Performance evaluation of conventionally reinforced concrete (CRC) bridge superstructure elements with diagonal cracks is of interest to the bridge engineering community. Standardized methods to predict service-level stress magnitudes in cracked bridge girders under combined bending and shear forces are not available. An analysis procedure was developed to determine the response of CRC bridge girders with existing diagonal cracks. The method estimates the stirrup stress range without prior knowledge of the previous stirrup strain history. Modified Compression Field Theory (MCFT), a sectional analysis procedure, is used in the current specifications to predict capacity of CRC and prestressed concrete beams and relies on equilibrium and compatibility conditions based on an initial uncracked section. In this research, the method was used to predict service-level performance for cracked sections. Effective response prediction of a previously cracked beam was achieved through alteration of the constitutive relationships to permit softening behavior of the concrete to begin much earlier in the loading history. Validation of the procedure was performed through comparison of analytically predicted
stirrup strains with those from full-size laboratory specimens. Empirical data were found to correspond well with the predicted responses. The cracked section analysis procedure was then utilized to estimate the potential for low-cycle fatigue damage of an in-service bridge. A moment and shear interaction surface corresponding to stirrup yielding was estimated and compared with the load effects produced from a data set of over 14,000 permitted trucks to estimate the anticipated life of the bridge based on low-cycle fatigue damage.
Analysis of Diagonally Cracked Conventionally Reinforced Concrete Girders in
the Service Load Range

by
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Melissa J. Robelo, Author
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Analysis of Diagonally Cracked Conventionally Reinforced Concrete Girders in the Service Load Range

Introduction

The AASHTO 2003 LRFD Specification contains a relatively new procedure for design and analysis of reinforced concrete beams and beam-columns for shear. The method, developed by Collins and Vecchio at the University of Toronto is called Modified Compression Field Theory (MCFT). MCFT contains relationships that reasonably predict the behavior of cracked reinforced concrete elements in combined shear and bending [Collins and Vecchio, 1986]. In particular, MCFT relationships tend to agree well with the softening behavior after first cracking and reasonably account for the post-cracking strength contribution of reinforced concrete. MCFT is used in the current specifications to predict capacity of conventionally reinforced concrete (CRC) and prestressed concrete girders based on an initial uncracked condition. However, MCFT does not predict the service level performance of CRC sections starting from a cracked condition. Analysis of previously cracked sections prior to reloading requires that MCFT be adjusted to account for the concrete material reloading at a more softened state than in the initially uncracked condition.

Through analysis of laboratory data of multiple reinforced concrete beam specimens, in which strain gages were used to record stirrup strains during the entire loading history, a factor was derived to modify the value of the cracking stress, thus allowing the softening behavior of the concrete to begin at an earlier loading stage. Using this adjustment factor
for the cracking stress, analysis using MCFT relationships was conducted to produce a moment-shear (M-V) interaction curve at the onset of yielding for the stirrups. This M-V interaction curve was then compared to the ultimate capacity M-V interaction curve produced by Response 2000™ [Bentz, 2000] and the AASHTO-99 MCFT curve. The fraction of ultimate at which yielding occurs was calculated. This relationship may be used, based upon laboratory data, to determine if a reinforced concrete deck girder (RCDG) bridge has experienced loading above the M-V yield surface. The number of loadings above the yield threshold may be calculated based on historical data to produce an estimate of the number of high magnitude events for low-cycle fatigue evaluation. This in turn, may give a much better estimate of the remaining life for a cracked RCDG bridge girder. To demonstrate the use of the M-V yield surface developed from MCFT and the adjustment factor for previously cracked sections, the McKenzie River Bridge was analyzed at the section with the lowest $\beta$, where $\beta$ is the number of standard deviations away from the mean capacity [Daniels, 2004].
Background

The performance of conventionally reinforced concrete (CRC) bridge superstructure elements with diagonal cracks is of interest to bridge engineers. Diagonal cracks may result in the premature failure of the member if web and longitudinal reinforcement are not sufficiently provided. This type of failure is called a shear failure and may occur suddenly in a non-ductile fashion [Collins and Mitchell, 1991].

Cracks form perpendicular to the principal stress trajectories which are dependent on the relative quantities of flexural, shear, and axial load applied to the section (Fig. 1).

![Fig. 1 - Principal stress trajectories [MacGregor, 1997].](image-url)

The principal stress angles can be found from Mohr’s circle for stress and strain (Fig. 2). Given an applied shear stress, $\tau_{xy}$, and normal stresses in the x and y-directions, $\sigma_x$, and $\sigma_y$, respectively, the principal tensile stress, $\sigma_1$, principal compressive stress, $\sigma_2$, and the angle of inclination for the principal stresses, $\theta_p$, can be derived. Principal stresses are calculated as [Ugural and Fenster, 1995]:
Fig. 2 - Mohr's circle of stress.

\[ \sigma_{\text{max,min}} = \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \] \hfill [1]

The angle of inclination of the principal stresses is calculated by:

\[ \theta_p = \frac{1}{2} \tan^{-1} \left( \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right) \] \hfill [2]

Early methods to design reinforced concrete beams in shear were developed by Ritter and Morsch, who used a truss analogy for the internal force carrying mechanisms [Ritter, 1899]. The truss analogy became the basis for the strut-and-tie model and describes the cracked reinforced concrete beam as an equivalent truss where the web is made up of diagonal concrete struts, the stirrups are transverse ties, and the compression zone and flexural steel make up the compression and tension chords (Fig. 3).
The angle for the compression struts has commonly been assumed to be 45°. However, using this steep angle conflicts with many tests and is overly conservative, especially for beams with small amounts of web reinforcement [Collins and Vecchio, 1988]. As a result, the ACI Building code [ACI 318-83] adopted an empirical correction factor to the truss equations to provide better correlation with experiments [Collins and Vecchio, 1988]. However, the ACI approach is known to provide wide scatter between predicted and empirical capacities for shear.

To better characterize the behavior of diagonally cracked reinforced concrete, Collins and Vecchio developed the relationships for MCFT from laboratory experiments as well as solid mechanics principles of equilibrium and material compatibility. The method relates average strains in a cracked section and Mohr’s circle for stress and strain (Fig. 4 and Fig. 5). Relationships for stress and strain were derived from panel testing of large-sized reinforced concrete elements loaded in various combinations of shear and normal stresses (Fig. 6).
Fig. 4 - Average strains of a cracked element [Collins and Mitchell, 1991].

Fig. 5 - Average stresses of a cracked element [Collins and Vecchio, 1988].

Fig. 6 - Panel tests performed by Collins and Vecchio [Collins and Mitchell, 1991].
The stresses and strains were measured across multiple cracks to determine average values for stress and strain across the section. Based on these tests as well as those of other researchers, constitutive relationships were proposed.

The average principal compressive stress in the concrete, based on a parabolic-shaped constitutive relationship, depends on the strain magnitude in the longitudinal direction, $\varepsilon_X$, as well as the principal tensile strain, $\varepsilon_1$. The average principal compressive stress, $f_2$, at a point along the height of the section is:

$$f_2 = f_{2\text{max}} \left[ 2 \frac{\varepsilon_X}{e'_c} - \left( \frac{\varepsilon_X}{e'_c} \right)^2 \right]$$  \hspace{1cm} [3]

where

$$f_{2\text{max}} = \frac{f'_c}{0.8 - 0.34 \frac{\varepsilon_1}{e'_c}} \leq f'_c$$  \hspace{1cm} [4]

which is based on experiments of maximum compressive stress for cracked concrete (Fig. 7). The maximum compressive stress, $f'_c$, is based on cylinder tests at 28 days and $e'_c$ is the corresponding strain.
Alternatively, the average principal compressive stress over the whole section being analyzed, as opposed to at each point along the height of the section, was derived from Mohr's circle as:

$$f_2 = (\tan \theta + \cot \theta) \frac{V}{b_wjd} - f_1$$  \[5\]

where $\theta$ is principal angle of strain, $V$ is the shear force, $b_w$ is width of the web, $jd$ is shear depth of the section, and $f_1$ is principal tensile stress. The recommended relationship between the average tensile stress and the average tensile strain is illustrated in Fig. 8.
The average tensile stress-strain relationship is linear until the principal tensile strain reaches the cracking strain, $\varepsilon_{cr}$. Therefore, if $\varepsilon_1 \leq \varepsilon_{cr}$ then:

$$f_1 = E_c \varepsilon_1$$  \[6\]

where $E_c$ is the modulus of elasticity of the concrete and $\varepsilon_{cr}$ is:

$$\varepsilon_{cr} = \frac{f_{cr}}{E_c}$$  \[7\]

An estimate for the stress at which cracking occurs, $f_{cr}$, is [Collins and Mitchell, 1991]:

$$f_{cr} = 4 \lambda \sqrt{f'_c} \text{ psi}$$  \[8\]

where $\lambda$ is a factor that accounts for the density of the concrete [Collins and Mitchell, 1991]:

$$\lambda = 1.00 \text{ normal-weight concrete}$$
\[ \lambda = 0.85 \text{ for sand-lightweight concrete} \]
\[ \lambda = 0.75 \text{ for all-lightweight concrete} \]

After cracking, \( \varepsilon_1 > \varepsilon_{cr} \), the average tensile stress-strain relationship is a function of the reinforcing bond characteristics, the principal tensile strain, and the cracking stress. However, average tensile stresses will differ with variations of stress along the section, mostly at crack locations. Tensile concrete stresses at the crack approach zero, while the tensile stress in reinforcement crossing the crack increases. The ability of the member to transmit forces across the crack may limit the shear capacity. As a result, a limit on the principal tensile stress is required to maintain equilibrium [Collins and Mitchell, 1991].

The relationship for principal tensile stress after cracking is:

\[ f_1 = \frac{\alpha_1 \alpha_2 f_{cr}}{1 + \sqrt{500 \varepsilon_1}} \leq v_{ci} \tan \theta + \frac{A_y}{s b_w} (f_{yy} - f_y) \tag{9} \]

Where \( \alpha_1 \) is a factor that accounts for the type of reinforcement bond.

\[ \alpha_1 = 1.0 \text{ for deformed reinforcing bars} \]
\[ \alpha_1 = 0.7 \text{ for plain bars, wires, or bonded strands} \]
\[ \alpha_1 = 0 \text{ for unbonded reinforcement} \]

The parameter \( \alpha_2 \) accounts for the type of loading.

\[ \alpha_2 = 1.0 \text{ for short-term monotonic loading} \]
\[ \alpha_2 = 0.7 \text{ for sustained and/or repeated loads} \]

The limiting value of shear stress that can be transmitted along the crack is a function of the concrete compressive strength, \( f'_c \), crack width, \( w \), and the maximum aggregate size, \( a \).

\[ v_{ci} = \frac{2.16 \sqrt{f'_c}}{0.31 + \frac{24 w}{a + 0.63}} \text{ psi and in.} \tag{10} \]
where the crack width is derived as a function of principal tensile strain, \( \varepsilon_1 \), and the maximum diagonal crack spacing.

\[
w = \varepsilon_1 s_m \theta
\]  \[11\]

The maximum diagonal crack spacing, \( s_m \theta \), is a function of the maximum longitudinal crack spacing, \( s_{mx} \), the maximum transverse crack spacing, \( s_{mv} \), and the angle of principal strain, \( \theta \):

\[
s_m \theta = \frac{1}{\sin \theta + \cos \theta} = \frac{s_{mx}}{s_{mv}}
\]  \[12\]

The maximum longitudinal and transverse crack spacing indicates the ability of the longitudinal and transverse reinforcement to control cracking as:

\[
s_{mx} = 2(c_x + \frac{s_x}{10}) + 0.25k_1 \frac{d_{bx}}{\rho_x}
\]  \[13\]

where \( c_x \) is the maximum distance to the reinforcement, \( s_x \) is the spacing of the longitudinal reinforcement, and \( d_{bx} \) is the diameter of the longitudinal reinforcement (Fig. 9). The adjustment factor for bond, \( k_1 \), is 0.4 for deformed bars or 0.8 for plain bars or bonded strands. The longitudinal reinforcement ratio, \( \rho_x \), is:

\[
\rho_x = \frac{A_{sx}}{A_c}
\]  \[14\]

where \( A_{sx} \) is the area of the longitudinal reinforcing steel and \( A_c \) is the gross area of the concrete.

Similarly, the maximum transverse crack spacing is computed as:

\[
s_{mv} = 2(c_v + \frac{s}{10}) + 0.25k_1 \frac{d_{by}}{\rho_v}
\]  \[15\]
where $c_v$ is the maximum distance to the transverse steel, $s$ is the stirrup spacing, $d_{sv}$ is the
diameter of the stirrups, and $\rho_v$ is the transverse reinforcement ratio:

$$\rho_v = \frac{A_v}{b_w s}$$  \[16\]

where $A_v$ is the area cross-sectional area of the stirrups.

**Fig. 9 - Crack spacing parameters [Collins and Mitchell, 1991].**

The resulting shear force is expressed in terms of the concrete and steel contribution for
any angle of principal strain, $\theta$ as:

$$V = \left(\frac{A_v f_y}{s} + f_t b_w\right) j d \cot \theta$$  \[17\]

The principal compressive strain is derived from Eqn. [1] as:
Longitudinal and transverse strains are derived based on Wagner's expression for the angle of inclination for the diagonal compression and Mohr's circle [Wagner, 1929].

\[ \tan^2 \theta = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_l - \varepsilon_2} \]  

From Mohr's circle:

\[ \varepsilon_l = \varepsilon_x + \varepsilon_t - \varepsilon_2 \]

From these two equations, the longitudinal and transverse strain can be found. The longitudinal strain is described by:

\[ \varepsilon_x = \frac{\varepsilon_l \tan^2 \theta + \varepsilon_2}{1 + \tan^2 \theta} \]

and the transverse strain is described by:

\[ \varepsilon_t = \frac{\varepsilon_2 \tan^2 \theta + \varepsilon_l}{1 + \tan^2 \theta} \]

Stirrup stress is computed assuming a bi-linear, elastic perfectly-plastic stress-strain relationship. Therefore, the stress in the stirrups, \( f_s \), is linear until yielding occurs. After reaching the yield strain, \( f_{sy} \), the stress remains constant at yield:

\[ f_v = E_s \varepsilon_t \leq f_{sy} \]

The MCFT can be used in two different ways. Either a detailed dual-sectional analysis or an approximate analysis may be conducted [Collins and Vecchio, 1988]. The detailed analysis (Fig. 10) involves dividing the specimen into sublayers, for which each sublayer must satisfy static equilibrium. For this type of analysis, the principal angle of stress,
which in MCFT is assumed to be equal to the principal angle of strain, is found at each sublayer. As a result, the principal angle of strain varies throughout the section height. The advantage of the detailed sectional analysis is that the results better predict the capacity and behavior, maintaining the compatibility and equilibrium conditions, but also requires more computational resources.

Fig. 10 - Detailed analysis [Collins and Mitchell, 1991].

The approximate method of sectional analysis using MCFT sets the angle of principal strain to be constant throughout the height of the cross-section. Thus every sublayer utilizes the same angle of principal strain in this approximate analysis (Fig. 11).

Fig. 11 - Approximate analysis [Collins and Mitchell, 1991].

Collins and Vecchio (1988) demonstrated that the approximate method using a constant angle of principal strain provides reasonable results compared with the detailed sectional analysis. Example moment and shear interaction diagrams comparing the approximate with the detailed analysis are shown in Fig. 12.
Several computer programs have been developed based on MCFT for analysis of reinforced concrete sections. *Response 2000™* is the most well-known program and was developed by Evan Bentz under the direction of Michael Collins at the University of Toronto [Bentz, 2000]. Previously developed programs include *Shear* (1990) and *Response* (1990). *Shear* calculates the resulting shear force, stress in the stirrups, and the principal stresses based on a value of principal strain and concrete section properties. *Response* does this but also incorporates a greater number of options for sectional analysis. The applied loading is provided as input and the program iterates for the resulting average stresses, average strains, and corresponding average principal angle. It is important to note that these are averages over the entire section as compared to results from *Response 2000™*. *Response 2000™* uses a detailed analysis approach while *Response* uses the approximate sectional analysis. However, as described previously, the approximate analysis approach has been shown to correspond well with the detailed analysis approach.
Analysis Approach

The procedure outlined by Collins and Mitchell (1991) for the programs Shear and Response was used to create a spreadsheet for implementation of MCFT. The spreadsheet results are referred to here as MR. This allowed for a greater flexibility in iterating for solutions using the MCFT relationships. The procedure outlined for Shear was used to calculate the shear force as well as the average stresses and strains across the section by varying the principal tensile strain. The Shear program's output included average stresses and strains including the transverse strain, $\varepsilon_t$, the longitudinal strain, $\varepsilon_l$, the stress in the stirrups, $f_s$, principal compressive stress, $f_2$, principal compressive strain, $\varepsilon_2$, principal tensile stress, $f_1$, and any stresses and strains in prestressing tendons if included. Axial force equilibrium was obtained by iteratively changing the angle of the principal strain. Moment equilibrium was added according to suggestions in Appendix A of Collins and Mitchell (1991). A plane-section analysis method was conducted by numerically integrating concrete sublayers. Any concrete in tension was neglected and concrete in compression was analyzed using the parabolic model (Eqn. 3). Step-by-step implementation of the methodology is illustrated in the appendix.

The output for this spreadsheet was compared with output from Response and with instrumented behaviors for three laboratory specimens: a T-beam with 12 in. stirrup spacing, T12 (Fig. 13 and Fig. 16), an inverted T-beam with 12 in. stirrup spacing, IT12 (Fig. 14 and Fig. 17), and a T-beam with 24 in. stirrup spacing, T24 (Fig. 15 and Fig. 18). The spreadsheet results were found to correspond well with the Response 2000™ capacity
curve, the AASHTO-99 MCFT capacity curve, and the *Response* capacity curve (Figs. 19 - 21).

In general, MR provides results above *Response* and below *Response 2000™*. This was likely due to improved convergence with the spreadsheet implementation, whereby the user, to a greater extent than with *Response*, could control the degree of precision of convergence. The analysis performed at low M/V ratios has been found to provide unreliable results. In addition, the procedure is only valid for monotonic loading and does not allow for unloading and reloading of cracked specimens.

![Fig. 13 - Specimen T12 used for analysis correlation.](image)
Fig. 14 – Specimen IT12 used for analysis correlation.

Fig. 15 – Specimen T24 used for analysis correlation.
Fig. 16 – T12 specimen laboratory setup and crack map.
North End

P/2

7 in

14 in

P/2

44 in

North End

P/2

10@12 in.

4@6 in.

Failure mode: Shear Tension/Anchorage
Peak Load: 358 kips

0.5 in. disp. sensors located from nearest corner of stem.

Note: All equipment is on west side of beam unless otherwise noted.

4 in. disp. sensor

0.5 in. disp. sensor

Strain gage (embedded)

Added strain gage

5 in. string pot (on both sides of beam)

Tilt sensor

Fig. 17 – IT12 specimen laboratory setup and crack map.
Fig. 18 - T24 specimen laboratory setup and crack map.
Fig. 19 - M-V capacity for specimen T12.

Fig. 20 - M-V capacity for specimen IT12.
Amplification Factor for Yielding of the Stirrups

Predicting the moments and shears that cause yielding of stirrups is required to determine the potential for low-cycle fatigue on the stirrups. Steel is fatigued when subjected to repeated yielding. Therefore, it is important to be able to predict the moments and shears that may cause yielding of stirrups in a reinforced concrete girder.

To estimate the onset of yielding in the transverse reinforcement, an amplification factor was used to relate the average transverse strains over the section to the maximum stirrup strain at a diagonal crack location (Fig. 22).
Fig. 22 - Average versus maximum transverse strain.

The MCFT analysis uses the average strains over a section height, which was taken to be the section of concrete between the compression and flexural reinforcement, referred to here as $DV$. The transverse strain distribution would be better represented by a parabolic shape over the actual shear depth, $h$. The amplification factor is derived from simple geometry and calculus. The relationship between the average strain over the whole section and the maximum strain of a corresponding parabolic distribution was found to be:

$$\varepsilon_{\text{parabola max}} = \frac{3}{2} \frac{DV}{h} \varepsilon_{\text{average}}$$  \[24\]

where $h$, assumed to extend from the flexural tensile reinforcement to the edge of the deck/stem interface for a T-section is:

$$h = d - t_{\text{deck}}$$  \[25\]

and $h$ for an inverted T-section, which was assumed to be the distance between the flexural tensile reinforcement and the edge of the compression zone, $a$, is:

$$h = d - a$$  \[26\]
The depth of the compression zone, \( a \), is computed based on an equivalent rectangular stress block as:

\[
a = \frac{f_s A_s}{0.85 f'_c b_w} \tag{27}
\]

where \( f_s \) is the stress in the reinforcement, and \( A_s \) is the cross-sectional area of the reinforcement.

A potential limitation of using this strain amplification approach arises in high moment to shear (M/V) ratios. At high M/V ratios, the strain in the flexural steel at the bottom of the section may increase significantly permitting larger transverse strains at the level of the reinforcement. This may distort the strain distribution. The new shape, containing an offset at the bottom where the flexural strains have increased, causes the real strain distribution to approach the average strain distribution. Therefore, for high M/V ratios, the amplification factor may be less reliable (Fig. 23).

![High Strain Due to Flexure](image)

**Fig. 23 - Average versus maximum strain at high M/V.**
Reduced Cracking Stress

The stress at first cracking used by Response was calculated using Eqn. 8. This is the stress at initial cracking, when concrete material that was previously uncracked begins to crack and thus soften. However, this value of cracking stress no longer functions for the onset of material softening for a concrete member that already contains diagonal cracks. Reloading of a cracked specimen induces material softening behavior once crack surfaces have decompressed. The beam load increases without an increase in strain until a new, reduced, value of $f_{cr}$ is reached. In order to account for this phenomenon, laboratory data from specimens that are typical of Oregon’s 1950’s vintage reinforced concrete deck girder (RCDG) bridges were analyzed. The load versus strain data collected from strain gages placed on stirrups during the tests to failure were used to assess concrete softening behavior. For the laboratory specimens, the value for $\varepsilon_{cr}$ (the onset of material softening behavior on the reloading branch) tended to be in the range between 33 percent and 50 percent of the original $\varepsilon_{cr}$ value with 33 percent being the more typical for strain gages near diagonal cracks. Therefore, cracking stress of 1/3 the initial value was used to analyze the cracked reinforced concrete sections for reloading response prediction (Figs. 24 - 26).
Shear vs. Stirrup Strain @ 4' from Support
Specimen 2T12, stirrup spacing=12"
North Side

Fig. 24 - Cracking strain, $\varepsilon_{cr}$, determination of specimen 2T12 north.

Shear vs. Stirrup Strain @ 4' from Support
Specimen 2T12, stirrup spacing=12"
South Side

Fig. 25 - Cracking strain, $\varepsilon_{cr}$, determination of specimen 2T12 south.
Fig. 26 - Cracking strain, $\varepsilon_{cr}$, determination of specimen 21T12 south.
Results

$V-\varepsilon_t$ Prediction

Laboratory data were employed to verify the validity of using reduced cracking stress for analyzing the effects of reloading a cracked reinforced concrete section. Three laboratory specimens were evaluated that were characteristic of vintage cracked 1950's RCDG Oregon bridges. Two T-beams with stirrup spacings of 12 in. and 24 in. as well as one inverted T-beam (IT) with stirrups spaced at 12 in were investigated. The softened cracking stress was taken as one-third of the original cracking stress, as discussed previously. The results of this analysis corresponded well with the reloading branches of the laboratory specimens demonstrating that the reduced cracking stress provided a suitable modification to model the reloading behavior of cracked concrete specimens (Fig. 27 - 46). The model for reloading based on MCFT, adjusted with a reduced cracking stress, can be seen as the dashed curves on each plot. In some cases, the curve has been shown both at the origin and offset to a distance where softening behavior is more readily evident.

In addition, the amplification factor from Eqn. 24 was applied to the model based on MCFT (spreadsheet MR) and compared with the laboratory data. In general, the results showed that the amplified models corresponded more reasonably to the initial loading branch, while the unamplified model provided a better correlation with the softened reloading branches after cracking. A possible explanation is that the stirrups, on initial loading, are fully bonded. Therefore, the strains vary locally throughout the height of the
stirrup, reaching maximum magnitude at the crack location. This corresponds well with the purpose of the amplification factor, which is to modify the average transverse strain to account for the maximum transverse strain over the section. On the reloading branch after cracking, the concrete softening results in diminished bond to the steel. With less bond to cause local variations in strain, the strain over the stirrup height approaches the average. Stirrup strains for a fully unbonded stirrup are constant (average) over the stirrup height. Consequently, the strain in the stirrup for the reloading branch is predicted more realistically by the average strain over the section height (the unamplified model) due to reduced bond between the softened concrete and the stirrup.

One additional factor affecting the degree of correlation between the data and the MCFT model (spreadsheet MR) is the location of the strain gage relative to the crack(s). The model assumes that the strain is measured at the crack location. Comparing the model predictions with a strain gage located at the crack produces good results. Conversely, the comparison of the model predictions with a strain gage located a distance away from the nearest crack shows results that are not as well correlated. An example of this can be seen in Fig. 33.
Shear-Stirrup Strain @ M/V=4 and S=12"
3T12 Precrack
South Side

Fig. 27 - V-εₜ at south side, M/V=4 for specimen 3T12.

Fig. 28 - Strain gage location relative to crack at south side, M/V=4 for specimen 3T12.
Shear-Stirrup Strain @ M/V=4 and S=12''
2T12
South Side

Fig. 29 - V-ε, at south side, M/V=4 for specimen 2T12.

Fig. 30 - Strain gage location relative to crack at south side, M/V=4 for specimen 2T12.
Shear-Stirrup Strain @ M/V=4 and S=12"  
2T12  
North Side

Fig. 31 - V-ε1 at north side, M/V=4 2T12 North

Fig. 32 - Strain gage location relative to crack at north side, M/V=4 for specimen 2T12.
Shear-Stirrup Strain @ M/V=4 and S=12"
2IT12
South Side

Fig. 33 - V-ε_t at south side, M/V=4 for specimen 2IT12.

Fig. 34 - Location of strain gage at south side, M/V=4 for specimen 2IT12.
Shear-Stirrup Strain @ M/V=4 and S=24"
10T24
South Side

Fig. 35 - V-ε_t at south side, M/V=4 for specimen 10T24.

Fig. 36 - Location of strain gage at south side, M/V=4 for specimen 10T24.
Shear-Stirrup Strain @ M/V=6 and S=12"
2T12
North Side

Fig. 37 - V-ε₁ at north side, M/V=6 for specimen 2T12.

Fig. 38 - Strain gage location relative to crack at north side, M/V=6 for specimen 2T12.
Shear-Stirrup Strain @ M/V=6 and S=12"
2T12
South Side

Fig. 39 - V-\(\varepsilon_1\) at south side, M/V=6 for specimen 2T12.

Fig. 40 - Strain gage location relative to crack at south side, M/V=6 for specimen 2T12.
Shear-Stirrup Strain @ M/V=6 and S=12"

2IT12
South Side

Fig. 41 - V-ε_t at south side, M/V=6 for specimen 2IT12.

Fig. 42 - Location of strain gage at south side, M/V=6 for specimen 2IT12.
Shear-Stirrup Strain @ M/V=6 and S=12"  
2IT12  
North Side

Fig. 43 - V-\varepsilon_t at north side, M/V=6 for specimen 2IT12.

Fig. 44 - Location of strain gage at north side, M/V=6 for specimen 2IT12.
Shear-Stirrup Strain @ M/V=6 and S=24"
10T24
North Side

Strain Gage
yield strain
MR
MR amplified by 1.5
MR ε amplified by 3*DV/(2h)=1.63
MR ε=1/3ε_cr_original
MR ε=1/3ε_cr_original offset (for demonstration)

Fig. 45 - V-ε, at north side, M/V=6 for specimen 10T24.

Fig. 46 - Location of strain gage at north side, M/V=6 for specimen 10T24.
M-V Interaction Yield Surfaces

Low-cycle fatigue of reinforced concrete girders may occur when load effects (M and V) combine to produce yielding of stirrups. The moment-shear (M-V) interaction yield surface is needed to perform a low-cycle fatigue evaluation. Using the MCFT with the adjustment factor for cracking stress, as well as an amplification factor to achieve the maximum strain in the section based on the average strains, an M-V interaction curve at yielding of the stirrups was developed. The yield M-V interaction diagrams were developed for the three laboratory specimens and two critical sections of the McKenzie River Bridge, located in Lane County, Oregon. The ratio between yielding and capacity values were calculated with respect to the AASHTO-99 MCFT capacity curve. Yielding was conservatively based on amplified strains corresponding to well bonded stirrups. The amplified yield curve exhibited lower moments and shears to produce the yield condition than that based on average strains.

Through analysis of the M-V interaction yield surfaces, it was found that for higher M/V ratios, there is a point at which the stirrups do not yield before capacity is reached for the section. As a result, sections at high M/V ratios do not appear to be prone to low-cycle fatigue.

Laboratory Specimens

The three laboratory specimens analyzed were the T12, IT12, and T24. On average, the shear and corresponding moment to cause yielding in the stirrup was approximately 67% of the ultimate capacity as predicted by the AASHTO-99 MCFT for the T12 specimen for M/V ratios of 3 to 14, approximately 69% for IT12 for M/V ratios of 3 to 10, and for T24 was about 63% for M/V ratios
from 3 through 20. These results are displayed in Tables 1 to 3, and graphically displayed in Figs. 47 to 49.

**Fig. 47 - M-V yield surface and predicted capacities for specimen T12.**

**Table 1 - Shear to produce yield as % of AASHTO-99 MCFT for specimen T12.**

<table>
<thead>
<tr>
<th>M/V</th>
<th>Moment at $V_y$ (k-ft)</th>
<th>$V_y$, kips</th>
<th>% $V_n$ by AASHTO-99 MCFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>80</td>
<td>44%*</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>100</td>
<td>55%*</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>133.3</td>
<td>73%</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>125</td>
<td>70%</td>
</tr>
<tr>
<td>6</td>
<td>625</td>
<td>104</td>
<td>61%</td>
</tr>
<tr>
<td>8</td>
<td>800</td>
<td>100</td>
<td>61%</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
<td>100</td>
<td>64%</td>
</tr>
<tr>
<td>12</td>
<td>1200</td>
<td>100</td>
<td>68%</td>
</tr>
<tr>
<td>14</td>
<td>1350</td>
<td>96</td>
<td>69%</td>
</tr>
</tbody>
</table>

*Low M/V values not well predicted by method.
Fig. 48 - M-V yield surface and predicted capacities for specimen IT12.

Table 2 - Shear to produce yield as % of AASHTO-99 MCFT for specimen IT12.

<table>
<thead>
<tr>
<th>M/V</th>
<th>Moment at $V_{xy}$ (k-ft)</th>
<th>$V_{xy}$ kips</th>
<th>% $V_{xy}$ by AASHTO-99 MCFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>120</td>
<td>65%*</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>150</td>
<td>81%*</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>133.3</td>
<td>72%</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>125</td>
<td>69%</td>
</tr>
<tr>
<td>6</td>
<td>700</td>
<td>116.7</td>
<td>68%</td>
</tr>
<tr>
<td>8</td>
<td>900</td>
<td>112.5</td>
<td>68%</td>
</tr>
<tr>
<td>10</td>
<td>1100</td>
<td>110</td>
<td>70%</td>
</tr>
</tbody>
</table>

*Low M/V values not well predicted by method.
Fig. 49 - M-V yield surface and predicted capacities for specimen T24.

Table 3 - Shear to produce yield as % of AASHTO-99 MCFT for specimen T24.

<table>
<thead>
<tr>
<th>M/V</th>
<th>Moment at $V_{ys}$ (k-ft)</th>
<th>$V_{ys}$ kips</th>
<th>% $V_n$ by AASHTO-99 MCFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>250</td>
<td>83.3</td>
<td>60%</td>
</tr>
<tr>
<td>4</td>
<td>350</td>
<td>87.5</td>
<td>64%</td>
</tr>
<tr>
<td>6</td>
<td>500</td>
<td>83.3</td>
<td>63%</td>
</tr>
<tr>
<td>8</td>
<td>625</td>
<td>78.1</td>
<td>62%</td>
</tr>
<tr>
<td>10</td>
<td>740</td>
<td>74</td>
<td>61%</td>
</tr>
<tr>
<td>12</td>
<td>875</td>
<td>72.9</td>
<td>62%</td>
</tr>
<tr>
<td>14</td>
<td>1000</td>
<td>71.4</td>
<td>63%</td>
</tr>
<tr>
<td>20</td>
<td>1400</td>
<td>70</td>
<td>69%</td>
</tr>
</tbody>
</table>
The two sections of the McKenzie River Bridge analyzed were an IT and T beam, sections A and B (Fig. 50) with stirrups spaced at 9 in. and 12 in., respectively (Fig. 51 and Fig. 52). The McKenzie River Bridge section tended to yield at higher percentages of the AASHTO-99 MCFT ultimate capacity than the laboratory specimens. For \( \frac{M}{V} \) between 1 and 4, the McKenzie River Bridge section A yielded on average approximately 81% of AASHTO-99 MCFT capacity while for the same range of \( \frac{M}{V} \) ratios, section B on average yielded about 86% of ultimate capacity predicted by AASHTO-99 MCFT. This data can be seen in Tables 4 and 5, and is plotted in Figs. 53 and 54.

---

**Fig. 50** – Sections A and B from McKenzie River Bridge.
Fig. 51 - McKenzie River Bridge at section A.

Fig. 52 - McKenzie River Bridge at section B.
Fig. 53 - M-V yield curve for McKenzie River Bridge girder section A.

Table 4 - Shear to produce yield as % of AASHTO-99 MCFT for McKenzie River Bridge girder section A.

<table>
<thead>
<tr>
<th>M/V</th>
<th>Moment at $V_y$ (k-ft)</th>
<th>$V_y$, kips</th>
<th>% $V_y$ by AASHTO-99 MCFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>115</td>
<td>115</td>
<td>81%</td>
</tr>
<tr>
<td>2</td>
<td>228</td>
<td>114</td>
<td>80%</td>
</tr>
<tr>
<td>3</td>
<td>340</td>
<td>113.3</td>
<td>80%</td>
</tr>
<tr>
<td>4</td>
<td>440</td>
<td>110</td>
<td>84%</td>
</tr>
</tbody>
</table>
As can be seen from comparing the yielding shear level with the capacity, the laboratory specimens yielded between approximately 60% and 73% of the nominal AASHTO-99 MCFT capacity, while the sections analyzed for the McKenzie River Bridge yielded between about 80% and 89% of the nominal AASHTO-99 MCFT capacity.
The load effects produced by service level trucks combined with dead load at the sections of interest were determined and compared with the predicted yield shear force and moment. To estimate service level truck loads at the McKenzie River bridge, weigh-in-motion data was used which had been collected for southbound truck traffic on interstate highway I-5 during the period of one year, that being 2003. The truck data was collected at the Wilbur weigh station, located just North of Roseburg, OR. In total, 900,000 trucks passed through Wilbur weigh station going southbound in 2003 [Daniels, 2004]. Of those, approximately 14,000 trucks were classified by ODOT as being permit table 3, 4, and 5 vehicles [Daniels, 2004], plotted in Fig. 55 and Fig. 56. These are vehicles that carry loads which exceed the legal limits. Load effects calculated using AASHTO LRFD live load distribution factors combined with AASHTO impact factors [Daniels, 2004] caused some trucks to induce yielding of the stirrups. For the location of section A, approximately 5 of the 14,000 permit table 3, 4, and 5 trucks caused yielding in the stirrups.
Fig. 55 - McKenzie River Bridge at section A using AASHTO LRFD LL distribution factors.

Fig. 56 - McKenzie River Bridge at section B using AASHTO LRFD LL distribution factors.
This procedure may be used to determine the yield moments and shears for any bridge. However, the iterative process of finding these moment and shear combinations may be bypassed if conservatively one were to choose a percentage of yielding to use as an initial screening for possible low-cycle fatigue damage accumulation. Additional analyses are needed for other bridge spans and indeterminacies to develop a reliable recommendation, but based on the analyses performed, yielding occurs between 60% and 89% of ultimate. A percentage within this range may be acceptable for roughly estimating low-cycle fatigue potential.
Conclusions

The Modified Compression Field Theory (MCFT) is currently used in the AASHTO LRFD specification to compute the shear capacity of reinforced concrete and prestressed beams and beam-columns. It uses strain compatibility and equilibrium to predict the capacity of sections under combined flexure and shear. Currently, the method is used to analyze beams assuming an initial uncracked condition. However, analysis of cracked sections is needed to evaluate potential for LCF in bridge girders.

MCFT was used to predict the behavior of girders with existing diagonal cracks. The constitutive relationships were modified to realistically predict stirrup strains for girders in the previously cracked condition. Analyses were performed for laboratory specimens and the method was applied to an existing bridge with diagonal cracks. Based on these analyses, the following conclusions are presented:

- Stirrup strain behavior for reloading of previously cracked girders can be reasonably predicted by MCFT and incorporating an adjustment factor to the cracking stress, $f_{cr}$.
- Transverse strains in the cross-section were amplified to account for higher stresses in the stirrups at diagonal cracks to provide a better correlation with experimental results.
- For high $M/V$ ratios, there is a limit at which the stirrups do not yield when the section reaches capacity. There is no risk for low-cycle fatigue at these sections.
- For very low $M/V$ ratios, the MCFT does not predict the shear and moment capacity very well.
• Assessment of low-cycle fatigue for a bridge requires comparison of the load effects from service level trucks and dead load with the yielding M/V interaction curve of the section. The analysis technique described provides a method to predict this threshold.
References


ACI Committee 318-83. (1983). *Building Code Requirements for Structural Concrete*. American Concrete Institute, Detroit, Michigan.

ACI Committee 318-02. (2002). *Building Code Requirements for Structural Concrete*. American Concrete Institute, Farmington Hills, Michigan.


Appendix
MCFT Procedure
Following is the algorithm for shear, moment, and axial load as outlined by Collins and Mitchell in *Prestressed Concrete Structures* (1991) for the program *Shear* (1990).

**LOOP FOR ITERATING ON $\varepsilon_1$**

Step 1: Choose a value of $\varepsilon_1$ at which to perform the calculations.

**LOOP FOR ITERATING ON $\theta$**

Step I: Estimate $\theta$

Step II: Calculate the crack width, $w$, and the shear along the crack, $v_	ext{cr}$

**LOOP FOR ITERATING ON VERTICAL EQUILIBRIUM**

Step a: Estimate $v_1$

Step b: Calculate the principal stress $f_1$

Step c: Calculate the shear, $V$, as a resultant of the shear stress.

Step d: Calculate the average compressive stress, $f_2$

Step e: Calculate $f_{2\text{max}}$, the maximum compressive stress

Step f: Check that $f_2 \leq f_{2\text{max}}$. If $f_2 > f_{2\text{max}}$, the concrete crushes

and the solution is not possible. Return to Step 1 and choose a
smaller $\varepsilon_1$. 

**Step g:** Calculate compressive strain, $\varepsilon_2$

**Step h:** Calculate the average longitudinal and transverse strains, $\varepsilon_x$ and $\varepsilon_y$.

**Step i:** Calculate the stress in the stirrup utilizing the transverse strain.

**Step j:** Check if the stirrup stress matches the initial guess for the stirrup stress. If the values do not match, return to Step a with an improved estimate of the stress in the stirrup.

_END LOOP FOR ITERATING ON VERTICAL EQUILIBRIUM_

**Step III:** Calculate the axial force, $N_v$, on the section resulting from

the concrete stresses in the effective shear area, $b_wjd$, and the moment caused by $N_v$.

$$N_v = \frac{-V}{\tan \theta} + f_1 b_v d_v \tag{A1}$$

$$M_v = N_v y \tag{A2}$$

where $y$ is the distance from the center of the effective shear area to the moment axis.
Step IV: Using a plane-section analysis procedure, find the stress
resultants for the section corresponding to the longitudinal strain,
\( \varepsilon_x \), and a curvature, \( \phi \), that corresponds with the applied moment
and longitudinal strain.

\[ N_p = f(\varepsilon_x, \phi) = N_{gross} - N_{b_{w,jd}} \tag{A3} \]

\[ M_p = f(\varepsilon_x, \phi) = M_{gross} - M_{b_{w,jd}} \tag{A4} \]

Step V: Calculate the axial resultant and moments

\[ N = N_p + N_v \tag{A5} \]

\[ M = M_p + M_v \tag{A6} \]

Step VI: Change the principal angle of strain \( \theta \) until the axial
force resultant equals the applied axial load.

\textit{END LOOP FOR ITERATING ON } \theta

Step VII: Check that the steel demand (right side of inequality) has not exceeded its strength (left side of inequality).

\[ \sum_{i=1}^{n} \left( f_{sy} - f_{si} \right) A_{si} + \sum_{i=1}^{m} \left( f_{pu} - f_{pi} \right) A_{pi} \geq f_1 b_{w,jd} + \left( f_1 - \frac{A_v}{b_{ws}} \right) \left( f_{vy} - f_v \right) \frac{b_{w,jd}}{\tan^2 \theta} \tag{A7} \]
Step 2: Verify that the applied shear force equals the resultant shear force. If the two are not close enough, revise the estimate of $\varepsilon_1$.

*END LOOP FOR ITERATING ON $\varepsilon_1$*