Stability of accretion disks in binary black hole systems and their effects on merger parameters

By
Jesse Hanson

A THESIS

submitted to

Oregon State University

University Honors College

in partial fulfillment of
the requirements for the
degree of

Honors Baccalaureate of Science in Physics and Applied Computer Science
(Honors Scholar)

Presented May 26, 2017
Commencement June 2017

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Abstract approved:

Davide Lazzati

We analyze merger parameters of binary black hole systems (BBHSs), including orbital separation and time at which gravitational and viscous timescales intersect for all combinations of black holes (BHs) in the range 1-100$M_{\odot}$. Our initial examination of BBHS merger parameters using the assumption $R = 0.5r$ reveals suspicious trends in the data suggesting that this assumption may not be sufficient for all BBHSs. Through our simulation, we determine several stable accretion disk radii which demonstrate a positive linear relationship between the accretion disk radius and the ratio of the mass of the BH with the disk to the total mass of the BBHS. Using this linear trend, we produce a more accurate derivation for the accretion disk radius. Our recalculation of merger parameters using this derivation produces results more typical of what we would expect. The duration of the transient for each BBHS is larger than the transient observed by the Fermi satellite during the event which led to the discovery of gravitational waves. However, these durations fall within the range of observed gamma-ray bursts, and should the solution proposed by Perna *et al.* prove ultimately true, it could change our current understanding of electromagnetic emissions resulting from binary BH mergers.

Key Words: accretion, accretion disks, gamma-ray bursts, black holes, binary black hole systems

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I understand that my project will become part of the permanent collection of Oregon State University Honors College. My signature below authorizes release of my project to any reader upon request.

Jesse Hanson, Author

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1 Introduction

1.1 Motivation and Objective

Since their detection in the late 1960s, gamma-ray bursts have long been one of the biggest mysteries in high-energy astrophysics. While evidence from satellites such as Fermi and Swift suggest these gamma-ray bursts originate from matter collapsing into a black hole (BH), mysteries surrounding these high-energy phenomena continue to arise [5]. Until recently, short gamma-ray bursts (SGRBs) were thought to only occur during the merger of two neutron stars, or a neutron star and a BH [2]. The material provided by the neutron star has long been considered an essential component in powering the burst. However, during the recent discovery of gravitational waves via a merger consisting of two BHs, a SGRB may have also been detected [3]. One possible solution for how this result may have occurred involves a binary black hole system (BBHS) with an accretion disk surrounding one of the BHs, exhibited in Figure 1. The idea is that the material and gases in the neutral fallback disk can survive for a long period of time, until the merger of the two BHs occurs. The material in the fallback disk is revived upon merging with the naked BH, and its accretion results in the SGRB [6].

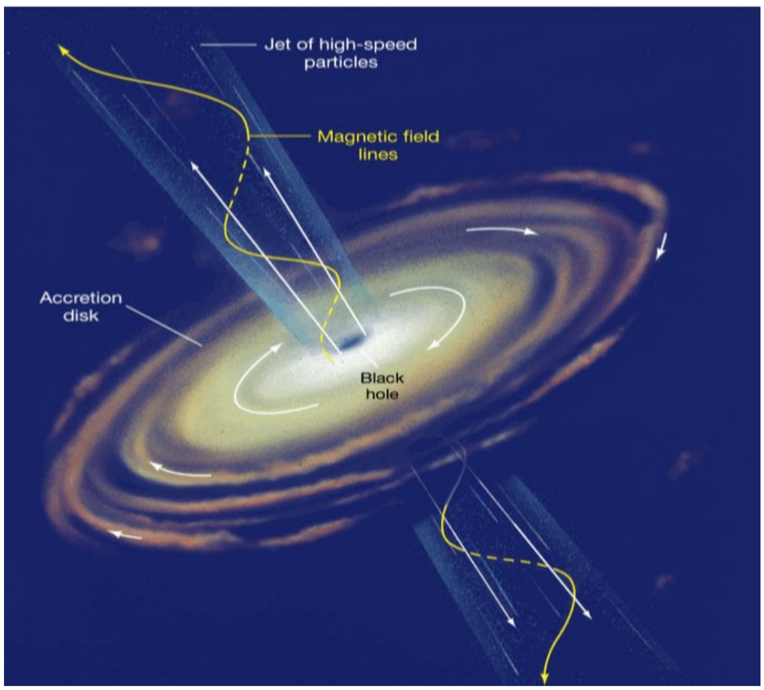


Figure 1: A black hole surrounded by an accretion disk [4].

The BHs used by Perna *et al.* [6] had masses of approximately 30 solar masses ($M_\odot$). Therefore, the purpose of our research is to determine the maximum outer radius for which a stable accretion disk could be formed for all combinations of BBHSs with BHs in the range of 1-100$M_\odot$. Our research explores how these radii effect merger parameters such as the orbital separation of the BBHS and the duration of a transient GRB just before the merger.

1.2 Short Gamma-ray Bursts

A gamma-ray burst is a very brief burst of gamma-ray light lasting anywhere from a couple of milliseconds to a few minutes. There are two different types of gamma-ray bursts, short duration gamma-ray bursts and long duration gamma-ray bursts. For our purposes, we will focus exclusively on SGRBs which can last anywhere from a few milliseconds to two seconds. The formation of these SGRBs appear to be affiliated with the merger of a binary system consisting of either two neutron stars, or a neutron star and a BH [5].

1.3 Accretion Disks

An accretion disk is a circular disk of material orbiting a central body such as a star or other stellar object. The accretion occurs as a result of the gravity of the central body, which causes the material in the disk to spiral inwards [1]. In our model, we assume the temperature of the accretion disk surrounding the BH is low enough that the material essentially freezes minimizing the accretion rate and creating a dead disk that can survive a long period of time. Then, as the BHs spiral inwards approaching a merger, an increase in temperature beginning at the outer rim of the disk will begin to gradually increase the rate of accretion. This condition allows for accretion to occur simultaneously with the merger of the two BHs, without which the possibility for a SGRB would disappear.

1.4 Timescales

For our purposes, a timescale can be thought of as a duration or quantity of time related to an event. We focus on two timescales for our research: gravitational and viscous timescales.

1.4.1 Gravitational Timescale

The gravitational timescale can be thought of as the expected time of merger for our BBHS, and can be written as

$$t_{GW} = \frac{5}{256}\frac{c^5}{G^3}\frac{r^4}{(m_1 m_2)(m_1 + m_2)} s, \tag{1}$$

where c is the speed of light, G is the gravitational constant, r is the orbital separation of the BBHS, m_1 is the mass of the BH surrounded by the accretion disk in grams, and m_2 is the mass of the naked BH in grams.

1.4.2 Viscous Timescale

The viscous timescale can be thought of as the evolution of the accretion disk surrounding one of the BHs in our BBHS. As the two BHs spiral inwards and the orbital separation between them decreases, the disk is forced to adjust. Therefore, the viscous timescale can be thought of as the time it takes the disk to adjust to the incoming naked BH, and is written as

$$t_{visc} = 160\alpha_{-1}^{+1} m_{30}^{\frac{1}{2}} R_{10}^{\frac{3}{2}} \left(\frac{R}{H}\right)^2 s, \tag{2}$$

where R_{10} is equal to radius of the accretion disk R divided by 10^{10}cm, $\alpha_{-1}^{+1} = 1$, $m_{30} = \frac{M_1}{30 M_\odot}$, and the height of the disk $H = \frac{1}{10}R$.

1.4.3 Relationship Between Timescales

How these timescales change as a function of the orbital separation of the two BHs plays an important role in our research, and can be seen in Figure 2. More specifically, we focus on the point at which these two timescales intersect, meaning the time it takes the accretion disk to adjust to the incoming black hole is equivalent to the time until merger. Following the time of intersection, the accretion disk is reactivated and the merger continues according to the gravitational timescale, followed by an electromagnetic emission which occurs according to the viscous timescale.

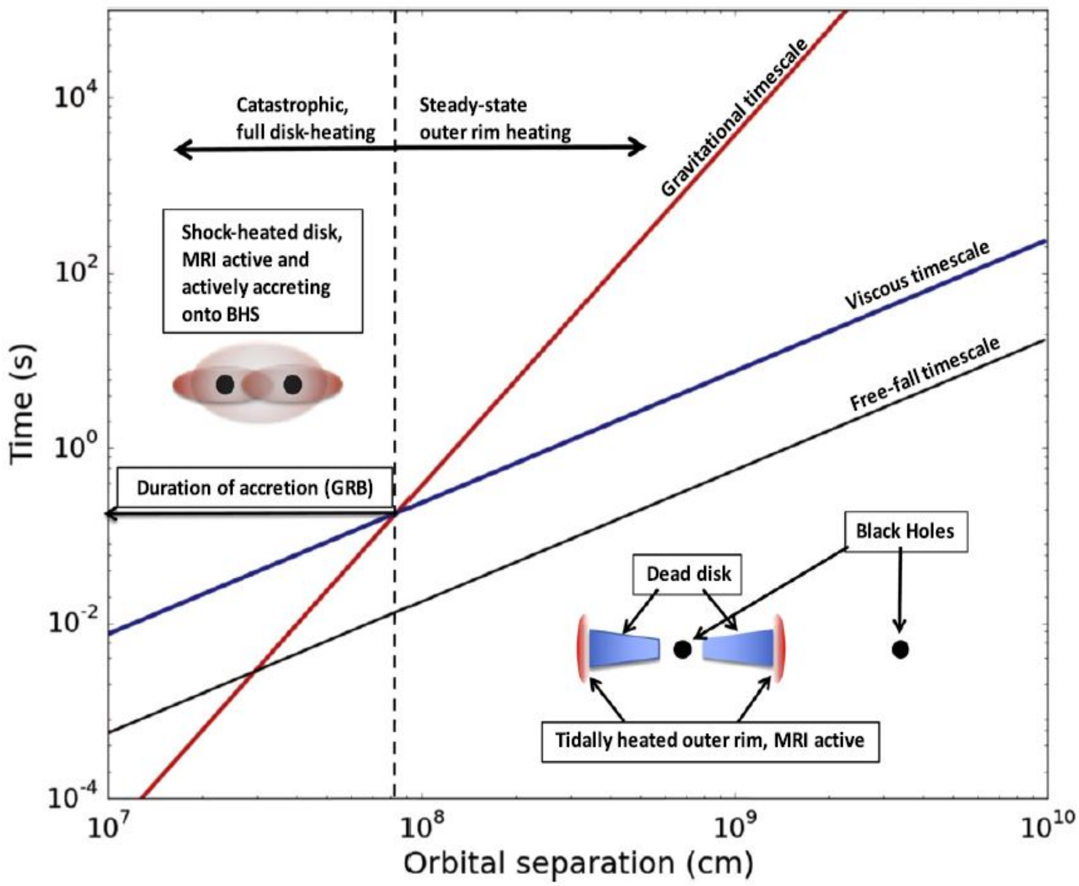


Figure 2: The evolution of the gravitational and viscous timescales as a function of the orbital separation [6].

2 Methods

2.1 Determination of Merger Parameters

The first step in our research is to determine the time and orbital separation at which the gravitational and viscous timescales intersect, as seen in Figure 2, for all combinations of BBHSs with BHs in the range of 1-100$M_{\odot}$. Setting Equations (1) and (2) equal to each other

$$160\alpha_{-1}^{+1} m_{30}^{\frac{1}{2}} R_{10}^{\frac{3}{2}} \left(\frac{R}{H}\right)^2 s = \frac{5}{256}\frac{c^5}{G^3}\frac{r^4}{(m_1 m_2)(m_1+m_2)} s, \tag{3}$$

we can solve for the orbital separation r. We assume the radius of the accretion disk R is equal to half of the orbital separation r. Now,

$$r = \left[4069 \frac{G^3}{c^5}\frac{(m_1 m_2)(m_1+m_2)}{10^{13}}\left(\frac{30M_{\odot}}{M_1}\right)^{\frac{1}{2}}\right]^{\frac{5}{2}} cm, \tag{4}$$

where m_1 is the mass of the BH with the accretion disk in grams, m_2 is the mass of the naked BH in grams, and M_1 is the mass of the BH with the accretion disk in solar masses. Once r has been calculated for each BBHS, it can then be placed into Equations (1) and (2) to verify that t_{GW} and t_{visc} are indeed equal, per our initial conditions. Finally, through the use of a Python program these results can be used to produce three-dimensional plots of the orbital separation and time at which these two timescales intersect for all combinations of BBHSs with BHs in the range of 1-100$M_{\odot}$.

2.2 Establishing Stable Accretion Disk Radii

In the above method, we made the assumption the radius R of the disk is equal to half of the orbital separation r. While this assumption was sufficient in allowing for the production of the aforementioned program, the next step in our research is to derive what fractional form of the orbital separation R will take for each BBHS. Therefore, we need to determine the maximum radius for which an accretion disk can be possible. To do this, we begin by considering a binary system in which two BHs are rotating about one another, and want to determine the equation of motion for a point mass orbiting the BHs. To simplify things, we choose our reference frame to be a rotating reference rotating with the BHs. Moreover, we choose a center of mass reference frame allowing us to describe the position of the particle in relation to the center of mass, as shown in Figure 3. According to Newton's Second Law, the equation of motion for a particle of mass m and position R in a rotating reference frame can be represented as

$$m\ddot{\vec{R}} = \vec{F} + 2m\left(\dot{\vec{R}} \times \vec{\Omega}\right) + m(\vec{\Omega} \times \vec{R}) \times \vec{\Omega}, \tag{5}$$

where the terms on the right side of equation (5) are the sum of forces in the inertial frame, Coriolis force, and centrifugal force, respectively. Applying this equation of motion to the test particle depicted in Figure 3, the sum of the forces on the particle in the inertial frame consist of simply the gravitational forces contributed by each BH.

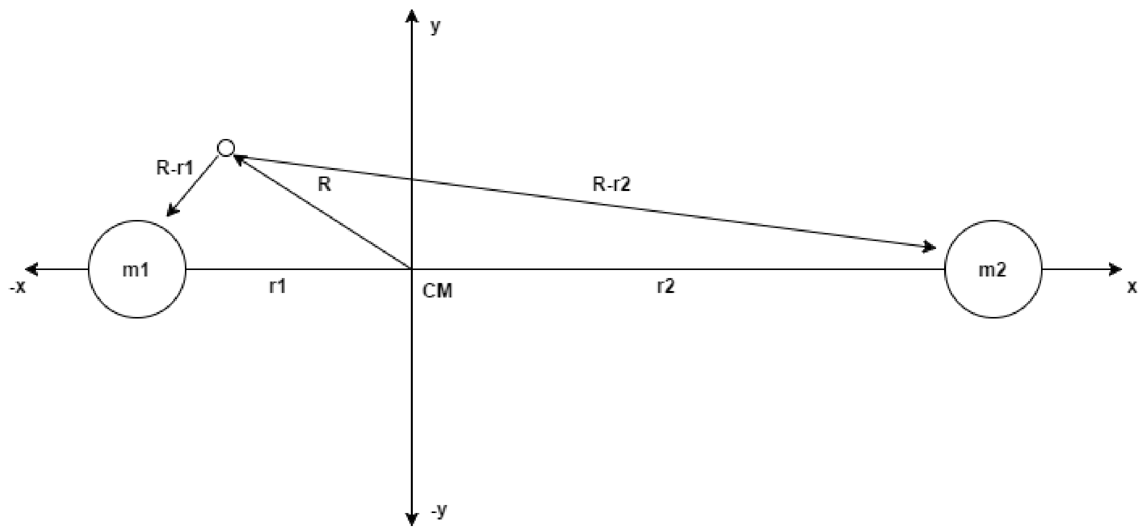


Figure 3: The orbit of the test particle about one BH in a stationary BBHS.

We can now write the equation of motion for the particle as

$$m\ddot{\vec{R}} = \left[-\frac{(Gm_1 m)}{||\vec{r_1} - \vec{R}||^3}(\vec{R} - \vec{r_1}) - \frac{(Gm_2 m)}{||\vec{r_2} - \vec{R}||^3}(\vec{R} - \vec{r_2}) \right] + 2m\dot{\vec{R}} \times \vec{\Omega} + m(\vec{\Omega} \times R) \times \vec{\Omega}, \tag{6}$$

where $\vec{\Omega}$ corresponds to the angular velocity vector. The angular velocity vector makes use of the reduced mass $\mu = (m_1 m_2)/(m_1 + m_2)$ and orbital separation d of the BBHS, and is represented as

$$\vec{\Omega} = \begin{pmatrix} 0 \\ 0 \\ \sqrt{G\frac{m_1 m_2}{\mu d^3}} \end{pmatrix}. \tag{7}$$

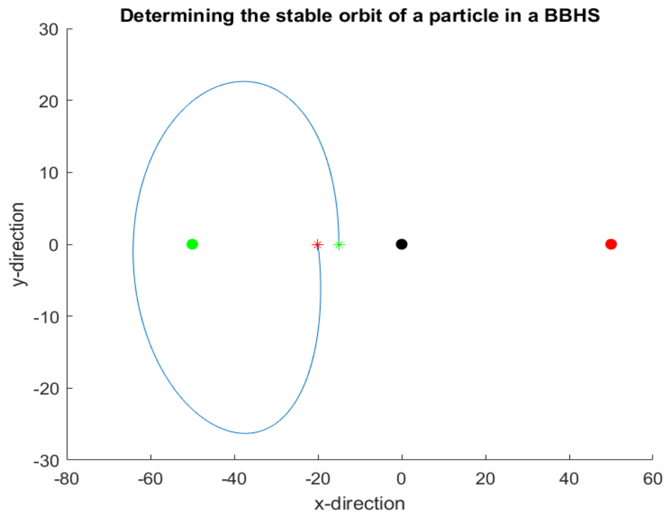
The mass of the particle falls out of equation (6), and the equation of motion can be simplified to

$$\ddot{\vec{R}} = \left[-\frac{(Gm_1)}{||\vec{r_1} - \vec{R}||^3}(\vec{R} - \vec{r_1}) - \frac{(Gm_2)}{||\vec{r_2} - \vec{R}||^3}(\vec{R} - \vec{r_2}) \right] + 2\dot{\vec{R}} \times \vec{\Omega} + (\vec{\Omega} \times R) \times \vec{\Omega}. \tag{8}$$

Given an initial position and velocity, we use Euler's method of numerical analysis to simulate the orbit of the particle. In other words, using the particle's current position and velocity, we calculate the acceleration of the particle according to Equation (8). This acceleration can then be used to calculate the particle's velocity some time step dt later. Similarly, this new velocity can be used to determine the particle's position after some time step dt. The pseudocode below illustrates this process.

```
vx_new    = vx_old + dt*ax;
vy_new    = vy_old + dt*ay;
x_new     = x_old + dt*vx_new;
y_new     = y_old + dt*vy_new;
```

Performing this numerical procedure over several iterations allows us to simulate the full orbit of the particle and observe its behavior. We want to examine complete orbits starting on the horizontal axis near the center of mass of the BBHS, and iteratively move the particle closer to the BH until we achieve an orbit for which an accretion disk could survive. Unfortunately, not all orbits are complete orbits unless the initial position and velocity are correctly provided. For example, Figure 4 demonstrates how the particle's orbit can be effected by an initial velocity that is either too slow or too fast.



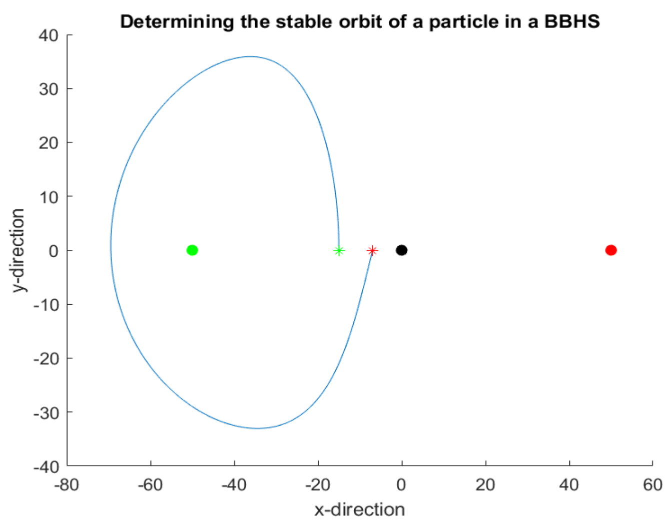


Figure 4: Simulated unstable single orbits of a particle in a BBHS with equal $50M_{\odot}$ BHs where the initial velocity is either too slow (top) or too fast (bottom). The green circle represents the BH with the disk, while the red circle represents the naked BH, and the black circle represents the center of mass of the BBHS. Moreover, the green and red stars represent the particle's starting and ending positions, respectively.

To combat this issue, we created a function called `find_stable_radius()` which takes the distance between the particle's starting and ending positions on the x-axis, and increases or decreases the initial velocity accordingly until the distance between these two locations is minimal. In other words, the function adjusts the initial velocity until the orbit is a complete orbit.

Next, we want to determine whether or not an accretion disk can survive for each complete orbit. In order for an accretion disk to be sustainable, the orbits of its particles should not intersect with neighboring orbits. Therefore, we want to find the maximum outer radius at which a single particle does not interfere with a neighboring orbit. To do this, we move the particle inward along the horizontal axis, keeping track of the x-value at which both the current and previous orbits cross the horizontal axis (180^0 through their orbit). Then, using a function called `find_stable_radius()`, we ensure the current x-value is outside the previous x-value, indicating they have intersected. Once this condition is no longer met, we know the current radius could support an accretion disk. This function notifies the program to record this radius, and stop running. This process is demonstrated in Figure 5.

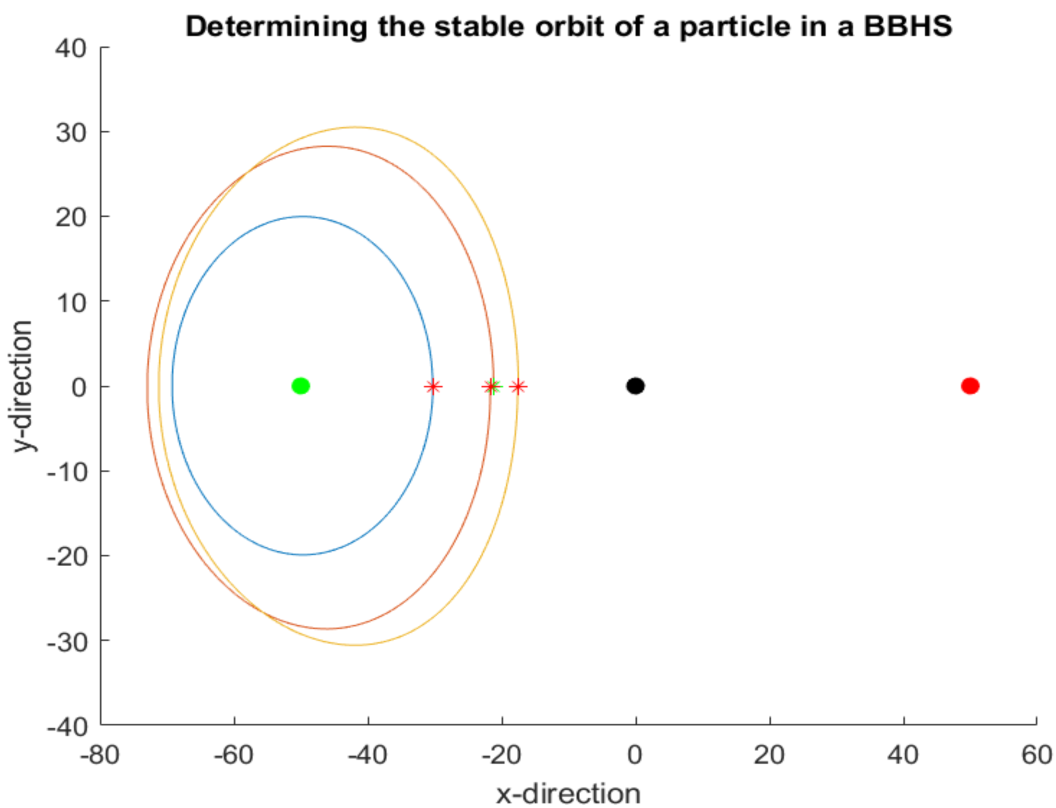


Figure 5: Determining the maximum radius for which an accretion disk could survive in a BBHS with equal 50 solar mass BHs. The green circle represents the BH with the disk, while the red circle represents the naked BH, and the black circle represents the center of mass of the BBHS. Moreover, the green and red stars represent the particle's starting and ending positions, respectively.

We then run this simulation for several combinations of BBHSs with total masses of $100M_{\odot}$, and fit a linear trend-line to the data. Using the equation of this trendline, we are able to determine what fraction of the orbital separation the radius of the disk will be for all combinations of BBHSs with BHs in the range of 1-$100M_{\odot}$. We then recalculate the orbital separation and time at which the gravitational and viscous timescales intersect using these radii as the radius R of the accretion disk. This allows us to provide a more complete picture as to how the size of the BHs involved in the merger of a BBHS effect the accretion disk radius, and in turn the orbital separation and duration of a transient GRB just before the merger.

3 Results

3.1 Merger Parameters

The orbital separation and time at which gravitational and viscous timescales intersect can be seen plotted in Figure 6 for all combinations of BBHSs with BHs in the range of 1-100$M_\odot$ using the assumption $R = 0.5r$. These three-dimensional plots can be read by locating the solar mass of the BH with the accretion disk on the x-axis, the solar mass of the BH without the accretion disk on the y-axis, and the corresponding orbital separation or time on the z-axis. Note the color in these plots represents the z-axis.

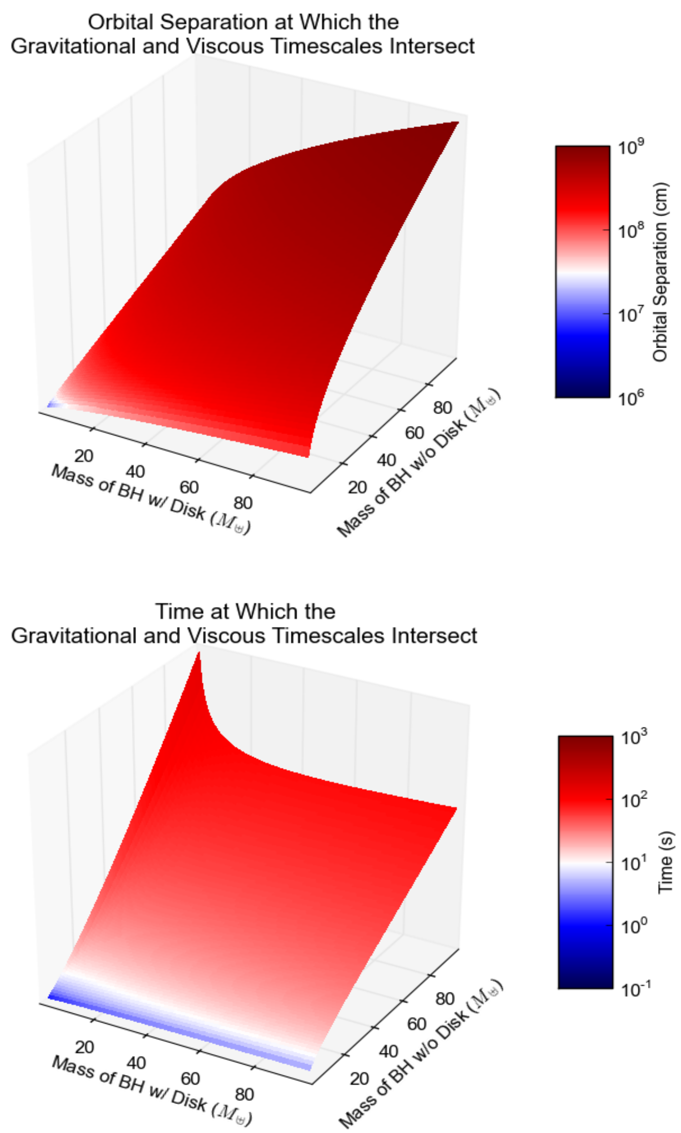


Figure 6: The orbital separation (top) and time (bottom) at which gravitational and viscous timescales intersect for all combinations of BHs in the range 1-100$M_\odot$ using the assumption that the radius of the accretion disk is equal to half of the orbital separation.

3.2 Stable Accretion Disk Radii

Using our simulation, we determine the maximum radius for which an accretion disk can survive for several combinations of BBHSs with total masses of $100M_{\odot}$, the results for which can be seen in Figure 7.

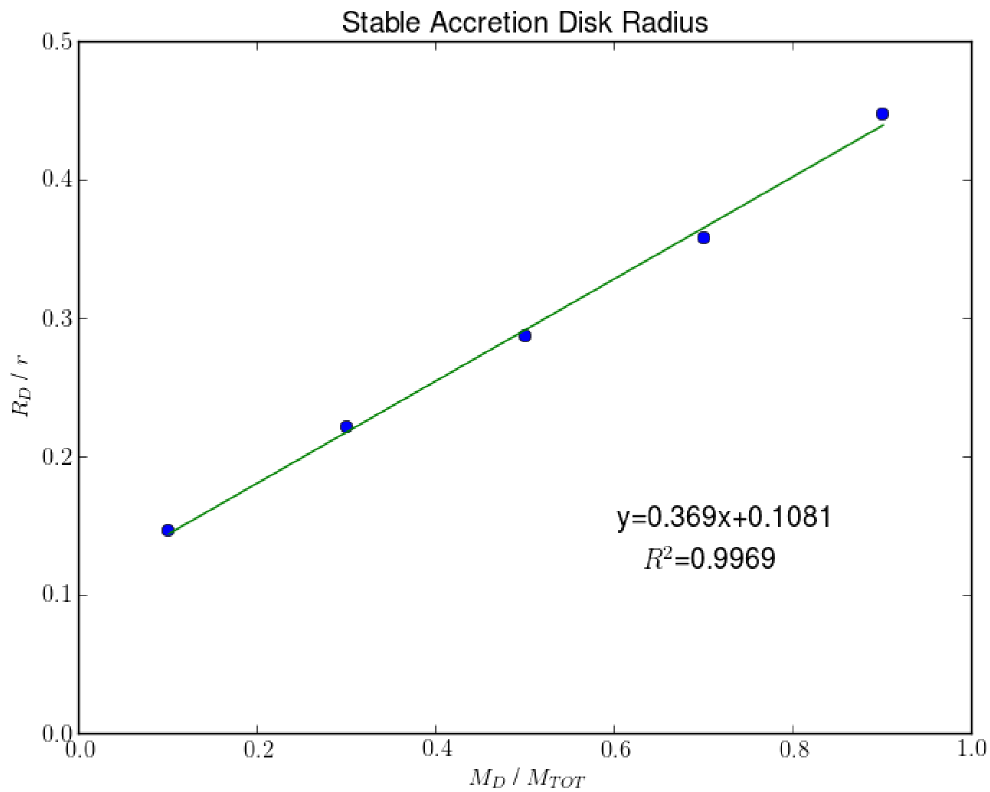


Figure 7: Stable accretion disk radii determined by our simulation for a BBHS with a total mass of $100M_{\odot}$ where R_D is the radius of the accretion disk, r is the orbital separation of the BBHS, M_D is the mass of the BH with the accretion disk, and M_{TOT} is to the total mass of the system.

The data displays a linear relationship which prompts us to fit a linear trend-line to the data, the equation for which can be written as

$$\frac{R_D}{r} = 0.369\left(\frac{M_D}{M_{TOT}}\right) + 0.1081. \tag{9}$$

Solving for R_D, we get

$$R_D = \left(0.369\left(\frac{M_D}{M_{TOT}}\right) + 0.1081\right) r, \tag{10}$$

allowing us to determine a more accurate assumption for the radius of an accretion disk given any BBHS.

3.3 Adjusted Merger Parameters

Using the assumption for the radius of the accretion disk described in Equation (10), we are able to recalculate the orbital separation and time at which gravitational and viscous timescales intersect for all combinations of BBHSs with BHs in the range of 1-100$M_\odot$, the results for which can be seen in Figure 8.

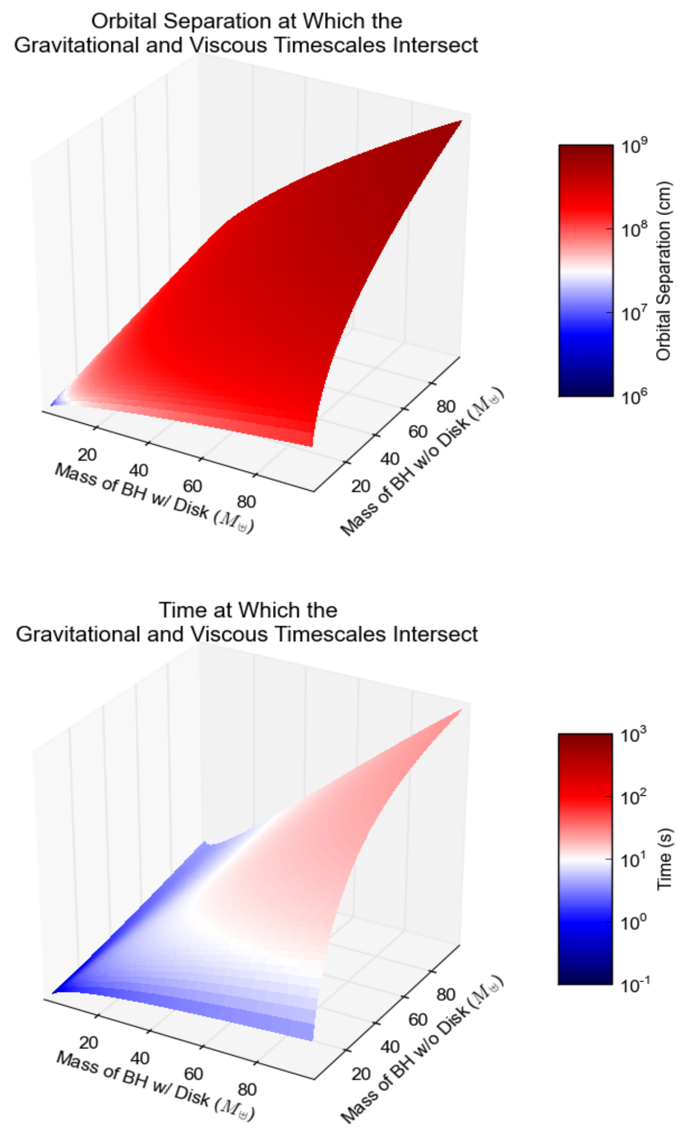


Figure 8: The orbital separation (top) and time (bottom) at which the gravitational and viscous timescales intersect for all combinations of BHs in the range 1-100$M_\odot$ using the assumption the radius of the accretion disk is equal to some fraction of the orbital separation determined from Equation (10).

4 Discussion

The calculations for orbital separation and time using the assumption that $R = 0.5r$ yields some rather suspicious results. In Figure 6, the orbital separation of the BBHS when the two timescales intersect appears roughly proportional to the total mass of the BBHS, which is typically what we would expect. However, the orbital separation for each BBHS has a magnitude of 10^8cm, which is slightly larger than the orbital separation of magnitude 10^7cm determined by Perna *et al.* [6] for a BBHS consisting of two $30M_\odot$ BHs. Alternatively, the time at which the two timescales intersect is almost completely dependent on the mass of the BH without the disk, and nearly independent of the mass of the BH with the disk, except when the mass of the BH with the disk approaches $1M_\odot$. This behavior is unexpected, as we would anticipate the time to increase relatively uniform with respect to the total mass of the BBHS. Moreover, these times - which correspond to the duration of a transient GRB - seem relatively large in comparison to the transient observed after the merger of the BBHS which led to the discovery of gravitational waves, which lasted around $1s$ [3].

In regards to the determination of stable accretion disk radii for all combinations of BBHSs with BHs in the range of 1-$100M_\odot$, our results are sufficient for more accurately determining the orbital separation and time at which the two timescales intersect. The R^2 value of 0.9969 suggests the linear trend-line is a good fit to the data. However, it is important to note our solution assumes the linear analysis of the data in Figure 7 is independent of the total mass and orbital separation of the BBHS.

Finally, using this linear trend-line represented in Equation (10) to determine the radius of the accretion disk for each combination of BBHSs with BHs in the range of 1-$100M_\odot$, we expect to see significant changes in the adjusted plots for orbital separation and time at which the two timescales intersect. According to Figure 7, the original assumption that $R = 0.5r$ is more accurate for BBHSs where the ratio of the mass of the BH with the disk to the total mass of the BBHS is larger. Therefore, we expect to see significant changes in merger parameters for BBHSs where this ratio is small. Evidently, our derivation for the radius of the accretion disk provides this adjustment, as can be seen in Figure 8. While the orbital separation of the BBHS remains roughly proportional to the total mass of the BBHS with magnitudes of 10^8cm, their individual values decrease in a relatively uniform way, with the most significant changes occurring in BBHSs where the ratio of the mass of the BH with the disk to the total mass of the BBHS is small. Moreover, the maximum orbital separation decreases from approximately 9e^{8}cm to around 6e^{8}cm. As for the duration of the transient GRB, our adjustment led to the behavior we initially expected, as the time increases relatively uniform with respect to the total mass of the BBHS. Moreover, their individual values also decrease substantially, with the maximum value going from approximately $140s$ to around $30s$.

5 Conclusion

The recent possible detection of a SGRB in the merger of a BBHS has challenged our current understanding of electromagnetic emissions resulting from binary BH mergers. Through our initial examination of BBHS merger parameters using the assumption $R = 0.5r$, suspicious trends in the data are revealed suggesting this assumption may not be accurate for BBHSs with BHs of varying masses. Through the use of our simulations, we are able to determine several stable accretion disk radii which demonstrate a linear relationship. Using this linear trend, we are able to produce a more accurate derivation of the radius of the accretion disk as a fraction of the orbital separation of the BBHS. Our recalculation of BBHS merger parameters using this derivation produces results more typical of what we would expect. The duration of the transient GRB for each BBHS is larger than the transient observed by the Fermi satellite during the same event which led to the discovery of gravitational waves. However, these durations do fall within the range of observed GRBs, and should this solution proposed by Perna *et al.* [6] prove ultimately true, it could change our current understanding of electromagnetic emissions resulting from binary BH mergers.

5.1 Future Research

Our research provides insight as to how the sizes of the BHs in a BBHS could affect the stability of an accretion disk, and in turn the orbital separation and time at which the gravitational and viscous timescales intersect. Future research might aim to use a better method of numerical analysis, such as Verlet integration, to devise a better derivation for the radius of the accretion disk as a fraction of the orbital separation of the BBHS.

6 Acknowledgements

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