BUCKLING LOADS OF FIAT SANDWICHPANELS IN COMPRESSION
Buckling of Flat Sandwich Panels withLoaded Edges Clamped and the
Remaining Edges Símply Supported
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No. 1525-C

Buckling of Flat Sandwich Panels with Loaded Edges Clamped and the Remaining Edges Simply Supported ${ }^{2}$
(Cores of End-grain Balsa or Cellular Cellulose Acetate and Facings of Aluminum or Glass Cloth Laminate)

## By

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## Summary and Conclusions

This report presents the results of edgewise compression tests to determine the critical buckling loads of flat sandwich panels, having the loaded edges clamped and the remaining edges simply supported. This investigation of the instability of sandwich panels was conducted at the Forest Products Laboratory to confirm equations (14) and (15) of the theoretical analysis presented in Forest Products Laboratory Report No. 15254 and subsequent modifications that were introduced to take into account the effect of transverse shear and of stresses greater than the proportional limit.
$\underline{1}_{\text {This }}$ report is one of a series of progress reports prepared by the Forest Products Laboratory relating to aircraft. Results here reported are preliminary and may be revised as additional data become available. Original report dated September 1947.
$\underline{2}$ This report is the fourth of a series of reports dealing with the buckling of flat sandwich panels in compression tested with various types of edge conditions.
${ }^{3}$ Maintained at Madison, Wis., in cooperation with the University of Wisconsin.
${ }^{4}$ March, H. W., and Smith, C. B. Buckling Loads of Flat Sandwich Panels in Compression--Various Types of Edge Conditions. Forest Products Laboratory Report No. 1525. March 1945. Information reviewed and reaffirmed 1962.

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The equations, as given in Report No. 1525, are applicable to sandwich panels that are clamped at the loaded edges and simply supported at the remaining edges. They were modified in this study to apply to panels that buckle at facing stresses above the proportional limit and also to take into account the effect of transverse shearing deformations in the core.

Tests were made on sandwich constructions that consisted of cores of either end-grain balsa or cellular cellulose acetate and facings of either aluminum or glass cloth laminate. The results of the tests show the experimental values of critical buckling loads to be approximately equal to the computed values as determined by the modified formulas.

## Introduction

This report is one of a series concerning experimental confirmation of the mathematical analysis that was previously developed ${ }^{4}$ at the Forest Products Laboratory for the critical buckling of flat panels supported by various types of edge conditions and modified to take into account the effect of transverse shear and of stresses greater than the proportional limit. It presents experimental data of edgewise compression tests on the buckling of flat sandwich panels clamped at the loaded edges and simply supported at the other edges. Information relative to materials, testing procedures, and results is arranged according to the outline used to present similar information in Report No. 1525-A. $\frac{5}{\text { Duplications of descriptions have been avoided by omitting them }}$ and referring to previous reports in this series, either Report No. 1525-A 5 or Report No. $1525-\mathrm{B}$. 6 Where variations from the previous reports were necessary, they are noted and fully described. The preparation and manufacture of specimens, the description of the test panels and coupons, and the procedure for determining the critical loads are identical to those described in Report No. $1525-B$. The scope of the work covered by this report and that of the work covered by Report No. 1525-B differ only by the difference in the edge conditions of the panels.

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## Methods of Test

A revision of the test apparatus used in the preceding studies was necessary to support the edges of the panels according to the requirements of this investigation. The loaded edges of the test panels were clamped and the remaining edges simply supported. The simple support was provided by vertical guide posts and rails, as shown in figure 13 of Report No. 1525-A. These fixtures and their function are described in that report.

The loaded edges were clamped by replacing the grooved end rods, rollers, and roller supports, shown in figure 24 of Report No. 1525-B, with the clamps shown in figure 33.7 These clamps consisted of 3 -inch angle irons, $3 / 8$ inch thick, drawn together by bolts passing through the test panel. The angle irons were reinforced between the holes by triangular plates, and surfaced on their faces. They were made in several short lengths, and combinations of them were used to support panels from 5 to 45 inches in width. In fastening the clamps to the panel, the bolts were not drawn tight enough to damage the core material. (The holes in the test panels did not produce sufficient stress concentrations or reductions in cross-sectional area to cause failures within this clamped portion of the panel.) The clamps and the bearing edges of the test panel rested against l/4-inch auxiliary loading plates, which were shimmed wherever necessary to provide uniform bearing.

Figure 34 shows a test panel supported in the apparatus. The vertical guide posts are those shown in figure 11 of Report No. 1525-A, and the loading beams are those shown in figure 26 of Report No. 1525-B.

The shear, flexure, and compression tests made on coupons for the determination of mechanical properties were conducted according to methods described in Report No. $1525-B$.

## Presentation of Data

Experimental data and results of computations are presented in tables 14 through 17 and figures 35 through 38. Most of the symbols used in this report are defined in the preceding reports in this series. Definitions of additional symbols are given in the appendix of this report. The tables are identical in form and have the same column headings as those of Report No. 1525-B. The descriptions of the columns are the same as presented in that report, except

[^1]that some of the values were obtained by methods that were different from those previously used. The variations from the methods used in Report No. 1525-B are as follows:

Column 11.--Computed by the method employed in Report No. 1525-A.
Column 19. --The values in column 19 of this report were computed as follows:
(1) For panels having glass cloth facings (tables 16 and 17)

$$
\begin{equation*}
P_{\text {cr comp }}=\frac{4 \pi^{2}}{a^{2}}\left[\frac{3}{16} D_{1} \frac{b^{2}}{a^{2}}+D_{2} \frac{a^{2}}{b^{2}}+\frac{1}{2}\left(\frac{E \sigma}{\lambda}\right)_{1,2} I+\mu_{1,2} I\right] \tag{1}
\end{equation*}
$$

(2) For panels having aluminum facings (tables 14 and 15)

$$
\begin{equation*}
P_{c r \operatorname{comp}}=\frac{4 \pi^{2}}{a^{2}} \frac{\left(D_{1}+D_{2}\right)}{2}\left(\frac{3 b^{2}}{16 a^{2}}+\frac{a^{2}}{b^{2}}+\frac{1}{2}\right) \tag{2}
\end{equation*}
$$

Column 20.--This column presents the values of the shear reduction ratio, $\frac{1}{1+\eta_{3}}$ in which

$$
\begin{equation*}
\eta_{3} \frac{8}{\left(h^{3}-c^{3}\right)\left[4\left(\mu_{y z}\right)_{c}+3\left(\mu_{x z}\right)_{c}\left(\frac{b^{2}}{a^{2}}\right)\right]} \tag{3}
\end{equation*}
$$

The values of $P_{\text {cr }}$ comp in this formula were computed according to either formula (1) or (2). The values of the shearing moduli that were substituted in this formula were 19,000 pounds per square inch for balsa and 3,500 pounds per square inch for cellular cellulose acetate.

Columns 21 and 22.--The values of M or N were obtained according to the definitions established in previous reports $\underline{5}, \underline{6}$ except that $P_{\text {cr comp }}$ and the constant $\eta_{3}$, in the respective formulas were computed in accordance with formulas (1) or (2) and (3), respectively.

[^2]\[

$$
\begin{align*}
& M=\frac{P}{c r \operatorname{comp}}  \tag{4}\\
& 2 f E_{f}  \tag{5}\\
& N=\frac{\eta_{3}}{E_{f}}
\end{align*}
$$
\]

Column 23. --This column presents the values for the critical loads obtained by the methods described in Report No. $1525-\mathrm{B}$ but using values of $\eta_{3}, \mathrm{M}$, and N as determined by formulas (3), (4), and (5) of the present report.

Figures 35 through 38. --Ratios of observed and computed values (cols. 24 through 26 of the tables) are presented graphically in figures 35 through 38. Figures 35 and 36 present the data obtained from panels having aluminum facings, and figures 37 and 38 present data obtained from panels having glass cloth facings. Figures 35 and 37 show the relationship of the observed values to the values computed by use of formulas (2) or (3) ( $P_{\text {cr comp }}$ ), and figures 36 and 38 show this relationship when the proper modifications of those formulas are employed. The ordinates and abscissas are the same as those used in Report No. 1525-B.

## Analysis of Results

The critical loads, $P_{\text {cr comp, }}$ of the sandwich panels tested are given by equations (14) and (15) of Report No. 1525 under the assumption that the stresses in the facings are less than the proportional limit and that the effect of transverse shear can be neglected. In the present report these equations are modified to apply when these assumptions are not warranted. The application of these modifications results in computed values of critical loads that represent the observed values with reasonable accuracy.

From equations (14) and (15) of Report No. 1525 the value of the critical buckling load for stresses in the facings below the proportional limit when no allowance is made for shear deformations is given by equation (1) for panels having glass cloth facings and equation (2) for aluminum facings. These equations apply to panels whose dimensions are such that they will buckle in a single half wave in the direction of loading.

Values of the elastic properties obtained from tests of coupons of each plate were substituted in equations (1) or (2). The critical loads thus calculated
are presented in column 19 of the tables. The relationship between these values and those observed in the tests is shown in figures 35 and 37 . For the greater values of the coordinates the plotted points fall to the right of the 45degree line indicating that the computed values are greater than the observed and that the modifications of formulas (1) and (2) subsequently described are necessary.

## Modification for Shear

The factor employed to modify formulas (1) and (2) so as to take into account the effect of transverse shearing deformations in the core is $\frac{8}{1+\eta_{3}}$, in which values of $\eta_{3}$ are obtained from equation (3), and $P_{c r}$ comp is obtained from equations (1) and (2).

The values of this factor for those panels that buckled at stresses below the proportional limit are given in column 20 of the tables. They range from 0.77 to 0.99 depending on the materials and dimensions of the panels. The lowest values are for the panels having cellular cellulose acetate cores, because of the low value of the shear modulus of this material. The effect of the use of this factor on the relation between the observed values and the computed values can be seen by comparing figure 37 with 38 . In figure 37 many points fall to the right of the $45-$ degree line, while in figure 38 corresponding points fall approximately on that line.

Modification for Shear and for Stresses

## Above the Proportional Limit

Panels having aluminum facings that buckled at stresses above the proportional limit required a combination of two modifications: (1) to allow for the effect of shear, and (2) to allow for a decrease in the modulus of elasticity of the facings. Both of these modifications are taken into account simultaneously. The graphical method described in Report No. 1525-A is employed using values of $M$ and $N$ determine from equations (4) and (5) of the present report.

The effect of the application of these two modifications to the experimental data can be seen by comparing figure 35 with figure 36. The points in figure 36 , corresponding to those in figure 35 that fall to the right of the 45-degree line, fall approximately on the line.

## Notation

The symbols used in this report are those of Report No. 1525 and supplements $A$ and $B$, with the additional ones redefined for the purposes of the present report listed here.

$$
\begin{aligned}
& \mathrm{M}=\frac{\mathrm{P}_{\mathrm{cr} \operatorname{comp}}}{2 \mathrm{fE} \mathrm{E}_{\mathrm{f}}} \\
& \mathrm{~N}=\frac{\eta_{3}}{\mathrm{E}_{\mathrm{f}}} \\
& \left.\eta_{3}=\frac{24 \mathrm{cfP}}{\left(\mathrm{~h}^{3}-\mathrm{c}^{3}\right)\left[4\left(\mu_{\mathrm{yz}}\right)_{c}+3\left(\mu_{\mathrm{xz}}\right) \mathrm{comp}\right.} \frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}\right] \\
& P_{\text {cr comp }}=\text { critical buckling load as computed by the formula of } \\
& \text { Report No. } 1525 \text { applying to panels with loaded edges } \\
& \text { clamped and remaining edges simply supported (pounds } \\
& \text { per inch of panel width). }
\end{aligned}
$$




[^3]Report No. 1525-C



[^4]Report No. 1525-C





[^5]Report No. $1525-\mathrm{C}$


Figure 33.--Sketch showing the clamping fixtures at the loaded edges of the panel.

ZM 74247 F


Figure 34.--Apparatus for testing sandwich panels having loaded edges clamped and other edges simply supported.

ZM 74249 F


igure 36．－－Observed critical load plotted against the computed critical load（modified for reduced modulus and shear）；both expressed as ratios of the compressive strength of the sandwich con－ struction．Critical buckling of panels faced with aluminum，clamped on the loaded edges and simply supported on the remaining edges．
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Figure 37.--Observed critical load plotted against the computed critical load (no modification); both expressed as ratios of the compressive strength of the sandwich construction. Critical buckling of panels faced with glass cloth laminate, clamped on the loaded edges and simply supported on the remaining edges.


Figure 38. - Observed critical load plotted against the computed critical load (modified for shear); oth expressed as ratios of the compressive atrength of the sandwich construction. Critical buckling of panels faced with glas supported on the remaining edges


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[^0]:    ${ }_{-}^{5}$ Boller, K. H. Buckling Loads of Flat Sandwich Panels in Compression-With Edges Simply Supported. Forest Products Laboratory Report No. 1525-A. February 1947. Information reviewed and reaffirmed 1960.

    - Boller, K. H. Buckling Loads of Flat Sandwich Panels in Compression-With Loaded Edges Simply Supported and the Remaining Edges Clamped. Forest Products Laboratory Report No. 1525-B. September 1947. Information reviewed and reaffirmed 1962.
    Report No. 1525-C.

[^1]:    ${ }^{7}$ The figures and tables in this report are numbered consecutively with those of Report No. 1525, and its supplements A and B.

[^2]:    $\underline{8}_{\text {Bhis }}$ approximate formula was developed by H. W. March at the Forest Products Laboratory.

[^3]:    

[^4]:    

[^5]:    Computed scooraing to formulas in Foreat Producta Laboretory Report 1525.
    ${ }^{\text {Compled }}$.

