Using the Graphical Method to Solve Linear Programs

J. Reeb and S. Leavengood

A key problem faced by managers is how to allocate scarce resources among activities or projects. Linear programming, or LP, is a method of allocating resources in an optimal way. It is one of the most widely used operations research (OR) tools. It has been used successfully as a decision-making aid in almost all industries, and in financial and service organizations.

Programming refers to mathematical programming. In this context, it refers to a planning process that allocates resources—labor, materials, machines, and capital—in the best possible (optimal) way so that costs are minimized or profits are maximized. In LP, these resources are known as *decision variables*. The criterion for selecting the best values of the decision variables (e.g., to maximize profits or minimize costs) is known as the *objective function*. The limitations on resource availability form what is known as a *constraint set*.

For example, let's say a furniture manufacturer produces wooden tables and chairs. Unit profit for tables is $6, and unit profit for chairs is $8. To simplify our discussion, let's assume the only two resources the company uses to produce tables and chairs are wood (board feet) and labor (hours). It takes 30 bf and 5 hours to make a table, and 20 bf and 10 hours to make a chair. There are 300 bf of wood available and 110 hours of labor available. The company wishes to maximize profit, so profit maximization becomes the *objective function*. The

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Decision variables. . .

"The resources available."

Constraint set. . .

"The limitations on resource availability."

Objective function. . .

"The criterion for selecting the best values of the decision variables."

resources (wood and labor) are the *decision variables*. The limitations on resource availability (300 bf of wood and 110 hours of labor) form the *constraint set,* or operating rules that govern the process. Using LP, management can decide how to allocate the limited resources to maximize profits.

The "linear" part of the name refers to the following:

• The objective function (i.e., maximization or minimization) can be described by a linear function of the decision variables, that is, a mathematical function involving only the first powers of the variables with no cross products. For example, $23X_2$ and $4X_{16}$ are valid decision variable terms, while $23X_2^2$, $4X_{16}^3$, and ($4X_1 * 2X_1$) are not. The entire problem can be expressed as straight lines, planes, or similar geometrical figures.

• The constraint set can be expressed as a set of linear equations.

In addition to the linear requirements, nonnegativity conditions state that the variables cannot assume negative values. It is not possible to have negative resources. Without these conditions, it would be mathematically possible to use more resources than are available.

This publication will introduce a small LP problem that can be solved graphically. In other words, we'll plot the appropriate information on a graph, and then use the graph to find a solution to the problem.

In EM 8720, *Using the Simplex Method to Solve Linear Programming Maximization Problems*, we'll build on the graphical example and introduce an algebraic technique known as the *simplex method*. This method lets us solve very large LP problems that would be impossible to solve graphically or without the analytical ability of a computer. In other publications, we'll define several special types of LP and use forest products examples to help explain their functions.

For more complex problems, many software packages are available for obtaining a solution. It's just a matter of choosing the one you wish to work with. In a later publication, we'll use commercial software to solve larger LP problems and to examine how to use sensitivity analysis to gain more information than that provided by a simple optimal solution.

Steps in solving an LP problem graphically

The steps in solving an LP problem graphically are introduced briefly below. In the remainder of this publication, we'll apply these steps to a simple LP problem.

Step 1. Formulate the LP problem. Formulation refers to translating the real-world problem into a format of mathematical equations that represent the objective function and the constraint set. Often, data gathering, problem definition, and problem formulation are the most important (and the most difficult, time-consuming, and expensive) steps when using any OR tool.
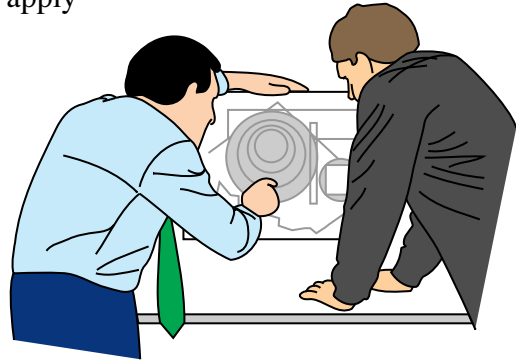
A thorough understanding of the problem is necessary in order to formulate it correctly. During the formulation stage, an OR specialist may discover new insights into the problem that may change the scope of the original problem. For this reason, it's very important to obtain the help of those who work most closely with the system being studied. They can help the OR specialist collect the necessary data and correctly define the problem so that he or she can create a valid model. In formulating an LP problem, it often helps to put all of the relevant information into a table (for example, see Table 1, page 4).

Step 2. Construct a graph and plot the constraint lines. Constraint lines represent the limitations on available resources. Usually, constraint lines are drawn by connecting the horizontal and vertical intercepts found from each constraint equation.

Step 3. Determine the valid side of each constraint line. The simplest way to start is to plug in the coordinates of the origin (0,0) and see whether this point satisfies the constraint. If it does, then all points on the origin side of the line are feasible (valid), and all points on the other side of the line are infeasible (invalid). If (0,0) does not satisfy the constraint, then all points on the other side and away from the origin are feasible (valid), and all points on the origin side of the constraint line are infeasible (invalid). There are two exceptions, which we will discuss later.

Step 4. Identify the feasible solution region. The feasible solution region represents the area on the graph that is valid for all constraints. Choosing any point in this area will result in a valid solution.

$$5(4) + 10X_2 = 110$$

$$30(4) + 20X_2 = 300$$

$$4(2.444) - X_2 = 0$$

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Step 5. Plot two objective function lines to determine the direction of improvement. Improvement is in the direction of greater value when the objective is to maximize the objective function, and is in the direction of lesser value when the objective is to minimize the objective function. The objective function lines do not have to include any of the feasible region to determine the desirable direction to move.

Step 6. Find the most attractive corner. Optimal solutions always occur at corners. The most attractive corner is the last point in the feasible solution region touched by a line that is parallel to the two objective function lines drawn in step 5 above. When more than one corner corresponds to an optimal solution, each corner and all points along the line connecting the corners correspond to optimal solutions. We'll use an example to illustrate optimal solutions later.

Step 7. Determine the optimal solution by algebraically calculating coordinates of the most attractive corner.

Step 8. Determine the value of the objective function for the optimal solution.

An LP example: Production of wooden tables and chairs

Let's look at the profit maximization problem that our furniture manufacturer faces. The company uses wood and labor to produce tables and chairs. Recall that unit profit for tables is $6, and unit profit for chairs is $8. There are 300 board feet (bf) of wood available, and 110 hours of labor available. It takes 30 bf and 5 hours to make a table, and 20 bf and 10 hours to make a chair. Table 1 contains the information for the LP problem. We will go through the step-by-step process of solving this problem graphically.

Table 1.—Information for the wooden tables and chairs linear programming problem.

Resource	Table (X_1)	Chair (X_2)	Available
Wood (bf)	30	20	300
Labor (hr)	5	10	110
Unit profit	$6	$8	

Step 1. Formulate the LP problem.

Table 1 will help us formulate the problem. The bottom row is used to formulate the objective function. Objective functions are developed in such a way as to be either maximized or minimized. In this case, the company's management wishes to maximize unit profit. The wood and labor rows are used to formulate the constraint set. Finally, the nonnegativity conditions are stated.

Maximize: $Z = 6X_1 + 8X_2$ (objective function)

Subject to: $30X_1 + 20X_2 \le 300$ (wood constraint: 300 bf available)

$5X_1 + 10X_2 \le 110$ (labor constraint: 110 hours available)

$X_1, X_2 \ge 0$ (nonnegativity conditions)

Since only two variables (wood and labor) exist in this problem, it can be solved graphically. If there were more than two variables, the graph would have to be more than two dimensions.

Step 2. Construct the graph and plot constraint lines.

Draw the graph with the *x* axis representing the number of tables and the *y* axis representing the number of chairs. Plot the two constraint lines by finding the *x* and *y* intercepts for the two constraint equations in the following manner.

First, rewrite the constraint inequalities as equalities and solve to obtain the intercepts:

Wood	**Labor**
$30X_1 + 20X_2 = 300$	$5X_1 + 10X_2 = 110$
Set $X_2 = 0$ and solve for X_1	Set $X_2 = 0$ and solve for X_1
$30X_1 = 300$	$5X_1 = 110$
$X_1 = 300/30$	$X_1 = 110/5$
= 10 tables	= 22 tables
(All the wood is used to make tables.)	*(All the labor is used to make tables.)*

Next:

Set $X_1 = 0$ and solve for X_2	Set $X_1 = 0$ and solve for X_2
$20X_2 = 300$	$10X_2 = 110$
$X_2 = 300/20$	$X_2 = 110/10$
= 15 chairs	= 11 chairs
(All the wood is used to make chairs.)	*(All the labor is used to make chairs.)*

Tip . .

In our example,
X_1 refers to tables,
X_2 refers to chairs, and
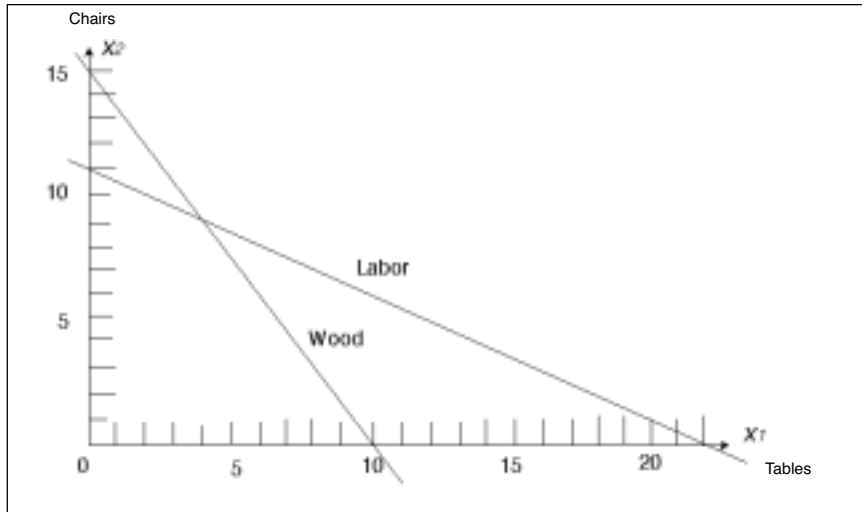Z refers to profit.
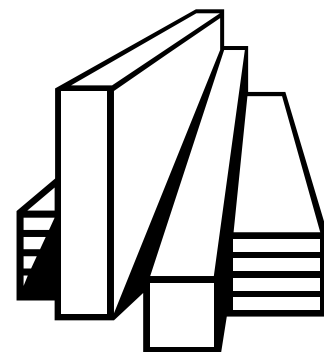
Figure 1.—Wood and labor constraint lines.

Now plot the wood constraint line, using the intercepts $X_1 = 10$ and $X_2 = 15$. Plot the labor constraint line, using the intercepts $X_1 = 22$ and $X_2 = 11$. See Figure 1.

Step 3. Determine the valid side of each constraint line.

We will use the origin (0,0) to check the valid side for both constraint lines. $30(0) + 20(0) \leq 300$ is valid, so we know the side toward the origin (0,0) is the valid side of the wood constraint line. $5(0) + 10(0) \leq 110$ also is valid, so we know the side toward the origin (0,0) is the valid side of the labor constraint line. We can draw arrows indicating the valid side of each constraint line. See Figure 2.

We could have chosen any point to test for the valid side of the line. For example, setting $X_1 = 20$ and $X_2 = 10$ (clearly on the other side, away from the origin) for the wood constraint line, we get $30(20) + 20(10) \leq 300$, which is not valid. In other words, there simply isn't enough wood to make 20 tables and 10 chairs.
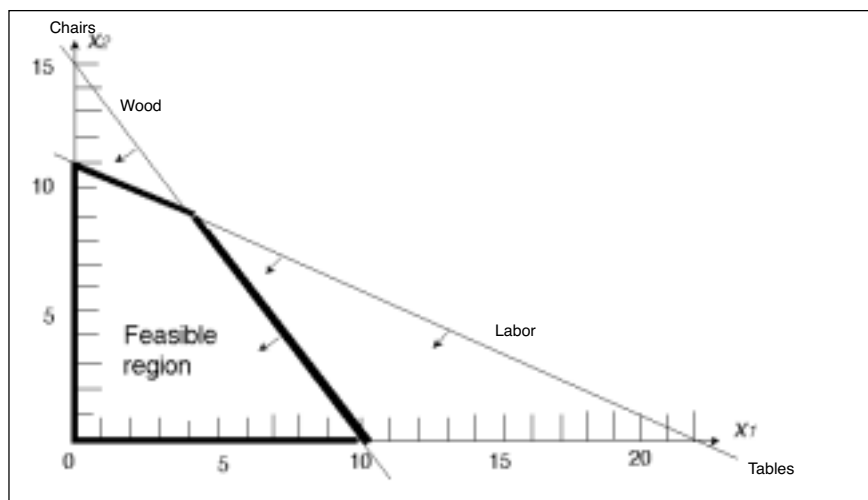

Figure 2.—Identification of the feasible region.

Step 4. Identify the feasible region.

The feasible region is the area on the valid side of both con-straint lines. Any point located on the invalid side of a constraint line is infeasible. Because of the nonnegativity conditions, the feasible region is restricted to the positive quadrant. See Figure 2.

Step 5. Plot two objective function lines to determine the direction of improvement.

First, we'll arbitrarily set profit, $Z = 48$, and then set profit, $Z = 72$. We'll find the x and y intercepts when $Z = 48$ and when $Z = 72$, and plot the two lines.

Set $Z = 48$

Set $X_2 = 0$ and solve for X_1

$48 = 6(X_1)$

$48/6 = X_1$

$X_1 = 8$

Set $Z = 72$

Set $X_2 = 0$ and solve for X_1

$72 = 6(X_1)$

$72/6 = X_1$

$X_1 = 12$

Next:

Set $X_1 = 0$ and solve for X_2

$48 = 8(X_2)$

$48/8 = X_2$

$X_2 = 6$

Set $X_1 = 0$ and solve for X_2

$72 = 8(X_2)$

$72/8 = X_2$

$X_2 = 9$

Now plot the objective function lines when $Z = 48$ and $Z = 72$. See Figure 3. We can see from the two objective function lines that as we move away from the origin (0,0), Z increases.

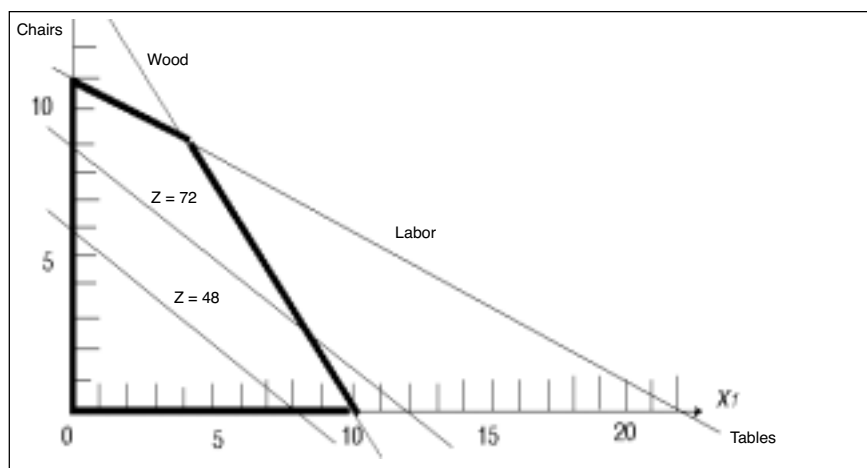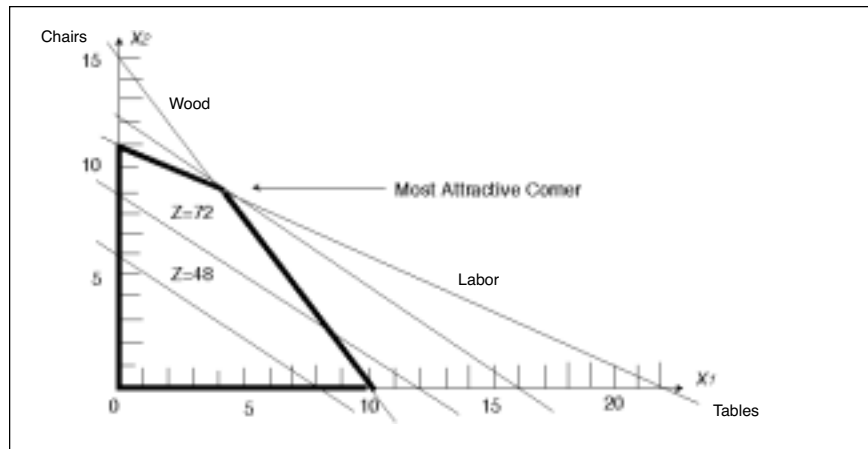*Figure 3.—Determining direction of increasing value.*

Figure 4.—Locating the most attractive corner.

Step 6. Find the most attractive corner.

Since we want to maximize Z, we will draw a line parallel to the objective function lines that touches the last point in the feasible region while moving away from the origin. This identifies the most attractive corner, which gives us the amounts of wood and labor that will result in the maximum profit (maximize Z). Thus, it represents the optimal solution to the problem (Figure 4).

Step 7. Determine the optimal solution by algebraically calculating coordinates of the most attractive corner.

The most attractive corner lies at the intersection of the wood and labor constraint lines. Therefore, coordinates for the most attractive corner can be found by simultaneously solving the constraint equations (wood and labor):

$$30X_1 + 20X_2 = 300 \quad \text{(wood)}$$

$$5X_1 + 10X_2 = 110 \quad \text{(labor)}$$

To do so, multiply the labor equation by -2 and add it to the wood equation so the X_2 variable becomes zero and we can solve for X_1.

$$30X_1 + 20X_2 = 300 \quad \text{(wood)}$$
$$-2(5X_1 + 10X_2 = 110) \quad \text{(labor)}$$
$$\overline{20X_1 + 0 \quad = 80}$$

$$X_1 \qquad = 4 \text{ tables}$$

Next, substitute the number of tables calculated above into either of the constraint equations to find the number of chairs. For this example, we will substitute into both equations to illustrate that the same value is found.

Wood constraint   **Labor constraint**

$30(4) + 20X_2 = 300$ $5(4) + 10X_2 = 110$

$120 + 20X_2 = 300$ $20 + 10X_2 = 110$

$\quad\quad 20X_2 = 300 - 120$ $\quad\quad 10X_2 = 110 - 20$

$\quad\quad 20X_2 = 180$ $\quad\quad 10X_2 = 90$

$\quad\quad X_2 = 180/20$ $\quad\quad X_2 = 90/10$

$\quad\quad X_2 = 9$ chairs $\quad\quad X_2 = 9$ chairs

Thus, the company's optimal solution is to make four tables and nine chairs. In this case, you could read this solution off the graph (Figure 4) by finding the values on the *x* and *y* axes corresponding to the most attractive corner. However, when the most attractive corner corresponds to an optimal solution with fractions, it is not possible to read directly from the graph. For example, the optimal solution to this problem might have been 3.8 tables and 9.2 chairs, which we probably would not be able to read accurately from the graph.

Step 8. Determine the value of the objective function for the optimal solution.

Plug in the number of tables and chairs and solve for Z:

$Z = \$6(4) + \$8(9) = \$96$

Thus, we find that maximum profit of $96 can be obtained by producing four tables and nine chairs.
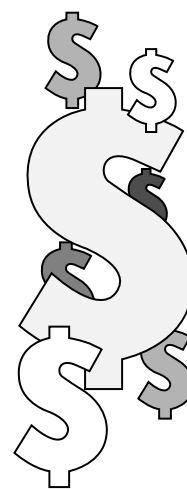
> ### *Mixture constraint. . .*
>
> "A special constraint occurring when a resource must be used in a fixed ratio with other resources."

Other types of constraints

Mixture constraints

A constraint line that passes through the origin (0,0) represents a special kind of constraint known as a *mixture constraint*. Mixture constraints arise when a resource must be used in a fixed ratio with other resources.

We can add a mixture constraint to our example by saying that tables and chairs are sold in sets of one table with four chairs. Some extra chairs can be made. We'll keep the other labor and wood constraints and add the following new constraint:

$4X_1 \leq X_2$ (The number of chairs must be at least 4 times the number of tables.)

To find the *x* and *y* intercepts of this new constraint line, first move all variables to the left side of the inequality:

$4X_1 - X_2 \leq 0$

Next, rewrite the inequality as an equality and solve to obtain the intercepts:

$4X_1 - X_2 = 0$

When $X_1 = 0$, then X_2 also is 0, so we see that the constraint line passes through the origin (0,0). In other words, if the company doesn't make any tables, it can't make any chairs and vice versa.

Now we can arbitrarily choose other values and solve the equation to find other points on the constraint line. If $X_1 = 3$, then substituting 3 into the equation we find:

$$4(3) - X_2 = 0$$

$$12 - X_2 = 0$$

$$X_2 = 12$$

Thus, if the company makes 3 tables, it needs to make 12 chairs. Point A represents where $X_1 = 3$ and $X_2 = 12$ in Figure 5.
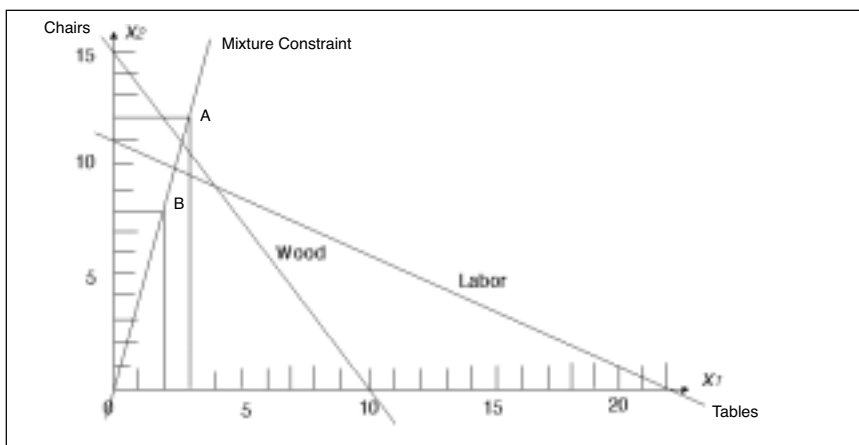

Figure 5.—Determining the mixture constraint line.

We will plot one more point for the constraint line by arbitrarily setting $X_1 = 2$ and solving for X_2. We find:

$$4(2) - X_2 = 0$$

$$8 \quad - X_2 = 0$$

$$X_2 = 8$$

Point B represents where $X_1 = 2$ and $X_2 = 8$. So, if the company makes two tables, it needs to make eight chairs.

We now can draw the mixture constraint line on the graph. See Figure 5.

To evaluate the valid side of the mixture constraint line, evaluate a point not on the line. We will arbitrarily choose $X_1 = 5$ and $X_2 = 10$ (Figure 6a). Substitute the values into the equation:

$$4(5) - 10 \leq 0$$

This statement is false, so this is not the valid side of the mixture constraint line.

When we substitute the values for a point on the opposite side of the mixture constraint line, we get a true statement. For example, let $X_1 = 2$ and $X_2 = 15$ (Figure 6b). Substituting the values into the equation:

$$4(2) - 15 \leq 0$$

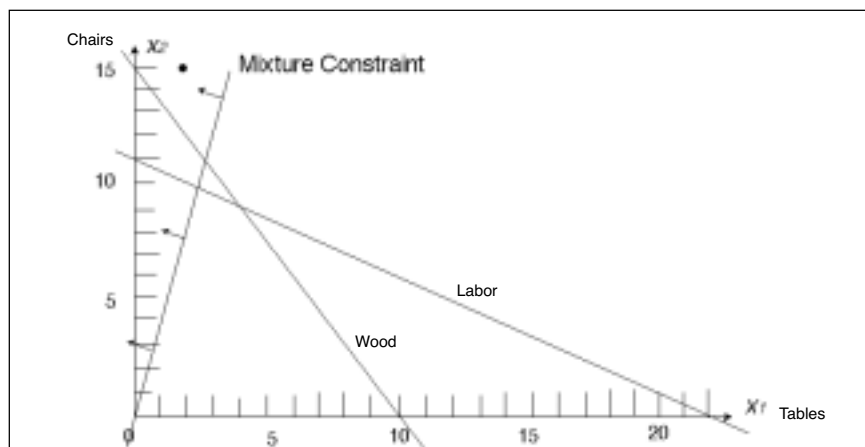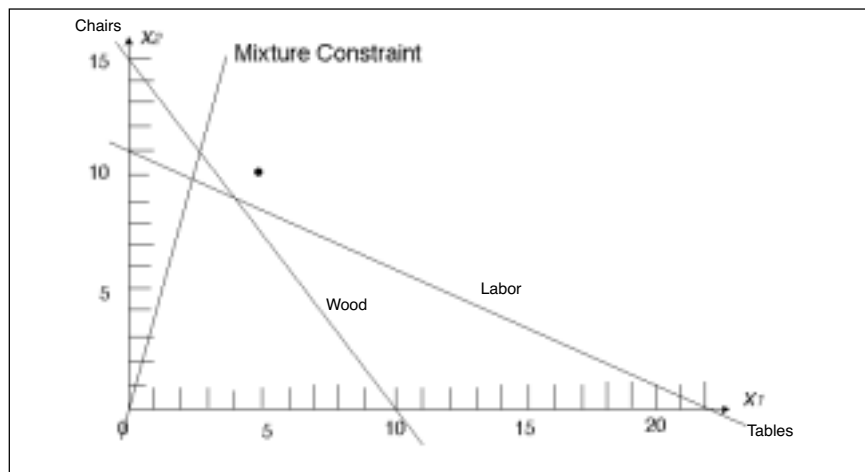This statement is true, thus signifying the valid side of the constraint line.

Figure 6a and 6b.—To determine the valid side of the mixture constraint line, evaluate a point not on the line.
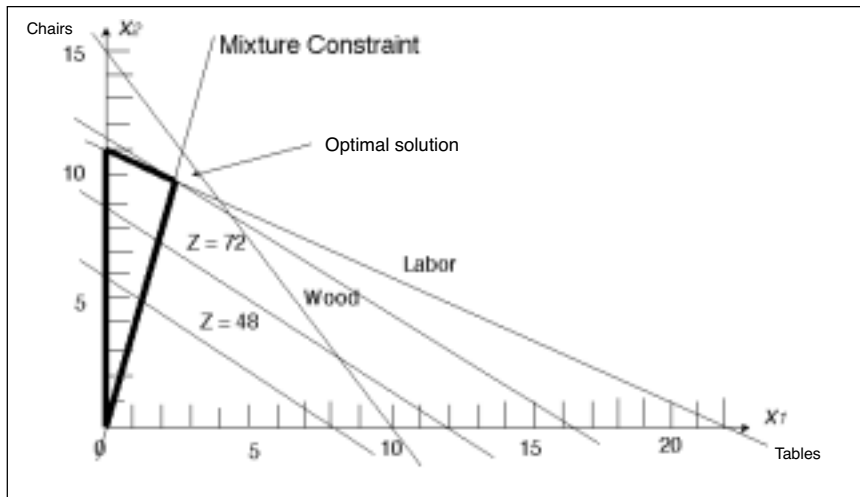
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Figure 7.—Locating the feasible region and the most attractive corner. In this case, the optimal solution is where the labor and mixture constraint lines intersect.

The boundary of the feasible region is defined by the mixture constraint and the labor constraint lines illustrated with the dark line in Figure 7. As before, plot two objective function lines to determine the direction of increase. Draw a line parallel to the two objective function lines that touches the last point in the feasible region (going in the direction of increasing value). The most attractive corner lies at the intersection of the mixture constraint and labor constraint lines (Figure 7).

The next step is to simultaneously solve the mixture and labor equations by multiplying the mixture constraint equation by 10 and adding it to the labor constraint equation so the X_2 variable becomes 0 and we can solve for X_1:

$$5X_1 + 10X_2 = 110 \quad \text{(labor constraint)}$$
$$\underline{10(4X_1 - X_2 = 0)} \quad \text{(mixture constraint)}$$
$$45X_1 + 0 = 110$$
$$X_1 = 110/45$$
$$X_1 = 2.444 \text{ (number of tables)}$$

Now, solve for X_2:

$$4X_1 - X_2 = 0$$
$$4(2.444) - X_2 = 0$$
$$9.777 - X_2 = 0$$
$$- X_2 = - 9.777$$
$$X_2 = 9.777 \text{ (number of chairs)}$$

This results in a non-integer (fractional) solution. Non-integer solutions are discussed later in this publication. Meanwhile, we can view the 0.444 tables and 0.777 chairs as work in progress.

From observing the graph, we see that the wood constraint line does not border the feasible region. This indicates that some wood

was not used in the optimal solution. The unused wood can be calculated as:

30 bf(2.444) + 20(9.777) = 268.88 bf of wood used out of 300 bf available

300 bf - 268.88 bf = 31.12 bf of wood unused

The unused wood can be thought of as *surplus* or *slack*. In EM 8720, *Using the Simplex Method to Solve Linear Programming Maximization Problems,* we'll use this concept in solving LP problems with the simplex method.

Note that it would be difficult to read this fractional optimal solution directly from the graph. Thus, this example is one where simultaneously solving the equations representing the intersecting lines at the most attractive corner is the most practical method for determining the optimal solution.

Equality constraints

Constraints can take the form of an equality. For example, the company may decide to sell all of its tables and chairs in sets (one table and four chairs). In this case, the new constraint is:

$$4X_1 = X_2$$

or

$$4X_1 - X_2 = 0$$

In this case, there is no valid side of the mixture line. Valid points lie exactly on the constraint line. The feasible region consists of the segment of the constraint line that meets the conditions of the other constraints, i.e., the part of the equality constraint line from the origin (0,0) to the point where the equality constraint line meets the labor constraint line (Figure 8).
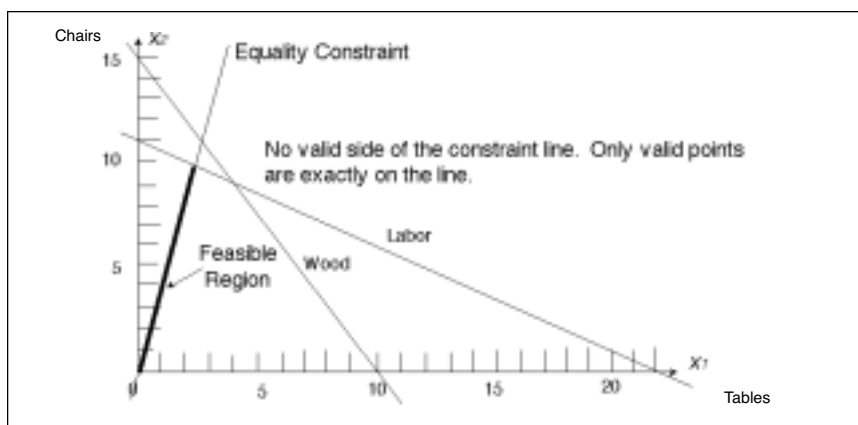

Equality constraint. . .

"A special constraint that causes all valid points to lie exactly on the constraint line."

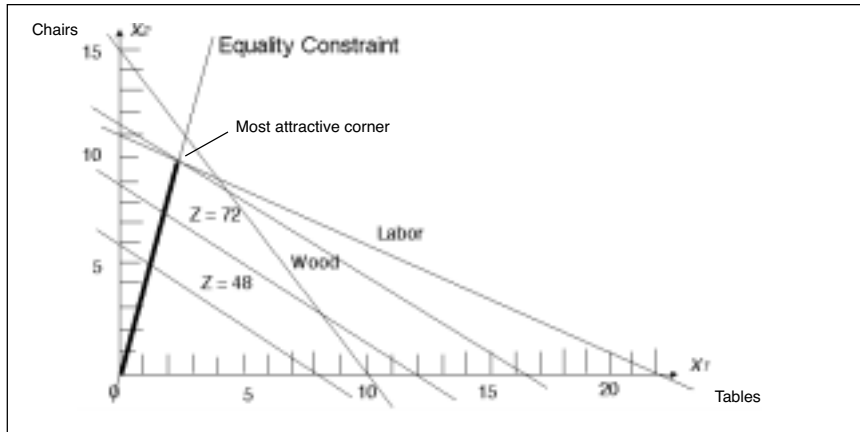*Figure 8.—Determining the feasible region for an equality constraint problem.*

Figure 9.—Identifying the most attractive corner for an equality constraint problem.

We set $Z = 48$ and $Z = 72$ in the same manner as above to see that Z increases in value as we move away from the origin. Drawing a line parallel to these objective function lines, we find the most attractive corner is where the equality constraint line and the labor constraint line intersect (Figure 9).

In this case, by simultaneously solving the equality and labor constraint equations, we get the same solution (and equation set) as in the mixture constraint example above.

If there is more than one equality constraint, the optimal corner is where the equality constraint lines intersect. We can add another equality constraint to our tables and chairs example by saying that exactly two tables need to be produced. This constraint is graphed with a vertical line intersecting the *x* axis at 2 (Figure 10). The optimal solution is where the table equality constraint and the set (four chairs for each table) constraint lines intersect. Substituting 2 into the equation gives:

$4X_1 - X_2 = 0$

$4(2) = X_2$

$X_2 = 8$ chairs

The optimal solution to the LP is:

$Z = \$6(2) + \$8(8) = \$76$ unit profit (Figure 10)
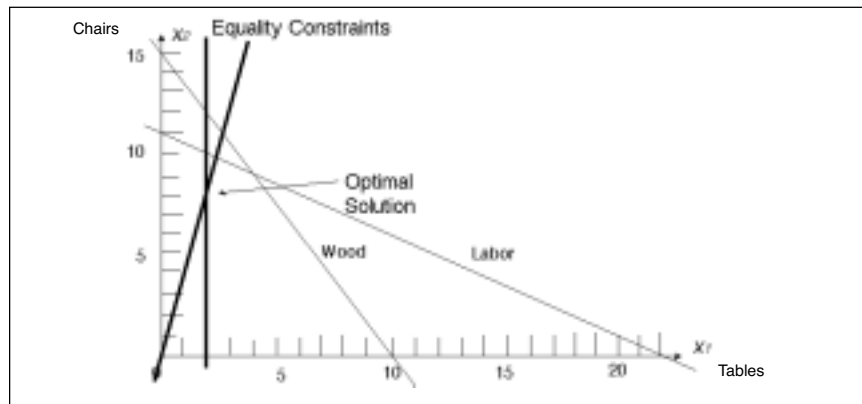

Figure 10.—Two equality constraints.

Multiple or alternative optimal solutions

Let's consider another maximization problem, unrelated to the tables and chairs problem, to illustrate *multiple*, sometimes called *alternative*, optimal solutions.

Step 1. Let Z = profit, and formulate the LP problem as:

Maximize $Z = 10X_1 + 12X_2$

Subject to $5X_1 + 6X_2 \leq 60$ (resource one)

$8X_1 + 4X_2 \leq 72$ (resource two)

$3X_1 + 5X_2 \leq 45$ (resource three)

$X_1,\ X_2 \geq 0$

In this case, there are multiple resources that can be combined in various ways. Our goal is to find the combination of resources that will maximize profits. It's possible that more than one resource mix will lead to equally optimal solutions.

Steps 2, 3, and 4. Construct the graph and plot the constraint lines. Determine the valid side of each constraint line. Identify the feasible region (Figure 11).
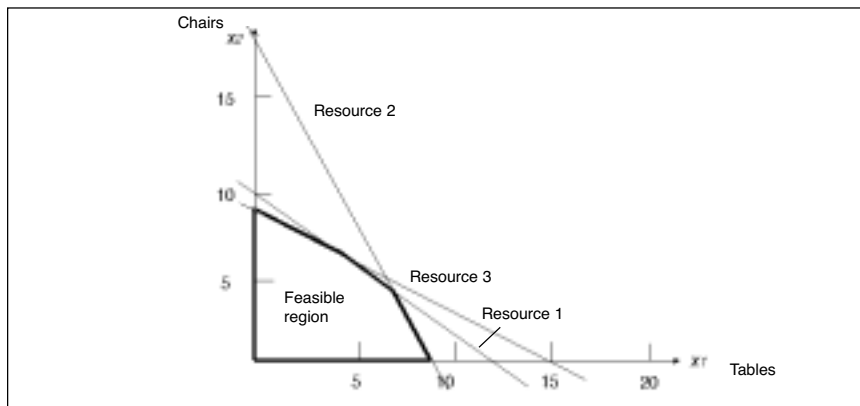
> With multiple resources that can be combined in various ways, it's possible that more than one resource mix will lead to equally optimal solutions.

Figure 11.—Identifying the feasible region with multiple possible resource mixes.
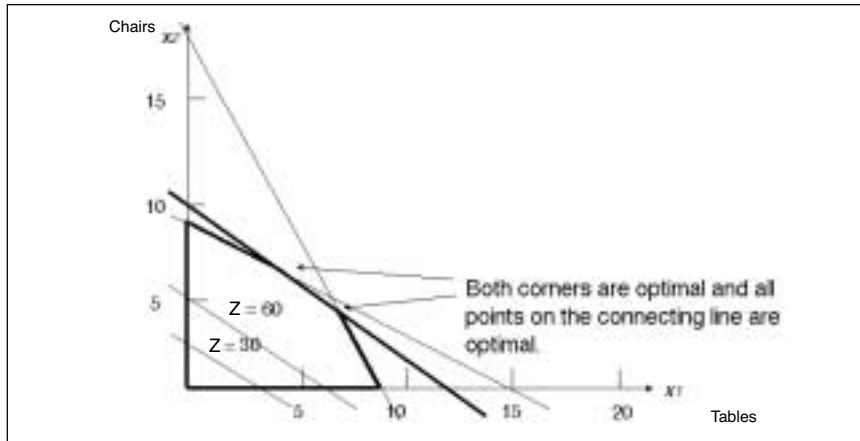
Figure 12.—Determining the most attractive corner with multiple optimal solutions.

Steps 5 and 6. Plot two objective function lines and determine the direction of increasing value (if a minimization problem, we would determine the direction of decreasing value). Identify the most attractive corner. For this example, we find that there actually are two most attractive corners, where resource one's constraint line intersects resource two's constraint line, and where resource one's constraint line intersects resource three's constraint line (Figure 12).

Step 7. Determine the optimal solution by algebraically calculating coordinates of the most attractive corners. By simultaneously solving both equation pairs for both points, we can choose the corner that corresponds to the highest objective function value (since this is a maximization problem). Solve for X_1:

$$5X_1 + 6X_2 = 60 \quad \text{(resource one)}$$

$$-6/4(8X_1 + 4X_2 = 72) \quad \text{(resource two)}$$

$$-7X_1 + 0 = -48$$

$$1/7(-7X_1 + 0) = -48$$

$$-X_1 = -48/7$$

$$X_1 = 6.86$$

Solve for X_2 by substituting into a constraint equation:

$$5(6.86) + 6X_2 = 60$$

$$34.28 + 6X_2 = 60$$

$$6X_2 = 60 - 34.28$$

$$6X_2 = 25.71$$

$$X_2 = 25.71/6$$

$$X_2 = 4.29$$

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Solve for Z in the objective function:

$Z = 10(6.86) + 12(4.29) = 120$

Solve for X_1:

$5X_1 + 6X_2 = 60$ (resource one)

$-6/5(3X_1 + 5X_2 = 45)$ (resource three)

$1.4X_1 + 0 = 6$

$X_1 = 6/1.4$

$X_1 = 4.29$

Solve for X_2 by substituting into a constraint equation:

$5(4.29) + 6X_2 = 60$

$21.45 + 6X_2 = 60$

$6X_2 = 60 - 21.45$

$6X_2 = 38.55$

$X_2 = 38.55/6$

$X_2 = 6.425$

Solve for Z in the objective function:

$Z = 10(4.29) + 12(6.425) = 120$

Both points are equally attractive, so there are multiple optimal solutions. As stated earlier, when more than one corner corresponds to an optimal solution, each corner and all points along the connecting line segment will correspond to optimal solutions.

Whenever all coefficients of one equation are the same multiple of another equation's coefficients, the lines are parallel. When the objective function line is parallel to a constraint line, there are multiple optimal solutions. In our example:

$Z = 10X_1 + 12X_2$ (objective function equation)

$5X_1 + 6X_2 = 60$ (constraint one equation)

The ratio of X_1 terms (10 and 5) is 10/5 or 2. The ratio of X_2 terms (12 and 6) is 12/6, also 2. Thus, the two lines are parallel and multiple optimal solutions exist. Multiple optimal solutions mean that more than one mix of resources leads to an optimal solution. In real-world problems, there may be reasons why one mix of resources would be easier to obtain than another, or there may be no reason to choose one solution over another.
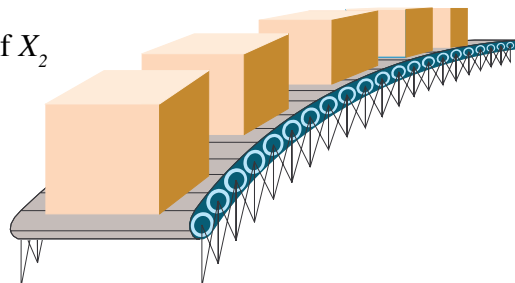
Unbounded solutions

An unbounded solution occurs when there are unlimited resources. A good way to explain unbounded solutions is to use a small problem that can be solved graphically. Consider the following LP problem:

Maximize: $Z = X_1 + 2X_2$

Subject to: $X_1 + X_2 \geq 2$

$X_2 \leq 4$

$X_1, X_2 \geq 0$

We can graph this problem as in Figure 13.

The constraint line intersecting the X_2 axis at 4 can increase infinitely along the X_1 axis. Thus, the feasible region is unbounded. The objective function can be improved infinitely by adding infinitely more resources. For real-life problems, of course, it is impossible to have unlimited resources. Thus, an unbounded solution signifies that the LP problem has been formulated incorrectly. Usually, one or more constraints has been omitted.

Unbounded solutions. . .

"A solution to a problem with unlimited resources." In the real world, such a situation is impossible.

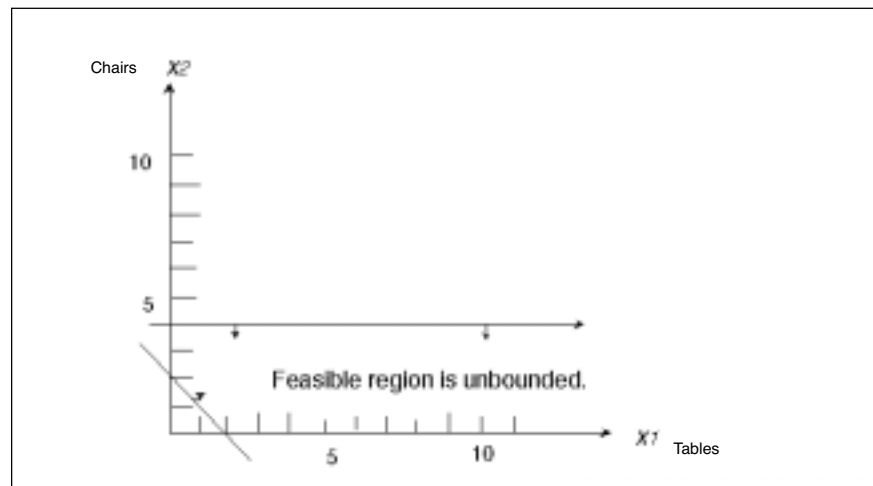
Figure 13.—Example of an unbounded solution, in which X_1 can be increased infinitely to increase the objective function.

Limitations to linear programming

As we saw in an earlier example, there is no guarantee of an integer (whole number) solution. The furniture company can't sell 0.444 of a table. We thought of this as work in progress for this particular problem. An optimal solution for another problem might be to buy 6.75 trucks. Obviously, you can buy either 6 or 7 trucks, but not 0.75 of a truck. In this case, rounding up (and for many non-integer answers, rounding either up or down) offers a practical solution. However, if you're studying the possibility of building a new plant addition, a variable can take on the value of 0 or 1, so a fractional answer is useless. Either you build the addition or you don't. Fortunately, methods called *integer programming* can handle these types of problems.

When using LP, there is no way to deal with uncertainty. We formulate the problem with the assumption that we know all of the values for costs, profits, constraints, etc. In real-life problems, these variables often are unknowns. Methods such as *chance constrained programming* or *linear programming under uncertainty* can be used in these situations.

Another limitation is the assumption of linearity. Often, the objective and constraints are not related linearly to the variables. In some problems, a nonlinear relationship can be forced to be a linear relationship without losing too much integrity in the original problem. For other problems, *nonlinear programming* techniques are available.

Integer programming, linear programming under uncertainty, and nonlinear programming are beyond the scope of these publications. If you wish to learn more about these topics, see the resources listed in the "For more information" section.

For more information

Bierman, H., C.P. Bonini, and W.H. Hausman. *Quantitative Analysis for Business Decisions* (Richard D. Irwin, Inc., Homewood, IL, 1977). 642 pp.

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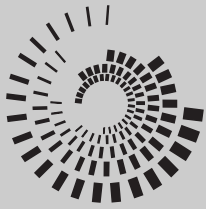
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