

$$\frac{\partial f}{\partial t} = \alpha f \quad (1)$$

with α a stationary random variable with zero mean. The solution is

$$f = f_0 \exp \left[\int_0^t \alpha(t') dt' \right]. \quad (2)$$

Taking the average of Eq. (2),

$$\langle f \rangle = f_0 \exp \left[\frac{1}{2} \int_0^t \int_0^t \langle \alpha(t') \alpha(t'') \rangle dt' dt'' \right]. \quad (3)$$

Now, consider solving Eq. (1) by an iteration process

$$\frac{f}{f_0} = 1 + \int_0^t \alpha(t') dt' + \int_0^t \alpha(t') dt' \int_0^{t'} \alpha(t'') dt'' + \dots \quad (4)$$

Now average Eq. (4). The n th term is

$$\langle f_n \rangle = \left\langle \int_0^t \alpha(t_1) dt_1 \int_0^{t_1} \alpha(t_2) \dots \int_0^{t_{2n-1}} \alpha(t_{2n}) dt_{2n} \right\rangle. \quad (5)$$

If we drew a vertex diagram and counted the number of pair correlations, we would have $(2n-1)!!$ diagrams to evaluate. However, there is an easier way. Consider an $2n$ -dimensional box with sides of length t . It is clear that the integration of Eq. (5) is $1/2n!$ of the full box integration. Thus,

$$\begin{aligned} \langle f_n \rangle &= \frac{1}{2n!} \left\langle \prod_{i=1}^{2n} \int_0^t \alpha(t_i) dt_i \right\rangle \\ &= \frac{(2n-1)!!}{2n!} \left[\int_0^t \int_0^t \langle \alpha(t_1) \alpha(t_2) \rangle dt_1 dt_2 \right]^n \end{aligned}$$

$$= \frac{1}{2^n n!} \left[\int_0^t \int_0^t \langle \alpha(t_1) \alpha(t_2) \rangle dt_1 dt_2 \right]^n.$$

Summing all terms,

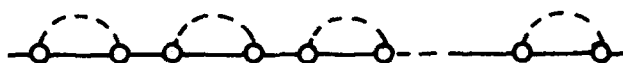
$$\langle f \rangle = f_0 \exp \left[\frac{1}{2} \int_0^t \int_0^t \langle \alpha(t_1) \alpha(t_2) \rangle dt_1 dt_2 \right],$$

in agreement with Eq. (3). Thus, it can be seen that counting diagrams is not the whole story.

Applying this method to the Vlasov equation is more difficult, since the time integration is nonlocal. Note that for the autocorrelation time τ much less than t , Eq. (3) goes over to

$$\langle f \rangle = f_0 \exp[\sigma^2 t],$$

where $\sigma^2 \equiv \int_0^\infty \langle \alpha(t) \alpha(t+t') \rangle dt'$. It can be shown by direct computation that this comes from only one term in the diagram, the nearest neighbor or "first bubble" term, given by



The reason is that any other diagram involves higher orders in τ . Thus, even in the case that Professor Orszag's result is correct, the irreducible diagrams, which form a power series in τ , not t , must be said to have a zero radius of convergence in autocorrelation time.

¹J. J. Thomson and G. Benford, Phys. Fluids 16, 1505 (1973).

Comments on "Variable fluid properties in Bénard convection"

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In a recent note,¹ Davenport ascribes the observation by Caldwell,² in a Bénard experiment on sea water, of critical Rayleigh numbers, R_c , much larger than the value calculated by linear perturbation for a pure fluid with constant properties, to the stabilizing effect of the curvature of the density profile. If the thermal expansion coefficient of a fluid is not constant, a linear temperature profile is accompanied by a nonlinear density profile, which resembles the nonlinear density profiles in experiments conducted with time-varying boundary temperatures, such as those of Davenport and King.³ In those experiments, an increase in critical Rayleigh number with increasing density profile curvature is found, unless an "effective Rayleigh number," Ra_i , in which the depth scale is related to the profile curvature, is used. In his note, Davenport draws on the plot given by Caldwell² of experimental values of R_c vs the fractional change of thermal expansion across the fluid layer, $\Delta\beta/\beta$, a line representing predicted val-

ues of R_c calculated under the assumption that convection begins in the sea water experiments at the same value of Ra_i as in his transient experiments (Fig. 1). The degree of agreement between experiment and this calculation is surprising in view of the fact that the stabilization has since been shown to be completely accounted for by salt stratification caused by Soret transport.

In Caldwell's paper,² both explanations for the seemingly anomalously high R_c were put forth, with no choice being possible between them because values for the Soret coefficient were not available and because no experiments on pure water with large curvature of the density profile had been done. Since then the Soret coefficient has been measured by Caldwell⁴ and quantitative theoretical calculations made by Hurle and Jakeman,⁵ among others, which account quite well for the observations, as shown by Caldwell.⁶ In the latter paper an experiment is described in which the temperature of the top surface of

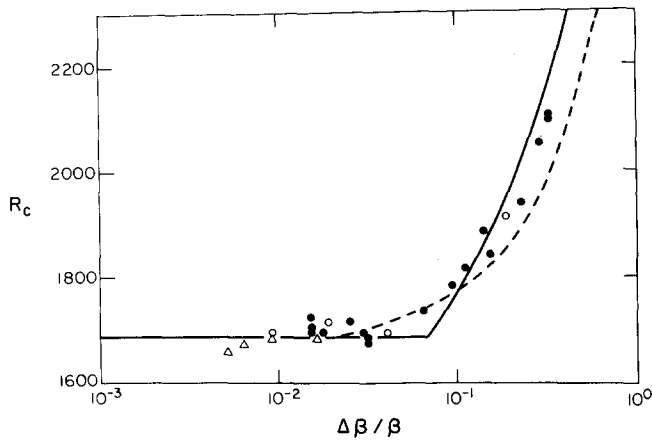


FIG. 1. Critical Rayleigh number R_c vs fractional change of thermal expansion coefficient, $\Delta\beta/\beta$, in a series of Rayleigh-Bénard experiments on sea water at various pressures and temperatures. The bath temperature was \circ 0°C , \bullet 1.7°C , and Δ 17.4°C .

a layer of pure water was 4.6°C . When the surface below the layer was heated to 8.2° , convection began. At this point R_c was observed to be less than 1708, as predicted by Jensen,⁷ although the curvature of the density profile was extreme, $\Delta\beta/\beta$ being 1.48, about five times its largest value in the sea water experiments. Thus, curvature of the density profile cannot be significant in stabilizing the layer, and since the calculations based on Soret-caused stratification quantitatively account for the observations, the salt stratification must be accepted as the significant element.

The question remaining is why such good agreement is found by Davenport between the experimental data and calculations made by assuming the wrong aspect of the experiments to be significant. The answer lies in the circumstance that the Soret coefficient, S_T , of aqueous NaCl solutions is highly correlated with the thermal expansion coefficient. When S_T is large and negative, as it must be to produce stabilization, β tends to be very small. When β decreases, the value of ΔT at a given value of R_c must increase if the other quantities in the experiment are not changed. Thus, $\Delta\beta = (\partial\beta/\partial T)\Delta T$ will increase, and since the Soret effect is stabilizing the layer, R_c also increases, adding to the effect.

The experiments under discussion were run at various pressures between 0 and 300 bars, at temperatures between 0 and 6°C . (Several values are plotted for high

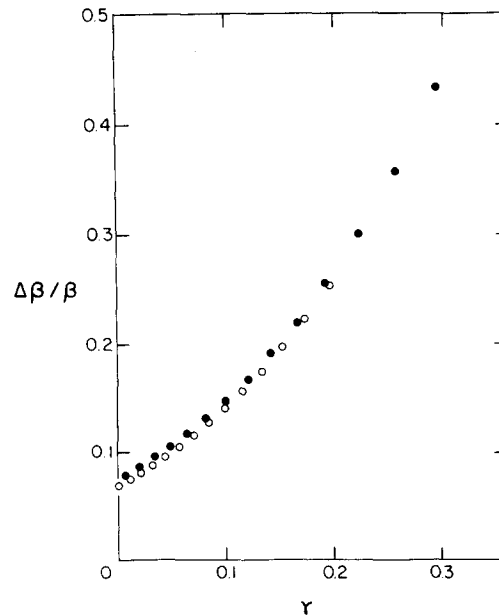


FIG. 2. Correlation between $\Delta\beta/\beta$ and γ at onset of convection assuming sea water as the fluid and Hurlé and Jakeman's theory for the critical Rayleigh number. γ is defined as $S_T C(1-C) \times \beta'/\beta$, where S_T is the Soret coefficient, C the concentration in mass fractions, and β' the saline expansion coefficient.

temperature, but these show no stabilization, as expected.) From Hurlé and Jakeman's theory, in which the Soret stratification is taken into account as the parameter γ , values of $\Delta\beta/\beta$ have been calculated, and are plotted vs γ as Fig. 2. The correlation, almost independent of pressure, is easily seen.

Using these values of $\Delta\beta/\beta$ we can calculate R_c by the Hurlé-Jakeman theory as an (apparent) function of $\Delta\beta/\beta$. This calculation, shown as a dashed line in Fig. 1, is not much different from Davenport's curve, but shows slightly better agreement with the experiments.

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