T H E S I S

ON

Calculations for Dimensions of Flywheel for Laboratory Motor Generator Set.

Submitted to the Faculty

of the

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Lorain Walter Haller

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APPROVED:

Signature redacted for privacy.

Department of

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Dean School of

It is a fact, recognized by electricians, and especially by those following illuminating engineering, that the intensity of the electric light varies greatly with very small voltage fluctuation.

In the practice of photometry, it is necessary, in order to get accurate measurements, that the voltage used, vary not more than .1 volt. To accomplish this with a single unit, with proper style of prime-mover, (such as steam or water turbine under constant head) would, perhaps, not be difficult, if the load under observation, constituted the connected load. However, to get such results from a commercial circuit, which is connected in parallel with hundreds of lights and motors that are being constantly cut in and out of service, is an entirely different matter.

The foregoing statement is a general outline of the problem which confronted the Electrical Engineering Department of the Oregon Agricultural College, when they were considering the installation of the apparatus needed in the study of photometry.

The laboratory was supplied with electrical power by the Oregon Power Co; at approximately 130 volts pressure, 3 phase, 60 cycles. The equipment available for this purpose, being in the nature of induction motors, A. C. and D. C. generators, and induction motor-generator sets.

This left open three possibilities of proceedure. First, that of installing a separate plant. Second, that of driving a generator by a belt from an induction motor. Third, that of using an induction motor-generator set.

First cost, made the installation of a separate plant impossible, at least for the present.

The room, necessarily taken by a belt driven generator, made this arrangement undesirable, besides, the slipping of the belt would make governing very difficult.

From the standpoint of economy in both cost and floor space, the motor-generator set seemed to offer advantages over the other arrangement, while the possibilities of governing, appeared to be no more difficult.

The necessity of governing, arises from the fact that the voltage output of a generator varies with the speed. The armature of the motor and generator being mounted on the same shaft, the speed of generator follows that of the motor. The speed of the motor varies with the voltage and frequency; i.e., the frequency remaining constant, the speed varies as the square of the voltage, and conversely the voltage remaining constant, the speed varies directly as the frequency. It is therefore evident that, with a supply voltage, varying in the neighborhood of 10 volts, to insure a fluctuation of output of less than .1 volt, requires pretty close governing.

Since an induction motor does not adhere rigidly to synchronous speed; i.e., an increase of load, being accompanied by an increase of slip, hence for any constant load, in order that the armature does not follow an increase or decrease of voltage by a corresponding increase or decrease of speed, it is necessary to impart to the rotating element such moment of inertia as would cause it to continue running under load, with a drop in speed of not more than .5 of 1%, per second, if the power were entirely shut off.

It was therefore decided to design a fly wheel of such proportions as would damp out the fluctuation in speed caused by a fluctuation in voltage of 10 volts for a period of 2 seconds, after which time, the change could be taken care of by a Tirrell regulator.

The machine decided upon, was a Westinghouse Motor-Generator set, The driving side of this machine was a 3.6 H. P. Induction motor, 110 volts, 19.5 amperes per terminal, 3 phase, 60 cycles and ran at 1700 R. P. M. at full load.

The generating side was a 2 K. W., D. C. generator, rated at 125 volts and 16 amperes.

For the purpose of obtaining information for computation, tests were made, one for the slip voltage curve with field open, (#1), and one to determine saturation curve of the generator over the entire range of the machine, (corrected to constant speed).

For the purpose of making this correction, curve #2 was plotted between the limits of open and maximum field. If intermediate readings had been taken, this curve would probably have resembled a porabola. However, the difference in results would have been so slight that it was thought safe to use the straight line. Values in slip R.P.M. column in Table #1 were obtained in this way and used in obtaining values for curve #3.

Table #1.

Voltage of Motors.			Ave.	Slip	Time	Slip
Phase 1	Phase 2	Phase 3	Voltage		in sec.	R.P.M.
124	128.00	127.0	126.3	10	38.2	15.7
104	108.50	107.0	106.5	10	27.4	21.9
97	99.80	98.5	98.1	10	23.0	26.1
81	88.50	90.5	86.6	10	16.0	37.5
62	78.00	80.0	73.3	10	12.5	48.0

Data for Slip voltage curve #

I_f	Volt	Slip R.P.M.	% Slip	E	E_t
3.450	190.0	25.00	1.4050	2.680	192.68
3.250	188.0	23.75	1.3380	2.500	190.50
3.000	184.0	21.75	1.2250	2.260	186.25
2.750	178.0	20.00	1.1250	2.000	180.00
2.500	174.0	18.25	1.0300	1.790	175.80
2.250	166.0	16.25	.9160	1.520	167.50
2.000	158.0	14.50	.8170	1.290	159.30
1.750	149.0	12.75	.7200	1.070	150.00
1.500	138.0	11.00	.6200	.856	138.80
1.250	122.0	9.00	.5070	.620	122.60
1.000	104.0	7.25	.4000	.425	104.40

Table #1. (Continued)

F	Volt	R.P.M.	% Slip	E	E_t
.750	84.0	5.50	.3100	.260	84.25
.500	60.0	3.50	.1980	.118	60.10
.250	33.5	1.50	.0845	.280	33.80
.200	28.0	1.40	.0790	.220	28.20
.150	24.0	1.00	.0565	.135	24.10
.125	22.0	.70	.0390	.087	22.10

Voltage machine would give if kept at 1775 R.P.M. with field current as indicated shown in column E_t

Table #2.

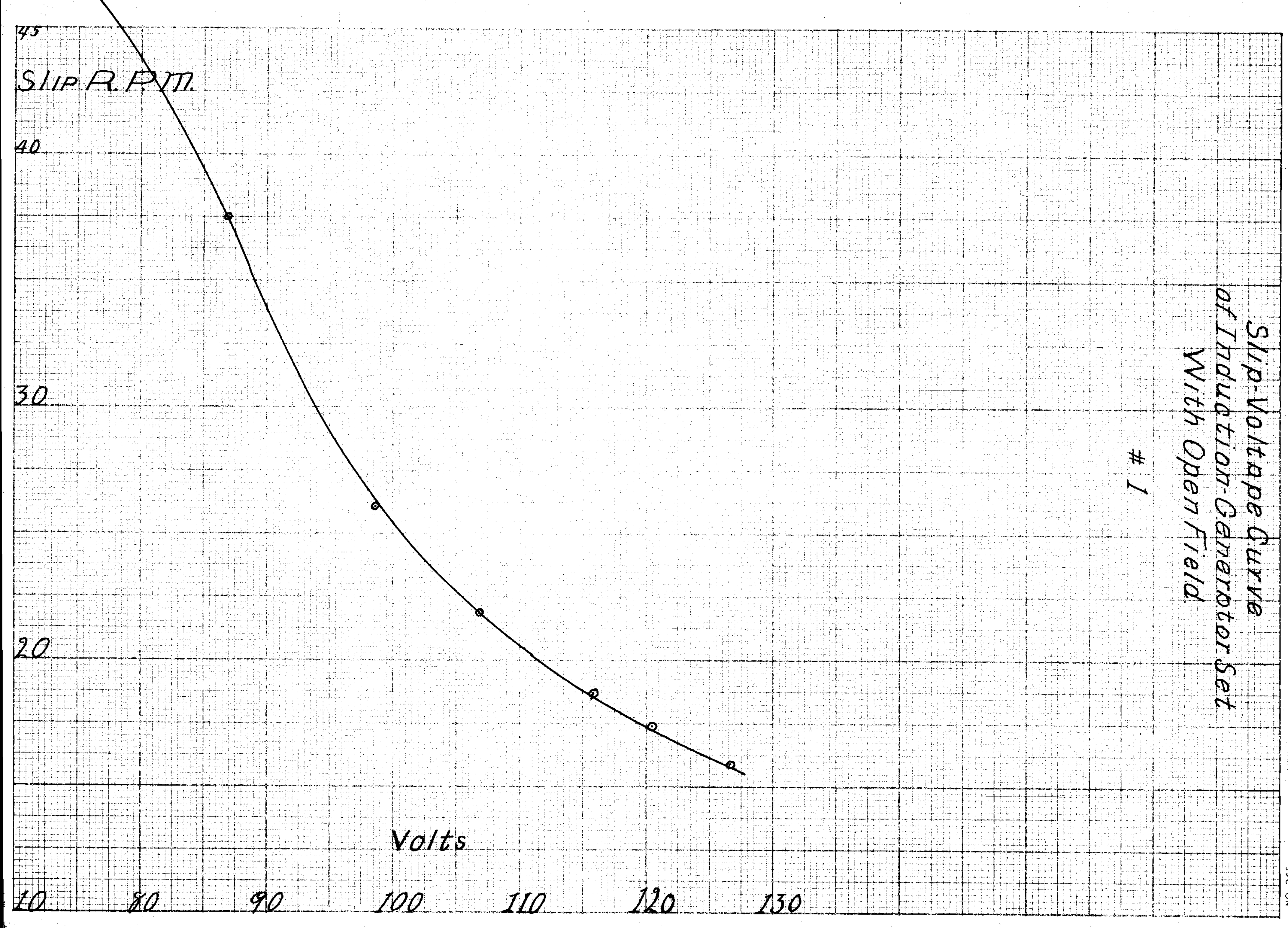
f	Volts	Time	Slip	Slip R.P.M.
56	130	15.0	3	12.00
varying	"	17.5	4	13.75
"	"	21.8	4	11.00
"	"	15.8	3	11.38
"	"	15.0	3	12.00
"	"	8.0	3	22.50
"	"	10.0	4	24.00
"	"	8.0	3	22.50
"	"	14.2	3	12.65
"	"	14.2	3	12.65
60	120	12.0	3	15.00
"	"	12.0	3	15.00
62	110	9.0	3	19.35
"	"	9.0	3	19.35
"	"	8.0	3	21.80
"	"	8.0	3	21.80

This test differs from the first, in that it was taken with 360 watts output from the generator. The frequency varied badly in this test.

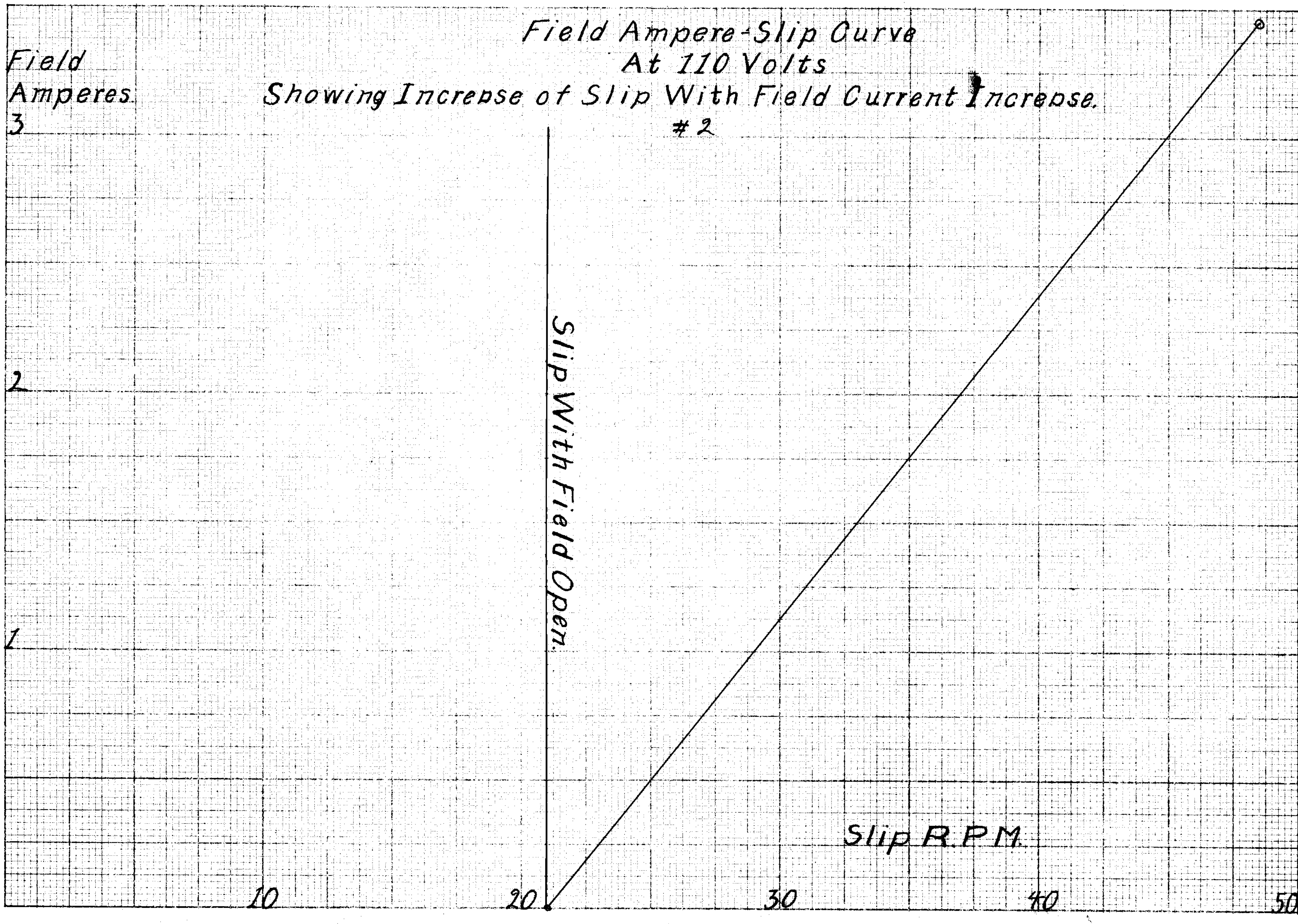
The purpose of plotting curve 1, was to get data from which to determine the change of speed per second, (caused by a sudden increase of 10 volts on the motor. This was providing for the maximum fluctuation) by the formula for kinetic energy, $KE = \frac{1}{2} I (\omega_1^2 - \omega_2^2) = 2\pi \int_{n_2}^{n_1}\int_{T_2}^{T_1} NT$, in which ω_2 was the value to be determined. This solution proved to be practically impossible, owing to the fact that both N & T were accelerated values, since the torque T varies as the square of the voltage and inversely as the speed. An attempt was made to get a solution by substituting in the formula $Y = AX^3 + BX^2 + CX + D$ from the slip voltage curve, but it was soon found that this led into complications, such as would make the method very laborious and results uncertain.

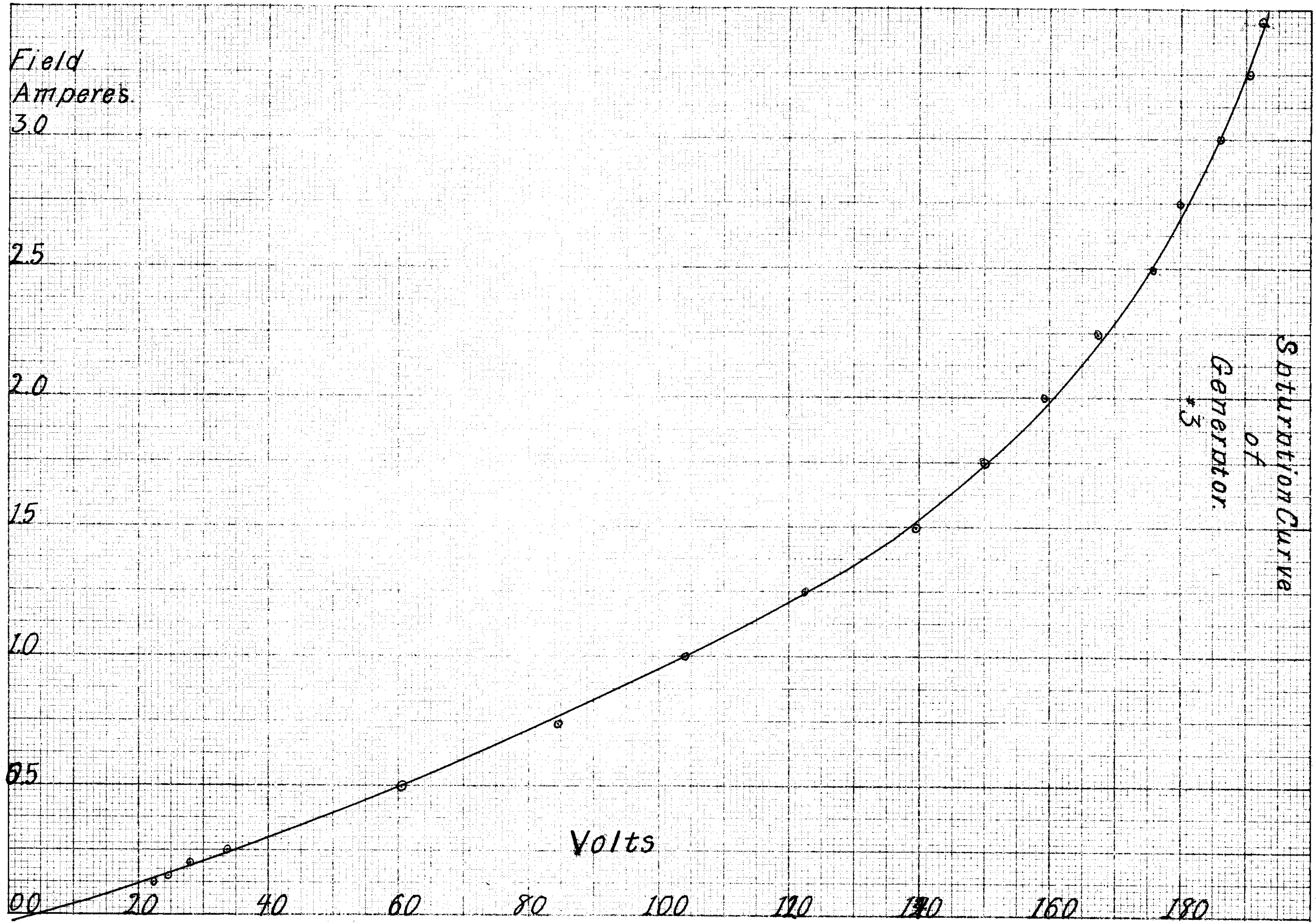
The object of plotting the saturation curve was to get the data, necessary for installing the Tirrill Regulator.

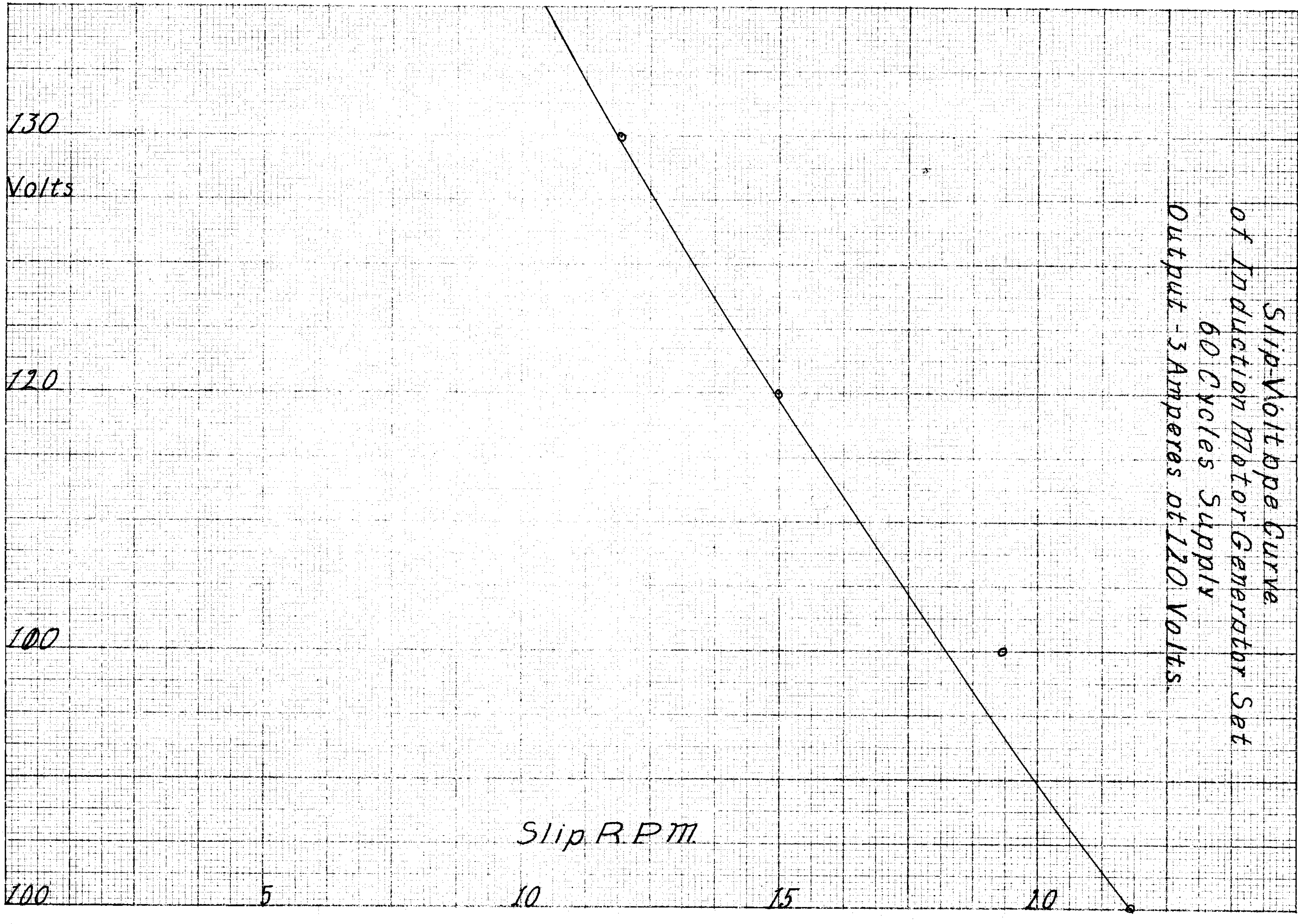
Owing to the unsatisfactory results obtained from the first test, a second test was made, (Table II), this time with 3 amperes load, at 120 volts on the D.C. side, (this being approximately the load used in photometry tests) and it was thought best, in taking the test, to take into account the retarding action, due to generating this power. Curve #4 was plotted from the data obtained



Slip R.P.M.
45
40
30
20
Volts
70
80
90
100
110
120
130
Slip-Voltage Curve
of Induction-Generator Set
With Open Field
1


Field Ampere-Slip Curve
At 110 Volts
Showing Increase of Slip With Field Current Increase.
#2
Field
Amperes.
3
2
1
Slip With Field Open.
Slip R.P.M.
10
20
30
40
50


Field
Amperes.
3.0
2.5
2.0
1.5
1.0
0.5
0.0
20
40
60
80
100
120
140
160
180
Volts
Saturation Curve
of
Generator.
#3


Slip-Voltage Curve
of Induction Motor Generator Set
60 Cycles Supply
Output - 3 Amperes at 120 Volts
130
Volts
120
100
100
5
10
15
20
Slip R.P.M.

in this test, but for the reason given above was worthless so far as obtaining results was concerned.

Table #4.

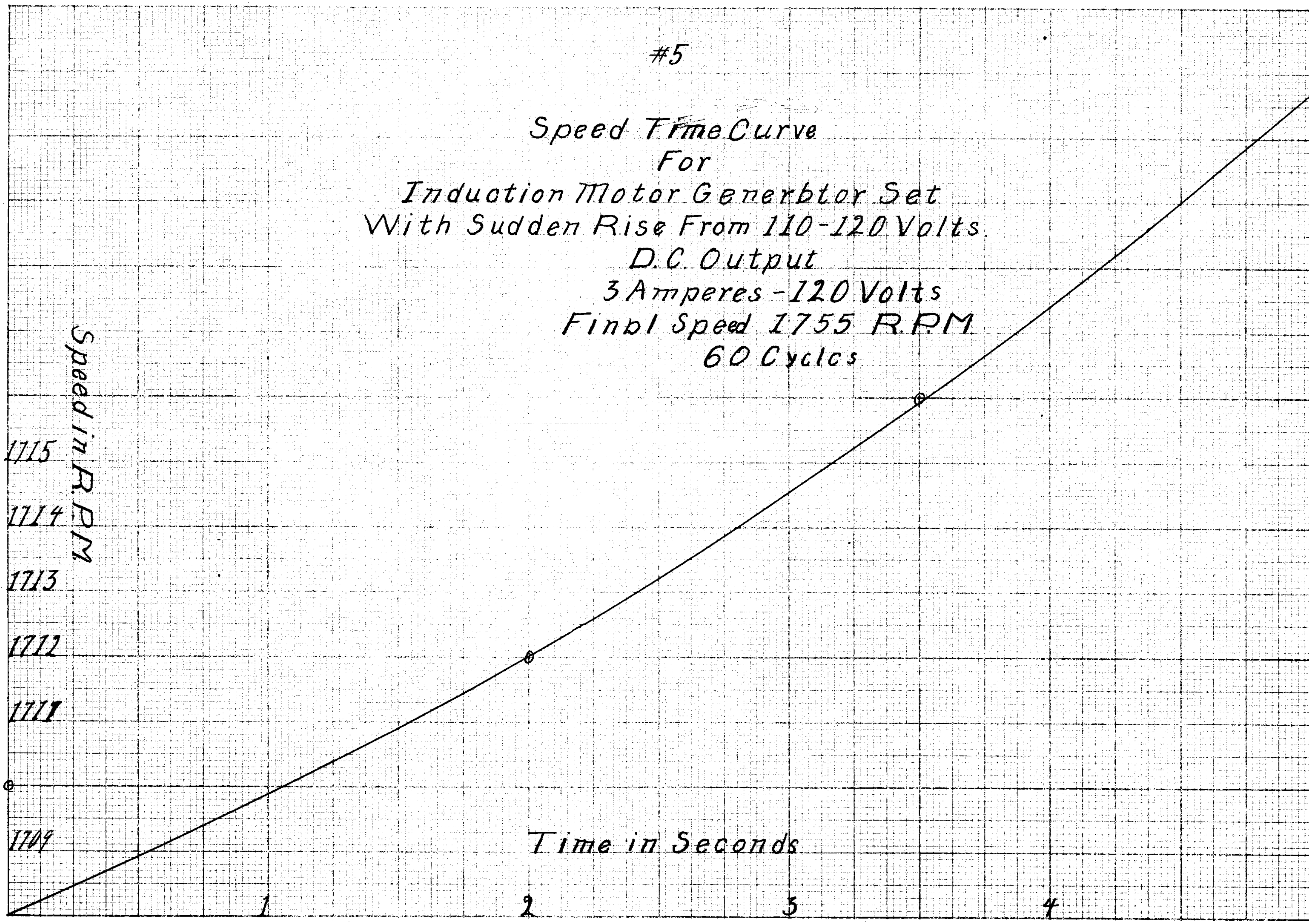
F	Time	Vib.	Cm.	Time in sec.	Belt travel Cm. per sec.	R.P.M.	D.C.
60	0	35	100.0	.2915	343.0	1780	118.0
"	1	"	100.7	. "	345.5	1790	118.0
"	1	"	100.0	. "	343.0	1780	118.0
"	2	"	100.5	. "	345.0	1790	118.5
"	2	"	100.4	. "	345.0	1790	118.5
"	3	"	100.6	. "	345.5	1792	119.5
"	4	"	100.6	. "	345.5	1792	120.0

It was now quite evident, that in order to obtain satisfactory results, that some other method of testing must be resorted to, since the above method gave one equation, containing 3 variables. Apparatus for measuring the increase of speed per second was nowhere available. The nearest approach to apparatus of this nature, to be had, was a tuning fork, which, when charged with 4 volts, direct current, made 120 vibrations per second. A large wheel was set up near the machine, (Fig.1) and driven by a belt, running from the generator shaft over the rim of the wheel, the surface of which was wide enough to permit a coating of Bon Ami to be spread at one side of the belt. 110 volts was then applied to the motor, and when it had reached a steady speed, the voltage was suddenly stepped up to 120, and the tuning fork applied to the surface of the large wheel one second after the rise in voltage. The machine was then shut down, the scratches, made on the rim of the wheel, counted, and the distance covered by the same, measured in centimeters. The apparatus was again started up and similar readings taken at 2, 3 and 4 seconds. These readings, together with the calculated speed, are shown in Table 3. Curve #5 was plotted from these results.

Table 3.

F	Time	Vib	Cm.	Time in sec.	Belt travel Cm. per sec.	R.P.M.
60	0.0	36	99.00	.300	330.0	1710
?	2.0	27	73.25	.225	325.5	1690
?	3.0	36	98.50	.300	328.0	1700
?	4.0	36	97.70	.300	326.0	1690
60	2.0	36	99.20	.300	330.0	1712
60	3.5	36	99.50	.300	331.5	1715
60		35	98.70	.291	339.0	1755

As a means of checking these results, a second experiment was performed, using the same connection the difference being that, instead of reading speed increase, the fluctuation in D;C. voltage was noted, when it was discovered that, immediately following the rise of motor voltage, there was a dip in the generator voltage. This lasted for a period of from ½ to ¾ second, after which it would rise to and above normal in a very short time. The arrangement thus far used had been to drive the alterna-


#5
Speed Time Curve
For
Induction Motor Generator Set
With Sudden Rise From 110-120 Volts
D.C. Output
3 Amperes - 120 Volts
Final Speed 1755 R.P.M
60 Cycles
Speed in R.P.M
1715
1714
1713
1712
1711
1709
Time in Seconds
1
2
3
4

tor, supplying power to the set, by a belt from a D.C. Motor, both of which were separately excited, the voltage feeding the set, being governed by field excitement. The purpose of such an arrangement was to insure constancy of voltage and frequency. Further investigation revealed the fact that the sudden rise in field current caused a slipping of the belt and likewise a falling off of frequency, which was regained usually before the expiration of a second. Thus it was evident that the results obtained in the test were sonewhat low, so it was decided to employ another method.

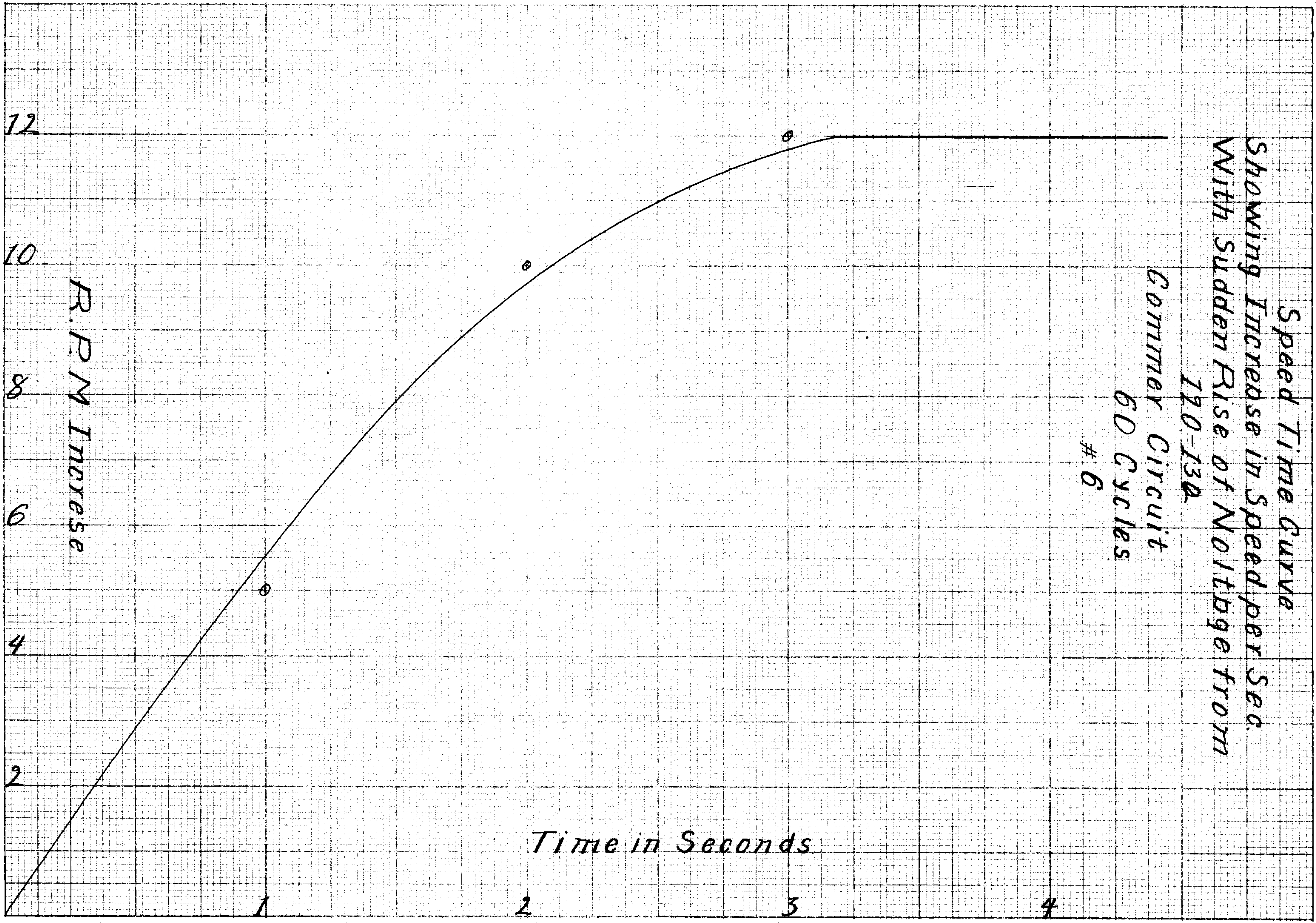
The last test, the data for which, appears in Table #4, was performed by driving the set directly off the commercial circuit. Resistances, sufficient to cause an I R drop of 10 volts, were connected to each phase around a three-pole switch. The machine was allowed to come to a steady speed. The switch was then closed and the tuning fork applied as previously described.

The results thus obtained showed little more than ½ of 1% increase in speed per second with load, stated above. With this information at hand, it was considered safe to make calculations from the basis of 4% drop in speed per second. Calculations were made as follows:-

The weight of the rotating element was 92.5#. Radius of gyration = 5.39 cm. $= \frac{5.39}{30.48} =$ .179 ft.

$I = MK^2 = \frac{92.5}{32.2} \times (.179)^2 =$ .092 biquadratic ft.

$I = 12.6 \times 10^5$ in metric units.


Speed Time Curve
Showing Increase in Speed per Sec.
With Sudden Rise of Voltage from
120-130
Commer Circuit
60 Cycles
#6
R.P.M Increse
Time in Seconds
12
10
8
6
4
2
1
2
3
4

Transferring $I = \frac{12.6 \times 10^5}{456.5 \times (2.54)^2 \times 144 \times 32.2} = .092$

$KE = \frac{1}{2} I \omega^2$, and for different values of ω,

K.E. given up $= \frac{1}{2} I(\omega_1^2 - \omega_2^2)$: With full switch-board voltage of 132 volts, the speed = 1790 R.P.M. with 360 watts output.

$\omega_1 = 2\pi N_1 = 2\pi \frac{1796}{60} = 187. \quad \omega_1^2 = 34970$

With 4% drop, $\omega_2 = .96 \times 187 = 179.5. \quad \omega_2^2 = 32{,}220$

Then $(\omega_1^2 - \omega_2^2) = 2{,}749$

Energy given up per second $= \frac{1}{2} I (2{,}749) = 126.5$ ft.lb

If weight is added to the shaft, the only retarding factors that are increased are the friction and windage, with a smooth wheel of not to exceed 200 lbs. This increase would be negligible, so that for constant output, the falling off of speed, varies inversely as the moment of inertia of the armature, if the driving power were entirely shut off.

Then to see the effect of 10 times present moment of inertia,

$126.5 = .46\ (34970 - \omega_2^2) = 16086 - .46\ \omega_2^2$

$\omega_2^2 = \frac{16086 - 126.5}{.46} = \frac{15960}{.46} = 34695.65$

$\omega_2 = 186.2$, showing a drop of $187 - 186.2 = .8$ radian per second or $\frac{.8}{187} = 0.428$ of 1%.

To take care of a retarding torque, caused by a momentary drop in frequency of the supply main, it was decided to build the wheel as large as the limited space would permit, provided the shaft was capable of carrying it. The amount of clearance necessarily limited the dimensions of the wheel to 3" $\times$ 16", the approximate weight as follows:-

Area in square feet $= \pi r^2 = \pi(\frac{2}{3})^2 = \frac{4\pi}{9}$

Volume $= \frac{1}{4}\,\frac{4\pi}{9} = .349$ Cu. Ft.

Since it was to be made of cast iron, the weight of which = 450 lbs. per Cu. ft., the wt. = .349 X 450 = 157lbs.

To ascertain the safety of placing this wt. on the shaft, calculations were made as follows:

$P_e = \frac{16}{\pi D^3}(.7\,M_b \pm 1.3\sqrt{M_b^2 + M_t^2})$., in which,

$M_b = Gd_2$ = Wt. times the distance from bearings in inches, in this case, 157 X 14.5 = 2280

$M_t = 63030\frac{H}{n}$, where H = maximum HP delivered by the shaft, in this case, 3.6 and N. = number of Revolutions per second, approximately 30.

D = diameter of shaft = 2.25".

Substituting these values.

$P_e = \frac{16}{\pi (2.25)^3}\left[.7 \times 2280 \pm \sqrt{(2280)^2 + (63030 \cdot \frac{3.6}{30})^2}\right] =$

$.447\,(1595 + 7900) = 4240.$

This shows a stress of 4240#per sq. inch on the shaft, which is allowing a factor of safety of 3. True, it does not require that amount of power to drive the wheel, but that amount of power will be exerted on the shaft in revolving the wheel and driving the generator at times.

The moment of inertia of a flat, circular plate $= \frac{\pi r^4}{4}$, and of a plate of thickness t and wt. e $= \frac{e t \pi r^4}{2g4}$, then for this case,

$I = \frac{450 \times \pi (\frac{2}{3})^4}{64.4 \times 4} \times \frac{1}{4} = 1.085.$

Then total I = 1.085 + .092 = 1.175 or 1.175 ÷ .092 = 12.8 times the original moment of inertia.

Up to this time, everything has been provided for except centrifugal force. The peripheral speed for a wheel 16" in diameter, running at 1800 R.P.M. = $2\pi(\frac{2}{3})1800$ = 7,550 ft. per minute. Cast iron fly wheels have been known to go to pieces at 11,000 to 25,000 ft. per minute on actual test. This made the factor of safety about 1.5, which was not considered safe. To obviate any danger from this source, it was decided to make the wheel 14" in diameter, of cast iron, and then shrink a wrought iron band 1" X 3" on the rim of the wheel.

The wrought iron, being slightly heavier than cast iron, the moment of inertia would be slightly increased, due to a lengthening of the radius of gyration.

Checking to determine the speed at the expiration of 1 second, with moment of inertia = 1.175.

$126.5 = 1.175/2\ (34,970 - \omega_2^2)$

$\omega_2^2 = (2054.87 - 126.5)/\ .5875 = 20418.375/5875 =$

34754.68 or $\omega_2 = 186.42$

% drop in speed = (187 − 186.42)/187 = .31 of 1%.

This result should cover any variation in speed which may arise, since the calculation in the first place, was made from a variation 4 times as large as that shown by the test.

For convenience in typewriting the Greek letter ȸ will be used instead of Omega, to represent the angular velocity, also ɵ for unit of wt. gamma.

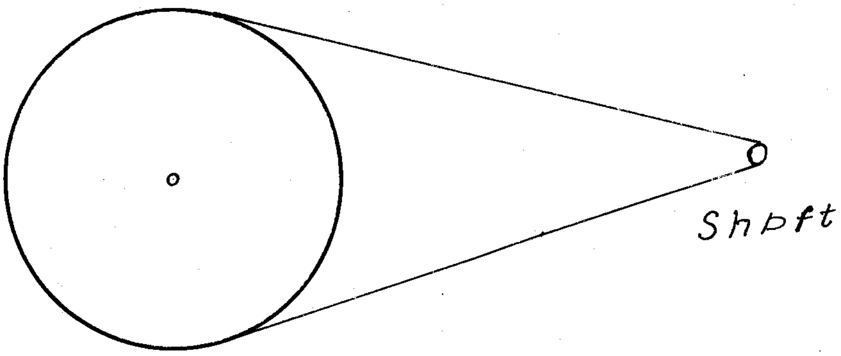


Fig. 1