AN ABSTRACT OF THE THESIS OF

Abdulhadi Ahmed Fatani for the degree of Doctor of Philosophy

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Title: An Analytical Model of Radiative and Convective Heat Transfer in Small and Large Particle Gas Fluidized Beds

Abstract approved: Redacted for privacy

Ronald L. Adams

A complete analytical model of radiative and convective heat transfer to a horizontal tube in small and large particle gas fluidized beds has been developed. In the model the bed of particles consists of an emulsion phase and a bubble phase. The emulsion phase is approximated by a series of transmitting alternate layers of gas and solid. The transmissivity of the solid slabs is obtained by treating the voids in the bed as windows through which radiation is transmitted to the immersed surface. The gas layer, the gas within the voids and the gas in the bubble are considered to be radiatively transparent.

Within the emulsion phase, radiosityes for all solid slab surfaces in addition to the heat transfer surface are obtained by an iterative scheme with the transmission effect included. The convective boundary condition is obtained by treating the bed as a series of contact resistances from particle to particle, starting with the initial resistance between the first layer of particles and the immersed surface. A one-dimensional unsteady conduction analysis is employed using an implicit finite difference technique to determine
the temperature distribution at the surfaces and inside the solid slabs. The gas convection contribution from the emulsion phase is based upon the Adams-Welty model and justified to be an additive term.

Gas convection heat transfer from the bubble phase is also included using an existing model. Radiation heat transfer through the bubble phase is obtained by considering a three-dimensional hemispherical bubble surface adjacent to a horizontal tube. The total heat transfer to the immersed surface is determined by adding the contributions of both emulsion and bubble phases, weighted by their contact fractions.

Results obtained with the model show that the radiant energy transmitted through the voids increases the heat transfer by about 25% on the average over the case when the solid slabs are opaque. At low bed temperatures, the model predictions are in qualitative agreement with experimental data for small particles, and in close agreement with experimental data for large particles. At high bed temperatures, the radiative heat transfer coefficients obtained with the model are higher than the experimental results for large particles. However, the experimental local and spatial average radiative heat transfer coefficients for small particles agree closely with model predictions.
An Analytical Model of Radiative and Convective 
Heat Transfer in Small and Large Particle 
Gas Fluidized Beds

by

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Typed by Mary Ann (Sadie) Airth for Abdulhadi Ahmed Fatani
In memory of my Mother,

Father, and

Abdulkaher Fatani, my brother.
ACKNOWLEDGMENTS

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\( f_0 \) Bubble contact fraction.

\( F_0 \) Fourier number.

\( g \) Acceleration due to gravity.

\( h \) Heat transfer coefficient, \( \dot{q}'/(T_B - T_w) \)

\( h_{gc} \) Heat transfer coefficient due to gas convection.

\( h_{pc} \) Heat transfer coefficient due to particle convection in the emulsion phase.

\( h_{rb} \) Radiative heat transfer coefficient from the bubble phase.

\( h_{rad} \) Heat transfer coefficient due to thermal radiation.

\( K_1, K_2 \) Parameters appearing in formula for Stokes region edge location.

\( K_g \) Thermal conductivity of gas.

\( K_g^* \) Gas thermal conductivity at mean temperature \( (T_w + T_B)/2 \).

\( K_s \) Thermal conductivity of solid.

\( K_B \) Pressure gradient parameter.

\( L \) The distance between the two planes containing \( dA_1, dA_2 \).

\( l_g \) Gas layer thickness.

\( l_s \) Solid layer thickness.

\( n_1, n_2 \) Unit normal to \( dA_1, dA_2 \).

\( \text{Nu}_{p, 2D} \) Two-dimensional Nusselt number for interstitial channel.

\( \text{Nu}_D \) Local instantaneous Nusselt number in the bubble phase.

\( \text{Nu}_{pm} \) Parameter appearing in formula for Stokes region edge location.

\( \text{Pr} \) Gas Prandtl number.
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<td>$S_o$</td>
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<td>$S_p$</td>
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<td>$t$</td>
<td>Time.</td>
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<td>$t_r$</td>
<td>Particle residence time.</td>
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<td>$T$</td>
<td>Temperature.</td>
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\( T_B \) Bed temperature.

\( T_w \) Immersed surface temperature.

\( u \) Boundary layer edge velocity.

\( u_{mf} \) Minimum fluidizing velocity.

\( u_0 \) Superficial velocity.

\( u' \) Interstitial turbulence intensity.

\( X \) Distance into the bed from the heat transfer surface.

\( x_1, y, z_1 \) Rectangular coordinates

\( x_2, y, z_2 \) Stokes region edge location from particle contact point.

**Greek Symbols**

\( \alpha' \) Exponent for gas thermal conductivity variation with temperature.

\( \alpha_s \) Thermal diffusivity of solid.

\( \alpha \) Absorptivity.

\( \alpha_o \) Absorptivity for an opaque surface.

\( \rho \) Reflectivity.

\( \rho_{eff} \) Effective reflectivity of solid slabs.

\( \rho_o \) Reflectivity for an opaque surface.

\( \rho_w \) Reflectivity of immersed surface.

\( \rho_s \) Particle density.

\( \tau \) Transmissivity.

\( \Delta X \) Solid slice thickness.

\( \Delta t \) Step size in time direction.
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**Superscript**
- $(-)$: Average value.
- $(^n)$: Value at $n$th time step.
- $(^*)$: Dimensionless parameter.

**Subscript**
- $i$: Index of a grid point.
- $j$: Index of a radiation surface.
- $w$: Immersed heat transfer surface.
- $r$: Radiation.
- $b$: Bubble phase.
- $e$: Emulsion phase.
AN ANALYTICAL MODEL OF RADIATIVE AND CONVECTIVE
HEAT TRANSFER IN SMALL AND LARGE PARTICLE
GAS FLUIDIZED BEDS

I. INTRODUCTION

In recent years, the use of fluidized bed combustion of coal to
generate steam to drive the turbine of a power plant has motivated
several investigators to acquire knowledge of the heat transfer
coefficient between the bed and the boiler tube for design purposes.
Such coefficients are dependent upon the operating conditions, bed
materials and tube dimensions.

The interest in the heat transfer process in fluidization tech-
nology is due to the high rate of heat transfer between the fluidized
solids and an immersed surface. Such high rates of heat transfer are
attributed to the extremely large area of contact between the solid
and fluid which results in a high overall heat transfer between the
solid and the fluid. These fluidized solids have high volumetric
heat capacity in comparison with that of the fluid which is usually
gas. Thus, the fluidized solids have most of the thermal mass, and
when they come into contact with the immersed surface, they exchange
heat through the interstitial gas phase.

The heat transfer process between a gas fluidized bed and an
immersed surface is dominated by convection due to both particles and
gas at low bed temperatures. At high bed temperatures, the thermal
radiation heat transfer contribution becomes significant. The rate of heat transfer can be described by the following equation.

\[ q''_w = q'_{pc} + q'_{gc} + q''_{rad} \]  

(1.1)

where

- \( q'_{pc} \) is the rate of heat transfer due to particle convection.
- \( q'_{gc} \) is the rate of heat transfer due to gas convection.
- \( q''_{rad} \) is the rate of heat transfer by thermal radiation.

and \( q''_w \) is the total rate of heat transfer to the immersed surface.

The separation of the particle convective component and gas convective component in equation (1.1) is justifiable according to Botterill [15], and further justification [5] will be discussed later.

Most experimental and theoretical research studies have been aimed toward a better understanding of the heat transfer process by unsteady state conduction from fluidized beds to immersed surfaces, at temperatures such that radiation can be neglected, and with small particles so that gas convection can also be neglected for unpressurized systems [16, 18, 29, 30]. Nevertheless, in the combustion of coal, large particles and high temperatures are encountered. Thus, the radiative and gas convective components must be considered in the analysis.

Numerous studies can be found in the literature [16, 18, 29, 46] which deal with the particle convective component. Some of these studies were based on the assumption that the packet of particles is a continuum. The continuum assumption considers a packet of
particles in some random configuration loosely packed with each other and immersed in the gas. This "packet" acting as an individual entity is the source or the sink for heat transfer in the bed. The thermal properties of the packet are the same as those of a quiescent bed. Mickley and Fairbanks [46] postulated that fluidized bed heat transfer occurs by unsteady diffusion of heat from the surface into the packets of solid particles and interstitial gas, when they come into contact with the immersed surface for a brief residence time.

Others [18, 29, 30] however have based their work on an analysis of individual particles within the packet. The individual particle approach is appropriate when residence times are short, as in Botterill and Williams [18]. As the residence time increases, the heat penetrates deeper into the bed, so that more than the row of particles adjacent to the surface are affected. Thus, additional layers of particles are more appropriate in this case.

There have also been several experimental and analytical investigations [2, 19, 27, 21] of the gas convective component, significant for large particles. The interstitial gas flow between particles near the heat transfer surface has a significant influence on heat transfer in this case. Botterill and Denloye [19] have developed a model to describe heat transfer between a cylindrical surface and packed as well as fluidized beds. The region considered was that of higher local voidage adjacent to the heat transfer surface extending about a half of a particle diameter. In their model, they also considered heat transfer through a boundary layer at
the wall which required numerical analysis. Adams and Welty [2] introduced a gas convection model by analyzing flow within channels between large particles contacting the surface of a horizontal tube. The heat transfer contribution of the gas flow within contacting bubbles was also obtained. Adams [4] presented an approximate model of bubble phase convective heat transfer to a horizontal cylinder immersed in a two dimensional fluidized bed. A single two dimensional slow bubble contacting a horizontal tube was considered, along with the one parameter integral method of Smith and Spalding [64] to obtain heat transfer to the tube.

For the radiation component there are several analytical studies [39, 48, 54, 57, 60] and experimental works [9, 11, 17, 26, 35, 59] that are also restricted to small particle fluidized beds. Kolar et al. [39] investigated the radiation contribution in small particles using the alternate slab model of Gabor [29, 30]. Kolar considered that radiation heat transfer takes place at the surface of each solid slab, and the surface was assumed to be opaque. Vedamurthy and Sastri [60] analyzed the radiation component by considering the emulsion packet (of 3 dp thick) to consist of a number of shields radiating as a black body, and radiation results were also obtained from the emulsion and the bubble phases. Thring [57] examined the effect of the radiative heat transfer by comparison of three models. Model (1) was the packet model using the mean thermal properties of the bed, model (2) was a layer of spherical particles adjacent to the heat transfer surface, and model (3) was a layer of cubical
particles. The spherical and cubical particle models of Thring were originally developed by Botterill and Williams [18]. Thring found that the radiation contribution by the spherical and cubical particle models to be higher than the packet model, since the temperature at the surface of the particle was used in the radiation calculations.

Vadivel and Vedamurthy [59] conducted an experiment for an immersed horizontal tube with small particles of coal ash. They measured local heat transfer coefficients as well as time-average heat transfer coefficients for the total radiative component and the total heat transfer coefficient. Their conclusion was that the radiation contribution is about 35% for the bed temperature of 750 C. Radiation from the bubble phase was studied by Yoshida et al. [66], who presented an expression to evaluate the radiative heat transfer coefficient from the bubble phase in terms of emissivities of surface and particle and bed and surface temperatures. In contrast to small particles, there have been fewer experimental and theoretical studies [7, 49] that deal with radiation in large particle fluidized beds.

Thus, the objective of this study is to formulate an analytical model which can be applied to small particles as well as to large particles, and for both cold and hot beds. Hence, the model is unique for its wide range of application. It can be used to calculate local and average values of heat transfer coefficients around immersed horizontal tubes in addition to its unified application for the complete particle size spectrum.
II. MODEL DESCRIPTION

In a gas fluidized bed, heat transfer from a surface into the bed takes place in a direction normal to the heat transfer surface according to Botterill and Williams [18]. The heat transfer by conduction to the particle point of contact with the heat transfer surface can be neglected, since it is theoretically zero for spherical particles. The principal resistance to heat transfer at the surface is then a gas film, and the moving fluidized particles scour the film to reduce this resistance. There have been models of heat transfer by unsteady conduction from the heat transfer surface to a single particle [18], and to a chain of particles [29]. The single particle model is found to be unsatisfactory for long residence time, since heat will penetrate into the second and third particles. The heat absorbed by other particles is neglected in the development of the single particle model, and the heat transfer will be underestimated for large residence times. The single particle model is however still satisfactory for short residence times. For these models, analysis of heat transfer to distinct particles is made, in contrast to the packet theory models. All these models require information regarding residence time. Botterill et al. [16] showed that heat transfer prediction for particles of different diameters could be reduced to a single curve for a particular gas-solid system if the parameters $h_d p$ and $t_r/d_p^2$ were plotted against each other as shown in
Figure (2.1). h is the heat transfer coefficient, \( d_p \) is the particle diameter, and \( t_r \) is the particle residence time at the heat transfer surface. He also demonstrated that, the effect of particle size and particle properties may be taken into account to produce a single curve for a particular gas-solid system if the Nusselt number \( \frac{h d_p}{k_p} \) is plotted against the Fourier number given by \( \frac{\alpha_p t_r}{d_p^2} \), where \( k_p \) and \( \alpha_p \) are the thermal conductivity and the diffusivity of the solid.

Gabor [29] examined heat transfer at the surface to a string of spherical particles, and utilized a similar set of equations as in Botterill and Williams [18]. The string of spheres model requires a large amount of computing time as was realized by Gabor, so he suggested a simpler model, while retaining the basic idea of Botterill and co-workers, with much less computing time which led to satisfactory predictions. The alternate slab model as proposed by Gabor [29] considers the transfer of heat to be through a series of alternate layers of solid and gas phases to represent a string of spherical particles perpendicular to the heat transfer surface. Then Kolar et al. [39] modified the alternate slab model of particle convective heat transfer to include radiation. Kolar et al. [39] considered the solid slab surface to be opaque so that the outgoing radiant energy from the surface consists of emitted and reflected radiant energies. Thus, the solid slab does not transmit any of the incident radiation on its surface.

In the present model the heat transfer surface is alternately exposed to an emulsion phase and a bubble phase. The heat transfer
Figure 2.1. The effect of particle diameter and residence time on the heat transfer coefficient.
from the emulsion phase is due to gas convection, particle convection and thermal radiation. The gas convection contribution is based upon the Adams-Welty model [2], in which a boundary layer analysis of the interstitial flow is used to obtain the convective heat transfer. In Adams-Welty model [2], the particle convective contribution to the heat transfer is obtained from a Stokes flow analysis near particle contact points and is weakly dependent on the Reynolds number based upon gas properties according to Adams [5]. Thus the gas and particle convective contributions are uncoupled. Therefore the particle convective and the radiative heat transfer to the immersed surface are obtained by separately modeling the emulsion phase as a series of transmitting alternate layers of gas and solid, as shown in Figure (2.2).

Heat conduction through the solid layers of the emulsion phase is treated as a one-dimensional transient conduction problem in which the experimental values of residence time are used. The thickness of the layers representing the solid is specified to be two-thirds of particle diameter ($d_p$), as suggested by Kunii and Smith [43]. The gas layer thickness is considered to be ($0.16 \, d_p$) for the gas phase between particles and ($0.08 \, d_p$) between the wall and the first layer of particles. This arrangement of gas layer thickness has been found to be satisfactory under all conditions of application [40].

For the purpose of establishing the transmissivity of these solid layers, a layer of opaque spheres in a simple cubic arrangement with a thickness of one particle diameter as shown in Figure (2.3) is
Figure 2.2. A transmitting alternate slab model.
Figure 2.3. Layers of opaque spheres in simple cubical arrangement
considered. Due to this cubical arrangement of particles, a unit cell is formed by four particles with a void at the center of the cell as shown in Figure (2.4). The void acts as a window through which radiation is transmitted through the layer. The void in each cell is enclosed by sections of solid spherical surfaces on the sides and imaginary planar openings at the ends of the cell. At the center of the unit cell the imaginary planar surface (shaded area) is as shown in Figure (2.5).

Hence, the surface of each solid layer is considered to be an emitting, reflecting and transmitting surface, and the effective emissivity, reflectivity and transmissivity of each solid slab surface is to be determined. The transmissivity of each solid slab surface is determined as a view factor between the square and cusped surfaces shown in Figure (2.5). The gas layer and the gas within the voids is considered to be a radiatively non-participating media.

In the bubble phase, the gas is also considered to be radiatively transparent. The gas convective heat transfer contribution is obtained by considering flow within a two-dimensional bubble of circular geometry contacting a horizontal tube according to Adams [4]. The radiation heat transfer contribution is obtained by analysis of the radiative heat transfer between a three-dimensional hemispherical surface and the tube surface according to Yoshida et. al. [66].
Figure 2.4. Particle arrangement for heat transfer model
Figure 2.5. Cusped cross section at the center of the solid slab.
III. METHODS OF ANALYSIS

3.1 General

The heat transfer process between a gas fluidized bed and an immersed surface is dominated by convection due to both particles and gas at low bed temperatures. At high bed temperatures, the thermal radiation heat transfer contribution becomes significant. The rate of heat transfer to the wall from the emulsion phase can be described by the following equation.

\[ q_w = q_{pc} + q_{gc} + q_{rad} \]  \hspace{1cm} (3.1.1)

where

- \( q_{pc} \) is the rate of heat transfer due to particle convection.
- \( q_{gc} \) is the rate of heat transfer due to gas convection.
- \( q_{rad} \) is the rate of heat transfer by thermal radiation.

The separation of particle and gas convection in equation (3.1.1) is justified by a previous study [5] which concluded that the particle convective contribution depends strongly upon gas-to-particle thermal conductivity ratio and Fourier number based upon particle properties and is weakly dependent upon Reynolds number based upon gas properties.

Heat is transferred between the bed and the immersed surface by (i) particle convection, gas convection and radiation through the emulsion phase, and by (ii) gas convection and radiation through the
bubble phase. Thus the total heat transfer coefficient to the immersed surface, $h_{\text{total}}$, can be written as,

$$h_{\text{total}} = (h_{\text{pc}} + h_{\text{gc}} + h_{\text{rad}}) (1 - f_0) + (h_{\text{gc}} + h_{\text{rad}}) f_0$$

(3.1.2)

where $h_{\text{pc}}$ is the particle convective heat transfer coefficient

$h_{\text{gc}}$ is the gas convective heat transfer coefficient

$h_{\text{rad}}$ is the radiative heat transfer coefficient

and $f_0$ is the bubble contact fraction

The physical model of heat transfer in a gas fluidized bed discussed in Chapter II is formulated into a mathematical model. This analytical model requires the development of a transient conduction analysis in the solid slab using an implicit finite difference technique with radiation and convection boundary conditions at its surfaces. The radiation boundary condition is developed using the radiosity analysis as in Siegel and Howell [52], in which the thermal radiant energy at the surface is composed of emitted, reflected and transmitted radiant energies. The convective boundary condition is obtained by treating the bed as a series of contact resistances from particle to particle, starting with the initial resistance between the heat transfer surface and the first layer of particles.

The thermal radiation in the bubble phase is developed according to Yoshida et al. [66], by considering three-dimensional hemispherical bubbles at the immersed surface. The gas convection analysis from the emulsion and the bubble phases are obtained according to Adams and Welty [2].
3.2 Particle Convective Heat Transfer

In order to determine the particle convective heat transfer to the immersed surface, the instantaneous temperature distribution within the solid slabs, and at their surfaces is required. For the determination of the temperature distribution, an unsteady conduction analysis is employed, using residence time information experimentally determined.

For the transient conduction inside the solid slabs, the temperature distribution is assumed to be one-dimensional and governed by,

\[ \frac{1}{\alpha_s} \frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2} \]  

(3.2.1)

with initial condition

\[ T(x,0) = T_B \]  

(3.2.2)

and boundary conditions

\[-K_s \frac{\partial T(x,t)}{\partial x} \bigg|_{\text{surface}} = q''_{\text{conv}} + q''_{\text{rad}} \quad \text{for } t > 0 \]  

(3.2.3)

The implicit finite difference representation of equation (3.2.1) is,

\[ \frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha_s \left( \frac{T_{i-1}^{n+1} - 2T_i^{n+1} + T_{i+1}^{n+1}}{(\Delta x)^2} \right) \]  

(3.2.4a)

Therefore,

\[ T_i^{n+1} - T_i^n = \lambda \left( T_{i-1}^{n+1} - 2T_i^{n+1} + T_{i+1}^{n+1} \right) \]  

(3.2.4b)

where \( \lambda = \frac{\alpha_s \Delta t}{(\Delta x)^2} \)
Hence,

$$- \lambda T_{i-1}^{n+1} + (1 + 2 \lambda) T_i^{n+1} - \lambda T_{i+1}^{n+1} = T_i^n$$  (3.2.4c)

The equation (3.2.4c) is valid only inside the solid slabs (i.e., not at the surface of the solid slabs), as shown in Figure (3.1).

Now, the finite difference approximation of the boundary condition in equation (3.2.3) using the central difference formula is

$$- \frac{K_s}{2\Delta x} (T_{-i+1}^{n+1} - T_{i-1}^{n+1}) = q_{pc}'' + q_{rad}''$$  (3.2.5)

where

$$q_{pc}'' = h_g (T_i^{n+1} - T_{i+1}^{n+1})$$  (3.2.6)

and

$$h_g = k_g/\rho_g$$

The (±) sign in equation (3.2.6) depends on whether the surface boundary faces the bed or the wall, respectively. The boundary condition is illustrated in Figure (3.2), where $T_{-(i+1)}$ is a fictitious node.

3.3 Radiation Heat Transfer From The Emulsion Phase

3.3.1 General

The radiation from the emulsion phase requires a radiosity analysis at the surface of the solid slabs, where the outgoing radiation from the surface consists of the emitted, reflected and transmitted radiant energies. The transmitted radiant energy is considered to be due to the holes in the solid slab.

The radiosity development, following Siegel and Howell [52], can be expressed as follows: if the surface faces the wall
Figure 3.1. Transient conduction inside the solid slab.
Figure 3.2. Surface boundary condition heat balance.
\[ q_{oj} = e_j \sigma T_j^4 + \rho_j q_{o(j-1)} + \tau q_{o(j+2)} \quad (3.3.1.1) \]

While if the surface faces the bed,

\[ q_{oj} = e_j \sigma T_j^4 + \rho_j q_{o(j+1)} + \tau q_{o(j-2)} \quad (3.3.1.2) \]

where \( e \) and \( \rho \) are the effective emissivity and reflectivity, and \( \tau \) is the transmissivity. Note that the effect of the radiant energy transmitted from further solid layers is included in the radiosity terms \( q_{o(j+2)} \), \( q_{o(j-2)} \) of the above two equations. Since the wall is considered to be opaque, the radiosity of the wall would be

\[ q_{ow} = e_w \sigma T_w^4 + \rho_w q_{o(j+1)} \quad (3.3.1.3) \]

The radiosity equations as defined above are related to each other since each radiosity equation is a function of its surface temperature and two other radiosities. One component of the radiosity is due to the reflected radiant energy from the opposite solid slab, while the other is due to the transmitted radiant energy coming from the next solid slab. Figures (3.3-4) are illustrations of equations (3.3.1.1-2). Thus, if the temperature of each solid slab surface is known, the radiosities of all slab surfaces as defined by equations (3.3.1.1 - 3) can be determined by an iterative procedure.

3.3.2 Radiosity Equations In Other Forms

The radiosity equations (3.3.1.1), (3.3.1.2) and (3.3.1.3) are modified in terms of the two bounding surfaces of the adjacent gas slab, and other radiosities. Thus, an elimination procedure was per-
Figure 3.3. Illustration of Equation (3.3.1.1).

Figure 3.4. Illustration of Equation (3.3.1.2).
formed between the radiosity equations in their original forms. This procedure is developed by assigning indices to the solid slab surfaces as shown in Figure (3.5) then, by writing all the radiosity equations for all slab surfaces as follows,

\[ q_{ow} = e_w \sigma T_w^4 + \rho_w q_{o1} \] (3.3.2.1)
\[ q_{o1} = e \sigma T_1^4 + \rho q_{ow} + \tau q_{o3} \] (3.3.2.2)
\[ q_{o2} = e \sigma T_2^4 + \rho q_{o3} + \tau q_{ow} \] (3.3.2.3)
\[ q_{o3} = e \sigma T_3^4 + \rho q_{o2} + \tau q_{o5} \] (3.3.2.4)
\[ q_{o4} = e \sigma T_4^4 + \rho q_{o5} + \tau q_{o2} \] (3.3.2.5)
\[ q_{o5} = \sigma T_B^4 \] (3.3.2.6)

Notice that all the radiative properties of all solid slab surfaces are considered to be the same (i.e., \( e_1 = e_2 = e_3 = e_4 \), \( \rho_1 = \rho_2 = \rho_3 = \rho_4 \) and \( T_1 = T_2 = T_3 = T_4 \)) and the bed surface is considered to be a black body with \( e = 1 \).

If we eliminate the common terms between equations (3.3.2.1) and (3.3.2.2), new forms of the radiosity equations are obtained,

\[ q_{ow} = \frac{1}{(1-\rho_w)} \left[ e_w \sigma T_w^4 + \rho_w e \sigma T_1^4 + \rho_w \tau q_{o3} \right] \] (3.3.2.7)

and

\[ q_{o1} = \frac{1}{(1-\rho_w)} \left[ e \sigma T_1^4 + \rho e_w \sigma T_w^4 + \tau q_{o3} \right] \] (3.3.2.8)
Figure 3.5. Sketch for developing alternate forms of radiosity equations.
Also, the pair of equations (3.3.2.3) and (3.3.2.4) are reduced to the following new forms:

\[ q_{o2} = \frac{1}{(1-p^2)} \left[ e \sigma T_2^4 + p e \sigma T_3^4 + p \tau q_{o5} + \tau q_{ow} \right] \] (3.3.2.9)

and,

\[ q_{o3} = \frac{1}{(1-p^2)} \left[ e \sigma T_3^4 + p e \sigma T_2^4 + p \tau q_{ow} + \tau q_{o5} \right] \] (3.3.2.10)

Finally,

\[ q_{o4} = e \sigma T_4^4 + p \sigma T_B^4 + \tau q_{o2} \] (3.3.2.11)

The net radiation exchange between two opposing solid slab surfaces is determined as follows [52],

\[ q_{rw} = q_{ow} - q_{o1} \]

\[ q_{rw} = \frac{1}{1-p_{w}} \left[ (1-p) e_w \sigma T_w^4 - (1-p_w) e \sigma T_1^4 \right. \]

\[ - (1-p_w) \tau q_{o3} \] (3.3.2.12)

and,

\[ q_{r1} = q_{o1} - q_{ow} \]

\[ q_{r1} = \frac{1}{1-p_{w}} \left[ (1-p_w) e \sigma T_1^4 - (1-p) e_w \sigma T_w^4 \right. \]

\[ + (1-p_w) \tau q_{o3} \] (3.3.2.13)

In addition,

\[ q_{r2} = q_{o2} - q_{o3} \]

\[ q_{r2} = \frac{1}{1+p} \left[ e \sigma T_2^4 - e \sigma T_3^4 + \tau q_{ow} - \tau q_{o5} \right] \] (3.3.2.14)

and

\[ q_{r3} = q_{o3} - q_{o2} \]

\[ q_{r3} = \frac{1}{1+p} \left[ e \sigma T_3^4 - e \sigma T_2^4 + \tau q_{o5} - \tau q_{ow} \right] \] (3.3.2.15)
Finally,

\[ qr_4 = q_{o4} - q_{o5} \]

\[ qr_4 = e \sigma T_4^4 - (1-\rho) q_{o5} + \tau q_{o2} \quad (3.3.2.16) \]

Now, recursion formulas can be obtained from the above equations for the net radiative energy supplied to each surface, if the number of solid slabs is to be arbitrary. Thus, for the wall surface, where \( j=0 \), the net radiative exchange equation is,

\[ qr_j = \frac{1}{(1-\rho_j)} \left[ (1-\rho) e_j \sigma T_j^4 - (1-\rho_j) e \sigma T_{j-1}^4 \right. \]
\[ \left. - (1-\rho_j) \tau q_o(j+3) \right] \quad (3.3.2.17) \]

For the first face of the first solid slab where \( j=1 \), the net radiative exchange is,

\[ qr_j = \frac{1}{(1-\rho_j-1)} \left[ (1-\rho_{j-1}) e_j \sigma T_j^4 - (1-\rho_j) e_{j-1} \sigma T_{j-1}^4 \right. \]
\[ \left. + (1-\rho_{j-1}) \tau q_o(j+2) \right] \quad (3.3.2.18) \]

For the solid slab surface facing the bed, the net radiative exchange is,

\[ qr_j = q_{o0} - q_o(j+1) \quad (3.3.2.19) \]

\[ qr_j = \frac{1}{(1+\rho)} \left[ e \sigma T_j^4 - e \sigma T_{j+1}^4 + \tau q_o(j-2) - \tau q_o(j+3) \right] \]

For the solid slab surface facing the wall, the net radiative exchange is,

\[ qr_j = q_{o0} - q_o(j-1) \quad (3.3.2.20) \]

\[ qr_j = \frac{1}{(1+\rho)} \left[ e \sigma T_j^4 - e \sigma T_{j-1}^4 + \tau q_o(j+2) - \tau q_o(j-3) \right] \]
And finally, for the last face of the last solid slab, the net radiative exchange equation is,

\[ q_{rj} = \varepsilon \sigma T_j^4 - (1-\rho) q_o(j+1) + \tau q_o(j-2) \quad (3.3.2.21) \]

Note that equations (3.3.2.17-21) will reduce to the appropriate form for the opaque case, if the factor \( \tau \) is allowed to take the value of zero in those equations. This means that all the holes in the solid slab disappear, and no radiant energy is allowed to pass through the solid slabs.

3.3.3 Radiosity Equations for the Unsteady Conduction Analysis

The radiosity equations (3.3.1.1-2) are modified for the purpose of determining the net radiative exchange between the surface of the solid slabs in the unsteady conduction analysis. The adjustment is obtained by subtracting the transmitted energy from the radiosity equations (3.3.1.1-2) due to the holes in the solid slabs.

\[ q_{oj}^c = q_{oj} - \tau q_o(j+2) \quad (3.3.3.1) \]

(if the surface faces the wall)

and

\[ q_{oj}^c = q_{oj} - \tau q_o(j-2) \quad (3.3.3.2) \]

(if the surface faces the bed)

This means that only the emitted and the reflected radiant energies are accounted for in the radiation boundary condition for the transient conduction analysis.
3.3.4 Net Radiative Exchange for the Transient Conduction Analysis

This section aims toward obtaining the final forms of the radiative boundary conditions for the transient conduction analysis.

From equations (3.3.3.1) and (3.3.3.2) the net radiative exchange can be determined, according to section (3.3.3), so that

\[ q_{rij} = q_{0j} - (1 - \tau) q_{o(j-1)} \]

or

\[ q_{rij} = [q_{oj} - q_{o(j-1)}] + \tau q_{o(j-1)} - \tau q_{o(j+2)} \quad (3.3.4.1) \]

(if the surface faces the wall)

and

\[ q_{rij} = q_{0j} - (1 - \tau) q_{o(j+1)} \]

or

\[ q_{rij} = [q_{oj} - q_{o(j+1)}] + \tau q_{o(j+1)} - \tau q_{o(j-2)} \quad (3.3.4.2) \]

(if the surface faces the bed)

For the first solid slab surface facing the wall, the net radiative exchange, where (j=1), would be

\[ q_{rij} = [q_{oj} - q_{o(j-1)}] - \tau q_{o(j+2)} + \tau q_{o(j-1)} \quad (3.3.4.3) \]

And finally, for the last solid slab surface facing the bed, the net radiative exchange would be,

\[ q_{rij} = [q_{oj} - q_{o(j+1)}] - \tau q_{o(j-2)} + \tau q_{o(j+1)} \quad (3.3.4.4) \]
Now, by substituting equation (3.3.2.20) into equation (3.3.4.1), we obtain the following equation for the net radiative exchange for the surface facing the wall

\[ q_{rj} = \frac{1}{(1+\rho)} \left[ e \sigma T_j^4 - e \sigma T_{j-1}^4 + (1+\rho) \tau q_0(j-1) 
- \rho \tau q_0(j+2) - \tau q_0(j-3) \right] \]  \hspace{1cm} (3.3.4.5)

And, by substituting equation (3.3.2.19) into equation (3.3.4.2), we obtain the following equation for the net radiative exchange for the surface facing the bed,

\[ q_{rj} = \frac{1}{(1+\rho)} \left[ e \sigma T_j^4 - e \sigma T_{j+1}^4 - \rho \tau q_0(j-2) 
+ (1+\rho) \tau q_0(j+1) - \tau q_0(j+3) \right] \]  \hspace{1cm} (3.3.4.6)

Also, by substituting equation (3.3.2.18) into equation (3.3.4.3), we obtain the following equation, for the net radiative exchange for the first face of the first solid slab.

\[ q_{rj} = \frac{1}{(1-\rho j-1)} \left[ (1-\rho j-1) e \sigma T_j^4 - (1-\rho) e_{j-1} \sigma T_{j-1}^4 
+ (1-\rho j-1) \tau q_0(j-1) - (1-\rho) \rho_{j-1} \tau q_0(j+2) \right] \]  \hspace{1cm} (3.3.4.7)

And finally, by substituting equation (3.3.2.21) into equation (3.3.4.4), we get the following equation for the net radiative exchange for the last face of the last solid slab.

\[ q_{rj} = e \sigma T_j^4 - (1-\rho-\tau) q_0(j+1) \]  \hspace{1cm} (3.3.4.8)

where

\[ q_0(j+1) = \sigma T_B^4 \]
In summary, the equations (3.3.4.5), (3.3.4.6), (3.3.4.7), and (3.3.4.8) represent a set of equations that describe the net radiative energy supplied to each solid slab surface to be incorporated into the transient conduction analysis. These equations take into account the radiant energy actually received by the surfaces, and allow the transmitted radiation to pass from the bed to the immersed surface through the holes. The temperature terms $T_j$ in equations (3.3.4.5-8) are evaluated at the current time step, as $T_j^{n+1}$.

The set of equations (3.3.4.5-8) are non-linear due to the fourth power in the temperature. These equations are linearized by expanding $[T_j^4(n+1)]$ into a Taylor series approximation as follows,

$$T_j^4(t+\Delta t) \approx T_j^4(t) + \Delta t \left[ 4 T_j^3(t) \frac{\Delta T}{\Delta t} + \frac{(\Delta t)^2}{2!} 12 T_j^2(t) \left( \frac{\Delta T}{\Delta t} \right)^2 \right] = T_j^4(t) + 4 T_j^3(t) \Delta T + O(\Delta T)^2$$

neglecting the higher order terms, we obtain

$$T_j^4(t+\Delta t) \approx T_j^4(t) + 4 T_j^3(t) [T_j(t+\Delta t) - T_j(t)]$$

$$= 4 T_j^3(t) T_j(t+\Delta t) - 3 T_j^4(t)$$
or

$$T_j^4(n+1) = 4 T_j^3(n) T_j(n+1) - 3 T_j^4(n) \quad (3.3.4.9)$$

### 3.4 System of Simultaneous Equations

The solid slab is divided into an arbitrary number of slices. The edges of these slices are the grid points to be considered for the transient conduction analysis. The temperatures on all grid
points are obtained by solving a set of simultaneous equations. These sets of equations are arranged in one tridiagonal matrix, with the grid points starting from the first face of the first solid slab to the last face of the last solid slab including the points inside the solid slabs as shown in Figure (3.6) for two solid slabs. An energy balance analysis at the boundary surfaces of each solid slab is used for the boundary nodes, while equation (3.2.4c) is used for grid points inside the solid slab and is rewritten below.

\[
- \frac{1}{M} T_{i-1}^{n+1} + (1 + \frac{2}{M}) T_i^{n+1} - \frac{1}{M} T_{i+1}^{n+1} = T_i^n
\]  

(3.4.1)

where \( \frac{1}{M} = \frac{\alpha_s \Delta t}{(\Delta x)^2} \)

Now, taking an energy balance at the first face of the first solid slab, we obtain the following,

\[
K_s \frac{T_{i+1}^{n+1} - T_{-(i-1)}^{n+1}}{2\Delta x} = q_r + h_g (T_i^{n+1} - T_w) + \rho c \frac{\Delta x}{2} \frac{(T_i^{n+1} - T_i^n)}{\Delta t}
\]  

(3.4.2)

By solving for \( T_{i-1}^{n+1} \) in equation (3.4.2) we obtain the following

\[
T_{-(i-1)}^{n+1} = T_{i+1}^{n+1} - \frac{2\Delta x}{K_s} q_r - \frac{2}{K_s} \frac{\Delta x}{\alpha_s \Delta t} (T_i^{n+1} - T_w)
\]  

(3.4.3)

Substituting equation (3.4.3) into equation (3.4.1) for \( T_{i-1}^{n+1} \) the following equation is obtained
Figure 3.6. Grid point arrangement
\[- \frac{2}{M} T_{n+1}^{i+1} + \left[ 2 + \frac{2}{M} + \frac{2N}{M} \right] T_{n+1}^{i} = - \frac{2 \Delta x}{M K_s} q_r \]

(3.4.4)

\[+ \frac{2N}{M} T_w + 2 T_i \]

where \( N = \frac{h g \Delta x}{K_s} \)

The term \( q_r \) in equation (3.4.4) is defined by equation (3.3.4.7), which represents the radiation boundary condition at that point. \( q_r \) is further linearized in temperature by using equation (3.3.4.9). Additional simplification is made by assuming the radiosity terms to be known values obtained from the previous time step. Dimensionless forms of equation (3.4.4) can be obtained by defining the following dimensionless parameters

\[ N = \frac{h g \Delta x}{K_s}, \quad M = \frac{(\Delta x)^2}{\alpha_s \Delta t} \]

\[ P = \frac{\sigma \Delta x T_B^3}{K_s}, \quad R = \frac{P (1-\rho)}{(1-\rho_w)} \]

\[ H = \frac{P (1-\rho)}{(1-\rho_w)}, \quad G = \frac{P e}{(1+\rho)} \]

\[ E = \frac{P T}{(1+\rho)} \]

Hence, after rearrangement and regrouping, equation (3.4.4) becomes

\[ [2 + \frac{2}{M} + \frac{2N}{M} + \frac{8eR}{M} T_i^3 n] T_i^* n+1 - \frac{2}{M} T_i^* n+1 = \frac{6eR}{M} T_i^* 4 n \]

\[+ \frac{2 e_w H}{M} T_w^* 4 + \frac{2 \rho H T}{M} q_o(1+2) - \frac{2 P T}{M} q_{ow}^* n \]

(3.4.6)

\[+ \frac{2N}{M} T_w^* \]
where $T$ and $q_{oi}$ are in dimensionless forms as,

$$T^* = \frac{T}{T_B}, \quad q_{oi}^* = \frac{q_{oi}}{\sigma T_B^4}$$

At the last face of each solid slab the energy balance is written

$$- K_s \frac{T_{i+1}^{n+1} - T_{i-1}^{n+1}}{2 \Delta x} = q_r - h_g (T_{i+1}^{n+1} - T_i^{n+1})$$

$$+ \rho c \frac{\Delta x}{2} \frac{T_{i+1}^{n+1} - T_i^n}{\Delta t} \quad (3.4.7)$$

where $q_r$ in equation (3.4.7) is defined by equation (3.3.4.6). Following the same procedure used to obtain equation (3.4.6) equation (3.4.7) is reduced to a dimensionless form for the last face of each solid slab.

$$- \frac{2}{M} T_i^{*n+1} + \left[ 2 + \frac{2}{M} + \frac{2N}{M} + \frac{8G}{M} T_i^{*3n} \right] T_i^{*n+1}$$

$$- \frac{2N}{M} + \frac{8G}{M} T_{i+1}^{*3n} \right] T_{i+1}^{*n+1} = \frac{6G}{M} T_i^{*4n} - \frac{6G}{M} T_{i+1}^{*4n}$$

$$+ \frac{2\rho E}{M} q_{ow}^{*n} - \frac{2PT}{M} q_{o(i+1)}^{*n}$$

$$+ \frac{2E}{M} q_{o(i+3)}^{*n} + 2 T_i^{*n} \quad (3.4.8)$$

At the first face of each solid slab, the energy balance is written as

$$K_s \frac{T_{i+1}^{n+1} - T_{i-1}^{n+1}}{2 \Delta x} = q_r + h_g (T_{i+1}^{n+1} - T_{i-1}^{n+1})$$

$$+ \rho c \frac{\Delta x}{2} \frac{T_{i+1}^{n+1} - T_i^n}{\Delta t} \quad (3.4.9)$$
where \( q_r \) in equation (3.4.9) is defined by equation (3.3.4.5). Equation (3.4.9) is reduced, following a similar procedure, to a dimensionless form for the first face of each solid slab,

\[
- \left[ \frac{2N}{M} + \frac{8G}{M} T_{i-1}^* \right] T_{i-1}^{n+1} + \left[ 2 + \frac{2}{M} + \frac{2N}{M} \right] T_{i-1}^* \\
+ \frac{8G}{M} T_{i-1}^* T_{i-1}^{n+1} - \frac{2}{M} T_{i+1}^{n+1} = \frac{6G}{M} T_{i-1}^* \Delta n
\]

\[
- \frac{6G}{M} T_{i-1}^* - \frac{2E}{M} q_{o(i)} - \frac{2P}{M} q_{o(i+1)} \\
+ \frac{2E}{M} q_{o(i+2)} + 2 T_{i-1}^*
\]

(3.4.10)

For the last face of the last solid slab, the formulation of the energy balance is

\[
- \frac{T_{i-1}^{n+1} - T_{i-1}^n}{K_s \Delta x} = q_r - h_g (T_B - T_{i-1}^{n+1}) \\
+ \rho c \frac{\Delta x}{2} \frac{(T_{i-1}^{n+1} - T_{i-1}^n)}{\Delta t}
\]

(3.4.11)

where \( q_r \) in equation (3.4.11) is defined by equation (3.3.4.8). Equation (3.4.11) is reduced, by applying the same procedure as above, to a dimensionless form.

\[
- \frac{2}{M} T_{i-1}^* + \left[ 2 + \frac{2}{M} + \frac{2N}{M} + \frac{8Pe}{M} T_{i-1}^* \right] T_{i-1}^{n+1} \\
= \frac{6Pe}{M} T_{i-1}^* + \frac{2P}{M} (1-\rho-\tau) q_{o(i+1)} + \frac{2N}{M} \\
+ 2 T_{i-1}^*
\]

(3.4.12)
Now the set of equations (3.4.1), (3.4.6), (3.4.8), (3.4.10) and (3.4.12) are solved simultaneously using a Gaussian "tri-diagonal matrix" elimination \cite{22} procedure to obtain the temperatures at all grid points for all time steps during the emulsion residence time.

Once the temperature distribution on all grid points is determined at every time step, the instantaneous particle convective flux and the instantaneous radiative heat flux from the emulsion phase are obtained as follows

\begin{align*}
\dot{q}^{''}_{pc} &= h_g (T_{1n+1} - T_w) \quad (3.4.13) \\
\dot{q}^{''}_{rad} &= \frac{1}{(1-\rho_w)} \left[ (1-\rho_w) e \sigma T_1^{4(n+1)} - (1-\rho_w) e \sigma T_w^4 \\
&\quad + (1-\rho_w) \tau q_{03}^{n} \right] \quad (3.4.14)
\end{align*}

where $h_g = k_g / (\varepsilon_g / 2)$

and $k_g$ is evaluated at $(T_{1n+1} + T_w)/2$

3.5 Determination of the Effective Emissivity and Reflectivity of the Slab Surface

In order to determine the effective emissivity and reflectivity of a transmitting solid slab surface, an examination of a transmitting and an opaque surface was made.

Consider the two surfaces shown in Figure (3.7) for the radiant energy received by the two surfaces. For the transmitting surface, the radiant energy balance would be

$$E_{in} = E_{abs} + E_{tr} + E_{ref}$$
a. Opaque surface

b. Transmitting surface

Figure 3.7. Opaque and transmitting surfaces
where $E_{in}$ is the incoming radiant energy

\[ E_{abs} = \alpha E_{in} \]
\[ E_{tr} = \tau E_{in} \]
\[ E_{ref} = \rho E_{in} \]

Therefore,
\[ \alpha + \tau + \rho = 1 \quad (3.5.1) \]

Using Kirchhoff's Law, we have $e = \alpha$

Thus,
\[ e + \tau + \rho = 1 \quad (3.5.2) \]

Hence,
\[ e + \rho = 1 - \tau \quad (3.5.3) \]

For the opaque surface, the radiant energy balance would be,

\[ E_{in} = E^{\circ}_{abs} + E^{\circ}_{ref} \]

where
\[ E^{\circ}_{abs} = \alpha^{\circ} E_{in} \]
\[ E^{\circ}_{ref} = \rho^{\circ} E_{in} \]

Therefore
\[ \alpha^{\circ} + \rho^{\circ} = 1 \quad (3.5.4) \]

Using Kirchhoff's Law, we get $e^{\circ} = \alpha^{\circ}$

Hence, $e^{\circ} + \rho^{\circ} = 1 \quad (3.5.5)$

Equations (3.5.3) and (3.5.5) have to be identical for the purpose of changing the opaque surface to a transmitting one. Therefore, equation (3.5.5) has to be multiplied by $(1 - \tau)$ to obtain the following

\[ (1 - \tau) e^{\circ} + (1 - \tau) \rho^{\circ} = 1 - \tau \quad (3.5.6) \]

Hence, the effective emissivity and reflectivity are

\[ e_{eff} = (1 - \tau) e^{\circ} \quad (3.5.7) \]
\[ \rho_{eff} = (1 - \tau) \rho^{\circ} \quad (3.5.8) \]
3.6 Determination of the Transmissivity Factor

The transmissivity factor \( T \), measures the amount of radiation that is transmitted through the slab. It is obtained by determining the view factor between the planar surface at one end of the slab and the cross sectional surface shown in Figure (2.5) at the center of the slab. The enclosure model for radiation is shown in Figure (3.8). The top planar surface is a square of one particle diameter in dimension, and the two surfaces are one-half particle diameter apart.

Due to the sections of solid spherical surfaces on the side of the enclosure shown in Figure (3.8), some parts of the top surface (square) do not see the cusped surface and vice versa. Considering the sections of solid spherical surfaces to be diffuse, the view factor value between the top surface (square) and the bottom surface (cusped cross section) would be a reasonable approximation.

The view factor between those plane surfaces mentioned earlier is determined according to Plamondon [50], for two parallel plates, as shown in Figure (3.9). The expression that describes the view factor, \( F_{12} \), between surface 1 and surface 2, is as follows

\[
F_{12} = \frac{1}{\pi A_1} \int_{b_2}^{b_1} \int_{a_2}^{a_1} f_2(x_2) g_2(x_1) f_1(x_2) g_1(x_1) dz_2 dx_2 dz_1 dx_1
\]

where

\[
F(z_2, x_2, z_1, x_1) = \frac{L^2}{[(x_2-x_1)^2 + L^2 + (z_2-z_1)^2]^2}
\]
Figure 3.8. Radiation enclosure for view factor calculations.
Figure 3.9. Geometry for radiant interchange between two parallel plates.
with

1) \( f_1(x_2) \) and \( f_2(x_2) \) written in terms of \( x_2 \) and \( z_2 \)

2) surface 1 in the \( y=0 \) plane

3) \( L \) and limits of integration to be supplied.

The two surfaces considered here are shown in Figure (3.10) and are separated by one particle radius. The lower boundary of surface 2 can be described by the following,

\[
y^2 + (x-r_p)^2 = r_p^2
\]  

(3.6.3)

So,

\[
y = \sqrt{r_p^2 - (x - r_p)^2} \quad 0 \leq x \leq r_p
\]  

(3.6.4)

\[
y = \sqrt{r_p^2 + (x + r_p)^2} \quad -r_p \leq x \leq 0
\]  

(3.6.5)

The upper boundary of surface 2 can be described by the following

\[
(y - 2r_p)^2 + (x - r_p)^2 = r_p^2
\]  

(3.6.6)

So,

\[
y = 2r_p - \sqrt{r_p^2 - (x - r_p)^2} \quad 0 \leq x \leq r_p
\]  

(3.6.7)

\[
y = 2r_p + \sqrt{r_p^2 - (x + r_p)^2} \quad -r_p \leq x \leq 0
\]  

(3.6.8)

Having determined the limits of integration for both surfaces, a multidimensional Gaussian Quadrature integration is performed to evaluate the view factor between surface 1 and surface 2 in Figure (3.10). The algorithm suggested by Freeman \[28\] has been adapted for the evaluation of equations (3.6.1) and (3.6.2) for the specified case. A list of the computer code is presented in Appendix (D). The
Figure 3.10. Sketch for the two surfaces involved in the numerical determination of their view factors.
results of the numerical integration for various cases are listed in Table (3.1).

3.7 Radiation Heat Transfer from the Bubble Phase

According to Yoshida et al. [66], the bubble is assumed to be a hemispherical surface above the tube surface as shown in Figure (3.11). The assumption given by Yoshida et al. [66] is just an approximation, while the true geometry is different. The difference is that the surface under the hemispherical surface is not a circular disk, but curved due to the curvature of the cylinder, and the bubble surface is not actually spherical.

The view factor between the wall and the bubble surface is,

\[ F_{w-b} = 1 \]  

(3.7.1)

From the configuration factor algebra, we get

\[ F_{b-w} + F_{b-b} = 1 \]  

(3.7.2)

and from the reciprocity relation, we have

\[ A_b F_{b-w} = A_w F_{w-b} \]

where \( A_b \) is the bubble surface area

\( A_w \) is a plane disk of bubble diameter \( d_b \)

Therefore,

\[ F_{b-w} = \frac{A_w}{A_b} F_{w-b} = \frac{A_w}{A_b} \]  

(3.7.3)

and

\[ F_{b-b} = 1 - \frac{A_w}{A_b} \]  

(3.7.4)

The radiative energy balance for the wall and the bubble surfaces would be
Table 3.1. View Factor Values Between Two Arbitrary Parallel Plates

<table>
<thead>
<tr>
<th>Surface 1</th>
<th>Surface 2</th>
<th>Dimension ((r_p))</th>
<th>Distance Separating them ((H))</th>
<th>View Factor obtained by the numerical integration (F_{1-2})</th>
<th>View Factor (Exact) Values (F_{1-2})</th>
<th>% Diff</th>
<th>Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>(r_p = 1)</td>
<td>(H = 1)</td>
<td>0.11075569</td>
<td>-</td>
<td>-</td>
<td>As shown in Fig. (3.10)</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram 1" /></td>
<td><img src="image4.png" alt="Diagram 2" /></td>
<td>(r_p = 1)</td>
<td>(H = 2)</td>
<td>0.19982506</td>
<td>0.2</td>
<td>-0.087</td>
<td>Both surfaces are opposite to each other in a cube</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram 1" /></td>
<td><img src="image6.png" alt="Diagram 2" /></td>
<td>(r_p = 1)</td>
<td>(H = 1)</td>
<td>0.38203108</td>
<td>0.38196601</td>
<td>0.017</td>
<td>The centers of both surfaces lie in the same axis</td>
</tr>
</tbody>
</table>
Figure 3.11. Radiation from the bubble phase.
Finally, the radiative heat flux in the bubble phase becomes

\[
\frac{Q_{rb}}{Q_{wb}} = 2
\]

with

\[
A_b = \pi d_b^2, \quad A_w = \pi d_b^2
\]

so that

\[
\frac{A_b}{A_w} = 2
\]

With further manipulation between the above equations, the following radiative energy flux for the bubble surface is obtained.

\[
Q_{rb} = A_b \left[ (1-e_b) + \frac{1}{e_b} \left( \frac{1}{e_w} \right) \right] (\sigma T_b^4 - \sigma T_w^4)
\]

Therefore, from equations (3.7.6) and (3.7.8), we have

\[
Q_{wb} = -Q_{rb}
\]

With further manipulation between the above equations, the following radiative energy flux for the bubble surface is obtained.

\[
Q_{rb} = A_b (\sigma T_b^4 - \sigma T_w^4 - q_{wb} + q_{wb})
\]

where \(e_w, e_b\) are the emissivities of the wall and bubble surfaces.

Therefore, from equations (3.7.6) and (3.7.8), we have

\[
Q_{wb} = A_b (q_{wb} - q_{wb})
\]

so that

\[
Q_{wb} = 2
\]

Finally, the radiative heat flux in the bubble phase becomes

\[
\frac{Q_{wb}}{Q_{wb}} = 2
\]
And the radiative heat transfer coefficient from the bubble phase is

\[ h_{rb} = \frac{Q_{rb}}{A_w (T_B - T_w)} \]

Hence,

\[ h_{rb} = \frac{2 \sigma (T_B^4 - T_w^4)}{(\frac{1}{e_b} + \frac{2}{e_w} - 1) (T_B - T_w)} \]  

(3.7.12)

3.8 Gas Convection Heat Transfer From The Emulsion Phase

For beds of large particles \((d_p > 1 \text{ mm})\), convective heat transfer by the fluidizing gas becomes significant. Heat transfer between large particle fluidized beds and immersed surfaces of different arrangements has been reported in the literature. Gabor [31], and Denloye and Botterill [27] have reported data for heat transfer to a vertical tube. Adams and Welty [2] reported a gas convective heat transfer model by analyzing flow within channels between large particles contacting the wall of a horizontal tube.

The gas convection component in this model is determined according to Adams [3, 5] by the following equation

\[ \dot{q}_{gc} = \frac{\text{Nu}_{p2D} K_g^* (T_B - T_w)}{d_p} (1 - \frac{x_s}{S_p}) \]  

(3.8.1)

where

\( \text{Nu}_{p2D} \) is the two-dimensional Nusselt number as shown in Figure (3.12).
Figure 3.12. Sketch for gas convection from the emulsion phase.
\( K^*_g \) is the gas thermal conductivity at mean temperature \((T_w + T_b)/2\).

\( X_s \) is the Stokes region edge location from particle contact point, as shown in Figure (3.12).

\( S_p \) is half the distance between particle centers, as shown in Figure (3.12).

The term \((1 - \frac{X_s}{S_p})\) on the right hand side of equation (3.8.1) is very dependent upon Reynolds number, \( Re_c \), and is determined [5] as follows.

\[
(1 - \frac{X_s}{S_p}) = \begin{cases} 
0.76 + \frac{S_o K_1}{\text{Nu}_{p2D}} & \text{for } \text{Nu}_{p2D} > \text{Nu}_{pm} \\
- K_2 \frac{S_o}{S_p} & \text{for } \text{Nu}_{p2D} < \text{Nu}_{pm} 
\end{cases} \tag{3.8.2}
\]

where \( K_1 \) and \( K_2 \) take on the packed bed values [5]

\[
K_1 \approx 5.26 - 7.72 \left[ \frac{S_o}{r_p} - 1 \right] \tag{3.8.3}
\]

\[
K_2 \approx 0.0127 + 0.0222 \left[ \frac{S_o}{r_p} - 1 \right] \tag{3.8.4}
\]

and

\[
\text{Nu}_{pm} = - \frac{2K_1 S_o}{[0.76 - (0.578 - 4 K_1 K_2)^{1/2}]} \tag{3.8.5}
\]

Also, the two-dimensional Nusselt number for the stagnation-like interstitial flow [5] is,

\[
\text{Nu}_{p2D} = 0.798 \text{ Pr} \left[ \frac{\text{Re}_c}{(0.2 + 0.8 e^{-0.0849 u'_{Re_c}^{1/2}})(r_p S_{r_p})} \right]^{\frac{1}{4}} \tag{3.8.6}
\]

with
\[ S_0 = -\frac{2\alpha'}{(\alpha' + 1)} \left[ 1 - \frac{T_w}{T_B} \right] \left[ 1 - \frac{T_w}{T_B} \right] \frac{\alpha'}{1 + \frac{T_w}{T_B}} \]  

(3.8.7)

where \( \text{Re}_c \) is the Reynolds number based upon the local interstitial velocity and particle diameter \((d_p)\), \(u'\) is the interstitial turbulence intensity and \(S_{p}/r_p = 0.75/\sqrt{1-\varepsilon} \).

The gas convective component is determined once the interstitial gas velocity is known. The interstitial gas velocity is obtained for a horizontal cylinder \([6]\) using the following equation.

\[ \frac{Q_g}{U_{mf}} = 2 \left[ \frac{K_b (\varepsilon)}{K_3 (\varepsilon)} \right] \frac{\sin \Theta}{\varepsilon} \]  

(3.8.8)

where

\[ K_3 (\varepsilon) = \frac{150}{\text{Re}_p} \left[ \frac{1 - \varepsilon}{\varepsilon} \right]^2 + \frac{1.75}{\varepsilon_\infty} \left( \frac{1 - \varepsilon}{\varepsilon_\infty} \right) \]  

(3.8.9)

At the lower stagnation point, where the voidage is high, the two-dimensional Nusselt number is obtained \([6]\) using the following equation

\[ \frac{\text{Nu}_{p2D}}{\text{Nu}_o} = 1.10 \left\{ \frac{K_3 (\varepsilon_\infty) \frac{d_p}{D} \text{Re}_p^{0.7} \frac{\Omega V}{\Omega_{V_o}} - 1}{\varepsilon \frac{K_3 (\varepsilon)}{\varepsilon_\infty} \left[ 1 + \left( \frac{\varepsilon}{\varepsilon_\infty} - 1 \right)^2 \left( 0.65 + \frac{d_p}{D} (12.13 + \frac{991}{\text{Re}_p}) \right) \right]} \right\} \]  

(3.8.10)

with

\[ \frac{\Omega V}{\Omega_{V_o}} = \left[ 0.2 + 0.8 e^{-0.0849u} \sqrt{\frac{\text{Re}_p}{\varepsilon_\infty} \varepsilon} \right] \]  

(3.8.11)

where \(d_p\) and \(D\) are particle and tube diameter, respectively.
To obtain the average $Nu_{p2D}$ around a horizontal tube, a trapezoidal approximation for integration of the local $Nu_{p2D}$, is used as follows

$$Nu_{p2D} = \frac{1}{\Theta} \int_{\Theta}^{\Theta} Nu_{p2D} (\Theta) \, d\Theta$$  \hspace{1cm} (5.8.12)

3.9 Gas Convection Heat Transfer From The Bubble Phase

The gas convection contribution from the bubble phase is dependent on the flow field within the bubble. For a single two-dimensional slow bubble of circular geometry contacting a horizontal tube (valid for large particle beds operating near $U_{mf}$), the boundary layer edge velocity for the flow at the tube surface is independent of bubble size as in Adams [4], and given by

$$\frac{U}{U_{mf}} = 4 \sin \Theta$$  \hspace{1cm} (3.9.1)

with $\Theta$ measured from the lower stagnation point on the tube. Equation (3.9.1) is used along with the method of Smith and Spalding [64] to calculate local instantaneous heat transfer to the tube surface within the contacting bubble as in [4] to yield the following

$$Nu_D (\Theta, \Theta_o) = [\frac{Pr_{0.7} Re_D Sin^b \Theta}{1.13 \int_{\Theta}^{\Theta} \sin^{b-1} \Theta_1 d\Theta_1 + \frac{0.297 \varepsilon_{\infty} d_p}{D \sin^{b-1} \Theta_o}}]^2$$  \hspace{1cm} (3.9.2)

where $\Theta_o$ locates the bubble trailing edge as shown in Figure (3.13)

- $d_p$ is particle diameter
- $\varepsilon_{\infty}$ is the bed voidage
Figure 3.13. Sketch for gas convection from the bubble phase.
The average bubble phase local Nusselt number is determined by integration over possible bubble locations according to [4].

\[
\overline{\text{Nu}_D(\Theta)} = \frac{1}{\Theta} \int_0^\Theta \text{Nu}_D(\Theta, \Theta_o) \, d\Theta_o \tag{3.9.3}
\]

with modification to account for the interstitial turbulence according to Adams [4]

\[
\frac{(\text{Nu}_D)_{u',i}'}{(\text{Nu}_D)_{u',i} = 0} = \left( \frac{\nu}{\nu_{\text{av}}} \right)^{1/2} = (0.2 + 0.8 e^{-0.0849 u} \sqrt{\frac{\text{Re}_D d_p}{\epsilon_{\infty} D}})^{-1/2} \tag{3.9.4}
\]

to yield the following

\[
\text{Nu}_D(0) = \frac{\sin b/2}{\sqrt{\text{Pr}^{0.7} \frac{d_p}{D} \text{Re}_p \frac{\text{Nu}_v}{\text{Nu}_v}}} \int_0^\Theta \frac{d\Theta_o}{1.13 \sin^{b-1} \Theta_o d\Theta_o + 0.297 \epsilon_{\infty} D \frac{d_p}{\epsilon_{\infty} D} \sin^{b-1} \Theta_o} \tag{3.9.5}
\]

where D is tube diameter

\[
b = 2.95 \text{ Pr}^{0.07} \text{(and for gas } b = 3 \text{ is a reasonable approximation)}
\]

For the lower stagnation point on the tube, the bubble phase gas convective contribution is determined using a two-dimensional stagnation point analysis, see Kays [37]. From Figure (3.14), we have \( R = D/2 \),

Thus,

\[
U_\infty = 4 U_{mf} \left( \frac{X_1}{D} \right) = 8 U_{mf} \left( \frac{X_1}{D} \right) \tag{3.9.6}
\]

From Kays [37] we have the following equation for the two-dimensional stagnation point Nusselt number,
Figure 3.14. 2-D stagnation point heat transfer from the bubble phase.
\[ \text{Nu}_x = 0.57 \text{Re}_x^{\frac{1}{2}} \text{Pr}^{0.4} \]  
\( (3.9.7) \)

or
\[ \frac{h_x X_1}{K_g} = 0.57 \left( \frac{U X_1}{\nu} \right)^{\frac{1}{2}} \text{Pr}^{0.4} \]

and
\[ \text{Nu}_D = \frac{h_x}{K_g} \]

So that equation (3.9.7) becomes
\[ \text{Nu}_D = 0.57 \left( \frac{8 U_{mf} D^{\frac{1}{2}}}{\nu} \right) \text{Pr}^{0.4} \]
\[ \text{Nu}_D = 1.61 \text{Re}_D^{\frac{1}{2}} \text{Pr}^{0.4} \]

or, with modification to account for the interstitial turbulence
\[ \frac{\text{Nu}_D}{\sqrt{\text{Re}_D \text{Pr}^{0.7} \frac{\text{Nu}_o}{\text{Nu}}}} = 1.61 \text{Pr}^{0.05} \]  
\( (3.9.8) \)

The average bubble phase gas convective heat transfer around the horizontal tube is obtained by integration, using a trapezoidal rule approximation of
\[ \overline{\text{Nu}}_D = \frac{1}{\Theta} \int_{\Theta}^{\Theta} \text{Nu}_D(\Theta) \, d\Theta \]  
\( (3.9.9) \)

Note that for (\( \Theta > 1.8 \)) separation of the boundary layer occurs, and equation (3.9.5) is not valid there. As an alternative procedure, equation (3.9.8) is used instead for (\( \Theta > 1.8 \) radians) in accordance with Catipovic's data.

3.10 Overall Analysis

The total heat transfer to the immersed surface is determined by obtaining the gas convection contribution from the emulsion and the bubble phases in addition to the particle convective and radiation.
from the emulsion phase and adding to them the radiation from the bubble phase, according to Equation (3.1.2). The gas convection contribution from the emulsion phase is obtained according to section (3.8), and the gas convective heat transfer coefficient is given by the following equation.

$$[h_{gc}] = (1 - \frac{X_S}{S_p}) \text{Nu} \frac{k^*}{\frac{(d_p)}{P_{2D}}}$$  \hspace{1cm} (3.10.1)

where \((1 - \frac{X_S}{S_p})\) is defined by Equation (3.8.2) and \(\text{Nu}_{P_{2D}}\) is defined by Equation (3.8.6).

The local value of the interstitial gas velocity given by Equation (3.8.8) is required for the calculations of \(\text{Nu}_{P_{2D}}\) and \((1 - \frac{X_S}{S_p})\), at any given location around the horizontal tube except at \(\Theta = 0\). At the lower stagnation point, where the voidage is high, the gas convection from the emulsion phase is determined according to Equation (3.8.10), and the gas convective heat transfer coefficient would be as follows

$$[h_{gc}]_{e} = \text{Nu}_{P_{2D}} \frac{k^*}{\frac{(d_p)}{P_{2D}}}$$  \hspace{1cm} (3.10.2)

where \(\text{Nu}_{P_{2D}}\) is defined by Equation (3.8.10).

The gas convection from the bubble phase is obtained by integrating Equation (3.9.5) from the lower stagnation point up to the given location \(\Theta\), except at \(\Theta = 0\) and \(\Theta > 1.8\) radian, where Equation (3.9.5) is not valid. The gas convective heat transfer from the bubble phase is as follows
where $Nu_p(\Theta)$ is defined by Equation (3.9.5). At the lower stagnation point ($\Theta = 0$) and for ($\Theta > 1.8$ rad), the heat transfer coefficient is obtained as follows,

$$[h_{gc}]_{b} = Nu_{p} \left( \frac{k}{d} \right)$$

(3.10.4)

where $Nu_D$ is defined by Equation (3.9.8).

The particle convective and the radiation heat transfer from the emulsion phase are determined once the temperature distribution on all grid points are obtained. This can be accomplished by obtaining the radiosities at all slab surfaces and the heat transfer surface by solving equations (3.3.1.1-3) by an iterative method with the temperature of each surface known.

The temperature distribution on all nodes shown in Figure (3.6) is obtained by setting up equations (3.4.1), (3.4.6), (3.4.8), (3.4.10) and (3.4.12), in a tri-diagonal matrix and solving them simultaneously. The results would be the instantaneous temperature values at all grid points. The temperature of the first node is used to obtain the particle convective and radiative heat transfer coefficients from the emulsion phase according to the following equations

$$h_{pc} = \frac{q_{pc}^{"}}{T_{B} - T_{W}}$$

(3.10.5)
and

\[ [h_{rad}]_{e} = \frac{\dot{q}_{rad}''}{(T_B - T_W)} \]  

(3.10.6)

where \( q''_{pc} \) is defined by equation (3.4.13)

and \( \dot{q}_{rad}'' \) is defined by equation (3.4.14).

The time average values of \( h_{pc} \) and \([h_{rad}]_{e} \) as defined by equations (3.10.5) and (3.10.6) are determined by integrating the instantaneous values over all time steps, using the trapezoidal rule approximation.

The radiative heat transfer coefficient from the bubble phase is determined according to equation (3.7.12).
IV. RESULTS AND DISCUSSION

The analytical model developed in Chapter III and written in Fortran IV has been used to obtain heat transfer coefficients between a fluidized bed and an immersed horizontal tube. Particles with mean diameter less than 1 mm are considered small, while particles with mean diameter greater than 1 mm are considered large. The results of the cold bed study are discussed first, while discussion of the parametric studies as well as the discussion of calculations obtained for hot bed results follow. The input data for these calculations are listed in Table (4.1).

The model was used to obtain cold bed results in which the time-average heat transfer coefficients were obtained for different particle sizes as shown in Figures (4.1-3). The model predictions were then compared with Catipovic’s [24] experimental results for cold bed temperature. For a small particle bed (\(d_p = 0.37\) mm), the similarity between the model predictions and the experimental results are shown in Figure (4.1). The values obtained through the analytical methods using the model developed are, however, higher than the experimental data by about (31-68%). Since the gas layer thickness for small particles is considered small in comparison to that of large particles (\(d_p = 1.3\) mm), this would result in high contact resistance with small particles which is about 3.5 times that of large particles (\(d_p = 1.3\) mm). The residence time data obtained by Catipovic [24] for small particles is large compared to the residence time data for
### Table (4.1). Properties Of Solids And Ranges Of Temperatures Used In The Calculations, Air

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho_s$ g/cm³</th>
<th>$K_s$ W/Cm. °K</th>
<th>$C_{ps}$ W. Sec/g. °K</th>
<th>$\varepsilon_W$</th>
<th>$\varepsilon_\infty$</th>
<th>Pr</th>
<th>$u'$</th>
<th>$T_B$ °K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>2.6</td>
<td>0.003981</td>
<td>0.7997</td>
<td>0.55</td>
<td>0.5</td>
<td>0.72</td>
<td>0.2</td>
<td>Room Temperature</td>
</tr>
<tr>
<td>Dolomite</td>
<td>2.75</td>
<td>0.013</td>
<td>0.879</td>
<td>0.55</td>
<td>0.5</td>
<td>0.72</td>
<td>0.2</td>
<td>Room Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00356</td>
<td>0.879</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1173</td>
</tr>
<tr>
<td>Ione Grain</td>
<td>2.7</td>
<td>0.014-</td>
<td>0.88-</td>
<td>0.55</td>
<td>0.5</td>
<td>0.72</td>
<td>0.2</td>
<td>$K_s$ (366-1589)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0164</td>
<td>1.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$C_{ps}$ (589-1589)</td>
</tr>
<tr>
<td>Coal Ash</td>
<td>0.8</td>
<td>0.00071</td>
<td>0.8368</td>
<td>0.55</td>
<td>0.5</td>
<td>0.72</td>
<td>0.2</td>
<td>1023</td>
</tr>
</tbody>
</table>
Figure 4.1. Variation of average heat transfer coefficients with superficial gas velocity for 0.37 mm sand at low bed temperature.
Figure 4.2. Variation of average heat transfer coefficients with superficial gas velocity for 1.3 mm sand at low bed temperature.
Figure 4.3. Variation of average heat transfer coefficients with superficial gas velocity for 4 mm dolomite at low bed temperature.
large particles. This implies that small particles approach the steady state condition rapidly, and the particle temperature approaches the surface temperature due to its long residence times. Thus, the particle convective heat transfer coefficients would be high. In addition the main heat transfer mechanism for small particles in cold beds is the particle convective component, since all other heat transfer components are insignificant. However, the model predictions for small particles are in good agreement with Baskakov's [9] correlations. Hence, the high model predictions for small particles in cold beds compared to the Catipovic's experimental data are justified.

The agreement between the results of calculations and Catipovic's Data is somewhat improved for very large particles such as $d_p = 4$ mm, shown in Figure (4.3). This is attributed to the stronger influence of the gas convection contribution from the emulsion and bubble phases and the decreasing importance of the particle convective contribution for large particles.

The results of calculations for local heat transfer in cold beds are shown in Figures (4.4-5) for small and large particles along with Catipovic's data. From Figure (4.4) the model predictions are qualitatively similar to the experimental values. However, for small particles the model predicts higher values than the experiment, and the previous discussion for heat transfer in beds of small particles would apply in this case. Figure (4.5) presents the comparison between the model calculations and the experimental data for large
Figure 4.4. Variation of time-average local heat transfer coefficients with superficial gas velocity for 0.37 mm sand at low bed temperature.
Figure 4.5. Variation of time-average local heat transfer coefficients with superficial gas velocity for 4 mm dolomite at low bed temperature.
particles. The agreement here between theory and experiment is excellent except at \((\Theta = 0 \text{ and } \Theta = 180^\circ)\) where the agreement is poorer.

In order to show the difference between the radiation results with and without transmittance at high bed temperatures, calculations of instantaneous radiative heat flux as a function of time are shown in Figure (4.6) for \(T = 0.0\) and \(T = 0.111\), according to Equation (3.3.2.12), along with Thring's [57] results for a bed of (1 mm) diameter sand. The correlations for residence time, bubble contact fraction, minimum and optimum fluidizing velocities used in these calculations (see Thring [57] and Kolar et al. [39]) are

\[
\tau_r = 8.932 \left\{ \frac{d_p}{U_{mf}} \left( \frac{U}{U_{o}} - 1 \right)^2 \right\}^{0.0756} \frac{d_p}{2.54}^{0.5} \tag{4.1}
\]

where \(\tau_r\) is the residence time in (sec) and \(d_p\) is in (cm)

\[
f_o = 0.08533 \left\{ \frac{U_{mf}}{d_p} \left[ \frac{U}{U_{o}} - 1 \right]^2 \right\}^{0.1948} \tag{4.2}
\]

\[
U_{mf} = \frac{g}{d_p} \left\{ \left(33.7\right)^2 + 0.0408 \text{ Ar} \right\}^{0.5} - 33.7 \tag{4.3}
\]

and

\[
U_{opt} = \frac{g}{d_p} \left[ \frac{\text{ Ar}}{18 + 5.22/\text{ Ar}} \right] \tag{4.4}
\]

where

\(f_o\) is the bubble contact fraction
Figure 4.6. Variation of instantaneous radiative heat flux with residence time for 1 mm sand at $T_B = 1173$ °K.
\( U_{mf} \) is the minimum fluidizing velocity

\( U_{opt} \) is the optimum fluidizing velocity

As expected, Figure (4.6) shows that the radiative heat flux is higher for \( \tau = 0.111 \) than for \( \tau = 0.0 \), due to the radiation received from the bed interior. The increase in radiation is about 25% on the average. However, Thring's spherical particle model, which was originally developed by Botterill and Williams [18], predicted higher values than those predicted by the present model when \( \tau = 0.111 \). This difference might be due to the individual particle approach of Thring's model, while in the present model predictions, three solid slabs were used to model three particle layers. Accordingly, the surface temperature of the particles facing the heat transfer surface would increase more rapidly for Thring's model than in the present model. Figure (4.7) shows the instantaneous radiative heat transfer coefficients for both \( \tau = 0.0 \) and \( \tau = 0.111 \). The increase in radiation is about 25% on the average, as for the instantaneous radiative heat flux calculations.

A study was conducted to establish the sensitivity of the present model to variations in the various bed parameters which influence the heat transfer process. Parameters such as particle diameter, superficial velocity, surface temperature, surface and particle emissivities, interstitial turbulence intensity and particle thermal conductivity were considered. For these parametric studies, the bed material was taken to be dolomite at \( (T_B = 1173^\circ K) \) and equations (4.1-4) were used to calculate residence time, bubble contact
Figure 4.7. Variation of instantaneous radiative heat transfer coefficients with residence time for 1 mm sand at $T_B = 1173$ °K.
fraction, minimum fluidizing velocity and the optimum fluidizing velocity.

The results indicate that radiative heat transfer coefficients increase with particle size up to 4 mm, then the radiative heat transfer coefficients are nearly invariant with particle size as shown in Figure (4.8). This result is due to the relatively small change in temperature of the surface facing the heat transfer surface, caused by the larger gas gap with large particles compared to smaller gas gap with small particles. The larger thermal mass of large particles also supports this trend.

The radiation component is directly proportional to the superficial velocity. As $U_o/U_{mf}$ increases from 1 to 2, the increase in radiation is about 11%, while for higher values of $U_o/U_{mf}$, the increase is about 3-5%. The total heat transfer coefficient is directly proportional to $U_o$, until it reaches a maximum value and then starts to decrease. This behavior is due to the decrease in the residence time, hence the cooling of particles takes place slowly, which enhances the radiation. Such behavior is also the result of the increase in the bubble contact fraction in addition to the decrease of residence time as $U_o$ increases, for the total heat transfer coefficient.

As the surface temperature increases (from 300-600 °K), the radiative heat transfer coefficient increases by about 50%. This is mainly due to the decrease in the difference between bed temperature and surface temperature. For the same condition, the total heat
Figure 4.8. Variation of average heat transfer coefficients with particle diameter for sand at $T_B = 1173 \, ^\circ\text{K}$. 

**Legend:**
- Sand
- $T_w = 303\,^\circ\text{K}$, $T_B = 1173\,^\circ\text{K}$
- $U = U_{\text{opt}}$
- $e_w = 0.9$, $e_p = 0.95$
transfer coefficient increases by about 20%, which is lower than the radiation increase. The increase in the total heat transfer coefficient is due to the increase in the particle convective heat transfer coefficient as a result of the increase of gas thermal conductivity at the gas layer adjacent to the immersed surface. As the bed temperature increases from (619-1051 °K), the radiative heat transfer coefficient increases by about 200%. This is due to the increase in the radiative heat flux across the gas layer adjacent the immersed surface.

The emissivity of the heat transfer surface has a great influence on the amount of the radiation received from both the emulsion and the bubble phases. As the heat transfer surface emissivity is changed from 0.4 to 1.0, the total radiation changes by 70%. In all cases the amount of radiation from the bubble phase is much greater than the amount of radiation from the emulsion phase. This is because the radiation calculation for the bubble phase is based on the bed temperature, while the radiation calculation for the emulsion phase is based on the temperature of the first face of the first solid slab. Similar arguments would apply to the particle emissivity influence on the radiation contribution which is directly proportional to the particle emissivity value. However, the emissivities of both the particle and the surface do not affect the mechanisms of other heat transfer components, since the gas convective components are additive components, while the particle convective component might have some effect due to the radiation boundary condition.
The interstitial turbulence intensity has no effect on the radiation and the particle convective components. The gas convection from the emulsion phase increases by about 5% if \( U' \) increases from 0 to 0.2, while the gas convection from the bubble phase would increase by 2% for the same change in \( U' \) of 1 mm diameter dolomite. The gas convection components are affected by the interstitial turbulence intensity through the velocity profile parameter calculation.

The particle thermal conductivity has some effect on the particle convective heat transfer coefficient in a direct way. An increase of 75% in the solid thermal conductivity causes an increase of 13% in the particle convective heat transfer coefficient, while the radiative heat transfer coefficient from the emulsion phase would increase by 15% for the same change in the thermal conductivity. This is due to the increase in the temperature of the first face of the first solid slab, as the thermal conductivity of the particle increases. A summary for these parametric studies is shown in Table (4.2).

Figure (4.9) demonstrates the effect of Fourier number \( (\alpha t_r/d_p^2) \) on the particle convective Nusselt number. The figure shows that \( \text{Nu}_{pc} \) decreases as Fourier number increases, and the maximum value of \( \text{Nu}_{pc} \) is 12.5. Additionally the figure indicates that for a particular gas-solid system, a single curve can be drawn for different particle sizes. The results shown in the figure are obtained using Catipovic's data for residence time.
Table (4.2). Summary of the Parametric Study Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>( h_{\text{rad (total)}} ) Effect</th>
<th>( h_{\text{g conv}} ) Effect</th>
<th>( h_{\text{total}} ) Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_p )</td>
<td>increase</td>
<td>increase slowly</td>
<td>-</td>
<td>decrease sharply and then increase slowly</td>
</tr>
<tr>
<td>( U_o )</td>
<td>increase</td>
<td>increase somewhat sharply and then increase slowly</td>
<td>-</td>
<td>increase until reach maximum and then decrease</td>
</tr>
<tr>
<td>( T_s )</td>
<td>increase</td>
<td>increase</td>
<td>-</td>
<td>increase but relatively less than the radiation increase</td>
</tr>
<tr>
<td>( T_B )</td>
<td>increase</td>
<td>increase sharply</td>
<td>-</td>
<td>increase</td>
</tr>
<tr>
<td>( e_w )</td>
<td>increase</td>
<td>increase</td>
<td>None</td>
<td>increase</td>
</tr>
<tr>
<td>( e_p )</td>
<td>increase</td>
<td>increase</td>
<td>None</td>
<td>increase</td>
</tr>
<tr>
<td>( u' )</td>
<td>increase</td>
<td>none</td>
<td>increase</td>
<td>increase</td>
</tr>
<tr>
<td>( K_s )</td>
<td>increase</td>
<td>increase</td>
<td>-</td>
<td>increase</td>
</tr>
</tbody>
</table>
Figure 4.9. Variation of particle convective Nusselt number with Fourier number for different bed materials.
The variation of maximum total Nusselt number with Archimedes number is shown in Figure (4.10). The values obtained by the present model are based on Catipovic's cold bed data for different sizes. The comparison between the present study results and Catipovic data are quite good at large particle sizes. However, for small particle sizes, the agreement between the model predictions and the Baskakov's [9] cold bed correlations are very good.

The results obtained by the model for large particles at high bed temperature are shown in Figures (4.11) and (4.12) along with preliminary experimental data of Alavizadeh [7] for comparison. The figures show the total and radiative time-averaged heat transfer coefficients. In the experiment the heat transfer surface is treated as a black body. The residence time data required for the calculations are obtained from the experiment, while the fraction of the surface exposed to bubbles is obtained using the correlation according to Catipovic [24].

\[
 f_o = 0.52 - \frac{0.065}{(U_o - U_{mf})} + 0.125 \tag{4.5}
\]

where \( U_o, U_{mf} \) are in (m/s)

Two approaches were used to determine the radiation from the bubble phase. In the first approach the radiation from the bubble phase is obtained according to equation (3.7.12), and the results of that correspond to the solid lines in Figures (4.11-12). For the second approach the radiation from the bubble phase is obtained using
Figure 4.10. Total maximum Nusselt number vs. Archimedes number for several studies.
Figure 4.11. Variation of average heat transfer coefficients with superficial gas velocity for 2.14 mm lone grain at $T_B = 847^\circ K$. 

Legend:
- Present model without modification
- Present model with modification

Parameters:
- Lone Grain $d_p = 2.14$ mm
- $T_B = 847^\circ K$
- $T_W = 365^\circ K$ (a)
- $T_W = 420^\circ K$ (b)
- $e_W = 1.0$, $e_p = 0.7$

Source: Alovizadeh (7)
Ione Grain $d_p = 2.14$ mm

$T_B = 1030$ °K

$T_W = 365$ °K (a)

$T_W = 420$ °K (b)

$e_w = 1.0$, $e_p = 0.7$

O Alavizadeh (7)

--- Present model without modification

--- Present model with modification

Figure 4.12. Variation of average heat transfer coefficients with superficial gas velocity for 2.14 mm Ione grain at $T_B = 1030$ °K.
equation (3.7.12) with some modification. The temperature at the end of the residence time of the first face of the first solid slab is used instead of the bed temperature in equation (3.7.12), and the results of that correspond to dashed lines in Figures (4.11-12). When the bubble comes into contact with the heat transfer surface to displace the old emulsion and bring new fresh emulsion from the core of the bed, it seems that the bubble surface probably contains a mixture of particles from the bed interior and the immersed surface. The results of both approaches indicate bounds.

Figures (4.11-12) show the comparison between the model predictions and the experimental data for large particles. The model predicts higher values than the experimental data for the radiative heat transfer coefficients. It may be noted from the figure that the quantitative disagreement between theory and experiment is mainly due to the difference in the radiative component. The analytical model predictions are brought closer to the experimental data, when the temperature at the end of residence time of the first face of the first solid slab is used in the calculations of the radiation from the bubble phase. The radiative heat transfer coefficients do get closer to the experimental data at low fluidizing velocities for the modified bubble radiation calculations, while at higher fluidizing velocities, they depart from the experimental values. This trend is attributed to the fact that the particles cool significantly at low fluidizing velocities due to the large residence time.
Other hot bed results were obtained by the present model for small particles and were compared with Vadivel and Vedamurthy's [59] experimental results shown in Figures (4.13-16). Equations (4.1-3) are used for the determination of residence time, bubble contact fraction and the minimum fluidizing velocity. The comparison between the model predictions and Vadivel and Vedamurthy's [59] experimental data shown in Figure (4.13), indicates that the radiative heat transfer coefficients obtained by the present model are lower than the experimental data, while the total heat transfer coefficients are substantially higher than the experimental results. This behavior is due to the low values of the radiative heat transfer coefficient, and the high values of the particle convective heat transfer coefficient from the emulsion phase.

Local heat transfer coefficients for a hot bed of small particles obtained by the present model were also compared with Vadivel and Vedamurthy's [59] experimental data shown in Figures (4.14-16). The minimum fluidizing velocity is obtained using equation (4.3), while the local residence time and bubble contact fraction are calculated by considering the local turbulent flow velocity along the tube surface up to the flow separation point, according to Vadivel [59].

\[ U_x = U_o \left[ 1.8155 \left( \frac{x}{r} \right) - 0.4094 \left( \frac{x}{r} \right)^3 - 0.005247 \left( \frac{x}{r} \right)^5 \right] \]  

(4.6)

where \( x \) is the distance along the tube circumference from the stagnation point.

\( r \) is the radius of the tube.
Figure 4.13. Variation of average heat transfer coefficients with superficial gas velocity for 0.82 mm coal ash at $T_B = 1023^\circ K$. 

- Total Coefficients
- Radiative Coefficients
- Present Model
- Vadivel and Vedamurthy (59)

- $T_B = 1023^\circ K$
- $e_w = 0.8$, $e_p = 1.0$

**Average Heat Transfer Coefficients**

$\frac{W}{h\left(\frac{^\circ C}{m^2}\right)}$

**Superficial Gas Velocity $U_0$ (m/s)**
Figure 4.14. Variation of local total and radiative heat transfer coefficients for 0.82 mm coal ash at $T_B = 1023 \, ^\circ \text{K}$, $U_o = 1.7 \, \text{m/s}$. 
Figure 4.15. Variation of local total and radiative heat transfer coefficients for 0.62 mm coal ash at $T_B = 1023 \, ^\circ K$, $U_o = 1.7 \, m/s$. 

---Vadivel and Vedamurthy (59) 
Present Model 

0.62 mm Coal Ash 
$T_B = 1023 \, ^\circ K$ 
$e_w = 0.8$, $e_p = 1.0$ 
$U_o = 1.7 \, m/s$
Figure 4.16. Variation of local total and radiative heat transfer coefficients for 0.82 mm coal ash at $T_B = 1023 \, ^\circ K$, $U_o = 0.87 \, m/s$. 

---Vadivel and Vedamurthy (59)

Present Model

0.82 mm Coal Ash

$T_B = 1023 \, ^\circ K$

e_w = 0.8, \ e_p = 1.0

$U_o = 0.87 \, m/s$
Thus, equation (4.6) along with equations (4.1) and (4.2) are used to obtain the local values around the horizontal tube for the residence time and the bubble contact fraction, up to the flow separation point, \( \Theta = 1.7 \) radian. For \( \Theta > 1.7 \) radian, the superficial gas velocity \( U_o \) is used in equation (4.1) and (4.2) instead of equation (4.6).

Figure (4.14) shows the variation of the total and the radiative heat transfer coefficients with angular locations around the horizontal tube. The model predictions for the radiative coefficients are lower than the experimental data, however, they are somewhat qualitatively similar. The total heat transfer coefficients obtained by the present analytical study are higher than the experimental results. The largest disagreement between the model predictions and the experimental results for the radiative coefficients are at the side section of the tube. Figure (4.15) demonstrates the same variation as in Figure (4.14) for even smaller particle size. For the radiative heat transfer coefficients the agreement between the present model results and the experimental data for particle size \( d_p = 0.62 \) mm shown in Figure (4.15) are much better than the results for the particle size \( d_p = 0.82 \) mm shown in Figure (4.14). From Figures (4.14-15), it can be seen that an increase in the particle size increases the radiative coefficient, but decreases the total coefficient. The variation of the local values of the total and radiative coefficients for \( d_p = 0.82 \) mm and \( U_o = 0.87 \) m/s are shown in Figure (4.16). The qualitative similarity between the present model
results and the experimental data is obvious, while the largest disagreement for the total coefficients occurs at the upper half of the horizontal tube.
V. CONCLUSIONS AND RECOMMENDATIONS

The analytical model described in Chapter II, and developed in Chapter III was evaluated for prediction of heat transfer components in small and large particle beds. Results were obtained for both cold and hot beds, and for local and time-average heat transfer around a horizontal tube. Comparisons were made with Catipovic's [24] experiment for cold bed, and with Alavizadeh [7] and Vadivel and Vedamurthy [59] experimental data for hot bed temperatures. Parametric studies were also made to investigate the influence of several bed parameters on the radiative component and the total heat transfer coefficients. The results suggest the following conclusions:

1. At low bed temperatures in small particle ranges, the particle convective component becomes the predominant mode of heat transfer. The gas layer thickness has a great influence on that component, and the thickness of \((0.16 \, d_p)\) is found to be satisfactory under most conditions.

2. The transmitted radiant energy through voids within the solid slabs increases the radiation contribution by about 25% on the average, over the case when the voids are absent.

3. The radiative heat transfer coefficient varies directly with particle diameter, surface and bed temperatures.
The radiative component is very sensitive to the surface temperature, and to the particle and surface emissivities.

4. The interstitial turbulence intensity has no effect on the radiation and the particle convective components, while it has some influence on the gas convective component.

5. The radiative and the total heat transfer coefficients vary directly with the thermal conductivity of the particles.

6. The radiation heat transfer from the bubble phase is quite significant, and in most cases is much higher than the radiation heat transfer from the emulsion phase.

7. At cold bed temperatures for small particles \((d_p < 1\text{mm})\), the total average heat transfer coefficients based upon the model are higher than those obtained by experiment, while for large particles \((d_p > 1\text{mm})\) the model predictions are in reasonable agreement with the experimental results.

8. The local values of the heat transfer coefficients obtained by the model are qualitatively similar to the experimental data for small particles at low bed temperatures, while the agreement between the model predictions and the experimental data are improved for large particles.
9. At high bed temperatures, the average radiative and the total heat transfer coefficients obtained by the model are higher than the experimental data for large particles of Ione grain.

10. The average radiative heat transfer coefficients predicted by the model are lower than the experimental data, while the total coefficients are higher than the experimental results for small particles at high bed temperature.

11. The agreement between the local radiative heat transfer coefficients predicted by the analytical model and the experimental data are reasonable for small particles at high bed temperatures.

Further studies could be pursued by considering the gas to be a radiatively participating media. This would account for the radiation attenuation in both the emulsion and the bubble phases, so that a more complete understanding of the radiation phenomena in gas fluidized beds can be obtained.
BIBLIOGRAPHY


APPENDICES
APPENDIX A

THE COMPUTER CODE
A computer code has been written in Fortran IV and developed to compute various values of the heat transfer coefficient between the bed and the heat transfer surface. The code is composed of one main program called XXXHEAT and ten subroutines. All heat transfer calculations are done in XXXHEAT, while most of the subroutines are called from XXXHEAT to obtain the necessary intermediate values. The input data to the main program is given and described on pages 109 and 110.

The subroutine FKGAS and VISCGAS are called frequently to compute the thermal conductivity and viscosity of the gas as a function of temperature using the formula in White [64].

The first stage of the calculations is the determination of the heat transfer by gas convection from the emulsion phase by calling the subroutine NUP2D in which the two dimensional Nusselt number, and \((1 - X_s/S_p)\cdot\) values are obtained. The calling of the subroutine NUP2D requires calling the subroutine GASVEL to obtain the local value of the interstitial gas velocity. At the lower stagnation point, the gas convection from the emulsion phase is obtained by calling the subroutine FNU2DST, in which the two dimensional Nusselt number is obtained.

The gas convection from the bubble phase is determined by calling the subroutine BUBHTC at any particular location around the horizontal tube except at \((\theta = 0\) and \(\theta > 1.8\) radians), in which the Nusselt number is obtained. For \((\theta = 0\) and \(0 \geq 1.8\) radians), the
Nusselt number is obtained by a simple expression derived according to equation (3.9.8).

The determination of the radiosities at all slab surfaces is accomplished by calling the subroutine RADIOST, which is essentially an iterative procedure, with all slab surface temperatures known. These radiosity values are used in the transient conduction analysis for the next time step.

The transient conduction calculation starts by setting up the coefficients A, B, C and D for all grid points in one tri-diagonal matrix, where the temperatures at those grid points are the unknown values. The subroutine TRIDIAG is then called to solve the system of equations simultaneously by the direct Gaussian elimination method to obtain the temperatures at all grid points at a particular time step. The procedure described above is repeated at every time step to the end of the residence time. The particle convective and the radiative heat flux are obtained also at each time step, using the temperature value at the first face of the first solid slab just obtained.

The determination of the radiative heat transfer coefficient from the bubble phase is obtained by an expression according to equation (3.7.12). The average values of the instantaneous particle convective and radiative heat transfer coefficients from the emulsion phase are obtained by integration using the trapezoidal rule approximation. A typical output of these calculations are the total heat transfer coefficient from the emulsion phase, the total heat transfer
coefficient from the bubble phase and the combined total heat transfer coefficient in addition to the various component heat transfer coefficients.
APPENDIX B

FORTRAN NOTATION FOR (XXXHEAT) PROGRAM
## FORTRAN NOTATION FOR (XXXHEAT) PROGRAM

<table>
<thead>
<tr>
<th>Program Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>Archimedes number</td>
</tr>
<tr>
<td>ALFA</td>
<td>Particle thermal diffusivity</td>
</tr>
<tr>
<td>DLG</td>
<td>Gas layer thickness</td>
</tr>
<tr>
<td>DXSSP</td>
<td>((1 - X_s/S_p))</td>
</tr>
<tr>
<td>RAUG</td>
<td>Gas density</td>
</tr>
<tr>
<td>VISCG</td>
<td>Kinematic viscosity of gas</td>
</tr>
<tr>
<td>REPMF</td>
<td>Reynolds number computed by a correlation at minimum fluidizing condition</td>
</tr>
<tr>
<td>REP</td>
<td>Reynolds number based on minimum fluidizing velocity</td>
</tr>
<tr>
<td>DX</td>
<td>Solid slice thickness</td>
</tr>
<tr>
<td>P</td>
<td>Dimensionless parameter as in Eq. (3.4.5)</td>
</tr>
<tr>
<td>PR</td>
<td>Dimensionless parameter ((R)) as in Eq. (3.4.5)</td>
</tr>
<tr>
<td>HR</td>
<td>Dimensionless parameter ((H)) as in Eq. (3.4.5)</td>
</tr>
<tr>
<td>G</td>
<td>Dimensionless parameter as in Eq. (3.4.5)</td>
</tr>
<tr>
<td>ET</td>
<td>Dimensionless parameter ((E)) as in Eq. (3.4.5)</td>
</tr>
<tr>
<td>DELT</td>
<td>Time step size</td>
</tr>
<tr>
<td>FM</td>
<td>Fourier number</td>
</tr>
<tr>
<td>FON</td>
<td>Fourier number based on &quot;RTIME&quot; and &quot;DIAM&quot;</td>
</tr>
<tr>
<td>GASK</td>
<td>Gas thermal conductivity at mean temperature</td>
</tr>
<tr>
<td>OMEGAR</td>
<td>Velocity profile parameters ratio</td>
</tr>
<tr>
<td>FNUSTG</td>
<td>Emulsion phase 2-D Nusselt No. at the stagnation point</td>
</tr>
<tr>
<td>NU2D</td>
<td>2-D Nusselt No. (Emulsion phase)</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>------------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td>BNUD</td>
<td>Bubble phase Nusselt number</td>
</tr>
<tr>
<td>GASBHTC</td>
<td>Bubble phase gas convection H.T.C.</td>
</tr>
<tr>
<td>VELG</td>
<td>Interstitial gas velocity (dimensionless)</td>
</tr>
<tr>
<td>QGC</td>
<td>Interstitial gas velocity</td>
</tr>
<tr>
<td>REC</td>
<td>Channel Reynolds number</td>
</tr>
<tr>
<td>SPRP</td>
<td>((S_p/r_p)) ratio</td>
</tr>
<tr>
<td>DXSSP</td>
<td>((X_s/S_p)) ratio</td>
</tr>
<tr>
<td>NUPM</td>
<td>Parameter appearing in formula for Stokes region edge location</td>
</tr>
<tr>
<td>HW2DEM</td>
<td>Emulsion phase gas convection H.T.C.</td>
</tr>
<tr>
<td>ENU2DL(I)</td>
<td>An array for local 2-D Nusselt number</td>
</tr>
<tr>
<td>BNUDL(I)</td>
<td>An array for local bubble phase Nusselt No.</td>
</tr>
<tr>
<td>TOLD(I)</td>
<td>An array for temperature at the previous time step</td>
</tr>
<tr>
<td>TNEW(I)</td>
<td>An array for temperatures at the current time step</td>
</tr>
<tr>
<td>QR(I)</td>
<td>An array for the surface radiosity</td>
</tr>
<tr>
<td>TR(I)</td>
<td>An array for temperatures at the radiation surfaces</td>
</tr>
<tr>
<td>NITER</td>
<td>Number of iterations at which radiosity eqns converge</td>
</tr>
<tr>
<td>QRADIO</td>
<td>Initial radiative flux from the emulsion phase</td>
</tr>
<tr>
<td>QCONVIO</td>
<td>Initial particle convective heat flux</td>
</tr>
<tr>
<td>HWREIO</td>
<td>Initial radiative H.T.C. from the emulsion phase</td>
</tr>
<tr>
<td>HWCEIO</td>
<td>Initial particle convective heat transfer coefficient</td>
</tr>
<tr>
<td>KGAS</td>
<td>Gas thermal conductivity</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------</td>
</tr>
<tr>
<td>BU</td>
<td>Biot number</td>
</tr>
<tr>
<td>A(I), B(I), C(I)</td>
<td>Coefficient arrays for the tri-diagonal matrix</td>
</tr>
<tr>
<td>D(I)</td>
<td>Right hand side array of the tri-diagonal matrix</td>
</tr>
<tr>
<td>QRAD</td>
<td>Instantaneous radiative heat flux from the emulsion phase</td>
</tr>
<tr>
<td>HWRE(I)</td>
<td>Instantaneous radiative H.T.C. from the emulsion phase</td>
</tr>
<tr>
<td>QCONV</td>
<td>Instantaneous particle convective heat flux</td>
</tr>
<tr>
<td>HWCE(I)</td>
<td>Instantaneous particle convective H.T.C.</td>
</tr>
<tr>
<td>HWRB</td>
<td>Radiative H.T.C. from the bubble phase</td>
</tr>
<tr>
<td>HWREM</td>
<td>Mean radiative H.T.C. from the emulsion phase</td>
</tr>
<tr>
<td>HWCEM</td>
<td>Mean particle convective H.T.C.</td>
</tr>
<tr>
<td>EMPNU</td>
<td>Emulsion phase particle convective Nusselt number</td>
</tr>
<tr>
<td>HTOTRAD</td>
<td>Total radiative H.T.C. from emulsion and bubble phases</td>
</tr>
<tr>
<td>HTOTEMU</td>
<td>Total H.T.C. from emulsion phase</td>
</tr>
<tr>
<td>HTOTBUB</td>
<td>Total H.T.C. from bubble phase</td>
</tr>
<tr>
<td>HWTOT</td>
<td>Total H.T.C. from emulsion and bubble phases</td>
</tr>
<tr>
<td>TOTNU</td>
<td>Total Nusselt number</td>
</tr>
</tbody>
</table>
APPENDIX C

LISTING OF THE COMPUTER PROGRAM (XXXHEAT)
PROGRAM XXXHEAT(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

THIS PROGRAM CALCULATES LOCAL AND AVERAGE TOTAL HEAT TRANSFER
COEFFICIENTS AS WELL AS OTHER SPECIFIC HEAT TRANSFER COEFFIC
IENTS, TO A HORIZONTAL TUBE IMMERSED IN GAS FLUIDIZED BEDS

COMMON /ANU/ NU2D,DXSSP,NUPM,SPRP,REC,TS,TB,PRN,UPR
COMMON /RAD/ EMISW,REFLW,EMIS,REFF,TAU
COMMON /VEL/ VELG,THET,VOID,VOIDE,SPHI,REP
COMMON /VISC/ TAIR,VISC0,RAUG
COMMON /BUB/ VOIDB,THET,DTUBE,DIAM,BINT,EDEL

DIMENSION A(150),B(150),C(150),D(150),TR(51),OR(51),NB(51),
TOLD(150),TNEW(150),HWRE(150),HWCE(150),ENU2DL(51),BNUDL(51)

REAL KSOLID,KGAS,NU2D,NUPM

READ INPUT DATA

ALL INPUT DATA SHOULD BE GIVEN IN THESE DIMENSIONS
TIME IN SECONDS
LENGTH IN CENTIMETERS
WEIGHT IN GRAMS
POWER IN WATTS
TEMPERATURE IN DEGREE KELVIN
ANGLE IN RADIANS

SIGMA=STEFAN-BOLTZMANN CONSTANT
PRN=PRANDTL NUMBER
UPR=INTERSTITIAL TURBULENCE INTENSITY
TS=SURFACE TEMPERATURE
TB=BED TEMPERATURE
DIAM=PARTICLE DIAMETER
DTUBE=TUBE DIAMETER
KSOLID=PARTICLE THERMAL CONDUCTIVITY
RAUS=PARTICLE DENSITY
CPS=PARTICLE SPECIFIC HEAT
VOID=LOCAL VOIDAGE
VOIDE=BED VOIDAGE
SPHI=PARTICLE SPHERICITY
EMISW=SURFACE EMITTANCE
EMISS=PARTICLE EMISSIVITY
REFLW=SURFACE REFLECTANCE
REFF=PARTICLE REFLECTANCE
TAU=TRANSMITTANCE
NOS=NO. OF SLAB SLICES
NOG=NO. OF GRIDS
NOR=NO. OF RADIATION SURFACES
NSLAB=NO. OF SOLID SLABS
NTSTEP=NO. OF TIME STEPS
THET=ANGULAR LOCATION AROUND THE TUBE
NFLAG=AN INTEGER ( 0, AVERAGE CALC.) ( 1, LOCAL CALC.)
NTHETA=AN INTEGER (TO DIVIDE ANGULAR SPACING INTO "N" SEGMENTS )
NBINTG=AN INTEGER ASSIGNED FOR BUBBLE INTEGRATION
NTNEW= (IF "0" DO NOT PRINT CURRENT TEMPERATURES)
IFREQ=AN INTEGER USED FOR PRINTING BETWEEN SUCCESSIVE ITERSATIONS)
RTIME=RESIDENCE TIME
BFO=BUBBLE CONTACT FRACTION
UMF=MINIMUM FLUIDIZING VELOCITY
UZERO=SUPERFICIAL VELOCITY
UOPT.=OPTIMUM FLUIDIZING VELOCITY
UXR=LOCAL VELOCITY AROUND A HORIZONTAL TUBE FOR OBTAINING
LOCAL RESIDENCE TIME AND BUBBLE CONTACT FRACTION

PAI=3.14159
SIGMA=5.729E-12
READ(*,*)PRN,UPR
READ(*,*)TS,TB,DIAM,Dtube
READ(*,*)KSOLID,RAUS,CPS
READ(*,*)VOID,VOIDE,SPM
READ(*,*)EMISW,EMISS,REFLW,REFL,TAU
READ(*,*)NOS,NSLAB,NTSTEP
READ(*,*)THET,NFLAG,NTHETA,NBINTEG
READ(*,*)NTNEW,IFREQ
READ(*,*)RTIME,BFO
READ(*,*)UMF,UZER

C
NGS=NSLAB*(NOS+1)
NR=2*NSLAB+1
NTHET=NTHETA+1
OTHET=PI/FLOAT(NTHETA)
NBUBINT=NBINTEG
C
ALFA=KSOLID/(RAUS*CPS)
TW=TS/TB
DLG=0.16*DIAM
C
TAIR=TB
CALL VISCGAS
C
C
C
C
C

C (UMF) IS OBTAINED HERE BY A CORRELATION
C (UZERO OR UOPT ) IS OBTAINED HERE BY A CORRELATION

AR=(980.67*(DIAM**3)/VISCG**2)*(RAUS/RAUG-1.0)
REPMPF=(33.7**2+0.0408*AR)**0.5-33.7
IF(UMF .NE. 0.0)60 TO 150
UMF=REPMPF*(VISCG/DIAM)
150
REP=UMF*DIAM/VISCG
C
IF(UZERO .NE. 0.0)00 TO 152
UZER0=(VISCG/DIAM)*(AR/(18.+5.22*SQRT(AR)))
152
CONTINUE
C
C
C
C
C

C (RTIME) IS OBTAINED HERE BY A CORRELATION
C (BFO) IS OBTAINED HERE BY A CORRELATION

RUUMF=(UMF**2)*((UXR/UMF-1.0)**2)
C
IF(RTIME .NE. 0.0)GO TO 162
C
RTIME=8.932*(((DIAM**980.67)/RUUMF)**0.0756)*((DIAM/2.54)**0.5)
C
162 IF(BFO .NE. 0.0)GO TO 165
BFO=0.08553*(((RUUMF/(DIAM**980.67)))**0.1948)
C
165 WRITE(6,555)
WRITE(6,230)SIGMA
WRITE(6,232)KSOLID,RAUS,CPS
WRITE(6,234)RAUG,VISCG
WRITE(6,242)AR
WRITE(6,392)UMF,UZER
WRITE(6,240)VOID, VOIDE, SPHI, REP
WRITE(6,236)NOS, N0G, NOR, NSLAB

EMIS = (1.-TAU) * EMISS
REFF = (1.-TAU) * REFL

WRITE(6,300)TS, TB, DIAM
WRITE(6,388)DTUBE, DLG
WRITE(6,304)RTIME, BFO
WRITE(6,306)EMISW, REFLW, EMISS, REFL
WRITE(6,308)EMIS, REFF, TAU

DX = (2./3.) * DIAM / FLOAT(NOS)
P = SIGMA * DX * (TB**3) / KSOLID
PR = P * (1.-REFLW) / (1.-REFF*REFLW)
HR = P * (1.-REFF) / (1.-REFF*REFLW)
G = P * EMIS / (1.+REFF)
ET=P*TAU/(1.+REFF)

DELT=RTIME/FLOAT(NTSTEP)
FM=ALFA*DELT/DX**2
FON=(RTIME/DELT)*(DX**2/DIAM**2)*FM

WRITE(6,238)NTSTEP,DELT,DX,FM
WRITE(6,252)FON

JADJ=2
TR(1)=TW
TR(2)=1.0
GASK=FKGAS(JADJ,TB,TR)
WRITE(6,390)GASK

IF(NFLAG .EQ. 0)GO TO 90
C
C LOCAL CALCULATIONS OF EMULSION AND BUBBLE GAS CONVECTIVE H.T.C.
C AROUND A HORIZONTAL TUBE
C
IF(THET .NE. 0.)GO TO 2
C
TAIR=0.5*(TS+TB)
CALL VISCGAS
C
REP=UMF*DIAM/VISCG
OMEGAR=0.2*0.8*EXP(-0.0849*UPR*SQRT(REP/VOID))
FNUSTG=FNU2DST(0.9,VOID,SPHI,DIAM,DTUBE,REP,PRN,UPR)
NU2D=FNUSTG
BCONST=SORT((PRN**0.7)*(DIAM/DTUBE)*REP*(1./OMEGAR))
BNUD=1.61*BCONST*(PRN**0.05)
WRITE(6,244)THET
GO TO 3
C
2
THETA=THET
VOIDB=VOID
WRITE(6,244)THET
C
TAIR=0.5*(TS+TB)
CALL VISCGAS
C
REP=UMF*DIAM/VISCG
OMEGAR=0.2*0.8*EXP(-0.0849*UPR*SQRT(REP/VOID))
BCONST=SORT((PRN**0.7)*(DIAM/DTUBE)*REP*(1./OMEGAR))
C
IF(THETA .GE. 1.8 )GO TO B
C
CALL BUBHTC
C
BNUD=BCONST*BNUD
GO TO 9
B
BNUD=1.61*BCONST*(PRN**0.05)
B
GASBHTC=BNUD*(GASK/DIAM)
C
CALL GASVEL
C
QGC=VELG*REP*(VISCG/DIAM)
REC=QGC*DIAM/VISCG
SPRP=0.75/SQRT(1.-VOID)
CALL NUP2D

C
WRITE(6,378)SPRP,REC,DXSSP,NUPM
HW2DEM= DXSSP*NU2D*(GASK/DIAM)
WRITE(6,384)NU2D,BNUD
GO TO 4
C

3
HW2DEM=NU2D*(GASK/DIAM)
GASBHTC=BNUD*(GASK/DIAM)
WRITE(6,384)NU2D,BNUD
GO TO 4
C
C
AVERAGE CALCULATIONS OF EMULSION AND BUBBLE GAS CONVECTIVE H.T.C
C
AROUND A HORIZONTAL TUBE
C
90 DO 93 I=1,NTHET
IF(THET .NE. 0.) GO TO 91

C
TAIR=0.5*(TS+TB)
CALL VISCGAS
C
REP=UMF*DIAM/VISCG
OMEGAR=0.2*0.8*EXP(-0.0849*UPR*SQRT(REP/VOID))
FNUSTG=FNU2DSTD(0.9,VOIDE,SPHI,DIAM,DTUBE,REP,PRN,UPR)
NU20=FNUSTG
ENU2DL(I)=NU2D
BCONST=SQRT((PRN**0.7)*(DIAM/DTUBE)*REP*(1./OMEGAR))
BNUD=1.61*BCONST*(PRN**0.05)
BNUDL(I)=BNUD
THET=FLOAT(I)*DTHET
GO TO 93
C
91
J=I-1
NBINTEG=J*NUBINT
THET=FLOAT(J)*DTHET
THETA=THET
VOIDB=VOIDE
C
TAIR=0.5*(TS+TB)
CALL VISCGAS
C
REP=UMF*DIAM/VISCG
OMEGAR=0.2*0.8*EXP(-0.0849*UPR*SQRT(REP/VOID))
BCONST=SQRT((PRN**0.7)*(DIAM/DTUBE)*REP*(1./OMEGAR))
C
IF(THETA .GE. 1.8 )GO TO 17
C
CALL BUBHTC
BNUD=BCONST*BNUD
GO TO 19
17
BNUD=1.61*BCONST*(PRN**0.05)
19
BNUDL(I)=BNUD
C
CALL GASVEL
QGC=VELG*REP*(VISCG/DIAM)
REC=QGC*DIAM/VISCG
SPRP=0.75/SQRT(1.-VOID)
C
CALL NUP2D
C
ENU2DL(I)=DXSSP*NU2D
93
CONTINUE
C
SUMNU=0.0
SUMBN=0.0
DO 96 I=1,NTHET
IF(I .EQ. 1 .OR. I .EQ. NTHET)GO TO 94
SUMNU=SUMNU+2.*ENU2DL(I)
SUMBN=SUMBN+2.*BNUDL(I)
GO TO 96
94
SUMNU=SUMNU*ENU2DL(I)
SUMBN=SUMBN*BNUDL(I)
96
CONTINUE
NU2D=SUMNU/(2.*FLOAT(NTHETA))
BNUD=SUMBN/(2.*FLOAT(NTHETA))

WRITE(6,250)NU2D,BNUD

HW2DEM=NU2D*(GASK/DIAM)
GASBHTC=BNUD*(GASK/DIAM)

SET INITIAL TEMPERATURES

4 DO 5 I=1,NOG
TOLD(I)=1.0
5 CONTINUE
TOLD(NOOG+1)=1.

SET INITIAL RADIOSITIES AT THE RADIATION SURFACES

DO 6 J=1,NOR
QR(J) = 1.0
CONTINUE
QR(NOR+1) = 1.0
C
DO 10 J = 1, NOR
IF (J .NE. 1) GO TO 7
TR(J) = TW
GO TO 10
7 TR(J) = 1.0
CONTINUE
TR(NOR+1) = 1.0
WRITE (6, 555)
C
IF (TAU .EQ. 0.) GO TO 97
CALL RADIOST (NOR, TR, QR, NITER)
WRITE (6, 210) NITER
WRITE (6, 217)
WRITE (6, 215) (TR(I), I = 1, NOR)
WRITE (6, 218)
WRITE (6, 215) (QR(I), I = 1, NOR)
97 WRITE (6, 554)
RRF = (1. - REFF) / (1. - REFF*REFLW)
RRFW = (1. - REFLW) / (1. - REFF*REFLW)
C
CALCULATE "INITIAL" EMULSION RADIATIVE AND PARTICLE CONVECTIVE
HEAT FLUX
QRADIO = 10. * SIGMA * (TB**4) * (RRFW * (EMIS + TAU * QR(4)) - RRF * EMISW * TW4**4)
HGAS = GAS / (0.5 * DLG)
OCONVIO = 10. * HGAS * (TB - TS)
C
CALCULATE "INITIAL" EMULSION RADIATIVE AND PARTICLE CONVECTIVE
H.T.C.
HWREIO = QRADIO / (TB - TS)
HWCEIO = HGAS * 10.
C
WRITE (6, 374) QCONVIO, QRADIO
WRITE (6, 382) HWCEIO, HWREIO
C
START TRANSIENT CONDUCTION CALCULATIONS TO THE END OF RESIDENCE
TIME
C
NCOUNT = 0
NFREQ = 0
TIME = 0.0
99 NCOUNT = NCOUNT + 1
NFREQ = NFREQ + 1
TIME = TIME + DELT
C
SET UP COEFFICIENTS ARRAY A, B, C, D AND PERFORM CALCULATIONS
C
ICOUNT = 1
DO 50 I = 1, NOG
IF (I .NE. 1) GO TO 12
J = I + 1
NB(J) = I
C JADJ = J

C KGAS = FKGAS(JADJ, TB, TR)

C HGAS = KGAS / (0.5 * DLG)
BU = (HGAS / KSOLID) * DX
B(I) = 1.0 + FM * (2 + 2 * BU + 8 * EMIS * PR * TOLD(I) ** 3)
C(I) = -2 * FM
D(I) = 6 * FM * EMIS * PR * TOLD(I) ** 4 + 2 * FM * EMISW * HR * TW ** 4 + 2 * FM * REF LW * HR * TAU * QR(J + 2) - 2 * FM * P * TAU * Q(R(J - 1)) + 2 * FM * BU * TW + TOLD(I)

GO TO 50

12 IF(ICOUNT .GT. 1) GO TO 30
15 A(I) = -FM
B(I) = (1 + 2 * FM)
C(I) = -FM
D(I) = TOLD(I)
IF(I .EQ. ICOUNT * (NOS + 1)) GO TO 20
GO TO 50
20 IF(I .EQ. NOG) GO TO 45
   J=1-ICOUNT*(NOS-1)
   NB(J)=I
   JADJ=J+1
   KGAS =FKGAS(JADJ,TB,TR)
   H=KGAS/DLG
   BU=(KGAS/KSOLID)*DX
   A(I)=-2.*Fm
   B(I)=1.+Fm*(2.*BU+8.*G*TOLD(I)**3)
   C(I)=Fm*(2.*BU+8.*G*TOLD(I)**3)
   D(I)=6.*Fm*G*TOLD(I)**4-6.*Fm*G*TOLD(I+1)**4+2.*Fm*REFF*ET*QR(J-2)
   +2.*Fm*TAGUR(J-1)+2.*Fm*ET*QR(J+3)
   ICOuNT=ICOUNT+1
   GO TO 50
30 NOFF=(ICOUNT-1)*(NOS+1)+1
   IF(I .EQ. NOFF) GO TO 40
   GO TO 15
40 J=I+1-(ICOUNT-1)*(NOS-1)
   NB(J)=I
   JADJ=J
   KGAS =FKGAS(JADJ,TB,TR)
   H=KGAS/DLG
   BU=(KGAS/KSOLID)*DX
   A(I)=-Fm*(2.*BU+8.*G*TOLD(I-1)**3)
   B(I)=1.+Fm*(2.*BU+8.*G*TOLD(I)**3)
   C(I)=-2.*Fm
   D(I)=6.*Fm*P*EMIS*TOLD(I)**4+2.*Fm*P*(1.-REFF-TAU)*OR(J+1)
   +2.*Fm*BU+ TOLD(I)
   ICOuNT=ICOUNT+1
   GO TO 50
45 J=I+1-ICOUNT*(NOS-1)
   NB(J)=I
   JADJ=J+1
   KGAS =FKGAS(JADJ,TB,TR)
   H=KGAS/DLG
   BU=(KGAS/KSOLID)*DX
   A(I)=-2.*Fm
   B(I)=1.+Fm*(2.*BU+8.*G*TOLD(I)**3)
   C(I)=Fm*(2.*BU+8.*G*TOLD(I)**3)
   D(I)=6.*Fm*P*EMIS*TOLD(I)**4+2.*Fm*P*(1.-REFF-TAU)*OR(J+1)
   +2.*Fm*BU+ TOLD(I)
   CONTINUE
50 CALL TRIDIAG(1,NOG,A,B,C,D,TNEW)
55 RNCOuNT=FLOAT(NCOUNT)/FLOAT(NTSTEP)
   DO 60 J=1,NOR
      TR(J)=Tw
      GO TO 60
   CONTINUE
IF (TAU .EQ. 0.) GO TO 65

CALL RADIOST(NOR, TR, QR, NITER)

DO 70 I = 1, NOG
  TOLD(I) = TNEW(I)
70  CONTINUE

QRADI = RRF * EMIS * TW**4 - RRFW * EMIS * TNEW(1)**4 - RRFW * TAU * QR(4)
QRAD = SIGMA * (TB**4) * QRADI * (-1.) * 10.
IC = NCOUNT
HWRE(IC) = QRAD / (TB - TS)

KGAS = FKGAS(2, TB, TR)

HGAS = KGAS / (.5 * DLG)
QCONV = HGAS * (TNEW(1) - TW) * TB * 10.
C
C
C

HWCE(IC) = QCONV / (TB-TS)
QTOTAL = QRAD + QCONV
C
IF(NFREQ .NE. IFREQ) GO TO 75
NFREQ = 0
C
WRITE(6,554)
WRITE(6,200) DELT, TIME, RNCOUNT
C
IF(NTNEW .EQ. 0) GO TO 72
C
WRITE(6,207)
WRITE(6,205) (TNEW(I), I=1,9)
WRITE(6,205) (TNEW(I), I=10,18)
WRITE(6,205) (TNEW(I), I=19,27)
C
IF(TAU .EQ. 0.) GO TO 73
C
WRITE(6,210) NITER
WRITE(6,208)
WRITE(6,215) (TR(I), I=1,NOR)
WRITE(6,209)
WRITE(6,215) (QR(I), I=1,NOR)
WRITE(6,554)
C
WRITE(6,222) OCONV, ORAD, QTOTAL
WRITE(6,350) HWCE(IC), HWRE(IC)
WRITE(6,204) TNEW(1)
C
END OF TRANSIENT CONDUCTION CALCULATIONS
C
CALCULATE BUBBLE RADIATIVE H.T.C. (3-D)
C
RWRB = (1./EMISS + 2./EMISW - 1.)*(TB-TS)
HWRB = 2.*SIGMA*(TB**4 - TS**4)/RWRB
HWRB = HWRB*10.
C
INTEGRATE THE INSTANTANEOUS EMULSION RADIATION H.T.C. AND OBTAIN
THE MEAN VALUE
C
SHWRE = 0.0
DO 80 I=1, NTSTEP
IF(I .EQ. NTSTEP) GO TO 79
SHWRE = SHWRE + 2.*HWRE(I)
GO TO 80
79
SHWRE = SHWRE + HWRE(I)
80
CONTINUE
HWREM = (HWRE10 + SHWRE) / (2.*FLOAT(NTSTEP))
C
INTEGRATE THE INSTANTANEOUS EMULSION PARTICLE CONVECTIVE H.T.C.
AND OBTAIN THE MEAN VALUE
C
SHWCE = 0.0
DO 85 I=1, NTSTEP
IF(I .EQ. NTSTEP) GO TO 84
SHWCE = SHWCE + 2.*HWCE(I)
GO TO 85
84  SHWCE = SHWCE + HWCE(I)
85  CONTINUE
    HWCEM = (HWCEIO + SHWCE) / (2. * FLOAT(NTSTEP))
    EMPNU = 0.1 * HWCEM * (DIAM/GASK)

C
    HW2DEM = HW2DEM * 10.
    GASBHTC = GASBHTC * 10.

C
    HTOTRAD = (1.0 - BFO) * HWREM + BFO * HWRB
    HTOTEMU = (HWCEM + HW2DEM + HWREM) * (1. - BFO)
    HTOTBUB = (GASBHTC + HWRB) * BFO
    HWTOT = HTOTEMU + HTOTBUB
    TOTNU = 0.1 * HWTOT * DIAM/GASK

C
    WRITE (6, 555)
    WRITE (6, 554)
IF(ABS(1.-TNEW(NOG)) .LT. 1.E-9)WRITE(6,376)
WRITE(6,386)HW2DEM,HWCEM,HWREM
WRITE(6,248)HTOTRAD
WRITE(6,394)TOTNU,EMPNU
STOP

C FORMAT SECTION

200 FORMAT(10X,"DELT =",E10.4,10X,"TIME =",E10.4,5X,"FRACTION OF RESIDENCE TIME =","F6.3/)"
204 FORMAT(5X,"TEMPERATURE AT 1ST FACE OF 1ST SLAB=".2X,E12.6/)"
205 FORMAT(5X,9(E12.6,2X)/)
207 FORMAT(15X,"TEMPERATURES AT ALL GRIDS")"
208 FORMAT(15X,"TEMPERATURES AT THE RADIATION SURFACES")"
209 FORMAT(15X,"RADIATIONS AT THE RADIATION SURFACES")"
210 FORMAT(15X,"ITERATION NO. =",2X,I3/)"
215 FORMAT(5X,9(E12.6,2X)/)
217 FORMAT(15X,"INITIAL TEMPERATURES AT THE RADIATION SURFACES")"
218 FORMAT(15X,"INITIAL RADIATIONS AT THE RADIATION SURFACES")"
222 FORMAT(5X,"PARTICLE CONV FLUX=".E10.4,1X,"(KW/M**2)",5X,"RAD FLUX=".1X,E10.4,5X,1X,"(KW/M**2)")"
230 FORMAT(10X,"SIGMA=",.2X,E12.6,2X,"(W/CM**2.K**4)")"
232 FORMAT(5X,"SOLID K =",.1X,E12.6,"(W/CM.K)",5X,"SOLID DENSITY=".E12.16,"(G/CM**3)",5X,"GAS DENSITY=".1X,E12.6,1X,"(G/CM**3)",5X,"GAS VISCOSITY")"
236 FORMAT(5X,"NO. OF SLICES=".1X,E12.6,5X,"NO. OF GRIDS=".1X,E12.6,5X,"NO. OF RADIATION SURFACES=".1X,E12.6,5X,"NO. OF SOLID SLABS=".1X,E12.6/)"
238 FORMAT(5X,"NO. OF TIME STEPS=".1X,E12.6,1X,"DELT=".1X,E12.6,1X,"(SEC)",5X,"DX=".1X,E12.6,1X,"(CM)",5X,"FOURIER NO.=".1X,E12.6/)"
242 FORMAT(5X,"ARCHIMEDES NUMBER=".1X,E12.6/)"
244 FORMAT(5X,"THETA=".2X,E12.6,1X,"RADIANS")"
246 FORMAT(5X,"TOTAL RADIATION H.T.C.=".1X,E12.6,2X,"(KW/M**2)")"
248 FORMAT(5X,"EMULSION TOTAL H.T.C.=".1X,E12.6,5X,"BUBBLE TOTAL H.T.C.=".1X,E12.6,5X,"BED TEMPERATURE=".1X,E12.6/)"
250 FORMAT(5X,"MEAN NUD=".1X,E12.6,5X,"MEAN BUB. NUD=".1X,E12.6/)"
252 FORMAT(5X,"FOURIER NO. BASED ON (DP) AND (RTIME)=".1X,E12.6/)"
300 FORMAT(5X,"WALL TEMPERATURE=".E10.4,1X,"K",5X,"PARTICLE DIAMETER=".E10.4,1X,"CM")"
304 FORMAT(5X,"RESIDENCE TIME=".E12.6,1X,"SEC",5X,"FRAC. HT SURFACE EXPOSED TO BUBBLES=".E10.4/)"
308 FORMAT(5X,"EFF. PARTICLE EMISS.".F4.2,3X,"EFF. PARTICLE REFLECT.".F4.2,3X,"TRANSMISSIVITY=".F4.2/)"
350 FORMAT(5X,"INST. PCONV. H.T.C.=".1X,E12.6,1X,"(KW/M**2)")"
370 FORMAT(5X,"BUB. RAD. H.T.C.=".1X,E12.6,5X,"(KW/M**2)")"
372 FORMAT(5X,"TOTAL H.T.C.=".1X,E12.6,1X,"(KW/M**2)")"
374 FORMAT(5X,"PARTICLE CONV FLUX AT TIME=0. IS=".2X,E12.6,5X,"RAD FLUX AT TIME=0. IS=".2X,E12.6,5X,"(KW/M**2)")"
376 FORMAT(5X,"THE TEMPERATURE OF LAST GRID DIFFERS BY A SMALL AMOUNT")
FROM BED TEMPERATURE

382 FORMAT(5X, "PCONV. H.T.C. AT TIME=0.", E12.6, 1X, "(KW/M**2.K)", 5X, "RAD. H.T.C. AT TIME=0.", 1X, E12.6, 1X, "(KW/M**2.K)"/)  
386 FORMAT(5X, "NU2D =", 1X, E12.6, 10X, "BUB. NU2D =", 1X, E12.6/)  
388 FORMAT(5X, "GCONV. H.T.C. =", 1X, E12.6, 5X, "MEAN PCONV. H.T.C. =", 1X, E12.6, 5X, "MEAN RAD. H.T.C. =", 1X, E12.6, 5X, "(KW/M**2.K)"/)  
390 FORMAT(5X, "TUBE DIAMETER =", 1X, E10.4, 1X, "CM", 5X, "GAS LAYER THICKNESS =", 1X, E10.4, 1X, "CM")  
392 FORMAT(5X, "GAS CONDUCTIVITY AT MEAN TEMP. =", 1X, E12.6, 1X, "W/CM.K")  
394 FORMAT(5X, "MIN. FLUIDIZATION VEL. =", 1X, E12.6, 1X, "CM/SEC", 10X, "SUPERFICIAL VEL. =", 1X, E12.6, 1X, "CM/SEC")  
396 FORMAT(5X, "NUSSELT NO. (TOTAL) =", 1X, E12.6, 10X, "MEAN PCONV. NUSSELT NO. =", 1X, E12.6/)  
555 FORMAT(1H1)
FUNCTION FKGAS(J, TB, TR)

FKGAS CALCULATES GAS THERMAL CONDUCTIVITY AS FUNCTION OF TEMPERATURE

DIMENSION TR(31)
REAL K0, NN
TBED=(9./5.)*TB
K0=0.01395*0.017296
T0=491.67
NN=0.81
TAVG=TBED*(TR(J)+TR(J-1))/2.
FKGAS=K0*(TAVG/T0)**NN
RETURN
END

SUBROUTINE TRIDIAG(IF,L,A,B,C,D,V)

DIMENSION A(150), B(150), C(150), D(150), BETTA(150), GAMMA(150), V(150)
BETTA(IF)=B(IF)
GAMMA(IF)=D(IF)/BETTA(IF)
IFP1=IFP+1
DO 1 I=IFP1, L
BETTA(I)=B(I)-A(I)*C(I-1)/BETTA(I-1)
GAMMA(I)=(D(I)-A(I)*GAMMA(I-1))/BETTA(I)
1 CONTINUE
V(L)=GAMMA(L)
LAST=L-IF
DO 2 K=1, LAST
I=L-K
V(I)=GAMMA(I)-C(I)*V(I+1)/BETTA(I)
2 CONTINUE
RETURN
END
SUBROUTINE NUP2D

COMMON /ANU/ NU2D, DXSSP, NUPM, SPRP, REC, TS, TB, PRN, UPR
REAL K1, K2, NUPM, NU2D

PR = PRN
TW = TS
TWB = TW / TB
K1 = 5.26 - 7.72 * (SPRP - 1.)
K2 = 0.0127 + 0.0222 * (SPRP - 1.)
ALFA = 0.8
ALFA1 = ALFA + 1.
BC1 = 2. * (1. - TWB) / (1. + TWB)
BC2 = 2. * TWB / (1. + TWB)
SO = -(BC1 + BC2) * ALFA + BC2 ** ALFA1) / (BC1 * ALFA1)
NUPM = -2. * K1 * SO / (0.76 - (0.578 - 4. * K1 * K2) ** 0.5)
AA = SPRP * (0.2 + 0.8 * EXP(-0.0849 * UPR * SQRT(REC)))
NU2D = 0.798 * PR ** 0.4 * REC / AA ** 0.5
IF (NU2D .LT. NUPM) GO TO 5
DXSSP = 0.76 + K1 * SO / NU2D
GO TO 10
5  DXSSP = -K2 * NU2D / SO
10  RETURN
END
SUBROUTINE RADIOST(NOR,TR,QR,NITER)

COMMON /RAD/ EMISW,REFLW,EMIS,REFF,TAU
DIMENSION QRI(31),TR(31),QR(31)
EPS=0.1E-09
ITER=30
DO 10 I=1,NOR
QRI(I)=QR(I)
10 CONTINUE
DO 50 I=1,ITER
NFLAG=1
DO 30 J=1,NOR
IF(J .NE. 1)GO TO 15
QR(J)=EMISW*TR(J)**4+REFLW*QR(J+1)
GO TO 30
15 IFLAG=NFLAG
IF(IFLAG .NE. 1)GO TO 20
QR(J)=EMIS*TR(J)**4+REFF*QR(J-1)+TAU*QR(J+2)
NFLAG=2
GO TO 30
20 IF(IFLAG .NE. 2)GO TO 30
QR(J)=EMIS*TR(J)**4+REFF*QR(J+1)+TAU*QR(J-2)
NFLAG=1
30 CONTINUE

TEST FOR CONVERGENCE

DO 40 K=1,NOR
QVALUE=ABS((QR(K)-QRI(K))/QR(K))
IF(QVALUE .GT. EPS)GO TO 44
40 CONTINUE
NITER=I
GO TO 60
44 DO 46 JJ=1,NOR
QRI(JJ)=QR(JJ)
46 CONTINUE
50 CONTINUE
WRITE(6,220)
60 RETURN
220 FORMAT(10X,"NO CONVERGENCE HAS BEEN REACHED")
END
SUBROUTINE GASVEL

COMMON /VEL/ VELG, THET, VOID, VOIDE, SPHI, REP
REAL KB1, KBE
KB1 = (150. / REP) * ((1. - VOID) / (VOID * SPHI))**2 + (1.75 / VOIDE) * (1. - VOID) / (VOID * SPHI)
KBE = (150. / REP) * ((1. - VOIDE) / (VOIDE * SPHI))**2 + (1.75 / VOIDE) * (1. - VOIDE) / (VOIDE * SPHI)
VELG = 2. * SIN(THET) * (KBE / KB1) / VOID
RETURN
END

SUBROUTINE VISCGAS

COMMON /VISC/ TAIR, VISCG, RAUG
FN = 0.666
TO = 491.6
U0 = 0.1716E-03
TA = TAIR * (9. / 5.)
UA = U0 * (TA / TO)**FN
RAUG = 1.325 * (29.921 / TA) / 62.43
VISCG = UA / RAUG
RETURN
END
FUNCTION FNU2DST(V, VE, SPHI, DP, DT, REP, PRN, UPR)
C
REAL KB1, KBE
S=SPHI
KB1=((150./REP)*((1.-V)/(V*S))**2+(1.75/VE)*(1.-V)/(V*S))
KBE=((150./REP)*((1.-VE)/(VE*S))**22+(1.75/VE)*(1.-VE)/(VE*S))
OMEGAR=0.2+0.8*EXP(-0.0849*UPR*SQRT(REP/VE))
TUP=KBE*(DP/DT)*REP*(PRN**0.7)*(1./OMEGAR)
TLO=VE*KB1*(1.+((V/VE-1.)**2)*(.65*(DP/DT)*(12.13+991./REP)))
FNU2DST=1.1*SQRT(TUP/TLO)
RETURN
END

SUBROUTINE BUBHTC
C
EXTERNAL TRAPI, FNUD
COMMON /BUB/ VOIDB, THETA, DTUBE, DIAM, NBINTEG, BNUD
VOID=VOIDB
EB=3.0
E=EB/2.
A=0.0
B=THETA
NI=NBINTEG
DP=DIAM
DT=DTUBE
BNU=TRAPI(A, B, NI, FNUD, VOID, DP, DT)
CONST=((SIN(B))**E)/B
BNU=CONST*BNU
RETURN
END
FUNCTION TRAPI(A,B,N,F,VOIDE,DP,DT)

H=(B-A)/FLOAT(N)
SUMEND=0.0
SUMMID=0.0
DO 5 K=1,N
  X=A+FLOAT(K-1)*H
  SUMEND=SUMEND+F(X,B,VOIDE,DP,DT)
  SUMMID=SUMMID+F(X+H,B,VOIDE,DP,DT)
5 CONTINUE
TRAPI=(H/2.)*(SUMEND+SUMMID)
RETURN
END

FUNCTION FNUD(X,XUP,VOIDE,DP,DT)

COF=0.297*VOIDE*(DP/DT)/SQRT(1.-VOIDE)
AA=1.13*(XUP/2.-SIN(2.*XUP)/4.)
AB=1.13*(X/2.-SIN(2.*X)/4.)
AC=COF*(SIN(X))**2
FNUD=1.0/SQRT(AA-AB+AC)
RETURN
END
APPENDIX D

LISTING OF THE COMPUTER PROGRAM (GAUSS) FOR DETERMINING THE TRANSMISSIVITY FACTOR
PROGRAM GAUSS (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)
EXTERNAL FN, FUP1, FLO1, FUP2, FLO2, FUP3, FLO3
DOUBLE PRECISION DX(16), DW(16)
DIMENSION X(40, 4), W(40, 4), MM(4)
CALL GTABLE (DX, DW, -1.0D0, 1.0D0)
DO 2 I = 1, 16
   XX = DX(I)
   X(I, 1) = XX
   X(I, 2) = XX
   X(I, 3) = XX
   X(I, 4) = XX
   WB = DW(I)
   W(I, 1) = WB
   W(I, 2) = WB
   W(I, 3) = WB
   W(I, 4) = WB
2 CONTINUE
MM(1) = 16
MM(2) = 16
MM(3) = 16
MM(4) = 16
A = -1.0
B = 1.0
PAI = 3.14159
RP = 1.0
AREA = (2.0 * RP)**2
Q = QMUL4(FN, A, B, FUP1, FLO1, FUP2, FLO2, FUP3, FLO3, X, W, MM)
Q = Q / (PAI * AREA)
WRITE(6, 200) Q
200 FORMAT (1H1, 10X, "VALUE OF INTEGRATION =", E14.8)
STOP
END
SUBROUTINE GTABLE(X,W,A,B)
DOUBLE PRECISION A,B,X(16),W(16),XX(8),WW(8)
DATA (XX(I),I=1,8)/
10.98940093499165000, 0.94457502307323300,
20.865831202387832D0, 0.75540440835500300,
30.61787624440264400, 0.4580167765722700,
40.26160355077925900, 0.09501250983763700/
DATA (WW(I),I=1,8)/
10.02715245941175400, 0.062253523938648100,
20.09515851160249300, 0.12462897125553400.
30.1495998816557700, 0.16915651939500300,
40.182603415044924D0, 0.18945061045506800/
BMA=0.500*(B-A)
BPA=0.500*(B+A)
 DO 2 I=1,8
  NI=17-I
  X(I)=-BMA*XX(I)+BPA
  X(NI)=BMA*XX(I)+BPA
  W(I)=BMA*WW(I)
  W(NI)=BMA*WW(I)
 CONTINUE
RETURN
END
FUNCTION QMULT4(FCN,A,B,FL1,FL2,FL3,X,W,MM)
DIMENSION X(40,4),W(40,4),MM(4)
M1=MM(1)
M2=MM(2)
M3=MM(3)
M4=MM(4)
H1=(B-A)/2.
G1=(B+A)/2.
Q1=0.0
DO 10 I=1,M1
UI=H1*X(I,1)+G1
WI=W(I,1)
D2=FU1(UI)
C2=FL1(UI)
H2=(D2-C2)/2.
G2=(D2+C2)/2.
Q2=0.0
DO 8 J=1,M2
VJ=H2*X(J,2)+G2
WJ=W(J,2)
D1=FU2(UI,VJ)
C1=FL2(UI,VJ)
H3=(D1-C1)/2.
G3=(D1+C1)/2.
Q3=0.0
DO 4 K=1,M3
VK=H3*X(K,3)+G3
WK=W(K,3)
D=FU3(UI,VJ,VK)
C=FL3(UI,VJ,VK)
H=(D-C)/2.
G=(D+C)/2.
Q=0.0
DO 2 L=1,M4
ZL=H*X(L,4)+G
Q=Q+W(L,4)*FCN(UI,VJ,VK,ZL)
Q3=Q3+WK*H+Q
Q2=Q2+WJ*H3*Q3
Q1=Q1+WI*H2*Q2
QMULT4=H1*Q1
RETURN
END
FUNCTION FN(X,Y,Z,W)
  H=1.0
  FCN=((Z-X)**2+H**2+(W-Y)**2)**2
  FN=H**2/FCN
RETURN
END

FUNCTION FUP1(X)
  RP=1.0
  FUP1=2.*RP
RETURN
END

FUNCTION FLO1(X)
  RP=1.0
  FLO1=0.0
RETURN
END

FUNCTION FUP2(X,Y)
  FUP2=1.0
RETURN
END

FUNCTION FLO2(X,Y)
  FLO2=0.0
RETURN
END

FUNCTION FUP3(X,Y,Z)
  RP=1.0
  FUP3=2.*RP-(RP**2-(Z-RP)**2)**0.5
RETURN
END

FUNCTION FLO3(X,Y,Z)
  RP=1.0
  FLO3=(RP**2-(Z-RP)**2)**0.5
RETURN
END