94.7	96.4	101.9	92.9
¾	101.8	103.0	100.1	95.6	100.0	91.1
1⅜						
Douglas fir:						
½	99.4	110.0	84.5	96.8	109.2	86.7
¾	97.6	101.1	93.5	96.3	98.7	93.4
1⅜						
Sitka spruce:						
½	95.2	97.5	91.7	92.9	96.8	88.1
Oak:						
½	99.1	107.4	95.1	96.2	98.9	93.0
All species and thicknesses	99.1	116.4	84.5	95.7	109.2	86.7

[1] Based on 2 tests. Each other line of data represents 4 tests for each ratio of radius to thickness.

Boards that are free from knots or bad cross grain, particularly those of coniferous species, when bent to a radius as small as 80 times their thickness, as for some of the members tested in this series, are strained almost to breaking. Many pieces containing knots or severe cross grain and occasional pieces free from defects and of relatively straight grain fail, but exceptional pieces can be bent much more severely. Bending to a radius 160 times the thickness produces stress approximately one-half the ultimate and about equal to the value at the proportional limit.

The moderate deficiency (maximum individual value barely over 50 percent for the maximum curvature and most values for this curvature less than 40 percent) in strength properties of the curved members, as shown by figures 13 to 15, indicates that the initial stresses have had much less effect than might have been expected from a consideration of their magnitude.

Several of the ratios shown in figures 13 to 15 and in table 1 are greater than 100 percent. This indicates that the strength of the curved members is affected by factors other than the curvature and the initial stress in the laminae. Hence it may be doubted that the strength deficiency is as great in any instance as is indicated by the lowest individual ratios. It is probable that the effect is more accurately represented by the average values shown in figure 15. The maximum deficiency in modulus of elasticity is less than 20 percent and the average less than 10 percent.

The data from tests of strength in compression parallel to grain, as listed in table 1, indicate a maximum strength deficiency in any instance of about 16 percent and an average for the most severe curvature of less than 5 percent.

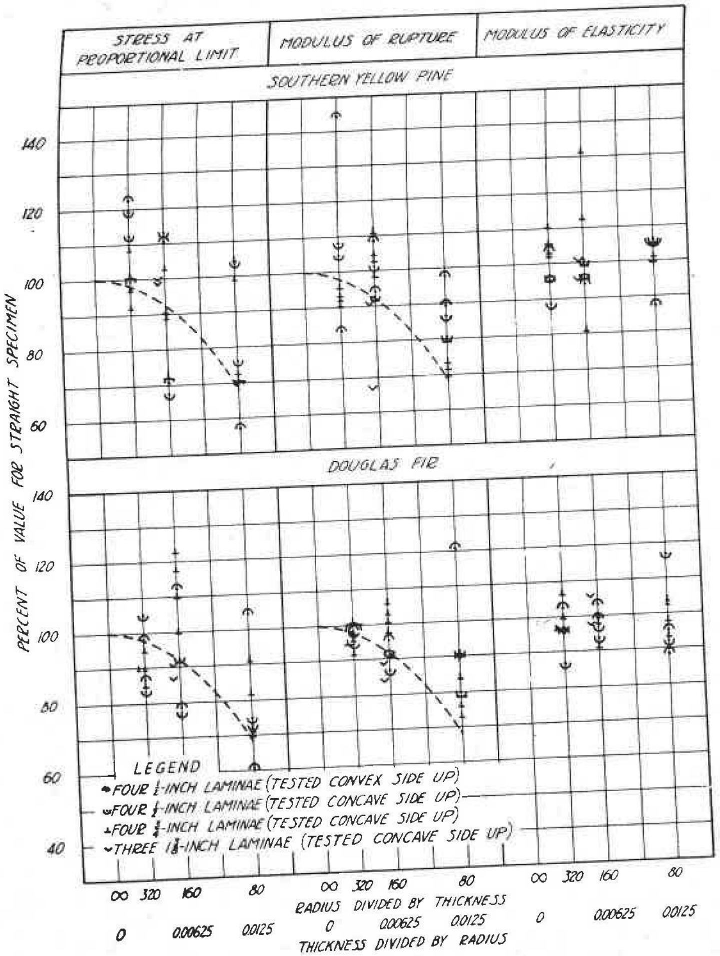

FIGURE 14.—Strength properties as related to curvature—from transverse tests on southern yellow pine and Douglas fir. The dotted curves represent the formula used for relating design stresses to curvature.

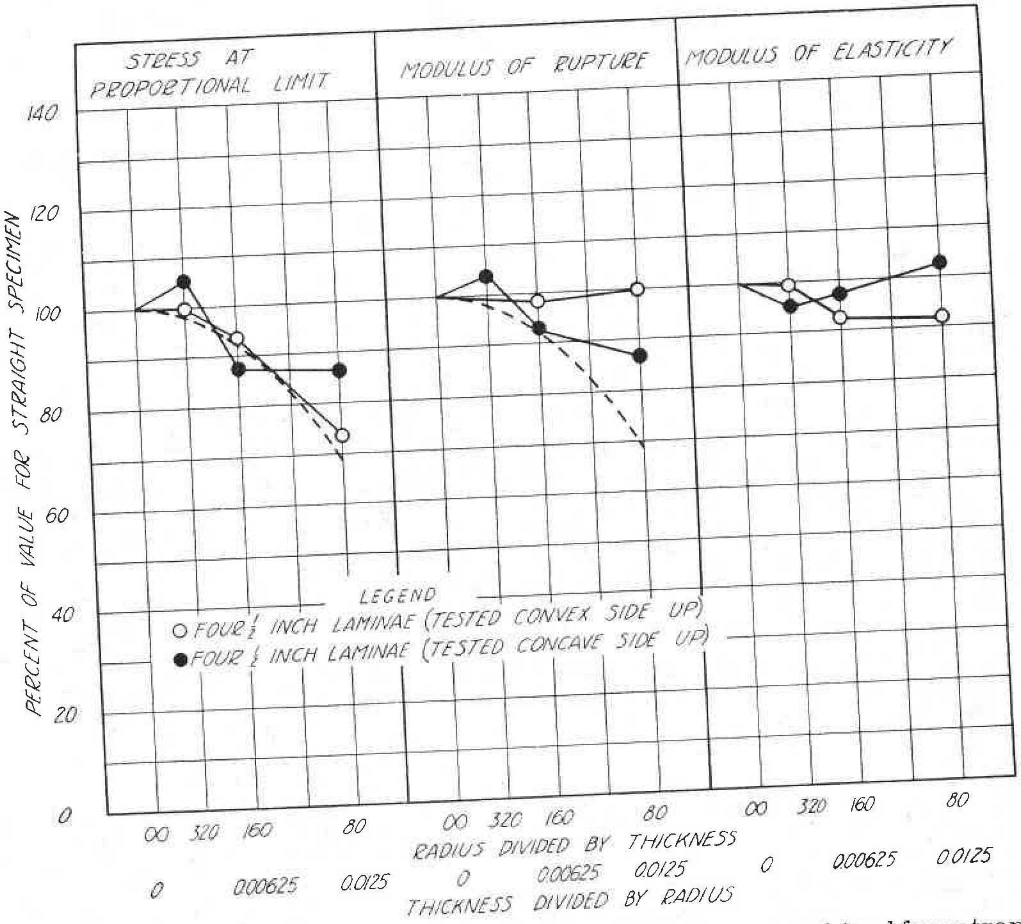

FIGURE 15.—Strength properties as related to curvature—combined from transverse tests of Sitka spruce, Douglas fir, and southern yellow pine. The dotted curves represent the formula used for relating design stresses to curvature.

TESTS OF CURVED MEMBERS UNDER END THRUST

For further study of the effect of initial stress, for investigating the effects of joints in laminations and of defects, and for obtaining data from which to derive values for design, curved members were subjected to thrust along the chord joining the centers of their ends, as indicated by figure 16, the load being applied through hinged bearings. Load, lateral deflection at the center of the length, and change in the

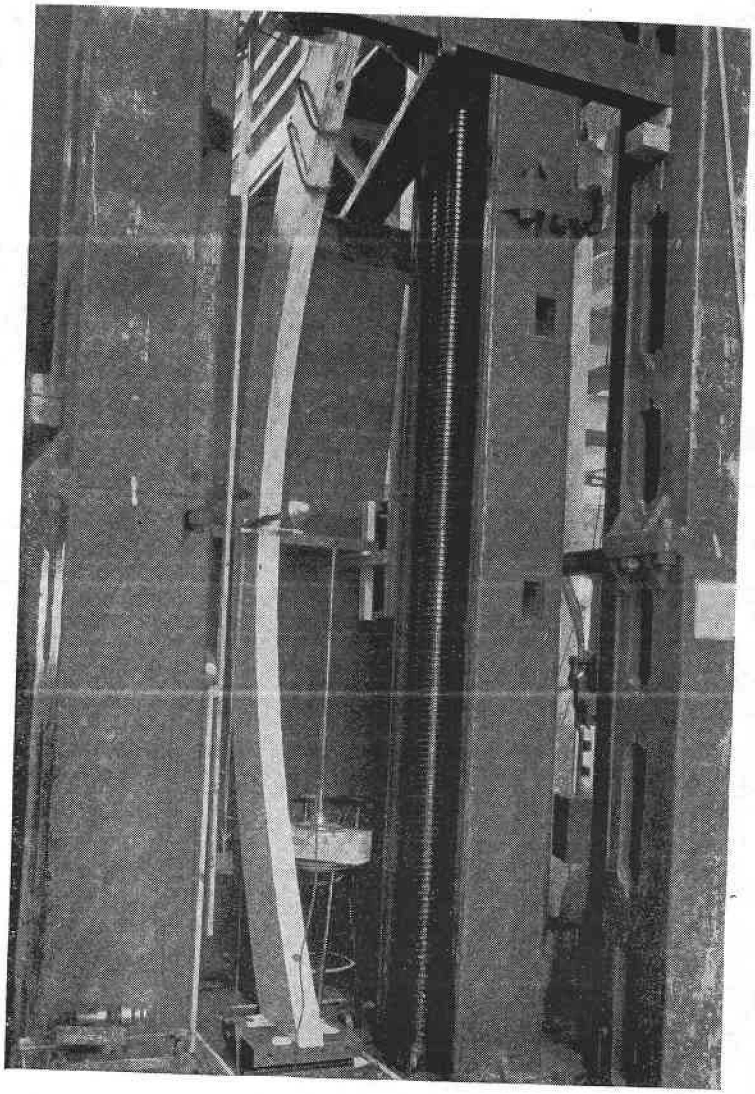

Figure 16.—Curved member in testing machine in position for test under end thrust.

length of the chord were observed simultaneously. A large number of members built up at the laboratory were tested in this way as were also half arches of two types as used in the laboratory service building previously mentioned.

The members built at the laboratory had parabolic axes with the vertex at the center of the length. The curves were such that the central offset from a chord of 20 feet was 1 foot, 3 feet, or 6 feet. The corresponding radii of curvature of the axes at the center of the length were 50, 16⅔, and 8⅓ feet (600, 200, and 100 inches). In nearly all instances, the thickness of laminae was three-fourths of an inch;

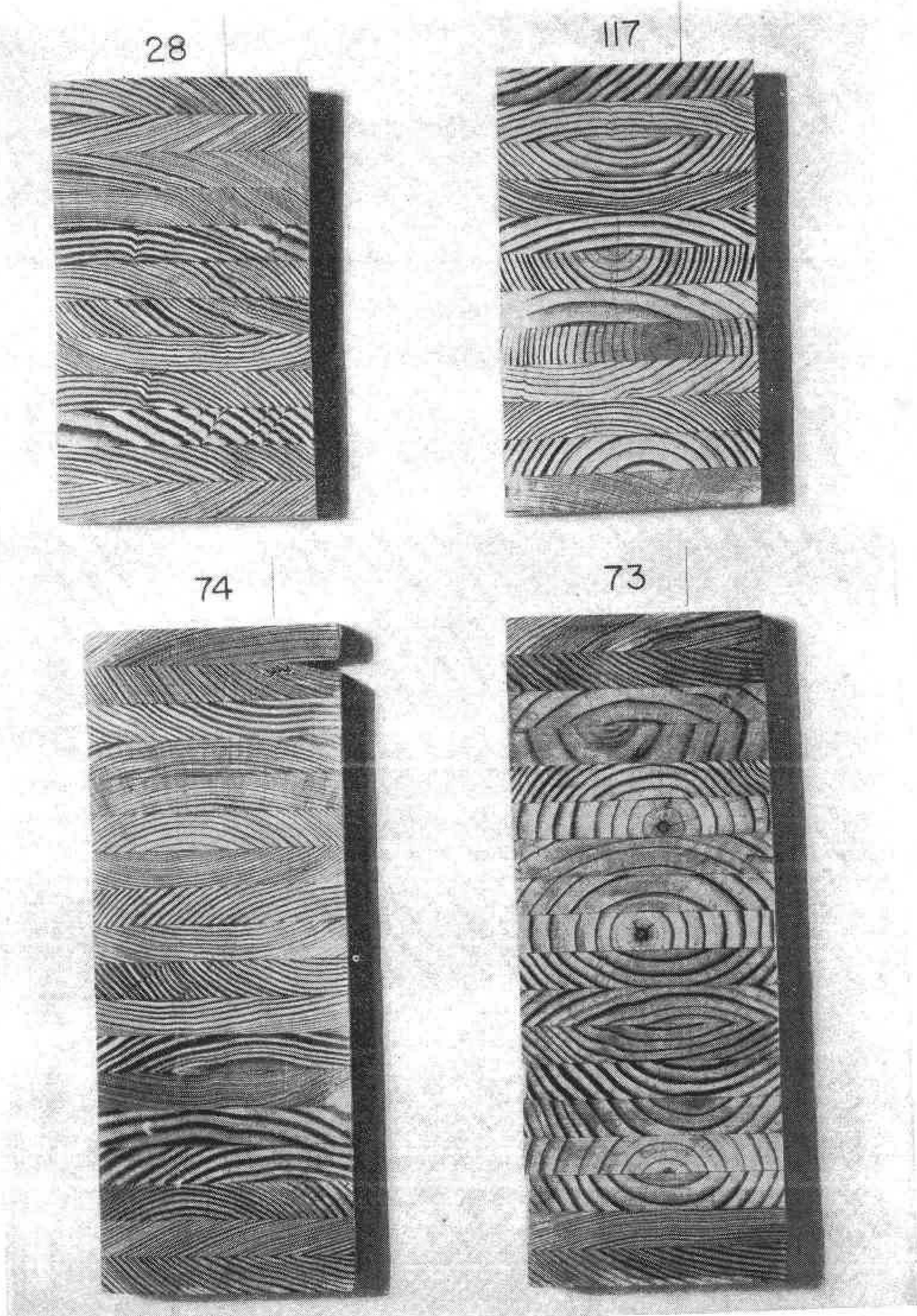

FIGURE 17.—Cross section showing character, with respect to rate of growth and percentage of summerwood of southern yellow pine used in experimental curved members. In No. 28, which has a total of 12 laminations, the 4 central laminations are from 6-inch No. 1 Common boards. In No. 117, which has 12 laminations, the 10 central laminations are from 6-inch No. 2 Common boards. In No. 74, which has 18 laminations, the 14 central laminations are from 6-inch No. 1 Common boards. In No. 73, which has 18 laminations, the 14 central laminations are from 6-inch No. 2 Common boards. All other laminations are from halves of 12-inch No. 1 Common boards.

the ratios of central radius of curvature to thickness were consequently 800, 266⅔, and 133⅓. These experimental arches are designated 1-foot, 3-foot, or 6-foot, respectively, according to the central offset from a 20-foot chord. The length of laminations was limited to 16 feet or less which resulted in the chord lengths being usually 15 feet for the 1-foot and 3-foot offsets, and 14 feet for the 6-foot offset. The actual offsets were the nominal ones multiplied by the factor, $\left(\frac{\text{chord length in feet}}{20}\right)^2$; usually then 6¾, 20¼, or 35⁷⁄₂₅ inches.

Some 300 curved members with parabolic axes as just described were made in the laboratory shops for test under end thrust. Cross sections were about 6 inches wide with depths of 6, 9, or 13½ inches composed of 8, 12, or 18 laminations, each of a ¾-inch finished thickness. Combining the 3 depths with the 3 curvatures enumerated previously made 9 differing combinations of depth of member and curvature. For the most part, well air-dried nominal 1-inch lumber was used. Boards, after being ripped to the required width, were prepared for gluing by being passed over a jointer for a light cut on 1 face, followed by a planer cut on the opposite face, after which a planer (single surfacer) cut was taken from each side leaving the faces smooth and true. Hide (hot) glue was used because mechanical spreaders for the convenient application of casein glue were not available.

SOUTHERN YELLOW PINE

Laboratory-built arches, which were tested to obtain further data on the effect of initial stress and to study the effect of knots and joints in laminations, were made principally of southern yellow pine.

The material used in the arches was purchased as No. 1 Common and No. 2 Common boards. The No. 1 Common was not so defective as the grading rules permit. For the most part it was nearly free from knots, those present being small and usually located near the ends of the boards. Laminations designated "clear" were from this stock. The No. 2 Common was typical material of this grade and supplied the laminae classed as "defective." The general character with respect to rate of growth and proportion of summerwood is illustrated in figure 17.

Sixty-three members built entirely of clear material were included.

For study of the effect of defects in laminae, clear boards were combined with defective ones in the same member. Defective boards formed the central part of the cross section, clear boards being used for the outer laminae on each side. Some of the members of this kind were matched to members consisting entirely of clear laminations. For the outer laminae of the matched members, 12-inch boards were ripped into two equal pieces, one of which was used in the outer portion of one assembly and the other in the same position in another. The central portion consisted of 6-inch clear boards in one assembly and 6-inch defective boards in the other. In all instances sketches of both sides of each lamination, made before gluing, recorded the direction of the grain and the sizes and locations of knots.

MEMBERS WITH LAMINATIONS FREE FROM DEFECTS

Tables 2, 3, and 4 present data on 63 members made from clear southern yellow pine lumber. The distribution of values, classified according to the number of laminae and the curvature, is shown.

TABLE 2.—*Distribution of bending plus compression stress at proportional limit as found in end-thrust tests of parabolic curved members of southern yellow pine with laminations free from defects—according to number of laminations and curvature* [1]

Class limits of combined stress (pounds/square inch)	1-foot offset				3-foot offset				6-foot offset				Tally, all offsets		
	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations
	Serial No.	*Serial No.*	*Serial No.*	*Number*	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Number*	*Number*	*Number*
3,000–3,199									100		74	2	1		1
3,200–3,399															
3,400–3,599	151			1					104			1	2		
3,600–3,799	150			1		124		1		70		1	1	2	
3,800–3,999	108; 110			2		128		1	60; 156	64	76; 140	5	4	2	2
4,000–4,199									94			1	1		
4,200–4,399							136	1			78	1			2
4,400–4,599	106	50	130	3	86			1	58; 155	68		3	4	2	1
4,600–4,799					82; 114	126		3	154		138	2	3	1	1
4,800–4,999	55; 149; 152	49	40; 132	6	88			1	102; 153		80	3	6	1	3
5,000–5,199		120		1	92; 116			2	96; 98			2	4	1	
5,200–5,399		51; 122		2	90		46	2	62			1	2	2	1
5,400–5,599										66		1		1	
5,600–5,799			38	1	84		134	2			72	1	1		3
5,800–5,999		28		1	112		44; 48	3					1	1	2
6,000–6,199	53	118		2		36		1					1	2	
6,200–6,399															
6,400–6,599															
6,600–6,799															
6,800–6,999															
7,000–7,199			42	1											1
Tally .... number	9	7	5	21	9	4	5	18	13	4	7	24	31	15	17
Average ..... pounds/square inch [2]	4,480	5,300	5,420	4,980	5,100	4,600	5,420	5,080	4,420	4,400	4,360	4,400	4,640	4,870	4,980
Average ratio $\left(\frac{\text{bending stress}}{\text{total stress}}\right)$	.904	.850	.780		.952	.931	.909		.974	.960	.941				

[1] All values adjusted to 12-percent moisture content.

[2] Averages from tests of 25 members (12 laminations) with glued scarf joints (slope 1 in 12) at the centers of 3 laminations next to the convex face (see p. 28)—for 3-foot offset, 4,460 pounds/square inch; for 6-foot offset, 3,780 pounds/square inch.

TABLE 3.—*Distribution of bending plus compression stresses at maximum moment as found in end-thrust tests of parabolic curved members of southern yellow pine free from defects—according to number of laminations and curvature* [1]

Class limits of combined stress (pounds per square inch)	1-foot offset				3-foot offset				6-foot offset				Tally, all offsets		
	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations
	Serial No.	*Serial No.*	*Serial No.*	*Number*	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Number*	*Number*	*Number*
6,000–6,299										68		1		1	
6,300–6,599									154			1	1		
6,600–6,899	55			1									1		
6,900–7,199									58; 156	66	74	4	2	1	1
7,200–7,499							46	1	60; 102			2	2		1
7,500–7,799									94	64; 70		3	1	2	
7,800–8,099									62; 100			2	2		
8,100–8,399									155			1	1		
8,400–8,699									104; 153			2	2		
8,700–8,999		49		1					98		72; 80	3	1	1	2
9,000–9,299					86			1					1		
9,300–9,599		50; 120		2	84; 114			2					2	2	
9,600–9,899			40; 132	2	88			1					1		2
9,900–10,199	149			1									1		
10,200–10,499			130	1	112	126		2	96			1	2	1	1
10,500–10,799	110; 152			2	82			1					3		
10,800–11,099					90; 92	36		3					2	1	
11,100–11,399			38	1											1
11,400–11,699	53; 151			2									2		
11,700–11,999															
12,000–12,299					116			1					1		
Tally........number	6	3	4	13	9	2	1	12	13	4	3	20	28	9	8
Average..pounds per square inch [2]	10,200	9,250	10,280	10,000	10,320	10,650	7,350	10,120	7,950	7,120	8,250	7,830	9,190	8,620	9,150
Average ratio.. $\left(\frac{\text{bending stress}}{\text{total stress}}\right)$	0.938	0.881	0.832		0.959	0.938	0.912		0.976	0.962	0.943				

[1] All values adjusted to 12-percent moisture content.

[2] Averages from tests of 25 members (12 laminations) with glued scarf joints (slope 1 in 12) at the centers of 3 laminations next to the convex face (see p. 28)—for 3-foot offset, 7,130 pounds per square inch; for 6-foot offset, 6,050 pounds per square inch.

TABLE 4.—*Distribution of modulus of elasticity as found in end-thrust tests of parabolic curved members of southern yellow pine free from defects—according to number of laminations and curvature* [1]

Class limits of modulus of elasticity (1,000 pounds/square inch)	1-foot offset				3-foot offset				6-foot offset				Tally, all offsets		
	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations	Tally, all laminations	8 laminations	12 laminations	18 laminations
	Serial Number	*Serial Number*	*Serial Number*	*Number*	*Serial Number*	*Serial Number*	*Serial Number*	*Number*	*Serial Number*	*Serial Number*	*Serial Number*	*Number*	*Number*	*Number*	*Number*
1,400–1,449									154			1	1		
1,450–1,499															
1,500–1,549															
1,550–1,599									153			1	1		
1,600–1,649	149; 150			2					102			1	3		
1,650–1,699	108; 151			2	88			1	100	66	80	3	4	1	1
1,700–1,749	152			1							140	1	1		1
1,750–1,799		49	40; 130	3					62	68; 70		3	1	3	2
1,800–1,849	55	50	42	3	82; 92		136	3			74; 138	2	3	1	4
1,850–1,899							46	1	96; 98; 104			3	3		1
1,900–1,949		28; 118; 122	38	4			48	1	58; 94		78	3	2	3	3
1,950–1,999	106	51		2	86; 90; 114	126	44	5	60		72; 76	3	5	2	3
2,000–2,049						124		1	156	64		2	1	2	
2,050–2,099					84; 116		134	3	155			1	3		1
2,100–2,149					112	36		2					1	1	
2,150–2,199	110	120	132	3									1	1	1
2,200–2,249		52		1		128		1						2	
2,250–2,299															
2,300–2,349	53			1									1		
Tally number	9	8	5	22	9	4	5	18	13	4	7	24	31	16	17
Average......1,000 pounds/sq. in. [2]	1,847	1,969	1,895	1,902	1,947	2,088	1,935	1,975	1,817	1,812	1,846	1,825	1,864	1,959	1,887

[1] All values adjusted to 12-percent moisture content.

[2] Averages from tests of 25 members (12 laminations) with glued scarf joints (slope 1 in 12) at the centers of 3 laminations, next to the convex face (see p. 28)—for 3-foot offset, 1,795,000 pounds/square inch; for 6-foot offset 1,724,000 pounds/square inch.

As may be noted, the strength values for any one combination of curvature and number of laminae, or depth of member, vary considerably. Figure 18, in which average values from tables 2, 3, and 4 are graphed, discloses no consistent relationship between strength values and number of laminations. It does show, however, that values for the 6-foot offset are lower than for the 1-foot and 3-foot offsets, with but little differentiation between the latter two.

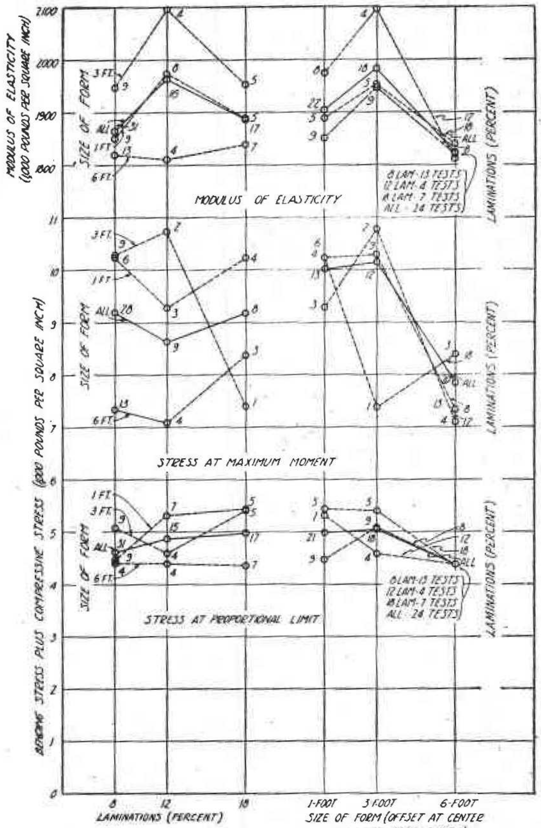

FIGURE 18.—Modulus of elasticity and stresses at proportional limit and maximum moment according to number of laminations and relative curvature for southern yellow pine.

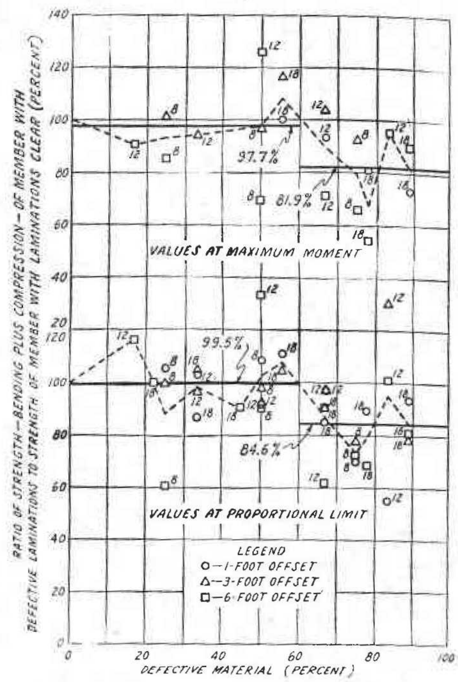

FIGURE 19.—Ratios of strength (bending plus compression) of members with defective laminations to strength of matched members with all laminations clear according to percentage of defective laminations for southern yellow pine.

MEMBERS WITH DEFECTS IN LAMINATIONS

Figure 19 presents data from tests on matched pairs of members with defective material in the central portion of the cross section of one individual, the other being made entirely of clear material. Each plotted point in this graph represents the ratio of the strength of a member with defective material to that of the matched member. The strength values used are the combined bending and compressive stresses at the proportional limit and at maximum moment.

Data from members consisting entirely of clear material, paired or matched by ripping 12-inch clear boards and placing the resulting 6-inch laminae in identical positions in the individuals of the pair, are useful in interpreting figure 19. These data are shown in table 5.

Considering, in connection with figure 19, the spread in strength values between matched members of clear material (table 5) it is concluded that up to 60 percent of defective material in the central portion of a laminated member does not significantly reduce the strength below that of members consisting entirely of clear laminations. Members with less than 60 percent defective material averaged 99.5 percent as high in proportional-limit values and 97.7 percent as high in values at maximum moment as the all-clear members with which they were matched. The corresponding ratios for members with more than 60 percent defective material are 84.6 and 81.9 percent.

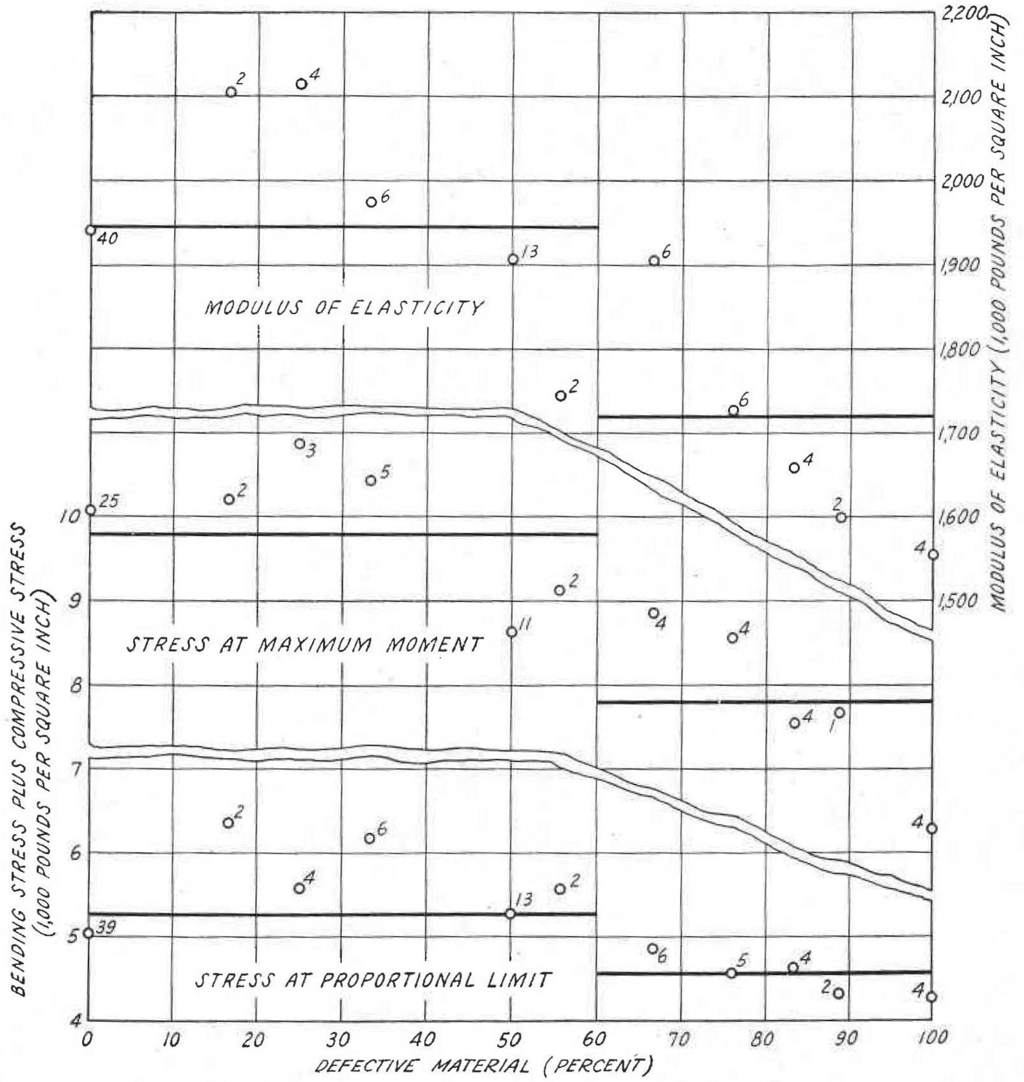

FIGURE 20.—Modulus of elasticity and stresses at proportional limit and at maximum moment for southern yellow pine members having 1-foot and 3-foot offsets with various percentages of defective material.

TABLE 5.—*Ratios between strength values of members of matched pairs composed entirely of clear material*

Pair No.	Laminations	Offset	Ratios between members of pair [1]		
			At proportional limit	At maximum moment	Modulus of elasticity
	Number	*Feet*	*Percent*	*Percent*	*Percent*
1	8	1	133	-----	100
2	8	1	71	106	95
3	8	6	96	80	89
4	8	6	86	84	96
5	12	1	107	94	96
6	12	1	-----	-----	88

[1] All ratios for a pair have as their base the strength values of the same individual.

A further evaluation of the effect of defective material is afforded by tables 6, 7, and 8 and figures 20 and 21. The tables show the distribution of strength values according to the percentage of defective material. Average strength values are plotted against the percentages of defective material; in figure 20 for 1- and 3-foot offsets combined and in figure 21 for the 6-foot offset. These tables and diagrams in-

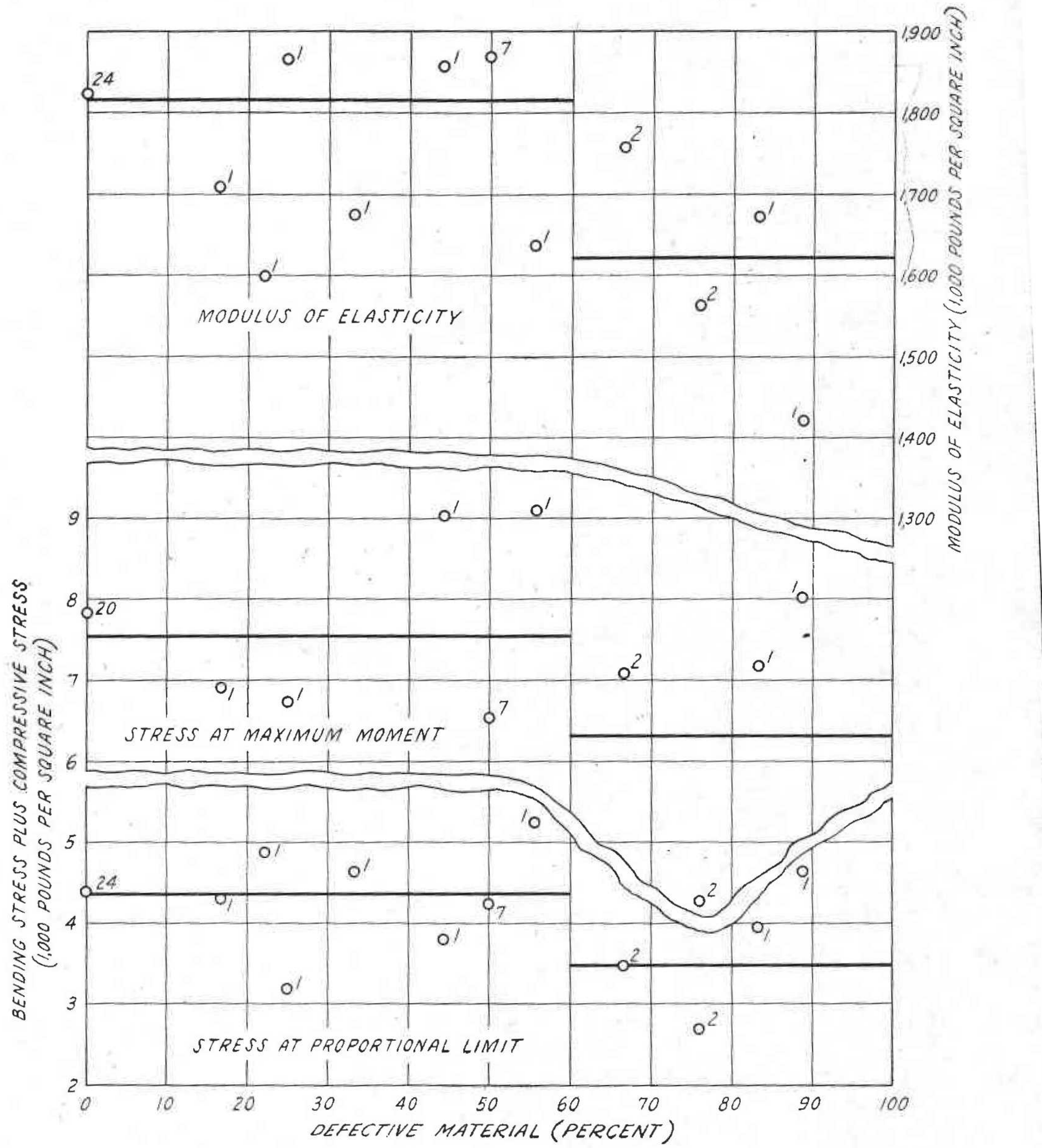

FIGURE 21.—Modulus of elasticity and stresses at proportional limit and at maximum moment for southern yellow pine members having a 6-foot offset with various percentages of defective material.

clude all available data on members with various percentages of defective laminations whereas figure 19 presents data on matched pairs of members only. Figures 20 and 21 like figure 19 indicate only moderately lowered average strength values for members with up to about 60 percent of defective laminations as compared with members consisting entirely of clear laminations.

TABLE 6.—*Distribution of bending plus compression at proportional limit as found in end-thrust tests of parabolic curved members of southern yellow pine, according to percent of defective material* [1]

Class limits of combined stress (Pounds per square inch)	0 percent	16⅔ percent	22⅔ percent	25 percent	33⅓ percent	44⅔ percent	50 percent	55⅔ percent	Tally, 0–55⅔ percent: 1-foot offset	Tally, 0–55⅔ percent: 3-foot offset	Tally, 0–55⅔ percent: 1- and 3-foot offset	Tally, 0–55⅔ percent: 6-foot offset	66⅔ percent	75, 77⅔ percent	83⅓ percent	88⅔ percent	100 percent	Tally, 66⅔–100 percent: 1-foot offset	Tally, 66⅔–100 percent: 3-foot offset	Tally, 66⅔–100 percent: 1- and 3-foot offset	Tally, 66⅔–100 percent: 6-foot offset
	Serial No.	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Number*	*Number*	*Number*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Number*	*Number*	*Number*
2,000–2,199														*73*							1
2,200–2,399																					
2,400–2,599																					
2,600–2,799																					
2,800–2,999																					
3,000–3,199	*74*; *101*			*61*								3		105				1		1	
3,200–3,399														*57*							1
3,400–3,599	*104*; 151						{*59*; 127; *242*; *246*}		1	1	2	4	*65*; *75*		117			1		1	2
3,600–3,799	*70*; 124; 150					*77*	240		1	2	3	2					17		1	1	
3,800–3,999	*60*; *64*; *76*; 108; 110; 128; *140*; *156*.								2	1	3	5	135		*63*		29		2	2	1
4,000–4,199	*94*			109			107; *250*		2		2	2	131	18				1	1	2	
4,200–4,399	*78*; 136	*69*					*248*			1	1	3				129; 133		1	1	2	
4,400–4,599	50; *58*; *68*; 86; 106; 130						113		3	2	5	2		111					1	1	
4,600–4,799	82; 114; 126; *138*; *154*; *155*.				*139*		*244*			3	3	5	125		30;123	*71*	21	1	3	4	1
4,800–4,999	40; 49; 55; *80*; 88; *102*; 132; 149; 152; *153*.		*79*				121		7	1	8	4	119				13	2		2	
5,000–5,199	92; *96*; *98*; 116; 120			115			{232; 234; 236; 238}		1	7	8	2		37				1		1	
5,200–5,399	*46*; 51; *62*; 90; 122							*137*	2	2	4	2									
5,400–5,599	*66*			20				39; 45	1	2	3	1									
5,600–5,799	38; *72*; 84; 134				47				1	3	4	1	31		22			1	1	2	
5,800–5,999	28; 44; 48; 112				27; 35		*67*		2	4	6	1									
6,000–6,199	36; 53; 118	26			24		25		5	1	6		23	14				2		2	
6,200–6,399					41				1		1										
6,400–6,599		34					19			2	2										
6,600–6,799																					
6,800–6,999					32					1	1										
7,000–7,199	42								1		1										
7,200–7,399							15; 33		1	1	2										
7,400–7,599																					
7,600–7,799				16					1		1										

[1] Serial numbers in italics indicate members with 6-foot offset; all others, 1-foot or 3-foot offset. All values adjusted to 12-percent moisture content.

TABLE 7.—*Distribution of bending plus compression at maximum moment as found in end-thrust tests of parabolic curved members of southern yellow pine—according to percentage of defective material* [1]

Class limits of combined stress (pounds per square inch)	0 percent	16⅔ percent	22 2/9 percent	25 percent	33⅓ percent	44 4/9 percent	50 percent	55 5/9 percent	Tally, 0–55 5/9 percent: 1-foot offset	Tally, 0–55 5/9 percent: 3-foot offset	Tally, 0–55 5/9 percent: 1- and 3-foot offset	Tally, 0–55 5/9 percent: 6-foot offset	66⅔ percent	75–77 7/9 percent	83⅓ percent	88 8/9 percent	100 percent	Tally, 66⅔–100 percent: 1-foot offset	Tally, 66⅔–100 percent: 3-foot offset	Tally, 66⅔–100 percent: 1- and 3-foot offset	Tally, 66⅔–100 percent: 6-foot offset
	Serial No.	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*					*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Number*	*Number*	*Number*
3,900–4,199														*73*							1
4,200–4,499																					
4,500–4,799														*57*							1
4,800–5,099							*59*; *250*					2	*65*				17		1	1	1
5,100–5,399							240			1	1						29		1	1	
5,400–5,699																					
5,700–5,999																					
6,000–6,299	*68*						*244*					2		18	30				2	2	
6,300–6,599	*154*						236			1	1	1									
6,600–6,899	55	*69*		*61*			*246*; *248*		1		1	4									
6,900–7,199	*58*; *66*; *74*; *156*											4	31		*63*		21	1	1	2	1
7,200–7,499	46; *60*; *102*						234			2	2	2									
7,500–7,799	*64*; *70*; *94*						67; 232			1	1	4	119		22; 117	129	13	5		5	
7,800–8,099	*62*; *100*											2				*71*					1
8,100–8,399	*155*											1									
8,400–8,699	*104*; *153*						*242*	45		1	1	3			123				1	1	
8,700–8,999	49; *72*; *80*; *98*						127		1	1	2	3		37					1	1	
9,000–9,299	86			20		*77*	19; 113; 238	*137*		5	5	2	*75*								1
9,300–9,599	50; 84; 114; 120				32				2	3	5			14; 111				1	1	2	
9,600–9,899	40; 88; 132	26						39	4	1	5		23					1		1	
9,900–10,199	149								1		1										
10,200–10,499	*3*; 112; 126; 130				35; 41		25; 33		3	4	7	1									
10,500–10,799	82; 110; 152	34			27				3	2	5										
10,800–11,099	36; 90; 92									3	3		125						1	1	
11,100–11,399	38			16			15		3		3										
11,400–11,699	53; 151				24				3		3										
11,700–11,999																					
12,000–12,299	116			115						2	2										

[1] Serial numbers in italics indicate members with 6-foot offset; all others, 1-foot or 3-foot offset. All values adjusted to 12-percent moisture content.

TABLE 8.—*Distribution of modulus of elasticity as found in end-thrust tests of parabolic curved members of southern yellow pine—according to percentage of defective material used* [1]

Class limits of modulus of elasticity (1,000 pounds per square inch)	0 percent	16⅔ percent	22 2/9 percent	25 percent	33⅓ percent	44 4/9 percent	50 percent	55 5/9 percent	Tally, 0–55 5/9 percent: 1-foot offset	Tally, 0–55 5/9 percent: 3-foot offset	Tally, 0–55 5/9 percent: 1- and 3-foot offsets	Tally, 0–55 5/9 percent: 6-foot offset	66⅔ percent	75–77 7/9 percent	83⅓ percent	88 8/9 percent	100 percent	Tally, 66⅔–100 percent: 1-foot offset	Tally, 66⅔–100 percent: 3-foot offset	Tally, 66⅔–100 percent: 1- and 3-foot offsets	Tally, 66⅔–100 percent: 6-foot offset
	Serial No.	Serial No.	Serial No.	Serial No.	Serial No.	Serial No.	Serial No.	Serial No.	Number	Number	Number	Number	Serial No.	Serial No.	Serial No.	Serial No.	Serial No.	Number	Number	Number	Number
1,400–1,449	*154*											1			117; 123	*71*	17; 29	1	3	4	1
1,450–1,499														*73*							1
1,500–1,549																					
1,550–1,599	*153*		*79*				234; 236			2	2	2				133			1	1	
1,600–1,649	*102*; 149; 150							*137*	2		2	2				129	21	2		2	
1,650–1,699	*66*; *80*; 88; *100*; 108; 151				*139*				2	1	3	4	135	18; 43; *57*; 105	*63*			1	3	4	2
1,700–1,749	152; *140*	*69*					107; 240; *246*	39	3	1	4	3	*65*	37			13	2		2	1
1,750–1,799	40; 49; *62*; *68*; *70*; 130						*67*; *250*	45	3	1	4	5	*75*	111	30				2	2	1
1,800–1,849	42; 50; 55; *74*; 82; 92; 136; *138*.				41		113		4	4	8	2									
1,850–1,899	46; *96*; *98*; *104*			*61*	27;47	*77*	25; 238; *248*		2	3	5	6	119; 125					1	1	2	
1,900–1,949	28; 38; 48; *58*; *78*; *94*; 118; 122.						121; 232; *244*		5	2	7	4	31	14				1	1	2	
1,950–1,999	44; 51; *60*; *72*; *76*; 86; 90; 106; 114; 126.				24		*59*		3	5	8	4									
2,000–2,049	*64*; *156*			109			*242*		1		1	3	131		22			2		2	
2,050–2,099	84; 116; 124; 134; *155*	26		115	35		127		1	7	8	1	23					1		1	
2,100–2,149	36; 112			16					1	2	3										
2,150–2,199	110; 120; 132	34					19		3	2	5										
2,200–2,249	52; 128			20	32		33		1	4	5										
2,250–2,299																					
2,300–2,349	53						15		2		2										

[1] Serial numbers in italics indicate members with 6-foot offset; all others, 1-foot or 3-foot offset. All values adjusted to 12-percent moisture content.

Table 9 is a resumé of the data in tables 2, 3, 4, 6, 7, and 8 and in figures 19, 20, and 21. It summarizes the results of end-thrust tests on southern yellow pine members and, in addition to average and minimum values, includes certain statistical measures to which reference will be made later.

TABLE 9.—*Resumé of end-thrust tests on parabolic curved members of southern yellow pine*

Test material	Tests	Average ($\bar{x}$)	Standard deviation (σ)	$\bar{x}-2.326\sigma$	$\bar{x}-3\sigma$	Minimum test value	Coefficient of variation ($\sigma \div \bar{x}$)
STRESS (BENDING AND COMPRESSION) AT PROPORTIONAL LIMIT							
	No.	*Lbs./sq. in.*	*Lbs./sq. in.*			*Lbs./sq. in.*	*Pct.*
All laminations clear:							
1-foot offset	21	4,980	859	2,980	2,400	3,560	17.2
3-foot offset	18	5,080	692	3,470	3,000	3,640	13.6
1- and 3-foot offsets combined	39	5,020	788	3,190	2,660	3,560	15.7
6-foot offset	24	4,400	698	2,780	2,310	3,020	15.9
Up to 56 percent of material in central part of cross section defective:							
1-foot offset	32	5,270	1,027	2,880	2,190	3,560	19.5
3-foot offset	34	5,250	876	3,210	2,620	3,540	16.7
1- and 3-foot offsets combined	66	5,260	953	3,040	2,400	3,540	18.1
6-foot offset	37	4,360	734	2,650	2,160	3,020	16.8
66 to 100 percent of material in central part of cross section defective:							
1-foot offset	11	4,770	935	2,600	1,960	3,100	19.6
3-foot offset	10	4,420	553	3,130	2,760	3,780	12.5
1- and 3-foot offsets combined	21	4,600	797	2,750	2,210	3,100	17.3
6-foot offset	6	3,500	775	1,700	1,180	2,130	22.1
STRESS (BENDING AND COMPRESSION) AT MAXIMUM MOMENT							
All laminations clear:							
1-foot offset	13	10,000	1,239	7,120	6,280	6,820	12.4
3-foot offset	12	10,120	1,163	7,410	6,630	7,380	11.5
1- and 3-foot offsets combined	25	10,060	1,204	7,260	6,450	6,820	12.0
6-foot offset	20	7,830	960	5,600	4,950	6,110	12.3
Up to 56 percent of material in central part of cross section defective:							
1-foot offset	21	10,240	1,093	7,700	6,960	6,820	10.7
3-foot offset	27	9,450	1,585	5,760	4,700	5,390	16.8
1- and 3-foot offsets combined	48	9,790	1,445	6,430	5,460	5,390	14.8
6-foot offset	31	7,550	1,194	4,770	3,970	5,000	15.8
66 to 100 percent of material in central part of cross section defective:							
1-foot offset	9	8,060	912	5,940	5,320	7,180	11.3
3-foot offset	8	7,480	1,945	2,960	1,640	5,020	26.0
1- and 3-foot offsets combined	17	7,760	1,574	4,100	3,040	5,020	20.3
6-foot offset	6	6,300	1,871	1,950	690	3,920	29.7
MODULUS OF ELASTICITY							
		1,000 lbs./sq. in.	*1,000 lbs./sq. in.*			*1,000 lbs./sq. in.*	
All laminations clear:							
1-foot offset	22	1,902	199	1,439	1,305	1,631	10.5
3-foot offset	18	1,975	131	1,670	1,582	1,678	6.6
1- and 3-foot offsets combined	40	1,935	176	1,526	1,407	1,631	9.1
6-foot offset	24	1,825	157	1,460	1,354	1,416	8.6
Up to 56 percent of material in central part of cross section defective:							
1-foot offset	33	1,919	192	1,472	1,343	1,631	10.0
3-foot offset	34	1,971	178	1,557	1,437	1,584	9.0
1- and 3-foot offsets combined	67	1,945	187	1,510	1,384	1,584	9.6
6-foot offset	37	1,817	149	1,470	1,370	1,416	8.2
66 to 100 percent of material in central part of cross section defective:							
1-foot offset	11	1,793	197	1,335	1,202	1,416	11.0
3-foot offset	11	1,657	170	1,262	1,147	1,408	10.3
1- and 3-foot offsets combined	22	1,725	196	1,269	1,137	1,408	11.4
6-foot offset	6	1,625	129	1,325	1,238	1,419	7.9

MEMBERS WITH END-TO-END JOINTS IN LAMINATIONS

Provision for laminated members longer than available boards requires consideration of how end-to-end joints in laminae can be made and of the effect of such joints on the strength of the member.

Obviously, butt joints without glue have no strength in tension, and tests have shown that when glued they are very erratic in strength, the best technique in using available glues affording no more than 25 percent of the tensile strength of wood (*29, pp. 59–60*). Hence glued butt joints are inadequate to resist the bending of laminae to shape or to resist tensile stress in a member after it is formed. Butt joints are undesirable in curved laminations because of their effect on interlamina

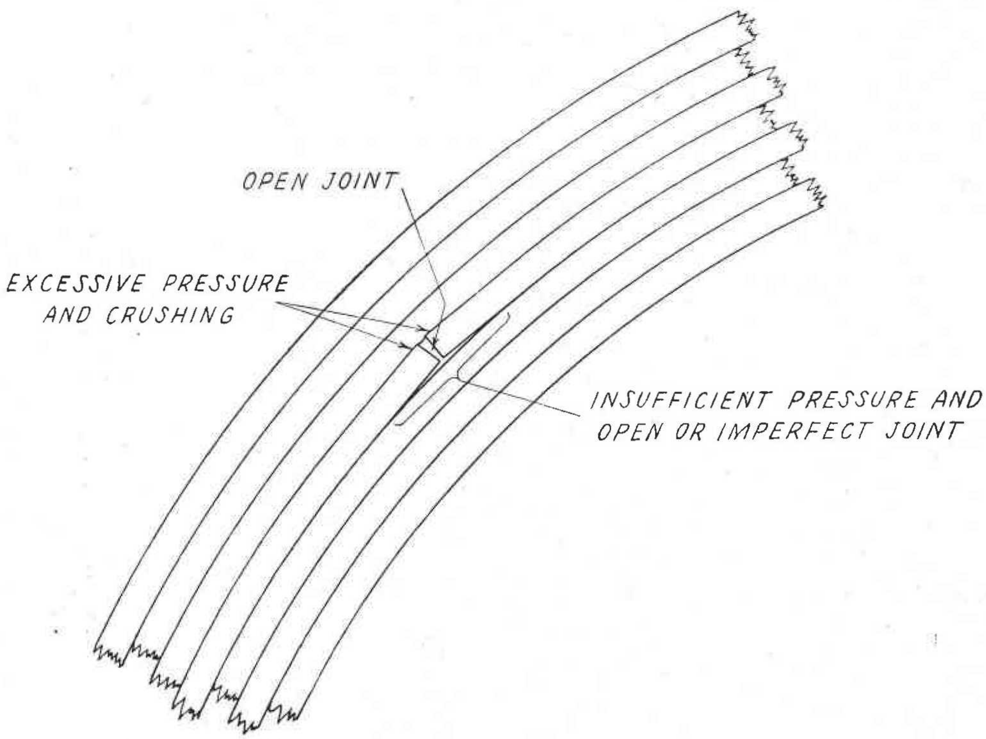

FIGURE 22.—Conditions at a butt joint in a lamination of a curved assembly.

contacts in their vicinity. It is impossible to produce curvature right to the end of a square-ended piece, hence, in the vicinity of butt joints, contact between adjacent laminae can result only from pressure sufficient to crush the wood and expel the glue, thus making the joints between laminae locally deficient in resistance to shear. Furthermore, effectiveness under longitudinal compression is lacking because the square ends of the two parts of the jointed lamination cannot be brought into contact for the full thickness. These conditions are illustrated by figure 22.

Tests on several species of wood (*10, 29*) have shown that a scarfed joint, if carefully prepared and well glued, has strength in tension or bending approximately as great as does the wood itself provided the length of the scarf is as much as 8 to 15 times the thickness of the material, depending on the kind of wood.

A slope of 1 in 12 is as long as seems practicable for scarf joints and tests of the effect of longer joints were not considered.

Table 10 presents the results of tests on members with 12 laminations each, matched in pairs, 1 of each pair having all laminations continuous, and the other having, at the centers of the 3 laminations next to the convex or tension face, glued scarf joints with slope of 1 in 12. The jointed laminations were made by preparing scarfs at each end of a board, which was then cut in the middle and the scarfed ends brought together and glued.

In making another series intended to be similar to that just described, an error resulted in joints being made in the laminae of both members of a pair. Three members of this series (Nos. 163, 186, and 189) did not fail in the joints and these are listed in table 10 as if all laminae were continuous, together with the matched members (Nos.

TABLE 10.—*Results of tests on matched pairs of parabolic curved members of southern yellow pine having 12 laminations—1 of each pair with 3 laminations next to convex face scarf-jointed at 1 in 12 slope; the other with all laminations continuous* [1]

Offset and serial No. of jointed and paired continuous members [2]	At proportional limit			Modulus of elasticity	At maximum moment			Kind of failure
	Bending stress	Bending + compression	Deflection		Bending stress	Bending + compression	Deflection	
	Lbs./sq. in.	*Lbs./sq. in.*	*Inches*	*1,000 lbs./sq. in.*	*Lbs./sq. in.*	*Lbs./sq. in.*	*Inches*	
3-foot offset:								
233	**5,485**	**5,890**	**2.135**	**1,825**	**6,175**	**6,625**	**2.42**	Tension. No failure in joints.
232	4,820	5,195	1.755	1,904	7,175	7,705	2.70	Tension.
235	**5,235**	**5,630**	**2.175**	**1,720**	**7,335**	**7,860**	**3.20**	Tension in half of joint in first and in all of joint in second lamination.
234	4,780	5,145	2.125	1,589	6,955	7,455	3.25	Tension.
237	**4,290**	**4,615**	**1.900**	**1,580**	**5,315**	**5,705**	**2.51**	Tension in half of joint in third lamination.
236	4,760	5,120	2.095	1,584	5,885	6,320	2.70	Tension.
239	**3,700**	**3,985**	**1.340**	**1,918**	**5,595**	**6,015**	**2.08**	Tension in half of joint in third lamination and in all of joint in second lamination.
238	4,805	5,175	1.785	1,870	8,680	9,290	3.62	Tension.
241	**3,740**	**4,025**	**1.475**	**1,820**	**5,975**	**6,400**	**2.68**	Tension in joint in first lamination and in half of joints in second and third laminations. Joint in first lamination partially open before test.
240	3,390	3,650	1.430	1,706	5,020	5,390	2.42	Tension.
164	**3,180**	**3,435**	**1.125**	**1,751**	**7,100**	**7,620**	**2.84**	Tension in half of joint in first, second, and third laminations.
163 [3]	3,950	4,260	1.390	1,755	7,000	7,510	3.09	Compression—no failure in joints.
Average	**4,270**	**4,595**	**1.691**	**1,769**	**6,250**	**6,705**	**2.62**	
	4,420	4,760	1.763	1,735	6,785	7,275	2.96	
Average of failing jointed members and paired mates.[4]	**4,030**	**4,340**	**1.603**	**1,758**	**6,265**	**6,720**	**2.66**	
	4,335	4,670	1.775	1,701	6,710	7,190	3.02	

[1] All strength values adjusted to 12-percent moisture content.
[2] Values for jointed members shown in bold-faced type.
[3] Nos. 163, 186, and 189 had glued scarf joints sloping 1 in 12 in the 3 laminations next to the convex face. No failures occurred in these joints and the 3 members are listed here with those having continuous laminations.
[4] Failures occurred in all jointed members except No. 233 (3-foot offset) and No. 243 (6-foot offset).

TABLE 10.—*Results of tests on matched pairs of parabolic curved members of southern yellow pine having 12 laminations—1 of each pair with 3 laminations next to convex face scarf-jointed at 1 in 12 slope; the other with all laminations continuous*—Continued

Offset and serial No. of jointed and paired continuous members	At proportional limit			Modulus of elasticity	At maximum moment			Kind of failure
	Bending stress	Bending + compression	Deflection		Bending stress	Bending + compression	Deflection	
	Lbs./sq. in.	*Lbs./sq. in.*	*Inches*	*1,000 lbs./sq. in.*	*Lbs./sq. in.*	*Lbs./sq. in.*	*Inches*	
6-foot offset:								
243	**3,505**	**3,670**	**1.235**	**1,670**	**5,140**	**5,380**	**1.98**	Progressive tension. No failure in joints.
242	3,360	3,525	.960	2,031	8,100	8,465	3.40	Progressive tension.
245	**3,690**	**3,865**	**1.140**	**1,929**	**5,215**	**5,455**	**1.68**	Tension failure in half of joint in first and third laminations.
244	4,470	4,680	1.365	1,925	5,765	6,035	1.79	Tension.
247	**2,761**	**2,895**	**.900**	**1,888**	**5,395**	**5,645**	**2.06**	Progressive tension failure in joint in first lamination and in half of joint in third lamination.
246	3,300	3,455	1.170	1,705	6,510	6,805	3.00	Progressive tension.
249	**3,010**	**3,150**	**1.030**	**1,840**	**4,980**	**5,205**	**2.16**	Progressive tension failure in joint in second lamination partially open before test.
248	4,120	4,315	1.340	1,872	6,320	6,610	2.14	Tension.
251	**3,815**	**3,995**	**1.405**	**1,715**	**4,910**	**5,135**	**2.00**	Progressive tension failure in part of joint in third lamination.
250	3,950	4,140	1.360	1,793	4,780	5,005	1.74	Tension.
185	**3,950**	**4,140**	**1.325**	**1,634**	**5,880**	**6,150**	**2.13**	Progressive tension in half of joint in first lamination.
186 [3]	3,900	4,085	1.320	1,634	5,790	6,055	2.22	Progressive tension—no failure in joints.
190	**4,310**	**4,500**	**1.065**	**2,150**	**5,560**	**5,800**	**1.55**	Progressive tension in joint in second lamination and in halves of joints in first and third laminations.
189 [3]	4,180	4,380	1.140	1,956	6,430	6,725	2.43	Progressive tension—no failure in joints.
Average	**3,580** 3,895	**3,745** 4,085	**1.156** 1.236	**1,832** 1,845	**5,295** 6,240	**5,540** 6,530	**1.94** 2.39	
Average of failing jointed members and paired mates.[4]	**3,590** 3,985	**3,755** 4,175	**1.144** 1.282	**1,859** 1,814	**5,325** 5,930	**5,565** 6,205	**1.93** 2.22	

[3] Nos. 163, 186, and 189 had glued scarf joints stoping 1 in 12 in the 3 laminations next to the convex face. No failures occured in these joints and the 3 members are listed here with those having continuous laminations.

[4] Failures occured in all jointed members except No. 233 (3-foot offset) and No. 243 (6-foot offset).

164, 185, and 190) which did fail in the joints. Average results of tests on members with scarf joints (slope 1 in 12) at the center of the length of three laminae next to the convex face have been appended to tables 2, 3, and 4. Table 11 presents the results of tests on a few members that include laminae jointed as described therein. In each instance, the member with one or more jointed laminations is compared to a matched member with all laminations continuous.

TABLE 11.—*Comparison of southern yellow pine members with jointed laminations and matched members with continuous laminations* [1]

Serial Nos. of members in matched pairs	Description of member with jointed laminations	Offset	Bending stress+stress in compression—At proportional limit: Continuous laminations (*A*)	At proportional limit: Laminations jointed (*B*)	At proportional limit: Ratio (B/A)	At maximum moment: Continuous laminations (*A*)	At maximum moment: Laminations jointed (*B*)	At maximum moment: Ratio (B/A)	Modulus of elasticity: Continuous laminations (*A*)	Modulus of elasticity: Laminations jointed (B)	Modulus of elasticity: Ratio (B/A)
		Feet	*Pounds per square inch*	*Pounds per square inch*		*Pounds per square inch*	*Pounds per square inch*		*1,000 pounds per square inch*	*1,000 pounds per square inch*	
81–82	Glued scarf joints 1:12 slope in outer lamination at center of length.	3	4, 675	5, 080	1. 09	10, 525	7, 660	0. 73	1, 810	1, 780	0. 98
83–84	Glued scarf joints 1:12 slope in outer lamination at center and in second lamination at quarter point of length.	3	5, 755	5, 375	. 94	8, 500	9, 585	1. 13	2, 010	2, 095	1. 04
93–94	Glued scarf joints 1:12 slope in outer lamination at center of length.	6	4, 020	4, 885	1. 22	7, 715	8, 020	1. 04	1, 916	1, 890	. 99
95–96	Glued scarf joints 1:12 slope in outer lamination at center and in center lamination at quarter point of length.	6	5, 040	4, 480	. 89	10, 445	9, 910	. 95	1, 860	2, 045	1. 10
85–86	Butt joints in second lamination at center and in third lamination at quarter point of length.	3	4, 550	5, 770	1. 27	9, 055	8, 500	. 94	1, 981	2, 010	1. 02
97–98		6	5, 085	5, 030	. 99	8, 850	6, 750	. 76	1, 887	1, 776	. 94
87–88	Unglued scarf joints 1:6 slope in second lamination at center and in third lamination at quarter point of length.	3	4, 840	4, 945	1. 02	9, 685	8, 365	. 86	1, 687	1, 861	1. 10
99–100		6	3, 020	5, 735	1. 90	8, 080	7, 505	. 93	1, 685	1, 643	. 97
89–90	Butt joints in third and fifth laminations 0, 3, and 6 feet from center and in fourth and sixth laminations 1½ and 4½ feet from center.	3	5, 350	5. 520	1. 03	10, 995	9, 470	. 86	1, 983	1, 824	. 92
101–102		6	4, 990	5, 246	1. 06	7, 380	8, 710	1. 18	1, 648	1, 815	1. 10
91–92	Unglued sawed scarfs 1:6 slope in third and fifth laminations 0, 3, and 6 feet from center and in fourth and sixth laminations 1½ and 4½ feet from center.	3	5, 010	4, 160	. 83	10, 985	7, 505	. 68	1, 806	1, 710	. 95
103–104		6	3, 430	3, 470	1. 01	8, 450	8, 600	1. 02	1, 858	1, 840	. 99

[1] Each member compos ed of 8 laminations.

The data presented in table 11 are somewhat erratic and furthermore do not cover all the possible combinations of type of joint and its position in the member. It is believed, however, that they, together with results of previous tests, justify the conclusion that the use of glued scarf joints sloping across the thickness of a lamina in not less than 12 times its thickness and reasonably well separated in adjacent laminae, will result in members whose strength, particularly at proportional limit, will not be significantly lower than that of members with all laminae continuous. It is indicated, however, by the data of table 10 and by comparing the values appended to tables 2, 3, and 4 with the other values in the same tables that such joints at the same point in the length of successive laminae are undesirable.

EASTERN HEMLOCK

Table 12 lists the results of tests on 15 curved members made from eastern hemlock (*Tsuga heterophylla*). The material used was 6-inch boards with knots varying in size from ½ to 4¼ inches. Outer laminations were sufficiently knotty that none of the members can be classed as grade I construction. (See p. 63 for definitions.) In fact, only three, the eighth, ninth, and tenth in the list, can be classed as grade II construction. (The last one is grade II except for oversized knots in central laminations.) All others have knots too large or cross grain of too steep a slope to be so rated.

TABLE 12.—*Results of end-thrust tests on individual curved members of eastern hemlock; ¾-inch laminations* [1]

Laminations (number)	Offset	Stress at proportional limit: Bending	Stress at proportional limit: Bending + compression parallel to grain	Modulus of elasticity	Stress at maximum moment: Bending	Stress at maximum moment: Bending + compression parallel to grain
	Feet	*Pounds per square inch*	*Pounds per square inch*	*1,000 pounds per square inch*	*Pounds per square inch*	*Pounds per square inch*
18	6	2,780	2,951	1,222	4,210	4,465
18	3	3,320	3,661	1,260	6,540	7,167
18	6	3,240	3,439	1,289	6,820	7,216
18	6	3,270	3,468	1,194	6,720	7,180
18	3	3,380	3,720	1,206	5,740	6,282
18	3	2,730	3,011	1,134	4,480	4,923
8	6	3,500	3,593	1,240	5,060	5,188
8	3	3,380	3,526	1,417	7,380	7,652
8	6	4,910	5,037	1,445	6,530	6,692
8	3	2,980	3,104	1,280	4,320	4,491
8	6	2,190	2,249	1,237	4,070	4,173
8	6	2,240	2,301	1,346	4,810	4,929
8	6	3,140	3,223	1,335	5,550	5,685
8	3	1,890	1,975	1,291	3,620	3,767
8	3	4,430	4,609	1,236	5,670	5,886
Average			3,324	1,275		5,708

[1] All strength values adjusted to 12-percent moisture content.

The number of tests is obviously insufficient to establish reliable averages on which to base working stress values for eastern hemlock in laminated arches. However, in view of the values obtained in these tests and the behavior of the members tested, the working stress values for eastern hemlock as obtained by the procedure later outlined are considered appropriate. These working stresses would, with grade I construction as later defined (p. 63), vary from 1,225 to 1,370 pounds per square inch for members of the curvatures and numbers of laminations listed in table 12.

DOUGLAS FIR

Tables 13, 14, and 15 show the distribution of strength values as found from a series of end-thrust tests on arches built of Douglas fir boards.

TABLE 13.—*Distribution of bending plus compression stress at proportional limit as found in end-thrust tests of parabolic curved members of Douglas fir, according to percent of defective material* [1]

Class limits for combined stress (pounds per square inch)	0 percent	25 percent	33⅓ percent	50 percent	Tally, 3-foot offset	Tally, 6-foot offset
	Serial No.	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Number*
3,000–3,199			302, *293*		1	1
3,200–3,399				*280*		1
3,400–3,599			*295*			1
3,600–3,799	*141*					1
3,800–4,399		*283*	296, *145*	*262*	1	3
4,000–4,199				*270, 274, 278*		3
4,200–4,399	142		304, *143*		2	1
4,400–4,599						
4,600–4,799		288, *291*			1	1
4,800–4,999			144, 294		2	
5,000–5,199		*285*	146		1	1
5,200–5,399	*147*		292		1	1
5,400–5,599		282	306		2	
5,600–5,799	148		*305*		1	1
5,800–5,999		290			1	
6,000–6,199						
6,200–6,399		286			1	
6,400–6,599						
6,600–6,799		284			1	
Tally ... number	2, *2*	5, *3*	8, *5*	0, *5*	15	15

[1] Members whose serial numbers are in italics were 6-foot offset; all others were 3-foot offset. All values adjusted to 12-percent moisture content.

TABLE 14.—*Distribution of bending plus compression stress at maximum moment as found in end-thrust tests of parabolic curved members of Douglas fir, according to percent of defective material* [1]

Class limits for combined stress (pounds per square inch)	0 percent	25 percent	33⅓ percent	50 percent	Tally, 3-foot offset	Tally, 6-foot offset
	Serial No.	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Number*
4,500–4,799				*280*		1
4,800–5,099				*264, 266, 276*		3
5,100–5,399				*268*		1
5,400–5,699				*278*		1
5,700–5,999		*291*		*274*		2
6,000–6,299						
6,300–6,599			292, *293*	*270*	1	2
6,600–6,899		288	296, 304, *143*		3	1
6,900–7,199		*285, 287*	*295*			3
7,200–7,499			294, 302	*262*	2	1
7,500–7,799		*283*	*145, 305*			3
7,800–8,099		286	*297*		1	1
8,100–8,399	142	284, 290, *289*			3	1
8,400–8,699	*147*					1
8,700–8,999			146		1	
9,000–9,299						
9,300–9,599	148	282			2	
Tally ... number	2, *1*	5, *5*	6, *6*	0, *9*	13	21

[1] Members whose serial numbers are in italics were 6-foot offset; all others were 3-foot offset. All values adjusted to 12-percent moisture content.

TABLE 15.—*Distribution of modulus of elasticity as found in end-thrust tests of parabolic curved members of Douglas fir, according to percent of defective material* [1]

Class limits for combined stress (pounds per square inch)	0 percent	25 percent	33⅓ percent	50 percent	Tally, 3-foot offset	Tally, 6-foot offset
	Serial No.	*Serial No.*	*Serial No.*	*Serial No.*	*Number*	*Number*
1,400–1,449	*141*					1
1,450–1,499		288		*268, 274, 280*	1	3
1,500–1,549			*295, 303*			2
1,550–1,599	142		304, *307*		2	1
1,600–1,649			294, *293, 305*	*266, 270*	1	4
1,650–1,699				*278*		1
1,700–1,749			306		1	
1,750–1,799		*283, 291*	292, *145*	*264, 272, 276*	1	6
1,800–1,849		282, 290	302, *143, 297*		3	2
1,850–1,899			146		1	
1,900–1,949		286			1	
1,950–1,999		*285, 289*				2
2,000–2,049	147		144, 296	*262*	3	1
2,050–2,099		284, *287*			1	1
2,100–2,149		*147*				1
Tally ... number	2, *1*	5, *6*	8, *8*	0, *10*	15	25

[1] Members whose serial numbers are in italics were 6-foot offset; all others were 3-foot offset. All values adjusted to 12-percent moisture content.

The stock used in the Douglas fir members had been kiln dried to a comparatively low moisture content and when the tests were made the average moisture content was 10 percent. The average moisture content when the boards were bent to shape was probably 8 percent or less. Because of this low moisture content, which rendered the material more subject to breakage, some pieces splintered slightly when bent to shape, particularly when bent to the 6-foot offset. This, together with the higher stress induced in bending the drier material, caused some of the members to fail prematurely when tested. In compiling tables 13 to 15, values that appeared to be lowered by these causes were omitted. The remaining values are probably affected similarly but to a lesser degree. The tabulated values in tables 13 to 15 have all been adjusted to a basis of 12-percent moisture content so that differences in strength due to differences in moisture content at time of test do not affect comparisons.

Results of tests on Douglas fir and southern yellow pine members are compared in table 16.

Considering the experience in making up and testing members of Douglas fir as related above, together with the comparisons shown in table 16 and other experience with and information on this species and on southern yellow pine, it appears acceptable to use for laminated glued members of Douglas fir the same working stresses as for southern yellow pine, provided such members are made up from stock with a moisture content appropriate to the curvatures to which laminations must be bent.

Experience with the tests of Douglas fir points definitely to the inadvisability of attempting to build up laminated members from stock dried to a low moisture content unless the curvature is very moderate (p. 60).

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TABLE 16.—*Comparison of results of end-thrust tests on curved members of southern yellow pine and Douglas fir* [1]

Item	Southern yellow pine		Douglas fir	
	3-foot offset	6-foot offset	3-foot offset	6-foot offset
Stress (bending + compression at proportional limit):				
Tests .... number	34	37	15	15
Average value, $\bar{x}$ .... pounds per square inch	5, 250	4, 360	5, 070	4, 180
Standard deviation, σ .... do	876	734	906	711
$\bar{x} - 2.326\sigma$	3, 210	2, 650	2, 960	2, 530
$\bar{x} - 3\sigma$	2, 620	2, 160	2, 350	2, 050
Minimum test value .... pounds per square inch	3, 540	3, 020	3, 020	3, 200
Coefficient of variation $\sigma \div \bar{x}$ .... percent	16. 7	16. 8	17. 9	17. 0
Stress (bending + compression at maximum moment):				
Tests .... number	27	31	13	21
Average value, $\bar{x}$ .... pounds per square inch	9, 450	7, 550	7, 830	6, 560
Standard deviation, σ .... do	1, 585	1, 194	988	1, 173
$\bar{x} - 2.326\sigma$	5, 760	4, 770	5, 530	3, 830
$\bar{x} - 3\sigma$	4, 700	3, 970	4, 870	3, 040
Minimum test value .... pounds per square inch	5, 390	5, 000	5, 210	4, 760
Coefficient of variation $\sigma \div \bar{x}$ .... percent	16. 8	15. 8	12. 6	17. 9
Modulus of elasticity:				
Test .... number	34	37	15	25
Average value, $\bar{x}$ .... 1,000 pounds per square inch	1, 971	1, 817	1, 812	1, 725
Standard deviation, σ .... do	178	149	181	195
$\bar{x} - 2.326\sigma$	1, 557	1, 470	1, 391	1, 271
$\bar{x} - 3\sigma$	1, 437	1, 370	1, 269	1, 140
Minimum test value .... 1,000 pounds per square inch	1, 584	1, 416	1, 458	1, 437
Coefficient of variation $\bar{x} \div \sigma$ .... percent	9. 0	8. 2	10. 0	11. 3

[1] All strength values adjusted to 12-percent moisture content.

TESTS OF BUILDING ARCHES

Roof supports in the service building at the Forest Products Laboratory (fig. 1) included three-hinged glued laminated arches of two types, designated C and D, as described on page 67. In addition to those required for the building, two half arches of each of these types were made. These were tested under end thrust.

TESTS OF BUILDING ARCHES—TYPE D

The half arches of the D type that were tested are designated D–1 and D–2.

Figure 23, which shows half-arch D–1 in position in the testing machine, illustrates the arrangement of the tests. The ends of both half arches were cut square with the chord joining the centers of their ends. Each half arch was marked with stations spaced 1 foot apart along this chord and designated alphabetically, *A* being at the lower end.

In half-arch D–1, which was tested in the full length, the principal failure occurred in the straight upper portion. Before testing D–2, about 11 feet of its upper end was cut off in order to encourage failure in the vicinity of the knee and thus get a measure of the strength of the most severely curved portion.

Horizontal deflection was measured at three stations by reading scales attached to the arch against fine wires fixed in a vertical position (fig. 23). Figure 24 shows five pairs of special nails as arranged across half-arch D–2. Nails of each pair were spaced to a gage length of 2 inches and changes in this gage length were measured by means of a strain gage reading to 0.0001 inch. Similar strain readings made on D–1 proved unreliable because the nails with slender stems as used were insecure. Nails used in D–2 had larger stems.

Stations at which horizontal deflections were read were:

For half-arch D–1 (fig. 23):

E—in the straight lower portion and 4 feet above the lower end.

I—in the curved portion and at the point where the axis is farthest from the line of application of the load, 8 feet above the lower end.

P—in the straight upper portion and 15 feet above the lower end.

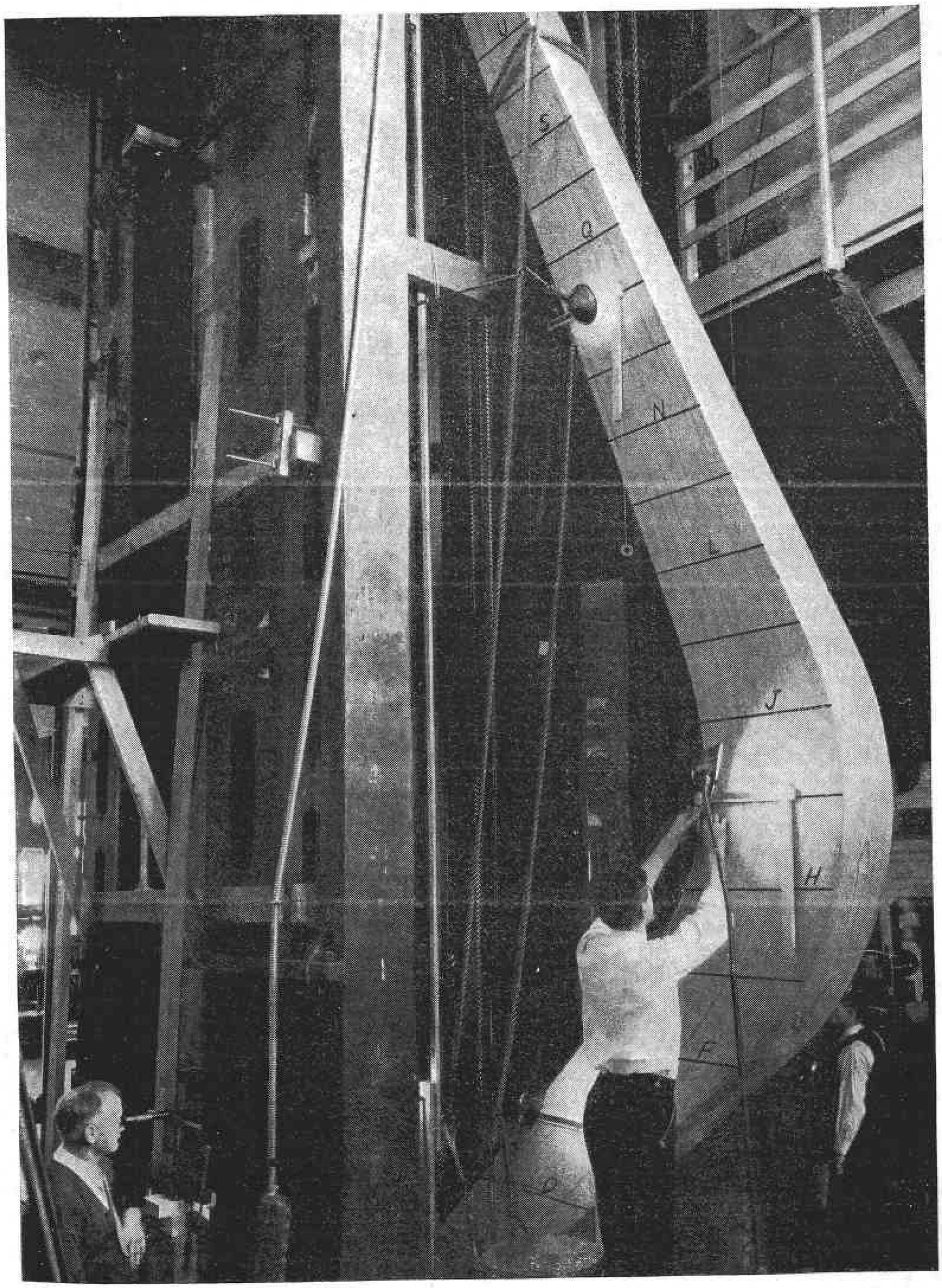

FIGURE 23.—Half-arch D–1 in position in machine with scales for reading horizontal deflections attached at stations *E*, *I*, and *P*.

For half-arch D–2:

F—in the straight portion and 5 feet above the lower end.

H+½—in the curved portion and at the point where the axis is farthest from the line of application of the load, 7½ feet above the lower end.

K—in the straight upper portion and 10 feet above lower end.

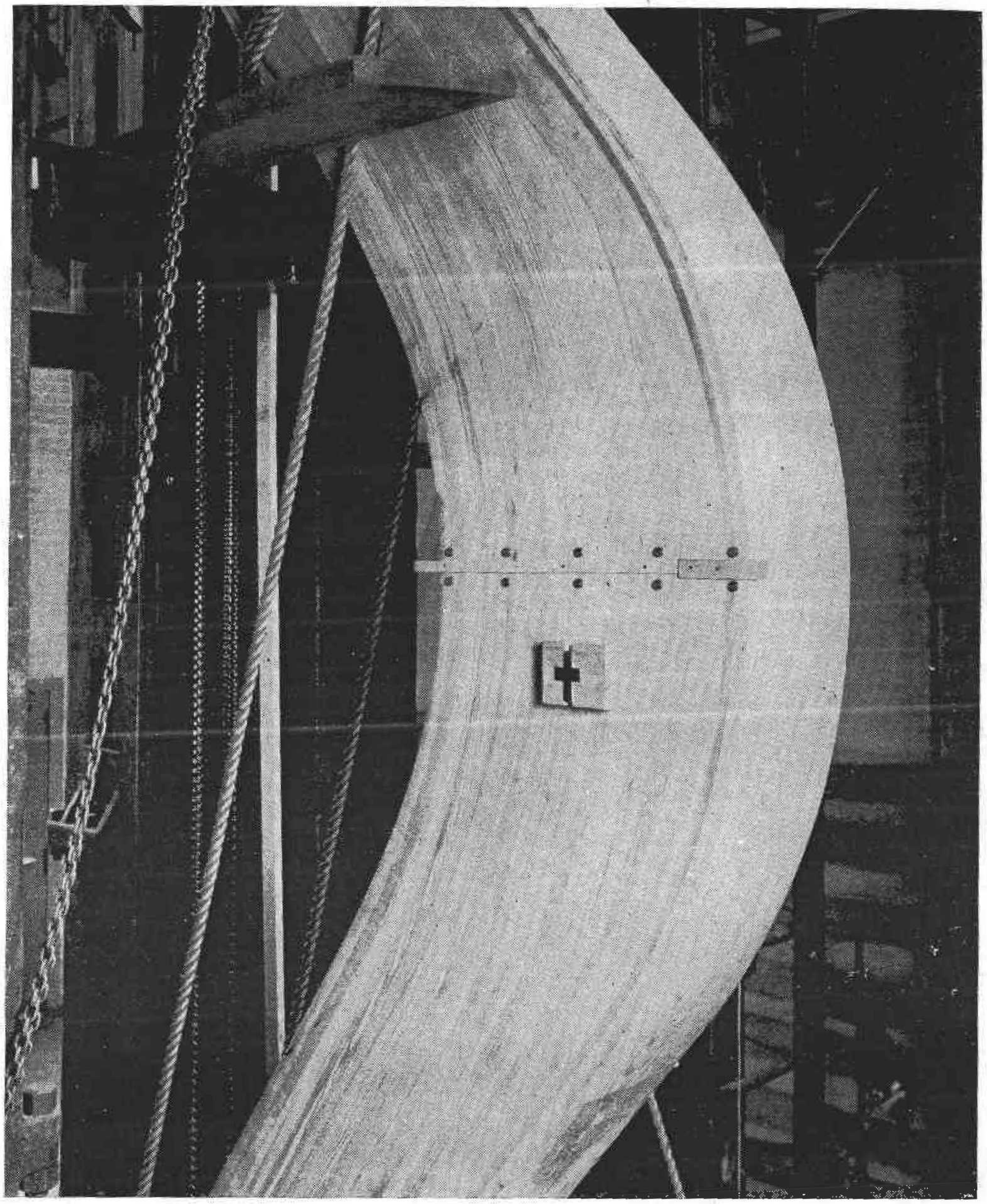

FIGURE 24.—Arrangement of pairs of special nails used in reading longitudinal strains at station *H*+½ of half-arch D–2. Nail pairs or gage lengths are numbered 2 to 6 from the convex to the concave face of the member. No. 1 was an exact 2-inch gage length on a steel bar fixed in the block below the special nails and was used to check the zero reading of the strain gage.

Tables 17 and 19 record readings on arches D–1 and D–2, respectively, together with brief notes describing the character and sequence of failures. Tables 18 and 20 list results of tests on small specimens cut from parts that were uninjured in the test of the member as a whole.

TABLE 17.—*Record of test on half-arch D–1* [1]

Load (pounds)	Horizontal deflection readings at station—			Shortening of chord	Remarks
	E	*I*	*P*		
	Inches	*Inches*	*Inches*	*Inches*	
0	0.06	0.07	0.07	0.14	
1,000	.12	.18	.20	.35	
2,000	.18	.26	.30	.47	
3,000	.24	.36	.42	.62	
4,000	.29	.45	.52	.75	
5,000	.34	.54	.64	.89	
6,000	.40	.62	.74	1.04	
7,000	.45	.71	.86	1.16	
8,000	.50	.80	.96	1.30	
9,000	.56	.89	1.07	1.43	
10,000	.61	.98	1.20	1.57	
11,000	.66	1.07	1.30	1.69	
12,000	.72	1.16	1.41	1.86	
13,000	.77	1.26	1.53	2.00	
14,000	.82	1.36	1.65	2.14	
15,000	.88	1.45	1.76	2.29	
16,000	.94	1.54	1.86	2.44	
17,000	.99	1.63	1.98	2.58	Slight tension between *E* and *F*.
18,000	1.04	1.72	2.10	2.73	
19,000	1.10	1.82	2.26	2.90	
20,000	1.16	1.92	2.34	3.03	
21,000	1.22	2.02	2.46		
22,000	1.28	2.13	2.59	3.37	
23,000	1.34	2.24	2.72	3.54	
24,000	1.40	2.34	2.83	3.71	
25,000	1.47	2.45	2.96	3.88	
26,000	1.54	2.56	3.09	4.05	
27,000	1.60	2.68	3.24	4.23	
28,000	1.67	2.80	3.37	4.43	
29,000	1.74	2.92	3.50	4.63	
30,000	1.80	3.04	3.65	4.83	
31,000	1.87	3.16	3.78	5.00	
32,000	1.94	3.28	3.91	5.21	
33,000	2.02	3.40	4.06	5.42	
34,000	2.09	3.52	4.20	5.63	
35,000	2.16	3.66	4.36	5.91	
36,000	2.24	3.78	4.50	6.06	
37,000	2.32	3.94	4.67	6.30	
38,000	2.40	4.08	4.84	6.53	
39,000	2.48	4.20	4.98	6.85	
40,000	2.58	4.38	5.16	7.06	
41,000	2.67	4.55	5.35	7.33	Tension on corner *R* to *Q*.
42,000	2.76	4.70	5.52	7.60	Compression first noted.
42,630	2.80	4.80	5.65		Tension on face *R* to *W*.
42,820					Tension on face *D* to *I*.
42,390	2.83	4.83	5.70	7.81	
43,000	2.88	4.93	5.80	7.98	
43,820	2.96	5.07	5.95	8.19	
43,890					Tension on face near *S*.
43,740	2.98	5.10	6.00	8.25	
43,890	3.00	5.12	6.00	8.36	
43,680	3.01	5.15	6.05	8.33	
44,000	3.06	5.22	6.12	8.45	
45,000	3.16	5.40	6.30	8.80	Increased compression.
45,920	3.27	5.60	6.52		
46,130	3.32	5.70	6.67		
46,610	3.38	5.80	6.75		
47,000	3.44	5.90	6.85		Tension on corner near *J*. Tension on face near *V*.
47,410	3.49	6.00	6.95		
47,820					Tension on face *N* to *J* and *Q* to *V*.
47,690	3.55	6.10	7.05		
47,970	3.60	6.20	7.16		Compression wrinkle at *G*.
48,060	3.61	6.24	7.22		
47,280	3.63	6.28	7.24		
47,810	3.70	6.40	7.39		
48,010	3.72	6.47	7.43		

[1] Length between ends 28.23 feet. Average width 10.84 inches. Weight before test 1,687 pounds. Average moisture content 10.7 percent.

TABLE 18.—*Results of bending tests on 2- by 2- by 30-inch specimens cut from arch D-1 following test of the arch*

Position from which taken	Moisture content	Fiber stress at proportional limit	Modulus of rupture	Modulus of elasticity
	Percent	*Pounds per square inch*	*Pounds per square inch*	*1,000 pounds per square inch*
Adjacent to convex face in straight portion below the knee	10.0	7,760	13,450	1,740
	11.2	7,350	11,230	1,510
	10.4	7,880	13,180	1,670
Same but concave face	10.5	7,920	13,320	1,860
	9.8	9,980	15,600	2,480
	9.9	9,980	15,150	2,170
Adjacent to convex face in straight portion near top end	10.8	10,500	14,180	2,285
	12.1	8,400	12,020	2,110
	10.9	8,400	11,660	2,080
Same but concave face	10.1	7,880	12,120	1,740
	11.3	7,920	9,420	1,800
	10.6	8,930	15,170	2,330
Adjacent to convex face in straight portion between top and knee	9.7	10,020	13,550	2,090
	10.5	10,500	15,090	2,325
	12.1	7,910	10,970	1,800
Same but concave face	12.0	8,400	13,020	2,110
	11.2	7,880	10,970	1,770
	10.8	8,930	14,500	2,135
Near convex face in most severely bent part	9.9	8,700	13,720	2,005
	11.0	6,670	11,490	1,560
	8.1	6,460	8,080	1,772
Same but concave face	9.3	5,970	10,390	1,955
	9.4	7,650	14,150	2,005
	9.9	7,880	11,390	1,895
Average	10.5	8,330	12,660	1,967

TABLE 19.—*Record of test on half arch D-2* [1]

Load	Horizontal deflection at station—			Strain gage readings at station $H+\frac{1}{2}$ on gage length No.—					Shortening of chord
	F	$H+\frac{1}{2}$	*K*	2	3	4	5	6	
Pounds	*Inches*	*Inches*	*Inches*	*Inch*	*Inch*	*Inch*	*Inch*	*Inch*	*Inches*
2,080	0.08	0.06	0.06	0.0004	0.0153	0.0102	0.0044	0.0150	0.33
4,000	.13	.12	.10	.0004	.0154	.0102	.0044	.0148	.40
6,000	.18	.17	.16	.0005	.0155	.0102	.0043	.0146	.48
8,000	.24	.22	.22	.0006	.0155	.0102	.0043	.0145	.56
10,000	.29	.28	.27	.0007	.0156	.0102	.0042	.0143	.63
12,000	.34	.34	.32	.0008	.0157	.0103	.0042	.0142	.71
14,000	.38	.40	.37	.0009	.0157	.0103	.0041	.0140	.80
16,000	.44	.45	.42	.0009	.0158	.0103	.0041	.0138	.89
18,000	.49	.51	.48	.0010	.0159	.0103	.0041	.0136	.97
20,000	.55	.57	.53	.0011	.0159	.0104	.0040	.0134	1.05
22,000	.60	.63	.58	.0012	.0160	.0104	.0040	.0132	1.14
24,000	.65	.69	.64	.0013	.0160	.0104	.0039	.0130	1.23
26,000	.70	.75	.70	.0014	.0161	.0105	.0039	.0130	1.32
28,000	.76	.81	.75	.0014	.0161	.0104	.0038	.0126	1.43
30,000	.82	.88	.80	.0015	.0162	.0105	.0038	.0124	1.54
32,000	.88	.94	.86	.0016	.0162	.0105	.0037	.0122	1.64
34,000	.94	1.00	.92	.0017	.0163	.0106	.0036	.0120	1.74
36,000	1.00	1.08	.98	.0018	.0164	.0106	.0036	.0118	1.84
38,000	1.07	1.15	1.05	.0019	.0164	.0106	.0036	.0115	1.93
[2] 40,000	1.14	1.23	1.13	.0020	.0165	.0107	.0035	.0113	2.03
42,000	1.21	1.31	1.20	.0021	.0166	.0107	.0034	.0111	2.14
44,000	1.27	1.38	1.26	.0022	.0166	.0107	.0034	.0108	2.26
[3] 46,000	1.34	1.46	1.32	.0023	.0167	.0107	.0033	.0105	2.39
48,000	1.42	1.54	1.41	.0024	.0167	.0108	.0032	.0102	2.55
50,000	1.50	1.64	1.50	.0026	.0168	.0108	.0031	.0108	2.69
[4] 52,000	1.60	1.74	1.58	.0026	.0168	.0108	.0030	.0093	2.83
[2] 54,000	1.68	1.84	1.67	.0028	.0169	.0108	.0029	.0089	2.94

[1] Length between ends 18.04 feet. Average width 10.97 inches.
[2] Compression at $H+\frac{1}{2}$.
[3] Slight tension at $G+\frac{1}{2}$.
[4] Slight tension at *F* and increase at $G+\frac{1}{2}$.

TABLE 19.—*Record of test on half arch D-2*—Continued

Load	Horizontal deflection at station—			Strain gage readings at station $H+\frac{1}{2}$ on gage length No.—					Shortening of chord
	F	$H+\frac{1}{2}$	*K*	2	3	4	5	6	
Pounds	*Inches*	*Inches*	*Inches*	*Inch*	*Inch*	*Inch*	*Inch*	*Inch*	*Inches*
56, 000	1. 77	1. 94	1. 76	. 0029	. 0169	. 0108	. 0029	. 0084	3. 10
57, 000	1. 83	2. 00	1. 82	----	----	----	----	----	3. 20
58, 000	1. 87	2. 06	1. 86	----	----	----	----	----	3. 28
59, 000	1. 91	2. 10	1. 90	----	----	----	----	----	3. 36
60, 000	1. 96	2. 16	1. 95	----	----	----	----	----	3. 45
61, 000	2. 01	2. 22	2. 00	----	----	----	----	----	3. 50
62, 000	2. 08	2. 28	2. 06	----	----	----	----	----	3. 60
63, 000	2. 14	2. 36	2. 13	----	----	----	----	----	----
64, 000	2. 21	2. 44	2. 20	----	----	----	----	----	3. 70
[5] 65, 000	2. 28	2. 52	2. 28	----	----	----	----	----	3. 80
66, 000	2. 36	2. 61	2. 36	----	----	----	----	----	3. 95
[6] 67, 000	2. 44	2. 70	2. 44	----	----	----	----	----	4. 11
[7] 68, 000	2. 53	2. 80	2. 53	----	----	----	----	----	4. 33
[8] 68, 860	2. 64	2. 93	2. 64	----	----	----	----	----	4. 48
[9] 68, 650	2. 64	2. 94	2. 65	----	----	----	----	----	4. 50
69, 000	2. 73	2. 98	2. 69	----	----	----	----	----	4. 55
[10] 69, 180	----	----	----	----	----	----	----	----	----

[5] Compression at *H*
[6] Increased compression at *H*.
[7] Further compression at *H*.
[8] Tension at *L*.
[9] Scarf joint opened slightly at *F*.
[10] Final failure longitudinal shear.

TABLE 20.—*Results of bending tests on 2- by 2- by 30-inch specimens cut from arch D-2 following test of the arch*

Position from which taken	Moisture content	Fiber stress at proportional limit	Modulus of rupture	Modulus of elasticity
	Percent	*Pounds per square inch*	*Pounds per square inch*	*1,000 pounds square inch*
Adjacent to concave face in straight portion below the knee	8. 6	11, 020	17, 750	2, 440
	7. 5	11, 780	18, 520	2, 240
	8. 8	8, 400	13, 760	2. 110
Adjacent to convex face in straight portion below the knee	10. 5	8, 930	11, 820	1, 650
	10. 7	10, 500	14, 180	1, 750
	8. 8	9, 450	14, 330	1, 715
Adjacent to concave face in straight portion above the knee	10. 5	8, 400	13, 440	2, 070
	8. 9	9, 640	11, 780	2, 545
	9. 6	12, 190	18, 980	2, 300
Adjacent to convex face in straight portion above the knee	9. 6	10, 070	12, 510	1, 885
	12. 1	7, 030	9, 630	1, 640
	11. 4	7, 500	11, 780	1, 625
Average	9. 8	9, 580	14, 040	1, 998

Figures 25 to 29 show the principal failures.

PROPORTIONAL LIMIT

Because of the curved and tapering form, neither horizontal deflection nor shortening of chord, as measured in the tests of half-arches D–1 and D–2 and listed in tables 17 to 20, can be expected to be directly proportional to loads or to bending moments. As the strain gage readings on half-arch D–1 proved unreliable, no acceptable method is available for locating the proportional limit in this instance. The strain gage readings afford a means of estimating the proportional limit of half-arch D–2. In figure 30, moments at station $H+\frac{1}{2}$, are plotted against the strain readings made near the two curved faces. From this figure the limit of proportionality between moment and the strain at gage length No. 6 (concave side of member) has been taken as a compressive strain of 0.0032 inch in the 2-inch gage length, corresponding to a moment of 2,325,000 inch-pounds

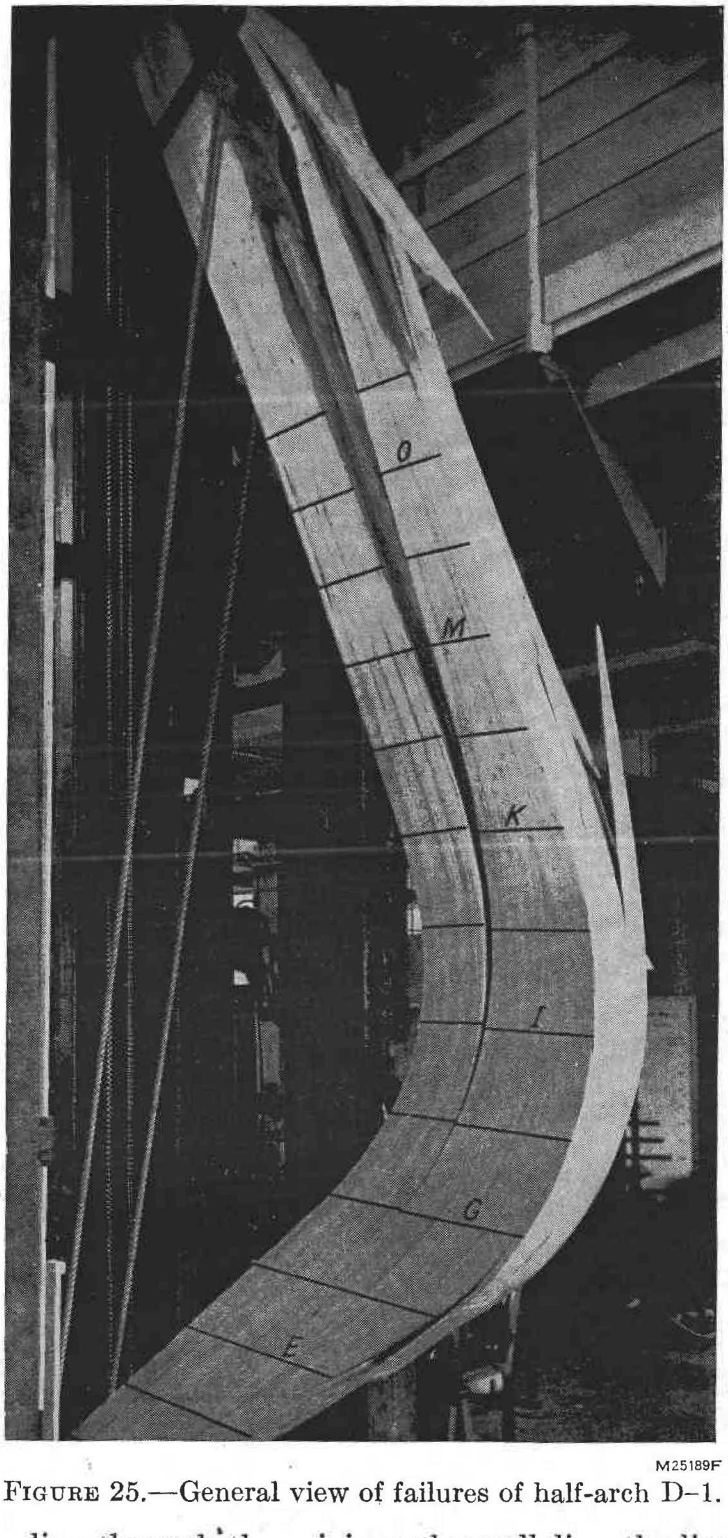

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FIGURE 25.—General view of failures of half-arch D-1.

(read from a line through the origin and paralleling the line averaging the moment-strain relation) with a corresponding tensile strain of 0.0014 inch. From these data, the stress at the proportional limit at station $H+\frac{1}{2}$, where the curvature of the member is approximately a maximum, is computed (table 21).

RESULTS OF TESTS AND COMMENTS

Table 21 is a summary of the tests on half-arches D–1 and D–2. As shown by figure 29, the upper end of D–2 had been notched and

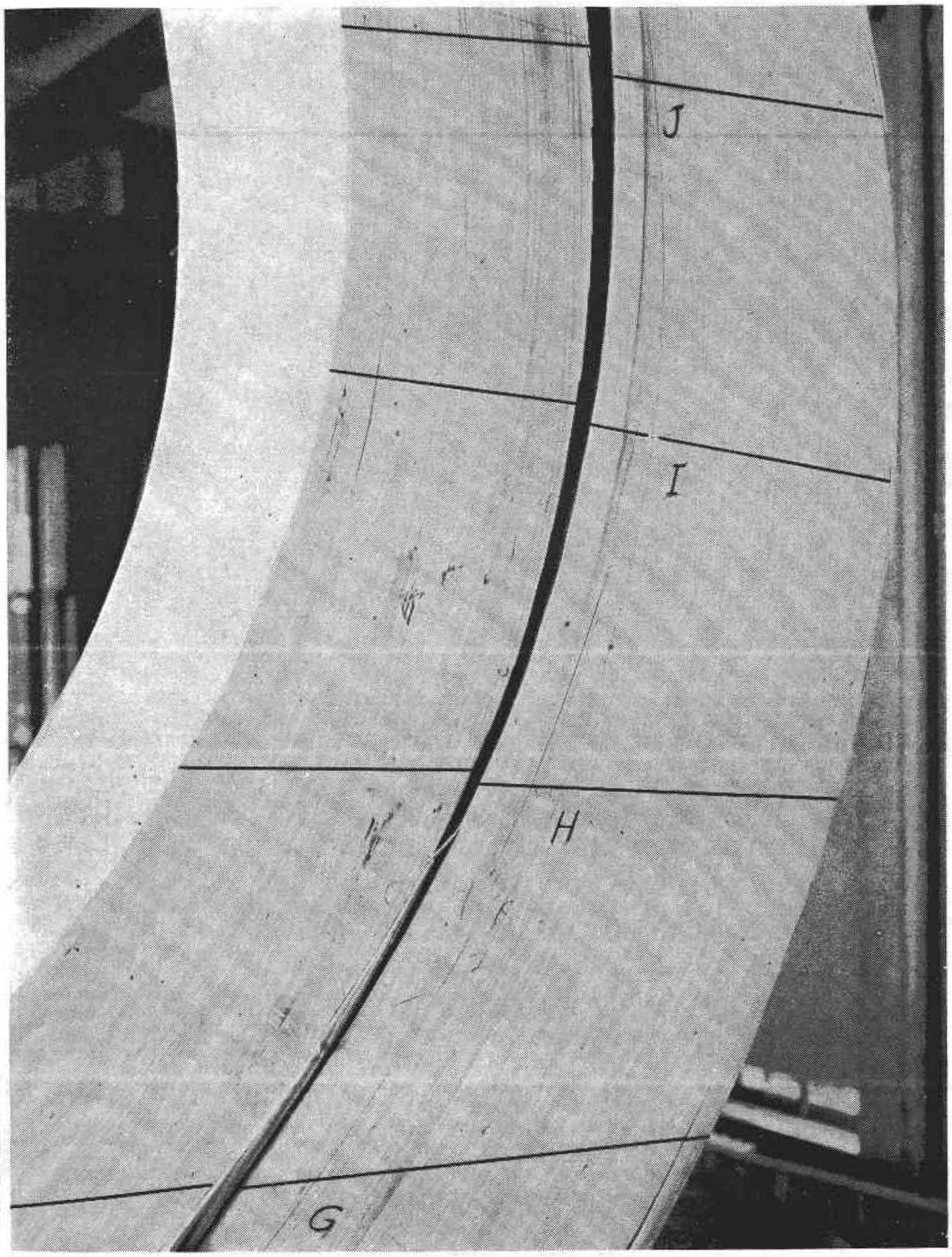

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FIGURE 26.—Compression failures on and near concave face of half-arch D–1 between stations *G* and *J*. The longitudinal split showing in this figure and in figure 25 is not a shear failure but a continuation of the tension failure that began in the upper leg of the half arch. The compression failures are shown to a larger scale in figure 27.

vertical and horizontal cuts made at each end to fit the arch to the end bearings. Similar fitting was done to the ends of D–1. The depths of cross section at the ends as listed in table 21 are measurements, perpendicular to the center lines or axes, that applied before the ends were fitted to the bearings. The computed stresses in shear and in compression parallel to grain take no account of the modifications of the cross sections at the ends, nor of the holes made at the lower ends for the bolt to which the rope shown in figures 28 and 29 was attached. These holes may have caused a substantial weakening in shear although, of the two members tested, only D–2 failed in shear.

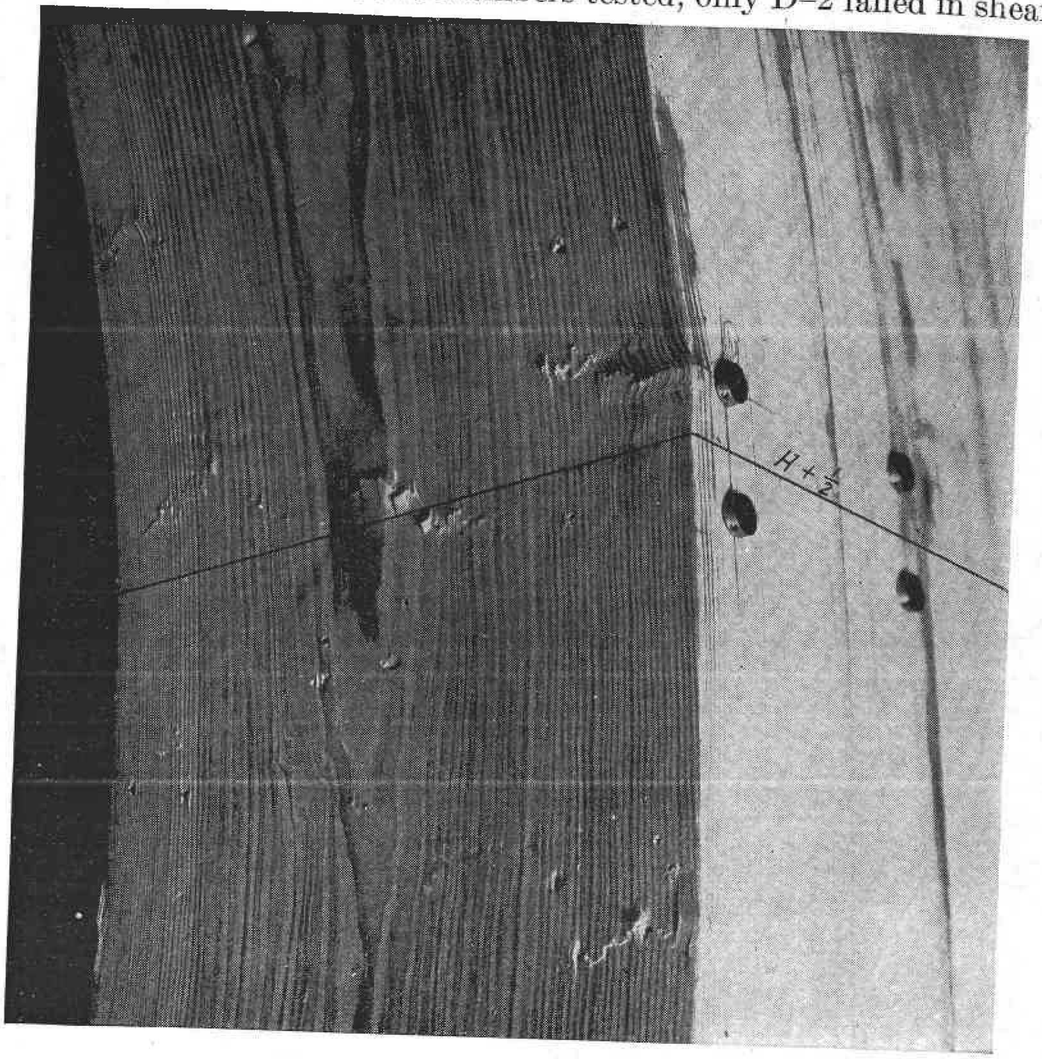

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FIGURE 27.—Compression failures on and near concave face of half-arch D–2 near station $H+\frac{1}{2}$.

In considering the stresses at maximum moment as listed in table 21, it should be noted that failure at the knee or point of maximum curvature did not occur in either D–1 or D–2. Although failure had begun at this point in both instances, considerably higher maximum moments would probably have been attained had the full strength in the region of the knee been developed. Bending moments in wooden members often increase after initial failure occurs and in many of the other tests of curved members the maximum moment was considerably greater than the value when failure began. Hence, stresses at maximum moment as listed in table 21 are too low to be a true measure of the strength at the point of maximum curvature.

TABLE 21.—*Dimensions and computed stresses for half-arches D–1 and D–2*

Dimension and stress	Half-arch D–1, at station—					Half-arch D–2, at station—				
	A	*E*		*P*	*CC*+ (top end)	*A*	*F*	*H*+1/2	*K*	*R*+ (top end)
Width of section ... inches	10.84	10.84	10.84	10.84	10.84	10.90	10.95	10.95	10.95	11.00
Depth of section ... do	13.15	19.07	23.70	16.78	8.95	13.30	20.27	24.34	20.94	15.00
Area of section ... do	142.5	206.8	256.9	181.9	97.0	145.0	222.2	266.7	229.5	165.0
Section modulus ... do	—	657	1,015	508.5	—	—	749	1,082	800	—
Distance from line of action of load to station before loading ... do	0	56.50	93.25	65.31	0	0	54.62	68.52	57.44	0
Approximate radius of curvature of axis ... do	—	—	43	—	—	—	—	43	—	—
At proportional limit at station *H*+1/2:										
Load ... pounds	—	—	—	—	—	—	—	33,500	—	—
Deflection ... inches	—	—	—	—	—	0	0.92	0.99	0.90	0
Moment ... inch-pounds	—	—	—	—	—	0	1,862,000	2,325,000	1,940,000	0
Bending stress ... pounds per square inch	—	—	—	—	—	0	2,485	2,150	2,425	0
Stress in compression perpendicular to grain at neutral surface ... do	—	—	—	—	—	—	—	302	—	—
Component of load parallel to axis [1] ... pounds	—	—	—	—	—	27,100	27,100	33,500	26,600	26,600
Stress in compression parallel to grain [1] ... pounds per square inch	—	—	—	—	—	186	122	124	116	161
Modulus of elasticity ... do	—	—	—	—	—	—	—	1,720,000	—	—
At maximum moment: [2]										
Deflection ... inches	0	3.61	6.24	7.22	0	0	2.75	2.98	2.72	0
Moment arm ... do	0	60.11	99.49	72.53	0	0	57.37	71.50	60.16	0
Moment ... inch-pounds	0	2,888,000	4,786,600	3,487,000	0	0	3,968,860	4,946,370	4,161,870	0
Bending stress ... pounds per square inch	0	4,390	4,715	6,860	0	0	5,300	4,550	5,200	0
Stress in compression perpendicular to grain at neutral surface ... do	—	—	650	—	—	—	—	646	—	—
Component of load paralles to axis [1] ... pounds	27,800	27,800	48,060	43,000	43,000	56,000	56,000	69,180	55,000	55,000
Stress in compression parallel to grain [1] ... pounds per square inch	195	134	187	236	443	386	251	257	240	333
Component of load perpendicular to axis [1] ... pounds	39,200	39,200	0	20,160	20,160	40,500	40,500	0	41,100	41,100
Shearing stress [1] ... pounds per square inch	413	285	0	166	312	420	273	0	268	373

[1] Computed from original shape of arch axis disregarding deformations in test.

[2] Load for D–1, 48,060 pounds; for D–2, 69,180 pounds.

TESTS OF BUILDING ARCHES—TYPE C

The building arches of the C type (p. 68) afford an interesting example of the use of glued laminated parts. Consideration of com-

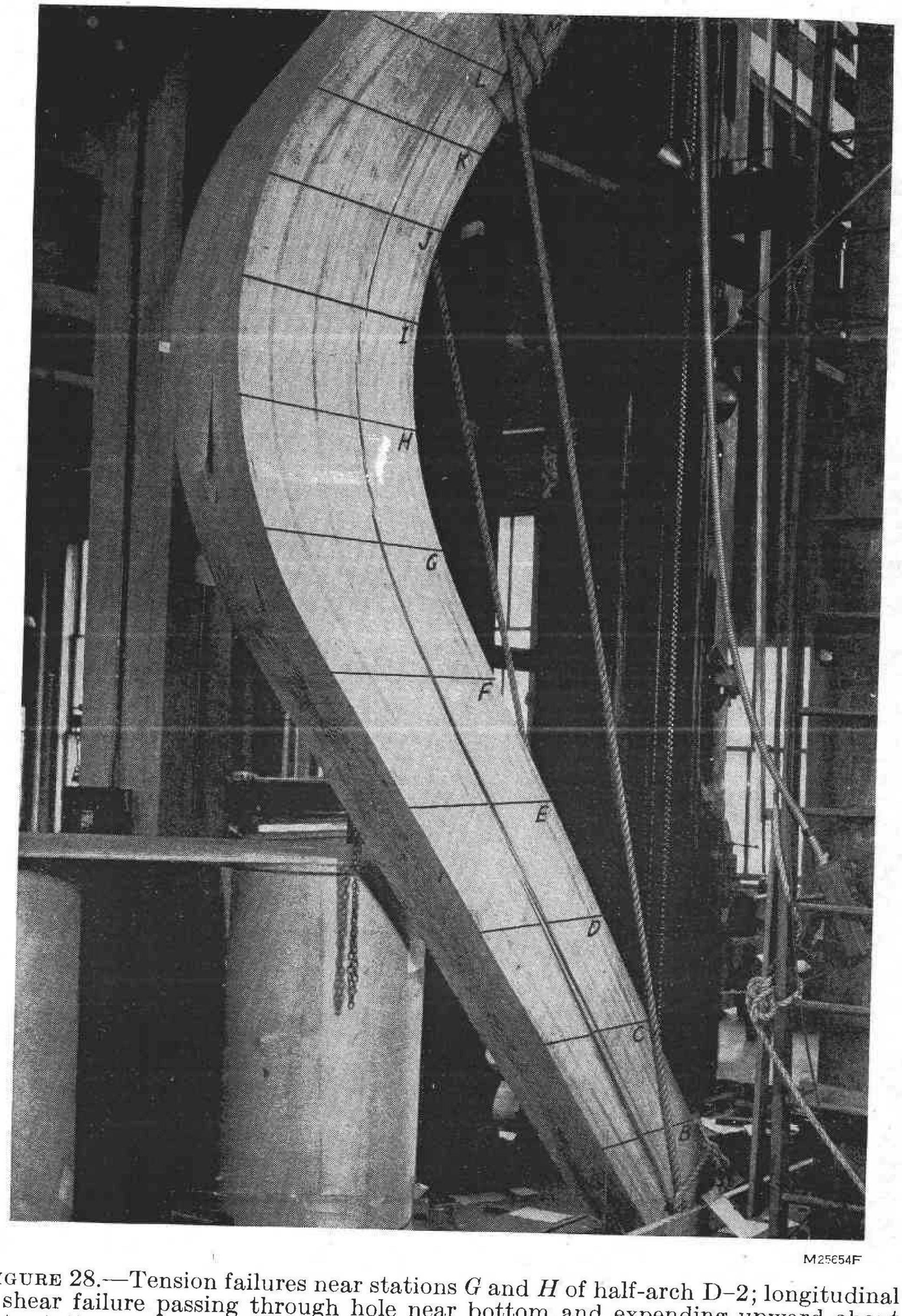

FIGURE 28.—Tension failures near stations *G* and *H* of half-arch D–2; longitudinal shear failure passing through hole near bottom and expending upward about to station *L*.

posite members of this type involves two new phases of the uses of wood in construction, namely, curved laminated parts and plywood. A detailed discussion of the C type arches is therefore beyond the scope of this bulletin. A brief account of the tests on two half arches of the C type is presented, however, but without attempting to generalize from the results or to set up a procedure for the design of such members.

The two half arches designated C–2 and C–5 were tested under end thrust applied along the chord connecting the crown and abutment hinge points.

The ends were cut square with the chord and load was applied through hinged bearing plates at each end.

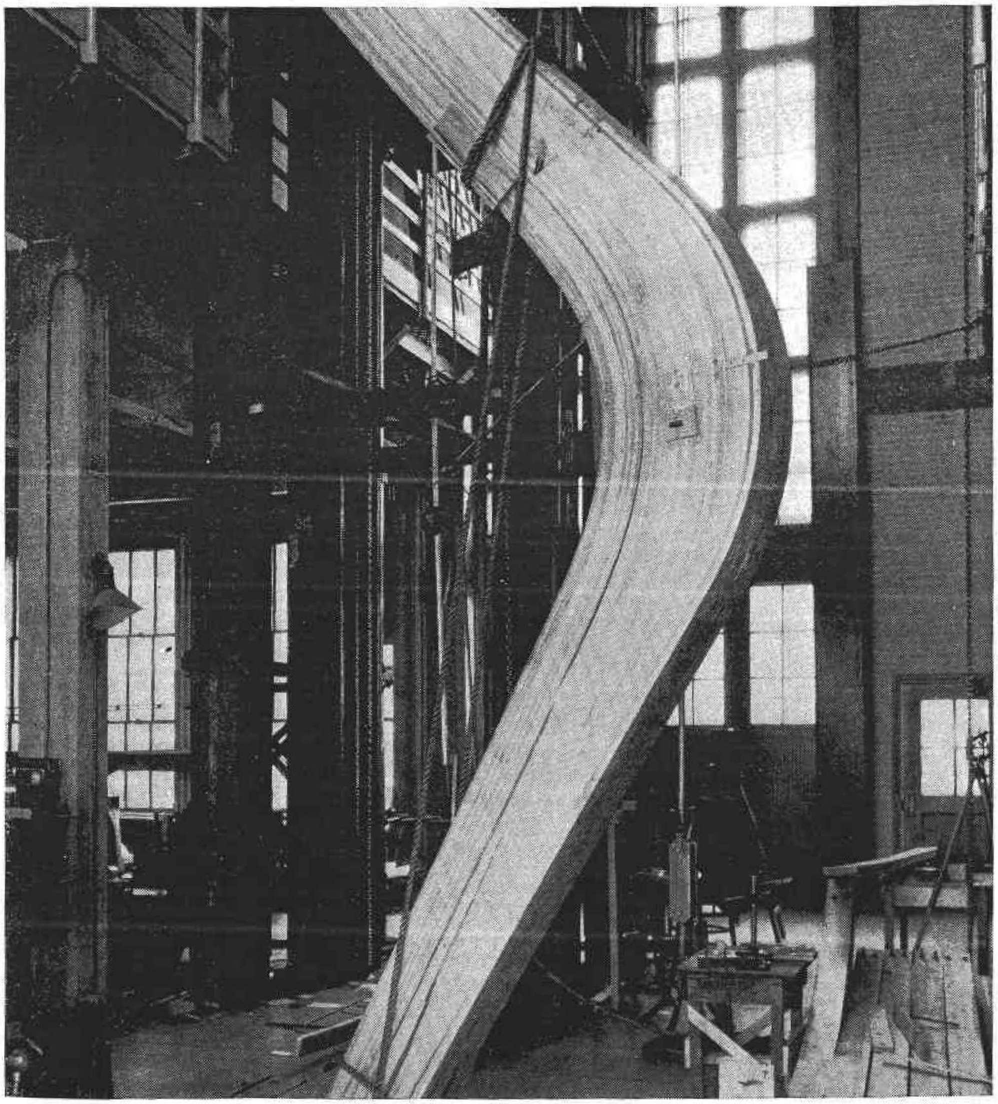

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FIGURE 29.—Longitudinal shear failure in half-arch D–2.

Tests on the two half arches were made in the same way except that the projecting parts of the plywood webs at the knee were left on C–5, but before the testing of C–2 were cut away along the outer curved border of the flange. Horizontal deflections were read at five points along the length (fig. 31, *A*) in the same way as in tests (p. 34) of half-arches D–1 and D–2. Strain was read between nails of each of four pairs located as shown by figure 31, *B*. Shortening of the distance between the two flanges was read near the point where the curvature is most severe. The webs of the type C half arches were stiffened at and approximately midway between splices (p. 70). At one point, about midway between stiffeners, readings were taken on one web to measure any buckling that might occur in it (fig. 31, *B*).

Forces tending to cause the flanges to approach each other are set up when, as in these tests, the applied moment is such that the

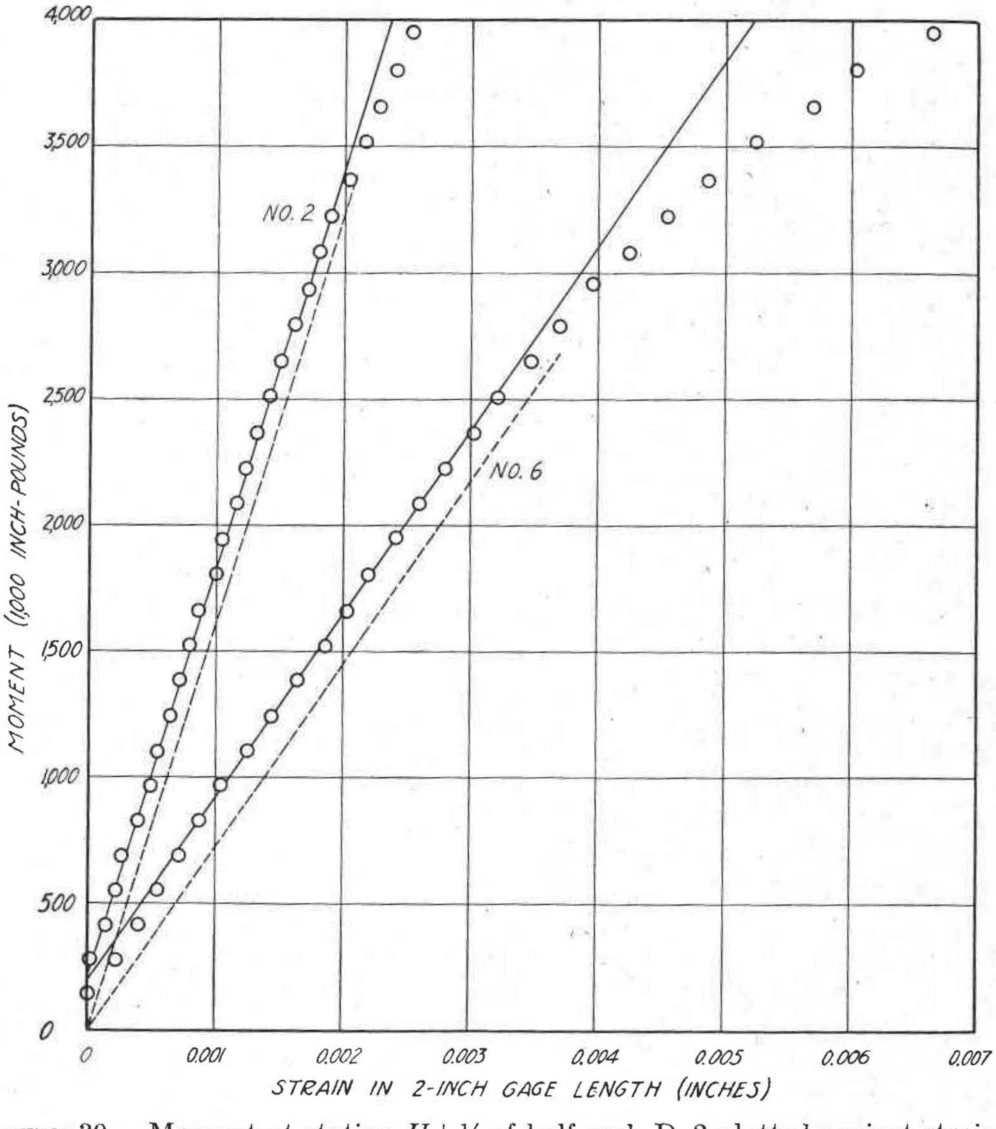

FIGURE 30.—Moment at station $H+\frac{1}{2}$ of half-arch D–2 plotted against strain in gage lengths Nos. 2 and 6 (near convex or tension and concave or compression faces of the member).

flange on the convex side is in tension. This results in a radially directed compression in webs and a radially directed shear between webs and flanges. The pieces inserted between the webs as stiffeners and at splices (p. 70) acting as struts between the inner faces of the flanges, lessened the shearing stress in the glued joints between the webs and the central parts of the flanges. There was no such reinforcement for the joints connecting the webs to the overhanging outer portions of the flanges, however, and stress in these joints was the primary cause of failure in both tests.

An early stage of the failure of C–5 is illustrated by figure 32, *A*. The fact that the glued joint connecting the overhanging part of the flange to the plywood web was stronger than the joints within the web caused the failure to be between plies in the web. A later stage is shown by figure 32, *B*, which is a view of the concave face of the mem-

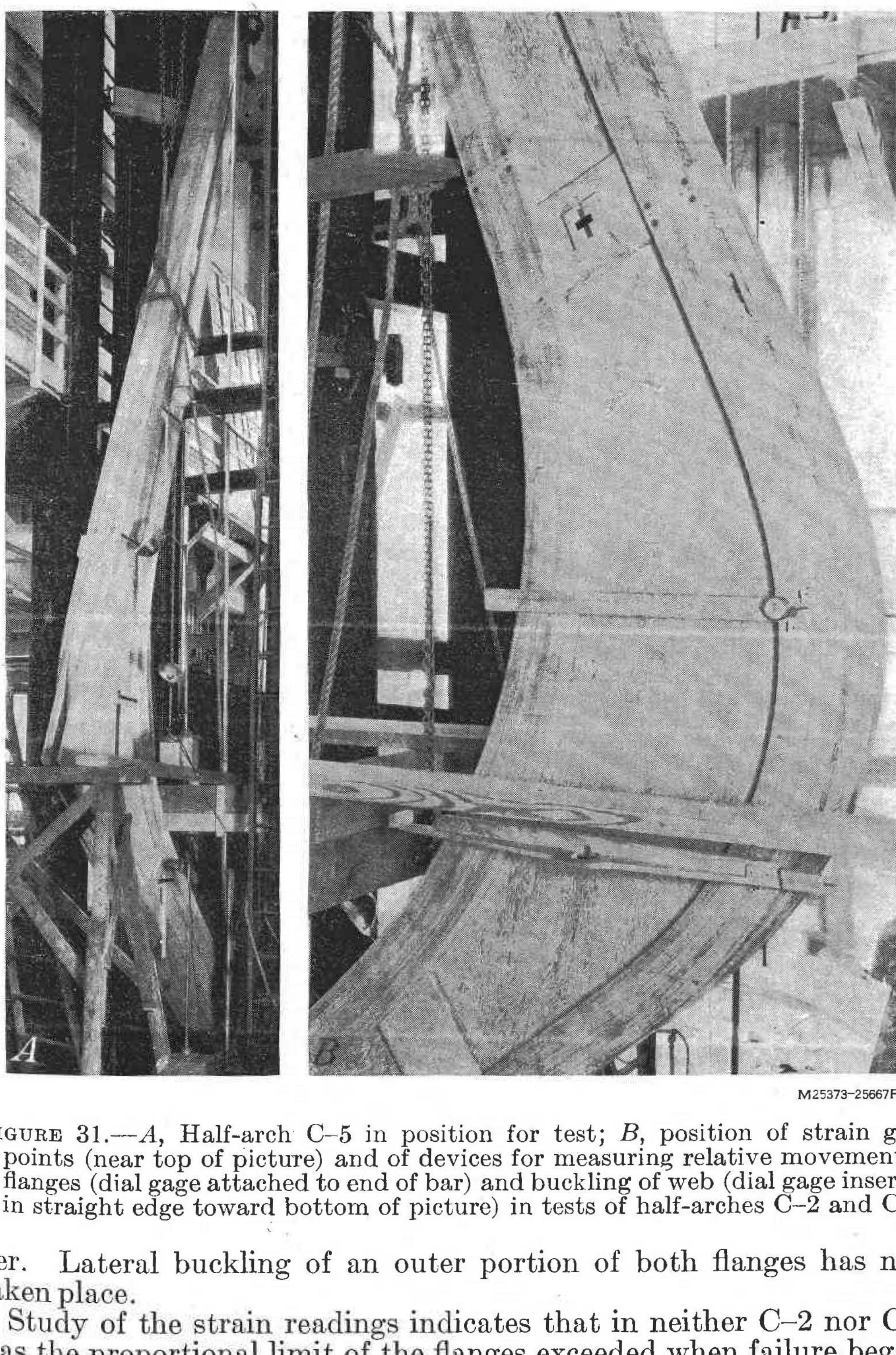

FIGURE 31.—*A*, Half-arch C–5 in position for test; *B*, position of strain gage points (near top of picture) and of devices for measuring relative movement of flanges (dial gage attached to end of bar) and buckling of web (dial gage inserted in straight edge toward bottom of picture) in tests of half-arches C–2 and C–5.

ber. Lateral buckling of an outer portion of both flanges has now taken place.

Study of the strain readings indicates that in neither C–2 nor C–5 was the proportional limit of the flanges exceeded when failure began.

The principal results of tests on half-arches C–2 and C–5 are shown in table 22.

After failure of C–2 had begun, clamps were placed to hold the parts of the tension flange together and restrain lateral buckling. The reinforcing action of the clamps, which were not used in the test of C–5, was probably the principal cause of the greater maximum load taken by C–2. Inspection after test showed that the gluing between the plies of the webs was not first class. Had this gluing been good,

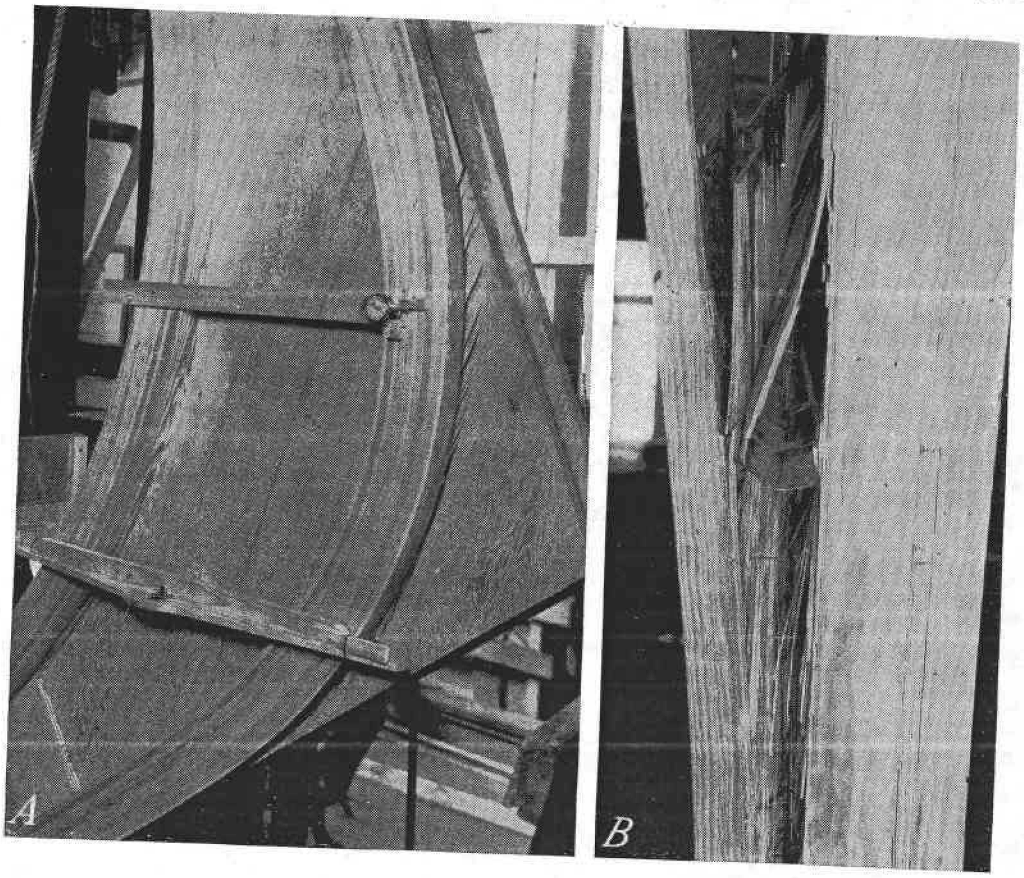

FIGURE 32.—Failure of half arch C–5: *A*, early stage; note devices for measuring relative movement of flanges and buckling of web; *B*, later stage, looking toward concave face of member.

and had there been struts between the outer overhanging portions as well as between the central portions of the flanges, the strength of the members would have been considerably higher. Either of these factors alone would probably have produced significant increases in strength. The strengthening effect of the struts is demonstrated by the fact that separation of the webs from the portions of the flanges between them did not occur and by the further fact that the load continued to increase after the outer overhanging portions of the flanges separated from the webs, buckled, and became quite ineffective.

TABLE 22.—*Principal results of tests on members C–2 and C–5, at first and maximum failure*

Item	C–2	C–5
Chord length ... feet and inches	27′	27′ 9½″
At first failure:		
Load ... pounds	32,000	32,800
At top end:		
Shear stress in web ... pounds/square inch	1,000	1,025
Shear stress in web ... do	[1] 750	[1] 770
Station $L+\frac{1}{2}$—point of maximum stress in flanges:		
Horizontal distance axis of member to chord (line of action of load) feet and inches	6′ 4.98″	6′ 7.80″
Horizontal deflection ... inches	1.805	2.075
Moment about center of inner flange ... inch-pounds	2,219,680	2,382,590
Moment about center of outer flange ... do	2,822,560	2,996,600
Total compression in inner flange ... pounds	149,800	160,080
Compression stress in inner flange ... pounds/square inch	2,760	2,990
Total tension in outer flange ... pounds	117,800	127,280
Tension stress in outer flange ... pounds/square inch	2,430	2,390
Shear stress in web ... do	340	350
Station I—point of maximum moment:		
Horizontal distance axis of member to chord (line of action of load) feet and inches	7′ 5.38″	7′ 5.34″
Horizontal deflection ... inches	1.475	1.660
Moment about center of inner flange ... inch-pounds	2,383,200	2,513,460
Moment about center of outer flange ... do	3,303,200	3,459,420
Total compression in inner flange ... pounds	114,890	119,950
Compression stress in inner flange ... pounds/square inch	2,200	2,330
Total tension in outer flange ... pounds	82,890	87,150
Tension stress in outer flange ... pounds/square inch	1,720	1,820
Radius of center line of outer flange ... feet and inches	3′ 8″	3′ 8″
Radial shear between outer flange and web ... pounds/square inch	88	93
At maximum load:		
Load ... pounds	47,000	42,890
At top end:		
Shear stress in web ... pounds/square inch	1,470	1,340
Shear stress in web ... do	[1] 1,100	[1] 1,010
At station $L+\frac{1}{2}$—point of maximum stress in flanges:		
Horizontal deflection ... inches	3.39	....
Moment about center of inner flange ... inch-pounds	3,322,900	3,169,000
Moment about center of outer flange ... do	4,208,380	3,970,000
Total compression in inner flange ... pounds	223,370	212,070
Compression stress in inner flange ... pounds/square inch	4,120	3,960
Total tension in outer flange ... pounds	176,160	169,290
Tension stress in outer flange ... pounds/square inch	3,640	3,180
Shear stress in web ... do	500	460

[1] Computed by the usual formula for shear at the neutral axis of a beam, that is, shear stress equals

$$\frac{VA\bar{y}}{It}$$

where V=external shear; $A\bar{y}$=statical moment of area above neutral axis; I=moment of inertia of section; t=total thickness of webs. Other computations are as follows: Load on flange=moment about other flange divided by distance C to C of flanges; stress in flange=load on flange divided by cross-sectional area of flange; shear stress in webs=V divided by distance C to C of flanges times t; radial shear between outer flange and web=tension stress in flange times width of overhanging portion of flange divided by radius of center line of flange.

RELATION OF STRENGTH PROPERTIES TO CURVATURE

The common assumption that bending stress is linearly distributed across the depth of a beam subjected to flexure does not hold true, even within the proportional limit, for curved members. The error of this assumption increases as the quotient, radius of curvature divided by depth of the member, decreases. For values of this quotient that apply to some of the members discussed herein, the error of this assumption is large. Common engineering formulas that involve the assumption of linearity of stress distribution are nevertheless employed throughout this bulletin in computations of stresses from tests, except in tests of members of **I** section with plywood webs (table 22), and are accordingly assumed to be used in design. With this procedure it is not believed that the errors of the assumption of linearity of stress dis-

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tribution result in any significant error in applying the data from tests to design. It is probable that some of the effects ascribed to the influence of stress induced in bending laminations (p. 52) are due to some small extent to the curvature of the members themselves.

Figures 33 and 34 bring together the principal data available for showing the relation of strength to curvature.

Data from the transverse tests are plotted on a percentage basis in figure 33, those for Sitka spruce, Douglas fir, and southern yellow pine being combined as in figure 15, values for laminated straight members to which the curved members were matched being the bases. Similarly plotted in figure 33 are data from end-thrust tests on laboratory-built members of southern pine, the average strength value for members with the least curvature (1-foot offset), relative curvature=0.00125 being taken as the base of percentages.

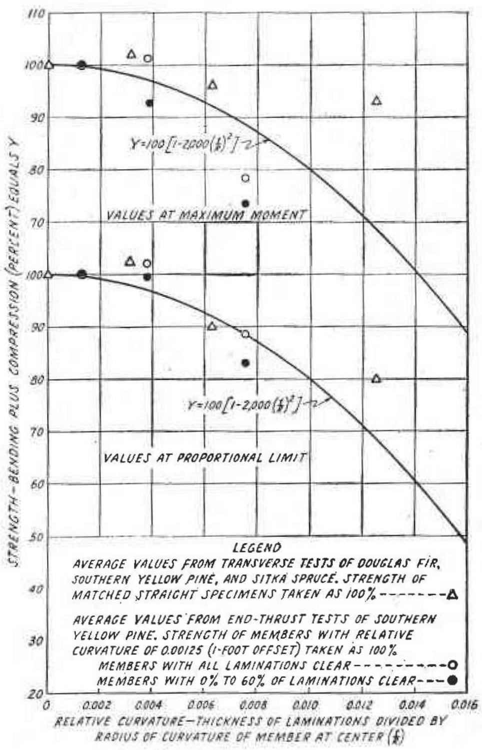

FIGURE 33.—Strength of laminated members as related to curvature. Strength values on percentage basis.

The plotted points in figure 34 represent average and minimum values as determined from end-thrust tests on laboratory-built members of southern yellow pine and individual test values for the two building arches D–1 and D–2, which were also of southern yellow pine.

The curves shown in figures 33 and 34 are essentially identical. In placing them, consideration was given to figures 33 and 34 and to figures 13 to 15, inclusive, to the number of tests represented by each plotted point, and to other pertinent information. Furthermore, more weight has been given to the values at the proportional limit than to those at the maximum moment. As has been noted, the full ultimate bending strength at the section of maximum curvature was not developed in the tests of half-arches D–1 and D–2. Consequently, the points representing these arches in the upper part of figure 34 are too low.

Considering half-arch D–2 alone, the curve for the proportional-limit values in figure 34 would seem to be too high. However, the value for D–2 is counterbalanced by values representing nearly as great curvature and lying, in general, well above the curve in figures 13, 14, 15, and 33.

A further factor for consideration is the relation between the depth of a wooden beam and the strength. Tests have shown that, as the depth increases, the fiber stress at the proportional limit, as well as the modulus of rupture, decreases. This relation has been put into the following formula

$$H = 1.07 - 0.07\sqrt{\frac{h}{2}}$$

where H is the depth factor for a beam of depth h, and is unity for a beam 2 inches deep. Values of H for members represented in figure 34 are as follows:

Laboratory-built members of 8, 12, and 18 laminations with depths of 6, 9, and 13½ inches have values of H of 0.95, 0.92, and 0.89, respectively. The D arches used in the service building with a depth at knee of 24 inches have a value of H of 0.83.

The strength values for the various beams would be made more nearly comparable by dividing each by the appropriate value of H. Inasmuch, however, as differences in height among the experimental members with 8, 12, and 18 laminations are insufficient to make any large differences in strength values, adjustments for height effect were not made. The foregoing tabulation suggests that an upward adjustment of approximately 10 percent might properly be applied to the values for D–1 and D–2 in figure 34 to make them more nearly comparable to the values for the laboratory-built members. Such adjustment would bring the values for D–1 and D–2 considerably nearer the curves, which might then be considered as corresponding to a height factor of about 0.92 (the average value for the arches with 8, 12, and 18 laminations).

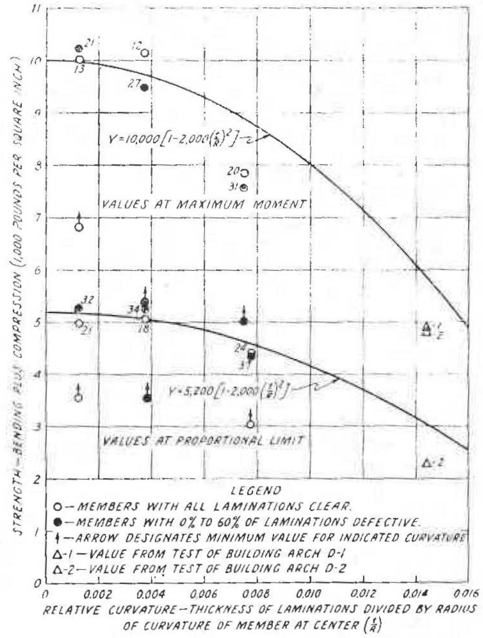

FIGURE 34.—Strength of laminated members of southern yellow pine as related to curvature. Strength values in pounds per square inch.

In the end-thrust tests, curved members were subjected to bending stress combined with stress in compression parallel to grain. Since stresses in compression parallel to the grain of wood, both at proportional limit and ultimate, are considerably lower than bending stresses, the validity of adding longitudinal compression to the compression resulting from bending might be questioned. However, in the tests, the bending stress has been from 78 to 98 percent of the total and a previous investigation (*19*) has shown that within this range the combined stress is not significantly lower at the proportional limit or at the ultimate than bending stress alone would be.

DISCUSSION OF STRENGTH DATA IN RELATION TO STRESSES FOR USE IN DESIGN

The data that have been discussed, together with some additional information and test results that will be introduced currently are the basis for deriving stresses for use in the design of glued laminated members.

BENDING COMBINED WITH COMPRESSION

Table 9 presents strength values from end-thrust tests on southern yellow pine members together with figures on standard deviation and coefficient of variation of these values.

From these data on variability it is estimated that the stress at the proportional limit will exceed 60 percent of the average value in 99 out of 100 members, the central three-fifths or less of whose laminae are defective. Values of stress at proportional limit lower than 3,120 $\left[1-2{,}000\left(\frac{t}{R}\right)^2\right]$, which is 60 percent of the value indicated by the average curve in figure 34, are accordingly to be expected no oftener than once in a hundred times. Failure of wooden members under long-continued loading is to be expected at a stress equal to that at the proportional limit. Consequently, if for such material as is represented in these tests, a value of 2,080 $\left[1-2{,}000\left(\frac{t}{R}\right)^2\right]$ were used in designing, the factor of safety would be approximately 1.5 under indefinitely prolonged loading with respect to a failure of 1-percent probability. Since, as previously stated, the curves in figure 34 may well be considered as corresponding to a height factor of approximately 0.92, the reduction of working stresses by the application of height factors in the manner later suggested increases the factor of safety somewhat. The factor of 1.5 is approximately the same as obtains with bending stresses recommended by the Forest Products Laboratory and widely accepted in designing timber beams. The recommended value for southern yellow pine beams, without any rate of growth or density requirement, is 2,000 pounds per square inch to which $\frac{1}{15}$ is added for pieces with not less than six growth rings per inch, making the value for such material 2,133 pounds per square inch. The material in the outer parts of the cross section of the laminated members tested under end thrust was of this character and it is concluded that the basic bending-stress values as used for determining design values for timber beams may be accepted for laminated members having each outer fifth of the depth of the cross section formed of practically clear material and with the interior part made up of laminae containing defects.

COMPRESSION PARALLEL TO GRAIN

The data of table 1 indicate that, up to a relative curvature of 0.0125 (laminations bent to a radius of 80 times their thickness), bent material has suffered no significant loss of strength as compared with straight material of the same character. Consequently no reduction for the effect of curvature seems necessary, and stress in compression parallel to the axis of curved members may be on the same basis as for straight members with appropriate reduction for the defects permitted in the laminations of the curved members.

COMPRESSION AND TENSION PERPENDICULAR TO GRAIN

Stress in compression perpendicular to the grain requires consideration because it is developed at the connection of arches to tie rods or to abutments. Conditions at such points do not differ essentially from those at the ends of beams and the same allowable stress values may be used as for the bearing of beams.

As may be demonstrated (p. 118) a radially acting compression exists in a curved member subjected to bending moments that tend to increase the curvature. For this radial compression, the same allowable stress values are suggested as for compression perpendicular to grain at the ends of beams.

When a curved wooden member is subjected to bending moments that tend to straighten it and decrease the curvature, the radially acting stress is tension perpendicular to the grain. The principal data available as a basis for determining allowable stresses in tension perpendicular to the grain in laminated curved members are the results of tests made as part of a standard series on some 164 species

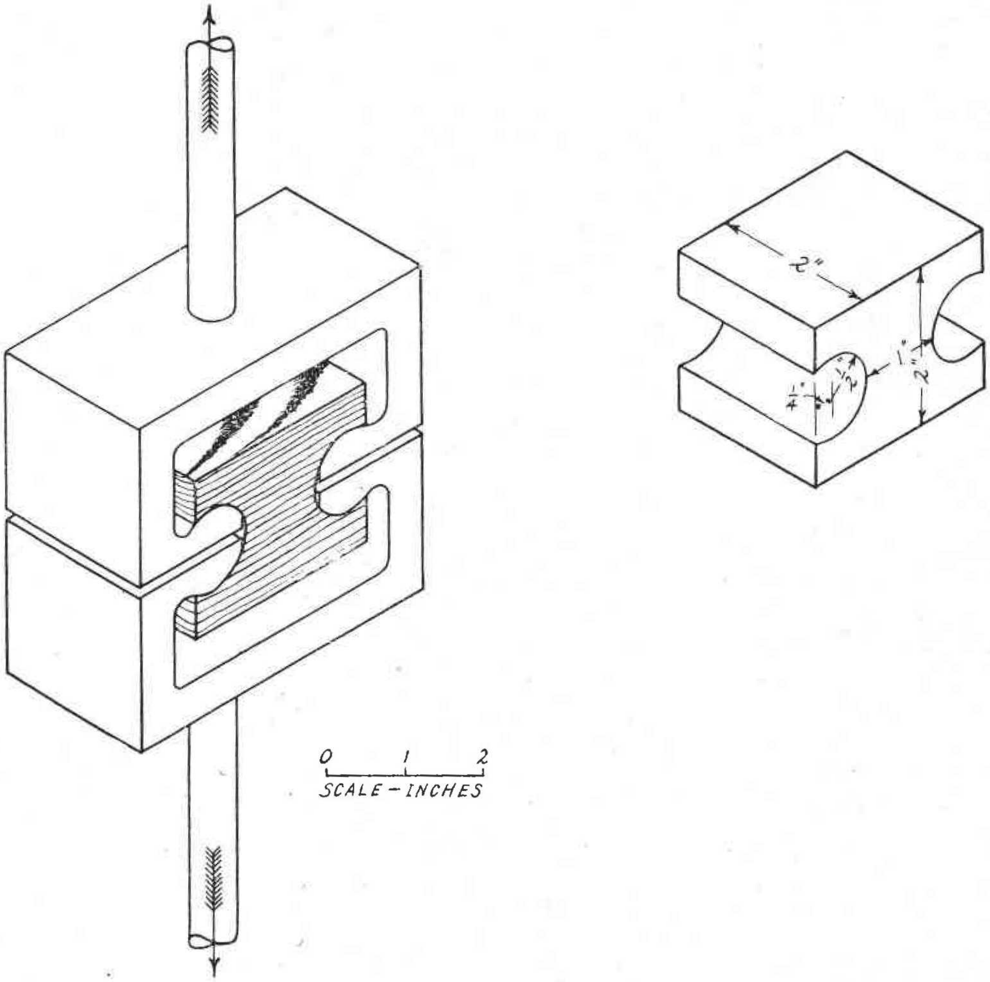

FIGURE 35.—Specimen and appliance for tests in tension perpendicular to grain.

(*16*). The type of specimen and testing appliance is illustrated in figure 35. Average values of tension perpendicular to grain for air-dried material (moisture content 12 percent) vary from as low as 180 for a lightweight, nondense softwood, to as high as 1,200 pounds per square inch for a very heavy, dense hardwood. In this test, the tensile stress is not uniformly distributed over the area by which the load is divided to get the unit value. Hence the values obtained are somewhat lower than the true tensile strength. They are roughly correlated with other mechanical properties, and a study of available data indicates that, for coniferous species, the tensile strength will rarely be less than 30 percent of the compression perpendicular value or less than 40 percent for hardwood species. It would seem proper then to permit a tensile stress 30 to 40 percent as great as the allowable stress in compression perpendicular to grain. However, unavoidable imperfections and discontinuities of gluing, together with checking during seasoning or in service, may result in less than full areas being available to resist tension with consequent concentration and nonuniform distribution of stress. It therefore seems advisable to apply further factors and to make the allowable tensile stress not greater than one-seventh of the stress in compression perpendicular

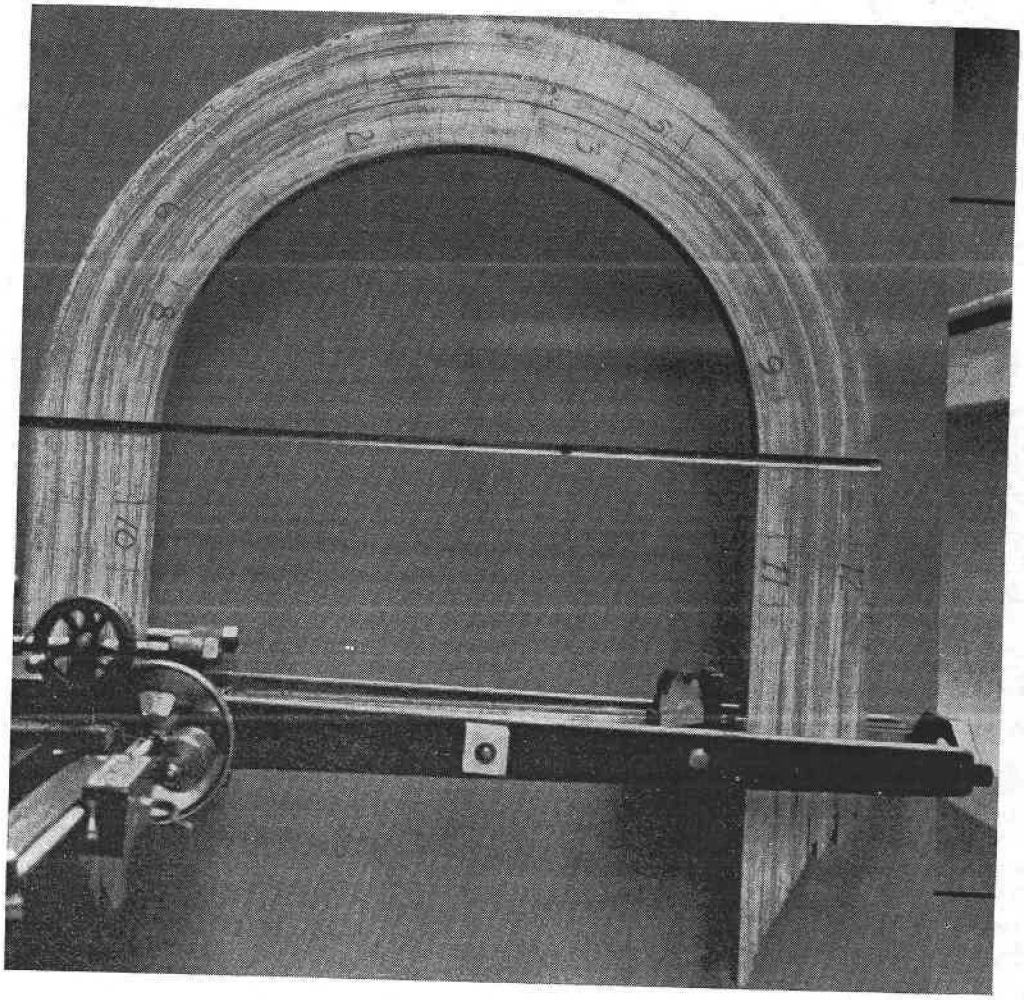

FIGURE 36.—Radial tension specimen in position, in machine after test. Test is made by pulling ends of specimen apart. Note circumferential cracks where tension failure has occurred.

to the grain for coniferous species and one-sixth of the stress in compression for hardwood species.

TESTS OF RADIAL TENSION

The radial tensile stress that could be expected in an actual curved member was checked in a few special tests. The form of specimen and method of test are shown in figure 36.

The specimens were of southern yellow pine with ⅛-inch laminations glued with casein glue. The radius of the concave face was 15 inches and the radial depth of the specimens was 2, 4, or 6 inches. Subsequent to the test of the curved specimen, minor specimens (these minor specimens were made so that there was a glued joint at the cross section of minimum area) cut from them were tested in tension perpendicular to the grain according to the method illustrated in figure 35. Results of all the tests, together with minor tests from straight specimens Nos. 198 and 200, which were matched lamina for lamina to curved specimens Nos. 197 and 199, are listed in table 23. All the curved specimens failed in radial tension. Owing to the thinness of the laminations and the use of hand clamps, the pressure applied in gluing was not well distributed, which probably is, at least in part, responsible in the minor tests for the erratic values in percentage of wood failure and in strength.

TABLE 23.—*Results from tests of radial tension in curved members and from tests in tension perpendicular to grain on minor specimens*

Curved member	Cross section		Maximum stress in radial tension	Curved member	Cross section		Maximum stress in radial tension
	Width	Depth			Width	Depth	
	Inches	*Inches*	*Pounds per square inch*		*Inches*	*Inches*	*Pounds per square inch*
197	2.23	2.16	206	202	2.24	2.07	163
198			(1)	203	2.24	4.08	192
199	2.27	6.22	218	204	2.21	4.18	186
200			(1)	205	2.25	6.25	182
201	2.24	2.14	269	206	2.23	6.25	240

Minor specimen	Minor specimens from member 197		Minor specimens from member 198[1]		Minor specimens from member 199		Minor specimens from member 200[1]	
	Stress in tension	Wood failure in joint	Stress in tension	Wood failure in joint	Stress in tension	Wood failure in joint	Stress in tension	Wood failure in joint
	Pounds per square inch	*Percent*	*Pounds per square inch*	*Percent*	*Pounds per square inch*	*Percent*	*Pounds per square inch*	*Percent*
1	106	0	211	100	121	60	272	35
2	377	90	394	100	312	95	368	50
3	355	5	451	15	402	95	442	30
4	366	100	348	97	325	5	391	70
5	393	80	562	40	420	5	389	100
6	345	100	366	95	357	80	490	25
7	240	15	312	80	275	35	396	95
8	520	50	245	20	285	15	365	35
9	445	100	425	18	158	90	538	20
10	256	0	293	25	437	70	500	20
11	138	2	426	35	523	2	368	40
12	345	100	327	60	470	60	434	90
General average	324		363		340		413	
Average 50+ percent failures	399		326		322		396	

Minor specimen	From member 201		From member 202		From member 203		From member 204		From member 205		From member 206	
	Stress in tension	Wood failure in joint	Stress in tension	Wood failure in joint	Stress in tension	Wood failure in joint	Stress in tension	Wood failure in joint	Stress in tension	Wood failure in joint	Stress in tension	Wood failure in joint
	Lbs. per sq. in.	*Percent*	*Lbs. per sq. in.*	*Percent*	*Lbs. per sq. in.*	*Percent*	*Lbs. per sq. in.*	*Percent*	*Lbs. per sq. in.*	*Percent*	*Lbs. per sq. in.*	*Percent*
1	87	3	273	15	126	5	356	70	472	75	395	40
2	189	5	307	5	300	20	215	40	399	100	487	95
3	274	5	376	15	242	0	328	60	513	70	340	75
4	370	40	328	20	280	2	263	25	350	50	414	25
5	365	60	262	10	280	50	351	65	394	95	291	50
6	376	13	123	0	262	92	250	20	258	1	421	75
7	118	25	146	0	189	50	345	25	82	0	466	40
8	292	10	202	2	246	35	364	70				
General average	259		252		241		309		353		402	
Average 50+ percent failures	365				244		350		426		385	

[1] Straight specimen; not tested in radial tension.

The values of radial stress when compared with the value of about 45 pounds per square inch that would be derived as allowable for southern yellow pine by the method already outlined, namely, division of the allowable stress in compression perpendicular to grain by 7, indicate factors of safety of 3½ or more. Factors at least this large are desirable because of the possibilities of imperfect glued joints and of checking of members in service.

LONGITUDINAL SHEAR STRESS

Longitudinal shear was the primary failure in tests of 29 out of 201 members of southern yellow pine. Values of the computed shearing stress at failure in these instances ranged from 162 to 786 pounds per square inch with eight values below 250. The average was 354 pounds per square inch.

The shear-test values, attained in the remaining 172 tests at the time the member failed from other causes, are distributed as follows: 5 below 160; 45 between 160 and 250; 77 between 250 and 350; 45 between 350 and 720 pounds per square inch.

Of the eight shear failures that occurred at a stress below 250 pounds per square inch, five were entirely in the glue line. In three instances, joints in which failure occurred were observed before test to be partly open.

These members were glued in the Laboratory shop where temperature is not subject to the regulation that is needed to insure consistently good gluing with animal glue, which was the type used. The large percentage of glue failures in some of the joints is attributed to the probability that unfavorable temperature conditions obtained at the time the joints were glued. In all probability, however, the gluing practice followed was better than obtains in the use of animal glue in the average commercial plant.

Some of the glue failures were probably due to one particular feature of the procedure in gluing. Each member was glued in two or more installments. Laminations were spread with glue (on both faces except that next to the form and the outer face of the last lamination of the installment or group) and placed in proper order in a stack, after which the group was turned so that the widths of the laminations were vertical. The group was then drawn against and clamped to the preceding group, the exposed face of which had been spread with glue. With this manipulation, joints between groups were open longer before clamping than were those between the laminations in a group. Thus there was more chance for glue to drain from the joint between groups and for the glue to be affected by unfavorable temperature or other conditions. The joints that were found to be imperfect before test were between groups of laminations. Casein and other types of glue suitable for use in building laminated members do not require such close regulation of temperature, and, with reasonable care in controlling gluing conditions, poor and weak joints are considerably less likely.

Material for glued laminated members, being of smaller dimensions, checks less in drying than do large timbers. Consequently it might be thought that higher values for horizontal shear could be used in designing such members. However, methods and procedure that are ordinarily available for gluing large members, particularly curved ones, are not such as to insure that glue joints will always be perfect and continuous. Tests have shown that discontinuities in shear-resisting areas (*17*, *20*) cause very high concentration of stress. Hence it is suggested that in the design of glued laminated members three-fourths the unit shear-stress values that have been set up as basic for structural timbers (table 24) be used.

TABLE 24.—*Basic stress values* [1] *for laminated wooden construction in pounds per square inch (to be used only after modification as indicated in the accompanying text)*

Species (1)	Combined bending and compressive stress in extreme fiber (2)	Compression perpendicular to grain (3)	Compression parallel to grain (4)	Maximum longitudinal shear (5)	Modulus of elasticity (6)
Softwoods:					
Cedar, Alaska	1, 466	250	1, 066	120	1, 200, 000
Cedar, northern and southern white	1, 000	175	733	93	800, 000
Cedar, Port Orford	1, 466	250	1, 200	120	1, 200, 000
Cedar, western red	1, 200	200	933	106	1, 000, 000
Cypress, southern	1, 733	300	1, 466	133	1, 200, 000
Douglas fir, coast region	2, 000	325	1, 466	120	1, 600, 000
Douglas fir, Rocky Mountain region	1, 466	275	1, 066	113	1, 200, 000
Fir, commercial white	1, 466	300	933	93	1, 100, 000
Fir, balsam	1, 200	150	933	93	1, 000, 000
Hemlock, eastern	1, 466	300	933	93	1, 100, 000
Hemlock, western [2]	1, 733	300	1, 200	100	1, 400, 000
Pine, western white,[3] northern white, sugar, and ponderosa	1, 200	250	1, 000	113	1, 000, 000
Pine, red	1, 466	300	1, 066	113	1, 200, 000
Pine, southern yellow [4]	2, 000	325	1, 466	146	1, 600, 000
Pine, southern yellow, dense	2, 333	380	1, 711	171	1, 600, 000
Redwood	1, 600	250	1, 333	93	1, 200, 000
Spruce, Engelmann	1, 000	175	800	93	800, 000
Spruce, red, white, and Sitka	1, 466	250	1, 066	113	1, 200, 000
Tamarack	1, 600	300	1, 333	126	1, 300, 000
Hardwoods:					
Ash, commercial white	1, 866	500	1, 466	167	1, 500, 000
Ash, black	1, 333	300	866	120	1, 100, 000
Beech	2, 000	500	1, 600	167	1, 600, 000
Birch, sweet and yellow	2, 000	500	1, 600	167	1, 600, 000
Chestnut	1, 266	300	1, 066	120	1, 000, 000
Elm, rock	2, 000	500	1, 600	167	1, 300, 000
Elm, American and slippery [5]	1, 466	250	1, 066	133	1, 200, 000
Gum, black and red	1, 466	300	1, 066	133	1, 200, 000
Hickory, true and pecan	2, 533	600	2, 000	187	1, 800, 000
Maple, sugar and black [6]	2, 000	500	1, 600	167	1, 600, 000
Oak, commercial red and white	1, 866	500	1, 333	167	1, 500, 000
Tupelo	1, 466	300	1, 066	133	1, 200, 000

[1] For members with laminations in volumes *A* and *C* (p. 64) of close-grained material of Douglas fir from the Pacific coast region, southern yellow pine, or redwood, values in columns (2) (3) and (4) may be increased 1/15. Close-grained material is defined as follows: Douglas fir from the Pacific Coast region and southern yellow pine shall average on one end or the other of the piece not less than 6 nor more than 20 annual growth rings per radial inch. Pieces averaging from 5 to 6 annual rings per inch to be accepted as the equivalent of close grained if having one-third or more summerwood. Redwood shall average on one end or the other not less than 10 nor more than 25 annual growth rings per radial inch. For members with laminations in volumes *A* and *C* of dense Douglas fir or dense southern yellow pine, values in columns (2), (3), (4), and (5) may be increased 1/6. Dense material of these species shall average on one end or the other not less than 6 annual growth rings per radial inch and in addition not less than one-third summerwood.

[2] Also sold as west coast hemlock.

[3] Also sold as Idaho white pine.

[4] Also sold as longleaf or shortleaf southern pine.

[5] Also sold as white elm or soft elm.

[6] Also sold as hard maple.

In the test of half-arch D–1, the maximum shearing stress when failure from other causes occurred was 413 pounds per square inch. In D–2 the maximum stress was 420 pounds per square inch when failure occurred by longitudinal shear through a bolt hole at the lower end of the member. Thus there was a factor of slightly less than 4 with respect to the design value of 110 pounds per square inch which would be obtained for southern yellow pine by multiplying the basic value of 146 pounds per square inch (column 5 of table 24) by three-fourths as suggested in the preceding paragraph.

MODULUS OF ELASTICITY

Available data indicate, in general, that modulus of elasticity decreases with increase in curvature (tables 9 and 21). The indicated decrease is so small as not to require consideration in computations of the deformation or deflection of members.

LIMITATIONS IN DESIGN

RATIO OF DEPTH TO WIDTH

The strength and stiffness of a member subjected to bending vary as the second and third powers, respectively, of the depth, whereas both vary as the first power of the width. Consequently, the amount of material required for strength and stiffness is minimized by making the depth as great as other considerations will permit. Because any change of curvature under load usually adds to the stress in a curved member, the maximum attainable stiffness is desirable.

Frequently, resistance to longitudinal shear, which in members of rectangular section is proportional to cross-sectional area, limits the depth of straight beams. Shear is less likely to be a limiting factor in curved members.

The extent to which strength and stiffness can be increased by increasing the depth and decreasing the width of a member is limited because large ratios of depth to width lead to lateral instability and to failure by twisting and lateral buckling at loads less than those computed by the usual formulas for flexural strength. The critical buckling load depends on the modulus of elasticity and modulus of rigidity (in torsion) of the material, the length and cross-sectional dimensions of the member, location or distribution of load, and the way in which the ends and edges of the member are supported and restrained. Formulas for critical load have been developed theoretically and checked experimentally for several combinations of these factors in straight beams (*30, pp. 159–162*). Adequate information along similar lines for curved members is lacking.

Consideration of some special tests on curved members and of the relation between dimensions and buckling loads as found in tests on beams and on columns with thin outstanding flanges (*28*) leads to the suggestion that the ratio of depth to width in a curved member with rectangular cross section should not exceed four when one edge is braced at frequent intervals, as by girts or roof purlins, and should not exceed three when such bracing is lacking. The length of members that are braced along one edge is probably not important. It is suggested that the combined bending and compressive stress in unbraced members be limited to one-third or less of the stress that, according to the Euler formula, would cause lateral buckling in straight members of the same cross section and of a length equal to the chord length of the curved member.

I- OR BOX-FORM OF CROSS SECTION

Use of cross sections of **I**- or box-form affords an apparently obvious method of obtaining the required strength and stiffness with a minimum amount of material in members subjected to bending stress, since they have greater moments of inertia and greater section moduli than rectangular sections of the same area. Their use is, however, limited by the resistance of webs to buckling, to radial compression or tension, and to longitudinal shear; by the resistance of overhanging parts of flanges to buckling; and by the "form factor" of the section.

FORM FACTOR

Tests (*18*) have demonstrated that in wooden beams the fiber stress at proportional limit and the modulus of rupture depend on the shape or form of cross section. Consequently, stress values appropriate to beams of rectangular section must be multiplied by a form factor to make them applicable to a beam with cross section of other form, and the usual formula for internal resisting moment becomes $M=F.f.S.$, in which F=form factor of section; f=fiber stress value for beam of rectangular section; and S=section modulus of section.

F is taken as unity for rectangular sections and is less than unity for **I**- and box-forms and greater than unity for a circular section (for which it is 1.18) or for a square section with transverse load acting along a diagonal of the section (for which it is 1.41).

The form factor relation (*18*) was developed from a series of tests on members similar in size to airplane wing beams. That it is applicable to such larger members as are here under consideration has not been checked by similar systematic tests but is indicated by numerous observations. The form factor at proportional limit[6] is given by the following equation:

$$F=0.58+0.42\ [K\ (1-t)+t],$$

where F is the form factor of a section composed of two flanges and one or more webs, the ratio of the total width of webs to the width of the section being t. Values of K depend on d, the ratio of the depth of the compression flange to the total depth according to the following tabulation:

d	K	d	K	d	K
0.10	0.085	0.45	0.660	0.75	0.970
.15	.155	.50	.740	.80	.985
.20	.230	.55	.810	.85	.995
.25	.315	.60	.875	.90	.998
.30	.400	.65	.920	.95	1.000
.35	.490	.70	.950	1.00	1.000
.40	.575				

Unlike stress, modulus of elasticity is unaffected by the shape or form of cross section. Consequently formulas for the deflections due to bending stress require no modification because of the shape of the cross section.

THICKNESS OF LAMINATIONS

The permissible or desirable thickness of laminations is affected by several considerations.

The stress induced in bending laminations for curved members is proportional to the ratio of thickness of lamination to radius of cur-

[6] The tests referred to above demonstrated that the form factor for modulus of rupture is less than that for stress at proportional limit.

vature. The tests discussed herein demonstrate that the effect of this stress on the strength of the member is less than might be expected from its magnitude but it is nevertheless necessary progressively to reduce working stress as this ratio increases. When the curvature is fixed it is possible to use thick laminations with comparatively low working values of flexural stress or thinner laminations with considerably higher stresses. If a large number of thicknesses of lumber were readily available, it would be possible to choose the most economical one in relation to the curvature and the cost of preparation, assembly, and gluing as affected by the number of laminations.

Standard thicknesses of lumber are, however, such that, without special sawing, choice is limited to nominal 1-inch or 2-inch stock which, when suitably surfaced, will have actual thicknesses of approximately ¾ or 1½ inches, or to material of nominal 2½ inches or greater thickness. Consequently, it is essential in design to adjust curvature to standard thicknesses unless it is found that special thicknesses obtained by resawing standard thicknesses or otherwise are economical.

Pressure applied in gluing should be correlated to the viscosity of the glue in order to insure close contact between surfaces joined and yet permit a thin and continuous film of glue to remain in the joint. Insufficient pressure and thick glue, or excessive pressure and thin glue, will cause inferior joints. Considerable warp, and twist in comparatively thin laminations can be overcome by pressure within acceptable limits but the stiffness and torsional rigidity of thick laminations make it difficult to bring warped or twisted surfaces together without excessive pressure at some points and insufficient pressure at others. Also, stiff laminations may, by retaining a tendency to straighten, put a strain on the glued joint when the clamping pressure is released. Changes in moisture content and in moisture distribution during seasoning subsequent to gluing and during service are more likely to cause objectionable and weakening checks if laminations are thick than if they are relatively thin. Consequently it is desirable that laminations be of reasonable thickness and with a minimum amount of twisting and warping. The maximum thickness of pieces that may be safely joined by gluing has not been determined but general experience suggests that the hazards of poor joints and of undesirable checking in service increase rapidly with increase in thickness. Most of the recent glued laminated construction in the United States has been built up from nominal 1-inch stock. In Europe, where material is often specially sawn for such construction, the thickness used has rarely exceeded 1¼ inches.

A ratio of radius to thickness of about 70 obtains in the glued laminated arches in the Laboratory's service building. Such a small value would not, however, be feasible with some species and grades of lumber, and a ratio approximately twice as great corresponding to a radius of 100 inches with nominal 1-inch lumber is perhaps desirable as a minimum.

MOISTURE CONTENT OF MATERIAL FOR GLUED LAMINATED CONSTRUCTION

Wood at any moisture content below 20 percent can be satisfactorily glued with casein glue. Hence no very close control of moisture is necessary to provide for good gluing and the principal determinants for the proper moisture content of glued laminated structural members at assembly are the effects of moisture on the bending of laminations and on the behavior of members in service.

The degree to which lumber can be bent depends on the moisture content. In general the curvature that can be produced without strain beyond the proportional limit increases as the moisture content is lowered. On the other hand, the curvature that can be attained without breakage decreases with decrease in moisture content. Hence for extreme curvatures, a moisture content as high as other considerations permit is desirable.

In order to minimize the effects of the shrinkage and swelling, which occur with evaporation and absorption of moisture, and to minimize changes in curvature (p. 119), the moisture content when a member is installed should be within the range that will obtain in service. Wood that is sheltered from precipitation but otherwise subjected to natural atmospheric conditions tends to assume a moisture content that varies from as low as 8 percent or less in the more arid to as high as 15 percent or greater in the more humid parts of the United States. In heated buildings the equilibrium moisture content may be as low as 4 or 5 percent during parts of the heating season. Changes in moisture content are consequently unavoidable and their effects will be minimized if laminated members when made up have a moisture content within the range of the equilibrium values to be expected in service. Consequently, a value of approximately 10 percent would seem to be most generally acceptable. It is doubtful, however, whether the necessity of close restriction of average moisture is critical.

Equality of moisture content among laminations incorporated into the same member is perhaps more important than is the average moisture content. If adjacent laminations differ widely in this respect when bonded together, subsequent changes will cause one to shrink or swell more than the other with consequent stress on the glue joint.

Observation of glued laminated construction does not indicate need for any extreme care either with respect to average moisture content or its spread among pieces in the same assembly. Avoidance of any abnormal spread is, however, desirable and the greater the uniformity and the closer the conformity to service conditions the less is the chance of disfiguring checks or other ill effects.

SPECIFICATIONS FOR MATERIAL AND CONSTRUCTION OF LAMINATED WOODEN MEMBERS

The following information for use in specifications is coordinated to the working stress values suggested on pages 65 to 67.

Two grades of construction are described; also, three alternative methods of distributing taper in members that vary in depth. In any case the grade of construction and method of distributing taper should be specified and if close-grained or dense material is to be used in outer portions of the cross section it should be so stated.

Comparatively low-grade material is permitted in laminations in the central part of the cross section in both grades of construction.

EQUIPMENT

The builder must have facilities for mixing and spreading glue properly, for maintaining proper temperatures in the shop where gluing is done, for clamping material to the specified pressure, for checking the pressure, and for completing the clamping within the specified period after spreading the glue.

SEASONING OF MATERIAL

Material must, before gluing, be seasoned to an average moisture content of not more than 15 and not less than 10 percent with a spread of not more than 5 percent of moisture content among pieces incorporated into a single built-up member.

DIVISION OF MEMBER INTO VOLUMES

For the purpose of specifying the selection and preparation of material, members are divided into three volumes as indicated in figure 37, where volume *A* extends from one face of the member to

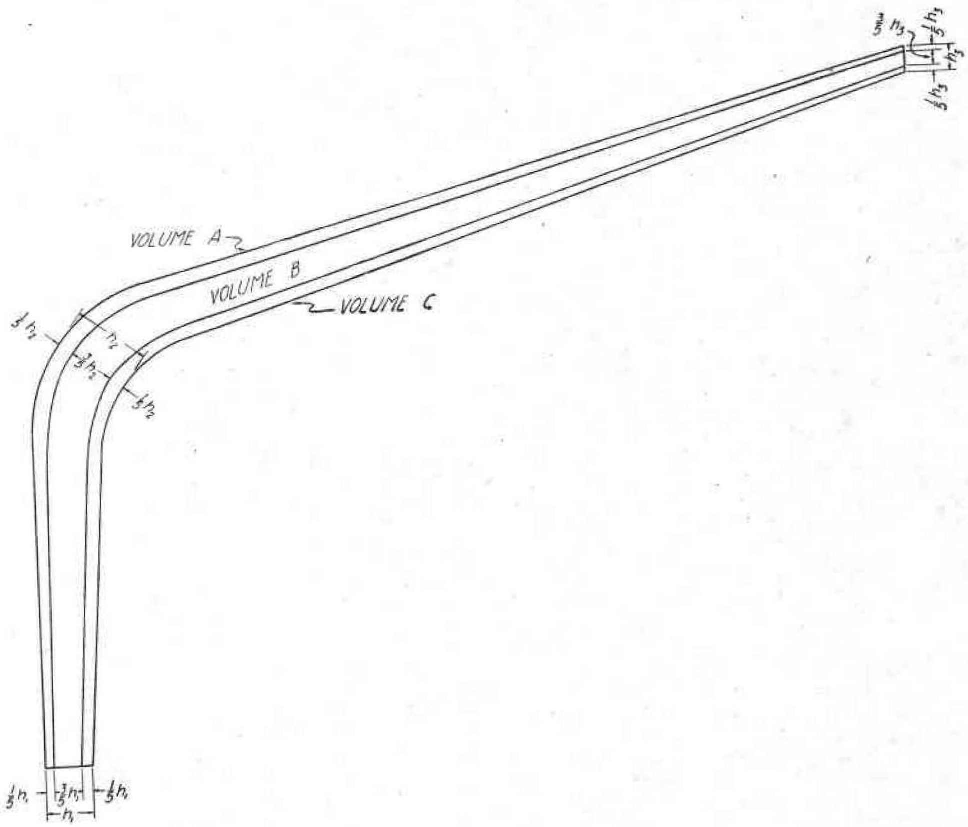

FIGURE 37.—Division of laminated member into volumes *A*, *B*, and *C* for specifying permissible defects in laminations according to their position in the assembly.

a surface that is at each point in the length at least one-fifth the depth of the member from that face. Similarly volume *C* extends from the opposite face to a surface that is at every point at least one-fifth the depth from that face. Volume *B* is the remaining central portion.

PERMISSIBLE DEFECTS

Limitations of knots apply to both sides and limitations of slope of grain to both sides and both edges of the lamination. All laminations shall be free from shakes or splits that when viewed from the ends of the piece make an angle of less than 30° with the wide faces. Size of a knot is to be taken as the dimension of the knot between lines touching it and parallel to the edges of the lamination. Direction of grain is to be measured over a distance sufficiently great to determine the general slope, local deviations being disregarded. Material that is obviously so resinous as to be likely not to hold glue shall be rejected.

LAMINATIONS IN VOLUMES *A* AND *C*

Laminations in volumes *A* and *C* must not contain any part of the pith of the tree.

When laminations of Douglas fir, southern yellow pine, or redwood are specified to be "close-grained" or "dense" they shall conform to the requirements for rate of growth or for rate of growth and percentage of summerwood as specified for close-grained or dense material of these species.[7] Regardless of the grade of construction, the outer lamination on each side of the member shall be of the character specified for volumes *A* and *C* in grade I construction. Further, these laminations shall have a thickness not greater than one one-hundred-and-fiftieth of the minimum radius to which they are bent.

GRADE I CONSTRUCTION

For grade I construction, full-width or part-width laminations in volumes *A* and *C* shall be free from knots whose size exceeds one-eighth the width of the piece—maximum size 1½ inches. The sum of the sizes of all knots in either face in any length equal to the width of the piece shall not exceed one-eighth the width. Laminations shall be free from diagonal or spiral grain whose slope is greater than 1 inch in 17 inches. Wane, whose greatest width does not exceed one-half the thickness of the piece, is permissible.

GRADE II CONSTRUCTION

For grade II construction, full-width or part-width laminations in volumes *A* and *C* shall be free from knots whose size exceeds one-fourth the width of the piece—maximum size 2½ inches. The sum of the sizes of all knots in either face within any length equal to the width of the piece shall not exceed one-fourth the width of the piece. Laminations shall be free from diagonal or spiral grain whose slope is greater than 1 inch in 15 inches. Wane, whose greatest width does not exceed the finished thickness of the piece, is permissible.

LAMINATIONS IN VOLUME *B*

GRADES I OR II CONSTRUCTION

Full-width or part-width laminations in volume *B* may have knot-holes or sound knots whose size does not exceed one-third the width of the lamination except that no defects shall be permitted that interfere with bending to the required curve without localized irregularities in the curvature, or that interfere with bringing laminations into close contact. Wane whose greatest width does not exceed the finished thickness of the lamination is permissible.

[7] See footnote 1, table 24.

LAMINATIONS IN VOLUMES *A*, *B*, AND *C*

It may be specified that all laminations shall be free from knots, knotholes, or wane that will be visible when the member is in place in the structure.

PREPARATION OF LAMINATIONS

Laminations will be single piece in length or will be built up to the full length of their runs by joining shorter pieces end to end, prior to the final surfacing of either side, by means of glued plain scarf joints that cross the thickness of the piece in a distance not less than 12 times the thickness for joints in volumes *A* and *C*, and not less than 6 times the thickness for joints in volume *C*. In volumes *A* and *C*, the sloping surfaces joined to form a scarf joint must be free from knots or pitch pockets.

DISTRIBUTION OF JOINTS

JOINTS IN THE WIDTH OF LAMINATIONS

Outer laminations shall be of one piece in width. Other laminations may be of two or more pieces provided their widths and arrangement are such that longitudinal joints in adjacent laminations are separated at least 1½ inches. Laminations or part laminations composed of two or more pieces edge glued to each other prior to the final surfacing of either side may be considered as one piece.

JOINTS IN THE LENGTH OF LAMINATIONS

Scarf joints in volumes *A* and *C* shall be so arranged that at any section perpendicular to the axis of the member the sum of the widths of the joints in any group of 3 successive laminations shall not exceed the width of the member. Furthermore, joints showing on either edge of a member shall not be closer together, center to center, in adjacent laminations in volumes *A*, *B*, or *C* than 24 times the thickness of a single lamination.

Joints in curved portions are to be avoided insofar as possible and laminations in volumes *A* and *B* shall not be jointed at any point where the radius of curvature to which they will be bent is less than 125 times the thickness of the lamination.

ARRANGEMENT OR DISTRIBUTION OF TAPER

The tapering of members that vary in depth shall be accomplished by whichever of the following methods is specified.

1. All laminations to parallel the center line of the member except the last one at each face, which shall follow the curvature of the face and be accurately fitted to the preceding laminations.

2. A group of outer laminations totaling at least one-fifth the depth of the member at the point of maximum depth or one-half the depth at the point of minimum depth to run parallel to each face of the member, the remaining laminations being so arranged that fitting their ends will not require cutting at a slope steeper than 1 in 12. Such fitting must be accurately done so that a good glue joint results.

3. The total taper to be approximately uniformly divided among all laminations, that is, each lamination to be tapered in the same proportions as the member itself.[8] For example, if there are 20 lami-

[8] Such tapering is probably not feasible with woodworking machines now available except for laminations that are so short that each can be fed through a planer on a properly tapered board.

nations, each will have at each point in its length a thickness approximately one-twentieth the depth of the member at the corresponding point.

MACHINING

All surfaces to be joined by gluing shall be carefully machined so that they are true and free from humps or depressions, and at the time of gluing shall be free of dust, dirt, or grease.

GLUE

Only a water-resistant casein glue, or other glue known to be equal in strength and moisture resistance to best quality casein glue, is to be used. Glue must be thoroughly mixed and be free from lumps and from excessive air bubbles.

Not less than 8 pounds of wet casein glue per 100 square feet of joint area, or equivalent amounts of other types used, shall be applied. Glue shall be applied to a uniform thickness and preferably to both of the faces meeting at a joint. Glue-coated pieces shall be laid together as soon as the glue is spread.

PRESSURE

All glued joints are to be subjected while the glue is setting to a pressure of not less than 100, nor more than 200 pounds per square inch by means of clamps, screw jacks, presses, or other similar appliances.

Clamping must be completed within 20 minutes after the first glue is spread if glue is applied to both faces meeting at a joint or within 15 minutes if glue is applied to only one of these faces. When a member is built up in installments, each group or installment of laminations shall remain under pressure for at least 3 hours before being released to add the next installment. Pressure shall be maintained at least 12 hours after the addition of the last installment.

Special care must be taken to assure good gluing between successive installments of laminations.

TEMPERATURE

The room in which gluing is done shall be maintained at a temperature of 50° F. or higher. Material is to be brought approximately to the temperature of the glue room before gluing is begun.

DESIGN STRESSES

The following is suggested as the procedure for determining from table 24, which is a tabulation of basic values adapted from a previous publication (*31*), stresses for use in the design of laminated members built in accordance with the specifications presented in the preceding section.

COMBINED BENDING AND COMPRESSION

The sum of the bending stress and the compressive stress parallel to the axis of the member shall not exceed the value in table 24, column 2 (increased in accordance with footnote 1 when this footnote is applicable to the material specified for use in volumes *A* and *C*) multiplied by grade of construction, curvature, and depth factors as follows:

Grade of construction factors:

For grade I construction	1.000
For grade II construction	.875
Curvature factor	$1.00-2000\left(\frac{t}{R}\right)^2$,

where $\frac{t}{R}$ is the maximum value of thickness of lamination divided by the radius to which the lamination is bent at that point in the length of the member at which the stress occurs. (No curvature factor is to be applied to stress in a straight portion of a member regardless of the curvature elsewhere.)

Depth factor	$1.07-0.07\sqrt{\frac{h}{2}}$

where h is the depth of the member in inches at the point under consideration.

If I- or box-sections are used further multiplication by the appropriate form factor should be made (p. 59).

COMPRESSION PARALLEL TO GRAIN

The allowable stresses in compression parallel to the grain are:

For laminations of the type specified for volumes *A* and *C* in grade I construction: The value given in table 24, column 4, increased for close-grained or dense material according to footnote 1.

For laminations of the type specified for volumes *A* and *C* in grade II construction: 80 percent of the value given in table 24, column 4, increased for close-grained or dense material according to footnote 1.

For laminations of the type specified for volume *B*: 75 percent of the value given in table 24, column 4, without increase for close-grained or dense material.

The total thrust at any cross section shall not exceed the sum of the products of the cross sectional areas of laminations of each type multiplied by the allowable stress in compression parallel to grain for laminations of that type.

COMPRESSION PERPENDICULAR TO GRAIN (AT BEARINGS OR RADIAL COMPRESSION)

For allowable stresses in compression perpendicular to grain, the value taken from table 24, column 3, increased for close-grained or dense material according to footnote 1, should be used.

TENSION PERPENDICULAR TO GRAIN (RADIAL TENSION)

For allowable stresses in tension perpendicular to grain, the values taken from table 24, column 3, should be used when multiplied by: One-seventh for softwood species or one-sixth for hardwood species and further multiplied by the following factors according to the type of material at the point under consideration:

1.00 for defects restricted as specified for volumes *A* and *C* in grade I construction; 0.70 for defects restricted as specified for volumes *A* and *C* in grade II construction; 0.60 for defects restricted as specified for volume *B*.

LONGITUDINAL SHEAR

For allowable stresses in longitudinal shear, three-fourths the values in column 5, table 24, should be used.

DESIGN, MANUFACTURE, AND ERECTION OF LAMINATED ARCHES IN FOREST PRODUCTS LABORATORY SERVICE BUILDING

Three types of roof supports were used in the Forest Products Laboratory service building (fig. 1). One was a framed truss designated type B. The other two were type C, a three-hinged arch with double-I section consisting of plywood webs and bent laminated flanges, and type D, a three-hinged bent laminated arch with rectangular cross sections. Principal dimensions of types C and D are shown in figure 38. Figure 39 shows a cross section of one arch of each of these types.

For purposes of design, a dead load of 15 pounds per square foot of horizontal area and a uniformly distributed live load of 30 pounds per square foot were assumed, it being further assumed that the live load could occupy the full span or either half span.

On the basis of the then existing information, the following stresses were taken as a guide to the design of the D-type arches:

	Pounds per square inch
Bending stress	1, 500
Compression stress parallel to grain	1, 000
Longitudinal shear	150

Owing to the fact that the computed required sizes of cross section were increased at several points to give even dimensions and the further fact that these sizes were not recomputed when some changes in height of roof peak were made, actual maximum stresses on the basis of the dimensions as built and with the assumed loadings are approximately 1,165 and 110 pounds per square inch for bending and shear, respectively. In an arch of this shape the stress in compression is very low because bending and shear stresses dictate the sizes of cross sections.

Because of a desire to conserve space within the building, the radii at the knees of the arches, particularly type D, were made very short necessitating the use of $\frac{5}{16}$ of an inch laminations or thinner. (As shown by fig. 38 the inner radius at the knee was 39 inches.) Material of this thickness was used in the flanges of type C also although the larger radius would have permitted the use of thicker material. The laminations were made from nominal 1-inch southern yellow pine boards that were practically free from knots. The moisture content was about 8 percent when the material was ready for gluing.

Laminations were surfaced to the required thickness after those that were longer than a single length of lumber had been made up of boards joined end to end by casein glued scarf joints having a slope of 1 in 12. (Fig. 40 shows a number of the joints under pressure in a press.) They were then spread on both sides with casein glue by a mechanical spreader, after which a number were assembled in a group in proper order on the gluing frames. Beginning at one end, pressure was applied by means of hand-operated screws. As far as possible, the time

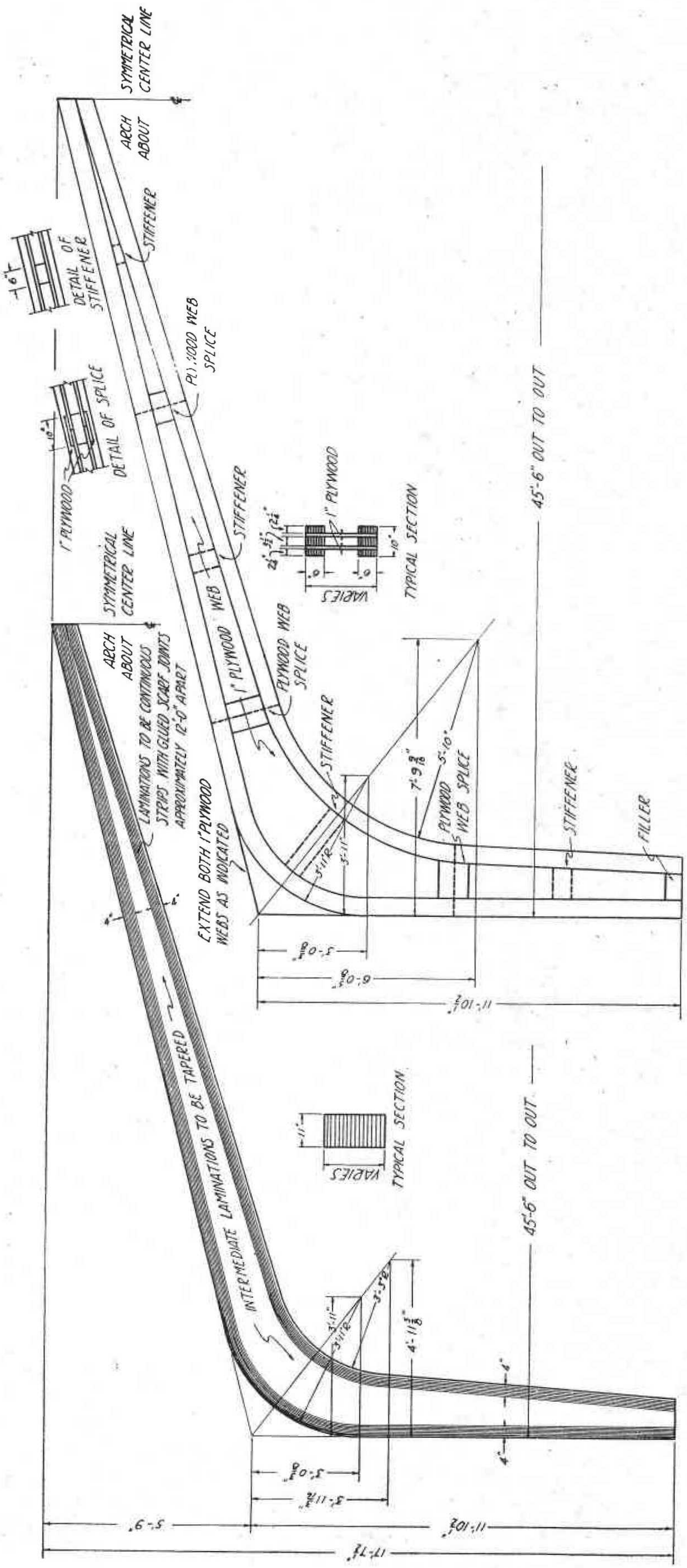

FIGURE 38.—Principal dimensions of arches of types C and D as used in Forest Products Laboratory service building.

from the spreading of the glue till the completion of the pressure on a group of laminations was restricted to 20 minutes. The procedure with the D-type was to build, at the first operation, about 4 inches

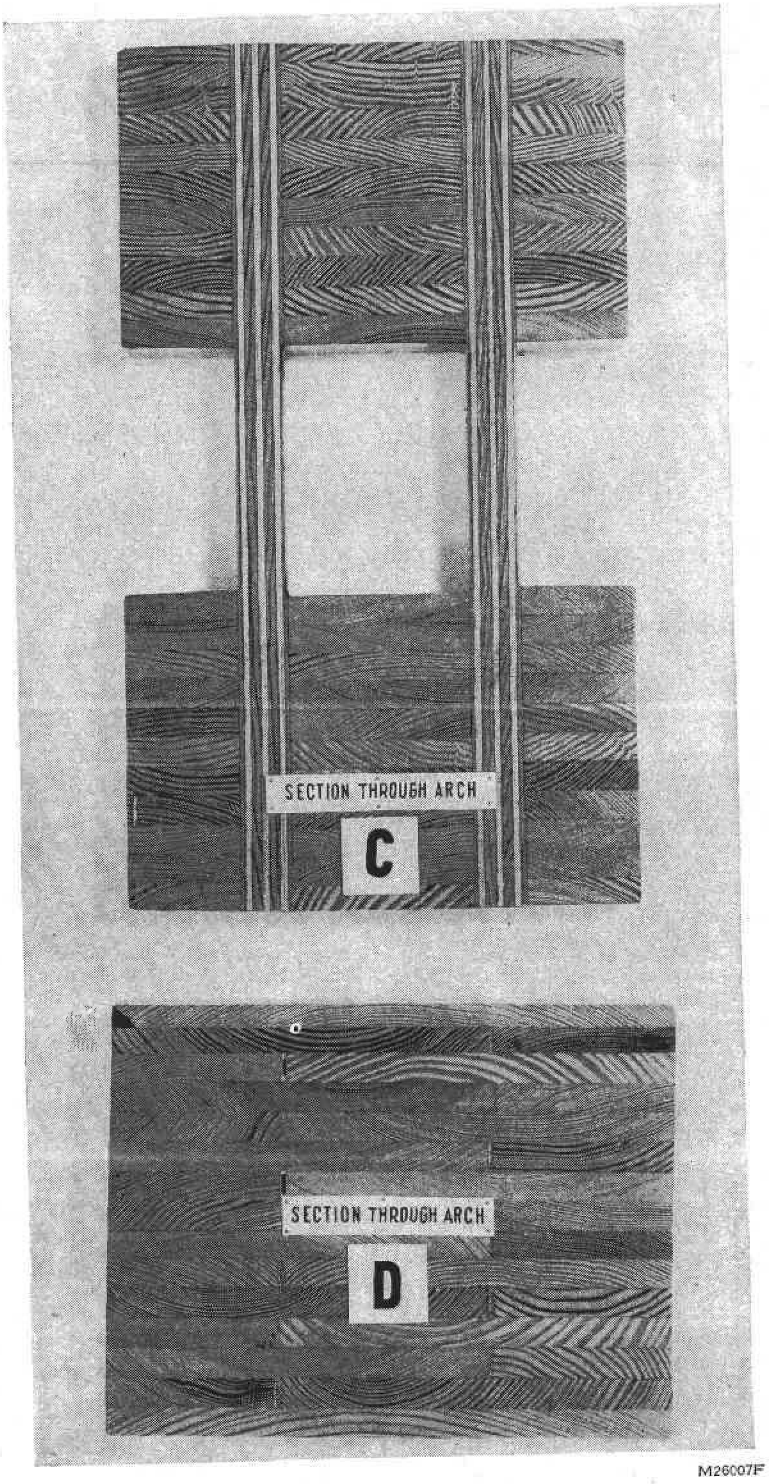

FIGURE 39.—Cross section of one arch of each of types C and D as used in the Forest Products Laboratory service building.

thickness at the concave side with full length laminations; then to add laminations shortened at one or both ends in installments of 7 to 10 until the assembly at all points extended beyond a line about 4 inches from and parallel to the convex face of the member. Next, the resulting stepped surface was planed to give a true face to which the group of eight remaining laminations was glued.

One complete flange for a type C arch was glued at one operation and was later ripped to form three pieces of the required sizes.

Measurements with hydraulic cylinders inserted in the clamping system showed that the average pressure applied in the gluing was 100 to 200 pounds per square inch of glued surface.

Six flange pieces and two plywood webs with stiffeners and splice plates were later assembled with casein glue to form a half arch of type C as shown in figure 38.

The outer laminations of type-D arches were full width, as shown in figure 39, while others were each of two pieces, the longitudinal joints

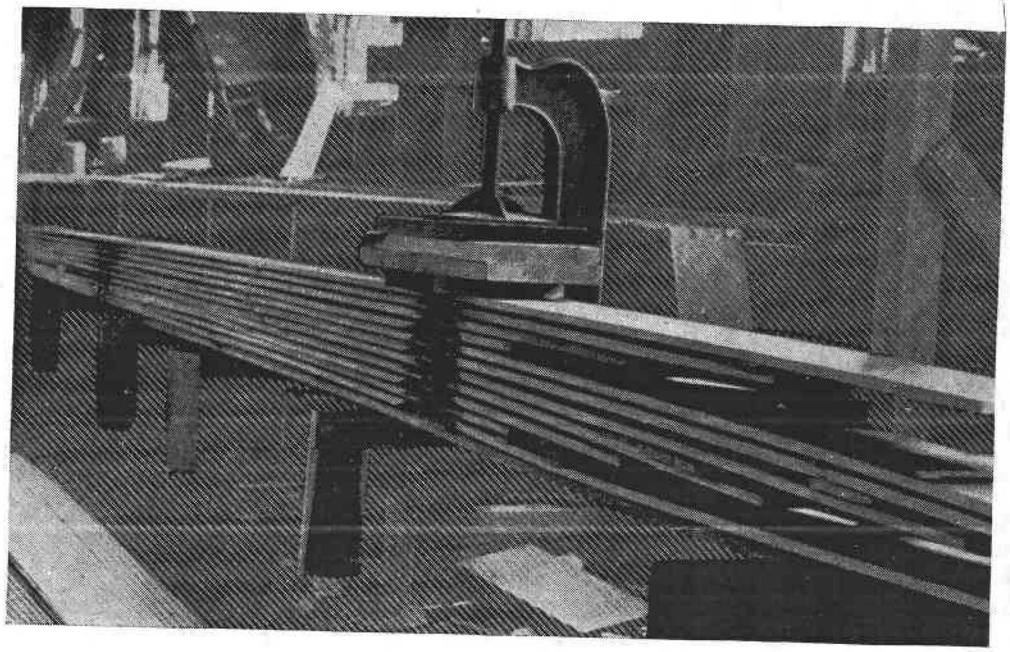

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FIGURE 40.—Scarf joints in laminations for the Forest Products Laboratory service building arches under pressure.

in adjacent laminations being staggered. No attempt was made to join the edges of adjacent parts of the width of a lamination with glue.

After attachment of the steel bases, the roof supports were erected by means of a portable crane (fig. 41). Later the bases were grouted to the same level.

The framed trusses and the types C and D arches for the service building were made by Unit Structures, Inc., of Peshtigo, Wis. As part of the contract, two half arches of each of types C and D were furnished in addition to the number required for the building. These extra half arches were tested at the Laboratory. The results of these tests are presented on pages 34 to 49.

TESTS OF GLUED LAMINATED MEMBERS UNDER CONTINUED LOADING

Tests on one arch of the D-type in the Laboratory service building and on a Laboratory-built member afford information on the behavior of laminated curved members under long continued loading.

TEST OF BUILDING ARCH

The center span of the service building was tested under long-continued loading. The loading was bags of sand and gravel stacked on the roof immediately above the arch and covered for protection from precipitation (fig. 42). The bags were uniformly distributed; at first

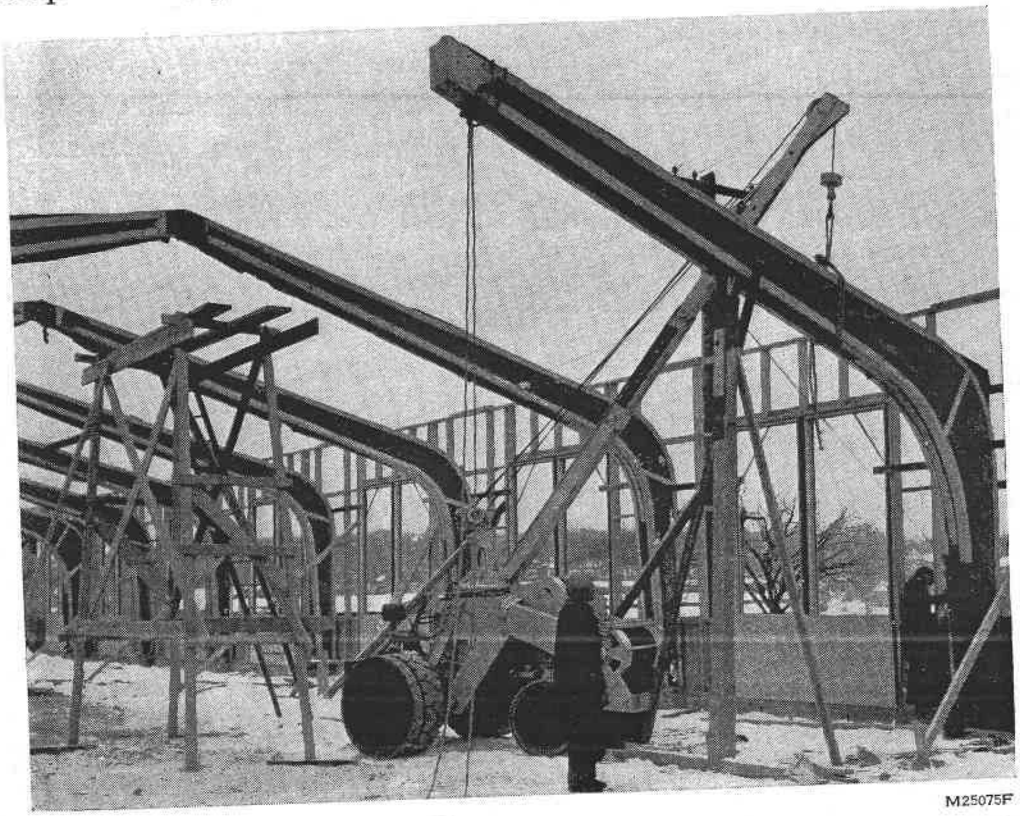

FIGURE 41.—Erection of arches in Forest Products Laboratory service building.

FIGURE 42.—Sandbags on central arch span of Forest Products Laboratory service building.

across the half span and later across the full span. The elevation of the peak of the arch and the horizontal distance between points on the arch 10 feet above the foundation piers were measured at frequent intervals. During the latter part of the period the temperature and relative humidity within the building were measured periodically.

The load was placed in increments the final total being 31,500 pounds, approximately uniformly distributed over the full span. This load is equivalent to 42.8 pounds per square foot on the roof area tributary to one arch, and thus about 42 percent in excess of the 30-pound-per-square-foot live load assumed in designing the building. Figure 42 shows the final load in place. Figure 43 is a record of the deflections during the test. Each load increment was, of course, accompanied by an increase of deflection; also deflection increased between placements of increments. Separating out and combining

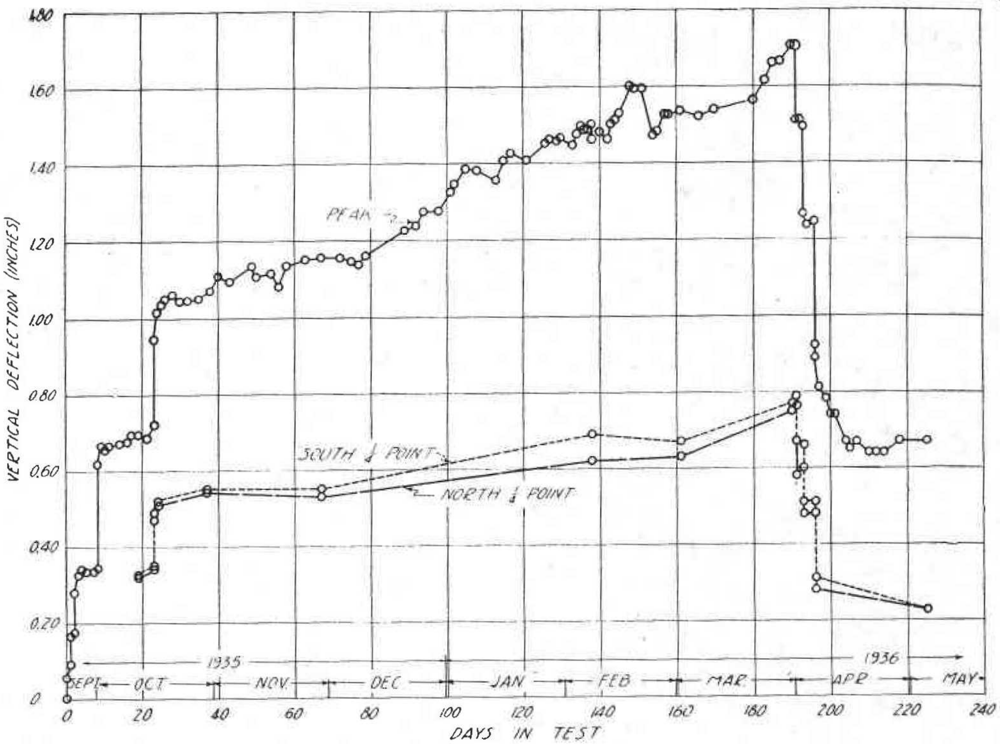

FIGURE 43.—Record of deflections of center arch of Forest Products Laboratory service building in continued loading test.

the deflections that occurred immediately on placing the several increments gives the data plotted in figure 44. It is not to be expected that an exactly linear relation will exist within the proportional limit of the material between deflection and the load on a member of such a shape. Figure 44 indicates an approximately linear relation between load and deflection with 0.74 inch as the deflection that would have occurred immediately had the entire load of 31,500 pounds been placed at one time. The actual deflection at the conclusion of the placing of the final load increment was 0.94 inch.

The graph of peak deflection (fig. 43) shows between the twenty-fifth and seventy-seventh day of the test three brief periods of 10 to 15 days each, during which there was little change. Beginning about the seventy-seventh day and continuing to the one hundred and fiftieth the deflection gradually increased with only minor depressions or dips in the curve, the total increase being about 0.45 inch and the increase per day three times as great as in the preceding 52 days. During this interval was a period of severely cold weather and the temperature within the building was maintained between 75° and 90° F., with the relative humidity averaging about 30 percent, an atmospheric condition under which wood tends to dry to a moisture content of 5 or 6 percent.

The moisture content of the arches when this test was begun is not accurately known but was probably 10 to 15 percent. Drying probably proceeded slowly in the early part of the test and at an increased rate during the later period of low humidity. Inasmuch as wood shrinks across the grain but not along the grain, a curved member

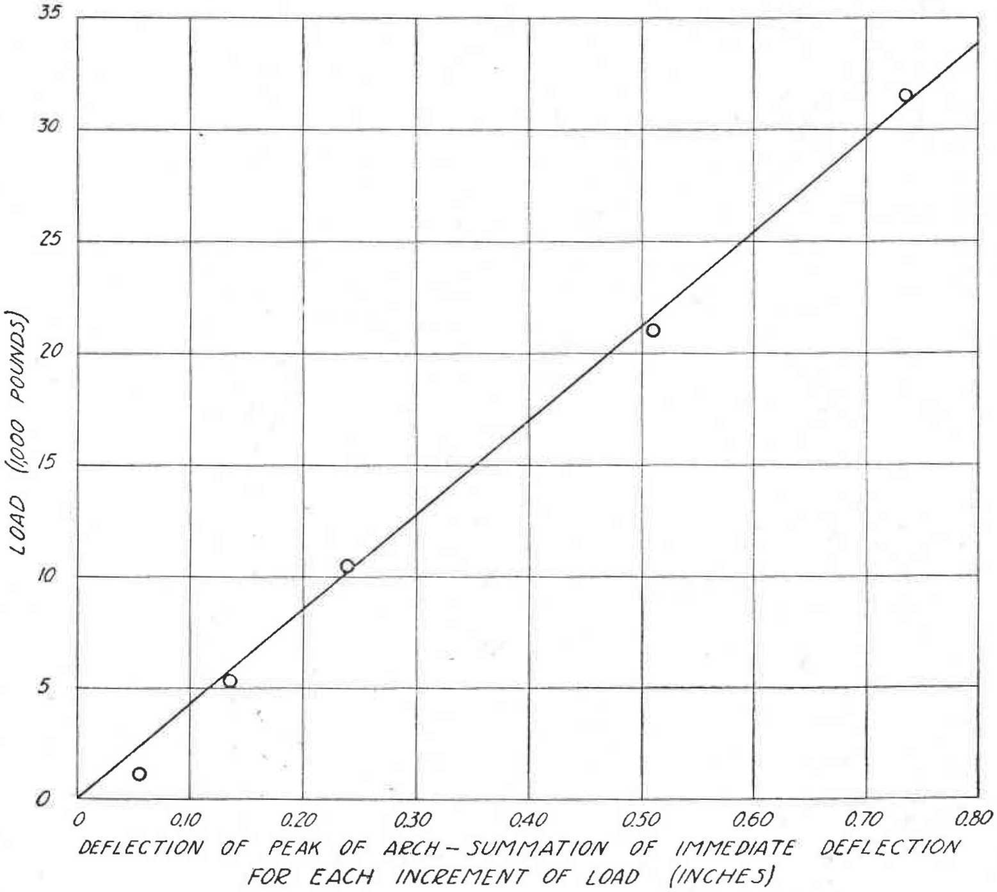

FIGURE 44.—Relation between loads on center arch of Forest Products Laboratory service building and the deflections at midspan resulting from their placement.

tends (p. 119) to become more curved as drying, accompanied by shrinkage, proceeds. Absorption of moisture being accompanied by swelling tends to decrease the curvature. It is computed from an equation developed later that a shrinkage of 0.5 percent across the grain, such as might result from a decrease of about 3 percent in moisture content, in these arches would of itself cause a depression of the peak of about an inch.

As shown by figure 43, a comparatively large (about ⅛-inch) decrease of deflection occurred between the one-hundred and fifty-first and one hundred and fifty-fourth day. This accompanied a slight increase in relative humidity but it does not seem that swelling of the wood from the absorption of moisture could have been sufficient to account for the upward movement of the peak.

The conclusion from a consideration of this information is that the possible effects of changes of moisture content on the deflection are such that it is impossible to determine whether increased deflection was caused by load during the 5 months it was in place. Only an inconsiderable amount of snow was on the roof at any time during the test.

A further factor affecting the deflection is that the construction of the roof was such that it added considerable support to the arch above which the load was placed. The roof panels were 4 feet wide and 16 long. Each had a nominal 2- by 6-inch joist at each edge and at the center of its width. Outer faces of the edge joists were grooved to receive splines holding faces of adjacent panels in alignment. For the most part these splines fitted the grooves very tightly. Glued to the lower edges of the joists was a sheet of three-ply ⅜-inch plywood and to the upper edges a sheet of five-ply ⅝-inch plywood. The panels

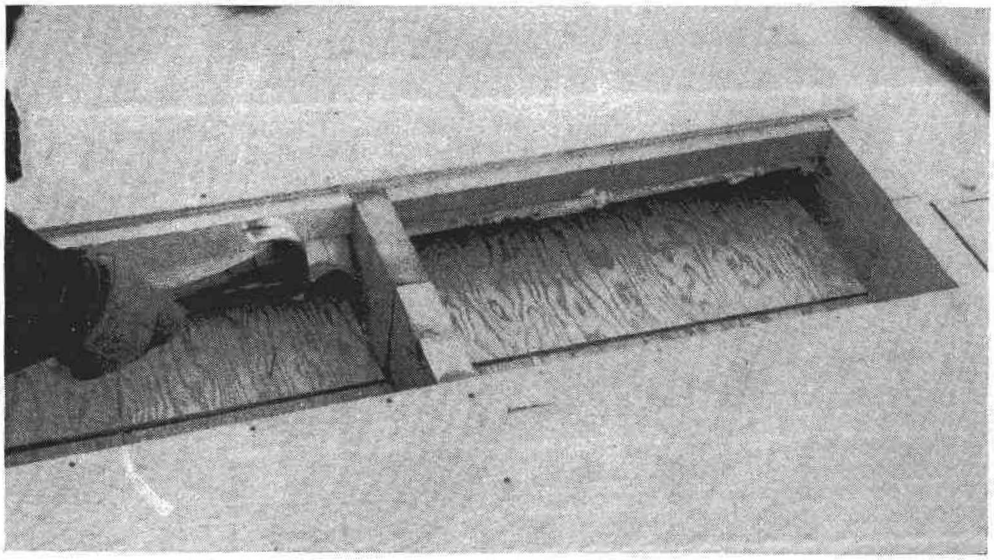

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FIGURE 45.—Nailing of roof panels to arches of Forest Products Laboratory service building.

thus formed box girders 4 feet wide, 16 feet long, and slightly more than 6 inches deep. The lower plywood sheet of each girder panel was nailed at each end to the upper face of one of the arches (fig. 45) with sixpenny nails spaced 6 inches. Also, the 2- by 6-inch joists within the girders were toe-nailed to the arches.

Vertical deflection of the loaded arch is, of course, due to bending and is accompanied by spreading near the junction of roof and side walls. Being nailed to the arches and being very stiff edgewise, the roof panels transmit forces causing spreading of the other arches, with consequent vertical deflections of their crowns.

The five central arches of the building are alike (*D* type) and the load was on the center one. Observations were made of the change in elevation of peaks of the five arches accompanying removal of the load at the conclusion of the test and accompanying replacement of the same load at a later date. Assuming that the change in elevation of each arch is a measure of the load carried by it and that only the five central spans participated in carrying the load on the building the ratio of the change of elevation of any one of these arches to the sum of the changes of elevation represents the proportion of the load that was carried by that arch. Data from the two sets of observations mentioned above and derived values of the ratio for the center arch are shown in table 25.

TABLE 25.—*Changes of elevation of the central 5 spans with removal and replacement of load of 31,500 pounds*

Span No.	At removal of load	At replacement of load	Span No.	At removal of load	At replacement of load
	Inches	*Inches*		*Inches*	*Inches*
3	0.06	0.07	7	−0.04	0.04
4	.27	.22			
5	.81	.88	Sum [1]	1.26	1.42
6	.16	.21	Ratio No. 5 to sum	.64	.62

[1] The sums listed in this table are the deflections that would have occurred had the load been carried by a single one of the D-type arches.

Obviously the ratios in the last column of table 25 taken as estimates of the proportion of the load actually carried by the center arch are only approximate. Nevertheless a substantial distributing effect of the roof panels is indicated. Such a finding means little in connection with load uniformly distributed over an entire roof or over one slope. It does, however, bring attention to the fact that roof panels, if sufficiently resistant to shearing forces acting in their plane, may have significant effect in distributing concentrated or nonuniformly distributed loads, or moving loads such as might be carried by a crane.

TEST OF LABORATORY-BUILT MEMBER

A curved member set up within the Laboratory was subjected to an end load during a period of 2½ years. This member consisted of eight ¾-inch southern yellow pine laminations free from defects. It was in the form of half an arch of 25-foot span and had a cross section 3¾ inches wide by 6 inches deep. Load was applied along the line connecting the ends of the half arch. The set-up is shown in figure 46. The load applied through the lever system shown was 3,175 pounds.

Prior to the period under load the member was tested for stiffness and finally it was tested to destruction. Determinations from these two tests showed that there had been about 4 percent increase in stiffness. In the final test the load at proportional limit was 8,750 pounds. The average moisture content was 5.6 percent at the final test and about 10 percent when the load was first placed. Adjustment for the difference in moisture content indicates that the value of load at proportional limit at the time the loading period began was about 6,750 pounds. Thus the applied load of 3,175 pounds was 47 percent of the estimated proportional limit at the beginning and 36 percent of the proportional limit as determined at the conclusion of the loading period.

The scale shown in figure 46 was attached at the point where the deviation of the axis of the member from the line of action of the load was a maximum (19.41 inches at the beginning of the test) and was read against a fine vertical wire to measure the deflection. Periodical weighings of a thin board that was kept alongside the member indicated the equilibrium moisture content which the member currently tended to approach. Readings of deflection and of equilibrium

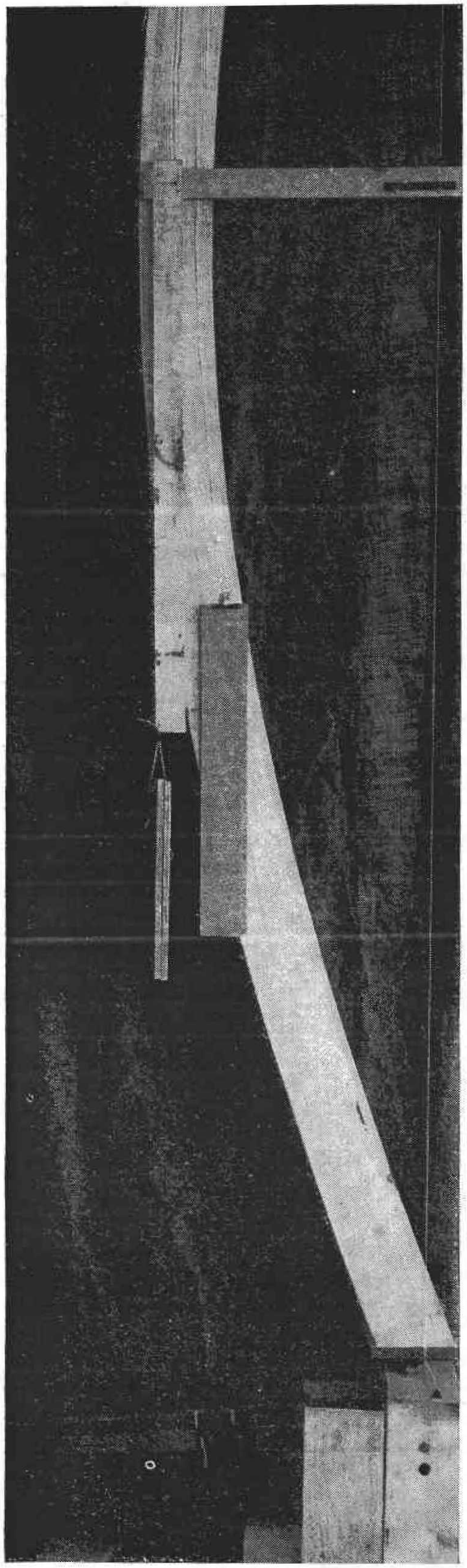

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FIGURE 46.—Laboratory-built curved member of southern yellow pine under load of 3,175 pounds applied to its ends through a lever system.

moisture content are graphed in figure 47. In this figure a correlation between atmospheric conditions and the deflection of the member can be traced. Up to about April 1, 1934, the deflection increased (but at a diminishing rate) while the equilibrium moisture content decreased. During the summer of 1934, the equilibrium moisture content rose and the deflection remained almost constant. With the beginning of the heating season (about October 1) lower atmospheric humidity returned and the deflection again increased till near the end of January 1935, when rising equilibrium moisture content was accompanied by decreased deflection till about August 1. Over the subsequent period to April 1, 1936, the drier atmospheric condition was accompanied by a slight increase in deflection. The evident effect of changes in moisture content makes it difficult in this case, as in the test of the building arch, to distinguish the purely time effect of the load from that of moisture changes and the resultant shrinkage and swelling and to determine when deflection resulting from the action of the load ceased to increase.

GLUED LAMINATED CONSTRUCTION IN FARM BUILDINGS

Glued laminated construction holds much promise as a means of improving the structural features of barns and other farm buildings. For example, although rafters, because of lower costs and less obstruction of space, are preferred to trusses for supporting barn roofs, previously available methods of construction have not in general provided rafters of adequate strength and of sufficient stiffness to avoid sagging. Two manufacturers are now making glued laminated rafters for the gothic-style dairy barn and other farm structures.

Designs include a member which is straight and vertical from the foundation to the mow floor level and thence curves uniformly to the roof peak, thus combining in a single piece the functions of stud and rafter (*5*).

The present research has not included specific study of glued laminated construction in farm structures but a few tests have been

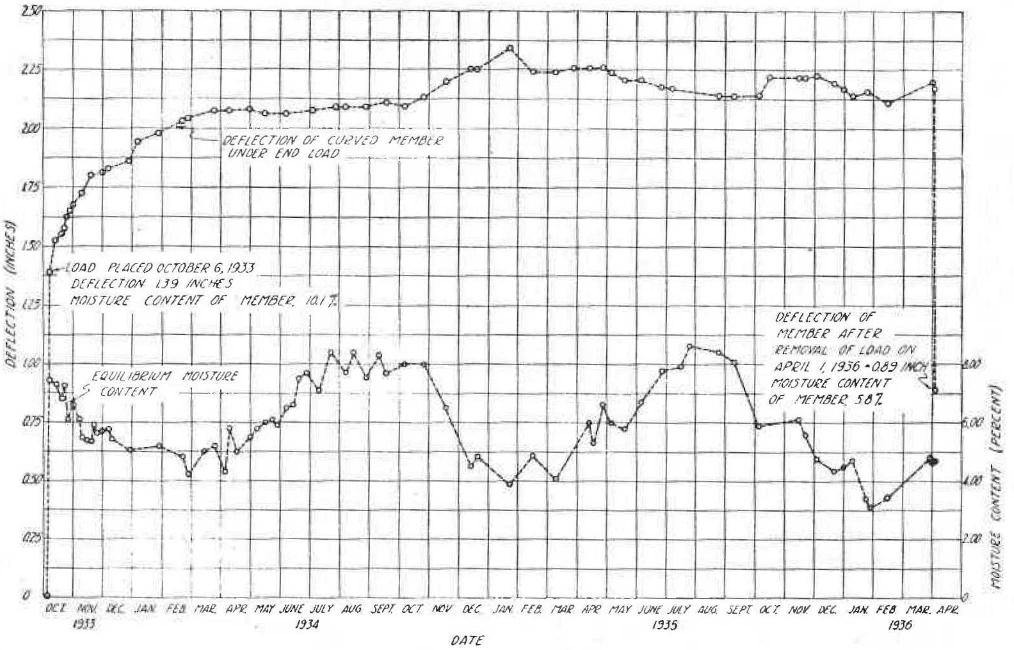

FIGURE 47.—Deflections and equilibrium moisture-content values in dead-load test of laboratory-built member.

made to demonstrate comparisons with barn rafters made according to current practice. A brief account of these tests follows.

COMPARISON OF COMMON TYPES OF CURVED BARN RAFTERS WITH GLUED LAMINATED MEMBERS

NAILED LAMINATED RAFTERS

One common type of curved barn rafter consists of a number of boards bent to the required curvature and nailed together. Comparison of members of this type with those made in the same way but glued instead of nailed is afforded by tests on five pairs of beams. Although the tests were on straight members, similar comparisons would result from tests on curved assemblies. In these tests, ¾-inch laminations cut from the same board occupied similar positions in nailed and glued assemblies constituting a pair. The laminations were horizontal (p. 2). The lumber used was at about 12-percent moisture content and after completion the beams were stored for a month under conditions of temperature and relative humidity to give an equilibrium moisture content of about 12 percent. They were then tested. Table 26 gives important details of construction and results of the tests.

These tests clearly demonstrate the superiority, with respect to both the strength and the stiffness, of glue over nails as a means of joining laminations. Experience, test, and observation show that nails decrease in effectiveness with fluctuations in the moisture content of the material nailed, and tests made after a period of service under usual conditions would no doubt be still more favorable to the glued construction.

Similar results were demonstrated in a series of tests made at Iowa Agricultural Experiment Station (*12, 13*).

TABLE 26.—*Comparison of beams with nailed laminae to those with glued laminae*

Detail of beams tested					Load		Deflection	
No.[1]	Laminae	Length	Span	Treatment	At proportional limit [2]	Maximum [3]	At proportional limit [2]	At maximum load [3]
	Number	*Inches*	*Inches*		*Pounds*	*Pounds*	*Inches*	*Inches*
1	8	96½	90	Glued	7,250	12,170	0.81	1.63
2 [4]	8	97⅛	90	Nailed		3,400		7.75
3	6	96	93	Glued	5,000	6,880	1.3	1.81
4 [5]	6	97	93	Nailed		2,270		4.90
						2,320		7.20
5 [6]	4	64½	60	do		1,990		5.00
						3,110		6.30
6	4	64⅛	60	Glued	3,250	6,080	.88	2.40
7 [6]	4	64⅜	60	Nailed		2,550		6.35
						2,990		6.40
8	4	64⅜	60	Glued	2,750	6,710	.72	2.70
9 [6]	4	64¼	60	Nailed		2,900		6.00
10	4	64⅝	60	Glued	3,500	6,725	.81	2.05

[1] On beams 4, 5, and 7 two tests were made.

[2] Load-deflection graphs for the nailed beams were rounded almost from the beginning and showed practically no direct proportionality.

[3] Tests of the nailed beams were discontinued when the deflection became extreme and the recorded maximum loads are final loads rather than true maxima.

[4] Beam No. 2.—An assembly of 3 laminae was first nailed together with sevenpenny cement-coated box nails driven from each side. 3 laminae were then added at the top and 2 at the bottom, each laminae being nailed to those previously assembled with sevenpenny cement-coated box nails. 182 nails passed through the neutral surface or joint between laminae at mid height of the beam.

[5] Beam No. 4.—An assembly of 3 laminae was first nailed together with about 64 sevenpenny cement-coated box nails driven from each side. 3 laminae were then added at the top, each being nailed to those previously assembled with 64 sevenpenny cement-coated box nails—129 nails through neutral surface. After the first test, the beam was reversed and the second test was made without the addition of nails.

[6] Beams Nos. 5, 7, and 9.—3 laminae nailed with sevenpenny cement-coated box nails driven from each side; top lamina then added and nailed with sevenpenny cement-coated box nails. The first test on No. 5 was made with this nailing of 102 sevenpenny cement-coated box nails through the neutral surface. After beam was straightened and 37 or 38 nails driven from the top and bottom of the beam (making 75 eightpenny nails through the neutral surface in addition to those present in first test), second test was made. First test on No. 7 made with the original nailing of 116 sevenpenny cement-coated box nails through the neutral surface. Beam straightened and second test made after driving 37 or 38 eightpenny common nails from the top and bottom of the beam, making 75 eightpenny common nails through the neutral surface in addition to those present in first test. 32 eightpenny common nails were driven from each face of beam No. 9 before the test, making 102 sevenpenny cement-coated box nails and 164 eightpenny common nails through the neutral surface.

SEGMENTAL RAFTERS

A second and widely used type of curved barn rafter is the segmental rafter made by band-sawing the edges of boards to the required curvature, assembling the boards, which must be in comparatively short lengths, and nailing them together with the necessary butt joints staggered in the adjacent layers. In this type the layers, or laminations, are vertical instead of being bent to follow the curvature. Sometimes the boards are not band-sawed but are left with straight edges and arranged with a polygonal outline approximating the desired curve. Tests to compare this type of construction with glued assemblies of bent laminations were made on members constructed as follows:

Nos. 53 and 55, bent laminated members 15 feet long consisting of seven ¾- by 5¼-inch laminae each and made on the form of 1-foot offset (p. 14).

No. 54, seven vertical laminae, ¾ by 5½ inches each from the same board as the one used in No. 53, cut and nailed as indicated by figure

48, *A* to form a member 15 feet long and with the same offset as Nos. 53 and 55.

No. 56 consisting of laminations cut from the same boards as those from No. 55, and made like No. 54 except that faces of laminations were spread with casein glue immediately before nailing the laminae together.

Results of tests on these two pairs of members are listed in table 27. Obviously, such a number of tests does not of itself justify any general conclusions. However, the indications from the tests are in line with engineering experience and judgment, and are:

TABLE 27.—*Comparison of bent laminated members with segmental members*

Arch No.	Construction	Proportional limit				Maximum load	Maximum moment[1]	Deflection at maximum moment
		Load	Moment[1]	Deflection	Moment[1] / deflection			
		Pounds	*Inch-pounds*	*Inches*		*Pounds*	*Inch-pounds*	*Inches*
53	Bent and glued	17, 500	186, 500	4. 13	45, 160	21, 550	308, 100	9. 05
54	Vertical laminae nailed	6, 000	43, 500 (41, 000)	1. 13	38, 500 (36, 280)	11, 160	106, 300 (100, 000)	3. 60
55	Bent and glued	13, 000	133, 300	3. 69	36, 120	15, 680	183, 900	5. 16
56	Vertical laminae nailed and glued	10, 000	77, 200 (76, 700)	1. 90	40, 630 (40, 370)	14, 650	151, 900 (151, 000)	4. 55

[1] A small difference in moisture content at test existed between Nos. 53 and 54 and between Nos. 55 and 56. Values in parentheses opposite Nos. 54 and 56 are adjusted to the moisture contents of Nos. 53 and 55, respectively.

1. That the stiffness (measured by $\frac{\text{moment}}{\text{deflection}}$ at the proportional limit) of the segmental members, particularly when glued in addition to the nailing, is comparable to that of members with continuous bent and glued laminations.

2. That the strength, both ultimate and that at the proportional limit, is low for nailed segmental members as compared to those with continuous bent laminations, but is very considerably enhanced by gluing. Possibly gluing under adequate pressure in a press would produce further improvement. However, the direction of grain in each lamination is at an angle to that in the adjacent lamination. With this arrangement there is less assurance of permanence of glued joints under such changes in the moisture content of the member as may occur in service than when direction of grain is parallel in adjacent laminations, as is true of bent laminated members.

Due to lack of lateral stiffness No. 54 (nailed member) bent sideways before final failure as shown by figure 48, *B*. Maximum load and maximum moment were affected but little, if at all, by this lateral bending. In service such bending or buckling is often considerably restrained by roof sheathing or other parts of the construction.

PROTECTION OF FRAMING MEMBERS IN BARNS

Condensation of moisture in walls is of frequent occurrence in barns housing livestock in the colder climates. This results in subjecting studs or other framing parts enclosed in the walls to conditions favorable to decay and numerous instances of such decay

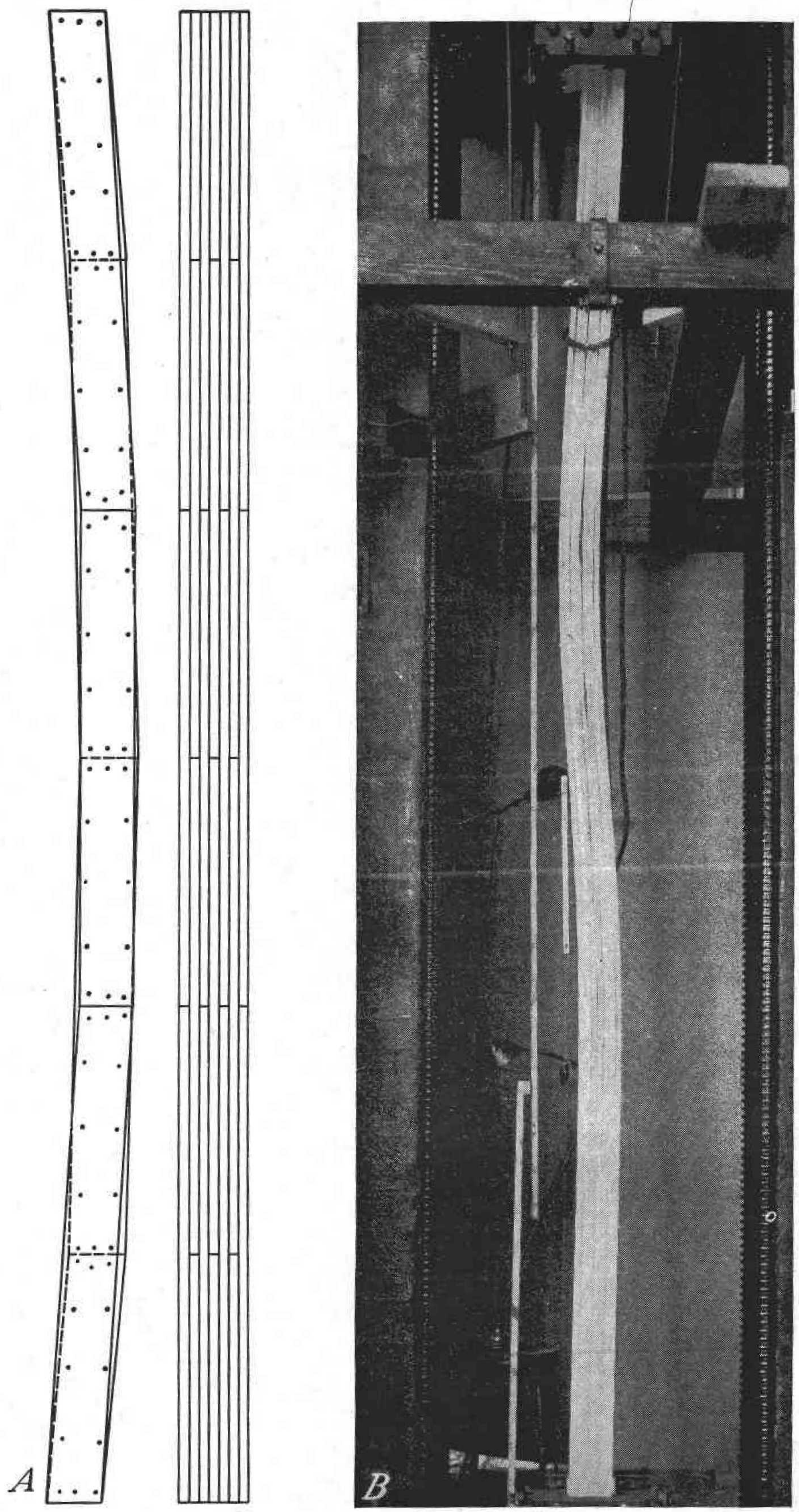

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FIGURE 48.—*A*, Segmental curved member, with vertical laminations; *B*, view of concave face of segmental curved member No. 54 showing lateral bending.

have been observed. The same conditions are favorable to the deterioration of glue. Regardless of the kind of members used in the walls, whether one-piece or laminated, effective moisture barriers to prevent the passage of water vapor from the interior of the building with resultant condensation in walls is desirable. Vapor is also likely to be condensed in the haymow or on the under side of the roof with similar hazards to mow joists and to rafters. Moisture barriers in the ceilings above livestock are consequently likewise desirable.

NAILING AS A SUBSTITUTE FOR PRESSING OR CLAMPING IN MAKING GLUED LAMINATED MEMBERS

The test data previously reported in this bulletin relate to glued laminated members in the making of which pressure on joints was produced by clamping. The use of clamps to produce such pressure has been the method principally practiced both in this country and in Europe with the exception of Sweden (p. 88). It is obvious, however, that if nailing of successive laminations without clamping or with but a minor amount of clamping would produce glued joints of adequate and dependable strength, manufacture of glued laminated members would be simplified and cheapened. Also, members could be built in place or near the building site thus making possible the use of sizes too large for convenient transportation over long distances.

The possibilities of nailing were explored in a preliminary way in two series of tests. The first consisted of shear tests on blocks cut from small assemblies glued up in the straight and referred to as "shear-block tests." In the second series curved laminated glued members were built by nailing glue-coated laminations together and were tested under transverse load in such a way as to cause failure by longitudinal shear. These are designated as "longitudinal shear tests under transverse load."

SHEAR-BLOCK TESTS

For the shear-block tests assemblies each consisting of four three-quarter-inch laminae about 5½ inches wide and 24 inches long were made up. The total number of such assemblies was 32, one-half of which was of southern yellow pine and one-half of Douglas fir. One-half the assemblies (A–H) of each species was glued in a press under a pressure of 200 pounds per square inch and the other half (I–P) was, after being spread with glue, nailed together. Laminations cut from end-to-end positions in a board occupied similar positions in a pressed and in a nailed assembly. Casein glue was used. Figure 49 shows the nailing and the positions from which specimens for the shear tests were obtained. These specimens were so cut that in test the shearing stress would come on the central glued joint.

As shown in figure 49, 8 eightpenny common nails were driven from each side. Nails 2½ inches long passed through three laminations and penetrated one-quarter inch into the fourth. There were then 16 nails through the central glued joint and, the joint having an area of approximately 132 square inches, there was on the average 1 nail for each 8¼ square inches of area of the central glued joint.

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TABLE 28.—*Summary of shear-block tests*

Species and assemblies	Nailed and glued specimen Nos.									Pressed and glued	
	1 and 2		3		4		5 and 6[1]	All			
	Shear	Wood failure	Shear	Wood failure	Shear	Wood failure	Shear	Shear	Wood failure	Shear	Wood failure
	Pounds per square inch	*Percent*	*Pounds per square inch*	*Percent*	*Pounds per square inch*	*Percent*	*Pounds per square inch*	*Pounds per square inch*	*Percent*	*Pounds per square inch*	*Percent*
Southern yellow pines:											
A, I	488	14	1,141	80	1,089	70	820	808	44	1,170	98
B, J	857	8	1,015	25	1,055	10	695	862	12	1,171	86
C, K	1,040	58	1,065	30	836	70	765	918	54	1,186	100
D, L	868	30	993	30	916	15	784	868	26	1,170	96
E, M	842	12	1,289	5	532	0	995	900	7	1,661	88
F, N	1,109	5	894	10	751	5	758	896	6	1,543	99
G, O	1,140	6	1,171	4	974	5	1,010	1,067	6	1,373	75
H, P	1,125	5	863	0	852	3	768	917	3	1,472	71
Average	934	17	1,054	23	876	22	824	904	20	1,343	89
Douglas fir:											
A, I	884	60	1,150	60	1,181	60	694	916	60	1,269	93
B, J	1,038	90	856	60	1,082	75	1,058	1,022	79	1,245	100
C, K	1,007	45	1,007	90	724	20	822	865	50	1,091	94
D, L	951	96	962	50	962	50	1,036	983	73	1,181	76
E, M	901	55	1,262	90	1,229	97	1,031	1,059	74	1,245	91
F, N	1,319	55	1,281	60	1,016	50	1,092	1,186	55	1,468	94
G, O	850	38	848	45	1,187	60	1,002	982	45	1,422	94
H, P	1,071	57	1,175	20	1,086	40	1,016	1,106	44	1,439	78
Average	1,003	62	1,068	59	1,058	56	969	1,015	60	1,295	90

[1] The percentage of wood failure in these specimens was not determined.

As may be noted from figure 49, specimens Nos. 1 and 2 were centered on a longitudinal gage line and midway between nails, No. 3 was on a transverse gage line and midway between nails, No. 4 was centered longitudinally and transversely between nails, and Nos. 5 and 6 each included one nail driven from each side of the assembly. Results of the tests are listed separately in table 28 for each of the types of location in the nailed assemblies. Location in the pressed

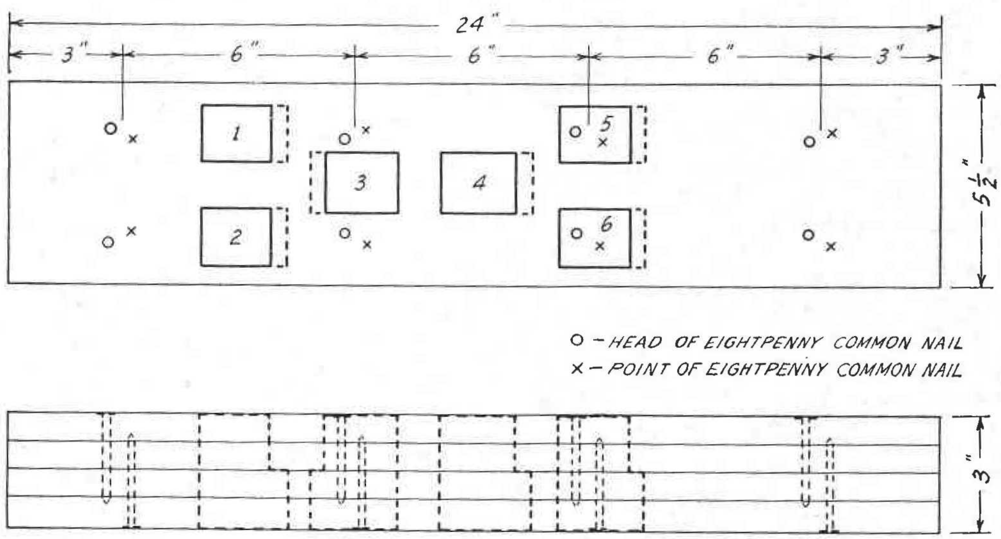

FIGURE 49.—Glued and nailed assembly for shear-block tests—showing locations of shear specimens Nos. 1 to 6.

assembly has no similar significance and the values entered in table 28 are averages for the six specimens from each pressed assembly. On the basis of the averages shown in table 28, the specimens from nailed assemblies of southern yellow pine had 68 percent as great shearing strength as those from the pressed assemblies, the corresponding figure for Douglas fir being 78 percent. The percentage of wood failure observed in the shear area was likewise less in the nailed specimens than in the pressed, the deficiency being much less for Douglas fir than for southern yellow pine. In addition to the lower average values, specimens from the nailed assemblies were considerably more variable in strength than those from the pressed assemblies. This is illustrated by table 29, which shows the distribution of test results. As may be noted, strength values for specimens Nos. 2 and 6, each of which included two nails, are not in general higher than for other specimens. Nails add no strength to a glued joint except by producing pressure during the gluing.

TABLE 29.—*Distribution of results of shear-block tests*

Class limits (Pounds per square inch)	Southern yellow pine specimens						Douglas fir specimens					
	Pressed	Nailed specimens No.—					Pressed	Nailed specimens No.—				
		1, 2	3	4	5, 6	All		1, 2	3	4	5, 6	All
200–249		1				1						
250–299												
300–349												
350–399												
400–449												
450–499												
500–549				1		1						
550–599								1				1
600–649					1	1					1	1
650–699					3	3		1			1	2
700–749					1	1		1		1	2	4
750–799		1		1	1	3						
800–849	1	3		1	5	9		1	1		1	3
850–899		2	2	1	1	6			1		1	2
900–949		2		1		3		1				1
950–999			1	1	2	4	2	1	1	1	3	6
1,000–1,049		2	1		1	4	3	2	1	1	1	5
1,050–1,099	1	2	1	2		5	1	2		2	2	6
1,100–1,149	6	2	1			3	3	2			1	3
1,150–1,199	12		1			1	6		2	2	1	5
1,200–1,249	6	1				1	4	2		1	1	4
1,250–1,299			1			1	7	1	2			3
1,300–1,349	1	1				1	3					
1,350–1,399	1						2	1				1
1,400–1,449							4				1	1
1,450–1,499	2						7					
1,500–1,549	4						5					
1,550–1,599	5						1					
1,600–1,649	5											
1,650–1,699	2											
1,700–1,749	1											
1,750–1,799	1											
Total	48	17	8	8	15	48	48	16	8	8	16	48

LONGITUDINAL SHEAR TESTS UNDER TRANSVERSE LOAD

Four parabolically curved members (made with a 3-foot offset at the center of 20-foot chord) supplied eight specimens for test of longitudinal shear under transverse load. Each of these curved members consisted of twelve ¾-inch laminations 5½ inches wide, the six central laminations being of southern yellow pines and the remainder of Douglas fir. Casein glue was used. The curved members were made

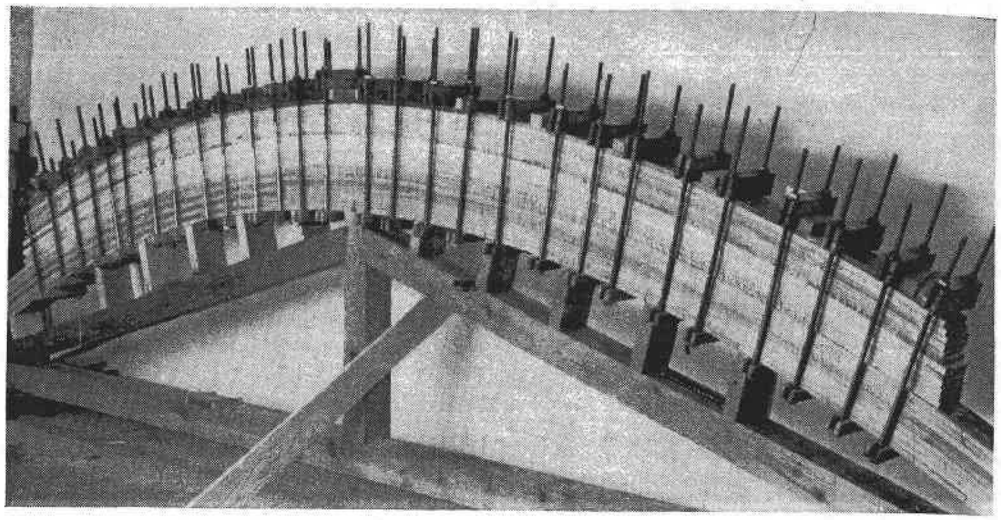

FIGURE 50.—Curved member for longitudinal shear test under transverse load on form with clamps.

up in two matched pairs and one of each pair was clamped in gluing while the other was nailed together.

The procedure in clamping was the same as in making the other parabolic curved members except that the form was placed vertically instead of horizontally. Figure 50 shows one of these members on the form with the clamps in place. Four laminations were glued at a time.

In producing the nailed members the first lamination was attached to the form; the second was spread with glue on its lower face and nailed to the first—nailing proceeding from the center of the length toward each end. Assembly proceeded continuously in this manner until all laminations were in place. A clamp, placed loosely at each end, held each lamination in place while it was being nailed and was tightened when the nailing was complete. When the assembly was

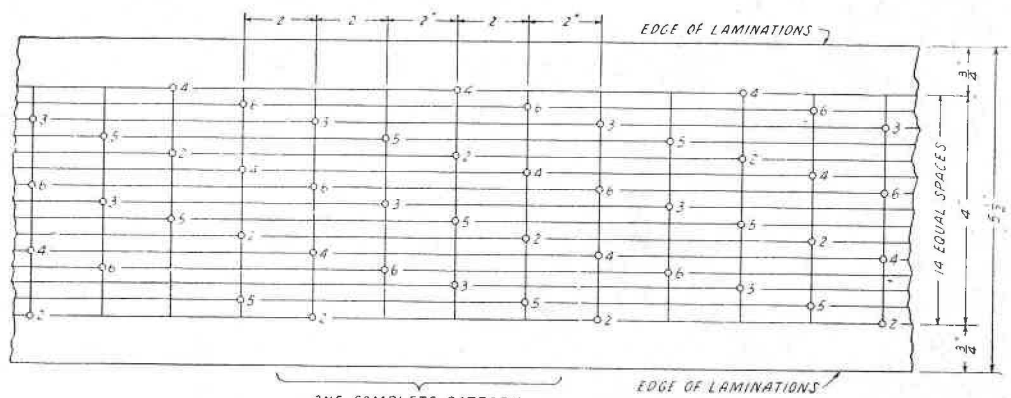

FIGURE 51.—Arrangement of nailing in members for longitudinal shear test under transverse load. Numerals indicate positions of nailheads in laminations of corresponding number—counting from side next to form. Laminations Nos. 7 and 12 same pattern as No. 2; lamination No. 8 same pattern as No. 3; lamination No. 9 same pattern as No. 4; lamination No. 10 same pattern as No. 5; lamination No. 11 same pattern as No. 6; size and kind of nails: threepenny common in lamination No. 2, sevenpenny cement-coated box in lamination No. 3, tenpenny common in lamination No. 4, twelvepenny cement-coated box in laminations Nos. 5 to 12.

complete each end was firmly clamped to the form. Assembly of each nailed member required about an hour.

Nailing was in accordance with figure 51.

After the four curved members had seasoned for about a month, each was cut in the middle and at points 7½ feet (along the chord) from the center, forming two specimens each of which was tested on a span of 7 feet with equal transverse loads 1½ feet on each side of its center. One specimen of each pair was tested with its convex side up; the other with convex side down. The set-up for the tests is shown in figure 52.

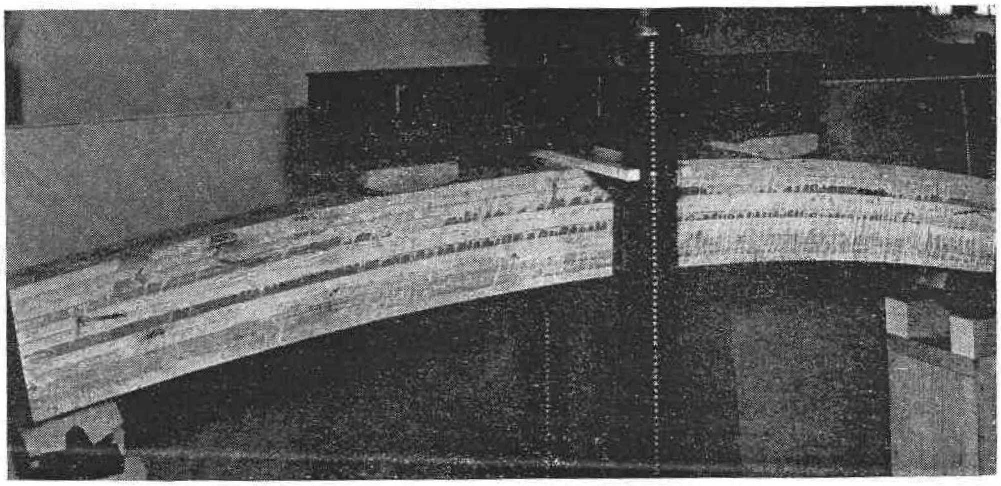

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FIGURE 52.—Curved laminated member in machine in position for longitudinal shear test under transverse load.

TABLE 30.—*Results of longitudinal shear tests under transverse load comparing nailed with clamped assemblies*

Manner of test, specimen No., and kind of assembly	Maximum load	Computed shear stress	Stress ratio, nailed to clamped	Wood failure [1]
	Pounds	*Pounds per square inch*	*Percent*	*Percent*
Convex side down:				
312A, clamped	46,040	670		[2]+60
313A, nailed	34,230	495	74	−30
314A, clamped	46,000	670		+95
315A, nailed	36,770	530	79	50
Average clamped	46,020	670		75
Average nailed	35,500	515	77	40
Convex side up: [3]				
312B, clamped	30,540	440		−25
313B, nailed	26,180	380	86	+0
314B, clamped	27,720	400		65
315B, nailed	30,500	440	110	+70
Average clamped	29,130	420		45
Average nailed	28,340	410	98	35

[1] Observed after splitting specimen with wedges, starting in fissure produced in test.
[2] Tension, no shear.
[3] In tests with the convex side of the member up there is a radial tension (perpendicular to the glued joint and to the grain of the wood) equal to approximately 22 percent o fthe computed longitudinal shear stress.

In considering the results (table 30), it may be noted that the specimens tested with the convex side up are consistently lower in longitudinal shear than those tested with the convex side down. With the combination of curvature and proportions of specimen and arrangement of loads, there was a radial stress equal to about 22 percent of the longitudinal shear stress. In the specimens with convex side up this radial stress is tension perpendicular to the glued joint and to the grain of the wood, which tends to lower the shear resistance. With the other placement, the radial stress is compression, which in the magnitude that existed in this instance probably does not affect the resistance to shear.

It may also be noted from table 30 that, in three of the four comparisons, the shearing stress and the percentage of wood failure is less for the nailed than for the clamped specimens.

SIGNIFICANCE

Considering the data of tables 28, 29, and 30, it seems unlikely that as strong and dependable glued joints can be produced by nailing as by clamping, except possibly by the use of so many nails that splitting would result and that the nailing would be uneconomical. However, in many instances members can be so designed that only moderate shearing resistance will be necessary. Furthermore, it may in some instances be economical to increase the size of members to make up for the lower shear resistance of joints in nailed members and thus provide for glued laminated construction under circumstances such that clamping during assembly is not feasible or in members that are too large to be transported to the building site following assembly by clamping.

On the basis of the data presented in tables 28, 29, and 30, it is suggested that the allowable stress in longitudinal shear for members assembled by gluing and nailing should not exceed two-thirds of the value for clamped members. A lesser amount of nailing than is represented by figure 51 and used in the members listed in table 28 is not to be recommended. As may be calculated from figure 51, each lamination had three nail-heads to each 44 square inches. The twelvepenny cement-coated nails, 3¼ inches long, extended completely through four of the ¾-inch laminations. Thus, for each 44 square inches of joint area, there were 9 nails that extended through one or more laminations on each side of the joint or approximately 5 square inches of joint area for each nail that extended through one or more laminations on each side of the joint. Assembly started at the concave side of the member and the first three joints had the same number of nails but some of them were smaller as indicated by figure 51. The last two joints toward the convex side had a smaller number of the twelvepenny cement-coated nails.

In assembling with nails, each nail that penetrates a joint and into one or more laminations beyond the joint adds to the pressure on the glue and, provided it is driven before setting of the glue begins, is effective in improving the adhesion of the glue. Hence, the maximum benefit from nailing results only when the assembly proceeds continuously from beginning to completion and is at such a rate that the number of laminations placed and nailed within the working life of the glue is at least as great as the number whose aggregate thickness equals the length of the nails.

GLUED LAMINATED CONSTRUCTION IN EUROPE

Glued laminated construction was originated by Otto Hetzer of Weimar, Germany, and is known in many parts of Europe as "Hetzer construction" or "Hetzer system" (*1*, *3*, *8*). The term is applied to any member composed of laminations joined with a water-resistant glue regardless of the shape or use of the member. Cross sections are I-shaped or rectangular. Beams often taper in depth from the center to each end. They may also be of uniform depth and may be straight or cambered. In structures sheltering railway passenger platforms, laminated columns and curved braces, and curved or straight rafters are used.

Hetzer construction was first used in Germany where it attained considerable popularity before the World War. War conditions, however, resulted in a shortage of casein for use as glue and stimulated the invention of mechanical devices that very greatly increase the efficiency of wood in jointed or framed structures such as trusses. Building with these devices has largely superseded Hetzer construction in Germany because of less cost in structures in which the appearance is acceptable. A further factor has perhaps been the expiration of patents on Hetzer construction, whereas many of the mechanical fasteners are protected by patents, and royalties may be an incentive to their promotion.

Hetzer construction was introduced in Switzerland about 1909 and and has since continued popular in that country. Numerous builders are skilled in its use. A probable reason for its success in competition with other types is a scarcity of timber of suitable size for framed construction. Structures with Hetzer members are common in all parts of Switzerland.

The use of Hetzer construction in Denmark began about 1913 with its introduction by a builder in Copenhagen. Structures were built by this system as recently as 1929 but it is said not to be economical at the present time in comparison with other types.

A plant was built in southern Norway in 1918 and a company formed to operate under the original Hetzer patents. A catalog issued the following year stated that this concern had furnished Hetzer members for structures with some 2½ acres of roof area. The firm continued in business for some years until the burning of its factory and other misfortunes caused its discontinuance.

A company with a plant on the bank of the Götha Canal at Töreboda, Sweden, began operation under Hetzer patents in 1919 and has continued in business. It appears to enjoy a monopoly on this type of construction in Sweden. This is perhaps due to the advantage of having pioneered under the original patents. Certain modifications made in the original construction may afford continued patent protection. These modifications which were introduced several years ago led to "Töreboda construction" as the name of the Swedish product. Water transportation to the factory door for materials and from it for the finished products is perhaps a factor in the company's success. Their product has, however, been used in many parts of Sweden besides those reached by waterways.

METHODS OF MANUFACTURE

Hetzer members are almost exclusively made in shops, manufacture at the site apparently not having been practiced.

GENERAL PRACTICE

Elsewhere than in Sweden the manufacturing practice is much the same. It consists of spreading boards with glue, assembling a number of boards in order after spreading the glue, and then clamping the group against a convex form. Forms are continuous. The usual clamping device consists of two flat steel rods rounded for a short distance from one end to pass through holes in a clamp plate and

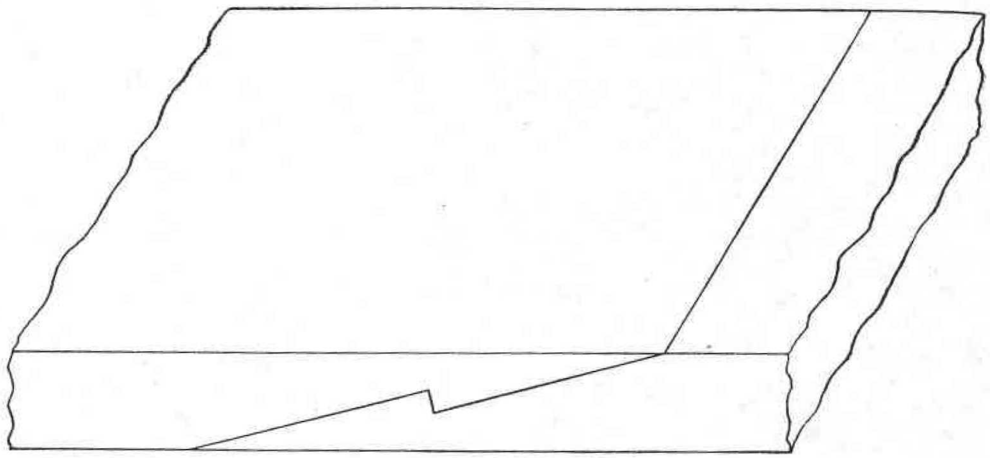

FIGURE 53.—Hooked scarf joint.

threaded to receive nuts. Rectangular holes through the flat part of the rods permit the placement of cross bars according to the depth of the member being clamped. Pressure is produced by tightening the nuts. Laminations in the outer portions of a member are usually full length, being sawed when necessary from specially cut logs. Those in the central portion are often of two or more pieces simply squared and placed end to end. Coniferous woods are used principally. Boards that have only sound knots of comparatively small size are preferred and resinous material is avoided. The lumber is usually dried, often in kilns, to a moisture content said to be between 12 and 20 percent, the degree of dryness depending on the builder's preference. All material is planed on the surfaces to be glued.

Members are made in no greater lengths than can be handled as a single piece. Thus beams, two-hinged arches, and the halves of three-hinged arches are in a single length.

SWEDISH PRACTICE

At the Swedish plant the selection, drying, and surfacing of material are done in the same way as elsewhere but special long boards are not used for outer laminations. Procedure in assembly differs from that practiced elsewhere.

Each part-length lamination is provided with a hooked scarf (fig. 53) at each end formed by passing it sideways under a cutter head equipped with properly shaped knives and revolving about an axis parallel to the length of the piece. The cutting is thus across the grain of the wood. A properly shaped "fence," or guide, follows the board and prevents fraying or splintering as the knife leaves the cut.

In assembling, the first lamination is bent and nailed to the convex side of a skeleton form. The second is then spread with glue on one face, placed against the first and nailed to it, the nailing following the bending closely. This is repeated until the desired number of laminations have been placed. The assembly is then, after placing clamps of the type previously described, removed from the form. The scarfs at the ends of part-length laminations are glued and nailed as assembly proceeds, the stepped, or hooked, form permitting the two parts to be quickly and accurately positioned so that they neither stand apart nor overlap too far. The hooked scarf joints obviously have less tensile strength than the pieces joined. However, when accurately made, they are superior in both tension and compression to butt joints and have the further advantage that each piece can be bent all the way to its end to make intimate contact with adjacent laminations, thus avoiding such openings in longitudinal joints between laminations as may result with butt joints in laminations (fig. 22).

The product of the firm at Töreboda is predominantly deep members with I-form cross section. Webs are fabricated as just described after which wider laminations to form the flanges are added to the edges of the web. Elsewhere stiffeners for the webs of members with I-sections are placed with their grain radial to the member and hence at right angles to the grain of laminations in the web. In Töreboda practice, stiffeners are short sections of assemblies made up in the same way as the web and have their grain parallel to that in the web. With this arrangement, high shearing stress between web and stiffener is avoided if swelling or shrinkage occurs but the stiffeners are less effective in their primary function of reinforcing the webs against buckling.

Some of the Töreboda constructions are designed as two-hinged or hingeless arches, the ribs being made in two or more sections for convenience in transportation. The sections are made with square ends and are provided with a wooden splice plate on each side of the web and metal splice plates on top and bottom of each flange. A typical Töreboda arch rib of 125-foot span is made in three sections and one of 154-foot span is made in four sections.

Bending by the procedure used in the Swedish plant probably cannot be done if the curvature is severe. The foregoing two-arch ribs have lamella thicknesses of 1.10 and 1.25 inches combined with radii of curvature of 42.65 and 58.25 feet, respectively, thus giving ratios of radius to thickness of 464 and 554.

DESIGN STRESSES

Stresses reported by European engineers as used in the design of glued laminated members are 1,280 to 1,710 pounds per square inch for bending and 115 to 170 pounds per square inch for shear. Stress induced in bending laminations is in general disregarded, no reduction in design stresses below those for straight members being made.

INSPECTION OF STRUCTURES

During the summer of 1936 the author visited in Germany, Switzerland, Denmark, Norway, and Sweden, a total of about 50 structures embodying glued laminated construction and varying in age up to 25 years. Careful inspection was made of the glued laminated members to determine the extent of decay, checking, opening of glued joints, deterioration of glue, or change of shape of the member. The following paragraphs relate to the most significant observations:

Footbridge over railway track, Adolfsberg, Sweden, built in 1923.—The deck is supported by four Töreboda beams of I-shaped cross section and 36-foot span. Water, retained in a joint at the junction of one of these beams with a stair string, has caused decay of the wood and complete deterioration of the glue near the junction. Otherwise the structure is in excellent condition.

Tower about 50 feet square, University of Zürich, Zürich, Switzerland, built in 1913.—Roof structure is of Hetzer construction (fig. 54).

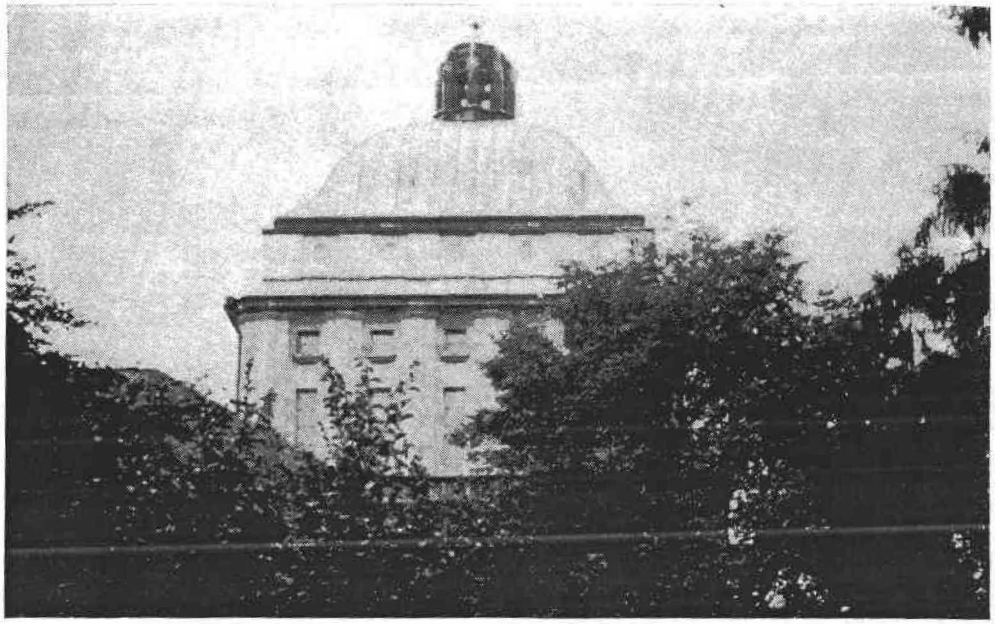

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FIGURE 54.—Tower of University of Zürich, Zürich, Switzerland. The roof framing is of glued laminated construction.

The hip rafters are 4¼ inches wide by 13½ inches deep, and have a total of 15 laminations; other rafters are 3¾ inches wide by 7 inches deep, and have 8 laminations. The surfacing of laminations is only moderately good. Continuous exposure to warm and dry atmosphere has resulted in only slight opening of the glued joints.

Hangar at military flying field, Dübendorf, near Zürich, Switzerland, built in 1918.—The building is 65 by 265 feet. Three-hinged Hetzer arches of 65-foot span spaced about 14¾ feet are used. The ribs are of uniform section (6¼ inches wide by 26½ inches deep) throughout their length, except at crown ends where they are reduced to about 5 by 20 inches, and pass through a vertical post whose lower end connects with the attic floor system. Three laminations at the top and bottom of each rib are full length of the half arch. The attic of the building is now used as quarters for the personnel stationed at the field. The arch ribs show considerable checking and some openings in glued joints, but none of sufficient extent to be serious.

Station platform roof structure at Trondhjem, Norway, built in 1921.—This structure (figs. 11 and 55), has 17 bents spaced about 25 feet.

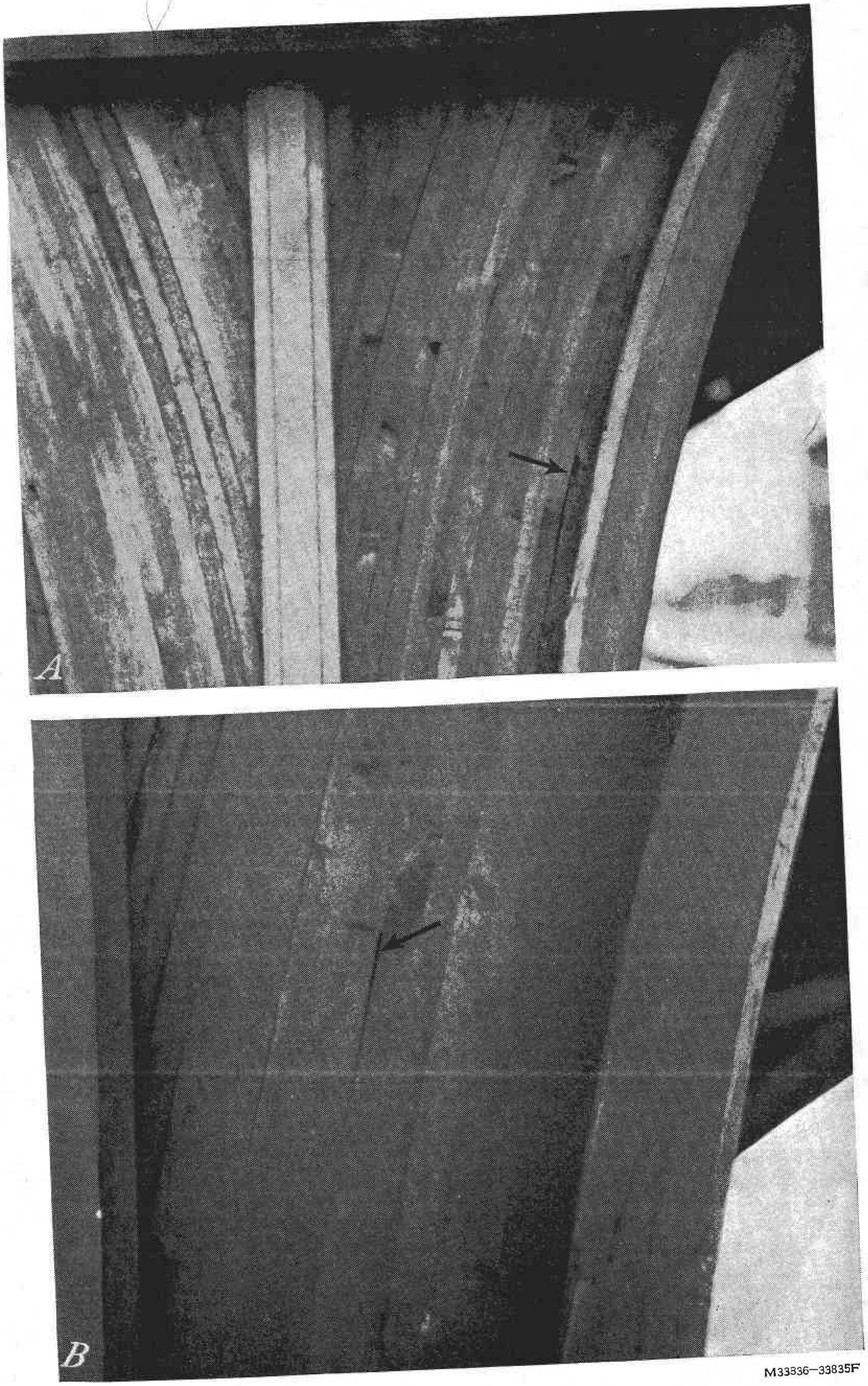

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FIGURE 55.—Outer face of end bent railway station platform roof, Trondhjem, Norway (same structure as shown in fig. 11): *A*, slight opening of longitudinal joints; *B*, imperfect longitudinal joint near butt joint in lamination.

The joints were slightly open in outer face of end bent as indicated in figure 55, *A*. Such opening of the joints is typical of many structures of this type. Note the open longitudinal joint adjacent to butt joint between parts of a lamination in figure 55, *B*.

Locomotive barns of Federal Railways at Bern, Switzerland, built in 1912.—Tapered Hetzer three-hinged arches carry the roof over four halls varying in width from 66 to 80 feet. The ribs are of rectangular cross section, 8½ inches wide with radial depths of about 18½, 40, and 16 inches at base, knee, and roof peak, respectively. The spacing of the arches is 16 feet. The building was used for about 10 years for

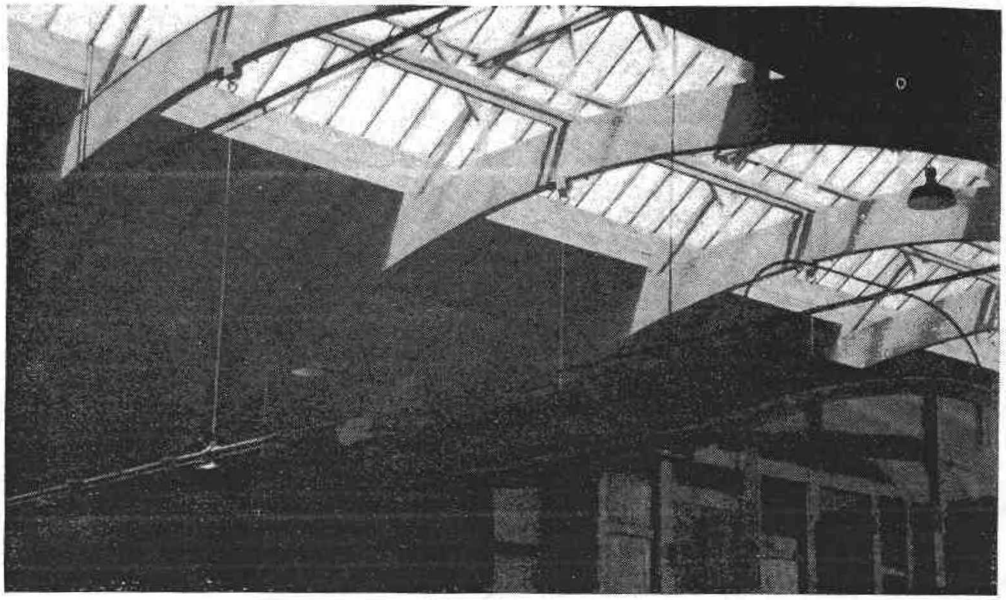

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FIGURE 56.—Glued laminated arches supporting roof of car-repair shop of Lötscherberg Railway at Böningen, Switzerland. Span 102 feet, arches about 6 inches wide by 20 inches deep.

steam engines, subsequently it has housed electric locomotives. Originally the bases of the arches rested on concrete piers a few inches above the floor level. Decay began some years ago at the intermediate piers and about 4 feet of the lower ends of arch ribs were cut off and replaced by concrete pedestals. Several of the arches show severe checking at the knee section. The engineer under whose supervision the building was designed and erected attributed this checking to less care in seasoning material than is now practiced. Severe checking obtains also at points along the roof near the sheet-metal ducts formerly used to conduct smoke and hot gases.

Locomotive barn, Stuttgart, Germany, built in 1913.—The span of this barn is about 80 feet. Hetzer three-hinged arches, about 31 inches deep at foundation and 43 inches deep at knee, are employed. The members are in excellent condition with no serious checking and no evidence of failure of glued joints.

Car-repair shop of Lötscherberg Railway, Böningen, near Interlaken, Switzerland, built in 1915.—Hetzer arches with tie rods are used in this structure (fig. 56). The span is 102 feet. The sections of the arches are about 6 by 30 inches. Some checking occurs in parts of the arches that are exposed through skylights to the sun.

Growing house for plants, St. Gallen, Switzerland, built in 1913.—Three-hinged arches of about 30-foot span resting on concrete piers about 2 feet high support the structure (fig. 57). The arch ribs exhibit no sign of checking, opening of glued joints, or other deterioration although conditions within the building are at times warm and very humid.

Factory building, Copenhagen, Denmark, built in 1913.—Hetzer arches carry the roof (fig. 58) of the building over masonry walls.

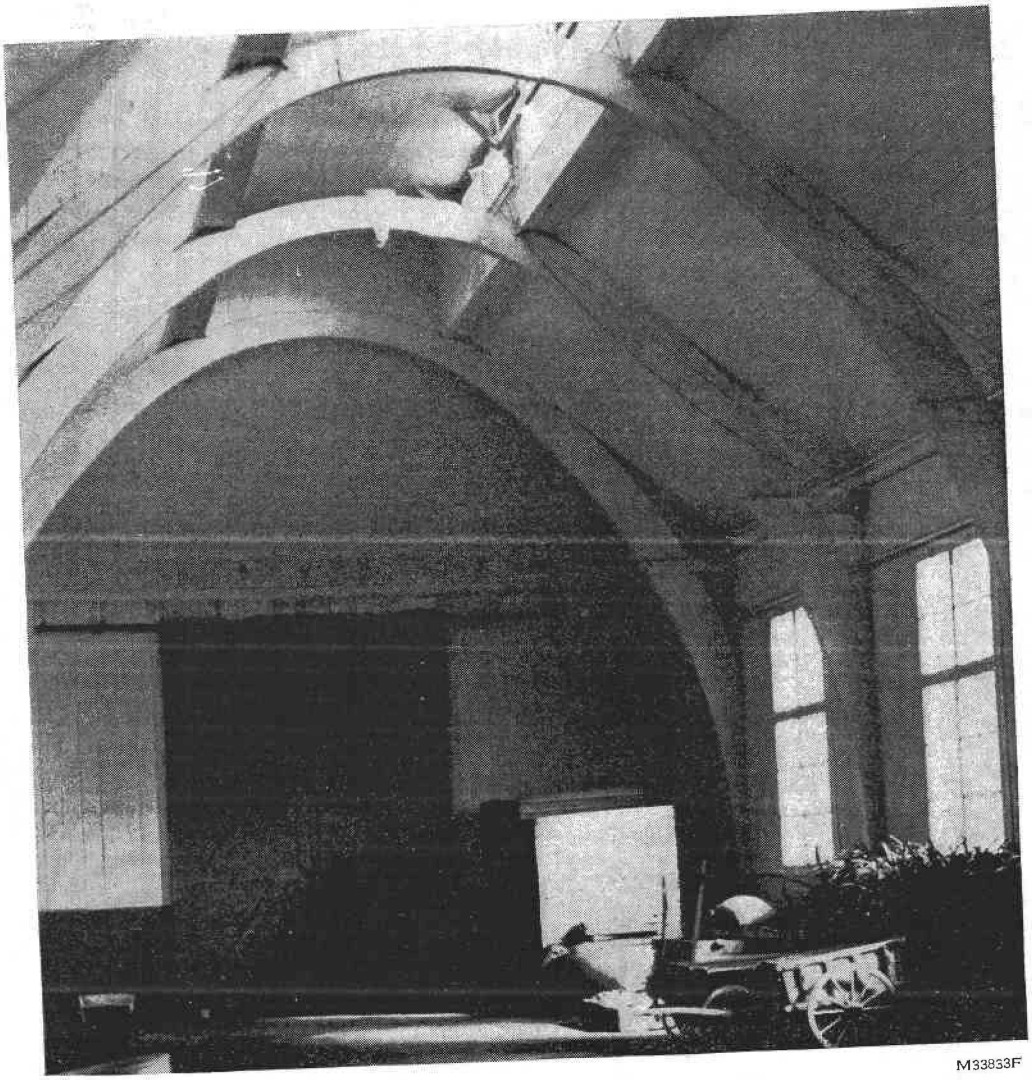

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FIGURE 57.—Growing house for plants for city gardens, St. Gallen, Switzerland.

The span is 60 feet. Because of a strike during erection these arches stood unprotected during wet weather from February to April, and the following summer was hot and dry. The arches are in good condition with no serious checking or opening of glued joints.

Footbridge over Karthausstrasse, Zürich, Switzerland, built in 1928.—The span of this bridge is about 26 feet (fig. 59). It was designed as a two-hinged arch, but it has no actual hinges. The ribs bear at the ends against an inclined face of a stone abutment which has a steel plate, slightly smaller than the cross section of arch, interposed. There was no visible deterioration at the time of inspection.

Bridge for vehicular traffic across canal, Copenhagen, Denmark, built in 1929.—The span of this bridge is 52 feet (fig. 60). It is carried by four full-length Hetzer arch ribs. A sheet of roofing paper on

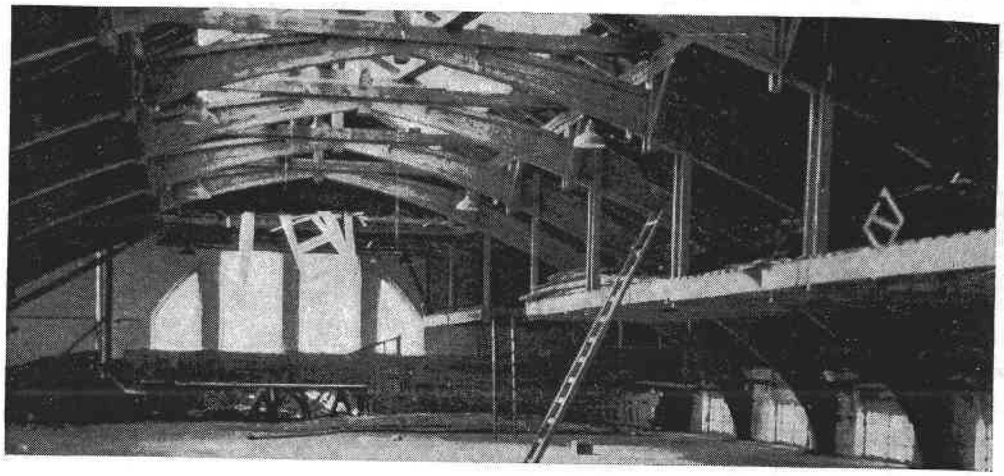

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FIGURE 58.—Hetzer arches carrying roof over masonry-walled factory building, Copenhagen, Denmark.

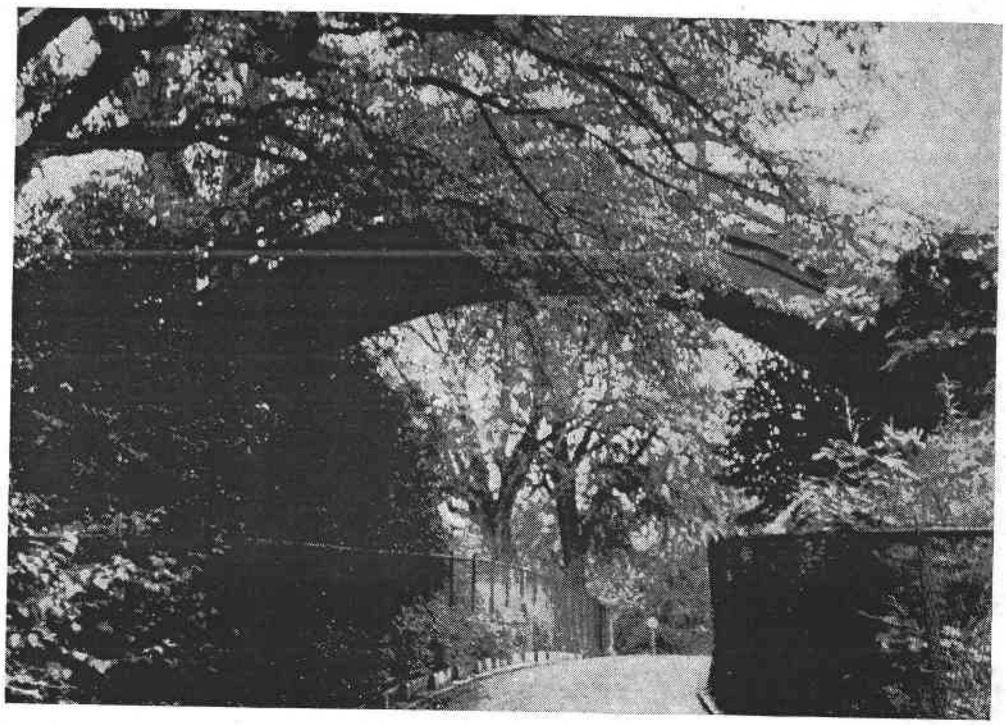

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FIGURE 59.—Footbridge over Karthausstrasse, Zürich, Switzerland.

top of each rib forms a roof extending about an inch each side of rib. A bent timber about 2 inches thick resting on this paper affords bearing for short spandrel columns supporting the bridge deck. The structure has been kept well painted and the arches show no sign of deterioration or of opening of glued joints.

DURABILITY OF GLUED LAMINATED CONSTRUCTION

The observations related in the preceding paragraphs point to long life for members laminated with casein glue when used in buildings in which normal atmospheric conditions prevail. Lack of examples of members that have failed or have seriously deteriorated under such exposure during the third of a century of the history of this type of construction precludes any but optimistic estimates

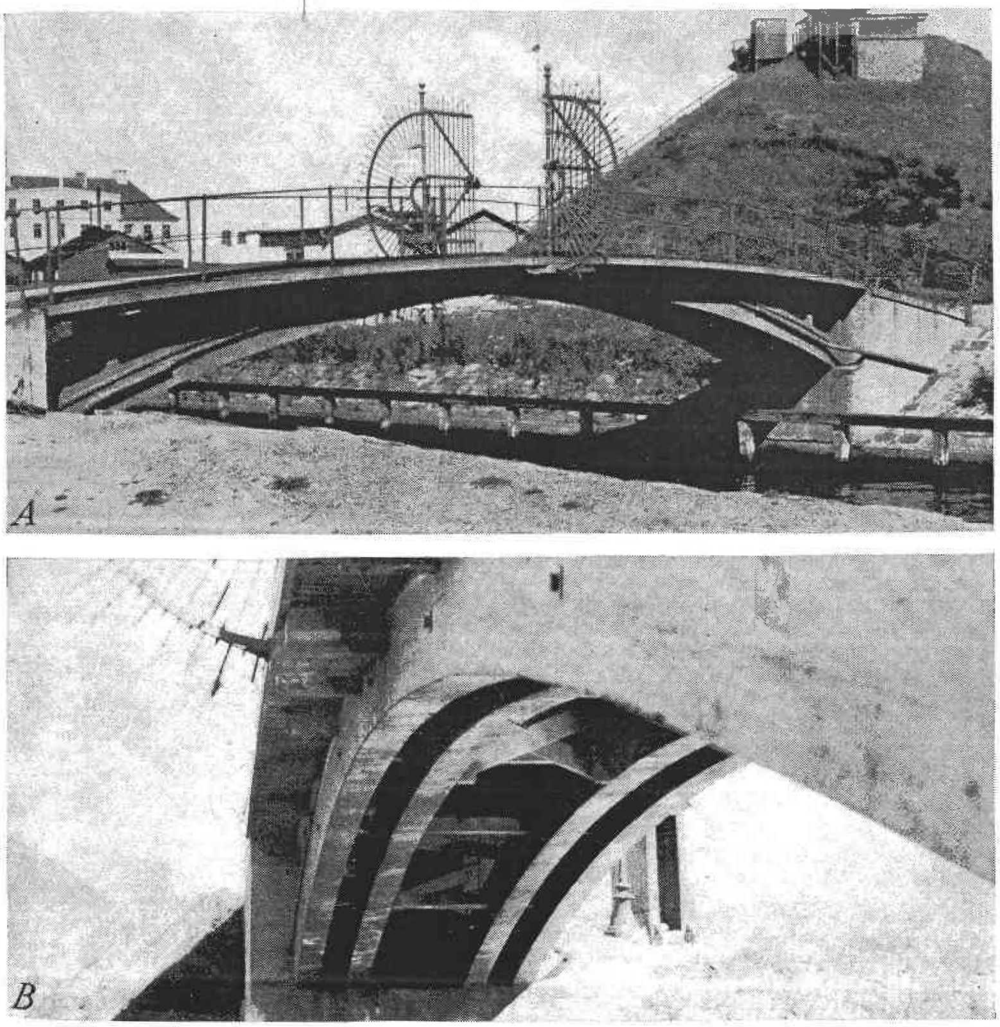

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FIGURE 60.—Bridge over canal, Copenhagen, Denmark: *A*, side view; *B*, under view.

of length of life and permanence. From experience to date it seems safe to assume that casein-glued laminated construction will last as long as solid wooden members of any but the more durable species or of preservatively treated material. The characteristics of casein glue render it unsuited for use in members in contact with damp earth or where the moisture content of the wood may repeatedly exceed 20 percent, such as in greenhouses, conservatories, or pulp mills.

The observed good condition of locomotive storage and repair buildings and other railway structures confirms the claim that casein-glued laminated construction is not attacked by fumes from coalfired locomotives. The reported good record in chemical factories in Switzerland over a period of years is furthermore indicative of excellent resistance to chemical fumes.

Observation of railway-passenger platform structures and the soundness of two bridges that had been in service for 7 and 8 years indicate that excellent performance may be expected from casein-glued laminated members in outdoor structures, provided they are built so that drainage and ventilation will cause the rapid run-off and evaporation of moisture. Observed decay of wood and deterioration of glue adjacent to a poorly drained and poorly ventilated joint after only 13 years service of the structure emphasizes the need for careful design of details.

Availability of casein glues of fully as great durability as those used in some of the European structures together with superior practice in conditioning material and carrying out gluing operations are factors that should add to confidence in American-built casein-glued laminated construction.

Glues made from synthetic resins and urea are much more resistant to moisture and to fungus attack than are casein glues. These types are being increasingly used in the manufacture of plywood. For the most part, high temperatures are required for setting them and they are consequently not, in general, economically usable for building up large members. Further developments that will render them applicable are to be expected. Such developments together with advances in methods of excluding moisture and in gluing material treated for resistance to fire, insect, and fungus attack may be expected to increase the durability of glued laminated construction and to extend the field of its applicability.

DESIGN AND ANALYSIS OF ARCHES

The purpose of this section is to give a simplified outline of the theory of the arch as it relates to the design and analysis of glued laminated arches.

Design of an arch usually involves "cut-and-try" procedure to some extent because the moments and forces on it cannot be computed until the position of the axis has been determined and this cannot be decided until some of the other dimensions are known. For example, the Laboratory service building was specified to have an external width of 46 feet. The span of the three-hinged arch could not be known until the dimensions of the rib at the foundation were decided and the required dimensions depend in turn on the span. Also the reaction and moments in a two-hinged arch cannot be computed until the size of cross section and position of the axis is known. Hence it is necessary in designing a two-hinged arch to assume the position of the axis, estimate the necessary sizes of cross section, check the stresses and, if they are found to be unsatisfactory, to revise the design.

Two-hinged and three-hinged arches are the types to which glued laminated construction is best adapted. Because of its simpler structural action, the latter type is discussed first.

THREE-HINGED ARCHES

The three-hinged arch consists of two parts pin-connected or hinged to their supports and to each other. In the more common symmetrical case, the two supports are at the same level, and the parts are alike and meet at the high point or crown of the arch.

The three-hinged arch is statically determinate, that is, its abutment reactions under any loading can be determined from the three basic equations of static equilibrium of forces in a plane: (1) The sum of all vertical forces must be zero; (2) the sum of all horizontal forces must be zero; (3) the sum of the moments of all forces about any point must be zero. Application of these equations to the arch as a whole determines the vertical reactions and establishes the fact that horizontal reactions or thrusts caused by vertical loads are equal and oppositely directed. Determination of the magnitude of the horizontal thrust requires application of the equations to parts of the arch separately. The parts selected must be free from moment at either end, otherwise additional unknowns that cannot be determined are introduced. Inasmuch as no moment can exist at a hinge, the intermediate or crown hinge qualifies as the point of separation. Applying the equations of equilibrium to all forces acting on the parts to the right and left of this hinge determines the horizontal thrust.

Design and analysis of a three-hinged arch are facilitated by a consideration of the equilibrium polygons for the specified loadings. The equilibrium curve or polygon is the moment diagram for the specified loading drawn in such a position and to such a scale that it passes through the hinges. Furthermore, it is the "linear arch" that under this loading would be subjected only to longitudinal forces in any of its parts, that is, no bending moment would exist in any part of such an arch.

THREE-HINGED ARCH UNDER VERTICAL LOAD

Equilibrium polygons, horizontal thrust, and vertical reactions for three arrangements of vertical loading are shown in figure 61 *A*, *B*, and *C*.

The following properties and uses of the equilibrium polygon for vertical loads are readily demonstrated:

1. The bending moment at any point on the arch axis equals Z, the vertical distance from that point to the equilibrium polygon, multiplied by H, the horizontal thrust of the arch, namely,

$$M=H\times Z$$

If the arch axis is below the equilibrium polygon, the moment is positive (such as to cause compression in the upper and tension in the lower sides of the arch rib), and conversely. Thus in figure 61, *A* and *C* the moment is positive between O and S and negative between L and O and between S and R. In figure 61, *B* it is negative throughout. It is zero, of course, at the hinges L, O, and R and at points where, as at S in figure 61, *A* and *C*, the equilibrium polygon crosses the arch axis.

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2. The thrust parallel to the arch axis, and the shear at right angles to it, are the components in those directions of a force whose horizontal component is H, and whose direction is parallel to the direction of the

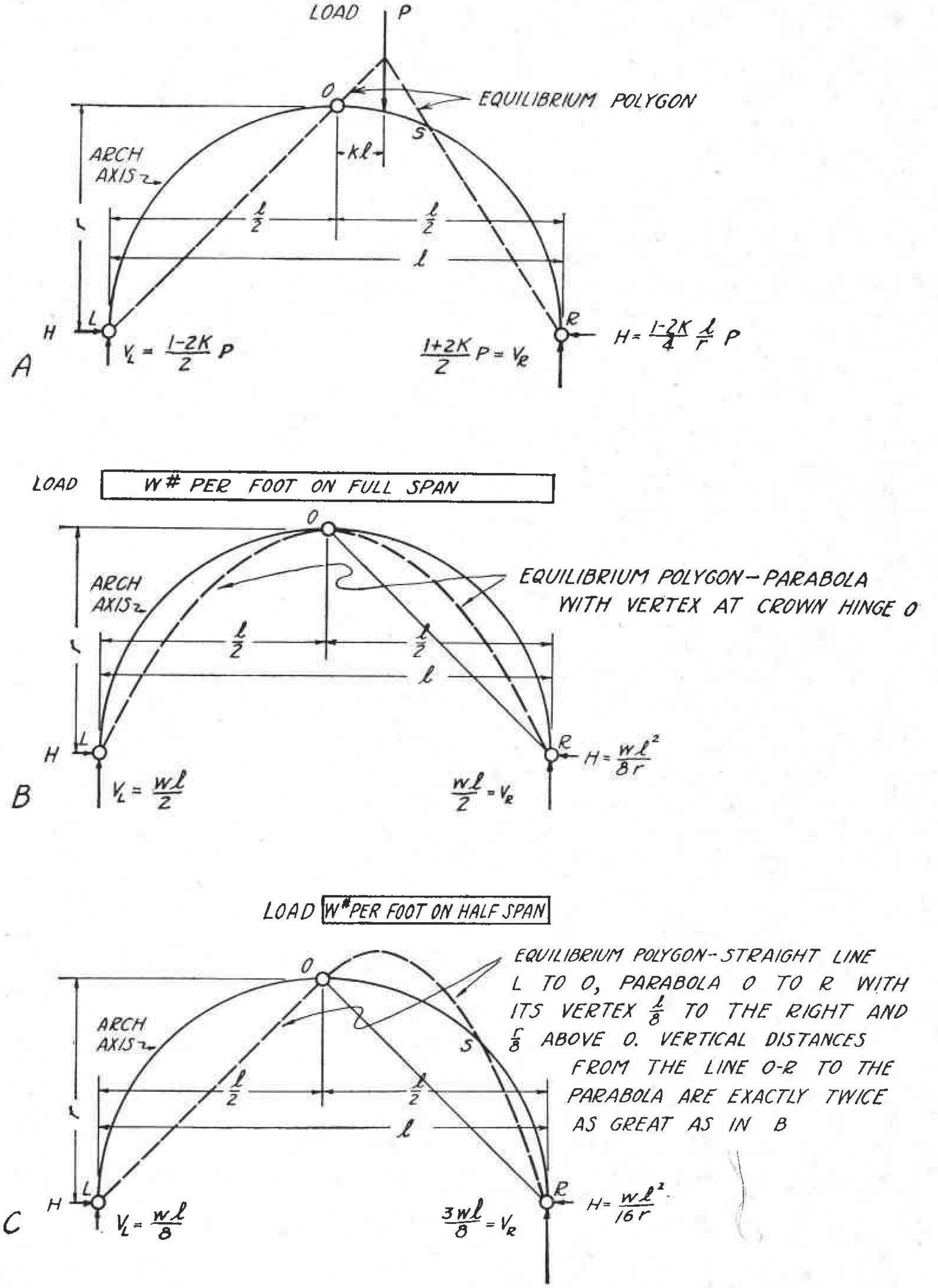

FIGURE 61.—Equilibrium polygons, horizontal thrust, and vertical reactions of three-hinged arch under vertical load: *A*, Single concentrated load; *B*, load uniformly distributed over full span; *C*, load uniformly distributed over one-half span.

equilibrium polygon at a point vertically in line with the point in the arch axis that is under consideration.

In accordance with this relation the thrust and the shear at any point is found by simple graphics, as illustrated for point U in figure 62.

In the diagrams at the right in figure 62, $a-b$ represents H, the horizontal thrust of the arch, to scale and $b-c$ is perpendicular to $a-b$. A line through a parallel to MN (which represents the direction of the equilibrium polygon at U' in the same vertical as U) intersects $b-c$ in d. The direction of the arch axis at U is represented by $a-e$, which is parallel to QW. Then $d-f$ is drawn perpendicular to $a-e$, and $a-f$ and $f-d$ represent, in amount and direction, the thrust and shear acting at U on the portion of the arch to the right of U.

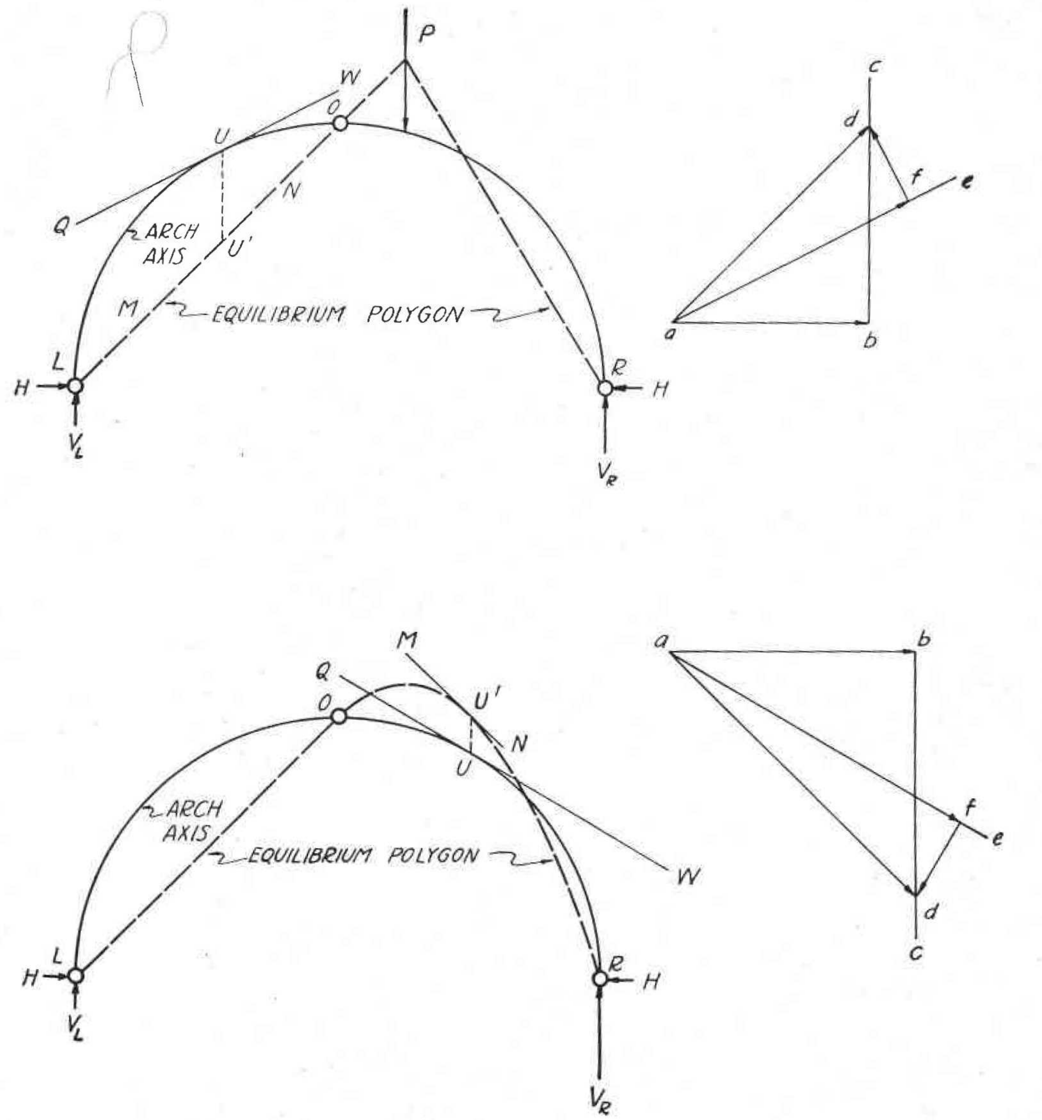

FIGURE 62.—Graphical method for finding thrust and shear at a point in the axis of a three-hinged arch.

Consideration of such diagrams as figures 61 and 62 and of the properties of the equilibrium polygon as just stated enables one to readily visualize the action of a three-hinged arch under any condition of loading.

Figure 61, B and C illustrates equilibrium polygons for load uniformly distributed on the full arch and on the right half, respectively. Load that is uniformly distributed on a roof is not ordinarily carried directly by the arch but is transmitted to it at panel points by purlins or other members. It is common in the design of roof arches to investigate stresses for load uniformly distributed over the entire span and over one-half of the span only. For either of these conditions, the equilibrium polygon consists of straight-line segments with its vertices[9] at points where verticals through the panel points intersect the parabola shown in figure 61, *B* or *C*. Thus the moments at panel points are exactly the same as if the loads were carried directly by the arch. The correct thrust and shear[10] at the left (or right) of a panel point can be found by using as the direction of such a line as *a—e* in figure 62, the slope of the parabola at a point half a panel length to the left (or right) instead of *MN*, the slope at a point vertically in line with

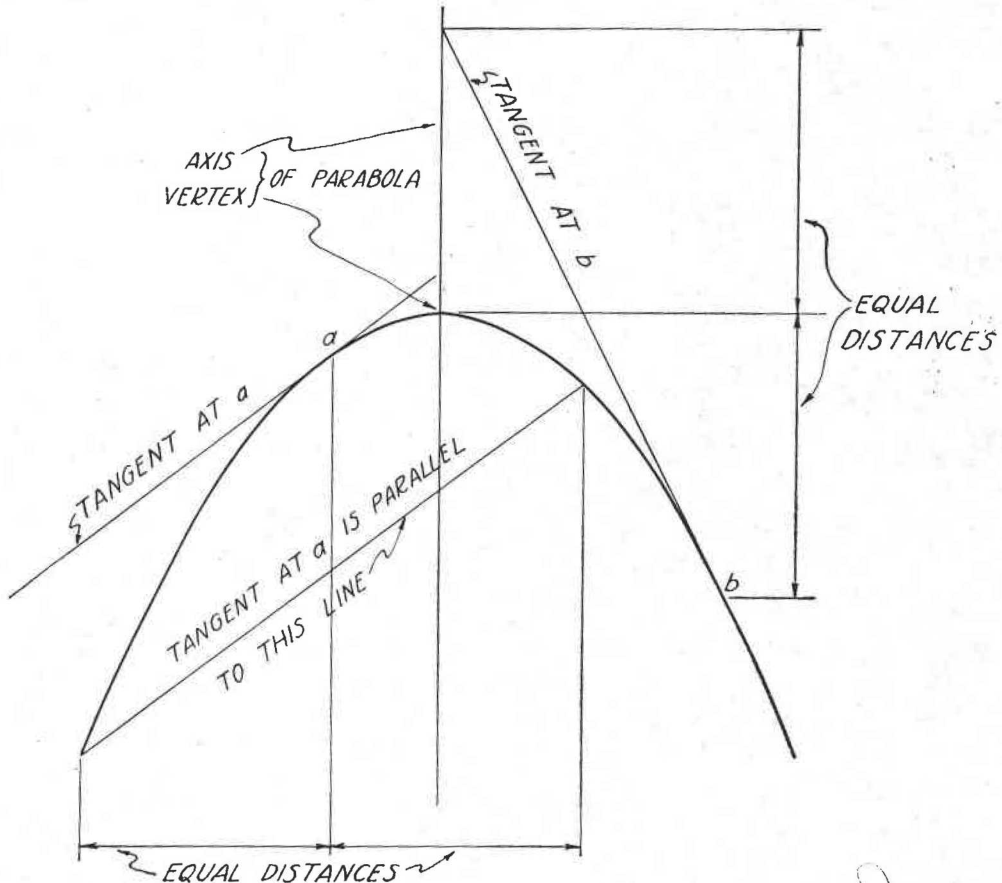

FIGURE 63.—Two methods of drawing a tangent at a specified point on a parabola.

the panel point. These relations make it feasible to use the equilibrium polygon for uniformly distributed load instead of that for the actual panel point loading and afford a basis for very simple analysis of stresses.

The determination of thrust and shear requires finding the tangent to parabolic parts of the equilibrium polygon at specified points. The axes of these parabolas being vertical and their vertices being known, tangents can be found by either of the two simple methods illustrated in figure 63.

3. In addition to the two properties of the equilibrium polygon as already discussed, there is a further important relation between the

[9] The fact that half a panel load is carried directly to the abutment, and thus has no effect on the arch itself, causes the equilibrium polygon to be inscribed in the parabola that is the equilibrium curve for uniform distribution of load. If the panel loads acted on the arch at the centers of panel widths, the equilibrium polygon would be circumscribed about this parabola.

[10] The values of thrust and shear will be different on the two sides of the panel point.

arch rib and the equilibrium polygon for vertical loads; under load the arch rib curves away from the equilibrium polygon for that load. Consequently the arch axis when under load is at critical points farther from the equilibrium polygon than in its unstrained position. This results in increased stress which is discussed later under the heading of secondary stresses.

PROPERTIES OF ARCH WITH PARABOLIC AXIS

As previously stated, if an arch is so built that its axis coincides with the equilibrium polygon or linear arch for a particular loading, this loading will cause only longitudinal stress in the arch rib and

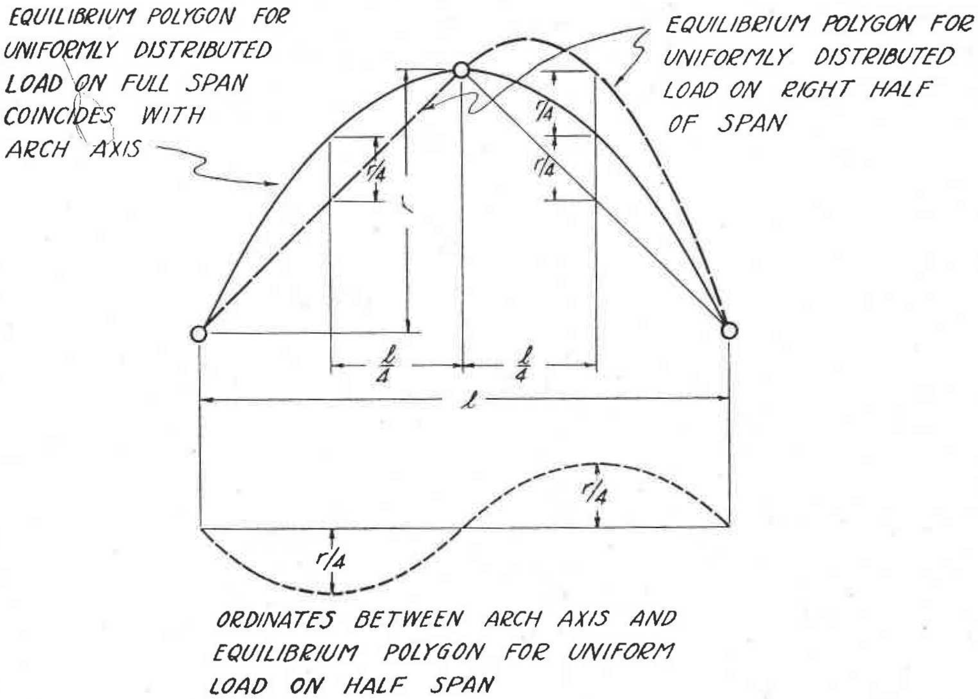

FIGURE 64.—Three-hinged parabolic arch with equilibrium polygon for uniformly distributed load on right half of span and diagram of ordinates between arch axis and equilibrium polygon.

there will be no bending moment or shear at any point. An arch with parabolic axis fulfills this condition for load uniformly distributed across the span.[11] This arch is unique also in that, under uniformly distributed load on half the span, the maximum bending moment is less than with any other form of axis. This may be realized from a consideration of figure 64. At any point to the right of the center the parabolic arch axis is just as far below the equilibrium polygon for uniformly distributed load on the right half as it is above it at a point the same distance to the left of the center. Consequently, if the form of the axis be changed, while keeping it symmetrical about the vertical center line, one of these distances, and consequently the moment, will be increased.

For the condition indicated in figure 64 the maximum vertical distance from the arch axis to the equilibrium polygon is at the quarter

[11] For load uniformly distributed along the axis, the linear arch or equilibrium polygon is a catenary curve.

point of the span where it is $r/4$. Since the thrust under half load is $\frac{wl^2}{16r}$, the moment is $wl^2/64$, or just one-eighth as great as the maximum bending moment in a beam under uniform load on the same span. This leads to the interesting fact that for the same bending stress the arch rib with parabolic axis will be just one-half as large in each cross-sectional dimension as would a simple beam on the same span.

A further feature, in which the arch with parabolic axis is unique, is that under uniformly distributed load over the full span or over the half span its behavior is very much the same whether it is three-hinged or two-hinged (a hinge at each abutment only). That it is the same under full-span loading is evident from the fact previously stated, that this loading causes only longitudinal thrust in the arch rib and there is no shear or bending moment at any point. Under the half loading the moments at points on opposite sides of the center of the three-hinged arch and equidistant from the center are numerically equal but of opposite sign. Consequently, the downward deflection of each point on the loaded side is exactly equal to the upward deflection of a point the same distance from the center on the other side and at the crown the tangent to the axis on the loaded side will turn through exactly the same angle as that on the other side. In other words, the two tangents are in the same line after loading just as they were before and as they would necessarily be after loading if the arch were continuous at the crown (no crown hinge). This reasoning is partially invalidated by the fact that the chord length of the half arch on the side without load shortens and that on the loaded side lengthens. Nevertheless the actions of the three-hinged and the two-hinged arch under uniformly distributed load on one side is so nearly the same that for such loading the two-hinged arch probably has little advantage over the three-hinged. Under other conditions, however, such as concentrated loads, rolling loads, or wind, the two-hinged arch may have considerable advantage.

THREE-HINGED ARCH WITH HORIZONTAL FORCES

Reactions and equilibrium polygons for a single horizontal force and for horizontal pressure uniformly distributed on the rise of the arch are shown in figure 65, *A* and *B*.

In partial contrast as well as in partial analogy to the condition for vertical loads, it is to be noted that, with horizontal forces, the two vertical reactions are equal and oppositely directed while the two horizontal reactions are unequal but in the same direction. In this case the moment at any point in the arch ring is equal to the vertical reaction multiplied by the horizontal distance from the arch axis to the equilibrium polygon. (Actually horizontal reactions could be used in this case with vertical distances; also the corresponding procedure for vertical loads as previously stated could be similarly reversed. However, the procedures stated here and in paragraph 1 under the discussion of vertical loads are simpler because they do not involve choice between two differing H's or V's.) Thus, referring to figure 65; for the single load, moment at $a=V$ multiplied by distance $a-b$, moment at $c=V$ multiplied by $c-d$, and moment at $e=V$ multiplied by $d-e$; while for the distributed load the moments at a and d are V multiplied by $a-b$ and $c-d$, respectively.

The second property of the equilibrium polygon or curve, as stated under the discussion of vertical loads, is also applicable to the case of horizontal forces with appropriate changes of wording that will be readily recognized.

For downward-acting vertical loads, all parts of the equilibrium polygon, considering it as a structure supporting the loads, are subject to longitudinal compression or thrust. With horizontal forces, on the other hand, parts of the equilibrium polygon, when it is similarly considered, are in tension and parts are in compression. Thus, for the

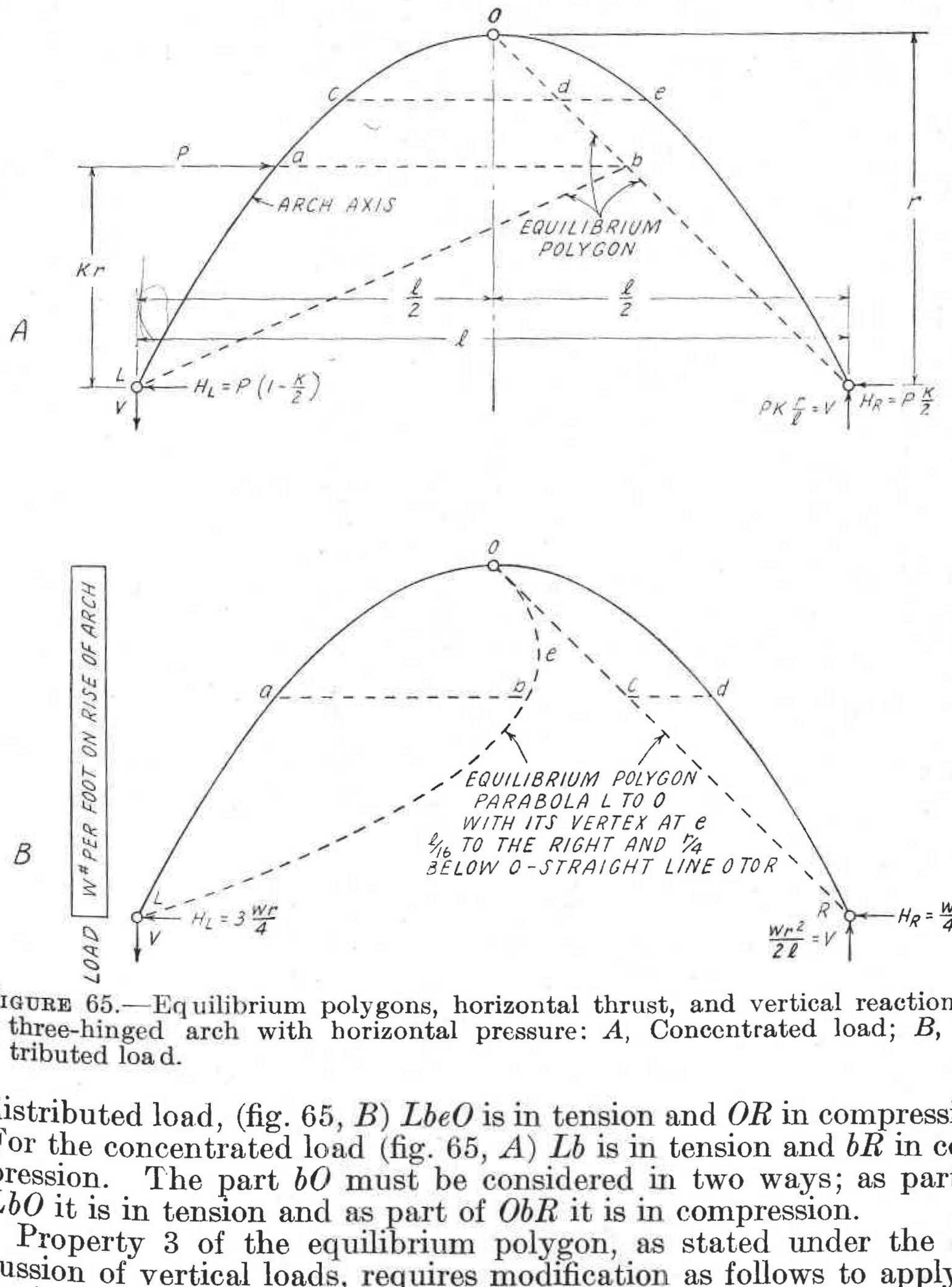

FIGURE 65.—Equilibrium polygons, horizontal thrust, and vertical reactions of three-hinged arch with horizontal pressure: *A*, Concentrated load; *B*, distributed load.

distributed load, (fig. 65, *B*) *LbeO* is in tension and *OR* in compression. For the concentrated load (fig. 65, *A*) *Lb* is in tension and *bR* in compression. The part *bO* must be considered in two ways; as part of *LbO* it is in tension and as part of *ObR* it is in compression.

Property 3 of the equilibrium polygon, as stated under the discussion of vertical loads, requires modification as follows to apply to horizontal forces:

The arch axis tends to curve away from the corresponding part of the equilibrium polygon if, when the equilibrium polygon is considered as a linear arch resisting the forces, that part is under compression, and to curve toward the corresponding part if under the same circumstances that part is in tension.

Usually the only horizontal pressure to which an arch will be subjected is that due to wind. Figure 65 (*B*) applies, as indicated, to force uniformly distributed vertically and, hence, might represent the condition resulting from wind pressure on a vertical wall that is braced against the arch at frequent intervals.

THREE-HINGED ARCH WITH INCLINED FORCES

Wind pressure on a curved or an inclined surface is ordinarily assumed to act perpendicular to the surface and, hence, inclined to the vertical and horizontal, and to vary with the inclination. A convenient procedure in such a case is to resolve the pressure into its vertical and horizontal components, find the effects of each, and then combine the results. Neither components will be uniformly distributed; hence, the equilibrium polygons for uniformly distributed load, as shown in figure 61, *B* and *C* for vertical load and in figure 65, *B* for horizontal load will not be directly applicable. They are, however, in general, illustrative of the types to be expected.

THREE-HINGED ARCH—GRAPHICAL METHODS

Graphical methods can often be effectively employed in arch analysis. The preceding discussion has included graphical methods for finding values of thrust and shear after the arch axis had been drawn and the equilibrium polygon plotted by analytical methods following the computation of the reactions. The reactions and the position of the equilibrium polygon can often be found more simply by graphics. The procedure is illustrated by figure 66, which shows the axis of an arch carrying five vertical loads with the magnitudes and positions indicated. An unsymmetrical arch with abutment hinges at different levels is used in this illustration in order to show the generality of the method.

The first step is to lay off the loads to a convenient scale and in order along a vertical line. A trial pole point as H' is taken. From L a line is drawn parallel to $H'-0$ to intersect the line of action of P_1 in a'. From a' a line parallel to $H'-1$ is drawn intersecting the line of action of P_2 in b'. This process is continued until the final line parallel to $H'-5$ intersects the vertical through R in R'. $L-R'$ is then drawn intersecting the vertical through the intermediate hinge at S. A line from H' parallel to $L-R'$ intersects the vertical load line in x. Then $o-x$ and $x-5$ represent to scale R_L and R_R the vertical reactions at L and R, respectively, and $La'b'c'O'd'e'R'$ is the trial equilibrium polygon. The desired or proper equilibrium polygon, however, must pass through L, O, and R. To make it do so a new position for the pole point is required. Use of a pole point anywhere to the left of the load line and on a line drawn through x parallel to $L-R$ will bring the equilibrium polygon through L and R. To make it pass through O the horizontal distance from H to the load line is so taken that when multiplied by r, the rise of the arch (or the height of the intermediate hinge above $L-R$, the line joining the abutment hinges) the product equals the horizontal distance from H' to the load line multiplied by the distance from O' to S. The distance from H to the load line then represents the horizontal thrust of the arch to the same scale to which the loads were laid off on the line *o—5*. Using *H* as the pole point, the same procedure as in drawing the trial equilibrium polygon is followed, and *LabcOdeR* the proper equilibrium polygon for the arch is located.

The further procedure in finding moments, thrust, and shears is as previously outlined.

TWO-HINGED ARCH

The two-hinged arch is continuous between its abutments to which it is pin-connected or hinged. When abutments are at the same level the vertical reactions caused by vertical loads are found and the equality of horizontal thrusts established by applying the equations of equilibrium to the arch as a whole just as with a three-hinged arch. The procedure used for finding the magnitude of the horizontal thrust

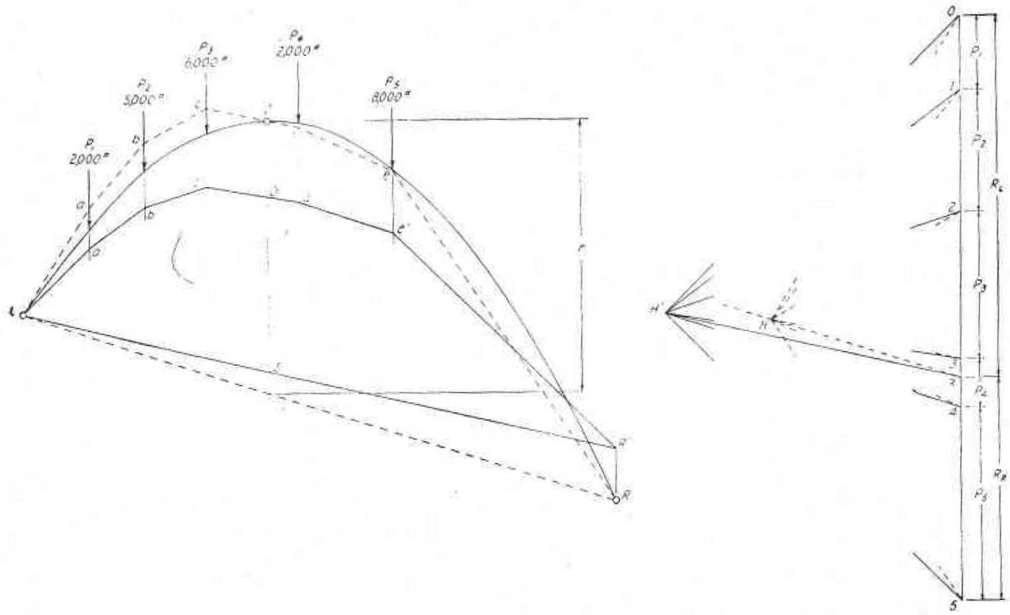

FIGURE 66.—Graphical method for finding equilibrium polygon, horizontal thrust, and vertical reactions of three-hinged arch under vertical loads.

of the three-hinged arch is not available, however, because there is no point at which the arch can be separated into parts without introducing the moment, of unknown magnitude and direction, that may exist at the point of separation. Because of this the two-hinged arch is said to be statically indeterminate. Consideration of the elastic behavior of the arch rib makes it possible to determine the horizontal thrust. It is necessary accordingly that the position of the axis and any variation in cross-sectional dimensions of the arch rib be known.

The two-hinged arch is likely to be used with comparatively small rise so that the effect of such horizontal forces as result from wind action will ordinarily be comparatively small. Consequently the further discussion relates to vertical load only.

INTERSECTION LOCUS

Computations of a two-hinged arch are facilitated by the use of the intersection locus, which is the path traced by the peak of the equilibrium polygon for a single load as the load moves across the arch span. Referring again to the three-hinged arch; any equilibrium polygon passes through the hinges and that for a single vertical load is, as shown by figure 61, *A*, formed by drawing a line from the abutment hinge farther from the load through the crown hinge to an intersection with the vertical through the load and connecting this intersection point to the other abutment hinge. As the load passes across the span, the intersection point or peak of the equilibrium polygon traces as an intersection locus the V-shaped figure that would be formed by prolonging lines *LO* and *RO* (fig. 61, *A*). Omitting the center hinge

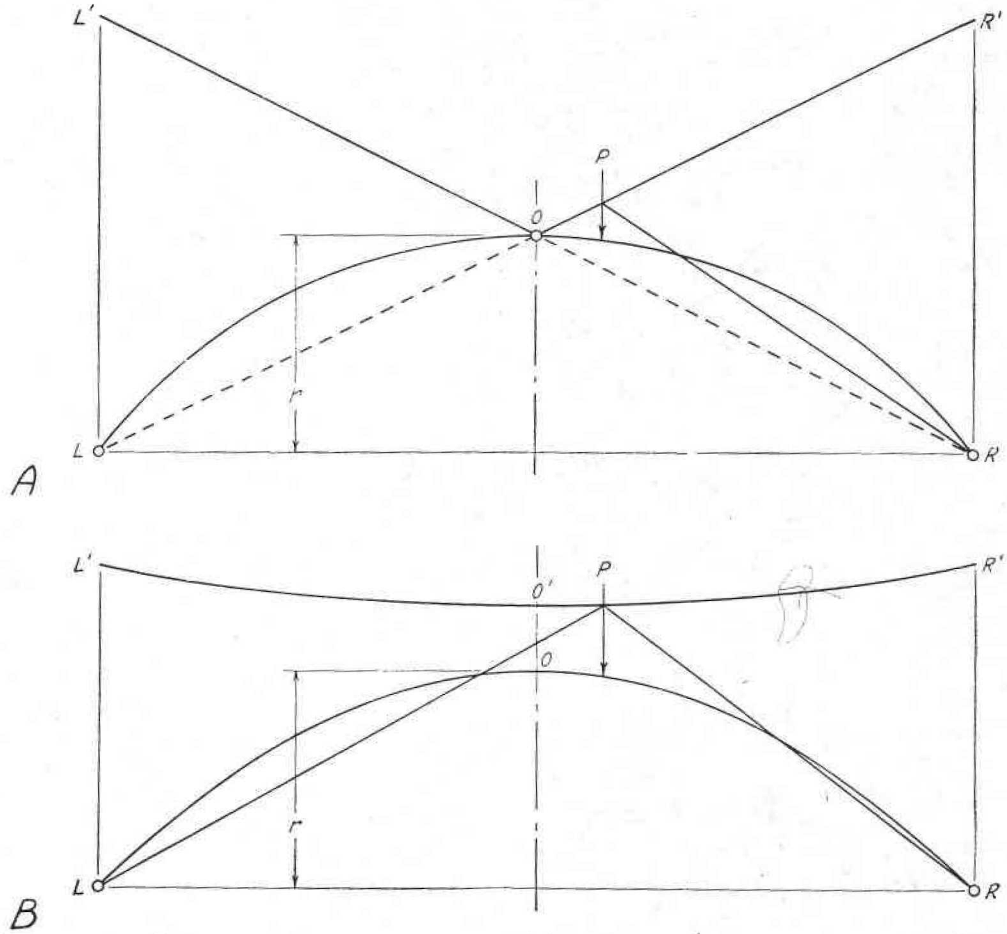

FIGURE 67.—Intersection loci for a three-hinged and for a two-hinged arch: *A*, Three-hinged arch with constant or variable cross section and any form of axis—$L'OR'$ is the intersection locus; *B*, two-hinged arch with constant cross section and parabolic axis—$L'O'R'$ is the intersection locus.

of a three-hinged arch and making the rib continuous converts it into a two-hinged arch and the intersection locus becomes a continuous curve extending across the span and lying entirely above the arch axis.

Intersection loci for a three-hinged and for a two-hinged arch are shown in figure 67.

It will be readily recognized that, when the intersection locus has been plotted in proper relation to the arch axis, the reactions of the arch for any single load can be readily found, since, as indicated by figure 67, the equilibrium polygon consists of the two straight lines connecting the abutment hinges to the point where the intersection locus crosses the vertical through the load. Reactions for single loads can then be combined to get values for any desired combination or system of loads. When the equation of the intersection locus is simple it can be incorporated in equations for reactions, moments, and the like, and these quantities computed algebraically.

Equations for the intersection loci of two-hinged arches of several types follow. In these equations r is the rise of the arch and y_o is the height of the intersection locus above the hinges at a point whose horizontal distance from the center is k. The half span is the unit of measurement for y_o, k, and r.

A simplifying assumption often used in the design of arches is that the cross section of the arch rib varies in such a manner that $I = I_o \sec \alpha$, where I and α are the moment of inertia of the cross section and the angle of inclination of the arch axis at any point and I_o is the moment of inertia at the crown. Masonry and metal arches are often so built as to conform approximately to this assumption, which leads to the following as the equation of the intersection locus of a two-hinged arch with parabolic axis:

$$y_o = \frac{1.28r}{1.00 - 0.20k^2} \tag{1}$$

Ribs with cross sections constant are in some instances more feasible for laminated wooden arches and the following equations are for arch ribs with constant cross section.

Two-hinged arch with parabolic axis:

$$y_o = (1-k^2)\frac{\dfrac{128r^5 - 40r^3 - 3r}{24}\sqrt{1+4r^2} + \dfrac{128r^4 + 16r^2 + 1}{16}\sinh^{-1} 2r}{\dfrac{1}{15}\Big[(56r^4 - 47r^2 - 4)\sqrt{1+4r^2} + (24k^4r^4 - 80k^2r^4 + 7k^2r^2 + 40r^2 + 4)\sqrt{1+4k^2r^2}\Big] + \dfrac{r}{2}(1+16r^2)(\sinh^{-1} 2r - k\sinh^{-1} 2kr)} \tag{2}$$

This equation is represented with reasonable accuracy by the following empirical form:

$$y_o = \frac{1.30r}{1.00 - 0.13k^2} \tag{2a}$$

For a two-hinged arch with axis in the form of a circular arc and with cross section constant:

$$y_o = (1-k^2)\frac{(3 - 2r^2 + 3r^4)\sin^{-1}\dfrac{2r}{1+r^2} - 6r(1-r^2)}{4r(1-k^2) - 4r(1-r^2)\left(\sin^{-1}\dfrac{2r}{1+r^2} - k\sin^{-1}\dfrac{2kr}{1+r^2}\right) - 2(1-r^2)^2 + 2(1-r^2)\sqrt{(1+r^2)^2 - 4k^2r^2}} \tag{3}$$

When the arch is semicircular, that is, when $r=1$ (rise=half span) equation (3) reduces to

$$y_o = \sin^{-1} 1 = \pi/2$$

and the intersection locus is a horizontal line whose height above the hinges is $\pi/2$, or 1.57, times the half span.

Table 31 lists values of y_o as computed from equations (1), (2), (2a), and (3) for several values of r and k.

TABLE 31.—*Values for y_o as computed from equations (1), (2), (2a), and (3)*

r	Equation (1)			Equation (2)			Equation (2a)			Equation (3)		
	$k=0$	$k=\frac{1}{2}$	$k=1$	$k=0$	$k=\frac{1}{2}$	$k=1$	$k=0$	$k=\frac{1}{2}$	$k=1$	$k=0$	$k=\frac{1}{2}$	$k=1$
⅕	0.256	0.269	0.320	-------	-------	-------	0.260	0.269	0.299	0.248	0.260	0.307
¼	.320	.337	.400	0.321	0.337	0.397	.325	.336	.374	-------	-------	-------
½	.640	.674	.800	.646	.671	.782	.650	.672	.747	.674	.695	.795
1	1.280	1.347	1.600	1.317	1.330	1.504	1.300	1.344	1.494	1.571	1.571	1.571
2	2.560	2.694	3.200	2.714	2.625	2.855	2.600	2.687	2.989	-------	-------	-------

The equations for intersection loci as presented above do not take into consideration the shortening of the arch axis resulting from compression stress and are based on the assumption that the arch bears against unyielding abutments. Inasmuch as dimensions of arch ribs are likely to be determined by requirements for resisting bending moments, with consequent low stress in compression, neglect of the effect of compression shortening will probably seldom be serious. Any spreading of abutments or stretch of tie rods when these are used renders the formulas for intersection locus as given here somewhat inaccurate as may be observed in discussions of these phases in textbooks on arches. These equations also do not take into account stress due to thermal expansion, or contraction of the arch rib. Because of the low coefficient of thermal expansion of wood, such stresses also are unlikely to be serious.

When the arch axis is other than a simple curve, such as a parabolic or a circular arc, equations for intersection loci are difficult to derive and in such instances numerical integration or summarization of certain quantities are necessary to a solution of the two-hinged arch.

COMPARISON OF TWO-HINGED AND THREE-HINGED ARCHES

As was pointed out previously, primary stresses in a parabolic arch under uniformly distributed vertical load on the full span or on one-half the span are the same regardless of whether the arch is two-hinged or three-hinged. For the parabolic arch under other loadings and for arches with axis in the form of other curves under any arrangement of vertical loads, maximum moments are in general less for the two-hinged than for the three-hinged arch. For example, in a two-hinged arch, with parabolic axis and constant cross section under a movable vertical load of P, the maximum moment to be provided for is approximately 0.084 Pl, in which l equals the span, whereas in a three-hinged arch with the same form of axis, the maximum moment is about 0.096 Pl. For other forms of arch axis the differences are likely to be greater.

DEFORMATION AND SECONDARY STRESSES IN ARCHES

The primary stresses in an arch, that is, the stresses that would exist if the arch axis occupied exactly the same position after loading as before, can be determined by application of the principles previously outlined. However, loading changes the position of the axis, usually in such a way as to increase the stresses. Consequently in checking a design, estimates of the deformations and of the resulting additional stresses or secondary stress are desirable.

Under the action of bending moments, a portion of a member with curved axis, such as is illustrated in figure 68, will take a new shape. The dotted curve in this figure indicates the position of the axis after bending has taken place and after it has been moved so that one end (A) is in its original position and the axis at that end is in its original direction. The other end, which was originally at B, is now in the

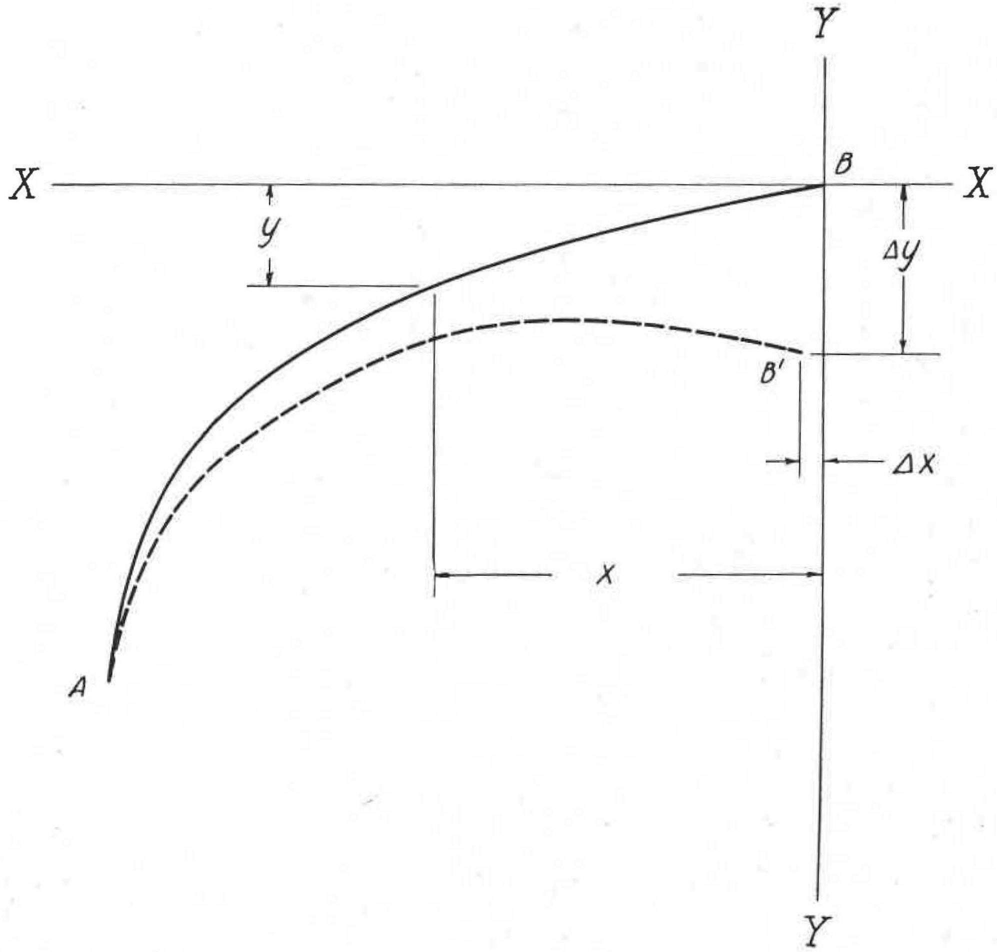

FIGURE 68.—Axis (solid curve $A-B$) of curved member before bending and (dotted curve $A-B'$) after being placed, subsequent to bending, so that at one end (A) it coincides with its original position and direction.

position B'. The displacements (Δx and Δy) of B parallel to the x and y axis, respectively, are given by the equations (*9, pp. 448–450; 15, pp. 128–133*).

$$\Delta x = -\int_B^A \frac{Myds}{EI}$$

$$\Delta y = \int_B^A \frac{Mxds}{EI}$$

Also the change in the angle between the ends of the member equals

$$\int_B^A \frac{Mds}{EI}$$

For the application of these formulas, the origin of coordinates must be taken at that end of the member (or at that point on the member) whose displacements are to be computed.

The expressions for Δx and Δy are usually most conveniently evaluated by numerical integration or summation. Their evaluation and use will be illustrated by applying them to the D-type arches used in the Laboratory service building. The movement of the crown of the arch relative to the abutment and the original direction of the axis at

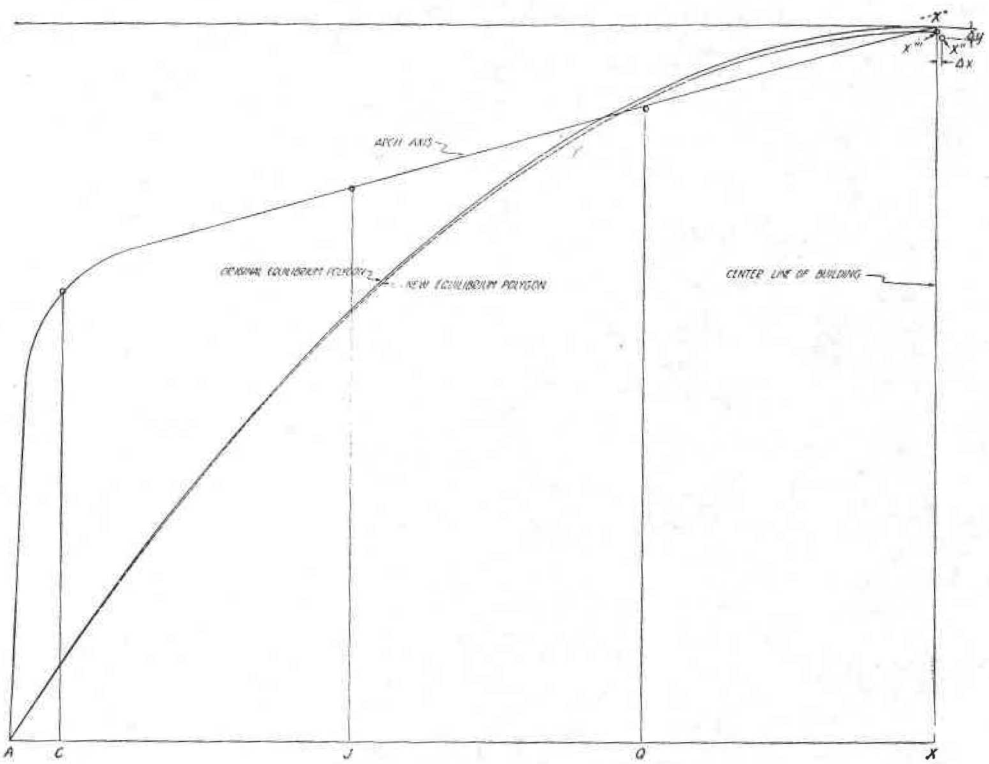

FIGURE 69.—Original shape and position of axis of D-type arch for Forest Products Laboratory service building and computed position of four points on the axis after application of load of 31,500 pounds uniformly distributed across the span.

the abutment, and the vertical deflection of the crown when the arch is uniformly loaded as in the dead-load test described on page 71 will be computed.

COMPUTATION OF DEFLECTIONS OF BUILDING ARCH

The shape of the axis of the D-type arches for the service building is shown in figure 69 together with a solid curve representing the equilibrium polygon for the loading under consideration, that is, uniformly distributed load over the entire span.

The moment at any point in the arch axis is given by the equation

$$M = H \times Z$$

where H is the horizontal thrust of the arch and Z is the vertical distance from the arch axis to the equilibrium polygon. The arch rib has a different depth at each point and this must be taken into account in the computation. Since $I=\frac{1}{12}bh^3$, then $\frac{M}{EI}=\frac{12HZ}{Ebh^3}$ and inserting this value in the equations quoted above:

$$\Delta x=-\frac{12H}{Eb}\int_{X'}^{A}\frac{Zy}{h^3}ds$$

$$\Delta y=\frac{12H}{Eb}\int_{X'}^{A}\frac{Zx}{h^3}ds$$

$$\text{and the change of angle}=\frac{12H}{Eb}\int_{X'}^{A}\frac{Z}{h^3}ds$$

Now $\int_{X'}^{A}\frac{Zy}{h^3}\,ds$ is equivalent to the area under a curve [11] whose base line is the length of the arch axis from X' to A and whose height above the base at each point is the value of $\frac{Zy}{h^3}$ at that point. A similar relation holds for the other two quantities under the integral signs. The computation of these three integrals is carried out in table 32. Inasmuch as the process is actually summation rather than integration, the sign of summation (Σ) is used instead of the integral sign ($\int$) and Δs instead of ds in this table. The stations are so chosen that their horizontal distances from the center line (x, column 10) are expressed in even feet. The distance y (column 15) is the vertical distance of a station from the horizontal line through X' and Δs is the distance between stations measured along the axis of the member. $\sum\frac{Z}{h^3}\Delta s$ is evaluated in columns 7 to 9, inclusive. Each value in column 7 is the average of two consecutive values of $\frac{Z}{h^3}$ in column 5. Each entry in column 8 is the product of the value in column 7 by the value of Δs from column 6. Values in column 9 are the sum of the values in column 8 to the station in question and the final value in column 9 is the summation from X′ to A of $\frac{Z}{h^3}\Delta s$. Summations of $\frac{Zx}{h^3}\Delta s$ and $\frac{Zy}{h^3}\Delta s$ are similarly computed in columns 10 to 14, inclusive, and in columns 15 to 19, inclusive, respectively. The final value in column 14 is to be multiplied by $\frac{12H}{Eb}$ to get Δy and the final value of column 19 by $\frac{-12H}{Eb}$ to get Δx. With the load of 31,750 pounds as applied in the dead load test H is 10,060 pounds. The width (b) of the member is 11 inches (fig. 38). E, as found by applying similar methods of computation to observed deformations in the test of arch D–1, is 1,500,000 pounds per square inch. Assuming that this value of E applies to the building arch that was subjected to dead load tests, values of Δy and Δx are computed [12] as 3.27 and -1.20 inches, respectively. Computation of similar values for any other station, as Q for example, requires that the summations be performed with values of x and y measured from that station. Such computations were made for three stations in addition to X'.

[11] The integral could be evaluated by drawing such a curve, measuring the area under it with a planimeter, and multiplying by suitable scale factors.

[12] Values in columns 14 and 19 of table 32 are dimensionless, hence when b, H, and E are expressed in units of inches and pounds the computed deflections are in inches.

TABLE 32.—*Computation of deflection of D-type arch under load uniformly distributed over the span*

Station	Z	h	h^3	$\frac{Z}{h^3}$	Δs [1]	Evaluation of $\left(\frac{Z}{h^3}\right)\Delta s$: Average $\frac{Z}{h^3}$	(Average $\frac{Z}{h^3}$) Δs	Σ (Average $\frac{Z}{h^3}$) Δs	x	$\frac{Zx}{h^3}$	Evaluation of $\left(\frac{Zx}{h^3}\right)\Delta s$: Average $\frac{Zx}{h^3}$	(Average $\frac{Zx}{h^3}$) Δs	Σ (Average $\frac{Zx}{h^3}$) Δs	y	$\frac{Zy}{h^3}$	Evaluation of $\left(\frac{Zy}{h^3}\right)\Delta s$: Average $\frac{Zy}{h^3}$	(Average $\frac{Zy}{h^3}$) Δs	Σ(Average $\frac{Zy}{h^3}$) Δs
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
	Feet	*Feet*	*Feet*^{3}	*Feet*$^{-2}$	*Feet*	*Feet*$^{-2}$	*Feet*$^{-1}$	*Feet*$^{-1}$	*Feet*	*Feet*$^{-1}$	*Feet*$^{-1}$			*Feet*	*Feet*$^{-1}$	*Feet*$^{-1}$		
X′	0.00	------	------	0.0000				0.0000	0.00	0.0000			0.0000	0.00	0.0000			0.0000
					1.04	−0.2828	−0.2941				−0.2828	−0.2941				−0.0792	−0.0824	
W′	−.22	0.73	0.3890	−.5656				−.2941	1.00	−.5656			−.2941	.28	−.1584			−.0824
					1.04	−.6986	−.7265				−1.1144	−1.1590				−.3079	−.3202	
V′	−.41	.79	.4930	−.8316				−1.0207	2.00	−1.6632			−1.4531	.55	−.4574			−.4025
					1.04	−.8392	−.8728				−2.1018	−2.1859				−.5801	−.6033	
U′	−.52	.85	.6141	−.8468				−1.8934	3.00	−2.5404			−3.6390	.83	−.7028			−1.0059
					1.04	−.8006	−.8326				−2.7792	−2.8904				−.7739	−.8049	
T′	−.55	.90	.7290	−.7545				−2.7260	4.00	−3.0180			−6.5293	1.12	−.8450			−1.8108
					1.04	−.6805	−.7077				−3.0252	−3.1462				−.8471	−.8810	
S′	−.52	.95	.8574	−.6065				−3.4338	5.00	−3.0325			−9.6755	1.40	−.8491			−2.6917
					1.04	−.5023	−.5224				−2.7106	−2.8190				−.7590	−.7893	
R′	−.41	1.01	1.0300	−.3981				−3.5962	6.00	−2.3886			−12.4946	1.68	−.6688			−3.4810
					1.04	−.2848	−.2962				−1.7942	−1.8660				−.5024	−.5225	
Q′	−.21	1.07	1.2250	−.1714				−4.2524	7.00	−1.1998			−14.3605	1.96	−.3359			−4.0035
					1.04	−.0857	−.0891				−.5999	−.6239				−.1680	−.1747	
P′	.00	------	------	.0000				−4.3415	8.00	.0000			−14.9844	2.25	.0000			−4.1782
					1.04	.0913	.0950				.8217	.8546				.2301	.2393	
O′	.30	1.18	1.6430	.1826				−4.2465	9.00	1.6434			−14.1299	2.52	.4602			−3.9389
					1.04	.2840	.2954				2.7492	2.8592				.7717	.8026	
N′	.70	1.22	1.8160	.3855				−3.9512	10.00	3.8550			−11.2707	2.81	1.0833			−3.1363
					1.04	.4670	.4857				4.9437	5.1414				1.3889	1.4445	
M′	1.15	1.28	2.0970	.5484				−3.4655	11.00	6.0324			−6.1292	3.09	1.6946			−1.6919
					1.04	.6354	.6608				7.3512	7.6452				2.0647	2.1473	
L′	1.70	1.33	2.3530	.7225				−2.8047	12.00	8.6700			1.5160	3.37	2.4348			.4554
					1.04	.7856	.8170				9.8522	10.2463				2.7622	2.8727	
K′	2.28	1.39	2.6860	.8488				−1.9876	13.00	11.0344			11.7623	3.64	3.0896			3.3281
					1.04	.9184	.9551				12.4332	12.9305				3.4862	3.6257	
J′	2.95	1.44	2.9860	.9880				−1.0325	14.00	13.8320			24.6928	3.93	3.8828			6.9538
					1.04	1.0392	1.0808				15.0940	15.6978				4.2367	4.4062	
I′	3.68	1.50	3.3750	1.0904				.0483	15.00	16.3560			40.3906	4.21	4.5906			11.3600
					1.04	1.1494	1.1954				17.8452	18.5590				5.0142	5.2148	
H′	4.50	1.55	3.7240	1.2084				1.2436	16.00	19.3344			58.9496	4.50	5.4278			16.5747
					1.04	1.2610	1.3114				20.8320	21.6653				5.8582	6.0925	

G′	5.38	1.60	4.0960	1.3135	1.04	1.3476	1.4015	2.5550	17.00	22.3295	23.6000	24.5440	80.6149	4.78	6.2785	6.6350	6.9004	22.6672
F′	6.32	1.66	4.5740	1.3817	1.04	1.4238	1.4808	3.9566	18.00	24.8706	26.3623	27.4168	105.1589	5.06	6.9914	7.4099	7.7063	29.5676
E′	7.33	1.71	5.0000	1.4660	1.06	1.3628	1.4446	5.4373	19.00	27.8540	26.5230	28.1144	132.5757	5.34	7.8284	7.5041	7.9543	37.2739
D′	8.37	1.88	6.6450	1.2596	1.28	1.1923	1.5261	6.8819	20.00	25.1920	24.4085	31.2429	160.6900	5.70	7.1797	7.2349	9.2606	45.2282
C′	9.00	2.00	8.0000	1.1250	4.84	1.3232	6.4049	8.4080	21.00	23.6250	28.5468	138.1665	191.9329	6.48	7.2900	12.0121	58.1388	54.4889
B′	6.00	1.58	3.9440	1.5213	6.38	.7606	4.8526	14.8123	22.00	33.4686	16.7343	106.7648	330.0994	11.00	16.7343	8.3672	53.3824	112.6277
A′	.00	------	------	.0000				19.6650	22.2083	.0000			436.8643	17.625	.0000			166.0101

[1] Δs=the distance between successive stations, measured along the arch axis.

In table 32 the plus sign indicates moments (Z's) that cause tension in the convex side of the member. Such moments tend to cause clockwise rotation about A and downward motion of X'. The fact that Δy has the same sign as these moments indicates that Δy is to be measured in the direction in which y's were taken as positive, which is downward from X'. The Δx being negative is to be measured in the opposite direction of the x's, that is, Δx is to be measured to the right.

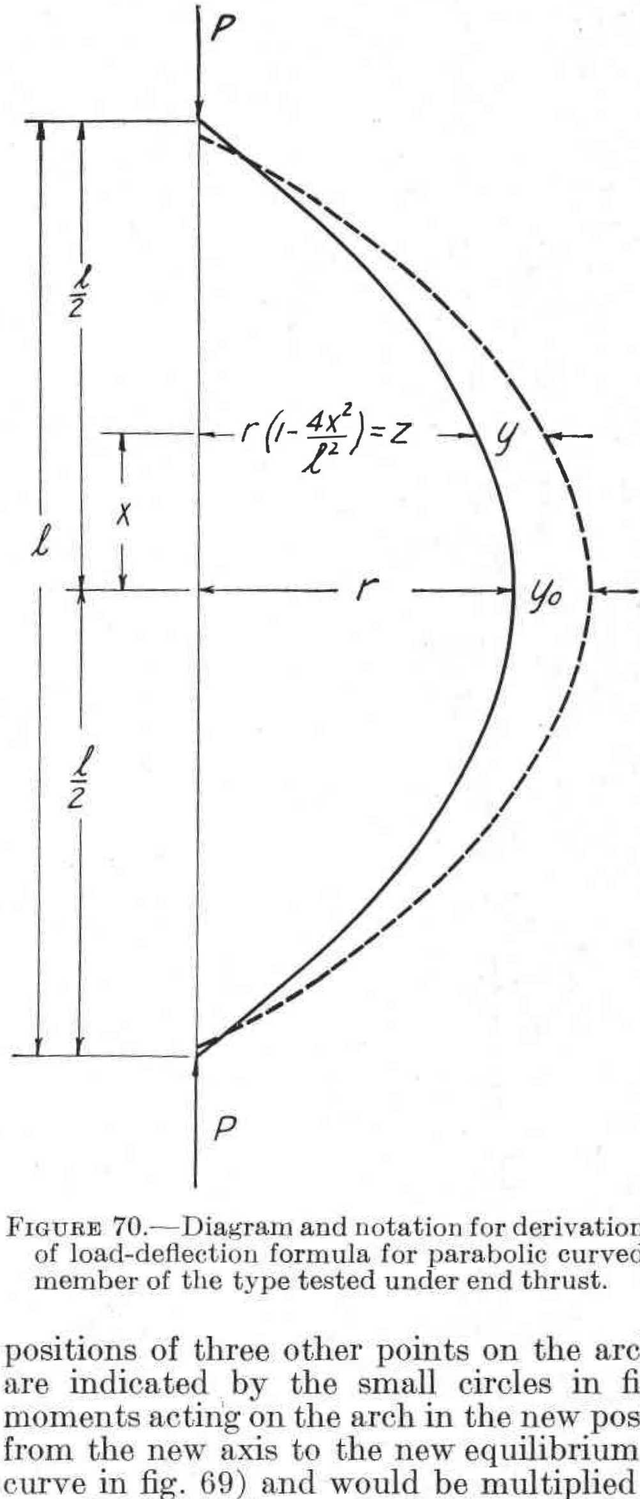

FIGURE 70.—Diagram and notation for derivation of load-deflection formula for parabolic curved member of the type tested under end thrust.

Measuring off the computed values of Δx and Δy gives X'' (fig. 69) as the new position of X'. This is the position X' would take if, after the member was bent by the imposed loads, it was rotated about point A so that the direction of its axis at A is the same as originally. Actually, under the loading considered, rotation does not take place because X', being common to the two half-arches, can have only vertical motion. The actual position of X' after loading must, therefore, be found by rotating X'' about A to X''' on the vertical through X. The position of X''' can be found by computing the diagonal distance from A to X'' and then computing the height of X''' above X from the fact that the diagonal distance from A to X''' must be the same as from A to X''. Such a computation gives 1.61 inches as the vertical deflection of X'. Similar computations give the positions of three other points on the arch axis under load and these are indicated by the small circles in figure 61. To compute the moments acting on the arch in the new position, Z's would be measured from the new axis to the new equilibrium polygon (shown as a dotted curve in fig. 69) and would be multiplied by a new value of H corresponding to the lowered position of X'. The new value of H is 10,140 pounds or 0.8 percent greater than the original value. Consideration of figure 61 indicates that the new $H \times Z$ products will not be significantly greater at any critical point than their original values. A similar computation using E as 750,000, thus making allowance for a doubling of the distortions through time effect, gives a peak deflection of 3.24 inches, which would increase H by 1.4 percent of its original value. Even under this assumption the new $H \times Z$ products, and consequently the moments and stresses at critical section, would not be more than about 3 percent greater than for the unstrained position of the arch rib.

It is interesting now to compare the computed deflection (1.61 inches) with the sums of the deflections of the five central arches of the D type as observed in the dead-load tests and recorded in table 25. (The sum of deflections is used because, as is explained in the discussion of the dead-load tests on the Laboratory service building, this sum is approximately equal to the deflection that would have resulted had the load been carried by a single arch.) The values from this table are 1.26 and 1.42 inches or 78 and 88 percent as great as the deflection of 1.61 inches computed above. The comparison indicates reasonable accuracy in the computation and suggests that the building arches at the time the deflections were measured may have been somewhat stiffer than the value of E (1,500,000 pounds per square inch) as derived from the test of arch D–1 and used in computing the deflection of 1.61 inches, indicates.

SECONDARY STRESSES IN THREE-HINGED PARABOLIC ARCH

Provided the cross section is constant or varies by some simple rule, it is possible to derive for a three-hinged arch with parabolic axis an equation that expresses the deflections under uniform load on the half span and to set up formulas such that secondary as well as primary bending stress may be considered in designing. The relations are developed following:

The ordinates between the arch axis and the equilibrium polygon for half-uniform load (fig. 64) are for each half span the ordinates of a parabola with vertex at the quarter point of the span and height at that point of $r/4$. Hence the moments and deflections in each half are those of a parabolic member with length $l/2$ and center ordinate $r/4$, under an end load equal to the horizontal thrust, H, which equals $\frac{wl^2}{16r}$. On page 117 a formula is given which, when proper substitutions are made to adapt it to the present use and the notation of figure 70, becomes

$$y_o + r/4 = \frac{128 EIr^2}{wl^4}\left(\sec \frac{l}{4}\sqrt{\frac{wl^2}{16EIr}} - 1\right),$$

where y_o is the vertical deflection at the quarter point of the span.

Now $y_o + r/4$ multiplied by the horizontal thrust H, or $\frac{wl^2}{16r}$, is the moment at the quarter point of the span (maximum moment), which, with bending stress f, is $\frac{1}{6} fbh^2$ where b and h are the width and depth, respectively, of the arch rib. Multiplying the right-hand member of the foregoing equation by $\frac{wl^2}{16r}$, setting it equal to $\frac{1}{6}fbh^2$, substituting $\frac{1}{12}bh^3$ for I, and simplifying

$$\text{Cos }\phi=\frac{4\frac{h}{l}\cdot\frac{r}{l}}{4\cdot\frac{h}{l}\cdot\frac{r}{l}+\frac{f}{E}}$$

$$\text{where }\phi=\frac{l}{8h}\sqrt{\frac{3}{E}\cdot\frac{w}{b}\cdot\frac{l}{h}\cdot\frac{l}{r}}$$

From the equation above Cos ϕ, and consequently ϕ, can be found for any combination of l, r, h, f, and E and after finding ϕ,

$$w=\frac{64}{3}\phi^2Eb\frac{r}{l}\left(\frac{h}{l}\right)^3 \text{ or}$$

$$\frac{W}{b}=256\phi^2E\frac{r}{l}\left(\frac{h}{l}\right)^3$$

where w is the load in pounds per inch and W the load in pounds per foot of half span that will cause a maximum bending stress of f pounds per square inch.

This formula cannot be used directly for finding all dimensions of an arch rib. It, however, affords a ready means for finding the necessary width of the rib after finding the value of bh^2 required to resist the primary moment and assuming a value for h. This is illustrated by the following example: Let it be required to find the cross-sectional dimensions of an arch rib to resist the bending moment induced by a load of 512 pounds per foot (W) on the half span of an arch whose span is 40 feet and rise 8 feet, the allowable bending stress being 1,600 pounds per square inch and E being taken as 1,000,000 pounds per square inch. The primary moment is $\frac{wl^2}{64}$ to resist which it is found that a value of 576 for bh^2 is required. Assuming h as 12 inches (which would make $b=4$ inches to take care of primary moment only), Cos ϕ is found from the relation just given to be 0.926, whence $\phi=22°11''=0.387$ radians. Then $\frac{W}{b}=256\times(0.387)^2\times1{,}000{,}000\times\frac{8}{40}\times\left(\frac{1}{40}\right)^3=120$. Whence for $W=512$, b, the required width of the arch rib, is $\frac{512}{120}$ or 4.27.

It must be remembered that the formula just given for W or $\frac{W}{b}$ applies only to an arch with parabolic axis and constant cross section.

FORMULA FOR DEFLECTION OF PARABOLICALLY CURVED MEMBER UNDER END THRUST

For evaluation of modulus of elasticity and determination of the proportional limit from data on the parabolic curved members tested under end thrust, an expression relating the dimensions of the specimen to its load, deflection, and modulus of elasticity is required. This is derived as follows, starting with the well-known relation $M=EI\,\frac{d^2y}{dx^2}$ and using the notation indicated by figure 70.

$$M=EI\frac{d^2y}{dx^2}=-P(Z+y)=-P\left(y+r\frac{l^2-4x^2}{l^2}\right)$$

$$\frac{d^2y}{dx^2}=-\frac{P}{EI}\left(y+r\frac{l^2-4x^2}{2}\right)=-n^2\left(y+r\frac{l^2-4x^2}{2}\right),$$

where $n^2=P\div EI$

The solution of this differential equation, subject to the conditions that $\frac{dy}{dx}=O$ when $x=O$ and $y=O$ when $x=\pm\frac{1}{2}$, is:

$$y=r\left[\frac{8\cos nx}{n^2l^2\cos\frac{nl}{2}}+\frac{4x^2}{l^2}-\frac{n^2l^2+8}{n^2l^2}\right]$$

When $x=O$, y_o=deflection at center under load P,

$$=r\left[\frac{8}{n^2l^2}\left(\sec\frac{nl}{2}-1\right)-1\right]$$

or with the original notation

$$y_o=r\left[\frac{8EI}{Pl^2}\left(\sec\frac{l}{2}\sqrt{\frac{P}{EI}}-1\right)-1\right]$$

This equation cannot be solved directly for E when concurrent numerical values of y_o and P, as found from test, are substituted. It can, however, be solved indirectly in the following way. Rewriting it:

$$\phi=\frac{y_o}{r}=\frac{2(\sec\theta-1)}{\theta^2}-1$$

where $\theta=\frac{l}{2}\sqrt{\frac{P}{EI}}$ and $\theta^2=\frac{Pl^2}{4EI}$

Then values of ϕ computed from a series of values of θ are plotted against θ^2 and a curve is drawn through the resulting points. The value of θ^2 corresponding to any experimentally determined ϕ, or y_o/r, can be read from this curve. Since $\theta^2=\frac{Pl^2}{4EI}$, then $E=\frac{Pl^2}{4\theta^2I}$ and $P=\frac{4\theta^2EI}{l^2}$; hence up to the proportional limit $P=K\theta^2$, where K is a constant equal to $\frac{4EI}{l^2}$. If then values of P from a test are plotted as ordinates with values of θ^2 read from the ϕ–θ^2 curve as abscissas, the resulting points follow a straight line to the proportional limit beyond which they depart from the linear relation. The P–θ^2 diagram thus serves the same function as the load-deflection diagram for a simple beam subjected to transverse load. The values of P and θ^2 at the propor-

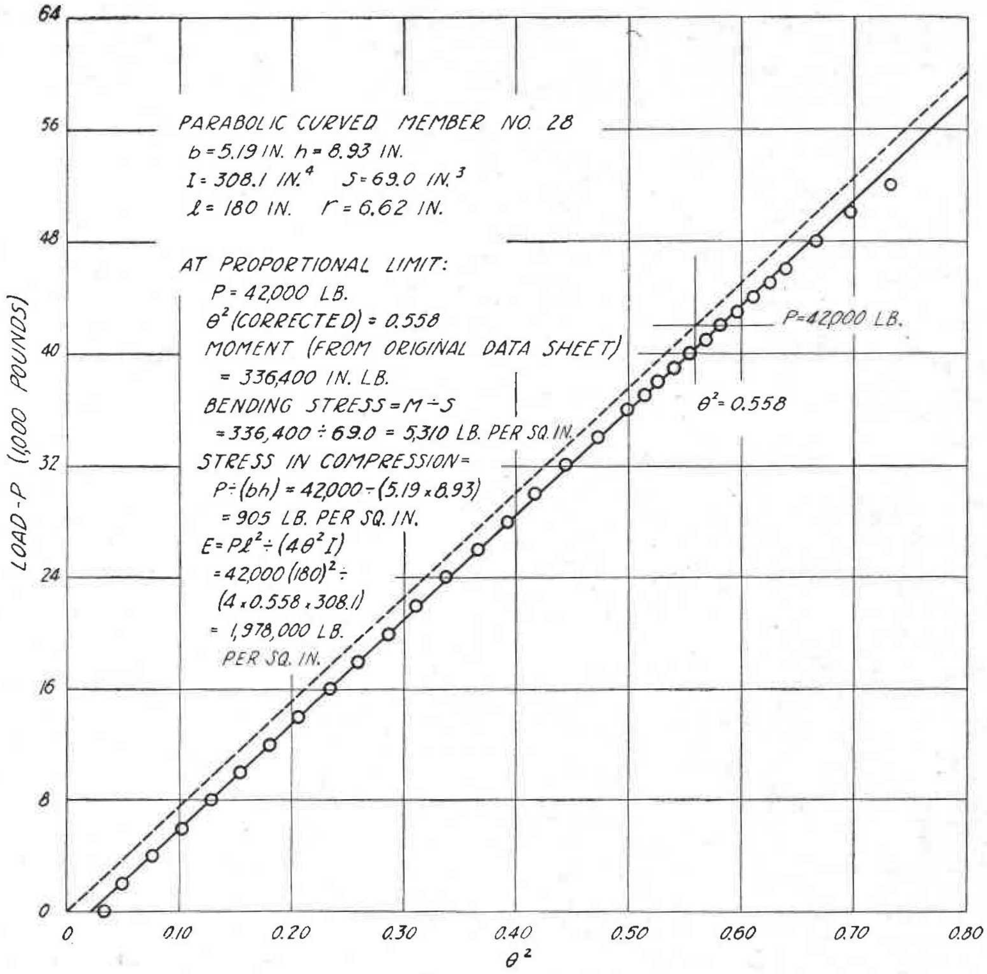

FIGURE 71.—$P-\theta^2$ Diagram for curved member No. 28.

tional limit are found from such a diagram, an example of which is shown in figure 71.

RADIAL STRESS IN A CURVED MEMBER

Assuming linear distribution of bending stress and referring to the notation shown in figure 72:

$$M=\frac{1}{6}fbh^2 \text{ or } f=\frac{6M}{bh^2}$$

and

$$f_z=\frac{6M}{bh^2}\cdot\frac{z}{h/2}=\frac{12Mz}{bh^3}$$

Total tension in a strip of width b and depth from z to convex surface

$$T=\frac{12M}{h^3}\int_z^{h/2} z\,dz=\frac{12M}{h^3}\left(\frac{h^2}{8}-\frac{z^2}{2}\right)=\frac{3M}{2h^3}(h^2-4z^2)$$

Now the total tension in a length b of a cylindrical shell of radius R under an internal pressure acting normal to the shell and of intensity p per unit of area is pbR, whence we may set

$$T=\frac{3M}{2h^3}(h^2-4z^2)=pbR$$

$$\therefore p=\frac{3M}{2Rbh^3}(h^2-4z^2)$$

and when $z=O$, $p=\frac{3M}{2Rbh}$.

Also if z is small compared to r, that is, r is very nearly equal to R,

$$p=\frac{3M}{2rbh^3}(h^2-4z^2)$$

nearly, and p is radially distributed similarly to the vertical distribution of longitudinal shear in a beam, which is in accordance with the

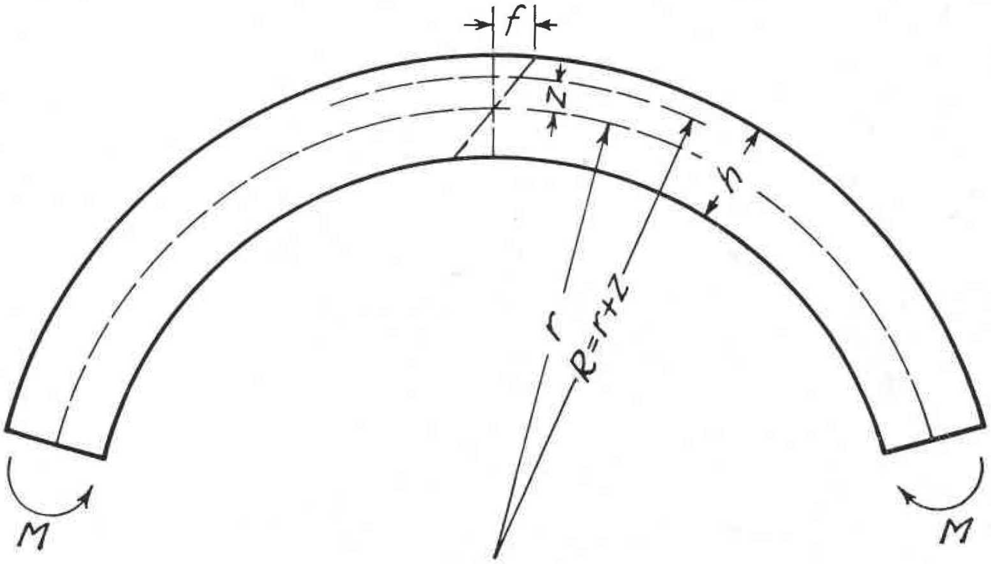

FIGURE 72.—Diagram showing notation used in deriving formula for radial stress in a curved member.

ordinates of a parabola whose vertex is at the neutral axis. The radial stress p has its maximum value at the neutral axis where it equals $\frac{3M}{2rbh}$.

If the moment is as shown in figure 72, p is stress in compression, while if moment were in the opposite direction p would be stress in tension.

EFFECT OF SHRINKAGE OR SWELLING ON THE SHAPE OF A CURVED LAMINATED WOODEN MEMBER

A curved laminated wooden member tends to swell or shrink radially (across the grain) with absorption or loss of moisture, but practically no change in longitudinal (along the grain) dimension occurs. Figure

73 shows the notation used in deriving the formula for change of curvature produced in a wooden arch by radial swelling or shrinkage. In this figure the length of convex face, $L_{,}=R\alpha$; the length of concave face, $l=r\alpha$; the difference in length, $L-l=(R-r)\alpha=t\alpha$.

Whence $t=\frac{L-l}{\alpha}$

Now suppose that, as a result of moisture changes, the thickness is changed by a factor k (positive for swelling and negative for shrinkage)

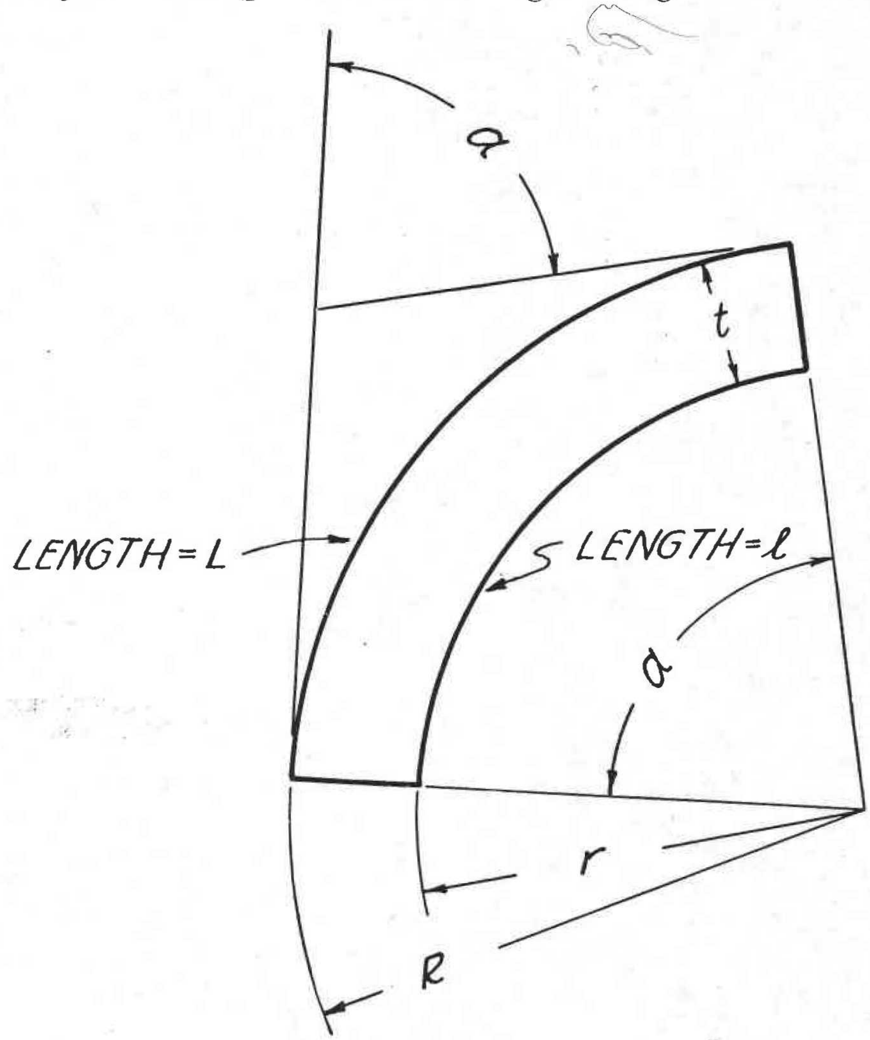

FIGURE 73.—Diagram showing notation used in deriving the formula for change of curvature produced in a wooden member by radial swelling or shrinkage.

so that the thickness becomes $t\ (1+k)$. This results in a change by a factor q in the angle α which then becomes $\alpha(1+q)$. Then since L and l are practically unchanged the new thickness equals

$$t(1+k)=\frac{L-l}{(1+q)\alpha}$$

but the original thickness was

$$t=\frac{L-l}{\alpha}$$

and dividing the new by the original thickness

$$1+k=\frac{1}{1+q}$$

whence $q=\frac{-k}{1+k}$. Since k is usually very small in comparison with unity, $q=-k$, approximately, and hence when t is changed by a small percentage k, α will be changed by approximately the same percentage but in the opposite direction; that is, radial swelling causes a decrease and radial shrinkage an increase in the angle between the ends of a curved member. It may be noted that the percentage change in angle is independent of the length or the dimensions of the cross section of the piece.

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