

AN ABSTRACT OF THE THESIS OF

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AND VOLTAGE BALANCE

Abstract approved

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The purpose of this thesis is to investigate the necessity of transposition in the construction of overhead transmission lines. Line currents, line voltages and line losses of both the transposed and untransposed lines were calculated and compared for two different voltage classes. The conventional method of symmetrical components was used to determine both the negative- and zero-sequence currents produced by the inductive and capacitive unbalance in the untransposed lines.

Both system voltages and line currents are affected when a transmission line is not transposed. This is a result of the two phenomena, (1) electromagnetic fields, and (2) electrostatic fields. To account for these effects, the transmission was modeled with a nominal-T network. The conductor configuration represented a single

circuit, three-phase, three conductor system with a flat horizontal spacing. Shielding was represented by two ground-wire conductors. The necessary iterative calculations were performed on a digital computer.

The unbalance generated by 100 miles of unbalanced line at a nominal voltage of either 230 Kv or 500 Kv did not produce an unbalance factor that exceeded the limits imposed by electrical machinery operation. The major effect of line unbalance is the influence of zero-sequence quantities on relaying systems and the additional power loss of the transmission lines. Both of these are functions of the length of the untransposed line.

The calculated unbalance factors for 100 miles of untransposed lines are 0.0474 and 0.0626 for the 230 Kv and 500 Kv lines respectively. The above factors include the effect of the terminating equipment, transformers and equivalent machinery.

Curves are presented to illustrate the phase currents, phase voltage drops, and the difference in transmission line power losses as functions of the length of the transposed and untransposed lines, for each voltage level.

Theoretical Influence of Untransposed Transmission Line  
Construction on System Losses and Voltage Balance

by

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## LIST OF SYMBOLS

C	shunt capacitance of conductor, farads per mile.
d	distance between parallel conductors, feet.
E	input voltage, volts.
f	frequency, Hertz
G	shunt conductance, mhos per mile
GMR	geometric mean radius of the conductor, feet.
h	height of the conductor above ground, feet.
I	current, amperes.
I*	conjugate current, amperes.
K	ratio of currents.
L	inductance of conductor, henrys per mile.
P	power dissipated, watts.
r	radius of conductor, inches.
R	resistance of conductor, ohms per mile.
s	spacing between one conductor and an image, feet.
UF	unbalance factor.
V	voltage drop, volts.
X	reactance of conductor, ohms per mile.
Z	complex impedance, ohms.
$\alpha$	unit vector of 120 degrees angular.
$\theta$	phase angle between voltage and current, radians.

# THEORETICAL INFLUENCE OF UNTRANSPOSED TRANSMISSION LINE CONSTRUCTION ON SYSTEM LOSSES AND VOLTAGE BALANCE

## I. INTRODUCTION

The electric energy demands have rapidly increased in every country in the world, for example, in the United States, the energy output rose at an average annual rate of 7.5 percent for the last 53 years (6). To meet the electrical energy demand, hydro-power plants, thermo-power plants and nuclear-power plants have been widely developed. Research is being performed on power transmission methods, system protection, system control and system economics in order to improve the electrical energy delivery to the ultimate consumer.

It is the purpose of this thesis to investigate the magnitude of system unbalance created by untransposed transmission lines of two different voltage classes. As most engineering decisions involve compromises, the cost of the transposition must be compared to an allowable amount of system unbalance. The results of this investigation should provide some basis for evaluating the effect of neglecting transposition.

Due to the iterative nature of the calculations involved in using the system model, the investigation was performed with the aid of a digital computer. The computer program was written in FORTRAN

IV for use on the CDC 3300 computer at Oregon State University.

The determination of transmission unbalance for two nominal system voltages, 230 Kv and 500 Kv, are considered with various transmission line lengths up to 100 miles. The results show that the unbalance factor does not exceed the limit imposed by electrical machinery operation. The system unbalance factors were determined by including the effects of terminating equipment; transformers and equivalent machinery. Effects which influenced the system when the lines were not transposed were 1) zero-sequence currents which affected the relaying system, and 2) the additional losses in the transmission line.

Curves that show the different magnitudes of system unbalance parameters are presented and can be used to determine if the transmission line should be transposed at a required length according to a particular operating policy.

## II. DESIGN OF THE GENERAL TRANSMISSION LINE SYSTEM

In designing the overhead transmission line which is, even nowadays, still the most economical means of transmitting energy from remote generating sources to users at a distance, the following factors have to be considered (7, 9, 15, 19):

- 1) An insulation system that withstands steady-state and transient voltage effects.
- 2) Supporting structure.
- 3) Appropriate conductor sizes, conductor per phase, and conductor configuration within the limits imposed by radio influence and corona loss.
- 4) The energy loss of the line as related to the total economic problem of line construction and energy transmitted.
- 5) Right-of-way acquisition and maintenance.

In addition to the factors mentioned above, another item that must be taken into account in designing a transmission line is the unbalance factor. Different conductor configurations cause the unbalance factor to vary over a considerable range when the lines are not transposed.

In overhead transmission line systems, for any conductor configuration, a certain amount of dissymmetry will always exist. Even for equilateral spacings, the conductor arrangement is not in

balance with respect to the ground. Such a dissymmetry affects the generation of the voltage and current unbalance in the power system (13, 16).

The unbalance of high voltage line leads to circulating residual ground currents in a system which is solidly grounded (10). If the system is ungrounded, a zero-sequence voltage will appear between true ground and the neutral of the system. The negative-sequence current resulting from positive-sequence current and charging current will flow through the lines and also through the windings of terminal equipments, such as transformers and rotating machines (11, 12, 18).

In order to bring the geometric unbalance into an effective balance condition, the transmission lines should be transposed. Power lines, in the early day, were transposed primarily to bring their reactance into balance and to minimize inductive interferences with parallel communication circuits, long distance telephone and telegraph lines. Transposition is affected by changing the position of conductors so that an individual conductor occupies each of the respective phase conductor positions for approximately the same length, Figures 1 and 2. The physical structure of transposition is usually called a "barrel of transposition." It should be kept in mind that the transposition of transmission line conductors represents a higher cost due to the transposition tower and also lowers the

mechanical and electrical strengths (8, 20).

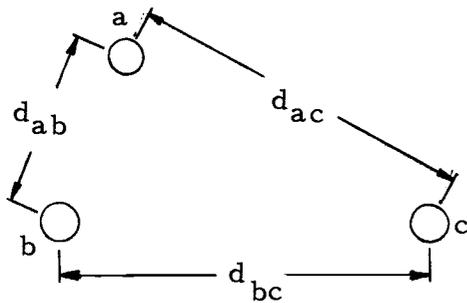


Figure 1. Cross section of three-phase line with unsymmetrical spacing.

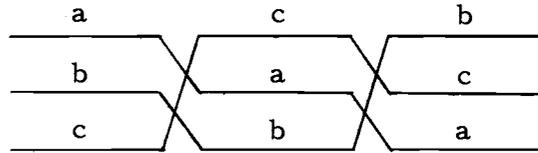


Figure 2. Transposition cycles for a three-wire, three-phase line.

### Transmission Line Parameters

From the point of view of power engineers, the transmission line characteristics can be expressed in terms of the following parameters:

- 1) Line inductance, henrys per mile
- 2) Line shunt capacitance, farads per mile
- 3) Line resistance, ohms per mile
- 4) Line shunt conductance, mhos per mile

In practice, these are expressed as values for a unit length of one mile. These parameters will be, respectively, symbolized by  $L$ ,  $C$ ,  $R$  and  $G$  throughout this thesis.

In the calculation of transmission line impedances, many inconsistent factors are inevitable. Fortunately, the errors caused by

them are usually less than three percent. To save labor and time, this error can be ignored and still produce acceptable results. Some of the factors causing the inaccuracy of the transmission line parameters are (4, 7):

- 1) The conductor sag changes with temperature and line loading.
- 2) The resistivity changes with temperature.
- 3) The capacitance is affected by the contour of the surrounding ground, foliage, etc.
- 4) The presence of shunt leakage conductance.

Herein, the values of resistance, radius and geometric mean radius of conductors are obtained directly from the tables of conductor characteristics.

### Line Impedance

In the design of a transmission line, the series line impedance is the most important factor. Line resistance will be included with the value of line inductive reactance to form the complex impedance of the line.

The two equations used in this analysis are directly simplified from Carson's formula (2). It is separated into two parts: 1) the determination of the self-impedance of one conductor with earth return, and 2) the mutual impedance between two conductors with common earth return.

$$Z_{nm} = R + 0.00159f + j 0.004657 f \log_{10} \frac{2160}{GMR} \sqrt{\frac{\rho}{f}} \quad (1)$$

ohms per mile

$$Z_{nm} = 0.00159 f + j 0.004657 f \log_{10} \frac{2160}{d_{nm}} \sqrt{\frac{\rho}{f}} \quad (2)$$

ohms per mile

where:

R = resistance of the conductor, ohms per mile

f = frequency, Hertz

$\rho$  = earth resistivity, ohms-meters

GMR = geometric mean radius of the conductor, feet

$d_{nm}$  = distance between parallel conductors, feet.

### Line Shunt Capacitance

The two fundamental equations used in this determination are derived from the classical method of calculating the capacitance of a transmission line, Maxwell's coefficients. Similar to the inductance calculation, the calculation for capacitance is separated into two parts: 1) the self-potential coefficient, and 2) the mutual-potential coefficient (1, 3, 13, 17).

$$P_{nn} = 25.753 \times 10^6 \log_{10} \frac{2h}{r} \quad \text{darafs per mile} \quad (3)$$

$$P_{nm} = 25.753 \times 10^6 \log_{10} \frac{S_{nm}}{s_{nm}} \quad \text{darafs per mile} \quad (4)$$

where:

$r$  = radius of the conductor, feet

$h$  = height of the conductor above ground, feet

$s_{nm}$  = spacing between two conductors, feet

$S_{nm}$  = spacing between one conductor and an image conductor,  
feet

Then, it follows that the capacitance is the reciprocal of the potential coefficient, and capacitive reactance can be, therefore, obtained.

$$C_{nn} = \frac{1}{P_{nn}}, \quad C_{nm} = \frac{1}{P_{nm}} \quad \text{farads per mile} \quad (5)$$

and

$$X_{c_{nn}} = \frac{1}{2\pi f C_{nn}}, \quad X_{c_{nm}} = \frac{1}{2\pi f C_{nm}} \quad \text{ohms per mile} \quad (6)$$

### Line Current

To find the transmission line current, the system will be assumed linear, and the method of superposition will be used. The problem is separated into two parts:

- 1) Current due to electromagnetic phenomenon only.
- 2) Current resulting from electrostatic effects.

## Electromagnetic Current

To find the current components due to electromagnetic effects, the equations of voltage drop along the 3-phase line with two ground wires will first be considered (1, 13, 14).

$$V_a = I_a Z_{aa} + I_b Z_{ab} + I_c Z_{ac} + I_x Z_{ax} + I_y Z_{ay} \quad (7)$$

$$V_b = I_a Z_{ab} + I_b Z_{bb} + I_c Z_{bc} + I_x Z_{bx} + I_y Z_{by} \quad (8)$$

$$V_c = I_a Z_{ac} + I_b Z_{bc} + I_c Z_{cc} + I_x Z_{cx} + I_y Z_{cy} \quad (9)$$

$$V_x = I_a Z_{ax} + I_b Z_{bx} + I_c Z_{cx} + I_x Z_{xx} + I_y Z_{xy} \quad (10)$$

$$V_y = I_a Z_{ay} + I_b Z_{by} + I_c Z_{cy} + I_x Z_{xy} + I_y Z_{yy} \quad (11)$$

$V_x$  and  $V_y$  are set to zero because there is no active source in the loop described by the ground-wire circuits. Thus the above equations can be solved for  $I_x$  and  $I_y$ . These solutions are then substituted into Equations (7), (8) and (9). This reduces the total number of equations to the following three which describe each phase voltage:

$$V_a = I_a Z'_{aa} + I_b Z'_{ab} + I_c Z'_{ac} \quad (12)$$

$$V_b = I_a Z'_{ab} + I_b Z'_{bb} + I_c Z'_{bc} \quad (13)$$

$$V_c = I_a Z'_{ac} + I_b Z'_{bc} + I_c Z'_{cc} \quad (14)$$

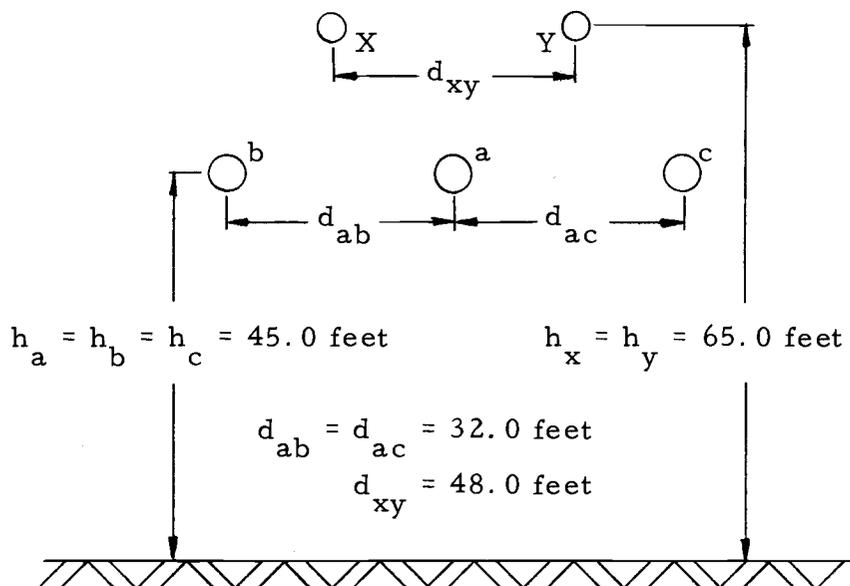


Figure 3. Configuration of conductors, horizontally spaced, single circuit line, with two ground wires, for 230-Kv system.

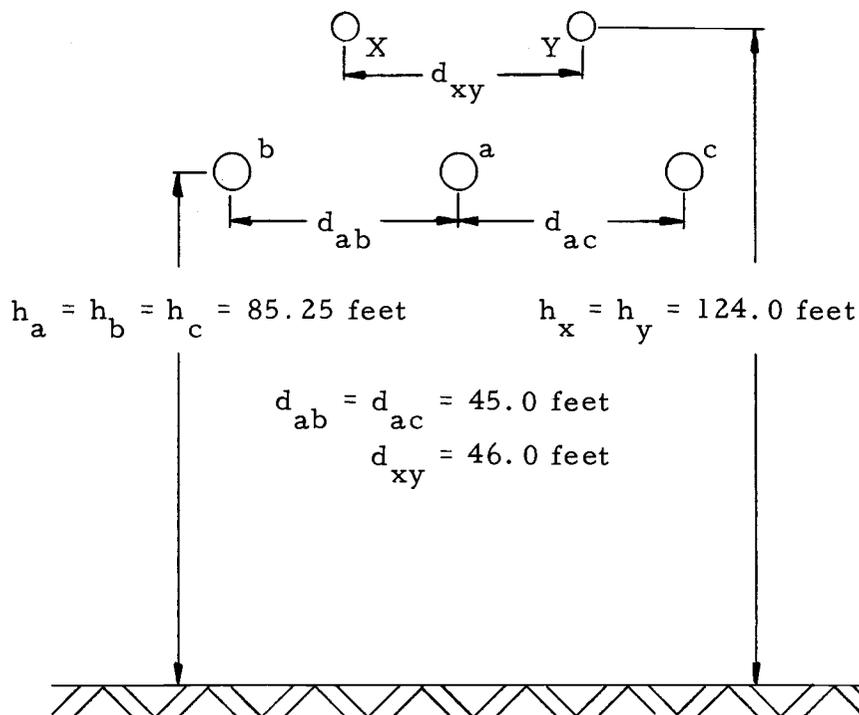


Figure 4. Configuration of conductors, horizontally spaced, single circuit line, with two ground wires, for 500 Kv system.

where:

$$Z'_{aa} = Z_{aa} + \frac{1}{\Delta} (2Z_{xa} Z_{ya} Z_{xy} - Z_{xa}^2 Z_{yy} - Z_{ya}^2 Z_{xx}) \quad (15)$$

$$Z'_{bb} = Z_{bb} + \frac{1}{\Delta} (2Z_{xb} Z_{yb} Z_{xy} - Z_{xb}^2 Z_{yy} - Z_{yb}^2 Z_{xx}) \quad (16)$$

$$Z'_{cc} = Z_{cc} + \frac{1}{\Delta} (2Z_{xc} Z_{yc} Z_{xy} - Z_{xc}^2 Z_{yy} - Z_{yc}^2 Z_{xx}) \quad (17)$$

$$Z'_{ab} = Z_{ab} + \frac{1}{\Delta} (Z_{xa} Z_{yb} Z_{xy} - Z_{xa} Z_{xb} Z_{yy} + Z_{xb} Z_{ya} Z_{xy} - Z_{ya} Z_{yb} Z_{xx}) \quad (18)$$

$$Z'_{ac} = Z_{ac} + \frac{1}{\Delta} (Z_{xa} Z_{yc} Z_{xy} - Z_{xa} Z_{xc} Z_{yy} + Z_{xc} Z_{ya} Z_{xy} - Z_{yc} Z_{ya} Z_{xx}) \quad (19)$$

$$Z'_{bc} = Z_{bc} + \frac{1}{\Delta} (Z_{xb} Z_{yc} Z_{xy} - Z_{xc} Z_{xb} Z_{bb} + Z_{xc} Z_{yb} Z_{xy} - Z_{yc} Z_{yb} Z_{xx}) \quad (20)$$

$$\Delta = Z_{xx} Z_{yy} - Z_{xy}^2 \quad (21)$$

By assuming that only positive-sequence current is flowing in the system, the following sequence impedances can be obtained.

$$Z_{11} = \frac{1}{3} [(Z_{aa} + Z_{bb} + Z_{cc}) - (Z_{bc} + Z_{ac} + Z_{ab})] \quad (22)$$

$$Z_{21} = \frac{1}{3} [(Z_{aa} + aZ_{bb} + a^2 Z_{cc}) + 2(Z_{bc} + aZ_{ac} + a^2 Z_{ab})] \quad (23)$$

$$Z_{01} = \frac{1}{3} [(Z_{aa} + a^2 Z_{bb} + aZ_{cc}) - (Z_{bc} + a^2 Z_{ac} + aZ_{ab})] \quad (24)$$

The first subscript of the sequence impedance represents the sequence of voltage drop while the second subscript represents the sequence current causing the voltage drop.

By the same assumption, if only negative- and zero-sequence currents are flowing in the system, the following sequence-impedances can be obtained.

$$Z_{12} = \frac{1}{3} [(Z_{aa} + a^2 Z_{bb} + a Z_{cc}) + 2(Z_{bc} + a Z_{ab} + a^2 Z_{ac})] \quad (25)$$

$$Z_{22} = \frac{1}{3} [(Z_{aa} + Z_{bb} + Z_{cc}) - (Z_{bc} + Z_{ac} + Z_{ab})] \quad (26)$$

$$Z_{02} = \frac{1}{3} [(Z_{aa} + a Z_{bb} + a^2 Z_{cc}) - (Z_{bc} + a Z_{ac} + a^2 Z_{ab})] \quad (27)$$

$$Z_{10} = \frac{1}{3} [(Z_{aa} + a Z_{bb} + a^2 Z_{cc}) - (Z_{bc} + a Z_{ac} + a^2 Z_{ab})] \quad (28)$$

$$Z_{20} = \frac{1}{3} [(Z_{aa} + a^2 Z_{bb} + a Z_{cc}) - (Z_{bc} + a^2 Z_{ac} + a Z_{ab})] \quad (29)$$

$$Z_{00} = \frac{1}{3} [(Z_{aa} + Z_{bb} + Z_{cc}) - 2(Z_{bc} + Z_{ac} + Z_{ab})] \quad (30)$$

The transmission line termination, e.g., transformers, are static components. Their positive- and negative-sequence impedances are identical while the mutual impedances between the sequence networks are unaffected (7, 17). Therefore, to find the total line impedances, the terminal impedances have to be added.  $X_1$ ,  $X_2$  and  $X_0$  are the total positive-, negative- and zero-sequence terminal impedances, as shown in Figure 4. Therefore:

$$Z'_{11} = Z_{11} + X_1 \quad (31)$$

$$Z'_{22} = Z_{22} + X_2 \quad (32)$$

$$Z'_{00} = Z_{00} + X_0 \quad (33)$$

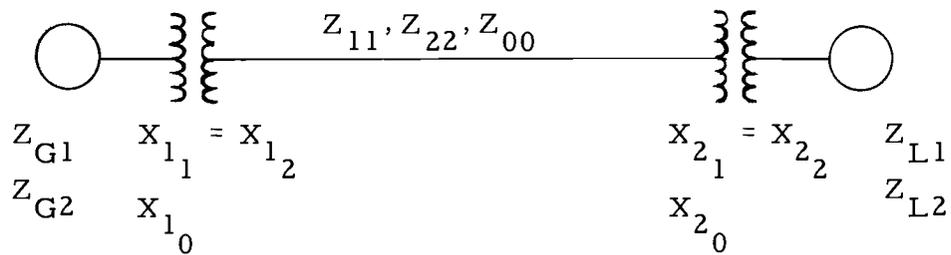


Figure 5. System showing terminal impedances and sequence impedance of a transmission line..

where:

$$X_1 = Z_{G1} + X_{11} + X_{21} + Z_{L1} \quad (34)$$

$$X_2 = Z_{G2} + X_{12} + X_{22} + Z_{L2} \quad (35)$$

$$X_0 = X_{10} + X_{20} \quad (36)$$

The input voltage to the combined transmission line and equivalent load can be equated to the total potential drop in terms of sequence currents and unbalanced impedances. This is shown in the following equation set.

$$\Sigma E_{a_1} = I_{a_1} Z'_{11} + I_{a_2} Z_{12} + I_{a_0} Z_{10} \quad (37)$$

$$\Sigma E_{a_2} = I_{a_1} Z_{21} + I_{a_2} Z'_{22} + I_{a_0} Z_{20} = 0 \quad (38)$$

$$\Sigma E_{a_0} = I_{a_1} Z_{01} + I_{a_2} Z_{02} + I_{a_0} Z'_{00} = 0 \quad (39)$$

Equations 38 and 39 are set to zero because they are not present as generated voltages in an ordinary system. By using Cramer's rule, currents  $I_{a_1}$ ,  $I_{a_2}$  and  $I_{a_0}$  can be determined

$$I_{a_1} = \frac{\Sigma E_{a_1}}{D} (Z'_{22} Z'_{00} - Z_{02} Z_{20}) \quad (40)$$

$$I_{a_2} = \frac{\Sigma E_{a_1}}{D} (Z_{20} Z_{01} - Z'_{00} Z_{21}) \quad (41)$$

$$I_{a_0} = \frac{\Sigma E_{a_1}}{D} (Z_{12} Z_{02} - Z'_{22} Z_{01}) \quad (42)$$

where:

$$D = Z'_{11} Z'_{22} Z'_{00} + Z_{12} Z_{20} Z_{01} + Z_{10} Z_{21} Z_{02} - Z_{01} Z'_{22} Z_{10} - Z_{01} Z_{20} Z'_{11} - Z_{12} Z_{21} Z'_{00} \quad (43)$$

### Electrostatic Current

To calculate the current components in the transmission line due to electrostatic effects, the procedure is identical to the calculation

for the currents due to the electromagnetic effects. The values of impedances in the first calculation part are replaced by the values of shunt capacitive reactance. Therefore, the detailed procedure will not be reanalyzed but the final results are shown as follows:

$$I_{c_{a_1}} = \frac{\Sigma E_{a_1}}{D_c} (X_{22}X_{00} - X_{02}X_{20}) \quad (44)$$

$$I_{c_{a_2}} = \frac{\Sigma E_{a_1}}{D_c} (X_{20}X_{01} - X_{00}X_{21}) \quad (45)$$

$$I_{c_{a_0}} = \frac{\Sigma E_{a_1}}{D_c} (X_{12}X_{02} - X_{22}X_{01}) \quad (46)$$

where:

$$D_c = X_{11}X_{22}X_{00} + X_{12}X_{20}X_{01} + X_{10}X_{21}X_{02} - X_{01}X_{22}X_{10} \\ - X_{02}X_{20}X_{11} - X_{12}X_{21}X_{00} \quad (47)$$

$I_{c_{a_1}}$  of Equation (44) is called the normal charging current whereas  $I_{a_1}$  from Equation (40) is called normal full load current.

Again, by the method of superposition, the total negative- and zero-sequence currents along the line under consideration can be expressed as:

$$IT_{a_2} = I_{a_2} + I_{c_{a_2}} \quad (48)$$

and

$$IT_{a_0} = I_{a_0} + I_{c_{a_0}} \quad (49)$$

By using symmetrical components, the line currents  $I_a$ ,  $I_b$  and  $I_c$  are expressed as follows (3, 18).

$$I_a = IT_{a_1} + IT_{a_2} + IT_{a_0} \quad (50)$$

$$I_b = a^2 IT_{a_1} + a IT_{a_2} + IT_{a_0} \quad (51)$$

$$I_c = a IT_{a_1} + a^2 IT_{a_2} + IT_{a_0} \quad (52)$$

### Line Drop

In order to improve the line representation, the nominal-T circuit will be used. The currents at both sending and receiving ends must be found. The transmission line drop is determined by reusing Equations (7), (8) and (9) consecutively. The total line drop of the transmission line can be obtained by summing the line drops in both parts of the "T" circuit.

$I_S$  and  $I_R$  in Figure 6 are sending and receiving end currents whereas  $E_S$  and  $E_R$  are sending and receiving end voltages.

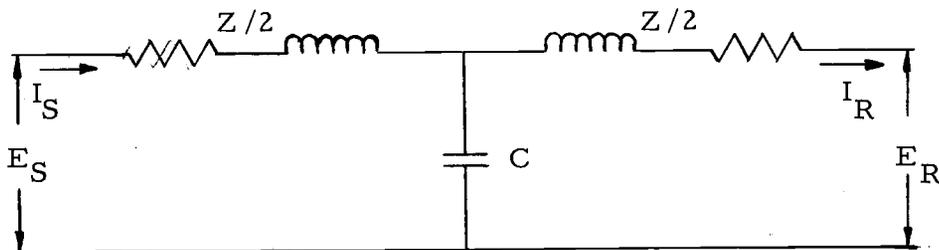


Figure 6. Nominal-T circuit.

A transposed transmission line is constructed so that each phase wire will occupy all positions in the conductor arrangement for equal distances over the entire length of the line. Figure 2 illustrates a transposed line using three conductors. The transmission line drop is found by calculating the total voltage drop of each phase; 'a', 'b' and 'c' ; by representing each phase line as a nominal-T circuit. The transposed line drop is obtained by averaging the total voltage drop for each line.

### Transmission Line Loss

The general equation used to find the transmission line power loss is (7).

$$P_i = V_i I_i^* \cos \theta_i \quad (53)$$

then

$$P_{\text{tot}} = \sum_{i=1}^n P_i \quad (54)$$

where

$P_i$  = power dissipated by the  $i^{\text{th}}$  impedance, watts

$V_i$  = voltage drop across  $i^{\text{th}}$  impedance, volts

$I_i^*$  = conjugate current through  $i^{\text{th}}$  impedance, amperes

$\theta_i$  = phase angle between voltage and current at the  $i^{\text{th}}$  impedance, radians

$P_{\text{tot}}$  = total power dissipated by the system, watts

### Unbalance Factor

The unbalance factor of the transmission line can be determined when the line is not in balance. It varies over a considerably wide range, depending on the conductor configuration. The value of the unbalance factor is equal to

$$UF_{\text{max}} \left\{ Z_{01}/Z'_{00}, Z_{21}/Z'_{22}, P_{01}/P_{00}, P_{21}/P_{22} \right\}. \quad (55)$$

The Equation (55) means that the unbalance factor; 'UF'; is the value of  $Z_{01}/Z'_{00}$ ,  $Z_{21}/Z'_{22}$ ,  $P_{01}/P_{00}$  or  $P_{21}/P_{22}$  whichever is the largest.

The allowable unbalance current in a three-phase synchronous or induction machines is determined by using Figure 7 (13).

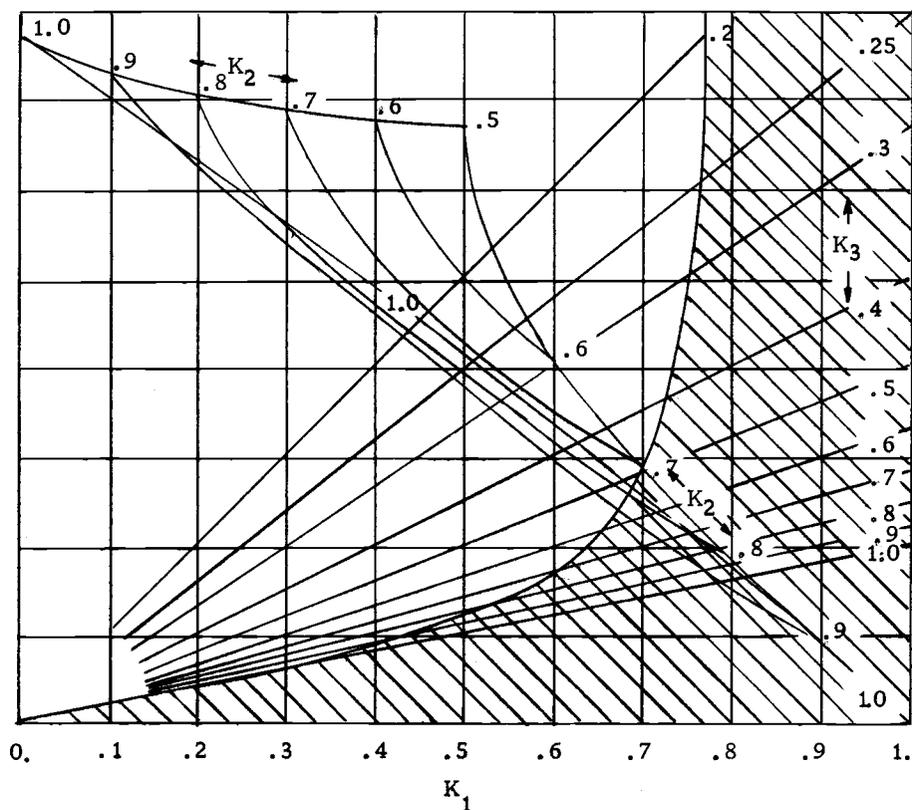


Figure 7. Determination of allowable current unbalance in 3-phase synchronous and induction machines.

Where:

$K_1$  = ratio; smallest phase current/largest phase current

$K_2$  = ratio; second largest phase current/largest phase current

$K_3$  = ratio; largest phase current/rated current

All phase currents are determined at a specific load condition.

The shaded area represents the unacceptable system unbalance.

### III. DIGITAL COMPUTER PROGRAMMING

#### Program Description

The computer program was written in FORTRAN IV for use on the CDC 3300 computer at Oregon State University. The whole computer program has been divided into a main program with several subroutines. The functions of the main program and subroutines are briefly described below:

Main Program: The main program is an executive type, therefore it controls all input, output, and all of the logical flow of the computations. Data which are read by the main program will be subsequently used in the subroutines. The logical sequence of calculations relative to the combination of impedances, superposition of currents, summation of line voltage drops, line losses and the unbalance factor of the transmission lines for both transposed and untransposed lines are also controlled by this main program.

Subroutine, Sub 1: This subroutine will be used to calculate all the required conductor spacings, spacing between two conductors and spacing between one conductor and the images of the other conductor, which are required by other programs.

Subroutine, Sub 2: This subroutine uses the spacings from the first subroutine and will calculate the self, and mutual-impedance of inductive reactances. This subroutine is also used to calculate the

self- and mutual-potential coefficients of the conductors and the self- and mutual-capacitive reactances.

Subroutine, Sub 3: After calculating all the previously mentioned values, the impedance coefficient values for both the inductive- and capacitive-reactance currents  $I_a$ ,  $I_b$  and  $I_c$  will be determined. The effect of two ground wires will be considered by this subroutine, see Equations 12, 13 and 14 for the definition of the coefficients.

Subroutine, Sub 4: This subroutine will be used to calculate the values of the self- and mutual-sequence impedances of the conductors. This includes both the inductive and capacitive reactance values.

Subroutine, Sub 5: After the values of the current coefficients have been found, this subroutine calculates the zero-sequence current components in the series impedance elements. Charging current, negative- and zero-sequence current components for the shunt branch elements are also calculated. This is illustrated by Equations 40, 41, 42, 44, 45 and 46.

Subroutine, Sub 6: This subroutine is used to find the line currents by using the method of symmetrical components. These sequence currents are calculated by superimposing the negative- and zero-sequence currents from subroutine, Sub 5.

Subroutine, Sub 7: Because the equivalent 'T' circuit is used in this thesis, the total transmission line is separated into two parts,

the sending end and receiving end. The main program which controls the logical calculation sequence will be used to determine the line currents for both the sending and receiving ends. This subroutine will be used to calculate the transmission line drop, part by part, and the main program will calculate the total line drop of both the transposed and untransposed transmission lines.

Subroutine, Sub 8: This subroutine performs in the same manner as the aforesaid subroutine, Sub 7. The main program controls the flow and this subroutine calculates the line losses of the sending and receiving ends. The total line loss of the transposed and untransposed transmission lines is calculated in the main program.

### Computer Operation

After the data and conductor configurations are read by the main program, the subroutine, Sub 1, is called to calculate all the conductor spacings. Subroutine, Sub 2, is called next to find the self- and mutual-line impedances and reactances. The self- and mutual-line impedances and line reactances, then, will be used to perform the calculation of the line impedance and reactance coefficients for currents  $I_a$ ,  $I_b$  and  $I_c$  including the effect of two ground wires by calling subroutine, Sub 3. The subroutine, Sub 4, is called next to calculate the self- and mutual-sequence impedances and reactances. The units for the above calculations are ohms per unit mile length.

Before calling further subroutines, the main program is used to set the first initial length of transmission line and find the total positive-, negative- and zero-sequence impedances and reactances for that length of line. The subroutine, Sub 5, is called to calculate the positive-, negative- and zero-sequence current components for the impedance elements and the negative- and zero-sequence currents for the shunt reactance branch. The subroutine, Sub 6, is called next to calculate the total line currents after the currents in the series and shunt branches have been superimposed by the main program.

Because the nominal-T circuit is used in this thesis, the transmission line is separated into two parts, the sending end and receiving end. The calculation of currents in the sending and receiving ends for both the transposed and untransposed transmission lines is controlled by the main program. The subroutine, Sub 7, is called to calculate the transmission line drop in both end sections. To find the transmission line loss, the subroutine, Sub 8, is called by the main program.

Before the next iteration is considered, the main program will calculate the unbalance factor of the system. If the unbalance factor is higher than the allowable value, the main program will be stopped and all the desired results printed. Otherwise, the next iteration will increase the line length and recalculate the unbalance factor until the required length is obtained. All the required results will be printed at each iteration of the transmission line distance.

#### IV. CONCLUSION

At the present time, the first purpose for transmission line transposition, to reduce or eliminate the interference with parallel communication circuit is becoming gradually less important. In modern practice, many communication systems have been greatly improved. Some are now underground and others are being replaced by microwave systems.

The second purpose, to bring the line currents and line voltages into balance, still plays a great role in the design of transmission lines. The unbalance of the transmission line leads to the negative- and zero-sequence current circulation in the lines. The zero-sequence current which flows in the terminal equipment connections should be taken into consideration with respect to sensitive relaying which sometimes operates on as little as 0.5-amperes of secondary current. The negative-sequence current will flow through the generators and transformers at the terminals of the line. Synchronous machines of either salient-pole or solid-rotor type and induction motors are subjected to additional heat generation in the stator and the rotor. Because the transformer or winding rating is based on the maximum current in any phase, the maximum current must not exceed the rated phase current (10, 13).

Results from the calculations indicate that the unbalance factor

for the 100 mile-long transmission lines considered are 0.0474 for the 230 Kv system and 0.0626 for the 500 Kv system. These values are below the limits recommended for electrical machinery operation; i.e., 0.07.

Line currents, from Figure 8 for 230 Kv and Figure 11 for 500 Kv, vary directly with the transmission line length. For the 230 Kv system, the current in the 'a' phase conductor increases from 603.0 to 612.2 amperes when the transmission line length is increased from 10 to 100 miles. Under the same condition, the current in phase 'b' increases from 602.5 to 604.2 amperes but, on the contrary, that of phase 'c' decreases from 601.8 to 591.5 amperes. For the 500 Kv system, the current in the 'a' phase conductor increases from 1157.2 to 1178.5 amperes when the transmission line length is increased from 10 to 100 miles; the current in phase 'b' changes slightly from 1154.4 to 1154.3 amperes, and the current in phase 'c' decreases from 1152.6 to 1132.3 amperes. These results occur because of the unbalanced line phenomena.

Figures 9 and 12 show that transmission line voltage drops vary with transmission line length. For the 230 Kv line system, the drop of phase 'a', phase 'b' and phase 'c' are 48.932, 53.482 and 52.406 Kv, respectively, when the transmission line length is increased to 100 miles. For 500 Kv system, these line drops are 86.890, 93.359 and 91.185 Kv, respectively, when the transmission

line length is increased to 100 miles.

Figures 10 and 13 indicate that the difference between transposed and untransposed line losses increase in a parabolic curve form. This means that losses of a transmission line increase very rapidly with transmission line length. For the 230 Kv line, it is evident that the difference in power loss ranges from 0.109 to 24.768 Kw when the distance is increased to 100 miles. For the 500 Kv line, the difference is increased from 0.735 to 61.949 Kw. This considerable amount of loss should be considered in the economical design of transmission lines. Comparison of examples and calculated results in this thesis, for both the 230 Kv and 500 Kv lines, will enable an engineer to visualize the results of an untransposed transmission line for any given length when making an economic evaluation of a design.

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## APPENDICES

## APPENDIX I

Program Testing

The calculations were performed for representative transmission lines as shown in Figure 5 with the conductor configurations as shown in Figure 3. These data are for the 230-Kv system. Other necessary data for this program are shown below:

Input Data for 230-Kv Line

		<u>Unit</u>
Line voltage	230	Kv
Line frequency	60	Hertz
Load transmitted	240	Mw
Power factor at sending end terminal	100	percent
Earth resistivity	100	ohm-meter
Positive-sequence impedance of generator	120	percent
Negative-sequence impedance of generator	20	percent
Positive-sequence impedance of transformer #1	10	percent
Negative-sequence impedance of transformer #1	10	percent
Zero-sequence impedance of transformer #1	10	percent
Positive-sequence impedance of transformer #2	10	percent
Negative-sequence impedance of transformer #2	10	percent
Zero-sequence impedance of transformer #2	10	percent

		<u>Unit</u>
Positive-sequence impedance of load	100	percent
Negative-sequence impedance of load	10	percent
	<u>Phase</u> <u>Conductor</u>	<u>Ground</u> <u>Wire</u>
		<u>Unit</u>
Radius of conductor	1.1080	0.492 inches
Resistance of conductor	0.1288	0.382 ohms/mile
Geometric mean radius	0.0375	0.01599 feet

### Computer Output

The computer will print out information for each distance iteration of the transmission line calculation. Line parameter, phase currents, phase voltage drops, and losses are printed. Computer output for the 230 Kv transmission line is shown in Tables 1, 2 and 3. Results of this calculation sequence are plotted and shown in Figures 8, 9 and 10.

Table 1. Sequence and phase currents of 230-Kv system transposed and untransposed transmission lines for different line lengths.

SYSTEM 230-KV LINE  
 LOAD = 240000 KW  
 DISTANCE, MILES  
 CURRENT, AMPERES

DISTANCE	TRANSPOSED PHASE CURRENTS						UNTRANSPOSED PHASE CURRENTS					
	---SEQUENCE CURRENT---			---PHASE CURRENT---			---SEQUENCE CURRENT---			---PHASE CURRENT---		
	POS	NEG	ZERO	A	B	C	POS	NEG	ZERO	A	B	C
10.00	612.5	0	0	602.5	602.5	602.5	602.5	2.5	2.0	603.0	602.5	601.8
20.00	612.5	0	0	602.5	602.5	602.5	602.5	4.8	3.1	604.1	602.6	600.8
30.00	612.5	0	0	602.5	602.5	602.5	602.5	6.8	3.9	605.3	602.6	599.6
40.00	612.5	0	0	602.5	602.5	602.5	602.5	8.6	4.9	606.5	602.8	598.3
50.00	612.5	0	0	602.5	602.5	602.5	602.5	10.2	4.9	607.6	602.9	597.1
60.00	612.5	0	0	602.5	602.5	602.5	602.5	11.6	5.2	608.7	603.1	595.9
70.00	612.5	0	0	602.5	602.5	602.5	602.5	13.0	5.5	609.7	603.3	594.8
80.00	612.5	0	0	602.5	602.5	602.5	602.5	14.2	5.7	610.6	603.6	593.6
90.00	612.5	0	0	602.5	602.5	602.5	602.5	15.3	5.9	611.4	603.9	592.6
100.00	612.5	0	0	602.5	602.5	602.5	602.5	16.4	6.1	612.2	604.2	591.5

Table 2. Phase voltage drops of 230-Kv system transposed and untransposed transmission lines for different line lengths.

SYSTEM 230-KV LINE  
 LOAD = 24000 KW  
 DISTANCE, MILES  
 PHASE VOLTAGE DROP, VOLTS

DISTANCE	----TRANPOSED PHASE VOLTAGE DROPS----			---UNTRANPOSED PHASE VOLTAGE DROPS----		
	PHASE-A	PHASE-B	PHASE-C	PHASE-A	PHASE-B	PHASE-C
10.00	4994.7327	4994.7327	4994.7327	4934.5119	5223.9621	5342.7853
20.00	9989.9388	9989.9388	9989.9388	9811.3820	10488.5336	10680.6256
30.00	14986.0912	14986.0912	14986.0912	14674.1312	15782.4738	15993.7864
40.00	19983.6629	19983.6629	19983.6629	19538.4824	21100.7497	21277.7104
50.00	24983.1256	24983.1256	24983.1256	24410.5732	26441.4984	26532.0651
60.00	29984.9546	29984.9546	29984.9546	29292.8131	31804.2766	31757.8833
70.00	34989.6189	34989.6189	34989.6189	34186.0433	37189.2576	36956.6408
80.00	39997.5911	39997.5911	39997.5911	39090.4721	42596.8698	42129.5184
90.00	45009.3423	45009.3423	45009.3423	44006.0337	48027.6348	47279.2707
100.00	50025.3429	50025.3429	50025.3429	48932.6036	53482.0945	52406.1762

Table 3. Line losses and difference in losses of 230-Kv system transposed and untransposed transmission lines for different line lengths.

SYSTEM 230-KV LINE  
 LOAD = 24000 KW  
 DISTANCE, MILES  
 LINE LOSS, KW

<u>DISTANCE</u>	<u>TRANSPOSED LINE LOSS</u>	<u>UNTRANSPOSED LINE LOSS</u>	<u>DIFFERENCE IN LOSS</u>
10.00	1403.123	1403.231	.109
20.00	2806.777	2807.535	.757
30.00	4211.495	4213.495	2.000
40.00	5617.808	5621.611	3.803
50.00	7026.248	7032.381	6.133
60.00	8437.346	8446.304	8.958
70.00	9851.634	9863.888	12.254
80.00	11269.643	11285.643	16.000
90.00	12691.906	12712.082	20.176
100.00	14118.953	14143.721	24.768

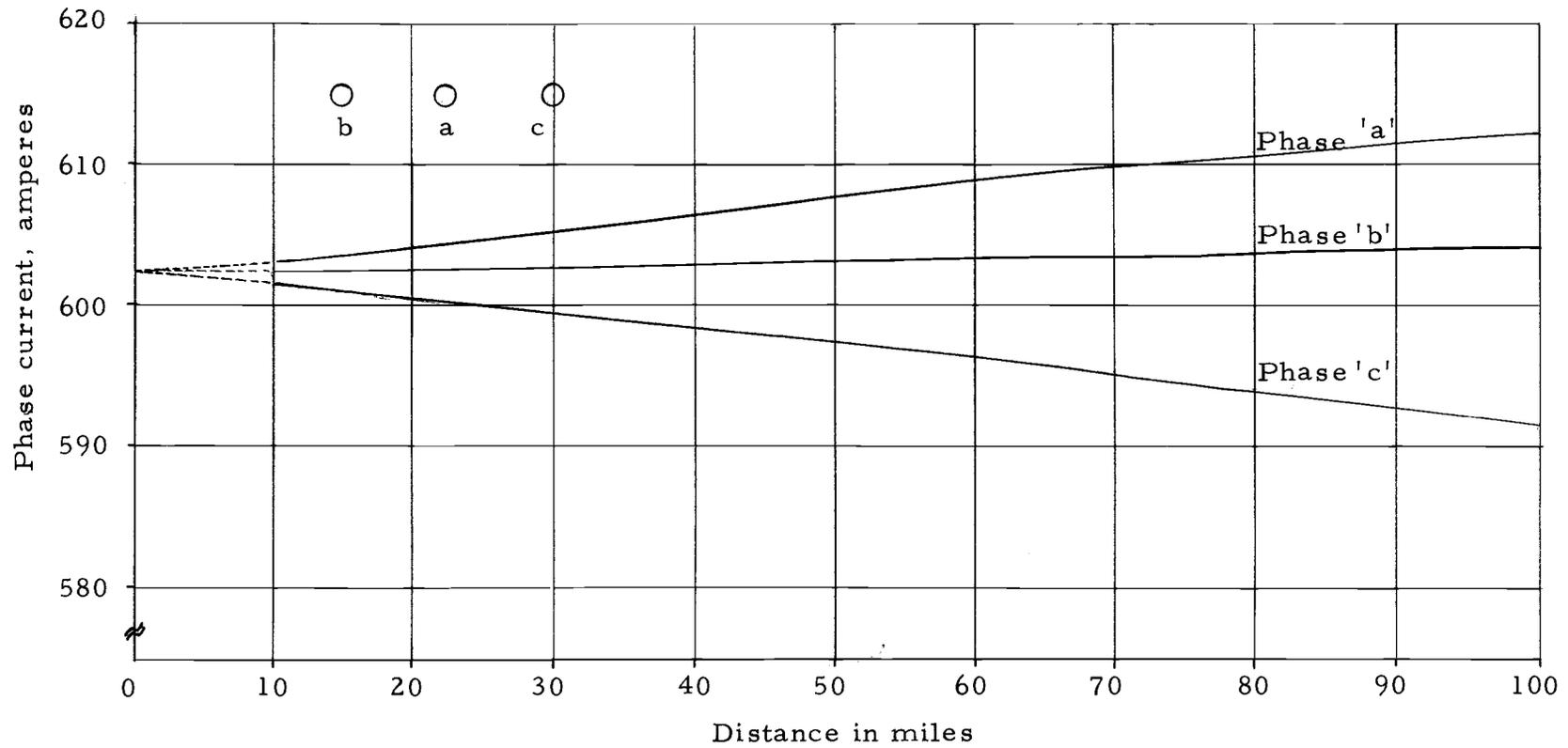


Figure 8. Curve of untransposed phase currents versus distance for 230-Kv line.

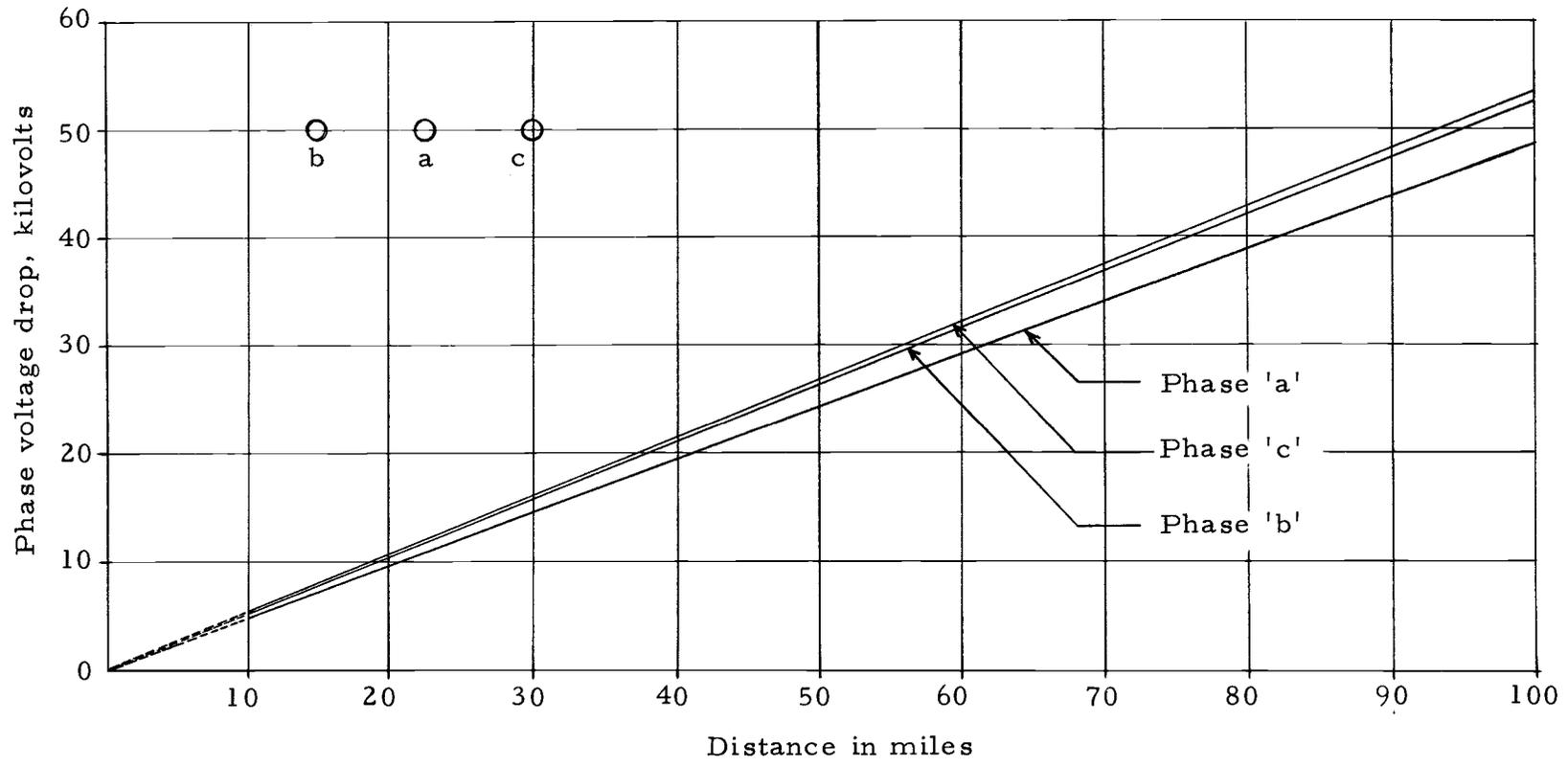


Figure 9. Curve of untransposed phase voltage drops versus distance for 230-Kv line.

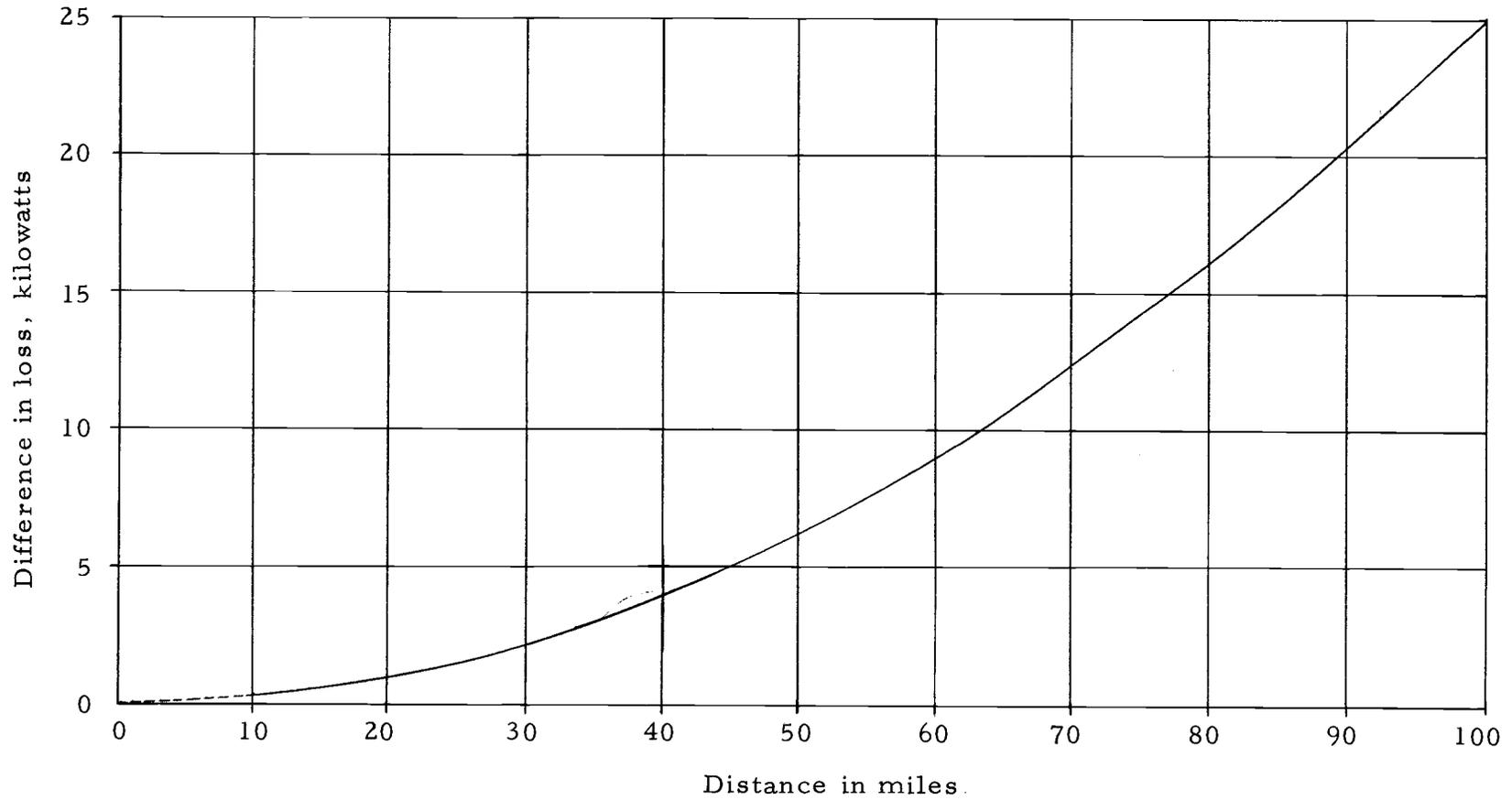


Figure 10. Curve of the difference in losses between transposed and untransposed lines versus distance for 230-Kv line.

## APPENDIX II

Program Testing for 500-Kv

The conductor configuration of Figure 4 will be used for the 500-Kv system (5). The other necessary data for this program are as follows:

Input Data

		<u>Unit</u>
Line voltage	500	Kv
Line frequency	60	Hertz
Load transmitted	1000	Mw
Power factor at sending end terminal	100	percent
Earth resistivity	100	ohm-meter
Positive-sequence impedance of generator	120	percent
Negative-sequence impedance of generator	20	percent
Positive-sequence impedance of transformer #1	10	percent
Negative-sequence impedance of transformer #1	10	percent
Zero-sequence impedance of transformer #1	10	percent
Positive-sequence impedance of transformer #2	10	percent
Negative-sequence impedance of transformer #2	10	percent
Zero-sequence impedance of transformer #2	10	percent
Positive-sequence impedance of the load	100	percent
Negative-sequence impedance of the load	10	percent

	<u>Phase</u> <u>Conductor</u>	<u>Ground</u> <u>Wire</u>	<u>Units</u>
Radius of conductor	2.5	0.492	inches
Resistance of conductor	0.0362	0.382	ohms/mile
Geometric mean radius	0.09	0.01599	feet

### Computer Output

Phase currents, phase voltage drops and losses are printed at each iteration. The computer output is shown in Tables 4, 5 and 6. The results have been plotted and shown in Figures 11, 12 and 13.

Table 4. Sequence and phase currents of 500-Kv system transposed and untransposed transmission lines for different line lengths.

SYSTEM 500-KV LINE  
 LOAD = 1000 MW  
 DISTANCE, MILES  
 CURRENT, AMPERES

DISTANCE	TRANSPOSED PHASE CURRENTS						UNTRANSPOSED PHASE CURRENTS						
	---SEQUENCE CURRENT---	----			----			---SEQUENCE CURRENT---	----			----	
-----	POS	NEG	ZERO	A	B	C	POS	NEG	ZERO	A	B	C	
-----	---	---	---	-----	-----	-----	---	---	---	-----	-----	-----	
10.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	4.8	2.2	1157.2	1154.4	1152.6	
20.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	9.1	3.6	1160.0	1154.0	1150.2	
30.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	13.0	4.6	1162.8	1153.8	1147.7	
40.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	16.5	5.3	1165.5	1153.6	1145.3	
50.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	19.7	5.9	1168.1	1153.5	1142.9	
60.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	22.7	6.3	1170.5	1153.5	1140.7	
70.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	25.4	6.7	1172.7	1153.6	1138.5	
80.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	27.9	7.0	1174.8	1153.8	1136.4	
90.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	30.3	7.3	1176.7	1154.0	1134.3	
100.00	1154.7	0	0	1154.7	1154.7	1154.7	1154.7	32.4	7.6	1178.5	1154.3	1132.3	

Table 5. Phase voltage drops of 500-Kv system transposed and untransposed transmission lines for different line lengths.

SYSTEM 500-KV LINE  
 LOAD = 1000 MW  
 DISTANCE, MILES  
 PHASE VOLTAGE DROP, VOLTS

DISTANCE	----TRANSPPOSED PHASE VOLTAGE DROPS----			---UNTRANSPPOSED PHASE VOLTAGE DROPS----		
	PHASE-A	PHASE-B	PHASE-C	PHASE-A	PHASE-B	PHASE-C
10.00	8718.1343	8718.1343	8718.1343	8669.3939	9214.2625	9289.0891
20.00	17437.4709	17437.4709	17437.4709	17296.5647	18450.6050	18561.4088
30.00	26159.2114	26159.2114	26159.2114	25922.0950	27709.6679	27794.1739
40.00	34884.5567	34884.5567	34884.5567	34561.9138	36993.0577	36981.1301
50.00	43614.7072	43614.7072	43614.7072	43222.6229	46303.2996	46121.4828
60.00	52350.8618	52350.8618	52350.8618	51906.9013	55643.3062	55216.4640
70.00	61094.2183	61094.2183	61094.2183	60615.7411	65016.0611	64268.0851
80.00	69845.9728	69845.9728	69845.9728	69349.4018	74424.4667	73278.6348
90.00	78607.3195	78607.3195	78607.3195	78107.8430	83871.2796	82250.4672
100.00	87379.4503	87379.4503	87379.4503	86890.9838	93359.0917	91185.9097

Table 6. Line losses and difference in losses of 500-Kv system transposed and untransposed transmission lines for different line lengths.

SYSTEM 500-KV LINE  
 LOAD = 1000 MW  
 DISTANCE, MILES  
 LINE LOSS, KW

DISTANCE	TRANSPOSED LINE LOSS	UNTRANSPOSED LINE LOSS	DIFFERENCE IN LOSS
-----	-----	-----	-----
10.00	1447.531	1448.265	.735
20.00	2895.859	2898.357	2.498
30.00	4345.785	4351.327	5.542
40.00	5798.105	5807.992	9.887
50.00	7253.618	7269.126	15.508
60.00	8713.122	8735.501	22.379
70.00	10177.416	10207.691	30.275
80.00	11647.297	11687.076	39.779
90.00	13123.564	13173.838	50.274
100.00	14607.015	14668.964	61.949

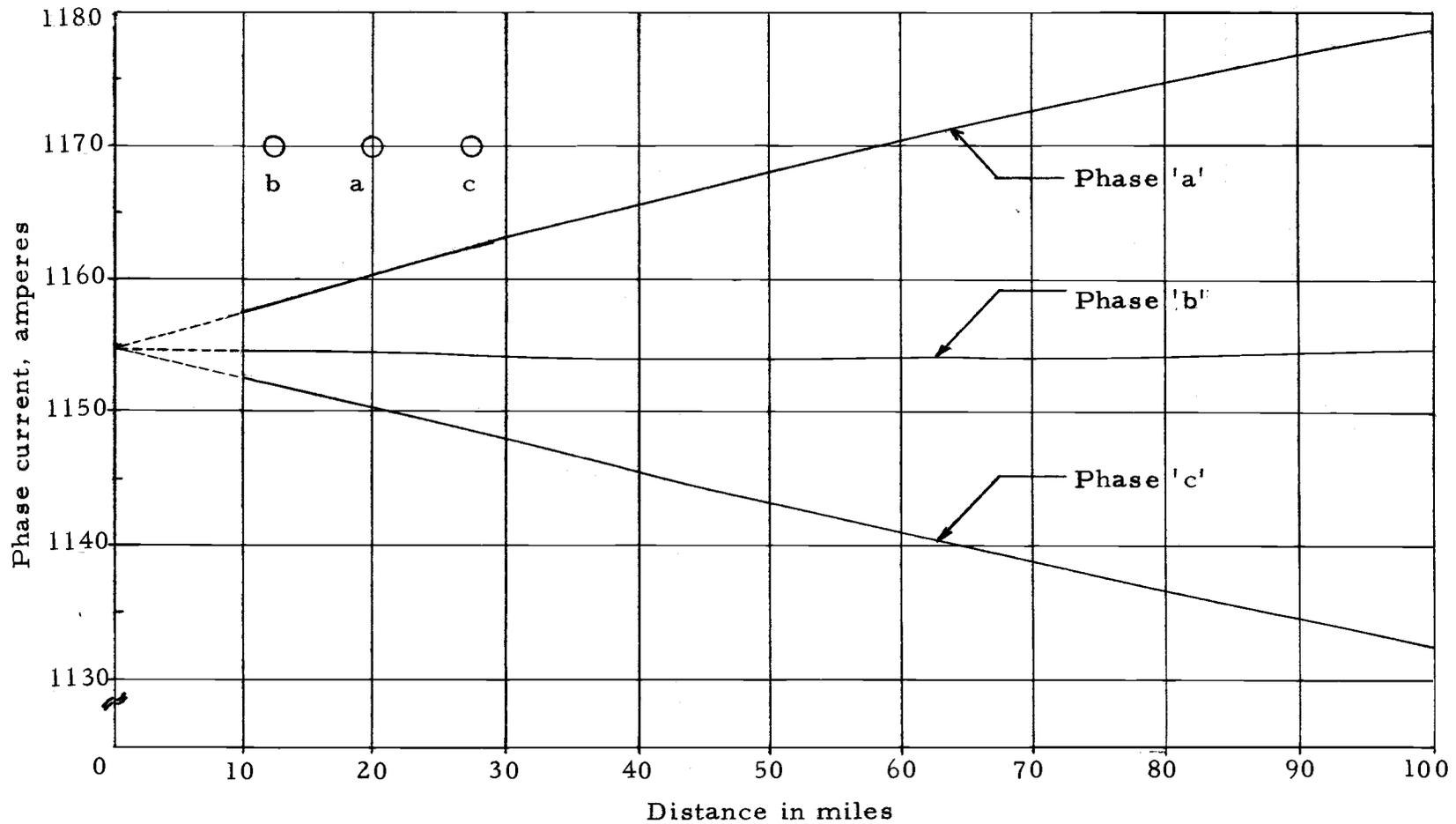


Figure 11. Curve of untransposed phase currents versus distance for 500-Kv line.

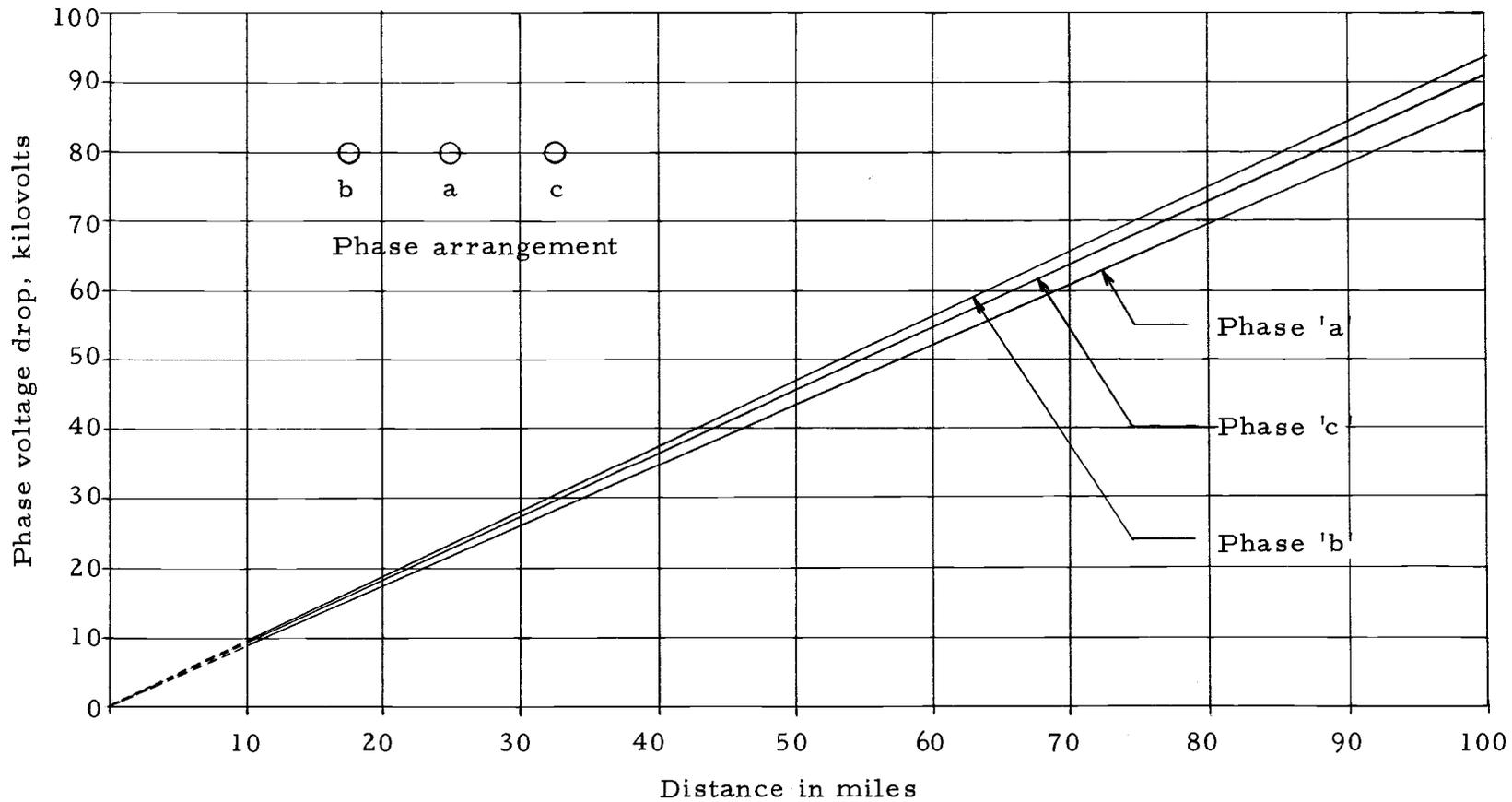


Figure 12. Curve of untransposed phase voltage drops versus distance for 500-Kv line.

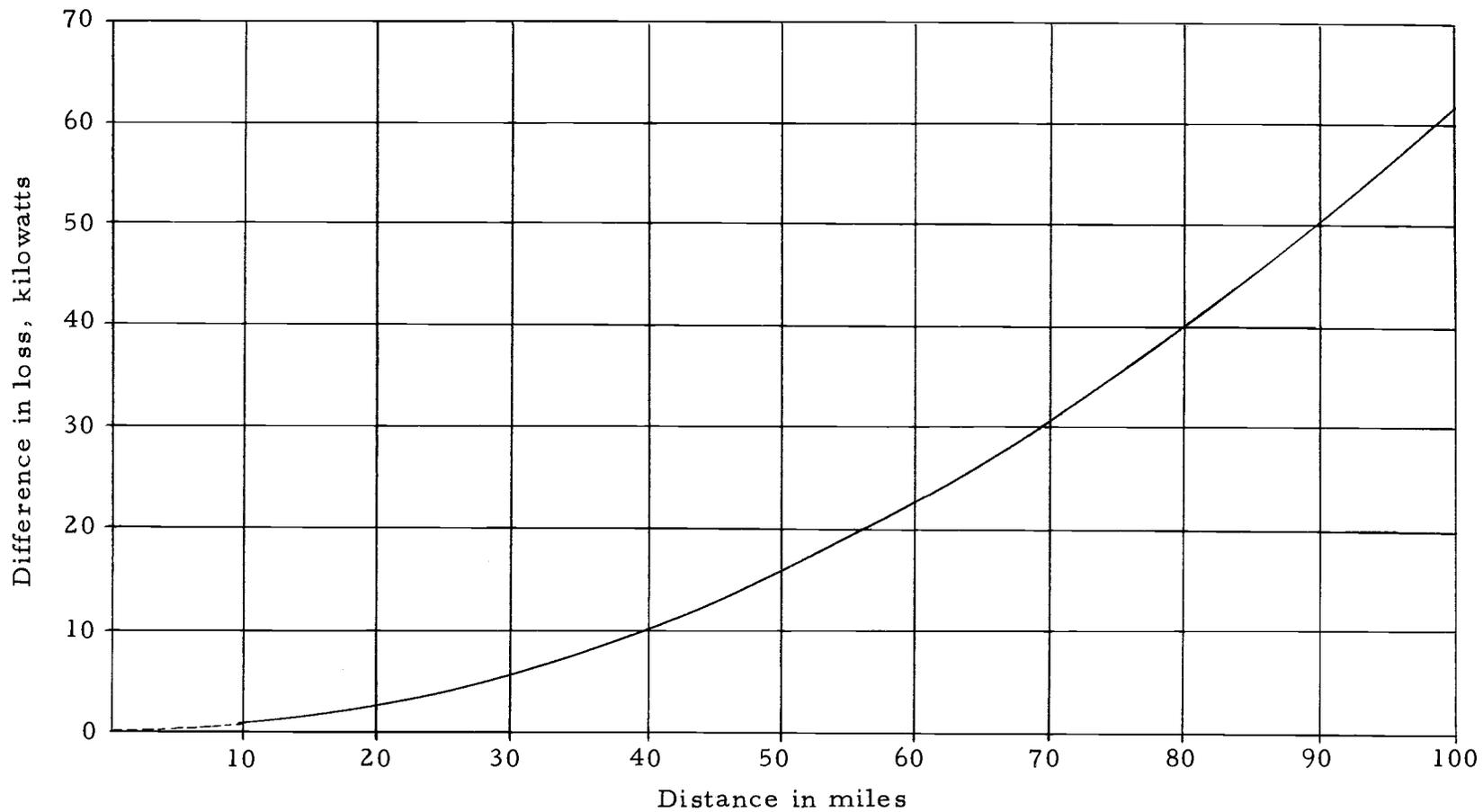


Figure 13. Curve of the difference in losses between transposed and untransposed lines versus distance for 500-Kv line.

