Geoffrey W. Frost for the degree of Master of Science in Mechanical Engineering presented on December 2, 1999. Title: Two Projectiles Connected by a Flexible Tether Dropped in the Atmosphere.

Redacted for Privacy

Abstract approved: ___________________________

Mark F. Costello

This study investigates the atmospheric flight dynamics of a munition system and the effect of a tether reel resistance mechanism for limiting the impact that the unreeling process has on the munition system. The munition system consisting of two projectiles connected by a tether line is released from an aircraft at altitude and drops toward a target on the ground. Initially the two projectiles are rigidly attached. At a specified time, the projectiles separate and subsequently unreel the tether line. After the tether line is fully payed out, the system settles toward a steady state as it approaches the ground. Two different computation procedures are compared for modeling the tether unreeling process, namely, the pop-out and all-out methods. The all-out method requires significantly higher computation whereas the pop-out method induces spurious vibration into the tether line as line is released. It is shown that while projectile position results converge for a relatively low number of tether line elements, the maximum tether loads require a significantly larger number of elements. Parametric studies indicate that increases in tether stiffness contribute to increases in maximum tether line load and maximum
follower projectile acceleration while having very little effect on the range of the lead and follower projectiles. An increase in the drag coefficient ratio increases the maximum tether line load and the maximum acceleration on the follower projectile. However, increasing the drag coefficient ratio also causes a decrease in the speed of the lead projectile, which leads to a decrease in range of the system. A follower projectile equal in weight to the lead projectile results in an increase in tether deployment time while having little effect on the range of the lead projectile. For a low follower-to-lead projectile-mass ratio, the tether line unreeling process is predominantly due to the follower and lead projectile separation. Conversely, for a high follower-to-lead projectile-mass ratio, the tether line tends to billow and subsequently unreels itself independent of lead and follower projectile motion. The parametric studies led to the determination of reel configurations which decrease tether line loads, maximum acceleration on the follower projectile, time to reach a steady state condition, and the terminal miss distance. Reel resistance functions based on feasible mechanisms were determined for various drop speeds, mass ratios, drag coefficient ratios and tether line stiffness. High end, low end, and mid-range optimal resistance functions were chosen from the range of each category. Each selected optimal resistance function was utilized in the above computation procedure for modeling the tether unreeling process to determine and compare the deployment characteristics produced over the entire range of each category. The deployment characteristics produced by each optimal resistance function were used to determine envelopes for successful deployment. Reel resistance based on either line out or line out rate provided two powerful means to reduce line loads. Resistance as a function of line out yields the overall best performance, however,
resistance based on line out rate provided suitable performance over a wider band of drop speeds, mass ratios, and drag coefficient ratios.
Two Projectiles Connected by a Flexible Tether
Dropped in the Atmosphere

by

Geoffrey W. Frost

A THESIS
Submitted to
Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

Completed December 2, 1999
Commencement June 2000
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ACKNOWLEDGEMENT

I would like to thank Dr. Mark F. Costello for his guidance, assistance and patience during this project. I would also like to thank my Father and Mother for their belief in me, and their unending support, which has provided me with a plentiful and gratifying scholastic experience. Further appreciation goes to Brian Daniels, Tim Glover and the Richard Hedders for contributing to my mental wellbeing throughout my graduate career.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 BACKGROUND INFORMATION</td>
<td>1</td>
</tr>
<tr>
<td>1.2 IMPROVED DEPLOYMENT CHARACTERISTICS</td>
<td>1</td>
</tr>
<tr>
<td>1.3 LITERATURE REVIEW</td>
<td>3</td>
</tr>
<tr>
<td>2. DYNAMIC MODELING</td>
<td>10</td>
</tr>
<tr>
<td>2.1 DYNAMIC MODELING OF A FULLY DEPLOYED TETHER</td>
<td>10</td>
</tr>
<tr>
<td>2.2 TETHER LINE DEPLOYMENT MODEL</td>
<td>14</td>
</tr>
<tr>
<td>2.3 DETERMINATION OF RESISTANCE PARAMETERS</td>
<td>17</td>
</tr>
<tr>
<td>3. BASIC SIMULATION RESULTS</td>
<td>19</td>
</tr>
<tr>
<td>3.1 NUMBER OF TETHER BEADS CONVERGENCE STUDY</td>
<td>19</td>
</tr>
<tr>
<td>3.2 EFFECT OF PROJECTILE DRAG COEFFICIENT RATIO FOR A LOW MASS RATIO CONFIGURATION</td>
<td>27</td>
</tr>
<tr>
<td>3.3 EFFECT OF PROJECTILE DRAG COEFFICIENT RATIO FOR A HIGH MASS RATIO CONFIGURATION</td>
<td>35</td>
</tr>
<tr>
<td>3.4 EFFECT OF TETHER STIFFNESS FOR A LOW MASS RATIO CONFIGURATION</td>
<td>41</td>
</tr>
<tr>
<td>4. REEL RESISTANCE SIMULATION RESULTS</td>
<td>48</td>
</tr>
<tr>
<td>4.1 EXAMINATION OF A TYPICAL MUNITION SYSTEM</td>
<td>48</td>
</tr>
<tr>
<td>4.2 DEPLOYMENT ENVELOPES</td>
<td>58</td>
</tr>
<tr>
<td>4.2.1 EFFECTS OF REEL RESISTANCE FOR A RANGE OF DROP SPEEDS</td>
<td>59</td>
</tr>
<tr>
<td>4.2.2 EFFECTS OF REEL RESISTANCE FOR A RANGE OF PROJECTILE MASS RATIOS</td>
<td>64</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS, CONTINUED

<table>
<thead>
<tr>
<th>Table/Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.3 EFFECTS OF REEL RESISTANCE FOR A RANGE OF PROJECTILE DRAG RATIOS</td>
<td>70</td>
</tr>
<tr>
<td>4.2.4 EFFECTS OF REEL RESISTANCE FOR A RANGE OF TETHER STIFFNESS</td>
<td>76</td>
</tr>
<tr>
<td>5. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>83</td>
</tr>
<tr>
<td>6. FURTHER WORK</td>
<td>86</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>87</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Flight Phases for the Weapon System Concept</td>
<td>2</td>
</tr>
<tr>
<td>2. Model Diagram and Inertial Reference Frame</td>
<td>11</td>
</tr>
<tr>
<td>3. Aerodynamic Velocity Diagram</td>
<td>15</td>
</tr>
<tr>
<td>4. Maximum Position Error of Follower</td>
<td>21</td>
</tr>
<tr>
<td>5. Maximum Line Load</td>
<td>22</td>
</tr>
<tr>
<td>6. Total Number of Time Steps</td>
<td>23</td>
</tr>
<tr>
<td>7. Number of Steps versus Time for the Pop-Out Method</td>
<td>24</td>
</tr>
<tr>
<td>8. Number of Steps versus Time for the All-Out Method</td>
<td>26</td>
</tr>
<tr>
<td>9. Range of Lead and Follower (Lead/Follower Mass Ratio 1%)</td>
<td>28</td>
</tr>
<tr>
<td>10. Lineout (Lead/Follower Mass Ratio 1%)</td>
<td>29</td>
</tr>
<tr>
<td>11. Pop-Out Method Tether Shape Sequence</td>
<td>30</td>
</tr>
<tr>
<td>12. Speed of Lead Projectile (Lead/Follower Mass Ratio 1%)</td>
<td>31</td>
</tr>
<tr>
<td>13. Speed of Follower Projectile (Lead/Follower Mass Ratio 1%)</td>
<td>32</td>
</tr>
<tr>
<td>14. Maximum Line Load (Lead/Follower Mass Ratio 1%)</td>
<td>34</td>
</tr>
<tr>
<td>15. Maximum Follower Acceleration (Lead/Follower Mass Ratio 1%)</td>
<td>35</td>
</tr>
<tr>
<td>16. Range of Lead and Follower (Lead/Follower Mass Ratio 1%)</td>
<td>36</td>
</tr>
<tr>
<td>17. Lineout (Lead/Follower Mass Ratio 100%)</td>
<td>37</td>
</tr>
<tr>
<td>18. Pop-Out Method Tether Shape Sequence</td>
<td>38</td>
</tr>
<tr>
<td>19. Speed of Lead Projectile (Lead/Follower Mass Ratio 1%)</td>
<td>40</td>
</tr>
<tr>
<td>20. Speed of Follower Projectile (Lead/Follower Mass Ratio 100%)</td>
<td>41</td>
</tr>
<tr>
<td>21. Range of Lead and Follower (Lead/Follower Mass Ratio 1%)</td>
<td>42</td>
</tr>
</tbody>
</table>
LIST OF FIGURES, CONTINUED

22. Lineout (Lead/Follower Mass Ratio 1%) ................................................................. 43
23. Speed of Lead Projectile (Lead/Follower Mass Ratio 1%) ....................................... 44
24. Speed of Follower Projectile (Lead/Follower Mass Ratio 1%) ............................... 45
25. Maximum Line Load (Lead/Follower Mass Ratio 1%) ............................................. 46
26. Maximum Follower Acceleration (Lead/Follower Mass Ratio 1%) ......................... 47
27. Range of Lead and Follower Projectiles .................................................................. 48
28. Body Forward Velocity of Lead Projectile ............................................................... 49
29. Body Forward Velocity of Follower Projectile ......................................................... 50
30. Tension in the Line Element Adjacent to the Follower versus Time ..................... 51
31. Tension in the Line Element Adjacent to the Follower versus Time ..................... 52
32. Tension in the Line Element Adjacent to the Follower versus Time ..................... 53
33. Lineout ...................................................................................................................... 54
34. Lineout Rate ........................................................................................................... 55
35. Maximum Snatch Load .......................................................................................... 56
36. Maximum Acceleration of Follower Projectile ....................................................... 57
37. Time to Reach Steady State ..................................................................................... 58
38. Maximum Snatch Load versus Forward Drop Speed ............................................. 59
39. Maximum Snatch Load versus Forward Drop Speed ............................................. 60
40. Maximum Acceleration of Follower versus Forward Drop Speed ......................... 61
41. Maximum Acceleration of Follower versus Forward Drop Speed ......................... 62
42. Time to Reach Steady State versus Forward Drop Speed ..................................... 63
LIST OF FIGURES, CONTINUED

43. Time to Reach Steady State versus Forward Drop Speed ........................................... 64
44. Maximum Snatch Load versus Mass Ratio ................................................................. 65
45. Maximum Snatch Load versus Mass Ratio ................................................................. 66
46. Maximum Acceleration of Follower versus Mass Ratio ............................................. 67
47. Maximum Acceleration of Follower versus Mass Ratio ............................................. 68
48. Time to Reach Steady State versus Mass Ratio .......................................................... 69
49. Time to Reach Steady State versus Mass Ratio .......................................................... 70
50. Maximum Snatch Load versus Drag Coefficient Ratio .............................................. 71
51. Maximum Snatch Load versus Drag Coefficient Ratio .............................................. 72
52. Maximum Acceleration of Follower versus Drag Coefficient Ratio .......................... 73
53. Maximum Acceleration of Follower versus Drag Coefficient Ratio .......................... 74
54. Time to Reach Steady State versus Drag Coefficient Ratio ........................................ 75
55. Time to Reach Steady State versus Drag Coefficient Ratio ........................................ 76
56. Maximum Snatch Load versus Tether Stiffness .......................................................... 77
57. Maximum Snatch Load versus Tether Stiffness .......................................................... 78
58. Maximum Acceleration of Follower versus Tether Stiffness ..................................... 79
59. Maximum Acceleration of Follower versus Tether Stiffness ..................................... 80
60. Time to Reach Steady State versus Tether Stiffness ................................................... 81
61. Time to Reach Steady State versus Tether Stiffness ................................................... 82
LIST OF TABLES

Table                                                                 Page
1. Constraints .......................................................................................... 18
2. Nominal Simulation Values ...................................................................... 20
LIST OF SYMBOLS

\( x_j, y_j, z_j \) Position vector components of the \( jth \) bead in the inertial reference frame.

\( X_A, Y_A, Z_A \) Drag force in the inertial reference frame for fully deployed model.

\( X_{T_i}, Y_{T_i}, Z_{T_i} \) Elastic line force of the \( ith \) element in the inertial reference frame.

\( W_j \) Weight of the \( jth \) bead.

\( \rho_n \) Air density at the altitude of the follower object.

\( A_n \) Projected area of the follower object.

\( A_{w_i} \) Wetted area of the \( ith \) element.

\( A_{c_i} \) Cross-sectional area of the \( ith \) element.

\( V_{sf, j, i} \) Velocity of the \( jth \) bead normal to \( ith \) tether element.

\( V_{sf, j, 0} \) Velocity of the \( jth \) bead normal to \( ith-1 \) tether element.

\( V_{fp, j, i} \) Velocity of the \( jth \) bead parallel to \( ith \) tether element.

\( V_{fp, j, 0} \) Velocity of the \( jth \) bead parallel to \( ith-1 \) tether element.

\( D_{sf, j, i} \) Skin friction drag force applied to the \( jth \) bead due to the \( ith \) tether element.

\( D_{sf, j, 0} \) Skin friction drag force applied to the \( jth \) bead due to the \( ith-1 \) tether element.

\( D_{fp, j, i} \) Flat plate drag force applied to the \( jth \) bead due to the \( ith \) tether element.

\( D_{fp, j, 0} \) Flat plate drag force applied to the \( jth \) bead due to the \( ith-1 \) tether element.
LIST OF SYMBOLS, CONTINUED

\( C_{D_f} \) Coefficient of drag for the follower object.

\( C_{sf} \) Coefficient of the skin friction drag.

\( C_{fp} \) Coefficient of the flat plate drag.

\( r_i \) Unit vector of the \( ith \) element.

\( \Delta x_j, \Delta y_j, \Delta z_j \) Position difference between each node for the \( jth \) element in the inertial reference frame.

\( \Delta \dot{x}_j, \Delta \dot{y}_j, \Delta \dot{z}_j \) Velocity difference between each node for the \( jth \) element in the inertial reference frame.

\( \Delta l_i \) Distance between each node for the \( ith \) element.

\( \Delta v_i \) Magnitude of the velocity difference between each node for the \( ith \) element.

\( k_i \) Stiffness coefficient for the \( ith \) element.

\( c_i \) Damping coefficient for the \( ith \) element.

\( m_j \) Mass of the \( jth \) bead.

\( m_r \) Mass of the reel.

\( ml \) Tether line mass per unit length.

\( l_b \) Length of tether assigned to each mass.

\( l_i \) Unstretched length of the \( ith \) element.

\( F_{T_i} \) Magnitude of the elastic line force for the \( jth \) element.

\( s \) Total length of tether line out.
LIST OF SYMBOLS, CONTINUED

\( \ddot{s} \) \hspace{1cm} \text{Acceleration of the exiting tether line.}

\( F_r \) \hspace{1cm} \text{Resistance force of the reel.}

\( m_r \) \hspace{1cm} \text{Mass of the reel.}

\( I_r \) \hspace{1cm} \text{Mass moment of inertia for the reel.}

\( r \) \hspace{1cm} \text{Radius of the reel.}

\( n \) \hspace{1cm} \text{Number of the follower object.}

\( i \) \hspace{1cm} \text{Bead index.}

\( j \) \hspace{1cm} \text{Line element index.}
1. INTRODUCTION

1.1 BACKGROUND INFORMATION

Connecting two bodies by means of a tether has been utilized in many aerospace applications including tethered spacecraft,\textsuperscript{1,2} aircraft air refueling,\textsuperscript{3,4} and atmospheric balloons.\textsuperscript{5} More recently, designers have concepted weapon systems with two projectiles connected by a tether line.\textsuperscript{6} In these concepts, the lead projectile is generally a munition and the follower projectile is a sensor platform. The scenario investigated here assumes the weapon is released from an aircraft at altitude and drops to a target. Initially, both projectiles are rigidly attached. The projectiles separate and at a pre-specified time the tether line starts unreeling. When the tether line is completely payed out, the system approaches a steady state as the projectiles and tether line approach the target. A schematic of the various flight phases for the weapon system concept is shown in Figure 1. The work presented first develops a flight dynamic model suitable for simulating the event described above.

1.2 IMPROVED DEPLOYMENT CHARACTERISTICS

Designers must balance the need to unreel the tether line in a specified period of time while at the same time limiting line loads and follower projectile acceleration. The maximum tether line loads occur shortly after the tether fully deploys and this point is called the snatch load. Snatch loads are typically large to the point where line failure is
Figure 1. Flight Phases for the Weapon System Concept
an important concern. Two types of reel resistance mechanisms are considered to improve system performance, namely, reel resistance proportioned to the length of line unspooled from the reel and reel resistance proportional to the rate at which line unspools from the reel. Reel resistance proportional to line out can be realized by mounting the reel on a threaded shaft such that when the reel unspools the reel displaces along the reel axis of symmetry. A helical compression spring connected to the reel and casing provides a resistant moment linearly related to the length of line released from the reel. Reel resistance proportional to line out rate can be realized with a centrifugal clutch. For both devices, resistance parameters are selected by minimizing a cost function containing pertinent performance characteristics. Performance characteristics such as tether line loads, follower projectile acceleration, terminal miss distance, and time for the system to reach steady state are computed using the nonlinear flight dynamic model discussed above.

1.3 LITERATURE REVIEW

There have been several analytical methods developed to investigate the dynamics of cable-body systems, however, little research has considered the actual tether line deployment process. The following is a review of related work in this area.

Choo and Casarella\textsuperscript{7} conducted a survey of analytical methods for the dynamic simulation of cable-body systems. They describe the objectives of the dynamic analysis and the merits and demerits of each method for meeting these objectives. The four most practiced techniques are method of characteristics, finite element method, linearization method, and equivalent lumped mass method. The main difference between these methods lies in the manner the cable is treated. In the method of characteristics and the
linearization method, the cable is regarded as continuum, while in the finite element method, it is represented as a series of segments. Of the four methods, the finite element method is the most versatile.

Winget and Huston\textsuperscript{8} discussed a nonlinear, three dimensional, finite-segment, dynamic model of a cable or chain. Later, Huston and Kamman\textsuperscript{9} presented several sets of data validating this cable model. The validation consisted of a comparison of results obtained from the model with analogous results obtained from a two-dimensional multi-link pendulum model, a comparison of data from the above models with the displacement and natural frequencies of a hanging cable with data obtained analytically from a linear partial differential equation model; and a comparison of model data for a submerged pendulum with experimental data recorded at the Civil Engineering Laboratory at Port Hueneme, California.

Costello and Frost\textsuperscript{10} studied the atmospheric flight mechanics of two projectiles connected by a flexible tether. Both projectiles were individually modeled with six degrees of freedom. The tether was split into a finite number of nodes, with each node possessing three translational degrees of freedom. Forces acting on the nodes included weight, line stiffness, line damping, and aerodynamic drag. The tether line deployment process was modeled with a single degree of freedom that permitted unreeling resistance to be incorporated. The tether deployment system was assumed to consist of a rotating reel acted on by an elastic line force, which acted to pay out the tether line, and a resistance force.

Janssens, Poelaert, and Crellin\textsuperscript{11} developed equations to show the correct relationship between the braking or retracting force and the tension at the feed out point
of a continuous space tether. When the equations of motion were derived from a
variational principle, non-conservative generalized forces associated with the generalized
coordinate describing the deployment/retraction point of the tether were introduced in the
corresponding Lagrange equation. Crellin, Janssens, and Poelaert\textsuperscript{12} show that treating the
mechanical model where the motion of the cable at the satellite is not prescribed but is
itself an unknown, results in a number of fundamental problems, especially if a
variational formulation of the equations of motion is used. They suggest that a
variational formulation is convenient for a complicated satellite system, because it also
supplies the boundary conditions for the cable, that is, the equations of motion of the
satellites. To make their point they treated a vertically hanging in-extensible string with a
body modeled as a point mass moving up and down under the action of a force acting
between body and string by retrieving the string in the body or deploying it from the
body. For the variational formulation of the equations of motion of such a system where
the change of its mass distribution is modeled by plastic impacts, it was necessary to
include the Carnot energy loss concept.

A simulation of a thin wired deployed from an aircraft was conducted by Dekel
and Pnueli\textsuperscript{13}. In this case one end of the wire was fixed to a stationary point on the
ground, while the other end was unwound from a spool carried in the aircraft. There was
a coupling between the tension of the wire at the spool, the geometry and the general
design of the spool, and the rate at which the wire was released. Their simulation
assumed that the transverse rotational whipping motion caused by the unwinding from
the spool, with its associated accelerations, decayed a short distance behind the aircraft.
In a passive deployment procedure for bringing a payload connected to an orbiting spacecraft by a cable into a permanent locally vertical position, Kane and Levinson\textsuperscript{14} avoid the effects of the unreeling process by starting with an unreeled cable. The payload is then in a free flight until the cable becomes taut. At this time the motion is altered drastically and a new free flight begins. Impacts and free flights thereafter occur alternately until the cable becomes permanently taut.

Doyle\textsuperscript{5} developed a mathematical model for the ascent and descent of a high-altitude tethered balloon. The mathematical model consisted of a spherical balloon and a cable consisting of an arbitrary number of links. The effect of the winching rate on the balloon launch trajectory was demonstrated.

Djerassi and Bamberger\textsuperscript{15} point out that a method suitable for the simulation of motions of systems deploying a cable from two platforms has to allow the cable to assume an arbitrary configuration. They state that one has to resort to an approach where cables are regarded as dynamical systems with a chain topology.

Taking advantage of the special properties of systems consisting of chain connected bodies Rosenthal\textsuperscript{16} established specific formulations for the generation of equations governing the motions of such systems. These were developed into extremely efficient algorithms known as order-$n$ algorithms. The number of operations required by these algorithms in the context of numerical solutions of the associated equations is proportional to $n$, the number of degrees of freedom. Rosenthal’s order-$n$ algorithm for a chain topology multi-body system was used by Banerjee\textsuperscript{17} to simulate motions of a cable deployed from or retracted into a platform having a prescribed motion, so that $n$ became a function of time. Djerassi and Bamberger required that the algorithm be revised when
the motion of the endpoint of the cable was prescribed, as when the cable was connected to a second moving platform. The force exerted by the latter on the cable endpoint had to be determined before the motion variables could be evaluated. In their study of the deployment of a cable from two moving platforms, the number of links was assumed to be constant, but the length of the links was time dependent, enabling deployment (or retraction). The masses of the platforms were assumed to significantly exceed that of the cable, so that the deployment process did not affect their motion. Furthermore, the platforms were regarded as moving along predetermined trajectories with known velocities.

Cochran and Innocenti\textsuperscript{18} developed a general simulation of a system consisting of a towing aircraft, a tow cable reel, the cable, and a maneuverable towed vehicle. In the development of the model they assumed that the towing vehicle was much larger than the towed vehicle so that the towing aircraft’s motion was unaffected by the towed vehicle and was specified. The tow cable was modeled as a system of \( n \) point masses connected by massless, straight cable segments. Aerodynamic and gravitational forces that acted on the masses were determined from physical and aerodynamic characteristics of cable segments. The number of masses used was arbitrary to allow for variations in the length of the cable during deployment and retrieval. The segments between point masses on the cable were assumed to be inextensible. However, the length of the segment between the mass closest to the aircraft could vary in length.

The process of removing yarn from a package by over-end withdrawal through a guide eye is fundamental to many operations in the textile industry, examples being winding and unwinding. The portion of the yarn between the package and the guide eye,
which is situated on the axis of symmetry of the package, attains an angular velocity about the package axis. The forces acting on the yarn, among them tension, air drag, centrifugal force, Coriolis force, and gravity, cause the yarn to fly out and form a balloon. Kothari and Leaf\cite{Kothari19} use a linearization method to develop equations of motion of yarn in a balloon and produce numerical results to demonstrate the balloon's properties.

Holzschuh and Hightower\cite{Holzschuh20} patented a system for passively deploying an optical fiber without active control of tension or of deployment rate. The system uses concatenated spools of optical fiber to assure that failure inducing stresses are avoided as a missile and launch platform are deployed.

Huffman and Genin\cite{Huffman21} formulated a non-linear mathematical model for the study of the dynamics of an extensible cable subjected to aerodynamic forces generated by a uniform flow field. Solutions were found considering large displacements caused by suddenly applied loads for a range of flow speeds and cable lengths.

Just prior to the publication of this thesis, Kamman and Huston\cite{Kamman22} presented an algorithm for modeling the dynamics of towed and tethered cable systems with fixed and varying lengths. The systems may have one or many open branches, but must be towed from a single point. The modeling uses finite-segment elements. Cable length changes (reel-in/pay-out) are modeled by having a link near the towing vessel change length. The physical properties of the cable may change from link to link. Effects of fluid drag, lift, and buoyancy are included. Added mass forces and moments are included for the towed bodies but not for the cable itself. An illustrative application is presented for a system with three different pay-out rates.
Matteis and Socio\textsuperscript{23} studied the equations of motion of a subsatellite and its tether to show the influence of the aerodynamic forces on the equilibrium states of the system and the corresponding perturbed motion. It is shown that aerodynamic forces can play a major role in determining the stability of the system equilibrium states in situations of practical interest.

Bannerjee and Van N. Do\textsuperscript{24} describe the development of an underwater cable dynamics model and a realistic control system that allows deployment, regulation, and retrieval of an unmanned underwater vehicle tethered to a ship. An order-n algorithm for a variable-mass cable subject to hydrodynamic forces and motion constraints is used to simulate the dynamics of the system. The dynamics of the underwater vehicle is separately given with the cable tension nonlinearly affecting the vehicle speed. This creates a constraint on the cable motion that depends nonlinearly on the constraint force, a problem that is iteratively solved using constraint stabilization.

Several similar papers were investigated and are included in the references for the readers benefit.\textsuperscript{25,26,27,28,29,30,31,32}
2. DYNAMIC MODELING

2.1 DYNAMIC MODELING OF A FULLY DEPLOYED TETHER

As previously mentioned, the weapon system consists of two projectiles connected by a flexible tether. This is modeled as a series of nodes or beads connected by springs and dampers arranged in parallel as depicted in Figure 2. The lead and follower projectiles are assumed to be stable and are modeled as point masses with three translational degrees of freedom. Likewise, each tether bead is modeled as a point mass, also with three translational degrees of freedom. Gravitational, aerodynamic, and elastic forces act upon both projectiles and the tether beads. The lead and follower objects are designated as the 0th and nth nodes, respectively. The tether is split into n – 1 point mass beads designated by j, and n line elements designated with i. The Earth's surface is used as an inertial reference frame. Air density is computed using a standard atmosphere model.33

When the tether is fully deployed, the equations of motion for the follower projectile (n) and the jth tether bead are written in the inertial reference frame and given in Equations 1 and 2.

\[
\begin{align*}
\begin{pmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{z}
\end{pmatrix}_n &=
\begin{pmatrix}
X_D \\
Y_D \\
Z_D
\end{pmatrix}
- 
\begin{pmatrix}
X_T \\
Y_T \\
Z_T
\end{pmatrix}_{n-1}
+ 
\begin{pmatrix}
0 \\
0 \\
W_n
\end{pmatrix} \\
\begin{pmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{z}
\end{pmatrix}_j &=
\begin{pmatrix}
X_T \\
Y_T \\
Z_T
\end{pmatrix}_{j+1}
- 
\begin{pmatrix}
X_T \\
Y_T \\
Z_T
\end{pmatrix}_i
+ 
\begin{pmatrix}
X_D \\
Y_D \\
Z_D
\end{pmatrix}_j
+ 
\begin{pmatrix}
0 \\
0 \\
W_T
\end{pmatrix}_j
\end{align*}
\]
The lead projectile (0) equations are identical in form to Equation 1. The elastic forces are due to the spring and damping characteristics of the tether. These forces are always

Figure 2. Model Diagram and Inertial Reference Frame
parallel to the direction of the line. In order to express the tether bead applied loads concisely; the position and velocity matrices shown in Equations 3 through 6 prove useful.

\[
\begin{bmatrix}
\Delta x_0 & \Delta y_0 & \Delta z_0 \\
\Delta x_1 & \Delta y_1 & \Delta z_1 \\
\vdots & \vdots & \vdots \\
\Delta x_{n-1} & \Delta y_{n-1} & \Delta z_{n-1}
\end{bmatrix}
= 
\begin{bmatrix}
x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\
x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\
\vdots & \vdots & \vdots \\
x_n - x_{n-1} & y_n - y_{n-1} & z_n - z_{n-1}
\end{bmatrix}
\]  

(3)

\[
\begin{bmatrix}
\Delta \dot{x}_0 & \Delta \dot{y}_0 & \Delta \dot{z}_0 \\
\Delta \dot{x}_1 & \Delta \dot{y}_1 & \Delta \dot{z}_1 \\
\vdots & \vdots & \vdots \\
\Delta \dot{x}_{n-1} & \Delta \dot{y}_{n-1} & \Delta \dot{z}_{n-1}
\end{bmatrix}
= 
\begin{bmatrix}
\dot{x}_1 - \dot{x}_0 & \dot{y}_1 - \dot{y}_0 & \dot{z}_1 - \dot{z}_0 \\
\dot{x}_2 - \dot{x}_1 & \dot{y}_2 - \dot{y}_1 & \dot{z}_2 - \dot{z}_1 \\
\vdots & \vdots & \vdots \\
\dot{x}_n - \dot{x}_{n-1} & \dot{y}_n - \dot{y}_{n-1} & \dot{z}_n - \dot{z}_{n-1}
\end{bmatrix}
\]  

(4)

\[
\begin{bmatrix}
\Delta l_0 \\
\Delta l_1 \\
\vdots \\
\Delta l_{n-1}
\end{bmatrix}
= 
\begin{bmatrix}
\sqrt{\Delta x_0^2 + \Delta y_0^2 + \Delta z_0^2} \\
\sqrt{\Delta x_1^2 + \Delta y_1^2 + \Delta z_1^2} \\
\vdots \\
\sqrt{\Delta x_{n-1}^2 + \Delta y_{n-1}^2 + \Delta z_{n-1}^2}
\end{bmatrix}
\]  

(5)

\[
\begin{bmatrix}
\Delta v_0 \\
\Delta v_1 \\
\vdots \\
\Delta v_{n-1}
\end{bmatrix}
= 
\begin{bmatrix}
\Delta x_0 \Delta \dot{x}_0 + \Delta y_0 \Delta \dot{y}_0 + \Delta z_0 \Delta \dot{z}_0 \\
\Delta x_1 \Delta \dot{x}_1 + \Delta y_1 \Delta \dot{y}_1 + \Delta z_1 \Delta \dot{z}_1 \\
\vdots \\
\Delta x_{n-1} \Delta \dot{x}_{n-1} + \Delta y_{n-1} \Delta \dot{y}_{n-1} + \Delta z_{n-1} \Delta \dot{z}_{n-1}
\end{bmatrix}
\]  

(6)

Stiffness, damping, mass and unloaded element length matrices are given below as Equations 7, 8, 9, and 10.

\[K = \begin{bmatrix} k_0 & k_1 & \cdots & k_{n-1} \end{bmatrix}\]  

(7)

\[C = \begin{bmatrix} c_0 & c_1 & \cdots & c_{n-1} \end{bmatrix}\]  

(8)

\[M = \begin{bmatrix} m_0 & m_1 & \cdots & m_n \end{bmatrix}\]  

(9)
\[ L = [l_0 \ l_1 \ \cdots \ l_{n-1}] \]  

As shown in Equation 11, the length of each tether element is equal to the total length of the tether divided by the number of tether beads, with the exception of the elements directly connected to the lead and follower projectiles which are given by Equation 12.

\[ l_i = \frac{l_i}{n-1} \]  

\[ l_0 = \frac{l_0}{2} \quad ; \quad l_{n-1} = \frac{l_{n-1}}{2} \]  

With the above definitions, the magnitudes of the tether forces are given in Equations 13 and 14.

\[ F_{T_i} = \begin{bmatrix} F_{r_0} & F_{r_1} & \cdots & F_{r_{n-1}} \end{bmatrix} \]  

\[ F_{T_i} = \begin{cases} k_i (\Delta l_i - l_i) + c_i \Delta v_i, & \Delta l_i \geq l_i \\ 0, & \Delta l_i < l_i \end{cases} \]  

The elastic tether forces expressed in inertial coordinates are shown in Equation 15.

\[ \begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} = F_{T_i} \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix} \]  

The tether line forces exerted on adjacent masses are equal in magnitude and opposite in direction.

The follower projectile drag force is given in Equation 16.

\[ \begin{bmatrix} X_D \\ Y_D \\ Z_D \end{bmatrix}_{n} = -\frac{1}{2} \rho_n \sqrt{\dot{x}_n^2 + \dot{y}_n^2 + \dot{z}_n^2} A_n C_{D_n} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_{n} \]  

The projectile drag coefficients are Mach number dependent and are computed by linear interpolation of tabulated data.
The aerodynamic force on the tether line includes skin friction drag along the tether line and flat plate drag perpendicular to the tether line. To determine the tether drag it is useful to define a unit vector with inertial frame components shown in Equation 17.

\[
\begin{bmatrix}
  r_x \\
  r_y \\
  r_z
\end{bmatrix} = \frac{1}{\Delta l_i} \begin{bmatrix}
  \Delta x_i \\
  \Delta y_i \\
  \Delta z_i
\end{bmatrix}
\]  

As shown in Figure 3, the tether line unit vectors are used to express the velocity of each bead into components along and normal to the adjacent elements. Note that the aerodynamic force acts on a bead even when the tether line is slack. The skin friction and flat plate drag for each element are given by Equations 18 and 19.

\[
D_{sf,i,j} = -\frac{1}{2} \rho_j \cdot A_{w,i} \cdot C_{sf} \cdot V_{sf,i,j} \cdot |V_{sf,i,j}|
\]  

\[
D_{fp,j,i} = -\frac{1}{2} \rho_j \cdot A_{w,i} \cdot C_{fp} \cdot \left(V_{fp,j,i}\right)^2
\]  

The tether bead aerodynamic forces expressed in the inertial frame are shown in Equation 20.

\[
\begin{bmatrix}
  X_D \\
  Y_D \\
  Z_D
\end{bmatrix}_j = D_{fp,j,i} \begin{bmatrix}
  r_x \\
  r_y \\
  r_z
\end{bmatrix}_i + D_{fp,j,0} \begin{bmatrix}
  r_x \\
  r_y \\
  r_z
\end{bmatrix}_i + \frac{D_{sf,i,1}}{V_{sf,i,1}} \begin{bmatrix}
  V_{sf,i,x,1} \\
  V_{sf,i,y,1} \\
  V_{sf,i,z,1}
\end{bmatrix} + \frac{D_{sf,j,0}}{V_{sf,j,0}} \begin{bmatrix}
  V_{sf,i,x,0} \\
  V_{sf,i,y,0} \\
  V_{sf,i,z,0}
\end{bmatrix}
\]

2.2 TETHER LINE DEPLOYMENT MODEL

As the lead and follower projectiles separate the tether line pays out. There are two aspects to modeling this process, namely, the pay out of the tether line from the lead
Figure 3. Aerodynamic Velocity Diagram

projectile and the motion of released tether line. Two methods for modeling the released tether line motion were examined, the pop-out and all-out deployment methods.

The pop-out tether deployment model initially places all tether beads on the lead projectile. As the tether line is payed out, beads are released from the lead projectile into the atmosphere. A bead is not placed into the atmosphere until a sufficient length of line has been unreeled. For this reason, during deployment using the pop-out method only a
fraction of the tether beads are dynamically active in the atmosphere. When a bead is placed into the atmosphere, it is placed along the line from the release point to the last bead released and initial conditions are established such that the elastic force across the line is unchanged. This tends to prevent a discontinuity in the line out rate due to bead release. However, because aerodynamic forces act on the bead immediately after it is released, a slight perturbation is generally observed when a bead is released. When a bead is released, the mass of the lead projectile is reduced by the released bead weight, the length from the release point to the last tether bead released is reset along with the stiffness and damping coefficients of the exiting tether line.

The all-out tether deployment model places all beads into the atmosphere immediately after the projectiles are separated. The mass, stiffness, and damping characteristics of the tether line elements are continuously updated as the line is payed out. Initially, the mass of each bead is small and as line is released from the reel, the mass of each bead increases.

The tether reel is assumed to consist of a rotating reel acted upon by the exiting bead elastic force. The elastic force between the lead object and the neighboring bead acts on the reel to pay out the tether line. The reel has a resistance force, \( F_r \), which opposes the unreeling process. The equation governing the dynamics of the tether line unreeling process is shown in Equations 21 and 22.

\[
\ddot{s} = \frac{(F_m - F_r)r^2}{I_r} \tag{21}
\]

\[
I_r = \frac{(m_r - ml \cdot s)r^2}{2} \tag{22}
\]
When the full length of tether line has been reached the acceleration and the velocity of the reel are set to zero. The functional form of the line out rate and line out reel resistance force is given by Equations 23 and 24 respectively.

\[ F_r = R_1(s - R_2)^2 + 1, \quad s \geq R_2 \quad (23) \]

\[ F_r = R_1(s - R_2) + 1, \quad s \geq R_2 \quad (24) \]

In Equations 23 and 24, the variables \( R_1 \) and \( R_2 \) are design parameters of the reel resistance device.

2.3 DETERMINATION OF RESISTANCE PARAMETERS

To compute the resistance parameters \( R_1 \) and \( R_2 \), an objective function producing a cost \( J \), given by Equation 25, is minimized.

\[ J = C_1 f_1(T_g) + C_2 f_2(t_{ss}) + C_3 f_3(T_t) + C_4 f_4(d) \quad (25) \]

In Equation 25, \( T_g \) is the maximum acceleration of the follower projectile, \( t_{ss} \) is the time for the system to reach steady state, \( T_t \) is the maximum tether line load, \( d \) is the maximum deviation of the lead projectile impact point, and \( C_j \) are objective function weights. Time to reach steady state is defined as the amount of time it takes, measured from the beginning of the simulation, for the oscillations that result due to the snatch load to die out. The barrier functions \( f_1, f_2, f_3 \), and \( f_4 \) all take on the form shown in Equation 26.

\[ f_i(x) = e^{A_ix + A_0} \quad (26) \]
The barrier function constants, \(A_i\) and \(A_o\), were created based on the constraints found in Table 1. Using a non-linear simplex method the resistance parameters are varied and the objective function cost is minimized.\(^{34}\) The objective function is set up so that the cost dramatically increases as the design variables (maximum tether line load, maximum projectile acceleration, time to reach steady state and deviation from target) near their maximum constraints. The advantage of this technique is that it requires only function evaluations and the disadvantage is slow convergence.

<table>
<thead>
<tr>
<th>Maximum Constraint - Line Out</th>
<th>Conservative Constraint - Line Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_g \leq 80) (g's)</td>
<td>(T_g \leq 40) (g's)</td>
</tr>
<tr>
<td>(t \leq 30) (sec)</td>
<td>(t_{ss} \leq 28) (sec)</td>
</tr>
<tr>
<td>(T_t \leq 800) (lbs)</td>
<td>(T_t \leq 600) (lbs)</td>
</tr>
<tr>
<td>(-1000 \leq d \leq 1000) (ft)</td>
<td>(-750 \leq d \leq 750) (ft)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum Constraint - Line Out Rate</th>
<th>Conservative Constraint - Line Out Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_g \leq 80) (g's)</td>
<td>(T_g \leq 40) (g's)</td>
</tr>
<tr>
<td>(t \leq 30) (sec)</td>
<td>(t_{ss} \leq 28) (sec)</td>
</tr>
<tr>
<td>(T_t \leq 1000) (lbs)</td>
<td>(T_t \leq 600) (lbs)</td>
</tr>
<tr>
<td>(-1000 \leq d \leq 1000) (ft)</td>
<td>(-750 \leq d \leq 750) (ft)</td>
</tr>
</tbody>
</table>
3. BASIC SIMULATION RESULTS

3.1 NUMBER OF TETHER BEADS CONVERGENCE STUDY

A key question for simulating the weapon system described above is how many elements should be used to model the tether. As the number of degrees of freedom increases linearly with the number of beads, it is obviously desirable to use relatively few beads. To investigate this matter, the equations documented above were simulated for varying tether discretizations. Typical values were selected for a 2000-lb bomb lead projectile released from a fighter aircraft and a follower projectile that is a sensor platform. Table 2 lists the nominal values used in the simulation.

Figure 4 plots the maximum position error from a reference trajectory of the follower projectile as a function of the number of beads used to model the tether line. When determining the number of beads to use for the reference trajectory, it was argued that the greater the number of beads the closer the system is to being continuous. However, using more beads increases the computational time. Beyond 200 beads the computational time becomes unreasonably high, therefore the simulated trajectory using 200 beads was chosen as the reference trajectory. Because the lead projectile is much heavier than the follower projectile, its trajectory is modified much less than the follower projectile's trajectory with the addition of tether line coupling. For a low projectile mass ratio, follower projectile trajectory deviations represent the worst case. As shown in Figure 4 even for a small number of beads \((n \leq 10)\), the maximum error in the follower trajectory is small.
Table 2. Nominal Simulation Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projectile Follower Frontal Area</td>
<td>1.77 ft²</td>
</tr>
<tr>
<td>Projectile Follower Weight</td>
<td>19.62 lbs</td>
</tr>
<tr>
<td>Projectile Follower Initial Forward Velocity</td>
<td>500 ft/s</td>
</tr>
<tr>
<td>Projectile Follower Initial Vertical Velocity</td>
<td>0 ft/s</td>
</tr>
<tr>
<td>Projectile Follower Initial Side Velocity</td>
<td>0 ft/s</td>
</tr>
<tr>
<td>Projectile Lead Frontal Area</td>
<td>1.77 ft²</td>
</tr>
<tr>
<td>Projectile Lead Weight</td>
<td>19.62 lbs</td>
</tr>
<tr>
<td>Projectile Lead Initial Forward Velocity</td>
<td>500 ft/s</td>
</tr>
<tr>
<td>Projectile Lead Initial Vertical Velocity</td>
<td>0 ft/s</td>
</tr>
<tr>
<td>Projectile Lead Initial Side Velocity</td>
<td>0 ft/s</td>
</tr>
<tr>
<td>Drag Coefficient Ratio (Follower/Lead)</td>
<td>2/1</td>
</tr>
<tr>
<td>Reel Radius</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Reel Weight</td>
<td>5 lbs</td>
</tr>
<tr>
<td>Tether Length</td>
<td>1000 ft</td>
</tr>
<tr>
<td>Tether Weight per Unit Length</td>
<td>0.01 lbs</td>
</tr>
<tr>
<td>Tether Diameter</td>
<td>0.0082 ft</td>
</tr>
<tr>
<td>Tether Stiffness</td>
<td>62500 lbs-ft/ft</td>
</tr>
<tr>
<td>Tether Damping Constant</td>
<td>0.30</td>
</tr>
<tr>
<td>Skin Friction Drag Coefficient</td>
<td>0.007</td>
</tr>
<tr>
<td>Flat Plate Drag Coefficient</td>
<td>1.100</td>
</tr>
<tr>
<td>Release Altitude</td>
<td>25000 ft</td>
</tr>
<tr>
<td>Separation Time</td>
<td>0 sec</td>
</tr>
<tr>
<td>Total Simulation Time</td>
<td>45 sec</td>
</tr>
</tbody>
</table>
Using both tether deployment schemes, Figure 5 plots the maximum tether line force versus the number of tether beads. For the configuration considered here, where the tether line is released from the lead projectile, the maximum tether line load occurs shortly after the tether has fully deployed. Figure 5 shows a significant difference in the maximum tether line load between the all-out and pop-out methods. The all-out tether deployment scheme predicts significantly lower maximum loads than the pop-out method. The trend for the maximum acceleration of the follower is similar in nature to the maximum tether line load trend. In the pop-out method, beads are released into the atmosphere continuously during tether deployment. When a bead is released...
aerodynamic forces immediately act on the bead and subsequently induce vibration throughout the tether line.

![Maximum Line Load](image)

**Figure 5. Maximum Line Load**

This vibration wave along the tether line increases the maximum tether line load experienced during the event. Furthermore, Figure 5 shows that to predict the maximum line load, a large number of tether beads is required compared to predicting follower projectile position. Moreover, using a low number of tether beads to predict the maximum tether line force creates a non-conservative estimate. Because the all-out tether deployment method does not introduce spurious tether line vibration as the pop-out
method does when releasing a bead, it is seen as a superior technique to model tether line deployment. However, Figure 6 shows that the all-out tether deployment model incurs significantly higher computation time. Figure 6 plots the total number of time steps to perform a simulation versus the number of tether beads. A 5\textsuperscript{th} order Runge-Kutta, adaptive time step, numerical integration scheme was used to integrate the equations of motion.

**Figure 6. Total Number of Time Steps**
Figures 7 and 8 show the density of integration (adaptive time) steps taken versus time for the pop-out and all-out methods using 100 beads.

**Figure 7. Number of Steps versus Time for the Pop-Out Method**

In the all-out method, all beads are released into the atmosphere when the projectiles separate. Initially, only a small amount of line has been released from the reel, thus the bead mass is small and the equations of motion are relatively stiff. Typically, the numerical integrator significantly slows immediately after the beads are released and also at the snatch load point. On the other hand, the pop-out method gradually releases beads,
as line is payed out. Hence, during deployment the pop-out method integrates fewer equations of motion and has larger bead masses compared to the all-out method. The increase in steps near the end of the simulation is a result of the lead projectile impacting the ground.

The pop-out method, using 100 beads, was chosen for the following simulations used to predict the basic response of the system for various design parameters. 100 beads were chosen because the maximum tether line load error levels out at this number and the amount of computational is reasonable. The pop-out method was chosen because the maximum tether line loads are conservative, the position error is modest, and the computational time is significantly less than that of the all-out method.
Figure 8. Number of Steps versus Time for the All-Out Method
3.2 EFFECT OF PROJECTILE DRAG COEFFICIENT RATIO FOR A LOW MASS RATIO CONFIGURATION

The separation dynamics are driven in large part by the difference between the drag forces on the lead and follower projectiles. One of the primary questions designers are faced with is how to shape the follower projectile to unreel the tether line over a specified duration of time and at the same time limiting the tether line maximum loads and the follower projectile acceleration at the snatch point. This section shows the basic response for various projectile drag ratios when the follower projectile weight is 1% of the lead projectile.

Figure 9 plots the range of the lead and follower projectiles for five different lead-to-follower drag coefficient ratios (1.25, 1.50, 1.75, 2.00, and 5.00). The shape of the drag coefficient curve versus Mach number is identical for both projectiles. For a given drag coefficient ratio, the lead and follower trajectories overlay one another. As would be expected, a decrease in range is noticed when the follower projectile drag coefficient is increased. As shown in Figure 10, when the follower drag coefficient is increased the tether line pays out more rapidly so that the tether line tension on the lead projectile is higher over a longer portion of the trajectory contributing to decreased range. For a drag coefficient ratio of 5.0, the decrease in range of 15% is substantial, however, the corresponding decrease in the tether deployment time is approximately 1 second.
Figure 9. Range of Lead and Follower (Lead/Follower Mass Ratio 1%)
Figure 10. Lineout (Lead/Follower Mass Ratio 1%)

Figure 11 plots a sequence of frames that show the tether shape during a typical event for a low projectile mass ratio configuration. Notice that these results are for the pop-out method with a projectile drag coefficient ratio of 2. The all-out method produces similar trajectory results with lower tether line loads. The lead projectile is on the right and the follower projectile is on the left side of the tether line. The diamond on the tether line indicates where the maximum load in the tether line is located at that time instant. The line fully deploys in less than 5.67 seconds and hits the snatch load in frame 3. Notice the maximum line load is at the lead projectile. After the first snatch condition the line goes slack and bunches as shown in frames 4, 5, and 6. The line encounters a
<table>
<thead>
<tr>
<th>Frame 1</th>
<th>Frame 2 - Line Out</th>
<th>Frame 3 - Snatch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time = 2.205 sec</td>
<td>Time = 5.535 sec</td>
<td>Time = 5.67 sec</td>
</tr>
</tbody>
</table>

- **Tether**
- **Maximum Force**

<table>
<thead>
<tr>
<th>Frame 4</th>
<th>Frame 5</th>
<th>Frame 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time = 6.93 sec</td>
<td>Time = 7.29 sec</td>
<td>Time = 7.515 sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frame 7 - Snatch</th>
<th>Frame 8</th>
<th>Frame 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time = 7.83 sec</td>
<td>Time = 7.92 sec</td>
<td>Time = 8.19 sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frame 10</th>
<th>Frame 11</th>
<th>Frame 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time = 8.955 sec</td>
<td>Time = 26.955 sec</td>
<td>Time = 44.505 sec</td>
</tr>
</tbody>
</table>

**Figure 11. Pop-Out Method Tether Shape Sequence**

(Mass Ratio 1% ; Drag Ratio 2/1)
second snatch condition at \( t = 7.83 \) seconds as shown in frame 7. At the second snatch load condition, the maximum tether line load is at the follower projectile. The projectile combination eventually settles into a steady state drop by approximately \( t = 8.955 \) seconds as shown in frame 10. In frame 12, the lead projectile has already impacted the ground. The maximum tether line load moves back and forth along the tether line throughout the event. However, for this configuration the overall maximum line load occurs at the first snatch condition at the tether line and the lead projectile connection point.

Figure 12 plots the magnitude of the inertial velocity of the lead projectile. For
the configuration analyzed the steady state drop velocity is larger than the release velocity so the lead projectile increases its speed over the trajectory until it impacts the ground and its velocity is zero. As expected, when the follower projectile drag is increased, the lead projectile’s speed is reduced and it takes longer to reach the ground. Figure 13 shows the speed of the follower projectile over the trajectory for different drag coefficient ratios.

Notice that all traces show the same characteristics. Because the steady state drop velocity of the follower projectile is lower than the aircraft release speed, the follower

![Figure 13. Speed of Follower Projectile (Lead/Follower Mass Ratio 1%)](image-url)
projectile initially slows down. The difference in speed between the lead and follower projectiles pays out the line. When the tether line is fully deployed the tether line grabs the follower and rapidly increases its speed. The follower then rebounds toward the lead projectile, so much that the tether line goes slack. With the line slack, the follower projectile again slows down to seek its steady state drop velocity. This oscillation continues until a steady state condition is arrived at where the lead and follower projectiles possess the same speed. At the end of the trajectory, the lead projectile impacts the ground and shortly after the tether line goes slack and again the follower projectile slows down and approaches its steady state drop velocity. Figure 14 shows a bar graph of the maximum tether line tension for different drag coefficient ratios using both tether deployment methods. Notice that increasing the drag coefficient ratio increases the maximum tension. Thus, one must take care in selecting the drag coefficient ratio and avoid exceeding the ultimate line strength. Corresponding to Figure 14, Figure 15 shows the maximum acceleration of the follower projectile. The maximum acceleration also increases with increased drag coefficient ratio.
Figure 14. Maximum Line Load (Lead/Follower Mass Ratio 1%)
3.3 EFFECT OF PROJECTILE DRAG COEFFICIENT RATIO FOR A HIGH MASS RATIO CONFIGURATION

This section considers the system response for various projectile drag ratios when the follower and lead projectile weights are equal. The range of the lead and follower projectiles for the same five lead-to-follower drag coefficient ratios considered above are plotted in Figure 16. For drag ratios of 1.75 and less the tether line never becomes fully extended before the lead projectile contacts the ground and the lead and follower trajectories overlay one another. As with the 1% mass ratio study, a decrease in range is noticed when the follower projectile drag coefficient is increased. For drag coefficient
ratios of 5.0 and 2.0 the lead and follower trajectories do not overlay one another because oscillations from the snatch point have not died out before the lead projectile comes in contact with the ground.

Figure 16. Range of Lead and Follower (Lead/Follower Mass Ratio 1%)

As shown in Figure 17, when the follower drag coefficient is increased the tether line pays out more rapidly. Except for the high drag coefficient ratios this does not have the effect of reducing the range of the lead projectile as the 1 % mass ratios did. Since the
tether line for the low drag coefficient ratios never fully extend, the lead and follower projectiles approach their steady state drop velocities with a slack tether line.

Figure 17. Lineout (Lead/Follower Mass Ratio 100%)

Figure 18 plots a sequence of frames that show the tether shape during a typical event for a high projectile mass ratio configuration. Notice that these results are for the pop-out method with a projectile drag coefficient ratio of 5.0. As before, the lead projectile is on the right and the follower projectile is on the left side of the tether line.
Figure 18. Pop-Out Method Tether Shape Sequence

(Mass Ratio 100% ; Drag Ratio 5/1)
The diamond on the tether line indicates where the maximum load is on the tether line at that instant. In frame 1, the two projectiles begin to separate due to the drag on the follower projectile. As the tether is affected by aerodynamic drag it begins to billow out in the shape shown in frame 2. Frame 3 and 4 show that tether line is pulled out mainly due to the aerodynamic load on the exposed tether and not from the position difference of the follower and lead projectiles. It requires a relatively long time for the drag of the follower to overcome its momentum and consequently snatch does not occur until approximately 23.94 seconds in frame 5. As with the 1% mass ratio case the maximum line load occurs between the connection point of the lead projectile and the first bead. The tether line, however, after snatch reacts differently than the 1% mass ratio configuration. Instead of the tether bunching, a whipping action is imparted to the tether and the follower increases in velocity as it swings downward and then again upward through the line as shown in frames 6 and 7. This action creates the potential for the entanglement of the tether line. Snatch occurs at approximately 34.2 seconds and the maximum line load is again at the connection point as shown in frame 8. This is the point in time at which the overall maximum line load occurs and not at the initial snatch point. Frame 9 shows that the snatch load causes the follower to fly forward in an upward swing. As the projectiles approach the ground in frame 10 the follower is significantly ahead of an extended trailing position that is achieved for the 1% mass ratio configuration. In frame 11 it is shown that the follower projectile lands past the lead when the two projectiles come in contact with the ground. Figure 19 plots the magnitude of the inertial velocity of the lead projectile. Unlike the low projectile mass ratio case, the lead projectile’s speed is greatly affected by the snatch load for the drag coefficient
ratio of 5.0. The other traces do not exhibit this characteristic because snatch does not occur or occurs just prior to the lead projectile hitting the ground.

![Graph showing speed of lead projectile](image)

**Figure 19. Speed of Lead Projectile (Lead/Follower Mass Ratio 1%)**

Figure 20 demonstrates the increases in the follower’s speed due to the whipping actions presented in Figure 18 for the drag coefficient ratio of 5.0. The lead and follower projectiles never enter into a steady state condition as the 1% mass ratio case.
3.4 EFFECT OF TETHER STIFFNESS FOR A LOW MASS RATIO CONFIGURATION

This section shows the basic response for various tether stiffness values when the follower projectile weight is 1% of the lead projectile and the drag coefficient ratio of the follower projectile to lead projectile is 2/1. Figure 21 plots the range of the lead and follower projectiles for five different stiffness values (10,000 lb-ft/ft, 25,000 lb-ft/ft, 50,000 lb-ft/ft, 62,500 lb-ft/ft, and 75,000 lb-ft/ft). As shown in Figure 21, all the trajectories coincide indicating that the tether stiffness has no noticeable effect on the range of the lead and follower projectiles.
Figure 21. Range of Lead and Follower (Lead/Follower Mass Ratio 1%)

Figure 22 shows that the same is true for the lineout rate. Figure 23 shows the speed of the lead projectile for various tether stiffness values. Except for a slight decrease at the point of snatch the tether stiffness has no effect on the lead projectile’s speed either. The speed of the follower projectile is plotted in Figure 24. A decrease in tether stiffness allows the oscillations caused by snatch to persist for a slightly longer period of time. However, even for a stiffness value of 10000 lb-ft/ft the oscillations die out and a steady state condition is achieved at approximately 18 seconds into the flight. Figure 25 shows a
bar graph of the maximum tether line tension for different tether stiffness values using both deployment methods.

Figure 22. Lineout (Lead/Follower Mass Ratio 1%)
Figure 23. Speed of Lead Projectile (Lead/Follower Mass Ratio 1%)
Figure 24. Speed of Follower Projectile (Lead/Follower Mass Ratio 1%)
Notice that increasing the tether stiffness increases the maximum tension. A corresponding graph is shown in Figure 26 for the maximum acceleration of the follower projectile. These two graphs demonstrate that in order to decrease maximum line loads and maximum follower projectile acceleration tether stiffness should be minimized.
Figure 26. Maximum Follower Acceleration (Lead/Follower Mass Ratio 1%)
4. REEL RESISTANCE SIMULATION RESULTS

4.1 EXAMINATION OF A TYPICAL MUNITION SYSTEM

The following simulation results examine a typical munition system using the nominal physical properties shown in Table 2 with and without reel resistance. The reel resistance parameters for the line out rate mechanism are $R_1 = 1697.26$ and $R_2 = 42.62$ while the parameters for the line out mechanism are $R_1 = 0.2511$ and $R_2 = 314.83$.

Figure 27 plots the range of the lead and follower projectiles.

Figure 27. Range of Lead and Follower Projectiles
As shown in Figure 27 previously, reel resistance has no significant effect on the range of the projectiles. However, range is effected by parameters such as drag coefficient ratio, forward drop velocity, and mass ratio. The body forward velocity of the lead projectile is shown in Figure 28.

![Figure 28. Body Forward Velocity of Lead Projectile](image)

At the time of snatch for the nominal case \((t = 6\, \text{sec})\), the velocity of the lead projectile is slightly decreased due to the impact of the snatch load. There are no noticeable effects on the lead projectile velocity for the systems with reel resistance mechanisms. Figure 29 shows the body forward velocity of the follower projectile. The nominal configuration
shows a rapid increase in forward velocity as the tether line grabs the follower during snatch.

![Graph showing body forward velocity of follower projectile](image)

**Figure 29. Body Forward Velocity of Follower Projectile**

The follower rebounds toward the lead projectile so much that the tether line goes slack. With the line slack, the follower projectile slows down to seek its steady state drop velocity. This oscillation continues until a steady state condition is arrived at where the lead and follower projectiles fall at the same speed. Using a mechanism that creates reel resistance proportional to line out rate drastically reduces the rapid speed increase caused by snatch and consequently the oscillations cease at an accelerated rate. However, line is
payed out at a much slower rate to achieve decreased speed growth and the snatch point occurs at approximately 23.5 seconds into deployment. Therefore, a steady state condition is not achieved until approximately 27.0 seconds compared to approximately 15 seconds for the nominal case. Generating reel resistance proportional to the line out practically eliminates the rapid increase in speed due to snatch and the oscillations die out at approximately 10.5 seconds. Figures 30 through 32 show the tension in the line element adjacent to the follower projectile over the duration of the simulation.

![Figure 30. Tension in the Line Element Adjacent to the Follower versus Time (No Reel Resistance)](image-url)
In Figure 30, it can be seen that deploying the system with no reel resistance induces a maximum tether force of roughly 2000 lbs. at the snatch point. The curve shown in Figure 31 demonstrates adding reel resistance proportional to line out rate decreases the snatch load roughly 81%.

Figure 31. Tension in the Line Element Adjacent to the Follower versus Time
(Reel Resistance Proportional to Line Out Rate)

Figure 32 demonstrates that employing reel resistance proportional to line out reduces the tension in the line element at snatch approximately 99%.
Figure 32. Tension in the Line Element Adjacent to the Follower versus Time

(Reel Resistance Proportional to Line Out)

The line out and line out rate as functions of time are shown in Figure 33 and 34. Reel resistance proportional to line out rate allows the line to accelerate to roughly 42 ft/s and then limits the speed at this value until all the line is deployed. This type of resistance forces the tether line to payout linearly. Reel resistance proportional to line out allows the line to payout quickly up to a rate of approximately 215 ft/s and then reduces the payout rate to almost 0 ft/s at snatch.
Figure 33. Lineout

Figure 34 shows the line out resistance pays the line out at an almost equivalent rate to that of no resistance and then drastically slows the rate just prior to the snatch point resulting in greatly reduced snatch loads. Figure 35 shows the maximum line loads at snatch. These loads occur in the line element adjacent to the lead projectile and are comparable to the loads experienced in the element adjacent to the follower that were shown in Figures 30 through 32. It is apparent that adding a reel resistance mechanism significantly reduces the snatch loads from that of a system with no resistance.
Figure 34. Lineout Rate

Figure 36 shows that the maximum acceleration of the follower is greatly reduced creating a safeguarded platform for any applicable sensory mechanisms. Reel resistance proportional to line out rate serves to increase the time it takes the system to reach a steady state condition from the nominal case by approximately 13 seconds as shown in Figure 37. However, not only does reel resistance proportional to line out decrease the acceleration of the follower and the tension in the line at snatch but it decreases the time to reach steady state by roughly 3.5 seconds.
Figure 35. Maximum Snatch Load
Figure 36. Maximum Acceleration of Follower Projectile
4.2 DEPLOYMENT ENVELOPES

Figures 38 through 61 show how system performance changes when a reel resistance mechanism that is designed for a specific point is employed under off design conditions. Reel resistance mechanisms tailored to specific drop speeds, projectile mass ratios, projectile drag coefficient ratios, and tether stiffness are considered. Maximum values for line tension, follower acceleration, and time to reach steady state were estimated based on a known system as a means to demonstrate how deployment
envelopes can be created for the system under off design conditions. These values are feasible limits used to create a workable design environment.

4.2.1 EFFECTS OF REEL RESISTANCE FOR A RANGE OF DROP SPEEDS

In Figures 38 and 39 the maximum snatch loads generated for reel resistance functions optimized about 400 ft/s, 650 ft/s and 900 ft/s drop speeds are shown. For a reel resistance proportional to line out rate any of the optimized functions will result in a snatch force that is less than the maximum allowable for drop speeds up to approximately

![Figure 38. Maximum Snatch Load versus Forward Drop Speed (Reel Resistance Proportional to Line Out Rate)](image-url)
760 ft/s. Beyond that, however, reel resistance proportional to line out rate is ineffectual in limiting the snatch load. A reel resistance proportional to line out produces smaller drop speed envelopes, but a function optimized about 900 ft/s is capable of creating snatch loads less than the maximum allowable. It can be seen that three different line out mechanisms would be needed to achieve a successful deployment for the entire drop speed range, whereas one line out rate mechanism could effectively control the snatch load up to a drop speed of 760 ft/s.

Figure 39. Maximum Snatch Load versus Forward Drop Speed
(Reel Resistance Proportional to Line Out)
In Figures 40 and 41 the maximum accelerations created for the optimized reel resistance functions are shown. Any of the resistance functions proportional to line out rate are able to control the follower acceleration to values far below the maximum allowable.

![Graph showing maximum acceleration of follower versus forward drop speed](image)

**Figure 40. Maximum Acceleration of Follower versus Forward Drop Speed**

(Reel Resistance Proportional to Line Out Rate)

For reel resistance proportional to line out, only the function optimized for a drop speed of 900 ft/s is capable of limiting the follower acceleration to values below the allowable for the entire drop speed range.
Figure 41. Maximum Acceleration of Follower versus Forward Drop Speed

(Reel Resistance Proportional to Line Out)

The times to reach a steady state condition for the same optimized reel resistance functions stated above are shown in Figures 42 and 43. All reel resistance functions, except for the function optimized around 900 ft/s for proportional to line out resistance, are capable of allowing the follower and lead projectile to enter into a steady state condition before the allotted time. The 900 ft/s case fails to meet this standard for only a very small range of drop speeds (400-425 ft/s).
Figure 42. Time to Reach Steady State versus Forward Drop Speed

(Reel Resistance Proportional to Line Out Rate)
Figure 43. Time to Reach Steady State versus Forward Drop Speed
(Reel Resistance Proportional to Line Out)

4.2.2 EFFECTS OF REEL RESISTANCE FOR A RANGE OF PROJECTILE
MASS RATIOS

Figures 44 through 49 present similar information as above for reel resistance functions optimized about mass ratios of 1%, 15% and 30%. From Figure 44 it can be seen that reel resistance proportional to line out rate is ineffective at decreasing the snatch loads below the allowable of all but a very small range of mass ratios. Figure 45 shows that reel resistance proportional to line out can reduce the snatch loads below the allowable but only for very small mass ratio ranges around each optimal function.
Figure 44. Maximum Snatch Load versus Mass Ratio

(Reel Resistance Proportional to Line Out Rate)
Figures 46 and 47 show that for both types of reel resistance the maximum follower accelerations are well below the allowable for the entire range of mass ratios. This is due to the increase in the mass of the follower rather than the influence of the reel resistance.
Figure 46. Maximum Acceleration of Follower versus Mass Ratio

(Reel Resistance Proportional to Line Out Rate)
Figure 47. Maximum Acceleration of Follower versus Mass Ratio
(Reel Resistance Proportional to Line Out)

As the mass of the follower increases, its steady state drop speed becomes closer to that of the lead projectile and consequently the separation time increases. Because reel resistance proportional to line out rate is only effective at limiting the speed at which the line is paid out it is not capable of decreasing the separation time of the two projectiles. It actually works to increase the time to reach steady state for the larger mass ratios as shown in Figure 48. Figure 49 shows that reel resistance proportional to line out can pay out line at a rapid rate and then reduce the speed of the exiting line as it approaches snatch for localized ranges of mass ratios about each optimized function.
Figure 48. Time to Reach Steady State versus Mass Ratio

(Reel Resistance Proportional to Line Out Rate)
4.2.3 EFFECTS OF REEL RESISTANCE FOR A RANGE OF PROJECTILE DRAG RATIOS

For reel resistance proportional to line out rate, as the drag ratios of the follower projectile to the lead projectile increases, snatch loads are produced prior to all the line being out. These snatch loads are a result of the follower separating from the lead projectile at an increased rate due to the drag and then being brought to a halt by the line out rate limiting action of the reel resistance. In Figure 50 it can be seen that reel
resistance proportional to line out rate is incapable of limiting these snatch loads for higher drag ratios.

Figure 50. Maximum Snatch Load versus Drag Coefficient Ratio  
(Reel Resistance Proportional to Line Out Rate)

Figure 51 shows that because reel resistance proportional to line out allows the line to pay out at a rapid rate and then gradually reduces this rate to approximately zero at the snatch point, the maximum line load can be greatly reduced for a resistance function optimized about a drag ratio of 5. A resistance function optimized around a drag ratio of 2 is also effectual in limiting the snatch loads up to ratios as high as 3.25.
Figure 51. Maximum Snatch Load versus Drag Coefficient Ratio
(Reel Resistance Proportional to Line Out)

In Figure 52, because the speed of separation is limited by reel resistance proportional to line out rate, the maximum acceleration of the follower is a result of the initial separation of the projectiles and not a result of snatch. Figure 54 shows that the time to reach steady state is diminished as the drag ratios are increased for all optimal resistance functions. In Figure 53, the acceleration of the follower projectile is minimized at the drag ratios that the reel resistance functions were optimized. These accelerations are also produced by the initial separation of the two projectiles and not at snatch. The accelerations for the optimized resistance functions of a drag coefficient
ratio of 1.25 grow in magnitude because the reel resistance proportional to line out can
not create enough resistance to limit the line out rate for the higher drag ratios.

Figure 52. Maximum Acceleration of Follower versus Drag Coefficient Ratio
(Reel Resistance Proportional to Line Out Rate)
Whereas for the optimized function of a drag coefficient ratio of 2.00 the accelerations are greater in magnitude for lower drag ratios because the resistance becomes too great and a snatch condition is induced prior to all the line being deployed. The resistance function optimized at a drag coefficient ratio of 5.00 produces enough resistance to limit the line out rate through the entire separation, producing low accelerations at the snatch point. However, the time to reach steady state is increased because the time for all the line to pay out is increased due to the resistance, as shown in Figure 55.
Figure 54. Time to Reach Steady State versus Drag Coefficient Ratio

(Reel Resistance Proportional to Line Out Rate)
Figure 55. Time to Reach Steady State versus Drag Coefficient Ratio

(Reel Resistance Proportional to Line Out)

4.2.4 EFFECTS OF REEL RESISTANCE FOR A RANGE OF TETHER STIFFNESS

Figures 56 and 57 show that all optimized functions are effective in reducing the snatch load below the allowable for the entire range of tether stiffness.
Figure 56. Maximum Snatch Load versus Tether Stiffness

(Reel Resistance Proportional to Line Out Rate)
Figures 58 and 59 show that all optimized functions are sufficient for reducing the follower accelerations below the allowable as well.
Figure 58. Maximum Acceleration of Follower versus Tether Stiffness

(Reel Resistance Proportional to Line Out Rate)
The time to reach steady state is roughly the same for both types of reel resistance over the range of tether stiffness for all optimized functions, as shown in Figures 60 and 61. This demonstrates that in order to reduce snatch loads, and follower projectile maximum acceleration, and maintain an acceptable time to reach steady state reduced tether stiffness along with a reel resistance mechanism should be employed.
Figure 60. Time to Reach Steady State versus Tether Stiffness

(Reel Resistance Proportional to Line Out Rate)
Figure 61. Time to Reach Steady State versus Tether Stiffness

(Reel Resistance Proportional to Line Out)
5. CONCLUSIONS AND RECOMMENDATIONS

A dynamic model of two projectiles that are connected by a tether line developed in this thesis demonstrated physically realistic results for two methods of tether line deployment. The pop-out method is numerically more efficient, but has the disadvantage of introducing tether vibration when beads are released. The all-out method requires significant computation, however this problem can be alleviated by initially deploying enough line so that the equations can be efficiently integrated. While the position trajectories converge for less than 10 tether bead elements, the convergence for maximum tether line force requires more than 100 elements. If this munition system is being simulated solely to obtain trajectory information, then a model that utilizes a low number of beads (< 10) is sufficient. On the other hand, if the simulation tool is going to be used for trajectory calculations and internal loads estimation, then a much higher order system must be used (> 100).

Simulation results show that for a low follower to lead projectile mass ratio, the tether line is unreeled by the difference in position between the projectiles. For a high mass ratio, the tether line aerodynamic force unreels the tether line. Hence, the tether line unreels itself.

The tether line stiffness has very little effect on the position dynamics but does strongly influence dynamic loading. From a design standpoint, a low stiffness, high ultimate strength tether material is most desirable. Proper tether material selection must consider both ultimate line strength and tether stiffness, which effect loads. For a low-mass ratio configuration, an increase in the follower-to-lead projectile drag coefficient
ratio has the expected effect of decreasing tether line deployment time and increasing tether line loads and follower projectile maximum acceleration.

Examination of load limiting mechanisms revealed that exceedingly high tether line loads and follower projectile acceleration could be dramatically reduced by reel resistance mechanisms that use a function of either the amount of the tether line release from the reel or the tether line release rate.

The line out rate mechanism forces the tether line to unspool at a constant rate. Because the unspooling rate is limited, the tether line requires a significantly longer time to fully deploy. Furthermore, while the snatch load is reduced with respect to a system with no reel resistance, a notable snatch load is still present because of the abrupt change in line out rate when the line is fully deployed. Thus, a fundamental trade exists between the time to fully deploy the tether line and the maximum tether line load. Relative to the line out mechanism, the line out rate mechanism can be utilized over a wide drop speed range and should be designed for the highest design drop speed. For low projectile mass ratios, the maximum snatch loads are directly proportional to mass ratio and the maximum follower acceleration is inversely proportional to mass ratio. The maximum snatch loads are also directly proportional to the projectile drag coefficient ratio and tether line stiffness.

The line out mechanism produces a large line out rate initially, which releases the bulk of the line. When the line is nearly fully deployed, the reel resistance is increased to reduce the line out rate, which in turn reduces the maximum tether line loads. Thus, the line out mechanism has the advantage of quickly deploying the tether while dramatically reducing maximum tether line loads and follower projectile acceleration. The main
drawback of the line out mechanism is that it must be tuned to a specific drop speed. A line out reel mechanism that is designed for a particular drop speed, projectile mass ratio, or a projectile drag ratio does not perform well under conditions away from the design point. Hence, for systems with a tightly controlled operational environment the line out reel resistances perform best, but for systems that operate under significantly varying operational conditions, the line out rate unreeling mechanism should be employed.
6. FURTHER WORK

Further work is recommended to improve and validate the existing atmospheric flight model.

Angle of attack of the follower projectile is an important concern in many of the applications of a munition system of this type. This flight characteristic should be addressed by increasing the degrees of freedom of the lead and follower projectiles to six in the atmospheric flight model. This modification would also enhance the model by increasing the accuracy of the aerodynamic effects.

The current tether dynamic model uses a Voigt spring and damper finite element. Performing tests with an actual tether line to determine a more appropriate modeling element under various conditions and implementing it would further refine the atmospheric flight model.

Dynamic models of "off the shelf" and contrived resistance mechanisms should be incorporated into the atmospheric flight model and the results compared with the reel resistance functions produced by minimization of the deployment characteristics cost function.

Scaled testing of a munition system with no resistance, a line out resistance mechanism, and a line out rate resistance mechanism should be performed to validate the results shown in this thesis.
REFERENCES


