Decentralized Model Predictive Control for Wave Energy Converter Arrays


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Decentralized Model Predictive Control for Wave Energy Converter Arrays

Daniela Oetinger, Mario E. Magaña, Senior Member, IEEE and Oliver Sawodny

Abstract—Most of the research on wave energy conversion has been focused on the characterization of the dynamic behavior of arrays of uncontrolled WECs in specific configurations, in order to quantify changes in the wave fields and absorbed power without active control. To maximize wave energy conversion, however, it is necessary to apply active control techniques to the WECs that conform the array. In this paper we propose the application of decentralized model predictive control (MPC) to the elements of an array by considering each individual WEC as a subsystem. Each decentralized MPC optimizes the absorbed power of its own WEC under the same input and state constraints that a centralized MPC otherwise would.

Index Terms—Model predictive control, point absorber, wave energy converter arrays

I. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a, m$</td>
<td>Radius (m), mass (kg) of one buoy</td>
</tr>
<tr>
<td>$a_3, m_4$</td>
<td>Radius (m), mass (kg) of center body</td>
</tr>
<tr>
<td>$A$</td>
<td>Added mass (kg)</td>
</tr>
<tr>
<td>$A_\omega, \eta$</td>
<td>Wave amplitude, surface elevation (m)</td>
</tr>
<tr>
<td>$B$</td>
<td>Viscous damping coefficient (N/(m/s))</td>
</tr>
<tr>
<td>$F$</td>
<td>Bottom mooring force (N)</td>
</tr>
<tr>
<td>$F_B$</td>
<td>Buoyancy force (N)</td>
</tr>
<tr>
<td>$F_{exc}$</td>
<td>Wave induced excitation force (N)</td>
</tr>
<tr>
<td>$F_{gen}$</td>
<td>Generated force (N)</td>
</tr>
<tr>
<td>$F_h$</td>
<td>Hydrodynamic force (N)</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration of gravity (m/s$^2$)</td>
</tr>
<tr>
<td>$G$</td>
<td>Buoy to center body cable length (m)</td>
</tr>
<tr>
<td>$H_s$</td>
<td>Significant wave height (m)</td>
</tr>
<tr>
<td>$J$</td>
<td>Objective function</td>
</tr>
<tr>
<td>$K$</td>
<td>Mooring cable stiffness of one cable (N/m)</td>
</tr>
<tr>
<td>$L$</td>
<td>Buoy to center of triangle distance (m)</td>
</tr>
<tr>
<td>$N$, $T_{hor}$</td>
<td>Length of time horizon, Time horizon (s)</td>
</tr>
<tr>
<td>$R$</td>
<td>Buoy to center mass tension force (N)</td>
</tr>
<tr>
<td>$T$</td>
<td>Simulation time (s)</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Wave period (s)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Sample time for discretization (s)</td>
</tr>
<tr>
<td>$w$</td>
<td>Weighting factor slack</td>
</tr>
<tr>
<td>$x_i, y_i, z_i$</td>
<td>Position buoy $i$ (m)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Cable angle to surface (°)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle between mooring line and surface (°)</td>
</tr>
<tr>
<td>$c_{x_i}, c_{y_i}$</td>
<td>state and input upper bounds</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Slack variable</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of sea water (kg/m$^3$)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angle of incident wave with x-axis (°)</td>
</tr>
<tr>
<td>$\phi, \psi$</td>
<td>Phase angle/shift (rad)</td>
</tr>
</tbody>
</table>

II. INTRODUCTION

The ocean is an almost inexhaustible source of energy and represents one of the most reliable and promising forms of renewable energy. However, wave energy is not as well developed as other resources like wind or solar. Most of the work in this field has been done for different types of single devices converting the sea motion into energy, i.e. floating point absorbers transforming the heave motion into electricity. For a considerable contribution to the world energy market and its commercialization, however, the deployment of very few and isolated wave energy converters (WECs) is not enough. Some research has been done on the dynamic behavior of an array of uncontrolled WECs arranged in a specific configuration and the resulting changes in the wave field and absorbed power have been described. Analyses as in [1] have shown that an array configuration with $n$ WECs does not necessarily convert $n$ times the energy of a single device. Nevertheless, such WEC arrays will play a more important role in the future, not only because of the amount of power. In [2] it is suggested that wave energy farms could be used to reduce and equalize output fluctuations for a smooth supply of the power grid. Further advantages consist in reduced costs for moorings, grid connections and maintenance [3].

The first wave energy farm deployed was the Aguçadoura Wave Farm in Portugal in 2008, generating about 2.25 MW [5]. Further projects have been announced and are under development by Pelamis Wave Power at different sites, e.g. in Scotland [6].

The challenges that arise when several WEC devices are arranged in a group can be best understood after having a look at what happens to a body subjected to a sea wave. In [7] different forms of wave-body interactions are analyzed in detail. This analysis characterizes the hydrodynamics of an oscillating body and tells us how the wave field is influenced by its presence. Since the objective of a WEC consists of extracting energy from the ocean, this energy is absorbed from the waves which are then either reduced or completely canceled. When several WECs are placed far apart from each other, however, these changes do not affect too much the power absorption of a single device. Nevertheless, it is desirable to limit the space between them, in order to absorb more power in a limited area.

In order to maximize wave energy conversion, optimal model predictive control (MPC) has been applied to point absorber WECs [8], [9], [10], and [11]. In each of these cases, various performance criteria and system constraints have been used to optimise wave energy conversion of a
single device. In [8] the performance criterion (cost function) is the maximum energy absorption with constraints on the heave motion and the machinery force. In [8], Hals et al also present a two-stage optimisation problem for the regular sea state. Moreover, they propose an MPC algorithm for irregular sea states using real-time prediction of the wave motion horizon. Abraham and Kerrigan [9] investigate a general optimal active control problem for a heaving point absorber whose objective is to maximise the energy extracted from ocean waves, subject to control forces constraints. The hallmark of this paper is that it applies a projected gradient method that is globally convergent to stationary points using a small number of iterations, to obtain the optimal solution. The works presented in [8] and [9] deal with one-body models and do not consider mooring forces. Moreover, their MPC approaches utilise an available optimal velocity trajectory which can be calculated for one-body WECs. In [10] and [11] a special formulation of the optimal MPC problem which allows the application of MPC without a reference trajectory is applied to maximise the energy extracted from ocean waves subject to machinery constraints of a two-body heaving WEC.

As stated previously, our objective is to maximize wave energy absorption of an array of WECs using active control. To this end, we propose to use a simple form of decentralized MPC. Decentralized and/or distributed control has been applied to large scale dynamic systems for many years [13]. When controlling a large scale dynamic system using decentralized control, however, the global control performance may not be satisfactory if the subsystems interact strongly. To address these shortcomings, cooperative and non-cooperative distributed MPC techniques which try to achieve the performance of global centralized MPC have been devised [14], [15]. In [16] Venkat et al apply a couple of distributed MPC algorithms to a power system that is decomposed into subsystems with strong coupling, each with its own MPC. The subsystem-based MPCs work iteratively to achieve the desired system-wide control objectives.

Depending on the nature of the dynamic coupling, different practical MPC techniques have been successfully applied to large scale systems. In [12], Scattolini suggests a decentralized MPC architecture for weakly coupled subsystems. For strongly coupled subsystems, Scattolini [12] also suggests, for example, to use either distributed MPC where information is shared between the individual MPC controllers of each subsystem via a communication system or one of several hierarchical control architectures.

In so far as wave energy farms is concerned, one envisions a large scale dynamic system composed of many subsystems (WECs) that may be either strongly or weakly hydrodynamically coupled. Depending on the type, proximity and the hydrodynamic reaction forces generated by the WECs, the interactions may be weak or strong. In the case of point absorber WECs, which is the subject of this paper, they are relatively small and their hydrodynamic reaction forces may be viewed as decaying disturbances [17] by the other WECs.

This paper applies active decentralized MPC to control an array configuration of three point absorber WECs [18] to optimize the power absorption. We use this type of control approach because the WECs under consideration are small and the distances between them are such that the reaction forces interactions are relatively small and we can, as a first approximation, ignore the coupling between them. Based on the derivation of the mathematical model of a WEC array configuration designed by a group of Portuguese researchers in [18], we develop an appropriate state model for the controller approach presented herein. Performance of the proposed decentralized MPC algorithm is evaluated when the array of WECs is excited by irregular waves. A summary of the findings is presented in the conclusion.

**III. WEC Array State Model**

In this paper, we consider an array of three inter-body moored WECs that are arranged in an equilateral triangular grid. Each buoy is modeled as a sphere and moored to the sea-floor with a cable at an angle $\beta$ with respect to the water surface. Additionally, each buoy is connected to a center mass with a cable of length $G$ at an angle $\alpha$ with respect to the surface. The top view as well as a perspective view of this configuration in calm sea state where $\theta$ represents the angle of the incident wave is shown in Fig. 1, which was adapted from [18] and [19].

![Figure 1: WEC array Plan view (a) and perspective view (b)](image)

For system modeling, we need to identify all the forces acting on each buoy in the three directions of motion surge ($x$), sway ($y$) and heave ($z$), knowing that the generator and buoyancy forces are only relevant in the heave direction.

The equations of motion of the WEC array are derived in [18] and can be rewritten in state-space form (see details of derivation in [20]) as

$$
\dot{x} = Ax + Bu + d_{pert} \\
y = Cx
$$

(1)

where

- $x$: $18 \times 1$ state vector composed of the buoys displacements $x_j$, $y_j$, $z_j$, $j = 1, 2, 3$ and their first derivatives
- $A$: $18 \times 18$ system matrix
- $B$: $18 \times m$ input distribution matrix
• \( d_{pert} \): \( 18 \times 1 \) perturbation vector that contains the influence of the perturbations with respect to a calm sea state
• \( C \): \( p \times 18 \) output distribution matrix
• \( y \): \( p \times 1 \) output vector

In order to better explain the system modeling approach, the state matrix is decomposed into two matrices, namely, 
\[ A_{\text{correct}} = A + A_{\text{corr}}, \]
where

1) Matrix \( A \) contains only the individual behavior of the buoys ignoring the interactions with the other buoys. The resulting matrix is diagonal and represents a dynamically decoupled array.

2) Matrix \( A_{\text{corr}} \) contains the interactions between the buoys and the center mass and is non-diagonal.

The diagonal decoupled state matrix is
\[
A = \begin{bmatrix}
A_1 & 0 & 0 \\
0 & A_2 & 0 \\
0 & 0 & A_3
\end{bmatrix}
\]  
(2)

where \( A_1, A_2, \) and \( A_3 \) are explicitly shown in the appendix.

To better understand the decentralized MPC strategy, let us rewrite the state equation as follows:
\[
\begin{align*}
\dot{x} &= Ax + \sum_{i=1}^{3} B_i u_i + \sum_{i=1}^{3} \left( F_{i}^{H,exc} S_{i}^{H,exc} + F_{i}^{S,exc} S_{i}^{S,exc} \right) + d_{pert} \\
y &= Cx = x
\end{align*}
\]  
(6)

This connecting matrix \( A_{\text{conn}} \) is then multiplied by \( N_{\text{coupling}} \) in order to describe the influence of the center mass in buoy coordinates. The actual correction term has the form
\[
A_{\text{corr}} = A_{\text{conn}} N_{\text{coupling}} = \begin{bmatrix}
A_c & A_c & A_c \\
A_c & A_c & A_c \\
A_c & A_c & A_c
\end{bmatrix},
\]
(5)

where * represent non-zero entries. In what follows, we will use this matrix representation for both the WEC model and the controller. The inputs in our system can be broken down in two groups: uncontrollable and controllable forces. The first group is comprised of the wave excitation forces in vertical and horizontal directions influencing the system’s heave and surge/sway dynamics, respectively.

All state-dependent terms are included in the dynamic matrix \( A \), the generator force \( F_{\text{gen}} \) is the control input. In our work, all three generator forces depend on the wave velocity. Separating horizontal and vertical excitation forces, indices \( S \) (sway) and \( H \) (heave), are used for each buoy \( i \).

The state-space model is then described by
\[
\begin{align*}
x_1(t) &= [A_{11} A_{12} A_{13}] [x_1(t)] + [B_1 B_2 B_3] [u_1(t)]  \\
x_2(t) &= [A_{21} A_{22} A_{23}] [x_2(t)] + [B_4 B_5 B_6] [u_2(t)]  \\
x_3(t) &= [A_{31} A_{32} A_{33}] [x_3(t)] + [B_7 B_8 B_9] [u_3(t)]  \\
&+ [B_{10} B_{11} B_{12}] [F_{H,exc}^{1} S_{H,exc}^{1} F_{S,exc}^{1} S_{S,exc}^{1}]  \\
&+ [B_{13} B_{14} B_{15}] [F_{H,exc}^{2} S_{H,exc}^{2} F_{S,exc}^{2} S_{S,exc}^{2}]  \\
&+ [B_{16} B_{17} B_{18}] [F_{H,exc}^{3} S_{H,exc}^{3} F_{S,exc}^{3} S_{S,exc}^{3}] + d(t)
\end{align*}
\]  
(7)
surge and sway.

\[
\begin{align*}
B_1 &= \begin{bmatrix} b_{\text{gen}} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} b_{\text{gen}} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, & B_3 &= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_{\text{gen}} \end{bmatrix},
\end{align*}
\]

where \( b_{\text{gen}} = \frac{1}{m + A^2} \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T \).

The wave excitation forces input distribution vectors \( B_i^S \)'s take into account the dependence on the incident angle \( \theta \) of the waves with the \( x \)-axis, i.e.

\[
\begin{align*}
B_i^H &= \begin{bmatrix} b_i H \\ 0 \\ \vdots \\ 0 \end{bmatrix}, & B_i^S &= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, & B_i^H &= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_i H \end{bmatrix},
\end{align*}
\]

where \( b_i H = \frac{1}{m + A^2} \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \) and \( b_i S = \frac{1}{m + A^2} \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \).

The vector \( \mathbf{d}_{\text{pert}} \) represents the influence of the perturbations \( r_i \) [20] on the tension forces \( R \) along the inter-buoy connection cables and thus the influence on the buoys’ positions. The matrices and vectors in the state-space representation depend on the frequency of the incident wave because they contain hydrodynamic coefficients. Therefore, we recompute them for each \( \omega \) before every simulation to assess system performance. In this paper we consider the frequency-dependent controller parameters.

\section*{A. Discretization}

For controller design and system simulation purposes, the discrete time state-space representation of equation (6) is needed, namely,

\[
\begin{align*}
\mathbf{x}_{k+1} &= A_d \mathbf{x}_k + B_d \mathbf{u}_k + \sum_{i=1}^{3} \left( B_{d_1} f_{\text{exc},i} \mathbf{H}_{i} + B_{d_2} f_{\text{exc},i} \mathbf{S}_{i} \right) + \mathbf{d}_d \\
\mathbf{y}_k &= C_d \mathbf{x}_k = \mathbf{x}_k
\end{align*}
\]

where, under the assumption that the input forces remain constant during the sampling interval (\( \Delta t \) is either 0.1 s or 0.2 s), the discrete versions of the system, input distribution, and output distribution matrices, respectively, are given by

\[
\begin{align*}
A_d &= e^{A \Delta t}, \\
B_d &= (\int_0^{T_s} e^{A \tau} \text{d} \tau)B = A^{-1}(A_d - I)B, \\
C_d &= C
\end{align*}
\]

where \( A \) and \( B \) are the same system and input distribution matrices used in (6), \( T_s \) is the sample time, and \( B_d = [B_{d_1}^1 \ B_{d_2}^2 \ B_{d_3}^3] \). The input distribution matrices \( B_{d_1}^S \), \( B_{d_2}^H \), and \( B_{d_3}^S \) are the discrete versions of \( B_1^S \), \( B_2^H \), and \( B_3^S \), respectively. The \( f_{\text{exc},k} \)'s and \( f_{\text{exc},i} \)'s represent the vertical and horizontal wave excitation forces on buoy \( i \) at time instant \( k \).

For the decentralized MPC approach, we consider the three buoys as three interconnected subsystems and (11) is rewritten as

\[
\begin{align*}
\begin{bmatrix} \mathbf{x}_{k+1}^1 \\ \mathbf{x}_{k+1}^2 \\ \mathbf{x}_{k+1}^3 \end{bmatrix} &= \begin{bmatrix} A_{d_1} & 0 & 0 \\ 0 & A_{d_2} & 0 \\ 0 & 0 & A_{d_3} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k}^1 \\ \mathbf{x}_{k}^2 \\ \mathbf{x}_{k}^3 \end{bmatrix} + \sum_{i=1}^{3} \begin{bmatrix} B_{d_1} f_{\text{exc},i} \mathbf{H}_{i} \\ B_{d_2} f_{\text{exc},i} \mathbf{S}_{i} \end{bmatrix} + \mathbf{d}_d \\
\mathbf{y}_d^i &= \mathbf{C}_d \mathbf{x}_d^{i} = \mathbf{x}_d^{i}, i = 1, \ldots, 3
\end{align*}
\]

\section*{B. Excitation forces}

The excitation forces depend on the wave elevation \( \eta(t) \) and the incident wave frequency \( \omega \), and can be calculated using the WAMIT coefficients for the magnitude and the phase. For example, for buoy 3, the frequency domain description of the heaving and the sway forces are [18]

\[
\begin{align*}
F_{\text{exc},3}^H(\omega) &= F_H^H(\omega)H(\omega) \\
F_{\text{exc},3}^S(\omega) &= F_S^S(\omega)H(\omega)
\end{align*}
\]

where \( F_{\text{exc},3}^H(\omega) \) is the Fourier transform of \( f_{\text{exc},3}^H(t) \), \( F_{\text{exc},3}^S(\omega) \) is the Fourier transform of the heaving impulse response \( f_{\text{exc},3}^H(t) \), \( H(\omega) \) is the Fourier transform of the wave elevation \( \eta(t) \), and \( F_{\text{exc},3}^S(\omega) \) is the Fourier transform of the sway impulse response \( f_{\text{exc},3}^S(t) \) [7]. Because of the different positions of the buoys, they experience the excitation forces with the following phase shift:

\[
\psi_j = \text{exp}(i \sqrt{3} k L \sin(\pm \theta + \frac{\pi}{3})) j = 1, 2
\]

where the minus and plus signs are for buoys 1 and 2, respectively. However, all simulations in this paper only consider an incident wave angle of \( \theta = 0^\circ \). Thus, the phase angle will be the same of both buoys. This means that these two buoys experience the same excitation forces at exactly the same time.

\section*{IV. DECENTRALIZED MPC DESIGN}

In our decentralized MPC design an optimization with states and inputs constraints problem is solved in real time every sample time [21].

In the work presented in [22] the idea of decomposing a system into interconnected subsystems and designing a decentralized controller for each of them is presented. Here we also model every buoy in the array as a subsystem and design three MPCs, one for every buoy. Each MPC
optimizes the absorbed power of its own buoy under input and state constraints. Since the three MPCs follow common but not contradictory objectives, the overall result is suboptimal.

Camponogara et al. present in [23] a situation where the different local MPCs, called agents, are able to communicate with each other either several times during the solution of the optimization problem or only once afterwards. In our case, we assume that the local MPCs are only able to send their states and results to their neighbor agents when they have finished the optimization process using a simple wireless mesh network [24]. In [25] the authors propose an algorithm for Robust Distributed Model Predictive Control (RMPC), which will be adapted to the agents when they have finished the optimization process or only once after wards. In our case, we assume that the local MPCs are considered problem in this paper. For example, instead of Predictive Control (RMPC), which will be adapted to the agents when they have finished the optimization process or only once afterwards. In our case, we assume that the local MPCs are considered problem in this paper. For example, instead of Predictive Control (RMPC), which will be adapted to the agents when they have finished the optimization process or only once afterwards.

To design the decentralized MPC, we use the discrete state-space form and ignore the perturbations \(d\) for the controller design, but include them in the actual array set \(A\). considering the possible delay \(d\) time step delay in order to model the communication at every instant of time and the states of its neighbors with one time step delay in order to model the communication process. We also consider the dynamics and interactions to be perfectly modeled by the same system equations as in the previous section.

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The objective of the \(i\)-th MPC is the maximization of the absorbed power

\[
\begin{align*}
\sum_{i=1}^{3} F_{\text{gen},i}(t) \dot{z}_i(t)
\end{align*}
\]

where \(F_{\text{gen},i} = u_i\) and \(\dot{z}_i(t)\) are the MPC calculated control input and heaving velocity of buoy \(i, i = 1, 2, 3\). To compare different control approaches, we use the average power absorption during the simulation interval \(I = [0, 50]\) s, i.e.

\[
\begin{align*}
\overline{P} = \frac{1}{50} \int_{0}^{50} P_{\text{gen}}(t) dt \approx \frac{1}{N} \sum_{k=1}^{N} P_{\text{gen}}(k)
\end{align*}
\]
where \( N = \frac{56}{T_p} \) is the number of samples.

In the time horizon \( k = 1, 2, \ldots, N \), the cost function evolves into [20]

\[
P_{gen,i} = -\left( \mathbf{u}_i^T \mathbf{J}_i \mathbf{T}^T \mathbf{U}^T \right) - \left( \mathbf{z}_i^T \mathbf{J}_i \mathbf{T}^T \mathbf{H}^T \mathbf{J}_i \mathbf{T} + \mathbf{z}_i^T \mathbf{H}^T \mathbf{J}_i \mathbf{T}^T + \mathbf{z}_i^T \right) \mathbf{T}^T + \sum_{j=1, j \neq i}^{3} \mathcal{X}_i \mathbf{J}_i \mathbf{T}^T \mathbf{U} \mathbf{U}^T \mathbf{G}^T \mathbf{U}^T \mathbf{U}^T \right),
\]

where

\[
\mathbf{T} = \begin{bmatrix} t_1 & \cdots & t_j & \cdots & t_N \end{bmatrix}^T,
\]

and \( t_j \) is a row vector with a 1 in the \((6 \cdot j)\)-th column and 0 everywhere else. This means that it selects the heave velocity of the \( i - \text{th} \) buoy at each optimization step \( j \).

Again, this is the only component contributing to power generation.

Finally, the optimization problem is reformulated for each buoy as

\[
\min_{\mathbf{U}} J(z_i, \mathbf{U}, \mathbf{U}^T, \mathbf{z})
\]

where

\[
J(z_i, \mathbf{U}, \mathbf{U}^T, \mathbf{z}) = -\frac{1}{2} P_{gen,i} + \frac{1}{2} \mathbf{U}^T \mathbf{R} \mathbf{U}^T + \sum w_{ii} \mathbf{z}_i^2
\]

subject to:

\[
\begin{align*}
\mathbf{x}_{k+1} &= \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k + \mathbf{B}_h \mathbf{f}_k + \mathbf{B}_s \mathbf{f}_s^i + \mathbf{f}_k^i \\
&+ \sum_{j=1, j \neq i}^{3} \mathbf{A}_d \mathbf{z}_j, i = 1, 2, 3 \\
|z_i| - \varepsilon &\leq \varepsilon_0, k = 1 \ldots N, \dim z_i = \dim \varepsilon = 6 \\
|u_i| &\leq c_u, k = 1 \ldots N \\
0 &\leq \varepsilon_l \leq c_{\varepsilon, l}, l = p, v
\end{align*}
\]

where \( \mathbf{R} \) is the weighting matrix that penalizes the control inputs and \( w_l \) are the weighting factors for the slack variables \( \varepsilon_l \), in our case \( \varepsilon_p \) and \( \varepsilon_v \) for the heave position and velocity, respectively.

## V. Performance evaluation

System performance is evaluated in an irregular wave scenario using computer simulation, since this is the most realistic ocean scenario. The parameter values listed in Table I are used in all simulations, unless otherwise noted. Subindexes \( h \) and \( z \) indicate horizontal and vertical directions, respectively. Subindex 4 is associated with the center mass.

Additionally, the input constraint is chosen as the maximum generator force of all three buoys if we model a linear feedback \( u_i = F_{gen,i}(t) = -C \ddot{z}_i(t) \) with a constant damping coefficient \( C = 3.7072 \times 10^5 \text{ N/(m/s)} \):

\[
c_u = \max_i u_i = \max_i F_{gen,i} = \max_i [-C \ddot{z}_i]
\]

The state and control force constraints are

\[
\begin{align*}
\varepsilon_z &= [10 \ 5 \ 10 \ 2 \ 1 \ 1.2]^T \text{ m} \\
c_u &= 3.58 \times 10^5 \text{ N}
\end{align*}
\]

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>( L )</td>
<td>30 m</td>
</tr>
<tr>
<td>( G )</td>
<td>\frac{L_{\text{cost}}}{m} \approx 34.6410 \text{ m}</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>30°</td>
</tr>
<tr>
<td>( \beta )</td>
<td>30°</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0°</td>
</tr>
<tr>
<td>( a )</td>
<td>7.5m</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>1.5m</td>
</tr>
<tr>
<td>( m )</td>
<td>1.0986 \times 10^6 \text{ kg}</td>
</tr>
<tr>
<td>( m_4 )</td>
<td>1.7671 \times 10^4 \text{ kg}</td>
</tr>
<tr>
<td>( B_{th,4z} )</td>
<td>0 N/(m/s)</td>
</tr>
<tr>
<td>( \lambda_{4z} )</td>
<td>0 N</td>
</tr>
<tr>
<td>( K )</td>
<td>2.1648 \times 10^5 \text{ N/m}</td>
</tr>
<tr>
<td>( T_s )</td>
<td>0.1 s</td>
</tr>
<tr>
<td>( T_{\text{hor}} )</td>
<td>6 s</td>
</tr>
<tr>
<td>( T_{\text{sim}} )</td>
<td>50 s</td>
</tr>
<tr>
<td>( N )</td>
<td>\frac{2\pi}{T_p} 60</td>
</tr>
<tr>
<td>( \xi_s )</td>
<td>[15 2 10 2 11] T m</td>
</tr>
<tr>
<td>( c_u )</td>
<td>2.5 \times 10^5 \text{ N}</td>
</tr>
<tr>
<td>( c_r )</td>
<td>1 m/s</td>
</tr>
<tr>
<td>( c_s )</td>
<td>1 m/s</td>
</tr>
<tr>
<td>( w_p )</td>
<td>10^8</td>
</tr>
<tr>
<td>( w_v )</td>
<td>10^6</td>
</tr>
</tbody>
</table>

For the parameters that are partially dependent on \( \omega \) the values in Table II are used.

Contrary to a simple sine wave, the incident wave is a composite of several sine waves with Pierson-Moskowitz spectrum [26], [27], i.e.

\[
S(\omega) = 263H_s^2 \omega^{-4} \exp(-1054T_p^{-3} \omega^{-4})
\]

where \( H_s \) is the significant wave height and \( T_p \) is the dominant period.

System performance shown in Figures 2-6 assumes \( T_s = 0.1 \text{ s} \), \( H_s = 1 \text{ m} \), and \( T_p = 12 \text{ s} \). The generator forces of the linear feedback controller and the decentralized MPC are compared in Fig. 2. In the case of the decentralized MPC, the input constraints are satisfied during the entire simulation interval of 50 s. The linear damping feedback controlled force follows the general form of the incident wave.

Fig. 3 depicts a comparison between the absorbed power produced by the linear fixed damping and decentralized MPC controllers. This figure shows that the absorbed power of buoys 1 and 2 is the same when \( \theta = 0 \). Also, peak power absorption of buoy 3 is about 100 kW higher.
Let us now compare how decentralized MPC fares against centralized MPC and linear fixed damping control over a practical range of dominant incident wave frequencies $\omega_p$. Fig. 4 shows that indeed centralized MPC outperforms both decentralized MPC and fixed damping control. However, decentralized MPC is much easier to implement than centralized MPC and it performs better than fixed damping control for the given damping coefficient. This is very important, since power absorption in [18] and [19], which are the basis of our research work, assume a PTO based on linear fixed damping control for each WEC.

Fig. 5 shows system performance when $T_{hor}$ and $T_s$ are varied as follows: Case 2 - $T_{hor} = 6$ s, $T_s = 0.1$ s, Case 3 - $T_{hor} = 6$ s, $T_s = 0.2$ s, Case 4 - $T_{hor} = 8$ s, $T_s = 0.1$ s, and Case 5 - $T_{hor} = 10$ s, $T_s = 0.1$ s. Cases 2 and 3 show the effect of changing the sample time (for the same time horizon of 6 s), whereas cases 4 and 5 show the effect of varying the time horizon (for the same sample time of 0.1 s).

Fig. 6 shows the absorbed energy performance for the incident wave frequency of $\omega = 0.5$ rad/s, where the peak power absorption occurs, and the damping coefficients 0.3, 0.6, 0.9, and 1.2 Newtons/rad/sec, when using linear feedback, centralized, and decentralized MPC. It is clear that the centralized MPC outperforms both decentralized MPC and linear damping control. Finally, the power absorbed performance of our active MPC design is much higher than that of the same uncontrolled WEC array (see Fig. 5 of [18], where the authors use the same parameter values, except $H_s = 2$ meters). After denormalizing their results and accounting for the difference in $H_s$, one can conclude that active control results in considerably increased power generation.
VI. Conclusion

In this paper we investigated the potential of the application of decentralized MPC to an array of 3 wave energy converters to maximize energy absorption. We evaluated system performance in an irregular wave environment using computer simulation. We observed that energy absorption can be increased using active MPC. While our decentralized MPC approach is suboptimal because it ignores the hydrodynamic reaction forces interaction between WECs, it is simple to implement and it is a first good step in demonstrating the practical application of MPC to an array of WECs deployed in a specific configuration. The next logical step in this research would be to incorporate the interactions between WECs and the communication between local MPCs (truly distributed MPC) to optimize system performance.

REFERENCES

VII. Appendix

\[
A_1 = \begin{bmatrix}
\frac{-K\cos^2 \beta + \frac{3\beta}{\alpha} \cos \alpha}{A_h + m} & -B_h & \frac{\sqrt{3}K \cos^2 \beta - \frac{3\beta}{\alpha} \cos \alpha}{A_h + m} & 0 & -\frac{K}{A_h + m} & 0 & 0 \\
\frac{\sqrt{3}K \cos^2 \beta - \frac{3\beta}{\alpha} \cos \alpha}{A_h + m} & 0 & -\frac{3L}{A_h + m} & \frac{\sqrt{3}B}{A_h + m} & 0 & 0 \\
-\frac{K \cos \beta \sin \beta}{2(A_z + m)} & 0 & \frac{\sqrt{3}K \cos \beta \sin \beta}{2(A_z + m)} & 0 & -\frac{K}{A_z + m} & 0 & 0 \\
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
\frac{-B_h}{A_h + m} & 1 & 0 & 0 & 0 & 0 \\
0 & -\frac{B_k}{A_h + m} & 0 & 1 & 0 & 0 \\
0 & 0 & -\frac{3L}{A_h + m} & 0 & 1 & 0 \\
-\frac{K \cos \beta \sin \beta}{2(A_z + m)} & 0 & 0 & 0 & 0 & -\frac{K}{A_z + m} \\
\end{bmatrix}
\]

and

\[
A_3 = \begin{bmatrix}
\frac{-K\cos^2 \beta}{A_h + m} & -B_h & \frac{\sqrt{3}K \cos \alpha}{2L(A_h + m)} & 0 & \frac{K \cos \beta \sin \beta + \frac{3\beta}{\alpha} \sin \alpha}{A_h + m} & 0 \\
0 & 0 & 0 & -\frac{K}{A_h + m} & 0 & 0 \\
0 & 0 & -\frac{L \cos \alpha}{L(A_h + m)} & -B_h & 0 & 0 \\
K \cos \beta \sin \beta & 0 & 0 & 0 & -\frac{K}{A_z + m} & 0 \\
\end{bmatrix}
\]

\[
N_{\text{coupling}} = \begin{bmatrix}
\frac{1}{5} & 0 & -\frac{\sqrt{3}}{\beta} & 0 & \frac{1}{3} & 0 & \frac{\sqrt{3}}{6\alpha} & 0 & -\frac{h}{3L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\sqrt{3}}{\beta} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & -\frac{h}{3L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{h}{6\alpha} & 0 & \frac{\sqrt{3}L}{6\alpha} & 0 & 0 & -\frac{1}{3} & 0 & \frac{h}{3L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]