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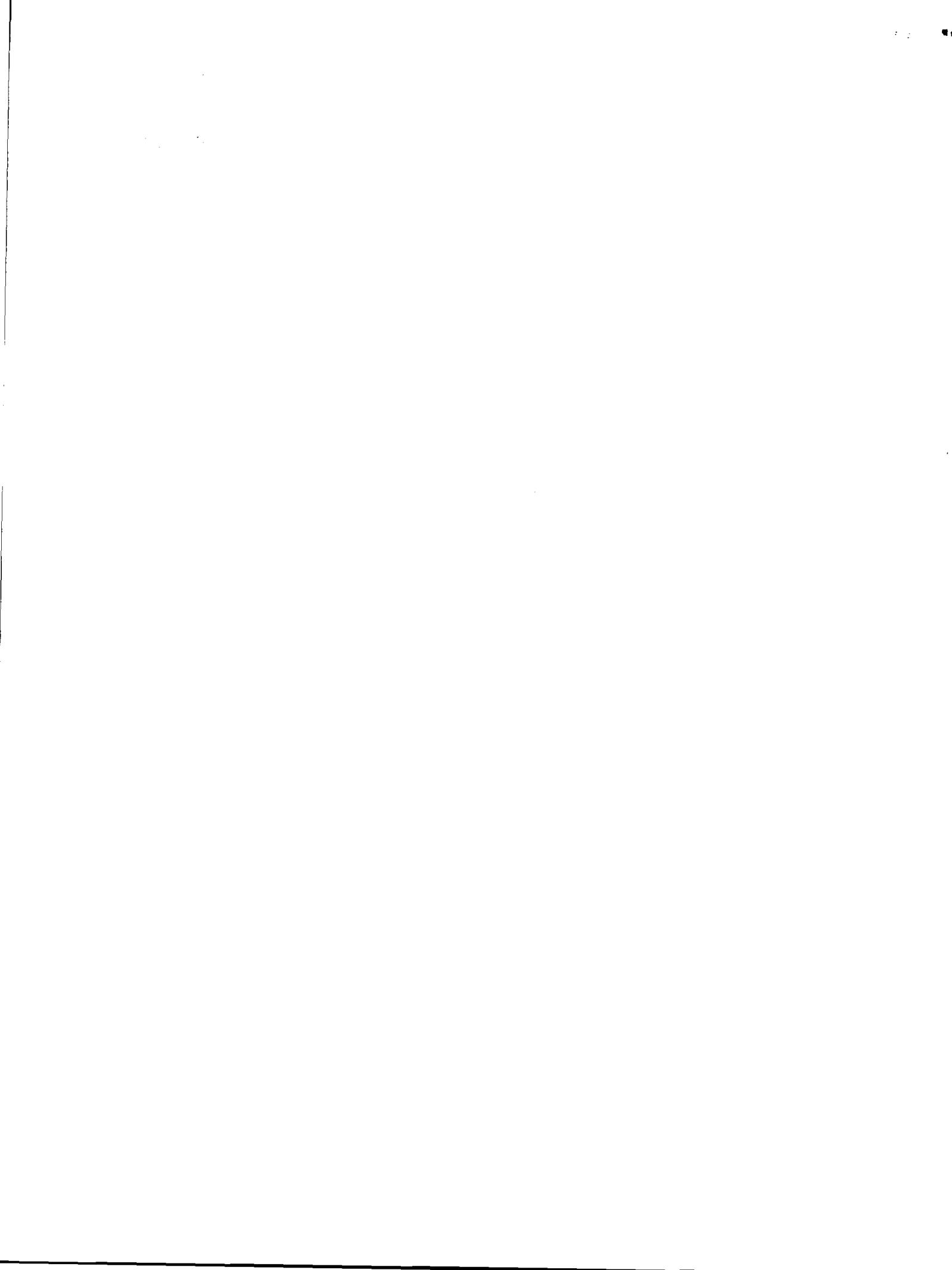
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STATISTICAL DESCRIPTION OF SHEAR STRENGTH  
IN RESIDUAL SOILS

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STATISTICAL DESCRIPTION OF SHEAR STRENGTH IN  
RESIDUAL SOILS<sup>1</sup>

M.R. Schoenemann and M.R. Pyles<sup>2</sup>

The endpoints of most forest roads are dictated by the requirements of the timber harvesting section of the forest management plan. The general route is also dictated by timber harvesting requirements, but some flexibility typically exists in the location of the specific route. It is the challenge of the road designer to develop the specific route and design that will result in the lowest construction, transportation, and maintenance costs. Selecting the specific route and design to minimize slope failures will typically result in the lowest-cost road. Minimizing slope failures should also result in the smallest adverse impacts on soil, water, and fishery resources.

The design of stable slopes by the geotechnical engineer is typically approached in a site-specific deterministic manner. When virtually all the cut and fill slopes along a forest road alignment are candidates for failure, this approach is unworkable. A probabilistic approach to the design of slopes appears to offer promise in dealing with forest road cut and fill slopes. In order to develop a probabilistic design for a given forest road, statistical descriptions of the parameters that influence stability must be developed. The two parameters whose variabilities have the greatest influence on the stability of a slope are soil strength and groundwater conditions (Gray and Megahan, 1981). The objectives of the work reported here were to develop a rational method of statistically treating site-specific soil strength, and to examine the use of "geostatistical" techniques (Journel and Huijbregts, 1978) to describe the spatial variability of soil strength. The focus of the work was on residual soils since they are commonly encountered in the forest environment, and appear to exhibit a greater variability in properties than do most transported soils (Sowers, 1963).

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**Abstract:** Any probabilistic approach to slope stability assessment requires an adequate statistical description of soil strength. Current statistical methods of strength description are unsuitable for this purpose. A statistical technique of soil strength description is developed here which is appropriate for the probabilistic assessment of slope stability. The technique is applied to two sets of strength test results, representing soils derived from a variety of initial materials. Geostatistical techniques are used to characterize the spatial variability of soil strength, and to assess the adequacy of the sample data sets for representing each population.

CURRENT APPROACH TO STATISTICS OF SOIL STRENGTH

The typical approach to obtaining soil strength values for use in a particular analysis involves two basic steps. First, a failure theory is selected to describe the stress state that corresponds to failure. Second, strength test data from a particular type of testing believed to be appropriate are correlated with the conditions of the failure theory. The choice of the failure theory used may be on the basis of personal preference, adaptability to a particular method of analysis, or the apparent suitability of the theory to the strength test data. Typically, the selection is not made as a result of a rigorous analysis of particular strength test data.

The most common failure theory used today is the Mohr-Coulomb theory, illustrated in figure 1. The general statement of the theory specifies that the admissible states of normal stress ( $\sigma$ ) and shear stress ( $\tau$ ) lie within an envelope symmetric about the  $\sigma$  axis in  $\sigma, \tau$  space. Further, in order for the failure envelope for a particular soil to be unique, it must be defined in terms of the effective normal stress,  $\sigma'$ . The variation of the theory that is used most often in practice considers the boundary of the admissible-stress region (the failure envelope) to be linear, expressed by the equation,

$$\tau = c' + \sigma' \tan \phi', \quad (1)$$

where  $c'$  is the cohesion intercept of the envelope, and  $\phi'$  is the angle of that envelope from the horizontal.

In the most typical current method of statistical description of strength test results, a series of samples is obtained which is presumably representative of the area of interest. For the purpose of assessing the strength behavior of natural soils, these samples should be relatively undisturbed. The most common and the most versatile soil strength test is the triaxial test. For triaxial testing, cylindrical test specimens are trimmed from the samples. These specimens are then grouped on the basis of similarity inferred from sample location or appearance, or the results of index tests.

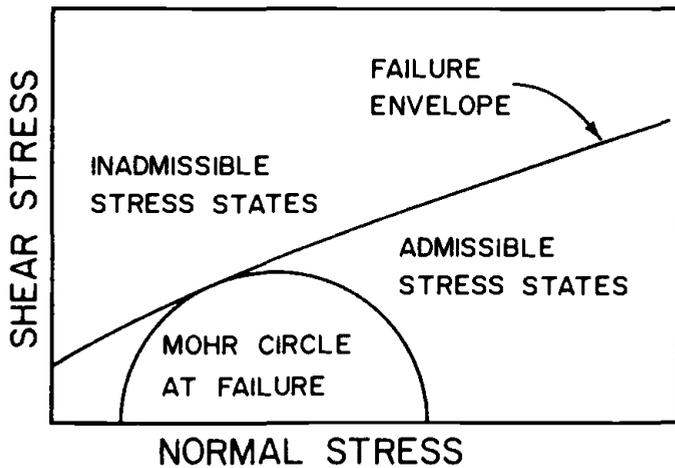


Figure 1--Schematic diagram representing the Mohr-Coulomb failure theory.

Within groups, each containing perhaps three or four specimens, each specimen is subject to a confining pressure different from the others, and then loaded axially to failure. For each specimen, a Mohr circle representing stress state at failure is plotted. For each group of specimens, one linear Mohr strength envelope is constructed tangent, ideally, to all of the failure Mohr circles of the group (fig. 2). This envelope has an angle  $\phi'$  and a cohesion intercept  $c'$ . The cohesion intercept may be equal to zero. Over the area of interest, strength is described by the means and standard deviations or ranges of the  $\phi'$  and  $c'$  values determined for the various groups of specimens.

There are several objections to this procedure. First, soils within "similar" groups may not be very similar in their strength behavior. The difficulty associated with fitting a tangent concurrently to three or more Mohr circles (fig. 2) reflects such within-group dissimilarity. In such cases (by no means rare), some sort of "averaging"

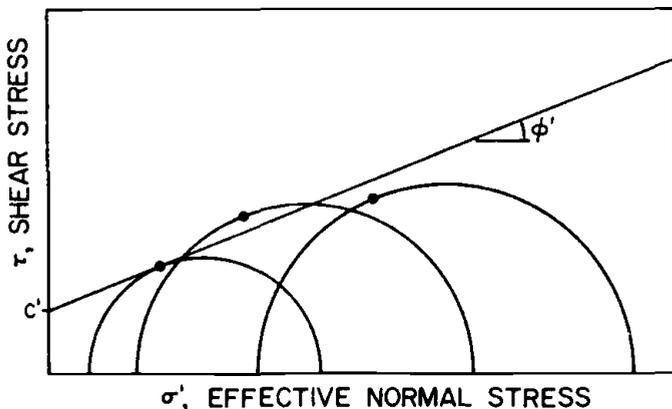


Figure 2--Construction of a Mohr envelope from the Mohr circles at failure of three triaxial strength tests.

results from the tangent-fitting process. As Handy (1976) suggests, the resulting "average" may be quite poor. This is illustrated in figure 3. Test specimens 1, 2, and 3 are all "similar", but have different strength. Specimen 1 has the largest strength, while specimen 3 has the smallest. If the specimens are tested at different confining pressures, one "average" strength envelope is obtained (fig. 3a). If however, the confining pressures for specimens 2 and 3 are reversed (fig. 3b), a very different "average" strength envelope is obtained. As can be seen, substantially different Mohr envelopes result from the chance assignment of these soils to the test confining pressures used.

If drawing one envelope tangent to several test circles is difficult, specimens are sometimes regrouped or disregarded on the basis of test results. Such second-guessing is another source of error.

The typical statistical approach to soil strength yields a confused picture of the strength relationships appropriate to the area of interest. As figure 4 illustrates, the typical expression of strength as  $\phi'_m \pm s_\phi$ ,  $c'_m \pm s_c$  may yield greatly differing pictures, depending on one's interpretation. Such a representation of strength cannot be used to develop confidence limits for purposes of probabilistic slope stability assessments.

This approach to the statistics of strength generates numbers which are easy to incorporate into stability equations, producing "limits" on the factor of safety. Such an approach is often acceptable in the design of engineering works, where past experience guides the selection of design safety factors. Such an approach is not suitable for the probabilistic assessment of the stability of forest roads. A.-Grivas (1977) points out that the relationship between factor of safety and probability of failure is a tenuous one, and cannot be generalized from one case to another.

Other statistical approaches to soil strength have been suggested. Well-founded proposals are advanced by A.-Grivas (1979), Vanmarcke (1977), and Wu (1974), among others, but they suffer from one or more of the following limitations for the present purpose:

- 1) These methods typically deal with soils whose strength behavior can be described by a single parameter. Usually, these are clays being analyzed for undrained failure, where  $S=c$  (that is,  $\phi=0$ ). As such, they are of little use in characterizing residual soils, whose strength behavior frequently must be described with two parameters,  $\phi'$  and  $c'$ .

- 2) Several of these approaches employ non-normal statistical distributions to describe the variability of soil strength. There are reasons for favoring such distributions. In particular, use of the normal distribution implies that there is

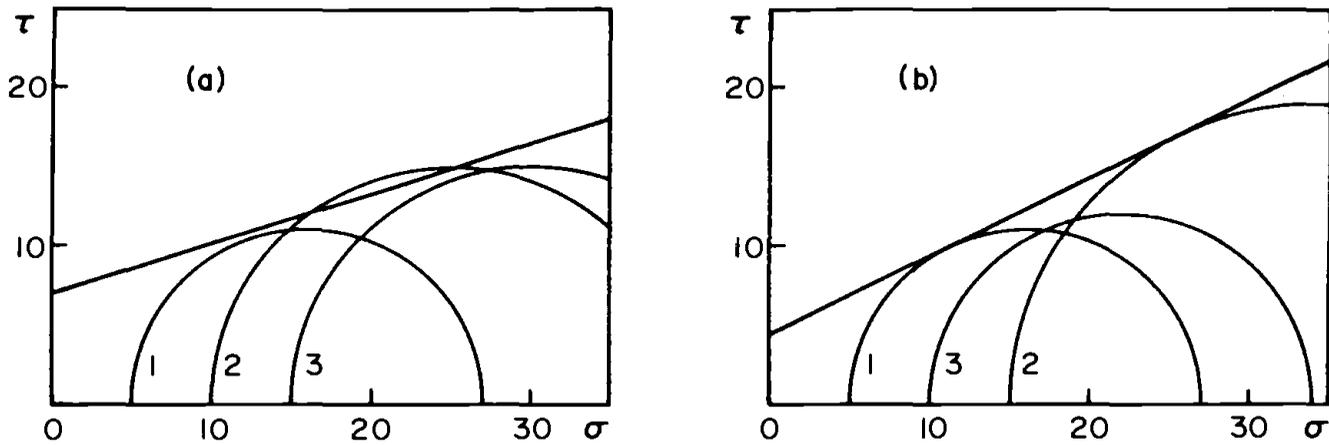


Figure 3--Hypothetical strength test results illustrating errors in strength envelope estimation which may arise from the grouping of test specimens. (After Handy, 1976)

a small but finite probability that a soil will exhibit a negative strength. Such an implication is inconsistent with our model of soil strength behavior. Accordingly, "tail-less" distributions would be preferred. Unfortunately, the properties of these distributions make them more complicated to work with. While this by itself is not enough to discard methods which use such distributions, it does render them less attractive than simpler methods. In many cases, normal statistics should suffice. Those approaches which do go to the trouble of using non-normal statistics deal only with single-parameter soils, rendering them unsuitable for characterizing the behavior of residual soils.

To be useful in the probabilistic assessment of forest road failure, a method of statistically describing soil strength should consider all strength data from an area of interest as a representative sample of the population, rather than data that are grouped by deliberate bias or an arbitrary method. Further, it should avoid forcing the data to fit into any preconceived model of strength behavior until the data themselves suggest such a model. Finally, it should allow the generation of prediction bands which, for a new observation, predict the range of strengths expected at a given normal stress. The approach presented in this paper accomplishes these objectives.

No standard statistical approach to soil strength currently exists. From 1958 to 1970, the American Society for Testing and Materials (ASTM) suggested a method involving the regression of  $\sigma_1'$  on  $\sigma_3'$  (effective major and minor principal stresses, respectively) to find a "best fit" Mohr envelope (Handy, 1981; Holtz and Noell, 1964). This procedure differs from that described in this paper in several respects. In particular, no

provision is made for generating confidence or prediction limits for the strength envelope.

#### PROPOSED STATISTICAL APPROACH

For the probabilistic assessment of forest slope stability, a statistical picture comparable to that presented in figure 5 is required. The traditional statistical approach to the strength of two-parameter soils (both  $\phi'$  and  $c'$ ) does not produce such a representation of strength. The approach proposed here provides the type of statistical description of shear strength required for probabilistic assessment of slope stability.

In brief, the proposed approach consists of several steps:

- 1) All of the test specimens obtained from the area of interest are examined as one group. Specimens are not grouped prior to statistical treatment on the basis of a presumed similarity. In this manner, the variability of strength over the area is more accurately accounted for.
- 2) Based on an appropriate failure criterion, one value each of  $p$  (average principal stress) and  $q$  (half deviator stress) at failure is determined for each test.
- 3) Regression of  $q$  on  $p$  is used to estimate a  $K_f$  line appropriate to the entire data set. If linear regression is not appropriate to the data set, curvilinear methods may be used.<sup>3</sup>
- 4) From the slope of the regression  $K_f$  line, a value of  $\phi'$  appropriate to the entire

<sup>3</sup>Throughout this paper, the term "linear regression" refers to linear regression in real (untransformed) space, while "curvilinear regression" refers to linear regression in transformed space.

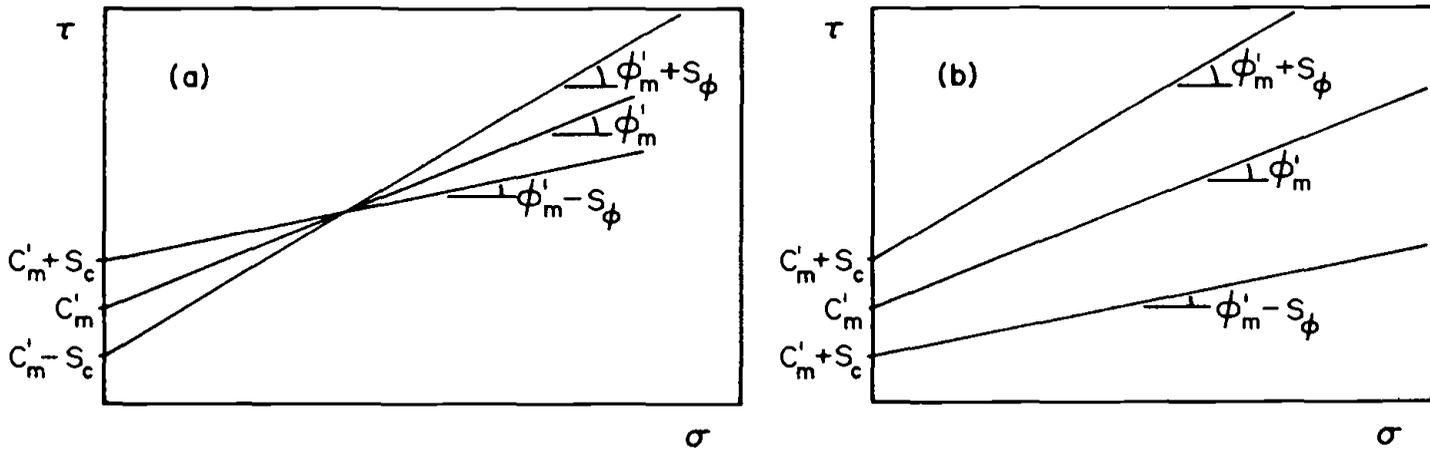


Figure 4--Depiction of the ambiguity associated with the traditional statistical approach to soil strength data. The mean and standard deviation values of  $\phi'$  and  $c'$  are equal between panels (a) and (b), but the two interpretations of these values yield radically different pictures.

data set is computed. If curvilinear regression was employed in step 3, this computation must be done on a piecewise basis.

5) The regression-estimated value of  $\phi'$  is used to compute a failure  $\sigma', \tau$  point associated with each failure p,q point. If curvilinear regression was employed in step 3, the computation of  $\sigma', \tau$  points must be done on a piecewise bases.

6) Regression of  $\tau$  on  $\sigma'$  produces an estimated Mohr envelope for the area of interest, and serves as the basis for generating prediction bands. Again, regression may or may not be linear.

This technique for obtaining a mean estimate of strength and prediction limits about that mean forces all variability into  $c'$ . In reality, it is likely that both  $c'$  and  $\phi'$  will have some variability. The magnitude of the difference

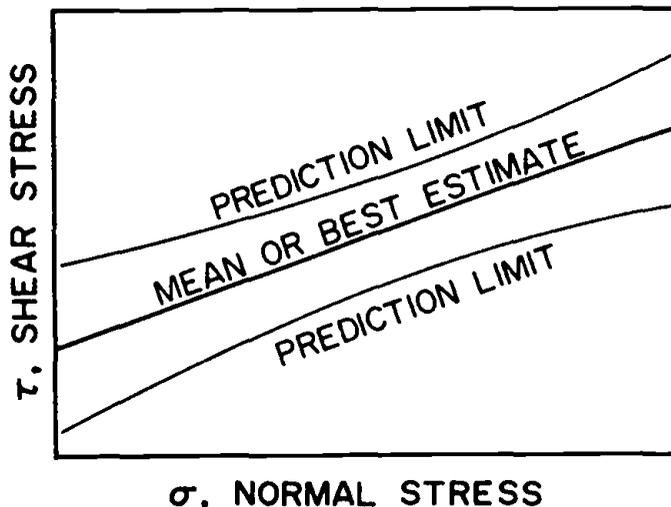


Figure 5--Schematic diagram of a statistical representation of soil strength suitable for probabilistic slope stability assessments.

between forcing all variability into  $c'$  and correctly partitioning variability between  $c'$  and  $\phi'$  is unclear. However, such effects should be of a second-order nature. Further research is pursuing this question.

An additional source of possible error is the regression of  $q$  on  $p$ . Handy (1976;1981) has noted that linear regression of  $q$  on  $p$  assumes that the direction of variability of  $q$  is vertical (perpendicular to the  $p$  axis). This is not true in general. Handy's proposed solution is to rotate the  $p, q$  axis system prior to regression, aligning the  $q$  axis with the direction of the stress paths of the tests. Following regression, the coordinate system is rotated back to its original configuration. This produces the proper regression  $K_f$  line.

While this approach is rigorous, it is only workable when all stress paths are aligned in the same direction. In practice, this is true of drained triaxial tests, only. In particular, undrained tests which straddle the preconsolidation pressure will produce radically different stress paths. There appears to be no means of accounting for the errors which may arise from a  $p, q$  regression for such tests.

#### GEOSTATISTICAL APPROACH

Geostatistics is the study of "regionalized variables", which are continuously distributed variables whose geographic variation is too complex to be described functionally (Journel and Huijbregts, 1978; Campbell, 1978). The behavior of a regionalized variable is both deterministic and random, in that the value of the variable at one point will be similar to the values at neighboring points, but the value at a point cannot be precisely determined from neighboring values.

Geostatistical techniques are widely employed in the mining industry, and for several years have been used by researchers in soil science (e.g.

Campbell, 1978; Bresler et al., 1984). The use of geostatistics in geotechnical engineering appears to be relatively new (Soulie et al., 1983).

A useful portion of geostatistics involves the graphical representation of the behavior of a regionalized variable on a plot called a "semivariogram". Campbell (1978) presents a useful summary of the generation and use of these plots. His explanation serves as the basis for the following discussion.

The variation of a regionalized variable is so complex that it cannot be completely described. The semivariance estimates the average rate of change of the variable over spatial distance. For a series of measurements of the variable,  $Y$ , the semivariance at lag or distance,  $h$ , between points is:

$$\gamma_h = \frac{1}{2N} \sum (Y_i - Y_{i+h})^2 \quad (2)$$

where  $N$  is the number of sample pairs at the spacing,  $h$ .

The semivariogram is a plot of semivariance vs. lag or distance (fig. 6). Ideally, at some distance (the "range"), the semivariance reaches a maximum value (the "sill"), which is maintained at larger lags. This sill value should approximate the variance of the population (Journel and Huijbregts, 1978).

The semivariogram reveals characteristics of the geographic distribution of the regionalized variable. The range indicates the size of a "zone of influence" in which values of the variable are statistically similar to the value at the center.

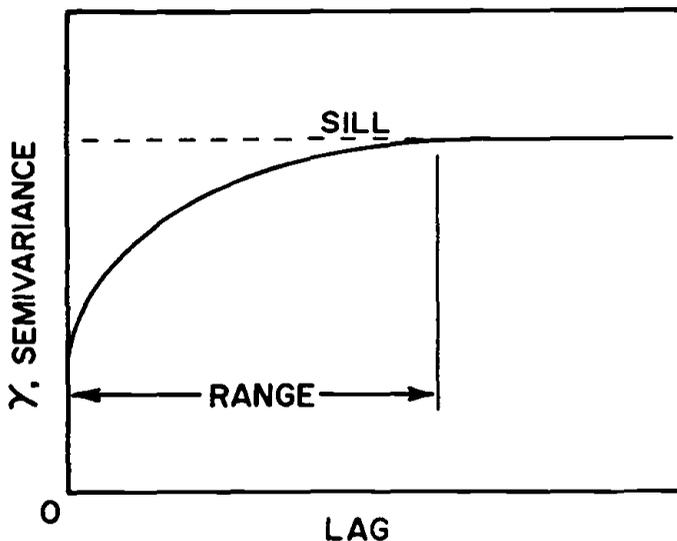


Figure 6--Idealized semivariogram. The semivariance  $\gamma$  increases with lag, reaching a plateau (sill) at some value of lag (the range).

To obtain statistically independent samples, sampling intervals should be no smaller than the range (Campbell, 1978).

In a rectangular sampling array with regular spacing, semivariance is typically computed by taking points selected to form a straight line. When samples are numerous, this approach allows the detection of anisotropic variance behavior. Such practice is routine in mining applications of the semivariance technique (Journel and Huijbregts, 1978). However, provisions can be made for the use of irregularly spaced and non-rectangular sampling.

When samples are not numerous (fewer than 30-50, perhaps), detection of anisotropy is unlikely (Bresler et al., 1984), and the small-size sample is best treated by examining the every pairing between points, regardless of direction. Although this produces semivariance values at many discrete lags, lags can be grouped into classes of similar lag values, each represented by a single semivariance value. This increases the number of sample pairs contributing to each semivariance value, increasing its precision (Journel and Huijbregts, 1978).

It should be noted that the number of sample pairs available to calculate a semivariance value is not the sole criterion of precision. In particular, one sample point far removed (geographically) from the main body of sampling can be paired with every other sample point. For convenience, all of these sample pairs would likely be grouped into one semivariance value. If the isolated point differed appreciably from the bulk of the data, that semivariance value would be markedly different from the rest of the semivariogram. In this case, the semivariance will have failed to achieve a sill value, and would indicate non-homogeneity of the sample.

## METHODS AND MATERIALS

### Site Description

The field study area consisted of 10 ha (25 ac) in the central Coast Range of Oregon. Slopes range from nearly level to 80% on south and east aspects. The area makes up a large portion of a clear-cut some 7 years old. The surrounding second-growth forest is principally Douglas-fir (*Pseudotsuga menziesii*) with some red alder (*Alnus rubra*). Present vegetation on the site is mainly herbaceous, but includes such low shrubs as salal (*Gaultheria shallon*) and blackberry (*Rubus spp.*), and patches of vine maple (*Acer circinatum*).

The site is located near the southwest end of a 380 km<sup>2</sup> (150 mi<sup>2</sup>) region of marine basalt. Soils surrounding the site are generally shallow, consisting of less than 1 m of stony or very stony loam and clay loam over weathered basalt.

Soils over much of the study area are somewhat deeper (1.3 m) and substantially less stony than is typical of the region, improving the possibility of obtaining relatively undisturbed samples for

strength testing. This admittedly introduced bias into the sampling process. However, current testing technology is not capable of producing high-quality test results from the stony soils typical of the region. At present, it appears to be impossible to determine the consequences of such sampling bias on the resulting statistical description of strength.

Although winters in the area are quite moist ( $\sim 200$  cm/y [80 in/y]), summers are dry. Whether vegetated or cleared, soils on all but North-facing slopes become quite dry, producing a substantial apparent over-consolidation.

### Sampling

In order to facilitate geostatistical analysis, a rectangular reference grid was established. Where possible, samples were taken at the nodes of the grid. However, many potential sample points proved to be unsuitable, due to the presence of large roots, stones, and large voids. Avoidance of such features resulted in an irregular sampling pattern which was not random.

The sampling procedure involved excavating a pit to the depth of interest. In general, this was the B horizon of the profile, below most of the roots and organic matter but above the C horizon, where rock fragments typically precluded sampling. Thin-wall Shelby tubes, 30 cm long x 7.11 cm diameter, were pushed into the soil by hand. Step-wise hand excavation of the surrounding soil aided the advance of the tube. In some cases, the soil below the tube's cutting edge was trimmed to approximate size by hand. Recovered samples were sealed, capped, and stored in their original sampling tubes. Storage times ranged from three to ten months in a humid room at 8°C.

### Triaxial Testing

Each relatively undisturbed soil sample was extruded from its Shelby tube directly into a stretched rubber membrane. Only the ends of each specimen were trimmed. In all cases, filter paper strips were included as side drains. All specimens were saturated with the aid of backpressures ranging from 275 to 450 kPa (40 to 65 lb/in<sup>2</sup>). All tests were multiple-stage consolidated undrained tests with pore pressure measurements, conducted at axial strain rates of about 0.04 percent per minute, a fairly slow rate. Three test specimens were isotropically consolidated. The remaining specimens were anisotropically consolidated, with consolidation ratios chosen to minimize radial strain during consolidation.

Because of the need to match the small field confining pressures of these specimens, test confining pressures were smaller than those commonly employed in triaxial testing. A special pressure regulator network provided constant confining stresses at these low pressures, and

allowed the use of house compressed air as the pressure source. This avoided the hazards associated with the use of mercury columns, a typical method of pressurizing a triaxial test cell. An internal load cell eliminated loading ram friction as a source of measurement error. Ram friction is often neglected in routine testing at higher confining pressures (Bishop and Henkel, 1962), but may cause significant and unpredictable errors at low pressures. The test apparatus allowed accurate testing at confining stresses as low as 6 kPa (0.8 lb/in<sup>2</sup>).

### Additional Field Data

Schroeder and Alto (1983) have reported the results of triaxial strength tests on several soils in the Oregon Coast Range and on Washington's Olympic Peninsula. Raw strength test data from that study have been used to provide an additional data set on which to demonstrate the approach developed here. These data are from soils developed from initial materials different from the weathered basalt of the Klickitat Road site. These data were also of value in examining the geostatistical approach to soil strength, since they were obtained from a much larger geographic area than were the Klickitat data.

One set of samples was obtained in the Mapleton District of the Siuslaw National Forest. The area is characterized by shallow, steeply-sloping residual and colluvial soils overlying interbedded sandstone and siltstone. The other set of samples represents soils of variable slope and depth developed from glacial till and outwash deposits on the Olympic Peninsula.

Strength data are the results of single-stage isotropically-consolidated undrained triaxial tests. Pore pressure measurement throughout each test permitted determination of effective stresses. Specimens were saturated by means of backpressure. The effective confining pressures employed ranged from 34 to 103 kPa (5 to 15 lb/in<sup>2</sup>), values more typical of those commonly employed in triaxial testing.

### DATA REDUCTION AND STATISTICAL ANALYSIS

For each triaxial test or stage, a failure criterion was used to determine a failure p,q point. For the multiple-stage test of the Klickitat Road specimens, the stress path method (Lambe, 1967) was employed (fig. 7). After plotting the stress path of each stage of a test, a single  $K_f$  line was drawn tangent to all stress paths for that specimen. The point on a stress path which best represented a point of tangency was taken as the failure p,q point of that path. For some specimens, one stress path deviated slightly from the  $K_f$  line which best fit the other stages. In such instances, the slope of the  $K_f$  line was transferred until tangency was achieved. Where

more than one point appeared to be equally suited, the suitable point representing the largest axial strain was selected.

For their work, Schroeder and Alto (1983) also used the stress path method to define failure for each test. The method was applied to their single-stage tests, grouped on the basis of site location. Although this procedure served their intended purpose (developing conservative estimates of strength parameters for design purposes), it may have introduced a bias into the statistical analysis. For the present analysis it was desired that no arbitrary grouping of test specimens take place before statistical analysis.

For purposes of the present analysis, failure points for the Schroeder and Alto single-stage triaxial tests were determined by means of the maximum principal stress ratio criterion (Holtz, 1947). The point at which the ratio  $\sigma'_1/\sigma'_3$  reached its maximum value was taken as failure. Although there are reasons for preferring the stress path approach, particularly for overconsolidated soils (Kenney and Watson, 1961), the difference in failure points given by the two procedures was not large for this data set.

For each data set, all of the p,q points were plotted together. Since these points lie close to a straight line (fig. 8a; fig. 9a), linear regression was used to find that line. The best-fit  $K_f$  line obtained makes an angle  $\alpha$  with the horizontal. By the simple relation

$$\sin \phi' = \tan \alpha, \quad (3)$$

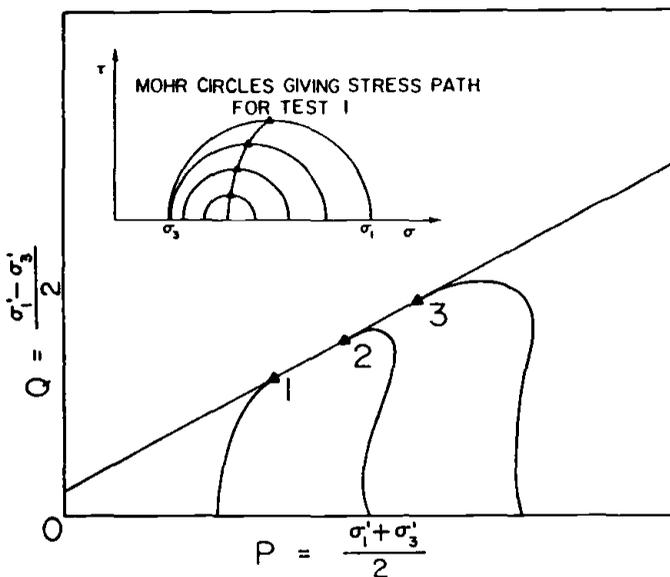


Figure 7--The stress path method for determining failure p,q points. In multiple-stage testing, each numbered stress path results from a separate stage of the test. Inset shows the growth of the Mohr circle during an individual test stage.

an angle  $\phi'$  appropriate to the entire group was computed. This value was used to compute an estimated failure  $\sigma', \tau$  point associated with each failure p,q point, following equations (4) and (5):

$$\sigma' = p - (q \sin \phi') \quad (4)$$

$$\tau = q^2 - (\sigma' - p)^2 \quad (5)$$

These failure  $\sigma', \tau$  points, too, lie close to a straight line (fig. 8b; fig. 9b). Linear regression of  $\tau$  on  $\sigma'$  produced one Mohr envelope estimating the strength behavior of the population. The scatter about the regression line served as the basis for computing prediction bands, representing the confidence associated with predicting the strength of a specimen from the population at a given normal stress (Neter and Wasserman, 1974). Confidence bands, representing the confidence associated with estimating the mean strength of the population at a given normal stress, could also be easily computed. These bands would lie closer to the regression line than do the prediction bands of the same degree of confidence.

The Mapleton and Olympic data were originally treated as separate sets. Since the regression results for the two small sets were virtually identical, they were combined for the analysis performed here. In addition to improving the precision of the regression-generated prediction region, the larger size of this combined data set also improved the chances for meaningful results from the geostatistical analysis. Based on their statistical treatment, Schroeder and Alto (1983) also concluded that these data sets could be combined.

If linear regression had not been appropriate for a p,q data set, a curvilinear regression would have been used. This would have forced the subsequent computation of failure  $\sigma', \tau$  points to be done on a piecewise basis, with various values of  $\phi'$  used to compute different points.

If the value resulting from the  $\sigma', \tau$  regression had differed from the value resulting from the p,q regression, an iterative computation and regression procedure would have been used. Each successive regression-derived value of  $\phi'$  would have been used to compute new failure  $\sigma', \tau$  points. For the data sets studied, this was not necessary.

#### Geostatistical Analysis

Because geostatistics can most easily be used to analyze univariate data, analysis of soil strength is not straightforward--the dependence of the shear stress  $\tau$  on the normal stress  $\sigma'$  confounds the analysis of the change in variation with spatial distance. To overcome this difficulty, the variable employed in the geostatistical analysis was the deviation of individual failure  $\sigma', \tau$  points from the regression Mohr envelope (residuals). Similar approaches have been employed elsewhere (Bresler et al., 1984), although not with soil

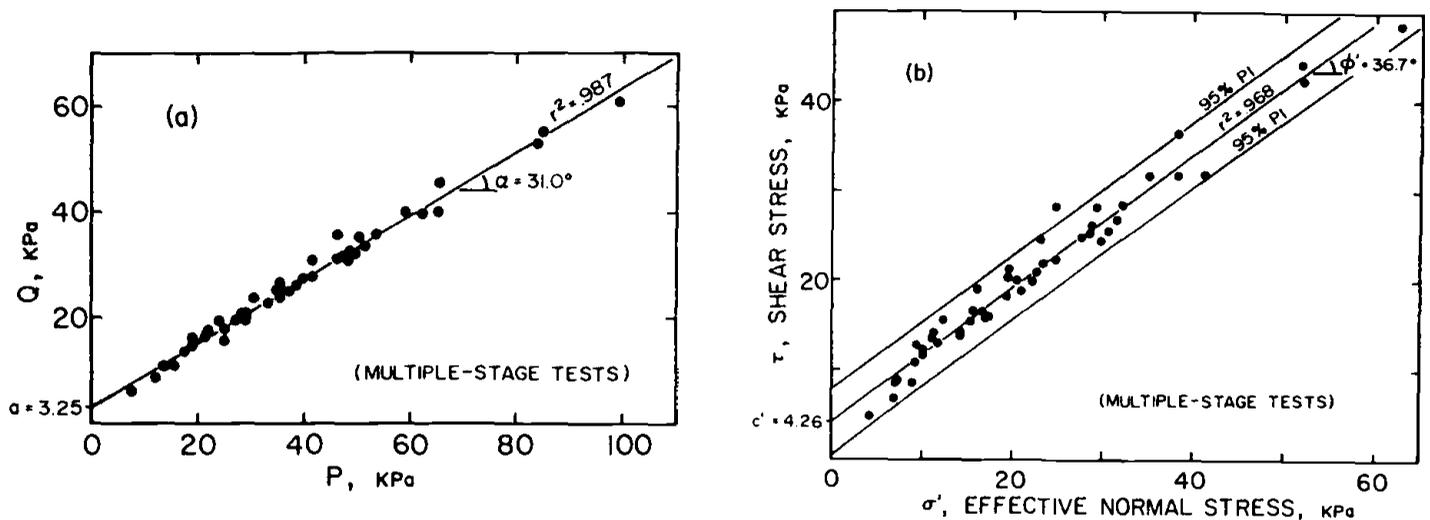


Figure 8--Klickitat Road site. a) Failure  $p, q$  points from multiple-stage tests on 16 specimens. b) Failure  $\sigma', \tau$  points computed from  $p, q$  points of (a), using  $\phi'$  value computed from  $p, q$  regression. Value of  $\phi'$  shown is result of subsequent regression of  $\tau$  on  $\sigma'$ .

strength data. For each failure  $\sigma', \tau$  point, its residual was compared with the residual of every other point.

For each pair of residuals, the difference between them was computed, then squared. Each squared difference was assigned to a lag class on the basis of the spatial distance between the points from which its constituent specimens were obtained. By equation (2), the sum of all squared difference values within a lag class was divided by twice the number of pairs summed. This produced a semivariance value for each of the lags considered (fig. 10; fig. 11). Because of the irregular nature of the sampling pattern, not all lags are represented in the semivariogram, and the number of pairs used is not constant from lag to lag.

#### DISCUSSION AND CONCLUSIONS

If probabilistic assessments of forest slope stability are to be feasible, they must be based upon adequate statistical descriptions of soil strength. Traditional statistical methods of soil strength description are not suitable for this purpose. The regression method developed here appears to provide a workable statistical description of soil strength suitable for use in probabilistic slope stability assessments. The statistical descriptions developed for the two data sets examined here (fig. 8b; fig. 9b) suggest the method may be suitable for a wide range of residual forest soils.

However, for both data sets there is no certainty that strength variability of each population has been adequately represented. This uncertainty is indicated by two portions of the analysis. The semivariogram of the Klickitat Road

data set (fig. 10) displays no definite sill value. Although several of the semivariance points can be questioned on the basis of the small number of sample pairs used in their calculation (most notably the very high value at a lag of 50 m), there is no assurance that the semivariance has stopped increasing over the lags studied. It is interesting to note that the variance of the 47 regression residuals ( $3.0 \text{ kPa}^2$ ) reasonably approximates the semivariance at a lag of 388 m ( $2.6 \text{ kPa}^2$ , fig. 10). This suggests that the semivariance may be stabilizing at this lag. However, this semivariance value results from the inclusion of just two specimens obtained from points well removed from the main sampling area. This value is thus suspect, even though many sample pairs were included in its calculation.

The semivariogram for the Mapleton and Olympic data may display a sill (fig. 11). Because of the relatively small size of these data sets, these semivariance values were computed from relatively few sample pairs. However, nearly equal values of semivariance make it tempting to conclude that the variability in strength between the Mapleton and Olympic areas (at 380 km) has already been achieved within each area (at 100 km).

If the data set adequately represents the population from which it is drawn, the semivariance value at any lag should reach a stable value with increasing number of sample pairs considered. In general, this was not true for the Klickitat Road data set. At many lags, semivariance values fluctuated substantially as new sample pairs were considered. However, this result may be an artifact of the non-random manner in which the sample pairs were considered.

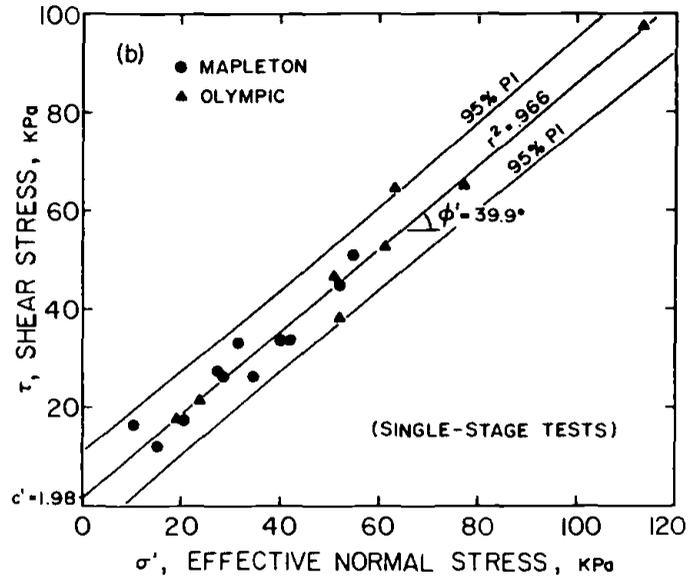
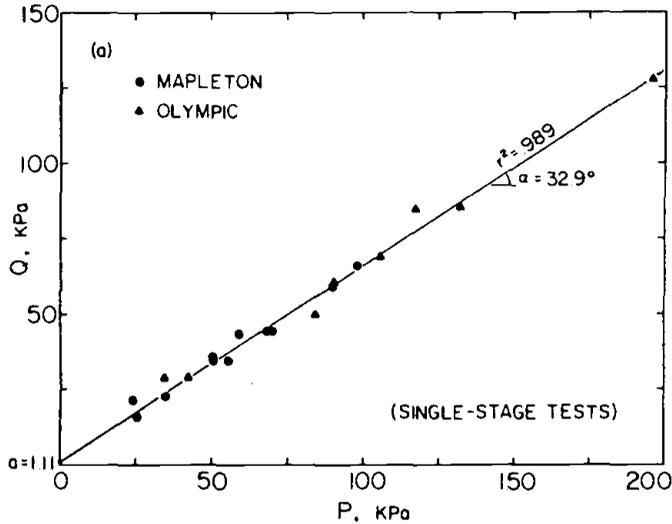


Figure 9--Combined Mapleton and Olympic data. a) Failure  $p, q$  points from single-stage tests on 20 specimens. b) Failure  $\sigma', \tau$  points computed from  $p, q$  points of (a), using  $\phi'$  value computed from  $p, q$  regression. Value of  $\phi'$  shown is result of subsequent regression of  $\tau$  on  $\sigma'$ .

An examination of regression residuals may provide a second indication of the degree to which a data set represents the population from which it is drawn. If the range in the variance of the residuals from independent random subsets, drawn from the entire data set, is small relative to the variance of the residuals from the entire data set, then it is reasonable to assume that the population is fully represented. Randomly selected subsets of the Klickitat Road data were analyzed by the regression method described here. The variance of the regression residuals was calculated. The range in variance of the subsets was large in relation to the variance of the entire data set. This indicates that the Klickitat Road data may not fully represent the population at the site.

Independence of the Klickitat Road data subsets could not be assured, but even with some dependence, the range in variance of the subsets was large relative to the variance of the entire data set. The Mapleton and Olympic data set was felt to be too small to be tested in this manner.

Thus, these data sets may have been inadequate on two levels. They may not have adequately represented changes in the variability of strength with increasing lag, the sample numbers may have been insufficient to characterize the variability of strength at the lags studied, and sample numbers may have been insufficient to characterize the area when considered as a homogeneous population. These findings reiterate the notions that the properties of residual soils are typically quite variable, and that many samples may be required to properly characterize those properties.

Rigorous probabilistic assessment of forest slope stability will require careful statistical treatment of the controlling variables, of which

strength is only one. From the analyses and discussion of soil strength statistics presented here, four concluding statements can be made:

- 1) The proposed method for describing soil strength provides the type of statistical description required for probabilistic assessment of slope stability.
- 2) Despite a significant number of tests, and a very good linear relationship for strength, it cannot be shown that the data sets presented, fully represent the populations from which they were drawn.

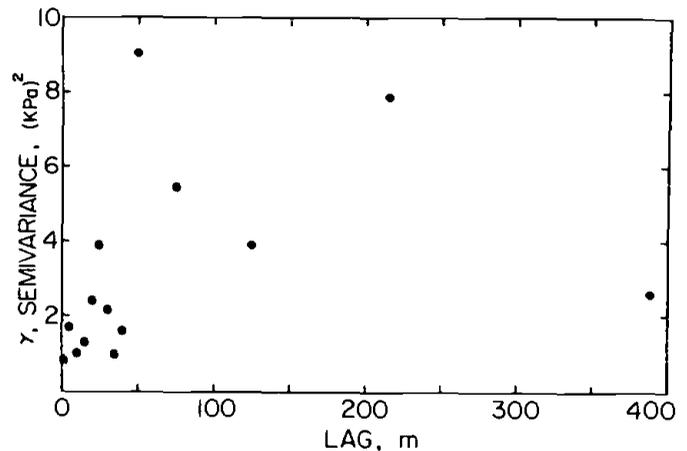


Figure 10--Klickitat Road data. Semivariogram showing semivariance of residuals from  $\sigma', \tau$  regression.

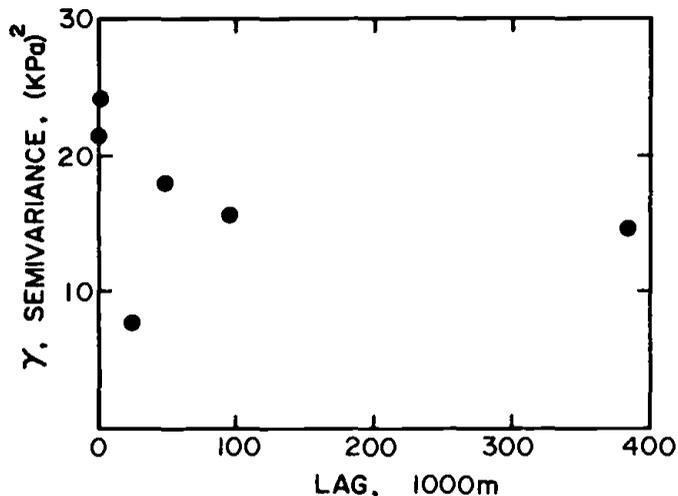


Figure 11--Combined Mapleton and Olympia data. Semivariogram showing semivariance of residuals of  $\sigma'$ ,  $\tau$  regression.

3) Geostatistical techniques, notably the semi-variogram may be useful in characterizing the spatial variation of soil strength, but a large data set appears to be required.

4) Certainty in the statistical description of soil strength can be assured only if additional data does not significantly change either the mean strength relationship or the variance of the residuals to the strength relationship. Obtaining certainty about the strength relationship appears to require a much larger data set than is necessary to obtain the mean strength relationship.

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