AN ABSTRACT OF THE THESIS OF

Ming-Kuang Hsu for the degree of Doctor of Philosophy in Civil Engineering presented on October 16, 1986.

Title: Random Wave Forces on Cylinders

Abstract approved: Redacted for Privacy

John H. Nath

The in-line force power spectrum is predicted by the frequency domain maximum force coefficient \[ C_{\mu n}(z,f_n) \] vs. Keulegan-Carpenter number \[ K_n(z,f_n) \] relationship. The frequency domain Keulegan-Carpenter number is defined as the velocity from the amplitude spectrum at \( f_n \) divided by the respective frequency and diameter of the cylinder \( K_n(z,f_n) = u_n(z,f_n)/(f_n \times D) \). When \( K_n(z,f_n) \) is small, the frequency domain maximum force coefficient closely follows the relation \( C_{\mu n}(z,f_n) = 2\pi^2/K_n(z,f_n) \) for a smooth cylinder. The force power spectrum can be estimated from a given wave spectrum by using linear wave theory and the above relation for \( C_{\mu n}(z,f_n) \). The non-dimensionalized rms errors between measured and predicted spectra are used to evaluate the predictions.

The force time history can be predicted from the wave profile and an inverse Fast Fourier Transform method. This method is valid when \( K_n(z,f_n) \) is less than 10 for smooth cylinder. Force time histories predicted by using this method compare reasonably well with measurements.
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Typed by Ming-Kuang Hsu
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## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>2</td>
</tr>
<tr>
<td>2.0</td>
<td>5</td>
</tr>
<tr>
<td>2.1</td>
<td>9</td>
</tr>
<tr>
<td>2.1.1</td>
<td>11</td>
</tr>
<tr>
<td>2.1.2</td>
<td>16</td>
</tr>
<tr>
<td>2.2</td>
<td>23</td>
</tr>
<tr>
<td>2.2.1</td>
<td>27</td>
</tr>
<tr>
<td>2.2.2</td>
<td>28</td>
</tr>
<tr>
<td>2.3</td>
<td>28</td>
</tr>
<tr>
<td>2.4</td>
<td>31</td>
</tr>
<tr>
<td>2.5</td>
<td>31</td>
</tr>
<tr>
<td>2.5.1</td>
<td>32</td>
</tr>
<tr>
<td>2.5.2</td>
<td>32</td>
</tr>
<tr>
<td>3.0</td>
<td>36</td>
</tr>
<tr>
<td>3.1</td>
<td>36</td>
</tr>
<tr>
<td>3.1.1</td>
<td>39</td>
</tr>
<tr>
<td>3.1.2</td>
<td>39</td>
</tr>
<tr>
<td>3.1.3</td>
<td>39</td>
</tr>
<tr>
<td>3.2</td>
<td>43</td>
</tr>
<tr>
<td>3.2.1</td>
<td>43</td>
</tr>
<tr>
<td>3.2.2</td>
<td>46</td>
</tr>
<tr>
<td>4.0</td>
<td>49</td>
</tr>
<tr>
<td>4.1</td>
<td>49</td>
</tr>
<tr>
<td>4.1.1</td>
<td>51</td>
</tr>
<tr>
<td>4.1.2</td>
<td>60</td>
</tr>
<tr>
<td>4.1.3</td>
<td>62</td>
</tr>
<tr>
<td>4.2</td>
<td>66</td>
</tr>
<tr>
<td>4.2.1</td>
<td>68</td>
</tr>
<tr>
<td>4.2.2</td>
<td>74</td>
</tr>
<tr>
<td>4.2.2.1</td>
<td>74</td>
</tr>
<tr>
<td>4.2.2.2</td>
<td>78</td>
</tr>
<tr>
<td>4.3</td>
<td>78</td>
</tr>
<tr>
<td>4.3.1</td>
<td>80</td>
</tr>
<tr>
<td>4.3.2</td>
<td>80</td>
</tr>
<tr>
<td>4.3.3</td>
<td>89</td>
</tr>
<tr>
<td>4.3.3.1</td>
<td>89</td>
</tr>
<tr>
<td>4.3.3.2</td>
<td>94</td>
</tr>
<tr>
<td>4.4</td>
<td>101</td>
</tr>
<tr>
<td>Chapter</td>
<td>page</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>5.0 CONCLUSIONS</td>
<td>107</td>
</tr>
<tr>
<td>6.0 REFERENCES</td>
<td>108</td>
</tr>
<tr>
<td>APPENDIX A Constructing the $C_\mu(K)$ Hyperbola</td>
<td>112</td>
</tr>
<tr>
<td>APPENDIX B The Derivative of $C_\mu(K)$ with Respect to $\alpha$</td>
<td>115</td>
</tr>
<tr>
<td>APPENDIX C Comparison of Force Coefficients Obtained through Different Methods</td>
<td>123</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.1-1</td>
<td>Wave induced hydrodynamic loading regimes as determined by individual size (from Shield and Hudspeth, 1985)</td>
</tr>
<tr>
<td>2.0-1</td>
<td>Definition sketch of wave force on a vertical cylinder</td>
</tr>
<tr>
<td>2.0-2</td>
<td>Definition sketch for the phases and magnitudes of the wave profile, velocity and local force measurements on a vertical cylinder in periodic waves (from Nath, 1985b)</td>
</tr>
<tr>
<td>2.0-3</td>
<td>The Wave Force Project II wave record</td>
</tr>
<tr>
<td>2.1-1</td>
<td>Maximum force coefficient for smooth cylinders as a function of K (from Sarpkaya, 1976)</td>
</tr>
<tr>
<td>2.1-2</td>
<td>Maximum force coefficients for the VSMC12 and VSMC8 Cylinders (from Nath, 1985a)</td>
</tr>
<tr>
<td>2.1-3</td>
<td>Non-dimensionalized phase shift for the VSMC12 and VSMC8 cylinders (from Nath, 1985a)</td>
</tr>
<tr>
<td>2.1-4</td>
<td>Values of $C_{d\phi}$ and $C_{m\phi}$ from Eqs.(2.1-9) and (2.1-10) using the curves shown in Figs. 2.1-2 and 2.1-3. The data points are the least squares values of $C_d$ and $C_m$ for the VSMC12 and VSMC8 cylinders (from Nath, 1985a).</td>
</tr>
<tr>
<td>2.1-5</td>
<td>The maximum force coefficients data points and their hyperbola for the VSMC12 and VSMC8 cylinders</td>
</tr>
<tr>
<td>2.1-6</td>
<td>Non-dimensionalized phase data points and the curve of predicted nondimensionalized phase</td>
</tr>
<tr>
<td>2.1-7</td>
<td>Drag and inertia force coefficients $C_{d\phi}$ and $C_{m\phi}$ calculated by TFPS method then converted into $C_{dL}$ and $C_{mL}$ by using Eqs. (C-8) and (C-9). The data points are the least-squares values of $C_d$ and $C_m$ for the VSMC12 and VSMC8 cylinders</td>
</tr>
<tr>
<td>2.2-1</td>
<td>The amplitude spectrum of WPII record-6887</td>
</tr>
<tr>
<td>2.5.1</td>
<td>Definitions of forces on vertical and horizontal cylinders</td>
</tr>
<tr>
<td>3.1-1</td>
<td>The OSU wave flume (from Nath, 1985a)</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>3.1-2</td>
<td>Sectional elevation of a VSMC12 test cylinder (from Hudspeth et al., 1984)</td>
</tr>
<tr>
<td>3.1-3</td>
<td>Sectional elevation of a vertical 8.625-in. test cylinder (from Nath, 1983)</td>
</tr>
<tr>
<td>3.1-4</td>
<td>Side view of a horizontal 8.625-in. test cylinder (from Nath, 1985a)</td>
</tr>
<tr>
<td>3.2-1</td>
<td>NMI Christchurch Bay Tower side view (from Bishop, 1984a)</td>
</tr>
<tr>
<td>3.2-2</td>
<td>NMI Christchurch Bay Tower top view (from Bishop, 1984a)</td>
</tr>
<tr>
<td>3.2-3</td>
<td>Platform layout of Wave Project II (from Blank, 1969)</td>
</tr>
<tr>
<td>3.2-4</td>
<td>Location of WPII dynamometers and wave staff (from Blank, 1969)</td>
</tr>
<tr>
<td>4.1-1</td>
<td>The $C_{\mu n}(K_n)$ hyperbola and their asymptotes</td>
</tr>
<tr>
<td>4.1-2</td>
<td>The velocity-force cross-amplitude spectrum</td>
</tr>
<tr>
<td>4.1-3</td>
<td>The velocity-force cross-phase spectrum</td>
</tr>
<tr>
<td>4.1-4</td>
<td>The amplitude filter: (a) before filter, (b) after filter</td>
</tr>
<tr>
<td>4.1-5</td>
<td>The band pass frequency filter: (a) before filter, (b) after filter</td>
</tr>
<tr>
<td>4.1-6</td>
<td>The $C_{\mu n}(K_n)$ data points and their regression line (before filter)</td>
</tr>
<tr>
<td>4.1-7</td>
<td>The $C_{\mu n}(K_n)$ data points and their regression line, after passing through an amplitude filter (cut-off amplitude = 0.01 spectral peak)</td>
</tr>
<tr>
<td>4.1-8</td>
<td>The $C_{\mu n}(K_n)$ data points and their regression line, after passing through an amplitude filter (cut-off amplitude = 0.1 spectral peak)</td>
</tr>
<tr>
<td>4.1-9</td>
<td>The $C_{\mu n}(K_n)$ data points and their regression line, after passing through an band pass (0.1 Hz to 1.0 Hz) filter</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>4.1-10</td>
<td>In-line force coefficient vs. K (from Bearman et al., 1985)</td>
</tr>
<tr>
<td>4.1-11</td>
<td>Maximum force coefficient for smooth cylinders as a function of K (from Sarpkaya, 1986b)</td>
</tr>
<tr>
<td>4.1-12</td>
<td>The $C_{un}(K_n)$ data points and their regression lines for NMI large and small cylinder (11 records at three locations)</td>
</tr>
<tr>
<td>4.1-13</td>
<td>Maximum force coefficient for rough cylinders (from Sarpkaya, 1986b)</td>
</tr>
<tr>
<td>4.2-1</td>
<td>In-line force coefficients and phases for the VSMC12 cylinder (Run 32)</td>
</tr>
<tr>
<td>4.2-2</td>
<td>Transverse force coefficients of the VSMC12 cylinder (Run 32)</td>
</tr>
<tr>
<td>4.2-3</td>
<td>In-line force coefficients and phases for the API84 VSRC.02 cylinder (Run 23)</td>
</tr>
<tr>
<td>4.2-4</td>
<td>Transverse force coefficients for the API84 VSRC.02 cylinder (Run 23)</td>
</tr>
<tr>
<td>4.2-5</td>
<td>Horizontal force coefficients for the API84 HSRC.02 cylinder (Run 55)</td>
</tr>
<tr>
<td>4.2-6</td>
<td>Vertical force coefficients for the API84 HSRC.02 cylinder (Run 55)</td>
</tr>
<tr>
<td>4.2-7</td>
<td>In-line force coefficients for the NMI large cylinder level-3 (Record 71)</td>
</tr>
<tr>
<td>4.2-8</td>
<td>In-line force coefficients for the NMI small cylinder level-3 (Record 71)</td>
</tr>
<tr>
<td>4.2-9</td>
<td>In-line force coefficients and phases for the WPII cylinder position-7 (Record 6887)</td>
</tr>
<tr>
<td>4.3-1</td>
<td>Raw in-line force power spectra for the VSMC12 cylinder (pseudo prediction)</td>
</tr>
<tr>
<td>4.3-2</td>
<td>Smoothed in-line force power spectra for the VSMC12 cylinder (pseudo prediction)</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES
(continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3-3</td>
<td>Raw in-line force power spectra for the VSRC.02 cylinder (pseudo prediction)</td>
<td>83</td>
</tr>
<tr>
<td>4.3-4</td>
<td>Smoothed in-line force power spectra for the VSRC.02 cylinder (pseudo prediction)</td>
<td>84</td>
</tr>
<tr>
<td>4.3-5</td>
<td>Raw horizontal force power spectra for the HSRC.02 cylinder (pseudo prediction)</td>
<td>85</td>
</tr>
<tr>
<td>4.3-6</td>
<td>Smoothed horizontal force power spectra for the HSRC.02 cylinder (pseudo prediction)</td>
<td>86</td>
</tr>
<tr>
<td>4.3-7</td>
<td>Raw vertical force power spectra for the HSRC.02 cylinder (pseudo prediction)</td>
<td>87</td>
</tr>
<tr>
<td>4.3-8</td>
<td>Smoothed vertical force power spectra for the HSRC.02 cylinder (pseudo prediction)</td>
<td>88</td>
</tr>
<tr>
<td>4.3-9</td>
<td>Raw in-line force power spectra for the VSMC12 cylinder (true prediction)</td>
<td>90</td>
</tr>
<tr>
<td>4.3-10</td>
<td>Smoothed in-line force power spectra for the VSMC12 cylinder (true prediction)</td>
<td>91</td>
</tr>
<tr>
<td>4.3-11</td>
<td>In-line force power spectra for the NMI large cylinder level-3, record-71 (pseudo prediction)</td>
<td>92</td>
</tr>
<tr>
<td>4.3-12</td>
<td>In-line force power spectra for the NMI small cylinder level-3, record-71 (pseudo prediction)</td>
<td>93</td>
</tr>
<tr>
<td>4.3-13</td>
<td>In-line force power spectra for the NMI large cylinder level-3, Record-71 (true prediction)</td>
<td>95</td>
</tr>
<tr>
<td>4.3-14</td>
<td>In-line force power spectra for the NMI small cylinder level-3, Record-71 (true prediction)</td>
<td>96</td>
</tr>
<tr>
<td>4.3-15</td>
<td>Raw in-line force power spectra for the WPII cylinder position-7, record-6887 (pseudo prediction)</td>
<td>97</td>
</tr>
<tr>
<td>4.3-16</td>
<td>Smoothed in-line force power spectra for the WPII cylinder, position-7 record-6887 (pseudo prediction)</td>
<td>98</td>
</tr>
<tr>
<td>4.3-17</td>
<td>Raw in-line force power spectra for the WPII cylinder, position-7, record-6887 (true prediction)</td>
<td>99</td>
</tr>
</tbody>
</table>
LIST OF FIGURES
(continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3-18</td>
<td>Smoothed in-line force power spectra for the WPII cylinder, position-7, record-6887 (true prediction)</td>
</tr>
<tr>
<td>4.4-1</td>
<td>The wave profile for the VSMC12 cylinder test</td>
</tr>
<tr>
<td>4.4-2</td>
<td>The measured and predicted in-line force time history for the VSMC12 cylinder (true prediction)</td>
</tr>
<tr>
<td>4.4-3</td>
<td>The wave profile record during Hurricane Carla (record-6887)</td>
</tr>
<tr>
<td>4.4-4</td>
<td>The measured and predicted in-line force time history for the WPII cylinder at position-7 (true prediction)</td>
</tr>
<tr>
<td>A-1</td>
<td>The $C_{\mu}$ hyperbola and the coordinate systems</td>
</tr>
<tr>
<td>B-1</td>
<td>$C_{df}$ and $C_{mf}$ versus $K$ for $\beta = 4480$ (from Sarpkaya, 1976)</td>
</tr>
<tr>
<td>B-2</td>
<td>$C_{df}$ and $C_{mf}$ versus $K$ for $\beta = 5260$ (from Sarpkaya, 1976)</td>
</tr>
<tr>
<td>B-3</td>
<td>$C_{d\phi}$ and $C_{m\phi}$ versus $K$ for $\beta = 4480$, converted from $C_{df}$ and $C_{mf}$ curves (Fig B-1) by using Eqs. (C-12) and (C-13)</td>
</tr>
<tr>
<td>B-4</td>
<td>$C_{d\phi}$ and $C_{m\phi}$ versus $K$ for $\beta = 5260$, converted from $C_{df}$ and $C_{mf}$ curves (Fig B-2) by using Eqs. (C-12) and (C-13)</td>
</tr>
<tr>
<td>C-1</td>
<td>Drag coefficients from different methods</td>
</tr>
<tr>
<td>C-2</td>
<td>Inertia coefficients from different methods</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>2.5-1</td>
<td>Summary of frequency domain maximum force coefficients, Keulegan-Carpenter numbers and phases</td>
</tr>
<tr>
<td>3.0-1</td>
<td>Summary of the test conditions</td>
</tr>
<tr>
<td>B-1</td>
<td>The values of the derivative of ( C_\mu(K) )</td>
</tr>
<tr>
<td>C-1</td>
<td>Force coefficients from different methods</td>
</tr>
<tr>
<td>NOTATION</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>$C_d(z)$</td>
<td>Drag coefficient for Morison equation</td>
</tr>
<tr>
<td>$C_{ds}$</td>
<td>Drag force coefficient of steady state flow</td>
</tr>
<tr>
<td>$C_{dF}(z)$</td>
<td>Drag force coefficient determined by Fourier analysis</td>
</tr>
<tr>
<td>$C_{dL}(z)$</td>
<td>Drag force coefficient determined by the least-square method</td>
</tr>
<tr>
<td>$C_{d\phi}(z)$</td>
<td>$[= C_{d\phi}(K)]$ Drag force coefficient determined by the force-phase method</td>
</tr>
<tr>
<td>$C_{Frms}(z)$</td>
<td>Root mean square of the maximum force coefficient</td>
</tr>
<tr>
<td>$C_m(z)$</td>
<td>Inertia coefficient for Morison equation</td>
</tr>
<tr>
<td>$C_{mF}(z)$</td>
<td>Inertia force coefficient determined by Fourier analysis</td>
</tr>
<tr>
<td>$C_{mL}(z)$</td>
<td>Inertia force coefficient determined by the least-squares method</td>
</tr>
<tr>
<td>$C_{m\phi}(z)$</td>
<td>$[= C_{m\phi}(K)]$ Inertia force coefficient determined by the force-phase method</td>
</tr>
<tr>
<td>$C_T(z)$</td>
<td>Total force coefficient, see Eq. (2.1-1)</td>
</tr>
<tr>
<td>$C_{xy}(f_n)$</td>
<td>Coincident spectral density function</td>
</tr>
<tr>
<td>$C_\mu(z)$</td>
<td>$[= C_\mu(K)]$ Maximum in-line force coefficient of vertical cylinder</td>
</tr>
<tr>
<td>$C_{\mu n}(z,f_n)$</td>
<td>$[= C_{\mu n}(K_n)]$ Frequency domain maximum force coefficient for frequency $f_n(K_n)$</td>
</tr>
<tr>
<td>$C_{H\mu n}(z)$</td>
<td>Maximum horizontal force coefficient of a horizontal cylinder</td>
</tr>
<tr>
<td>$C_{T\mu n}(z)$</td>
<td>Maximum transverse force coefficient of a vertical cylinder, calculated by in-line horizontal velocities</td>
</tr>
<tr>
<td>$C_{V\mu n}(z)$</td>
<td>Maximum vertical force coefficient of a horizontal cylinder</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter of a smooth cylinder</td>
</tr>
<tr>
<td>$\bar{E}^2$</td>
<td>Mean square error between measured and predicted forces</td>
</tr>
</tbody>
</table>
\( E_{\text{rms}} \) Non-dimensionalized rms errors

\( F(z,t) \) Force per unit of length

\( \hat{F}(z,t) \) Non-dimensionalized force

\( F_m(z) \) Measured force (Appendix C)

\( \hat{F}_m(z,t) \) Non-dimensionalized measured force (Appendix C)

\( F_n(z,f_n) \) Amplitude of in-line force amplitude spectrum for frequency \( f_n \)

\( F_P(z,t) \) Predicted force time history

\( \hat{F}_P(z,t) \) Non-dimensionalized measured force (Appendix C)

\( F_{\mu}(z) \) Maximum force during a wave cycle

\( F^H_n(z,f_n) \) Force amplitude in the horizontal direction (horizontal cylinder)

\( F^V_n(z,f_n) \) Force amplitude in the vertical direction (horizontal cylinder)

\( F^T_n(z,f_n) \) Force amplitude in the transverse direction (vertical cylinder)

\( F_{\mu}(z) \) Nondimensionalized maximum force

\( G_x(f_n) \) Power spectral density function for \( X(t) \)

\( G_{xy}(f_n) \) Cross-amplitude spectrum for \( X(t) \) and \( Y(t) \)

\( G_y(f_n) \) Power spectral density function for \( Y(t) \)

\( H \) Wave height

\( H_n(f_n) \) Transfer function

\( K(z) \) Keulegan-Carpenter number

\( K_i \) K value of the intersection of two asymptotes \( L_1 \) and \( L_2 \)

\( K_n(z,f_n) \) Frequency domain Keulegan-Carpenter number

\( L \) Wave length

\( L_0 \) Deep water wave length

\( L_1 \) Slant asymptote
\[ L_2 \] Horizontal asymptote

\[ N \] Total number of components

\[ P = \frac{d C_d}{d \alpha} \]

\[ Q = \frac{d}{d \alpha} \left( \frac{C_{d' \phi}^2}{C_{d \phi}} \right) \]

\[ Q_{xy}(f_n) \] Quadrature spectral density function

\[ R \] Reynolds number

\[ S_{AA}(z,f_n) \] Water particle acceleration power spectrum

\[ S_{FF}(z,f_n) \] Wave force power spectrum

\[ [S_{FF}(z,f_n)_m]_\mu \] Wave force power spectral peak

\[ S_{FF}(z,f_n)_m \] Measured force power spectrum

\[ S_{FF}(z,f_n)_p \] Predicted force power spectrum

\[ S_{uu}(z,f_n) \] Horizontal water particle velocity power spectrum

\[ S_{vv}(z,f_n) \] Water particle velocity power spectrum

\[ S_{uF}(f_n) \] Velocity-force power spectrum

\[ T \] Wave period

\[ X(t) \] Time series

\[ Y(t) \] Time series

\[ X \] Axis (Appendix A)

\[ Y \] Axis (Appendix A)

\[ \overline{X} \] Axis (Appendix A)

\[ \overline{Y} \] Axis (Appendix A)

\[ X' \] Axis (Appendix A)

\[ Y' \] Axis (Appendix A)

\[ a \] Wave amplitude [transverse axis (Appendix A)]
amplitude of wave amplitude spectrum

b

transverse axis (Appendix A)

d

water depth

f

frequency

f_1

fundamental frequency

f_n

n \times f_1

j

imaginary number

m(K)

see Eq. (2.1-18)

p

\[-(1 - \sin^2(\phi)) \left[ \frac{d}{dK} \left[ \frac{2}{\pi^2} \right] \right] \] (Appendix B)

q

\[-\left[ \frac{d}{dK} \right]\left[ \frac{2}{\pi^2} \right] \] (Appendix B)

s

water particle orbit size

t

time

u(z,t)

water particle velocity in x direction

\ddot{u}(z,t)

water particle acceleration in x direction

u_{\mu}(z)

maximum water particle velocity in x direction during a wave cycle

u_n(z,f_n)

water particle velocity spectrum in x direction

v(z,t)

water particle velocity in y direction

v_n(z,f_n)

water particle velocity spectrum in y direction

w(z,t)

water particle velocity in z direction

w_n(z,f_n)

water particle velocity spectrum in z direction

x

axis

y

axis

z

distance upward from the still water surface

\Gamma(z)

phase, see Eq. (2.1-2)
\( \Delta f \) Frequency increment

\( \phi_n(f_n) \) Phase

\[ \frac{1}{4} \pi \]

\( \alpha(K) \) [\( = \frac{K}{K^2} \)]

\( \beta \) [= R/K] Frequency parameter

\( \delta \) Angle (Appendix A)

\( \varepsilon \) Relative roughness of the cylinder

\( \eta \) Wave profile

\( \theta \) [= \( \omega t \)]

\( \theta_n(f_n) \) Phase of wave phase spectrum with frequency \( f_n \)

\( \theta_{xy}(f_n) \) Cross-phase spectrum between \( X(t) \) and \( Y(t) \)

\( \tau \) Down-crossing wave period

\( \nu \) Kinematic viscosity

\( \rho \) Mass density of fluid

\( \phi(z) \) Phase angle between peak velocity and peak force in a wave cycle

\( \phi_a(z) \) Phase angle between peak velocity and peak acceleration in a wave cycle

\( \hat{\phi}(z) \) [= \( \phi/(-90^\circ) \)] Non-dimensionalized phase

\( \tilde{\phi}(z) \) [= \( \phi/\phi_a \)] Non-dimensionalized phase

\( \phi_n(z,f_n) \) Phase between \( F_n(z,f_n) \) and \( u_n(z,f_n) \)

\( \phi_n^H(z,f_n) \) Phase between \( F_n^H(z,f_n) \) and \( u_n(z,f_n) \)

\( \phi_n^T(z,f_n) \) Phase between \( F_n^T(z,f_n) \) and \( v_n(z,f_n) \)

\( \phi_n^V(z,f_n) \) Phase between \( F_n^V(z,f_n) \) and \( w_n(z,f_n) \)

\( \omega \) [= \( 2\pi/T \)] Angular frequency

\( \omega_n \) [= \( 2\pi f_n \)]
### Superscripts

<table>
<thead>
<tr>
<th>Superscript</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>F</td>
<td>Fourier analysis method</td>
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<tr>
<td>L</td>
<td>Least-squares method</td>
</tr>
<tr>
<td>d</td>
<td>Drag force</td>
</tr>
<tr>
<td>m</td>
<td>Inertia force (or measured quantity)</td>
</tr>
<tr>
<td>n</td>
<td>nth component</td>
</tr>
<tr>
<td>p</td>
<td>Predicted (forces, velocities, etc.)</td>
</tr>
<tr>
<td>u</td>
<td>Maximum value (forces, velocities, etc.)</td>
</tr>
<tr>
<td>φ</td>
<td>Force-phase method</td>
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</tbody>
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### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>API84</td>
<td>American Petroleum Institute Project-84</td>
</tr>
<tr>
<td>ARMC</td>
<td>Artificial Marine Roughened Cylinder</td>
</tr>
<tr>
<td>ASCE</td>
<td>American Society of Civil Engineering</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FFPS</td>
<td>Frequency Domain Force-Phase-Slope method</td>
</tr>
<tr>
<td>FPPφ</td>
<td>Frequency domain Force-Phase-φ method</td>
</tr>
<tr>
<td>NCEL</td>
<td>Navy Civil Engineering Laboratory</td>
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<tr>
<td>NMI</td>
<td>National Maritime Institute</td>
</tr>
<tr>
<td>OSU</td>
<td>Oregon State University</td>
</tr>
<tr>
<td>OTC</td>
<td>Offshore Technology Conference</td>
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<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>TFPS</td>
<td>Frequency domain Force-Phase-Slope method</td>
</tr>
<tr>
<td>TFPφ</td>
<td>Frequency domain Force-Phase-φ method</td>
</tr>
<tr>
<td>VSMC8</td>
<td>Vertical 8.625-in. Diameter Smooth Cylinder</td>
</tr>
<tr>
<td>VSMC12</td>
<td>Vertical 12.75-in. Diameter Smooth Cylinder</td>
</tr>
</tbody>
</table>
V(H)SP3-55D Vertical (Horizontal) Dried Cylinder from the South Pass region of Gulf of Mexico. It was placed 55 feet below the sea surface for 3 years.

V(H)SRC.02 Vertical (Horizontal) Sand Roughened Cylinder ($c = 0.023$)

WPII Wave Force Project II
1.0 INTRODUCTION

1.1 Predicting Ocean Wave Forces

Offshore drilling platforms were first used in the Gulf of Mexico in the late 1940's. Since then a wide variety of platforms has been built which are located in increasing depths and subjected to severe environmental conditions. Forces acting on the platforms must be predicted accurately to design safe, economical platforms for these severe environments.

Wave forces on offshore structures are generally calculated by two different methods; (1) the Morison equation, and (2) the diffraction theory (Sarpkaya and Isaacson, 1981; Dean and Dalrymple, 1984; Shields and Hudspeth, 1985; Chakrabarti, 1985).

The Morison equation is used when the structure exhibits no sensible effect on the wave field, and waves passing the structure remain essentially unmodified by the presence of that structure. The Morison equation assumes that the force on the structure is the sum of a drag force and an inertia force which are 90° out of phase for linear waves. Experimentally determined drag and inertia coefficients are required.

When the size of the structure is relatively large compared with the wave length, the body will substantially alter the wave field in the vicinity of the structure, thus influencing the forces on the structure. In this case, the diffraction of the waves from the surface of the structure must be taken into account, and the diffraction
theory should be applied. The effects of vortex shedding are usually neglected.

Shields and Hudspeth (1985) provided a useful plot Fig. 1.1-1 to help engineers quickly assess the appropriate technique for determination of the wave-induced hydrodynamic loads.


In this thesis only cylinders which belong to the Morison equation regime are studied (i.e. cylinders with a diameter to wave length ratio of 1/5, or smaller). A new method which can be used to estimate the random wave forces is introduced. Using this method, the force spectra and force time series can be predicted for a given wave profile.

1.2 Scope

Three problems are addressed: 1) the estimation of force power spectra; 2) the prediction of force time history; and 3) the determination of force coefficients for the frequency domain Morison equation.
Fig. 1.1-1 Wave induced hydrodynamic loading regimes as determined by individual size (from Shield and Hudspeth, 1985).
The time domain force-phase method described by Nath (1985a) is extended to the frequency domain to solve these problems. The basic assumptions used in this method are: 1) the cylinder is rigid; 2) the Morison equation regime; 3) linear wave theory; 4) linear superposition for water particle velocities, accelerations, and forces; 5) the forces in one frequency were induced by the wave kinematics in that frequency only; 6) no current; and 7) the axes of vertical (horizontal) cylinders are perpendicular (parallel) to the wave crest.

Laboratory and field data are used in this study. Laboratory data were obtained from wave tank experiments performed at Oregon State University. Smooth and rough cylinders in both the vertical and horizontal positions were studied. Field data came from the Wave Force Project II and Second Christchurch Bay Tower projects. The field cylinders were vertical, and they were all considered to be smooth.

The predicted force power spectra are compared to actual laboratory and field data, and they are also compared with predictions obtained by the Borgman (1967a) method. The predicted force time histories are compared to measured data.
2.0 THEORETICAL CONSIDERATIONS FOR OCEAN WAVE FORCES

A definition sketch of wave forces on a small diameter vertical cylinder is shown in Fig. 2.0-1. The coordinate system is located at the still water level. The y axis is horizontal and perpendicular to both the x and the z axes. Water particle velocities in the x, y, and z directions are denoted as u, v, and w. The cylinder is located at x = 0, and the wave is traveling to the right. The force shown in the figure is the in-line force.

The in-line force per unit length at position z can be expressed approximately with the Morison equation (Morison et al., 1950):

\[ F(z,t) = C_d(z) \frac{\rho D}{2} u(z,t) u(z,t) + C_m(z) \frac{\rho D^2}{4} \frac{\partial u(z,t)}{\partial t} \]  

(2.0-1)

where D is the diameter of the cylinder, \( \rho \) is the mass density of the fluid, \( u(z,t) \) is the horizontal water particle velocity, \( \dot{u}(z,t) \) is the time derivative of the water particle velocity, and \( C_d(z) \) and \( C_m(z) \) are drag and inertia force coefficients, respectively. Relationships between the wave profile, the horizontal water particle velocity, and the in-line force are shown in Fig. 2.0-2 for an idealized record.

One problem involved in using the Morison equation is the determination of the force coefficients \( C_d(z) \) and \( C_m(z) \). The least-squares method is usually used with short data samples. Data samples can be the whole wave cycle or only a part of it, such as the wave trough or the wave crest. Coefficients are then determined by minimizing the mean square error between predicted and measured forces. In regular waves, the force coefficients determined in this way often
Fig. 2.0-1 Definition sketch of wave force on a vertical cylinder.
Fig. 2.0-2 Definition sketch for the phases and magnitudes of the wave profile, velocity, and local force measurements on a vertical cylinder in periodic waves (from Nath, 1985b).
vary from wave cycle to wave cycle. They are often averaged over several wave cycles.

Force coefficients are sensitive to the variation of the phase, $\phi(z)$, which is the time lag between peak velocity and peak force. A small change in $\phi(z)$ induces a large change in the force coefficients (see Hudspeth et al., 1986). Small phase variations from wave cycle to cycle cause variations in vortex shedding in regular laboratory wave tests. Nonlinear wave effects, the return current, and other extraneous effects also cause variations. Small phase variations are impossible to prevent, and so are the relatively large variations of force coefficients.

To improve force predictions, a phase-insensitive parameter, the maximum force coefficient, $C_{\mu}(z)$, is introduced.\(^1\) For a unit length of cylinder, it is defined as

$$ C_{\mu}(z) = \frac{F_{\mu}(z)}{\frac{1}{2} \rho Du_{\mu}^2(z)} $$

where $F_{\mu}(z)$ is the maximum force per unit length on a cylinder in a wave cycle and $u_{\mu}(z)$ is the maximum ambient undisturbed horizontal velocity in the wave cycle.

The Keulegan-Carpenter (1958) number $K(z)$ is an important parameter in this thesis. The time domain Keulegan-Carpenter number is defined as

$$ K(z) = \frac{u_{\mu}(z) T}{D} $$

\(^1\)The subscript, $\mu$, will be used to indicate maximum values throughout this thesis.
for regular waves, where \( T \) is the wave period. For random waves the time domain Keulegan-Carpenter number is defined as

\[
K_T(z) = \frac{u(z) \tau}{\mu D}
\]  

(2.0-4)

where \( \tau \) is the down-crossing wave period of an individual wave selected from a random wave record. It may also be defined as the time interval between two up-crossings. The \( K_T(z) \) values determined by this definition usually yield large numbers. For example, the largest individual wave recorded during Hurricane Carla for Wave Force Project II (Blank, 1969) had a wave height of about 38 ft with a period of about 13 sec, Fig. 2.0-3. The water depth is about 98 ft. For a 1-foot diameter riser, the \( K_T \) of this individual wave is approximately 160. For 3-ft and 8-ft cylinders, the \( K_T \) of this individual wave is about 53 and 20, respectively.

2.1 The Time Domain Force-Phase Method

When Starsmore (1981) studied field data from the Christchurch Bay Tower and laboratory data collected by Keulegan and Carpenter, he found that the peak force correlated well with the Reynolds number and the Keulegan-Carpenter number. In order to obtain good correlations for the phases, he had to average several cycles. He also found that the force coefficients of the Morison equation could be replaced by total force coefficient \( C_T(z) \) and phase angle \( \Gamma(z) \). The \( C_T(z) \) was defined as

\[
C_T(z) = \sqrt{C_m^2(z) + \left( \frac{C_d(z)K(z)}{\pi^2} \right)^2}.
\]  

(2.1-1)
Fig. 2.0-3 The Wave Force Project II wave record.

WPII record-6887

$H = 38 \text{ ft.}$
$
\tau = 13 \text{ sec.}$
The $C_T(z)$ is closely related to the $F_U(z)$. The $\Gamma(z)$ was defined as

$$\Gamma(z) = \tan^{-1} \left[ \frac{C_m(z) \pi^2}{C_d(z) K(z)} \right] = \tan^{-1} \left[ 2 \cos \left( \frac{\pi}{2} - \frac{2\phi(z) u(z)}{s} \right) \right]$$

(2.1-2)

where $s$ is the water particle orbit size. Thus, the effects of phase $\phi(z)$ can be examined separately from the effects of peak force $F_U(z)$. If the force coefficients $C_d(z)$ and $C_m(z)$ are needed, they can be obtained by recombining the peak force with a statistically stationary phase angle $\Gamma(z)$.

Nath (1984b, 1985a) used $C_\mu(z)$ and $\phi(z)$ to develop the force-phase method. His method is more straightforward and it has been used to analyze regular wave forces on vertical cylinders with surfaces of various roughness. Nath's ideas are extended here and applied to the analysis of random wave forces in the frequency domain.

Two time domain force-phase methods for simple harmonic waves will be reviewed before extending them to the frequency domain.

2.1.1 The Time Domain Force-Phase-$\phi$ Method

From linear wave theory (Ippen, 1966), for the coordinate system shown in Fig. 2.1-1, the wave profile $\eta(t)$ can be expressed as

$$\eta(t) = a \cos \theta$$

(2.1-3)

where $a$ is the amplitude, $\theta = \omega t$, and $\omega = 2\pi/T$. (Where $t$ is time).

When the cylinder is absent, linear wave theory shows that horizontal water particle velocity is
The horizontal water particle acceleration is

\[ u(z,t) = u_\mu(z) \cos \theta \]  \hspace{1cm} (2.1-4)

\[ \dot{u}(z,t) = -\frac{2\pi}{T} u_\mu(z) \sin \theta \]  \hspace{1cm} (2.1-5)

Substituting Eqs. (2.1-4) and (2.1-5) into Eq. (2.0-1), and nondimensionalized by dividing both sides by \( \frac{1}{2} \rho D u_\mu^2(z) \), we obtained the nondimensionalized force \( \hat{F}(z,t) \) as

\[ \hat{F}(z,t) = C_d(z) \cos \theta \cos \theta - \frac{\pi^2}{K(z)} C_m(z) \sin \theta \]  \hspace{1cm} (2.1-6)

The phase of \( \theta \) at which \( \hat{F}(z,t) \) is maximum is designated as \( \phi(z) \), as shown in Fig. 2.0-2. It can be obtained by setting \( d\hat{F}/d\theta = 0 \) and solving for the phase. Thus

\[ \phi(z) = -\sin^{-1}\left[ \frac{\pi^2 \left( C_m(z) \right)}{2K(z) C_{d\phi}(z)} \right] , \]  \hspace{1cm} (2.1-7)

where \( C_{m\phi}(z) \) and \( C_{d\phi}(z) \) are force coefficients determined by the force-phase method. When positive maximum forces are considered, \( \phi(z) \) is between 0° and -90°. In this thesis, phases shown in the figures have usually been nondimensionalized by divided by -90°. They are designated as \( \hat{\phi}(z) \).

Substituting Eq. (2.1-7) into Eq. (2.1-6) and letting \( \cos^2 \phi(z) = 1 - \sin^2 \phi(z) \), the nondimensionalized maximum force can be expressed as

\[ C_\mu(z) = C_{d\phi}(z)[1-\sin^2 \phi(z)] - \frac{\pi^2}{K(z)} C_{m\phi}(z) \sin \phi(z) \]  \hspace{1cm} (2.1-8)

The force coefficients \( C_{d\phi}(z) \) and \( C_{m\phi}(z) \) can be obtained from Eqs. (2.1-7) and (2.1-8) as follows:
\[ C_d(\phi) = \frac{C(z)}{1 + \sin^2 \phi(z)} \quad (2.1-9) \]

and

\[ C_{m\phi}(z) = \frac{2K(z) C(z)}{\pi^2} \frac{-\sin\phi(z)}{1 + \sin^2 \phi(z)} . \quad (2.1-10) \]

Note that \( C_u(z), \phi(z), C_d(\phi), \) and \( C_{m\phi}(z) \) are functions of \( K(z) \) and \( R(z) \). To emphasize their relationships with \( K(z) \), it is sometimes more suitable to express them in terms of \( K(z) \) as \( C_u(K), \phi(K), C_d(\phi)(K), \) and \( C_{m\phi}(K) \).

Now consider the extreme conditions. When \( K(z) \) is very small, e.g. less than 8, the drag term of Eq. (2.1-8) is small compared to the inertia term, and it can be neglected. The maximum force occurs when \( \phi(K) \) approaches \(-90^\circ\), and Eq. (2.1-8) can be simplified to

\[ C_u(K) = \frac{1}{2} C_{m\phi}(K) \quad (2.1-11) \]

Assume that when \( K \) is small the flow field of an oscillating flow is similar to a wavy flow. Then according to Sarpkaya (1986a), the Stokes force on a fixed circular cylinder in a sinusoidally oscillating flow can be expressed in terms of a linearized Morison equation. That is, when \( K \ll 1 \) and \( \beta (= R/K) \gg 1 \) the inertia force coefficient can be expressed as

\[ C_m = 2 + 4(\pi\beta)^{-1/2} + (\pi\beta)^{-3/2} \quad (2.1-12) \]

When \( \beta \) is large, (e.g. larger than 5000), \( C_m \) approaches 2. Herein it is assumed that when \( \beta \) is large and \( K \) is small, \( C_{m\phi}(K) \) is equal to 2. By taking the log of both sides of Eq. (2.1-11), we obtain
\[
\log C_u(K) = \log(2\pi^2) - \log K .
\]  
\[(2.1-13)\]

In a log-log plot, \( C_u(K) \) is a straight line with a slope of \(-1\).

When \( K \) is very large, the inertia term of Eq. (2.1-8) is small compared to the drag term. Therefore, it can be neglected. Equation (2.1-8) can be simplified to

\[
C_u(K) = C_{d\phi}(K) = C_{ds} ,
\]  
\[(2.1-14)\]

where \( C_{ds} \) is the steady-state drag coefficient. By taking the log of both sides, it becomes

\[
\log C_u(K) = \log C_{ds} .
\]  
\[(2.1-15)\]

In a log-log plot, \( C_u(K) \) is a straight line parallel to the log \( K \) axis.

To prevent a sharp transition \( C_u(K) \) in a log-log plot, a hyperbola with Eqs. (2.1-13) and (2.1-15) lines as asymptotes is used. Referring to Sarpkaya's (1976) U tube results\(^2\) (Fig. 2.1-1), this is a good approximation for large \( \beta(R/K) \) when the cylinder is smooth and \( \beta \) is greater than 3500. It is assumed that the log\( C_u(K) \) curve of a wavy flow is similar to that of oscillatory flow. The laboratory tests and field tests studied in this thesis had a values greater than 3500. Thus in this thesis, the measured log\( C_u(K) \) data points are fitted by a hyperbola. (The procedures are detailed in Section 4.1 and Appendix A.)

\(^2\)The \( C_u \) and \( K \) values of U tube tests are not functions of \( z \).
Fig. 2.1-1 Maximum force coefficient for smooth cylinders as a function of $K$ (from Sarpkaya, 1976).
Peak velocities and peak forces can be measured from strip chart recordings. The $C_u(K)$ data points are then calculated from Eq. (2.0-2). By using the procedures described in Appendix A, a $C_u(K)$ hyperbola can be determined. The phase $\phi(K)$ can be measured directly from the strip chart or determined from a cross-correlation function (Bendat and Piersol, 1971). The nondimensionalized phases $\hat{\phi}(K) = \phi(K)/(-90°)$ are then plotted on log-log paper, and a smoothly fitting curve is drawn.

With measured values of $C_u(K)$ and $\phi(K)$, the force coefficients can be obtained from Eqs. (2.1-9) and (2.1-10). Data examples from Nath (1985a) with curves fitted by eye are shown in Figs. 2.1-2, 2.1-3, and 2.1-4. In this example velocities are obtained from the wave profile by using stream function theory (Dean, 1965). Because this method is related to the measured phase $\phi(z)$, it is designated as the Time Domain Force-Phase-\phi Method (TFP\phi).

2.1.2 The Time Domain Force-Phase-\phi Method

In this section it will be shown that under some assumptions only the $C_u(K)$ relationship is required to determine $C_{d\phi}(K)$, $C_{m\phi}(K)$, and $\phi(K)$.

Equation (2.1-8) can be rewritten as

$$C_{u}(K) = C_{d\phi}(K) + \frac{C_{m\phi}^2(K)}{C_{d\phi}(K)} \alpha(K)$$

(2.1-16)

where $\alpha(K) = \pi^4/4K^2$. By taking the derivative of Eq. (2.1-16) with respect to $\alpha(K)$, we obtain
Fig. 2.1-2 Maximum force coefficients for the VSMC12 and VSMC8 cylinders (from Nath, 1985a).

Fig. 2.1-3 Non-dimensionalized phase shift for the VSMC12 and VSMC8 cylinders (from Nath, 1985a).
Fig. 2.1-4 Values of $C_{d\phi}$ and $C_{m\phi}$ from Eqs. (2.1-9) and (2.1-10) using the curves shown in Fig. 2.1-2 and 2.1-3. The data points are the least squares values of $C_d$ and $C_m$ for the VSMC12 and VSMC8 cylinders (from Nath, 1985a).
\[
\frac{dC_\mu(K)}{d\alpha(K)} = \frac{dC_{\phi}(K)}{d\alpha(K)} + \alpha(K) \frac{d}{d\alpha(K)} \left[ \frac{C_{m\phi}(K)}{C_{d\phi}(K)} \right] + \frac{C_{m\phi}(K)}{C_{d\phi}(K)}
\]

\[= P(K) + Q(K) + m(K) \quad (2.1-17)\]

Appendix B illustrates that \(P(K) + Q(K) \ll m(K)\) is acceptable when \(K\) is less than 40 and when \(K\) is large. The data for \(40 < K < 90\), (Rodenbusch and Gutierrez, 1983), were not available when \(P(K) + Q(K) \ll m(K)\) was tested. When \(K\) is small, Eq. (2.1-17) can be approximated as

\[
\frac{dC_\mu(K)}{d\alpha(K)} = m(K) = \frac{[C_{m\phi}(K)]^2}{C_{d\phi}(K)}. \quad (2.1-18)
\]

An explicit hyperbolic function for \(C_\mu(K)\) can be determined from experimental data (see Section 4.1 and Appendix A). Equations (2.1-16) and (2.1-18) can then be used to solve for \(C_{d\phi}(K)\) and \(C_{m\phi}(K)\):

\[
C_{d\phi}(K) = C_\mu(K) - m(K) \alpha(K) \quad (2.1-19)
\]

and

\[
C_{m\phi}(K) = \sqrt{m(K)C_\mu(K) - m^2(K) \alpha(K)}. \quad (2.1-20)
\]

The phase, \(\phi(K)\), can be determined by substituting Eqs. (2.1-19) and (2.1-20) into Eq. (2.1-7), to yield

\[
\phi(K) = -\sin^{-1} \sqrt{\frac{m(K) \alpha(K)}{C_\mu(K) - m(K) \alpha(K)}} \quad (2.1-21)
\]
Examples are shown in Figs. 2.1-5 through 2.1-7. All but the velocity data used in this and previous examples are the same (Hudspeth et al., 1984). The velocities used in this example are from measurements. In Fig. 2.1-6 the phase \( \phi(K) \) is nondimensionalized by dividing by phase \( \phi_a(K) \) and denoting it as \( \tilde{\phi}(K) \). Because this method is related to the slope of the \( C_p(K)-\alpha(K) \) curve, it is designated as the Time Domain Force-Phase-Slope method (TFPS).

One of the reasons for the scatter in the force coefficients is the variation of the phase. In the ocean, however, waves come from all directions, the spatial phase shift between the axes of the vertical cylinder and wave staff vary from time to time. This additional phase shift also affects the scatter of force coefficients if it is not taken into account.

To minimize the influence of spatial separation, one can install the wave staff (or current meter) as close to the cylinder as possible. When the instruments are too close to the cylinder, however, the vortex-shedding effects from the presence of the cylinder alters the wave field and the measured wave profile, distorting the wave kinematics.

The location of the wave staff (or current meter) is no longer important when the TFPS method is used (except that it must not be in the cylinder wake), because the force coefficients can be determined without knowing the phase. Theoretically, if measuring instruments are installed outside of the region influenced by vortex shedding, more accurate wave kinematics can be recorded and more accurate force coefficients can be calculated. More studies are needed to test the
Fig. 2.1–5 The maximum force coefficients data points and their hyperbola for the VSMC12 and VSMC8 cylinders.

Fig. 2.1–6 Nondimensionalized phase data points and the curve of predicted nondimensionalized phase.
Fig. 2.1-7 Drag and inertia force coefficients $C_{d\text{a}}$ and $C_{m\text{a}}$ calculated by TFPS method then converted into $C_{d\text{L}}$ and $C_{m\text{L}}$ by using Eqs. (C-8) and (C-9). The data points are the least-squares values of $C_d$ and $C_m$ for the VSMC12 and VSMC8 cylinders.
validity of this approach and to determine the optimal location of wave staffs and current meters.

2.2 The Frequency Domain Force-Phase Method

In this thesis random waves are studied in the frequency domain. At \( x = 0 \), the random wave profile can be expressed as:

\[
\eta(t) = \sum_{n=1}^{N} a_n(f_n) e^{j[\theta_n(f_n) - \phi_n(f_n)]} \quad (2.2-1)
\]

where \( j \) is the imaginary number, subscript \( n \) denotes the \( n \)th frequency component, \( f_n = n f_1 \) (\( f_1 \) is the fundamental frequency), \( a_n(f_n) \) is the ordinate of the wave amplitude spectrum for frequency \( f_n \), \( \theta_n(f_n) = 2\pi f_n t \), \( \phi_n(f_n) \) is the frequency-dependent phase relative to an arbitrary origin of time, and \( N \) is the number of data points. The amplitude \( a_n(f_n) \) and phase \( \phi_n(f_n) \) can be obtained from the amplitude and phase spectra by using the Fast Fourier Transform (FFT).

For random linear waves, the horizontal water particle velocity at elevation \( z \) can be written as

\[
u(z,t) = \sum_{n=1}^{N} u_n(z,f_n) e^{j[\theta_n(f_n) - \phi_n(f_n)]} \quad (2.2-2)
\]

where \( u_n(z,f_n) \) is the amplitude of the horizontal water particle velocity and \( \phi_n(f_n) \) is the phase for frequency \( f_n \).

By FFT methods, the measured random wave force, \( F(z,t) \), can also be decomposed as
\[ F(z, t) = \sum_{n=1}^{N} F_n(z, f_n) e^{j[\theta_n(f_n) - \phi_n(f_n) + \phi_n(f_n)]} \]  \hspace{1cm} (2.2-3)

where \( F_n(z, f_n) \) is the amplitude of the in-line force for frequency \( f_n \), and \( \phi_n(f_n) \) is the phase lag between in-line force and horizontal water particle velocity for frequency \( f_n \). The phase \( \phi_n(K_n) \) is obtained from velocity-force cross spectrum analysis. The cross spectrum analysis is described as follows.

The cross-spectral density function \( G_{xy}(f_n) \) of a pair of time history records \( X(t) \) and \( Y(t) \) is the Fourier transform of the cross-correlation function (Bendat and Piersol, 1971). The cross-spectral density function is generally a complex number such that

\[ G_{xy}(f_n) = C_{xy}(f_n) - jQ_{xy}(f_n) \] \hspace{1cm} (2.2-4)

where the real part \( C_{xy}(f_n) \) is called the coincident spectral density function and the imaginary part \( Q_{xy}(f_n) \) is called the quadrature spectral density function. It is convenient to express the cross-spectral density function in complex polar notation such that

\[ G_{xy}(f_n) = |G_{xy}(f_n)| e^{-j\theta_{xy}(f_n)} \] \hspace{1cm} (2.2-5)

where the cross-amplitude spectrum \(|G_{xy}(f_n)|\) and the cross-phase spectrum \( \theta_{xy}(f_n) \) are related to \( C_{xy}(f_n) \) and \( Q_{xy}(f_n) \) by

\[ |G_{xy}(f_n)| = \sqrt{C_{xy}^2(f_n) + Q_{xy}^2(f_n)} \] \hspace{1cm} (2.2-6)

and
\[ \theta_{xy}(f_n) = \tan^{-1} \left[ \frac{Q_{xy}(f_n)}{C_{xy}(f_n)} \right]. \] (2.2-7)

Assume that the power spectral density functions for \( X(t) \) and \( Y(t) \) can be expressed as \( G_x(f_n) \) and \( G_y(f_n) \). Then without smoothing

\[ |G_{xy}(f_n)|^2 = G_x(f_n)G_y(f_n). \] (2.2-8)

In this thesis \( X(t) = u(z,t), Y(t) = F(z,t), |G_{xy}(f_n)| = S_{uF}(f_n), \)
\[ \theta_{xy}(f_n) = \phi_n(z,f_n), \text{ and } S_{uF}(f_n) \text{ is the velocity-force cross-amplitude spectrum.} \]

The time domain Keulegan-Carpenter numbers have already been defined in Eqs. (2.0-3) and (2.0-4). The frequency domain Keulegan-Carpenter number, \( K_n(z,f_n) \) is defined in a different way. It is

\[ K_n(z,f_n) = \frac{u_n(z,f_n)}{D f_n}. \] (2.2-9)

According to linear wave theory, \( K_n \) at the still water level \( z = 0 \) can be expressed as \( K_n(0,f_n) = 2\pi a_n/D \) where, in this case, \( a_n \) is the amplitude of the amplitude spectrum at the frequency \( f_n \). The amplitude spectrum for the Hurricane Carla wave profile shown in Fig. 2.0-3 is shown in Fig. 2.2-1. The largest \( K_n(0,f_n) \) for 1-ft diameter riser is about 3.4. The largest \( K_n(0,f_n) \) for 3 and 8-ft diameter cylinders are about 1.13 and 0.43, respectively. Thus, it is that the largest \( K_n(z,f_n) \) calculated from a random wave amplitude spectrum is much smaller than the largest \( K_r \) calculated from the same wave record.

The frequency domain maximum force coefficient, \( C_{\mu n}(z,f_n) \), is defined as
Fig. 2.2-1 The amplitude spectrum of WPII record-6887.

\[ a_n(0, f_n) = 2.0 \text{ ft} \]
\[ f_n = 0.102 \text{ Hz} \]
\[ k_n(0, f_n) = 3.4 \]
\[ C_{un}(z, f_n) = \frac{F_n(z, f_n)}{\frac{1}{2} \rho D u_n^2(z, f_n)} \quad (2.2-10) \]

where \( F_n(z, f_n) \) and \( u_n(z, f_n) \) are taken from the respective amplitude spectra. When \( K_n(z, f_n) \) is small \( C_{m\phi}(K_n) = 2.0 \), \( C_{un}(K_n) \) may be expressed as

\[ C_{un}[K_n(z, f_n)] = \frac{\pi^2}{K_n} C_{m\phi}[K_n(z, f_n)] = \frac{2\pi^2}{K_n} \quad (2.2-11)^3 \]

A set of Morison force relations can be obtained by using Eq. (2.1-6) if one assumes that the force components at any one frequency result from wave components at only that frequency. Then the time domain formulas can be used in frequency domain to determine the force coefficients.

2.2.1 The Frequency Domain Force-Phase-0 Method

When experimental \( C_{un}(K_n) \) and \( \phi_n(K_n) \) values are both available, the frequency dependent \( C_{d\phi}(K_n) \) and \( C_{m\phi}(K_n) \) can be solved as

\[ C_{d\phi}(K_n) = \frac{C_{un}(K_n)}{1 + \sin^2 \phi_n(K_n)} \quad (2.2-12)^4 \]

and

\[ C_{m\phi}(K_n) = \frac{2 K_n C_{un}(K_n) - \sin \phi_n(K_n)}{\pi^2} \quad (2.2-13)^5 \]

This method is designated as Frequency Domain Force-Phase-\( \phi \) Method (FFP\( \phi \)).

---

3 Similar to Eq. (2.1-11).
4 Similar to Eq. (2.1-9).
5 Similar to Eq. (2.1-10).
2.2.2 The Frequency Domain Force-Phase-S Method

When experimental $C_{mn}(Kn)$ values only are available, the force coefficients can be estimated by

$$C_{d\phi}(Kn) = C_{mn}(Kn) - m(Kn) a(Kn)$$  \hspace{1cm} (2.2-14)\textsuperscript{6}

and

$$C_{m\phi}(Kn) = \sqrt{m(Kn) C_{mn}(Kn) - m^2(Kn) a(Kn)}$$  \hspace{1cm} (2.2-15)\textsuperscript{7}

The phase can be determined by using Eq. (2.2-11). That is,

$$\phi(K_n) = -\sin^{-1} \left[ \frac{m(K_n) a(K_n)}{C_{mn}(K_n) - m(K_n) a(K_n)} \right].$$  \hspace{1cm} (2.2-16)\textsuperscript{8}

This method is designated as the Frequency Domain Force-Phase-Slope Method (FFPS).

2.3 The Prediction of Force Power Spectra

By using the Morison equation and regression analysis, a pair of force coefficients can be obtained from measured wave kinematics and forces. Then, by substituting these force coefficients back into the Morison equation, the predicted forces are obtained and compared to the measured forces in order to check the method of determining the coefficients. Herein this process is designated as pseudo prediction. If $C_{mn}(Kn)$ is obtained from Eq. (2.2-11) and $\phi_n(K_n)$ is

\textsuperscript{6}Similar to Eq. (2.1-19).
\textsuperscript{7}Similar to Eq. (2.1-20).
\textsuperscript{8}Similar to Eq. (2.1-21).
assumed to be \(-90^\circ\) or \(C_n(K_n)\) and \(\phi_n(K_n)\) are taken from other test results, the prediction is designated as a true prediction.

Borgman (1965,1967a,1967b,1972a) proposed a method to predict the force power spectrum from a given wave spectrum, which is described as follows:

Let \(S_{FF}(z,f_n)\) be the power spectral density for measured forces and \(S_{VV}(z,f_n)\) and \(S_{AA}(z,f_n)\) are the power spectral densities for water particle velocities and accelerations. The velocity and acceleration power spectra are calculated from the wave amplitude spectrum by using linear wave theory. Borgman found that force power spectral densities can be approximated by

\[
S_{FF}(z,f_n) = \frac{8}{\pi} \left[ \frac{C_d(z) \rho D \omega_n}{2} \right]^2 S_{VV}(z,f_n) + \left[ \frac{C_m(z) \rho D \pi}{4} \right]^2 S_{AA}(z,f_n) .
\]

(2.3-1)

where \(\omega_n = 2\pi f_n\). A least-square fitting of the theoretical force power spectrum against the actually measured force power spectrum yields \(C_d(z)\) and \(C_m(z)\). This method is designated as the Borgman method. The Borgman method assumes that the force coefficients, \(C_d(z)\) and \(C_m(z)\), are constant for the whole spectrum frequency range and thus for all \(K_n(z,f_n)\) values. Because the Borgman method needs only a pair of force coefficients, it is useful in true predictions.

Brown and Borgman (1965) obtained frequency-dependent force coefficients by using the cross-spectral density between the force and wave profile. This method is designated as the Brown-Borgman method. The details will not be described here. For one example studied, they found that force coefficients exhibited a distinct
tendency to be functions of frequency. But the difficulty of explaining the negative force coefficients in the high frequency region, and the difficulty of estimating the frequency-dependent force coefficients makes it less useful for true predictions.

The method introduced in this thesis is simple. Only the $C_{un}(K_n)$ relationship is used (Hsu and Nath, 1986). By rearranging Eq. (2.2-10), the force amplitude $F_n(z,f_n)$ can be obtained with

$$F_n(z,f_n) = C_{un} [K_n(z,f_n)] \frac{\rho D}{2} u_n^2(z,f_n). \quad (2.3-2)$$

If the regression line of the measured $C_{un}(K_n)$ values is available from another test, the force amplitude $F_n(z,f_n)$ can be obtained by using Eq. (2.3-2) and the wave force power spectrum can be predicted by

$$S_{FF}(z,f_n) = \frac{F_n^2(z,f_n)}{\Delta f}, \quad (2.3-3)$$

where $\Delta f$ is the frequency increment.

If the surface of the cylinder is smooth and all of the decomposed $K_n(z,f_n)$ values are less than 8, then $C_{un}(K_n)$ can be replaced by Eq. (2.2-11). The force amplitude can be expressed as

$$F_n(z,f_n) = \frac{\rho \pi^2 D}{K_n(z,f_n)} u_n^2(z,f_n). \quad (2.3-4)$$

or

$$= \rho \pi^2 D^2 f_n u_n(z,f_n). \quad (2.3-5)$$
2.4 The Prediction of Force Time History

The wave grouping effects are usually studied in the time domain.

By comparing Eqs. (2.2-2) and (2.2-3), the relationship between velocity and force for frequency $f_n$ can be expressed as

$$F_n(z,f_n) = H_n(f_n) u_n(z,f_n).$$

(2.4-1)

Where $H_n(f_n)$ is the transfer function, which can be expressed as

$$H_n(f_n) = |H_n(f_n)| e^{j\phi_n(z,f_n)},$$

(2.4-2)

and $|H_n(f_n)|$ can be derived from Eq. (2.3-5) to yield

$$|H_n(f_n)| = \rho \pi^2 D^2 f_n = \frac{1}{2} \rho \pi D^2 \omega_n.$$  

(2.4-3)

For pseudo prediction $\phi_n(z,f_n)$ can be estimated from velocity-force cross spectrum analysis. For smooth cylinders when $K_n$ is small, the true prediction can be achieved by using $\phi_n(z,f_n) = -90^\circ$ and $C_{\mu n}(z,f_n) = 2\pi^2/K_n$. An example will be shown in section 4.4.

2.5 Force-Phase Method Extension

The force-phase method was only applied to the in-line forces on vertical cylinders in previous sections. The transverse forces on vertical cylinders are of about the same magnitude, however, and there are no reasons to ignore them. Horizontal cylinders play an important role in offshore structures, and there are no reasons to ignore them either. Application of the FFP and FFPS to the transverse forces on vertical cylinders, and the horizontal and vertical forces on horizontal cylinders will be described as follows.
The wave forces acting on a vertical cylinder consist of in-line forces and transverse forces. Similarly, the forces acting on a horizontal cylinder consist of horizontal in-line forces and vertical forces (see Fig. 2.5-1).

The transverse, horizontal, and vertical force power spectra can be predicted (pseudo) by substituting maximum respective force coefficients and correlated phases into the formulas derived in previous sections. Those formulas are tabulated in Table 2.5-1.

2.5.1 Vertical Cylinders

The symbol $C_{\mu n}^T(K_n)$ is reserved for the frequency domain maximum in-line force coefficient. The frequency domain maximum transverse force coefficient is described as follows:

Theoretically, there is no flow in the transverse direction in a unidirectional wave field when the cylinder is not there. Therefore, the maximum transverse force coefficient is defined by using the undisturbed in-line horizontal velocities $u_n(z,f_n)$ as

$$C_{\mu n}^T(K_n) = \frac{F_T^T(z,f_n)}{\frac{1}{2} \rho D U_n^2(z,f_n)}$$

(2.5-1)

where $F_T^T(z,f_n)$ is the amplitude of the transverse force for frequency $f_n$. The correlated phase $\phi_T^T(z,f_n)$ is defined as the phase between $F_T^T(z,f_n)$ and $u_n(z,f_n)$.

2.5.2 Horizontal Cylinders

The maximum horizontal force coefficient in the frequency domain, $c_{\mu n}^H(z,f_n)$, is defined as:
Fig. 2.5.1 Definitions of forces on vertical and horizontal cylinders.
Table 2.5-1 Summary of frequency domain maximum force coefficients, Keulegan-Carpenter numbers and phases

<table>
<thead>
<tr>
<th>Force</th>
<th>Vertical Cylinder</th>
<th>Horizontal Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In-line</td>
<td>Transverse</td>
</tr>
<tr>
<td>Maximum-Force coefficient</td>
<td>$C_{\mu n} = \frac{2 F_{\mu n}(z,f_n)}{\rho D u_{\mu n}^2(z,f_n)}$</td>
<td>$C_{\mu n} = \frac{2 F_{\mu n}(z,f_n)}{\rho D u_{\mu n}^2(z,f_n)}$</td>
</tr>
<tr>
<td>Keulegan-Carpenter number</td>
<td>$u_{\mu n}(z,f_n)$</td>
<td>$u_{\mu n}(z,f_n)$</td>
</tr>
<tr>
<td></td>
<td>$D f_n$</td>
<td>$D f_n$</td>
</tr>
<tr>
<td>Phase</td>
<td>$\phi_n(z,f_n)$</td>
<td>$\phi_n(z,f_n)$</td>
</tr>
<tr>
<td></td>
<td>phase lag between $F_{\mu n}(z,f_n)$ and $u_{\mu n}(z,f_n)$</td>
<td>phase lag between $F_{\mu n}(z,f_n)$ and $u_{\mu n}(z,f_n)$</td>
</tr>
</tbody>
</table>
\[ C_{\mu n}^H (k_n) = \frac{F_n^H(z,f_n)}{\frac{1}{2} \rho D u_n^2(z,f_n)} \] (2.5-2)

where \( F_n^H(z,f_n) \) is the amplitude of the horizontal force for frequency \( f_n \). The correlated phase \( \phi_n^H(z,f_n) \) is defined as the phase between \( F_n^H(z,f_n) \) and \( u_n(z,f_n) \).

The maximum vertical force coefficient in the frequency domain is defined as:

\[ C_{\mu n}^V (k_n) = \frac{F_n^V(z,f_n)}{\frac{1}{2} \rho D w_n^2(z,f_n)} \] (2.5-3)

where \( F_n^V(z,f_n) \) is the amplitude of the vertical force for frequency \( f_n \), and \( w_n(z,f_n) \) is the amplitude of the vertical water particle velocities for frequency \( f_n \). The correlated phase \( \phi_n^V(z,f_n) \) is defined as the phase between \( F_n^V(z,f_n) \) and \( w_n(z,f_n) \).
3.0 DESCRIPTION OF EXPERIMENTS

With some exceptions, full scale measurements on offshore structures offer little assistance in the systematic study of wave forces, because of difficulties in quantifying the real sea environment. Laboratory tests allow us to measure fluid loading on a cylinder under controlled conditions. However, in model testing it is difficult to extrapolate the fluid loading to full scale conditions. The scaling factors such as \( R \) and \( K \) can not match the ocean situation at the same time. Nevertheless, laboratory tests and field tests are necessary to check the validity of the theory.

The experimental data from Oregon State University (OSU), Wave Project II (WPII), and the National Maritime Institute Second Christchurch Bay Tower (NMI) were used to check the new method. Only a brief review will be given to summarize general aspects of the experiments. More information can be found from cited references. The test conditions are summarized in Table 3.0-1.

3.1 Laboratory Experiments

The Laboratory experiments described in this thesis were conducted in the OSU wave tank (Fig. 3.1-1). The overall length of the wave flume is 340 ft. The width is 12 ft and the depth is 15 ft, of which 3.5 ft is freeboard. Each test cylinder was located 136 ft from the wave board (Hudspeth et al., 1984).

Three cylinders were tested.
### Table 3.0-1 Summary of the test conditions

<table>
<thead>
<tr>
<th>Date (year)</th>
<th>Project</th>
<th>Test Site</th>
<th>Water Depth (ft)</th>
<th>Spectrum #</th>
<th>Cylinder Diameter (in.)</th>
<th>Orientation</th>
<th>Roughness*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1933</td>
<td>NCEL</td>
<td></td>
<td></td>
<td>JONSWAP</td>
<td>12.75</td>
<td>Vertical</td>
<td>SMC12</td>
</tr>
<tr>
<td>1984</td>
<td>AP184</td>
<td>OSU Wave Tank</td>
<td>11.5</td>
<td>Scott</td>
<td>8.625</td>
<td>Vertical</td>
<td>SRC.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bretschneider</td>
<td>Vertical</td>
<td>SRC.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Scott</td>
<td>8.625</td>
<td>Vertical</td>
<td>SP3-55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bretschneider</td>
<td>Vertical</td>
<td>SP3-55</td>
<td></td>
</tr>
<tr>
<td>1961</td>
<td>WPII</td>
<td>Gulf of Mexico</td>
<td>97.5</td>
<td></td>
<td>44.0</td>
<td>Vertical</td>
<td></td>
</tr>
<tr>
<td>1982</td>
<td>NMI</td>
<td>English Channel</td>
<td>28.5</td>
<td></td>
<td>18.9</td>
<td>Vertical</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>110.2</td>
<td>Vertical</td>
<td></td>
</tr>
</tbody>
</table>

*SMC12: Smooth aluminum cylinder - 12.75 in.
SRC.02: Sand roughened cylinder - $\epsilon = 0.023$ (D = 8.625 in.)
SP3-55 D: South Pass platform, 3-year old, -55 ft.

#JONSWAP spectrum (Hasslemann et al., 1973)
Scott spectrum (Scott, 1965)
Bretschneider spectrum (Bretschneider, 1959)
Fig. 3.1-1 The OSU wave flume (from Nath, 1985a).
3.1.1 The SMC12 Cylinder

The SMC12 cylinder was a 12.75-in. diameter SMooth Cylinder (Hudspeth et al., 1984). It had a 12-in. long local force transducer. This cylinder is detailed in Fig. 3.1-2. It was tested as a Vertical cylinder and is designated the VSMC12 cylinder.

3.1.2 The SRC.02 Cylinder

The SRC.02 cylinder was a Sand-Roughened Cylinder with $\varepsilon = 0.023$, where $\varepsilon$ is the sand grain size divided by smooth cylinder diameter. It is designated as the VSRC.02 and HSRC.02 for Vertical and Horizontal tests, respectively. The positions of the VSRC.02 and HSRC.02 cylinders are shown in Figs. 3.1-3 and 3.1-4.

3.1.3 The SP3-55D Cylinder

The SP3-55D cylinder is from the South Pass region of the Gulf of Mexico. It was placed -55 ft below the sea surface for 3 years and became covered with rough natural marine organisms (Nath, 1985b). Before it was used to do the tests, it was Dried. It is designated as the VSP3-55D and HSP3-55D for Vertical and Horizontal tests, respectively. The arrangement of the HSP3-55D cylinder is the same as for the HSRC.02 cylinder.

For all of the vertical and horizontal cylinders, the center of the force transducers was 3.7 ft below the still water surface.

All of the cylinders were tested in regular waves and random waves. For periodic wave tests, the wave periods ranged from 2.5 sec. to 6.0 sec. with maximum wave heights of about 5 feet. The maximum Reynolds number reached was $3.1 \times 10^5$. The maximum Keulegan-
Fig. 3.1-2 Sectional elevation of a VSMC12 test cylinder (from Hudspeth et al., 1984).
12 x 8 rectangular steel tube. 39.7 plf

same roughened “skins” as for the horizontal cylinder tests

0.77

2 guys ea. connection
8 total

Fig. 3.1-3 Sectional elevation of a vertical 8.625-in. test cylinder (from Nath, 1983).
Fig. 3.1-4  Side view of a horizontal 8.625-in. test cylinder (from Nath, 1985a).
Carpenter number was 27, and maximum values of $\beta (= R/K)$ were about $4 \times 10^4$. More details can be found in Nath (1985b).

For the random wave tests, each cylinder test comprised one run of a simulated Scott spectrum and one run of a Bretschneider spectrum. Each spectrum had a target significant wave height of 3.2 ft and a modal period of 3.9 sec. The actual values ranged from 3.0 to 3.21 ft and 3.72 to 3.85 sec., respectively (Nath and Yeh, 1986).

3.2 Field Experiments

Two sets of field data were used in this study: the WPII project and the NMI tests.

3.2.1 NMI Second Christchurch Bay Tower

The NMI Second Christchurch Bay Tower (Bishop, 1985) had a large column (cylinder) with diameter $D = 2.8$ m and a small column (cylinder) with diameter $D = 0.48$ m. The side view and plan view of the tower are shown in Figs. 3.2-1 and 3.2-2. The wave profile was obtained by a capacitance instrument Fig. 3.2-1.

The water particle velocities, influenced by both waves and current, were measured by perforated ball instruments (Bishop, 1984a). One perforated ball instrument measured the wave kinematics in two orthogonal directions. Near the small cylinder, perforated ball instruments were installed at four depth stations, Fig. 3.2-1. At each station, a pair of perforated ball instruments were used to give three velocity components. The predominant wave direction was determined from velocity measurements. Force measuring elements were located at 5 different depths (see Fig. 3.2-2, Bishop 1984a).
Fig. 3.2-1 NMI Christchurch Bay Tower side view (from Bishop, 1984a).
Fig. 3.2-2  NMI Christchurch Bay Tower top view (from Bishop, 1984a).
3.2.2 Wave Project II Tests

The platform layout for Wave Project II (Blank, 1969) is shown in Fig. 3.2-3. The force measurement device of WPII had 16 local pressure transducers\(^9\), with an active length of 6 inches (Blank, 1969). The devices were clamped onto one of the platform support piles. The outer diameter of each device was about 44 in. During hurricane season, the eight meters were spaced vertically along the pile\(^10\) as shown in Fig. 3.2-4. A wave staff was placed about 3 ft seaward from the instrumented pile (see Fig. 3.2-3).

During Hurricane Carla only the data at positions 7 and 8 were legible and were analyzed. The biggest individual wave recorded during Hurricane Carla was recorded in record-6887.

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\(^9\)The forces were obtained by integrating the pressure around the cylinder. The pressure force varied linearly in the interval between two successive pressure transducers.

\(^10\)During the hurricane season, waves were expected to be higher than during other seasons. Thus, near the free surface, the spacing between meters should be wider in order to record all possible waves.
Fig. 3.2-3 Platform layout of Wave Project II (from Blank, 1969).
Fig. 3.2-4 Location of WPII dynamometers and wave staff (from Blank, 1969).
4.0 RESULTS AND DISCUSSION

Determination of the $C_{un}(K_n)$ relationship is the major function of the FFP$\phi$ and FFPS methods; they will be studied in Section 4.1. Applications of the $C_{un}(K_n)$ relationship, such as for force coefficient determination, the force power spectrum, and force time history prediction, will then be studied in Sections 4.2, 4.3 and 4.4, respectively.

4.1 The $C_{un}(K_n)$ Regression Lines

The in-line force on the vertical cylinder will be used as an example to describe the procedures for obtaining a maximum force coefficient regression line from measured velocities and forces. Other forces, e.g. the transverse forces, vertical forces, and horizontal forces, can be analyzed by similar procedures.

The procedures for obtaining the $C_{un}(K_n)$ regression line are described below.

1. $C_{un}(K_n)$ data points are obtained by substituting measured velocities and measured forces into Eq. (2.2-10).

2. Scattered $C_{un}(K_n)$ data points are then removed. These procedures are described in sections 4.1.1. and 4.1.2.

3. The remaining $C_{un}(K_n)$ data points with small $K_n$ values (decomposed $K_n$ values less than about 8) are then fitted to a straight line, $L_1$. An ideal $L_1$ line is shown in Fig. 4.1-1.
Fig. 4.1-1 The $C_{\mu n}(K_n)$ hyperbola and their asymptotes.
(4) The line $L_2$ shown in Fig. 4.1-1 is then determined by setting $C_{\mu n}(K_n)$ equal to $C_{ds}$, the steady state drag coefficient. In this thesis $C_{ds}$ is set equal to 0.5 for smooth cylinders and 1.0 for rough cylinders. These numbers were obtained from OSU towing test results (Nath, 1986).

(5) A hyperbola, the $C_{\mu n}(K_n)$ curve, was constructed by using $L_1$ and $L_2$ as asymptotes. (See Fig. 4.1-1).

The details of procedures (5) are described in Appendix A. Data were plotted on 6 x 5 cycle log-log graph paper, which can accommodate all possible $K_n$ regions.

4.1.1 The Amplitude Filter

Figures 4.1-2 and 4.1-3 show a typical force-velocity cross-amplitude spectrum and cross-phase spectrum. From these two figures, it can be found that for those frequencies with large cross-amplitudes the correlated phases, $\phi_n(z,f_n)$, are near $-90^\circ$. When the amplitudes are small, however, the phases become random because noise is a large percent of the signal. Phase randomness may be caused by environmental noise and other unknown effects.

Line $L_1$ is determined by the least-squares method. In the least-squares method, all data points are weighted equally. If unimportant data points occur far from the regression line, the slope and intercept of the regression line may be greatly changed.

Note that the maximum transverse force coefficient, $C^{T}_{\mu n}(K_n)$, was only fitted by a straight regression line, $L_1$. The $L_2$ line did not exist.
Fig. 4.1-2 The velocity-force cross-amplitude spectrum.
Fig. 4.1-3 The velocity-force cross-phase spectrum.
Unimportant data points may thereby lead to inaccurate interpretation. Therefore, the unimportant data points which were calculated from small-amplitude components were removed before the regression analysis.

Not all of the velocity and force components are used in a regression analysis to determine $L_1$. The velocity-force cross-amplitude spectrum is used to help to determine which frequency will be used. The velocity-force cross-amplitude spectrum is first passed through an amplitude filter (Fig. 4.1-4), and then a band pass frequency filter (Fig. 4.1-5). The through frequencies that remain after passing through these two filters are used. This does not mean that the removed frequency components will be discarded. They are not used to determine line $L_1$ which will be used to construct the $C_{\mu n}(K_n)$ curve. All frequency components are used to predict the force power spectrum and the force time history (by inverse FFT).

The cut-off amplitude of the amplitude filter was selected by specifying a cut-off amplitude to peak amplitude ratio. The ratios were obtained by trial and error. Each trial ratio began at 0 and was then increased until most of the remaining data points were evenly distributed on both sides of the regression line. The VSMC12 in-line force data points and the regression lines corresponding to different cut-off amplitudes are shown in Figs. 4.1-6, 4.1-7 and 4.1-8. The results in Fig. 4.1-6 include all of the plotted $C_{\mu n}(K_n)$ data points. When the frequency components that have cut-off amplitudes of less than 1% and 10% are deleted, the results are like Figs. 4.1-7 and 4.1-8, respectively. This example illustrates that an
Fig. 4.1-4 The amplitude filter: (a) before filter, (b) after filter.
Fig. 4.1-5 The band pass frequency filter: (a) before filter, (b) after filter.
Fig. 4.1-6 The $C_{\mu n}(K_n)$ data points and their regression line (before filter).
Fig. 4.1-7 The $C_{\mu n}(K_n)$ data points and their regression line, after passing through an amplitude filter (cut-off amplitude = 0.01 spectral peak).
Fig. 4.1-8 The $C_{\mu n}(K_n)$ data points and their regression line, after passing through an amplitude filter (cut-off amplitude $= 0.1$ spectral peak).
increase in the cut-off amplitude of the amplitude filter decreases the data scatter.

It was found that a small cut-off amplitude, such as 1% of the spectral peak, was sufficient for the VSMC12 (smooth cylinder). The $C_{\mu n}(K_n)$ data points for the SRC.02 (rough cylinder) are more scattered, and a higher cut-off amplitude, such as 10% of the spectral peak, was necessary. In field tests the noise/signal ratio is high, and even for smooth cylinders a higher cut-off amplitude of at least 10% of the spectral peak may be necessary.

4.1.2 The Frequency Filter

Figure 4.1-9 shows the $C_{\mu n}(K_n)$ data points for the VSMC12 cylinder after passing them through a band-pass (0.1 to 1.0 Hz) frequency filter. A comparison of Fig. 4.1-9 with Fig. 4.1-6 indicates that some of the scattered data points in Fig. 4.1-6 have disappeared. The cut-off frequencies were determined from the bandwidth of the wave spectrum as follows. A lower bound of 0.1 Hz was used for the OSU laboratory tests, because only a small amount of energy existed in the region where frequencies are less than 0.1 Hz, they may be caused by surge and other effects. The upper bound was set at 1.0 Hz because only noise energy existed above that level. Peak frequencies in the WPII and NMI tests were lower than those measured in the OSU laboratory tests. Lower and upper bounds for the NMI tests were set at 0.1 Hz and 0.5 Hz, respectively. Lower and upper bounds for the WPII tests were set at 0.05 Hz and 0.5 Hz, respectively.
Fig. 4.1-9 The $C_{\mu n}(K_n)$ data points and their regression line, after passing through an band pass (0.1 Hz to 1.0 Hz) filter.
4.1.3 Reynolds Number Effects

The Reynolds number discussed in this section is calculated from $u_\mu D/v$. For regular waves and random waves, $u_\mu$ is the maximum water particle velocity during a wave cycle. For oscillatory flow, $u_\mu$ is the maximum water particle velocity during an oscillatory flow cycle.

Bearman (1985) and his co-workers studied the forces on cylinders of planar oscillatory flow generated in a U-tube in the small $K$ region ($K < 10$) with 2.56 to 7.48 cm smooth cylinders. They found

$$C_{F_{rms}} = \frac{F_{rms}}{\frac{1}{2} \rho D u_{rms}^2} = \frac{\sqrt{2} \pi^2}{K},$$

where $C_{F_{rms}}$ is the non-dimensionalized root-mean-square force. By comparing Eq. (4.1-1) with Eq. (2.1-11), it is evident that $C_{rms} = C_\mu \sqrt{2}$ when $C_{m\phi} = 2$. Thus, by substituting $C_\mu \sqrt{2}$ for $C_{F_{rms}}$, Eq. (4.1-1) is exactly the same as Eq. (2.1-11) when $C_{m\phi} = 2$.

Bearman's results are shown in Fig. 4.1-10, where the range of Reynolds numbers is between 350 and 16,650. Figure 4.1-10 shows that the test data fit the theoretical line very well. Sarpkaya's (1986a) U tube results are shown in Fig. 4.1-11. When $K < 8$, his results are also in good agreement with the theoretical line. The Reynolds numbers of Sarpkaya's tests were between $2 \times 10^3$ and $1.2 \times 10^4$ (for $K < 8$).

An example of the OSU random wave test results is shown in Fig. 4.1-6. The highest Reynolds number in the OSU tests was $3.1 \times 10^5$. For the large and the small cylinders of the NMI field tests, the $C_{\mu n}(K_n)$ data points from eleven records (superimposed) at three different depths are shown in Fig. 4.1-12. The highest Reynolds
Fig. 4.1-10  In-line force coefficient vs. K (From Berman, etc. 1985).
Fig. 4.1-11 Maximum force coefficient for smooth cylinders as a function of K (from Sarpkaya, 1986b).

- o, $\beta = 2300$
- x, $\beta = 3435$
- #, $\beta = 4720$
- *, $\beta = 6555$
- +, $\beta = 11525$
Fig. 4.1-12 The $C_{\mu n}(K_n)$ data points and their regression lines for NMI large and small cylinder (11 records at three locations).
number obtained in the NMI tests was about $4.2 \times 10^6$. It occurred on the large cylinder at level 3. Even though $R$ was high, the $C_{\mu n}(K_n)$ data points fit the Eq. (2.2-11) line quite well because $C_{\mu n}$ was determined from the frequency domain amplitude spectrum.

The $R$ values covered by these five different tests are between 350 and $4.2 \times 10^6$. It is suggested for in-line forces that the Reynolds number may not be such an important factor, whether in oscillatory flow, regular waves, or decomposed random waves when $K < 8$ for a smooth cylinder.

For roughened cylinders, the effective diameter should be used to determine $K_n$ and $C_{\mu n}(K_n)$. The effective diameter should be determined in a reasonable way. According to Nath (1986), the $C_{\mu}(K)$ regression lines are not only a function of $K$, but also a function of $R$. Sarpkaya’s (1986b) U tube results from a sand-roughened cylinder ($c = 0.02$) are shown in Fig. 4.1-13. For $K_n < 7$, the $C_{\mu n}(K_n)$ data points fit the Eq. (2.2-11) line very well. By comparing Figs. 4.1-11 and 4.1-13, it can be seen that there are no significant differences between the $C_{\mu n}(K_n)$ regression line and the Eq. (2.2-11) line when $c$ and $K_n$ are small.

4.2 Force Coefficients

The FFPS method is used here to determine force coefficients and phases. When the $C_{\mu n}(K_n)$ hyperbola is obtained, curves of drag coefficients, $C_{d\phi}(K_n)$, inertia coefficients, $C_{m\phi}(K_n)$, and phase, $\phi(K_n)$, can be calculated from the $C_{\mu n}(K_n)$ curve by using Eqs. (2.2-14), (2.2-15) and (2.2-16). To compare the force coefficients with those obtained from regular waves by using the least-squares method,
Fig. 4.1-13 Maximum force coefficient for rough cylinders (from Sarpkaya, 1986b).
$\text{C}_\text{d} \phi (K_n)$ and $\text{C}_\text{m} \phi (K_n)$ were converted into $\text{C}_\text{dL} (K_n)$ and $\text{C}_\text{mL} (K_n)$ by using Eqs. (C-8) and (C-9) described in Appendix C. Measured $\text{C}_\mu n (K_n)$ and measured $\phi_n (K_n)$ were also plotted on the same figure.

4.2.1 Laboratory Test Results

The $\text{C}_\mu n (K_n)$ hyperbola, curves of predicted force coefficients, curve of predicted non-dimensionalized phase, measured $\text{C}_\mu n (K_n)$ data points, and measured non-dimensionalized phases for the VSMC12 cylinder are shown in Fig. 4.2-1. The $\text{C}_\mu n (K_n)$ data points were tightly distributed along the hyperbola and the predicted non-dimensionalized phases are in good agreement with the measured phases (close to $-90^\circ$). The $C^T_{\mu n} (K_n)$ data points and their regression line for the VSMC12 cylinder are shown in Fig. 4.2-2.

The $\text{C}_\mu n (K_n)$ hyperbola, curves of predicted force coefficients, curve of predicted non-dimensionalized phase, measured $\text{C}_\mu n (K_n)$ data points, and measured non-dimensionalized phases for the VSRC.02 cylinder are shown in Fig. 4.2-3. The $C^T_{\mu n} (K_n)$ data points and their regression line for the VSRC.02 cylinder are shown in Fig. 4.2-4.

The $\text{C}_H (K_n)$ hyperbola, curves of predicted force coefficients, curve of predicted non-dimensionalized phase, measured $\text{C}_\mu n (K_n)$ data points, and measured non-dimensionalized phases for the HSRC.02 cylinder are shown in Fig. 4.2-5. The $C^T_{\mu n} (K_n)$ data points were tightly distributed along the hyperbola but the predicted non-dimensionalized phases are in poor agreement with the measured non-dimensionalized phases. The $C^V_{\mu n} (K_n)$ hyperbola, curves of predicted force coefficients, curve of predicted nondimensionalized phase,
In-line force coefficients and phases for the VSMC12 cylinder (Run 32).
Fig. 4.2-2 Transverse force coefficients of the VSMC12 cylinder (Run 32).
Fig. 4.2-3 In-line force coefficients and phases for the VSRC.02 cylinder (Run 23).
Fig. 4.2-4 Transverse force coefficients for the VSRC.02 cylinder (Run 23).
Fig. 4.2-5  Horizontal force coefficients for the HSRC.02 cylinder (Run 55).
measured $C_{\mu n}^V(K_n)$ data points, and measured nondimensionalized phases for the VSRC.02 cylinder are shown in Fig. 4.2-6.

4.2.2 Field Test Results

The field data studied here were from the NMI and WPII tests. The results of these tests are shown in sections 4.2.2.1 and 4.2.2.2, respectively.

4.2.2.1 NMI Second Christchurch Bay Tower

The wave profiles, velocities, and force time histories of the NMI tests were not available. Fortunately, NMI report-177 (Bishop, 1984a) provided figures for the velocity power spectra $S_{uu}(z,f_n)$ and force power spectra $S_{FF}(z,f_n)$. Values for $S_{FF}(z,f_n)$ and $S_{uu}(z,f_n)$ were digitized from the figures. The force amplitude $F_n(z,f_n)$ can be obtained by rearranging Eq. (2.3-3) as

$$F_n(z,f_n) = \sqrt{S_{FF}(z,f_n) \Delta f} . \quad (4.2-1)$$

Similarly, the velocity amplitude $u_n(z,f_n)$ can be calculated from

$$u_n(z,f_n) = \sqrt{S_{uu}(z,f_n) \Delta f} . \quad (4.2-2)$$

The velocity power spectra calculated from the perforated ball measurements were used and in this thesis it was assumed that the measured velocities were pure wave kinematics.

Record 71 level-3 is an example. The $C_{\mu n}^V(K_n)$ hyperbola, curves of predicted force coefficients, curve of non-dimensionalized phase, and $C_{\mu n}$ data points for the NMI large and small columns are shown in Figs. 4.2-7 and 4.2-8.
Fig. 4.2-6 Vertical force coefficients for the HSRC.02 cylinder (Run 55).
Fig. 4.2-7 In-line force coefficients for the NMI large cylinder level-3 (Record 71).
Fig. 4.2-8 In-line force coefficients for the NMI small cylinder level-3 (Record 71).
4.2.2.2 Wave Force Project II Tests

WPII had only one wave staff (Blank, 1969) and no velocity measurements. The velocities predicted from the wave profile were treated as $u_n(K_n)$. Those velocities were assumed to be in the same direction as the vector sum of the orthogonal force measurements. The $C_{\mu n}(K_n)$ hyperbola, curves of predicted force coefficients, curve of nondimensionalized phase, the $C_{\mu n}(K_n)$ data points, and nondimensionalized measured phases of position-7 during Hurricane Carla (record 6887) are shown in Fig. 4.2-9. The $C_{\mu n}$ data points and measured non-dimensionalized phases are quite scattered.

4.3 The Prediction of Force Power Spectra

The results of force power spectra predicted by pseudo and true predictions will be shown in the following sections. Both pseudo and true predictions use Eqs. (2.3-2) and (2.3-3) to predict the force power spectra. For pseudo prediction the maximum force coefficient vs. $K_n$ hyperbolas [e.g. $C_{\mu n}(K_n)$, $C_{\mu n}^V(K_n)$ or $C_{\mu n}^H(K_n)$ hyperbolas] are obtained from measured wave profiles (velocities) and measured forces by using the method described in section 4.1. Only the in-line force power spectrum for a smooth vertical cylinder can now be predicted by true prediction (when $K_n$ is small). Equation (2.2-11) was used in true prediction.

The NMI spectra were previously smoothed by a 14-point box car moving average method. For OSU and WPII tests, a 9-point box car moving average method was used to smooth both the measured and the predicted raw spectra (Borgman, 1972b).
Fig. 4.2-9 In-line force coefficients and phases for the WPII cylinder position-7 (Record 6887).
4.3.1 Evaluation Criteria for Predicted Force Power Spectra

The non-dimensionalized root-mean-square-error $E_{rms}$ of spectral amplitudes are used to evaluate the goodness of the pseudo prediction and true prediction. It is defined as

$$E_{rms} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left[ \frac{[S_{FF}(z, f_n)_m] - [S_{FF}(z, f_n)_p]^2}{[S_{FF}(z, f_n)_m]^2} \right]}$$

(4.3-1)

where $S_{FF}(z, f_n)_m$ is the measured spectral amplitude, $S_{FF}(z, f_n)_p$ is the predicted spectral amplitude, $N$ is the number of frequency components involved, and $[S_{FF}(z, f_n)_m]_\mu$ is the spectral peak of measured spectrum.

4.3.2 Laboratory Tests

In-line, vertical, and horizontal force power spectra predicted by the pseudo prediction for cylinders of various roughness will be described first. The pseudo predictions of transverse force power spectra are poor and they are not shown here. Then the true predicted in-line force power spectra for vertical smooth cylinders will be described.

Pseudo predictions of the in-line force raw spectra and smoothed spectra of the VSMC12 and VSR0.02 cylinders are shown in Figs. 4.3-1 through 4.3-4. The predicted smooth cylinder in-line forces power spectrum is in good agreement with the measured spectrum. The smoothed spectrum can be predicted even more precisely.

The raw and smoothed spectra for horizontal and vertical forces acting on HSRC.02 cylinders are shown in Figs. 4.3-5 through 4.3-8.
Fig. 4.3-1  Raw in-line force power spectra for the VSMC12 cylinder (pseudo prediction).
Fig. 4.3-2 Smoothed in-line force power spectra for the VSMC12 cylinder (pseudo prediction).
Fig. 4.3-3 Raw in-line force power spectra for the VSRC.02 cylinder (pseudo prediction).
Fig. 4.3-4 Smoothed in-line force power spectra for the VSRC.02 cylinder (pseudo prediction).
Fig. 4.3-5 Raw horizontal force power spectra for the HSRC.02 cylinder (pseudo prediction).
Fig. 4.3-6 Smoothed horizontal force power spectra for the HSRC.02 cylinder (pseudo prediction).
Fig. 4.3-7 Raw vertical force power spectra for the HSRC.02 cylinder (pseudo prediction).
VERTICAL FORCE POWER SPECTRA (SMOOTHED)

Fig. 4.3-8 Smoothed vertical force power spectra for the HSRC.02 cylinder (pseudo prediction).
Predicted horizontal force power spectra were in good agreement with measured spectra. The predicted vertical force power spectra were in poor agreement, however, especially in the low and high frequency region (Fig. 4.3-8).

The true prediction of in-line force power spectra and smoothed spectra for the VSMC12 cylinder are shown in Figs. 4.3-9 and 4.3-10, respectively. Equation (2.2-11) was used in this true prediction. The predicted and measured spectra are in good agreement.

4.3.3 Field Tests

The field data used here are from the NMI and WPII measurements. Only in-line force power spectra are shown here.

4.3.3.1 NMI Second Christchurch Bay Tower

Both pseudo and true predictions for the NMI cylinders are shown. The pseudo prediction are shown first, then the true predictions.

Pseudo predictions of the in-line force power spectra for small and large cylinders in record 71 are shown in Figs. 4.3-11 and 4.3-12. Measured and predicted spectra are shown in the same plot. The spectra that Bishop (1984b) predicted by using Borgman's method are also plotted for comparison. Bishop applied long term averaged force coefficients to predict the force power spectra for a short record. The method may be considered as midway between pseudo and true prediction. Measured velocities were used in all of the pseudo predictions. For a large cylinder, the power spectra predicted by the new method are in better agreement with theory than those
Fig. 4.3-9 Raw in-line force power spectra for the VSMC12 cylinder (true prediction).
Fig. 4.3-10 Smoothed in-line force power spectra for the VSMC12 cylinder (true prediction).
Fig. 4.3-11 In-line force power spectra for the NMI large cylinder level-3, record-71 (pseudo prediction).
Fig. 4.3-12  In-line force power spectra for the NMI small cylinder level-3, record-71 (pseudo prediction).
obtained by using Borgman's method. For small cylinder, the new method was more precise than Borgman's method in the high frequency regions, but Borgman's method was in better agreement with measured spectra in the low frequency regions.

The true predictions of in-line force power spectra for both the small and the large cylinders in record 71 are shown in Figs. 4.3-13 and 4.3-14. Measured velocities were used in true prediction. The true predictions are not as good as those obtained by using Borgman's method.

4.3.3.2 Wave Force Project II Tests

Both pseudo and true predictions for the WPII cylinder are shown here. The pseudo prediction will be shown first, then the true predictions.

Predicted raw and smoothed spectra, predicted by the pseudo prediction, for the WPII cylinder (record-6887) at position-7 are shown in Figs. 4.3-15 and 4.3-16. These predictions did not agree well with measured values. One reason may be that the velocities used to predict force power spectra were predicted from the wave record, and the effects of current were ignored.

The true predicted raw and smooth spectra are shown in Figs. 4.3-17 and 4.3-18. By comparing the $E_{rms}$ values, it can be found that the true predictions are better than the pseudo predictions.
Fig. 4.3-13 In-line force power spectra for the NMI large cylinder level-3, record-71 (true prediction).
Fig. 4.3-14 In-line force power spectra for the NMI small cylinder level-3, record-71 (true prediction).
In-line force power spectra (raw)

Fig. 4.3-15 Raw in-line force power spectra for the WPII cylinder position-7, record-6887 (pseudo prediction).
Fig. 4.3-16 Smoothed in-line force power spectra for the WPII cylinder, position-7, record-6887 (pseudo prediction).
Fig. 4.3-17 Raw in-line force power spectra for the WPII cylinder, position-7, record-6887 (true prediction).
Fig. 4.3-18 Smoothed in-line force power spectra for the WPII cylinder, position-7, record-6887 (true prediction).
4.4 The Prediction of Force Time Histories

Two true prediction examples will be shown here. The force time history for the VSMC12 cylinder (laboratory test) will be shown first, then the force time history for the WPII cylinder (field test) during Hurricane Carla.

In-line forces for the VSMC12 cylinder were predicted by the true prediction. The measured wave profile is shown in Fig. 4.4-1. The wave amplitude and phase spectrum are obtained by using FFT. The velocity phase spectrum is the same as the wave phase spectrum. The velocity amplitude spectrum is obtained from the wave amplitude spectrum by using linear wave theory. Equation (2.2-11) then was used to estimate $F_n(z, f_n)$. Phase shifts between the velocity phase spectrum and the force phase spectrum $\phi_n(z, f_n)$ were set equal to $-90^\circ$ in this prediction. The inverse FFT is used to obtain the in-line force time history. The predicted force time history and the measured force time history are shown in Fig. 4.4-2. They are in good agreement.

In-line forces for the WPII cylinder record-6887 at position 7 were predicted. The measured wave profile is shown in Fig. 4.4-3. The predicted force time history and the measured force time history are shown in Fig. 4.4-4. Most of the peak positions were shifted. However, the shape of the predicted force time history compares reasonably well with measurements. If the directional spectrum is considered the prediction may be improved.

For design, one usually develops a design wave spectrum, or it is specified. The phases of the frequency components can be determined from a random number generator and by assuming that the phases
Fig. 4.4-1 The wave profile for the VSMC12 cylinder test.
Fig. 4.4-2 The measured and predicted in-line force time history for the VSMC12 cylinder (true prediction).
Fig. 4.4-3 The wave profile record during Hurricane Carla (record-6887).
Fig. 4.4-4  The measured and predicted in-line force time history for the WPII cylinder at position-7 (true prediction).
are uniformly distributed between 0 and $2\pi$. Thus, the amplitude and phase spectra of the forces can be determined with linear wave theory, Eq. (2.2-11), and the assumption at all the $\phi_n = -90^\circ (-\pi/2)$. Thence the time domain realization of the force can be determined with inverse FFT.
5.0 CONCLUSIONS

1. The in-line force power spectrum for smooth vertical cylinders can be estimated with surprising accuracy by using only the inertia term in the Morison equation.

2. By adding another assumption that the phase lag between the velocity spectrum and the force spectrum is $-90^\circ$, the force time history can be predicted with surprising accuracy.

3. The new method is most suitable for preliminary design, because there is no need to worry about how to choose the usual force coefficients.

4. The new frequency domain Keulegan-Carpenter number was defined as $K_n(z,f_n) = u_n(z,f_n)/(f_n x D)$, which turned out to be smaller than 4 even during Hurricane Carla in the Wave Force Project II measurements.

5. In small $K_n(z,f_n)$ regions, maximum force coefficients closely follow the relation $C_{\mu n}(z,f_n) = 2\pi^2/K_n(z,f_n)$ for a smooth cylinder.

6. For small $K_n$, the phase lag between $u_n(z,f_n)$ and $F_n(z,f_n)$ approaches $-90^\circ$ for smooth cylinders.

7. When the force power spectra pseudo prediction method was applied to forces on horizontal cylinders, the results were in good agreement with measured values. Smooth cylinder predictions were more precise than the predictions for rough cylinders.
6.0 REFERENCES


APPENDICES
APPENDIX A

CONSTRUCTING THE $C_{u}(K)$ HYPERBOLA

The method of constructing a $C_{u}(K)$ curve described here is valid for both periodic waves and decomposed random waves. In order to simplify notations, the log(K) and log$C_{u}(K)$ axes are replaced by X and Y axes. Procedures for constructing the hyperbola are shown below.

The hyperbola with respect to $\overline{X}$ and $\overline{Y}$ axes (Fig. A-1) can be expressed as

$$\frac{\overline{X}^2}{a^2} - \frac{\overline{Y}^2}{b^2} = 1 \quad (A-1)$$

where $a$ and $b$ are transverse axes. The value of $a$ is set equal to 0.1 in this thesis. The relationship between $a$ and $b$ can be expressed as

$$\frac{a}{b} = \tan \delta \quad (A-2)$$

where $\delta$ is the angle between asymptote $L_1$ and the $\overline{Y}$ axis. The angles $\delta$ and $\theta$ can be determined from asymptotes $L_1$ and $L_2$ (Fig. A-1). The asymptotes $L_1$ and $L_2$ are the same as those described in section 4.1.

Relationships between the $\overline{X}$ - $\overline{Y}$ axes and the $X'$ - $Y'$ axes can be expressed as

$$\overline{X} = X'\cos\theta + Y'\sin\theta \quad (A-3)$$

and

$$\overline{Y} = -X'\sin\theta + Y'\sin\theta \quad (A-4)$$
Fig. A-1 The $C_\mu$ hyperbola and the coordinate systems.
Relationships between the $X' - Y'$ axes and the $X - Y$ axes can be expressed as

$$X' = X - X_1$$  \hspace{1cm} (A-5)$$

and

$$Y' = Y - Y_1$$  \hspace{1cm} (A-6)$$

where $(X_1, Y_1)$ is the intersection of the two asymptotes $L_1$ and $L_2$.

By substituting Eqs. (A-5) and (A-6) into Eqs. (A-3) and (A-4), then substituting Eqs. (A-3) and (A-4) back into Eq. (A-1), we can obtain a hyperbola with respect to the $X - Y$ axes. The ordinates of the hyperbola are obtained by using a digital computer.
APPENDIX B

THE DERIVATIVE OF $C_\mu(K)$ WITH RESPECT TO $\alpha$

The $C_\mu$ can be expressed as

$$C_\mu = C_{d\phi} + \frac{C_{m\phi}}{C_{d\phi}} \alpha$$  \hspace{1cm} (B-1)

where $\alpha = \pi^4/4K^2$. The derivative of $C_\mu(K)$ with respect to $\alpha$ can be expressed as

$$\frac{d C_\mu}{d \alpha} = \frac{d}{d \alpha} \left[ C_{d\phi} + \frac{C_{m\phi}}{C_{d\phi}} \alpha \right]$$

$$= \frac{d C_{d\phi}}{d \alpha} + \frac{d}{d \alpha} \left[ \frac{C_{m\phi}}{C_{d\phi}} \right] + \frac{C_{m\phi}}{C_{d\phi}}$$

$$= P + Q + m.$$  \hspace{1cm} (B-2)

By substituting $\alpha = \pi^4/4K^2$ into Eq. (B-2), after rearrangement Eq. (B-2) can be rewritten as

$$\frac{d C_\mu}{d \alpha} = p + q + m$$  \hspace{1cm} (B-3)$^{12}$

where

$$p = - [1 - \sin^2(\phi)] \frac{d C_{d\phi}}{d K} \frac{2K^3}{\pi^2}$$  \hspace{1cm} (B-4)

and

$$q = - \frac{d C_{m\phi}}{d K} \frac{2K^3}{\pi^2}$$  \hspace{1cm} (B-5)

$^{12}$Note that $P + Q = p + q$, but $P \neq p$ and $Q \neq q$. 
The range of $K$ is roughly divided into four $K$ regions: (1) $K < 10$, (2) $10 < K < 40$, (3) $40 < K < 100$, (4) $100 < K$.

(1) $K < 10$

In this region, the phase, $\delta$, approaches $-90^\circ$, $C_{d\phi} = \frac{\pi^2}{2K}$ and $C_{m\phi}$ approaches 2, so

$$\lim_{K \to 0} \frac{d \ C_{d\phi}}{d \ K} = -\frac{\pi^2}{2K^2}$$  \hspace{1cm} (B-6)

and

$$\lim_{K \to 0} \frac{d \ C_{m\phi}}{d \ K} = 0$$  \hspace{1cm} (B-7)

Substituting Eqs. (B-6) and (B-7) into Eqs. (B-4) and (B-5), we find that when $K$ approach 0, $p = 0$, $q = 0$, and $(p+q)/m = 0$.

(2) $10 < K < 40$

For $10 < K < 40$, both inertia force and drag force are important, but no theoretical conclusions about $p$ and $q$ can be made. Experimental results are therefore used to evaluate the value of $p + q$. Unfortunately, no periodic wave results are available in this region of $K$ with large $\beta$. The only data available with large $K$ and $R$ are Sarpkaya's U tube results (Sarpkaya, 1976b). His tests are not as close to reality as real wave tests, but they can give us a rough idea about the value of $p+q/m$.

Sarpkaya used the Fourier analysis method to estimate $C_{dF}$ and $C_{mF}$, Figs. B-1 and B-2. In order to maintain consistency, his results was converted into $C_{d\phi}$ and $C_{m\phi}$ by using the formulas in Appendix C. The converted $C_{d\phi}$ and $C_{m\phi}$ were then plotted on log-log
Fig. B-1  $C_d$ and $C_m$ versus $K$ for $\beta = 4480$ (from Sarpkaya, 1976).
Fig. B-2 \( C_d \text{F} \) and \( C_m \text{F} \) versus \( K \) for \( \beta = 5260 \) (from Sarpkaya, 1976).
paper and smooth fitting curves were drawn, Figs. B-3 and B-4. From these curves the slopes of \( \log(C_{d\phi}) \) and \( \log(C_{m\phi}) \) were estimated.

Since

\[
\frac{d C_{d\phi}}{d K} = \frac{C_{d\phi}}{K} \frac{d \log(C_{d\phi})}{d \log(K)} \tag{B-8}
\]

and

\[
\frac{d C_{m\phi}}{d K} = \frac{C_{m\phi}}{K} \frac{d \log(C_{m\phi})}{d \log(K)} \tag{B-9}
\]

Then \( p \) and \( q \) can be written as

\[
p = -\left[1 - \sin^2(-\phi)\right] \frac{2 C_{d\phi} K^2}{\pi} \frac{d \log(C_{d\phi})}{d \log(K)} \tag{B-10}
\]

and

\[
q = -\frac{C_{m\phi} K}{\pi} \frac{d \log(C_{m\phi})}{d \log(K)} \tag{B-11}
\]

Finally \( p, q, \) and \( (p+q)/m \) are determined by Eqs. (B-10) and (B-11).

The results are shown in Table B-1.

<table>
<thead>
<tr>
<th>( K )</th>
<th>( \beta = 4480 )</th>
<th>( \beta = 5260 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p + q )</td>
<td>( m )</td>
</tr>
<tr>
<td>10</td>
<td>-0.13</td>
<td>3.44</td>
</tr>
<tr>
<td>20</td>
<td>0.082</td>
<td>7.00</td>
</tr>
<tr>
<td>40</td>
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<td>12.60</td>
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It is found that \( 10 < K < 40 \) the values of \( (p+q)/m \) were small.
Fig. B-3  $C_{d\phi}$ and $C_{m\phi}$ versus $K$ for $\beta = 4480$, converted from $C_d$ and $C_m$ curves (Fig. B-1) by using Eqs. (C-12), and (C-13).
Fig. B-4  $C_d\phi$ and $C_m\phi$ versus $K$ for $\beta = 5260$, converted from $C_{dF}$ and $C_{mF}$ curves (Fig. B-2) by using Eqs. (C-12), and (C-13).
(3) \( 40 < K < 100 \)

No data are available to examine the values of \((p+q)/m\) at the time this study is carried out.

(4) \( 100 < K \)

When \( K \) is large it is assumed that \( C_\mu(K) = C_{d\phi}(K) = C_{ds} \), the maximum force is total drag force, so \( C_{m\phi}(K) \) is equal to 0. Since \( C_{ds} \) is a constant, the values of the derivative of \( C_{d\phi}(K) \) and \( C_{m\phi}(K) \) with respect to \( K \) must be 0. Thus both \( p \) and \( q \) are equal to 0.

After examining all four regions of \( K \), it can be found that \((p+q) \ll m\) is acceptable for \( K < 40 \) and for when \( K \) is very large, if \( C_\mu(K) = 2\pi^2/K \) is true when \( K \) is small and \( C_\mu(K) = C_{ds} \) is true when \( K \) is large.
APPENDIX C

COMPARISON OF FORCE COEFFICIENTS OBTAINED THROUGH DIFFERENT METHODS

In the Morison regime, wavy flow data are often analyzed by the least-squares method, because least squares minimize the root mean square error between predictions and measurements. The Fourier analysis method is often used for oscillating flow data, such as U tube tests (Sarpkaya, 1976) or cylinders oscillated in still water. Although the data base are the same, the values of the drag coefficients and the inertia coefficients are different from method to method. In order to compare the force coefficients obtained from other publications, the relationships of force coefficients from different methods should be established.

The relationships of force coefficients are very complex before the completed formula is derived. Simplified formulas will be used in this thesis, involving only the first harmonic component of force.

When only the first harmonic component of the force is considered, the measured force $F_m$, with phase lag $\phi$ in respect to peak velocity, can be written as

$$F_m = F_p \cos(\theta + \phi) \quad (C-1)$$

where $\theta = \omega t$. Dividing by $\frac{1}{2} \rho D u^2$ will non-dimensionalize $F_m$, so that

$$\hat{F}_m = C_u \cos(\theta + \phi) \quad (C-2)$$
(a) **Derivation for the Least-Squares Method**

The force coefficients determined by the simplified least-squares method shown here are designated as $C_{dL}$ and $C_{mL}$. The mean square error between the measured and predicted forces is defined as

$$ E^2 = \frac{1}{2\pi} \int_{0}^{2\pi} (F_m - \hat{F}_p)^2 \, d\theta $$  \hspace{1cm} (C-3)

where $\hat{F}_m$ is the measured force and $\hat{F}_p$ is the predicted force.

To determine $C_{dL}$ and $C_{mL}$, the mean square error is minimized by taking the derivative with respect to each coefficient:

$$ \frac{d}{dC_{dL}} E^2 = 0 $$  \hspace{1cm} (C-4)

and

$$ \frac{d}{dC_{mL}} E^2 = 0 $$  \hspace{1cm} (C-5)

By substituting Eq. (C-3) into Eq. (C-4) and Eq. (C-5), and rearranging the terms,

$$ C_{dL} = \frac{4}{3\pi} \int_{0}^{2\pi} \hat{F}_m \cos\theta \cos\theta \, d\theta $$  \hspace{1cm} (C-6)

and

$$ C_{mL} = \frac{K}{2\pi} \int_{0}^{2\pi} \hat{F}_m \sin\theta \, d\theta $$  \hspace{1cm} (C-7)

Equation (C-2) is now substituted in Eq. (C-6) and Eq. (C-7) to obtain
and
\[ C_{dL} = \frac{32}{9} \pi \cos \phi \, C_{\mu} \]  \hfill (C-8)

and
\[ C_{mL} = \frac{K}{\pi^2} \sin \phi \, C_{\mu} \]  \hfill (C-9)

(b) Derivation for the Fourier Analysis Method

The force coefficients determined by Fourier analysis are designated as \( C_{dF} \) and \( C_{mF} \). The drag coefficient can be obtained by replacing \( \hat{F} \) by \( \hat{F}_m \) then multiplying both sides of Eq. (2.1-6) by \( \cos \theta \) and integrating over one wave cycle of 0 to \( 2\pi \) (Sarpkaya, 1976):

\[ C_{dF} = \frac{3}{4} \int_0^{2\pi} \hat{F}_m \cos \theta \, d\theta \]  \hfill (C-10)

The inertia coefficients can be obtained by multiplying Eq. (2.1-6) by \( \sin \theta \) and integrating from 0 to \( 2\pi \):

\[ C_{mF} = \frac{-2K}{3} \int_0^{2\pi} \hat{F}_m \sin \theta \, d\theta \]  \hfill (C-11)

By substituting Eq. (C-2) in Eq. (C-10) and Eq. (C-11) and rearranging the terms,

\[ C_{dF} = \frac{3}{8} \pi \cos \phi \, C_{\mu} \]  \hfill (C-12)

and

\[ C_{mF} = \frac{K}{\pi} \sin \phi \, C_{\mu} \]  \hfill (C-13)
The relationships between these force coefficients and the maximum in-line force coefficient for a vertical cylinder $C_u$ are shown in Fig. C-1 and Fig. C-2.

The phases and force coefficients obtained by different methods are tabulated in Table C-1. The results of extreme conditions are obtained by set $\phi = -90^\circ$, when $K$ is small, and set $\phi = 0^\circ$ when $K$ is large.
Fig. C-1 Drag coefficients from different methods.

\[
\frac{C_{dF}}{C_\mu} = \frac{3}{8} \cos \phi \\
\frac{C_{dL}}{C_\mu} = \frac{32}{9\pi} \cos \phi \\
\frac{C_{d\phi}}{C_\mu} = \frac{1}{1 + \sin^2 \phi}
\]

Fig. C-2 Inertia coefficients from different methods.

\[
\frac{C_{mF}}{C_\mu} = \frac{C_{mL}}{C_\mu} = \frac{K}{\pi^2} \sin \phi \\
\frac{C_{m\phi}}{C_\mu} = \frac{K}{\pi^2} \frac{2 \sin \phi}{1 + \sin \phi}
\]
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<tr>
<th>Keulegan Carpenter Number</th>
<th>Phase</th>
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<th>Maximum Force Coefficient</th>
<th>Force-Phase Method</th>
<th>Least-Squares Method</th>
<th>Fourier Analysis Method</th>
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<td>$\frac{2\sin\phi C_u}{1 + \sin^2\phi}$</td>
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<td>$C_{d\text{S}}$</td>
<td>$C_u$</td>
<td>$1.13 C_u$</td>
</tr>
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</table>

Table C-1  Force coefficients from different methods