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Report

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METHOD OF MEASURING THE SHEARING MODULI IN WOOD[1]

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Formulas for the flexure and the buckling of rectangular plywood plates involve the stiffness factors of strips of unit width, parallel and perpendicular, respectively, to the grain of the face plies and a shearing modulus of the wood associated with shearing strains in planes parallel to the faces of the plate. Wood has three principal shearing moduli corresponding to shearing strains in the three planes of elastic symmetry, viz., the longitudinal-tangential, longitudinal-radial, and radial-tangential planes. In the usual torsion tests[3] made on rectangular bars to determine these moduli, the effects of two of the three moduli are combined. In order to obtain either modulus it is necessary to combine results of tests on two or more specimens of different dimensions or to make use of inconveniently thin specimens. A method of testing, based on a well-known result in the theory of the bending of thin plates, has been developed by means of which any one of the three principal shearing moduli can be measured directly by using a test specimen in the form of a square flat plate. The planes of the faces and edges of the plate are as nearly as possible parallel to the planes of symmetry of the wood. The faces are chosen parallel to the plane of the shearing strain corresponding to the desired shearing modulus.

Suitable arrangement is made as in figure 1, to apply four equal loads at the corners of the square plate, the two loads at the extremity of one diagonal acting downward and the two at the extremity of the other diagonal acting upward. In making the first square test specimens small projections were left at the corners as shown in figure 1 in order that the loads could be applied directly to the corners of the square. These projections should not be made larger than necessary. Experience has shown that the projections may be omitted and metal pieces used at the corners as illustrated in figure 2. The preparation of test specimens is much simpler with this arrangement. A number of tests showed an increase of only about 1 percent in the measured modulus when specimens were tested first with projections at the corners and then with metal corner pieces after the projections were sawed off.

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[1] This mimeograph is one of a series of progress reports issued by the Forest Products Laboratory to aid the Nation's war program. Results here reported are preliminary and may be revised as additional data become available.

[2] J. A. Liska, assistant engineer, helped make the first series of tests.

[3] Carrington, H., Phil. Mag. 41, 206, 1921 (London).

The shear modulus is obtained from a load-deflection curve. The deflection is the deflection relative to the center of the plate of a point on a diagonal. In actual practice it is advisable to take the mean of the deflections at four points equally distant from the center on the two diagonals. A convenient device for obtaining this mean from the reading of a single dial is shown in figure 1. The dial readings give twice the average deflection relative to the center of the four points of support of the device, if the plate is deflecting according to the theory. The surface of the plate should be an hyperbolic paraboloid, a saddle-shaped surface. The sections of the theoretical surface made by vertical planes through the two diagonals are two identical parabolas with their vertices at the center of the plate. One of the parabolas is concave upward, the other concave downward. Small adjustments in the lengths of the arms of the device can be made by means of the nuts and screw-threads shown in the figure. For larger adjustments another set of arms can be used.

From the theory to which reference will be made later, it is known that if the origin is taken at the center of the plate and rectangular axes are chosen parallel to the edges as in figure 3, the deflection, w, relative to the center, of any point with coordinates x and y is given by the equation

$$w = \frac{3P}{Gh^3} xy \qquad (1)$$

where

P = the load applied to each corner
h = the thickness of the plate
G = the shearing modulus associated with a shearing strain referred to the axes x and y. For a point on a diagonal at a distance u from the center, $x = y = u/\sqrt{2}$. The equation for the deflection then becomes

$$w = \frac{3Pu^2}{2Gh^3} \qquad (2)$$

It follows that

$$G = \frac{3\ u^2}{2h^3} \frac{P}{w} \qquad (3)$$

The experimental procedure is to plot a load deflection curve and from its slope determine G with the aid of equation (3).

A plate loaded in the manner here described was used as a basis for the discussion of the edge conditions in the theory of the flexure of flat plates by Thomson and Tait.[4] It was shown that concentrated forces at the corners are required to maintain the torsional couples that exist along the edges of a plate deformed in accordance with equation (1). Other discussions of the theory can be found in books on the Theory of Elasticity.[5,6]

[4] Thomson and Tait. Treatise on Natural Philosophy, Part II, (1890), Section 656.

[5] S. Timoshenko, Theory of Plates and Shells, p. 91.

Nadai[6] made a series of experiments with square plates loaded at the corners to check the accuracy of the theory of isotropic plates by comparing the observed deflections along the diagonals with those calculated from equation (2).

The extension of the theory to plates of wood or of plywood can be readily made with the aid of Section 2 and Appendix 2 of Mimeograph No. 1312, "Flat Plates of Plywood under Uniform and Concentrated Loads," Forest Products Laboratory, March 1942. The theory assumes that the faces of the plate are parallel to one or another of the planes of the symmetry of the wood; the longitudinal-tangential, longitudinal-radial, or radial-tangential planes. It also assumes that the plane of each edge of the plate is parallel to a plane of symmetry.

To avoid appreciable effects of shear deformation in vertical planes, the ratio of length of edge to thickness should be as great as 25:1. When the deflections relative to the center are of the order of the thickness of the plate, direct stresses (membrane stresses) begin to produce an appreciable effect and the load deflection curve bends upward. Consequently, the early linear portion of the curve should be used. It is found that the effects of direct stresses make it inadvisable to use test plates whose ratio of side to thickness is greater than 50:1.

The method of measurement described above has been used to obtain satisfactory results for flat-grain wood and plywood. No difficulty is to be anticipated with edge-grain material. To obtain the shear modulus in a plane perpendicular to the grain of the wood, specimens can be cut from the ends of blocks made by gluing together flat-grain or edge-grain planks or veneer.

In plywood, if the plies are of different material or elastic properties, the method gives the effective shearing modulus that appears in the equation governing the bending of a plate. This is the modulus that is needed in discussing the deflection and buckling of plates.

Considerable errors are introduced if the grain of the wood is markedly inclined to the faces or to the edges in portions of the plate. Two groups of tests were run to determine a possible effect of small inclinations of the grain of the wood to the edges of the plate. Two plates of plywood were tested first with the grain of the wood as nearly as possible parallel or perpendicular to the edges. Then each plate was successively cut down to obtain plates with the grain inclined to one pair of edges with slopes of 1 in 20, 1 in 10, 3 in 20, and 1 in 5. The moduli measured for these plates showed no definite trend and no greater variability than could reasonably be attributed to experimental errors. If the test plates are cut with the edges as nearly parallel or perpendicular to the grain of the wood as possible it appears that the effects of inevitable variations in the direction of the grain are not important. If the plate is deflecting according to theory, lines drawn on the faces parallel to the edges remain straight during the experiment. A straight edge may be used to determine whether gross

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[6] A. Nadai, Elastische Platten, p.34-45.

departures from theoretical assumptions exist in a given plate. Also the deflections of the four gage points with respect to the center should be approximately equal. Thus it is readily shown that a plate in which the grain of the wood makes an angle of 45° with the edges, does not deflect in accordance with equation (1).

When possible, fairly thick specimens should be used to avoid effects associated with initial curvature. It is believed that the best ratio of side to thickness is in the neighborhood of 30:1. For plates that are quite flat the ratio of side to thickness may be allowed to become as great as 50:1. The effect of small initial curvature can be minimized by obtaining two load deflection curves, one for the loads acting downward on two given corners, the other with the plate rotated through 90° so that the loads act upward on these corners. When there is large initial curvature a large difference in the two calculated shear moduli will result and the average cannot be accepted as a good value of the actual shear modulus.

Two series of tests were run to discover possible effects of the ratio of the length of edge to the thickness of the specimen. Two plates of commercial 5-ply Douglas-fir plywood 0.5 inch thick were cut down to successively smaller dimensions and tested. The results of the tests are shown in table 1 and in figure 4. The agreement of the values of the modulus for the two plates is to be regarded as accidental. Considerable variations in the values of the modulus measured on different plates are to be expected. For each specimen tests were made on two smaller plates than those shown. The shear moduli calculated from those tests were considerably lower than those found for the larger plates. But the values were erratic and are not considered reliable because the gage points of the available device for measuring the deflections were too near the points of application of the loads at the corners. The readings could thus be influenced not only by the effects of shear deformation in vertical planes, but also by local irregularities of the plywood in the vicinity of the corners. An effect of shear deformation in vertical planes is to be expected for specimens for which the ratio of length of edge to thickness is not great enough. The results of these tests confirm the statement previously made that suitable values of this ratio appear to lie between 25 to 1 and 50 to 1.

It is known that the values of shearing moduli are affected by the rate of loading. The test offers a convenient method of determining the effect of rate of loading on each of the shearing moduli. However, under present circumstances, this effect can be considered to be of secondary importance. A modulus determined at a moderate rate of loading appears to be adequate for use in buckling problems. In the tests listed in table 1, the speed of the head of the testing machine ranged from 0.08 inch per minute for the larger specimens to 0.03 inch per minute for the smaller.

Table 1.--Results of tests on plates with different ratios of length of side to thickness[1]

Ratio of length of side to thickness	Modulus of rigidity x 10^{-4}			
	Specimen 1 Diagonal 1	Specimen 1 Diagonal 2	Specimen 2 Diagonal 1	Specimen 2 Diagonal 2
20	10.47	10.43	10.54	10.20
24	10.34	10.48	10.57	10.74
28	10.53	10.54	10.53	10.61
32	10.49	10.41	10.46	10.48
36	10.37	10.37	10.62	10.49
40	10.34	10.45	10.52	10.61
44	....	....	10.56	10.66
45	10.55	10.70	....	....
47	....	....	10.55	10.60

[1] Two plates of 5-ply Douglas-fir plywood 0.5 inch thick were tested at successively smaller sizes. In each case, the load was applied downward first at the extremities of one diagonal and then at the extremities of the other diagonal. The moisture content was approximately 6 percent.

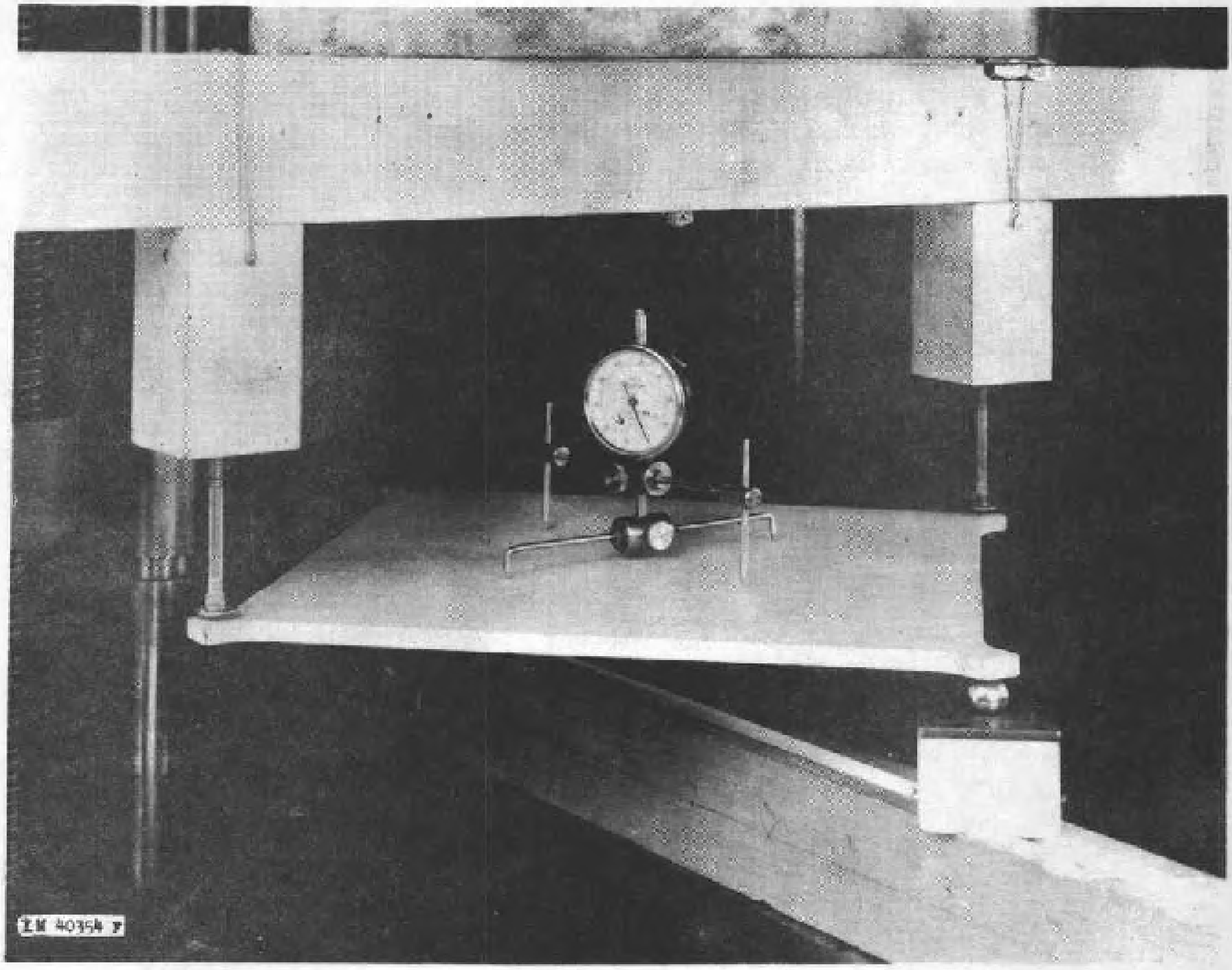

Fig. 1.--Loading of test specimen and device for measuring deflections.