NOTES AND CORRESPONDENCE

The Quest for K_ρ—Preliminary Results from Direct Measurements of Turbulent Fluxes in the Ocean

J. N. MOUM

College of Oceanography, Oregon State University, Corvallis, Oregon

16 February 1990 and 15 June 1990

ABSTRACT

Simultaneous measurements of vertical velocity fluctuations, w', and temperature fluctuations, T', on scales of three-dimensional turbulence yield a direct measure of the turbulent heat flux, J_q. The scales contributing most significantly to J_q are tens of centimeters or about 10 times larger than the scales contributing to the turbulent kinetic energy dissipation for the data examined. A limited sample indicates that, instantaneously, J_q is almost as frequently positive (countergradient) as negative (downgradient). In contrast, instantaneous estimates of J_q from microstructure measurements of ϵ or χ are always, by definition, downgradient.

1. Introduction

A primary goal of ocean microstructure measurements has been to estimate the vertical fluxes of mass, heat, momentum, etc., due to three-dimensional turbulence processes in the water column. Although *direct* measurements of the turbulent fluxes (or transports) have not previously been made, estimates based on the measurements of χ (dissipation of temperature variance) and ϵ (dissipation of turbulent kinetic energy) have been widely used. It has been almost twenty years since Osborn and Cox (1972) proposed estimating the turbulent diffusion coefficient for heat from temperature microstructure measurements of χ and ten years since Osborn (1980) suggested a method by which velocity microstructure measurements of ϵ could be used to estimate the turbulent diffusion coefficient for mass K_ρ. These estimates have been compared to each other (Oakey 1982; Moum et al. 1989) but not to *direct* measurements of the turbulent transport; the applicability of the methods remains untested.

Recent estimates of K_ρ from microstructure measurements in the main thermocline indicate that there is a considerable range of variability. These estimates vary by a factor of 20, from being insignificant for numerical models (Gregg and Sanford 1988) to being definitely important (Moum and Osborn 1986). What is the source of this discrepancy? The possibilities include:

(i) limited sampling—measurements in the deep ocean are considerably more difficult and time-consuming than upper ocean measurements. It is quite possible that the variability of turbulent mixing in the main thermocline has not been resolved.

(ii) lack of understanding of the applicability of our estimates of K_ρ.

The purpose of this paper is to present preliminary results of an attempt to directly measure turbulent heat fluxes from simultaneous measurements of the vertical velocity fluctuation, w', and temperature fluctuation, T'. The results are compared to estimates from measurements of ϵ and the presentation is prefaced by a discussion of the methods of estimating K_ρ from ϵ and χ. These methods have been reviewed recently by Caldwell (1983) and Gregg (1987); it is important to note precisely how these relate to the turbulent heat flux, $J_q \equiv \rho C_p \overline{w'T'}$.

a. Estimating K_ρ from χ: Osborn and Cox (1972)

The equation describing the evolution of temperature variance in a three-dimensional stratified turbulence field is

$$\frac{\partial \overline{T'^2}}{\partial t} + U_i \frac{\partial \overline{T'^2}}{\partial x_i} + \frac{\overline{u_i' \partial T'^2}}{\partial x_i} + 2\overline{u_i' T'} \frac{\partial T}{\partial x_i}$$

$$= D \frac{\partial}{\partial x_i} \frac{\partial \overline{T'^2}}{\partial x_i} - \chi \quad (1)$$

where T' refers to the small scale fluctuations of temperature, D is the thermal diffusivity due to molecular processes, $\chi = 2D(\partial T'/\partial x_i \cdot \partial T'/\partial x_i)$ and the overbar denotes an appropriate average. From profiler measurements, only the vertical component $(\partial T'/\partial z)^2$ is obtained so that by necessity it is assumed that $(\partial T'/\partial x_i \cdot \partial T'/\partial x_i) = (1 - 3)(\partial T'/\partial z)^2$, the coefficient $(1 - 3)$ being a function of the degree of isotropy

---

of the turbulence. The turbulent eddy coefficient for heat is defined by $\overline{w'T'} \equiv -K_T \partial T / \partial z$. Since the only measurable terms are χ and the vertical temperature gradient, it is conveniently assumed that all of the other terms are unimportant and that the balance reduces simply to

$$\overline{w'T'} \frac{\partial T}{\partial z} = -\frac{\chi}{2}$$

so that

$$K_T = \frac{\chi/2}{(\partial T / \partial z)^2}.$$

The quantity $\overline{(\partial T' / \partial z)^2} / (\partial T / \partial z)^2$ is termed the Cox number, C_x, so that

$$K_T = (1 - 3)DC_x. \tag{2}$$

b. Estimating K_ρ from ϵ: Osborn (1980)

The turbulent kinetic energy (TKE) equation is given by

$$\frac{\partial}{\partial t} \overline{q^2} + U_j \frac{\partial}{\partial x_j} \overline{q^2} + \frac{\partial}{\partial x_j} \overline{u_j' q^2} + \frac{1}{\rho} \frac{\partial}{\partial x_i} \overline{(u_i' p')}$$

$$= -\overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j} - \frac{g}{\rho} \overline{w' \rho'} - \epsilon = P + J_b - \epsilon \tag{3}$$

where TKE is defined by $q^2 \equiv \overline{u_i u_i}/2$; U represents mean or large scale velocities and u represents fluctuations. Terms on the lhs are neglected for practical purposes; they have not been measured. Some scaling arguments have been proposed to support this neglect. It is not clear, however, under what range of conditions the lhs may be considered unimportant. The rhs is a combination of mechanical production of TKE, P; buoyancy flux suppression (or in some cases, production) of TKE, J_b; and the rate of viscous dissipation of TKE, ϵ. The flux Richardson number, R_f, is defined as the ratio of buoyancy flux to mechanical production; $R_f \equiv J_b/P$. The eddy diffusivity for mass, K_ρ, is defined as

$$K_\rho \equiv -\frac{\overline{w' \rho'}}{\partial \rho / \partial z},$$

yielding $J_b = -K_\rho N^2$, where $N^2 = -g/\rho \cdot \partial \rho / \partial z$ is the local buoyancy frequency. A practical assumption has been that R_f has some constant value. Osborn (1980) intended only to define an upper bound for R_f based on laboratory measurements leading to an upper bound for K_ρ. However the estimate is not really useful without using *some* value for R_f. Of course, R_f has not been measured directly in the ocean, since measurements of $\overline{w' \rho'}$ and $\overline{u_i' u_j'}$ have not been made. An indirect estimate follows from a comparison of χ- and ϵ-estimates of K_ρ. With the above forms for R_f and K_ρ, the reduced form of the TKE equation, $P + J_b = \epsilon$ yields

$$K_\rho = \gamma \frac{\epsilon}{N^2}; \quad \gamma = \frac{R_f}{1 - R_f}. \tag{4}$$

With an assumed upper bound for R_f of 0.17, the form for K_ρ becomes $K_\rho < 0.2 \, \epsilon/N^2$. If we further assume that the mixing of temperature and density are described by the same eddy diffusivities, i.e., $K_\rho = K_T$, then we can equate the χ-estimate [Eq. (2)] to the ϵ-estimate [Eq. (4)] to get

$$\gamma = (1 - 3) \frac{DC_x N^2}{\epsilon} \tag{5}$$

Simultaneous measurements of χ (or C_x) and ϵ have indicated that

$0.05 < \gamma < 0.47; \quad 0.05 < R_f < 0.32$ (Oakey 1982)

$0.12 < \gamma < 0.48; \quad 0.11 < R_f < 0.32$

(Moum et al. 1989).

This large range of variation is troubling and it is difficult to determine what the source of the variability is, given the numerous assumptions made to arrive at our estimates. Undoubtedly, some of the problem lies in the natural intermittency of the turbulence. How much? Perhaps γ is simply not constant, in which case this form for estimating K_ρ is not a useful prescription. Recently, Oakey (1988) has suggested that γ may depend on the strength of double-diffusive processes.

2. The direct approach—measuring $\overline{w'T'}$

a. Method

Velocity microstructure measurements of horizontal velocity components u', v' are now made routinely using airfoil shear probes (Osborn and Crawford 1980). However, the critical measurement required to determine the vertical transports due to turbulence is the vertical component, w'. Moum (1990) has recently demonstrated a technique for detecting w' in the inertial subrange of ocean turbulence. The measurement is of dynamic pressure using a highly sensitive differential piezoresistive pressure transducer. The sensor, essentially a pitot tube, is mounted on the nose of our profiler, the RSVP (Caldwell et al. 1985), separated spatially from shear probes and fast response thermistor by approximately 1.5 cm. Spectral analysis of the signal indicates that, although hydrodynamic noise limits the range to frequencies less than 30 Hz in its present incarnation, the sensor is able to resolve the inertial subrange. The equivalent spatial range of scales covered is 2 m to 2 cm, the upper limit due to the highpass filtering effect of the 1.2 m long vehicle itself as it responds (imperfectly) to vertical velocities in the water column.

The calibration procedure is described by Moum (1990) as is the comparison to simultaneous airfoil shear probe velocity microstructure measurements.

Briefly, independent calibrations were applied separately to the outputs of both pitot sensor and shear probes. Power spectra were computed and scaled in the inertial subrange. The excellent agreement of the independently scaled spectra is taken to demonstrate the validity of the measurement.

b. Preliminary results

The measurements discussed here are from a group of 20 consecutive profiles made on 30 April 1987 at the equator and 110°W over a period of approximately two hours at sunrise. In the thermocline below the nearly mixed surface layer, a continually mixing turbulent patch of ~10 m vertical scale was observed in almost every profile. This feature is qualitatively similar to the mixing events observed in the thermocline at 140°W in 1984 (Moum et al. 1989). At 140°W the mean depth of the core of the equatorial undercurrent (EUC) is about 120 m and thermocline mixing events in November 1984 were observed over the depth range 35 to 85 m. In contrast, at 110°W in April 1987, the EUC core was observed to be located at about 50 m depth; thermocline mixing events were observed at depths of 10 to 30 m. Detailed plots of microstructure quantities from this sequence of profiles are discussed by Moum (1990).

At present, there is no good model of vehicle response to vertical velocity in the water column. There is undoubtedly a response to scales greater than about 2 m that acts as a highpass filter on the measured vertical velocity. Hence, only velocities on scales smaller than 2 m were considered for further analysis.

Profiles of temperature and vertical velocity were highpass filtered at ½ Hz (or 1.8 m vertically at our nominal fall speed). A typical vertical profile of w' and T' (Fig. 1) indicates three distinct regions:

1) Surface to 8 m—well-mixed in temperature, ±6 cm s^{-1} variations in w' due to the effects of vehicle instabilities during initial acceleration and possibly surface wave orbital velocities.

2) 8 to 16 m—within the actively mixing patch, variations in T' are ±0.05°C over vertical scales of centimeters to 10s of centimeters. Similarly, variations in w' are ±4 cm s^{-1} over vertical scales of centimeters to 10s of centimeters.

3) 16 to 32 m—variations in w' and T' are relatively small and occur over a smaller range of vertical scales. The noise level in w', approximately 2–3 mm s^{-1}, is the source of high frequency variability below 16 m.

The instantaneous turbulent heat flux is defined by $J_q \equiv \rho C_p w'T'$. Considering only contributions due to scales of 2 m down to 2 cm, vertical profiles of $w'T'$ were obtained by simply multiplying the two series, w' and T'. Our example profile shows clearly the effect of the turbulent patch (Fig. 1). Although variability in T' and w' occurs in all three regions discussed in Fig. 1, only within the turbulent patch do they correlate to effect significant vertical heat transport.

Instantaneous values of $w'T'$ are both negative and positive. Negative values are due to either vertical advection of warm water downwards or cool water upwards. In the case of a mean temperature profile that decreases with depth this results in downgradient
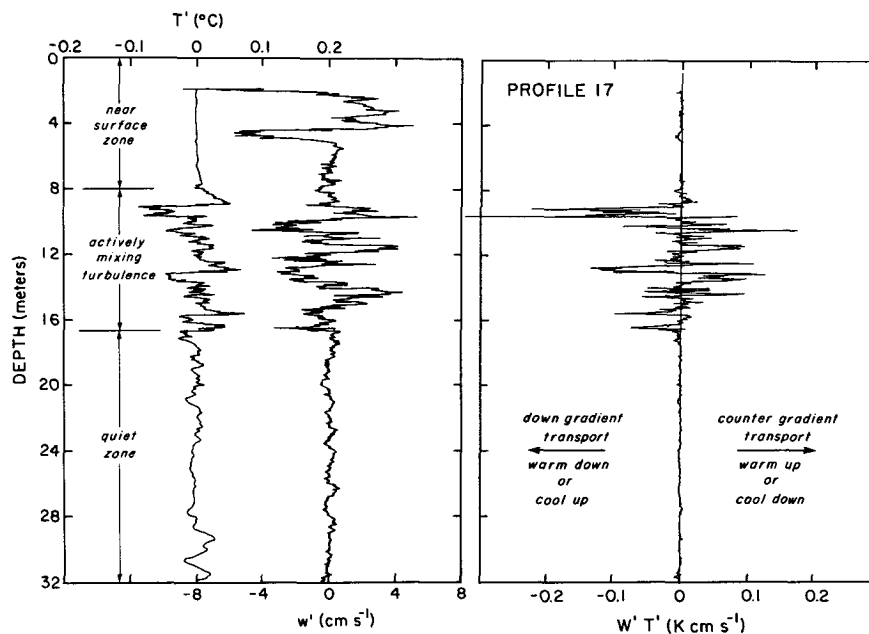

FIG. 1. High pass-filtered (½ Hz) temperature and vertical velocity records (right panel) from a profile taken at the equator and 110°W in May 1987. The left panel shows the result of multiplying the two series.
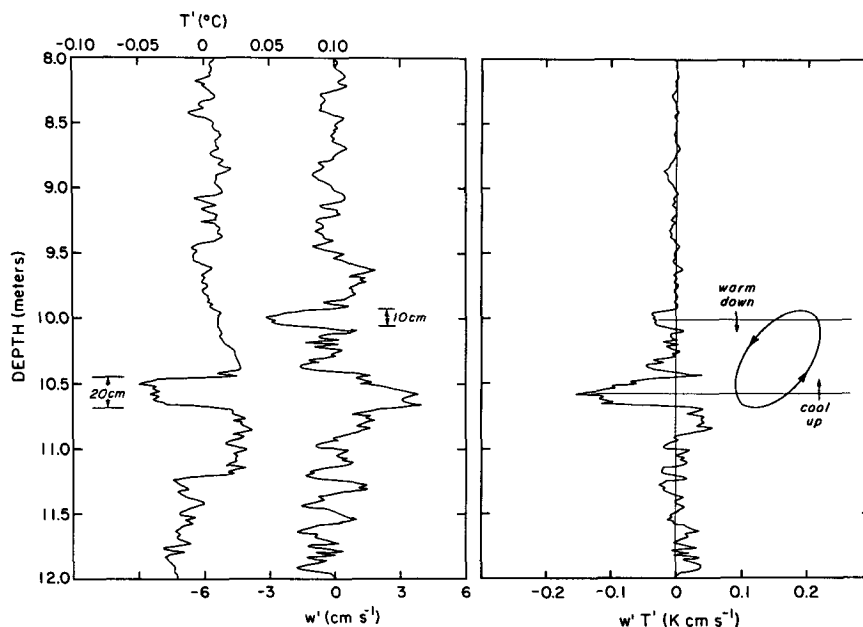
FIG. 2. As in Fig. 1. but for an expanded vertical section from another profile
taken shortly after that shown in Fig. 1.

transport. However, numerous instantaneous examples of countergradient transport (due to vertical advection of either warm water upwards or cool water downwards) are also present. The largest contributions to $w'T'$ occur over vertical scales of tens of centimeters.

Detailed examination of $w'T'$ leads to some interesting small scale structure (Fig. 2). This particular profile was made approximately 20 minutes after that shown in Fig. 1. The plot has been expanded to focus on the structure between 9.5 and 11 m. A 20 cm thick patch at 10.5 m is approximately 0.05°C cooler than ambient and is being rapidly advected upwards at 3 cm s^{-1}. A 10 cm thick patch at 10 m is slightly warmer than ambient and advected downwards at about 3 cm s^{-1}. Each of these two parcels contribute to a downgradient (negative) heat transport (Fig. 2). The structure suggests a coherent overturning eddy, as depicted in Fig. 2.

3. Discussion

The data considered for this first examination of the statistics of $w'T'$ come from a particularly energetic mixing regime. The range of ϵ was 10^{-7} to 10^{-6} m^2 s^{-3} for the 40 patches evaluated here and the mean value of $\epsilon/\nu N^2$ was in excess of 70 000, which I believe suggests to all that the turbulence is isotropic. The corresponding Kolmogoroff wavelength, $\eta = 2\pi(\nu^3/\epsilon)^{1/4}$ ranged from ½ to 1 cm (with kinematic viscosity, $\nu = 10^{-6}$ m^2 s^{-1}). The peak in the dissipation spectrum is associated with scales of ~ 5–10η (Townsend 1976) or 2 to 10 cm. In comparison to the scales which appear to dominate the turbulent heat transport, the dissipa-

tion scales are significantly smaller (Fig. 3). Apparently, the dissipation is dominated by structures having scales of centimeters whereas the turbulent transports are dominated by scales of many tens of centimeters. If this can be consistently demonstrated with more extensive datasets, it relaxes the spatial resolution re-
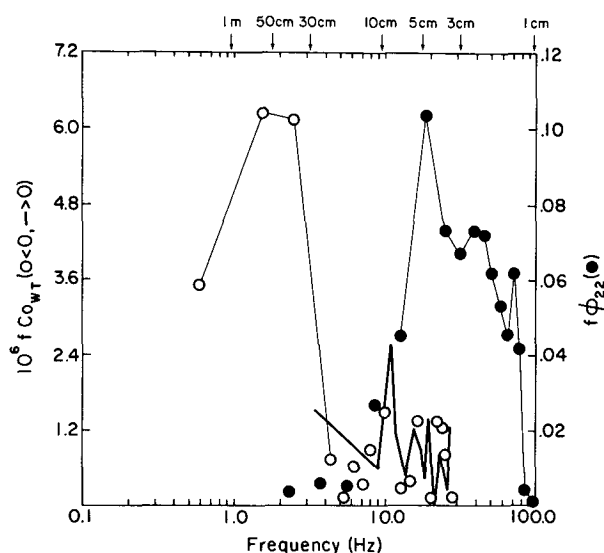

FIG. 3. Averaged cospectrum of $w'T'$ (open circles; negative values are represented by the dark line) in variance-preserving form. Averaged variance-preserving dissipation spectrum (closed circles) coincident with Co_{wT}. The sample size was 40. Conversion from frequency to length scales (top axis) uses the mean flow speed. The scales which dominate ϵ are O(1 cm) while those dominating the turbulent heat flux are O(10 cm).

quirements for instrumentation designed to sense the vertical transports. Rather than requiring full resolution of the dissipation spectrum of the turbulence, it may only be necessary to resolve the inertial subrange.

To a large extent, these profiles represent instantaneous snapshots of the turbulent fields of vertical velocity and temperature fluctuations. The largest distinct overturns, on scales of tens of centimeters, are associated with vertical velocities of several centimeters per second. A corresponding overturning time scale might be estimated as $\tau \sim l/u \sim O(10 \text{ s})$. This time scale is large compared to the time required for the profiler to traverse tens of centimeters (9 seconds to traverse the entire 8 m patch of Fig. 1). By comparison, the local buoyancy period was many hundreds of seconds.

These snapshots of turbulent patches indicate that, at any particular instant in time, there is a considerable contribution to countergradient transport by the turbulence. As a measure of this contribution, it is noted that a value of $w'T' = 0.1 \text{ K cm s}^{-1}$ (Figs. 1, 2) corresponds to a turbulent heat flux $J_q \sim 4000 \text{ W m}^{-2}$, where $\rho \sim 1025 \text{ kg m}^{-3}$ and $C_p \sim 4000 \text{ J kg}^{-1} \text{ K}^{-1}$.

The heat flux resulting from a single turbulent patch is the patch-averaged value $\langle w'T' \rangle$ (equivalently, this is the integrated variance of the cospectrum of w' and T'). For the patch from 8 to 16 m in Fig. 1, $J_q = -240$ W m^{-2}, a significant downgradient transport.

The degree of irreversible heat transfer in a single estimate of $\langle w'T' \rangle$ is not clear. Individual occurrences of countergradient transport are likely a result of restratification following the initial overturn. The patch-averaged transport $\langle w'T' \rangle$ was computed for each of

forty patches observed over approximately the same depth range from 20 consecutive profiles. The values are presented as a histogram in Fig. 4. The range of $\langle w'T' \rangle$, although concentrated at low values, is large. Averaged over all 40 patches, the turbulent heat flux was -50 W m^{-2}. In contrast, the eddy coefficient estimate based on ϵ and $R_f = 0.17$ [Eq. (4)], leads to an average value of

$$J_q = -\rho C_p K_p (dT/dz) \sim -420 \text{ W m}^{-2}.$$

Based on individual patch estimates of the heat flux, the ϵ-estimate gives *downgradient* transport when significant *countergradient* transport acutally occurs. This estimate, of course, is always negative instantaneously and does not include restratification effects. It is only meant to hold in an *averaged* sense. The $w'T'$ correlation, on the other hand, represents the true turbulent heat flux at the time and place of the measurement. Unfortunately, I think that the number of data points is as yet too small to allow a critical examination of the apparent disparity in heat flux estimates. Obtaining an adequate determination of $\overline{w'T'}$ will require many more realizations.

REFERENCES

Caldwell, D. R., 1983: Small-scale physics of the ocean. *Rev. Geophys. Space. Phys.,* **21,** 1192–1205.
——, T. M. Dillon and J. N. Moum, 1985: The Rapid-Sampling Vertical Profiler: An evaluation. *J. Atmos. Oceanic Technol.,* **2,** 615–625.
Gregg, M. C., 1987: Diapycnal mixing in the ocean. *J. Geophys. Res.,* **92,** 5249–5286.
——, and T. B. Sanford, 1988: The dependence of turbulent dissipation on stratification in a diffusively stable thermocline. *J. Geophys. Res.,* **93,** 12 381–12 392.
Moum, J. N., 1990: Profiler measurements of vertical velocity microstructure in the ocean. *J. Atmos. Oceanic Technol.,* **7,** 323–333.
——, and T. R. Osborn, 1986: Mixing in the main thermocline. *J. Phys. Oceanogr.,* **16,** 1250–1259.
——, D. R. Caldwell and C. A. Paulson, 1989: Mixing in the equatorial surface layer and thermocline. *J. Geophys. Res.,* **94,** 2005–2021.
Oakey, N. S., 1982: Determination of the rate of dissipation of turbulent energy from simultaneous temperature and velocity shear microstructure measurements. *J. Phys. Oceanogr.,* **12,** 12 256–12 271.
——, 1988: Estimates of mixing inferred from temperature and velocity microstructure. *Small-Scale Turbulence and Mixing, in the Ocean,* Elsevier Oceanography series, vol. 46, J. C. J. Nihoul and B. M. Jamart, Eds., Elsevier, 239–247.
Osborn, T. R., 1980: Estimates of the local rate of vertical diffusion from dissipation measurements. *J. Phys. Oceanogr.,* **10,** 83–89.
——, and C. S. Cox, 1972: Oceanic fine structure. *Geophys. Fluid Dyn.,* **3,** 321–345.
——, and W. R. Crawford, 1980: An airfoil probe for measuring turbulent velocity fluctuations in water. *Air-Sea Interaction: Instruments and Methods,* F. Dobson, L. Hasse and R. Davis, Eds., Plenum, 369–386.
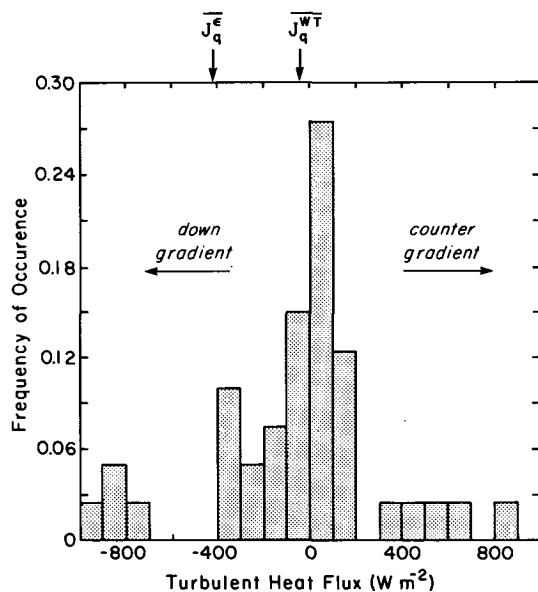Townsend, A. A., 1976: *The Structure of Turbulent Shear Flow.* Cambridge.

FIG. 4. Histogram of patch-averaged estimates of J_q from $\langle w'T' \rangle$. The sample size was 40 (as in Fig. 3). The average value $J_q^{w'T'}$ is -50 W m^{-2} compared to the average value obtained using the dissipation method, J_q^ϵ of -420 W m^{-2}.