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Title: Model of Student Understanding of Probability in Modern Physics.

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This study aimed to investigate students' models of probability in a modern physics context. The study was divided into three phases. The first phase explored student pre-knowledge about probability before modern physics instruction. The second phase investigated student understanding of concepts related to probability such as wave-particle behavior, the uncertainty principle, and localization. The third phase probed how students used the wave function to interpret probability in potential energy problems. The participants were students taking modern physics at Oregon State University. In the first phase, we developed a diagnostic test to probe mathematical probability misconceptions and probability in a classical physics content. For the mathematical probability misconceptions part, we found that students often used a randomly distributed expectancy resource to predict an outcome of a random event. For classical probability, we found that students often employed an object's speed to predict the probability of locating it in a certain region, which we call a classical probability reasoning resource. In the second and the third phases, we interviewed students in order to get more in-depth data. We also report the findings from Fall 03 preliminary interviews which indicated the need for a more detail theoretical framework to analyze student reasoning.

Therefore, we employed the framework proposed by Redish (2003) to analyze the interview data into two perspectives—reasoning resources and epistemic resources. We found that most students used a classical probability resource to interpret the probability from the wave function. Additionally, we identified two associated patterns that students used to describe the traveling wave function in the potential step and barrier. Finally, we discuss some teaching implications and future research that the findings suggested.

# Model of Student Understanding of Probability in Modern Physics

by
Pornrat Wattanakasiwich

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I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes the release of my dissertation to any reader upon request.

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Pornrat Wattanakasiwich, Author

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# Model of Student Understanding of Probability in Modern Physics

# Chapter 1 Introduction

Quantum Mechanics (QM) was created in between 1900 and 1925 in order to explain physical phenomena at an atomic level that classical mechanics was not able to do. The quantum theory is fascinating as well as remarkably complicated. In the development of quantum mechanics, there had been astonishing efforts by many great physicists in the early twentieth century. The amazing efforts to change from classical mechanics thinking to quantum mechanics thinking have been extremely challenging, often against the intuition of not only inventors of the quantum theory, but also those who study it today.

## 1.1 Difficulties in Learning and Teaching Quantum Mechanics

As is well known in the physics education community, the most difficult and significant subject in the pursuit of a career in physics is quantum mechanics. Most students have difficulty understanding quantum mechanics because it is highly abstract, almost impossible to visualize and not intuitive to them, so it is hard for students to understand QM conceptually (Bao & Redish 2002; Singh 2001). QM is highly abstract because it deals with extremely small particles, and when these tiny particles move, their behaviors are very different from what we experience everyday. Observing these infinitesimal particles has lead to the discovery of a new type of behavior—their probabilistic nature. In the early days of the development of quantum mechanics, Einstein was worried and did not fully accept this new idea. He suggested, "But, surely God does not throw dice in determining how electrons should go!" (Feynman, 1963). Even Einstein was not comfortable with this new philosophy about the probabilistic nature of atomic particles, so it is not surprising that students first taking QM have a hard time comprehending it as well.

Not only do students have difficulties in learning QM, but instructors also have a hard time teaching QM. Since QM introduces a new philosophy that is entirely different from what students have learned from classical physics, the instructor has a hard time explaining and teaching concepts that are abstract and different from the students' daily experiences. These difficulties in teaching QM are confirmed through the lack of analogies or metaphors to explain quantum concepts. Therefore, the instructor must struggle to communicate these abstract concepts effectively to students. As a result, many students get overwhelmed when they first learn quantum mechanics.

Students majoring in physics need to develop their expertise in QM in order to pursue their career as a physicist. If students have difficulties understanding QM at a beginning level course, they will have an even harder time at an upper level QM course. According to Singh (2001), students in upper level classes also have difficulties with conceptual understanding regarding QM because of their lack of fundamental physics knowledge. As a result, students did not understand the mathematical solutions conceptually, so they resorted to memorizing the solutions.

Vosniadou and Ioannides (1998) described the process by which children learn science as "a gradual process during which initial conceptual structures based on children's interpretations of everyday experience are continuously enriched and restructured." Therefore, if students can understand the core concepts of QM early in their undergraduate career, then they might be able to better build their QM understanding from these concepts. Also, students might be able to understand the QM mathematical representations in their upper level courses at a deeper conceptual level.

In order to help students learn concepts of QM efficiently, the Physics Education Research (PER) community needs to gain more knowledge about the difficulties that prevent students from learning QM. However, only a handful of research has been done with students taking higher-level physics courses (McDermott & Redish, 1999). Most physics education research has been focused on teaching and learning at introductory university and high school level physics courses. Recently, the PER community has done more work in the area of quantum physics.

Within the small amount of research dealing with modern physics or QM, only a few studies have investigated students in upper-level courses. Most of the early studies concentrated on students' misconceptions about the atomic model at introductory university or high school level physics (Bethge & Niedderer, 1995; Johnston, Crowford, & Fletcher, 1998; Mashhadi & Woolnough; 1996; Petri & Niedderer, 1998). Most recent studies are concentrated around concepts of modern physics such as the wave-particle paradox, the uncertainty principle (Johnston *et al.*, 1998), de Broglie wavelength and electron diffraction (Vokos *et al.*, 2000). Only a few studies have investigated upper-division college students. For example, a study by Wittmann, Steinberg, & Redish (2002) focused on upper-division students taking modern physics/quantum mechanics classes. This study observed how students reasoned in terms of conductivity models about material properties of conductors, insulators, and semiconductors, and how these properties lead to different conductive behavior. In addition, a few studies have been involved with more upper-division concepts of QM, such as the quantum measurement process (Muller & Wiesner, 2002; Singh, 2001), and quantum probability (Bao & Redish, 2002; Singh, 2001; Muller & Wiesner, 2002). These studies will be discussed in more detail in Chapter 2.

The results from PER in QM are still far behind compared to PER in classical mechanics, which has some fully developed standard instruments like the Force Concept Inventory (FCI) (Hestenes, Wells, & Swackhammer, 1992). Therefore, to help improve the ways of teaching and learning QM, the PER community is still in need of more significant evidence regarding students' learning and understanding of quantum core concepts.

## 1.2 Statement of Problem

According to Bao and Redish (2002), probability plays a fundamental and significant role in making sense of quantum mechanics. Therefore, helping students understand probability in a context of modern physics can assist them in developing a more coherent understanding of the core concepts of QM. As mentioned earlier, a few studies have been done on certain areas of basic modern physics concepts. The researchers studied the basic QM concepts, which are usually covered in a modern physics class. As a whole, these concepts explain the quantum behaviors of small particles, such as electrons or protons, and the studies created a new paradigm and philosophy to describe nature in terms of probability. Therefore, probability is a significant concept that students need to master in order for them to learn QM successfully.

On the other hand, only a small number of physics education studies have attempted to investigate student conceptions about probability. The study by Bao and Redish (2002) contributed noteworthy findings about many aspects of student concepts of probability. They explored student reasoning in terms of probability when solving potential well, potential step, and potential barrier problems. In order to further our understanding about students' conceptions about probability in a modern physics class, more studies about this particular concept have to be conducted but in different contexts. Findings about student concepts of probability in different contexts will help the PER community to acquire a more complete understanding of how student concepts of probability are constructed and developed. Therefore, our study investigated student understanding of probability and how they related probability and basic QM concepts such as wave-particle duality, the Heisenberg uncertainty, wave function, etc. Our study also employed Redish's (2003) theoretical framework which explains student models of thinking to analyze our qualitative data such that our findings provide a different and more in-depth study of students' probability conceptions.

## 1.3 Statement of Purpose and Research Questions

The double-slit experiment is used as the main context of interviews conducted in this study. The double-slit experiment with electrons is considered to be a simple way to introduce the wave nature of an electron as well as to convey a probability concept to students. Compared to other earlier experiments that were used to introduce the idea of wave nature to students, the double-slit and single-slit experiment are much simpler. For example, the Davisson-Germer experiment, which was an early experiment used to study electron diffraction by investigating the reflection of the electron beam from the surface of a crystal, is complicated and requires extensive prerequisite knowledge. The double-slit experiment, on the other hand, requires less physics background for students to comprehend the probability concepts. Also, students have seen the double-slit experiment with light previously in an introductory physics course. This provides enough basic knowledge to comprehend the probability concepts.

This study has three goals—to survey, to describe, and to compare students' conceptions of probability at a modern physics level. The first goal of this study is to survey students' conceptions of probability before they take a modern physics class. The second goal is to describe students' conceptions of probability in a double-slit experiment with electrons. The third goal is to compare students' conceptions about probability in contexts of the double-slit and potential well, step, and barrier problems. Here are more specific research questions:

1. What is the nature of students' conceptions of probability before taking a modern physics class?
2. What is the nature of students' understanding of probability?

## 1.4 Significance of the Study

In spite of its difficulties, QM is important in understanding almost every aspect of physics from superconductivity to black hole radiation. The profound impact on our present and future lives can be identified from cutting-edge technologies such as semiconductors, nanotechnology, quantum computing, etc. In order to get more qualified people involved in development of these new technologies, the quality of teaching QM has to be improved. As a result, college students will learn more effectively and be able to appreciate the quantum theory, so they do not feel overwhelmed by the extraordinary quantum phenomena but rather be fascinated by them.

This study aims to contribute evidence of students' probability understanding in a single-slit and a double-slit experiment. The findings from this study will provide a better defined framework of student conceptions of probability at a modern physics level. This can help further the development of any future PER in order to improve the ways we teach and learn QM.

## 1.5 Limitations of the Study

The study has two limitations in terms of the participants and the content. First, the study was limited to students taking a modern physics course at Oregon

State University (OSU). Learning is a complex process and depends on an instruction or an input from an instructor; therefore, students taking modern physics in other places might develop different understanding as a result of different instructional setting. Therefore, we could not generalize the findings in our study to describe students taking a modern physics class somewhere else. Second, the probability concepts discussed in this study are limited in a context of mathematical and modern physics probability.

## 1.6 Thesis Overview

Chapter 2 contains a review of the literature that is part of the theoretical framework in this study. The literature references are from three fields of research—students' misconceptions of mathematical probability, PER about QM and modern physics, and the coordination classes as a model of a knowledge system.

In Chapter 3, we describe a research design for this study. The research design involves focus groups and research methodologies used in the study. We describe the focus groups which include a class in which we conducted the study and its student participants. The overall characteristics of the modern physics class, the research participants and a recruitment process are discussed. In term of the research design, we used both qualitative and quantitative methodologies in order to obtain different kinds of evidence. We employed a quantitative approach in a probability diagnostic test, questionnaire about probability in modern physics context, and conceptual quizzes. However, the main source of evidence came from a qualitative approach in two in-depth interviews. The qualitative method dominated this study. Therefore, we provide a description of the researcher because the researcher is considered as the most important instrument in a qualitative study.

Chapter 4 contains results of the diagnostic test about students' conception of mathematical and classical probability. We discuss the statistical analysis used to analyze the test which consisted of two parts—multiple-choice part and open-end part.

Chapter 5 discusses findings from the preliminary interview conducted in Fall 03. We analyzed the data based on students' difficulties. In Chapter 6, we used the findings from the preliminary interview to modify the interview questions. The Fall 03 results suggested that we need a better theoretical framework to analyze students' reasoning. First, we used the coordination class proposed by diSessa and Sherin (1998). However, the model could not fully describe the interview data, so we implemented a conceptual model proposed by Redish (2003). We discuss and report the use of this model and the findings based on Redish's model.

Finally, in Chapter 7, we discuss and propose hypotheses that emerge from out study. We also discuss the possible implementation to improve our teaching of modern physics and suggest possible future research.

# Chapter 2

# Literature Review

This chapter reviews the literature that is relevant to this study and provides theoretical frameworks, which are obtained from three different fields—mathematics education, physics education, and cognitive psychology. The first framework includes previous literature about student misconceptions in the field of mathematical probability. Both the mathematics education and the cognitive psychology literature provide us with a background of students' understanding of certain aspects of mathematical probabilities. The second framework is devoted to previous physics education research about students' difficulties in modern physics or quantum mechanics, especially in the topics of probability, the single-slit experiment, and the double-slit experiment. The last framework involves the discussion of the theoretical literature that provides a model explaining the reasoning process.

## 2.1 Mathematical Probability Framework

The mathematics education and cognitive psychology literature has not only provided evidence about students' misconceptions of probability but also established cognitive theories explaining why students have these misconceptions.

Before 1970, education and psychology were viewed separately until the breakthrough studies conducted by Jean Piaget, who is well-known for his remarkable works in the field of developmental psychology. Piaget and Inhelder (1975) studied people as they reasoned about uncertainty events and found three age-dependent stages in the development of the idea of chance in children. The first stage is found in six- or seven-year-olds, who usually do not understand the word "chance." In the second stage, the six- to nine-year-olds are able to explain the idea of chance in particular events. Finally, at age twelve children correctly reacted and defined chance as "the interaction or interference of independent causal series" (Piaget & Inhelder, 1975). Through their experiments, using a variety of random-generating devices as a tool to interview children about probability, Piaget and Inhelder concluded that children can reason about probability by the age of 12.

In contrast to Piaget's findings, Green (1983) found that students who did not reach the level of Piagetian formal operations by age 16 left school at a concrete operations level. The subjects in his study consisted of 2930 British students age 11-16. The instrument, called the Probability Concepts Test, had been developed over a two-year period. The test was first administered as six pilot tests before achieving the final version. Most of the tasks in the test were conceptual, such as tree diagrams, visual representations of randomness, and questions on the language of probability. Green concluded that students had difficulty understanding and using the language of probability, such as "at least," "certain," or "impossible." Also, students showed weakness in concepts of randomness, stabilities of frequencies, and inference.

### 2.1.1 Probabilistic Misconceptions due to Judgmental Heuristic

The work of Piaget opened up a new field of psychology; however, his work did not provide much explanation or theory about how people reason in the probabilistic or uncertainty tasks. In contrast, Kahneman and Tversky's (1972) works have provided a theoretical framework for mathematics educators in researching learning probability and statistics (Shaughnessy, 1992).

In the late twentieth century, Kahneman and Tversky (1972) did a series of studies about people who were reasoning about uncertainty events, and they provided a theoretical framework to support their findings, called a *judgmental heuristic*. Their theory is that people who are statistically naïve or who have not been trained about probability theory make estimates for the likelihood of events by using certain judgmental heuristics such as a *representativeness* or *availability heuristic,* which will be examined later.

### 2.1.1.1 Representativeness Heuristic

Kahneman and Tversky's definition of representativeness heuristic is that people estimate likelihoods of events based on how well an outcome represents the distribution of the parent population or the random process (Kahneman & Tversky, 1972). Misconceptions that occur due to the representativeness heuristic include randomness distribution, ignoring sample size, and Gambler's Fallacy.

- ***Random Distribution Expectancy***

People who don't have training in probability theory or are still naïve statistically tend to think of events distributed randomly as being "most likely" to occur in any given situation. For example, Shaughnessy (1977) asked undergraduates to compare two families of six children: BGGBGB and BBBBGB, assuming that the probability of having a boy in any birth is ½. Before the subjects received the experimental teaching activities, 50 out of 70 students indicated that BGGBGB was more likely to occur, and after the course, 22 of 40 students still chose the same way. He also had subjects compare BGGBGB and BBBGGG offering an additional option: "about the same chance." On this item, while 28 out of 70 subjects still chose BGGBGB, another 25 of the subjects reasoned that they were "about the same chance" since both families contained three boys and three girls. This example confirms that statistically naïve people intuitively predict random events as "most likely to occur" compared to all the possible events.

- ***Ignoring Sample Size***

People who use the representativeness heuristic inappropriately tend to ignore the sample size or the magnitude of the sample; they think that either a small or a large sample represents the same characteristics of a population in a random event. For example, Shaughnessy (1997) replicated Kahneman and Tversky's hospital problem[1] study and found that 48 of 80 undergraduates said that the size of a hospital makes no difference—it could be either small or large.

In a different setting, Bao and Redish (2001) also found that college students taking physics classes at an introductory or advanced level still believed small samples would replicate the probabilistic trends expected from a very large number of trials. This study consisted of interviews and open-end surveys with students taking a modern physics class or an upper-division quantum mechanics class. This study was conducted to investigate students' understanding regarding probabilistic interpretations of physical systems. Bao and Redish's study investigated both quantum and mathematics probability, but we are only concerned about the mathematics probability in this chapter. At first, students were administered the concept quizzes. Only one question was designed to probe students' understanding of fundamental ideas in probability—the independence of events (sample size) and the Gambler's Fallacy. In order to investigate further about students' understanding, the interviews were conducted with 16 volunteer students. The findings showed that students in a modern physics class had two

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[1] A certain town is served by two hospitals. In the large hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50 percent of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50 percent, sometimes lower. For a period of 1 year, each hospital recorded the days on which more than 60 percent of the babies born were boys. Which hospital do you think recorded more such days?

misconceptions regarding the sample size: 1) small samples would replicate the probabilistic trends expected from a very large number of trials, and 2) the specific result of any single measurement could be affected by the previous sequence of the outcome. These indicate that students still use the judgmental heuristic even at the college level.

In another study, Fischbein and Schnarch (1997) investigated the evolution of probabilistic misconceptions ranging from the intuitions in children to pre-service secondary teachers. Significantly, they found that the miscalculations of the sample size remained a strong misconception across the ages, with only one student from the entire group answering correctly. Subjects included five groups of Israeli students without any previous instruction on probability—20 students in each grade level (5, 7, 9, and 11) and 18 pre-service secondary mathematics teachers. The instrument was a questionnaire consisting of seven multiple-choice items. Each item represented a different misconception including representativeness, negative and positive recency effects, simple and compound events, conjunction fallacy, effects of sample size, availability and time-axis fallacy. Each item contained a description of an event with three forced-choice possible answers—the correct response, the common incorrect misconception response, and a distracter. By comparing the average percentage of students who answered correctly with students who chose a misconception choice, the results indicated that the misconceptions, in terms of representativeness, negative and positive recency, and the conjunction fallacy decreased with age. In contrast, the availability and the time-axis misconceptions increased with age, while the positive recency, compound and simple events remained the same with age.

Cox and Mouw (1992) studied the misconception about sample size even further. They hypothesized that if students were trained to decrease the inappropriate use of the representativeness heuristic, then they should score higher on the probability test. They investigated senior-level and graduate students' understanding of probability concepts in an introductory inferential statistics course. Fifty-four participants were randomly assigned to either one of two experimental groups or a control group. Each group was pre- and post-tested. The pre-test had 14 questions that particularly assessed the students' use of representativeness in determining probability. The post-test contained 10 mutiple-choice items to assess the subjects' use of the representativeness heuristic. Three of the 54 participants did not complete the study. The experimental groups were exposed to short-term experiences designed by the researchers to decrease the inappropriate use of the representativeness heuristic. As a result, mean post-test scores from each group indicated that the experimental groups performed significantly better than the control group. Therefore, with proper training or teaching, students can learn to avoid using the judgmental heuristic.

- ***Gambler's Fallacy***

People who have a Gambler's Fallacy misconception believe that future results will compensate for previous (short term) results in order to bring things back to the average (Bao & Redish, 2001). Gambler's Fallacy is sometimes referred to as the positive and negative recency. For example, Bao and Redish asked students to predict the probability of getting a head (greater than, less then, or equal to 50%) after tossing the coin three times and getting three heads in a row. They found that 61% of students predicted that the probability of getting a head should be less than 50% because there was already three heads in a row. This finding indicated that a majority of college students thought that the result of a single coin-flipping event depends on the results of previous coin-flipping activity—a probabilistic misconception due to the Gambler's Fallacy. However, the study across ages by Fischbein and Schnarch (1997) indicated that the misconceptions of the Gambler's Fallacy (negative and positive recency) decreased with ages. The conflict between these two findings leaves the relationship between the misconception due to Gambler's Fallacy and students' ages inconclusive.

### 2.1.1.2 Availability Heuristic

Kahneman and Tversky (1972) also mentioned that people based their prediction of the likelihood of an event on their previous experiences. In other words, many people believe that events that are easier for their mind to list are more numerous than events that require longer thought processes to list. They called this misconception the *availability heuristic*. The judgment that comes from this availability heuristic can induce significant biases. More recently, Bao and Redish mentioned that students seemed to hold a deterministic, empirical intuition of probability. They later reported that small numbers of students had done probability analysis in math class and none of them had used probability to describe a real physical event. This might explain why students lack correct availability heuristic judgment because they did not gain the probability experience that they needed.

### 2.1.2 Outcome Approach

The judgmental heuristic has served as the theoretical framework for most of the research in probability and statistical learning. However, Konold (1989) found that the judgmental heuristic, especially the representativeness heuristic, is rather limiting. Konold did not refute the validity of past research conducted on this heuristic. He claimed that students showed a significant misconception when they were asked to solve probability tasks different from the tasks used in the judgmental heuristic based research.

Konold (1989) expanded the judgmental heuristic further by formulating a new model of informal reasoning under uncertainty, called an *outcome approach*. He conducted qualitative interviews with 16 undergraduates regarding informal conceptions of probability. The interview was to observe student predictions if theirs were based on a deterministic model of the situation. Then, the follow-up interviews with a different set of problems were conducted in order to investigate the responses of students if their predictions were outcome-oriented. The results were scored based on the outcome-oriented responses. Then, the outcome scores from the first interview were correlated with the scores from the second interview. The correlation between two outcome scores was $r = 0.797$ ($p < 0.005$, one-tailed), so the outcome scores from both interviews were positively correlated. This suggested that students' non-normative responses were fairly consistent. Konold also claimed that the correlations between the first interview performance and problem performance indicated the power of the "outcome approach" as a predictive model. He also clarified that the outcome approach was different from the representativeness heuristic. For example, people tend to use the representativeness heuristic when asked to predict a "most likely" outcome of a random event. In contrast, people tend to use the outcome approach when asked to predict a "least likely" outcome of the same random event. Thus, the outcome approach is used to explain a different error compared to the error often explained by the representativeness heuristic.

Another study by Konold, Pollatsek, Well, and Lipson (1993) investigating why people's reasoning about probabilistic and statistical concepts seems to be inconsistent also confirmed the outcome approach. In this study, Konold and his colleagues first administered a questionnaire that contained two items—one related to the fifth outcome of a coin if it resulted in heads on each of four previous tosses, and the other involved head-tail sequences in which a coin is tossed five times. These two questions appeared with other probability and statistics questions. The questions were multiple-choice to see if the subjects would choose the correct answer. The subjects were 16 high school students participating in a workshop on probability. They were asked to predict whether a head or tail was the most likely outcome or if the two outcomes were equally likely in the first case. They were given five choices for the second question, one of which was that all of the other four listed sequences were equally likely. The heads-tails sequence question contained a second part that asked participants to predict the least likely outcome.

As a result, 86 % of students chose the correct answer, and most of them indicated that the sequences listed for tossing coins five times were all equal likely. In contrast, when asked about the least likely outcome, only 38% responded that each of the four sequences was equally likely. Konold *et al.* (1993) cited one reason for students inconsistently answering the question about the most likely and least likely sequences differently is that the students "reason about the two questions from different perspectives" (p. 399). Some persons seem to use an outcome approach when asked to answer a question such as which sequence of coin tosses is most likely. On the other hand, when asked which sequence of coin tosses is least likely, people answer the question using "the more probabilistically-oriented representativeness heuristic" (p. 401).

Konold *et al.* also performed a second study in which they investigated how 20 undergraduate students enrolled in psychology courses would answer questions similar to those used in the first study. These 20 individuals participated in an hour-long videotaped interview. The purpose of this study was to document why there was an inconsistency in the participants' responses to the second question. Results indicated that some of those who selected the correct response for the head-tail sequence problem did not assign equal probabilities to each sequence when asked about the probabilities of the sequence.

### 2.1.3 Probabilistic Misconceptions outside the Limit of Judgmental Heuristic

Even though the judgmental heuristic is the main theoretical framework in explaining how statistically naïve people reason about uncertainty events, there are some misconceptions in the domain of probability and statistics about which this theory cannot provide explanation. The following references studied misconceptions outside the limit of the judgmental heuristic such as sample space and mean.

- ***Sample Space***

Jones, Langrall, Thornton, and Mogill (1999) investigated the thinking of third-grade students in relation to an instructional program in probability. The main goal of this study was to develop and evaluate an instructional program in probability. The areas of focus in this program included sample space, probability of an event, probability comparisons, and conditional probability. To achieve their goal the researchers presented the instructional program to two third-grade classes, one in the fall and the other in the spring. Thirty-seven students were included in the study. Students received instruction on probability problems over 8 weeks. Data were collected from pairs of students on their probabilistic thinking as mentors worked with the pairs. Interview and observational data were obtained from researcher-generated assessments conducted at the beginning, midway, and at the end of the school year; mentor evaluation and ratings on each of their two students for each instructional session; and researcher explanations of observations on each of four target students and their mentors. Twenty tasks were used in assessing the students—five on sample space, four on the probability of an event, seven on probability comparisons, and four on conditional probability. Students could respond to tasks using written or verbal answers and they could also use concrete materials to demonstrate their thinking. Observational data obtained by the mentor and researcher were considered for four target students while a quantitative analysis of data obtained from all of the students was performed. Analysis of the data revealed that the number of students in both groups who increased their level of probabilistic understanding after instruction was significant.

- ***Mean***

Pollatsek, Lima, and Well's (1981) study included seventeen undergraduate volunteers who were mostly psychology majors at the University of Massachusetts. Seventeen participants were interviewed and audio-taped individually. The results showed that many of them could not weight or combine two group means into a single mean correctly. Participants were interviewed about one or two of three weighted mean problems. Only two of the 15 students computed the overall mean involving a grade point average correctly on their own. Thirteen students gave the unweighted mean as the answer. Pollatsek *et al.* confirmed their preliminary finding that many college students cannot correctly weight and combine two means into a single mean. Similarly, Bao and Redish (2001) also found that the majority of their subjects, which were college students, had a misconception about mean. They asked a question concerning probable values of students' SAT scores:

> Suppose the student average SAT score at Enormous State University is 1000. Your friend is in a writing class of 10 students. Her score was 1100. What is the most probable average of the other 9 students? Explain your reasoning.

As a result, more than 67% of the students thought that knowing one student's score would affect the probable average score of the other students.

### 2.1.4 Summary

Studies about how people reason on uncertain events were started by Jean Piaget's research on children's ideas of chance. Piaget and Inhelder conducted a series of studies and concluded that children by the age 12 can explain the uncertainty that was generated from the random generating gadget. However, the study by Green (1983) indicated otherwise. With questions challenging visualization and conceptual understanding of probability, Green found that students did not reach the level of Piagetian formal operations by age 16, and seemingly left school at "concrete operations level" (Green, 1983).

Then the cognitive psychology studies of Kahneman and Tversky (1972) have changed the perspective of mathematics educators in conducting research about learning and teaching probability. Kahneman and Tversky introduced the theoretical framework that people use to predict events of chance when they do not formally understand probability concepts, called the judgmental heuristic. According to the judgmental heuristic, two common strategies that statistically naïve people uses in solving probability tasks have been identified as the representativeness and the availability heuristics. When people use the representativeness heuristic, they tend to look for the random distribution of an event (Shaughnessy, 1997; Bao & Redish, 2001), ignoring the sample size (Cox & Mouw, 1992; Fischbein & Schnarch, 1997; Shaughnessy, 1997; Bao & Redish, 2001), or show the Gambler's Fallacy (Bao & Redish, 2001). All of these factors contributed to their intuitive incorrect responses or misconceptions about uncertainty events or tasks. Not only the representativeness heuristic as mentioned before, but the availability heuristic also causes misconception among stochastically naïve people. Therefore, judgmental heuristic plays an important role in people's misconceptions about uncertainty events.

Despite the growing studies based on the judgmental heuristic, Konold (1989) suggested that people use an additional heuristic, which he named as the outcome-based approach, as a strategy for solving probabilistic tasks. People using the outcome approach "do not see their goal as specifying probability that reflects the distribution of occurrences in a sample but as predicting the results of a single trial" (Konold *et al.*, 1993, p. 394).

The judgmental and outcome approach heuristics have served as the theoretical framework for most of the studies about people's learning and solving probabilistic tasks. However, there are some misconceptions that the judgmental heuristic cannot contribute to students' misconceptions such as sample space (Jones *et al.*, 1998) and mean (Pollatsek *et al.*,1981; Bao & Redish, 2001).

From the literature about learning and teaching probability, the findings suggest that when dealing with uncertainty events, most statistically naïve people have seven misconceptions. Three misconceptions are due to people using the representativeness heuristic—randomness distribution, sample size, and Gambler's Fallacy. Two misconceptions are due to using the availability and outcome-approach heuristic, respectively. The last two misconceptions cannot be explained by any cognitive theory including sample space and mean. These misconceptions will be used as a basis in developing the quantitative instrument, in order to investigate students' probabilistic concepts prior to any modern physics instruction.

## 2.2 PER in Modern Physics and Quantum Mechanics

Most previous research in physics education has been in the area of classical mechanics. However for the last fifteen years, the focus of PER has been shifted toward modern physics and quantum mechanics because of its immense importance in new cutting-edge technology (Johnston *et. al.*, 1998; Bao, 1998). Therefore, understanding modern physics and quantum mechanics became a more common requirement for most science majors or engineers. The basic understanding about quantum mechanics can make a great impact on a person's perspective about the physical world (Bao, 1998); however, learning it is not easy. Therefore, it is our responsibility, as a physics education community, to make learning quantum mechanics easier for most of our students. In order to do that, we need to know what difficulties most students have or encounter. In this section, I summarize the literature about modern and quantum physics according to the significant findings from previous research.

### 2.2.1 Bohr Atomic Model

Previous research regarding modern physics or QM concepts often was conducted at a pre-college level. The early studies about modern physics focused on disadvantages of teaching quantum physics based on classical physics, especially about atomic models and their influence on students' thinking. The significant finding is that the Bohr model makes it harder for students to accept an electron cloud model or the probability model of atom.

Bethge and Niedderer (1995) found that 50% of students still stuck to the Bohr model even after they had been taught about atoms and electron orbitals using a Schrödinger approach. Their study involved Grade 13 students in German high schools attending quantum mechanics class. They also found that students tend to use a model in a variety of meanings, ranging from "true pictures" to "tools of thinking" and "visualization." Since the Bohr model is so concrete and easy to visualize, students easily become attached to it whenever they think of an atomic structure.

Petri and Niedderer (1998) also studied about students' using atomic models, and they found that after instruction a student combined three models to explain an atom. Their study was a case study investigating one student's (Carl) learning pathway or cognitive system[2] while enrolled in a quantum atomic physics course in grade 13 of German gymnasium (secondary school). The authors generated Carl's cognitive system from their data analysis. Then, they used his cognitive system as a hypothetical model to describe, analyze, and explain his thinking and learning while interacting with an instructor. They found that Carl's learning pathway could be described as a progression of using several conceptions of an atom, starting from the Bohr planetary model. His final cognitive system of the atomic model after the instruction was displayed as a co-existing of three models—his initial planetary model, a state-electron model and an electron-cloud model.

These studies suggested that students relied on the Bohr model when asked to discuss an atom. The Bohr model does not correctly describe the probabilistic

[2] Cognitive system, proposed by Niedderer's group from Institute of Physics Education, University of Bremen, Germany, is a model of student's mind constructed by the researcher. Accordingly, a cognitive system consists of stable deep structure and topical current constructions. The subunits of the deep structure are called 'stable cognitive elements.'

nature of electrons in an atom, and it could possibly cause student difficulties later when they learn modern physics. Therefore, many physics education researchers suggest that we should eliminate the Bohr model altogether from the science curriculum.

Fischler and Lichtfeldt (1992) created a new teaching approach that avoided teaching the phenomena of quantum physics in terms of the conceptions of classical physics. Their teaching approach consisted of avoiding reference to classical physics, especially the Bohr atomic model; beginning the teaching unit with electrons; using the statistical interpretation instead of dualistic descriptions for quantum phenomena; and introducing the uncertainty at the beginning. They created a teacher guide and held several workshops for physics teachers at the German Gymnasiums[3] in Berlin. In evaluation of their teaching approach, 240 students in eleven Gymnasium classes during the spring semester of 1988-1989 were selected and separated into a control group and a treatment group, which received a new teaching approach. Students' conceptions of a hydrogen atom and its stability were analyzed from students' answers to a questionnaire given before and after the teaching unit. As a result in terms of conceptual change, most students conceptualized a hydrogen atom as circle and shell. After the instruction, 68% of students in the treatment group leaned toward the conception of localization energy[4]. In contrast, only 7% of students in the control group developed a conceptual change; 60% of the control group had the same conception as they started with. The results of this study seem promising in terms of a possible solution to help students move from Bohr atomic model, which represents a classical perspective, to an electron-cloud model, which represents a quantum perspective.

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[3] Gymnasium is a level of school in German, which is similar to an academic secondary school.
[4] From their study, Fischler and Lichtfeldt defined a conceptual pattern emerging from students' answers as *localization energy*, which was the conception that students related the stability of atoms with the Heisenberg uncertainty principle. According to this conception, students thought that the constraint of space inside the atom resulted in an uncertainty of electron kinetic energy, which caused a statistical distribution of locating an electron inside the atom.

Instead of avoid teaching the Bohr model altogether, Budde *et al.* (2002) suggested a new atomic model for teaching, called 'Electronium'. They argued that students' attachment to the Bohr model could not be resolved if we still taught energy levels without presenting any visual atomic models. Their Electronium model differs from the probability model in terms of a distribution of electrons that are not represented a cluster of dots but a smooth distribution of a kind of liquid substance, called Electronium. They argued that this model should help students overcome difficulties that could emerge when teaching with the probability model.

### 2.2.2 Wave-particle Duality and the Uncertainty Principle

Besides studies about atomic models, physics education researchers conducted several studies which involved topics of wave-particle paradox and the Heisenberg uncertainty principle.

Johnston, Crowford and Fletcher (1998) studied students' mental models of basic quantum concepts—wave-particle paradox and Heisenberg uncertainty principle. Thirty-three third year students at the University of Sydney were participants. The instrument was a survey quiz consisting of open-end questions. Only questions about wave-particle paradox and the uncertainty principle were analyzed. The authors claimed that results of the study revealed categories of description, which could be interpreted in terms of mental models. However, they only found fragmented mental models consisting of isolated facts that could not be fitted into an internally consistent framework. The authors mentioned that they believed these were all indications of the fragmented nature of the underlying mental models students were using.

In another work, Muller and Wiesner (2002) reported an evaluation of a research-based course on QM, a product from their research about students' difficulties with concepts. All studies in the series were conducted with Gymnasium students in Germany. Although they had done many previous studies on quantum concepts, in this report, they emphasized the results of the evaluation, which was conducted with 60 students from five Gymnasium schools. From their previous work in Germany, there were some interesting findings in terms of localization and the uncertainty principle. When they asked students, "Does an electron in an atom have a definite position at each moment of time?", the responses consisted of (i) the electron is localized in a certain region with some probability (25%), (ii) the electron has a definite but unknown position (21%), (iii) no definite position because of uncertainty (18%), and (iv) the electron has a position but no trajectory (7%). When students were asked, "What is the meaning of $\Delta x$ and $\Delta p$?", they responded (i) position measurement affects the momentum measurement (21%), (ii) regions of localization (18%), (iii) interval that the exact value is in with some probability (18%), (iv) measurement uncertainty (15%), and (v) standard deviation (13%).

### 2.2.3 Wave Nature and de Broglie Wavelength

Vokos, Shaffer, Ambose, and McDermott (2000) studied students' difficulties regarding the wave nature of matter, especially the de Broglie wavelength. The study included students enrolled in courses ranging from an introductory to more advanced level courses at University of Washington. Written problems on the diffraction and interference of electrons were administered to more than 450 students in first-, second-, and third-year physics courses. Fourteen third-year volunteer students taking a quantum mechanics course were interviewed. The common difficulties were found and divided into three categories: (a) failure to recognize the relevance of the de Broglie wavelength to the interference or diffraction pattern of particles, (b) failure to relate the de Broglie wavelength to the momentum of particles, and (c) failure to treat particles with and without mass differently (in the non-relativistic limit).

### 2.2.4 Probability and Wave Function

Bao (1999) investigated students' understanding of the wave function in relation to energy and probability while solving certain fundamental potential problems. His study was an iterative process of research, development and instruction as a part of the University of Maryland physics education research group. The interesting findings resulted from his interviews with student participants, which came from two courses—an introductory physics course and an upper-division undergraduate QM course. During the interview, students needed to solve potential problems in different conditions and construct the potential diagram. In terms of students' difficulties, Bao found that many students often fail to interpret the potential energy diagram correctly because they intuitively thought of the potential diagram as the real system like a gravitational well. Also, students tended to see energy as a measurable physical quantity, so they expected the energy to be an absolute and positive value. As a result, they had a hard time to determine the discrete value of total energy in the finite square well. Bao also investigated students' understanding of the probabilistic interpretation of the wave function. He found that students had difficulties with interpreting potential energy diagrams, shape of the wave function, bound states, and using inappropriate classical models for quantum systems. Students failed to determine the correct kinetic energy in different regions of the potential energy diagram. Most students seemed to have a difficult time to relate the wave function to the local kinetic energy. Bao gave an example of one student who did not know what the wave function, as a solution to the tunneling problem, represented and the conditions to which it applied. Moreover according to Bao's findings, students could not understand the meaning of the probability density represented by the squared amplitude of the wave function. In summary, students' difficulties from Bao's study were mostly results of student incorrect understanding of the potential-energy diagram and the wave function.

### 2.2.5 Summary: Implication for Teaching and Further Research

From the literature review, most students preferred the Bohr model because of its simple structure. Instead of eliminating the Bohr model from the modern physics or science curriculum, Muller and Wiesner (2002) suggested if the model limitation was discussed, students tended to have less connection or use it more appropriately. Therefore, when the Bohr model is introduced in the classroom, an instructor must discuss its limitations explicitly. Alternatively, Budde *et al.* (2002) suggested the Electronium atomic model for teaching, which can possibly help students construct visualization coherent with basic quantum concepts.

Quantum mechanics is not intuitive to students, so they have many difficulties not only about atomic models but also interpreting basic concepts such as wave-particle duality, the Heisenberg uncertainty principle, wave natures, and the de Broglie wavelength. Students seem to construct these concepts independent of each other; as a result, their understanding or their mental models of these concepts are fragmented or unstructured (Johnston, Crowford and Fletcher, 1998).

Since their knowledge is fragmented, students cannot apply their knowledge in problem-solving situations. Consequently, they are more likely to memorize the solutions to the problems, for example the step potential problems, and cannot interpret a meaning or construct a graphical representation of the wave function (Bao, 1999). Therefore, students develop certain mathematics skills but lack conceptual understanding of the basic concepts.

Most previous studies in modern physics had focused on certain particular topics instead of the relationship or correlation between topics in modern physics. Muller and Wiesner (2002) suggested that the strange and counter-intuitive phenomena of quantum mechanics could be organized and incorporated into a coherent cognitive picture with carefully chosen basic concepts. Therefore in this project, efforts were made to investigate students' understanding of probability and students' reasoning resources that indicate their comprehensive understanding of probability. This terminology of reasoning resources will be explained in the next section about the theoretical framework.

## 2.3 Coordination Classes: A Model of a Knowledge System

### 2.3.1 Motivation for using Coordination Classes

Many physics education research works involved studying the differences between novices and experts in problem solving or comparing students' concepts to experts' concepts, which are acceptable in the physics community. The main finding from these works is that novices often recognize physics problems by surface features, compared to experts who recognize physics problems by physics principles or laws associated with the problems. In addition, students who solved as many as 1000 mechanics problems still had many of the well-known conceptual difficulties (Kim & Pak, 2002). If our goal of physics instruction is to help students learn physics both conceptually and quantitatively, this finding indicates that students who can solve problems successfully might not actually learn concepts of physics. Many researchers suggest that novices just memorize steps to solve specific problems, so their responses usually are context-dependent, especially when they first learn the material. In other words, students are unclear how and when to apply conditions in different types of problems (Bao & Redish, 2001). Can this behavior count as learning?

To answer the above question, let's discuss the process of learning. There are many theories of learning in education. The most well known in the physics education community is called *conceptual change theory* (Posner *et al.*, 1982), which is the theoretical model of a process of learning in order to separate two types of learning—fact memorization and gradual changes in beliefs (DiSessa & Sherin, 1998). The conceptual change often is considered to involve changes in core concepts. If we do not know what a concept is, statements about changes in concepts are questionable. Most previous researches involving conceptual change treated concepts as a black box. They have described only the before and after states of students' thinking rather than observing how the concepts work and develop or fitting data from observation into a theoretical model (DiSessa & Sherin, 1998). In addition, empirical studies on physics education have focused on student difficulties and misconceptions, of which the latter is ambiguous because of the same reason, the lack of an explicit model of concept. Without a better model of concept we cannot conclude anything general about conceptual change from any empirical researches (DiSessa & Sherin, 1998). Therefore, many educators and cognitive psychologists have developed a theoretical cognitive model to represent the knowledge structures of a learner.

Many models or structures attempt to represent insights into student reasoning and student knowledge structures such as phenomenological primitives or p-prims (DiSessa, 1993), facets (Minstrell, 1992), reasoning resources (Hammer, 1996), schema (Chi *et al.*, 1994), and mental model (Vosniadou, 1994). These models will be discussed in detail later. The focus of this study will be on the recently proposed model by DiSessa and Sherin (1998), the coordination class, a model of how people "see" or recognize information in the world by coordinating directly observed information to infer implicit information.

In the following sections, I will describe coordination classes in detail, including a structure of coordination classes, criteria of performance, an example that DiSessa and Sherin gave in their paper but with my own interpretation, utility of coordination class, and limitations of coordination class.

## 2.3.2 Structure of Coordination Classes

DiSessa and Sherin (1998) began their ideas of coordination classes with the contention that 'seeing things'—metaphorically referring to interpreting information—in the world is a complex cognitive execution. Therefore, they attempted to describe what generates this ability to 'see things.' As a result, they created a model concerning a class of concepts, called *coordination classes*. They defined coordination classes as "systematically connected ways of getting information from the world," which include strategies of selecting attentions, determining and integrating observations into the essential information. The coordination class consists of two structural elements—a collection of *readout strategies* and a *causal net*. Readout strategies are strategies used in selecting our attention on meaningful information. The information is, then, coordinated with a set of inferences or reasoning strategies to determine implicit information or to draw a conclusion. The set of inferences or the reasoning strategies is called the *causal net*. The process of coordination between readout information and causal net inferences can be repeated or continued on a 'conceptual bootstrapping' process (DiSessa & Sherin, 1998) if the observer cannot conclude on any reliable information. DiSessa and Sherin explained the process of conceptual bootstrapping quite well as follows:

> The relations between readout strategies and the causal net are intimate. One looks for things that are related (via the causal net) in order to determine some quantity. Indeed, even seeing those secondary features may involve addition inferences from the same or another causal net. In general, readout strategies and the causal net should co-evolve as learning occurs. There should be episodes of 'conceptual bootstrapping', where causal assumptions drive the learning of new readout strategies. On the occasions, 'noticings'—for example, that something surprisingly affects something else—may drive reformulations in the causal net. In general, characteristics of one will have important influences on how the other behaves and develops. (p.1177)

DiSessa and Sherin (1998) also indicated that from this coordination class perspective, the conceptual change is possible to occur as a result of changes of readout strategies and the causal net. They explained further that the changes can be a small change in organization, development, or even a total change by constructing a new causal net or readout strategies.

### 2.3.3 Coordination Classes Performance Criteria

In order to determine the reliability of concluded information, DiSessa and Sherin suggested that the coordination classes have two performance criteria—invariance and integration. The two criteria are used to check the reliability of inferred information or a conclusion. When multiple readout strategies are used to observe one situation and then arrive at a coherent set of inferences as the outcome, the process is called *integration.* When different types of strategies or observations are used in different circumstances and a stable conclusion is reached, the process is called *invariance.*

### 2.3.4 Examples of Coordination Classes

#### 2.3.4.1 Perspective of Coordination Classes

The perspective of coordination classes is to extract the meaningful information from the situation; therefore, the coordination depends on purpose. DiSessa and Sherin (1998) gave one example: You are at a party, and you meet John for the first time. You, then, try to figure out John's personality, for example, is he sincere? In order to determine his sincerity, you need to pay attention to his tone, his eye contact, his gesture, and whatever one defies as readout strategies or signals for sincerity. Then, you need to gather the information from several observations in order to conclude or infer if he is sincere. This is one example of coordination of multiple readout strategies and application to the 'sincerity' causal net. The 'sincerity' causal net for each individual is different depending on their experience. Also, people from different cultures might have quite different 'sincerity' causal nets. This is just an example of the coordination class in everyday experiences. The next example steps into the differences between experts and novices from the perspective of coordination classes.

### 2.3.4.2 Expert and Novice Differences in Coordination Classes

DiSessa and Sherin (1998) believed that the coordination class "is necessary to describe specific ways in which a learner's concept system behaves like an expert's, and the ways and circumstances in which it behaves differently." Wittmann (2002) gave a really interesting example of differences in the causal nets structure between an expert and a novice driver when they are driving and seeing a plastic bag in the road in front of a car. He indicated that a driver's reaction depended on the ability to estimate the bag's location, size, weight, etc. For example, the ability to estimate the bag's location is usually the one that we already have and use in everyday experience when we interact with people or objects around us. In comparison to the novice drivers, the expert drivers could evaluate these choices and act immediately. Wittmann explained that what differentiates the expert from the novice is not the existence of reasoning resources[5] in their thinking, but rather the structure and organization of the reasoning resources. The experts have a more coherent and organized structure of reasoning resource; as a result, the experts can activate these reasoning resources or causal nets quickly as a whole in a particular situation, compared to the novices. From my own interpretation, the expert's quick reaction can also be a result of

[5] In his article, Wittmann (2002) used reasoning resources, which he referred to be the same as the causal net by DiSessa and Sherin (1998).

more experiences or more practice, so the expert's causal net is always activated and ready to use.

Although this example is not from physics education, it gives some aspects of coordination classes in understanding expert-novice differences in a general context. In order to present the important aspect of the coordination class in its application to how people construct their science learning, the following are examples in learning physics.

### 2.3.4.3 Coordination Classes in Physics Learning

In order to help understand coordination classes in physics learning, I discuss DiSessa and Sherin (1998)'s example of J, a freshman student who had done well in high school physics and did well in introductory physics. In one of the episodes, the interviewer had introduced the problem of pushing a book along a table and asked J to describe the situation. J explained the motion of the book as the result of imbalanced forces; in DiSessa and Sherin's words, J used the phenomenological primitives or p-prims[6] *balance implies rest* and *imbalance implies motion*. Later, J found her naïve causal net or the above p-prims in contradiction with an equation $F = ma$. She noticed the constant speed and understood that from the equation, unbalanced forces resulted in acceleration not constant speed. J carefully re-examined the situation if there was any acceleration of the book or not, and she confidently concluded that there was no acceleration. J resolved this conflict by deciding that $F = ma$ did not apply to this situation, pushing a book. DiSessa and Sherin claimed that J's problems were not misclassifying a force, but rather inability to select correct attributes, like motion or rest, to coordinate. In addition, this example confirmed their idea that equations cannot be equated with the causal net.

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[6] *Phenomenological primitives* (p-prims) or reasoning primitives proposed by DiSessa in 1993. P-prim is an irreducible cognitive element in order to "understand the intuitive sense of mechanism which accounts for commonsense predictions, expectations, explanations, and judgments of mechanical situations."

DiSessa and Sherin claimed that the coordination class is important in science education, so they explained the role of equations from a perspective of coordination classes. They specified that equations are not coordination classes. However, a learner can use equations to help coordinate observations to needed information. They also suggested that a qualitative use of any equation is more important than a quantitative use.

### 2.3.5 Limitation of Coordination Classes

DiSessa and Sherin (1998) were careful not to interchange concepts with the idea of coordination class because they stated that the coordination class is only a type of concept, so some concepts, such as category-like concepts, cannot be explained by this model. However, they claimed that the coordination class is important in learning science and in explaining the conceptual change theory.

Although the coordination class helps clarify the conceptual change theory, the coordination can be accounted for only by one form of conceptual change theory. For other type of concepts, other forms of conceptual change might be needed in order to understand the process.

### 2.3.6 Coordination Classes and Previous Cognitive Models

Many theoretical frameworks from either education or cognitive psychology describe student learning or student reasoning processes by focusing on a fine-grain structure model of knowledge, for example phenomenological primitives or p-prims (DiSessa, 1993), facets (Minstrell, 1992), reasoning resources (Hammer, 1996), schemas (Chi *et al.*, 1994), mental models (Vosniadou, 1994), and physical models (Bao & Redish, 2001). We will briefly discuss these theoretical structures of cognitive elements, and we will then compare and contrast these models with the coordination classes.

As mentioned before, the *phenomenological primitives* (p-prims) or reasoning primitives are the smallest unit of cognitive element in order to understand the intuitive sense of mechanism. DiSessa found that people describe how a thing works mechanically in the real world, often, by using simple and brief statements. When they were asked to explain further, they cannot give any reasons beyond it with typical response as, "That's just the way things work" (Bao & Redish, 2001). Hammer (1996) indicated further that how students respond depends on which p-prim is activated. For example, students often use a statement "closer means stronger" to explain a higher temperature when closer to a heat source, or to explain a stronger light intensity when closer to a light source. However, this "closer means stronger" p-prim accounts for students' incorrect responses when they explain a relationship between the Earth's rotation and season changes. Students often use *closer means stronger* to explain that as the Earth gets closer to the Sun, the Earth gets more sun light, as a result it becomes hotter or the season changes to summer.

Minstrell (1992) proposed a similar fine-grain model, called a *facet*, which is a convenient unit of thought, a piece of knowledge, a pre-existing idea, or a strategy (Niedderer & Schecker, 1992). Students use the facets in order to explain particular physical situations. For example, students come to the introductory physics class with a pre-existing idea that a heavier object falls faster than a lighter one. This idea is there before any physics instruction because students construct the idea from their everyday experiences. Minstrell's facets and DiSessa's p-prims both support the idea of "knowledge in pieces" (DiSessa, 1993). Although facets and p-prims seem similar, p-prims are more refined than facets and contextually independent. Therefore, we can think of facets as a mapping of p-prims into a physical situation (Bao & Redish, 2001), in other words, as relating p-prims to an appropriate context or physical situation.

*Reasoning resources*, proposed by Hammer (1996), is a small grain size model of reasoning, which can be described by either p-prims or facets (Wittmann, 2002).

The term *schema* has been used often in the physics education literature, especially problem-solving studies. Schema is defined as a theoretical construct describing the way knowledge is organized in memory. In other words, Maloney (1994) explained schema as "a model we have in memory for an object, situation, event, and so forth." Bao and Redish (2001) gave a similar definition as, "a cognitive element or set of cognitive elements that activate together in a cascading chain of activation or priming in responses to a stimulus or situation presented to student." A mental model is considered one type of schema that has a well-constructed structure (Bao & Redish, 2001).

The physical model is introduced by Bao and Redish (2001), as a type of mental model that is used or activated depending on particular population, context, and concept. For example, when asking about an atomic model, a middle school student might come up with the Bohr model, while a condensed-matter physicist might think of electrons and ions in a microscopic level involving an energy gap.

DiSessa and Sherin (1998) mentioned that p-prims can be considered as one basic structure of the causal net. From carefully analyzing the definitions of facets and reasoning resources, it can be concluded that these two types of fine-grain structure should be fitted into fundamental structure of the causal net as well.

In comparing schema with the element of coordination classes, it is similar to the causal net. However from its definition, the schema role is fairly passive compared to the causal net. This is because the lack of 'conceptual bootstrapping' (DiSessa and Sherin, 1998) between readout strategies and the causal net. Therefore from review and synthesis of previous cognitive models, the coordination classes are the most complete and well defined to explain students' reasoning processes.

### 2.3.7 Previous PER using the Coordination Classes

The coordination class is considered innovative among the PER community, so there are a few studies that employed this model to analyze data. In order to clarify the role of coordination class in physics education research, I review two recent empirical studies that analyzed data by using the coordination class as a guided theory.

Wittmann (2002) used the coordination class to analyze student responses on a semi-structured interview about wave physics. The interviews were conducted at the University of Maryland between 1996 and 1998 as a part of physics education research. The participants were students taking calculus-based introductory physics in the second semester of a three-semester sequence. Wittmann indicated that students who participated in the interviews were doing well in the class. The author pointed out that the physics of waves involves the description of propagating disturbances in media, independent of a length of wavepulses or wavetrains. The interview questions, focused around this main concept of wave, consisted of two contexts: the creation and propagation of wavepulses on a string and a superposition of wavepulses. Wittmann analyzed the interview responses using the elements of coordination class—readout strategies and the causal net. He found that students only paid attention to the peak of the wavepulses, so he suggested that students saw the wavepulse as an object with its peak as a representation point. Regardless of object-like description of wavepulses, students employed a causal net in terms of event-like descriptions such as Actuating Agency [7](Hammer, 1996), Maintaining Agency [8](Hammer, 1996), and Working Harder [9](DiSessa, 1993). In summary, Wittmann found evidence from students' responses that students employed an object coordination class intuitively

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[7] Actuating Agency describes an initial cause of some effect, for example, the force that sets an object in a motion.
[8] Maintaining Agency describes a continuing cause to maintain a motion.
[9] Working Harder describes when the stronger force is applied and makes the wavepulses move faster.

to reason about the motion of wavepulses in different situations. The object coordination class consisted of two readout strategies and two resources, which Wittmann defined to be the same as the causal. The readout strategies included *object as point* and *wave as solid.* The two resources included motion resources, consisting of *Actuating Agency*, *Working Harder* p-prim, *Smaller is Faster* p-prim, and interaction resources, consisting of *Adding*, *Bouncing*, and *Canceling.* The author claimed that the coordination class helped to observe students' reasoning in wave physics in terms of inappropriately linked reasoning resources, instead of ambiguous misconceptions.

In another study using the coordination class in a classical mechanics context, Thaden-Koch *et al.* (2004) used the coordination class to analyze interviews in which college-level students judged the realism of ball-rolling simulations. The authors claimed that the coordination class was useful in understanding students' decision-making processes. However, the authors used the term coordination system instead of the coordination class because the *coordination system* does not necessarily meet the performance criteria of either integration or invariance. Individual interviews were conducted with two groups of students taking an honors calculus-based physics course and an educational psychology course. In the interviews, students were asked to judge the realism of one ball and two balls rolling on a pair of metal tracks A and B. Both tracks begin with an initial incline; then, Track A is flat while Track B begins and ends at the same height as Track A but includes a V-shaped valley. The first animation presents one ball rolling on Track B only. The second animation presents both balls released simultaneously, and ball B wins the race. For each animation, there were 5 versions in which ball B's motion was not realistic, except animation 5. However, ball A was rolling at a constant speed after the initial incline in all animations. Twelve interviews with physics students and twenty four interviews with psychology students were recorded, transcribed, and analyzed by the coordination system. The authors claimed that in order for students to decide which animation was more realistic than the other, they needed to develop certain expectations and compare their observation with those expectations. The authors referred to common student expectations for realistic motion as causal net elements and students' observation of speed change as readout strategies.

### 2.3.8 Roles of the Coordination Classes in this Study

Most physics education researches analyze students' responses from interviews or open-end questions according to misconceptions or students' difficulties. However, both speculations are too broad and do not provide any useful information in terms of instructional implications (Wittmann, 2002; Hammer, 1996). In order to provide useful results beyond students' difficulties about probability concepts, I also apply the coordination class in analyzing students' responses on both open-end questions and interviews. In this study, the data were analyzed not only in terms of student difficulties but also common elements involved in students' coordination when they were reasoning about probability. These two approaches in the data analysis should provide us with a more fruitful and complete picture of students' understanding of probability.

### 2.3.9 Summary

Coordination classes include strategies of selecting attention—readout strategies—and strategies of determining and integrating observations into the requisite information—the causal net. The coordination classes also involved two performance functions: coordinating observations within a single situation (integration), and coordinating different features in different situations to find the same information (invariance).

The utility of coordination class is not only to understand student reasoning but also to help us gain understanding of differences between experts and novices. Thaden-Koch (2003) indicated that the coordination class theory can help us gain more understanding of "how knowledge and observations interact, and of what it means to learn and use scientific concepts." The coordination classes can help explain student reasoning and help us see in more details of what students are able to do and how they do it, instead of what they are unable to do.

Comparing the coordination class to the existing theories of cognitive structure, what is unique about this model are the readout strategies and two performance functions. The previous fine-grain cognitive structure, such as p-prims, facets, or reasoning resources, can be described as one node in a complex structure of the causal net. Also, the coordination classes model is more complete than the idea of schema because of its 'conceptual bootstrapping' between readout strategies and the causal net.

The coordination classes can only be used to explain certain types of concepts. However, most of coordination class concepts are important in learning science (DiSessa and Sherin, 1998). Therefore in this study, the data about students' understanding of probability were analyzed by two approaches—students' difficulties and students' reasoning processes based on the coordination classes. The purpose of using two approaches is to give a way to understand and convey the richness of student reasoning (Wittmann, 2002).

# Chapter 3

# Research Design

In this chapter, we describe the focus group, the research design, and the researcher. The focus group includes a modern physics class (PH314) and characteristics of students taking this class. We will describe in more detail those students who participated in interviews in Chapters 5 and 6. For a research methodology, we used both a qualitative and a quantitative approach in order to collect both in-depth and in-breadth data. Since the study involved a qualitative methodology, we provide some background of the researcher who did the data analysis of the interview data.

## 3.1 Focus groups

This study is a descriptive study to investigate students' conception of probability in a modern physics context. Therefore, only one group is enough to provide descriptive data. The population was undergraduate students taking a modern physics course (PH314) at Oregon State University (OSU) in Fall 2003 and Spring 2004. The following subsections describe the focus group in terms of the structure of the modern physics class and the characteristics of population or students taking this class.

### 3.1.1 Modern Physics Class (PH314)

The modern physics class is a requirement for electrical engineering and physics majors. This class is a 10-week class and is usually offered in fall and spring quarters. The class meets three times a week for an hour lecture and once a week for a three-hour lab. The class covers general topics about modern physics including the special theory of relativity, wave and particle properties of atomic particles, Schrödinger equation, and introduction to statistical physics.

Both the Fall 2003 and Spring 2004 classes had been taught with the same instructor, who has a lot of experience in teaching physics classes, especially modern physics. He has been teaching this class for several years, and he wrote a modern physics textbook. His interest also includes physics education, so he uses many teaching techniques to improve the class and help students learn more effectively.

The teaching techniques include Peer Instruction and in-class group activities. At the beginning of each lecture, students are required to hand in a homework assignment. The homework includes materials that were covered from the previous lecture. During the first five minutes of the class, the instructor gives students a reading quiz. Students, then, answer the quiz by using their Personal Response System (PRS)[10]unit, which is a popular device used in Peer Instruction classrooms. Then, the instructor gives a brief lecture summarizing the new materials. The instructor follows his lecture by discussion questions in order to see how well students have learned the new materials. Depending on the fraction of correct responses students may be asked to discuss with their neighbors and re-enter their answers. Most of the questions aim to help students gain conceptual understanding of topics covered in that lecture.

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[10] PRS is a tool for voting or surveying with immediate results. After students submit their answer through a transmitter, which looks similar to a remote control, receivers in the lecture room pick up the signal and send it to the instructor's notebook, which displays histograms of the class answers.

The in-class activities focus on major conceptual difficulties and are based on the instructor's teaching experience or physics education research findings. For example, most students have a hard time with graphical representation of a wave and a wave packet. The instructor developed an in-class activity to help students gain more understanding about relationship between momentum and wavelength in different situations.

### 3.1.2 Participants

The participants were undergraduate students, who took a modern physics class (PH314) at OSU. Most of the students taking this class are electrical engineering, nuclear engineering, computer science/engineering, physics, and mathematics majors. About half of them are electrical engineering students. Levels of students ranged from sophomore to senior, with most of students in their senior year. They already took an introductory physics with calculus sequence and an ordinary differential equations class in their sophomore year as their requirement and prerequisite for PH314 course. The demographic information of participants in each term is presented in Table 3.1.

| Terms | Freshman | Soph. | Junior | Senior | Non-deg. undergrad | Post-bac. | Total |
|---|---|---|---|---|---|---|---|
| Fall 03 | 1 | 7 | 9 | 34 | - | 1 | 52 |
| Spring 04 | - | 3 | 31 | 47 | 1 | 2 | 84 |

**Table 3.1: Statistics of students enrolled in each term**

## 3.2 Research Design

The research design consisted of two parts—quantitative and qualitative methods. We used both types of methodology because we would like to collect different types of students' responses in order to answer our research questions. In educational research, using different methods to collect data to check for consistency of students' response is called *triangulation*. The idea of triangulation is the same as an agreement of findings either from simulations in computational physics research or from laboratory results in experimental physics research. Also, collecting and analyzing qualitative data requires a lot of time and effort; therefore, we employed mixed methodology to reduce time and still obtain valid data. Figure 3.1 shows a diagram representing the order of data collecting (in Spring 2004) with different methods.

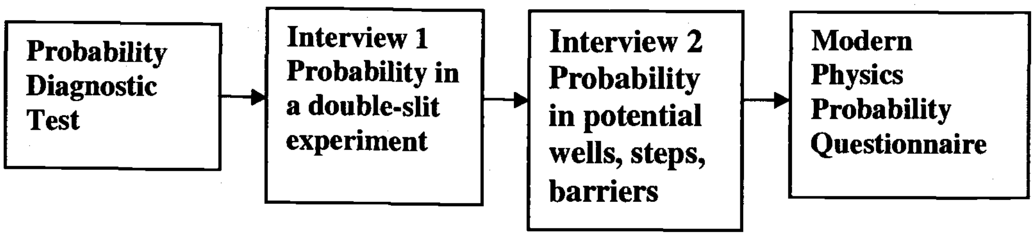


**Figure 3.1: Data collection order for Spring 2004 term**

## 3.2.1 Quantitative Method

Quantitative data was collected by using 3 tests—a probability diagnostic test, a modern physics probability questionnaire, and conceptual quizzes.

### 3.2.1.1 Probability Diagnostic Test

The main purpose of this test is to probe students' probability conceptions prior to taking a modern physics class. From the literature, it appears that most of the studies of students' misconceptions usually employed a qualitative approach, especially interviews (Treagust, 1988). However, Treagust (1988) suggested that a quantitative approach could also be used to probe students' conceptions; for example, Halloun and Hestenes (1985) developed a physics diagnostic test assessing students' basic knowledge of mechanics. Treagust used the term "diagnostic test," which is the multiple- choice test, being used to probe students' misconceptions. The diagnostic test is different from typical multiple choice tests, because the distracters are based on findings from previous qualitative research about students' misconceptions in that particular topic or from the answers of students to open-end or essay questions. From the previous literature, however, the diagnostic test contains either the multiple-choice test alone or the multiple-choice question with an open-end question asking students to explain their reasons for that choice. Therefore, in this study we used a two-tier multiple choice test, as presented in Figure 3.2, in order to obtain not only students' misconceptions about probability but also their reasons for making certain choices in the test. For example:

Which of the following is the most likely result of five flips of a fair coin?

a) HHHTT
b) THHTH
c) THTTT
d) HTHTH
e) All four sequences are equally likely.

**Please explain your reasons:**

**Figure 3.2: An example of two-tier format used in the diagnostic test.**

Based on the synthesis of mathematical probability misconception researches discussed in Chapter 2, items in the instrument were categorized according to seven misconceptions, which are presented in Table 3.2. Three misconceptions came from people using the representativeness heuristic — random distribution expectancy, sample size, and Gambler's Fallacy. The other two misconceptions were due to people using availability and outcome-approach heuristic, respectively. The last two misconceptions cannot be explained by any cognitive theory--sample space and mean. From the literature, the researcher selected 19 multiple-choice items regarding these seven misconceptions. After discussing with an expert in physics education, three items were eliminated because of an inappropriate content. The expert also suggested adding another four items regarding probability in a classical physics context, which is known as classical probability in the physics community. The complete diagnostic test may be found in Appendix A.

| Misconceptions | Items |
|---|---|
| **Judgmental Heuristic** | |
| Representativeness heuristic | |
| • Random distribution expectancy | 1, 12, 20 |
| • Ignoring sample size | 9, 10, 16 |
| • Gambler's fallacy | 5, 14 |
| Availability | 15 |
| Outcome approach | 2, 11 |
| **Other misconceptions** | |
| • Sample space | 4, 8, 17 |
| • Mean | 6, 7 |
| **Classical Probability** | 3, 13, 18, 19 |

**Table 3.2: Items corresponding to seven misconceptions**

In order to understand how we developed the test, the procedure of test development is presented below.

Step 1: Obtained the seven misconceptions about probability from the previous literature in both physics education and mathematics education.

Step 2: 19 items regarding the misconceptions were gathered from the previous literature.

Step 3: The items were reviewed by an expert in the physics education field. In order to verify the content validity, the expert eliminated three items and suggested to add four items about classical probability in the physics context.

Step 4: (Pilot study) The pilot test was administered to two graduate students to verify the readability of the test.

Step 5: The test was corrected according to suggestions from the graduate students.

Step 6: The test was administered to the subjects in the first laboratory.

Step 7: The same test was also given to 12 physics graduate students. The test results from graduate students were used as a known group result, which was calculated a known-unknown difference to verify the construct validity of the test (see Chapter 4 for more detail).

The diagnostic test was administered in laboratory sections of a modern physics class (PH314). The diagnostic test was administered only in Spring 04 because we needed more information about students' conceptions of probability after reviewing collected data from Fall 03. The test was given right after students finished the first lab. Teaching assistants for each lab administered the test. The students were asked to spend no more than 45 minutes to complete the test.

### 3.2.1.2 A Modern Physics Probability Questionnaire

The questionnaire is designed to obtain students' concepts of probability in a modern physics context. There are two versions of the questionnaire—Fall 2003 version and Spring 2004 version. Statements from both questionnaires were developed from students' interview results. The Spring 2004 version also came from revising the Fall 2003 version of the questionnaire. Students were asked to indicate whether they agreed or disagreed with the given statements. Then, they were required to give an explanation of why they agreed or disagreed with each statement. There are 17 statements in the Fall 2003 version, and 19 statements in the Spring 2004 version. The questionnaire may be found in Appendix B. From discussion with the PH314 instructor, the questionnaire statements were based on the following concepts, as presented in Table 3.3.

| Constructs | Items |
|---|---|
| Probability wave and localization | 1, 8 |
| Principle of complementarity | 4 |
| Single-slit and uncertainty principle | 2, 7, 12, 16 |
| Double-slit and probability | 17, 13, 15, 6, 3, 10 |
| Probability of finding an electron in potential energy well | 9, 14, 18 |
| Probability of finding an electron in tunneling | 5 |
| Probability of finding an electron in encountering a step | 11 |
| Probability | 19 |

**Table 3.3: Constructs for the modern physics probability questionnaire (Spring 2004 version)**

We discuss procedures of developing the questionnaire below, so the reader can understand the procedure and we can establish the validity of the questionnaire content.

Step 1: Brainstorming concepts related to probability in a modern physics context.

Step 2: Use initial results from both Fall 2003 interviews and come up with statements that students usually made during the interviews.

Step 3: Create a list of items or statements.

Step 4: Discuss with the PH314 instructor and select items from the list to finalize the first draft of questionnaire.

Step 5: Proof-read the first draft by the instructor, the researcher, and one graduate student.

Step 7: The Fall 2003 version of the questionnaire was administered to the subjects.

Step 8: The results from the Fall 2003 questionnaire were analyzed.

Step 9: Items that seemed unclear to students were eliminated.

Step 9: Use initial results from Spring 2004 interviews to construct more items.

Step 10: Discuss with the PH314 instructor and revise or eliminate items that were unclear or would create some conflicted statements within the questionnaire.

Step 11: The Spring 2004 version of the questionnaire was administered to the subjects.

Step 12: The questionnaire also was also given to a physics senior to compare the results.

Both questionnaires were administered in the last laboratory period. Students were given about 45 minutes to complete the questionnaire. However, some students did not finish the questionnaire, or they did not explain their reasons why they agreed or disagreed with each statement. This might be because completing the questionnaire would not affect their grades for the class. Because of the small numbers of complete questionnaires, we did not report the results from the questionnaire.

### 3.2.2 Qualitative Method: Interviews

The qualitative method consisted of two interviews. The preliminary interview was conducted in Fall 03. We used the results from Fall 03 to modify the Spring 04 interviews. There were two interviews for each term that we collected data. The first interview aimed to get students' general concepts about electrons and probability in a single and double slit experiment context. The second interview aimed to obtain students' conceptions about probability when they were asked to explain the wave function solutions to potential well, barrier, and step problems.

At the first class, students were informed that they could receive extra credit if they participated in the interviews. Students also received a handout letter explaining the purpose, procedure, and brief details of the interviews. The sign-up sheet for the interviews was distributed in the second laboratory. Students were asked to write their name in the time slot that they were available. Students also had an option to write a term paper to receive extra credits if they decided not to participate in the interviews.

Students were informed in a letter that the interview would be video taped. Most of the students who participated in the first interview also participated in the second interview if they did not withdraw from the class. The number of students who participated in Fall 2003 and Spring 2004 interviews are presented in Table 3.4.

| Terms | Interview I (only) | Interview II (only) | Both |
|---|---|---|---|
| Fall 03 | 6 | 7 | 7 |
| Spring 04 | 3 | - | 12 |

**Table 3.4: Numbers of students who participated in Fall 03 and Spring 04 interviews**

## 3.3 Researcher

In a qualitative study, the researcher is a "main" data collection instrument; therefore, one potential threat to the validity of qualitative study is researcher bias. Johnson and Christensen (2004) stated in their book, "The problem with qualitative research is that the researchers 'find' what they want to find, and then they write up their results." In order to avoid researcher bias, the key strategy used to understand one's own bias is called *reflexivity* (Johnson & Christensen, 2004). Here is the description of my personal background, how it may affect the research, and strategies I used to address potential problems.

Originally, I came from Thailand. I received pre-college and one year of college education there. I received a scholarship to study physics in United Stated during my freshman year at Chiangmai University, Chiangmai, Thailand. I came here during June 1995, and continued my undergraduate study at Lehigh University, Bethlehem, Pennsylvania. I studied physics at Lehigh for four years before I applied for the masters and PhD program at Oregon State University (OSU). I received a teaching assistant (TA) appointment during my study for the masters and PhD degrees at OSU.

Being a TA not only has influenced my perspectives about learning and teaching but also my interest in physics education. Gaining more experience in teaching, I have become more curious about how people learn and form concepts, especially physics concepts. I started taking science education classes to serve my own curiosity at first. I realized that I enjoy learning about how people learn, and my interest in physics education has developed to a point that I decided to conduct my PhD dissertation in physics education.

I determined to conduct a physics education research project about modern physics because of my own struggle to understand the subject when I first learned it. Although I got an A from the class, I did not quite understand it enough. As a result, I had even more difficulties learning quantum mechanics in upper-level classes. As recalled from my experience, I could solve problems mathematically, but I had a hard time understanding problems and solutions conceptually. This experience influences my belief that we need to help students understand physics at a conceptual level not only at a problem-solving level. Also from my experience, my inability to grasp the concept of probability inhibited me from further understanding of the class. This is because probability is a profound concept in modern physics and quantum mechanics (Bao & Redish, 2001). Therefore, I decided to conduct physics education research focused on students' conceptual understanding of probability.

In conducting physics education research, I enrolled in many science-education classes. These classes influenced my constructivism perspective about how knowledge is structured. In the early 1900s, a newborn's mind was viewed as a blank slate. Beginning in the 1920s, experimental findings by Jean Piaget, a well-known experimental cognitive psychologist, suggested that the young human mind (even the newborn) can best be described in terms of complex cognitive structures. Piaget's work on cognitive development supported the epistemology view that knowledge is constructed. Based on the constructivism perspective, my research was based on students' conception of probability in a double-slit and a single-slit experiment, which they had seen before in an introductory physics class.

"Probability" in my understanding is the chance of a specific event to happen under a limited condition. Therefore, in order to compare different chances of similar event under similar conditions, you somehow need to normalize both systems. Or you need to find the ratio between the event and the condition in order to be able to compare the probability. This idea of normalization is also important in wave mechanics.

# Chapter 4

# Results of Mathematical and Classical Probability Misconceptions

Quantum mechanics (QM) and modern physics deal with phenomena not directly observable in everyday experience. Thus quantum misconceptions are different from misconceptions found in introductory physics courses, which often are results of an incorrect or naïve "world view" (Singh, 2001). Even though modern physics is not easily observed in everyday experiences, students might have some experience or develop some basic understanding about probability, which is a fundamental concept of QM. Students might construct prior probability knowledge from mathematics classes or from introductory physics classes. Consequently, this knowledge will have an effect on students' learning probability in a modern physics context. This chapter presents the results of the diagnostic test, which aims to measure students' misconceptions regarding mathematical and classical physics probability. From literature reviews, there are many types of mathematical probability misconceptions as shown in Figure 4.1.

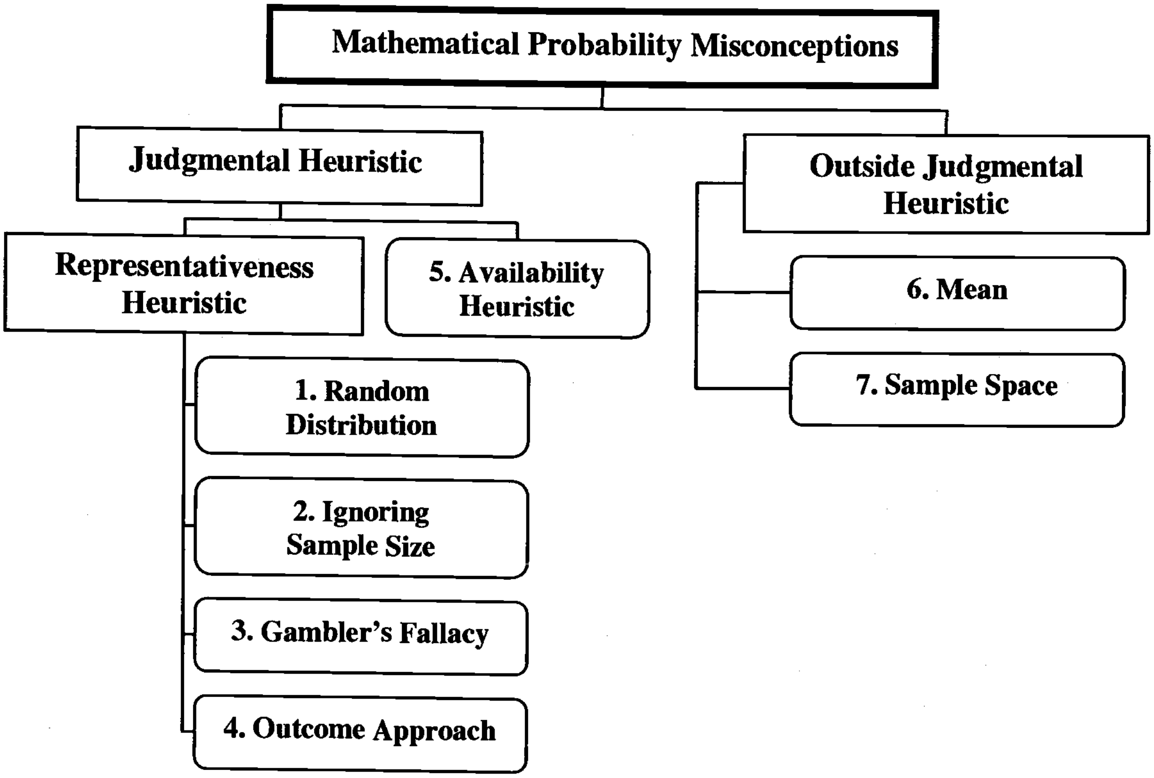


**Figure 4.1: Seven mathematical probability misconceptions**

## 4.1 Statistical Analysis

The probability diagnostic test is a multiple-choice test; thus, the results were analyzed in terms of item analysis, item difficulty, and item discrimination. The issues of validity and reliability are discussed according to the results of statistical analysis. There were total of 75 students taking the test. However, one test was disregarded because it was less than 50% complete, so only 74 tests were used in the data analysis.

### 4.1.1 Item Analysis

Item difficulty and discrimination were determined in order to establish the credibility of the test. Item difficulty was determined by calculating the average score for the question divided by the maximum achievable score. Later this was converted into percentages and expressed in terms of low (> 80% or > 0.8), medium (20% - 80% or 0.2-0.8) or high (< 20% or < 0.2) difficulty level.

Item discrimination indicates the extent to which success on an item corresponds to success on the whole test. Since all items in a test are intended to cooperate to generate an overall test score, any item with negative or zero discrimination undermines the test. Positive item discrimination is generally productive, unless it is so high that the item merely repeats the information provided by other items on the test (Ebel *et al.*, 2002). To calculate the item discrimination, the tests were divided into two equal-sized high and low scoring groups. Using Truman Kelley's "27% of sample" group size (Ebel *et al.*, 2002), the top 27% of the high scoring group (U) and the bottom 27% of the low scoring group (L) were used in calculating the *Discrimination Index (D).* The index was calculated by subtracting L from U and dividing by half of the total number of students or the size of a group.

$$D = (U - L) / (0.5\ \mathrm{Total}) \tag{4.1}$$

The range of this index is +1 to -1. D-values of 0.4 and above are regarded as high and less than 0.2 as low (Ebel *et al.*, 2002)

The process of calculating item difficulty and discrimination was repeated for each item and represented in a tabular format. As presented in Table 4.1, the items 4, 15, and 17 that had too high (below 0.2) or too low (above 0.8) item difficulty level were eliminated from the test.

| Items | U | L | Difficulty | Discrimination |
|---|---|---|---|---|
| 1 | 20 | 3 | 0.548 | 0.810 |
| 2 | 16 | 0 | 0.381 | 0.762 |
| 3 | 17 | 14 | 0.738 | 0.143 |
| **4** | **20** | **18** | **0.905** | **0.095** |
| 5 | 20 | 13 | 0.788 | 0.333 |
| 6 | 11 | 15 | 0.619 | -0.191 |
| 7 | 19 | 14 | 0.786 | 0.238 |
| 8 | 16 | 3 | 0.452 | 0.619 |
| 9 | 17 | 10 | 0.643 | 0.333 |
| 10 | 13 | 1 | 0.333 | 0.571 |
| 11 | 18 | 1 | 0.452 | 0.810 |
| 12 | 20 | 4 | 0.571 | 0.762 |
| 13 | 6 | 10 | 0.381 | -0.191 |
| 14 | 21 | 12 | 0.786 | 0.429 |
| **15** | **1** | **7** | **0.191** | **-0.286** |
| 16 | 11 | 11 | 0.524 | 0.000 |
| **17** | **21** | **13** | **0.810** | **0.381** |
| 18 | 18 | 15 | 0.786 | 0.143 |
| 19 | 14 | 5 | 0.452 | 0.429 |
| 20 | 21 | 8 | 0.691 | 0.619 |

**Table 4.1: Difficulty and Discrimination of Each Test Item**

### 4.1.2 Validity

Reliability and validity of the instrument can be used to determine the credibility of quantitative research. Validity refers to the accuracy of the instrument, while reliability refers to the consistency of the measurement results. In other words, it is the degree to which the test scores are free from errors of measurement (Tangmongkollert, 1994). Validity is the most important quality to be considered in developing the instrument, as Krathwohl (1998) mentioned, "We seek validity first, because if there is evidence of adequate validity for our intended purpose, we don't need to worry about reliability." In this study, I determined two types of validity—content validity and construct validity. Content validity is used to exhibit the extent to which a sample of items, tasks, or questions on a test are representative of a domain of content that we would like to measure. The content validity was determined by experts in the field of physics and mathematics in order to confirm that the instrument measures an appropriate content of probability in both the physics and mathematics contexts.

Construct validity refers to the degree to which inferences can legitimately be made from the operationalizing[11] in a study to the theoretical constructs on which those operationalizings were based (Trochim, 2002). Construct validity is used to demonstrate that a test is a valid measure of constructs or focused concepts in this study (Krathwohl, 1998). Construct validity is important especially in the test aiming to measure misconceptions (Tangmongkollert, 1994). According to Tangmongkollert (1994), the known-group difference can be used to confirm the construct validity. The known-group difference procedure identifies two groups of subjects, those who understand the concept and those who have misconceptions concerning the concept (Tangmongkollert, 1994). Then, the t-test is used to calculate a statistically significant difference between the known concept group and the unknown group. If there is a significant difference ($p < 0.05$) between those two groups, then the test is considered to have construct validity or to provide an accurate measurement.

[11] Mertens (1997) defined operationalizing as a process of determining what to collect data about and how to do it.

### 4.1.3 Reliability

Not only validity is important in developing an instrument, but reliability is also necessary for the developer to consider. The reliability of the diagnostic test will be determined using the Spearman-Brown split-half technique (Huck, 2004) to find an internal consistency.

The test was divided into two sub-tests, even items and odd items tests, and then the findings from the two sub-test scores were correlated. The reliability coefficient was calculated using the Spearman-Brown formula. The reliability of the 20-item instrument was found to be 0.713. After eliminating items 4, 15 and 17, the reliability increased to be 0.754[12]. Since the diagnostic test was administered only at the beginning of the modern physics class, so the stability of the test or test-retest could not be established in this study.

## 4.2 Results and Discussion

This instrument consists of 20 questions with two parts—a multiple-choice part and an open-end part. The test aims to investigate students' conceptions of probability before taking a modern physics class (PH314). The first part contained a conceptual question, each with a set of multiple-choice responses. The second part asked students to provide reasons why they made that choice.

### 4.2.1 Known-Group Difference Procedure

The instrument was analyzed using a known-group difference procedure to determine whether this instrument could differentiate between students who had and those who did not have misconceptions of selected probability concepts. Twelve students out of 74 students total were randomly selected for this procedure as an unknown group. The known group consisted of twelve physics graduate

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[12] If the reliability is higher than 0.7, the test is considered to be reliable or consistent. If we give the test to a similar group of students, the results should not vary that much.

students who volunteered to take the test. Using the t-test, a statistically significant difference ($p < 0.01$) was found between those students who had and those who did not have misconceptions of selected concepts in probability, as presented in Table 4.2. This result indicates that the test can be used to measure probability misconception. This also indicates the construct validity of the test.

| Group | N | Mean Score | t | p-value |
|---|---|---|---|---|
| Unknown | 12 | 10.08 | -5.066 | 0.0004* |
| Known | 12 | 15.167 | | |

*p < 0.01

**Table 4.2: The known-group difference t-test**

### 4.2.2 Results of the Diagnostic Test

Students' reasoning was analyzed according to two viewpoints—misconceptions and coordination classes. The results will be presented categorically according to each type of the seven mathematical misconceptions, as presented in Figure 4.1, and a classical probability. For each item, the statistics of student responses on the multiple-choice part are presented and followed by the summary of student responses on the open-end part about their reasoning. At the end of each misconception, a summary of student responses on each type of misconception is presented in both misconceptions and coordination classes.

#### 4.2.2.1 Representativeness Heuristic: Random Distribution Expectancy

This misconception is about an expectation that random events should have random distributions. People who don't have training in probability theory or are still naïve statistically tend to think of events distributed randomly as being "most likely" to occur in any given situation. When considering the random selection of a certain number of events from a given sample space, it seems very likely that the events chosen will be from *all over* the sample space. Similarly, it seems very unlikely that the selection will consist only of a *portion* of the sample space. For that reason, people usually unknowingly *force* statistical arrangements to represent their beliefs about a random event. For example a set of random numbers will be carefully mixed up so no similar numbers are near one another. This misconception is called random expectancy distribution, which is a part of the representativeness heuristic (Kahneman & Tversky, 1972), and it was tested in items 1, 12, and 20.

**Item 1**

1. Which of the following is the most likely result of five flips of a fair coin?

    a) HHHTT

    b) THHTH

    c) THTTT

    d) HTHTH

    *e) None of the above*

Explain why you chose this answer.

*There is a 50/50 chance of getting either a head or a tail on each toss. However, each toss is independent of the previous toss or the next one. Therefore, any outcome has the same probability of happening.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 1 | 1.4 |
| b) | 12 | 16.2 |
| c) | 1 | 1.4 |
| d) | 24 | 32.4 |
| ***e)*** | ***36*** | ***48.6*** |
| Total | 74 | 100.0 |

**Table 4.3: Frequency and percent of students' responses on item 1**

**Summary of student reasoning for item 1**

We report the data based on the distracter that most students chose, and then we analyze students' reasoning about why they chose that particular answer. As presented in Table 4.3, thirty-eight students incorrectly chose answer d). Three out of twenty-four students chose d) because they thought that choice d) most represented a random distribution of five coins tossing. They intuitively thought that for random events the outcome should be distributed randomly, in this case alternating between head and tail, as one student mentioned:

> *"Although it is totally random, more likely to alternate from H to T."*

However, not all students who believed in random distribution of random events picked d). Twelve students picked b) because they were too skeptical to choose d), which they referred to as being perfectly random. Some typical student responses are:

> *"The coin flip would be random, but not perfectly random."* and *"A fair coin has a 50% chance of H & 50% of T. There is a lower chance of flipping alternate perfect sequence."*

Not only did students pick the wrong choices because of random distribution, but also most students seemed to base their answers on the 50-50 chance of getting head or tail. For examples:

> *"There is a 50/50 chance on the first flip but there is less than 50/50 chance one outcome will repeat,"* and
> *"The chance is 50% so it's more likely to alternate."*

This argument seemed to make sense at first because students who picked the correct answer, choice e) also provided the same argument about 50-50 chance of getting head or tail. However, the only difference between their reasoning is that students answering e) mentioned the independence of each flip. One student gave a typical response as:

*"Each flip has a 50% chance of heads or tails. Each flip is statistically independent of the others. For this reason each of the first four answers is equi-probable."*

**Item 12**

12. In a family of six children, which birth order of children is more likely?

a) BBBBGB

b) BBBGGG

c) BGGBGB

d) GGGBBB

*e) None of the above*

Explain why you chose this answer.

*We usually assume that the probability of getting a boy or a girl is 50/50. Also, each birth is independent of the others. Therefore, all of the above outcomes have an equal probability, so there is NO birth order of the children that is more likely.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 1 | 1.4 |
| b) | 0 | 0 |
| c) | 34 | 45.9 |
| d) | 0 | 0 |
| *e)* | *39* | *52.7* |
| Total | 74 | 100 |

**Table 4.4: Frequency and percent of students' responses on item 12**

**Summary of student reasoning for item 12**

When the misconception about the representativeness heuristic was presented again in item 12 but in a different context, more students (52.7%)

responded correctly, as presented in Table 4.4. Thirty-four students chose choice c) because of a random distribution. They provided common reasons for choice c):

*"Most variation,"* and *"It's the most random distribution."*

One student also presented an explanation revealing his thought process that the more variation or more random distribution of an event is more probable. He wrote:

*"There is more variation compared to the others so in my opinion it is more probable."*

However, students who chose choice c) might not be satisfied with their answer because they felt that choice c) did not present enough randomness, as one student explained:

*"Equal B and G, as well as regular variation. (BGBGBG is most likely, but not listed)"*

Most students who answered e) provided similar reasons as when they answered item 1, in terms of the equal-chance of having a boy or a girl and the independency of each birth. As one student explained:

*"Assume the birth probability between boys & girls is 50%, and each birth is statistically independent of the other births, all these answers are equiproable."*

Only two students stated that they guessed on their answers. As one student wrote:

*"There are an equal number of boys and girls in some of the answers, so any those are possible, plus, I have no idea."*

**Item 20**

20. One has to choose six numbers from a total of forty. John has chosen 1, 2, 3, 4, 5, 6. Ruth has chosen 39, 1, 17, 33, 8, 27. Who has a greater chance of winning?

a) John b) Ruth ***c) Both have equal chance of winning.***

Explain why you chose this answer.

*The probability of getting either set of six numbers is the same.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 1 | 1.4 |
| b) | 19 | 25.7 |
| ***c)*** | ***53*** | ***71.6*** |
| No response | 1 | 1.4 |
| Total | 74 | 100 |

**Table 4.5: Frequency and percent of students' responses on item 20**

### Summary of student reasoning for item 20

For item 20, more students (71.6%) answered correctly, as shown in Table 4.5. Students who answered incorrectly explained their reasons that Ruth's numbers seemed more random than John's. Some typical responses are:

*"I chose Ruth because her numbers characterize a more random distribution, which is more typical of the winning numbers,"*

*"Her answers vary throughout the whole number pool. More random,"* and

*"I don't know. It just seems a range of #'s would be better to pick."*

Students who picked the correct choice provided reasons that both sets of numbers have the same probability. Some common explanations are:

*"Since both are predictions of future events & each single event has an equal opportunity of occurring, then both groups have equal chance of occurring,"*

*"The odds are 6/40 for each person,"* and

*"They both picked 6 numbers out of 40, so same chances of winning."*

## Summary: Expectancy of Random Distribution

From the analysis of student reasoning on their responses, the findings suggested two characteristics of the expectancy of random distribution that emerged from the data.

***Random distribution***

The findings agreed with the mathematical probability misconception literature (Tversky & Kahneman, 1974; Kahneman & Tversky, 1982) that people usually expect a random distribution of an event that they consider to be random. According to their reasoning, students who picked incorrect answers often explained that their choice best represented a random distribution. Their reasons were a sign of their belief that an outcome of any random event should represent randomness or distribute randomly.

***Expectancy of random distribution is probably context-dependent.***

From the analysis of these three items about random distribution expectancy, item 1 had the most incorrect responses when compared to item 12 and item 20. This might be a result of its context-dependent nature. However instead of context-dependent, the alternative explanation of this finding might be that students started to reflect more on their thinking when answering item 12 and item 20, so their answers did not depend mostly on their intuition as in item 1. Also, another possible explanation is that the most random choice (BGBGBG) was not available in the item 12, so the results between item 1 and item 12 were different. In order to confirm the context-dependent characteristic of the expectancy of random distribution reasoning, a further study requires making more thoroughly investigation.

#### 4.2.2.2 Representativeness Heuristic: Ignoring Sample Size

When we look at a small sample of events that occur we sometimes mistake the *empirical* results for that of the true *theoretical probability* of the event. Intuitively, people expect that a small sample will resemble exactly the characteristics of the larger population. This has been identified as the law of small numbers (Cox & Mouw, 1992; Tversky & Kahneman, 1971). delMas (2002) clarified the law with two perspectives. The first perspective is that a person may have prior notions about the characteristics of a population and expect a small sample to have identical characteristics, proportionally scaled to the size of the sample. Alternatively, a person may treat a small sample as highly representative of a larger population and believe that all samples, regardless of size, will have characteristics similar to the individual sample. Items 9, 10, and 16 investigated a misconception associated with this law, which is called *ignoring sample size*. This misconception indicated a common student behavior of neglecting a sample size when dealing with random events.

**Item 9**

9. The likelihood of getting "heads" at least twice when tossing three coins is ______ the likelihood of getting at least 200 "heads" when tossing 300 coins.

a) less than b) equal to *c) greater than*

Explain why you chose this answer.

*When the sample size is larger, the outcome probability will be closer to the theoretical probability. Therefore, when tossing three coins the likelihood that the outcome will be 2 heads out of 3 coins is higher than the likelihood of getting 200 heads out of 300 coins.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 11 | 14.9 |
| b) | 45 | 60.8 |
| *c)* | *18* | *24.5* |
| Total | 74 | 100 |

**Table 4.6: Frequency and percent of students' responses on item 9**

**Summary of student reasoning for item 9**

60.8 % of students responded incorrectly by choosing b) the most, as shown in Table 4.6. Students who answered b) provided reasoning that the sample size should not affect the chances, so the chance of getting 2 heads out of 3 tosses should be the same as the chance of getting 200 heads out of 300 tosses. Here are some representative student responses:

*"the 200 heads tossing 300 coins seems like a good sample to base the assumption of 2 heads out of 3,"* and

*"H and T have equal probability, sample size does not matter."*

Some students also based their reasons in terms of proportionality, for example:

*"The ratio same 2/3 = 200/300,"* and

*"The individual likelihood of 2H's is proportionally equal to a larger sample group."*

Students picked a) because they expected that there should be more distribution with a larger sample; therefore, it increased the chance of getting more heads than tails. Here are some common student responses:

*"Larger sample increases likelihood,"* and

*"3 coins has less chance of getting heads/ more probability 200 heads out of 600 chances."*

For the correct response or c), students mostly provided reasons that larger sample size should increase a chance to make the outcome approaching the theoretical probability. For examples:

*"Because you are adding more events making more choices available and that reduces the probability. 2 heads in 3 tosses is more probable,"*

*"smaller flips much greater likelihood. The higher you go in general the closer to 50% it will be,"* and

*"More possible variation with 300 coins than with 3 coins."*

**Item 10**

10. When choosing a committee composed of two members from among 10 candidates the number of possibilities is __________ the number of possibilities when choosing a committee of 8 members from among 10 candidates.

a) less than    ***b) equal to***    c) greater than

Explain why you chose this answer.

*Committees of 2 people come to mind more easily than committees of 8 people because they are simpler to form. However, there are just as many possible committees of 8 people. One way to think about it is that every time you choose a committee of 2 people, you have automatically selected a "partner" committee of 8 people. This "partner" committee is composed of everyone that you did not choose for the 2-person committee. As a result, for every unique 2-person committee that is possible, there is a unique 8-person "partner" committee.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 15 | 20.3 |
| ***b)*** | ***31*** | ***41.9*** |
| c) | 27 | 36.5 |
| No response | 1 | 1.4 |
| Total | 74 | 100 |

**Table 4.7: Frequency and percent of students' responses on item 10**

**Summary of student reasoning for item 10**

As presented in Table 4.7, most students or about 57% chose incorrect answers either a) or c) because they did not consider the sample size of either case to be the same. Therefore, they thought that the possible outcomes of the two cases would be different. For example, students who selected a) explained:

*"Least requirement of being member. 2 members is more selective."*

Therefore, they typically concluded,

*"There are more selections to be made thus more ways to select."*

Otherwise, they gave reasons in terms of probability ratio:

*"2/10 < 8/10"*

Similarly, students who selected c) concentrated on the numbers of committee positions, so they commonly explained,

*"There are more choice = more possibilities,"* and

*"For 2 member positions, you can choose from 10. But choose 8 members, which is 4x[times] more people, you still have only 10 to choose from."*

Students who correctly chose b) recognized that the sample size is the same in both cases. Therefore, they commonly responded,

*"Omitting a combination of 2 members is the same as choosing a combination of 2."*

## Item 16

16. Assume that the chance of having a boy or girl baby is the same. Over the course of a year, in which type of hospital would you expect there to be more days on which at least 60% of the babies born were boys?

a) In a large hospital

***b) In a small hospital***

C) It makes no difference

Explain why you chose this answer.

*The empirical probability is likely to agree with theoretical probability as the number of events increases. Therefore, a smaller hospital which has a low number of births per week does in fact have a higher chance of deviating from the theoretical probability than a large hospital.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 5 | 6.8 |
| ***b)*** | *43* | *58.1* |
| c) | 24 | 32.4 |
| No response | 2 | 2.7 |
| Total | 74 | 100 |

**Table 4.8: Frequency and percent of students' responses on item 16**

### Summary of student reasoning for item 16

This item is a classic problem, the hospital problem (Kahneman & Tversky, 1972) for investigating the ignoring sample size misconception, which sometimes is called the law of small numbers. For incorrect answers, most students chose c), as shown in Table 4.8, and their written responses implied that they did not use the sample size for figuring the problem. Here are some typical student responses:

*"chance are the same, sample size does not matter,"* and

*"same chance regardless of location"*

A few students who chose a) provided their reasons in terms of the sample size. However, they correctly stated the law but incorrectly applied it. Here are some examples of student responses:

*"The larger the pool, the greater the chances for random things to happens,"* and

*"The more trials the better the probability approaches the theoretical probability."*

The above responses implied that students might consider 60% boy is close to 50%, which is the theoretical probability.

Students who chose b) consider an effect of either a large or a small sample size. Here are some general student responses:

*"The larger hospital has a greater chance of balancing out and assuming the 50/50 assumed characteristics,"*

*"The smaller the sample size, the more likely it will deviate from the expected values,"*

*"small sample size are often prone to inequalities of average. 'large' samples are far more even, if s[sample] $\rightarrow \infty$, all are equal,"* and

*"sample size makes a difference"*

**Summary: Ignoring Sample Size**

From an analysis of student reasoning, the following patterns emerged from the data.

***Proportional Reasoning***

From incorrect student responses, it can be concluded that they used proportional reasoning to relate the outcome of a large sample size to the outcome of the small one. This finding agrees with delMas's (2002) first perspective of the law of small numbers. He suggested that most people with prior knowledge about probability expect a small sample to have identical characteristics, proportionally scaled to the size of the sample. This indicated that most students considered the event to be random, such as coin tosses; they only thought of the probability of the outcome to be 50-50 without thinking of other conditions such as the sample size.

***The larger the sample size, the more varied the outcomes***

Many students thought that outcomes should be more varied in the large sample size. For example, students expected getting 200 heads out of 300 coin tosses was greater than 2 heads out of 3 tosses because they believed that a larger sample increased a likelihood of getting any outcomes. A few students also thought that the larger sample size, the greater the chances of getting random outcomes. This claim was supported by student responses on item 16.

#### 4.2.2.3 Representativeness Heuristic: Gambler's Fallacy

From the review of the literature, there are two perspectives on the Gambler's Fallacy definition. The first perspective came from Fischbein and Schnarch (1997). They explained that the Gambler's Fallacy stems from people's lack of knowledge of *independent* and *dependent events.* They also referred to this as the *negative and positive recency effects.* Often, students are misguided to believe that recent events will have an influence on the outcome of future events. The concept that the students must understand in order to correct this fallacy is that some events, such as the gender of a baby, are *independent events.* The outcome of one event does not affect the outcome of the next. In other words, the birth of a girl does not increase the odds of a birth of a boy the next time.

Alternatively, Kahneman and Tversky (1972) referred to the Gambler's Fallacy as the law of averages. In this perspective, the Gambler's Fallacy represents a belief that chance processes are self-correcting or average out over very small runs (delMas, 2002). For example, consider a person who is informed that there are 5 heads in row from five coin tosses. Then the person is asked to state a possible outcome of the next toss. A person who believes in the law of averages will answer tails because a balance between heads and tails is expected (i.e., representative) (delMas, 2002). The Gambler's Fallacy misconception was investigated in items 2 and 11. Both perspectives of the Gambler's Fallacy were investigated in items 5 and 16. Also, this misconception was indirectly investigated in items about the outcome approach and mean misconceptions.

**<u>Item 5</u>**

5. Suppose you tossed a coin five times and get five heads in a row. The probability of getting a head on the next toss is __________.

a) greater than 50%  b) less than 50%  *c) equal to 50%*

Explain why you chose this answer.

*Each coin toss is independent of the previous toss or the next toss.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 1 | 1.4 |
| b) | 8 | 10.8 |
| *c)* | *65* | *87.8* |
| d) | 0 | 0 |
| e) | 0 | 0 |
| Total | 74 | 100.0 |

**Table 4.9: Frequency and percent of students' responses on item 5**

### Summary of student reasoning for item 5

As presented in Table 4.9, only a few students incorrectly answered choice b). Here are their common reasons for choosing b):

*"It just seems that it will be less likely to occur due to previous flips,"* and

*"Each time you get heads, it's less likely to happen again."*

One interesting finding is that all students who chose b) for this item also picked choice a) in item 2.

Most of the students correctly answered c), and their reasoning indicated that they quite understood that each flip is independent of the previous flip, for example,

*"The chance of getting a head or tail is equal and not influenced by previous tosses,"* and

*"Each toss is independent of all others."*

**Item 14**

14. Ronni flipped a coin three times and in all cases heads came up. Ronni intends to flip the coin again. Is the chance of getting heads the fourth time ____________the chances of getting tails?

a) less than *b) equal to* c) greater than

Explain why you chose this answer.

*For the fourth tossing, the chance of getting heads is equal to the chance of getting tails. The chance for each toss is independent of the previous tosses.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 9 | 12.2 |
| ***b)*** | ***64*** | ***86.5*** |
| c) | 1 | 1.4 |
| Total | 74 | 100 |

**Table 4.10: Frequency and percent of students' responses on item 14**

**Summary of student reasoning for item 14**

As shown in Table 4.10, only a few students answered incorrectly. Students who answered incorrectly intuitively thought that the previous flip could influence the current flip. One typical student responses:

*"happened 3 times, less likely to happen again."*

Some student answered with uncertainty because they had conflict in their reasoning. For example one student indicated,

*"It seems that he's already flipped heads many times already, and even though this answer conflicts with the reasoning in so many of the other questions I have already answered, that's ok, because I don't know."*

Most of students answered correctly and provided similar reasons as in item 5. For example:

*"Each flip is statistically independent of the others. The flip of a coin has two options each w/ 50% probability. The chance of getting heads is equal to chance of getting tails."*

One student answered c) because he was not sure if the coin was fair, as he wrote,

*"There is a chance the coin is not fair but there should be 19 trails before we assume 1:1 is not H:T"*

**Summary of Gambler's Fallacy**

According to the results from items 5 and 16, most students seemed not to present the Gambler's Fallacy misconception. Only a few students indicated a law of average in their reasoning when answering item 5 and 16. Most of them made the correct choice and provided justified reasoning. However, students used the Gambler's Fallacy misconceptions on items about the outcome approach and mean, especially the law of average perspective. Therefore, the analysis of the

Gambler's Fallacy will be discussed further in the section of the outcome approach and mean misconceptions.

#### 4.2.2.4 Availability

The availability heuristic is a more general misconception. Many people believe that events that are easier for their mind to list are more numerous than events that require longer thought processes to list.

**<u>Item 15</u>**

15. Suppose a word is randomly picked from an English dictionary. Is it more likely that the word begins with the letter K, or that K is its third letter?

a) The word beginning with the letter K is more likely picked.

***b) The word having K in its third letter is more likely picked.***

c) Both are likely equal to be picked.

Explain why you chose this answer.

*It is more likely that the word with K as its third letter is to be picked, because there are more words with K in the third letter than K in the first letter.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 39 | 52.7 |
| ***b)*** | ***14*** | ***18.9*** |
| c) | 16 | 21.6 |
| No response | 5 | 6.8 |
| Total | 74 | 100 |

**Table 4.11: Frequency and percent of students' responses on item 15**

**Summary of student reasoning for item 15**

This item is eliminated because its difficulty is too high or it is too difficult. As presented in Table 4.11, most students, more than 50%, answered it incorrectly. Most of the students could not answer this question correctly. Also from student reasoning, the majority of them answered mainly by guessing. Therefore, this item did not provide sufficient data for investigating the availability heuristic.

### 4.2.2.5 Outcome Approach

Konold (1989) found the judgmental heuristic, including representativeness heuristic and availability heuristic, to some extent limiting. Konold did not reject the validity of past research conducted on this heuristic; however, he claimed that students used a different heuristic when they were solving probability tasks outside the tasks normally used in the judgmental heuristic based researches. Konold (1989) expanded the judgmental heuristic further by formulating a new model of informal reasoning under uncertainty, called the *outcome approach.* He argued that students displayed another type of misconception if the wording of the probe question was changed. He explained that this misconception was a result of their misinterpretation of the goal in reasoning about the uncertain events. The outcome approach misconception was investigated in items 2 and 11.

## Item 2

2. Which of the following is the least likely result of six flips of a fair coin?

a) HHHHHT

b) THHTTH

c) THTTHT

d) HTHTHT

*e) None of the above*

Explain why you chose this answer.

*All the above events have the same probability to happen.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 41 | 55.4 |
| b) | 4 | 5.4 |
| c) | 1 | 1.4 |
| d) | 5 | 6.8 |
| *e)* | *23* | *31.1* |
| Total | 74 | 100.0 |

**Table 4.12: Frequency and percent of students' responses on item 2**

### Summary of student reasoning for item 2

As shown in Table 4.12, most students who answered a) reasoned in terms of an unusual result of coin tossing to have five heads in a row. These are a few example of students' explanation on answering a):

*"Theoretically, we should get 3 Hs and 3 Ts. And probability of getting 5 Hs in a row is very slim,"*

*"Difficult to flip with that consistency,"* and

*"With a 50% of either H or T 5/6 heads seems unlikely."*

A few students who answered a) also provided reasons, which indicated their Gambler's Fallacy misconception or their lack of knowledge of independent and dependent events. Here are some common responses:

*"The odds of continuing to get heads in a roll is least,"* and

*"It is 'unlikely' to get the same side many times in a row if both sides will turn up equal probability."*

Comparing the statistics of responses from item 2 to item 1, more students had the outcome approach misconception than the representativeness heuristic misconception. This also indicated that many students who answered correctly on item 1 failed to apply the same concept to this item.

**<u>Item 11</u>**

11. In a family of six children, which birth order of children is less likely to occur?

a) BBBBGB

b) BBBGGG

c) BGGBGB

d) GGGBBB

*e) None of the above*

Explain why you chose this answer.

*All of the above birth orders have the same chance.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 39 | 52.7 |
| b) | 1 | 1.4 |
| c) | 1 | 1.4 |
| d) | 1 | 1.4 |
| *e)* | *32* | *43.2* |
| Total | 74 | 100 |

**Table 4.13: Frequency and percent of students' responses on item 11**

### Summary of student reasoning for item 11

As shown in Table 4.13, more than 50% of students answered a), which is an incorrect answer. Some of these students are students who also answered correctly on item 1. Some students provided reasons, implying that they used a representativeness heuristic kind of reasoning. For example:

*"It's a least random distribution,"* and

*"deviates from 50/50 the most."*

Some students' reasoning indicated Gambler's Fallacy misconception, as one student stated,

*"While there is a slight inequity in birthrates among families, it is rather rare to generate 4 of either gender in consecutively."*

Most students who answered correctly often gave reasons that the sex of one child is statistically independent from the sex of another, so all of these orders have equal probability of happening. For example,

*"Assuming the birth probability between boys & girls is 50%, and each birth is statistically independent of the other births, all these answers are equiprobable,"* and

*"Equally probable and the next event doesn't depend on the first."*

### Summary: Outcome Approach

Results from the previous research stated that although students answered correctly on item 1, the representativeness heuristic, they might not be able to answer correctly on item 2, an outcome approach. The research considered that these students still have an outcome approach misconception. From the data analysis, our results agreed with the previous empirical findings that more students seemed to present an outcome approach because they misinterpreted the goal in reasoning about the uncertain events (Konold, 1989). Also, student reasoning indicated the influence of the Gambler's Fallacy on their reasoning processes. Here are two common themes emerging from analyzing student responses.

***Random Distribution Expectancy***

When asked students similar questions about coin tossing and birth order of children, more than 50% of students chose incorrect answers, corresponding to more ordered outcomes. Most students explained that their choice was a least random distribution, so it was *less likely* to occur. This implied their expectation that the outcomes of stochastic events should distribute randomly.

***Gambler's Fallacy***

From most of the incorrect student responses to these three items, their reasoning demonstrated a basic misunderstanding that stems from their lack of knowledge of *independent* and *dependent* events or the Gambler's Fallacy. Students are often misguided to believe that recent events will have an influence on the outcome of future events. Although students answered and responded correctly on items about the Gambler's Fallacy, they intuitively activated this misconception when dealing with uncertainty events such as coin tossing or birth orders.

#### 4.2.2.6 Sample Space

Sample space is a set of ALL possible outcomes for a random experiment. It can be derived by employing the fundamental counting principles, permutations, and combinations. The following three items investigated student misconceptions of sample space.

**Item 4**

4. From a batch of 3000 light bulbs, 100 were selected at random and tested. If 5 of the light bulbs in the sample were found to be defective, how many defective light bulbs would be expected in the entire batch?

a) 50 b) 100 *c) 150* d) 200 e) None of the above

Explain why or show how you arrived at this answer.

*Simply calculate the ratio between 5 deflected bulbs out of 100 sample bulbs and compare to 3000 bulbs.* $5/100 = x/3000 \rightarrow x = 150$

| Responses | Frequency | Percent |
|---|---|---|
| a) | 0 | 0 |
| b) | 0 | 0 |
| *c)* | *70* | *94.6* |
| d) | 0 | 0 |
| e) | 4 | 5.4 |
| Total | 74 | 100.0 |

**Table 4.14: Frequency and percent of students' responses on item 4**

## Summary of student reasoning for item 4

As shown in Table 4.14, almost all students answered correctly. Therefore, this item is eliminated because its difficulty is too low or it is too easy. It does not provide any information in terms of students' misconceptions.

## Item 8

8. If you roll two dice at the same time, what is the probability that the sum of the numbers rolled is less than 9?

a) 16/36 b) 20/36 *c) 26/36* d) 30/36 e) None of the above

Explain why or show how you arrived at this answer.

*Here is a summary of all the possible combinations:*

| | | | | | |
|---|---|---|---|---|---|
| **1 - 1** | **1 - 2** | **1 - 3** | **1 - 4** | **1 - 5** | **1 - 6** |
| **2 - 1** | **2 - 2** | **2 - 3** | **2 - 4** | **2 - 5** | **2 - 6** |
| **3 - 1** | **3 - 2** | **3 - 3** | **3 - 4** | **3 - 5** | 3 - 6 |
| **4 - 1** | **4 - 2** | **4 - 3** | **4 - 4** | 4 - 5 | 4 - 6 |
| **5 - 1** | **5 - 2** | **5 - 3** | 5 - 4 | 5 - 5 | 5 - 6 |
| **6 - 1** | **6 - 2** | 6 - 3 | 6 - 4 | 6 - 5 | 6 - 6 |

*There are 26 possible combinations of getting the sum less than 9.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 7 | 9.5 |
| b) | 8 | 10.8 |
| *c)* | *26* | *35.1* |
| d) | 20 | 27.0 |
| e) | 5 | 6.8 |
| No response | 8 | 10.8 |
| Total | 74 | 100 |

**Table 4.15: Frequency and percent of students' responses on item 8**

### Summary of student reasoning for item 8

As presented in Table 4.15, many students (about 27%) answered incorrectly. Most students who answered used their knowledge about probability to answer this question. Only a few people just guessed at their answer, and often they selected choice d). For example, one student stated his reason for choosing d):

*"cause it makes sense"*

Most students who answered d) did mistakes on their calculation. For example, one student miscounted the events as,

*"6+6, 6+5, 6+4, 5+6, 5+5, 4+6 > 9   so 30/36 for < 9"*

He forgot to include the event of getting the sum to be 9—6+3, 3+6, 5+4, and 4+5.

For students who answered correctly, they used the same approach of counting the events of getting a sum more than or equal to 9 and subtracting that from the total possible events. For example, one student showed his work as:

*"10/36 for 9 or greater. So, 26/36 for < 9"*

However, there were eight students who did not respond. One of the students mentioned:

*"I never take probability."*

**Item 17**

17. Two bags have black and white counters.

Bag J: 3 black and 1 white    Bag K: 6 black and 2 white

Which bag gives the better chance of picking a black counter?

***a) Same chance***

b) Bag J

c) Bag K

d) Don't know

Explain why you chose this answer.

*The two bags have the same ratio between back and white counters. Therefore, the chances of picking a black counter from either bag are the same.*

| Responses | Frequency | Percent |
|---|---|---|
| *a)* | ***63*** | ***85.1*** |
| b) | 3 | 4.1 |
| c) | 5 | 6.8 |
| d) | 2 | 2.7 |
| No response | 1 | 1.4 |
| Total | 74 | 100 |

**Table 4.16: Frequency and percent of students' responses on item 17**

**Summary of student reasoning for item 17**

As shown in Table 4.16, majority of students about 85% answered correctly, so this item is eliminated because its difficulty is too low or it is too easy.

**Summary: Misconception regarding Sample Space**

Results of two out of three items about sample space misconceptions were eliminated because their difficulties were too low. This can be inferred as either the questions were too easy, or students already had a good understanding about the sample space. Then when considering the result and student reasoning from item 8, most students answered incorrectly because of their mistakes in calculation. Therefore, this can be concluded that students at college level already have developed a good understanding about the concept of sample space.

**4.2.2.7 Mean**

Pollatsek, Lima, and Well's (1981) study found that many college students could not correctly weight and combine two means into a single mean. Similarly, Bao and Redish (2001) also found that majority of their subjects, who were college students, had a misconception about the mean. Therefore, items 6 and 7 were included in the test to investigate students' misconception about mean.

**Item 6**

6. Suppose the student average SAT score at Enormous State University is 1000. Your friend is in a writing class of 10 students. Her score was 1100. What is the most probable average of the other 9 students?

a) 850 b) 900 c) 989 ***d) 1000*** e) None of the above

Explain why or show how you arrived at this answer.

*Assuming independence among the ten SAT scores, the mean for the other nine students is expected to be 1000.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 0 | 0 |
| b) | 3 | 4.1 |
| c) | 50 | 67.6 |
| ***d)*** | ***17*** | ***23*** |
| e) | 3 | 4.1 |
| No response | 1 | 1.4 |
| Total | 74 | 100 |

**Table 4.17: Frequency and percent of students' responses on item 6**

## Summary of student reasoning for item 6

Although this item was to investigate the misconception about the mean, it also relates to the misconception of ignoring sample size and the Gambler's Fallacy. Since students had to make assumptions at the beginning that each student's score was independent and the sample size was large, which could be inferred from the university name.

As shown in Table 4.17, most students answered incorrectly by choosing c). They provided their reasons as follows:

*"If you make the 10 person sample a 9 person sample w/o her, the avg is only 989,"* and

*"1000 x10 = 10,000 -1100 = 8900 / 9 = 988.889 ≈ 989,"*

One student picked the wrong answer c), but he reasoned as:

*"Without any other evidence, the average of a sample group must be assumed to be the average of the entire control group (989x9 + 1100)/10 = 1000"*

For students who chose the correct response explained as following:

*"The average SAT score of the other 9 students is statistically independent to the SAT score of the one student,"* and

*"average has already determined"*

Most students, who answered incorrectly, used mathematics to calculate an average, but they did not consider that the university is enormous and an SAT score for each student is independent. For example, one student stated his reasoning quite clear,

*"The average should not depend on the sample size."*

**<u>Item 7</u>**

7. A professor teaches two physics classes. The morning class has 25 students and their mean on the first test was 82. The evening class has 15 students and their mean on the same test was 74. What is the mean on this test if the professor combines the scores for both classes?

a) 76 b) 78 *c) 79* d) 80 e) None of the above

Explain why or show how you arrived at this answer.

*Combining the classes, the mean can be calculated by combining the means as:*

*(25(82) + 15(74)) / (25 + 15) = 79*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 0 | 0 |
| b) | 8 | 10.8 |
| *c)* | *57* | *77.0* |
| d) | 4 | 5.4 |
| e) | 1 | 1.4 |
| No response | 4 | 5.4 |
| Total | 74 | 100 |

**Table 4.18: Frequency and percent of students' responses on item 7**

## Summary of student reasoning for item 7

As shown in Table 4.18, most students answered correctly, and only a few students chose incorrect answers. A few students who answered b) either guessed on their answers or made mistakes on their calculations. For example, one student responded:

*"it should be the avg[average] of the 2 scores."*

Students who answered c) calculated the average mean between the two classes. For example, one student typically responded as,

*"(25(82) + 15(74))/40 = 79"*

Also, some students provided the conceptual reason why they selected c) without doing much calculation. Here are some common student responses:

*"the larger class is weighted heavy so it must be 79 or 80 but not to a 2:6 ratio so 79 seems be suited,"* and

*"closer to 82 (25 students) than 74 (15 students)"*

Students who chose either d) or e) provided their reasons indicating their confusions of how to calculate the total mean. Some typical student responses are:

*"more students did better in the larger class, so the avg of class 1 will be reduced slightly by class 2,"* and

*"I guessed cause I have no idea how to find it."*

**Summary: Misconceptions about Mean**

From the test results, most students were able to calculate the mean in a simple situation, but they used the representative heuristic when a situation was more sophisticated. Students could correctly weight and combine means into a single mean when a question was straightforward, as in item 7. From analysis of item 7, more than 70% of students made a correct choice and provided well justified reasons.

However students had a hard time when a question was more complicated and involved other misconceptions—ignoring the sample size and the Gambler's Fallacy. More than 50% of students answered item 6 incorrectly. From analyzing their reasoning, students did not consider the sample size or numbers of students in the Enormous State University. This indicated students' ignoring the sample size. Also most students did not assume the independency of each score, so they just used the score of one person to determine the average score of nine students. This implied that students believed in the law of averages. This finding disagreed with the previous findings (Pollatsek, Lima, & Well, 1981; Bao & Redish, 2001).

#### 4.2.2.8 Classical Probability

The term *classical probability* refers to concepts of probability applied in a context of classical mechanics. The concept of a trajectory is fundamental to classical mechanics. Given a particular mass with a given initial velocity and knowledge of the forces acting on it, classical mechanics is used to predict the exact position and velocity of the particle at any future time. Thus in the classical view, the trajectory of the particle can be calculated to any preferred degree of accuracy. It is also possible, within the framework of classical mechanics, to measure the position and velocity of a particle at any given instant of time. Classical probability is about the probability of finding an object at a certain range compared to a total possible location. Items 3, 13, 15, and 19 were used to investigate student understanding of classical probability.

**Item 3**

| |
|---|
| 3. A ball is rolling at speed v on a flat frictionless horizontal surface in the positive x direction. When the ball reaches x = +L, it collides with a wall and rebounds with no loss at speed. Moving now in the opposite direction, it strikes another wall at x = -L and again rebounds with the same speed. It continues to bounce back and forth between the walls at x = +L and x = -L always with speed v. What is the probability to find the ball moving in either direction between x = -L/2 and x = +L/2? |
| a) 25% *b) 50%* c) greater than 50% d) less than 50% |
| Explain why you chose this answer. |
| *The surface is frictionless; therefore, there is no loss in energy, so the ball is always moving with the same speed. If the probability of finding the ball anywhere in between the two walls is 100%, then the probability to find the ball moving in either direction between x = -L/2 and x = +L/2 is half of the total probability or 50%.* |

| Responses | Frequency | Percent |
|---|---|---|
| a) | 5 | 6.8 |
| *b)* | *58* | *78.4* |
| c) | 3 | 4.1 |
| d) | 7 | 9.5 |
| e) | 1 | 1.4 |
| Total | 74 | 100.0 |

**Table 4.19: Frequency and percent of students' responses on item 3**

**Summary of student reasoning for item 3**

As presented in Table 4.19, most students answering wrong were likely to chose either d) or a). Here is some typical student response for choosing a).

*"probability of finding the ball between +L & -L = 50%, so P of finding the ball between +L/2 & -L/2 = 50/2 = 25%"*

Students choosing d) thought that the speed of the ball changes, faster through the center region and slower near both walls. Here are some common student responses for choosing d):

*"Because the ball spends more time when it is turning around @ the wall,"*

*"Ball travels faster through center region,"* and

*"There are two very short periods where the ball is stopped and not moving in any direction. But the periods are too short to be 25%"*

In contrast, three students who answered c) reasoned the opposite as,

*"It spends more time in the middle than at the edges."*

Some students, who chose the correct answer or choice b), reasoned in terms of constant speed, so the probability of finding it between –L/2 and L/2 is 50%. For example:

*"Because there is no loss of speed, you have an equal chance of finding it moving in either direction."*

A few students analyzed even further that there was no loss in energy because the ball travels with the same speed. One student wrote,

*"It always goes at the same speed. It would take longer while at the walls but there was no loss of energy so it must not have compressed at all."*

**Item 13**

13. A block of mass m is attached to an ideal spring and oscillates on a frictionless horizontal surface. The amplitude of the oscillation is A, so that the block oscillates between x = +A and x = -A. What is the probability to find the block moving in either direction between x = -A/2 and x = +A/2?

a) 25% b) 50% c) greater than 50% ***d) less than 50%***

Explain why you chose this answer.

*Between x = -A/2 and x = +A/2, the block speed is at maximum, so the probability to find the block is less than a region -A < x < -A/2 and +A/2 < x < +A. Therefore, the probability to find the block moving in either direction between x = -A/2 and x = +A/2 is less than 50%.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 2 | 2.7 |
| b) | 20 | 27.0 |
| c) | 10 | 13.5 |
| *d)* | *41* | *55.4* |
| No response | 1 | 1.4 |
| Total | 74 | 100 |

**Table 4.20: Frequency and percent of students' responses on item 13**

## Summary of student reasoning for item 13

As shown in Table 4.20, 27% of students who answered incorrectly chose b). Students who answered a) argued in terms of the total distance that the total distance is 2A and A/2 is 25%, so they chose choice a) or 25%. As one student wrote,

*"Total distance of travel is 2A...2A . 25% = A/2 "*

Students chose b) because they interpreted this problem to be similar to problem 3. As one student mentioned:

*"see explanation for #3 replace L with A"*

Therefore, they pictured the same situation that the block is moving at a constant speed, so the chance of finding it in between –A/2 and A/2 is 50%. Here are common responses:

*"The sections on the surface are all covered an equal amount of times, -A/2 ≤ x ≤ A/2 is equal to 50%,"* and

*"It moves at a constant speed so equal time in all areas. –A is 50% of 2A so time between is 50% of total time."*

These responses implied that students did not understand simple harmonic motion, so they forgot that the block's kinetic energy exchanges with the spring potential energy. In other word, they forgot that the spring exerted a force on the block. As a result, the block speed can be represented by a sinusoidal function, where the maximum speed is in the middle. Most students thought that there was no force or no acceleration acting on the block. For example, one student mentioned:

*"with no acceleration the time spent in the region of interest would be ½"*

Students who chose d) understood that there is acceleration so the speed changed. However, they did not apply the concept correctly. They thought that the block slowed down in the middle. Here are some student responses:

*"Because of the deceleration that occurs at –A/2 > x ; x > A/2,"* and

*"not sure if right reasoning but I think that as the particle is getting close to A or –A it goes slower, and possibly spends less time there in A/2 the v is greater and would..."*

The students whose responses revealed their confusion between $x$ (the block location) and the compression of spring $x$ (in the Hooke's law equation: $F = -kx$) reasoned that the acceleration increases as $x$ increases. Their reasoning might be a result of this misunderstanding, as one student explained,

*"Oscillation velocity is not constant. It is faster as x increases. The spring spends more time near x=0."*

Students who answered correctly reasoned that the block was moving faster between –A/2 and A/2 and spent less time in that region, so it was less likely to be found there. Here are some typical student responses:

*"block spends less time in this region due to it's increased speed,"* and *"The blocks highest velocity is between –A/2 & A/2 & lowest at –A & A therefore it will spend more time near –A & A then between –A/2 & A/2"*

**Item 18**

18. A boy ties a ball to a string and whirls it at a constant angular speed in a horizontal circle above his head. Let the position of the ball be represented by the angular coordinate $\phi$, with the direction of the boy's nose indicating $\phi=0$. What is the probability to find the ball between $\phi = 45^{\circ}$ and $\phi = 90^{\circ}$?

a) 25% b) greater than 25% *c) less than 25%*

Explain why you chose this answer.

*Let the probability of finding the ball between $\phi = 0^{\circ}$ and $\phi = 360^{\circ}$ or a full circle to be 100%. The ball is moving at a constant angular speed, so the probability to find the ball between $\phi = 45^{\circ}$ and $\phi = 90^{\circ}$ is considered as one eighth the probability of the whole circle or 12.5%.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 11 | 14.9 |
| b) | 1 | 1.4 |
| *c)* | *61* | *82.4* |
| No response | 1 | 1.4 |
| Total | 74 | 100 |

**Table 4.21: Frequency and percent of students' responses on item 18**

**Summary of student reasoning for item 18**

As presented in Table 4.21, most students who answered incorrectly chose a). Students' reasons on choosing a) did not reveal much of their thinking. One student provided a right reason but he answered a).

*"constant angular speed means distribution of probability equal at all angles."*

Therefore, it is possible that many students answered a) because they miscalculated the probability. For example, one student wrote:

*"90-45 = 45, 45/180 = 25%"*

Most students answered correctly, and their reasons implied that they pictured the whole circle as 100% and the probability of finding the ball between 45° and 90° is one eighth of 100% or 12.5%. Here are some student responses:

*"An angle of π/4 is only 1/8of the circle."*

*"Constant angular velocity makes for equal chance at all points*

*(90-45)/360 = 12.5%"*

*"no 45° slice is preferred, and there are 8 slices, so p = 1/8"*

**Item 19**

19. A steel ball is dropped from a height y = +H. It strikes a horizontal steel surface at y = 0 and bounces back to height y = +H. It continues to bounce, reaching height y = +H after every bounce. What is the probability to find the ball anywhere between y = +H/2 and y = +H?

a) 25% b) 50% *c ) greater than 50%* d) less than 50%

Explain why you chose this answer.

*The ball has a lowest speed at the height between y = +H/2 and y = +H. Therefore, it spends more time when reaching the highest height. The probability to find the ball at the height between y = +H/2 and y = +H is more than between y = +H/2 and y = 0.*

| Responses | Frequency | Percent |
|---|---|---|
| a) | 4 | 5.4 |
| b) | 19 | 25.7 |
| *c)* | *38* | *51.4* |
| d) | 11 | 14.9 |
| No response | 2 | 2.7 |
| Total | 74 | 100 |

**Table 4.22: Frequency and percent of students' responses on item 19**

## Summary of student reasoning for item 19

As shown in Table 4.22, many students (about 52%) answered correctly. For those who answered incorrectly, students who answered a) interpreted this problem to be the same as problem 3, as one student mentioned his reason to be:

*"same as no. 3"*

Students answered b) because they thought that the same distance is traveled so the probability should be the same. Here are some typical responses in terms of the equal distance:

*"The vertical height covered by each bounce is equal for all sections. $H/2 < y < H$ is 50%,"* and

*"$H-H/2 = 1/2\ H$ ...$(1/2\ H)/H = 50\%$"*

Also, a few students thought of this problem to be the same as problem 3. For example:

*"see #3 for explanation"*

Interestingly, some students thought of the difference in acceleration but an idea of equal distance seemed to dominate their reasoning process. For example, two students explained as:

*"Ball accelerates on way down, decelerates on way up, same acceleration, same distances, so 50%,"* and

*"As the ball goes up it gets slow but it goes faster as it goes down. Therefore equaling the amount of time spent in each section since equal length in each sector 50%."*

Students who answered correctly mostly reasoned in terms of speed decreasing as the ball reaches the highest height. Therefore, the ball spends more time in the top half of the motion. Here are examples of student responses:

*"Gravity makes it fall fast at bottom, bounce back up first, and slow down as it reaches top again,"*

*"Based on the area lone it should be 50% but the ball slows as it reaches the top and travels slower through that half of space,"*

*"it goes its slowest during that time due to gravities negative acceleration on it,"* and

*"same as #13, it has less K & more U so it is in that interval longer."*

Student responses for choosing d) implied that they thought that the ball moved faster before it hit the ground, so the probability of finding the ball should be more when it was near the top. Student responses implied that they only consider the moving up motion, as one student explained:

*"Caused by the deceleration due to gravity as the ball gets closer to +H"*

When they considered the way up, they thought that the ball was slowing down. Here are some typical student responses:

*"It is moving slower higher up,"* and

*"And for the same reason as the other questions but it will be less than 25%."*

**Summary: Classical Probability**

Most students illustrated some understanding the relationship between an object's speed and a probability of locating it in a certain region, especially for constant speed. However students had difficulties when the problems involved acceleration, as in item 13 and 19. Most students did not quite understand simple harmonic motion because many students misinterpreted the motion of a block attached to a spring, as in item 13. For item 19, a few students did not even consider an effect of acceleration or deceleration due to gravity. Many figured out the effect of gravity on the motion of the bouncing ball, but they incorrectly related the ball's motion to the probability of finding it in a top half region.

## 4.3 Discussion

In order to observe student prior probability knowledge, the multiple-choice diagnostic test was developed. The reliability and validity of the test were established. The test reliability increased when the items having either too low or too high discrimination index and difficulties level were eliminated. The test was validated in two perspectives—content validity and construct validity. The content validity was established from expert panelists. The construct validity was determined from the known-group difference, the ability to differentiate between those who had and those who did not have misconceptions.

The item analysis indicated that students hold misconceptions similar to those suggested in the literature. The representativeness heuristic and the outcome approach were commonly found in student responses. Often, students intuitively expected a stochastic event to distribute randomly. Furthermore, analysis of student responses indicated that this misconception might be context-dependent which seemed to activate more in coin-toss situations than the others. Randomness expectancy is so dominant in student thinking that they neglect a sample size and independency of individual events. From the analysis of these three items about random distribution expectancy, item 1 had the most incorrect responses when compared to item 12 and item 20. This might be a result of its context-dependent nature. However instead of context-dependent, the alternative explanation of this finding might be that students started to reflect more on their thinking when answering item 12 and item 20, so their answers did not depend mostly on their intuition as in item 1. Also, another possible explanation is that the most random choice (BGBGBG) was not available in the item 12, so the results between item 1 and item 12 were different. In order to confirm the context-dependent characteristic of the expectancy of random distribution reasoning, a further study requires making more thoroughly investigation.

In additional to the mathematical probability misconceptions, the test also investigated the classical probability or probability in the classical mechanics context. Insufficient understanding of velocity and acceleration concepts resulted in student incorrect responses about probability of finding an object in a certain region.

In summary, students hold several naïve mathematical probability concepts, possibly p-prim type, and a classical probability reasoning resource prior to modern physics instruction. Further results and analysis of student probability concepts will be presented and discussed in Chapter 5 and 6.

# Chapter 5

# Preliminary Findings

> When we instead analyze the outcomes in terms of probabilities, we are really admitting our inability to do the analysis exactly...According to this interpretation, we could predict exactly the behavior of the electron in our atom if only we knew the nature of a set of so called 'hidden variables' that determine its motion. However, experimental evidence disagrees with this theory, and so we must conclude that the random behavior of a system governed by the laws of quantum physics is a fundamental aspect of nature and not a result of our limited knowledge of the properties of the system.
>
> Kenneth S. Krane (1996, p. 127)

Quantum mechanics (QM) is an interesting but nevertheless challenging subject matter in physics. Learning and teaching QM is not a simple job. One reason that makes QM a difficult subject to teach is that QM is based on probability. When students first learn basic quantum mechanics, they require understanding of wave-particle duality, de Broglie wavelength, and Heisenberg uncertainty principle. Then they move on to solve the Schrödinger equation for a particular basic quantum system. The solution of a quantum calculation, the wave function, can be interpreted based on the probability density (amplitude squared of the wave function). All these ideas are based on probability and are bound by the uncertainty principle, making an exact observation in classical physics become a rather blurry complication. Therefore, a good understanding of probability is necessary in comprehending QM.

To develop a concrete understanding of probability in QM, students should understand where it comes from. This chapter reports results of preliminary interviews during Fall 03. The first interview was to observe how students interpreted the wave nature of electrons and related this idea to probability. The second interview was to investigate student interpretations of the wave functions as a solution to the Schrödinger equation for particular problems. Before reporting and discussing the preliminary results, a discussion of probability and the wave-like nature of particles is provided to prompt the reader about this aspect of QM. The data collection process and the interview protocol are discussed to confirm the validity of a qualitative method.

## 5.1 The Matter Wave and Probability

Before reporting the interview results, it is necessary to discuss the definitions of chance and probability. An idea of chance is commonly used in everyday living. For example, a weatherman forecasts that there might be 60% chance of raining tomorrow. What do we mean by chance? When using chance, we make an estimate or a guess. Why do we, including us scientists, make a guess? Usually, we make a guess when we have incomplete information or uncertain knowledge but need to make a prediction or a decision. Any physics theory is rather a guess, but it is based on a systematic guess (Feynman, 1977). The probability is an approximation or a systematic guessing to predict the possible outcomes of an event about which we do not have enough information. For example, coin tossing has a 50-50 chance of getting either a head or a tail on each toss.

According to QM theory, there is always some uncertainty in the specification of positions and velocities. This uncertainty is inherent in the nature of measurement. In QM, we also assume that there is an observable wave, called the de Broglie wave, associated with any moving particles of atomic or nuclear size. Therefore, the best we can say about the location of any moving particle is limited to a certain probability. The focus of these interviews was to observe if students could use physics principles or relationships associated with the concept of probability appropriately and if they could understand the correct relations between these physics concepts such as wave-particle duality, Heisenberg uncertainty principle, and the wave function. This information could help us realize a more complete picture of student QM thinking models.

## 5.2 Data Collection and Interview Protocol

The results reported in this chapter are from interview I and II conducted during Fall 2003. Interview I was conducted from the fifth to the sixth week right after the first PH314 midterm. Interview II was conducted from the eighth to the ninth week. Both interviews were volunteer-based, and the compensation was extra credits on the homework scores for participating students.

Each interview was one-on-one. Before the interview began, each participant was asked to read and sign an informed consent document, which included purposes of the project and the participant's right as a human subject. Then the researcher encouraged the participant to answer all questions and to take time to think each question through. If the participant had some difficulties answering questions at some point, the participant was instructed to inform the researcher. The researcher stated that she would ask questions that were intended to elicit more information about the participant's thinking process and should not be taken as guiding or a hint. Then the researcher asked students some general questions regarding their academic achievement and prerequisite courses, such as introductory physics sequences.

During the interview, participants were not allowed to use reference materials including textbooks and lecture notes. Participants were asked to make an effort to think and explain their answers. If they were stuck at a certain part of the interview, the researcher asked if they could explain what they were thinking and if they needed certain clarification. The researcher then asked some questions to help clarify students' thinking. If asked, the researcher would provide an equation that the participants were not sure about or could not remember. Interviews usually lasted about half an hour. The Interview I questions are listed in Figure 5.1.

## 5.3 Interview I Results and Analysis

Student initial states are always important factors affecting their learning of new materials. The students coming to physics classes are not blank slates. They bring in with them their understandings of the physical world. A good understanding of student initial states of knowledge is the starting point for better understanding of student difficulties. The interviews were analyzed based on student initial knowledge and their difficulties. The emerging themes of student initial knowledge, as a result of the analysis, are reported here, including concepts of an electron, the wave-particle duality, the wave function (sometimes called the probability wave), and probability.

### I. Concept about electron in general

#### A negatively charged particle

When students were asked to describe what an electron is, most students depicted an electron as a negatively charged particle, which is acceptable. This perception

### Interview I protocol (Fall 2003)

**I. General concepts of electrons, wave-particle duality, and probability wave**

- What is an electron?
- Please explain the wave-particle behavior of electrons
- What is the probability wave?

**II. General concepts about the double-slit experiment**

- Please explain how the double-slit works with light.
- Explain how the double-slit works for electrons.

**III. Probability**

- What do you think would happen on the screen if only two electrons are going through the slits?
- If only one electron goes through the slits, where should it land on the screen?
- What do you think determines that position on the screen?
- What do you think the pattern on the screen means in terms of physics concepts?

**IV. De Broglie wave**

- What would you think would happen if the energy of the electron beams were decreased by half?
- How does the energy of electrons affect the interference pattern?
- How does the momentum affect the interference pattern?
- Please explain why.
- How are these differences in patterns related to any physics concepts that you have learned so far in this class?

**V. Superpostion principle**

- What do you think would happen if one of the slits is closed?
- What do you think would happen if I open this one and close another one?
- What do you think the pattern should look like if I combine the pattern from closing each one of the slits?

**Figure 5.1: The interview I protocol for Fall 2003 preliminary study**

perception of the electron is a result of what students had seen and learned since middle school, or even in an introductory physics course. Here are some typical responses:

> *"a basic unit of electricity, a carrier, it transfer of electric energy,"* or
> *"It is always has been something that is just sort of given and you build off it… It has a fairly small mass with a certain defined charge."*

Students viewed an electron as a point particle because they had learned that in introductory physics, during their study of electromagnetism.

**Associating an electron with the Bohr model**

In an addition, a few students indicated that they visualized an electron orbiting around a nucleus like in a Bohr model. Josh (codename) provided a common response:

> *"An electron is a charged particle that orbits or goes around the nucleus of an atom."*

The Bohr model is a common topic covered in most middle school science classes. Moreover, students had solved problems based on the Bohr model both in introductory physics and introductory chemistry classes. Because of the concrete structure of Bohr model, it is easy for students to visualize and retain. Therefore it was not surprising to see students associating the Bohr model with an electron.

**II. Concepts of the wave-particle duality**

Students then were asked to explain what the wave-particle duality was. Here are common patterns of student explanations.

**The wave nature of a particle is associated with its mass.**

Students reasoned that if the mass of particle were increased, the wave nature of the particle diminishes. Max (codename) also related the mass of particle to its wavelength, *"Because the mass of the particle determines wave length ... So when you have a more massive particle you have less wave length."*

This is a reasonable explanation for the wave-particle nature of quantum objects. This way of thinking is analogous to the concept of a de Broglie wave.

**Attempt to visualize the wave-particle behavior**

Because of its counter-intuitive behavior, some student sought out a visualization to represent the wave-particle behavior of electrons. Here is Max's visualization of this strange behavior of quantum objects:

*"As ph314, it is both a particle and a wave at the same time. It is kind of hard to reconcile. There is this picture that has kind of stuck with me of. . ."* (see Figure 5.2)

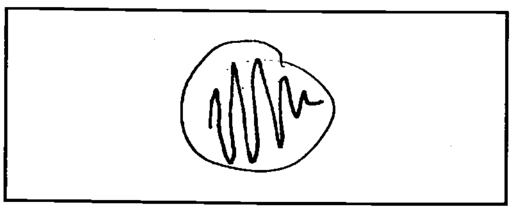


**Figure 5.2: Max's pictorial representation of wave particle behavior of electrons**

**Accept it without formulating reasons**

Not every student could provide an explanation for this weird and wonderful behavior. A few students admitted that they did not understand; as a result, they could not explain or answer why quantum objects had this behavior. Danny (pseudonym) mentioned that he just accepted this strange behavior without understanding it. As he mentioned:

*"I don't really understand it, I just accept it. It is like 2 plus 2 is 4 because that is what you have been told. The sky is blue because that is what you have been told."*

**III. Concepts of the wave function or the probability wave**

Subsequently, the participating students were asked to explain the meaning of a probability wave. The term probability wave has the same meaning as the de Broglie wave and the wave function. From observation of the class lectures, it appears that the instructor used these three terms interchangeably. Here are a couple of interpretations that students provided.

**The probability wave related to the de Broglie wave**

Some students understood correctly that the de Broglie wave relates to the probability wave. Eddy indicated his understanding as:

*"The probability wave existing, related to the de Broglie wave."*

Later in the interview, his reasoning revealed why he thought that the probability wave was related to the de Broglie wave, as he pointed out:

*"It has, like a helium atom has a lot more mass than an electron does. I would think that the wave function describing its probability is going to change somewhat. I guess it would depend on how fast it is going, how much mass it has."*

His explanation indicated his understanding of the de Broglie relationship between the wavelength and the momentum of a particle. However, he did not specify about how the wave function should change when the particle became more massive.

**Reason for existence of the probability wave**

Students mentioned that the probability wave existed because we could not measure the behavior of electrons with exact 100% accuracy. One student explained his reasoning:

*"Because it is a wave, but we cannot measure with 100 percent accuracy. We are left to measure larger groups of them and measure the entire group to a certain percentage, rather than exact number."*

Some students thought that the probability wave could not exist for one electron. In order to have a probability wave, a large number of electrons is required. As Nathan (codename) explained,

*"I think you would if you did the probability for each, in other words, for the total electrons you sent. But if you do the probability for each single electron, no."*

**IV. Probability concept**

**Energy relates to the amplitude squared**

In modern physics, the amplitude of the wave function squared is equal to probability if the wave function is normalized. However, from the interview, a few students confused energy with the amplitude of the wave squared. Eddy (codename) mentioned that he remembered this from his previous physics class:

**Eddy:** Just my own pride, I guess. What I thought was going on is this function is describing where the particles, this is the wave function of the particle, like the probability wave, and this is like where it has, this dotted line, the region up here is like the square of this one. Then the particle has the probability of existing or being in some location according to this square of the probability wave function.

**Interviewer:** Then somehow this dash line represents energy also?

**Eddy:** Yeah, yeah.

**Interviewer:** Why you relate it to that?

**Eddy:** I don't know. I think that is something from classical physics that I remember.

**Interviewer:** That energy has something to do with something squared?

**Eddy:** Well, in a wave, because like in classical, a standing wave or something, this spot, this crest would have more energy than like a node or something.

Eddy's explanation indicated that he used the classical wave as knowledge building blocks to understand the wave function. Although he had a correct understanding of the probability wave, he was still confused about what the amplitude squared should represent.

**Probability and intensity**

In a double-slit experiment, students viewed the probability of finding electrons on the screen corresponding to the intensity of the fringe pattern as,

*"...the amplitude [of the function showing the distribution of the fringes pattern] represents probability and intensity."*

This is an acceptable interpretation of the probability. Students who were able to relate the probability to the fringe pattern seemed to understand that a large number of events is essential to determine the probability.

**The probability wave of electron (s)**

When students were asked to predict a pattern after sending two electrons through the double-slit, some of them predicted that there is sill the fringe pattern on the screen because the two electron probability wave can interfere with each other. Even with one electron, some students also predicted that the same fringe pattern will appear. Eddy provided his reason:

> *"Because every electron has its own probability wave for where it should exist. That wave goes through the two slits, it is going to interfere with itself and produce the same."*

His response indicated that he did not relate the number of electrons sent with the intensity of the fringe pattern. This can be interpreted that when he thought of the wave function of electrons passing through the slits, he only thought of it as a pure classical wave instead of the probability wave.

In contrast to Eddy's response, some students predicted that if ten or two electrons were sent through the double slits, there would be equal or smaller numbers of dots observed on the screen. They understood that when some small numbers of electrons were sent through the slits, the distribution of the same number of dots would be observed on the screen. When students were asked to predict the result of sending two electrons, Max further explained as,

> *"With only two electrons, then again, if I were to make a bet, I would bet on the middle, just because that has the greatest probability. Again, you could get one that shoots of at almost 90 degrees. It is a really, really, really low chance that that is going to happen."*

## 5.4 Interview II Results and Analysis

The Interview II protocol is shown in Figure 5.3. In the Interview II results, student responses indicated many difficulties regarding their interpretation of the wave function and the energy diagram. A correct interpretation of the energy diagram is essential to understand the physical system because a potential energy diagram is used to represent many quantum systems. Here are various students' difficulties when they were asked to draw and interpret the wave functions for an infinite potential well problem.

### I. Difficulties with interpreting the energy diagram

**Associating the energy diagram with a physical well**

Students thought of the infinite potential well as an actual physical well. They thought of an electron as bouncing back and forth between two walls of the well. They still used classical mechanics to explain the motion of the electron. They thought of the wave function as the actual physical wave describing the motion of electron. A student, Jeremy (codename), gave a typical response:

**Interview II Fall 2003 Protocol**

Please draw the wave function for the following potential problems. Make sure that you draw it carefully.

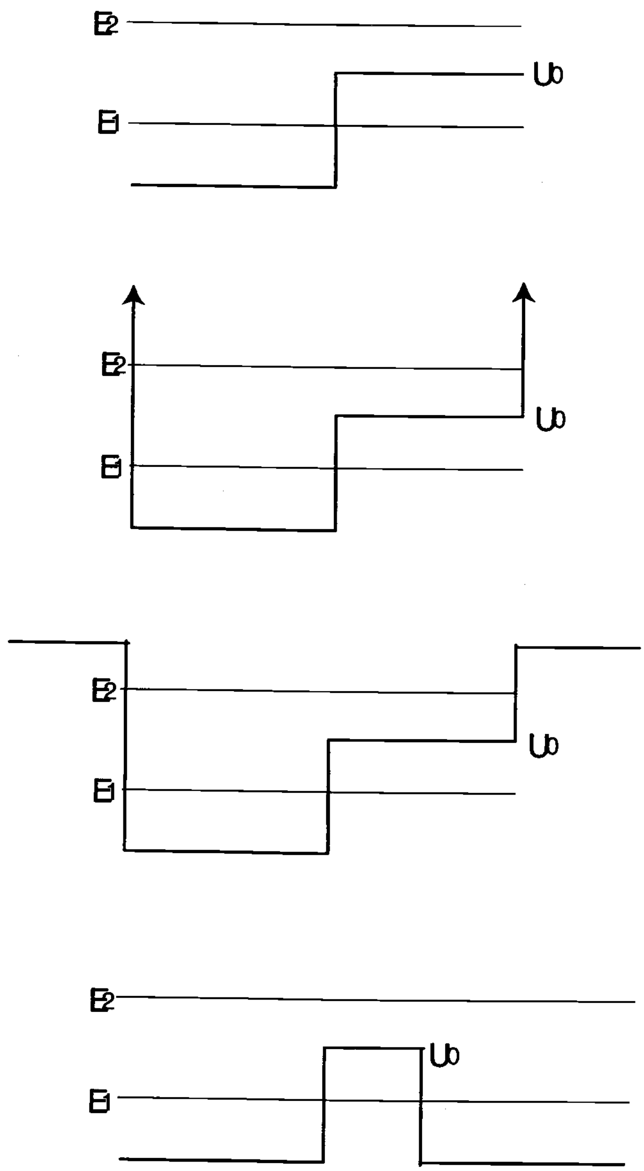



**Figure 5.3: The interview II protocol for Fall 2003 preliminary study**

**Jeremy:** Umm…you're more likely to find it in, um, the center region.

**Interviewer:** Why?

**Jeremy:** Well, it spends more time there [in the center]…Um…

**Interviewer:** What time? What do you mean by that?

**Jeremy:** Well…I'm not sure I can explain it, but...[silence]…It's not real likely to, um, be…outside of it. And I don't think it's as likely to be closer to the sides of the infinite well, than it is to be in the inside.

**Interviewer:** Uh huh. Why is that? Why you think that?

**Jeremy:** I just keep thinking of, uh, something in an actual well, if um, the electron was going back and forth, then um…I'd say overall in this region it'd be spending more time traveling through that region than it would closer to the walls.

This finding agreed with Bao (1998), who found in his study that students often took the potential energy diagram as a picture of a 2-D gravitational well. Since most students were more familiar with a gravitational well or an actual well where the y-axis indicates the position and also is associated with the gravitational potential energy, it is easier for students to visualize and make sense of the potential energy diagram using the analogy of the 2-D gravitational well.

### Confusion that the energy of a particle should stay constant

However there were a couple of students confused about energy. They intuitively thought the energy should drop because the potential energy changed when encountering the step, but they were bothered by the line representing the energy. They interpreted the line to represent the constant energy, so if the energy is constant, the wavelength could not be the only thing that changed. There should be something changing to oppose the wavelength changes. Thus they thought that the amplitude of the wave should change to oppose the wavelength and made the energy constant.

**Interviewer:** I don't know! [laughing] You tell me. What energy you mean? Like when you say that the energy is drop?

**Matt:** Um…I, actually it looks like the energy remains constant [pointing to the line representing energy].

**Interviewer:** Uh huh.

**Matt:** But uh…but uh the, the uh, wavelength increases and so the amplitude—or the waveplate, yeah waveplate, uh wavelength increases, so the amplitude increases, so the energy remains constant, so that'd be…

## II. Difficulties with interpreting the amplitude of the wave function

### Confusing the amplitude of the wave function with energy

This is the typical misunderstanding that most students had when drawing the solution to both the potential energy step and the potential energy barrier problems. When asked to draw and explain the wave function of particles of energy $E$ encountering a potential energy step of height $U_0$, students thought that the amplitude indirectly related to the energy. Mary provided a typical response:

**Interviewer:** So, so…from what you said it seem like wavelength change? Somehow the wavelength affect the amplitude. Is that…what you think it is, like from what you say that's what, how I understand it.

**Mary:** Um…not so much the wavelength, but this, um…I would think, let me think what would it affect…the amplitude?…I would think it'd be…the…energy would affect the amplitude. I wouldn't think it would be the wavelength. But it does, on both of these it does change when the wavelength changes, but I would think it'd be the energy.

With a similar response, Danny gave a more detailed explanation that the amplitude should decrease because the kinetic energy or $E-U_0$ decreased as the step increased in height. Therefore, the electron was moving slower. Danny indicated:

*"You would most likely find an electron in this section [where the potential energy increases to $U_0$] because there is less kinetic energy and the electron is moving slower."*

This difficulty was commonly found when students were asked to draw not only the wave function for the potential energy step, but also the wave function for the potential energy barrier problem. When the energy of the wave function is lower than the potential energy barrier, the amplitude of the tunneling wave decreased because some portion of the wave was reflected back. Students correctly drew the amplitude of the wave function getting smaller as it tunneled through. However they reasoned incorrectly that the amplitude was decreased because the electrons lost some of its energy during the tunneling process. Robert gave a common response:

**Interviewer:** Mmm….okay. So, what, what the amplitude represents?

**Robert:** The probability. But if the energy is already, if it's enough to overcome it, then they should all get through, just have less energy.

**Interviewer:** Mhmm, mhmm…and…so you seem like you have problem with energy and probability? Is that true? Because you keep saying energy, but you cannot draw the amplitude.

**Robert:** Uh…I don't really know, I guess what I, what I would say is that when it's energy is above the potential energy step, that the wavelength will get longer, just because it has less energy there, but the amplitude, which is the probability shouldn't change, just because its energy is greater than this energy.

**Interviewer:** So…it…all the particles will be able to make it through, so there should be the same number, so they're the same probability.

This reflected their classical mode of thinking. They thought of the potential barrier as the real physical barrier, so when the electron wave tunneled through it lost some of its kinetic energy so the amplitude decreased.

### Relating the wave function directly to the probability

In modern physics, the wave function is used to describe the probability wave as a solution of the Schrödinger equation with different potential energy conditions. There are two characteristics of potential energy that results in two different types of wave function—traveling waves and normalized waves. The traveling wave function, which is not normalized, is a solution to unbounded potential energy problems. The normalized wave function is a solution to bounded potential energy problems. Confusing or unclear interpretation of this difference can affect students' interpretation of the amplitude of the wave function. From the second interview, a few students indicated this confusion of amplitude by relating the probability directly to the amplitude of the wave function. When asked about where it was most likely to find electrons, students often pointed to the exact location on the wave function. Max gave a typical response:

> **Interviewer:** Okay. So, um, so how 'bout let's try to pick the region that you would be able to find electrons—that have the highest probability of finding electrons.
> **Jeremy:** Okay, um…the region where they'll most likely be?
> **Interviewer:** Mhmm, yeah, where it's most likely be, yep.
> **Jeremy:** Uh…the regions around here [pointing to the peak of the wave] I believe that's what the peaks mean.
> **Interviewer:** Mhmm, I see. So…you, when you say region, is that mean it's just this one…right here? Just this region, or the entire region?

He thought that the location of the probability wave is related to the actual position of an electron.

### Confusing the solutions to a potential step and a potential well

Students could not distinguish solutions of the potential energy step and the potential energy well that represented different situations. The wave function of the potential well relates to the probability of locating an electron trapped inside a well. In contrast, the wave function to the potential step is a traveling wave and represents only the real part of electron currents. Therefore these two solutions require different interpretations. Most students thought of both wave function solutions either representing a stream of electron flow or an electron. For example, Matt (codename) thought the solution for a potential well problem represented a whole bunch of electrons:

**Interviewer:** Mhmm. So, is that probability of find one electron or a whole bunch of electrons?

**Matt:** Uh, yeah.

**Interviewer:** Which one? One electron or a whole bunch?

**Matt:** Oh, a whole bunch.

In contrast to Matt, Jeremy thought of solutions to both potential energy well and step representing a wave function of one electron, as he explained:

**Interviewer:** Okay, okay good. So…so far all these case, um, like this one right here and this one [pointing to the potential step diagram], you draw the wave function of one electron or many electrons?

**Jeremy:** One?

**Interviewer:** One…And this one [pointing to the infinite potential well diagram] also for one electron?

**Jeremy:** Yes.

The inability to distinguish or point out the difference between these two solutions indicated that most students still did not understand the conditions of probability wave. As a result, most students drew identical wave functions as a solution for the potential energy step when $E > U_0$ and for the infinite potential well when $E > U_0$. For example in Figure 5.4, Danny drew the amplitudes for both wave functions—the potential step and the infinite potential well to be bigger as the waves encountered the increasing potential energy step.

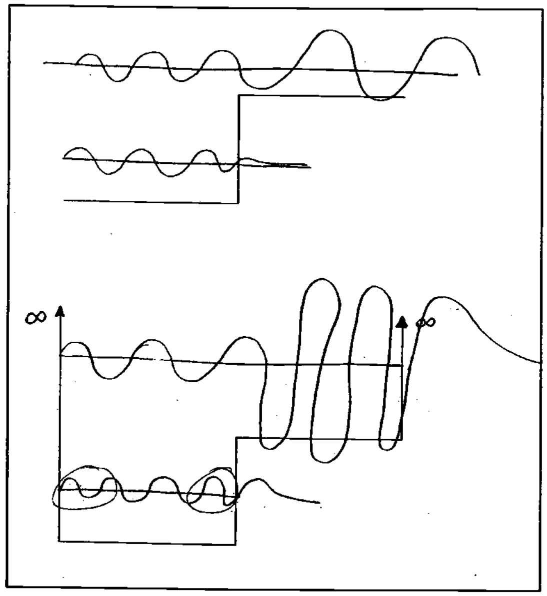



**Figure 5.4: From Danny's drawing, solutions to the potential energy step and the infinite potential energy well are nearly identical.**

## 5.5 Summary of Preliminary Findings

From the interview analysis, we learned that students coming to a modern physics class are not a blank slate. Because the QM building blocks are unavailable for them to construct QM concepts, students often use classical ideas or what they had learned from their previous science or physics classes. For example, using their previous knowledge, most students thought of an electron as a negatively charged particle, and they used the Bohr model to visualize an electron orbiting around a nucleus. Because of the concrete structure of Bohr model, it is easy for students to visualize and retain; as a result, students often associated the Bohr model with an electron or electrons (Fischler & Lichtfeldt, 1992; Petri & Niedderer, 1998; Muller & Wiesner, 2002; Wittmann, Steinberg, & Redish, 2002; Kalkanis, Hadzidaki, & Stavrou, 2003).

Students mentioned that the wave-particle behavior is counter-intuitive; however, they attempted to understand this strange behavior either conceptually or visually. Some students interpreted it conceptually by associating the wave nature of a quantum particle with its mass, which is similar to the idea of the de Broglie wave. A few students attempted to use visualization to model this behavior. However, many students just accepted it as a fact without formulating their own understanding.

Students correctly related the probability wave with the de Broglie wave because they understood that the particle mass influenced the wavelength. In addition, students often reasoned the probability wave existed because of the uncertainty in measurement. Therefore a large number of electrons should be present in order to use the probability wave.

In the second interview, the main purpose was to observe student difficulties with the energy diagram and the wave functions as solutions to different potential problems. For the energy diagram, students often took the potential well as an actual physical well and the electron as a ball bouncing back and forth between two walls of the well. This finding agreed with the Bao (1998) study. In addition, students misunderstood that the particle energy should stay constant at all times because of the straight line representing the energy of particles, as shown in Figure 5.3.

In interpreting the wave function, students had three major difficulties regarding the amplitude. First, in the potential energy step, most students confused the amplitude of wave function with energy because they thought that an electron was moving slower, so it was more likely to find the electron in the region with a higher potential energy.

Second, in the potential energy barrier, students correctly drew the amplitude of the wave function getting small as it tunneled through. However, they gave an incorrect reason that the electrons lost some of their energy during the tunneling process, so the amplitude decreased. Third, a few students related the probability directly to the amplitude of the wave function. When asked about where it was most likely to find electrons, students often pointed to the exact location on the wave function. These findings indicate students' difficulties in distinguish the difference between the traveling wave function and the normalized wave function.

Comparing responses to different potential problems, most students could not distinguish solutions of the potential energy step and the potential energy well. Most students thought of both wave function solutions either representing a stream of electron flow or an electron. The inability to distinguish or point out the difference between these two solutions indicated that most students still did not understand when and how to apply the normalized probability wave. This difficulty could also relate to student misinterpretations of the wave function amplitude. However, further study and more evidence are needed to verify this speculation.

# Chapter 6

## Final Interview Results

In an effort to investigate student understanding of probability in modern physics, an immense amount of data was collected. In Chapter 4, the results already helped us answer the first research question about students' pre-knowledge of probability before learning modern physics. Chapter 5 discussed a preliminary effort to collect in depth data through interviewing students. However the analysis in terms of student difficulties was rather limited. It disregarded much fruitful information that could be used to understand student probability knowledge structures. Also reporting results in terms of student difficulties did not suggest much implication to improve teaching. Therefore, in this chapter, we introduce a theoretical framework or model that was used in analyzing the interview data. This theoretical framework is based on two proposed models of student reasoning/ thinking—the coordination class by DiSessa and Sherin (1993) and a model of student thinking by Redish (2003). To help the reader understand the nature of participating students, we discuss in detail the sampling method and characteristics of participants. Then we gave details of the data collection and interview protocol. Also, we report on the data analysis process and also establish its validity. Finally, we report results from both interviews in two perspectives suggested by Redish's model.

## 6.1 Theoretical Framework: Student Thinking Model

In physics education research, certain terms such as "concepts" or "misconceptions" are used with ambiguity assuming that "every one knows what it means" (Redish, 2003; Wittmann, 2002). To understand student thinking better, we need a model to describe such complex processes. In addition, models of student thinking and reasoning should help us understand how students develop their understanding over the course of time. In this section, we introduce a rather new model of student thinking proposed by Redish (2003) and compare it with the coordination class that was discussed in Chapter 2. This model is based on the fundamental principle of *constructivism*—one constructs new knowledge based on the pre-existing knowledge.

### 6.1.1 Student Thinking Model: A Two-Level System

From his synthesis of the literature in neuroscience and cognitive science, Redish (2003) has developed a theoretical framework to understand student thinking, especially in physics teaching and learning. The human brain consists of millions of neurons, and they form networks or connections as shown in Figure 6.1. When there is an activation of at least one neuron, this can lead to activation of other neurons. Redish claimed that this fact suggested the idea of *activation*. Learning appears to be associated with the growth of connections between neurons. Redish asserted that to create learning, the brain should function by *association*. Also, one neural connection can enhance or inhibit another neural connection. Redish stated that this fact suggested the idea of *enhancement* and *inhibition*.

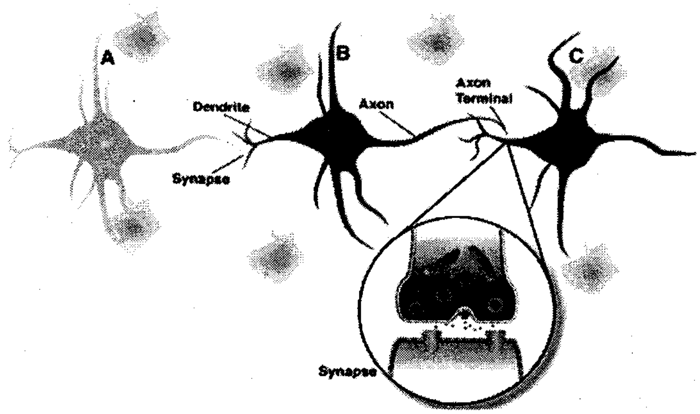



**Figure 6.1: Neurons and Neural Connections (Introduction, n.d.)**

In his paper, Redish also wrote about findings from cognitive science that suggested a structure of memory, as presented in Figure 6.2. He suggests three interesting issues about working memory. The first issue relates to a limited size of working memory, so it can only process *chunks* of knowledge or groups of knowledge elements that are tightly connected. The second issue is that working memory does not function independently from long-term memory, as presented in Figure 6.2. The working memory encodes activated information and sends it to the long-term memory which sends back or retrieves *chunks* of knowledge associated with the activated information. The activation of one item from the long-term memory can create activation of other associated items more easily. This process is called *priming*, which can occur even if the stimuli are presented very quickly or not consciously noted (Redish, 2003). The last idea is that the effective number of chunks taken up in the working memory depends on the individual's state of knowledge. For example, students' responses to a certain piece of information depend on their pre-existing knowledge and what information they are cued to access. In another words, Redish explained that, when a specific bit of knowledge is activated, other associations are also enhanced or inhibited. This process is called *spreading activation.*

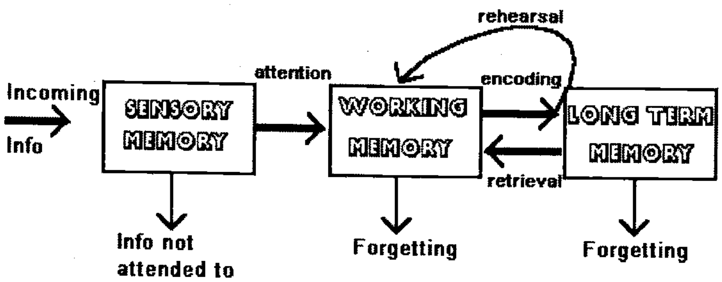



**Figure 6.2: Basic structures of human memory**
**(Human memory, n.d.)**

Up to this point the findings from neuroscience and cognitive science underline two similar concepts—*activation* and *association*, explaining how a knowledge structure should work. Redish cited Baddley's work in 1998 that the brain had a mental executive to manage activations in the working memory. He suggested that this finding indicated that there should be a *control* structure that controls the activation in the knowledge structure.

Based on these three concepts—*activation*, *association*, and *control*, Redish proposed a student thinking model consisting of two-level structures—association and control, as shown in Figure 6.3. Redish defined the association structure as where associated patterns of *resources* dominate and the control structure as where one can describe expectations or *epistemology*[1] *resources.*

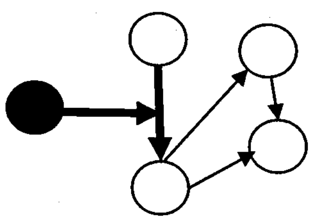


**Figure 6.3: The control structure (a dark circle) controls the activation in the associated pattern (a link of light circles).**

[1] Epistemology means the study or a theory of the nature and grounds of knowledge especially with reference to its limits and validity.

#### 6.1.1.1 The Pattern of Association

Redish indicated that the knowledge structure of the model should consist of a pattern of associations of resources. He defined resources as a "compiled or activated knowledge element that appears irreducible to the user. Because different persons have different constructs for the same knowledge, different levels of structure may be used as resources by different users. For example, when asking a student who has not had any proper physics instruction to explain what a force is, that student may not be able to provide any information at all. However when asking a physicist the same question, we should expect to receive a well constructed answer about a force. Therefore, in the student's knowledge structure, force is stored as a resource, a fundamental level. In contrast, in the physicist's knowledge structure, force is an associated pattern of resources. Redish compared his idea of resources with the previous proposed model of student thinking. He suggested that a resource is similar to a phenomenological primitive (p-prim) and a facet, both of which were explained in Chapter 2. Wittmann (2002) specified the resources that a person used to explain a certain phenomena as *reasoning resources*. In this study, we use reasoning resources to specify resources that students used in order to provide reasoning for their answers to our interview questions. The reasoning resources are considered to consist of either the p-prim or the facet. In order to help the reader understand the meaning of reasoning resources and how we use that to analyze our data, an example of reasoning resources in terms of the p-prim and the facet that were found in a previous PER is provided in Table 6.1.

| |
|---|
| **Why is it hotter in the summer than in the winter?** |
| ■ "It is because the earth is closer to the sun." |
| ■ "The seasons occur when the earth is tilting on its axis and the northern or southern hemisphere whatever one is closest to the sun is hotter." |
| **Reasoning resources:** |
| ■ p-prim—closer is stronger |
| ■ Facet—when it is warmer on the earth we are closer to the sun. |

**Table 6.1: An example of reasoning resources in terms of the p-prim and the facet.**

#### 6.1.1.2 Control Structure

Redish (2003) stated, "Separating control from association allows us to describe knowledge structures and their context dependence separately and has powerful implications both for our understanding of our students' responses and for our understanding of what produces those responses" (p.16). For a given situation, association of resources provides the appropriate structure of knowledge, while the control structure is linked with attention, context dependence, and goal-oriented decisions. There is something "between" our coding of sensory input data and our making sense of data—a control filters that and decides what knowledge structures are to be activated, as shown in Figure 6.3. Most instructors are aware that student expectations about what they are supposed to do in class can play a powerful and destructive role in creating "selective attention," which limits what students actually do in class and what they learn. Redish proposed three components in this control structure:

- Epistemic resources—the processes or tools that an individual uses to decide how they know something or construct knowledge.
- Epistemic frames—an individual develops certain expectations helping them to make sense of complex information that they interact with.
- Epistemic messages—an individual perceives a message from an external environment and uses it to develop certain expectations.

Because of the nature of our data collection, we are only interested in epistemic resources that are found in student responses to specific interview questions. For example, when the researcher asked one student if he wondered why electrons have wave-particle behavior, his response indicated he constructed the knowledge of wave-particle behavior as *knowledge by authority*, which is defined in Table 6.2 with an interview excerpt from Bill.

| |
|---|
| **Interviewer:** In your opinion, do you ever wonder why electrons have wave/particle behavior?<br>**Bill:** I do wonder why they have a wave and particle behavior, <u>but you accept it and say, okay, they do, and you go on.</u><br>**Epistemology resources:**<br>*Knowledge by authority*—accept the knowledge given from an instructor or a textbook |

**Table 6.2: Bill interview I excerpt indicates his epistemology resources as *knowledge by authority*.**

### 6.1.2 Coordination Classes versus Two-Level System

As mentioned in Chapter 2, the coordination class is the model that an individual uses to 'see' the world. Therefore each individual constructs his or her own reality in seeing the world; this underlying idea also agrees with principle 4 that Redish (2003) synthesized from neuroscience research[2]. Also, the causal net element of the coordination classes is defined as the set of relevant inferences about the relevant information. This definition indicates the association of relevant knowledge elements, which is similar to the pattern of association. These two models shared many similarities; however, there are some differences that should be discussed.

The coordination class is useful in understanding student learning of science, especially physics. However when we attempted to use the coordination class to analyze our interview data, we could not find that many readout strategies that students used in answering our interview questions. Redish clarified the definition of readout strategies as "a set of resources that translate sensory information into meaningful and processable terms" (p. 22). From Redish's definition, the readout strategies are hardly found in student responses because of the nature of modern physics concepts based solely on abstraction, so students' readout strategies could not be identified easily in the modern physics context compared to the classical mechanics context.

## 6.2 Sampling Method and Interview Participants

The process of recruiting students and selecting participating students is discussed in order to clarify characteristics of interview participants. The recruitment started in the first week of PH314 class; the instructor announced in class and gave brief information about earning extra credits by participating in the interview. Also the recruitment letter was distributed in class to give more detailed information about both interviews. An interview sign-up sheet was available for students to sign up in laboratory. For Spring 2004, 31 students signed up for the interviews. From 31 students, interview participants were selected by a purposeful

[2] In his paper, Redish (2003) stated principle 4: "There is a real world out there and every individual creates his or her own internal interpretation of that world based on sensory input."

sampling method according to their physics and overall achievements. Because of the undisclosed information of student grades and their GPA, the researcher was not permitted to see students' records. The Physics Department secretary was asked to select 12 students according to their overall GPA and their grades from PH213 (the last course in an introductory physics sequence covering topics in electromagnetism, electromagnetic waves, and optics). According to student achievement, the secretary was asked to rank students into three groups—top, middle, and low achievement groups, and to pick five students from each group. The secretary, then, gave 15 student names to the researcher, but the researcher had no information about which student among the 15 students was from which group. Later in the class, two students from 15 selected participants withdraw from the class, and one student did not show up for the second interview. Therefore only 12 students participated in both interviews, and their interview data were analyzed and reported here. Table 6.3 represents each student's overall performance and their midterms and final scores only on the problem about probability and related concepts, compared with each exam total scores.

### 6.3 Data Collection and Interview Protocol

Each interview was one-on-one, conducted in a closed room and video-recorded. In average, the first interview took about 40-50 minutes to complete, and the second interview was shorter, 20-30 minutes. Before each interview, a participant was asked to read and sign the informed consent form in order to inform the students of the human subject rights. The researcher, then, explained the overall project and its purpose, and told the participants to try to explain their thinking as much as they could. If they could not answer the question at some point and had no idea, then they should notify the researcher. The researcher clarified that the questions she would ask were attempting to get accurate information about the participants' thinking processes, so the questions should not be taken as guiding questions. During both interviews, participants were not allowed access to a class textbook or lecture notes. If they became unsure and wanted to know about a certain equation, then they were encouraged to ask but the researcher might not be able to answer questions that could lead participants' thinking.

| Student Codename | PH314 Grade | Midterm I | | Midterm II | | Final | |
|---|---|---|---|---|---|---|---|
| | | Score (30) | Total (100) | Score (30) | Total (100) | Score (30) | Total (150) |
| **Ben** | A | 17 | 77 | 27 | 96 | 27 | 127 |
| **Ethan** | A | 15 | 85 | 18 | 82 | 27 | 122 |
| **Jim** | A | 17 | 77 | 28 | 87 | 26 | 137 |
| **Mac** | A | 16 | 86 | 24 | 78 | 30 | 130 |
| **Nik** | A | 20 | 80 | 25 | 83 | 21 | 131 |
| **Paul** | B+ | 8 | 51 | 25 | 65 | 23 | 113 |
| **Tom** | B+ | 11 | 63 | 24 | 87 | 18 | 109 |
| **Sam** | B | 25 | 74 | 16 | 70 | 14 | 103 |
| **Bill** | C | 3 | 50 | 14 | 66 | 7 | 90 |
| **Brian** | C | 11 | 63 | 24 | 70 | 14 | 103 |
| **Ken** | C | 12 | 57 | 9 | 58 | 9 | 91 |
| **Owen** | C | 15 | 51 | 21 | 64 | 17 | 98 |

**Table 6.3: Student codenames and their class grades and performances on exam problems relating to probability compared to their total scores for each exam**

After the introduction, the interview worksheet (the worksheet for both interviews can be found in the appendix) was available for participants to write down any equations or any diagrams that helped clarify their thinking processes. For most of the participants, the questions were asked according to the interview worksheet except Mac for his first interview. Mac struggled to answer question C and D in part I, which were about drawing the probability wave for a free electron moving at a speed v and 2v. Mac asked if he could move on to the next one and come back to these two questions in the end. The researcher allowed that because the later questions did not relate to questions C and D. After finishing the interview, students were allowed to ask questions if they were curious about the answers if there was enough time to do so. These questions were answered and discussed with the researcher off the video camera.

## 6.4 Analysis Process and Its Validity

Both interviews aimed to investigate student thinking and reasoning resources in terms of six concepts— the electron, wave-particle duality, de Broglie wave, Heisenberg uncertainty principle, wave function, and probability. However to get a better understanding of student probability knowledge structures, we also need to look at the epistemology resources, as mentioned in Section 6.1. Therefore I analyzed both interviews in terms of reasoning resources and epistemology resources.

The researcher first read the interviews from 12 students, and then identified themes that emerged from the whole data. The themes were converted into codes, and the researcher went through each interview again and coded it. During this step, the code descriptions were revised to best describe the nature of excerpts associated with the codes. Occasionally, a new code was created to represent student responses that indicated an existence of a new reasoning resource. An example of the codes used in analyzing question A of the first interview is presented in Table 6.4. In order to establish the validity of the interview analysis, the researcher compared coding with the physicist who had been teaching PH314 class many times, including both in Fall 03 and Spring 04. The instructor suggested adding certain codes to represent student reasoning about the de Broglie wavelength. The coding scheme was given to the PH314 instructor, and he coded some samples of interview excerpts. Then the codes used by the instructor and the researcher were compared, and both codings were in agreement. An example of the coding scheme is shown in figure 6.4.

**A. What is an electron?**

| Label | Brief Definition |
|---|---|
| **General resources** | |
| **R-ChargedParticle** | An electron is a negatively charged particle. |
| **C-Current** | An electron flow produces a current. |
| **C-WaveParticle** | An electron can behave as either wave or particle. |
| **Atomic Model** | |
| **R-BohrModel** | An electron orbits around a nucleus. |
| **R-ProbModel** | An electron cloud atomic model. |

**Table 6.4: An example of coding scheme used in analyzing question A in the first interview.**

## 6.5 Spring 04 Results

I describe here how student responses indicate consistent use of the same reasoning resources in a variety of situations. Also I discuss the results in terms of epistemology resources as mentioned in the theoretical framework section.

### 6.5.1 The Electron

When students were asked to explain what an electron is, 10 out of 12 students mentioned that an electron was a negatively charged particle. An example of a student quotation explaining an electron in terms of a negatively charged particle is presented in Table 6.5. Four students also associated an electron with a current flow. Once again, I found students relating an electron with the Bohr model. Only one out of 6 students that thought of an electron in an atom referred to the probability model of an atom or an electron cloud model. This finding agreed with the preliminary results that students strongly associated the Bohr model with an electron. Only one student mentioned about the wave-particle behavior of an electron. Here are resources that we found students used to describe an electron:

- A negatively charged particle
- A current—an electron flow creates a current.
- Bohr model— an electron orbits around a nucleus.
- A probability model—an electron cloud model
- A wave-particle behavior—an electron has both particle and wave behaviors.

| |
|---|
| Ben (codename) provided his idea of what an electron is:<br>*"An electron is one of the fundamental particles. It carries electric charge. Electrons circle around the atoms, which contain a nucleus. A nucleus contains protons and electrons."* |

**Table 6.5: Transcript excerpt indicating an electron as a negatively charged particle and behaving according to the Bohr model**

### 6.5.2 Wave-particle Duality

When asked to explain the wave-particle duality of electrons, most students gave the explanation that electrons were able to present wave behaviors—diffraction and interference, so an electron also is a wave, as well as a particle.

When asked to explain what characteristics of electrons create this wave behavior, five students said that they just accepted the fact without much understanding it. However when asked why a baseball did not behave like a wave compared to an electron, one out of five could explain why an electron had such behavior but a baseball did not have. The overall analysis of students' responses in terms of reasoning resources and epistemic resources is shown in Table 6.6.

| Codename | Reasoning Resources | Epistemic Resources |
|---|---|---|
| **Ethan**<br>**Ben**<br>**Ken**<br>**Paul**<br>**Bill** | Mentioned the experiments that prove the electron has a wave behavior—single/double-slit experiments. Only Bill explained the infinitesimal size of electron caused the wave behavior. | Using ***Knowledge by Authority***, except Ethan who did not show enough information to conclude anything. |
| **Jim**<br>**Mac**<br>**Nik**<br>**Sam**<br>**Brian**<br>**Owen**<br>**Tom** | • Provided the experiments that prove the electrons have a wave behavior.<br>• The wave behavior is a result of an infinitesimal size or mass of the electron.<br>• The wave behavior is just the probability wave of locating the electron. | Using ***Knowledge by making sense*** |

**Table 6.6: Summary of students' reasoning resources and epistemic resources for the wave-particle behavior**

**Reasoning Resources**

From the overall results, seven students related the de Broglie wave with the wave-particle duality. However, only six students indicated their understanding of the de Broglie wave. They reasoned that the electron is infinitesimal, so the wave behavior is so significant and can be observed. Sam provided a typical response relating the wave-particle behavior to the electron mass:

> **Interviewer:** Why do you think an electron has wave and particle behaviors?
> **Sam:** Because all particles all naturally have a wave and a particle behavior. It is just there.
> **Interviewer:** Why do you think the electron can be able to observe better? Can you see the wave behavior for an electron better than another matter?
> **Sam:** Because an electron is small enough that it has a load of mass that it can be affected by diffraction, whereas something with a larger mass is much harder to deflect by diffraction unless you have it moving at an extremely low velocity, and that is almost impossible to attain.

Sam's reason also indicated his understanding of the de Broglie wavelength when he suggested that a larger mass had to move at an extremely low velocity for its wavelength to be observable. Sam did not only provide an evidence for the wave-particle behavior but also the reason for its existence. Students' explanations that wave behaviors of electrons could be observed by means of diffraction or interference indicated that they to some extent thought of an electron as a wave. Also, some students related the wave nature of a particle to the probability wave of locating it. Here is a common explanation in terms of the probability wave that Jim provided:

> "*Electrons, like other particles, behave like waves which can be described using the de Broglie wavelength and the wave function, which you need a potential energy to ascribe how it is going to behave. The wave property of it gives the probability of locating it when you look for it. that is kind of its wave aspect.*"

From Table 6.6, it seems that students who associated the wave behavior of the electron with its mass could also think of the de Broglie wave as the probability wave of locating an electron.

**Epistemic Resources**

Beside what students do know about wave-particle duality, we are also interested in a source or a process by which students came to "know" this modern physics knowledge, which Redish defined as the epistemic resource. As shown in Table 6.6, students who could explain how and why an electron has a wave behavior constructed their knowledge by making sense. When students construct new knowledge based on their pre-existing knowledge, we call the epistemic resources for this process *knowledge by making sense*. For example, Owen provided his interpretation of wave-particle behavior:

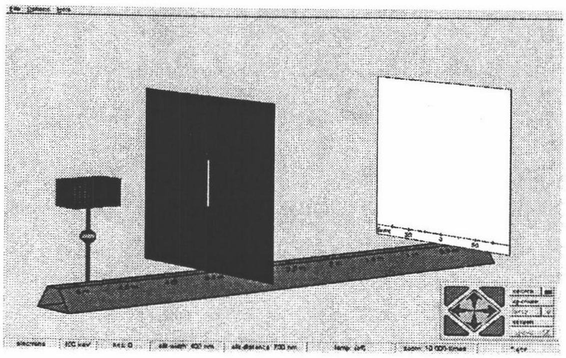
> "*<u>The best way I can grasp</u> is that it is a particle and then it has got wave properties that are determined – like the probability of finding the particle at any given location is the wave associated with that. <u>I try to stick with my classical concepts</u> of an electron being a single particle, only it just happens to do things like a wave, occasionally.*"

In contrast, students, who could not provide much reasoning resources to explain the wave-particle behavior, indicated in their interview responses that they just accepted the fact or they were not told why. When students view learning as receiving knowledge from an authority or a textbook, we call *knowledge by authority*. Bill's interview excerpt provided a good example for this epistemic resource, as presented before in Table 6.2. There is one student, Ethan, for whom we could not identify his epistemic resources because he did not provide enough information in his answer.

### 6.5.3 Heisenberg Uncertainty Principle

From the preliminary interview (Fall 03), there was little information about student reasoning in terms of the uncertainty principle. Therefore I added one question related to the uncertainty principle in Interview I (Spring 04). The first question was to investigate how students applied the Heisenberg uncertainty principle in an unfamiliar situation. Also this question indirectly observed the idea of localization, which relates to the uncertainty principle. The question and its acceptable answer are presented in Figure 6.4. Because of the nature of this question, the information about student epistemic resources was not available from the interview responses.

A. Explain how the single-slit experiment works using the language of the UNCERTAINTY relationship, and also draw the pattern that you should observe on the screen.



(The electron gun sends out electrons with certain energy, so the electrons have a definite momentum. When they pass through the slit, the electrons are localized in a certain region, $\Delta x$. From the uncertainty principle, the electrons also have an uncertainty in momentum, so they spread out with various velocities and create a diffraction pattern.)

**Figure 6.4: A question to investigate student resources for the Heisenberg uncertainty principle.**

**Reasoning resources**

Most students could not identify that the width of the slit indicated the uncertainty in the x direction ($\Delta x$), as presented in Table 6.7. This leads to their inability to relate it to the uncertainty principle.

- **Relate $\Delta x$ with the distribution of the diffraction pattern**

Students used the elements in the uncertainty principle incorrectly. Three students associated $\Delta x$ with the distribution of the diffraction pattern. Bill gave a typical response as:

> *"If you increase the velocity of the protons or the velocity of the electrons, you are going to have a better diffraction pattern – no, if you slow them down, you are going to have a clearer diffraction pattern and more spaced out. If you increase it, the patterns are going to be closer. They are going to be more. ."*

From Bill's response, he did not apply the uncertainty in momentum, but he thought that the velocity is inversely proportional to $\Delta x$, or the distribution of the diffraction pattern.

- **Using the uncertainty principle and the de Broglie wavelength**

Quite a few students correctly explained how the single slit worked using the uncertainty principle. They correctly identified that $\Delta x$ related to the width of the slit and the distribution of the fringes related to $\Delta p$. Mac gave a typical response:

> **Mac:** I think it would look something like that. By making the slit narrowerer, it will spread out, the distance between the peaks. That is because by making the delta x smaller, you increase the uncertainty, and changing momentum, so it can different velocities and do different things.
>
> **Interviewer:** When you decrease the slit, then what happens?
>
> **Mac:** The distance between maxima becomes greater, like the interference pattern will spread out across the screen.
>
> **Interviewer:** Why do you think this, why is it spread out more?

**Mac:** There are different relationships. Isn't there like a delta x, delta lambda equals to H bar, or something like that. So by changing the different x, you get a different possible value of the wavelength, and that will allow it to get different patterns on the screen. It will be more spread out.

In Mac's case, he also explained in two different reasoning resources—using the uncertainty principle and using the relationship between the wavelength and the slit width. Student responses to the question about the uncertainty principle are summarized in Table 6.7.

| Codename | Reasoning Resources |
|---|---|
| **Ethan**<br>**Brian**<br>**Ken**<br>**Paul** | Cannot explain in terms of the uncertainty principle.<br>Only Ethan said that he only learned that "*the smaller the aperture, the bigger the diffraction angle*" for the single-slit experiment |
| **Jim**<br>**Tom**<br>**Nik**<br>**Mac** | Relate the slit width to Δx and Δp to the spreading out of the pattern and correctly used the uncertainty principle to explain the relationship between the slit width and the pattern. |
| **Bill**<br>**Sam**<br>**Owen** | Relate Δx to the distribution pattern |
| **Ben** | Relate the slit height to Δy and explain the uncertainty correctly but using Δy Δp, instead. |

**Table 6.7: A summary of student responses to the uncertainty principle in the single-slit experiment**

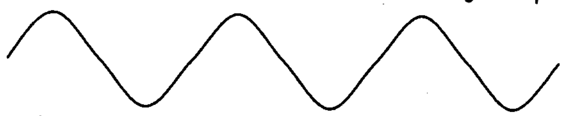
### 6.5.4 Probability and Wave Function

In this section we investigate student understanding of the probability wave or the wave function. We explored students' reasoning resources of the wave function in different conditions such as the wave function for a free electron, a potential energy step, a potential energy well, etc. Also, we asked students to interpret the probability from their wave functions. In this section, not only the reasoning resources of the wave function were found but also of the probability.

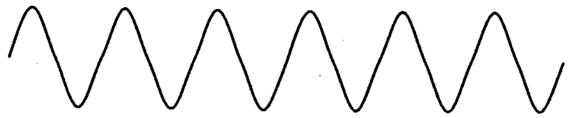
#### 6.5.4.1 A Free-moving Electron Wave Function

In Spring 04 interview I, the questions C and D, as presented in Figure 6.5, were added in order to observe student understanding of a probability wave (also known as the de Broglie wave or a wave function) of a free electron. The probability wave is a hypothetical wave, not a physical wave like students learned in an introductory class. The amplitude of the probability wave does not relate to any physical quantity such as displacement or pressure. In modern physics, the probability wave is a graphical representation of probability in finding a localized particle. *The probability of finding the particle at any point depends on the amplitude of its de Broglie wave at that point* (Krane, 1996). This statement can be applied only when the particle is trapped or localized. The idea of localization relates to the normalized wave function (or the probability wave).

C. Draw the probability wave of a free electron moving at a speed v.



D. Draw the probability wave of a different free electron moving at a speed 2v.



*(The amplitude is the same for both waves. This indicates that the probability of locating a free electron is same through out the space because the free electron is not localized or the normalization of a wave function (probability wave or the de Broglie wave) is applied when the electron can be localized in a certain region. In terms of wavelength, the wave for the faster electron should have a shorter wavelength according to the de Broglie wavelength relationship,* $\lambda = \frac{h}{p}$*)*

**Figure 6.5: Questions (with acceptable answers) investigating the probability wave of a free-moving electron.**

## Reasoning Resources

- **The x-axis indicates the position of an electron.**

Table 6.8 shows the summary of each student's response to Question C and D. Most students correctly identified the x-axis as position of the electron. Only one student, Paul, identified the x-axis as time. However, his language indicated that he thought of the x-axis as position:

> *"No, I think less. If you are comparing C and D, it should be less probability to find the electron in region 2 than in region 1. If I were to compare this, that is what I would say. It is moving twice as fast, so finding it in that interval* [pointing to the region along the x-axis in the drawing for question C], *it would only be in this interval* [pointing to the region along the x-axis in the drawing for question D] *for half as much time as it would be in this interval."*

He was not so sure what the x-axis represented, but after he compared the waves that he drew for questions C and D, he came to conclude that the x-axis should represent time. All the students used the x-axis to represent the position of the moving electron.

- **The representation of the y-axis**

Students used different reasoning resources when asked them what the y-axis represented. Five students, who correctly stated that the y-axis represented the amplitude of the probability, also mentioned that the amplitude squared was equal to the probability of locating the electron.

For five students who indicated that the y-axis represented the probability, some of them drew the probability wave as if it was a probability density or $|\psi|^2$. The word "probability" in the probability wave might just bring attention to their thinking, so they drew the probability density instead of the wave function. Here is the probability wave that Bill drew:

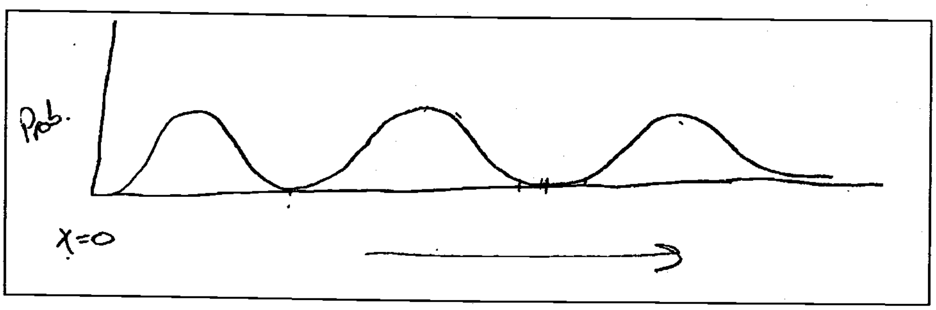



**Figure 6.6: Bill's drawing of the probability wave for the electron moving at speed v.**

Only two students reasoned that the y-axis represented the vertical position. The reasoning resources indicated that these students thought of the probability wave as the actual physical wave. They thought that the electron actually was moving up and down, literally, as Bill mentioned:

> **Bill:** The y axis would represent the position.
> **Interviewer:** So the electron is moving along up and down?
> **Bill:** Yes.

- **Comparing the probability between two electrons**

When asked to compare the probability wave between the electron moving at speed v and another one moving at speed 2v, most students reasoned that the electron moving slower had more probability of being located because it spent more time in a given region. Therefore, they related this inference to the amplitude of the probability wave, so they drew the amplitude of faster moving electron to be less than the slow moving one, as presented in Figure 6.7.

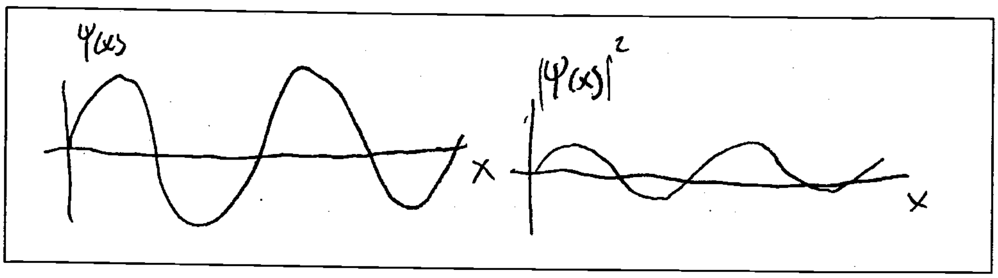



**Figure 6.7: Ethan's drawing of the probability waves, the moving electron at speed v is on the left and one at speed 2v is on the right.**

Most students stated that the probability of locating the faster moving electron was lower than the slow moving one because the electron with lower speed spent more time in a certain region, so it was more probable. This reasoning resource indicated a facet that ***a slower moving object is more observable***. We call this facet, the classical probability facet. This facet is dominated in students' reasoning processes. Owen gave a typical response:

> **Interviewer:** It would have half the wavelength? How about the amplitude?
> **Owen:** The amplitude, I don't know that that would change. It might change, but I don't know how.
> **Interviewer:** Why do you think it would change? It would be bigger, smaller?
> **Owen:** The energy changes? Actually I would venture to guess that the amplitude would actually shrink, because if this is the probability of finding it in a given region of space, as <u>the particle moves faster, it will spend less time</u> in any given region of space. So I guess I would then come to the conclusion that <u>it would have a lower amplitude</u> than over here [point to the picture of the electron with speed v].

This facet dominated students' thinking processes because students have seen it before in classical mechanics or in everyday experiences. Students learned this facet, which is the same idea as $v = x/t$, early in their physics classes. Also, from their own experience, they have seen this facet in the real world. When there are two cars approaching, they can see the slower moving car in more detail than the fast moving car. This type of everyday observation strengthens the association of this facet with the concept of probability.

Besides using the facet, a few students reasoned in terms of the uncertainty principle, localization, or the normalization of wave. One student, Bill, incorrectly applied the uncertainty principle, which made him make the wrong conclusion at the end. Nik and Mac correctly associated the localization to the probability, as Nik explained:

> **Interviewer:** What do you think should change?
> **Nik:** I think the wavelength should change, probably the amplitude. It is hard to say by the amplitude, because we don't have any <u>potential energy well</u>, but the wavelength should definitely change.

From the interview excerpt, Nik was not sure with her answer because she only associated the probability with the potential energy well, which represents that the electron has to be localized or trapped. For the free-moving electron, she was not certain how to apply the probability because the electron was not trapped. In contrast, Mac also associated the probability with the localization. He stated the amplitude did not change because the probability could be calculated when the wave function is a standing wave, which only occurred when the electron was trapped. Jim used the concept of normalization to come to his conclusion that the free electron probability wave was not normalized, so its probability could not be determined, as he explained,

> **Jim:** Well, the amplitude, I guess, if you are thinking about the probability, it depends if it is normalized or not. You can't normalize a free electron, the probability of a free electron, because the sine wave goes up to infinity. To be able to set the probability equation to 1, you wouldn't be able to, so you can't normalize it.
> **Interviewer:** You cannot draw the probability?
> **Jim:** Right. I guess you could draw its wave function, but it wouldn't be normalized, so maybe then you wouldn't be able to draw probability.

For a probability of locating the free moving electron, students used several reasoning resources to interpret the probability wave. Here is the summary of the reasoning resources:

1. **The classical probability facet**—the slow-moving electron is more observable or more probable to locate.
2. **The uncertainty principle**
3. **The localization of the electron**—students associated it with the potential energy well.
4. **The normalization of the probability wave**—Jim reasoned that the probability can be determined from the normalized wave function only.

**Epistemic Resources**

Most students' response did not indicate any epistemic resources that they used in construct or retrieve this knowledge. However, one student, Ethan, indicated an interesting mix between *knowledge by making sense* and *knowledge by authority.*

> **Ethan:** I didn't mean to change the period at all. I was trying to have the period the same, but the amplitude, because it being the probability at any position, I would think would be less if it was moving faster, the probability for it to be in any position.
> **Interviewer:** Why are you thinking this?
> **Ethan:** I think <u>he [ the instructor] said along those lines, and it kind of made sense to me</u> at the time, that if you <u>**had a small area**, that if it was going slower, the probability of it being in that area is more than if it was going faster through that.</u>

Ethan claimed that he learned about this facet from the instructor, and the idea made sense to him. This indicated that the facet was constructed based on two epistemic resources—*knowledge by authority* and *knowledge by making sense.* However, he seemed to misinterpret the information that he got from the instructor. He repeated the information and mentioned *"had a small area"* However, he did not quite understand that the small area in the sentence can be anywhere along the x-axis. As a result, the electron could be anywhere on the x-axis or the probability of finding it anywhere along the x-axis would be the same. Table 6.8 summarizes the students' responses to questions C and D and gives the reasoning resources they used in making their answers.

| Codename | Axes of the wave | | Comparison between an electron traveling at speeds v and 2v | | | |
|---|---|---|---|---|---|---|
| | **x-axis** | **y-axis** | **wavelength** | **Reasoning Resources** | **Probability** | **Reasoning Resources** |
| **Ben** | position | Probability | shorter | Velocity increases so the kinetic energy increases. From the de Broglie wave relationship, the wavelength gets shorter. | decrease | **Facet :** ***slow-moving object is more obserable*** The electron with speed 2v is moving faster so there is less chance to locate it or it is less likely to find it, compared to the slow moving electron. |
| **Ethan** | | Amp. of $\Psi$ | | | | |
| **Owen** | | Amp. of $\Psi$ | | | | |
| **Tom** | | Probability | | | | |
| **Sam** | | Amp. of $\Psi$ | | | | |
| **Paul** | time | Probability | --- | ---- | | |
| **Ken** | position | position | shorter | Velocity increase using the de Broglie equation. | | |
| **Brian** | | Probability | longer | Forgot the equation | | Forgot the equation |
| **Bill** | | position | shorter | Velocity increase using the de Broglie equation. | decrease | Start using the uncertainty principle but use momentum instead of $\Delta p$, so end up using the facet reasoning. |
| **Mac** | | Amp. of $\Psi$ | | | same | The probability does not depend on the amplitude because the electron is not localized or confined in a certain area (like in a potential step). |
| **Nik** | | Probability | | | Not so sure | |
| **Jim** | | Amp. of $\Psi$ | | | Cannot say | The wave is not normalized because it can go to infinity. (similar to not being localized) |

**Table 6.8: A summary of each student's response to Questions C and D**

### 6.5.4.2 A Traveling Wave Function Encountering a Potential Energy Step

In PH314 class, students are required to solve the potential energy step problem, as shown in Figure 6.9. Many students are able to solve the complicated mathematical solution to this problem using the Schrödinger equation; however, most of them did not understand the solution to this problem conceptually (Bao, 1998). In this section and the following two sections, we investigated students reasoning resources for applying the wave function in different conditions.

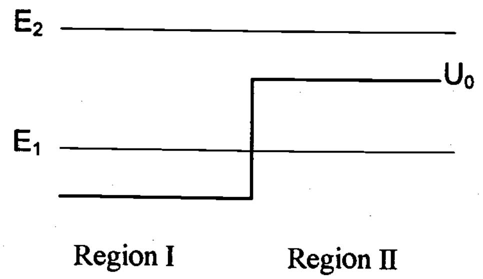



**Figure 6.9: A diagram for the potential energy step question.**

**Reasoning Resources for $E_2$ or $E > U_0$**

As shown in Table 6.9, almost all students understood that the wavelength of the wave function decreased as it moved to region II. Most of them explained the decrease in wavelength using the de Broglie wavelength relationship. Only one student could not predict or explain the change in wavelength. One student, Tom, thought that the wavelength was directly proportional to the kinetic energy (KE), so he drew the wavelength shorter when KE was less. His reasoning the wavelength in terms of KE was consistent throughout.

For determining the probability, students used a greater variety of reasoning resources. Here is the summary of reasoning resources that students used.

- **Using the facet—*a slow-moving object is more observable***

In terms of the amplitude and the probability, six students were still using the facet to explain why it was more likely to find electrons in Region II. Students were liable to use the facet because it was based on classical motion and it ***makes sense*** to students. Owen gave an explanation indicating that this facet did make sense to him:

> *"Mmmm, well, the wave function, let's see the square of the wave function is the probability of finding something which is related to the, to the velocity, so... Well it's not so much the velocity that it's related to, it's the likelihood of finding it. It's the...but because of the, the, what's it called, the, you can't know two things at the same time, for that short equation, well this is region anyway, umm because of that mutual exclusiveness, like when you have more velocity you're going to have less knowledge of where it's going to be. And less knowledge means like a more erratic wave function, so with higher amplitude I guess? That's the best I can come with. I could be entirely wrong, I'm not entirely sure."*

He did think in terms of the facet when he mentioned *"that mutual exclusiveness...of where it's going to be."* He said that *"when you have more velocity you're going to have less knowledge,"* which has the same underlined idea as the facet. Then he related the lack of knowledge to mean that the probability is higher, which indicated his thinking of the probability function as an error function. This idea is opposite from the probability that we intended students to learn.

- **The probability related to number of electrons**

One student thought that the probability related to the number of electrons, so when the energy of the particle was higher than the potential energy, all electrons could move through. Therefore the probability was the same throughout.

- **A reflection of the wave resulted in more probability in Region I.**

One student, Mac reasoned that the wave reflected back when it encountered the step, so there was more likely to find an electron in Region I.

- **The probability can be determined only from the normalized wave function or the wave function in a confined space.**

Two students who used this reasoning resource could not determine the probability because they were not sure how to apply it in this case. Nik gave a common response:

> *"Yeah. About the amplitude...amplitude represents the probability of finding particle in specific region. And since this region's not completely defined, I mean, we can't say anything about the probability of finding the electrons."*

<table>
<tr><th rowspan="2">Codename</th><th colspan="4">The electrons have energy higher than the potential energy step ($E > U_0$)</th></tr>
<tr><th>$\lambda$</th><th>Reasoning Resources</th><th>Amplitude</th><th>Reasoning Resources</th></tr>
<tr><td>Brian</td><td>longer</td><td>The KE decreases.</td><td>----</td><td>Cannot explain what the amplitude relates to. He drew the amplitude higher in Region I. But he could not relate it to anything.</td></tr>
<tr><td>Ethan</td><td>Same</td><td>Not so sure</td><td>decrease</td><td>$E > U_0$ gets smaller so the amplitude is smaller. Relate the amplitude to energy by using the Schrödinger eqn. The probability is high where the peaks are because Probability = $|amp|^2$</td></tr>
<tr><td>Tom</td><td>shorter</td><td>KE is less so $\lambda$ is less.</td><td rowspan="6">increase</td><td rowspan="4">Facet: a slow-moving object is more observable.<br>The probability increases because the electrons have less KE, so they are moving slower in Region II.</td></tr>
<tr><td>Bill</td><td rowspan="9">longer</td><td rowspan="9">The KE is less because the potential energy increases. The energy is inversely proportional to wavelength, so less KE makes the wavelength shorter.</td></tr>
<tr><td>Paul</td></tr>
<tr><td>Ken</td></tr>
<tr><td>Ben</td><td rowspan="2">Probability = $|amp|^2$, so the high prob. is where the peaks are. The prob. is higher in Region II because the electrons are moving slower, as a result of having less KE.</td></tr>
<tr><td>Owen</td></tr>
<tr><td>Sam</td><td>same</td><td>Because it's the same number of electrons and the same wave going thru, so the probability is the same.</td></tr>
<tr><td>Mac</td><td>decrease</td><td>Probability is bigger in Region I because the wave is reflected back when encountering the step.</td></tr>
<tr><td>Nik</td><td>Cannot say</td><td>Cannot say anything about the amplitude. The amplitude is related to the prob. when the electron is confined but in this case it is not.</td></tr>
<tr><td>Jim</td><td>Cannot say</td><td>The wave is not normalized, so the probability cannot be determined.</td></tr>
</table>

**Table 6.9: A summary of each student's response to the potential energy step ($E > U_0$)**

**Reasoning Resources for $E_1$ or $E < U_0$**

For the wave function of electrons having energy less than the potential energy, students could draw the wave function correctly in terms of the exponential decay when the wave hit the wall. They also gave acceptable reasoning that the electron sneaked in to this region because the wave function was decaying.

In terms of the probability, all students could draw the wave function correctly, but many of them could not explain why the probability was higher in Region I. However, it seemed intuitively obvious that the electrons could not exist inside the step. More than half of students could gave different reasoning resources when explaining the probability. Here are the identified reasoning resources that students used.

- **Using probability $= |\psi|^2$ or $|\text{amplitude}|^2$**

Students using this reasoning resource answered that the high probability was in where the peaks were. They applied this equation inappropriately. They seemed unaware that this equation can only apply for the normalized wave function or the wave function in a confined space.

- **Using the standing wave**

Two students used the standing wave as their reasoning resources to explain why the peaks had higher probability. Jim gave an explanation in terms of the standing wave:

*"This one you will have two waves, so you will have one wave going this direction – I'll try to draw it the same – and then once it hits this wall, then it is going to bounce back, and then you will get another wave traveling in this direction with the same wavelength. Then these will superimpose on each other and create another, a standing wave. This isn't a traveling wave, so the amplitude, so you will have high and low probabilities finding the particle here*

*and here [pointing to the peaks or anti-node]. You will have more probability here and here and here [at the peaks], and less probability here [the node]."*

Despite using different reasoning resources, they reached the same answer as the group using the probability = $|\psi|^2$ or $|\text{amplitude}|^2$.

- **Reflection of wave**

  Two students recognized that the wave function would be reflected back when encountering the step. However, they did not associate the superimposed wave to the standing wave idea. One student thought that the probability should be the same throughout region I, while another one could not say anything because the wave function was not confined.

**Epistemic Resources**

- **The overlap of epistemic resources**

  From student responses for determining the wave function when $E > U_0$, many students used the *slow-moving object, more observable* facet. Interestingly, students using this facet indicated that they processed this facet based on ***knowledge by making sense.*** Paul gave a typical response showing his epistemic resources:

> *"...the way I think of it, like a particle is moving slower, so it is easier to grab it. That's why it has a higher probability of finding it."*

Also, Bill gave an analogy comparing the motion of an electron going through the potential step with a motion of a car going through a speed bump:

> *"Because here, the energy of the electrons is considerably different from how it was here, because it's passed through the step. And so...we can actually say...we,* we can imagine *as if it's this electron which is going*

> *very fast and then it passes through a speed bump. And then it's slowing down before it wants to or doesn't want to gain, um, start moving fast again…so, it's slower at that point, as soon as it passes the potential energy barrier. And so it's, it's became, there is a more probable. There is more probability to find electron here [after the step] rather than here [before the step]."*

However, some students using the facet could not elicit any example or any analogy to explain the facet further. The use of a facet to answer or to explain questions is an epistemic resource that we call ***knowledge by faceting***. Students often use the facet or p-prim as a valid reasoning. Redish (2003) referred to the process of choosing to use related p-prims to reason as making common sense. Similarly, we also suggest that the making common sense process is not only choosing to use related p-prims but also related facets. Therefore, students used epistemic resources *knowledge by faceting* are also processed *knowledge by making sense*. Student responses to this question are summarized in Table 6.10.

| Codename | The electrons have energy lower than the potential energy step (E < $U_0$) | | | |
|---|---|---|---|---|
| | Wavelength when encountering the step | Reasoning Resources | High Probability in ________ | Reasoning Resources |
| Ethan | Exponential decay | The energy is less so it's only going to sneak thru. | Region I, the probability is the same through out the region. | Write down the solution and use probability = $\lvert\psi\rvert^2$ to reason that the probability is high at the peaks. |
| Paul | | | | Because the electrons cannot penetrate through the step. |
| Sam | | | | |
| Bill | | | | |
| Ken | | | | |
| Ben | | | Region I where the peaks are. | Find an electron where the peaks are because probability = $\lvert amp\rvert^2$ |
| Owen | | | | |
| Brian | | | | |
| Jim | | | | Reflect back and form a standing wave. The regions where the peaks are or where the anti-nodes of standing wave are located. |
| Tom | | | | |
| Mac | | | Region I, the probability is the same through out the region. | The wave reflected back when it encountered the potential energy step higher than its energy. Therefore, the reflected wave combined with the original wave give more probability. Did not mention about the standing wave. |
| Nik | | | Cannot really say | Only mention the reflection of wave when the wave function hit the wall. Did not mention about the standing wave. |

**Table 6.10: A summary of each student's response to the potential energy step (E < $U_0$)**

### 6.5.4.3 A Standing Wave Function of an Electron Trapped in a Potential Well

As shown in Figure 6.10, the potential infinite well problem might look similar to the potential energy step, but they can be explained with different wave functions. Answering the potential well problem conceptually requires an understanding of the normalization, standing waves, the probability density, etc. Student reasoning resources for the wave functions when the electron has an energy $E_1$ and $E_2$ are menioned below.

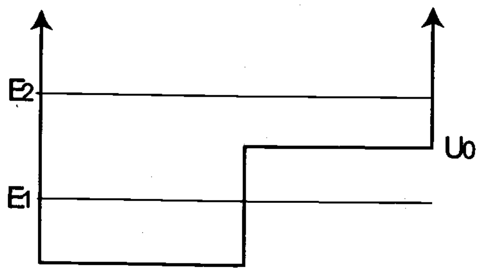



**Figure 6.10: The infinite potential energy well problem**

**Reasoning Resources for $E_2$ or $E > U_0$**

From Table 6.11, most students could use the de Broglie wavelength to draw the wavelength shorter when the wave ran into the potential step. Most students correctly predicted that the probability would be higher in region II, especially where the peaks were. However, they used different resources to reach this answer. Here are all types of student reasoning resources.

- **Using the *slower the motion, the more observable* facet**

From Table 6.11, eight students used this facet to explain the probability. However, only two out of nine students realized the condition of using this facet. Jim and Nik stated first that the wave function was localized so the probability is high where the electron was moving slower. Interestingly, the six students, who did not mention about the normalization of the wave function, were those using the facet to explain the potential energy step ($E > U_0$). Some students even stated that the wave function should be the same as the potential step wave function. This indicated their inability to distinguish between the potential energy step and potential well.

- **Using the standing wave**

Mac and Ethan used a reasoning resource in terms of a reflected wave to explain the amplitude and the probability. However, Mac applied it correctly that the wave reflected not only when reaching the potential $U_0$ but also when hitting the infinite wall, so the amplitude in region II was bigger because there were all waves reflected back when it hit the wall. In contrast, Ethan thought that the wave reflected only when it ran into the increased potential, so the amplitude in region I was bigger.

- **The probability related to numbers of electron**

Sam who used this reasoning resource in the potential step problem also applied the same resources to this situation. He saw the potential energy step and the potential energy step inside the infinite well to be the same system.

**Reasoning Resources for $E_1$ or $E < U_0$**

From Table 6.12, all students correctly identified that the wave should not penetrate through the infinite wall because the energy difference was too much. Similar to the previous finding, students could draw the wave function in region I for $E < U_0$, but most of them could not explain why. They could only give the probability is equal to the amplitude square. Four students could explain why the probability was more at the peaks of the wave. Here are two interesting resources that they used.

- **Using the standing wave**

Jim and Mac again reasoned by using the reasoning resources of the standing wave. They mentioned that the incoming wave traveled to the right and hit the potential energy step. As a result, the wave reflected back, combined with the incoming wave, and so resulted in the standing wave, where the anti-nodes indicated the high probability.

- **The probability depends on the level of discrete energy.**

Nik and Sam did not directly mention about the standing wave. However, they explained in terms of the reflected wave and the amplitude of the wave increased at the node. They made an interesting point that the probability depended on what discrete energy level the electron had. They associated the probability of a confined electron with the discrete energy levels.

| Codename | The electrons have energy higher than the step the infinite potential energy well ($E > U_0$) | | | |
|---|---|---|---|---|
| | **Wavelength After the step** | **Reasoning Resources** | **Higher prob. in ______** | **Reasoning Resources** |
| **Brian** | Longer | The KE decreases. | Region I | More space for the electron to exist [pointing to the space between $U_0$ and E in the diagram] |
| **Ethan** | Not sure | Cannot explain. | | Reflection: the wave is reflected back when it encountered the higher potential, so the probability is higher in Region I. |
| **Mac** | Longer | The KE decreases. | Region II | Reflection: The wave reflects from the wall so the amplitude is more where $U_0$. |
| **Sam** | | | Same in both regions | Refer back to his reason in the potential energy step where $E > U_0$. The probability is the same through out because the numbers of electron are the same. |
| **Tom** | Shorter | KE is less so $\lambda$ is less. | Region II at the peaks are | The same probability in Region II because the electron moves slower in that region. The KE is less so the velocity is less, so there is more probability of finding electron in region II. The probability is more at the peaks.<br>Note: Only Nik and Jim mentioned that the normalized wave or the confined wave function could now be used to determine the probability. |
| **Owen** | Longer | The KE is less because the potential energy increases. The energy is inversely proportional to wavelength, so less KE makes the wavelength shorter. | | |
| **Ken** | | | | |
| **Ben** | | | | |
| **Bill** | | | | |
| **Paul** | | | | |
| **Nik** | | | | |
| **Jim** | | | | |

**Table 6.11: A summary of each student's response to the potential infinite well ($E > U_0$)**

<table>
<tr><th rowspan="2">Codename</th><th colspan="4">The electrons have energy lower than the step inside the infinite potential energy well ($E < U_0$)</th></tr>
<tr><th>Wave near the well</th><th>Reasoning Resources</th><th>Amplitude/ probability</th><th>Reasoning Resources</th></tr>
<tr><td>Ben</td><td rowspan="12">No decay</td><td rowspan="12">The energy well is too high so the electron cannot sneak through.</td><td rowspan="12">Region I where the peaks are</td><td rowspan="8">Probability = $|amp|^2$. High probability where the peaks are.</td></tr>
<tr><td>Ethan</td></tr>
<tr><td>Bill</td></tr>
<tr><td>Ken</td></tr>
<tr><td>Brian</td></tr>
<tr><td>Paul</td></tr>
<tr><td>Owen</td></tr>
<tr><td>Tom</td></tr>
<tr><td>Nik</td><td>Depending on the energy. The peak presents the most probable region to find an electron.</td></tr>
<tr><td>Sam</td><td>The probability of locating the electron depending on what energy that it has because the discrete energy will generate different wave resulting in different probability.</td></tr>
<tr><td>Mac</td><td rowspan="2">Standing wave: The high probability is in regions where the peaks or the anti-nodes are. Use the analogy of the standing wave but did not relate the standing wave with the energy level.</td></tr>
<tr><td>Jim</td></tr>
</table>

**Table 6.12: A summary of each student's response to the potential infinite well ($E < U_0$)**

#### 6.5.4.4 A Traveling Wave Function When Encountering a Step Barrier

This problem aimed to investigate students' understanding the traveling wave and how they related the probability to the traveling wave. We used the diagram similar to Figure 6.11 to interview students. We report on student reasoning resources used in explaining the wave function when the energy is higher and lower than the potential energy barrier.

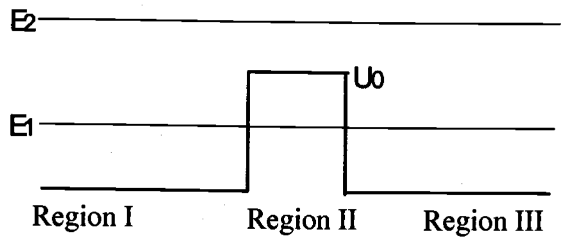



**Figure 6.11: The potential energy barrier problem**

**Reasoning resources when E <$U_0$**

- **Relate the amplitude to the energy**

Many students related the amplitude to the energy, as shown in Table 6.13. They thought when the wave tunneled through the barrier, it lost some of its energy. This indicated their lack of understanding for the uncertainty principle in terms of $\Delta E \Delta t$. Owen gave a common response:

> *"The amplitude indicated that it lost energy in tunneling through this, so yeah."*

However, two students found conflicts in their reasoning. Paul found the amplitude = energy conflicted with the *slow-moving is more observable* facet. When he thought that the energy decreased, he also activated the facet and got confused. Later, he changed his answer to that the amplitude should remain the same. Brian also conflicted the amplitude = energy reasoning resources with the amplitude = numbers of electron. He first thought that the amplitude should decrease because it lost energy. Later, when he answered and mentioned about the number of electrons, he started to click and questioned himself that the amplitude should represent the number of electrons. Finally, he changed the amplitude to be decreased in region III because the number of electrons in that region was less than in region I.

- **The amplitude represents the number of electrons**

Students using this reasoning resource explained first in terms of the wave being reflected back, and then the number of electrons in region III would be less. Jim gave a typical explanation why the amplitude in region III decreased:

> *"That is because the particles are coming in here and most of them are reflected off, but some can tunnel through. The amplitude decreases, because only some of the particles come out the side."*

Students seemed to think in terms of the wave behavior first, and then associated the wave with the number of electrons.

**Reasoning resources when E >$U_0$**

- **Using the facet**

Again students used the facet as their reasoning resources to claim that the probability was higher in Region II because the electrons were moving slower. The summary of students' responses is presented in Table 6.14. Ben gave a typical response:

> **Interviewer:** Mmmm...okay. So, so it's gonna' be greater why? Like this amplitude [in region II] should be bigger than the other one [region I and III]?
> **Ben:** Yeah.
> **Interviewer:** Why?
> **Ben:** Umm...cuz the...electron is gonna' be going slower, so it'll be easier to find in that area...

However, a few students activated conflicted reasoning resources at the same time, and then they became confused. Finally, they picked the reasoning resources that more dominated in their thinking processes than another one. For example, Ethan had conflicted with using two different reasoning resources:

> **Interviewer:** So, um, the amplitude is getting smaller. Why?
> **Ethan:** Um...yeah, the difference between the potential energy and the total energy.
> **Interviewer:** So how bout, let's talk about the speed. Where, where, where you think, um, the particle will moving faster?
> **Ethan:** Um...let's see here did I...equals k plus u, so if you got your e minus u equals k, so when that's the greatest...so it'd be moving faster here, and here, than it would be in there.
> **Interviewer:** Okay. So, isn't that when subject moving, um, slower is easier to find it?
> **Ethan:** [silence] Yeah... Uh oh. Right. So that way it would mean that here, or wait wait, because yeah that would mean that it would be moving faster here, because you'd want it, it would be easier to find when it's moving slower.[laughs]... [silence]...okay.
> **Interviewer:** So...
> **Ethan:** So yeah I think, I think that this isn't right, just maybe because of this u, u-naught thing or somethin'. That maybe that's not right and that, cuz it does seem, yeah you would...if it's moving slower you'd have a higher probability of finding it in that region. So...I guess I'm gonna' reverse that and say that it's moving faster in, in the middle here.

Finally he chose the classical probability facet because it made more sense to him at the time of interviewing.

- **Using the wave reflection**

As presented in Table 6.14, only two students answered in terms of the reflection of wave. Then they associated the amplitude with how much the wave was left after the reflection. As a result, region I should have the biggest amplitude because it had more waves in that area compared to region II and III. Then region II had the second biggest amplitude because it had the wave reflected from region III. Jim gave a common response:

**Interviewer:** Why is the amplitude decreasing?

**Jim:** The amplitude decreases because waves are being reflected off here [between region I and II] and from there [between II and III]. So I guess the wave is reflected, so the wave amplitude decreases. I think the wavelength is smaller in this portion because it has less kinetic energy.

| Codename | The electrons have energy lower than the potential energy barrier ($E < U_0$) | | | |
|---|---|---|---|---|
| | Compare $\lambda_1$ and $\lambda_3$ | Reasoning Resources | Compare prob. b/w I and III | Reasoning Resources |
| Brian | same | Same kinetic energy before and after the barrier. | | The amplitude decreases because it lost energy. Later having a conflict in reasoning that the numbers of electron indicated the amplitude. |
| Ken | | | | Cannot relate the amplitude to anything. |
| Paul | | | | There is some energy lost during the tunneling. The amplitude is more because it moving slower in Region III. Later, change his answer that the probability should remain the same, there is no energy lost. |
| Owen | | | Prob in III > Prob in I | The amplitude decreases because it lost energy through the tunneling process. From using probility = $\|amp\|^2$, they thought that the probability is higher in region III where the amplitude is higher. |
| Ethan | longer | KE is smaller, losing energy through the tunneling. | | |
| Sam | | | | |
| Bill | same | The KE is the same before and after the barrier. | High probability in region I | The amplitude relates to the numbers of electron, so there are more likely to find electrons before the barrier. The wave reflected back, so numbers of electrons could not go through the barrier. |
| Ben | | | | |
| Mac | | | | |
| Tom | | | | |
| Jim | | | | |
| Nik | | | | |

**Table 6.13: A summary of each student's response to the potential energy barrier ($E < U_0$)**

| Codename | The electrons have energy higher than the potential energy barrier ($E > U_0$) | | | |
|---|---|---|---|---|
| | **Compare $\lambda 1, \lambda 2, \lambda 3$** | **Reasoning Resources** | **Compare prob. in region I / II/ III** | **Reasoning Resources** |
| **Ben** | longer | KE decreases<br>The kinetic energy decreases thru the barrier so $\lambda$ is longer there.<br><br>Note: Tom thought that KE and the wavelength are directly proportional. | II > I=III | Same amplitudes in I and III. The amplitude is greater in II because the electron is moving slower in region II. Paul and Tom said that the wave function should be the same as the potential step case.<br><br>Note: Ken answered that the electrons are moving slower, but amplitude decreases because he recalled that the amplitude should decrease, so he relates it to the KE. |
| **Bill** | | | | |
| **Paul** | | | | |
| **Owen** | | | | |
| **Sam** | | | | |
| **Ken** | | | | |
| **Tom** | Shorter | | | |
| **Ethan** | Same | Not so sure | First → II < I=III<br>Finally→ II > I=III | First, using amp. = KE. Then he used the facet and got confused because the amp. in II should be more from the facet. So, he went with the facet. |
| **Brian** | longer | The KE decreases. | First→ II< I=III<br>Finally→ I=II=III | First, drawing the smaller amplitude in II. Then changed it to be the same in all regions because of same numbers of electron. |
| **Nik** | | | Cannot say | Cannot say anything about the amplitude because it is infinite. She thought about the numbers of electron will go thru, but not so sure to relate that to the amplitude. |
| **Mac** | | | I>II>III | Reflection. The wave got reflected when it moves from region I to II and II to III. So the numbers of electron decreases and the amplitude decreases. |
| **Jim** | | | | |

**Table 6.14: A summary of each student's response to the potential energy barrier ($E > U_0$)**

# Chapter 7

# Discussion

In this study, we conducted three separated but related investigations in order to observe students' models for understanding probability. In this chapter, we combine and summarize the overall findings from these investigations to describe the student thinking model. Finally, we propose possible hypotheses and recommendations that our findings suggested.

## 7.1 Randomly Distributed Expectancy Resource

First, we investigated student pre-knowledge of probability. We found that most students had the representativeness and the outcome approach misconceptions. Students were more likely to intuitively think that a random event should give a random outcome (Tversky & Kahneman, 1974; Kahneman & Tversky, 1982). For example, when asking students what patterns of tossing five coins are more likely to occur, most students will choose the randomly distributed pattern. Students explained that they chose that pattern because it was the most random one. When we changed the question's format slightly by asking for the least likely event, more students chose the randomly distributed pattern. Some of these students were those who answered correctly and used a statistical explanation when asking for the most likely event. Also, students who had this misconception could not explain further why a random event should have a random outcome. From a cognitive perspective, this finding suggests that students used the "a random event has a random outcome" reasoning at a fundamental level, which is easily activated and irreducible. Students using this reasoning as being irreducible suggested that they possibly operated this "a random event has a random outcome" reasoning at a reasoning resource level according to Redish's thinking model.

> **Hypothesis 1:** When students are asked to predict an outcome of a random event, they tend to use a *randomly distributed expectancy* resource.

This resource might be so strongly associated with a resource for recognizing a random event that students used it intuitively. This finding is in agreement with the representativeness and the outcome approach misconceptions (Tversky & Kahneman, 1974; Kahneman & Tversky, 1982). However, we avoid using the term "misconception" because it is ambiguous (Redish, 2003; Wittmann, 2002) and does not give much insight into how students operate this idea. In contrast, we use the term "reasoning resource," which gives us more ground to understand student reasoning processes at a cognitive level.

## 7.2 The Classical Probability Reasoning Resource

In Chapter 4, we not only investigated students' mathematical misconception but also classical probability, which is probability in the classical mechanics content. We found that students performed exceptionally well in this area. From the data analysis, most students often provided a reason that in a region where an object is moving slower and spending more time, there is a higher probability of finding it compared to a region where the object is moving faster. We define this reasoning as *classical probability*. This reasoning is acceptable to use in the classical probability, but in modern physics, it can be used to describe the amplitude of the wave function only under certain conditions. From the findings in Chapter 5, we found in the preliminary interview that students could not distinguish the wave functions for the potential energy step and the infinite potential energy well. In the infinite potential energy well, a particle is confined or localized, so the wave function is a standing wave. In the potential energy step, the wave function is a traveling wave and describes a beam of particles, so it cannot be described with the same reasoning as the one in the potential step. Also, we noticed the significance of this problem during the Fall 03 classroom observation when one student asked a question regarding the change in amplitude of the wave function when it encountered the potential step. He explained his understanding that the amplitude should be bigger using the classical probability reasoning.

We took that particular observation seriously and used a more profound framework to analyze the final interview during Spring 04. We employed Redish's theoretical framework (2003), a two-level system to describe students' thinking model. We found that students perhaps operated a *classical probability* reason at a fundamental level, which we identify as a reasoning resource or a facet (as mentioned in Chapter 6). They automatically applied this reasoning resource when they recognized that the kinetic energy decreased; the speed then decreased, so the particle was more observable. As a result, they drew increasing amplitude and explained that the probability increased. Bao (1998) found a similar result in his study, but he identified the finding in a more general term as student difficulties of understanding the bound state.

> **Hypothesis 2:** Most students are likely to use the *classical probability* reasoning resource to explain the amplitude of the wave function because they perhaps "see" the problem in terms of the decreasing speed or the decreasing kinetic energy.

Based on Redish's framework, he mentioned that when presented with complicated information, we direct our attention only to a certain piece of information. This idea is similar to the readout strategies, an element in the coordination class (diSessa and Sherin, 1993). Redish (2003) defined the readout strategies in his own terms as a resource for an interpretation of sensory input. Based on this theoretical framework, we can speculate that when students were presented with the potential energy step diagram, most probably saw, or selected their attention to, the increasing potential energy which resulted in decreasing kinetic energy. When the attention was on the kinetic energy, the classical probability reasoning resource was activated in order to predict the probability.

Only some students seemed to recognize the limitation of the classical probability reasoning resource. In Chapter 6, a few students did not use the classical probability reasoning resource to explain the amplitude change in the potential step. They not only stated that the probability could not be determined because the wave function was not normalized or bounded, but also associated either the localization or the normalization knowledge to the limit of using this reasoning resource. They seemed to use this reasoning resource just when they realized that the wave function was normalized or localized such as in the infinite potential energy well. Therefore, when presented with the potential energy step problem, these students not only saw the change in kinetic energy but also the unbounded potential energy. From the finding in Chapter 6, students tended to correctly apply the classical probability resource to explain the amplitude when they saw the infinite potential energy well. This appeared to guide their cognitive activations of concepts like normalization and localization.

**Hypothesis 3:** Students might correctly use the classical probability resource when they "see" the potential problem in both the bounded condition and the kinetic energy change.

This hypothesis suggests that to help students understand the wave function conceptually, we need to help them understand the condition of using the wave function. From the quantum postulate, the wave function can only be used to determine the probability when it is normalized. The concept of normalization might be too complicated for students to completely comprehend at this level. However, we can help them recognize a situation in which the normalization can be applied. The situation associated with the normalization is the localization or the bound potential energy function. Therefore, when teaching about the potential energy problem, we need to emphasize the bounded/unbounded potential energy function and how it relates to the wave function.

## 7.3 Two Associated Patterns for the Amplitude of the Wave Function

In Chapter 6, we used four different potential energy functions to investigate student models of probability. The finding, especially in the case of the potential energy barrier, suggests that students appeared to use two different patterns of association—an object pattern and a wave pattern to describe the amplitude.

Most students tended to use the object pattern of association when reasoning about the wave function of the potential barriers, and they seemed to use two conflicted reasoning resources to explain the change in amplitude. When the particle energy was lower than the potential energy, most students explained that the amplitude in region I should be higher than in regions II and III because most of the particles were "bounced" back. Therefore, there were fewer particles that could tunnel through the potential barrier. When the particle energy was higher than the potential energy, most students answered that the probability of finding particles was the same in all three regions because all particles were able to move pass the barrier. Figure 7.1 presents a possible mental model when students used the object pattern to explain their reasoning.

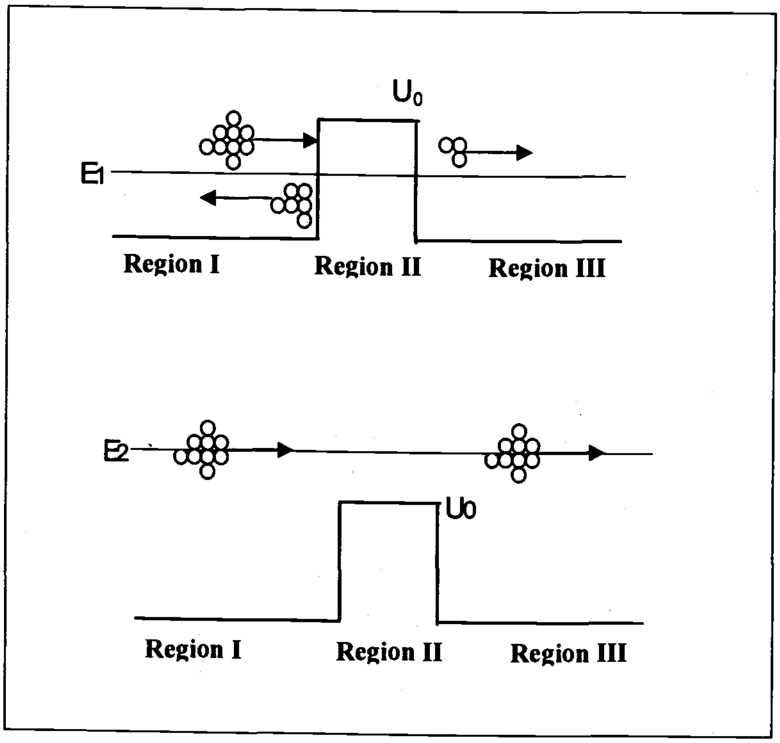



**Figure 7.1: A possible student mental model when using an object pattern of association.**

In contrast, a few students used the wave pattern of association to explain the probability in the potential barrier problem. They explained that when the particle energy was lower than the potential energy, the probability was highest in region I because the wave "reflected" back and only some portion could tunnel through the barrier. Also, students' reasoning when the particle energy was higher than the potential energy indicated their interpretations of the wave function behaviors in terms of the wave pattern. Students possibly using the wave pattern answered that the wave reflected back when it went from region I to II and from II to III, so there was more wave amplitude in region I. As a result, the probability was highest in region I. Their reasoning emphasized the wave behavior that the wave reflects when moving from one region to another.

**Hypothesis 4:** Students interpret behaviors of the wave function into two different patterns of association—an object pattern and a wave pattern.

Wittmann (2002) found similar results that students employed the *object* coordination class to describe wave phenomena. In other words, students used behaviors of an object to reason about the wave phenomena. Therefore, our findings and Wittmann's findings made a strong case that students are more likely to interpret the wave function as an object in stead of a wave.

We already presented the possible hypotheses of students' probability thinking model that were suggested from our data analysis. We recognize that what physics education researchers have identified as students' difficulties can be described and understood with a thinking model. Then, the next question is how we can help students to construct an appropriate knowledge pattern and be aware of the *classical probability* resource. We answer this question in the next section, as well as explain the significance of epistemic resources toward learning.

## 7.4 Epistemic Resources

From the results in Chapter 6, we identified three epistemic resources *knowledge by authority*, *knowledge by making sense*, and *knowledge by faceting*. Students constructing their knowledge by authority are more likely to accept the information from an instructor or a textbook without understanding in their own terms. From the interview results, students expressing the source of their knowledge by authority often did not present an association of their knowledge, or could not explain the idea in their own words. For example, students viewing the knowledge by authority with the wave-particle behavior could not explain the wave behavior, so they just recalled the definition that they heard in the lecture or read from the textbook.

In contrast, students viewing the knowledge by making sense could explain how and why the electron behaves as a wave, and some students could come up with an analogy explaining the wave behavior. When a facet was used as the source for their knowledge, we call this epistemic resource, *knowledge by faceting*. This is similar to knowledge by p-priming that Redish (2003) defined in his paper, except that we emphasize a facet as the source instead of a p-prim. The reason is that we found one facet, the classical probability facet, in this study. Therefore, we do not want to use the term knowledge by p-prim to describe a facet as a source of knowledge.

The knowledge by faceting and the knowledge by making sense might appear to be the same for some, but there is a difference. When students experience a particular problem for the first time and they are trying to make sense of it, the reasoning process or the knowledge that they revealed came from knowledge by making sense. If these students are presented with similar problem again, and the same knowledge pattern is activated, they find themselves using the same reasoning resources or the same facet. Then, their pattern of association for that facet is strengthened. When they see a problem as similar to the problem that they used the facet to explain before, they are going to use the facet as their reasoning resource, almost automatically. We call the process by which students 'know' their answer based on a facet as *knowledge by faceting*.

## 7.5 Implications for Teaching

We have identified the reasoning resources and the pattern of association that possibly explained why students have difficulties understanding probability in modern physics. Also, we have discussed different epistemic resources that influence students' knowledge construction. To help students construct appropriate reasoning resources, we not only have to emphasize the right condition for using particular reasoning resources, but also create a classroom that promotes acquiring knowledge by masking sense.

First to emphasize the right condition for using particular reasoning resources, we have to be monitor student reasoning progresses. This is hard to do in a large class. However, we have a technology that can help us adapt the idea. In PH314 at OSU, we have been using Peer Instruction and an electronic response system to engage students in making sense of presented materials. We can design questions that create conflicts in students' reasoning or activate different reasoning resources at the same time. When students have conflicts in their thinking, it might bring their attention to their inappropriate use of certain reasoning resources. To help students construct an appropriate pattern to reason about the wave function, we suggest providing students with a visualization of the wave behaviors. Static pictures of the wave function encountering a potential energy step or barriers as in the modern physics textbook do not help students to overcome the object pattern of association. Using some animation or visualization should help students see the reflection of wave better and construct a better mental model associated with the wave function.

Second, students also have to be in the *knowledge by making sense* mode to be able to reflect on their reasoning processes. We need to provide an instructional environment to promote *knowledge by making sense*. Thus, we need to encourage a class discussion, and also assess students by asking them to explain their reasoning not just show their calculations. This suggests the need for conceptual test questions in addition to numerical problems.

## 7.6 Suggestion for Future Research

The findings in this study are somewhat limited because of the small number of participants. Moreover in education, we are dealing with a complex system, students' cognitive processes, so in order to confirm our findings; we suggest a repetition of research on a similar setting. Therefore, a comparable study should be conducted in order to confirm the results. The findings from this study can be used to design a test item such as a multiple-choice multiple-response problem which can then be administered to a larger population. Also, the interview questions from this study can be modified to probe students' understanding better and more thoroughly.

In addition, this study aimed to observe students' understanding of probability, which is a broad concept and associated with many other concepts. We only observed some basic concepts associating with probability at a superficial level. Our findings seemed to indicate that understanding mathematical probability does not help students understand probability in modern physics better. Therefore, more detailed investigations could be done separately in terms of students' understanding of either mathematical probability or modern physics probability. Moreover, we suggest that more studies should be done to explore students' understanding of normalization or localization, which we observed to strongly relate to better students' probability understanding. Finally, we hope that our findings would help expand PER in modern physics and guide the future research in this topic.

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# Appendix A: The Diagnostic Test

## PH314: The Diagnostic Test

### Direction

Please circle your answer and *write two or three sentences explaining how you arrive at the answer.*

---

1. Which of the following is the most likely result of five flips of a fair coin?

   a) HHHTT

   b) THHTH

   c) THTTT

   d) HTHTH

   e) None of the above

Explain why you chose this answer.

2. Which of the following is the least likely result of six flips of a fair coin?

   a) HHHHHT

   b) THHTTH

   c) THTTHT

   d) HTHTHT

   e) None of the above

Explain why you chose this answer.

3. A ball is rolling at speed v on a flat frictionless horizontal surface in the positive x direction. When the ball reaches x = +L, it collides with a wall and rebounds with no loss at speed. Moving now in the opposite direction, it strikes another wall at x = -L and again rebounds with the same speed. It continues to bounce back and forth between the walls at x = +L and x = -L always with speed v. What is the probability to find the ball moving in either direction between x = -L/2 and x = +L/2 ?

a) 25%  b) 50%c) greater than 50%  d) less than 50%

Explain why you chose this answer.

4. From a batch of 3000 light bulbs, 100 were selected at random and tested. If 5 of the light bulbs in the sample were found to be defective, how many defective light bulbs would be expected in the entire batch?

a) 50  b) 100  c) 150  d) 200  e) None of the above

Explain why or show how you arrived at this answer.

5. Suppose you tossed a coin five times and get five heads in a row. The probability of getting a head on the next toss is __________.

a) greater than 50%  b) less than 50%  c) equal to 50%

Explain why you chose this answer.

6. Suppose the student average SAT score at Enormous State University is 1000. You friend is in a writing class of 10 students. Her score was 1100. What is the most probable average of the other 9 students?

a) 850 b) 900 c) 989 d) 1000 e) None of the above

Explain why or show how you arrived at this answer.

7. A professor teaches two physics class. The morning class has 25 students and their mean on the first test was 82. The evening class has 15 students and their mean on the same test was 74. What is the mean on this test if the professor combines the scores for both classes?

a) 76 b) 78 c) 79 d) 80 e) None of the above

Explain why or show how you arrived at this answer.

8. If you roll two die at the same time, what is the probability that the sum of the numbers rolled is less than 9?

a) 16/36 b) 20/36 c) 26/36 d) 30/36 e) None of the above

Explain why or show how you arrived at this answer.

9. The likelihood of getting "heads" at least twice when tossing three coins is ______ the likelihood of getting at least 200 "heads" when tossing 300 coins.

a) less than b) equal to c) greater than

Explain why you chose this answer.

10. When choosing a committee composed of two members from among 10 candidates the number of possibilities is __________ the number of possibilities when choosing a committee of 8 members from among 10 candidates.

a) less than  b) equal to  c) greater than

Explain why you chose this answer.

11. In a family of six children, which birth order of children is less likely to occur?

a) BBBBGB
b) BBBGGG
c) BGGBGB
d) GGGBBB
e) None of the above

Explain why you chose this answer.

12. In a family of six children, which birth order of children is more likely?

a) BBBBGB
b) BBBGGG
c) BGGBGB
d) GGGBBB
e) None of the above

Explain why you chose this answer.

13. A block of mass m is attached to an ideal spring and oscillates on a frictionless horizontal surface. The amplitude of the oscillation is A, so that the block oscillates between x = +A and x = -A. What is the probability to find the block moving in either direction between x = -A/2 and x = +A/2?

a) 25% b) 50%c) greater than 50% d) less than 50%

Explain why you chose this answer.

14. Ronni flipped a coin three times and in all cases heads came up. Ronni intends to the coin again. Is the chance of getting heads the fourth time ____________the chances of getting tails?

a) less than b) equal to c) greater than

Explain why you chose this answer.

15. Suppose a word is randomly picked from an English Dictionary. Is it more likely that the word begins with the letter K, or that K is its third letter?

a) The word beginning with the letter K is more likely picked.

b) The word having K in its third letter is more likely picked.

c) Both are likely equal to be picked.

Explain why you chose this answer.

16. Assume that the chance of having a boy or girl baby is the same. Over the course of a year, in which type of hospital would you expect there to be more days on which at least 60% of the babies born were boys?

a) In a large hospital

b) In a small hospital

C) It makes no difference

Explain why you chose this answer.

17. Two bags have black and white counters.

Bag J: 3 black and 1 white   Bag K: 6 black and 2 white

Which bag gives the better chance of picking a black counter?

a) Same chance
b) Bag J
c) Bag K
d) Don't know

Explain why you chose this answer.

18. A boy ties a ball to a string and whirls it at a constant angular speed in a horizontal circle above his head. Let the position of the ball be represented by the angular coordinate $\phi$, with the direction of the boy's nose indicating $\phi$=0. What is the probability to find the ball between $\phi$ = 45º and $\phi$ = 90º ?

a) 25%   b) greater than 25%   c) less than 25%

Explain why you chose this answer.

19. A steel ball is dropped from a height y = +H. It strikes a horizontal steel surface at y = 0 and bounces back to height y = +H. It continues to bounce, reaching height y = +H after every bounce. What is the probability to find the ball anywhere between y = +H/2 and y = +H?

a) 25% b) 50% c) greater than 50% d) less than 50%

Explain why you chose this answer.

20. One has to choose six numbers from a total of forty. John has chosen 1, 2, 3, 4, 5, 6. Ruth has chosen 39, 1, 17, 33, 8, 27. Who has a greater chance of winning?

a) John b) Ruth c) Both have equal chance of winning.

Explain why you chose this answer.

# Appendix B: Questionnaire (Fall 2003 Version)

Name____________________

**PH314 Questionnaire:** In the following statements, please indicate whether you agree or disagree with each statement. *If you disagree, please explain your reasons.*

---

1. An electron is trapped in a region of space of a finite potential energy well. In its ground state, the electron is most likely found near the edges of the well because its velocity is almost zero there.
   **Agree** **Disagree**
   Reasons:

2. An atom has a similar structure as the solar system (planets that orbit the sun).
   **Agree** **Disagree**
   Reasons:

3. Individual electrons are fired towards a very narrow slit. On the other side is a photographic plate. What happens is that the electrons strike the plate one by one and gradually build up a diffraction pattern.
   **Agree** **Disagree**
   Reasons:

4. In the double-slit experiment, the wave function determines the distribution of electrons on the screen.
   **Agree** **Disagree**
   Reasons:

5. Electrons move about the nucleus in definite orbits with a high velocity.
   **Agree** **Disagree**
   Reasons:

6. The atom is stable due to a 'balance' between an attractive force and the movement of the electron.

   **Agree** **Disagree**

   Reasons:

7. In passing through a small gap, electrons continue to move along a straight line.

   **Agree** **Disagree**

   Reasons:

8. It is possible for a single electron to constructively and destructively interfere with itself.

   **Agree** **Disagree**

   Reasons:

9. Electrons move randomly around the nucleus within a certain region or at a certain distance.

   **Agree** **Disagree**

   Reasons:

10. Coulomb's law, electromagnetism and Newtonian mechanics cannot explain why atoms are stable.

    **Agree** **Disagree**

    Reasons:

11. If we know an electron's instantaneous position and velocity, then we can calculate and predict exactly how it will behave as the time goes by.

    **Agree** **Disagree**

    Reasons:

12. If an electron is trapped between two infinite walls, a slow-moving electron has a greater probability density than a fast-moving electron.

    **Agree** **Disagree**

    Reasons:

13. Whether one labels an electron a ‘particle’ or ‘wave’ depend on the particular experiment being carried out.

**Agree** **Disagree**

Reasons:

14. The photon is a ‘lump’ of energy that is transferred to or from the electromagnetic field.

**Agree** **Disagree**

Reasons:

15. If we send a free electron through a slit, then the uncertainty in position is reduced or the electron is more localized.

**Agree** **Disagree**

Reasons:

16. An electron is trapped in a region of space between two walls. If the walls are moved further apart, then the average kinetic energy of the electron will decrease.

**Agree** **Disagree**

Reasons:

17. If we knew the initial conditions precisely enough, we could predict where each electron will be found on the screen in a double-slit experiment.

**Agree** **Disagree**

Reasons:

# Appendix C: Questionnaire (Spring 2004 Version)

Name_______________________

**PH314 Questionnaire**: In the following statements, please indicate whether you agree or disagree with each statement. Please explain your Reasons.

| | | |
|---|---|---|
| 1. A pure sine wave, which extends from $-\infty$ to $+\infty$, represents a free electron moving at a certain speed $v$, so the highest probability of finding it is where the peak of the sine wave occurs.<br>**Reasons:** | **Agree** | **Disagree** |
| 2. If we send a free electron through a slit, then the uncertainty in position is reduced or the electron is more localized.<br>**Reasons:** | **Agree** | **Disagree** |
| 3. If we knew the initial conditions precisely enough, we could predict where each electron will be found on the screen in a double-slit experiment.<br>**Reasons:** | **Agree** | **Disagree** |
| 4. Whether one labels an electron a 'particle' or 'wave' depend on the particular experiment being carried out.<br>**Reasons:** | **Agree** | **Disagree** |
| 5. The wave function of a particle of energy $E < U_0$ encounters a barrier of height $U_0$ and thickness $L$. The amplitude of the wave function beyond the barrier is much less than the original amplitude because the wave loses some energy when it tunnels through the barrier.<br>**Reasons:** | **Agree** | **Disagree** |

| | | |
|---|---|---|
| 6. In the double-slit experiment, if the kinetic energy of the electrons in the beam were decreased by half, then the probability of finding electrons in the center maximum will decrease by a factor of four because the probability depends on the energy squared.<br>**Reasons:** | **Agree** | **Disagree** |
| 7. Individual electrons are fired towards a very narrow slit. On the other side is a photographic plate. What happens is that the electrons strike the plate one by one and gradually build up a diffraction pattern.<br>**Reasons:** | **Agree** | **Disagree** |
| 8. Because of its infinite extent, the pure sine wave is of no use in localizing particles.<br>**Reasons:** | **Agree** | **Disagree** |
| 9. An electron is trapped in a region of space of a finite potential energy well. In its ground state, the electron is most likely found near the edges of the well because its velocity is almost zero there.<br>**Reasons:** | **Agree** | **Disagree** |
| 10. Suppose the wave function gives a 50% probability of finding electrons in the central maximum of a double slit experiment. Firing electrons one at a time at the apparatus, we find that the first 10 electrons all land in the central maximum. The $11^{th}$ electron will most likely land <u>outside</u> the central maximum.<br>**Reasons:** | **Agree** | **Disagree** |

| | | |
|---|---|---|
| 11. A beam of electrons of energy $E$ is moving in a region where $U = 0$ from $x = -\infty$ toward $x = 0$. The particles encounter a step of height $U_0$ where $E > U_0$. The step extends from $x = 0$ to $x = +\infty$. The amplitude of the electrons' wave function increases as the wave crosses the step because the electrons' kinetic energy decreases so the probability of finding the electrons in this region is higher.<br>**Reasons:** | **Agree** | **Disagree** |
| 12. In passing through a small gap, electrons continue to move along the same straight line in the same direction.<br>**Reasons:** | **Agree** | **Disagree** |
| 13. Interference experiments can be done by passing one electron at a time through a double-slit apparatus, so it is possible for a single electron to constructively or destructively interfere with itself.<br>**Reasons:** | **Agree** | **Disagree** |
| 14. If an electron is trapped between two infinite walls, a slow-moving electron has a greater average probability density than a fast-moving electron.<br>**Reasons:** | **Agree** | **Disagree** |
| 15. In a double-slit experiment, the interference pattern relates to the probability density of finding electrons on the screen.<br>**Reasons:** | **Agree** | **Disagree** |

| | | |
|---|---|---|
| 16. If we know an electron's instantaneous position and velocity, then we can calculate and predict exactly how it will behave as time goes by.<br>**Reasons:** | **Agree** | **Disagree** |
| 17. In the double-slit experiment, the wave function determines the distribution of electrons on the screen.<br>**Reasons:** | **Agree** | **Disagree** |
| 18. An electron is trapped in a region of space between two walls. If the walls are moved further apart, then the average kinetic energy of the electron will decrease.<br>**Reasons:** | **Agree** | **Disagree** |
| 19. Probability in the step and barrier tunneling problems relates to numbers of electrons, while probability in the potential well problem relates to how fast one electron is moving.<br>**Reasons:** | **Agree** | **Disagree** |

# Appendix D: Interview (I) Worksheet

**Direction: Please answer the following questions and provide explanations as best as you can.**

**Part I: General Concepts about an electron and its behaviors**

A. What is an electron?

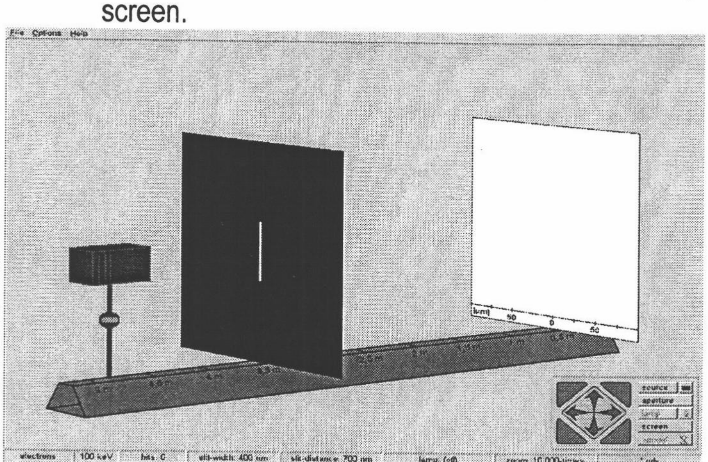
B. Please explain the wave-particle behavior of electron.

C. Please draw the probability wave of a free electron moving at a speed v.

D. Please draw the probability wave of another free electron moving at a speed 2v.

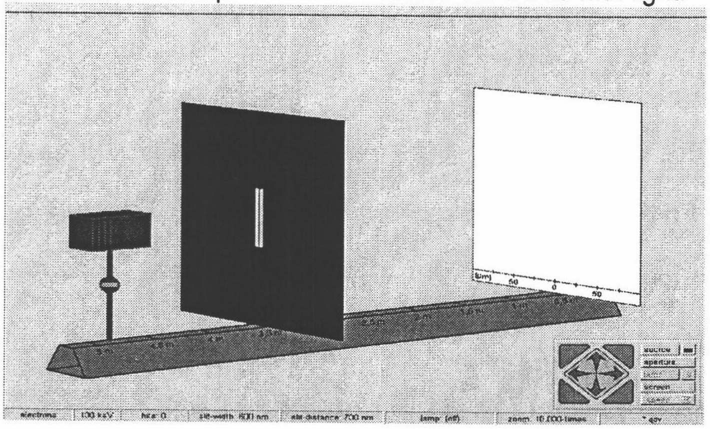
**Part 2: General concepts about the single slit and the double-slit experiment.**

A. Please explain how the single slit works using the language of the UNCERTAINTY relationship, and also please draw the pattern that you should observe on the screen.

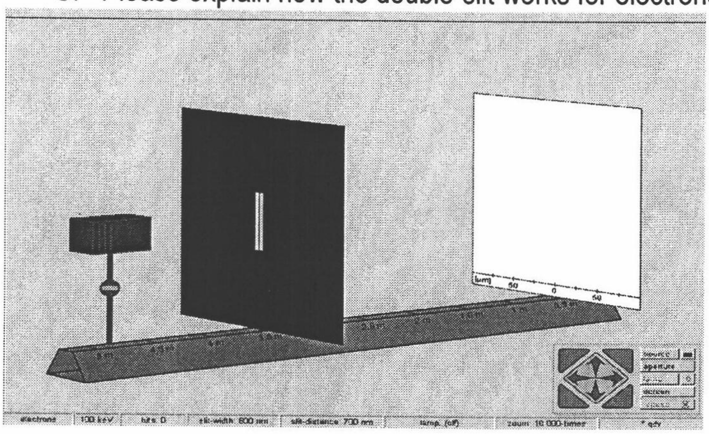


(source: http://www.physik.uni-muenchen.de/didaktik/Computer/Doppelspalt/dslit.html)

B. Please explain how the double-slit works with light.

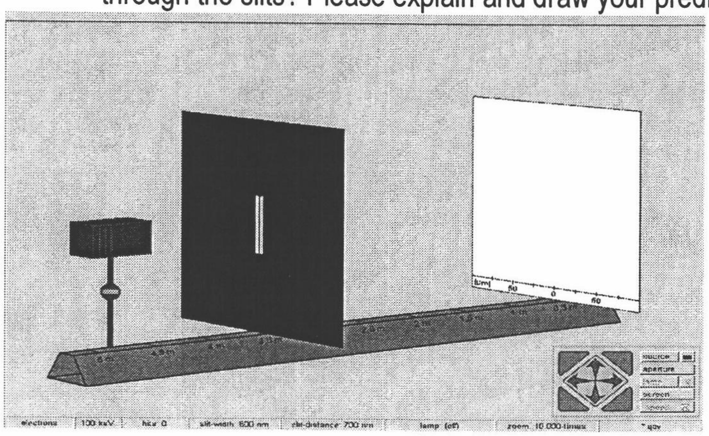


C. Please explain how the double-slit works for electrons.

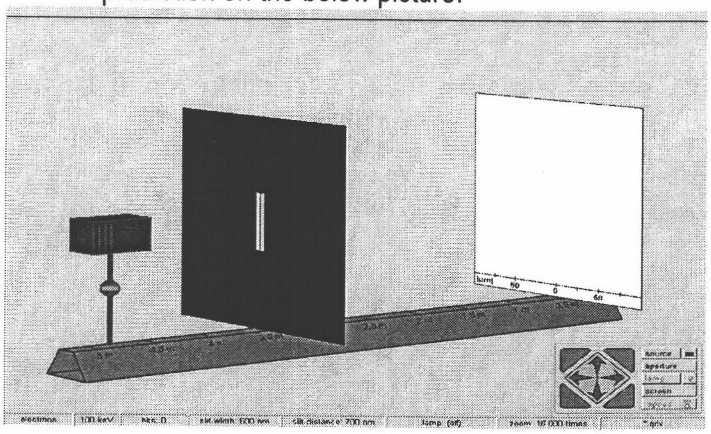


**III. Probability**

A. What do you think would happen on the screen if only two electrons are going through the slits? Please explain and draw your prediction on the below picture.



B. If the electron gun is sending out one electron at a time for some period of time, what pattern should you observe on the screen? Please explain or draw your prediction on the below picture.

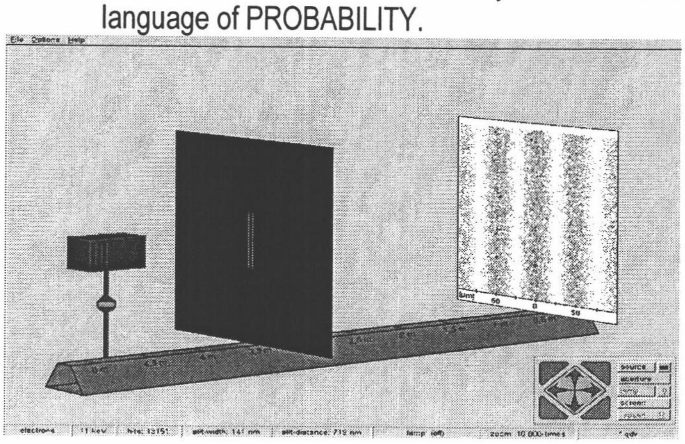


C. What do you think determines that position of electron on the screen?

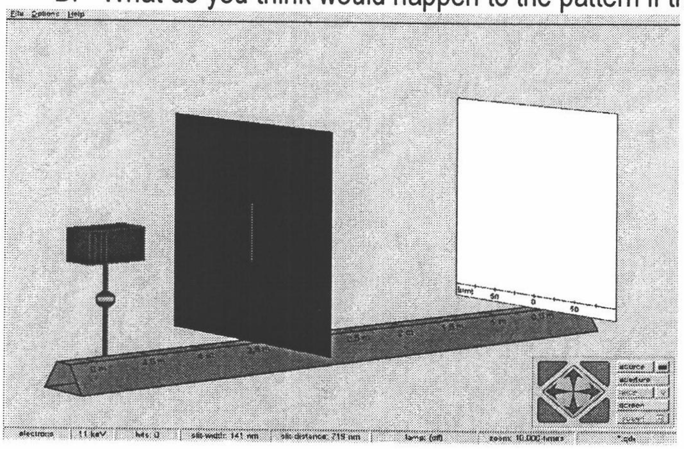
D. What do you think the pattern on the screen means in terms of physics concepts?

**IV. The double-slit experiment with electrons**

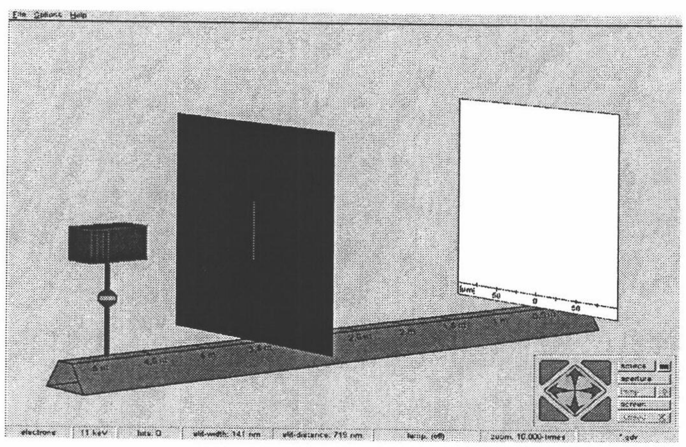
A. What do you think would happen to this interference pattern if the energy of the electron beams were decreased by halved? Please explain your answer using the language of PROBABILITY.



B. What do you think would happen to the pattern if the left slit is closed?



C. What do you think would happen if I open this one and close another one?



D. If you combine the pattern that you drew from part B and C, will the result pattern be the same as the interference pattern? Why or Why not?

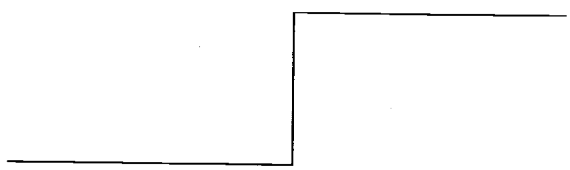
# Appendix E: Interview (II) Worksheet

**Direction: Please answer the following questions and provide explanations as best you can.**

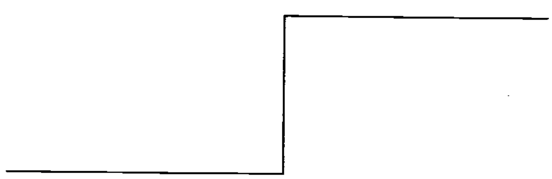
I. Please draw the wave function of an electron of energy for each potential energy function.

II. Please indicate where in each diagram you most likely find an electron. EXPLAIN YOUR ANSWER.

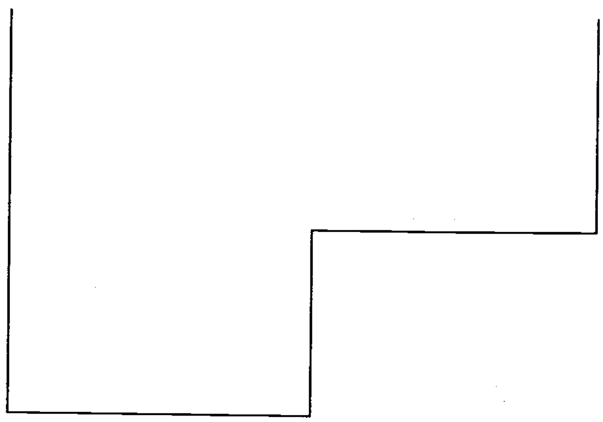
**Potential Energy Step: $E > U_0$**



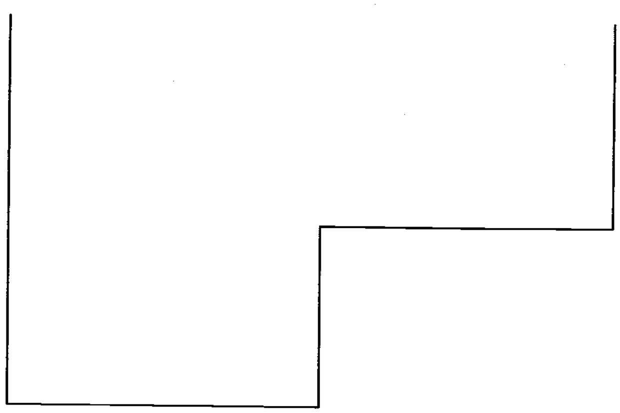
**Potential Energy Step: $E < U_0$**



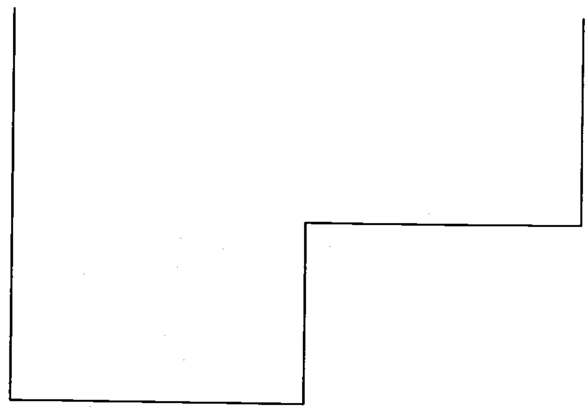
**Infinite Potential Well: $E > U_0$**



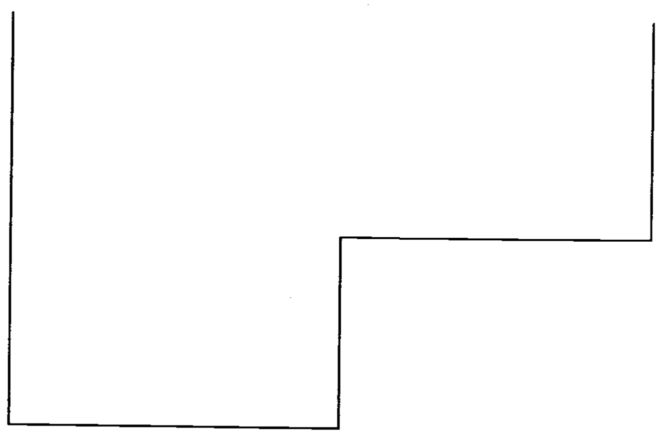
**Infinite Potential Well: $E > U_0$**



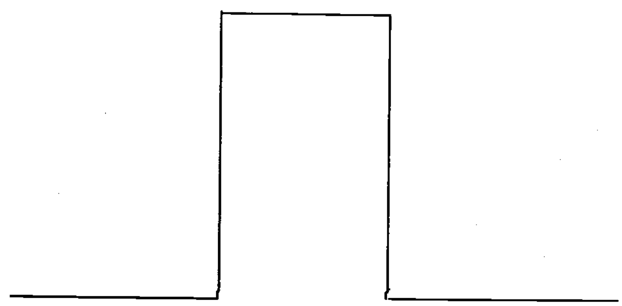
**Potential Well: $E > U_0$**



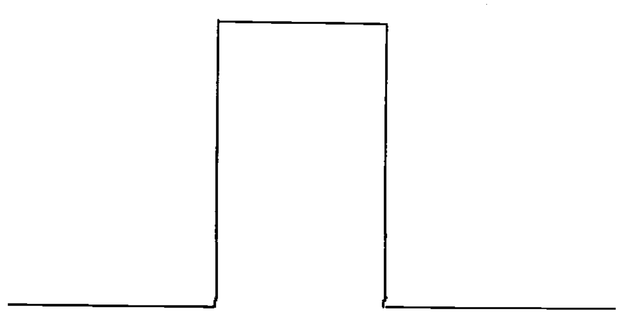
**Potential Well: $E > U_0$**



**Step Barrier: $E < U_0$**



**Step Barrier: $E > U_0$**



# Appendix E: Jim's Full Transcripts (Interview I and II)

## Interview I: Jim (4-5 pm, 4/27/2004)

**Interviewer:** What is an electron? You don't need to write it down, you can tell me.

**Jim:** What is an electron? I guess it would be a fundamental particle. It has a negative charge and a very small mass, but it does have mass. I guess even though it is a particle like everything else, it behaves like a wave. That is one thing about it. Was that good enough for electron? That is all I can think about it.

**Interviewer:** What is your major?

**Jim:** Electrical engineering.

**Interviewer:** What year are you?

**Jim:** I am a senior, fifth-year senior.

**Interviewer:** Let's talk about Part B. Can you explain what is the wave-particle behavior of electrons?

**Jim:** Electrons, like other particles, behave like waves which can be described using the De Broglie wavelength and the wave function, which you need a potential energy to ascribe how it is going to behave. The wave property of it gives the probability of locating it when you look for it. that is kind of its wave aspect.

**Interviewer:** How about the particle aspect?

**Jim:** The particle aspect is a little bit more well known, I guess, in that the particle has mass, carries momentum, and can interact with other particles through collisions. Let's see, something about it? That's all I can think of.

**Interviewer:** Why do you think electron has both wave and particle behavior?

**Jim:** How I see it, I just don't see that electrons have wave and particle behaviors, but everything does. It is just kind of another particle, and since all particles have both wave and particle duality. . .

**Interviewer:** How come we talk about electron wave and particle behavior more often than like a baseball or anything like that?

**Jim:** It is because when you are getting down and looking at electron behavior, the wavelength of it is, let's see, I guess since you can look at it and see its momentum isn't very large, then its wavelength is longer, which is more observable, whereas things like a baseball have such short wavelengths that you can't really study it, because you don't have good tools to do that. By going down to the size of an electron, you can study its wave behavior within the means of the instruments that we have.

**Interviewer:** So for Part C, can you draw the probability wave of a free electron moving at a speed v.

**Jim:** So a free electron, moving at a speed v, would just have like a sine wave. It has constant amplitude, which means it would have a constant probability of finding it.

**Interviewer:** Is this two-dimensional wave that you are drawing?

**Jim:** Yes.

**Interviewer:** What is the horizontal represent then?

**Jim:** OK, this would be x and then this would be, I guess, probability, so the probability wave is wave function, I guess.

**Interviewer:** You think that probability is the same throughout the whole thing?

**Jim:** Yes, so that is because you know it is moving at a speed v, and since you know its mass, therefore you know its momentum. Since you know its momentum, therefore you know this, which is that. So since you know this exactly, then this is zero, and therefore this has to be infinite for this to work.

**Interviewer:** What is meant by delta if it is infinite?

**Jim:** That means that you have no idea where the particle is at. It has an equal probability to be anywhere along this axis.

**Interviewer:** So the probability anyplace here and here is going to be the same?

**Jim:** Right, because the amplitude of the sine wave is constant.

**Interviewer:** I see. Good. How about the Part D. Can you draw another one with is moving at a speed of 2v, instead of v, so the speed is doubled?

**Jim:** The speed is doubled, let's see. The probability wave, how would that change it, so it has twice the kinetic energy and is still a free electron. Hmm, let's see. I'd draw it the same and I'm not sure how its amplitude would change. Its wavelength, I'm not sure how it would look compared to the other one, but I think it would be the same.

**Interviewer:** Nothing would change. It would look exactly the same?

**Jim:** I don't know what the scale would be between the two.

**Interviewer:** Let's draw the dash line, this one, I'll draw it right here. So draw another wave.

**Jim:** How it would look like compared. . .

**Interviewer:** Compared to this one, but instead of drawing a solid line, draw a dash-line here.

**Jim:** On this one?

**Interviewer:** Yes, corresponding to the probability wave of 2v.

**Jim:** I am thinking about different equations I could use to describe it. The De Broglie wavelength and p = mv. I guess if this is, let's say that is mv and if you had twice the velocity, you have half of the wavelength, so I guess we could go and do it like this, and just do twice the frequency, like that. Then the amplitude, I'm not sure how the amplitude would change compared to that, but I think the wavelength would be half.

**Interviewer:** Would be half, but you are not so sure about the amplitude of the second one compared to the first one?

**Jim:** Right.

**Interviewer:** What is the amplitude? They are the same again?

**Jim:** Well, the amplitude, I guess, if you are thinking about the probability, it depends if it is normalized or not. You can't normalize a free electron, the probability of a free electron, because the sine wave goes up to infinity. To be able to set the probability equation to 1, you wouldn't be able to, so you can't normalize it.

**Interviewer:** You cannot draw the probability?

**Jim:** Right. I guess you could draw its wave function, but it wouldn't be normalized, so maybe then you wouldn't be able to draw probability.

**Interviewer:** So you cannot look at the probability from the wave function? Is that what you are trying to say?

**Jim:** Yes.

**Interviewer:** Good. Let me go on to the next one. I'll have you read the problem. "Explain how the single-slit experiment works using the language of the UNCERTAINTY relationship." OK. I guess that is you look at this point here in the slit, well, I guess we can do this thing again. Let's see, if you talk about this in the y direction, I guess define it like that, so at this point you are defining delta x at this point, so some finite number, and this would be delta x in the y direction. Since you are defining where this delta x is, there has to be some finite delta py, which means that there is uncertainty in the momentum in the y direction. That means that the light coming through this slit is going to have some momentum in the y direction and therefore it is going to spread out. So instead of having just an image of the slit here, you will have like a distribution. Then there is also interference here.

**Interviewer:** Can you draw exactly what you will see on the screen?

**Jim:** I think you should see a bright fringe and then harmonics here, which are not as bright, and then they dies off.

**Interviewer:** What happens if I reduce the width of the slit?

**Jim:** If you reduce the width of the slit, then this term drops, and therefore this term has to go up. Therefore, you don't know, the uncertainty in the momentum is greater, so these should spread out. I am guessing that this is going to get maybe a little. . .

**Interviewer:** So the central maximum would get bigger?

**Jim:** Yes, and then these would move out farther.

**Interviewer:** Also the second maximum you have further apart, the second and third?

**Jim:** Yeah, here and then these go out.

**Interviewer:** So all the pattern is spread out more.

**Jim:** Yes, the interference patterns just spread out.

**Interviewer:** What will happen to the intensity then of the central maximum?

**Jim:** I think the intensity will drop because you are not really putting more light through the slit. You have to have the same number of photons hitting this total, so that you spread the area out so the intensity drops.

**Interviewer:** But it seems like you are saying that the photon will not be moving through these as much. Why is that, moving through the slit as much as before?

**Jim:** I was saying that like the rate of photons going through here would be the same. You haven't changed anything over here.

**Interviewer:** Yes, the change is the slit width.

**Jim:** The slit width, so the only thing that happens is this spreads out and the total number of photons going through is the same, so the intensity drops.

**Interviewer:** Because it is spread out?

**Jim:** Right.

**Interviewer:** This is the double slit. Can out explain how the double slit works with light?

**Jim:** To explain the double slit, you can't really look at the particle nature, so you have to look at the wave nature. Let's see. I will just draw it like a schematic view. You have the two slits here and have some distance in between them here. Then you have the screen back over here. If you are looking at the wave nature, you are going to get maxima and minima on the screen over here. Like patterns over here. For there to be a maximum, the difference in path length between these – let's see – for maximum the difference in path length needs to be a multiple of half the wavelength.

**Interviewer:** You get the maximum.

**Jim:** Let's see, is that right. So in order to get a minimum you have a half wavelength difference. So for a maximum you have the whole wavelength difference. Say if you are going from this slit here to there, x1 and x2, so then if you go x1 minus x2, this will be some difference of the wavelength.

**Interviewer:** Is that for maximum?

**Jim:** That is for maximum. Then for minimum, it would be that, so I guess it would be n plus ½ the wavelength, and this would be n goes from 1, 2, 3, integers, and then one would be 0, 1, 2. This is for minimum.

**Interviewer:** That is how it works for light?

**Jim:** Yes, and that is just looking at the wave nature of the waves coming out here and interfering at points out here.

**Interviewer:** So can you use the language of uncertainty to explain double slit?

**Jim:** I guess, I am trying to think. I understand how if you are kind of defining where the protons have to go through here, so there is some kind of delta xy here, which causes there to be a finite delta momentum in the y direction, so the light has to spread out basically at these two points. I am not sure exactly how to describe where the maximum and minimum are with respect to that. I guess I only know about how to describe its diffraction through the slit.

**Interviewer:** Through one of the slits but not both?

**Jim:** Well, they diffract through both. I guess there is diffraction happening on both, so I'm not sure. I think you have to look at each individually, but I'm not quite sure about that.

**Interviewer:** That's fine. That is for light. Can you explain the double slit for electrons.

**Jim:** OK, the double slit for electrons? Let's see, I think it would exactly the same way. The only difference would be to be able to see the patterns you have to get the distance between the slits and the slit themselves, very, very small compared to light, because the electrons are larger. Then their wavelength is much smaller.

**Interviewer:** You are saying that you should decrease the slit width and the separation between the two slits?

**Jim:** Right, to be able to observe, because if they are too big you are just going to have the electrons hitting in what seems like just a line on the wall, but just because everything is crunched together and it is not spread out enough. The smaller you make this, the more spread out it will be.

**Interviewer:** What pattern do you see on the screen?

**Jim:** It should be exactly the same as before, because the light. . .

**Interviewer:** Can you draw it?

**Jim:** It will be, let's see, I think for the double slit there are two maximums here and then you have the secondary and the higher orders out here.

**Interviewer:** So you have two maximum?

**Jim:** I think the single slit has one maximum and then the double slit has two.

**Interviewer:** Two maximum because there are two slits?

**Jim:** Yes.

**Interviewer:** As you go out, then the width is decreasing?

**Jim:** Yes, the width and the intensity decreases.

**Interviewer:** Decreasing as well, so it is like disappear as you go further?

**Jim:** Yes.

**Interviewer:** What do you think would happen on the screen if only two electrons passed through the slit for the double-slit experiment?

**Jim:** If only two electrons pass, then you will see two points on the screen, but where they are, you won't be able to tell, but the probability of them is given by that. Where this is more intense, there will be more probability to find it there.

**Interviewer:** If you reduce the rate and then you draw the two electrons on this path, where is it going to be?

**Jim:** Where they will be?

**Interviewer:** Yes, compared to this pattern, if you send two electrons?

**Jim:** They could be anywhere.

**Interviewer:** They could be anywhere, but which one has more probability?

**Jim:** I guess there would be most probable in the high intensity regions, so I guess here and here.

**Interviewer:** How about if I send a third one?

**Jim:** A third one?

**Interviewer:** Where is it likely to land on the screen?

**Jim:** High intensity, but I don't think it matters which side. It is only the intensity that determines the probability. It could be here, I guess.

**Interviewer:** Do the third one, where it is going to land, depending on the first two locations?

**Jim:** No, they should all just be independent. But statistically, they will form this pattern if you spend enough through the slits.

**Interviewer:** So if we send a lot of them?

**Jim:** Yes, they should make this pattern.

**Interviewer:** Look at this.

**Jim:** Sends out one electron at a time for a long period, what pattern should you observe on the screen? I think just like I drew before, you should see the exact same thing. You would see the maxima and then you should see the minima, and they drop off out here.

**Interviewer:** What factor determines the location of electron on the screen? What is the factor?

**Jim:** What factor determines its location?

**Interviewer:** Yes, on the screen?

**Jim:** I guess it would be its wave function. Since its wave function determines its probability of where it will be, so I guess this would be an amplitude of its wave function, squared on this plane.

**Interviewer:** What is the physics concept that is related to the thing that you have seen on the screen?

**Jim:** I guess this would be, so if you just plotted the intensity across here, and you would have minima and then you would get maximum and I guess you would have maxima here and then they would drop off. So this on here, you could say this would be its wave function squared, and then you would can integrate over like a small part of this, and that would give you its probability.

**Interviewer:** If I integrate over a small part?

**Jim:** I guess also you have to take this squared x, dx and then over some interval, that is 2, and also to normalize it, you have to go from there to infinity...

**Interviewer:** To get the intensity?

**Jim:** To get the probability, which is the intensity.

**Interviewer:** You think that intensity and probability are the same thing?

**Jim:** Yes.

**Interviewer:** So the intensity of?

**Jim:** The image.

**Interviewer:** Then the probability of locating the electrons? OK. Why do you think that is?

**Jim:** Because the higher the intensity is, the more electrons have hit at that point, which means that is the most probable location for them, since that is where they landed.

**Interviewer:** What do you think would happen to this interference pattern if the kinetic energy of the electrons decreased by half?

**Jim:** Let's see. Since the kinetic energy is going down, so we know that is kinetic energy and how it affects momentum. Therefore, the momentum of the particles is going down. We know that kinetic energy is going down and that goes down, that goes down, so the length goes up. I think if the wavelength goes up, then I think this would spread out so the interference patterns would spread out more.

**Interviewer:** You can use the red pen to draw the new one.

**Jim:** So lambda equals d sine theta. So wavelength is going up, and distance stays the same, so that means theta is going up. The spacing in between, so I guess the probability would, so if you look at this as kind of a probability distribution, I guess you could say that these would shift out and they would just spread out. Let's see, the kinetic energy is going up, so I think intensity would drop down and these would spread apart, spread out and apart. If you have a high intensity here, that it would be kind of like this. Then these ones would spread out from here and out to there.

**Interviewer:** The height, the peak of the part that you are doing would be dropped and the base would expand more?

**Jim:** I think so. Let's see. I am trying to think if the intensity would drop or how that would work, because it depends on if they are getting more kinetic energy and they are still coming out at the same rate, then I think that the intensity would drop.

**Interviewer:** If the delta x between here and here, so this is delta x, so which one will have higher probability? The one with the original kinetic energy or the one with half of the kinetic energy?

**Jim:** In that region. I am thinking it is going to be more probability, but I'm not quite sure.

**Interviewer:** More probability on which case?

**Jim:** The second.

**Interviewer:** The second one, when you reduce it.

**Jim:** Because in the first case you have a high probability and you are looking in this region. You have to integrate this area, but in the second one, it is coming out further, but it is also lower, I think. I think it could have more, but it might not be true. I think it depends, maybe.

**Interviewer:** Why you think the second case would be more? Don't you think the area would be more?

**Jim:** Yeah.

**Interviewer:** If the area of the second case is more?

**Jim:** Then it would have higher probability, otherwise it wouldn't.

**Interviewer:** So it doesn't depend on the peak? The first case seemed like it more peak here.

**Jim:** Right. In the first case. . .

**Interviewer:** It had more peak there, so that peak doesn't really matter? It just depends on the total area?

**Jim:** Yeah. The peak here just determines how much area you have under here. Even though the peak is higher in the first case, it is spread out farther. Even if that comes down this way, it is also spreading out that way.

**Interviewer:** Here is the next one. Now this one is what happens if you close the left slit and then let the electron beams go through. What should be the pattern?

**Jim:** You should just have a single slit interference pattern, so it would look like a big one in the center and then it will have some smaller ones on the side, like before.

**Interviewer:** So it should look like the one with the single slit?

**Jim:** Yes.

**Interviewer:** Now what happens if I open the left one and close the right one?

**Jim:** I think it should look the same as this one. It might be offset.

**Interviewer:** It seems like you draw this one in scale to the zero. So what should be this one?

**Jim:** It will be pretty much just the same, from zero again. It would still have these things.

**Interviewer:** Some one could say if you combine the pattern, this is one, let's say this is pattern 1, and this is pattern 2, and then you overlap it, then you should end up with this pattern. Do you agree with that statement?

**Jim:** No, I don't.

**Interviewer:** Why not?

**Jim:** Because each one of these is a function of the wave function squared. When you square it, you lose the negative components. To overlap, you would have to go back and know what the wave function was, and then overlap the wave functions. Then you do that, so it is kind of like this, if you had psi 1 squared and then add that to this [psi 2 squared], that is not equal to psi 1 plus psi 2 and then square that.

**Interviewer:** What does this represent then?

**Jim:** This represents this one, yes.

**Interviewer:** What is phi 1 and phi 2 standing for?

**Jim:** These are the wave functions of just the individual slits.

**Interviewer:** They can go to each slit and have phi 1 and phi 2?

**Jim:** Yes, this is the electrons going through. Each one has their own wave function for each slit. Then you have to superimpose the wave functions, and then square them to get the probability.

**Interviewer:** That is why it shouldn't be the same, right?

**Jim:** Yes.

**Interviewer:** Good. That's it.

## Interview II: Jim (1-2 pm, 6/2/2004)

**Interviewer:** Please draw the wave function as a solution for each potential energy function.

[For a potential energy step E > Uo]

**Jim:** [silence] . . . each of these, indicate where you are most likely to find an electron. So wave function, I think that you have like this, and then I think it is here then, at this point that you have a larger amplitude and a longer wavelength. So the amplitude, so if this is amplitude 1 and this is amplitude 0, amplitude 1 is greater than amplitude 0, and so therefore the. . .

**Interviewer:** Why the amplitude 1 and the amplitude 0?

**Jim:** This is because the particle is moving with a slower velocity, so you want to use 0, because it is moving at a slower velocity. . .

**Interviewer:** How do you know that it is moving with slower velocity?

**Jim:** I was assuming that the particles are coming from this direction, so it is because the energy in this one right here, let's just say that the particles have the kinetic energy here equal to e, because u is 0. Then in this one they have a kinetic energy v minus. . .

**Interviewer:** So the kinetic energy decreases?

**Jim:** Yes, so because the kinetic energy decreased, the velocity is less. Because there is a smaller velocity, that means that the probability density is higher, because they are moving slower in this region.

**Interviewer:** How about the wavelength?

**Jim:** The wavelength is shorter in this region than in this region. I think, let's see, the wavelength is equal to h bar momentum. I think that is right. So because in this region the velocity is smaller, so this is smaller, therefore this is larger in this region. The wavelength is larger.

**Interviewer:** Then you are more likely to find electron in which one?

**Jim:** In this region, the probability is higher?

**Interviewer:** Is there anywhere in particular that you are more likely to find electron in this region, any specific location?

**Jim:** I don't know. Let's see. Since it goes out to infinity, you can't normalize it. I don't know if you could know the probability exactly. I don't know if the probability, you could just find the ratio of the probabilities, maybe.

**Interviewer:** So you are not sure of any specific location that you are more likely to find electrons, because it is not normalized?

**Jim:** Right.

**Interviewer:** Now, how about this one?

**Jim:** This one you will have two waves, so you will have one wave going this direction – I'll try to draw it the same – and then once it hits this wall, then it is going to bounce back, and then you will get another wave traveling in this direction with the same wavelength. Then these will superimpose on each other and create another, a standing wave. This isn't a traveling wave, so the amplitude, so you will have high and low probabilities finding the particle here and here. You will have more probability here and here and here, and less probability here.

**Interviewer:** Then you are not going to find it here?

**Jim:** No. Well, the particle will go in here for a short time, so there will be a wave. I guess like here. If I draw it like this, let me shift this over a bit so it will be easier to draw, so let's say this is the wall there. It will go in a little bit. I guess it goes towards, something like that. It will extend a small ways in.

**Interviewer:** You are more likely to find somewhere here and where the peak is at, right? The location you are more likely to find an electron?

**Jim:** Yes.

**Interviewer:** Where the peak is?

**Jim:** Yes, you are more likely to find it there, and less likely here.

**Interviewer:** The next one?

**Jim:** On this one you should have, so the wave function should look like – I'll use this as an axis, I guess – so you will have this, and there will be a node here. It hits here and it will go like this. There will be a node there. For the same reasons as this one, the particle will be moving slower in this area, so the wave function has a large amplitude, just like this one. Except this one is different in that you can normalize it.

**Interviewer:** Can you tell where you are more likely to find an electron?

**Jim:** There will be a higher probability to find it in this region.

**Interviewer:** Any specific location?

**Jim:** No. This amplitude is the same all along here, so that means that there is an equal probability here. There is just a probability difference because the amplitudes are different – I think. They are standing waves, I think, so actually there should be a higher probability to find it at the peaks, I think. You have a wave coming in this direction, so these are standing waves, so, yeah, I think there is a higher probability to find it at the peaks. To find the probability you have to square the wave function, so these negative peaks will go up.

**Interviewer:** You think you are more likely to find it where the peak is at?

**Jim:** Yes.

**Interviewer:** Does that matter? For example, in these two areas, A and B, which one has a higher probability of finding an electron?

**Jim:** Area A and B? Let's see. This is one dimensional, right? I guess I am kind of confused by what you mean by the two areas, because they are in the same X region.

**Interviewer:** That means they are going to have the same probability?

**Jim:** Yes. They should have the same probability.

**Interviewer:** How about this one?

**Jim:** On this one, you can't have negative kinetic energy, so there won't be any probability to find it over here. You should just have this here, like that. This wavelength is the same. The probability of finding it all along here, you will again find it more at the peaks of the square of this wave function.

**Interviewer:** So at the peak of the square?

**Jim:** Yes.

**Interviewer:** Like before?

**Jim:** Yes.

**Interviewer:** Now this one?

**Jim:** This is now not infinite and so it decays but less than this. I think this should be the same as this one. I think as long as the energy is below the top of this, then it will be the same as this.

**Interviewer:** Could you draw that?

**Jim:** I am going to try to make everything match up near the edges.

**Interviewer:** So you are more likely to find electron where?

**Jim:** In this region.

**Interviewer:** Where the peak at again?

**Jim:** Yeah, when you square it, the wave function, where the peaks are at.

**Interviewer:** How about this one and this one?

**Jim:** This one will be the same as this, so you will have this here, like that. Then here, I think that if the electrons are coming from this direction, you will have waves reflected at each of these boundaries. You will have a wave coming forward here and a reflection back here, back here, and back here. You will get standing waves at each of those, and the super positions of all those waves. Let's see. I am just going to guess. I'm not really sure if this is correct. I think that you will have like a large amplitude here and then you will have potential energy. The kinetic energy will be highest here, so you will have smallest wavelength here and then a little bit longer here and then back to this.

**Interviewer:** It seems like the averages here and here are the same?

**Jim:** Well, let me think about this. Maybe it won't. Maybe since there is – actually let me redraw this. I think that the particle, again reflected at each of these points, and so maybe since they are getting reflected, I am thinking that the amplitude at each of these areas is just going to decrease, because there are particles being reflected, but you can have like reflection. You can have waves coming in this way being reflected this way as well, up this.

**Interviewer:** It is kind of complicated.

**Jim:** It is kind of complicated. I am not quite sure how exactly it is going to look.

**Interviewer:** Let's go region by region.

Jim: Let's just keep this one here, this could be the reference. Let's see, there is going to be a wave reflected and a wave transmitted. I am not sure if the particles move faster in there, so the wavelength decreases – I'm not sure – but I think that the amplitudes might just step down at each of these points. If the particles are traveling in this direction, then I guess this is sort of like a discontinuity. You will have particles reflected. I am going to draw it like a little bit lower here and then the wavelength, I think, is going to change, because they move faster, and then smaller here, and then maybe wavelength will get a little big bigger, and then here, I think it is going to be shallowest here, because particles are coming from this direction and they get reflected off of each of these discontinuities back toward this way.

**Interviewer:** That is why the amplitude becomes decreasing?

**Jim:** Right.

**Interviewer:** How about the wavelength?

**Jim:** I think that the wavelength should depend on the kinetic energy of the particle, so that the wavelength here is the smallest and then here and here should be the same. Then here is the middle of the barrier. So you have a wavelength here and wavelength here, so you will have the wavelength 1 less than the wavelength 2, which is less than wavelength 0 and 3, I think. Because the wavelength depends on the kinetic energy.

**Interviewer:** And you are most likely to find it here?

**Jim:** Yes. Well, I think. Again, you can't normalize this because it goes out to infinity. I'm not quite sure.

**Interviewer:** That means the amplitude doesn't really tell you the probability?

**Jim:** Right.

**Interviewer:** This?

**Jim:** OK, on this one there is some wave coming here and then when it comes in here it should have exponential decay here and then we will have this and then that goes off.

**Interviewer:** Why the amplitude decreasing?

**Jim:** That is because the particles are coming in here and most of them are reflected off, but some can tunnel through. The amplitude decreases, because only some of the particles come out the side.

**Interviewer:** How about the wavelength?

**Jim:** The wavelength should be the same, actually.

**Interviewer:** Why do you think it should be the same?

**Jim:** I think it should be the same because the particles – well, they have the same energy on both sides. Since the potential is the same, they should have the same kinetic energy. The kinetic energy on both sides should be equal.

**Interviewer:** So you lose the energy when it is tunneling?

**Jim:** Right.

**Interviewer:** How can it tunnel through the barrier?
**Jim:** It is because, I think it is just because the wave function, the wave extends through here, so that there is a probability that there are particles on the side. I think, let's see, I think what happens is that it can borrow energy. If it gives it back in the time given by the uncertainty principle that it can get over here.

**Interviewer:** The uncertainty principle?

**Jim:** Correct.

**Interviewer:** What happens if I reduce the width of this to be half, or not quite half?

**Jim:** That means, I think that the amplitude here would be larger. I guess on here, if you make this like this, it should be small, and then this here would be larger than the other one. I will draw below it.

**Interviewer:** You can draw the back line over here.

**Jim:** So a little bit, it will be a little bit larger because more particles are able to tunnel through.

**Interviewer:** Where are you most likely to find the electrons?

**Jim:** Between the two sides, I think, I don't know if you can find the probability exactly, but there should be more electrons on this side, I would think.

**Interviewer:** But you cannot normalize it?

**Jim:** Right, you can't normalize it.

**Interviewer:** How about this one? This one is energy of both barriers.

**Jim:** On this one, there will be reflections at this point and this point. You will have some coming in here, and then here the kinetic energy decreases. I think the wavelength decreases as well, and then I think that it should go like that.

**Interviewer:** Why is the amplitude decreasing?

**Jim:** The amplitude decreases because waves are being reflected off here and from there. So I guess the wave is reflected, so the wave amplitude decreases. I think the wavelength is smaller in this portion because it has less kinetic energy.

**Interviewer:** It has less kinetic energy so the wavelength is longer in this portion?

**Jim:** Yes.

**Interviewer:** So the wavelength is longer here?

**Jim:** In these it is longer. Wait, sorry. I made that backward. The wavelength should be longer so these have more kinetic energy here. It is like this. It goes through here and then. . . it has less kinetic energy here, so that the wavelength is longer.

**Interviewer:** Where do you think you are more likely to find electrons?

**Jim:** I think on this side. Again, you can't normalize it, so I don't know if you can find the probability exactly.

**Interviewer:** How is the probability connected to the wave function?

**Jim:** The probability, wave function squared, so this is the normalized wave function. This is proportional to probability. If you normalize the wave function, it basically means to be able to know what this amplitude is exactly, because I think when you solve for the wave function, you just get like a constant out in front. To be able to normalize it, you have to be able to integrate it across the entire thing.

**Interviewer:** You will be able to integrate it in a certain region?

**Jim:** Or you have to integrate it from negative infinity to infinity.

**Interviewer:** I see.

**Jim:** That allows you to normalize the wave function.

**Interviewer:** That's it. Thank you.

# Appendix G: Solutions to Interview II Questions

We provide solutions to the interview problems in this appendix. We solve the Schrödinger equation for each potential energy function.

## I. The potential energy step problem

In this problem, we solve the Schrödinger equation for a beam of particles moving from $x \to -\infty$ and encoutering the potential energy step at $x = 0$, as presented in Figure G.1.

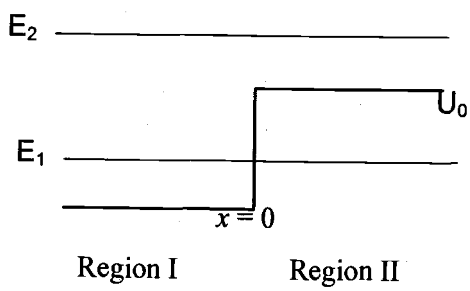



**Figure G.1: A diagram for the potential energy step question.**

**Case 1: $E_2 > U_0$**

Start with the Schrödinger equation, and apply the potential energy and the energy of the particle.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U_0\psi = E\psi \tag{G.1}$$

or

$$\frac{d^2\psi}{dx^2} = -k^2\psi \tag{G.2}$$

where

$$k^2 = \frac{2m(E-U_0)}{\hbar^2} \tag{G.3}$$

Whenever $E$ is greater than $U_0$, $k^2$ is always positive, the solution to the Schrödinger equation is

$$\psi(x) = A\sin kx + B\cos kx \tag{G.4}$$

Apply the condition for the potential energy step as shown in Figure G.1:

$$\begin{aligned} U(x) &= 0 \quad && x < 0 \\ &= U_0 \quad && x \geq 0 \end{aligned} \tag{G.5}$$

We can write down the solutions to the Schrödinger equations in the two regions:

$$\psi_I(x) = A\sin k_I x + B\cos k_I x \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}} \qquad x < 0 \tag{G.6}$$

$$\psi_{II}(x) = C\sin k_{II} x + D\cos k_{II} x \qquad k_{II} = \sqrt{\frac{2m(E-U_0)}{\hbar^2}} \qquad x \geq 0 \tag{G.7}$$

Apply conditions that the wave must be continuous at $x = 0$ to find the relationship among the four coefficients—$A$, $B$, $C$, and $D$.

$$\psi_I(0) = \psi_{II}(0) \tag{G.8}$$

$$A\,(0) + B(1) = C\,(0) + D\,(1)$$

So,

$$B = D \tag{G.9}$$

$$\psi_I'(0) = \psi_{II}'(0) \tag{G.10}$$

$$Ak_I(1) - Bk_I(0) = Ck_{II}(1) - Dk_{II}(0)$$

$$A = \frac{k_{II}}{k_I}C \tag{G.11}$$

**Case 2: $E_1 < U_0$**

Whenever $E$ is less than $U_0$, $k^2$ is always negative, the solution to the Schrödinger equation is

$$\psi(x) = Ae^{kx} + Be^{-kx} \tag{G.12}$$

where

$$k^2 = \frac{2m(U_0 - E)}{\hbar^2} \tag{G.13}$$

For $x < 0$, the solution is the same as in equation (G.6), but the solution $\psi_{II}$ for $x > 0$ becomes:

$$\psi_{II}(x) = Ce^{k_{II}x} + De^{-k_{II}x} \qquad k_{II} = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} \tag{G.14}$$

We must set $C = 0$ to keep $\psi_{II}$ from becoming infinite as $x \to \infty$. Apply the boundary conditions on $\psi(x)$ and $\psi'(x)$ at $x = 0$.

$$\begin{aligned} \psi_I(0) &= \psi_{II}(0) \\ B &= D \end{aligned} \tag{G.15}$$

$$\begin{aligned} \psi_I'(0) &= \psi_{II}'(0) \\ Ak_I(1) - Bk_I(0) &= -Dk_{II}(1) \\ A &= -\frac{k_{II}}{k_I}D \end{aligned} \tag{G.16}$$

**II. The infinite potential energy well problem**

Given the particle moving in the following potential energy well, as shown in Figure G.2:

$$\begin{aligned} &\text{Region I:} & U(x) &= 0 & 0 \le x \le L \\ &\text{Region II:} & U(x) &= U_0 & L \le x \le 2L \\ & & U(x) &= \infty & x < 0 \text{ and } x > 2L \end{aligned} \tag{G.17}$$

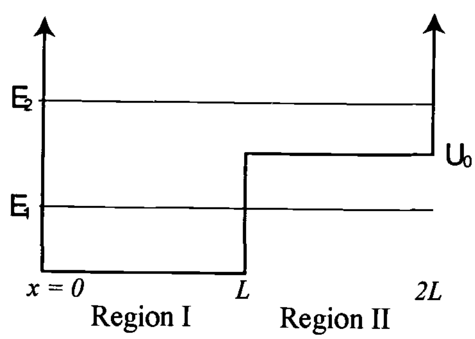



**Figure G.2: The infinite potential energy well problem**

**Case 1: $E_1 < U_0$**

The solutions for the wave function in each of the two regions are:

$$\psi_I(x) = A\sin k_I x + B\cos k_I x \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}} \tag{G.18}$$

$$\psi_{II}(x) = Ce^{k_{II}x} + De^{-k_{II}x} \qquad k_{II} = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} \tag{G.19}$$

Apply the boundary condition on the continuity of $\psi(x)$. The wave function is zero in the region where the potential energy is infinite.

At $x = 0$,

$$0 = \psi_I(0)$$

$$B = 0$$

So

$$\psi_I(x) = A\sin k_I x \tag{G.20}$$

At $x = 2L$,

$$\psi_{II}(x = 2L) = Ce^{k_{II}2L} + De^{-k_{II}2L} = 0$$

$$D = -Ce^{4k_{II}L}$$

So

$$\psi_{II}(x) = C(e^{k_{II}x} - e^{k_{II}(4L-x)}) \tag{G.21}$$

At $x = L$,

$$\psi_I(L) = \psi_{II}(L)$$

$$A\sin k_I L = C(e^{k_{II}L} - e^{3k_{II}L})$$

$$\psi_{II}(x) = \frac{A\sin k_I L}{e^{k_{II}L} - e^{3k_{II}L}}(e^{k_{II}x} - e^{k_{II}(4L-x)}) \tag{G.22}$$

We can apply another boundary condition on $\psi'(x)$ at $x = L$ to determine the energy $E$.

**Case 2: $E_2 > U_0$**

The solutions for the wave function in each of the two regions are:

$$\psi_I(x) = A\sin k_I x + B\cos k_I x \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}} \tag{G.23}$$

$$\psi_{II}(x) = C\sin k_{II} x + D\cos k_{II} x \qquad k_{II} = \sqrt{\frac{2m(E - U_0)}{\hbar^2}} \tag{G.24}$$

Apply the boundary condition on the continuity of $\psi(x)$. The wave function is zero in the region where the potential energy is infinite.

At $x = 0$,
$$0 = \psi_I(0)$$
$$B = 0$$

So
$$\psi_I(x) = A\sin k_I x$$

At $x = 2L$,
$$\psi_{II}(x = 2L) = C\sin k_{II}2L + D\cos k_{II}2L = 0$$
$$D = -C\tan k_{II}2L$$

So,
$$\psi_{II}(x) = C[\sin(k_{II}x) - \tan(k_{II}2L)\cos(k_{II}x)] \tag{G.25}$$

At $x = L$,
$$\psi_I(L) = \psi_{II}(L)$$
$$A\sin k_I L = C[\sin(k_{II}L) - \tan(2k_{II}L)\cos(k_{II}L)]$$

$$\psi_{II}(x) = \frac{A\sin k_I L}{[\sin(k_{II}L) - \tan(2k_{II}L)\cos(k_{II}L)]}[\sin(k_{II}x) - \tan(k_{II}2L)\cos(k_{II}x)] \tag{G.26}$$

We can apply another boundary condition on $\psi'(x)$ at $x = L$ to determine the energy $E$.

## III. The potential energy barrier

In this problem, we solve the Schrödinger equation for a beam of particles moving from $x \to -\infty$ and encoutering the potential energy barrier at $x = 0$, as presented in Figure G.3.

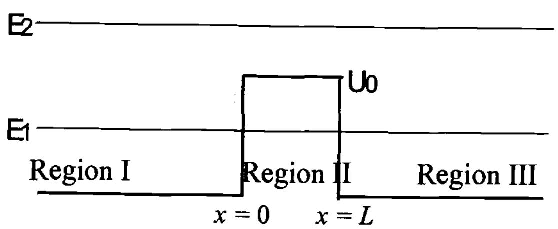



**Figure G.3: The potential energy barrier problem**

**Case 1: $E_2 > U_0$ (resonances)**

The solutions for the wave function in each of the three regions are:

$$\text{Region I:}\quad \psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x} \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}} \tag{G.27}$$

$$\text{Region II:}\quad \psi_{II}(x) = Ce^{ik_{II} x} + De^{-ik_{II} x} \qquad k_{II} = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} \tag{G.28}$$

$$\text{Region III:}\quad \psi_{III}(x) = Fe^{ik_I x} + Ge^{-ik_I x} \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}} \tag{G.29}$$

Let us choose the coefficient $G$ to be zero because the incident particle coming from $x = -\infty$. Apply the boundary condition on the continuity of $\psi(x)$ for all three regions.

At $x = 0$,
$$\psi_I(0) = \psi_{II}(0)$$
$$A + B = C + D$$

At $x = L$,
$$\psi_{II}(L) = \psi_{III}(L)$$
$$Ce^{k_{II}L} + De^{-k_{II}L} = Fe^{ik_I L}$$

Apply the boundary condition on the continuity of $\psi'(x)$. Thus we get:

$$A = \left[\cos k_{II}L - i\frac{k_I^2 + k_{II}^2}{2k_I k_{II}}\sin k_{II}L\right]e^{ik_I L}F \tag{G.30}$$

$$B = i\frac{k_{II}^2 - k_I^2}{2k_I k_{II}}\sin(k_{II}L)\,e^{ik_I L}F \tag{G.31}$$

Then calculate the reflection coefficient R and the transmission coefficient T of the barrier:

$$R = \left|\frac{B}{A}\right|^2 = \frac{(k_I^2 - k_{II}^2)\sin^2 k_{II}L}{4k_I^2 k_{II}^2 + (k_I^2 - k_{II}^2)\sin^2 k_{II}L} \tag{G.32}$$

$$T = \left|\frac{F}{A}\right|^2 = \frac{4k_I^2 k_{II}^2}{4k_I^2 k_{II}^2 + (k_I^2 - k_{II}^2)\sin^2 k_{II}L} \tag{G.33}$$

We know that $R + T = 1$ and we can substitute $k_I$ and $k_{II}$ in terms of $U_0$ and $E$, so we have:

$$T = \frac{4E(E-U_0)}{4E(E-U_0)+U_0^2\sin^2\left[\sqrt{\frac{2mL^2(E-U_0)}{\hbar^2}}\right]} \tag{G.34}$$

**Case 2: $E_1 < U_0$ (tunnel effect)**

The solutions for the wave function in each of the three regions are:

Region I: $\psi_I(x) = Ae^{ik_Ix}+Be^{-ik_Ix} \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}}$ (G.35)

Region II: $\psi_{II}(x) = Ce^{k_{II}x}+De^{-k_{II}x} \qquad k_{II} = \sqrt{\frac{2m(U_0-E)}{\hbar^2}}$ (G.36)

Region III: $\psi_{III}(x) = Fe^{ik_Ix}+Ge^{-ik_Ix} \qquad k_I = \sqrt{\frac{2mE}{\hbar^2}}$ (G.37)

Let us choose the coefficient $G$ to be zero because the incident particle coming from $x=-\infty$. Apply the boundary condition on the continuity of $\psi(x)$ for all three regions.

At $x = 0$,
$$\psi_I(0) = \psi_{II}(0)$$
$$A+B = C+D$$

At $x = L$,
$$\psi_{II}(L) = \psi_{III}(L)$$
$$Ce^{k_{II}L}+De^{-k_{II}L} = Fe^{ik_IL}$$

Apply the boundary condition on the continuity of $\psi'(x)$. Then follow the same steps as in case 1, so we have the transmission coefficient in term of $E$ and $U_0$ as:

$$T = \frac{4E(U_0-E)}{4E(U_0-E)+U_0^2\sinh^2\left[\sqrt{\frac{2mL^2(U_0-E)}{\hbar^2}}\right]} \tag{G.38}$$