AN ABSTRACT OF THE THESIS OF

Andrew P. Lennette for the degree of Master of Science in Forest Resources presented on July 9, 1999.

Title: Twenty-five-year Response of Larix occidentalis Stem Form to Five Stand Density Regimes in the Blue Mountains of Eastern Oregon.

Abstract approved: Signature redacted for privacy.

Douglas A. Maguire

Upper-stem diameter measurements from a levels-of-growing-stock study on Larix occidentalis in the Blue Mountains of eastern Oregon were collected over a 25 year period with an optical dendrometer. The following hypotheses were tested with these data: 1) stand density regime has a significant effect on stem form; and 2) crown size controls the response of stand density regime to stem form. The five stand density regimes corresponded to five different growing-stock levels (GSLs) defined by bole surface area. A variable exponent taper model was fit to the data from each individual tree and the resulting parameters were treated as indices of stem form for comparisons among density regimes. MANOVA performed on these parameters estimates confirmed statistically significant differences in stem form both between the two lowest levels-of-growing stock and between these two treatments and all three regimes with higher levels of growing stock. The individual tree parameter estimates were also tested in the context of a seemingly-unrelated regression (SUR) analysis. The two stem profile parameter estimates were closely related to diameter/height ratio and crown ratio, respectively. Indicator variables for the different density regimes did not contribute any explanatory power beyond diameter/height ratio for the first parameter or crown ratio for the second parameter. DBH, height and crown ratio are concluded to account for the variation in stem form imposed by differences in stand density management.
I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Signature redacted for privacy.

Andrew P. Lennette, Author
Twenty-five-year Response of *Larix occidentalis* Stem Form to Five Stand Density Regimes in the Blue Mountains of Eastern Oregon.

by

Andrew P. Lennette

A Thesis Submitted
to
Oregon State University

In Partial Fulfillment of the requirements for the degree of

Master of Science

Presented July 9, 1999
Commencement June 2000
**TABLE of CONTENTS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Problem Statement and Justification</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Levels-of-Growing-Stock Studies</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Past Evidence of Stand Density Effects on Stem Form and Taper</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Overview of Catherine Creek</td>
<td>7</td>
</tr>
<tr>
<td>1.5 Objectives and General Thesis Structure</td>
<td>9</td>
</tr>
<tr>
<td>2 MODELING STEM FORM AND TAPER</td>
<td>10</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>10</td>
</tr>
<tr>
<td>2.2 Dependent Variables</td>
<td>11</td>
</tr>
<tr>
<td>2.3 Variables and Model Form</td>
<td>12</td>
</tr>
<tr>
<td>2.4 Tree Measurements for Taper Modeling</td>
<td>16</td>
</tr>
<tr>
<td>3 ANALYSIS OF THE EFFECTS OF THINNING ON STEM FORM</td>
<td>18</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>18</td>
</tr>
<tr>
<td>3.2 Methods</td>
<td>20</td>
</tr>
<tr>
<td>3.3 Results</td>
<td>32</td>
</tr>
<tr>
<td>3.4 Discussion</td>
<td>47</td>
</tr>
<tr>
<td>4 CONCLUSIONS</td>
<td>52</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>53</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>3-1</td>
<td>Estimated $a_2$ vs. $a_1$ random profile model coefficients</td>
</tr>
<tr>
<td>3-2</td>
<td>GSL-average $a_1$ random coefficients in 1991</td>
</tr>
<tr>
<td>3-3</td>
<td>GSL-average $a_2$ random coefficients in 1991</td>
</tr>
<tr>
<td>3-4</td>
<td>GSL-average EDOH in 1991</td>
</tr>
<tr>
<td>3-5</td>
<td>GSL-average CR (%) in 1991</td>
</tr>
<tr>
<td>3-6</td>
<td>Relative stem profile of the largest 40 trees/acre by GSL in 1991</td>
</tr>
<tr>
<td>3-7</td>
<td>Absolute stem profile of the study-wide average tree with GSL-specific CR.</td>
</tr>
<tr>
<td>3-8</td>
<td>Relative stem profiles for the GSL-average tree in 1991</td>
</tr>
<tr>
<td>3-9</td>
<td>Absolute stem profiles for the GSL-average tree in 1991</td>
</tr>
</tbody>
</table>
### LIST of TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Past taper model work showing dependent variables, authors and date</td>
</tr>
<tr>
<td>3-1</td>
<td>Pretreatment average stand attributes (per acre) of live trees in spring 1966</td>
</tr>
<tr>
<td>3-2</td>
<td>Post-treatment average stand (per acre) attributes of live trees in spring 1966</td>
</tr>
<tr>
<td>3-3</td>
<td>Average stand attributes (per acre) of live trees in spring 1991</td>
</tr>
<tr>
<td>3-4</td>
<td>Average sample tree measurements by GSL for spring 1991</td>
</tr>
<tr>
<td>3-5</td>
<td>Stem Profile Predictor Variables for All-Subsets Regression Model Selection</td>
</tr>
<tr>
<td>3-6</td>
<td>Multivariate pairwise comparisons of between-GSL differences in stem profile model parameter estimates</td>
</tr>
<tr>
<td>3-7</td>
<td>Parameter estimates for the structural equation that predicts $a_1$ while accounting for cross-equation error correlation with a SUR approach for parameter estimation. P-values are in parentheses</td>
</tr>
<tr>
<td>3-8</td>
<td>Parameter estimates for the structural equation that predicts $a_2$ while accounting for cross-equation correlation with the SUR approach for parameter estimation. P-values are in parentheses</td>
</tr>
<tr>
<td>3-9</td>
<td>Residuals variance-covariance matrix for the SUR system with structural equations developed with GSL-indicator variables</td>
</tr>
<tr>
<td>3-10</td>
<td>Parameter estimates and their standard errors for the structural equations estimated independently by NLS and simultaneously in the SUR system-of-equations with GNLS</td>
</tr>
<tr>
<td>3-11</td>
<td>Cross-equation residuals correlation matrix for the SUR system with structural equations developed from exogenous variables EDOH and CR</td>
</tr>
<tr>
<td>3-12</td>
<td>Tukey's HSD multiple comparisons of EDOH by GSL</td>
</tr>
<tr>
<td>3-13</td>
<td>Tukey's HSD multiple comparisons of CR (%) by GSL</td>
</tr>
</tbody>
</table>
1 INTRODUCTION

1.1 Problem Statement and Justification

The type of thinning regime imposed after stand establishment is an integral component of stand density management. While thinning regimes are applied to influence stand growth and tree vigor, the response of stem shape and form is also subtly influenced by silvicultural regime. Because a wide range of levels and intensities is possible, it is uncertain what differences in stem form will result from a given thinning treatment. The questions addressed in this study are: 1) Does the long-term response of stem form differ significantly among thinning treatments implemented in a levels-of-growing-stock study? 2) If there are significant differences, how might they be predicted in an operational setting? The answer to these questions influence our strategy for accurately predicting and measuring responses to silvicultural treatment in both a timber and non-timber context.

Stem form here refers to the geometric shape of the stem at various intervals or on successive segments of the stem. Tree taper refers to the relative change in diameter per unit increase in height along the bole of the tree. In essence, stem form describes the approximate shape of the stem, while taper defines an average rate of change in diameter per unit increase in height along an arbitrary portion of the bole. Although stem form and taper have been studied in great detail for some species and regions, very little, if any, analysis has compared the stem form of western larch among differing silvicultural treatments. Likewise, some fundamental questions about mechanisms controlling stem form among stand density regimes remain for commercial species in general. Because past
research suggests that the residual growing-stock level may or may not elicit a long-term change in stem form, a quantitative analysis of stem form and taper responses to repeated, long-term stand density manipulations has both practical and scientific significance. As described below, numerous studies have documented stand density effects on tree dimensions, including diameter at breast height (DBH), total tree height (H) and crown size. Likewise, many studies provide implications for timber volume production. Fewer have documented effects on stem form, although the general results are consistent and widely accepted.

1.2 Levels-of-Growing-Stock Studies

Much of the timber produced in the Pacific Northwest is harvested from second-growth, intensively-managed stands. Because thinning is a primary tool for controlling stand density and promoting growth and vigor of individual trees, the effects of thinning and residual stand density on growth capacity, growth rate and tree mortality continue to interest forest managers. Numerous levels-of-growing-stock (LOGS) studies have been initiated to quantitatively investigate the effects of stand density manipulations on tree and stand features. Several publications exist that describe the design and results of North American LOGS studies (Williamson and Staebler 1971; Oliver 1979; King 1986; Curtis and Marshall 1986; Curtis 1992; Seidel 1987, Cochran and Seidel 1996).

In general, the LOGS studies were initiated to investigate the effects of thinning on the growth and yield of cubic and merchantable volume, tree diameter growth, height growth and stand mortality. Many of the first North American LOGS studies began in the 1930's and were established in older stands. Various other studies in younger stands have been initiated since the 1960's throughout Oregon, Washington and British Columbia, with cooperators including federal, state and provincial agencies, and private industry (Marshall
The associated studies have provided valuable information regarding the production ecology of these stands and allometrics of constituent trees. These study sites will also serve as the foundation for many other investigations of various other aspects of tree growth and development in intensively managed stands.

A summary of the generally consistent results of LOGS studies follows:

1) DBH increases with increased spacing or decreasing stand density. Wider initial spacings and heavier thinnings reduce inter-tree competition. The response to stand density reductions is increasing diameter growth, particularly in the lower portion of the stem (Oliver 1979; Reukema 1979; Tappeiner et. al. 1982; Seidel 1987; Morris et. al. 1994). Growth responses to stand density regime, however, can also be difficult to distinguish from increases in average residual diameter imposed by thinning from below (Seidel 1971; Oliver 1979; Curtis and Reukema 1970).

2) Total height is relatively insensitive to changes in stand density except extreme densities, although greater average DBH is sometimes associated with larger average total height. Average top height has generally been observed to be independent of stand density (Kozlowski 1971; Seidel 1971; Tappeiner et. al. 1982; Marshall 1990; Morris et. al. 1994), but several studies also indicate that differences in height among density levels may occur on sites of low quality (Reukema 1979; Tappeiner et. al. 1982; Seidel 1987). Likewise, results in initial spacing trials in Douglas-fir show greater height (and DBH) with increasing density (Scott et. al. 1998).

3) Cubic volume per acre increases with increasing residual stand density, although merchantable volume generally decreases at higher stand densities because the trees on average are smaller than in lower density stands. Total cubic volume, but not necessarily net volume growth, increases with increasing stand density because the higher density stands carry higher levels of growing stock (Curtis and Marshall 1986; Curtis 1992; Morris
et. al. 1994; Cochran and Seidel 1996). Significant differences in total cubic volume are generally observed between stands of the highest and of the lowest densities, but volume differences in intermediate stocking levels may not necessarily be observed (Seidel 1971; Oliver 1979). Oliver and Larson (1996) describe the “cross-over” effect of stand volume yield in which higher density stands reach higher yields sooner than lower density stands, but are then surpassed by the lower density stands which produce a larger yield of volume per unit area at a later time. Although the highest density stands produce the largest cubic volumes per acre, maximum merchantable volume production is generated in stands of intermediate and lower densities because of the minimum size (DBH) requirement for merchantable trees and the increase in bf/cubic foot ratio with increasing DBH (Seidel 1971; Oliver 1979; Curtis and Marshall 1986).

4) The rate of tree mortality increases with increasing stand density. Intraspecific competition, disease, insect attack, fire, drought, and ice or snow damage are examples of biotic and environmental factors that influence tree mortality, and each can operate in a density-dependent fashion. The increasing mortality expected with increasing stand density is generally consistent with the −3/2 law of self-thinning (Curtis 1970; Long 1985; Zeide 1987), and empirical mortality patterns in many thinning studies generally support this theory (Reukema 1979; Cochran and Seidel 1996; Marshall 1990). The trend in mortality with increasing stand density contributes to the “cross-over” effect mentioned above.

1.3 Past Evidence of Stand Density Effects on Stem Form and Taper

Stem form ultimately is controlled by relative diameter and height growth. As already noted, the effects of stand density on diameter growth at breast height are well documented. Although the rate of increase in growth may differ between species and
location (site), an increase in diameter growth is consistently promoted by a decrease in stand density (Kozlowski 1971; Seidel 1971; Reukema 1979; Oliver 1979; Curtis and Marshall 1986).

The influence of stand density on total height is somewhat more ambiguous than density effects on diameter. Absolute height growth is a factor of species and site, but in general, height growth is relatively independent of stand density (Kozlowski 1971). In several spacing studies, Seidel (1971) with larch, Oliver (1979) with ponderosa pine, Reukema (1979) and Marshall (1990) with Douglas-fir, and Seidel (1987) with grand and red fir, tree height growth was shown to be independent of stand density. Although some work has shown that height growth may not be completely independent of density, these results are typically found in stands that are located on poor sites (Reukema 1979; Tappeiner et. al. 1982; Seidel 1987; Cochran 1996). Depending on what type of average is computed (for example, average of all trees vs. the 40 largest per acre) differences in both diameter and height may be, at least in part, an artifact of leaving larger, faster-growing trees in thinnings from below stands (Curtis and Reukema 1970, Seidel 1971; Oliver 1979; Curtis and Marshall 1986).

The size of the live crown is influenced by stand density and has a direct effect on stem form, although the specific mechanisms are still incompletely understood (Gray 1956; Larson 1963; Kozlowski 1971). A systematic increase in crown length with decrease in stand density has been documented in many spacing studies for example, Oliver (1979), Curtis and Marshall (1986), Curtis (1992) and Cochran (1996). Curtis and Reukema (1970) showed that although no overall differences in crown length were detected among spacings, systematic increases in crown length were evident with increased spacing when trees of the same relative position, or crown class, but of different stand densities, are compared. The increased crown length in lower density stands can be attributed to delayed
crown recession where there is more available growing space; in contrast, crown base recedes earlier with increasing density (Curtis and Reukema 1970; Kozlowski 1971; Thompson and Barclay 1984). Larson (1963) states that “every silvicultural treatment that results in alteration of the growth of the live crown will be reflected by a concomitant change in the form of the stem”. Because crown size, particularly crown length, plays a decisive role in determining stem form, any environmental or silvicultural treatment that affects crown size, e.g., a thinning, will also affect stem taper.

Stem increment, in terms of cross-sectional area, varies at different heights along the stem, but this variation is systematic and predictable (Larson 1963; Kozlowski 1971). Areal increment is very small at the top of the tree and increases progressively down through the live crown to the height above which maximum leaf volume is supported (Kozlowski 1971). This point on the stem is generally in the region of the live crown base (Larson 1963). Cross-sectional increment below this point depends on the development of the crown, or the class of the tree. In dominant trees, the stem increment decreases down the length of the bole, but increases again near the base of the stem. In suppressed trees the cross-sectional increment is more or less constant, but the ring width decreases continuously down the bole with no increase at the stem base, and commonly discontinuous rings are produced (Reukema 1964; Kozlowski 1971). Because of this slower growth in the lower bole, suppressed trees have a more cylindrical form than the larger codominant and dominant trees in a stand (Larson 1963). However, the pattern of growth changes dramatically upon thinning. Thinning stimulates growth on the lower portion of the stem and decreases growth in the “crown wood” region of the tree (Kozlowski 1971; Yerkes 1960). In a spacing study with Douglas-fir, Yerkes (1960) observed that released trees grew faster along the lower 20% of the stem and slower in the upper 20% than unreleased trees, while the unreleased trees showed a uniform decrease in
radial growth increment over the entire bole. An increase in “butt flare” is more
pronounced after thinning in suppressed and intermediate trees than in large trees
(Kozlowski 1971; Thompson and Barclay 1984). Larson (1963) and Kozlowski (1971)
state that the increment in the lower bole is less stable than in the crown due to seasonal
changes in weather or environment. Yerkes (1960) observed yearly fluctuations of growth
increment within trees due to the effects of weather and environment. In fact, this response
of diameter growth to weather fluctuation forms the basis of dendrochronology (Cook and
Kairiukstis 1990).

Thinning stands to specified densities is perhaps the most common silvicultural
tool implemented to meet a variety of forest management objectives. Because a prescribed
stand density is the operational objective of thinning, forest managers must be able to
quantify the effects that residual stand density has on tree stem form and volume
production. This information is essential to determine an appropriate post-thinning stand
density that will yield trees of a desirable size and quality and produce stands of the target
structure. If diameters respond to stand density, and total height does not, then differences
in stem form and taper will result. In the unlikely event that diameters and heights are
uniform between densities, differences in stem form may yet be observed if crown sizes
differ, since the latter is the link between form and stand density.

1.4 Overview of Catherine Creek

The Catherine Creek LOGS study was initiated in western larch (Larix
occidentalis) stands in the Blue Mountains of eastern Oregon in 1966 by the USDA Forest
Service. The goals of this LOGS study were to determine optimal spacings of trees in
commercial stands, to investigate the relation between stocking levels and growth and yield
of commercial stands, and to determine the relation between stand density and mortality 
(Cochran and Seidel 1996).

The experimental units were ten 0.4-acre plots (with 30-ft. buffer strips). These ten plots 
include five treatment levels, each replicated twice. The thinning treatments were designed 
to leave five levels of bole surface area in the residual stands: 5,000 ft.$^2$, 10,000 ft.$^2$, 15,000 
ft.$^2$, 20,000 ft.$^2$ and 25,000 ft.$^2$ per acre. Thinning treatments were applied in 1966, 1976 
and 1986 as thinnings from below.

Bole surface area (ft.$^2$/acre) was selected as the measure of growing stock in this 
study as an approximation of the cambial surface area on which wood is produced (Lexen 
1943). The sampling units for determining bole area consisted of 134 trees (15 trees per 
plot on average) sampled from across the range of DBH on each plot (Cochran and Seidel, 
1996). The bole surface area inside bark, or cambial surface (CS) area, was determined for 
trees within each plot with the model

$$\ln \text{CS} = b_1 + b_2(\ln \text{DBH}) + b_3(\ln \text{DBH})^2$$  \hspace{1cm} [Eq. 1-1]

where:

DBH = diameter in inches at breast height

Unique coefficients for the model were determined with the fifteen sample trees on 
each plot during each measurement period. All samples trees were measured by 
dendrometer for total height and upper-stem diameters (Cochran and Seidel, 1996). In 
short, regulation of growing stock level by bole surface area necessitated periodic upper 
stem measurements, creating a unique opportunity to look at changes in stem form over a 
1.5 Objectives and General Thesis Structure

The objective of this study was to compare the effects of thinning regime on stem form and taper in western larch. Chapter 2 describes past work and some common techniques used for stem profile modeling and the procedures for obtaining upper-stem diameters. Section 3.1 describes the experimental study area and data collection methods. Section 3.2 describes the methodology used in the analysis of thinning effects on stem form, and section 3.3 presents the results of the analysis. A discussion of the results is provided in section 3.4, and the conclusions, as they pertain to LOGS studies in general and to the objective statement specifically, are presented in Chapter 4.
2  MODELING STEM FORM AND TAPER

2.1  Introduction

Stem form and taper are often indexed with conventional stem form measures such as the height-diameter ratio or Girard from class, or through whole-bole volume equations (whole bole systems). However, these indices and whole-bole volume models are generally not accurate enough for use in the comparison of stem form over varying stand densities (Marshall 1990; Groman 1972; Kozak 1988; and Newnham 1991). For example, Yerkes (1960) and Marshall (1990) found no significant difference in form class between thinning treatments, while some differences were noted for taper models that examine the trend of taper over the entire bole.

Any model developed for application in stands managed under a range in stand density regimes should account for the stand density factors that strongly influence stem form and taper, such as crown length. The indices that rely solely on lower stem measurements such as Girard form class may overlook important differences in upper stem taper, relative to equations that recognize variations in upper stem growth patterns (Kozak 1988; Marshall 1990; and Newnham 1991).

The first step in constructing high precision models for a comparison of taper is to select an equation that accurately predicts stem taper for individual trees. Ideally, this equation would describe taper by sectioning the subject tree into portions based on their geometric form. Equations that segment the tree into cylinders (the stump), neiloids (buttswell region near stem base), paraboloids (the crown-free bole) and cones (the crown covered portion of the bole) have been used with success in accurately quantifying bole taper (Marshall 1990; Groman 1972; Kozak 1988; and Newnham 1991). “Trees are described as a continuum of short, monotonic shapes such as the traditional geometric
forms” (Grosenbaugh, 1963). More recent stem taper models describe stem form as a continuously changing function of height. For example, Kozak (1988) developed a variable exponent taper model that fit the data significantly better than common stem form indices and whole-bole systems (Kozak 1988). Kozak (1988) and Newnham (1991) both observed that this form of taper equation described stem shape with very small bias from the ground level to the tip of the tree.

2.2 Dependent Variables

Taper models serve several purposes: 1) they provide a means of estimating upper-stem diameters so that merchantable heights and the volume of the whole bole or segments of the bole can be predicted; 2) they facilitate determination of optimal bucking strategies; 3) they allow analysis of relative wind firmness; and 4) they help improve quantitative descriptions of general stand structure. Several alternative dependent variables can be selected; however, the ability of a given dependent variable to meet the assumptions of regression modeling depends on the model form and set of independent variables used. Several dependent variables have been used successfully to model stem form with a high degree of accuracy and precision. Although all assumptions of regression are not always met, various methods have been developed to ameliorate violations.

Upper-stem diameters (dᵢ), both inside-bark (dᵢᵠ) and outside-bark (dᵢₒ), are often predicted from equations that use dᵢ², dᵢ/DBH, (dᵢ/DBH)², and other transformations of dᵢ as the dependent variable, but are easily modified to solve for dᵢ. Table 2.1 provides examples of past work in which a variety of transformations of upper-stem diameters (dᵢ) were predicted by various model forms. All predicted diameters are inside-bark.
Table 2-1 Past taper model work showing dependent variables, authors and date.

<table>
<thead>
<tr>
<th>Predicted Dependent Variable</th>
<th>Author(s) and Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>Omerod (1973); Bennett and Swindel (1972); Amidon (1984); Newberry and Burkhart (1986); Flewelling and Raynes (1993); Kozak (1997); Tassisa and Burkhart (1998)</td>
</tr>
<tr>
<td>$d_1^2$</td>
<td>Valenti and Cao (1986)</td>
</tr>
<tr>
<td>$d_1/DBH$</td>
<td>Max and Burkhart (1976); Czap!ewski and Bruce (1991)</td>
</tr>
<tr>
<td>$(d_1/DBH)^2$</td>
<td>Bruce et. al. (1968); Kozak et. al. (1969); Real and Moore (1985); Burkhart and Walton (1985); Thomas and Parresol (1991)</td>
</tr>
<tr>
<td>$d_1/DIB$ (where DIB=predicted from DBH)</td>
<td>Walters and Hann (1986)</td>
</tr>
<tr>
<td>$\ln(d_1)$</td>
<td>Kozak (1988); Muhairwe et. al. (1994)</td>
</tr>
<tr>
<td>$\ln((H-h_1)/(H-1.3))/\ln(d_1/DBH)$</td>
<td>Newnham (1992)</td>
</tr>
</tbody>
</table>

2.3 Variables and Model Form

The purpose of any taper model is to predict diameters at various heights on the stem. Dependent and independent variables are usually functions of these two measurements. For practical reasons, some models predict $d_1$ or $d_1^2$; however, it is quite common to predict relative diameter ($d_1/D$) as a function of relative height ($h/H$). This latter choice allows for the construction of only one profile curve to be fit for trees of different diameter ($D$) and total heights ($H$), and tends to homogenize the variance around the regression function. Historically, taper models have been constructed using equation forms such as linear, log-transformed and nonlinear models. Segmented polynomials represent a special case of linear models if the join points are known, and nonlinear models if the join points are estimated from the data (Max and Burkhart 1976). The goal of a taper model is to produce an equation that accurately and precisely estimates tree taper and has the simplest possible form. Many models have been constructed in the past; however, they all use similar variables to predict taper, including:
D = some diameter, inside or outside bark; sometimes referred to as d_i (inches)

DBH = diameter outside bark at 4.5 feet (1.3 m) from the base of the tree (inches)

H = total tree height (feet)

h = some height above ground

X = (H-h)/(H-4.5)

\[ X_p = 1-(h/H)^{1/2}/(1-p^{1/2}) \]

where \( p = h/H \) when \( h \) is the height of the inflection point

Z = h/H

CR = crown ratio

\( X^c \); where \( c \) is a linear function of other tree variables

Having multiple terms in a polynomial or a variable exponent model provides a means of accounting for the geometry of the different portions of the bole, such as the butt swell, main branch-free bole, and live crown. Although butt swell can generally be characterized as a frustum of a neiloid, the branch-free bole has been quantified as a frustum of a parabola, and the live crown region as a frustum of a cone (Larson 1963; Kozak 1988). These different portions of the stem are more efficiently and accurately modeled as segments of a polynomial or as varying exponent model to describe local form. The inflection point or points that represent a relatively distinct change in form are often included in models as a function of relative height (Max and Burkhart 1976; Newberry and Burkhart 1986; Kozak 1988; Newnham 1992).

The two-segmented and the three-segmented (Max and Burkhart 1976) polynomial taper functions can be fit in both linear (join point is known), or nonlinear (join point estimated from the data) forms. A quadratic-quadratic equation can be written as:
\[
d^2/DBH^2 = b_1(Z - 1.0) + b_2(Z^2 - 1.0) + b_3(a-Z)^2 [\text{Eq. 2-1}]
\]

where,

\( I = 1 \) for \( Z \leq a \) and \( I = 0 \) for \( Z > a \).

\( a \) = the join point where the polynomial segments are equal and have equal first derivatives.

A three-segmented (quadratic-quadratic-quadratic) polynomial can be written as:

\[
d^2/DBH^2 = b_1(Z - 1.0) + b_2(Z^2 - 1.0) + b_3(a_1 - Z)^2I_1 + b_4(a_2 - Z)^2I_2 [\text{Eq. 2-2}]
\]

where,

\( I_i = 1 \) for \( Z \leq a_i \) and \( I_i = 0 \) for \( Z > a_i \) for \( i = 1,2 \).

\( a_1 \) and \( a_2 \) = join points of the polynomial segments

An example of non-segmented polynomial taper model was developed by Bennett and Swindel (1972):

\[
d_i = b_1 DBH(H-h_i) / (H-4.5) + b_2 (H-h_i)(h_i - 4.5) + b_3 H(H-h_i)(h_i - 4.5) + b_4(H-h_i)(h_i - 4.5)(H+h_i+4.5) [\text{Eq. 2-3}]
\]

The variable-exponent (Kozak 1988) and variable-form (Newnham 1992) systems are examples of nonlinear taper models. These models use a continuously changing exponent to describe taper. Kozak's (1988) variable exponent model is:

\[
d_i = DI \cdot X^c [\text{Eq. 2-4}]
\]

where,

\( DI\) = diameter inside bark at an inflection point estimated with the function

\[
a_0DBH^a_1DBH^{a_2}
\]

\( c = b_0 + b_1z^2 + b_2\ln(z + 0.001) + b_3z^{0.5} + b_4\exp z + b_5(DBH/H) \)
The exponent describes the changing stem geometry as a function of relative height (Z) and the ratio DBH/H. The inflection point is set at an empirically estimated height (h1) that varies from 20% to 25% of the total tree height (H) (Demaerschalk and Kozak 1977). Newnham (1992) found that the relative height (p) of the inflection point varies from 15 to 35% with very little effect on the predictive properties of the model. This model form is attractive because the exponent c can be designed to incorporate various single tree or whole-stand parameters to describe taper (Real and Moore 1988; Muhairwe 1994). Newnham’s (1992) variable-form model is:

\[ d_i = DBH \times X^c \]  

[Eq. 2-5]

This model is similar to Kozak’s (1988) model with a few exceptions. First, the inflection point is assumed to occur at breast height, and second, the exponent (c) was developed through stepwise regression from the combinations of 25 transformations of X, DBH/H and H. Although it is not shown here, both of these nonlinear examples were logarithmically transformed to linearize the model for fitting.

Many additional models of alternative form and predictor variable sets have been suggested. Muhairwe (1994) and Real and Moore (1988) have recommended incorporating stand level parameters such as stand age, site quality, density, and the quadratic mean diameter into taper models. Crown size is of particular significance in taper systems, and is typically described as either crown length or live crown ratio. Walters and Hann (1986), Valenti and Cao (1986) and Newberry and Burkhart (1986) found that incorporating crown ratio (CR) into taper equations significantly enhances their predictive ability. Newnham (1992) and Kozak (1988) found the DBH/H ratio to be positively correlated with crown size and is a good surrogate for crown features in taper models.
2.4 Tree Measurements for Taper Modeling

The standard method for obtaining accurate upper-stem diameter and height measurements is stem analysis, as described by Husch et. al. (1983). This procedure consists of the direct measurement of stem diameter on cut sections of a tree. However, this approach requires that the observed trees be destructively sampled (felled) and is therefore not desirable when standing trees need to be measured.

Therefore some type of dendrometer is needed whose primary requirements for making accurate upper stem diameter measurements are: 1) the device has magnification capacity, which reduces ambiguity and personal bias; and 2) any bias created by tree noncircularity or lean is negligible for the device in question (Grosenbaugh 1963). The Barr and Stroud TYPE FP-12 optical dendrometer is probably the most widely used instrument for measuring upper-stem diameters on standing trees and was found to fulfill both of these requirements.

The Barr and Stroud TYPE FP-12 optical dendrometer is a magnified (5.5x), coincidental, short-based-rangefinder. Coincident dendrometers have the capability of aligning two lines-of-sight simultaneously, as opposed to sequentially, which minimizes user error (Grosenbaugh 1963). The TYPE FP-12 measures diameters from 1.5 to 200 inches, but the minimum range increases as diameters exceed 30 inches. The claimed uncertainty in diameter measurements by the manufacturer for the instrument is +/- 0.1 inches for diameters up to 10 inches and +/- 1% for diameters between 10 and 200 inches. The range uncertainty is 0.16% at 36 feet and increases to 0.79% at 210 feet. Heights may be normally determined with the elevation angle gauge between -25 and +45 degrees, or between -60 to +60 degrees with the clinometer fixture. The manufacturer states that
uncertainty in height is +/- 1.5% for vertical angles greater than 10 degrees (Groman 1969).

In a field test of the instrument, volumes computed from dendrometer measurements differed from felled check-tree volume estimates by only +/- 4 percent (Grosenbaugh 1963). For the Black Rock levels-of-growing-stock study, Bell and Groman (1971) observed an accuracy of +/- 3% for both diameters and heights over the entire length of the tree. In the Hoskins levels-of-growing-stock study, the Barr and Stroud dendrometer was found to differ from check-trees by only 1.5 percent, which is quite adequate for detecting differences in stem profile among stand density regimes (Marshall 1990). In short, precision dendrometers, such as the Barr and Stroud TYPE FP-12, may be used for measuring upper stem diameters with a high degree of accuracy and an acceptable amount of error (Grosenbaugh 1963; Bruce 1966; Groman 1969; Bell and Groman 1971; James and Kozak 1984).
3 ANALYSIS OF THE EFFECTS OF THINNING ON STEM FORM

3.1 Introduction

The form (geometric shape) and taper (decrease in diameter per unit increase in height) of trees in a stand have historically been of interest to forest managers, primarily from a timber production standpoint. If two trees of the same basal diameter and total height differ in total stem volume, this difference is attributable to differences in stem form. In addition to implications for log dimensions and recoverable timber volume, stem form is important for maintaining wind stability and is correlated with individual tree growth and vigor. Larson (1963) noted that the development of strong, wind-firm stems of "good form" should be an objective of stand management equal in importance to increased growth and vigor. Stem form and its correlates are frequently relevant aspects of general stand structure regardless of the management objective.

Several investigations have shown that the size of the live crown is functionally linked to the form and taper of an individual tree (Gray 1956; Larson 1963; Kozlowski 1971). Stem form parameters have been quantitatively related to crown size in numerous species (Burkhart and Walton 1985; Newberry and Burkhart 1986; Walters and Hann 1986; Valenti and Cao 1986; Hann et. al. 1987; Real and Moore 1988; Morris et. al. 1994; Muhairwe et. al. 1993; Tassisa and Burkhart1998). A number of hypotheses have been formulated to explain the effect of the live crown on stem form. In the mechanistic theory of Metzger, as discussed by Larson (1963), the bole of a tree develops to counter the vertical forces applied by the weight of the crown, including snow and ice, and the horizontal bending forces applied by wind. The water conduction theory of Jaccard, again described by Larson (1963) posits that the bole assumes the form necessary to move water from the roots through the stem to the live crown to meet the transpirational requirements.
of the foliage. Duff and Nolan (1953) present a nutritive hypothesis suggesting that the growth rate of a given internode is a function of the amount of foliage supporting that internode and the photosynthetic activity of that foliage.

Previous work indicates that a varying, but systematic, vertical pattern of wood deposition along the bole exists, and that it is controlled primarily by the live crown. In stand-grown trees, ring width has been shown to increase from the apex of the crown with increasing depth into the crown to a point of maximum growth at or near the base of the live crown. Below the crown, ring width decreases with decreasing height above the base of the tree. Ring width may then increase slightly in the region of butt swell in dominant trees, but continues to decrease or even disappear in suppressed trees unless they have been released (Duff and Nolan 1953; Farrar 1961; Smith and Wilsie 1961; Larson 1963; Kozlowski 1971).

Numerous thinning trials consistently document a decline in crown size with increasing stand density due to earlier onset of crown recession (Curtis and Reukema 1970; Oliver 1979; Curtis and Marshall 1986; Marshall 1990; Curtis 1992; Cochran and Seidel 1996). Conversely, decreasing stand density elicits an increase in crown length. Curtis and Reukema (1970) and Muhairwe (1994) note that trees of similar relative position in stands with differing densities will have much different crown structure and development. Larson (1963) concludes that the primary factors influencing crown development are stand density and relative position of the tree within the stand.

The influence that stand density has on crown size, and the influence of crown size on stem form and taper, leads to an expected correlation between thinning regime and stem form. While this correlation has been observed in some studies (Newnham 1965; Thompson and Barclay 1984; Morris et. al. 1994; Muhairwe 1994; Tassisa and Burkhart 1998), it has not been found in others. The latter studies typically include those that focus
on a simplified index of stem form such as Girard's form class, form quotient and cylindrical form factor (Behre 1932; Matte 1949; Yerkes 1960; Marshall 1990). Given that thinning is used to manipulate stand density, which in turn influences crown size and perhaps form, a reasonable question is: Do all levels of thinning induce noticeable changes in stem form and taper? The objectives of this study were: 1) to test whether there was a significant stand density effect on stem form of Larix occidentalis after 25 years of repeated thinning; and 2) to test whether differential crown development was sufficient to explain the effects of stand density on stem form.

3.2 Methods

3.2.1 Study Area

The Catherine Creek Levels-of-Growing-Stock study site is located at an elevation of 4000 feet, in Sections 26 and 27 of T. 5S., R. 11E., Willamette Meridian, on the Union District of the Wallowa-Whitman National Forest, about 15 miles southeast of Union, Oregon (45° 7.5' N, 117° 30' W). The even-aged larch stand was 33 years of age in 1966, with vegetation typical of the seral stage of the Abies grandis/Calamagrostis plant community (Cochran and Seidel 1996). The soil is Tolo silt loam (Typic Vitradent), a well-drained regosol developed from dacite pumice, which is underlain at three feet by a buried soil developed from basalt (Seidel 1971; Cochran and Seidel 1996). The average daily temperature in July is 66.5°F and 30.0°F in January. The average annual rainfall is 13.8 inches with 5.8 (42%) inches falling from May to September. The average annual snowfall is 22.5 inches, falling in the period from October to May.
3.2.2 Experimental Design and Stand Treatments

The Catherine Creek levels-of-growing-stock study conforms to a completely randomized design consisting of two replicates of five growing-stock levels (GSL). A replicate consists of a 0.4-acre square plot with a 33-foot buffer strip. Growing stock was regulated by bole surface area (BSA) per acre, where BSA was intended as an approximation of the cambial surface of the bole (Lexen 1943). The plots were treated by thinnings that reduced the BSA down to one of five levels: 5,000, 10,000, 15,000, 20,000 and 25,000 $\text{ft}^2$/acre. Table 3-1 summarizes pre-treatment stand conditions in spring of 1966. Table 3-2 and Table 3-3 summarize some post-treatment stand characteristics by GSL for 1966 and 1991 (Cochran and Seidel 1996).

Table 3-1 Pretreatment average stand attributes (per acre) of live trees in spring 1966

<table>
<thead>
<tr>
<th>GSL</th>
<th>Bole Area $\text{ft}^2$</th>
<th>Basal Area $\text{ft}^2$</th>
<th>Number of Trees</th>
<th>SDI Spacing $\text{ft}$</th>
<th>Average Diameter $\text{in}$</th>
<th>Average Height $\text{ft}$</th>
<th>CFV $\text{b.f.}$</th>
<th>BFV $\text{ft}^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25800</td>
<td>118.6</td>
<td>924</td>
<td>269</td>
<td>6.9</td>
<td>4.9</td>
<td>1995</td>
<td>98</td>
</tr>
<tr>
<td>2</td>
<td>31125</td>
<td>132.7</td>
<td>1161</td>
<td>303</td>
<td>6.1</td>
<td>4.6</td>
<td>2287</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>34180</td>
<td>139.2</td>
<td>1406</td>
<td>327</td>
<td>5.6</td>
<td>4.3</td>
<td>2367</td>
<td>193</td>
</tr>
<tr>
<td>4</td>
<td>32880</td>
<td>143.7</td>
<td>1377</td>
<td>333</td>
<td>5.6</td>
<td>4.4</td>
<td>2322</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>32700</td>
<td>135.6</td>
<td>1459</td>
<td>312</td>
<td>5.5</td>
<td>4.1</td>
<td>2200</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3-2 Post-treatment average stand (per acre) attributes of live trees in spring 1966

<table>
<thead>
<tr>
<th>GSL</th>
<th>Bole Area</th>
<th>Basal Area</th>
<th>Number of Trees</th>
<th>SDI</th>
<th>Average Spacing</th>
<th>Average Diameter</th>
<th>Average Height</th>
<th>CFV</th>
<th>BFV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ft²)</td>
<td>(ft²)</td>
<td></td>
<td></td>
<td>(ft)</td>
<td>(in)</td>
<td>(ft)</td>
<td>(ft³)</td>
<td>(b.f.)</td>
</tr>
<tr>
<td>1</td>
<td>4780</td>
<td>26.0</td>
<td>96</td>
<td>53</td>
<td>21.4</td>
<td>7.1</td>
<td>48.2</td>
<td>474</td>
<td>98</td>
</tr>
<tr>
<td>2</td>
<td>9524</td>
<td>49.6</td>
<td>215</td>
<td>102</td>
<td>14.3</td>
<td>6.5</td>
<td>47.4</td>
<td>902</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>14242</td>
<td>70.9</td>
<td>355</td>
<td>151</td>
<td>11.1</td>
<td>6.1</td>
<td>46.0</td>
<td>1268</td>
<td>193</td>
</tr>
<tr>
<td>4</td>
<td>19313</td>
<td>96.2</td>
<td>546</td>
<td>207</td>
<td>9.0</td>
<td>5.7</td>
<td>42.6</td>
<td>1616</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>24203</td>
<td>109.8</td>
<td>745</td>
<td>240</td>
<td>7.6</td>
<td>5.2</td>
<td>43.1</td>
<td>1847</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3-3 Average stand attributes (per acre) of live trees in spring 1991

<table>
<thead>
<tr>
<th>GSL</th>
<th>Bole Area</th>
<th>Basal Area</th>
<th>Number of Trees</th>
<th>SDI</th>
<th>Average Spacing</th>
<th>Average Diameter</th>
<th>Average Height</th>
<th>CFV</th>
<th>BFV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ft²)</td>
<td>(ft²)</td>
<td></td>
<td></td>
<td>(ft)</td>
<td>(in)</td>
<td>(ft)</td>
<td>(ft³)</td>
<td>(b.f.)</td>
</tr>
<tr>
<td>1</td>
<td>5885</td>
<td>44.4</td>
<td>32</td>
<td>73</td>
<td>37.2</td>
<td>16.1</td>
<td>83.0</td>
<td>1345</td>
<td>7434</td>
</tr>
<tr>
<td>2</td>
<td>11397</td>
<td>69.3</td>
<td>88</td>
<td>122</td>
<td>22.4</td>
<td>12.1</td>
<td>74.0</td>
<td>1991</td>
<td>9845</td>
</tr>
<tr>
<td>3</td>
<td>16909</td>
<td>92</td>
<td>148</td>
<td>166</td>
<td>17.2</td>
<td>10.7</td>
<td>72.0</td>
<td>2633</td>
<td>9351</td>
</tr>
<tr>
<td>4</td>
<td>22987</td>
<td>115.7</td>
<td>216</td>
<td>216</td>
<td>14.3</td>
<td>10.0</td>
<td>72.3</td>
<td>3384</td>
<td>7862</td>
</tr>
<tr>
<td>5</td>
<td>28868</td>
<td>126.6</td>
<td>310</td>
<td>244</td>
<td>11.9</td>
<td>8.7</td>
<td>71.6</td>
<td>3702</td>
<td>5042</td>
</tr>
</tbody>
</table>

Bole surface area (BSA), cubic-foot volume (CFV), and international ¼ inch board-foot volume (BFV) were determined for each sample tree with the STX program (Grosenbaugh 1964). The STX program contains an integral function which converts DBH to an inside-bark value and thus BSA, CFV and BFV are inside-bark estimates. These values were estimated for all other trees in each GSL by fitting the following models to the sample trees for a given GSL (Cochran and Seidel 1996):
\[
\ln(\text{BSA}) = \alpha_0 + \alpha_1[\ln(\text{DBH})] + \alpha_2[\ln(\text{DBH})]^2 \\
\ln(\text{CFV}) = \beta_0 + \beta_1[\ln(\text{DBH})] + \beta_2[\ln(\text{DBH})]^2 \\
\ln(\text{BFV}) = \delta_0 + \delta_1[\ln(\text{DBH})] + \delta_2[\ln(\text{DBH})]^2
\]  

[Eq. 3-1]  
[Eq. 3-2]  
[Eq. 3-3]

The treatments were generally thinnings from below, implemented with the objective of promoting the largest and most vigorous "crop" trees on the plot (Seidel 1971). The treatments were applied in the spring of 1966, fall of 1975, and fall of 1985, and slash was left on-site. After the initial thinning, about 40% of the basal area of Plot 3 and Plot 1 (GSL 4 and GSL 5), consisted of lodgepole pine (Pinus contorta). However, this species was discriminated against in future thinnings (Cochran and Seidel 1996).

### 3.2.3 Measurements and Data

The plots were remeasured every five years by USDA Forest Service PNW Research Station field crews. Diameter at breast height (DBH) was measured for all trees on the plots to the nearest 0.1-inch at the beginning and end of every five-year measurement period. Total heights (H) were measured for all plot trees only after the initial thinning in 1966. For each subsequent five-year measurement period upper stem diameters (d), their corresponding heights (h) and total tree height (H) were measured for an average of fifteen trees per plot by an optical dendrometer. The sample trees were selected proportionally across the DBH range of their respective plots. Total heights were estimated for the remaining trees on each plot by fitting the following model to the measured trees on that plot (Cochran and Seidel 1996):

\[
\ln H = \gamma_1 + \gamma_2 \frac{1}{\text{DBH}} + \gamma_3 \frac{1}{\text{DBH}^2}
\]  

[Eq. 3-4]

Because the DBH measurements were made with both diameter tape and optical dendrometer, all dendrometer measurements (d) were adjusted on a plot basis by multiplying the dendrometer diameter by the plot-average ratio \(\frac{\text{DBH}_{\text{Taped}}}{\text{DBH}_{\text{Dendrometer}}}.\)
Heights to green crown base (HTCB) were measured for the sample trees by dendrometer in 1966 and by clinometer in spring of 1993, the latter having been collected to provide western larch crown data for calibrating the growth model PROGNOSIS. The 1993 crown base measurements were applied to the 1991 sample trees with the assumption that there were no differences in height to crown base between spring 1991 and spring 1993. Crown ratios (CR) were therefore determined for 1991 as

\[ CR_{1991} = \frac{(H_{1991} - HTCB_{1993})}{H_{1991}} \]  

[Eq. 3-5]

Table 3-4 contains a summary of sample tree attributes in 1991 for the Catherine Creek study.

### Table 3-4 Average sample tree measurements by GSL for spring 1991

<table>
<thead>
<tr>
<th>GSL</th>
<th>No. of Diameter (d) Measurements/ Tree</th>
<th>Diameter Breast Height (DBH)</th>
<th>Total Tree Height (HT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>27</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

### 3.2.4 Data Analysis

#### 3.2.4.1 Scope of Inference

The Catherine Creek Levels-of-Growing-Stock Study was not replicated, and no specific target population was identified when the experiment was implemented. The scope of inference therefore strictly includes only the plots constituting the study, and more
loosely a very well-defined set of "similar" western larch stands in the same geographic locale.

Similarly, the specific thinning regimes imposed in the Catherine Creek study represent an arbitrary selection from many other possibilities, but probably cover the range of rationale residual densities under typical timber production. Hence, in this analysis the specific regimes are regarded as fixed factors for which stem profile responses were tested.

3.2.4.2 Autocorrelation and General Approach

The goal of the analysis was to develop an index representative of stem form for each tree in the study so that differences in 1991 stem form among the five levels-of-growing-stock could be tested. One approach would be to pool the data and construct a single regression model describing stem profile for all trees, and then introduce variables representing the different stand density treatments. Indicator variables representing discrete treatments, or a continuous stand density variable, could then be tested for significance, which would indicate a treatment effect. However, this approach to stem profile models presents a problem for hypothesis testing: that is, because profile models are constructed from sample units with multiple observations, the errors in the model are not independent. Ordinary, weighted, and nonlinear least-squares techniques produce unbiased and consistent parameter estimates when autocorrelation exists, but correlated error structures can create biased estimates of the standard errors of the parameter estimates. When the parameter variance estimates are biased, confidence intervals and significance tests are no longer valid (Kmenta 1986; Williams and Reich 1996). While the effects of autocorrelated data have little effect on the predictive capabilities of profile models (Kozak 1997), the undesirable properties of autocorrelation preclude statistical tests for treatment differences in stem profile parameters (Tasissa and Burkhart 1998).
One approach to testing for treatment effects on stem form in the presence of autocorrelation is to use multivariate statistical methods. The goal of a multivariate procedure would be to reduce the information for each tree down to a few variables representative of stem form and taper. If a large number of upper stem diameter measurements on each tree was reduced down to 2 or 3 parameters representative of stem form, then differences in form among GSLs could be examined with a multivariate analysis of variance (MANOVA). The MANOVA essentially tests the significance of the ratio of within treatment variation to between treatment variation for a vector of mean values (Stiteler 1978). Similar to the univariate ANOVA procedure, Wilk's lambda in the context of MANOVA tests whether at least two of the treatments are significantly different from one another (Johnson and Wichern 1992). Hotelling's $T^2$ statistic (Johnson and Wichern 1992) may then be applied to test for differences between mean vectors in several pairwise comparisons among GSLs.

Alternatively, generalized least squares (GLS) is capable of correcting for correlated error structures in data representing multiple observations per unit (Ferguson and Leech 1978; Davis and West 1981; West et. al. 1984; Gregoire et. al. 1995). Kmenta (1986) demonstrates that a correlated error structure may be accounted for in a least squares model by estimating the variance-covariance matrix of the error terms and incorporating this matrix into a GLS algorithm. Thus, if the covariance of observations within a tree are known or can be estimated, GLS can be applied to fit a taper model to the data pooled across trees. Although GLS in its simplest form is commonly used to account for autocorrelation in multilevel data (for example, see Goldstein 1995), GLS has also been applied to address cross-equation error correlation in systems-of-equations for predicting two or more endogenous variables (Kmenta 1986). This systems-of-equations technique has been applied by Biging (1985) for modeling height growth of individual trees with a
varying-parameters (random coefficients) approach. He showed that not only can the varying parameters approach avoid problems associated with correlated errors between observations within a tree, but that a single model fit by ordinary least-squares produces parameter estimates which are incorrect and significantly different from those produced with GLS.

An alternative to both of these approaches was to fit taper curves to each tree separately, and then estimate the taper function parameters as a function of tree and stand density variables in a systems-of-equations algorithm. This approach is more flexible than the MANOVA approach since it allows incorporation of continuous variables into the statistical model, analogous to an analysis of covariance in a univariate context, or to a regression model with a mix of indicator and continuous variables. Assuming the taper function contains two or more parameters, the parameter prediction system would have to contain two or more equations whose errors would be correlated. Borders (1989) introduced a technique initially presented by Zellner (1962) in an econometric context, in which GLS is used to account for cross-equation error correlation in a seemingly unrelated (SUR) system-of-equations.

In this study, profile models containing two parameters were fit to each tree, and the resulting parameter estimates were then treated as a bivariate stem form response to treatments (GSL) in a MANOVA. The two parameter estimates were then also treated as endogenous variables in a SUR system-of-equations (Zellner 1962; Borders 1989). Variables representing stand density regime were introduced as exogenous predictor variables to test for treatment effects on stem form, the latter being represented by the stem profile parameter estimates.
3.2.4.3 Taper Model

Of the numerous profile models available, the variable-exponent model proposed by Kozak (1988) has routinely performed well (Kozak and Smith 1991; Flewelling and Raynes 1993; and Tasissa and Burkhart 1998). The variable exponent model estimates taper with an acceptable level of precision (low SE) and accuracy (low bias), and accounts for changes in form over the entire length of the tree bole. Under log transformation the model is linear, as is the case with segmented polynomial systems with fixed join-points (Max and Burkhart 1976; Walters and Hann 1986). Additional independent variables, which can account for differences in general shape with height on the bole, are easily incorporated into the exponent (Muhairwe et. al. 1994; Maguire and Batista 1996; Williams and Reich 1996). This variable exponent model does not necessarily describe form better than any other taper systems, but, it is easily interpreted due to the relationship between the exponent and implied shape (Kozak 1988). The general model form is:

\[ Y_i = X^c \quad \text{[Eq. 3-6]} \]

where,

\[ Y_i = \frac{d_i}{DBH} \]

\[ d_i = \text{diameter inside bark at } h_i \text{ (inches)} \]

\[ DBH = \text{diameter at 4.5 feet (inches)} \]

\[ h_i = \text{height from ground, } 0 \leq h_i \leq H \text{ (feet)} \]

\[ H = \text{total tree height (feet)} \]

\[ X = \frac{(1- (Z)^{1/2})}{(1- p^{1/2})} \]

\[ Z = \frac{h_i}{H} \]

\[ p = \frac{4.5}{H} \]
\[ c = \text{a linear function of predictor variables} \]

The properties of this model are (1) \( d = 0 \) when \( h/H = 1.0 \); (2) \( d = DBH \) when \( h/H = p \) (Kozak 1988).

The subset selection for the variable-exponent was done in three parts using the equation \( \ln(d/DBH) = c \times \ln(X) \), where \( c \) is a linear function of one to five different predictor variables. First, all observations from all trees were pooled and an "all subsets" regression was performed to identify the best predictor variables from the class of relative height variables (Table 3-5).

Table 3-5  Stem profile predictor variables for all-subsets regression model selection

<table>
<thead>
<tr>
<th>Relative Height Variables</th>
<th>Tree Variables</th>
<th>Stand Density Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = 4.5/H )</td>
<td>DBH = d; h=4.5 feet</td>
<td>GSL = growing stock level</td>
</tr>
<tr>
<td>( Z = h/H )</td>
<td>H = Total height</td>
<td>TPA = trees acre(^{-1})</td>
</tr>
<tr>
<td>( Q = (h/H)^{1/2} )</td>
<td>DOH = DBH/H</td>
<td>QMD = quadratic mean</td>
</tr>
<tr>
<td>( X = [1 - (h/H)^{1/2}] / [1 - (4.5/H)^{1/2}] )</td>
<td>HOD = H/DBH</td>
<td></td>
</tr>
<tr>
<td>( A = \sin^{-1}(1 - (h/H)^{1/2}) )</td>
<td>EDOH = ( \exp(-DBH/H) )</td>
<td></td>
</tr>
<tr>
<td>( Q )</td>
<td>DX = DBH(^1! - (h/H)^{1/2} / (1 - (4.5/H)^{1/2}) )</td>
<td></td>
</tr>
<tr>
<td>( Z^2 = (h/H)^{1/2} )</td>
<td>RZ = 1/[(DBH/H) + (h/H)]</td>
<td>BA = basal area acre(^{-1})</td>
</tr>
</tbody>
</table>
| \( ZR = \ln(h/H) + 0.001 \) | DBH\(^2 = DBH^2 \) | SDI = stand density index; = TPA *
| \( LZ = \ln(h/H) + 0.001 \) | DBH\(^2 = DBH^2 \) | *(QMD/10)^{1/3} |
| \( X^1 = (1 - (h/H)^{1/2}) / (1 - (4.5/H)^{1/2}) \)\(^{1/10} \) | CR = [(H-HTCB)/H] | RD = relative density |
|                         | CL = H-HTCB | BA/ (QMD)^{1/2} |

Once the best subset of variables was determined for the exponent, weighted linear (WLS), non-linear (NLS), and weighted nonlinear least-squares (WNLS) regressions were fit to the pooled data to determine the best form of the profile model. Selection of the "best" model form was based on residual plots and Furnival's Index (Furnival 1961), which essentially compares the residual mean square error of the models, after they have
been adjusted for any transformations. After the model form was determined, a model containing only relative height variables was fit for each tree and the resulting parameter estimates and their variances were tabulated and grouped by growing-stock level.

3.2.4.4 Hypothesis Testing

The collection of stem form parameters were treated as multivariate random variables representative of tree form, and were examined for significant differences among GSL treatments in a MANOVA. Pairwise significant differences in form between any two GSLs were tested by Hotelling’s $T^2$ statistic.

The additional explanatory power of tree attributes and stand density was then explored with the pooled data set, temporarily ignoring the autocorrelation between observations within a tree. Once a set of strong predictors of the form parameters was determined through all-subsets model selection, parameters in a seemingly unrelated regression (SUR) system-of-equations were predicted simultaneously in equations that were weighted with parameter variance estimates obtained from the individual tree fits. Various combinations of tree and stand density variables were introduced as exogenous predictor variables to assess effects of stand density regime on stem form. Because many of the potential tree variables themselves were expected to respond to stand density regime, differences in these predictor variables among treatments were also tested by one-way analysis of variance.

To assess any direct treatment effects on stem form, indicator variables for each GSL were tested for significance in the system by forcing them into the structural equations both with and without the addition of any other tree or stand-level variables. First, the SUR system was fit with only GSL indicators ($I_i; i=1,2,3,4$ or 5) and GSL
continuous variables to determine if the individual tree profile model parameter estimates could be predicted as a function of the discrete treatments. The equations were

\[ a_1 = c_0 + c_1I_2 + c_2I_3 + c_3I_4 + c_4I_5 \]  \hspace{1cm} [\text{Eq. 3-7}] \\
\[ a_2 = c_5 + c_6I_2 + c_7I_3 + c_8I_4 + c_9I_5 \]  \hspace{1cm} [\text{Eq. 3-8}]

where

\[ I_2 = \{1 \text{ if } GSL=2, \ 0 \text{ otherwise} \} \]
\[ I_3 = \{1 \text{ if } GSL=3, \ 0 \text{ otherwise} \} \]
\[ I_4 = \{1 \text{ if } GSL=4, \ 0 \text{ otherwise} \} \]
\[ I_5 = \{1 \text{ if } GSL=5, \ 0 \text{ otherwise} \} \]

In the system illustrated above, GSL 1 is the reference level to which all of the other GSLs are compared. Because the goal was to determine if differences existed between all five GSLs, this system-of-equations was fit five different times with each successive (i=1-5) GSL used as the reference level as an alternative to computing the corresponding linear contrasts.

In addition to the tree- and stand-level effects on stem form, two possible sources of confounding were examined. First, a trend in initial height with pre-treatment conditions was observed before the initial thinning (Table 3-1). To account for possible influences of initial height or site differences in 1966 on the results in 1991, the height of the largest (by DBH) forty trees per acre in 1966 was introduced into the SUR system.

Second, one plot each in GSL 4 and 5 had an initial component of lodgepole pine which was discriminated against in subsequent thinnings. Although some lodgepole pine remained in 1991, the relative amount clearly changed over time. Thus, plot-level indicator variables for those plots containing any lodgepole were introduced into the SUR system to assess whether the competitive effects (on stem taper) of a given unit of
lodgepole pine bole area differed significantly from the effects of an equivalent unit of larch bole area.

3.3 Results

3.3.1 Stem Profile Model Construction

Over 90% of the variation in relative diameter (d/DBH) was explained by transformations of relative height (h/H) in the pooled dataset. Similar predictive ability has been observed by other investigators (Max and Burkhart 1976; Kozak 1988; Tasissa and Burkhart 1998). A two-variable exponent with the relative height variables AQZ and Z^2 (Table 3.5) was selected for the variable exponent model in this study (R^2 = 0.967).

Although transformations of the relative height variable Z explained much of the variation in relative diameter, the other all-subsets runs indicate that Z^2 yields similar results when tree variables DBH, DOH and CR (R^2 = 0.974, 0.972 and 0.971) were introduced into the model. The model was limited to two-variables because, in general, three-, four- and five-variable models provided little increase in R^2 (maximum R^2 of 0.98).

The best form for the variable exponent profile model was a nonlinear model. Initially, Furnival’s Index and residual plots indicated that the nonlinear model fit the data better than the log-linear and weighted nonlinear version. However, when the nonlinear model form was fit to each individual tree, the residual plots indicated the presence of heteroscedastic variance. Thus, a weighted nonlinear version was used to meet the assumptions of constant variance within a tree. The selected model, which was weighted by the quantity \( 1 / [(1 - (h/H)^{1/2}) / (1 - (4.5/H)^{1/2})] \), follows:
\[ d/DBH = \left[ \frac{1-\left(\frac{h}{H}\right)^{1/2}}{1-\left(\frac{4.5}{H}\right)^{1/2}} \right]^c \quad [Eq. 3-9] \]

where \( c = a_1 \sin^{-1}\left(\frac{h}{H}\right) + a_2 \left(\frac{h}{H}\right)^2 \) \quad [Eq. 3-10]

The two model parameters were significantly different from zero for all 134 sample trees (\( \alpha = 0.05 \)).

### 3.3.2 Comparisons of Stand-Level Effects on the Mean Difference Vector of Stem Form

The MANOVA indicated that 25 years of thinning has created significant differences in stem form between at least two of the growing-stock levels (Wilk’s Lambda = 0.546; p-value < 0.0001). Considering the estimated profile model parameters for trees within a treatment to be bivariate random variables, treatment effects were explored by MANOVA with the model

\[ a_{1ij} = f(GSL_i) \]
\[ a_{2ij} = g(GSL_i) \quad [Eq. 3-11] \]

where \( a_{1ij} \) and \( a_{2ij} \) are the profile model parameter estimates for the \( j^{th} \) tree of the \( i^{th} \) GSL.

Hotelling’s \( T^2 \) identified significant differences in stem form between GSL 1 and all other GSLs and between GSL 2 and all other GSLs (Table 3-6).

Because mean vectors are being compared, it is possible for significance to be detected between vectors if even only one of the elements of the mean vector is different from the corresponding element of the other mean vector (Johnson and Wichern 1992). Figure 3-1 illustrates the effects of thinning on stem form with a bivariate plot of \( a_2 \) on \( a_1 \) for each GSL.
Table 3-6 Multivariate pairwise comparisons of between-GSL differences in stem profile model parameter estimates.

<table>
<thead>
<tr>
<th>GSL Comparison</th>
<th>Hotelling's $T^2$ Statistic</th>
<th>Significant Difference ($\alpha=0.05$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSL1 vs. GSL2</td>
<td>17.815</td>
<td>***</td>
</tr>
<tr>
<td>GSL1 vs. GSL3</td>
<td>26.647</td>
<td>***</td>
</tr>
<tr>
<td>GSL1 vs. GSL4</td>
<td>30.563</td>
<td>***</td>
</tr>
<tr>
<td>GSL1 vs. GSL5</td>
<td>27.157</td>
<td>***</td>
</tr>
<tr>
<td>GSL2 vs. GSL3</td>
<td>10.792</td>
<td>***</td>
</tr>
<tr>
<td>GSL2 vs. GSL4</td>
<td>15.078</td>
<td>***</td>
</tr>
<tr>
<td>GSL2 vs. GSL5</td>
<td>12.996</td>
<td>***</td>
</tr>
<tr>
<td>GSL3 vs. GSL4</td>
<td>0.822</td>
<td>n.s.</td>
</tr>
<tr>
<td>GSL3 vs. GSL5</td>
<td>0.678</td>
<td>n.s.</td>
</tr>
<tr>
<td>GSL4 vs. GSL5</td>
<td>0.516</td>
<td>n.s.</td>
</tr>
</tbody>
</table>

Figure 3-1 Estimated $a_2$ vs. $a_1$ random profile model coefficients
3.3.3 *SUR System-of-equations Estimation for Stand-Level Effects*

The SUR system for predicting the $a_1$ and $a_2$ parameters for the profile model was fit with only indicator variables ($I_i$) representing GSL. The parameter estimates for each GSL indicator in the two structural equations resulted in three possible outcomes: neither parameters are significantly different from zero, both are significantly different from zero, or only one is significantly different from zero. Because the pair of variables is used to describe the form of the individual tree, differences between the reference GSL and the GSL in question exist if either one or both of the coefficients for that GSL are significantly different from zero. The parameter estimates and p-values for the structural equations predicting $a_1$ and $a_2$ are given in Table 3-7 and Table 3-8, respectively. The cross-equation correlation of residuals of the structural equations was considerable, reinforcing the desirability of the SUR system estimation method (Table 3-9). The average value of $a_1$ and $a_2$ for each GSL is shown in Figure 3-2 and Figure 3-3.

<table>
<thead>
<tr>
<th>G6L</th>
<th>G6L 1</th>
<th>G6L 2</th>
<th>G6L 3</th>
<th>G6L 4</th>
<th>G6L 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>G6L 1</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G6L 2</td>
<td>-0.1587</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G6L 3</td>
<td>-0.0896</td>
<td>0.0691</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0309)</td>
<td>(0.1065)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G6L 4</td>
<td>-0.1850</td>
<td>-0.0263</td>
<td>-0.0954</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.5280)</td>
<td>(0.0159)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G6L 5</td>
<td>-0.1830</td>
<td>-0.0243</td>
<td>-0.0934</td>
<td>0.0020</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.5747)</td>
<td>(0.0236)</td>
<td>(0.9597)</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 3-8 Parameter estimates for the structural equation that predicts $a_2$ while accounting for cross-equation correlation with the SUR approach for parameter estimation. P-values are in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>GSL 1</th>
<th>GSL 2</th>
<th>GSL 3</th>
<th>GSL 4</th>
<th>GSL 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSL 1</td>
<td>--</td>
<td>--</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GSL 2</td>
<td>-0.0576</td>
<td>--</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1205)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GSL 3</td>
<td>-0.2079</td>
<td>-0.1503</td>
<td>--</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GSL 4</td>
<td>-0.1750</td>
<td>-0.1174</td>
<td>0.0329</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0005)</td>
<td>(0.2339)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GSL 5</td>
<td>-0.2068</td>
<td>-0.1492</td>
<td>-0.0012</td>
<td>-0.0318</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.9694)</td>
<td>(0.2873)</td>
<td>--</td>
</tr>
</tbody>
</table>

Figure 3-2 GSL-average $a_1$ random coefficients in 1991
3.3.4 SUR System-of-Equation Estimation for Tree-Level Effects

All-subsets regression indicated that $a_1$ is best predicted by $EDOH$ (that is, $\exp(-DBH/H)$; $R^2=0.78$) and $a_2$ by $CR$ (crown ratio; $R^2=0.90$). In addition to $EDOH$, variation in the $a_1$ parameter was associated with $QMD$ as a second variable, and $SDI$, $RD$ and $BA$ as three alternative third predictors. Additional variation in the $a_2$ form parameter was predicted best by $H$, $DBH$, and $DOH$ as three alternative second variables. However,
extra-sums-of-squares F-tests indicated that none of the models with more than one variable provided significant additional predictive power for either of the structural equations.

The initial parameter estimates for the structural equations were determined using OLS. Models were then fit with NLS to estimate the cross-equation covariance matrix, and the SUR system was fit using generalized nonlinear least squares (GNLS) to correct for the cross-equation correlation of errors (Borders 1989). The final structural equations for predicting the parameter estimates of the profile model were

\[ a_1 = b_0 \times EDOH^{b_1} \]  

[Eq. 3-12]

\[ a_2 = b_2 \times CR^{b_3} \]  

[Eq. 3-13]

Parameters were significantly different from both zero and one (\( \alpha = 0.05 \); Table 3-10). Correlation between the two equations was considerable, as expected, again reinforcing the need for adjustment, which is accomplished by SUR (Table 3-11).

Table 3-10  Parameter estimates and their standard errors for the structural equations estimated independently by NLS and simultaneously in the SUR system-of-equations with GNLS

<table>
<thead>
<tr>
<th>Nonlinear Least Squares (NLS)</th>
<th>Generalized Nonlinear Least Squares (GNLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Estimate</td>
</tr>
<tr>
<td>( b_0 )</td>
<td>0.2616</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>-5.5701</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.1353</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>0.3915</td>
</tr>
</tbody>
</table>
Table 3-11  Cross-equation residuals correlation matrix for the SUR system with structural equations developed from exogenous variables EDOH and CR

<table>
<thead>
<tr>
<th></th>
<th>a₁</th>
<th>a₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₁</td>
<td>1.0000</td>
<td>-0.1645</td>
</tr>
<tr>
<td>a₂</td>
<td>-0.1645</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Analysis of variance for EDOH and CR across GSLs indicated that these variables differed significantly between at least two of the GSLs. To determine exactly which treatments differed from the others, Tukey’s HSD was done for ten comparisons, controlling for the experimentwise error. EDOH differed significantly (α = 0.005) among all GSLs except for GSL 5 vs. 4 and 3 vs. 2 (Table 3-9). CR differed significantly (α = 0.005) among all GSLs except 4 vs. 5 (Table 3.10).

The effects of treatment on the average values of EDOH and CR are apparent in Figure 3-2 and Figure 3-3. Because initial tree dimensions were suspect (Table 3-1), CR and EDOH were compared for the largest 40 trees per acre in 1966 to determine if differences existed initially. The comparison detected no significant difference in CR among GSLs, and EDOH differed significantly only between GSLs 3 and 5, but not among any of the other GSLs.
Table 3-12 Tukey's HSD multiple comparisons of EDOH by GSL

<table>
<thead>
<tr>
<th>GSL Comparison</th>
<th>Difference Between Mean EDOH</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 - 4</td>
<td>0.0094</td>
<td>0.1247</td>
</tr>
<tr>
<td>5 - 3</td>
<td>0.0232</td>
<td>0.0001</td>
</tr>
<tr>
<td>5 - 2</td>
<td>0.0327</td>
<td>0.0001</td>
</tr>
<tr>
<td>5 - 1</td>
<td>0.0569</td>
<td>0.0001</td>
</tr>
<tr>
<td>4 - 3</td>
<td>0.0138</td>
<td>0.0061</td>
</tr>
<tr>
<td>4 - 2</td>
<td>0.0233</td>
<td>0.0001</td>
</tr>
<tr>
<td>4 - 1</td>
<td>0.0474</td>
<td>0.0001</td>
</tr>
<tr>
<td>3 - 2</td>
<td>0.0095</td>
<td>0.1512</td>
</tr>
<tr>
<td>3 - 1</td>
<td>0.0337</td>
<td>0.0001</td>
</tr>
<tr>
<td>2 - 1</td>
<td>0.0242</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table 3-13 Tukey's HSD multiple comparisons of CR (%) by GSL

<table>
<thead>
<tr>
<th>GSL Comparison</th>
<th>Difference Between Mean CR</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 2</td>
<td>12.280</td>
<td>0.0001</td>
</tr>
<tr>
<td>1 - 3</td>
<td>21.600</td>
<td>0.0001</td>
</tr>
<tr>
<td>1 - 4</td>
<td>29.900</td>
<td>0.0001</td>
</tr>
<tr>
<td>1 - 5</td>
<td>32.859</td>
<td>0.0001</td>
</tr>
<tr>
<td>2 - 3</td>
<td>9.320</td>
<td>0.0001</td>
</tr>
<tr>
<td>2 - 4</td>
<td>17.620</td>
<td>0.0001</td>
</tr>
<tr>
<td>2 - 5</td>
<td>20.579</td>
<td>0.0001</td>
</tr>
<tr>
<td>3 - 4</td>
<td>8.300</td>
<td>0.0004</td>
</tr>
<tr>
<td>3 - 5</td>
<td>11.259</td>
<td>0.0001</td>
</tr>
<tr>
<td>4 - 5</td>
<td>2.959</td>
<td>0.5629</td>
</tr>
</tbody>
</table>
Figure 3-4 GSL-average EDOH in 1991

Figure 3-5 GSL-average CR (%) in 1991
3.3.5 *SUR for Testing Redundancy of Tree Covariates and Indicators for GSL*

To determine the marginal effect of treatment level after accounting for tree-level effects of EDOH and CR, the GSL indicator variables were included with EDOH and CR to produce the following SUR system:

\[
a_1 = d_0 + d_1 \text{EDOH} + d_3 I_2 + d_4 I_3 + d_5 I_5 \quad \text{[Eq. 3-14]}
\]

\[
a_2 = d_6 + d_7 \text{CR} + d_9 I_2 + d_{10} I_3 + d_{11} I_5 \quad \text{[Eq. 3-15]}
\]

The stand-level indicator variables provided no additional predictive power once the effects of EDOH and CR were included in the system.

The following SUR was performed to address the potential confounding effects of larger mean pre-treatment tree size in heavier thinning regimes. To account for possible initial height (or site quality) influences, the variable $H_{40(1966)}$ was added to with equations 3-12 and 3-13.

\[
a_1 = e_0 \times \text{EDOH}^{e_1} \times H_{40(1966)}^{e_2} \quad \text{[Eq. 3-16]}
\]

\[
a_2 = e_3 \times \text{CR}^{e_4} \times H_{40(1966)}^{e_5} \quad \text{[Eq. 3-17]}
\]

Insignificant parameter estimates observed for the height-site quality covariates indicated that the initial trends observed in Table 3-1 had little effect on the general relation between stem form and both DBH/H and crown ratio.

To account for differences in the stem form development of larch on the plots that contained lodgepole pine, an additional variable was introduced into the SUR to account for possibly different effects of lodgepole pine. PL1 and PL3 (PL=Plot Lodgepole) were indicators for the two plots containing lodgepole. Thus, equations 3-12 and 3-13 were modified as:
\[ a_1 = f_0 \cdot EDOH^{F_1} \cdot PL_1^{F_2} \cdot PL_3^{F_3} \]  
\[ a_2 = f_4 \cdot CR^{F_5} \cdot PL_1^{F_6} \cdot PL_3^{F_7} \]  

[Eq. 3-18]  
[Eq. 3-19]

The plot level indicator variables for those plots containing lodgepole were not statistically significant in this SUR system. This indicates that although some lodgepole pine existed on two plots in 1991, the stem form development of the larch was not significantly influenced by mixed species composition.

3.3.6 Summary of Stem Form Results

Since the larger trees in a stand typically have larger crowns than other trees from the same stand, the dominant trees are expected to show the greatest response in stem form changes induced by 25 years of thinning. Figure 3-6 shows the relative stem profile of the average of the largest (by DBH) 40 trees per acre. The curves reflect similarities of both form and taper among GSLs 4 and 5. A slight difference is noticeable between GSL 3 and GSLs 4 and 5, while differences between GSL 3 and GSLs 1 and 2 are obvious. The stem profile of trees in GSLs 1 and 2, the lower of the five density levels, are obviously different from each other, and are also different from GSLs 3, 4 and 5. The taper curve for GSL 1 has markedly different shape than the other curves, with acute taper beginning in the lower 20% of the bole. GSL 2 shows similar shape to those of GSLs 3-5 up to about 30% of total height, at which point the taper curve diverges.
Figure 3-6 Relative stem profile of the largest 40 trees/acre by GSL in 1991

Crown size is sensitive to stand density regime (Table 3-10), and is a significant predictor of stem form (Table 3-7). The effects of CR on stem form may be observed in a comparison of the stem profiles of a tree of a given DBH and H, but that would differ by CR in response to different stand density regimes. Figure 3-7 displays the absolute stem profile of the tree of the study-wide average DBH and H, 12 inches and 75 feet in this case, but with GSL-specific crown ratio. A tree with these dimensions would be of different relative position in stands of different densities; suppressed in the low-density stands and codominant or dominant in the medium and high density stands. Thus, although Figure 3-7 compares a tree of the “best” form in the lower density treatments with those of the “worst” form in the higher densities, absolute differences are still apparent. The insignificant difference of CR between GSLs 4 and 5 is reflected by almost identical profile curves.
Figure 3-7 Absolute stem profile of the study-wide average tree with GSL-specific CR.

Although obvious differences in form appear between the larger trees (Figure 3-6) and from the influence of crown size (Figure 3-7), perhaps the best overall comparison of the effects of thinning regime on stem form is provided by observing the relative profile curves of the GSL-specific average trees (Figure 3-8) rather than the 40 largest. This comparison reveals any differences between GSL-average form that may be obscured by only considering the larger trees (Figure 3-6), or by not accounting for relative position (Figure 3-7). Similar to the curves in Figure 3-6, GSLs 1 and 2 show obvious differences between each other and GSLs 3-5. However, when trees of all crown classes are considered, the profile curve for GSL 2 indicates that the rate of taper on the lower bole is less than in GSL 1, but that it increases to about the same rate at about 70% of total height.
Figure 3-8 Relative stem profiles for the GSL-average tree in 1991.

The effects of thinning regime on the volume produced by the average tree in each GSL is provided by observing differences in the curves of absolute stem profile. Figure 3-9 shows the differences in stem form and first-log scaling diameters (diameter at the top of 16- to 40-ft butt logs) of the GSL-specific average tree. The GSL 1 curve shows that trees at this low stand density have considerably larger basal diameters, volume and average rate of taper than trees from the other four GSLs. The only significant difference in final average height among GSLs was that of GSL 1. Although the rate of taper is most extreme in GSL 1, the increased growth capacity associated with the low density of this GSL has allowed a larger amount of growth to develop over the entire bole. GSLs 2-5 have similar absolute diameters above 60 feet, and thus similar rates of taper. Below 60 feet, the influence of stand density is evident in the differences in form among these GSLs.
3.4 Discussion

Although treatment effects on stem form were confirmed by two separate statistical methods, an important caveat concerning the interpretation of the results should be noted. Table 3-1 suggested that a correlation between initial tree size and final density regimes exists. Of greatest import were the trends in initial DBH and H. The causes of the trend in initial size differences among density regimes is unknown. However, a possible height or site quality influence on form was examined, but failed to demonstrate any effect. Thus the assumption was made that the pretreatment conditions had a relatively minor effect relative to thinning on the post-treatment development of stem form in the sample trees.

An additional potential problem was the fact that repeated entries were made into the plots between 1966 and 1991, hence, different thinning regimes could conceivably
yield different results. However, given that the null hypothesis of no treatment effects was rejected, the net conclusion is that stand density regime can affect stem form. Differences in stand density regime cannot be concluded to necessarily lead to differences in stem form (for example, GSL 4 and 5 did not differ). It is also conceivable that similar final stand densities arrived at under a different regime may produce greater or lesser stem profile differences than those shown here.

3.4.1 LOGS Volume Estimation

Numerous LOGS studies have examined tree and stand developmental differences under varying stand density regimes. For example, DBH and total height are common tree variables of interest, and QMD, volume/acre and mortality/acre are stand variables typically assessed. Stem volume production is typically quantified by applying plot-specific volume equations. Plot-wise estimation of individual tree volume explicitly recognizes the expected differences in stem form among stand density regimes. Since standard volume equations are a function of only diameter and height, expected stem form differences would not enter into volume estimates if one equation was applied across density regimes.

3.4.2 Crown Response and Relation to Stem Form

Many other important tree and stand structural features that may be important in either a timber or non-timber context have been shown to respond to stand density manipulation. For example, the control of crown size through changing stand density manipulations is well documented (Curtis and Reukema 1970; Oliver 1979; Curtis and Marshall 1986; Curtis 1992; Cochran and Seidel 1996). Crown size and canopy structure
are considered important parameters of wildlife habitat and of hydrological behavior, but they have implications for timber management as well. For example, numerous past volume and taper equations have successfully incorporated crown size as a predictor (Burkhart and Walton 1985; Newberry and Burkhart 1986; Walters and Hann 1986; Valenti and Cao 1986; Hann et. al. 1987; Real and Moore 1988; Morris et. al. 1994; Muhairwe 1994). Similarly, crown structure has important implications for several ecophysiologica processes that control net production.

3.4.3 *Stem Form Hypotheses*

Several theories regarding the mechanisms behind stem form-crown size correlations have been developed to explain the dynamics of wood increment on the bole. Two functional concepts attempting to explain the crown influence on stem form are the mechanistic and water conduction theories as summarized by Larson (1963). The mechanistic theory of stem development describes the stem as a beam of uniform strength in which the bole assumes a form that resists the vertical stresses produced by the weight of the live crown and attached snow and ice, and the lateral bending stresses produced by wind. The water conduction theory holds that the rate of water conductance at a given height on the stem is regulated by the transpirational requirements of the foliage mass above that height. In their study of stem growth, Duff and Nolan (1953) present a theory based on nutrient gradients to and from the point of maximum growth near the live crown base. The growth rate of the stem at a given height should be a function of the amount and type of foliage in the crown above that point. The hormonal theory describes radial growth as a function of the rate of change in auxin content through the bole, which in turn is controlled by the size, shape and vigor of the crown (Panshin and de Zeeuw 1980). Larson
(1963) indicates that these theories are all valid in many respects and under certain circumstances, but have also been disputed by other investigators.

Although a widely accepted theory regarding mechanisms by which crown size is linked to stem form does not exist, some general empirical results are consistent. Typically, trees that develop in denser stands, or that are of the suppressed crown class, have relatively shorter crowns and maintain a more cylindrical form below the live crown. Trees of the dominant and codominant crown classes, and those that grow in low-density stands, develop a larger amount of cambial growth increment down the bole below the live crown. In all cases, the maximum rate of taper occurs in the region of the live crown. The net result is that the shift of radial growth increment, and hence stem form and taper development, is “controlled” by the stand density regime, as mediated by live crown responses to changing stand density.

The fact that stand density directly regulates crown size and indirectly influences stem form prompts the question: If a functional relation exists between stem form and crown size, do crown size and its surrogates adequately account for stem form differences among stand density regimes? The lack of significance observed for the GSL indicator variables in both of the second-stage coefficient prediction equations (3-14 and 3-15), after accounting for CR and DBH/H, suggests that in fact density effects are adequately explained through crown size.

Stem profile models have usually been developed with the intent of predicting upper-stem diameters in an operational setting. Although these models produce unbiased parameter estimates, the estimated variances of parameters are unreliable due to autocorrelation, and thus inferences made about the statistical significance of the parameter estimates and the corresponding effects of different variables are inappropriate. Profile curves are preferable to simpler form indices for comparisons of stem form because they
depict the shape of the entire bole. The MANOVA and SUR procedures applied in this study accommodated the autocorrelation and identified significant differences in stem form among the tested stand density regimes, as mediated by the response of DBH, height, and crown length.
The stem form of western larch is responsive to stand density manipulations, but heavy thinnings are required to evoke a treatment response. Although heavy thinnings produced the largest average trees with the largest average per tree merchantable volumes, intermediate thinnings yield as much or more merchantable volume per unit area because these stands have more continuous site occupancy than heavily thinned stands. Thus, intermediate thinning levels produce substantial merchantable volume yields, and trees of slightly smaller mean size, but of better form, than heavier thinning intensities.

While mean differences in stem form may be detected as a function of discrete stand density levels, the same variation in stem form may be accounted for by continuous tree-level variables, in this case crown ratio and DBH/H. The former has great practical significance since it implies that crown ratio may be an effective integrator of past stand density regime and its cumulative effect on stem form.
BIBLIOGRAPHY


