SOLVING REAL OPTIONS MODELS OF FISHERIES INVESTMENT WHEN SALVAGE VALUE IS DIFFICULT TO ESTIMATE

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ABSTRACT

Real options models arguably provide a useful framework for understanding individual fishermen's investment and participation decisions, which play an important role in fisheries management issues ranging from capacity reduction to stock assessment. While these models appear to predict actual behavior reasonably well, one important difficulty in applying them is that the salvage value of the fishing enterprise (or the opportunity cost of fishing, which when capitalized can be treated as a salvage value) is often difficult to assess. For example, there may be few sales in a market for boats or licenses, and there may be little information on other employment opportunities available to fishermen. This paper adopts a fisherman's exit decision as a framework for exploring solution techniques for real options models when salvage value is difficult to estimate. We first present a simple model of the exit decision as a disinvestment problem, and then describe this model's solution by minimization and finite difference methods, including an iterative nonlinear least squares approach that allows rapid solution for a wide range of putative deterministic salvage values. We then allow salvage value to follow its own stochastic process, again exploring this example with both minimization and finite difference techniques. Our general conclusion is that minimization methods are much preferable when they apply, but that in most fisheries applications it will be necessary to resort to finite difference, finite element, or Monte Carlo methods.

Keywords: fisheries investment, real options, numerical solution, entry and exit

INTRODUCTION

Many important decisions in fisheries can be represented as optimal stopping problems, either from the fisherman's perspective (e.g., whether to buy or sell a boat, or whether to participate in a particular fishery) or the manager's perspective (e.g., whether to close a fishery or tear down a dam). The essense of these decision problems is that the decisionmaker chooses whether to stop one process and start another. If there is uncertainty about the effect of exchanging one process for another, and if there is a fixed cost or benefit to doing so, stochastic dynamic optimization models provide a framework for analyzing the decision problem. In this paper we are primarily concerned with issues raised by such lump-sum costs or benefits, examples of which include the price of a new engine, the price of a license to enter a limited-entry fishery, or the opportunity cost of permanently exiting such a fishery,. For the sake of concreteness, we consider this last case, a fisherman's exit decision, in which the fisherman can exchange the value of the fishing enterprise for a lump-sum benefit. This benefit, depending on the circumstance, might be the scrap value of a boat, the sale price of a tradable permit, or simply the capitalized expected value of entering alternative fisheries.

Our purpose in this paper is to describe and analyze some of the issues raised by the specification of this benefit, which we will refer to generically as 'salvage value.' Our model fisherman faces a choice between staying active in a limited-entry fishery, or permanently exiting that fishery. If he chooses exit, he is free to pursue other fisheries. Though each alternative fishery could be treated as a separate project with its own dynamics, that approach significantly complicates the analysis. Therefore we define the capitalized expected value of the next-most-profitable alternative fishery as the salvage value of leaving the limited-entry fishery permanently. In past work, we have found that this value is difficult to estimate and that model results are sensitive to the specification of salvage value. Even in cases where the definition of salvage value is more traditional, such as the sale price of a boat, there may be few boat sales and the information may not be publicly available. Additionally, the salvage value may be changing over time, and may be linked to other stochastic model variables.

We explore the implications of these issues in the context of a 'real options' model, a variety of stochastic dynamic optimization model. Real options models, which draw on the tools of financial economics, provide a representation of optimal stopping problems that is convenient for many purposes. They have become increasingly popular in corporate finance, and have found some application in natural resource economics. While the technical apparatus of these models is borrowed from financial economics, the underlying concept is the same as the discrete-time stochastic dynamic programming models more familiar to economists: agents make decisions (often stopping decisions, and often irreversible) by acting on random state variables evolving as stochastic processes, with the aim of maximizing the expected value of some objective function.

As with other dynamic optimization models, analytical solutions of real options models are not obtainable in most interesting applications. A variety of numerical techniques can be used, depending on the structure of the particular problem and on the degree to which analytical work can be done prior to the numerical work. In the context of stopping problems, model development ideally proceeds from specification of stochastic processes, to identification of partial differential equations obeyed by the optimal solution, to solution of those partial differential equations and identification of stopping thresholds (such as the revenue level at which it is optimal to enter or exit a fishery). When analytical solutions to the partial differential equations can be found, these solutions and their boundary conditions may be used to find the stopping values as the solution to systems of nonlinear equations. When the partial differential equations can be specified but not solved explicitly, finite difference or finite elements methods can be applied. Still more generally, when the dynamic optimization problem cannot be reduced to a partial differential equation or system of equations, simulation methods may be used.

In this paper, we focus on the first two techniques: the solution of real options models represented as systems of nonlinear equations and as partial differential equations. We begin by demonstrating the application of both methods to a very simple representation of the exit decision, then discuss some of the numerical problems that arise in more complicated models and describe a simple grid search algorithm that is helpful in some circumstances. Finally, we allow salvage value to follow a stochastic process, and again apply both nonlinear equations and finite difference solution techniques to this more general model.

Our general conclusions are that minimization methods provide significant advantages if analytical solutions to the partial differential equations can be obtained: they are faster, involve fewer parameters to be specified, and provide more precise estimates of optimal stopping values. They can also be applied to several important situations commonly arising in fisheries, such as the analysis of entry and exit decisions and latent capacity. In many cases, however—often as a result of the stochastic process chosen to represent a state variable or the presence of multiple state variables—finite difference, finite element, or Monte Carlo methods will be necessary. It is our hope that this paper gives fisheries researchers some insight into what to expect from two of these solution techniques, and, for those unfamiliar with options modeling, an appreciation of the utility of this framework.

SOLUTION METHODS WHEN SALVAGE IS DETERMINISTIC

We begin by laying out a model of a fisherman's exit decision, which will be the basis for the rest of the analysis in this paper[1].

Suppose a fisherman faces a choice between staying active in a limited-entry fishery or exiting that fishery permanently, and that the fisherman's goal is to maximize the expected value of the discounted profit stream. Suppose further that periodic revenues *R* follow a geometric Brownian motion:

$$1)\quad dR = \alpha R dt + \sigma R dz$$

where α is the instantaneous rate of change, σ is the instantaneous volatility, and dz is a standard Brownian motion. Given this stochastic process for revenue, the value *F* of a fishing enterprise with a perpetual option to cease operations is the solution to the following partial differential equation:

$$2)\quad R - C + \alpha R \frac{\partial F(R,t)}{\partial R} + \frac{1}{2}\sigma^2 R^2 \frac{\partial^2 F(R,t)}{\partial R^2} - \rho F(R,t) + \frac{\partial F(R,t)}{\partial t} = 0$$

where *C* is periodic operating cost and ρ is the fisherman's discount rate. If the fisherman elects to exit the fishery, he receives a salvage value *S*, which may be the sale price of a boat or license, or the capitalized value of earnings in a different fishery or other activity. That is, the salvage value is whatever the fisherman gains in exchange for giving up the current fishing enterprise. Salvage value can be used to impose boundary condition on (2) as follows:

$$3a)\quad F(R_x, t) = S$$

$$3b)\quad F(0,t) = S$$

$$3c)\quad F(\infty, t) = \frac{R}{\rho - \alpha} - \frac{C}{\rho}$$

The first condition (known as the value-matching condition) requires that, at the optimal exit threshold R_x, the value of the project with option must equal the salvage value for which it is exchanged. The second condition, which is germane to the finite difference approximation but not the minimization approach, simply requires that the value of the project when revenues are zero also be the salvage value. The third condition states that, at very high revenues, the expected value of the project with an exit option is the same as the expected value without the option: as revenues grow arbitrarily large, the exit option has no value. We assume here that *S* is not a function of *R*. Solution of the system (2-3) yields the value of the active firm *F* and also the value of R_x, the revenue level below which an expected-profit-maximizing fisherman would optimally exit.

The model presented here treats salvage value as deterministic, though it may be unknown to the analyst. This assumption simplifies the model, and is in many circumstances quite appropriate (e.g., a fisherman's reservation value for selling a tradable permit), though we will later allow salvage value to be a stochastic process itself. If we assume that the exit option is perpetual, so that the derivative with respect to time is zero, the system (2-3) may be solved analytically, which yields a system of nonlinear equations that must in turn be solved for R_x. Solution of these nonlinear equations will require numerical methods in all but the simplest cases. Alternatively, we may directly approximate the system (2-3) using numerical techniques such as finite differences. We next demonstrate these two approaches, and compare their results and performance.

Solution by Nonlinear Least Squares

The model (1-3) is equivalent to an American put option, which in general does not have a closed-form solution. However, if we treat the exit option as perpetual, which is not too strong an assumption in many fisheries, we can obtain the following solution:

$$4)\quad F(R) = \frac{R}{\rho-\alpha} - \frac{C}{\rho} + AR^{\beta}$$

where A is a constant yet to be identified and β is a function of the parameters α, σ, and ρ (see [1] for a complete derivation of the corresponding call option). Using this result, conditions (3b-3c) lead to following systems of two nonlinear equations in two unknowns (R_x, A_x):

$$5)\quad \frac{R_x}{\rho-\alpha} - \frac{C}{\rho} + A_x {R_x}^{\beta} - S = 0$$

$$5')\quad \frac{1}{\rho-\alpha} + \beta A_x {R_x}^{\beta-1} = 0$$

A system this simple may be solved algebraically for the exact solution. In general, systems of nonlinear equations may be solved by relying on extensions of Newton's method or conversion to minimization problems [2, 3]. We adopt the latter approach, finding {R_x, A_x} that minimizes the sum of least squared errors:

$$6)\quad \{R_x, A_x\} = \min_{R,A}\,[\{\frac{R}{\rho-\alpha} - \frac{C}{\rho} + AR^{\beta} - S\}^2 + \{\frac{1}{\rho-\alpha} + \beta AR^{\beta-1}\}^2\,]$$

Solution of this simple problem is extremely fast and reliable. We present results of an application below, but first introduce an alternative method, finite difference approximation.

Solution by Finite Difference Approximation

Another method that may be used to solve for the exit threshold R_x is direct approximation of (2) by finite differences [4]. The real benefit of this approach is when the partial differential equation(s) such as (2) cannot be reduced to a system of nonlinear equations, but we apply the method here for comparative purposes.

The finite difference method proceeds by replacing differentials with differences and then solving over a grid of time and state variables (here, revenue) subject to the boundary conditions. The easiest way to do this is to derive the partial differential equation for the exit option value alone, which we designate G:

$$7)\quad \alpha R\frac{\partial G}{\partial R} + \frac{1}{2}\sigma^2 R^2 \frac{\partial^2 G}{\partial R^2} - \rho F_1 + \frac{\partial G}{\partial t} = 0$$

The following difference approximations are used:

$$\frac{\partial G}{\partial R} = \frac{G_{t,i+1} - G_{t,i-1}}{2\Delta R}$$
$$\frac{\partial^2 G}{\partial R^2} = \frac{G_{t,i+1} - 2G_{t,i} + G_{t,i-1}}{(\Delta R)^2}$$
$$\frac{\partial G}{\partial t} = \frac{G_{t+1,i} - G_{t,i}}{\Delta t}$$

where *t* and *i* are indices of the time and revenue grid spacing, respectively. Substituting these difference approximations into (7) and rearranging yields a linear equation implicitly relating the values of the exit option G_t to its value at G_{t+1}:

8) $$aG_{t,i-1} + bG_{t,i} + cG_{t,i+1} = G_{t+1,i}$$
where
$$a = -\tfrac{1}{2}dt(-\alpha R + \sigma^2 R^2)$$
$$b = 1 + dt(\sigma^2 R^2 + \rho)$$
$$c = -\tfrac{1}{2}dt(\alpha R + \sigma^2 R^2)$$

There is such an equation describing the value of the option at each point on the grid. This system of equations, subject to boundary conditions analogous to (3), yields a numeric approximation to the function *G* and also the revenue value R_x at which the exit option is optimally exercised.

An Application and Comparison of Methods

Here we present results of the simple exit model, solved by both of the above methods for comparative purposes. Our example is based on data from the California salmon fishery. The data used were fleet average values for revenues and cost proxies (operating and license costs). Maximum likelihood estimates of the parameters α and σ were derived from the revenue series, and a real discount rate of 5% (continuously compounded) was assumed. We compared two putative real salvage values, $10,000 and $40,000. While these values are very low relative to those that would be expected in a heavily-capitalized fishery, most boats in the California salmon fishery are small and employ simple technologies. Further, because the fishery for which we are defining the exit option is only open from May to September, the relevant salvage value is what fishermen who leave the salmon fishery could obtain in other fisheries (or other employment) during the summer months. Based on our analysis of the catch of salmon fishermen in alternative fisheries (primarily albacore tuna) in years when they do *not* catch salmon, we believe that these two salvage values are reasonable for this fishery.

Table 1 shows the parameter estimates used, the solution by nonlinear least squares, and the solution by finite differences. To approximate the process of information arrival and expectations updating, the parameters were generated by maximum likelihood estimation using rolling 7-year data sets, e.g., the parameter estimates for 1988 were derived from data for 1982 to 1988. Because the discounted sum of future revenues is not bounded when α exceeds the discount rate, the thresholds in this case are set to zero. We have not included the exact solutions because in every case they were within $10 of the minimization results, i.e. for all practical purposes there was no meaningful difference between the nonlinear least squares and exact solutions.

Table 1: Optimal exit thresholds by year
(in $000s of real dollars, base year 2000)

			Nonlinear Least Squares		Finite Differences	
	α	σ	S=10	S=40	S=10	S=40
1988	0.18	0.67	0.00	0.00	0.00	0.00
1989	-0.02	0.83	1.07	1.30	1.30	1.50
1990	0.16	0.63	0.00	0.00	0.00	0.00
1991	0.04	0.58	0.27	0.34	0.40	0.40
1992	-0.06	0.56	2.02	2.57	2.20	2.80
1993	-0.06	0.56	2.26	2.83	2.50	3.10
1994	-0.10	0.51	2.94	3.67	3.20	4.00
1995	-0.10	0.52	2.95	3.68	3.20	4.00
1996	-0.01	0.32	2.24	2.88	2.40	3.00
1997	0.04	0.35	0.53	0.67	0.60	0.80
1998	-0.03	0.43	2.08	2.69	2.20	2.90
1999	0.12	0.54	0.00	0.00	0.00	0.00
2000	0.13	0.54	0.00	0.00	0.00	0.00
2001	-0.01	0.61	0.86	1.17	1.00	1.30
2002	-0.01	0.61	1.01	1.33	1.20	1.50

As Table 1 shows, the estimated exit threshold varies widely from year to year as parameter values change. The average difference in threshold values for S=10 and S=40 is 27% for the nonlinear least squares results and 25% for the finite difference results. The finite difference estimates of the exit threshold are consistently higher than those from the nonlinear least squares. This overestimation is partly due to the use of a finite-horizon (100-year) grid to approximate an infinite-horizon problem and partly due to approximation error. Both of these problems can be mitigated (by increasing the time horizon and by making the grid finer), but at the cost of increased computation time.

The nonlinear least squares approach is clearly preferable to the finite difference approach for the simple example given above. However, in more complicated models the partial differential equations corresponding to (2) above will often not have analytical solutions. For example, different specifications of the stochastic processes for state variables may preclude analytical solution, as may the presence of multiple state variables.

Iterative Nonlinear Least Squares for Harder Problems

In some cases, such as when a fisherman can choose to move back and forth among states (e.g., active to idle and back) by paying fixed costs for each move, the choice problem may be representable as a more extensive system of nonlinear equations. In the California salmon fishery, fishermen must pay each year to maintain their limited-entry permit, whether they fish or not: once the permit has lapsed, it cannot be renewed. Thus a fisherman with a permit can be active, receiving a stream of revenues from salmon; be idle but maintain the permit, in which case he pays a permit price but has neither operating costs nor revenues; or exit the fishery, in which case he saves the expense of the license fee but can never again take part in the salmon fishery. The value of being in each of these states is:

9) $V_A = \dfrac{R}{\rho - \alpha} - \dfrac{C}{\rho} - \dfrac{L}{\rho} + K_I R^{\beta_1}$

10) $V_I = K_A R^{\beta_2} + K_X R^{\beta_1} - \dfrac{L}{\rho}$

11) $V_X = 0$

where V_A is the value of the active boat with an option to suspend salmon fishing, V_I is the value of an idle boat with the options to either resume salmon fishing or to exit the fishery permanently, and V_X is the value of the salmon fishing enterprise once the boat has exited, i.e. zero. Our goal is to identify the thresholds R_I, R_A, and R_X, at which (respectively) the fisherman suspends fishing, resumes fishing, and exits the fishery for good. To accomplish this, we invoke value-matching and smooth-pasting conditions as follows:

12) $V_I(R_I) + S - T_I = V_A(R_I)$

12') $V_I^{'}(R_I) = V_A^{'}(R_I)$

13) $V_A(R_A) - S - T_A = V_I(R_A)$

13') $V_A^{'}(R_A) = V_I^{'}(R_A)$

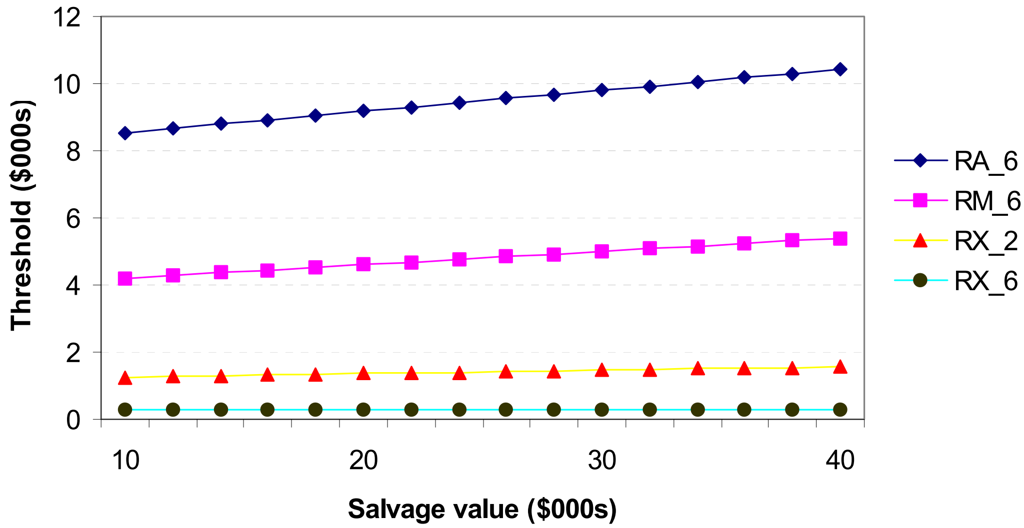
14) $V_X(R_X) - S_X = V_I(R_X)$

14') $V_X^{'}(R_X) = V_I^{'}(R_X)$

Here S represents the value of the alternative fishery, as before, T_I is the transition cost of moving from salmon into the other fishery, and T_A is the transition cost of moving from the alternative fishery back into salmon. At R_I, the idling threshold, the value of being active in the fishery is just equal to the value of being idle in the salmon fishery, plus the value of the alternative fishery, less the cost of making the transition to the alternative fishery. In our case, this transition cost is small, essentially a matter of a day spent in port while changing gear. At R_A, the resumption threshold, the value of being idle in the salmon fishery must equal the value of the active project less the foregone alternative fishery and the transition cost of re-entering the salmon fishery (again, a matter of changing gear). At R_X, the value of being idle is just equal to the value of exiting, which is to say the options held by the idle boat are exactly offset by cost of the salmon permit purchase. Notice that both the idle and the exited boat have access to the alternative fishery.

Making substitutions from the system (9-11) into the system (12-14) yields a system of six nonlinear equations in six unknowns. While this system can be solved efficiently given good starting values, it is often quite difficult to find those starting values, without which the solver can converge to incorrect solutions (including high residual values and imaginary numbers). We have found that, rather than solve for a few select salvage values (our initial strategy), it is much faster to solve for a large number of salvage values by creating a grid of salvage values and using the solution to (12-14) at a given S as the starting value set for the solver at the next increment on the salvage value grid. Figure 1 shows the result of applying this technique to both the simple exit model and the model with an idle option. The lines represent, from top to bottom, the resumption threshold R_{A_6}, the idling threshold R_{M_6}, the exit threshold from the simple exit model of the previous section R_{A_2}, and finally the exit threshold from the model with the option to suspend operations R_{X_6}. As intuition would suggest, the exit threshold is much lower in the model that includes this option.

Figure 1: Action Thresholds Compared



STOCHASTIC SALVAGE VALUE

In some contexts, it may be more appropriate to think of salvage value itself as following a stochastic process. For example, the sale price of boats is generally linked to trends and fluctuations in local fish price and catch. Here we follow a simple approach to this problem, suggested by [5], in which salvage value is treated as a numeraire. While this simplification is not appropriate in all circumstances, it will allow us to examine the implications of introducing stochastic salvage value without facing the numerical challenges raised by a more general specification in which the partial differential equation has multiple state variables. Because it makes more sense to normalize project value by salvage value than to normalize revenues by salvage value, we drop discussion of revenues and costs, and proceed with a single variable *V*, i.e, the expected capitalized value of profits from salmon fishing.

We let the value of the fishing enterprise and salvage value follow lognormal diffusions:

$$15a)\quad dV = \alpha_V V dt + \sigma_V V dz$$

$$15b)\quad dS = \alpha_S S dt + \sigma_S S dz$$

Following [5], we define a variable *Y* = *V/S* and let *H* be the value of the exit option normalized by salvage value (in the notation of the previous section, *H(Y,t)* = *G(V,S,t)/S)*. The partial differential equation governing the value of the exit option is then

$$16)\quad \frac{1}{2}\sigma_Y^2 Y^2 \frac{\partial^2 H(Y,t)}{\partial Y^2} + (\delta_S - \delta_V) Y \frac{\partial H(Y,t)}{\partial Y} + -\delta_S H(Y,t) + \frac{\partial H(Y,t)}{\partial t} = 0$$

where σ_Y^2 is the variance of the normalized variable, δ_S is the percentage by which the disount rate exceeds the drift rate of salvage value, and δ_V is the percentage by which the disount rate exceeds the drift rate of project value. We maintain the δ terms here only so that the reader may compare the exposition to that in [5]; the significance of the term $(\delta_S - \delta_V)$ is simply the percentage by which the drift rate of project value exceeds that of salvage value.

The boundary conditions on (13) reflect the normalization as well:

17*a*) $H(Y_X, t) = 1 - Y_X$

17*b*) $H(0, t) = 1$

17*c*) $H(\infty, t) = 0$

The first condition requires that at the optimal exit threshold, the option value be worth its net exchange value. The second condition is simply the normalized equivalent to condition (3b): even if the ratio *Y* falls to zero, the option *H* is still worth whatever can be had by exercising it. Since the abandonment option *G* at *V*=0 can be traded for *S*, the normalized abandonment option *H*=*G/S* can be traded for *S/S*=1. Finally, the third condition requires that the abandonment option become worthless as the ratio of project to salvage value becomes very large.

As before, if we assume the exit option is perpetual, we get an analytic solution, which in this case is

18) $H(Y) = AY^{\beta}$

which, with the boundary conditions, leads to the system of nonlinear equations

19) $A{Y_X}^{\beta} = 1 - Y_X$

19') $\beta A{Y_X}^{\beta-1} = -1$

Otherwise, we can again resort to a finite difference approximation, following the same steps as in the previous section. Because the exit thresholds thus identified are not directly comparable to those obtained under the deterministic salvage value scenario, we don't report results for all years. Instead, Tables 2 and 3 demonstrate the effect of varying the degree of correlation between project value and salvage value and of varying the volatility parameter for salvage, σ_S. The effect of increasing correlation between *V* and *S* is to increase the exit threshold. The effect of increasing the variance of salvage value is to decrease the exit threshold, except at very small values of σ_S, where the effect of correlation dominates the effect of σ_S.

Table 2: Optimal exit ratio Y_X, as a function of correlation(V,S)

Corr(V,S)	Nonlinear Least Squares	Finite Differences
0.00	0.21	0.26
0.25	0.25	0.28
0.50	0.30	0.34
0.75	0.37	0.40
1.00	0.50	0.54

σ_S = 0.25 for all values here.

Table 3: Optimal exit ratio Y_x as a function of σ_S

σ_S	Nonlinear Least Squares	Finite Differences
0.00	0.25	0.28
0.25	0.30	0.34
0.50	0.25	0.28
0.75	0.17	0.20
1.00	0.11	0.16

Correlation(V,S)=0.5 for all values here.

ASSESSMENT AND CONCLUSIONS

Above, we have demonstrated the significance of different assumptions about salvage value in the application of real options models to fisheries investment, and explored some of the numerical approaches that may be used in this context. To accomplish this, we have ignored several important complications, such as time-varying parameters, parameter uncertainty, multiple state variables (addressed in [6]), and alternative stochastic processes for state variables. Our general conclusions are as follows. Minimization techniques for systems of nonlinear equations solved the exit problem quickly and accurately, but may be difficult to apply to large systems of nonlinear equations. Finite difference methods did not perform as well, specifically they overestimated the exit thresholds and took much longer to generate results. However, they are a reasonable alternative for situations in which the partial differential equation has no explicit solution, as will often be the case. Finally, the reduction of a problem with stochastic salvage value to an equivalent single-factor model enables an exploration of the impact of stochastic salvage value on the estimated exit threshold, but in some cases it will be necessary to solve directly the model in which salvage value and project value both follow stochastic processes.

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ENDNOTES

1. More complete descriptions of similar models are available in [1]. [5] treats abandonment options generally, while [6] addresses fisheries exit with two state variables, price and catch.