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A systematic optimization of the Cohen class time-frequency transformation for detecting the parameters change is developed. The local moments approach to change detection is proposed and a general formula for the local moments is derived. The optimal kernel functions of the time-frequency transformation are determined based on the combined criteria of maximum sensitivity with respect to parameters change and minimum distortion of physical interpretation of the local moments. The sensitivity of the local moment with respect to a certain kind of inputs is analyzed and a most "convenient" and a "worst" input are identified. The results are presented in the form of the case studies for detecting parameters change in simple linear systems.

Detection and Diagnosis of Parameters Change in Linear System

Using Time-Frequency Transformation

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CHAPTER 1
INTRODUCTION

The problems of change detection arise in many areas of automatic control and signal processing, and may be classified as follows:

1) Segmentation of signals for the purpose of recognition, and monitoring of dynamical systems.

2) Failure detection in controlled systems.

3) Reinitialization of adaptive algorithms, for tracking quick variations of the parameters.

The change detection procedure essentially comprises two tasks:

a) Generating "residuals" or change indicating signal, and

b) Designing decision rules based on these residuals.

Both deterministic and stochastic approaches for solving these two tasks are discussed in the literature. There is an excellent survey by Willsky [1] of methods for the detection of abrupt changes in the state and output variables of a dynamical system. The survey deals mainly with sensors and actuators failure in dynamical systems. There is also a comprehensive survey by Basseville [2] on detection of parameters change in signals and systems. In [2], the focus is on the change of coefficients of AR or ARMA models, and the change of eigenstructure of system models in a random environment. Both [1] and [2] assume an abrupt change or jump in parameters as the change model.

Here the methods for detecting change in parameters of linear systems are studied. The parameters of a linear system are understood as constants or time-dependent coefficients in the system equations [3]. When the parameters of a continuous, linear, time-invariant systems change, the output of such a system is a "non-stationary" signal in the sense that it can be characterized by a varying-in-time power spectrum or varying-in-time energy spectrum.

To analyze "non-stationary" signals we can use so-called short-time Fourier transform (STFT). The Fourier transform provides a powerful tool for analysis of stationary signal whose spectral content does not change in time. The STFT can be used to analyze non-stationary signal by windowing the signal in time domain so that over the length of the window the signal is stationary (short time stationary signals). The Fourier transform of this windowed signal is used to characterize the energy distribution at a time that is given by the center of the window. Sliding the window over the entire signal displays the variations of the distribution in time. This approach yields so-called spectrogram [20], which is commonly used to analyze non-stationary signals. The major drawback of spectrogram based analysis is that the window length is directly related to the frequency resolution. To increase the frequency resolution, one has to take a longer window, which means that non-stationarities occurring during this interval will be smeared out in time and frequency.

The second approach to analyze non-stationary signals uses the notion of instantaneous power spectrum. In general, this approach consists of a signal transformation that depends on two variables: time and frequency. Various time-frequency distributions have been proposed, each with different properties. These transformations offer finer resolution in both, time and frequency, as compared to the short-time Fourier transformation. Cohen [10] introduced a general class of time-frequency transformations with each member of this class given by:

$$C_f(t,\omega;\phi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-j\tau\omega + j\xi t - j\xi\mu}\,\phi(\xi,\tau)\,f\!\left(\mu+\frac{\tau}{2}\right)f^*\!\left(\mu-\frac{\tau}{2}\right)d\tau\,d\xi\,d\mu. \tag{1-1}$$

In Eq.(1-1), $f(t)$ is the time signal, $f^*(t)$ is its complex conjugate, and $\phi(\xi,\tau)$ is an arbitrary function called kernel function. Various transformations are obtained by choosing a particular kernel. For example the Wigner, the rule of Born and Jordan, and Choi-Williams transformation are obtained by choosing $\phi(\xi,\tau) = 1$, $\phi(\xi,\tau) = 2\sin(\xi\tau/2)/\xi\tau$, and $\phi(\xi,\tau) = \exp(-\xi^2\tau^2/\sigma)$, respectively.

There are many applications of non-stationary signal analysis using the time-frequency transformation [19, 26, 27, 28, 29, 30, 31]. One approach is to calculate the transformation to see whether it reveals more information than the spectrogram. This technique was successfully applied by Janse and Kaizer [26] to a loudspeaker design. The Wigner transformation was calculated for a number of standard filters and found to be a particularly useful tool in handling the inherently non-stationary signals encountered in loudspeaker operation. Another method is to use particular properties of time-frequency transformation, such as those of local moments. For example, Boashash et al. [19] used Wigner-Ville transformation to extract the instantaneous frequency with application to geophysical exploration. Various transformations have their merits and drawbacks. An obvious objective in their application is to emphasize the merits and limit the drawbacks. This is accomplished by imposing proper constraints on the kernel function.

This thesis presents a systematic approach to selecting and optimizing kernel function for a given signal analysis problem. In particular the issue of detecting the parameter changes in linear time-invariant (LTI) system is addressed. To this end the time-frequency transformation is applied to the output of LTI system and the local moments of such transformation are obtained. The optimal kernel function is sought, such that the local moments are most sensitive to selected parameter changes.

A secondary goal accomplished in this research is to diagnose (identify) the type of parameter change using a particular kernel function.

The local moment approach to change detection provides a systematic, theoretical methodology. The kernel functions are determined based on the combined criteria of maximum sensitivity with respect to parameters change and minimum distortion of their physical interpretation. The local moments with some constraints on the kernel function have "physical" meaning, thus aiding the change diagnosis. The local moments often can be calculated directly without actually performing the time-frequency transformation. The latter is particularly important in on-line implementation of proposed detection algorithms. Although the study does not take into account noise effects in the signal measurement, the solution to the deterministic problem should provide a good basis for developing solution to the corresponding stochastic problems.

The main contribution of this work is in developing the methods of systematic optimization of the Cohen class transformations for a given non-stationary signal analysis problem. The results are presented in the form of the case studies for detecting parameter changes in linear system dynamics.

Thesis Outline:

Chapter 2 discusses the basic concepts of time-frequency transformation (TFT) and its properties which result from the kernel constraints. The introductory paper of Claasen and Mecklenbrauker [9] is used here as the main reference. Next the properties of a TFT applied to the output of a LTI system are studied. These properties depend on both; the kernel and the system parameters.

Chapter 3 introduces the criteria for selecting the best kernel for the detection of parameter changes in a LTI system. Optimizing the kernel with respect to these criteria results in the constraints of the kernel function. In particular, the maximum sensitivity of the local moments with respect to parameter change is sought. The derivation of a general local moment equation is presented. A criterion for the moment sensitivity is established and related to a general form of kernel function. Finally, the best kernel is characterized by the constraints on the initial value and derivatives of the kernel function.

In Chapter 4 an application of the proposed methodology to detect parameters change a continuous time LTI system is discussed and the computer simulation of change detection is presented. The possible extensions to a nth order linear system are discussed.

Chapter 5 summarizes results obtained in this thesis and discusses future development of the proposed methodology.

CHAPTER 2

BASIC CONCEPTS

2.1. Time-Frequency Representation

The general class of time-frequency transformations (TFT) is given by Eq.(1-1), rewritten here for convenience:

$$C_f(t,\omega;\phi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-j\tau\omega + j\xi t - j\xi\mu}\,\phi(\xi,\tau)\, f\!\left(\mu+\frac{\tau}{2}\right) f^{*}\!\left(\mu-\frac{\tau}{2}\right) d\tau\, d\xi\, d\mu. \tag{2-1}$$

In the above f(t) denotes a complex time signal defined for $t \in (-\infty,+\infty)$, f(t)* denotes its complex conjugate, and $\phi(\xi,\tau)$ represents the kernel function, which is either real or complex function of its arguments. The kernel function defines a particular member of the Cohen class. In terms of the Fourier transform of the signal, each member of the Cohen class can be expressed as

$$C_f(t,\omega;\phi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-j\xi t + j\tau\mu - j\tau\omega}\phi(\xi,\tau)\, F\!\left(\mu+\frac{\xi}{2}\right) F^{*}\!\left(\mu-\frac{\xi}{2}\right) d\tau\, d\xi\, d\mu, \tag{2-2}$$

where $F(\omega)$ denotes the Fourier transform of f(t). There are many well-known members of the Cohen class of transformations [14, 15, 16, 17, 18]. The properties of a particular transformation result from the constraints imposed on the kernel. In the sequel C_f is referred to as a time-frequency signal transformation or simply transformation.

2.2. General Description of Kernel Function

The kernel function can be a function of time t, frequency ω and, in general, a function of the signal f(t) [13]. In this thesis, unless otherwise stated, it is assumed that ϕ is not a function of time or frequency and that it is independent of the signal. As is shown later, independence of time and frequency of the kernel function assures that the transformation is time and shift invariant. Also if the kernel is independent of the signal, then the transformation is said to be quadratic in the signal.

An important subclass of the Cohen transformations are those transformations for which the kernel is a function of the product of its arguments, i.e., $\phi(\xi,\tau) = \phi(\xi\tau)$. This product form is particularly attractive for the analysis of non-stationary signals, i.e., the signals for which the spectral content varies significantly in time (e.g., "chirp" signals). If the signal to be analyzed is a stationary signal, then a convenient kernel function is a product of two functions, i.e., $\phi(\xi,\tau) = \phi_1(\xi)\phi_2(\tau)$.

For the time-frequency transformation to exist, the kernel function must be integrable in the domain of signal support, i.e.,

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |\phi(\xi,\tau)| \; d\xi \, d\tau < \infty. \tag{2-3}$$

Assuming that the kernel function is Fourier transformable in ξ and τ separately, Eq.(2-1) can be rewritten as

$$C_f(t,\omega;\phi) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f\left(\mu+\frac{\tau}{2}\right)f^*\left(\mu-\frac{\tau}{2}\right) h(y-\mu,\tau)\, e^{-j\tau\omega}\, d\mu\, d\tau, \tag{2-4}$$

and Eq.(2-2) can be rewritten as

$$C_f(t,\omega;\phi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F\left(\mu+\frac{\xi}{2}\right)F^*\left(\mu-\frac{\xi}{2}\right) H(\xi,\omega-\mu)\, e^{j\xi t}\, d\xi\, d\mu, \tag{2-5}$$

where h and H are the following Fourier-like transformations:

$$h(\theta,\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \phi(\xi,\tau)\, e^{j\xi\theta}\, d\xi, \tag{2-6}$$

$$H(\xi,\eta) = \int_{-\infty}^{+\infty} \phi(\xi,\tau)\, e^{-j\tau\eta}\, d\tau. \tag{2-7}$$

Eq.(2-4) shows that a time-frequency transformation is obtained through the convolution in time t of a kernel $\phi(t,\tau)$ with the signal "correlation" $f(t+\tau/2)f^*(t-\tau/2)$ and the Fourier transform in time variable τ. Similarly, Eq.(2-5) shows that a time-frequency transformation is obtained through the convolution in frequency ω of a kernel $H(\xi,\omega)$ with the spectrum "correlation" $F(\omega+\xi/2)F^*(\omega-\xi/2)$, followed by the inverse Fourier transform in frequency variable ξ.

2.3. Relationship Between Kernel Function and Signal Properties

Assume that $\phi(\xi,\tau) = \phi(\xi\tau)$, that the time signal $f(t) = a(t)\, e^{j\psi(t)}$, and that the Fourier transform of f(t) is $F(\omega) = A(\omega)\, e^{j\Psi(\omega)}$.

2.3.1. Instantaneous power and energy density spectrum

$C_f(t,\omega;\phi)$ represents a joint time-frequency signal power distribution, if it satisfies certain consistency conditions. For example, when the frequency variable is integrated out we expect to obtain the instantaneous power $|f(t)|^2$, and similarly when the time is integrated out we expect to obtain the energy density spectrum $|F(\omega)|^2$. Integrating Eq.(2-1) with respect to ω, we have

$$\int_{-\infty}^{+\infty} C_f(t,\omega;\phi) = \phi(0)\ |f(t)|^2. \tag{2-8}$$

In order for Eq.(2-8) to be equal to $|f(t)|^2$, we must have

$$\phi(0) = 1. \tag{2-9}$$

Therefore, with the constraint (2-9), we have

$$\int_{-\infty}^{+\infty} C_f(t,\omega;\phi)\, d\omega = |f(t)|^2. \tag{2-10}$$

Similarly, it can be shown that

$$\int_{-\infty}^{+\infty} C_f(t,\omega;\phi)\, dt = |F(\omega)|^2, \tag{2-11}$$

requires that condition (2-9) be satisfied. An immediate consequence is that if the constraint (2-9) is satisfied then the integral $C_f(\tau,\omega;\phi)$ over the whole (t,ω)-plane is equal to the total signal energy.

2.3.2. First order local moment and instantaneous frequency

The **nth order local moment at time t** is defined as

$$M_n(t) \equiv \frac{Z_n(t)}{Z_0(t)}, \tag{2-12}$$

where

$$Z_n(t) \equiv \int_{-\infty}^{+\infty} \omega^n \, C_f(t,\omega;\phi) \, d\omega. \tag{2-13}$$

For n = 1, Eq.(2-12) represents the **first order local moment at time t**:

$$M_1(t) \equiv \frac{\int_{-\infty}^{+\infty} \omega \, C_f(t,\omega;\phi) \, d\omega}{\int_{-\infty}^{+\infty} C_f(t,\omega;\phi) \, d\omega}$$

$$= \frac{2\,\phi'(0)}{\phi(0)} \frac{a(t)'}{a(t)} + \psi(t)', \tag{2-14}$$

where the prime denotes differentiation. The second equality results from the assumed form of $f(t) = a(t)\exp(j\psi(t))$. When an analytical signal is analyzed, the so-called **instantaneous frequency** is often defined as the derivative of the phase $\psi(t)$ [17]. Assuming that the signal is analytical, $M_1(t)$ is equal to $\psi(t)'$ if

$$\phi(0)' = 0. \tag{2-15}$$

Thus, we can obtain the instantaneous frequency under the constraint (2-15) from the first local moment of an analytical signal:

$$M_1(t) = \psi(t)'. \tag{2-16}$$

Similarly, the **first order local moment at frequency** ω is equal to $\Psi(\omega)'$ with the constraints (2-15), thus,

$$M_1(\omega) = -\Psi(\omega)', \tag{2-17}$$

where

$$M_1(\omega) \equiv \frac{\int_{-\infty}^{+\infty} t\, C_f(t,\omega;\phi)\, dt}{\int_{-\infty}^{+\infty} C_f(t,\omega;\phi)\, dt}. \tag{2-18}$$

For an analytical signal, the first local moment in frequency indicates **group delay**, which is defined as the phase derivative in frequency domain. Note that both, the instantaneous frequency and the group delay, are important characteristics of any dynamical behavior.

2.3.3. Second order local moment and spread

For n=2, Eq.(2-12) represents the **second order local moment at time t**:

$$M_2(t) \equiv \frac{\int_{-\infty}^{+\infty} \omega^2\, C_f(t,\omega;\phi)\, d\omega}{\int_{-\infty}^{+\infty} C_f(t,\omega;\phi)\, d\omega}$$

$$= \frac{2\phi''(0)}{\phi(0)}\left\{\left[\frac{a(t)'}{a(t)}\right]^2 + \frac{a(t)''}{a(t)}\right\} + \frac{2\phi'(0)}{\phi(0)}\left[\frac{2a(t)'}{a(t)}\psi(t)' + \psi(t)''\right]$$

$$+ \frac{1}{2}\left\{\left[\frac{a(t)'}{a(t)}\right]^2 - \frac{a(t)''}{a(t)}\right\} + [\psi(t)']^2. \tag{2-19}$$

The second equality results from the assumed form of $f(t) = a(t)\exp(j\psi(t))$. With the constraint (2-15), the second order local moment at time t can be expressed as

$$M_2(t) = \frac{1}{2}\left[1 + 4\frac{\phi(0)''}{\phi(0)}\right]\left[\frac{a(t)'}{a(t)}\right]^2 - \frac{1}{2}\left[1 - 4\frac{\phi(0)''}{\phi(0)}\right]\left[\frac{a(t)''}{a(t)}\right] + [\psi(t)']^2. \qquad (2\text{-}20)$$

The **spread at time t** is defined as

$$M_2(t) - M_1(t)^2 = \frac{1}{2}\left[1 + 4\frac{\phi(0)''}{\phi(0)}\right]\left[\frac{a(t)'}{a(t)}\right]^2 - \frac{1}{2}\left[1 - 4\frac{\phi(0)''}{\phi(0)}\right]\left[\frac{a(t)''}{a(t)}\right], \qquad (2\text{-}21)$$

where $M_1(t)$ and $M_2(t)$ represent the first and the second local moment at time t, respectively. Again the second equality is valid for $f(t) = a(t)\exp(j\psi(t))$. Eq.(2-21) may become negative for some of the TFT's. For the Wigner transformation, $\phi(\xi,\tau) = 1$, the spread is obtained by setting $\phi''(0) = 0$ in Eq.(2-22):

$$M_2(t) - M_1(t)^2 = \frac{1}{2}\left[\frac{a(t)'}{a(t)}\right]^2 - \frac{1}{2}\left[\frac{a(t)''}{a(t)}\right]. \qquad (2\text{-}22)$$

As pointed out by Classen and Mecklenbrauker [9], Eq.(2-22) can become negative and cannot be properly interpreted as the "variance" of the time-frequency distribution. Using the so-called Choi-Williams kernel, $\phi(\xi,\tau) = \exp(-\xi^2\tau^2/\sigma)$, the spread is obtained by setting $\phi''(0) = -2/\sigma$ in Eq.(2-21):

$$M_2(t) - M_1(t)^2 = \frac{1}{2}\left[1 - \frac{8}{\sigma}\right]\left[\frac{a(t)'}{a(t)}\right]^2 - \frac{1}{2}\left[1 + \frac{8}{\sigma}\right]\left[\frac{a(t)''}{a(t)}\right]. \qquad (2\text{-}23)$$

Eq.(2-23) produces negative values for the signal a(t) = exp(-αt) and $\sigma > 0$. In order for the spread to preserve its intuitive interpretation of "variance", Eq.(2-21) should stay always nonnegative. The following constraint

$$\phi''(0) = \frac{1}{4}, \tag{2-24}$$

yields:

$$M_2(t) - M_1(t)^2 = \left[\frac{a(t)'}{a(t)} \right]^2. \tag{2-25}$$

Therefore, with the constraints (2-15) and (2-24), a nonnegative value for the spread is secured. There are many transformations satisfying Eq.(2-9), (2-15), and (2-24), for example, the modified exponential kernels introduced by Cohen [11]:

$$\begin{aligned} \phi(\xi\tau) &= \varphi(\xi\tau)\, e^{-\frac{\xi^2\tau^2}{\sigma}} \\ &= (c_0 + c_1\xi\tau + c_2\xi^2\tau^2 + \dots)\, e^{-\frac{\xi^2\tau^2}{\sigma}}. \end{aligned} \tag{2-26}$$

where c_0, c_1, c_2 .... are constant coefficients. Rewriting Eq.(2-26) in terms of x = $\xi\tau$, we have:

$$\phi(x) = (c_0 + c_1 x + c_2 x^2 + \dots)\, e^{-\frac{x^2}{\sigma}}. \tag{2-27}$$

Therefore the kernel function at the origin is

$$\phi(0) = c_0. \tag{2-28}$$

From constraint (2-9) we have $c_0 = 1$. Taking the first and second derivative with respect to x at the origin we have, respectively,

$$\phi'(0) = c_1, \tag{2-29}$$

and

$$\phi''(0) = 2c_2 - \frac{2}{\sigma}. \tag{2-30}$$

From constraints (2-15) and (2-24), we can solve for c_1 and c_2:

$$c_1 = 0, \tag{2-31}$$

$$c_2 = \frac{1}{8} + \frac{1}{\sigma}. \tag{2-32}$$

Thus one of the modified exponential kernels that satisfies the constraints (2-9), (2-15), and (2-24) is:

$$\phi(\xi\tau) = (1 + c_2\xi^2\tau^2)\, e^{-\frac{\xi^2\tau^2}{\sigma}}, \tag{2-33}$$

where c_2 is given by Eq.(2-32).

2.4. Selected TFT Properties

In general, the kernel function can be a function of time and frequency [9]. Therefore a general class of quadratic time-frequency transformations has the kernel function: $\phi(\xi,\tau,\omega,t)$, where ξ, τ, ω, and t denote the integration variable of frequency, the integration variable of time, frequency variable, and time variable, respectively.

Eq.(2-1) can be now generalized as follows:

$$C_{fg}(t,\omega;\phi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-j\tau\omega+j\xi t-j\xi\mu}\,\phi(\xi,\tau,\omega,t)\, f\left(\mu+\frac{\tau}{2}\right)g^*\left(\mu-\frac{\tau}{2}\right)d\tau\, d\xi\, d\mu, \tag{2-34}$$

where $C_{fg}(.)$ represents "cross-transformation" of signals f(t) and g(t). For $f \equiv g$, $C_{ff}(.)$ belongs to the Cohen class of transformations.

Claasen and Mecklenbrauker [9] related certain properties of general TFT's to the corresponding constraints on the kernel. In the following notation, "P_k" and "C_k" stand for properties and the corresponding kernel constraints, respectively.

P_1: If $g(t) = f(t-t_0)$ then $C_g(t,\omega;\phi) = C_f(t-t_0,\omega;\phi)$, provided that

C_1: $\phi(\xi,\tau,\omega,t) = \phi(\xi,\tau,\omega)$, i.e., ϕ does not depend on t.

P_2: If $g(t) = f(t)e^{j\omega_0 t}$ then $C_g(t,\omega;\phi) = C_f(t,\omega-\omega_0;\phi)$, provided that

C_2: $\phi(\xi,\tau,\omega,t) = \phi(\xi,\tau,t)$, i.e. ϕ does not depend on ω.

Properties P_1 and P_2 state that shifts in time or frequency of a signal result in corresponding shifts in the distribution. These properties are essential if we want time and frequency variables of the transformation to correspond to a signal and its spectrum independent variables, respectively. Constraints C_1 and C_2 demand the kernel to be independent of time and frequency.

P_3: If $f(t) = 0$ for $|t| > T$ then $C_f(t,\omega;\phi) = 0$ for $|t| > T$, provided that

C_3: $h(t,\tau) = 0$ for $|\tau| < 2|t|$.

P_4: If $F(\omega) = 0$ for $|\omega| > \Omega$ then $C_f(t,\omega;\phi) = 0$ for $|\omega| > \Omega$, provided that

C_4: $H(\xi,\omega) = 0$ for $|\xi| < 2|\omega|$.

In the above h(.) and H(.) are given by Eq.(2-6) and Eq.(2-7). The finite support properties P_3 and P_4 are important from the application point of view. They state that if a signal has a support region in time or frequency, then its transformation will have the same support region in time or frequency, respectively.

P_5: $C_f(t,\omega;\phi) = C^*_f(t,\omega;\phi)$, provided that

C_5: $\phi(\xi,\tau) = \phi^*(-\xi,-\tau)$.

Property P_5 is also very convenient from a practical point of view, stating that the TFT is real valued. This contrasts with the fact that the Fourier transform is, in general, complex valued.

Finally we can recover the signal from $C_f(t,\omega;\phi)$ uniquely up to a constant multiplier. To obtain the signal from a TFT we take the inverse Fourier transformation of Eq.(2-1) and obtain

$$f\left(\mu+\frac{\tau}{2}\right)f^*\left(\mu-\frac{\tau}{2}\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{C_f(\eta,\omega;\phi)}{\phi(\xi,\tau)}\, e^{-j\xi\eta+j\tau\omega+j\xi\mu}\, d\eta\, d\omega\, d\xi, \tag{2-35}$$

The above assumes that

$$\frac{C_f(\eta,\omega;\phi)}{\phi(\xi,\tau)} \tag{2-36}$$

is integrable in the variables ξ,η and ω. By setting $\mu = t/2$ and $\tau = t$, we have

$$f(t) = \frac{1}{2\pi\, f^*(0)} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{C_f(\eta,\omega;\phi)}{\phi(\xi,t)}\, e^{jt\omega + j\xi\left(\frac{t}{2}-\eta\right)}\, d\eta\, d\omega\, d\xi, \tag{2-37}$$

that is, f(t) is reconstructed uniquely up to the constant $f^*(0)$. Some of the properties of TFT's presented here are used in the sequel using the equation numbers as a reference. It should be pointed out that this chapter represents only a small portion of the properties of TFT. For a complete survey see [9].

CHAPTER 3

KERNEL DERIVATION FOR DETECTION

3.1. Introduction

The time-frequency transformations which are members of the generalized Cohen's class have received a considerable attention as the tools for analyzing non-stationary signals [9, 14, 15, 16, 17, 18, 25]. The review paper by Cohen [13] discusses the well known time-frequency transformations, their properties, and applications. Cohen's class is defined by Eq.(2-1), with various transformations obtained by choosing a particular kernel function ϕ. The properties of a particular transformation are controlled by choosing the constraints on the kernel. Choi and Williams [14] devised a very interesting method to analyze the effects of the kernel constraints, by examining the local autocorrelation function. They pointed out that since the main interest in a TFT is to study a "local" signal phenomena a relatively large weight should be given to $f(\mu-\tau/2)f^*(\mu-\tau/2)$ when the integration variable μ is close to t, to emphasize the events near time t. This concept was very effectively used in devising the so-called Choi-Williams kernel. Zhao, Atlas, and Marks [25] proposed a special type of "cone-shaped kernel" which produces good resolution in both time and frequency and at the same time suppresses transformation artifacts. The basic question in all these studies is how to derive the constraints on the kernel in order to obtain the desirable properties of the TFT. Some of the existing results on this subject are summarized in Chapter 2.

In this chapter the constraints for the kernel function are determined, such that the local moments become most sensitive for parameter change in a linear system. To achieve this objective, the following steps are taken:

1. A general formula for the local moments equation is derived.
2. A criterion is established for the moment sensitivity evaluation and it is expressed in terms of the kernel function values.
3. The optimized kernel function is proposed.

These results are applied to parameter change detection in linear time-invariant systems.

3.2. Derivation of the Best Kernel for Detection

3.2.1. Preliminary concepts

Let $u(t)$ and $y(t)$, $t \in (0,\infty)$ denote the input and output of a single input, single output (SISO), continuous-time linear system, respectively. Assume that the system parameters α change at unknown time $t^* \in [0,T]$, that is $\alpha \equiv \alpha_0$ for $t < t^*$ and $\alpha \equiv \alpha_1$ for $t \geq t^*$. The choice of the input plays an important role in parameter change detection analysis. We may for example want to study the detection in the "worst" input case (i.e., the change of system parameters are "masked" by the input characteristics). It may be also desirable to study the most "convenient" inputs (i.e., inputs which "expose" the system parameters change in the output waveform).

To emphasize the dependence of the output on the system parameters α, we write $y(\alpha,t)$ and express it in terms of the input $u(t)$ and the impulse response $h(\alpha,t,\tau)$:

$$y(\alpha,t) = \int_{-\infty}^{+\infty} h(\alpha,t,\eta)\, u(\eta)\, d\eta. \tag{3-1}$$

For a linear, time-invariant, and causal system, Eq.(3-1) becomes

$$y(\alpha,t) = \int_{-\infty}^{t} u(\eta)\, h(\alpha,t-\eta)\, d\eta. \tag{3-2}$$

For simplicity we assume that $u(\tau) \equiv 0$ for $\tau < 0$. Next, we transform $y(\alpha,t)$ through the time-frequency transformation and derive the local moments. Finally, we select a best kernel function which yields maximum sensitivity of these local moments with respect to the change in parameters α.

3.2.2. Criteria for the best kernel for detection

To define the criteria for the best kernel function for detecting parameters changes, we introduce a payoff function. This payoff function should measure the sensitivity of the TFT with respect to the parameter changes. In this thesis we propose the use of local moments for change detection. The local moments represent (under proper kernel constraints) physical properties of the signal and thus are convenient tools for parameter change diagnosis. Also it is worth noticing that the local moments can be calculated without performing entire TFT thus increasing the feasibility of implementation. Using the local moments the payoff function may take the following form:

$$J(\alpha,\alpha_0,\phi,u,t) = \| M_n(\alpha,\phi,u,t) - M_n(\alpha_0,\phi,u,t) \|, \tag{3-3}$$

where $u(t)$, ϕ, M_n and $\| \ \|$ denote input, kernel function, nth order local moment, and a norm, respectively. The parameter vector α consists of m elements:

$$\alpha = [\alpha_1 \ \ \alpha_2 \ \ldots \ \alpha_m]^T. \tag{3-4}$$

Assuming that $M_n(\alpha,\phi,u,t)$ is continuously differentiable with respect to α, Eq.(3-4) can be approximated by expanding $M_n(\alpha,\phi,u,t)$ into the Taylor series, around $\alpha = \alpha_0$:

$$M_n(\alpha,\phi,u,t) \cong M_n(\alpha_0,\phi,u,t) + \nabla M_n(\alpha,\phi,u,t)\,|_{\alpha=\alpha_0} (\alpha - \alpha_0), \tag{3-5}$$

where

$$\nabla M_n(\alpha,\phi,u,t) = \left[\frac{\partial M_n(\alpha,\phi,u,t)}{\partial \alpha_1} \ \ \frac{\partial M_n(\alpha,\phi,u,t)}{\partial \alpha_2} \ \ \cdots\cdots \ \ \frac{\partial M_n(\alpha,\phi,u,t)}{\partial \alpha_m} \right]^T. \tag{3-6}$$

The payoff function becomes the local **sensitivity function** as expressed by

$$J(\alpha,\alpha_0,\phi,u,t) = S(\alpha_0,\phi,u,t), \tag{3-7}$$

where

$$S(\alpha_0,\phi,u,t) \equiv \| \nabla M_n(\alpha,\phi,u,t)\,|_{\alpha=\alpha_0} \|. \tag{3-8}$$

Two approaches are proposed for obtaining an approximate payoff function as illustrated in Fig. 3.1 and Fig. 3.2. Since M_n is defined by Z_n/Z_0 (see Eq.(2-12) and (2-13)) we obtain an approximation of (3-8) by calculating the following gradient (Fig. 3.1)

$$\frac{\partial M_n(\alpha,\phi,t)}{\partial \alpha}\Big|_{\alpha=\alpha_0}$$

$$= \frac{\partial\left[\dfrac{Z_n(\alpha,\phi,t)}{Z_0(\alpha,\phi,t)}\right]}{\partial \alpha}\Big|_{\alpha=\alpha_0}$$

$$= \frac{\left[\dfrac{\partial Z_n(\alpha,\phi,t)}{\partial \alpha}\right] Z_0(\alpha,\phi,t) - \left[\dfrac{\partial Z_0(\alpha,\phi,t)}{\partial \alpha}\right] Z_n(\alpha,\phi,t)}{Z_0(\alpha,\phi,t)^2}\Big|_{\alpha=\alpha_0}. \qquad (3\text{-}9)$$

$\frac{\partial Z_n(\alpha,\phi,t)}{\partial \alpha}\Big|_{\alpha=\alpha_0}$ is expressed in terms of the system output in Appendix A.

For the second case (Fig. 3.2), we have (see Appendix A)

$$\frac{\partial M_{n_\varepsilon}(\alpha,\phi,t)}{\partial \alpha}\Big|_{\alpha=\alpha_0} = \frac{\partial\left[\dfrac{Z_{n_\varepsilon}(\alpha,\phi,t)}{\partial Z_0(\alpha,\phi,t)}\right]}{\partial \alpha}\Big|_{\alpha=\alpha_0}$$

$$= \frac{\left[\dfrac{\partial Z_{n_\varepsilon}(\alpha,\phi,t)}{\partial \alpha}\right] Z_0(\alpha,\phi,t) - \left[\dfrac{\partial Z_0(\alpha,\phi,t)}{\partial \alpha}\right] Z_{n_\varepsilon}(\alpha,\phi,t)}{Z_0(\alpha,\phi,t)^2}\Big|_{\alpha=\alpha_0} = 0, \qquad (3\text{-}10)$$

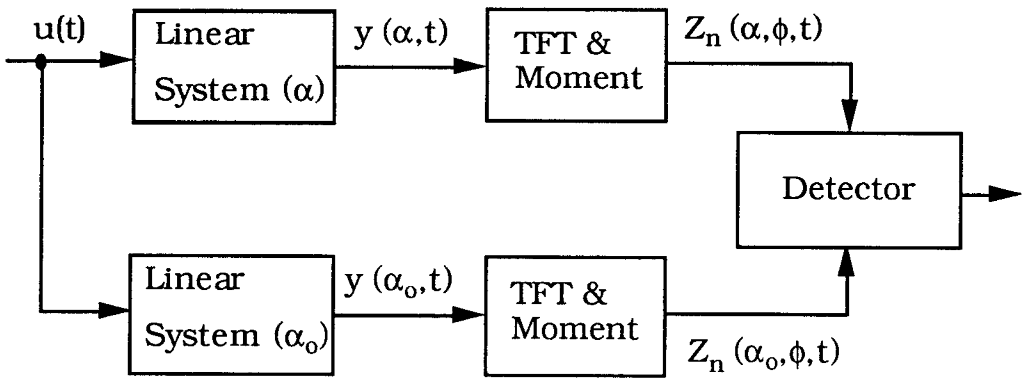


Fig. 3.1. Approach 1 for approximate payoff function

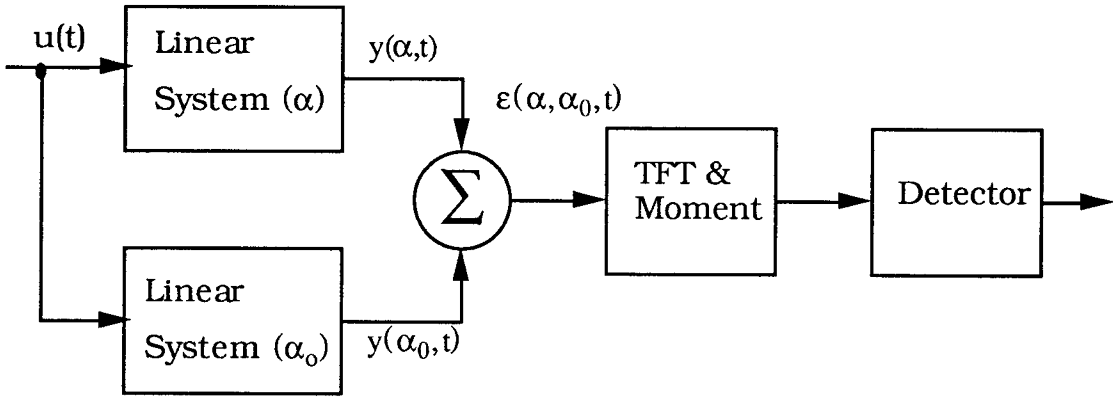


Fig. 3.2. Approach 2 for approximate payoff function

and thus an approximation of Eq.(3-3) calls for the second order terms in the Taylor series expansion and calculation for the corresponding Hessian are presented in Appendix A.

3.2.3. Derivation of local moments formula

The **nth order local moment at time t** from the system output $y(\alpha,t)$ is defined by Eq.(2-12). In order to express local moments in terms of kernel function, Eq.(2-12) can be rewritten as

$$M_n(\alpha,\phi,t) \equiv \frac{Z_n(\alpha,\phi,t)}{Z_0(\alpha,\phi,t)}, \tag{3-11}$$

where

$$Z_n(\alpha,\phi,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \omega^n\, e^{-j\tau\omega + j\xi t - j\xi\mu}\, \phi(\xi,\tau)\cdot$$

$$\cdot y\left(\alpha,\mu+\frac{\tau}{2}\right) y^*\left(\alpha,\mu-\frac{\tau}{2}\right) d\tau\, d\xi\, d\mu\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \omega^n\, e^{-j\tau\omega + j\xi t - j\xi\mu}$$

$$[\,\phi(\xi,\tau)\, g(\alpha,\mu,\tau)\,]\, d\tau\, d\xi\, d\mu\, d\omega, \tag{3-12}$$

and

$$g(\alpha,\mu,\tau) = y\left(\alpha,\mu+\frac{\tau}{2}\right) y^*\left(\alpha,\mu-\frac{\tau}{2}\right). \tag{3-13}$$

Assuming that the impulse response function is n-time continuously differentiable at zero we have:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \omega^n e^{-j\tau\omega} h(\tau)\, d\omega\, d\tau = 2\pi\, (-j)^n\, h^{(n)}(\tau)\,|_{\tau=0}, \tag{3-14}$$

where $h^{(n)}(\tau)$ denotes the nth order derivative of $h(\tau)$. Integrating Eq.(3-12) with respect to ω and τ variables and using Eq.(3-14) yields

$$Z_n(\alpha,\phi,t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (-j)^n\, [\phi(\xi\tau)\, g(\alpha,\mu,\tau)]^{(n)}\,|_{\tau=0}\, e^{j(t-\mu)\xi}\, d\xi\, d\mu, \tag{3-15}$$

where

$$\begin{aligned}
[\phi(\xi\tau)\, g(\alpha,\mu,\tau)]^{(n)}\,|_{\tau=0} &= \xi^n\, \phi^{(n)}(0)\, g(\alpha,\mu,0) \\
&+ (n)\, \xi^{n-1}\, \phi^{(n-1)}(0) \left[\frac{\partial g(\alpha,\mu,\tau)}{\partial\tau}\,|_{\tau=0} \right] \\
&+ \frac{(n)(n-1)}{2}\, \xi^{n-2}\, \phi^{(n-2)}(0) \left[\frac{\partial^2 g(\alpha,\mu,\tau)}{\partial\tau^2}\,|_{\tau=0} \right] \\
&+ \ldots. + \\
&+ (n)\, \xi\, \phi^{(1)}(0) \left[\frac{\partial^{n-1} g(\alpha,\mu,\tau)}{\partial\tau^{n-1}}\,|_{\tau=0} \right] \\
&+ \phi(0) \left[\frac{\partial^{n} g(\alpha,\mu,\tau)}{\partial\tau^{n}}\,|_{\tau=0} \right].
\end{aligned} \tag{3-16}$$

Integrating Eq.(3-15) with respect to the ξ and μ variables yields:

$$\begin{aligned}
Z_n(\alpha,\phi,t) &= \phi^{(n)}(0)\, \frac{\partial^n g(\alpha,\mu,\tau)}{\partial\mu^n}\,|_{\substack{\tau=0\\ \mu=t}} \\
&+ \phi^{(n-1)}(0)\, (n)\, (-j)\, \frac{\partial^n g(\alpha,\mu,\tau)}{\partial\tau\, \partial\mu^{n-1}}\,|_{\substack{\tau=0\\ \mu=t}} \\
&+ \phi^{(n-2)}(0)\, \frac{n(n-1)}{2}\, (-j)^2\, \frac{\partial^n g(\alpha,\mu,\tau)}{\partial\tau^2\, \partial\mu^{n-2}}\,|_{\substack{\tau=0\\ \mu=t}}
\end{aligned}$$

$$+ \ldots . +$$

$$+ \phi^{(1)}(0)\,(n)\,(-j)^{n-1} \frac{\partial^n g(\alpha,\mu,\tau)}{\partial\tau^{n-1}\,\partial\mu} \Big|_{\substack{\tau=0\\ \mu=t}}$$

$$+ \phi(0)\,(-j)^n \frac{\partial^n g(\alpha,\mu,\tau)}{\partial\tau^n} \Big|_{\substack{\tau=0\\ \mu=t}}, \tag{3-17}$$

and

$$Z_0(\alpha,\phi,t) = \phi(0)\, g(\alpha,t), \tag{3-18}$$

where

$$g(\alpha,t) \equiv g(\alpha,\mu,\tau) \Big|_{\substack{\tau=0\\ \mu=t}}. \tag{3-19}$$

From Eq.(3-11), Eq.(3-17), and Eq.(3-18), the local moments equation for $M_n(\alpha,\phi,t)$ is obtained as:

$$M_n(\alpha,\phi,t) = \sum_{i=0}^{i=n} \left[\frac{\phi^{(i)}(0)}{\phi(0)} \right] (-j)^{n-i} \binom{n}{i} \left[\frac{\partial^n g(\alpha,\mu,\tau)}{\partial\tau^{n-i}\,\partial\mu^i} \Big|_{\substack{\tau=0\\ \mu=t}} \right] \left[\frac{1}{g(\alpha,t)} \right]. \tag{3-20}$$

Eq.(3-20) can be rewritten conveniently as

$$M_n(\alpha,K,t) = K^T P + R, \tag{3-21}$$

where

$$K = [\, K_1\ K_2\ \ldots .\ K_n\ ]^T,$$

$$P = [\, P_1(\alpha,t)\ \ P_2(\alpha,t)\ \ldots .\ P_n(\alpha,t)]^T,$$

$$R = R(\alpha,t), \tag{3-22}$$

and

$$K_1 \equiv \frac{\phi^{(1)}(0)}{\phi(0)},\ K_2 \equiv \frac{\phi^{(2)}(0)}{\phi(0)},\ \ldots\ldots\ ,K_n \equiv \frac{\phi^{(n)}(0)}{\phi(0)}, \tag{3-23}$$

$$P_i(\alpha,t) \equiv (-j)^{n-i} \binom{n}{i} \left[\frac{\partial^n g(\alpha,\mu,\tau)}{\partial\tau^{n-i}\,\partial\mu^i} \Big|_{\substack{\tau=0\\ \mu=t}} \right] \cdot \left[\frac{1}{g(\alpha,t)} \right],$$

$$R(\alpha,t) \equiv (-j)^n \left[\frac{\partial^n g(\alpha,\mu,\tau)}{\partial\tau^n} \right] \cdot \left[\frac{1}{g(\alpha,t)} \right]. \tag{3-24}$$

From Eq.(3-21), it is seen that the nth order local moment is a linear function of K. When nth order moment is used as a change detector, the K vector relates the properties of this moment to the properties of the kernel ϕ. Note that K provides complete local characterization of ϕ as n goes to infinity. We can constrain the kernel function ϕ knowing the values of K_1 through K_n. The gradient of nth order local moment becomes:

$$\nabla M_n(\alpha,K,u,t) \big|_{\alpha=\alpha_0} = WK + V, \tag{3-25}$$

where

$$W = [\, W_1, W_2, \ldots\ , W_n \,],$$

$$W_i = \left[\frac{\partial P_i}{\partial\alpha_1}\ \frac{\partial P_i}{\partial\alpha_2}\ \cdots\cdots\ \frac{\partial P_i}{\partial\alpha_m} \right]^T \Big|_{\alpha=\alpha_0},$$

$$V = \left[\frac{\partial R}{\partial\alpha_1}\ \frac{\partial R}{\partial\alpha_2}\ \cdots\cdots\ \frac{\partial R}{\partial\alpha_m} \right]^T \Big|_{\alpha=\alpha_0}. \tag{3-26}$$

In the Eq.(3-26), W is a m x n matrix and V is a m x 1 column vector, elements of which are functions of time. The m and n represent the number of parameters and the order of local moments, respectively.

3.2.4. Determination of the best kernel function

In this section, we obtain the best kernel function for detection of parameter changes in a linear time-invariant system.

Eq.(3-25) shows that the sensitivity function is a linear function of K, i.e., the sensitivity is unbounded in K. Additional constraints on K come from the constraints on the properties of the kernel function ϕ. As an example consider problem of limiting the bias of local moments which constrains the kernel function and thus the values of K. We illustrate this approach by studying several special cases.

Method 1: Limiting the bias of the first moment as an estimator of instantaneous frequency

From Eq.(2-14) and Eq.(3-21), we can obtain $M_1(t)$ in terms of amplitude and phase of an analytical signal $f(t) = a(t) \exp[j\psi(t)]$:

$$M_1(t) = K_1 P_1 + R_1, \tag{3-27}$$

where

$$P_1 = 2\frac{a(t)'}{a(t)}, \tag{3-28}$$

$$R_1 = \psi(t)'. \tag{3-29}$$

Recall that the instantaneous frequency $M_1(t)$ is obtained under the assumption that $\phi(0) = 1$ and $\phi'(0) = 0$. Hence from Eq.(3-21), the bias of $M_1(t)$ is given by

$$K_1 P_1. \tag{3-30}$$

The sensitivity function becomes now

$$S_1 = \| K_1 W_1 + V_1 \|, \tag{3-31}$$

where W_1 and V_1 are given by Eq.(3-26) for n =1. Eq.(3-30) and (3-31) represent conflicting goals for minimizing the bias of M_1 and maximizing $M_1(t)$ sensitivity. The payoff function may take the following form:

$$g_1(K_1) = q_1 \int_{t_1}^{t_2} K_1^2 P_1^2 \, dt - q_2 S_1^2, \tag{3-32}$$

with the additional constraint:

$$\left| \frac{K_1 \int_{t_1}^{t_2} P_1 \, dt}{\int_{t_1}^{t_2} R_1 \, dt} \right| \le \varepsilon_1, \tag{3-33}$$

where q_1, q_2 denote weighting functions, and ε_1 represents bias tolerance limits. S_1^2 is represented by:

$$S_1^2 = \sum_{i=1}^{i=m} \int_{t_1}^{t_2} \left| \frac{\partial P_1}{\partial \alpha_i} K_1 + \frac{\partial R_1}{\partial \alpha_i} \right|^2 dt. \tag{3-34}$$

In the above equations $[t_1,t_2]$ defines the change detection interval. Minimization of g_1 belongs to the class of non-linear programing optimization problems. Obviously, the optimum value of K_1 depends on the interval t_1 and t_2, weights q_1, q_2, tolerance ε_1, and nominal value of α_0. Selection of t_1 and t_2 is guided by both the requirement for change detection time and the mathematical constraints (e.g., existence of the corresponding integrals of P_1 and R_1). When P_1 and R_1 are periodic functions, the t_1 and t_2 can be determined by their period. Thus, the determination of t_1 and t_2 depends on P_1 and R_1.

Method 2: Constraining the spread of the TFT

From Eq.(2-20) and Eq.(3-21), we obtain $M_2(t)$ in terms of amplitude and phase of an analytical signal:

$$M_2(t) = P_{21}K_1 + P_{22}K_2 + R_2, \tag{3-35}$$

where

$$P_{21} = 2\left[\frac{2a(t)'}{a(t)}\psi(t)' + \psi(t)''\right], \tag{3-36}$$

$$P_{22} = 2\left\{\left[\frac{a(t)'}{a(t)}\right]^2 + \frac{a(t)''}{a(t)}\right\}, \tag{3-37}$$

$$R_2 = \frac{1}{2}\left\{\left[\frac{a(t)'}{a(t)}\right]^2 - \frac{a(t)''}{a(t)}\right\} + [\psi(t)']^2. \tag{3-38}$$

A positive spread is obtained for $\phi(0) = 1$, $\phi'(0) = 0$, and $\phi''(0) = 1/4$. This defines $K_1 = 0$ $K_2 = 1/4$ thus not allowing to manipulate the sensitivity function of the second order local moment

$$S_2 = \| W_1 K_1 + W_2 K_2 + V_2 \| , \tag{3-39}$$

with W_1, W_2, and V_2 given by Eq.(3-26) for $n = 2$. Since both, the maximum sensitivity and the positiveness of the spread, are desirable, we propose the following constrained optimization problem:

$$\max_{K} S_2, \tag{3-40}$$

subject to the constraint:

$$M_2(t) - M_1(t)^2 \geq 0. \tag{3-41}$$

Obviously the optimum values of K resulting from Method 1 are not the same as that from Method 2. We can "cascade" Methods 1 and 2 by first securing (for example) K_1 which limits bias of M_1 and maximize its sensitivity. Next, using this K_1 as a fixed value in Method 2 we find an optimum value of K_2 which maximizes the sensitivity of $M_2(t)$ and ensures the positiveness of the spread function. Finally from the characterization of ϕ by K_1 and K_2 we can obtain (for example) the modified exponential transformation (Eq.(2-24)) by choosing proper c_1 and c_2. Of course many parametrized TFTs can be adjusted to yield proper K_1 and K_2.

Methods 1 and 2 can be easily extended to include higher than the second order local moments. The main concept is to introduce payoff functions which compromise between maximum moment sensitivity and minimum deterioration of the TFT properties. The payoff function proposed in Method 1 and Method 2 take advantage of a "physical meaning" of $M_1(t)$ and $M_2(t)$ as pointed out in Chapter 2.

CHAPTER 4

APPLICATION OF AN OPTIMIZED KERNEL FUNCTION TO CHANGE DETECTION IN LINEAR TIME-INVARIANT SYSTEMS

In Chapter 3 the procedure for obtaining an optimal kernel function was developed. In this chapter, the choice of an optimal kernel for change detection in the linear time-invariant (LTI) system is studied. We begin by deriving a formula for the first local moment as a function of the system input and impulse response. Next a payoff function is proposed, and the sensitivity of the first local moment with respect to the input parameters is analyzed. The most "convenient" and the "worst" input from the point of view of change detection are determined. Application of the obtained results to the second order LTI system is presented. The possible extensions to a nth order linear system are discussed.

4.1. The First Order Local Moment for Sensitivity Analysis

Using Eq.(3-13) and Eq.(3-20), we can express the first order local moment as

$$M_1(\alpha,K_1,u,t) = K_1 P_1 + R_1, \tag{4-1}$$

where

$$P_1 = \left[\frac{1}{g_1(\alpha,t)}\right]\left[\frac{\partial g_1}{\partial \mu}\right]\Big|_{\substack{\tau=0\\ \mu=t}} + \left[\frac{1}{g_2(\alpha,t)}\right]\left[\frac{\partial g_2}{\partial \mu}\right]\Big|_{\substack{\tau=0\\ \mu=t}},$$

$$R_1 = (-j)\left\{\left[\frac{1}{g_1(\alpha,t)}\right]\left[\frac{\partial g_1}{\partial \tau}\right] + \left[\frac{1}{g_2(\alpha,t)}\right]\left[\frac{\partial g_2}{\partial \tau}\right]\right\} \Big|_{\mu=t}^{\tau=0}, \tag{4-2}$$

and

$$g_1 = g_1(\alpha,\mu,\tau) = \int_0^{\mu+\frac{\tau}{2}} h\left(\alpha,\mu+\frac{\tau}{2}-\eta\right) u(\eta)\ d\eta,$$

$$g_2 = g_2(\alpha,\mu,\tau) = \int_0^{\mu-\frac{\tau}{2}} h^*\left(\alpha,\mu-\frac{\tau}{2}-\eta\right) u^*(\eta)\ d\eta,$$

$$g(\alpha,\mu,\tau) = g_1(\alpha,\mu,\tau)\ g_2(\alpha,\mu,\tau). \tag{4-3}$$

Setting $\mu = t$ and $\tau = 0$ in Eq.(4-3) yields

$$g_1(\alpha,t) = \int_0^t h(\alpha,t-\eta)\ u(\eta)\ d\eta,$$

$$g_2(\alpha,t) = \int_0^t h^*(\alpha,t-\eta)\ u(\eta)\ d\eta,$$

$$g(\alpha,t) = g_1(\alpha,t)\ g_2(\alpha,t). \tag{4-4}$$

Taking the first partial derivative of Eq.(4-3) with respect to μ and τ, and setting $\mu = t$ and $\tau = 0$ in Eq.(4-2), we obtain

$$P_1 = \left\{ \frac{h(\alpha,0)\ u(t) + \displaystyle\int_0^t \frac{\partial h\left(\alpha,\mu+\frac{\tau}{2}-\eta\right)}{\partial \mu} \Big|_{\mu=t}^{\tau=0} u(\eta)\ d\eta}{\displaystyle\int_0^t h(\alpha,t-\eta)\ u(\eta)\ d\eta} + \frac{h^*(\alpha,0)\ u^*(t) + \displaystyle\int_0^t \frac{\partial h^*\left(\alpha,\mu-\frac{\tau}{2}-\eta\right)}{\partial \mu} \Big|_{\mu=t}^{\tau=0} u^*(\eta)\ d\eta}{\displaystyle\int_0^t h^*(\alpha,t-\eta)\ u^*(\eta)\ d\eta} \right\}, \tag{4-5}$$

and

$$R_1 = \left(-\frac{j}{2}\right)\left\{ \frac{h(\alpha,0)\,u(t) + \int_0^t \frac{\partial h\left(\alpha,\mu+\frac{\tau}{2}-\eta\right)}{\partial\mu}\Big|_{\substack{\tau=0\\ \mu=t}} u(\eta)\,d\eta}{\int_0^t h(\alpha,t-\eta)\,u(\eta)\,d\eta} - \frac{h^*(\alpha,0)\,u^*(t) + \int_0^t \frac{\partial h^*\left(\alpha,\mu-\frac{\tau}{2}-\eta\right)}{\partial\mu}\Big|_{\substack{\tau=0\\ \mu=t}} u^*(\eta)\,d\eta}{\int_0^t h^*(\alpha,t-\eta)\,u^*(\eta)\,d\eta} \right\}. \tag{4-6}$$

The gradient of the first local moment can be expressed as follows:

$$\nabla M_1(\alpha,K_1,u,t)\,\big|_{\alpha=\alpha_0} = W_1 K_1 + V_1, \tag{4-7}$$

where

$$W_1 = \left[\frac{\partial P_1}{\partial\alpha_1}\ \frac{\partial P_1}{\partial\alpha_2}\ \cdots\ \frac{\partial P_1}{\partial\alpha_n}\right]^T \text{ and } V_1 = \left[\frac{\partial R}{\partial\alpha_1}\ \frac{\partial R}{\partial\alpha_2}\ \cdots\ \frac{\partial R}{\partial\alpha_n}\right]^T. \tag{4-8}$$

The sensitivity function is obtained as the norm of the gradient of the first local moment.

$$S(\alpha_0,K_1,u,t) = \|W_1 K_1 + V_1\|. \tag{4-9}$$

The sensitivity function depends on the input and system parameters. In order to obtain the best kernel, the following optimization problem is solved:

$$\max_{K\in\mathcal{K}}\ \min_{u\in\mathcal{U}} S(\alpha_0,K,u,t), \tag{4-10}$$

where $\mathcal{K}$ represents the constraint set for K_1. The minimization over $u \in \mathcal{U}$ represents the "worst" case input for detecting parameters change. The best kernel function can be obtained by solving the constrained minimax problem (4-10).

4.2. Choice of Input and Impulse Response

To emphasize a point made earlier that the choice of input u(t) plays an important role in detection, we concentrate here on complex exponential input $u(t) = t^q \exp(-\beta t + j\varpi t)$, parametrized by q, β, and ϖ. The values of q, β, and ϖ can be obtained by solving (4-10). Note that replacing $\min\limits_{u \in \mathcal{U}}$ with $\max\limits_{u \in \mathcal{U}}$ in Eq.(4-10) allows for study of the most "convenient" input from the point of view of change detection. The set $\mathcal{U}$ is defined now by the sets constraining q, β, and ϖ.

The impulse response of a linear time-invariant system can be modeled as

$$h(\alpha,t) = \sum_r C_r\, t^r e^{-\lambda_r t + j\omega_r t} + \sum_m C_m\, e^{-\lambda_m t + j\omega_m t} + \sum_n C_n\, e^{-\lambda_n t}. \tag{4-11}$$

For simplicity, we focus on the linear time-invariant system which has the impulse response $h(\alpha,t) = t^r \exp(-\lambda t + j\omega t)$. The analysis of such a system provides good basis for analyzing a higher order system. The parameters of this LTI system are r, λ, and ω. Given the input $u(t) = t^q \exp(-\beta t + j\varpi t)$ and the impulse response $h(\alpha,t) = t^r \exp(-\lambda t + j\omega t)$, the output can be expressed as follows:

$$y(\alpha,t) = \int_0^t h(\alpha,t-\eta)u(\eta)\, d\eta$$

$$= \int_0^t (t-\eta)^r \, e^{-\lambda(t-\eta)+j\omega(t-\eta)} \, \eta^q \, e^{-\beta\eta + j\varpi\eta} \, d\eta$$

$$= e^{-\lambda t + j\omega t} \int_0^t (t-\eta)^r \, \eta^q \, e^{-(\beta-\lambda)\eta + j(\varpi-\omega)\eta} \, d\eta \, . \qquad (4\text{-}12)$$

For the first order LTI system, the impulse response is $h(\alpha,t) = \exp(-\lambda t)$. From Eq.(4-5) and Eq.(4-6), P_1 and R_1 are obtained as:

$$P_1 = -2\lambda + (A+B) \ \text{ and } \ R_1 = -\left(\frac{j}{2}\right)(A-B) \, , \qquad (4\text{-}13)$$

where

$$A = \frac{t^q e^{-Z_1 t}}{\int_0^t \eta^q e^{-Z_1\eta} \, d\eta} \ \text{ and } \ B = \frac{t^q e^{-Z_2 t}}{\int_0^t \eta^q e^{-Z_2\eta} \, d\eta} \, , \qquad (4\text{-}14)$$

$$Z_1 = (\beta - \lambda) - j\varpi \ \text{ and } \ Z_2 = (\beta - \lambda) + j\varpi \, . \qquad (4\text{-}15)$$

Assume that q fixed. From Eq.(4-8) and Eq.(4-13), W_1 and V_1 are obtained as:

$$W_1 = \left[-2 + \left(\frac{\partial A}{\partial \lambda} + \frac{\partial B}{\partial \lambda}\right)\right]\Big|_{\lambda=\lambda_0} \ \text{ and } \ V_1 = \left(-\frac{j}{2}\right)\left(\frac{\partial A}{\partial \lambda} - \frac{\partial B}{\partial \lambda}\right)\Big|_{\lambda=\lambda_0}, \qquad (4\text{-}16)$$

where

$$\frac{\partial A}{\partial \lambda}\Big|_{\lambda=\lambda_0} = \frac{[t^{q+1} e^{-Z_1 t}]\left[\int_0^t \eta^q e^{-Z_1\eta} \, d\eta\right] - [t^q e^{-Z_1 t}]\left[\int_0^t \eta^{q+1} e^{-Z_1\eta} \, d\eta\right]}{\left[\int_0^t \eta^q e^{-Z_1\eta} \, d\eta\right]^2}, \qquad (4\text{-}17)$$

and

$$\frac{\partial B}{\partial \lambda}\Big|_{\lambda=\lambda_0} = \frac{[t^{q+1}\, e^{-Z_2 t}]\left[\int_0^t \eta^q e^{-Z_2\eta}\, d\eta\right] - [t^q e^{-Z_2 t}]\left[\int_0^t \eta^{q+1}\, e^{-Z_2\eta}\, d\eta\right]}{\left[\int_0^t \eta^q e^{-Z_2\eta}\, d\eta\right]^2}. \quad (4\text{-}18)$$

Since

$$\int_0^t \eta^r\, e^{-\alpha\eta}\, d\eta = \frac{r!}{\alpha^{r+1}} - e^{-\alpha t} \sum_{\kappa=0}^{r} \frac{r!}{\kappa!} \frac{t^\kappa}{\alpha^{r-\kappa+1}}, \quad (4\text{-}19)$$

we can simplify the expressions for W_1 and V_1. The optimal kernel selection problem is now formulated as

$$\max_{K_1 \in \mathcal{K}} \min_{q\,\beta\,\varpi} S(\lambda_0, K_1, q, \beta, \varpi, t), \quad (4\text{-}20)$$

where

$$S(\lambda_0, K_1, q, \beta, \varpi, t) = \| W_1 K_1 + V_1 \|. \quad (4\text{-}21)$$

For q=0 and q=1, Eq.(4-14), Eq.(4-17), and Eq.(4-18) can be further simplified:

For q=0:

$$A = \frac{Z_1\, e^{-Z_1 t}}{(1 - e^{-Z_1 t})}, \quad (4\text{-}22)$$

$$B = \frac{Z_2\, e^{-Z_2 t}}{(1 - e^{-Z_2 t})}, \quad (4\text{-}23)$$

and

$$\frac{\partial A}{\partial \lambda}\Big|_{\lambda=\lambda_0} = \frac{e^{-Z_1 t}\,(Z_1 t + e^{-Z_1 t} - 1)}{(1 - e^{-Z_1 t})^2}, \quad (4\text{-}24)$$

$$\frac{\partial B}{\partial \lambda}\Big|_{\lambda=\lambda_0} = \frac{e^{-Z_2 t}\,(Z_2 t + e^{-Z_2 t} - 1)}{(1 - e^{-Z_2 t})^2}. \tag{4-25}$$

For q=1:

$$A = \frac{Z_1^2 t\, e^{-Z_1 t}}{[1 - e^{-Z_1 t} - Z_1 t\, e^{-Z_1 t}]}, \tag{4-26}$$

$$B = \frac{Z_2^2 t\, e^{-Z_2 t}}{[1 - e^{-Z_2 t} - Z_2 t\, e^{-Z_2 t}]}, \tag{4-27}$$

and

$$\frac{\partial A}{\partial \lambda}\Big|_{\lambda=\lambda_0} = \frac{Z_1 t\, e^{-Z_1 t}\,[\,Z_1 t + Z_1 t\, e^{-Z_1 t} + 2e^{-Z_1 t} - 2]}{[\,1 - e^{-Z_1 t} - Z_1 t\, e^{-Z_1 t}]^2}, \tag{4-28}$$

$$\frac{\partial B}{\partial \lambda}\Big|_{\lambda=\lambda_0} = \frac{Z_2 t\, e^{-Z_2 t}\,[\,Z_2 t + Z_2 t\, e^{-Z_2 t} + 2e^{-Z_2 t} - 2]}{[\,1 - e^{-Z_2 t} - Z_2 t\, e^{-Z_2 t}]^2}. \tag{4-29}$$

For q=0, the first moment and its gradient with respect to λ are obtained using the following equations:

$$M_1(\lambda, K_1, u, t)\,\Big|_{\lambda=\lambda_0} = K_1 P_1 + R_1, \tag{4-30}$$

and

$$\nabla M_1(\lambda, K_1, u, t)\,\Big|_{\lambda=\lambda_0} = W_1 K_1 + V_1, \tag{4-31}$$

where P_1, R_1, W_1, and V_1 are given by Eq.(4-13), Eq.(4-16), Eq.(4-22), Eq.(4-23), Eq.(4-24), and Eq.(4-25). The sensitivity analysis is performed by inspecting the behavior of the gradient in terms of W_1 and V_1, (note that the sensitivity is proportional to the gradient norm). Due to the complexity of the corresponding equations for W_1 and V_1, a numerical analysis is performed. Computer simulation is performed by varying the input parameters β and ϖ, with fixed q and λ_0. Assume that all the parameters are positive. Fig. 4.1 shows the input, impulse response, and output for input parameters: $\beta = 1$(a) and $\beta = 3$(b) with fixed $\lambda_0 = 2$, $q = 0$, and $\varpi = \pi$. As β becomes larger the output settling time, approximated by $4/\beta$, becomes smaller. In this simulation, we use the observation interval of 5 seconds.

W_1 and V_1 are plotted separately to observe their individual behavior. Fig. 4.2 shows the gradient for various values of β with $\lambda_0 = 2$, $\varpi = 0$, and $q = 0$. In case of $\varpi = 0$, and $q = 0$, it is easy to calculate $y(t)$, $P_1(t)$, $R_1(t)$, $W_1(t)$, and $V_1(t)$ from the input and impulse response of the first order linear time-invariant system directly. From Eq.(3-2), y(t) is given by

$$y(t) = \frac{e^{-\lambda_0 t}}{\lambda_0 - \beta}[e^{-(\lambda_0 - \beta)} - 1]. \tag{4–32}$$

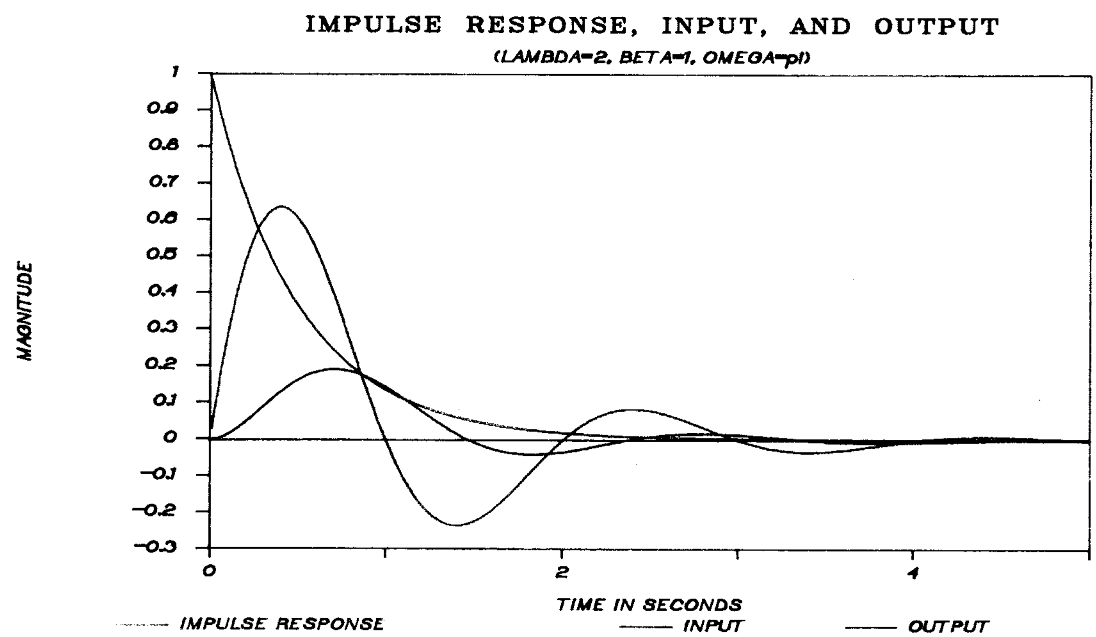
Let define $\rho \equiv \lambda_0 - \beta$, then from Eq.(3-28) and Eq.(3-29) $P_1(t)$ and $R_1(t)$ are:

$$P_1 = -2\lambda_0 + \frac{2\rho\, e^{\rho t}}{e^{\rho t} - 1}, \tag{4-33}$$

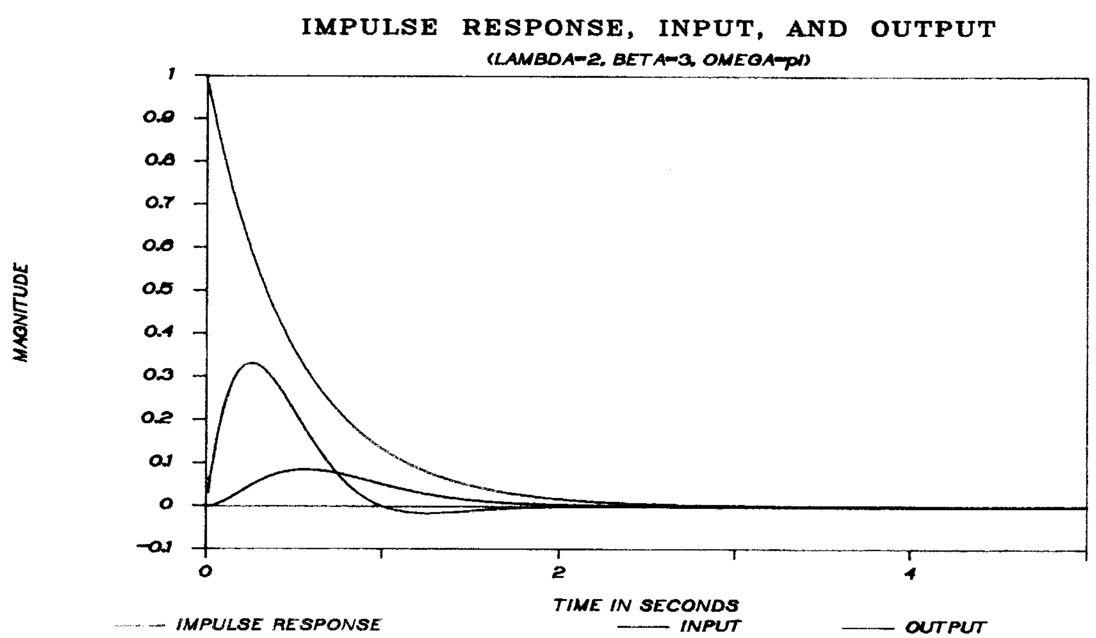
and

$$R_1 = 0, \tag{4-34}$$

since $y(t) = a(t)$.

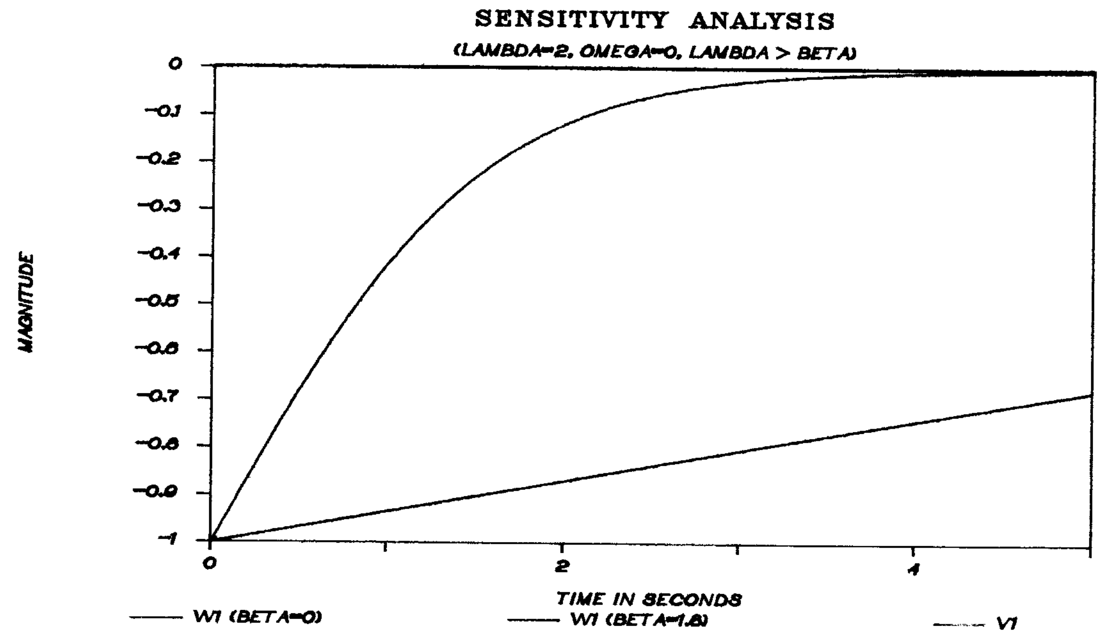


(a)

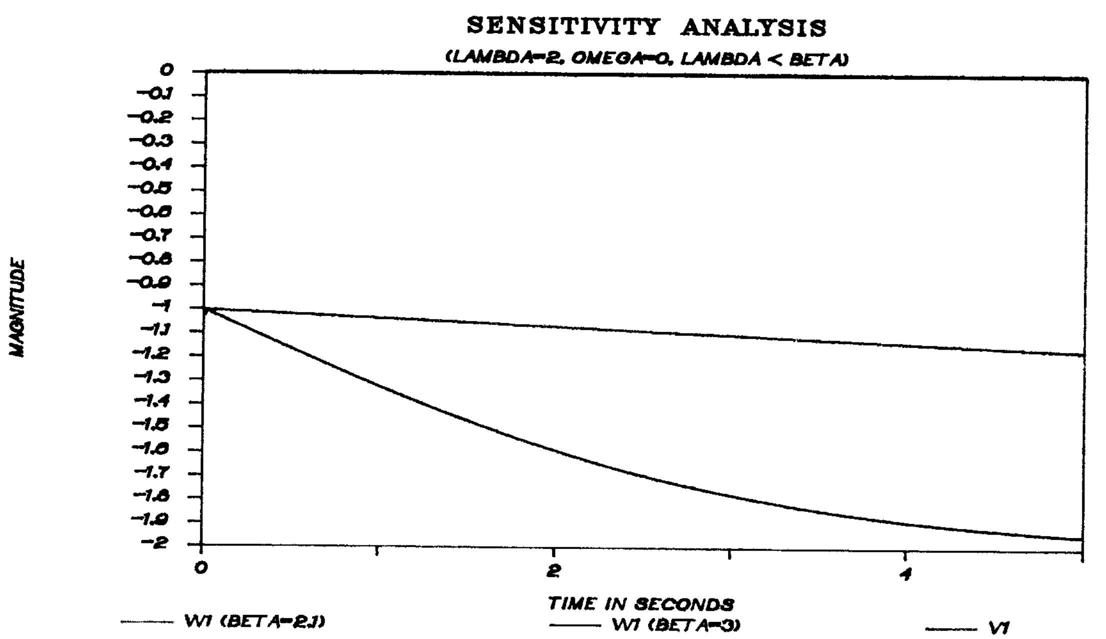


(b)

Fig. 4.1. Signal Analysis 1: Input, impulse response, and output for (a) $\beta = 1$ and (b) $\beta = 3$ with $\lambda_0 = 2$, $\varpi = \pi$, and $q = 0$



(a)



(b)

Fig. 4.2. Sensitivity Analysis 1: W_1 and V_1 of (a) $\rho > 0$ ($\beta = 0$, $\beta = 1.8$) and (b) $\rho < 0$ ($\beta = 2.1$, $\beta = 3$) for $\lambda_0 = 2$, $\varpi = 0$, and $q = 0$

From Eq.(3-26) $W_1(t)$ and $V_1(t)$ are:

$$W_1 = 2\left[\frac{e^{2\rho t} - \rho t\, e^{\rho t} - e^{\rho t}}{(e^{\rho t} - 1)^2} - 1\right], \tag{4-35}$$

and

$$V_1 = 0. \tag{4-36}$$

Therefore, in the limiting cases of ρ we have:

$$\lim_{\rho \to 0} \frac{e^{2\rho t} - \rho t\, e^{\rho t} - e^{\rho t}}{(e^{\rho t} - 1)^2} = \frac{1}{2}, \tag{4-37}$$

$$\lim_{\rho \to \infty} \frac{e^{2\rho t} - \rho t\, e^{\rho t} - e^{\rho t}}{(e^{\rho t} - 1)^2} = 1, \tag{4-38}$$

and

$$\lim_{\rho \to -\infty} \frac{e^{2\rho t} - \rho t\, e^{\rho t} - e^{\rho t}}{(e^{\rho t} - 1)^2} = 0. \tag{4-39}$$

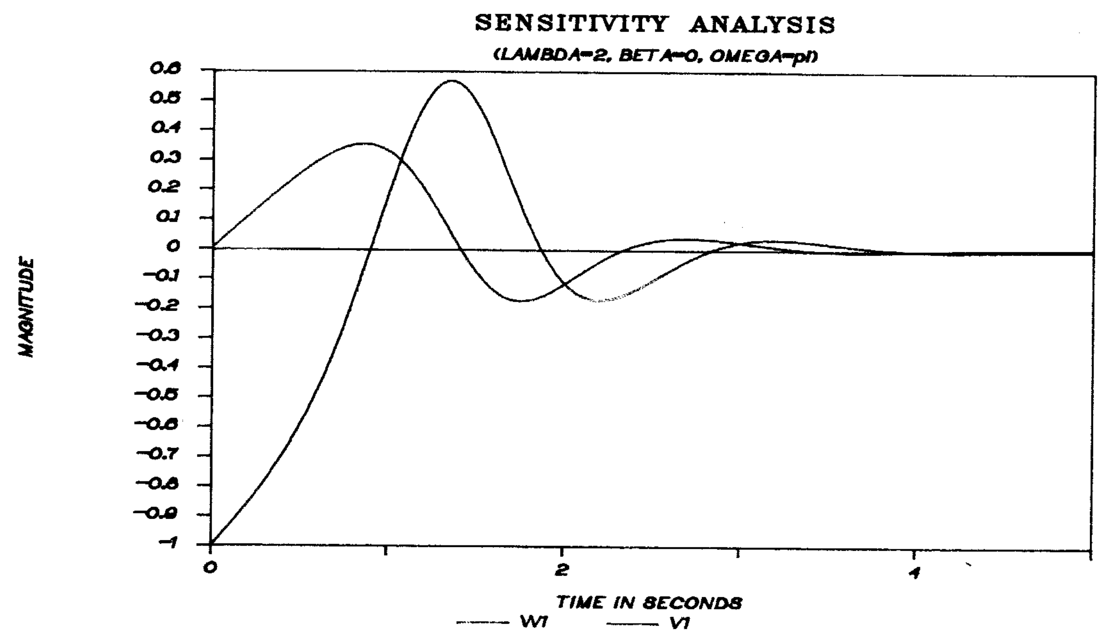
Similar limits are obtain for limits in t. The sensitivity is constant (equal to -1), for $\rho > 0$ the sensitivity decreases monotonically as $t \to \infty$ to zero and for $\rho < 0$ the sensitivity monotonically increases as $t \to \infty$ to a finite value (equal to -2).

Discussion: $\beta = 0$ corresponds to the step input. It is seen that for fixed λ this gives the least desirable sensitivity. As $\beta \to \infty$ the input approximates an impulse of a finite amplitude. For finite β the linearity of system dynamics allows to conclude that impulse input is the best from the point of view of maximum sensitivity.

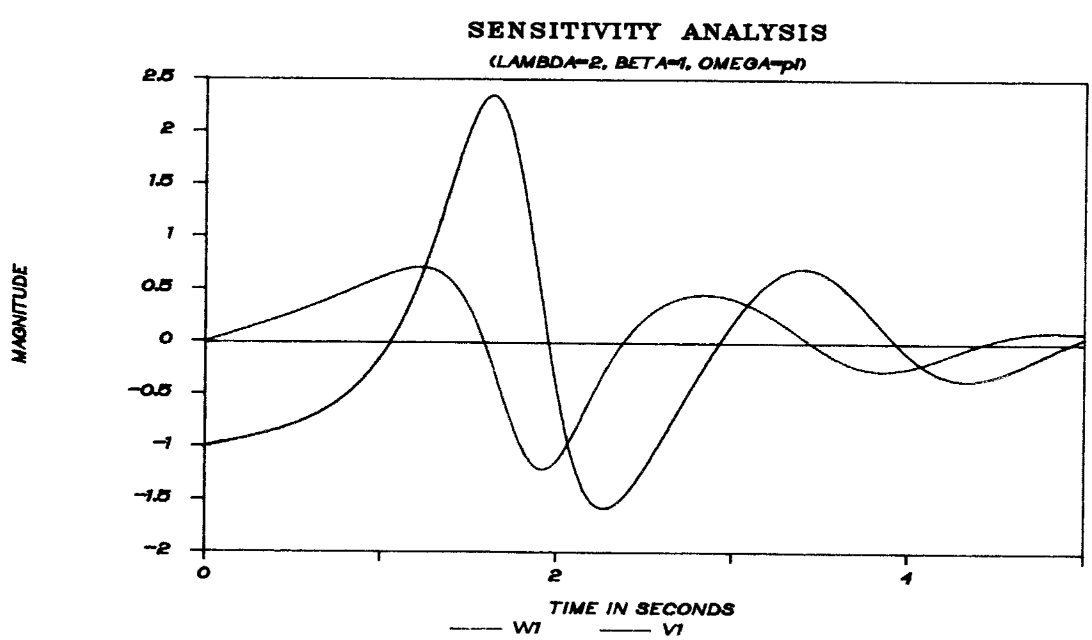
Fig. 4.3 and Fig. 4.4 show the W_1 and V_1 for various values of β with $\lambda_0 = 2$, $\varpi = \pi$, and $q = 0$. The gradient becomes unbounded as β approaches λ_0, and the maximum peak position of W_1 coincides with that of V_1. As β becomes larger for $\beta > \lambda_0$ the W_1 goes to finite value and V_1 to zero similar to the case of $\varpi = 0$. Therefore we have the maximum sensitivity when $\beta = \lambda_0$ with $\varpi \neq 0$ (see Fig. 4.4 (a)), thus defining the most "convenient" input. As shown in Fig. 4.3 and Fig. 4.4 non-zero ϖ causes fluctuations in sensitivity. Thus the increase in sensitivity comes at the expense of uniformity, while an uniform sensitivity is desired for better detection performance during the observation period. Next, we analyze the sensitivity for $q = 1$. Fig. 4.5 shows the input, impulse response, and output for $\beta = 1$ (a) and $\beta = 3$ (b) with $\lambda_0 = 2$ and $\varpi = \pi$. The variation of the gradient with respect to parameter in the case of $q = 1$ is similar to that of $q = 0$. We analyze the variation of gradient with varying β only for fixed $\varpi = \pi$. In Fig. 4.6 and Fig. 4.7, we have the maximum value of gradient at approximately $\beta = 2.6$ ($\beta \neq \lambda_0$, see Fig. 4.7 (b)) in the observation interval. As shown in Fig. 4.8 as β becomes larger the gradient becomes bounded for $\beta > \lambda_0$ similarly to the case of $q = 0$.

The simulation results for the first order LTI system can be summarized as follows:

1. As the β approaches λ_0 for $\varpi \neq 0$ and $q = 0$, the maximum sensitivity value becomes unbounded and has more fluctuation.

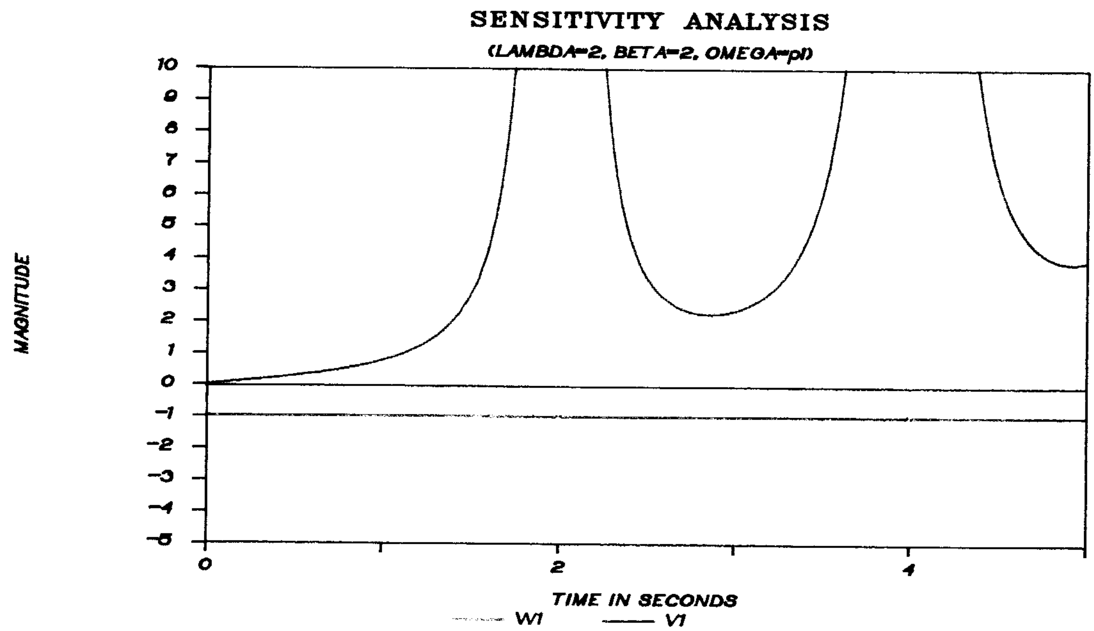


(a)

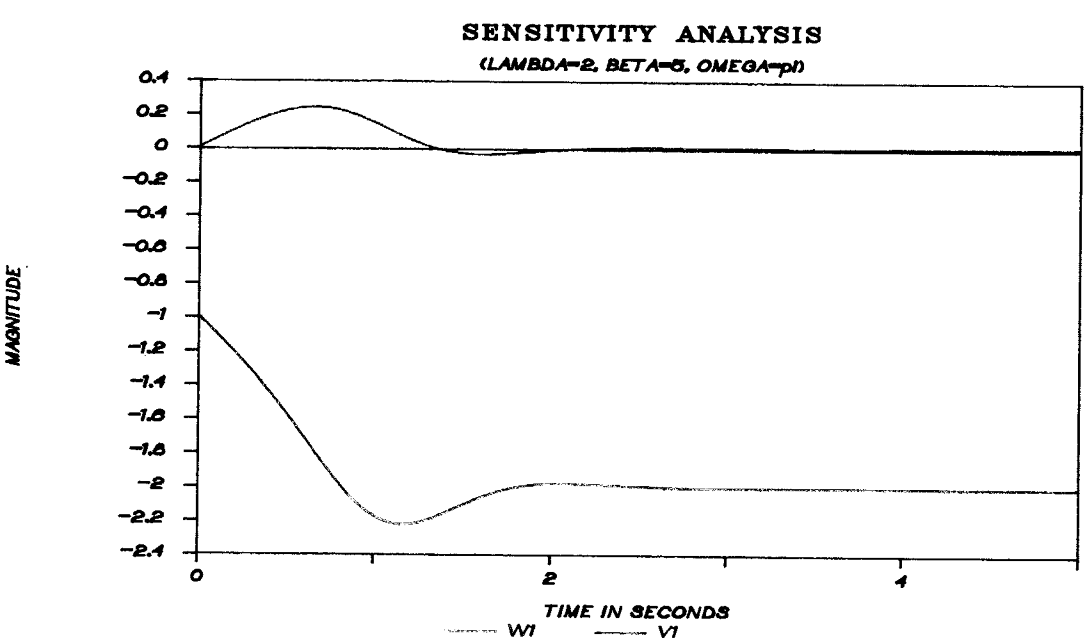


(b)

Fig. 4.3. Sensitivity Analysis 2: W_1 and V_1 of (a) $\beta = 0$ and (b) $\beta = 1$ for $\lambda_0 = 2$, $\varpi = \pi$, and $q = 0$

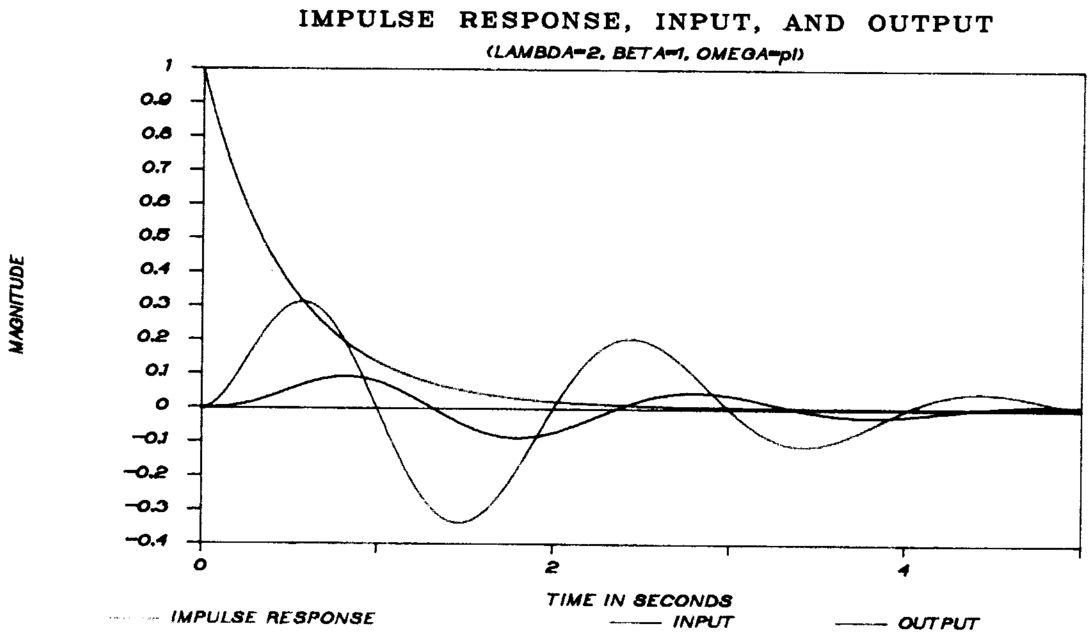


(a)

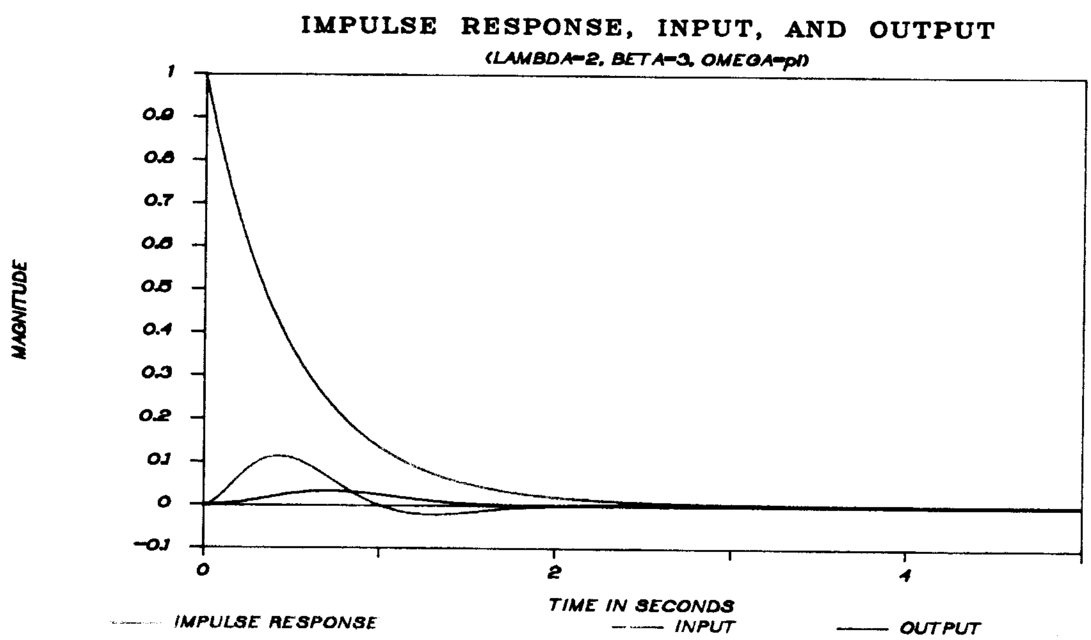


(b)

Fig. 4.4. Sensitivity Analysis 2: W_1 and V_1 of (a) $\beta = 2$ and (b) $\beta = 5$ for $\lambda_0 = 2$, $\varpi = \pi$, and $q = 0$

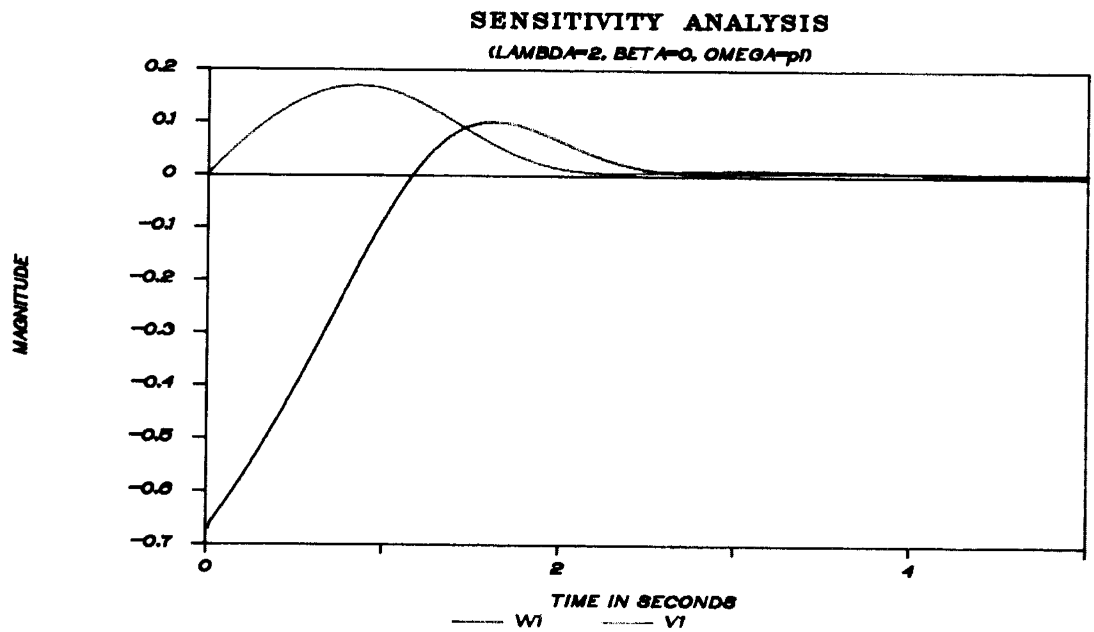


(a)

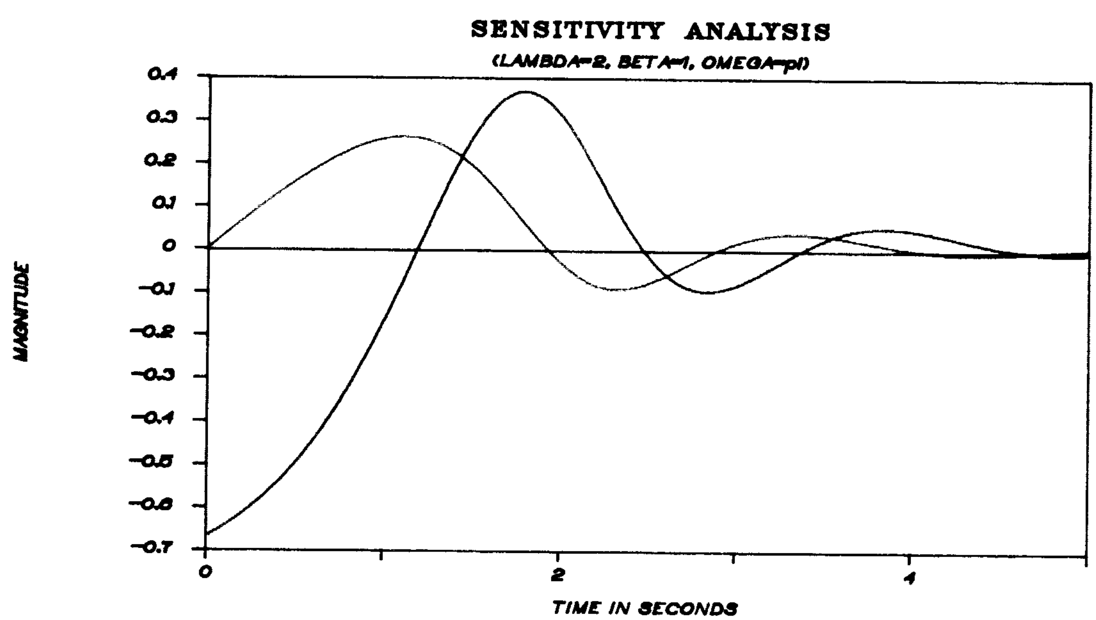


(b)

Fig. 4.5. Signal Analysis 2: Input, impulse response, and output for (a) $\beta = 1$ and (b) $\beta = 3$ with $\lambda_0 = 2$, $\varpi = \pi$, and $q = 1$

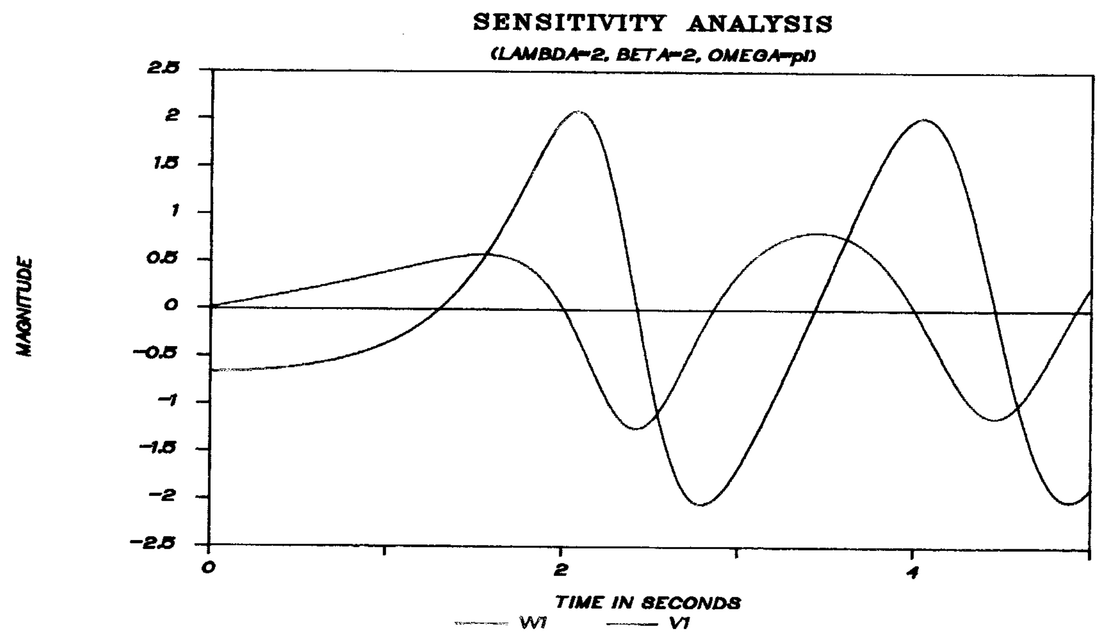


(a)

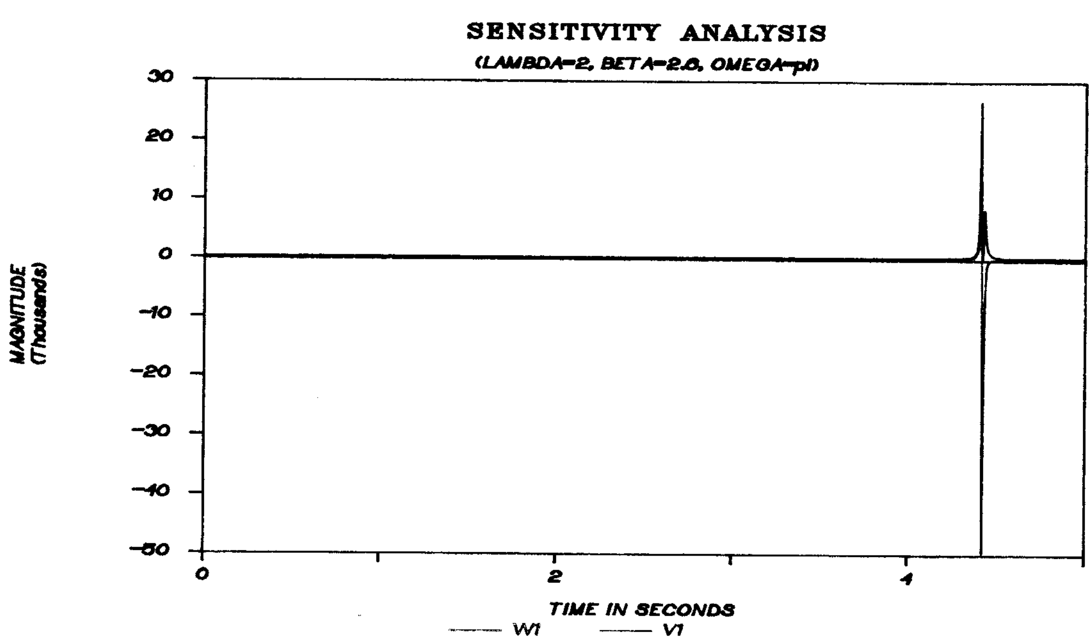


(b)

Fig. 4.6. Sensitivity Analysis 3: W_1 and V_1 of (a) $\beta = 0$ and (b) $\beta = 1$ for $\lambda_0 = 2$, $\varpi = \pi$, and $q = 1$

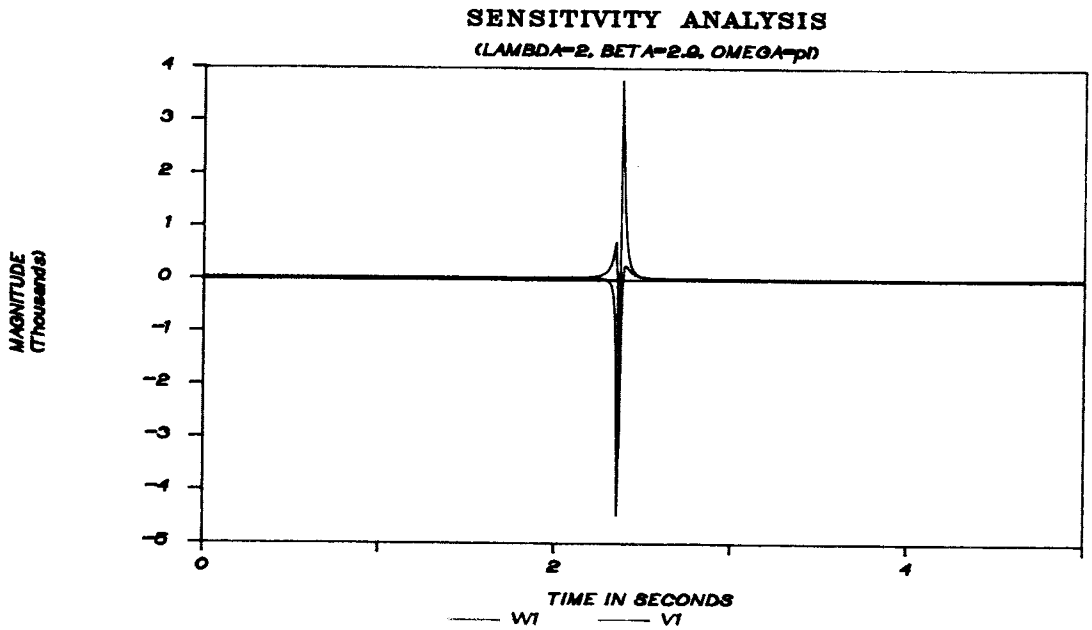


(a)

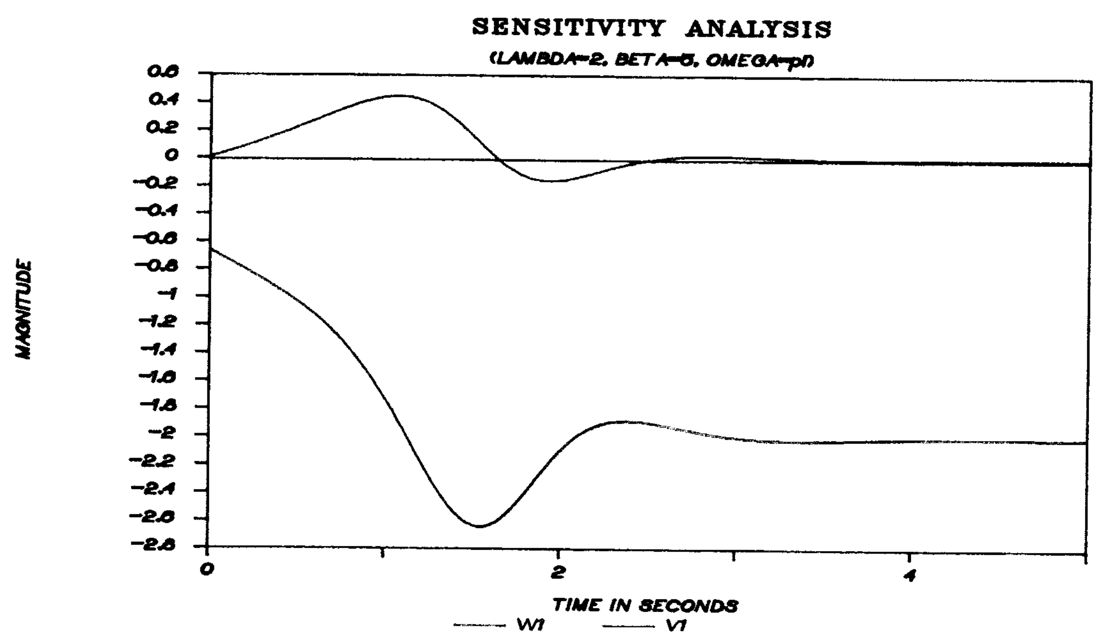


(b)

Fig. 4.7. Sensitivity Analysis 3: W_1 and V_1 of (a) $\beta = 2$ and (b) $\beta = 2.6$ for $\lambda_0 = 2$, $\varpi = \pi$, and $q = 1$



(a)



(b)

Fig. 4.8. Sensitivity Analysis 3: W_1 and V_1 of (a) $\beta = 2.9$ and (b) $\beta = 5$ for $\lambda_0 = 2$, $\varpi = \pi$, and $q = 1$

2. As β become larger for $\beta > \lambda_0$, then the sensitivity becomes uniform.

3. As the ϖ increases, the maximum sensitivity value increases but exhibits more fluctuations.

Conclusion: for the first order linear system, we can obtain the most "convenient" input by setting $\beta = \lambda_0$, $\varpi \neq 0$, and $q = 0$. We have the minimum sensitivity with $\beta = 0$, $\varpi = 0$, and $q = 0$. Also we have more uniform sensitivity with large β. The input signal $\exp(-\beta t)$ and $\beta \exp(-\beta t)$ give the same moments because of normalization (Eq.(2.12)) and the system linearity. Therefore we conclude that the impulse function:

$$\lim_{\rho \to \infty} \rho\, e^{-\rho |2t|} = \delta(t), \tag{4-40}$$

where $\rho = |\beta - \lambda_0|$, is the most convenient input for $\varpi = 0$ and $q = 0$.

4.3. Application to Second Order LTI System

In this section, we apply the methodology of Chapter 3 to find the best kernel for the second order LTI system. This provides further insight into the properties of the best kernel function of the nth order linear systems. The impulse response of a second order LTI system is obtained by setting $r = 0$ in $h(\alpha,t) = t^r \exp(-\lambda_0 t + j\omega_0 t)$. The first moment, second moment, and spread are:

$$M_1(t) = -2\,\lambda_0\, K_1 + \omega_0, \tag{4-41}$$

$$M_2(t) = -4\lambda_0\omega_0K_1 + 4\lambda_0^2K_2 + \omega_0^2, \tag{4-42}$$

$$M_2(t) - M_1(t)^2 = 4\lambda_0^2 K_2 - 4\lambda_0^2 K_1^2, \tag{4-43}$$

where $\lambda_0 \geq 0$ and $\omega_0 \geq 0$. Eq.(4-41), (4-42), and (4-43) do not depend on time. We find the optimal kernel functions for the three cases: (1) only λ changes, (2) only ω changes, (3) both, ω and λ change. All sensitivity functions are obtained from the Euclidian norm of the gradient. In Case 1, the sensitivity functions of first and second moment with respect to λ and the payoff function of Method 1 are given by:

$$S_{1(\lambda)} = 2\,|\,K_1\,|, \tag{4-44}$$

$$S_{2(\lambda)} = 4\,\omega_0\,|\,K_1 - 2\frac{\lambda_0}{\omega_0}K_2\,|, \tag{4-45}$$

$$g_1(K_1) = K_1^2(\,4\,q_{11}\lambda_0^2 - 4q_{12}), \tag{4-46}$$

where the subscripts $1(\lambda)$ and $2(\lambda)$ denote the sensitivity of the first and second moment,respectively. q_{11} and q_{12} denote the weights of the payoff function g_1 of Method 1 (3-32).

In Case 2, the sensitivity functions of the first and second moment with respect to ω and payoff function of Method 1 are

$$S_{1(\omega)} = 1, \tag{4-47}$$

$$S_{2(\omega)} = 2\omega_0\,|\,1 - 2\frac{\lambda_0}{\omega_0}K_1\,|, \tag{4-48}$$

$$g_1(K_1) = 4\,q_{11}\lambda_0^2\,K_1^2 - q_{12}. \tag{4-49}$$

Finally, in Case 3, the sensitivity function with respect to λ and ω and payoff function of Method 1 are

$$S_{1(\omega,\lambda)} = (4K_1^2 + 1)^{\frac{1}{2}}, \tag{4-50}$$

$$S_{2(\omega,\lambda)} = \left[16\omega_0^2 \left(K_1 - 2\frac{\lambda_0}{\omega_0}K_2 \right)^2 + 4\omega_0^2\left(1 - 2\frac{\lambda_0}{\omega_0}K_1 \right)^2 \right]^{\frac{1}{2}}, \tag{4-51}$$

$$g_1(K_1) = K_1^2(4q_{11}\lambda_0^2 - 4q_{12}) - q_{12}\ . \tag{4-52}$$

The constraint for K_1 is (see Eq.(3-33)):

$$-\frac{\varepsilon_1\omega_0}{2\lambda_0} \le K_1 \le \frac{\varepsilon_1\omega_0}{2\lambda_0}. \tag{4-53}$$

The optimal kernel function ϕ for each case is obtained by cascading Methods 1 and 2. From Eq.(4-43), we have $K_2 \ge K_1^2$ to yield a positive spread. From sensitivity function (4-45), we observe that K_1 and K_2 have to have opposite signs in order to maximize the sensitivity and therefore K_1 must have negative value. We introduce the bias of spread as follows:

$$M_2(t) - M_1(t)^2 - \left(\frac{P_1}{2}\right)^2, \tag{4-54}$$

where P_1 = $-2\lambda_0$ (see Eq.(3-28)). Substituting the expression for P_1 into Eq.(4-54) yields

$$4\lambda_0^2K_2 - 4\lambda_0^2K_1^2 - \lambda_0^2. \tag{4-55}$$

The payoff function which compromises between a minimum bias and maximum sensitivity takes the following form:

$$g_2(K_1,K_2) = q_{21}(4\lambda_0^2K_2 - 4\lambda_0^2K_1^2 - \lambda_0^2)^2 - q_{22}(S_2)^2, \tag{4-56}$$

where q_{21} and q_{22} denote weights. S_2's for each case are given by Eq.(4-45), (4-48), and (4-51), respectively. The constraints represent limits of the deterioration of the properties of time-frequency transformation. The payoff function g_2 (4-56) has the general quadratic form:

$$g_2(K_2) = a_1K_2^2 + a_2K_2 + a_3, \tag{4-57}$$

with K_1 obtained from Method 1. In order for g_2 to have minimum value with respect to K_2, a_1 has to be positive. If

$$-\frac{a_2}{2a_1} \geq K_1^2, \tag{4-58}$$

then the optimal K_2 becomes

$$K_2 = -\frac{a_2}{2a_1}, \tag{4-59}$$

otherwise $K_2 = K_1^2$, which provides zero spread.

<u>Case 1:</u>

The payoff function (4-46) has the minimum when:

$$K_1 = 0, \quad q_{11}\lambda_0^2 > q_{12}, \tag{4-60}$$

or

$$K_1 = \pm\varepsilon_1\frac{\omega_0}{2\lambda_0}, \quad q_{11}\lambda_0^2 < q_{12}. \tag{4-61}$$

For each case of K_1, the payoff functions g_2 (4-56) are:

$$g_2(K_2)\,|_{K_1=0} = (16\,\lambda_0^4\,q_{21} - 64\,\lambda_0^2\,q_{22})\,K_2^2 - 8\,\lambda_0^4\,q_{21}\,K_2 + \lambda_0^4\,q_{21}, \tag{4-62}$$

and

$$\begin{aligned} g_2(K_2)\,|_{\pm\varepsilon_1\frac{\omega_0}{2\lambda_0}} &= (16\,\lambda_0^4\,q_{21} - 64\,\lambda_0^2\,q_{22})\,K_2^2 \\ &+ [\,\pm 32\,\varepsilon_1\,\omega_0^2\,q_{22} - 8\,\lambda_0^2(\varepsilon_1^2\,\omega_0^2 + \lambda_0^2)\,q_{21}]\,K_2 \\ &+ \left[\,(\varepsilon_1^2\,\omega_0^2 + \lambda_0^2)^2\,q_{21} - \left(2\frac{\varepsilon_1\,\omega_0^2}{\lambda_0}\right)^2 q_{22}\right]. \end{aligned} \tag{4-63}$$

a_1 is positive if the following inequality is satisfied (see Eq.(4-57)),

$$\lambda_0^2\,q_{21} > 4\,q_{22}. \tag{4-64}$$

If Eq.(4-58) is satisfied then the optimal K_2 for each case of K_1 are:

for $K_1 = 0$,

$$K_2 = \frac{\lambda_0^2\,q_{21}}{4\,\lambda_0^2\,q_{21} - 16\,q_{22}}, \tag{4-65}$$

and for $K_1 = \pm\varepsilon_1\dfrac{\omega_0}{2\lambda_0}$,

$$K_2 = -\frac{\pm 4\varepsilon_1\omega_0^2\,q_{22} - \lambda_0^2(\varepsilon_1^2\omega_0^2 + \lambda_0^2)q_{21}}{4\lambda_0^2(\lambda_0^2\,q_{21} - 4\,q_{22})}. \tag{4-66}$$

For example, if we choose $\varepsilon_1 = 1/15\pi$ and $q_{11} < 1/5$, and $q_{21} = 0.906$ for $\lambda_0 = 4$ and $\omega_0 = 40\pi$ then the optimized kernel becomes $K_1 = -1/3$ and $K_2 = 1/2$.

Fig. 4.9 and Fig. 4.10 compare the sensitivity results of the new kernel function versus those of the unbiased kernel function for $\lambda_0 = 4$ and $\omega_0 = 40\pi$. The parameter $\lambda_0 = 4$ changes to $\lambda_1 = 5$ at time $t^*=$ 0.25 second. The new kernel is more sensitive to change in λ_0 which shows as more pronounced jumps in the first and second moment at $t = t^*$. The modified exponential kernel from Eq.(2-27) is

$$\phi(\xi,\tau) = (1 + c_1\xi\tau + c_2\xi^2\tau^2)\, e^{-\frac{\xi^2\tau^2}{\sigma}}, \tag{4-67}$$

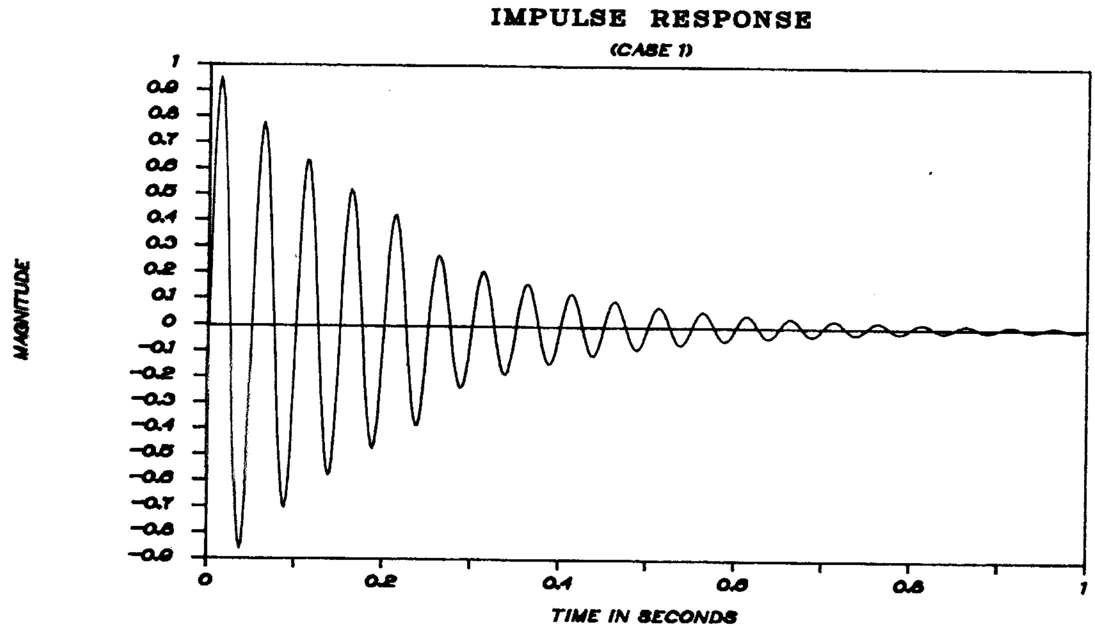
where $c_1 = -1/3$ and $c_2 = 1/4 + 1/\sigma$ to yield $K_1 = -1/3$ and $K_2 = 1/2$.

Case 2:

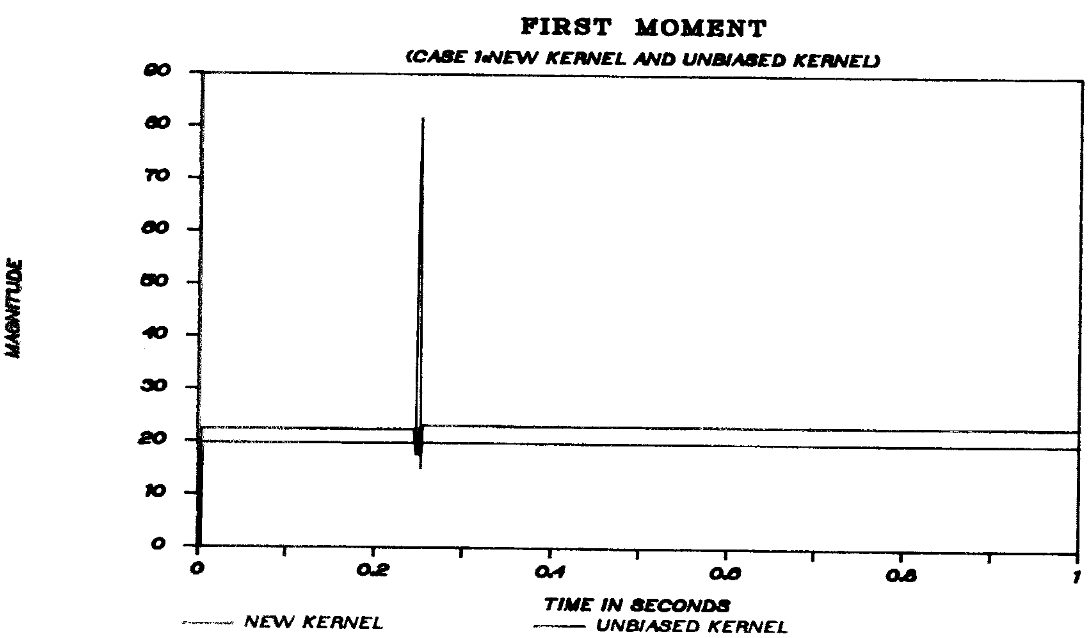
Next, we obtain the optimized kernel function when ω changes. From Eq.(4-47), the sensitivity function of the first moment is independent of K_1, and we obtain that $K_1 = 0$ from the payoff function (4-49). With $K_1 = 0$, we have always positive spread for $K_2 > 0$. The payoff function g_2 (4-57) becomes:

$$g_2(K_2)\,\big|_{K_1=0} = (4\,K_2 - 1)\,\lambda_0^4\, q_{21} - 4\,\omega_0^2\,(1+\varepsilon_1)^2\, q_{22}. \tag{4-68}$$

From Eq.(4-68), g_2 has the minimum value at $K_2 = 1/4$. Therefore the optimal kernel function satisfies $K_1 = 0$ and $K_2 = 1/4$ which coincides with the unbiased kernel function. Fig. 4.11 and Fig. 4.12 show signal and the first moment, the second moment and spread for $K_1 = 0$ and $K_2 = 1/4$. The parameter $\omega_0 = 40\pi$ changes to $\omega_1 = 48\pi$ at time $t^*=0.25$ second.

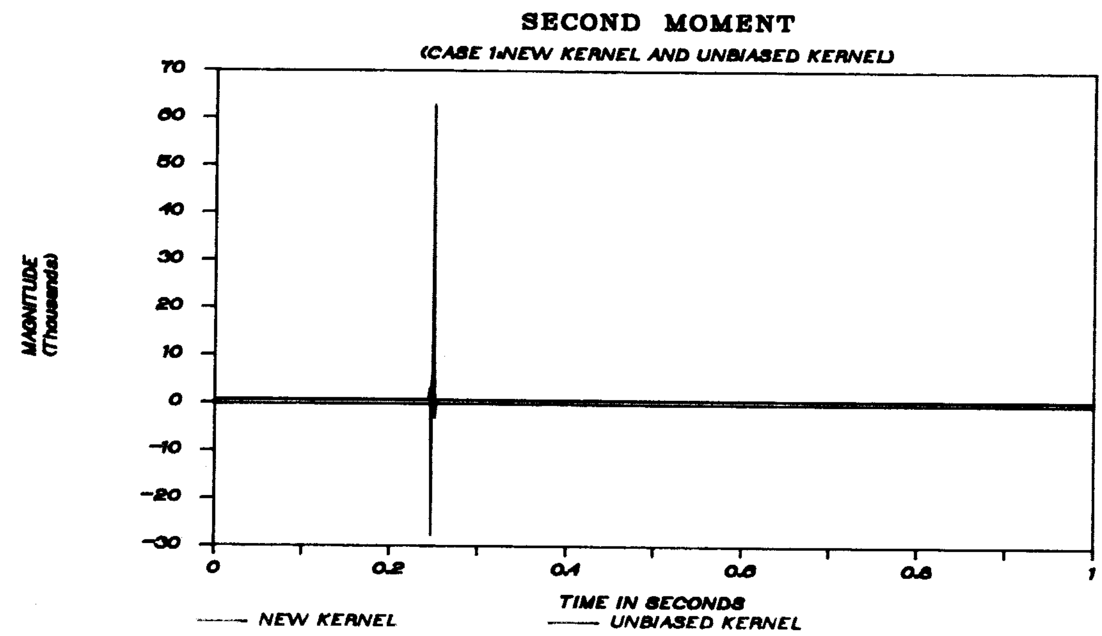


(a)

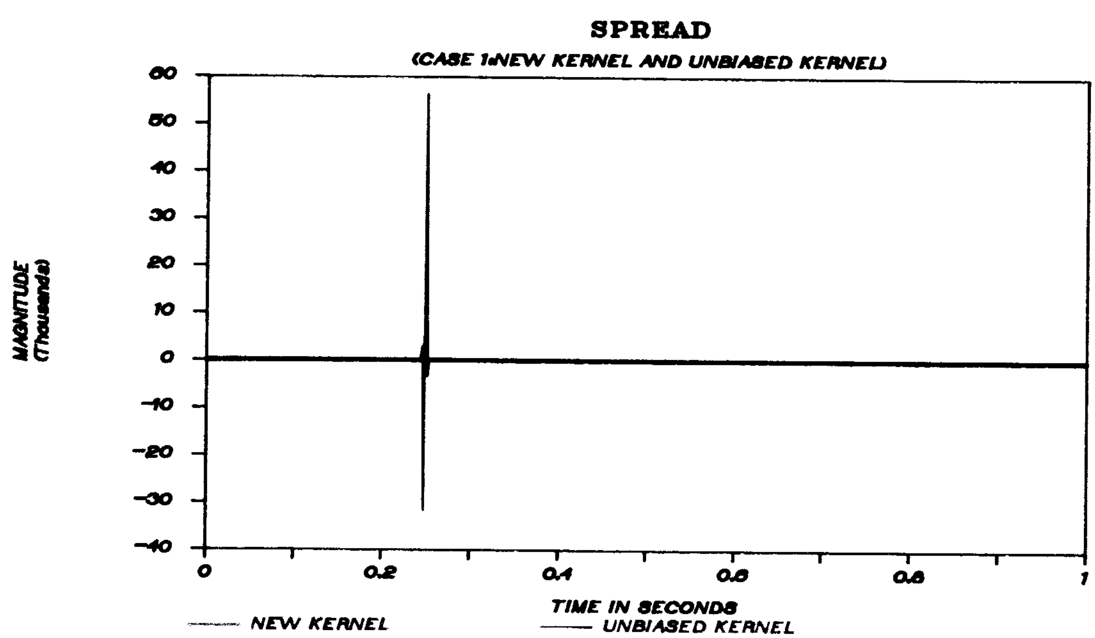


(b)

Fig. 4.9. Case 1: (a) signal and (b) the first moment for λ changes only

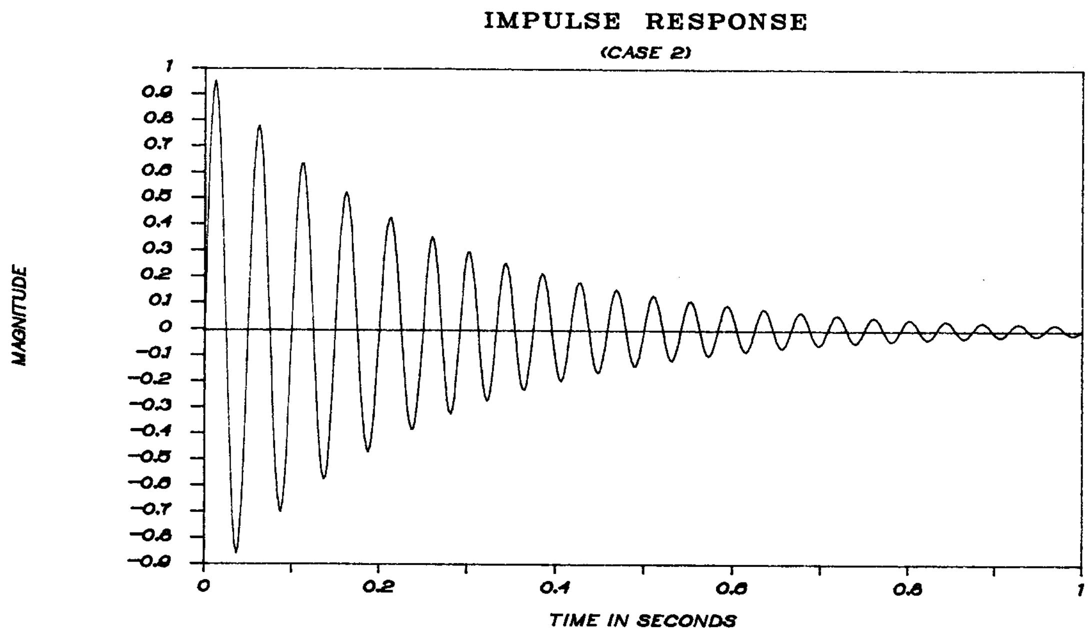


(a)

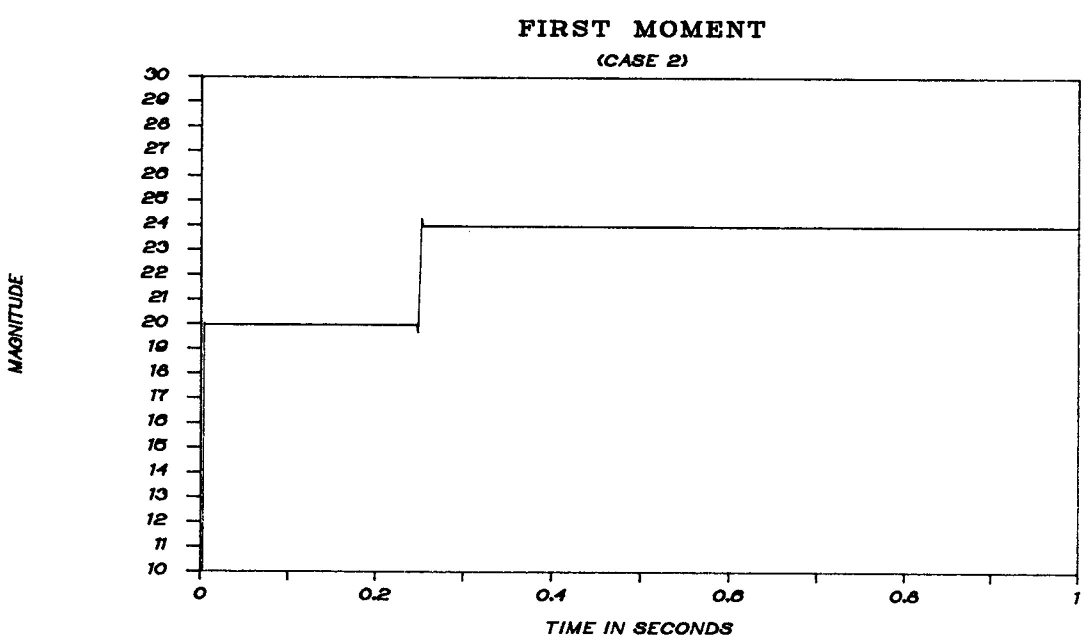


(b)

Fig. 4.10. Case 1: (a) second moment and (b) spread for λ changes only

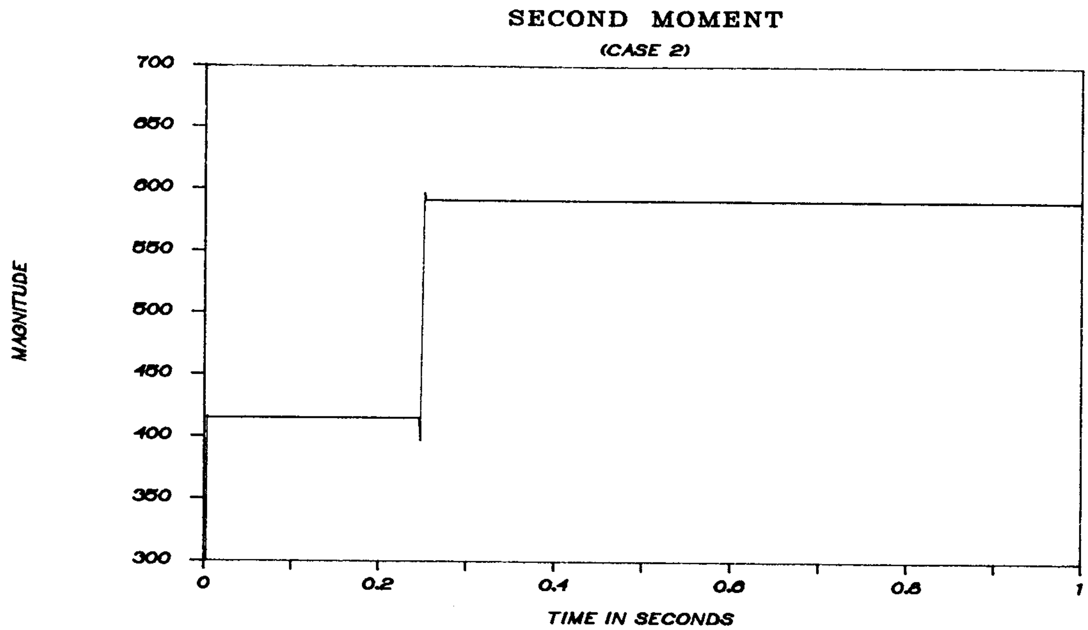


(a)

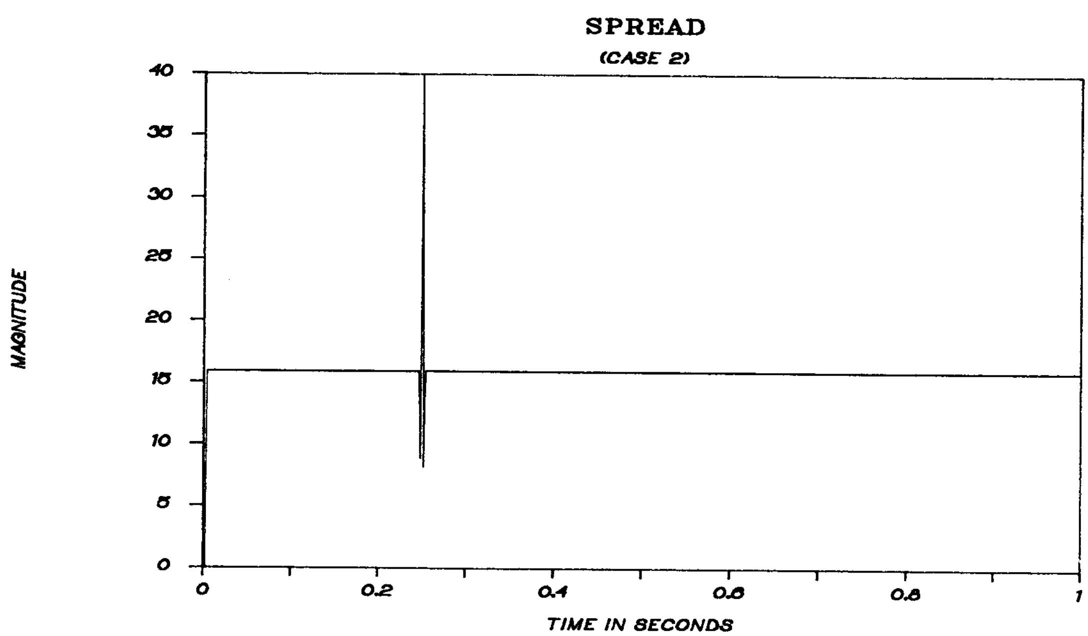


(b)

Fig. 4.11. Case 2: (a) signal and (b) the first moment for ϖ changes only



(a)



(b)

Fig. 4.12. Case 2: (a) second moment and (b) spread for ϖ changes only

As shown in Fig. 4.11 and Fig. 4.12, the first moment and square root of spread correspond to ω and λ, respectively. The modified exponential kernel function with $K_1 = 0$ and $K_2 = 1/4$ is given by Eq.(2-34).

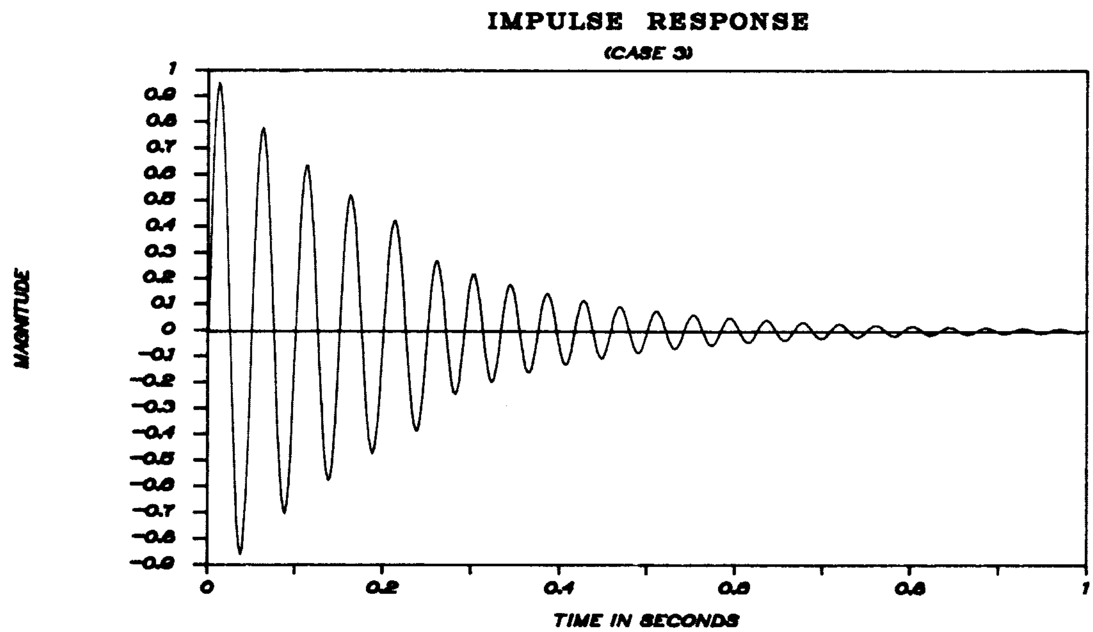
Case 3:

Finally, when the two parameters change at the same time we obtain different kernel function than in the first and second case. The criteria for the optimal kernel function is a compromise between the criteria of the first and second cases. If we choose $\varepsilon_1 = 1/10\pi$, $q_{11} < 1/5$, and $q_{21} = 0.816$ for $\lambda_0 = 4$ and $\omega_0 = 40\pi$ then the optimal kernel function becomes $K_1 = -1/2$ and $K_2 = 1$. Fig. 4.13 and Fig. 4.14 show the first and second moment with the optimal kernel function satisfying $K_1 = -1/2$ and $K_2 = 1$. The parameters $\lambda_0 = 4$ and $\omega_0 = 40\pi$ change to $\lambda_1 = 5$ and $\omega_1 = 48\pi$ at $t^* = 0.25$ second. We observe again that the optimal kernel function yields a bigger jump than the unbiased kernel function at $t = t^*$. The modified exponential kernel function with $K_1 = -1/2$ and $K_2 = 1$ is given by Eq.(4-57) where $c_1 = -1/2$ and $c_2 = 1/2 + 1/\sigma$.

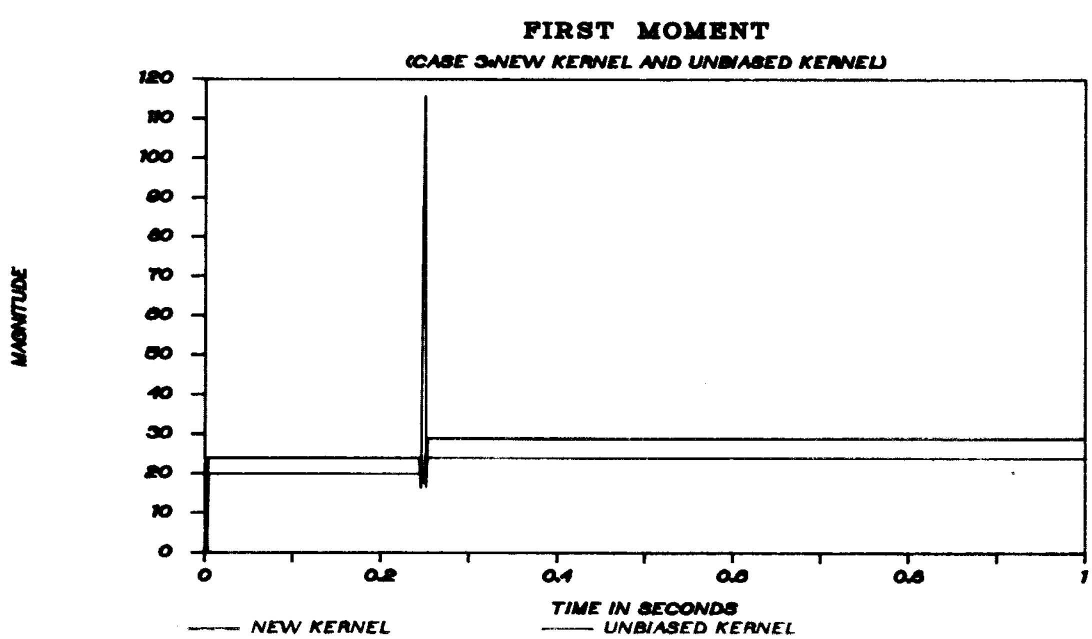
4.4. Diagnosis of Parameters Change in LTI System

4.4.1. The relationship between TFT moments and signal characteristics

We recall here the results presented in Chapter 2.

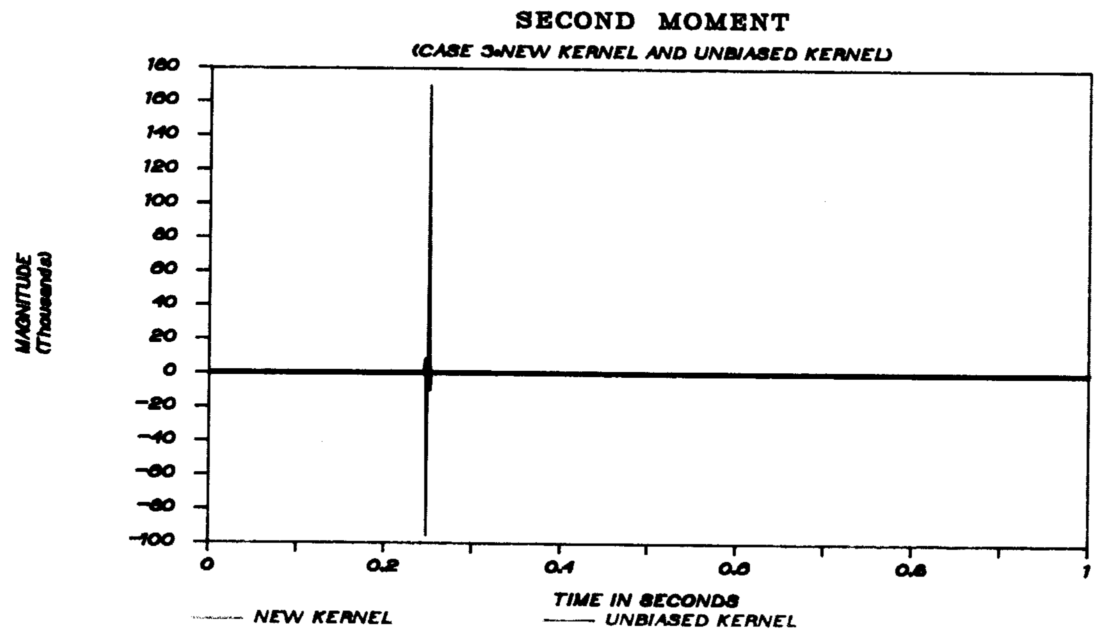


(a)

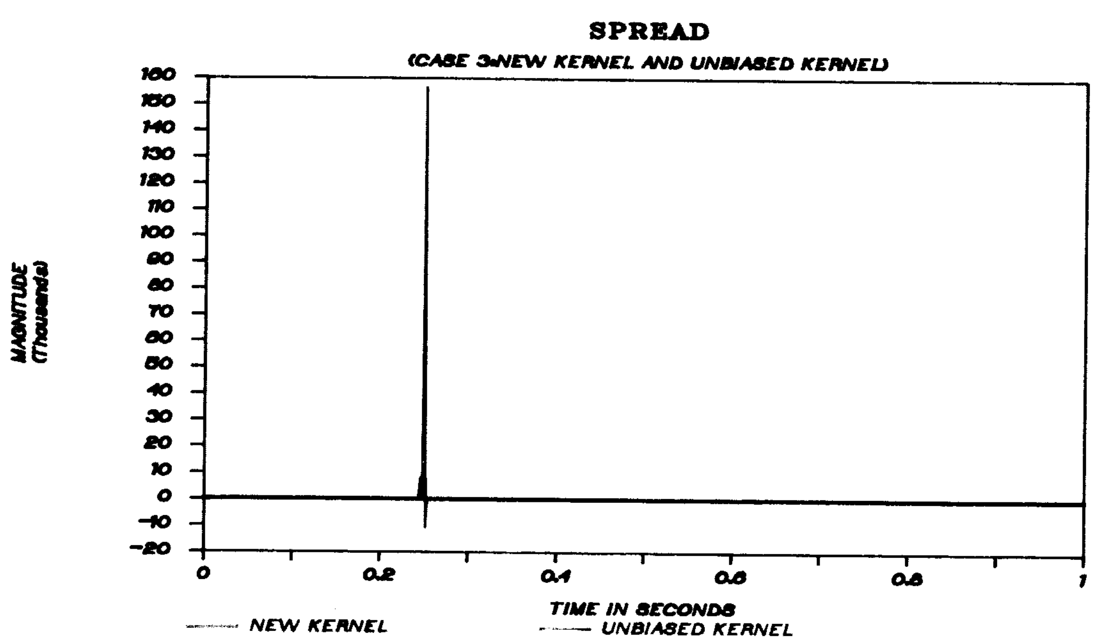


(b)

Fig. 4.13. Case 3: (a) signal and (b) the first moment for both λ and ω changes



(a)



(b)

Fig. 4.14. Case 3: (a) second moment and (b) spread for both λ and ω changes

Instantaneous power and energy density spectrum

If $\phi(0) = 1$, then

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} C_f(t,\omega;\phi)\, d\omega = |f(t)|^2, \text{ and } \int_{-\infty}^{+\infty} C_f(t,\omega;\phi)\, dt = |F(\omega)|^2.$$

First order local moment and instantaneous frequency

The instantaneous frequency $\psi'(t)$ equals the first order local moment $M_1(t)$ of an analytical signal with the constraint $\phi'(0) = 0$.

Second order local moment and spread

With constraint $\phi'(0) = 0$ and $\phi''(0) = 1/4$, we have a positive spread, which for an analytical signal is given by:

$$M_2(t) - M_1(t)^2 = \left[\frac{a(t)'}{a(t)}\right]^2.$$

Lemma 1

Under the constraints (2-9), (2-16) and (2-25) the first order local moment and the spread of a complex impulse response of a second order linear system represent the natural frequency and damping coefficient, respectively.

Proof:

If a signal is a complex exponential signal $f(t) = \exp(-\lambda_0 t + j\omega_0 t)$, then the first order local moment and spread of the first order local moment are λ_0 and ω_0, respectively. Using Eq.(2-14) and Eq.(2-20), we have

$$M_1(t) = \psi(t)' = \omega_0, \tag{4-69}$$

$$M_2(t) - M_1(t)^2 = \left[\frac{a(t)'}{a(t)} \right]^2 = \lambda_0^2 . \tag{4-70}$$

Note: the first order local moment and spread do not depend on time.

Lemma 2

If the first order local moment and spread do not depend on time, then the corresponding signal is a complex exponential signal.

Proof:

Assume that the first order local moment and variance are constant, that is,

$$\psi(t)' = \omega, \tag{4-71}$$

$$\left[\frac{a(t)'}{a(t)} \right] = \lambda, \tag{4-72}$$

where λ and ω are constants. Solving the differential equation Eq.(4-71), we have

$$\psi(t) = \omega t + c_1, \tag{4-73}$$

where c_1 is a constant. Eq.(4-72) can be expressed as

$$\frac{d\,a(t)}{a(t)} = \lambda\ dt\,. \tag{4-74}$$

Integrating both sides of Eq.(4-74) with respect to time, we have

$$a(t) = c_2\, e^{\lambda t}, \tag{4-75}$$

where c_2 is a constant. From Eq.(4-71) and Eq.(4-75), we derive a formula for complex exponential signal.

4.4.2. Diagnosis of parameters change

The local moments with constraints (2-16) and (2-25) can be used for the diagnosis of parameters change. Basically, these moments work very well for the monocomponent signal, where the monocomponent signal is defined as a signal which has concentrated energy in time-frequency domain. A signal having more than one energy concentration patterns is called a multicomponent signal. The energy maxima in monocomponent signals coincide with the instantaneous frequency. This interpretation is lost for the multicomponent signals. Lost of the instantaneous frequency interpretation makes diagnosis of parameter changes in higher dimensional systems less intuitive. Similar comments apply to the interpretation of the spread.

Remark: in order to test if the signal is coming from a linear system we can apply series of tests. The following example provides a test for second order linear system

$$M_3(t) = 3\,M_2(t)\,M_1(t) - 2\,M_1^3(t). \tag{4-76}$$

Such test gives a necessary condition for a signal to be equal to A $\exp(-\lambda_0 t + j\omega t)$. In general this test is not a sufficient condition for such a signal. Test for linearity is an important practical consideration since in the real measurement situation nonlinearity of the system may interfere with detection argorithm performance.

4.4.3. On-line implementation

An efficient use of time-frequency transformation for non-stationary real signal requires, in general, analytical signals. There are basically two reasons for using analytical signals in calculating the time-frequency transformation. First, some of the TFT's give the first order local moment equal to the instantaneous frequency, which in turn has some physical meaning. Second, while sampling at the Nyquist rate (twice the maximum bandwidth of signal) we can avoid aliasing. In general, the aliasing in TFT's is avoided by sampling at twice the Nyquist rate. In addition, Boashash [24] emphasized the use of analytical signal in Wigner transformation since it reduces the interaction between frequency components which in turn cause artifacts. The analytical signal f(t) is defined by [17].

$$f(t) = s(t) + js_h(t), \tag{4-77}$$

where $s_h(t)$ is the Hilbert transform of s(t):

$$s_h(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{s(\eta)}{t-\eta}\, d\eta . \tag{4-78}$$

The analytical signal has a spectrum given by

$$F(\omega) = \begin{cases} 2S(\omega), & \omega>0 \\ S(0), & \omega=0 \\ 0, & \omega<0 \end{cases} \tag{4-79}$$

where $S(\omega)$ is the Fourier transform of s(t). If the signal is analytical we have:

$$\begin{aligned} f(t) &= s(t) + js_h(t) \\ &= \sqrt{s(t)^2 + s_h(t)^2}\, e^{j\tan^{-1}\left(\frac{s_h(t)}{s(t)}\right)}. \end{aligned} \tag{4-80}$$

From Eq.(2-17), Eq.(2-26), and Eq.(4-80), the instantaneous frequency and the spread can be obtained as

$$M_1(t) = \frac{s_h(t)'s(t) - s_h(t)s(t)'}{s(t)^2 + s_h(t)^2}, \tag{4-81}$$

and

$$M_2(t) - M_1(t)^2 = \left[\frac{s(t)s(t)' + s_h(t)s_h(t)'}{s(t)^2 + s_h(t)^2}\right]^2. \tag{4-82}$$

Quadrature approximation to the Hilbert transform of modulated exponential signal

Let s(t), $s_h(t)$, and $s_q(t)$ denote $a(t)\cos(\omega_0 t)$, the Hilbert transform of s(t), and the quadrature version of s(t), which is $a(t)\sin(\omega_0 t)$, respectively. The quantitative measure of approximation of $s_h(t)$ by $s_q(t)$ is the energy in the difference function:

$$E = \int_{-\infty}^{+\infty} [s_h(t) - s_q(t)]^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |S_h(\omega) - S_q(\omega)|^2 \, d\omega, \tag{4-83}$$

where $S_h(\omega)$ denotes the Fourier transform of $s_h(t)$, and $S_q(\omega)$ denotes the Fourier transform of $s_q(t)$. In order for s(t) and $s_q(t)$ to be a Hilbert transform pairs, it is necessary and sufficient that the Fourier transform $A(\omega)$ of a(t) be zero for $\omega < -\omega_0$ [23]. Thus the following quantity becomes a measure of approximation of $s_h(t)$ by $s_q(t)$:

$$\int_{-\infty}^{-\omega_0} |A(\omega)|^2 \, d\omega. \tag{4-84}$$

If the quantity given by Eq.(4-84) is small then:

$$\text{Hilbert transform of} [\, a(t) \cos(\omega_0 t)] \cong a(t) \sin(\omega_0 t). \tag{4-85}$$

For f(t) which satisfies Eq.(4-85), we can calculate instantaneous frequency and the spread without performing the time-frequency transformation. Now we can implement Eq.(4-81) and Eq.(4-82) by using the FIR Hilbert transformer [22] and differentiator [21]. This significantly reduces the amount of computations as compared to calculating TFT.

4.5. Comments on Extending the Results to Nth Order LTI System

In general, the impulse response of a nth order LTI system is a multi-component signal. This means that the interpretation of moments becomes less physical. Also the expressions for the moments become highly complicated functions of system parameters. This makes the detection and diagnosis that much more difficult. However, the main concepts of defining sensitivity function and selecting the best kernel by maximizing the sensitivity under constraints imposed on the kernel are still valid. With loss of physical interpretation of the moments the concept of bias is also lost. This means that the constraints have to come from different sources, for example from the interpretation of TFT as an energy distribution function. Introduction of higher than second moments seems to be natural extension for nth order LTI system, but it complicates even more the corresponding expressions. A practical approach to analysis of the nth order LTI systems would be to pre-filter their output in order to separate its component and deal with each component similarly to the case of second order LTI system.

We conclude, that while the basic concepts of sensitivity and payoff function can easily be extended to the nth order LTI system, at the same time the analytical difficulties seems to be overwhelming.

Note that the basic linear-in-K form of the nth order local moment does not depend on the system order. With properly defined payoff functions, the optimal kernel selection problem will be that of quadratic programing with linear constraints. More generally the optimization of the kernel function as proposed in this thesis belongs to the nonlinear (static) programing.

CHAPTER 5

CONCLUSION AND FUTURE RESEARCH

5.1. Summary

The time-frequency transformation (TFT) is a good tool in analyzing non-stationary signals. Various members of Cohen-transformations are obtained by selecting a particular kernel function. The properties of time-frequency transformation are related to the constraints imposed on the kernel. Setting those constraints properly makes the applications of TFT even more attractive and efficient. In this thesis the concept of selecting the best kernel for a given signal analysis application is studied. In particular an application of TFT to parameter change detection in LTI systems is discussed. The underlying idea is to monitor the local moments of the TFT applied to the system output signal. The best kernel is selected to maximize the sensitivity of the local moments with respect to the parameters change.

The main contribution of this study can be divided into two parts: development of the TFT kernel optimization methodology and application to the change detection. This thesis provides systematic derivation of the general formula for local moments and their sensitivity functions. Next the special form of the LTI output is explored and the sensitivity is directly related to the TFT kernel. Examples of optimized kernels for simple case studies are provided. It is shown that the introduced concepts produce indeed useful results in the form of an efficient parameters change detector. The kernel optimization procedure is reduced to solving the problem of quadratic programing with linear constraints. As an additional result the change diagnosis is possible for monocomponent signals. The latter restricts the applications to first and second order LTI systems. Comments on efficient implementation of the proposed detection methodology are provided, and it is shown that the local moments can be calculated without performing actual TFT. It is expected that this study provides solid basis for extending the obtained results to multi-dimensional linear systems.

5.2. Future Research

Presented here results apply to the detection problems in deterministic LTI systems. In the presence of noise, the choice of the best kernel function must compromise between maximum sensitivity with respect to parameter changes and minimum sensitivity with respect to random errors. The TFT approach is known to be sensitive to noisy signal and some form of pre-filtering seems to be necessary. This pre-filtering may be combined with separation of components of a multi-component signal.

An interesting analogy between the classical correlation methods used in identification and the TFT ratios can be explored. The correlation methods have been applied widely in system identification [32]. For the LTI system we define:

$$R_{yu}(\tau) \equiv \int_{-\infty}^{+\infty} h(\tau - t)\, R_{uu}(t)\, dt, \tag{5-1}$$

where $R_{uu}(\tau)$ and $R_{yu}(\tau)$ are the **sample auto-correlation** of input u(t) and the **sample cross-correlation** of output y(t) and input u(t), respectively. Likewise $\Phi_{uu}(\omega)$ and $\Phi_{yu}(\omega)$ are the **auto-spectrum** of u(t) and **cross-spectrum** of y(t) and u(t) defined by

$$\Phi_{uu}(\omega) \equiv \int_{-\infty}^{+\infty} R_{uu}(\xi)\, e^{-j\xi\omega}\, d\omega, \tag{5-2}$$

$$\Phi_{yu}(\omega) \equiv \int_{-\infty}^{+\infty} R_{yu}(\xi)\, e^{-j\xi\omega}\, d\omega. \tag{5-3}$$

The Cohen's class time-frequency transformation can be expressed by **local correlation function** [13]

$$C_{fg}(t,\omega) = \int_{-\infty}^{+\infty} R_{f(t)\,g(t)}(\tau)\, e^{-j\tau\omega}\, d\tau, \tag{5-4}$$

where

$$R_{f(t)\,g(t)}(\tau) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f\left(\mu+\frac{\tau}{2}\right) g^*\left(\mu-\frac{\tau}{2}\right) \phi(\xi,\tau)\, e^{-j\xi(t-\mu)}\, d\xi\, d\mu. \tag{5-5}$$

Comparing Eq.(5-2) and (5-4) with (5-4) we define new detector function:

$$\tilde{H}(t,\omega) \equiv \frac{C_{yy}(t,\omega)}{C_{yu}(t,\omega)}, \tag{5-6}$$

where $C_{yy}(t,\omega)$ and $C_{yu}(t,\omega)$ represent the auto-time-frequency transformation of y(t) and cross-time-frequency transformation of y(t) and u(t), respectively. Eq.(5-6) represents the "time-frequency" transfer function, thus generalizing the well known methods of frequency domain. This "new" transfer function can be used for detection algorithm design by maximizing its sensitivity with respect to parameter changes. The maximization is again accomplished by properly adjusting the kernel function of C_{yy} and C_{yu}. The advantage of this approach over the old one is that the input affects the sensitivity both in explicit and implicit (thru y) fashion.

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APPENDICES

APPENDIX A

GRADIENT AND HESSIAN OF MOMENTS

Gradient of Output Signal Moment

$$\frac{\partial Z_n(\alpha,\phi,t)}{\partial\alpha}\Big|_{\alpha=\alpha_0}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \omega^n e^{-j\tau\omega+j\xi t-j\xi\mu}\,\phi(\xi,\tau)\cdot$$
$$\cdot\frac{\partial}{\partial\alpha}\left[y\left(\alpha,\mu+\frac{\tau}{2}\right)y^*\left(\alpha,\mu-\frac{\tau}{2}\right)\right]\Big|_{\alpha=\alpha_0} d\tau\, d\xi\, d\mu\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \omega^n e^{-j\tau\omega+j\xi t-j\xi\mu}\,\phi(\xi,\tau)\cdot$$
$$\cdot\left\{\left[\frac{\partial}{\partial\alpha}y\left(\alpha,\mu+\frac{\tau}{2}\right)\right]\left[y^*\left(\alpha,\mu-\frac{\tau}{2}\right)\right]+\right.$$
$$\left.\left[\frac{\partial}{\partial\alpha}y^*\left(\alpha,\mu-\frac{\tau}{2}\right)\right]\left[y\left(\alpha,\mu+\frac{\tau}{2}\right)\right]\right\}\Big|_{\alpha=\alpha_0} d\tau\, d\xi\, d\mu\, d\omega$$

Gradient of Error Signal Moment

$$\frac{\partial Z_{n_\varepsilon}(\alpha,\alpha_0,\phi,t)}{\partial\alpha}\Big|_{\alpha=\alpha_0}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \omega^n e^{-j\tau\omega+j\xi t-j\xi\mu}\,\phi(\xi,\tau)\cdot$$
$$\cdot\left\{\left[\frac{\partial}{\partial\alpha}\varepsilon\left(\alpha,\alpha_0,\mu+\frac{\tau}{2}\right)\right]\left[\varepsilon^*\left(\alpha,\alpha_0,\mu-\frac{\tau}{2}\right)\right]+\right.$$
$$\left.\left[\frac{\partial}{\partial\alpha}\varepsilon^*\left(\alpha,\alpha_0,\mu-\frac{\tau}{2}\right)\right]\left[\varepsilon\left(\alpha,\alpha_0,\mu+\frac{\tau}{2}\right)\right]\right\}\Big|_{\alpha=\alpha_0} d\tau\, d\xi\, d\mu\, d\omega$$

$$= 0$$

Hessian of Error Signal Moment

$$\frac{\partial^2 Z_{n_\varepsilon}(\alpha,\alpha_0,\phi\,,t)}{\partial\alpha^2}\Big|_{\alpha=\alpha_0}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \omega^n\, e^{-j\tau\omega+j\xi t-j\xi\mu}\,\phi(\xi,\tau)\cdot$$
$$\left[\frac{\partial}{\partial\alpha}\,\varepsilon\!\left(\alpha,\alpha_0,\mu+\frac{\tau}{2}\right)\right]\left[\frac{\partial}{\partial\alpha}\varepsilon^*\!\left(\alpha,\alpha_0,\mu+\frac{\tau}{2}\right)\right]\Big|_{\alpha=\alpha_0} d\tau\; d\xi\; d\mu\; d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \omega^n\, e^{-j\tau\omega+j\xi t-j\xi\mu}\,\phi(\xi,\tau)\cdot$$
$$\left[\frac{\partial}{\partial\alpha}\,y\!\left(\alpha,\mu+\frac{\tau}{2}\right)\right]\left[\frac{\partial}{\partial\alpha}y^*\!\left(\alpha,\mu+\frac{\tau}{2}\right)\right]\Big|_{\alpha=\alpha_0} d\tau\; d\xi\; d\mu\; d\omega$$

APPENDIX B

ADDITIONAL READINGS

The following publications furnish additional information that may be helpful to the reader.

WIGNER TRANSFORMATION

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