Robust viable management of a harvested ecosystem model

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Abstract

The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of the ecosystem approach by 2010. We propose a framework that deals jointly with i) ecosystem dynamics, ii) conflicting issues of production and preservation and iii) robustness with respect to dynamics uncertainties. More specifically, we define robust viability kernels as initial species biomasses such at least one appropriate harvesting strategy guarantees minimal production and preservation levels for all times, whatever the uncertainties. We apply our approach to the anchovy–hake couple in the Peruvian upwelling ecosystem. We comment on the management implications of comparing robust viability kernels with and without uncertainties.

Key words: control theory; uncertainties; robustness; sustainability; ecosystem management; Peruvian upwelling ecosystem.

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1 Introduction

There is a growing demand for moving from single species management schemes to an ecosystemic approach of fisheries management (Garcia, Zerbi, Aliaume, Chi, and Lasserre, 2003). The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of the ecosystem approach by 2010.

Furthermore, uncertainty inherent to fisheries is recognized to play a important role in the failure of management regimes. Fisheries modeling requires estimations of stock status and total withdrawal from stock. Such information remains imprecise and error prone. Uncertainty can also concern the structure and dynamics of ecosystems which are poorly known. At last, random fluctuations such as climatic hazards or technical progress are likely to affect fisheries productivity. (Lauck, Clark, Mangel, and Munro, 1998) explains that fishing decreases the resilience of fish populations, rendering them more vulnerable to environmental change. Not accounting for uncertainty can lead to excessive harvest of a resource (Hilborn and Walters, 1992), though it is obvious that complex system, in which uncertainty is overwhelming, cannot be perfectly predicted.

Our interest is to mitigate environmental risk adopting a robust framework. We propose a framework that deals jointly with i) ecosystem dynamics, ii) conflicting issues of production and preservation and iii) robustness with respect to dynamics uncertainties.

The robust viability theory (RVT) (De Lara and Doyen, 2008) is a relevant approach to address dynamic control problems under constraints with uncertainty. The objective of this approach is not to maximize a criterium as in optimum control theory, but rather to describe all evolution of a dynamic system under uncertainty, satisfying at each instant given objectives whatever the uncertainties. The set robust viable states is the set of states starting from which their exists a control strategy guaranteeing constraints describing given production and preservation, for all times over a given time span, and for all uncertainty scenarios. In other words, the RVT allows no trade-offs between pursued objectives or time periods: all constraints must be satisfied for all times, whatever uncertainties.

We apply this theory to a discrete time two-species dynamic model in which harvesting efforts act as controls. Uncertainty takes the form of additive disturbances affecting each species dynamics, and are assumed to take their values in a known given set. We consider two different sets in order to appraise the sensibility of our results to uncertainty scenarios. Constraints impose minimum safe biomass levels, usually identified by biologists, and minimum required harvesting levels assumed to ensure economic needs. Starting from a robust viable state, it is possible to drive the system on a sustainable path along which catches and biomasses stand above production and biological minimums, despite of uncertainties.

Reducing uncertainties to zero amounts to formulate the problem within the deterministic viability framework (Aubin, 1991). Comparison of these deterministic viable states with the sets of robust viable states shades light on the potential risks of not adopting a most precautionary approach in harvested ecosystem management.

The paper is organized as follow. Section 2 introduces a generic class of harvested nonlinear ecosystem models, the sustainability constraints, and presents the robust viability theory. The deterministic viability theory is also displayed for comparison purpose. In section 3, we proceed to an application of the robust and deterministic viability analysis to the Peruvian hake–anchovy upwelling ecosystem between 1971 and 1981. After exposing our assumptions on uncertainty sets, sets of robust viability states are computed numerically and compared to the set of deterministic viable states, whose expression can be obtained analytically. Section 4 concludes.

2 The Robust Viability Approach

In what follows, we present a class of generic harvested *nonlinear* ecosystem models with uncertainty. Next, we introduce the concept of robust viable state, that is, a state starting from which conservation and production constraints can be guaranteed over a given time span, despite of uncertainty. We compare it with the set of deterministic viable states− states guaranteeing conservation and production constraints in absence of uncertainty− for which we are able to provide an analytical expression.

2.1 A generic ecosystem model with uncertainty and the associated sustainability constraints

For simplicity, we consider a dynamic model with two species, but it can be easily extended to N species in interaction. Each species is described by its biomass: the two–dimensional state vector (y, z) represents the biomasses of both species. The two–dimensional control vector (v_y, v_z) comprises the harvesting effort for each species, respectively, each lying in $[0, 1]$. The two additive terms, ε_y and ε_z, correspond to uncertainties affecting each species biomass, respectively. The discrete–time control system we consider is given by

$$\begin{cases} y(t+1) &= y(t)R_y\big(y(t),z(t)\big)\big(1-v_y(t)\big)+\varepsilon_y(t)\ , \\ z(t+1) &= z(t)R_z\big(y(t),z(t)\big)\big(1-v_z(t)\big)+\varepsilon_z(t)\ , \end{cases} \tag{1}$$

where t stands for time (typically, periods are years), and ranges from the initial time t_0 to the time horizon T. The two functions $R_y : \mathbb{R}^2 \to \mathbb{R}$ and $R_z : \mathbb{R}^2 \to \mathbb{R}$ represent biological growth factors. The catches are given by $v_y y R_y\big(y,z\big)$ and $v_z z R_z\big(y,z\big)$ (measured in biomass). This model is generic in that no explicit or analytic assumptions are made on how the growth factors R_y and R_z indeed depend upon both biomasses (y,z).

An uncertainty scenario is defined by a sequence of uncertainty couples of length $T-t_0$

$$\big(\varepsilon_y(\cdot),\varepsilon_z(\cdot)\big) = ((\varepsilon_y(t_0),\varepsilon_z(t_0)),\dots,(\varepsilon_y(T-1),\varepsilon_z(T-1))) \in \Omega = \prod_{t=t_0}^{T-1} \mathbb{S}(t), \tag{2}$$

where uncertainties $(\varepsilon_y(t),\varepsilon_z(t))$ are assumed to take there values in a two-dimensional set:

$$(\varepsilon_y(t),\varepsilon_z(t)) \in \mathbb{S}(t) \subset \mathbb{R}^2. \tag{3}$$

We now propose to define sustainability as the ability to respect preservation and production minimal levels for all times, building upon the original approach of (Béné, Doyen, and Gabay, 2001). For this purpose we consider:

- on the one hand, *minimal biomass levels* $y^\flat \geq 0$, $z^\flat \geq 0$, one for each species,
- on the other hand, *minimal catch levels* $Y^\flat \geq 0$, $Z^\flat \geq 0$, one for each species.

2.2 The robust viability kernel

A strategy γ is defined as a sequence as follows:

$$\gamma = \{\gamma_t\}_{t=t_0,\dots,T-1}, \quad \text{with} \quad \gamma_t : \mathbb{R}^2 \to [0,1]^2. \tag{4}$$

A strategy γ as in (4) and the dynamic model (1) produce state paths by

$$\begin{cases} y(t+1) &= y(t)R_y\big(y(t),z(t)\big)\big(1-\gamma_t(y(t),z(t))\big)+\varepsilon_y(t)\ , \\ z(t+1) &= z(t)R_z\big(y(t),z(t)\big)\big(1-\gamma_t(y(t),z(t))+\varepsilon_z(t)\ , \end{cases} \tag{5}$$

and control paths by

$$(v_y(t), v_z(t)) = \gamma_t(y(t), z(t)) \ . \tag{6}$$

Notice that, under uncertainty, we look after strategies as solutions: as in (6), controls $(v_y(t), v_z(t))$ are determined by constantly adapting to the state $(y(t), z(t))$ of the system, itself affected by disturbances $(\varepsilon_y(t-1), \varepsilon_z(t-1))$.

The robust viability kernel $\mathbb{V}^R(t_0)$ (De Lara and Doyen, 2008), is the set of initial states $(y(t_0), z(t_0))$ starting from which there exists a control strategy γ as in (4) producing state paths $\{(y(t), z(t))\}_{t=t_0,\dots,T}$ as in (5), and control paths $\{(v_y(t), v_z(t))\}_{t=t_0,\dots,T-1}$ as in (6), such that, for all uncertainty scenarios $(\varepsilon_y(\cdot), \varepsilon_z(\cdot)) \in \Omega$ in (**??**), the following goals are satisfied:

- preservation (minimal biomass levels)

$$y(t) \geq y^\flat \ , \quad z(t) \geq z^\flat \ , \quad \forall t = t_0, \dots, T \tag{7}$$

- and production requirements (minimal catch levels)

$$v_y(t)y(t)R_y\big(y(t), z(t)\big) \geq Y^\flat \ , \quad v_z(t)z(t)R_z\big(y(t), z(t)\big) \geq Z^\flat \ , \quad \forall t = t_0, \dots, T-1 \ . \tag{8}$$

Characterizing robust viable states makes it possible to test whether or not minimal biomass and catch levels can be guaranteed for all time, despite of uncertainty. By *guaranteed* we mean that biomasses and catches never fall below the minimal thresholds as in the inequalities (7) and (8).

The robust viability kernel can be computed numerically by means of a dynamic programming equation associated with dynamics (1), state constraints (7) and control constraints (8) (De Lara and Doyen, 2008).

2.3 The deterministic viability kernel

The deterministic version of the framework exposed at § 2.2 corresponds to the case where $(\varepsilon_y(t), \varepsilon_z(t)) = (0, 0), \forall t = t_0, \dots, T-1$, that is $\mathbb{S}(t) = \{(0, 0)\}$ in (3). Then, the robust viability kernel coincides with the so called *viability kernel* (Aubin, 1991), defined in § A.

The following Proposition 1 gives an analytical expression of the viability kernel under some conditions on the guaranteed levels in (7) and (8). The proof is given in § A in the Appendix.

Proposition 1 *Consider $T \geq t_0+2$. If the minimal biomass thresholds $y^\flat$, $z^\flat$, and catch thresholds $Y^\flat$, $Z^\flat$, are such that*

$$y^\flat R_y\big(y^\flat, z^\flat\big) - y^\flat \geq Y^\flat \quad \text{and} \quad z^\flat R_z\big(y^\flat, z^\flat\big) - z^\flat \geq Z^\flat, \tag{9}$$

the viability kernel is given by

$$\mathbb{V}(t_0) = \Big\{ \ (y,z) \mid y \geq y^\flat, z \geq z^\flat, yR_y\big(y,z\big) - y^\flat \geq Y^\flat, zR_z\big(y,z\big) - z^\flat \geq Z^\flat \ \Big\} . \tag{10}$$

Conditions (9) mean that, at the point $(y^\flat, z^\flat)$, the surplus $y^\flat R_y\big(y^\flat, z^\flat\big) - y^\flat \geq Y^\flat$ and $z^\flat R_z\big(y^\flat, z^\flat\big) - z^\flat \geq Z^\flat$ are at least equal to the minimal catch thresholds $Y^\flat$ and $Z^\flat$, respectively.

3 Application to the anchovy–hake couple in the Peruvian upwelling ecosystem (1971–1981)

We now apply the above robust viability analysis to the Peruvian hake–anchovy fisheries between 1971 and 1981. For this, we extend the model in (De Lara, Ocaña, Oliveros-Ramos, and Tam, 2012) to the uncertain case. We compute the robust viability kernel numerically, testing different assumptions on the uncertainty sets $\mathbb{S}(t)$ in (**??**), to appraise the sensitivity of robust viable states to uncertainty scenarios.

3.1 Lotka-Volterra model with uncertainty and associated sustainability constraints

The Peruvian anchovy–hake system is modeled as prey-predator system, where the anchovy growth rate is decreasing in the hake population. We describe this interaction by the following discrete-time Lotka-Volterra system

$$\left\{ \begin{array}{rcl} y(t+1) & = & y(t)\overbrace{\big(R - \frac{R}{\kappa}y(t) - \alpha z(t)\big)}^{R_y\big(y(t),z(t)\big)}\big(1 - v_y(t)\big) + \varepsilon_y(t) \\ z(t+1) & = & z(t)\underbrace{\big(L + \beta y(t)\big)}_{R_z\big(y(t),z(t)\big)}\big(1 - v_z(t)\big) + \varepsilon_z(t), \end{array} \right. \tag{11}$$

where $R > 1$, $0 < L < 1$, $\alpha > 0$, $\beta > 0$ and $\kappa = \frac{R}{R-1}K$, with $K > 0$ the carrying capacity for the prey. The variable y stands for anchovy biomass and z for hake biomass. The purpose of this compact model is not to provide biological "knowledge" on the Peruvian upwelling ecosystem, but rather to capture the essential features of the system in what concerns decision making.

The 5 parameters of the model (11) have been estimated in (De Lara, Ocaña, Oliveros-Ramos, and Tam, 2012), based on 11 yearly observations of the Peruvian anchovy-hake biomasses and catches over the time period 1971-1981, and their values are given in Table 1.

Parameters	**Estimates**
R	2.25 $year^{-1}$
L	0.945 $year^{-1}$
κ	67113 10^3 $tons$
K	37285 10^3 $tons$
α	1.220 10^6 $tons^{-1}$
β	4.845 10^{-8} $tons^{-1}$

Table 1: Parameter estimates of the model (11)

3.2 Choice of uncertainty sets

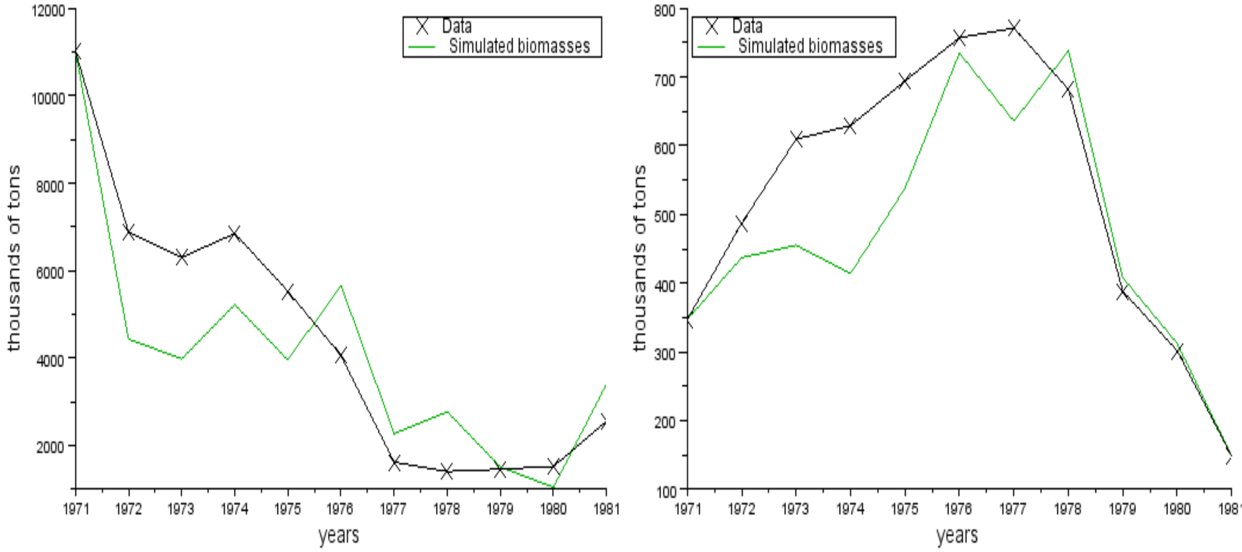
We will now be more specific about the uncertainties sets $\mathbb{S}(t)$ in (3).

Figure 1 represents the simulated biomasses with the deterministic version of the Lotka-Volterra model and the observed biomasses of Peruvian anchovy and hake over 1971-1981. The gap between these curves over the time span gives us indications on the ranges in which the uncertainties $\varepsilon_y(t)$ and $\varepsilon_z(t)$ in (11) may take there values.

The time period 1971-1981 is denoted by $t = t_0, \ldots, T$, with $t_0 = 0$, and $T = 10$. Let $(\overline{y}(t), \overline{z}(t))_{t=t_0,\ldots,T}$ and $(\overline{v}_y(t), \overline{v}_z(t))$ denote the observed biomass and effort trajectories. We set $\overline{\varepsilon}_y(t)$ and $\overline{\varepsilon}_z(t)$ defined by

$$\begin{cases} \overline{y}(t+1) &= \overline{y}(t)\big(R - \frac{R}{\kappa}\overline{y}(t) - \alpha\overline{z}(t)\big)\big(1 - \overline{v}_y(t)\big) + \overline{\varepsilon}_y(t) \\ \overline{z}(t+1) &= \overline{z}(t)\big(L + \beta\overline{y}(t)\big)\big(1 - \overline{v}_z(t)\big) + \overline{\varepsilon}_z(t), \end{cases}$$

so that (11) is satisfied.



(a) Anchovy (b) Hake

Figure 1: Observed and simulated biomasses over 1971-1981

Figure 2 displays the points $\{(\bar{\varepsilon}_y(t), \bar{\varepsilon}_z(t)) | t = t_0 + 1, \dots, T\}$, that is, the difference between simulated and observed biomasses (there are 10 points as 1971 observations are used as starting points for simulating biomasses). We denote $\bar{\varepsilon}_y^{min} = -1610.95$ kt, $\bar{\varepsilon}_y^{max} = 2441.35$ kt, $\bar{\varepsilon}_z^{min} = -55.64$ kt and $\bar{\varepsilon}_z^{max} = 215.24$ kt.

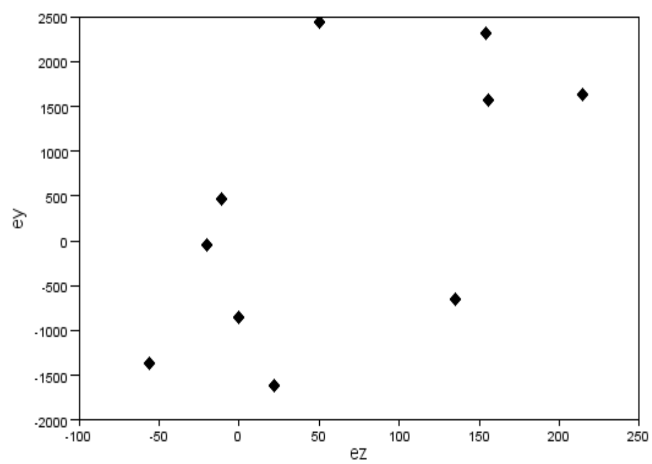


Figure 2: Empirical uncertainties

We will compute robust viable kernels corresponding to the following two cases of the uncertainty sets $\mathbb{S}(t)$ in (3).

- $\mathbb{S}_L$ is made of the 10 empirical uncertainty couples and the uncertainty couple $(\varepsilon_y, \varepsilon_z) = (0,0)$ (see diamonds in Figure 3):

$$\mathbb{S}_L = \{(\overline{\varepsilon}_y(t), \overline{\varepsilon}_z(t)) | t = 1, \ldots, 10\} \cup \{(0,0)\} \ . \tag{12}$$

- $\mathbb{S}_H$ is made of 400 uncertainty couples delineated by a 20×20 grid over the surface $[\overline{\varepsilon}_y^{min}, \overline{\varepsilon}_y^{max}] \times [\overline{\varepsilon}_z^{min}, \overline{\varepsilon}_z^{max}]$, including uncertainty couples in $\mathbb{S}_L$ (see the grid in Figure 3).

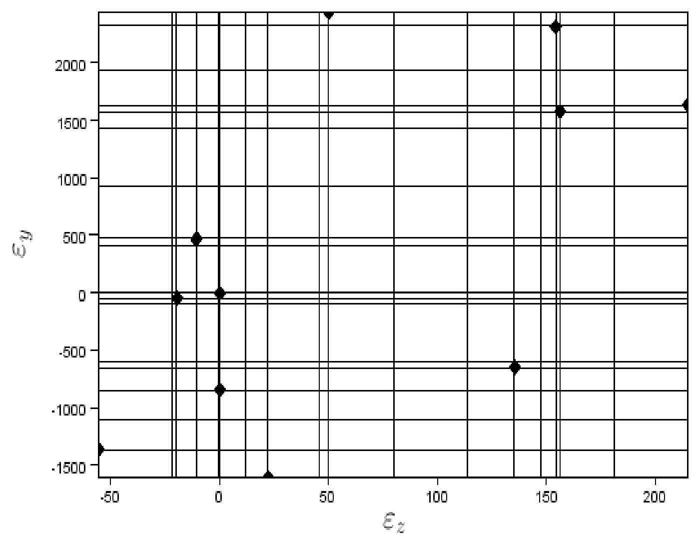


Figure 3: Uncertainty sets $\mathbb{S}_L$ (dimamonds) and $\mathbb{S}_H$ (grid)

Since $\{(0,0)\} \subset \mathbb{S}_L \subset \mathbb{S}_H$, the corresponding robust and deterministic viable kernels satisfy

$$\mathbb{V}_H^R(t_0) \subset \mathbb{V}_L^R(t_0) \subset \mathbb{V}(t_0) \ . \tag{13}$$

3.3 Robust and deterministic viability kernels

We consider the minimal biomasses $y^\flat = 7{,}000{,}000$ tons and $z^\flat = 200{,}000$ t, and minimal catches $Y^\flat = 2{,}000{,}000$ tons and $Z^\flat = 5{,}000$ tons (IMARPE, 2000, 2004). The condition (9) in Proposition 1 is satisfied for the above minimal threshold values and for the Lotka-Volterra model coefficient estimates in Table 1.

Figure 4 displays the robust viability kernels associated with dynamics (11), constraints (7) and (8), and with the uncertainty sets $\mathbb{S}_L$ and $\mathbb{S}_H$. As expected, the sets of robust viable states decreases as the number of scenarios increases as in (13).

Replacing R_y and R_z in (10) by their expressions (11) yields the expression of the viability deterministic kernel:

$$\mathbb{V}(t_0) = \left\{ \ (y,z) \mid y \geq y^\flat, z^\flat \leq z \leq \frac{1}{\alpha}[R - \frac{R}{\kappa}y - \frac{y^\flat + Y^\flat}{y}] \ \right\}. \tag{14}$$

This set $\mathbb{V}(t_0)$ is delineated by the outer (red) curve in Figure 4.

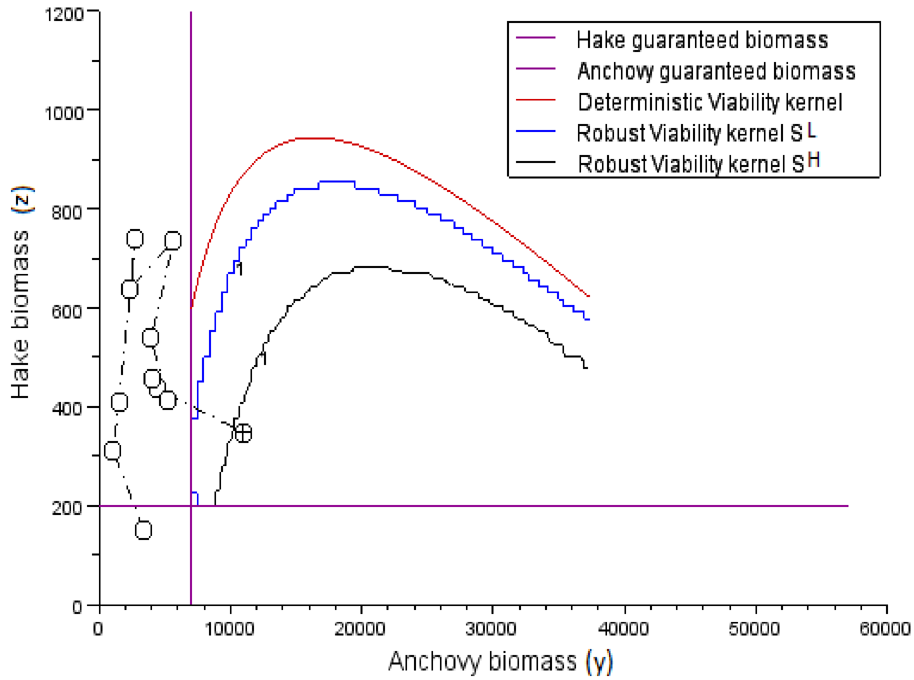


Figure 4: Comparing robust and deterministic viable states

The horizontal and vertical (purple) lines represent the sustainability constraints and the circles indicate the biomass observations of the anchovy-hake couple (black marks) over 1971-1981. Only one circle, marked by a cross, stands within the three delineated sets of states, corresponding to the initial biomass couple observed in 1971. Thus, starting from that date, there theoretically existed a strategy providing, for all times, at least the sustainable yields $Y^\flat$ and $Z^\flat$ and guaranteeing biomasses over the preservation thresholds $y^\flat$, $z^\flat$, for all times, whatever the scenario stemming from $\mathbb{S}_H$. In reality, the catches of year 1971 were very high, and the biomass trajectories were well below the biological minimal levels for 14 years.

4 Conclusion

This work is a theoretical and practical contribution to ecosystem sustainable management under uncertainties. The robust viable kernel that we introduced and computed in an application is an interesting mean to display the impact of uncertainty on management options.

A The deterministic viability kernel

The *deterministic viability kernel* is defined as the set of viable states as follow. A couple (y_0, z_0) of initial biomasses is said to be a *viable state* if there exist appropriate harvesting efforts (controls) $\big(v_y(t), v_z(t)\big) \in [0,1],\ t = t_0, \dots, T-1$, such that the state path $\{\big(y(t), z(t)\big)\}_{t=t_0,\dots,T}$, and control path $\{\big(v_y(t), v_z(t)\big)\}_{t=t_0,\dots,T-1}$, solution of (11), starting from $\big(y(t_0), z(t_0)\big) = (y_0, z_0)$ satisfy the following goals:

- preservation (minimal biomass levels)

$$y(t) \geq y^\flat \ , \quad z(t) \geq z^\flat \ , \quad \forall t = t_0, \dots, T \tag{15}$$

- and production requirements (minimal catch levels)

$$v_y(t)y(t)R_y\big(y(t), z(t)\big) \geq Y^\flat \ , \quad v_z(t)z(t)R_z\big(y(t), z(t)\big) \geq Z^\flat \ , \quad \forall t = t_0, \dots, T-1 \tag{16}$$

Here below is the proof of Proposition 1.

Proof. We set:

$$\mathbb{V}_0 = \{\ (y,z) \Big| y \geq y^\flat, z \geq z^\flat\ \},$$

and define $\mathbb{V}_1$ and $\mathbb{V}_2$ according to:

$$\begin{aligned} \mathbb{V}_k = \{(y,z)|\exists(v_y, v_z) \in [0,1]\,\text{such that}\, yv_yR_y(y,z) \leq Y^\flat, zv_zR_z(y,z) \leq Z^\flat \\ \text{and setting}\, y' = yR_y(y,z)(1-v_y),\ z' = zR_z(y,z)(1-v_z),\, \text{then}\, (y',z') \in \mathbb{V}_{k+1}\}\ . \end{aligned}$$

We obtain

$$\begin{aligned}
\mathbb{V}_1 &= \left\{ (y,z) \;\middle|\; \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ and, for some } (v_y, v_z) \in [0,1], \\ v_y y R_y(y,z) \geq Y^\flat, v_z z R_z(y,z) \geq Z^\flat, \\ y R_y(y,z)(1-v_y) \geq y^\flat, z R_z(y,z)(1-v_z) \geq z^\flat \end{array} \right\} \\
&= \left\{ (y,z) \;\middle|\; \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ for which there exist } (v_y, v_z) \text{ such that} \\ \frac{Y^\flat}{yR_y(y,z)} \leq v_y \leq \frac{yR_y(y,z)-y^\flat}{yR_y(y,z)} \quad \text{and} \quad 0 \leq v_y \leq 1, \\ \frac{Z^\flat}{zR_z(y,z)} \leq v_z \leq \frac{zR_z(y,z)-z^\flat}{zR_z(y,z)} \quad \text{and} \quad 0 \leq v_z \leq 1 \end{array} \right\} \\
&= \left\{ (y,z) \;\middle|\; \begin{array}{l} y \geq y^\flat, z \geq z^\flat, \\ \sup\{0, \frac{Y^\flat}{yR_y(y,z)}\} \leq \inf\{1, 1 - \frac{y^\flat}{yR_y(y,z)}\} \\ \sup\{0, \frac{Z^\flat}{zR_z(y,z)}\} \leq \inf\{1, 1 - \frac{z^\flat}{zR_z(y,z)}\} \end{array} \right\} \\
&= \left\{ (y,z) \;\middle|\; y \geq y^\flat, z \geq z^\flat, \frac{Y^\flat}{yR_y(y,z)} \leq \frac{yR_y(y,z)-y^\flat}{yR_y(y,z)}, \frac{Z^\flat}{zR_z(y,z)} \leq \frac{zR_z(y,z)-z^\flat}{zR_z(y,z)} \right\} \\
&= \left\{ (y,z) \;\middle|\; y \geq y^\flat, z \geq z^\flat, Y^\flat \leq yR_y(y,z) - y^\flat, Z^\flat \leq zR_z(y,z) - z^\flat \right\} \\
\mathbb{V}_2 &= \left\{ (y,z) \;\middle|\; \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ and, for some } (v_y, v_z) \in [0,1], \\ v_y y R_y(y,z) \geq Y^\flat, \; v_z z R_z(y,z) \geq Z^\flat, \; y' \geq y^\flat, \; z' \geq z^\flat, \\ Y^\flat \leq y' R_y(y',z') - y^\flat, \; Z^\flat \leq z' R_z(y',z') - z^\flat \end{array} \right\} \\
& \text{where } y' = yR_y(y,z)(1-v_y), \; z' = zR_z(y,z)(1-v_z).
\end{aligned}$$

We now make use of the property in (De Lara, Ocaña, Oliveros-Ramos, and Tam, 2012) that when the decreasing sequence $(\mathbb{V}_k)_{k\in\mathbb{N}}$ is stationary, its limit is the viability kernel $\mathbb{V}(t_0)$. Hence, it suffices to show that $\mathbb{V}_1 \subset \mathbb{V}_2$ to obtain that $\mathbb{V}(t_0) = \mathbb{V}_1$.

Let $(y,z) \in \mathbb{V}_1$, so that

$$y \geq y^\flat, \quad z \geq z^\flat \text{ and } \; yR_y(y,z) - y^\flat \geq Y^\flat, \quad zR_z(y,z) - z^\flat \geq Z^\flat \; .$$

Let us set $\hat{v_y} = \frac{yR_y(y,z)-y^\flat}{yR_y(y,z)}$. On the one hand, we have that $\hat{v_y} \leq 1$ since $y^\flat \geq 0$. On the other hand, since $yR_y(y,z) - y^\flat \geq Y^\flat \geq 0$, we deduce that $\hat{v_y} \geq 0$. Thus, $\hat{v_y} \in [0,1]$ and it is such that $y' = yR_y(y,z)(1-\hat{v_y}) = y^\flat$. The same holds for $\hat{v_z}$ in $z' = zR_z(y,z)(1-\hat{v_z}) = z^\flat$. By (9), we deduce that

$$y'R_y(y',z') - y^\flat = y^\flat R_y(y^\flat, z^\flat) - y^\flat \geq Y^\flat \text{ and } z'R_z(y',z') - z^\flat = z^\flat R_z(y^\flat, z^\flat) - z^\flat \geq Z^\flat \; .$$

The inclusion $\mathbb{V}_1 \subset \mathbb{V}_2$ follows, hence $\mathbb{V}(t_0) = \mathbb{V}_1$. □

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