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Title ENERGY DISSIPATION IN A TIDAL ESTUARY

Abstract approved W Bruce McAlister
(Major Professor)

The extent and mechanism of energy dissipation has been studied in Coos Bay, a coastal plain estuary. Coos Bay is located on the central Oregon coast, 184 nautical miles south of Astoria, Oregon, and 35 nautical miles north of Cape Blanco.

Past field work in Coos Bay and data obtained for this study are presented and summarized. These investigations include temperature, salinity, and current measurements, as well as tidal observations.

The circulation patterns observed in Coos Bay throughout the year have been analyzed, and Coos Bay has been classified as a well-mixed coastal plain estuary. The dynamics of the estuarine system have been investigated, and integrals of the equations of motion and continuity have been evaluated to show the magnitude of advection and diffusion in the estuary. The tide in Coos Bay has been shown to be a damped semi-diurnal wave.

Energy flow in the estuary has been followed from the energy source in the tides, through stages of turbulence, to final dissipation by viscosity into heat. The energy required to provide the observed mixing and increase of potential energy in the estuary has been observed and measured and compared with the source energy and energy lost through viscous dissipation.

A dissipation constant of 2.5 has been calculated for the incoming tidal wave as an index of the mean rate at which energy is extracted from the tidal motion into turbulent motion. The turbulent energy distribution has been represented as a function of eddy size. The turbulent energy spectrum has been obtained in Coos Bay from observed tidal velocities.

The amounts of energy involved in mixing and turbulent dissipation have been compared with each other and with the rate of tidal energy dissipation and viscous dissipation. Most of the tidal energy is lost directly into heat. The dissipation rate in Coos Bay was measured at 10^{-4} watts per kilogram. This value compares well with values determined by other investigators.

ENERGY DISSIPATION IN A TIDAL ESTUARY

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JACKSON ORIN BLANTON

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APPROVED:

W Bruce McAlister

Assistant Professor of Oceanography

In Charge of Major

Wayne V Burt

Chairman of Department of Oceanography

H P Hansen

Dean of Graduate School

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Typed by Rose Bethel

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LIST OF SYMBOLS

b........ width

c........ wave velocity

e........ 2.718

f........ Coriolis parameter ($2\ \omega \sin \lambda$)

g........ acceleration of gravity

h........ water depth

k........ $2\ \pi/L$

m........ mass

p........ pressure

s........ salinity or salt content

t........ time variable

u, v, w.. velocities in the x, y, z directions

A........ amplitude

A_o....... amplitude at $x = 0$

E........ energy

E_σ ...... energy per unit surface area

G........ rate of energy dissipation

G_m....... rate of energy dissipation per unit mass

J........ rate of potential energy gain per unit mass

K_x, K_y, K_z..components of the salt diffusivity

K_1....... diurnal solar-lunar tidal constituent

L........ wave length or eddy dimension

M_2....... semi-diurnal lunar tidal constituent

O_1....... diurnal lunar tidal constituent

P........ energy transport

S_2....... semi-diurnal solar tidal constituent

U........ amplitude of tidal velocity

U_f....... average fresh water velocity through the estuary

V........ volume

X, Y, Z.. components of the body forces

ϵ........ rate of energy dissipation per unit mass in eddies

η........ water surface elevation

η_H....... water surface elevation at high water

λ........ earth's latitude

μ........ damping modulus

ρ........ density

σ........ 2π/period

σt_H...... angular time of high water

τ_{xx}, τ_{xz}. components of turbulent stress

ϕ........ dissipation constant

ω .... the earth's angular velocity

$\overline{u}, \overline{h}, \overline{s}$, etc... denotes mean values

u', s', etc.... denotes turbulent values

<> .... denotes time averages

cm........ centimeters

kg........ kilograms

min....... minutes

rad....... radians

sec....... seconds

ENERGY DISSIPATION IN A TIDAL ESTUARY

INTRODUCTION

Geography of Coos Bay

This paper describes a study of the estuary of the Coos River on the central Oregon coast. Known as Coos Bay, it is located 184 nautical miles south of Astoria and the Columbia River and 35 nautical miles north of Cape Blanco.

Interest in coastal estuaries has mounted in recent years as conflicting interests, such as recreation and industry, have contended for their use. Lumbering is the largest industry in the region around Coos Bay. There are more than 30 sawmills in Coos County (37, p. 3), many of them located on the bay itself. The Coos Bay Pulp and Paper Company operates a large paper mill near the town of Empire, five miles up the estuary, which discharges waste effluent directly into Coos Bay. There is also a new mill approximately eight miles from the entrance which pipes its wastes to the ocean. There are four main centers of population on the bay. The towns of Charleston, Empire, North Bend and Coos Bay, populations of 900, 3781, 7512 and

7084, respectively (10, p. 18), discharge a mixed domestic and industrial sewage load in the estuary. Because of a large shipping tonnage of 3,821,000 tons per year (10, p. 149), the U.S. Army Corps of Engineers has conductes several harbor modification projects, and dredging is almost continuous.

Heavy use has been made of Coos Bay as a disposal source. As a consequence, the estuary's relatively large clam and oyster industries have ceased within the last decade. Sport fishing in the bay is now limited to fishing over the bar and deep-sea fishing.

The study and analysis of estuaries are of immediate interest and value to engineers, industrial managers, conservationists, and all others interested in the utilization of an estuary for any purpose. Queen (27, p. 1-16) conducted early studies in Coos Bay during 1930 and 1931. Further data were taken during 1955 and 1958 (4, p. 9; 5, p. 5-6). The State Sanitary Authority has performed several water quality studies in the past few years. The U.S. Public Health Service has sponsored a joint physical and biological study of Coos Bay. Dr. W. B. McAlister at Oregon State University has directed a physical oceanographic study, and Dr. J. A. Macnab of Portland State College has conducted a biological study from the Marine Biological Station at Charleston.

Estuarine Classifications

Estuaries have been variously defined. Pritchard (20, p. 245) has described an estuary as "a semi-enclosed coastal body of water having a free connection with the open sea and containing a measureable quantity of sea salt." This is a broad definition, and under this general definition, estuaries may be further separated into various classifications. Pritchard has proposed two main separations--positive and negative--based on fresh water addition and evaporation. When fresh water addition exceeds the evaporation, the estuary is called a positive estuary. On the other hand, a negative estuary is one in which the evaporation is greater than the addition of fresh water. Estuaries also have been classed according to their geomorphological structure (20, p. 244). The coastal plain estuary, the fiord estuary, and the bar-built estuary are the main structural divisions.

Coastal plain estuaries were once river valleys which have subsided and flooded. (Alternatively, one may consider that sea level has risen.) These are usually positive estuaries, although negative coastal plain estuaries may exist in regions of high evaporation. A positive coastal plain estuary may change to a negative estuary if stream flow is seasonally lowered or diverted sufficiently or if evaporation increases. Coastal plain estuaries have been studied extensively by Pritchard (20; 21; 22; 23; and 24) and Ketchum (12; 13; and 14).

Fiords are estuaries with deep, elongated basins. They may or may not have a sill at the mouth. Fiords are found along the Norwegian coast, the Canadian and Alaskan Pacific coast, and parts of New Zealand. They are associated with regions of glaciation, and the sill commonly represents a terminal moraine. Fiords are almost exclusively positive estuaries due to their location in areas of recent glaciation which also are climatic zones of high precipitation, although the connection is indirect. Oceanographic features of fiords in British Columbia and Alaska have been described by Pickard (19), Tully (30), and McAlister _et al._ (18).

Bar-built estuaries occur with the development of an offshore bar. These bars are normally deposited on shorelines having very small slopes; consequently, the enclosed bay is shallow. Between the shallow bay and the open sea, there is a narrow channel that depends upon tidal currents to keep it scoured (3, p. 3). Estuaries of this classification are found along the Texas Gulf coast and the coasts of Florida. Many bar-built estuaries are negative type estuaries containing water that is more saline than the adjacent sea, although positive bar-built estuaries also are found.

Numerous other schemes for classifying estuaries have been proposed; among these are salinity variations, ecology, and the physical causes of circulation and mixing, such as tidal action, wind or river flow.

The estuarine system investigated in this study is Coos Bay (Fig. 1), a coastal plain estuary of the positive type. For simplicity in the remainder of this paper, the term "estuary" will refer to a positive coastal plain estuary. Because of the rise in sea level since the late Pleistocene, the valley of the Coos River has been drowned, leaving the sloughs and marshes which are visible today (1, p. 34). The waters in the estuary are diluted by heavy

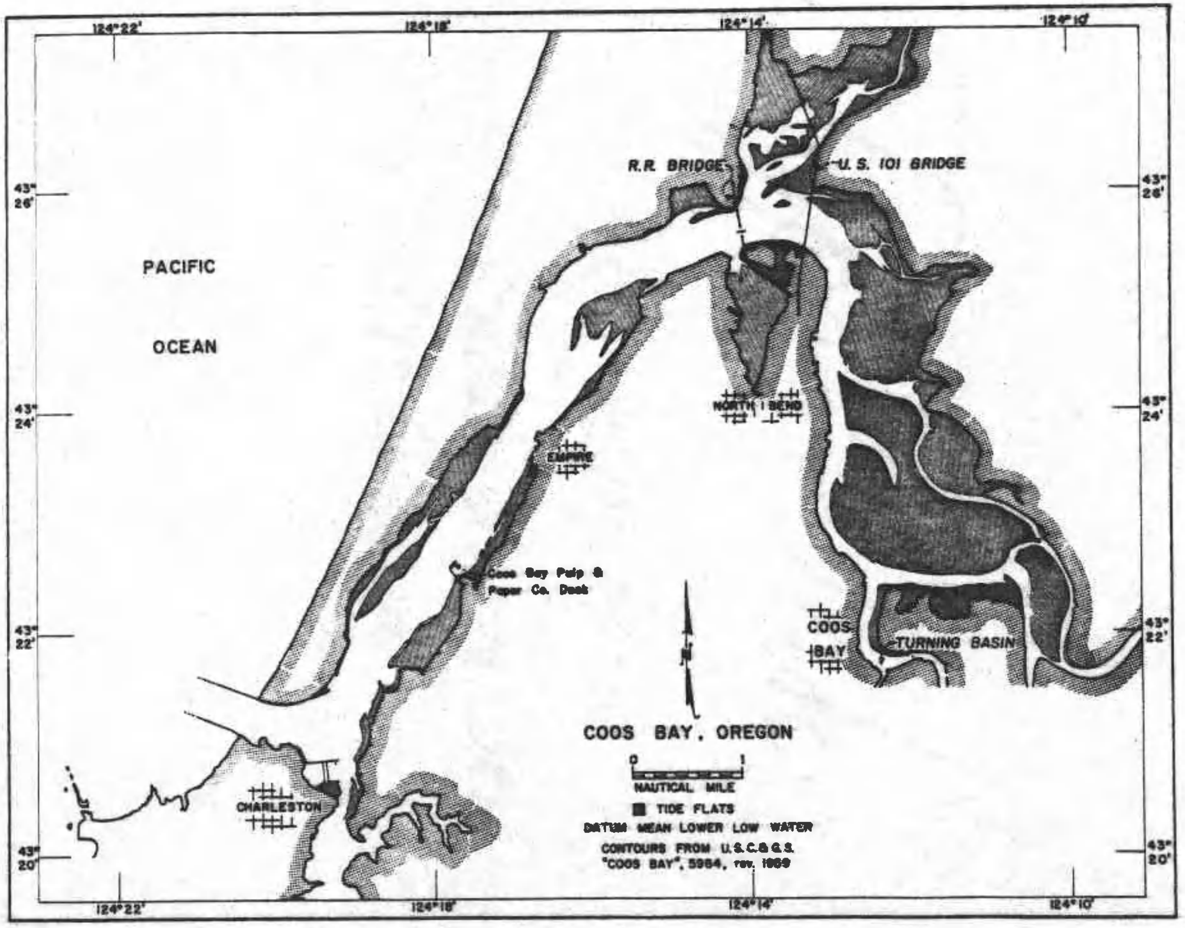


Figure 1. Map of Coos Bay showing location and extent of tide flats.

precipitation and runoff from the Coastal Mountain Range. In months of heavy rainfall during winter and spring, runoff into Coos Bay may reach 100,000 cubic feet per second from the Millicoma and Coos Rivers. This runoff may drop to around 100 cubic feet per second in the summer months (37, p. 239). This variation produces significant changes in the character and circulation within the estuary.

Scope of Problem

In estuary studies, it has been customary to work with the equations of motion and continuity to attempt to predict circulation and mixing patterns. These non-linear partial differential equations become tractable only by adopting a series of assumptions. Many of these assumptions are about processes which, at best, are poorly understood, especially those assumptions dealing with the nature of the turbulent processes. Since the motion and circulation observed bear a direct relationship to the energy supplied, this paper provides an analysis of the energy supplied to Coos Bay and the dissipation of this energy. The energy used to mix the waters of Coos Bay is primarily provided by the tides. Tidal ranges in excess of six feet are common, and they may reach ranges as great as nine or ten feet at certain times of the year (34, p. 172). Additional sources of energy due to winds and other meteorological phenomena have been neglected in this study by considering data only for periods of weak and variable winds.

CHARACTERISTICS OF A COASTAL PLAIN ESTUARY

Classification by Salinity Gradients

Pritchard (22, p. 1-11) has conveniently divided positive coastal plain estuaries into four types, depending upon the observed circulation and salinity distribution. Pritchard's classification of A, B, C and D estuaries has been widely accepted and can be used to describe the circulation in Coos Bay.

The type A estuary is a two-layered estuary. Fresh water forms the upper layer and the lower layer consists of a salty wedge of ocean water. Conditions may be nearly homogeneous within the respective layers. The estuary is vertically stratified between the two layers, but due to turbulence at the interface, a flux of saline water is assumed to occur from the salty wedge to the fresh upper layer. Although there may be some mixing of fresh water downward, there is no significant dilution of the salty wedge (Fig. 2). The upper layer becomes somewhat brackish as it moves seaward. The net flux is upwards, and this advective transport from the bottom to the upper layer is

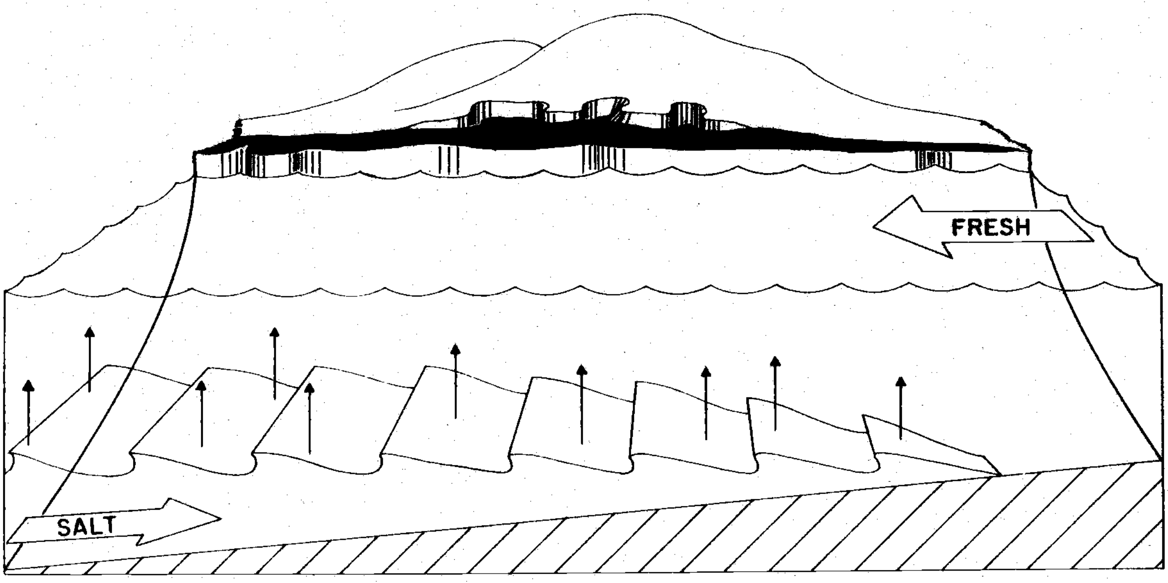


Figure 2. Circulation in a type A estuary.

Source: Pritchard (22, p. 10)

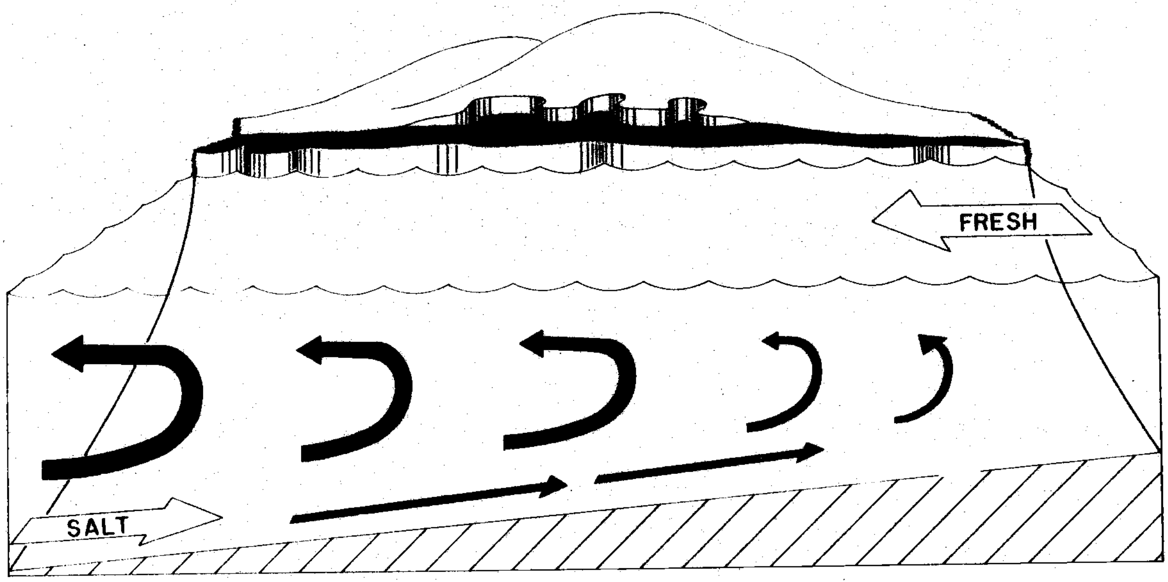


Figure 3. Circulation in a type B estuary.

Source: Pritchard (22, p. 10)

non-turbulent. (Turbulence involves an exchange, not a transport, of mass between the layers, and the resulting exchange of water mass may represent a transport of salt.) The salt advected into the upper layer becomes well mixed due to local turbulence within the upper layer. In order for a two-layer system to be maintained, the tidal action must be slight, the depth relatively large and the river flow large.

The type B estuary also is a vertically stratified estuary. At the same time, it is horizontally stratified since the salinity decreases with distance from the ocean at all depths. There may still be considered two layers defined by the net motion upstream and downstream (Fig. 3). Some of the more saline water in the lower layer is still advected into the upper layer as in the type A estuary. However, turbulent diffusion now accounts for a significant part of the salt transfer, and fresh water is exchanged downward. In place of the sharp boundary between the upper and lower layers, there is a halocline with a continuous salinity gradient from the lower to upper layer (31, p. 526). For the formation of type B estuaries, moderate to strong tides are important, together with strong river flow. Since tides are, in effect, long waves, tidal currents are constant with depth in the absence of any type of obstruction. These tidal velocities and associated eddies provide the energy source for turbulent mixing in the estuary.

The type C estuary has no vertical gradients; it is vertically homogeneous. In the absence of vertical gradients, there can be no vertical transport of salt. The longitudinal and lateral gradients remain. The salinity increases toward the ocean at all depths, and the estuary is wide enough for Coriolis effects to produce the lateral salinity gradients. The water on the side of lower salinity has a net movement downstream, and the high salinity water on the opposite side has a net movement upstream. Since there is no vertical salinity gradient, there is no vertical movement of salt either through turbulent diffusion or advective transport. There is horizontal advection and diffusion across the halocline separating the higher saline water from the lower saline water (Fig. 4).

The type D estuary is similar to type C, except that there is no lateral salinity gradient (Fig. 5). The only gradients are longitudinal gradients along the axis of the

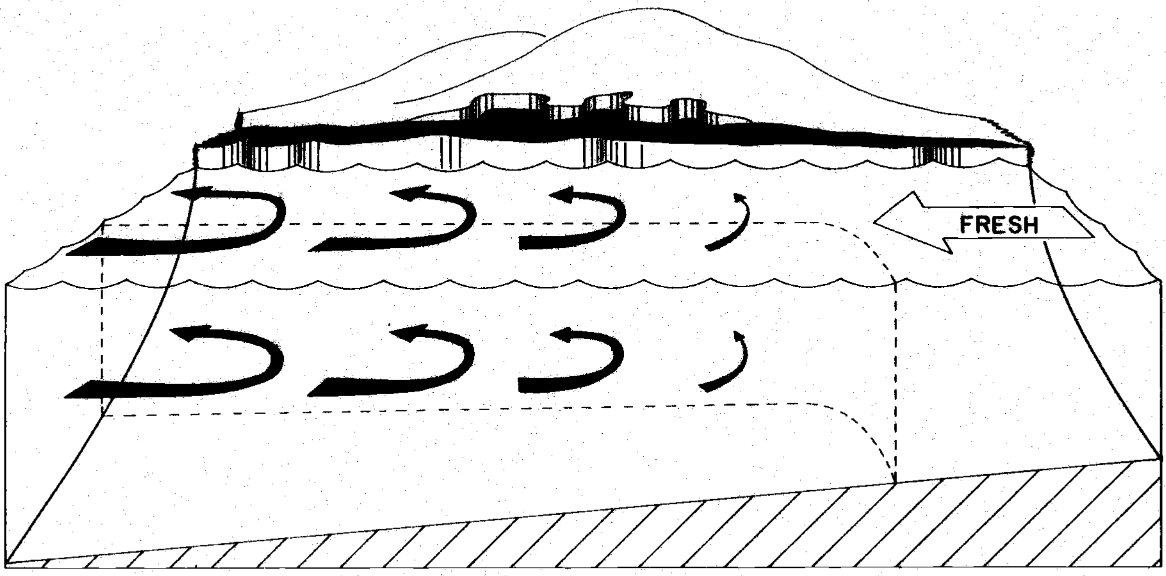


Figure 4. Circulation in a type C estuary.

Source: Pritchard (22, p. 11)

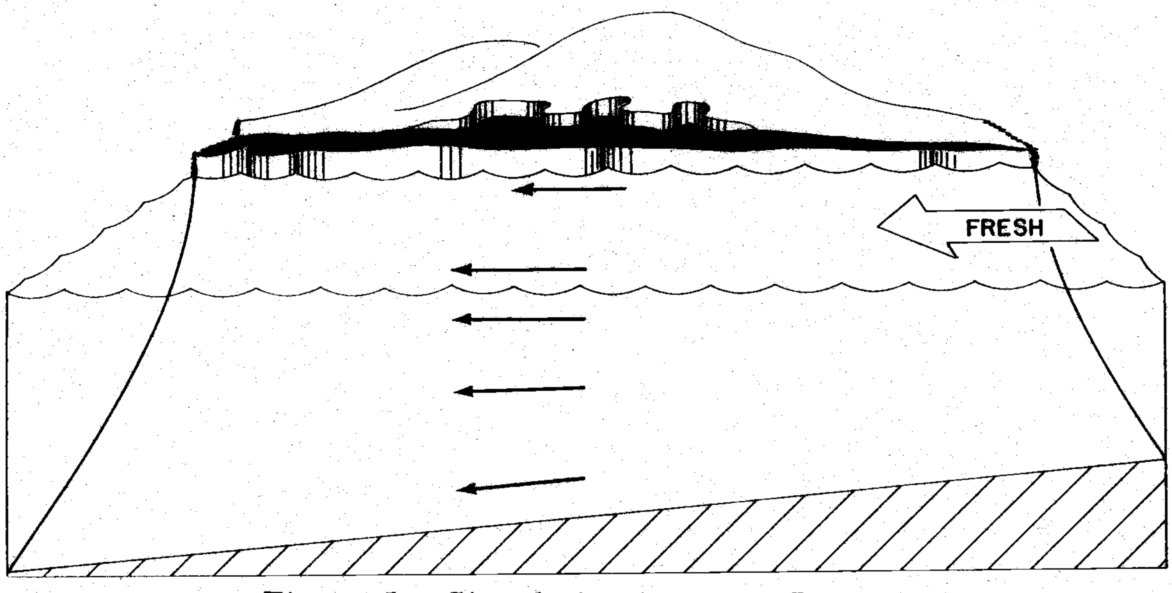


Figure 5. Circulation in a type D estuary.

Source: Pritchard (22, p. 11)

estuary. Estuaries of this type are sufficiently narrow that Coriolis effects are small, tidal currents are strong, and mixing is sufficient to obliterate any lateral salinity gradient. However, the estuary remains vertically homogeneous. Over a sufficient period of time, steady state conditions may be established in a type D estuary. Then there will be no net advection of salt upstream. The longitudinal salinity gradient will result in an upstream diffusion (turbulent diffusion) of salt. This must be balanced by the salt transport downstream associated with the net nontidal transport. This net movement must transport a mass of water downstream equal to the river flow and quantity of salt downstream equal to the upstream diffusion.

Several parameters are important in determining the "type" of an estuary. These are river flow, tidal velocities, width and depth (22, p. 6-7).

With variations in river flow and the other three parameters holding constant, a given estuary may assume any of the types discussed above. If river flow is large compared to tidal flow, one can expect a type A estuary. If the river flow decreases until it is smaller than the tidal flow, the estuary will be of type B. If the fresh water influx diminishes even further, the estuary will evolve to type C or D.

By varying the tidal velocities and assuming the other variables constant, we can again expect an estuary to evolve to any type. With no tides, the estuary should be of type A. With increasing tidal action, the estuary changes from type B to C or D as tidal velocities get very high.

A changing width, in effect, changes the ratio of tidal flow to river flow. A wide estuary increases the ratio by lowering the <u>velocity</u> related to river flow. Therefore, as width increases, one can expect an estuary to tend to less stratification and to go from type A to type C.

Depth also plays a role in determining the type of an estuary. An increase in depth results in the lowering of the effectiveness of tidal velocities to produce vertical mixing. This increasing depth also has the effect of confining the river flow to the upper layer. The estuary will tend to become more stratified, from type C to type A, as it deepens.

There are other physical parameters that may influence the characteristics of an estuary such as wind effects, air temperature, solar radiation and bottom roughness (22, p. 8).

Only two of these types of estuaries as classified above by Pritchard are found along the Oregon coast. These are types B and D. The two-layered estuary (type B) is found only seasonally when fresh water addition is sufficient to overcome the effects of tidal mixing. The salt-wedge, or two-layered estuary, occurs irregularly in the Columbia and Umpqua estuaries but is not found at any time in Coos Bay or other coastal estuaries (6, p. 18).

The partially mixed estuary is commonly found along the Oregon coast. It may be thought of as having characteristics intermediate between types B and D. In this type of estuary, there is a considerable amount of fresh water addition, but tidal action results in the partial mixing of the upper and lower layers.

During periods of low runoff, the tidal action may be great enough to change an estuary from the partially mixed to the well-mixed estuary (type D). None of the Oregon coastal estuaries are wide enough for significant lateral gradients to develop and be measured. Thus, type C estuaries with lateral gradients are not found on the Oregon coast.

The idealized salinity regimes of the partially mixed and the well-mixed estuary are shown in Figures 6 and 7. Coos Bay changes from a partially mixed to a well-mixed estuary. This has been previously indicated by Burt and McAlister (6, p. 22). However, Coos Bay appears to remain essentially a type D estuary (well-mixed) during most of the year. Only during the few winter and spring months of extremely high runoff does the estuary become somewhat stratified and, thus, partially mixed. This is the expected regime in view of the large tidal range of five to seven feet and the comparatively small runoff during most of the year. The estuary is also relatively shallow, which contributes to turbulence and mixing. It is sufficiently narrow that Coriolis effects are missing. The change from a well-mixed to a partially mixed system generally occurs during the months of March to June when there is heavy runoff from the Coast Range. Figures demonstrating seasonal changes in the estuary are presented later.

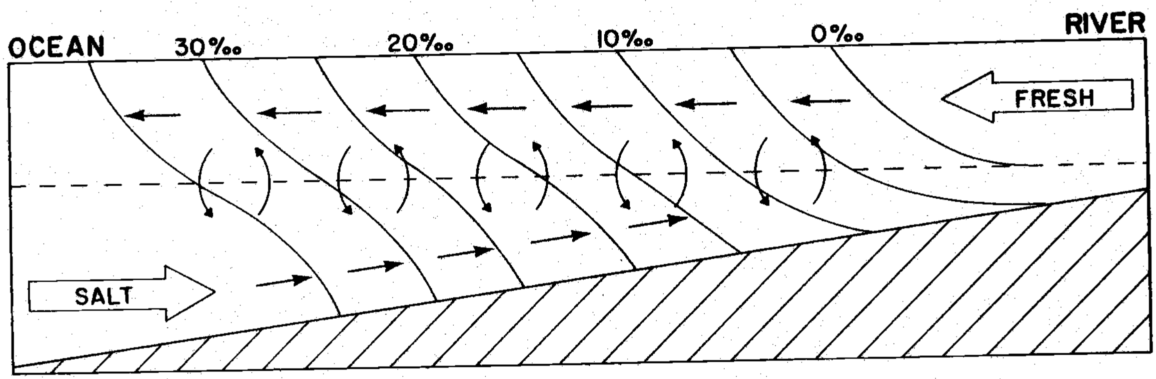


Figure 6. Salinity regime for a partially mixed estuary.

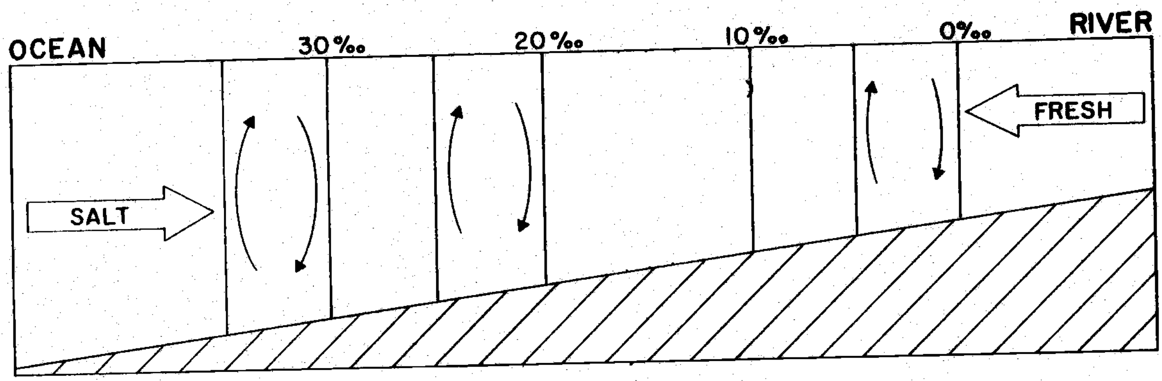


Figure 7. Salinity regime for a well-mixed estuary.

Tidal Effects

In order to mix waters of different densities, such as the waters in a two-layered estuary, energy must be supplied to these waters. The more energy supplied to an estuary, the better mixed is the estuary. By tabulating the energy supplied along the channel of the estuary, one should be able to classify the estuary by relating the energy supplied to the energy necessary to mix the estuary to varying degrees.

In general, the main source of energy for estuaries comes from the tides. Meteorological sources such as the winds are other possible sources, but, owing to the constancy of tidal action, these other sources are overshadowed under normal conditions. It is this incoming tidal energy that drives the circulation and mixing of the water. The energy per unit surface area of any tidal wave is

$$E\sigma = \tfrac{1}{2}\rho g A^2 .$$

In this equation and in the following equations, one should refer to the Table of Symbols where all mathematical symbols are defined. Any decrease in tidal range as the tide progresses up a uniform channel indicates a dissipation of energy; thus, there is less energy available to mix the water.

Specifically, tidal velocities enter into the equations of motion in the form of

$$u \frac{\partial u}{\partial x} = \frac{1}{2} \frac{\partial \overline{u}^2}{\partial x^2} .$$

(This will be shown in the discussion of the motion equations.) Over a tidal cycle, the average value of u is practically zero, but the average value of $\overline{u}^2$ is not zero since negative numbers squared have a positive value. Thus, any decrease in tidal velocities indicates a loss of energy from the mean motion to turbulent motion. This paper is an investigation of the transfer of tidal energy in a coastal estuary. The tides along the Oregon coast are mixed tides with a strong semi-diurnal component and a weaker diurnal component. A mixed tide has two high waters and two low waters as in any semi-diurnal tide, but there are strong inequalities in the ranges of the highs and lows in any 24-hour period.

The principal tidal harmonic constants for Coos Bay (33, p. 12) are (in feet) K_1= 1.15, O_1= 0.66, M_2= 2.43 and

$S_2 = 0.54$. Evaluating Defant's ratio (8, p. 306-307), it is found that

$$\frac{K_1 + O_1}{M_2 + S_2} = 0.61 .$$

This represents a mixed tide, mainly of the semi-diurnal type. The inequalities are at a maximum when the moon has passed its maximum declination. The mean spring tide range is $2(M_2 + S_2)$ or about six feet.

Equation of Motion

The equation of motion is simply a statement of Newton's second law of motion: F = ma where F and a are vector quantities of force and acceleration, respectively. Theoretically, this equation will give the accelerations imparted to a mass if the forces acting on the mass are known. From this, the circulation pattern of a body of water could be deduced.

The longitudinal equation of motion is written

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + \frac{1}{\rho}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right] + X$$

where the coordinate axes are located as shown in Figure 8. The velocities in the above equation represent instantaneous velocities.

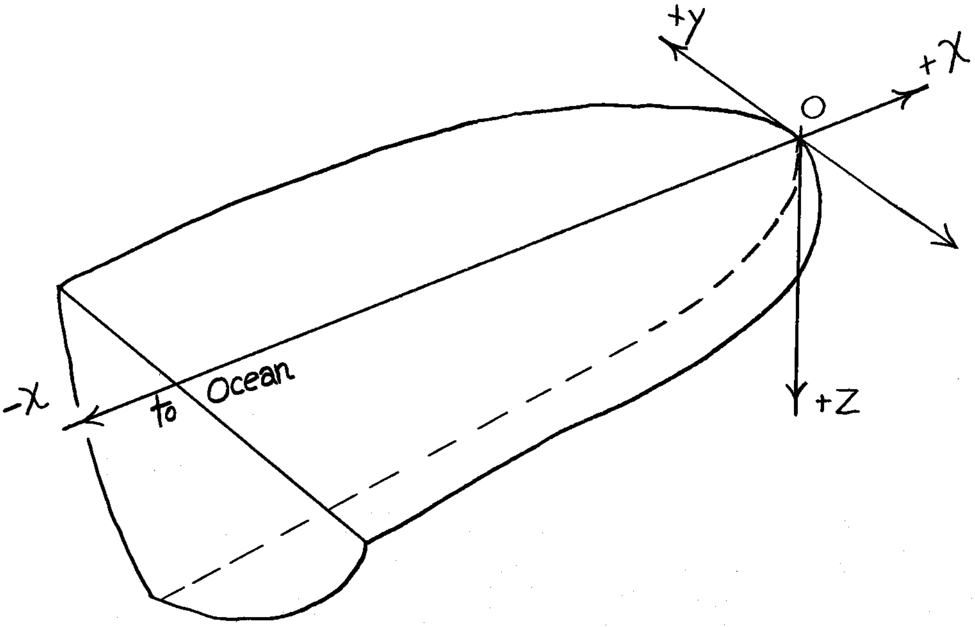


Figure 8. Location of coordinate axes.

In most cases, there are not enough synoptic data available to use in an instantaneous equation of motion. Therefore, the time mean of the equation is taken. The instantaneous velocity components are replaced by the sum of a mean non-tidal velocity and a deviation from the mean velocity and the tidal velocity. The instantaneous longitudinal component becomes

$$u = \bar{u} + u' + U.$$

As for the lateral velocity terms in an estuary, it is assumed that there is no mean velocity in the y-direction, but that there can be a turbulent term, $\underline{v'}$. Therefore, the instantaneous lateral term is replaced by

$$v = v'.$$

The vertical instantaneous velocity is replaced by the sum of the mean and turbulent velocities in the z-direction,

$$w = \bar{w} + w'.$$

In taking the time mean of the longitudinal equation, it is further assumed that steady state conditions exist or

Those terms in which the time averages of the products of $\underline{U}$ and $\underline{u'}$ or $\underline{v'}$ appear, say $\langle Uu' \rangle$, are also negligible. This assumption is made since there is no reason to expect any relationship between the tidal velocities $\underline{U}$ and the turbulent velocities. With these assumptions, the time mean equation of motion becomes

$$\bar{u}\,\frac{\partial \bar{u}}{\partial x} + \bar{w}\,\frac{\partial \bar{u}}{\partial z} + U\,\frac{\partial U}{\partial x} = -\left\langle \frac{1}{\rho}\,\frac{\partial p}{\partial x}\right\rangle - \frac{\partial}{\partial x}\langle u'u'\rangle - \frac{\partial}{\partial y}\langle u'v'\rangle$$

$$-\frac{\partial}{\partial z}\langle u'w'\rangle + X.$$

This equation may be further simplified for use in a coastal plain estuary. Pritchard (21, p. 138) has shown in the James River that the horizontal turbulent transport of salt is practically negligible when compared to the advective transport, and this also appears true for Coos Bay. If it is assumed that the horizontal turbulent transfer of momentum is analogous to the horizontal salt diffusion, horizontal variations such as $\frac{\partial}{\partial x}\langle u'u'\rangle$ and $\frac{\partial}{\partial y}\langle u'v'\rangle$ can be neglected when compared to advective momentum transfer terms in the equations of motion. However, the vertical variations of the turbulent terms must be kept in the equations.

The simplified longitudinal equation for a coastal plain estuary under these assumptions is

$$\bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{w}\frac{\partial\bar{u}}{\partial z} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho}\left\langle\frac{\partial p}{\partial x}\right\rangle - \frac{\partial}{\partial z}\langle u'w'\rangle.$$

This is essentially the same equation as obtained by Pritchard (23, p. 38) for the James River estuary.

By similar means, the lateral equation of mean motion can be obtained. In estuaries with curvature, centrifugal force must be considered as a body force, Y. The lateral equation, assuming steady state as before, becomes

$$\left\langle\frac{1}{\rho}\frac{\partial p}{\partial y}\right\rangle = f\bar{u} - Y$$

(18, p. 31). Given sufficient data to determine the lateral pressure gradient, one can test to see if centrifugal force plays a significant role. Since lateral effects have been neglected in Coos Bay, the lateral equation will not be considered further.

The vertical equation of motion becomes

$$\bar{u}\frac{\partial\bar{w}}{\partial x} + \bar{w}\frac{\partial\bar{w}}{\partial z} = -\left\langle\frac{1}{\rho}\frac{\partial p}{\partial z}\right\rangle + g.$$

where g represents the force per unit mass of gravity. Under most conditions, the inertial terms on the left side of the equation are negligible as compared to the pressure and gravity terms. The vertical equation then becomes simply the hydrostatic equation $0 = -\left\langle\frac{1}{\rho}\frac{\partial p}{\partial z}\right\rangle + g.$

Equation of Salt Continuity

Within any segment of an estuary, the change in salt content per unit of time can be determined by a consideration of the amount of salt that enters and leaves the segment in the x, y and z directions, respectively. Both turbulent and advective transport of salt is considered. A general equation for the salt balance, including mean and turbulent terms, is

$$\frac{\partial \bar{s}}{\partial t} + \bar{u}\frac{\partial \bar{s}}{\partial x} + \bar{v}\frac{\partial \bar{s}}{\partial y} + \bar{w}\frac{\partial \bar{s}}{\partial z} = \frac{\partial}{\partial x}\left\{Kx\frac{\partial \bar{s}}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{Ky\frac{\partial \bar{s}}{\partial y}\right\} + \frac{\partial}{\partial z}\left\{Kz\frac{\partial \bar{s}}{\partial z}\right\}$$

(26, p. 91). This equation can be simplified for the estuaries found along the Oregon coast.

In the two-layered estuary (Fig. 3), there is an advection of salt upwards through the halocline. Also, tidal currents introduce enough turbulence for there to be a turbulent exchange through the halocline. If it is assumed that the horizontal salinity gradient, $\frac{\partial \bar{s}}{\partial x}$, is fairly constant, then the second order term, $\frac{\partial}{\partial x}\left\{K_x\frac{\partial \bar{s}}{\partial x}\right\}$, is negligible when compared to the vertical turbulent term. For the two-layered estuary, the salt balance equation reduces to $\bar{u}\frac{\partial \bar{s}}{\partial x} + \bar{w}\frac{\partial \bar{s}}{\partial z} = \frac{\partial}{\partial z}\left\{K_z\frac{\partial \bar{s}}{\partial z}\right\}$ (22, p. 4).

In the partially mixed estuary (Fig. 6), increased vertical mixing tends to obscure the halocline more than in the two-layered system. In addition, due to the increased mixing, horizontal diffusion may be expected to account for a significant part of the salt exchange. A new term, therefore, must be added to the salt exchange equation. Thus,

$$\bar{u}\,\frac{\partial \bar{s}}{\partial x} + \bar{w}\,\frac{\partial \bar{s}}{\partial z} = \frac{\partial}{\partial z}\left\{K_z \frac{\partial \bar{s}}{\partial z}\right\} + \frac{\partial}{\partial x}\left\{K_x \frac{\partial \bar{s}}{\partial x}\right\}$$

where the added term represents the horizontal turbulent diffusion.

The well-mixed estuary (Fig. 7) is vertically homogeneous. Over a sufficient period, steady state conditions are established, and there will be no <u>net</u> advection of salt upstream. The longitudinal salinity gradient will result in an upstream turbulent diffusion of salt. This must be balanced by the salt transport downstream associated with the <u>net</u> non-tidal transport, which must transport a mass downstream equal to the fresh water addition and salt equal to the upstream transport. Under the above conditions, only two terms remain in the salt-balance equation:

$$\bar{u}\,\frac{\partial \bar{s}}{\partial x} = \frac{\partial}{\partial x}\left\{K_x\,\frac{\partial \bar{s}}{\partial x}\right\} \qquad (22,\ p.\ 5).$$

Energy Equation

An energy equation is simply the first integral of the momentum equation, or $\int m\ v)\,dv = \frac{1}{2}mv^2$. The equations of motion indicate the forces per unit mass acting on the water in an estuary. The rate at which these forces act is another way of describing the rate at which energy is transferred in the water. By multiplying the equation of motion by the velocity, the resulting expression has the units of energy rate or power. For the x-component of the equation of motion multiplied by u, the result is

$$\rho \frac{D(\frac{1}{2}\bar{u}^2)}{Dt} = - \frac{\partial(p\bar{u})}{\partial x} - \frac{\partial(\tau_{xx}\bar{u})}{\partial x} - \bar{u}\frac{\partial \tau_{xz}}{\partial z} .$$

This can be written as

$$\rho \frac{D(\frac{1}{2}\bar{u}^2)}{Dt} = - \frac{\partial(p\bar{u})}{\partial x} - \frac{\partial(\tau_{xx}\bar{u})}{\partial x} - \frac{\partial(\tau_{xz}\bar{u})}{\partial z} + \tau_{xz}\frac{\partial \bar{u}}{\partial z} .$$

The term on the left-hand side represents the rate at which energy is added to an estuary. As has already been said, this energy is added via the tides.

The energy added to an estuary may be used to increase the kinetic energy associated with the mean motion of the water. This is manifested in the tidal currents. The first three terms on the right side of the above equation indicate the rate of increase of kinetic energy.

Any energy that does not go toward increasing the kinetic energy may be used to mix the water, or the energy may be dissipated as heat. This energy used to mix the water is discussed in the section on energy flow. Let it suffice for now that the energy necessary to mix a water column must be extracted from the mean motion. It is the last term on the right-hand side of the energy equation that represents the rate at which energy is lost from the mean motion due to vertical turbulence (26, p. 99-101).

ENERGY FLOW

Energy is often defined as the ability to do work. Since work is required to mix and circulate the water masses within an estuary, energy must be supplied to the estuary. This energy is converted into several forms.

Incoming Tidal Energy

The primary source of energy supplied to an estuary is through tidal action. The tidal energy is later found in turbulent form in the estuary. The relationship of the estuarine tide to the ocean tide at the entrance has been investigated by Redfield (28, p. 5). The available energy at any section of an estuary is proportional to the square of the tidal height, or $E \propto A^2$.

As the tidal wave is damped by friction as it moves up the estuary, energy is withdrawn by turbulence and the tidal range decreases. The incident tidal wave moves up the estuary, undergoing partial reflection from the sand bars, tidal flats and bends in the channel, but the main body of the incident wave continues up the estuary and undergoes reflection at the head. The observed tides in an estuary are the combination of the primary and reflected tidal waves.

Tidal waves in estuaries may occur as either progressive waves or standing waves. A progressive wave is one in which maximum currents associated with the wave and the maximum height of the wave occur at the same time. The currents and heights are said to be in phase. In a standing wave, just the opposite effect is observed. At the time of maximum height, the currents are zero. The maximum currents occur midway between the crest and the trough of the wave. The heights and currents are 90^{o} out of phase in a standing wave. Standing waves are often seen at vertical barriers such as sea-walls and breakwaters. Therefore, these waves imply total reflection at a barrier as opposed to progressive waves which break and roll up on a sloping beach or barrier.

If the tide is a standing wave, the tidal currents will be at a maximum half-way between high and low water. In a progressive tidal wave, maximum tidal currents occur precisely at high and low water. As may be expected in the field, one rarely encounters tides in estuaries that are strictly either standing or progressive waves, but the observed tides usually fall intermediate between the two. A comparison of current and amplitude phases, together with phase changes along the estuary, enables one to estimate the extent of a standing and progressive wave at various locations throughout the estuary, and thus to estimate the strength of the incident and reflected waves plus their rate of attenuation. From the relative attenuation, the loss of energy from the mean tidal energy into estuarine turbulence may be calculated. Caldwell (7, p. 9) has found that the maximum currents at the entrance in Coos Bay precede high tide by 1 hour 28 minutes. This time lag was determined from a study of U.S. Coast and Geodetic Survey Tide Tables and Current Tables. Measurements by the Oregon State University Department of Oceanography have shown that the maximum currents at the U.S. 101 Bridge usually precede high tide between 1 hour 35 minutes and 1 hour 45 minutes.

A simple attenuation of height as the wave moves along an estuary would occur only in a uniform rectangular channel. In practice, this is never the case. The primary and incident waves are altered and their velocities are distorted by the irregularities in the channel's cross-section. The flow and tidal ranges affected by these irregularities can still be calculated by the equations of motion and continuity, and the wave velocities behave according to the long wave phase velocity

$$c = \sqrt{gh} .$$

These irregularities are, in part, responsible for the variations observed in the times and heights of high and low water in the different parts of the estuary. However, after accounting for changes to the shape of the water profile and flow along the channel, the attenuation of height is reflected by the phase change observed along the estuary. The greater the phase change along the estuary, the greater the withdrawal of energy into turbulence and the greater the damping of the wave. The phase change in degrees, kx, may be related to the tidal wave length, L, where

$$k = \frac{2\pi}{L} .$$

The wave length is a function of the mean channel depth, since the wave length changes with the phase velocity according to $\sqrt{gh}$.

The tidal wave, as it is propagated up the estuary, carries with it the energy associated with the range of the wave. Since the wave travels with a velocity, c, one can say that the energy is transported with the wave at this same velocity. The transport of energy across any section in an estuary is

$$P = E_{\sigma} cb$$

(11, p. 13), where E_{σ} is the energy per unit surface area, c is the phase velocity, and b is the channel width. Therefore, any slowing down of the tidal wave (a phase change) results in a loss of energy to turbulence.

The method in Ippen and Harleman (11, p. 7-28) was used to calculate the energy transported across four sections of Coos Bay. Since the tidal wave is a long wave, total reflection of the primary wave was assumed (36, p. 62). This results in apparent high damping in the last shoreward section. This is expected because the large amount of marshes and shallow tidal flats and erratically shaped bays and beaches in the upper reaches of the estuary (Fig. 1) would encourage the formation of turbulence and the dissipation of the tidal energy.

Turbulent Dissipation Rate

The energy extracted from the tidal energy manifests itself first in the form of large and small eddies. There is a hierarchy of eddies responsible for the transfer of kinetic energy down from the largest eddies to the smallest ones. The end result is the final dissipation of energy into heat by the action of viscosity in the very smallest eddies (16, p. 116-120). The following poem attributed to L. F. Richardson illustrates the concept:

> "Big whirls have little whirls
> which feed upon their velocity,
> Little whirls have smaller whirls
> and so on to viscosity."

Tidal energy is the prime source of the energy of the entire spectrum of eddies. Since the large eddies furnish the energy for the smaller ones (16, p. 119), energy must be continuously supplied to the large eddies in order to maintain the turbulence. If one assumes a steady state, the mean rate at which energy is supplied to the large eddies equals the rate of dissipation by the smallest eddies. The rate of dissipation is proportional to

$$\epsilon \propto \frac{\Delta u^3}{L}$$

(16, p. 119), where $\underline{\epsilon}$ is the rate of energy dissipation in ergs/(gram·sec), $\underline{\Delta u}$ is the order of magnitude of the velocity variation, and $\underline{L}$ is the eddy size or the distance over which the velocity varies. When dealing with tidal energy, however, the rate at which energy is supplied to the large eddies varies periodically. Thus, steady state does not exist and there may not be an equality between the energy supplied to the large eddies and the rate of dissipation by the small eddies. Strictly speaking, it is thought that there is no loss of energy from the energy flow between the large and small eddies. This energy is only lost as heat at the final stage of turbulent flow. This energy, however, is derived from the largest of the eddies and flows downward through the chain of continuously decreasing eddies.

Eddy sizes are defined by the distance over which velocity fluctuations take place. It is reasonably supposed that the largest of any series of eddies is restricted by the width of the channel. These eddies, of course, are superimposed on the mean velocity.

Energy Spectra

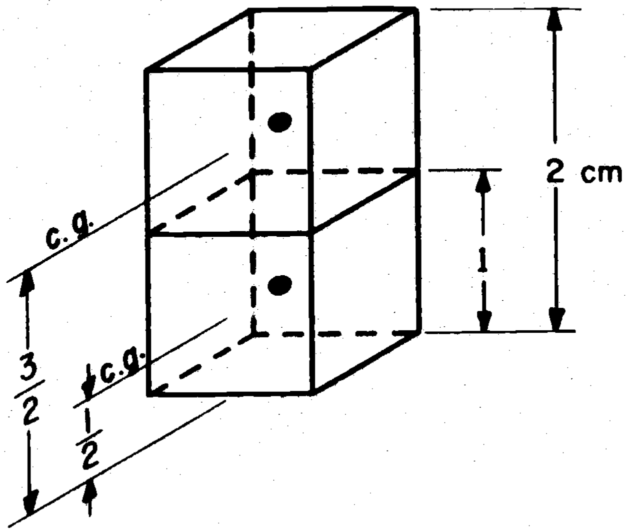
A meaningful way of showing the energy distribution throughout the range of eddy sizes is by means of an energy spectrum. A turbulence energy spectrum analogous to that used to illustrate the components of light or sound is a plot of the fraction of turbulent energy associated with the different frequencies or wave lengths. The spectrum of turbulence has been used by many (2; 9) to describe the distribution of energy, flow and dissipation in a turbulent velocity field. This method has also been used to describe the distribution of turbulent energy in Coos Bay.

Potential Energy Losses

Dissipation into heat is not the only form of energy loss in an estuary. The energy that is extracted from the tidal energy can also be used to mix the water in an estuary.

The processes that evolve in order to mix the layers in a two-layered estuary illustrate the concept of organized energy in the form of periodic tidal energy being converted into disorganized turbulent energy. The turbulent energy is sufficient to mix heavier (saltier) lower layer water with fresher, upper layer water, thereby increasing the potential energy of the mixture. That this is so is illustrated by Figure 9. Therefore, it requires energy to mix a water column, and this energy is extracted from the mean motion as said before (26, p. 99). The increased potential energy of the mixture is associated with further pressure gradients which provide the kinetic energy necessary to drive the <u>net</u> circulation of the estuary. All of these energies are withdrawn from the organized tidal energy. A schematic diagram of the energy flow (Fig. 10) summarizes the preceding discussion.

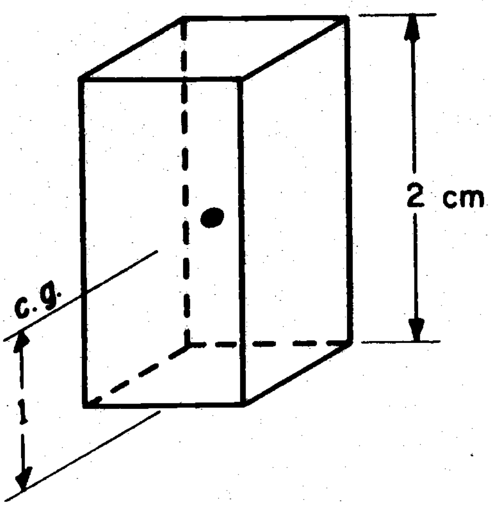
TWO-LAYERED COLUMN



ρ	V	h	g	PE
1	1	$\frac{3}{2}$	g	$\frac{3}{2}$g
3	1	$\frac{1}{2}$	g	$\frac{3}{2}$g

Total = 3g

WELL MIXED COLUMN



ρ	V	h	g	PE
2	2	1	g	4g

Total = 4g

[Potential energy (PE) = ρVhg]

Figure 9. A potential energy gain illustrated by the mixing of a two-layered column of fluid.

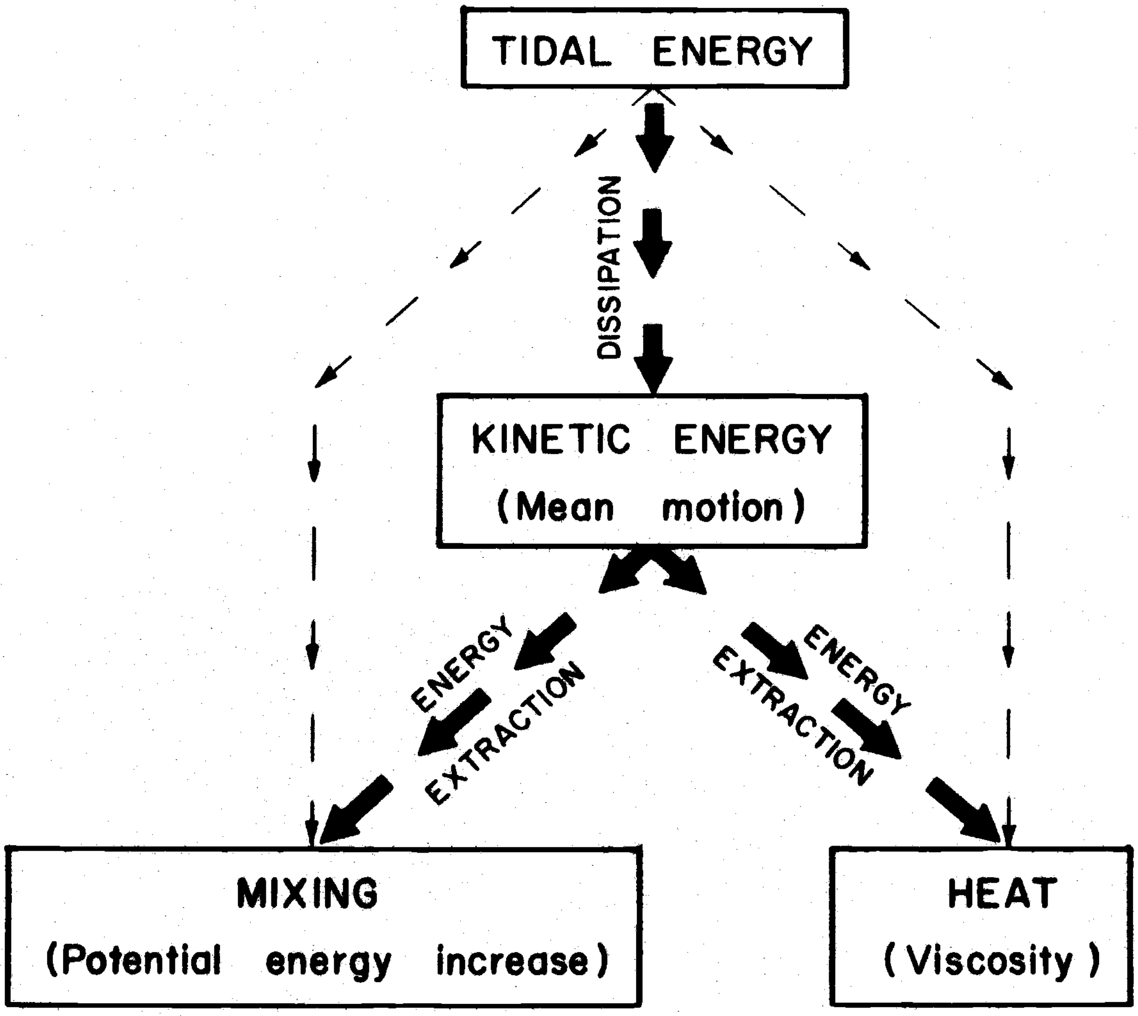


Figure 10. Schematic diagram of energy flow.

FIELD WORK

Field work in Coos Bay included measurements of tidal range and phase, currents, temperature and salinity profiles and dissolved oxygen, mostly since 1960. Measurements in Coos Bay prior to 1960 have also been used in this study.

Queen and Burt (27) list data taken by John Queen from 1930 to 1932 at 11 stations in Coos Bay. Measurements were made of temperature, salinity and currents at both the surface and bottom. Some data regarding tides, dissolved oxygen, H_2S and pH were included also. In additional data reports of the Oregon State University Department of Oceanography, Burt (4, p. 9) presents temperature and salinity figures for seven mid-channel stations for May 8 and September 7, 1955; and Burt and McAlister (5, p. 5-6) list temperature and salinity data at five-foot depth intervals for the dates of June 27, 1956; October 5, 1957; January 26, March 22, and June 27, 1958.

During the years 1960 to 1963, a series of 17 temperature, salinity and current measurements were made in Coos Bay as part of this study. The results have been presented separately as Data Report No. 10 (17). Sampling stations were located at the Coos Bay Pulp and Paper Dock, 4 miles from the ocean; beneath the U.S. 101 highway bridge, 10 miles from the ocean; and at the municipal dock at North Bend, 11½ miles from the ocean (Fig. 1). Occasionally, other stations along and across the channel were occupied.

When possible, simultaneous readings were taken at all three stations. Each station was occupied for one or more tidal cycles. Time intervals were different for the various stations, but in general, serial measurements were taken every one-half hour at the U.S. 101 bridge station and every hour at the other two regular stations. Temperatures were read to 0.1°C using a standardized bucket thermometer or a standardized temperature-conductivity indicator (CTI). Chlorinity and salinity were usually determined by titration. On several sampling days, salinity was determined at one station using the CTI. These were calibrated by additional *in situ* samples taken and titrated. Most current measurements were obtained with a current drag similar to that described by Pritchard and Burt

(25, p. 180-188). Data obtained before June 1961 were of surface currents and of currents two feet above the bottom. After this date, current measurements were taken at five-foot depth intervals. Some of the current data were obtained from Ekman and from Price meters. In March 1963, a series of current readings at intervals of one minute were taken with a cup-anemometer type current meter (Price meter). These currents were measured at the U.S. 101 bridge and at a depth of five feet below the surface. Instrumentation at the several stations is more fully described in Data Report No. 10 (17).

The data presented in Figure 11 were gathered in late September 1960 during a period of low fresh water addition of about 600 cubic feet per second. At this time, there were no vertical salinity gradients, and the values are oceanic throughout the estuary because of low runoff. This structure illustrates a well-mixed estuary.

Figure 12 shows the temperature and salinity structure during high runoff. The data were taken during December 1960 when the rate of fresh water addition had increased to an estimated 75,000 cubic feet per second. The salinity varied at a selected location from around

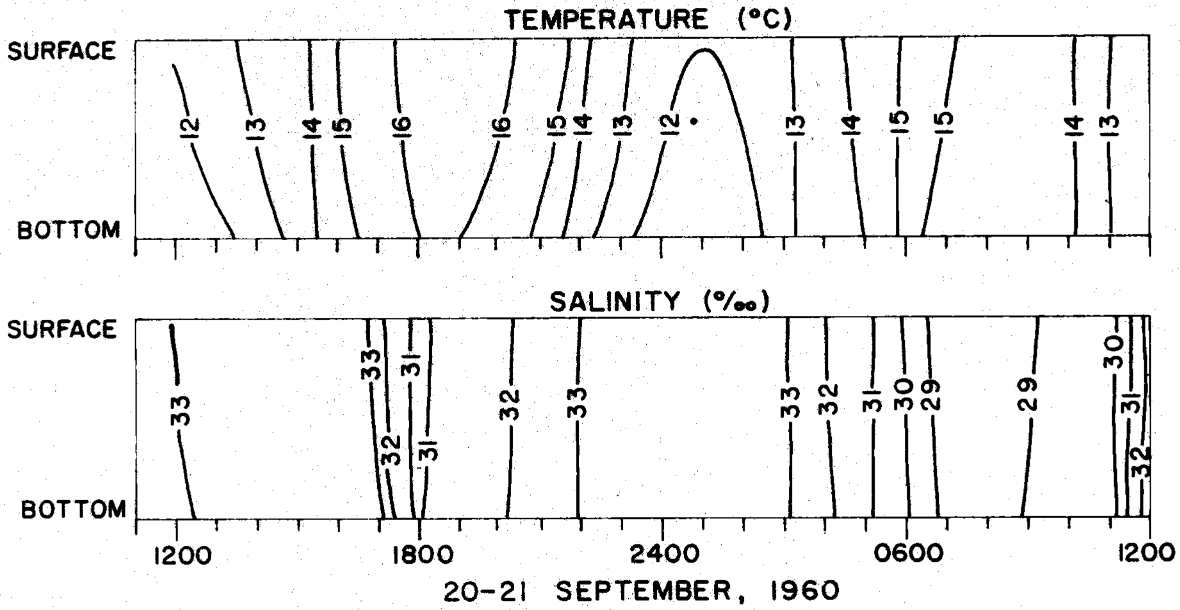


Figure 11. Well mixed conditions at the U.S. 101 bridge.

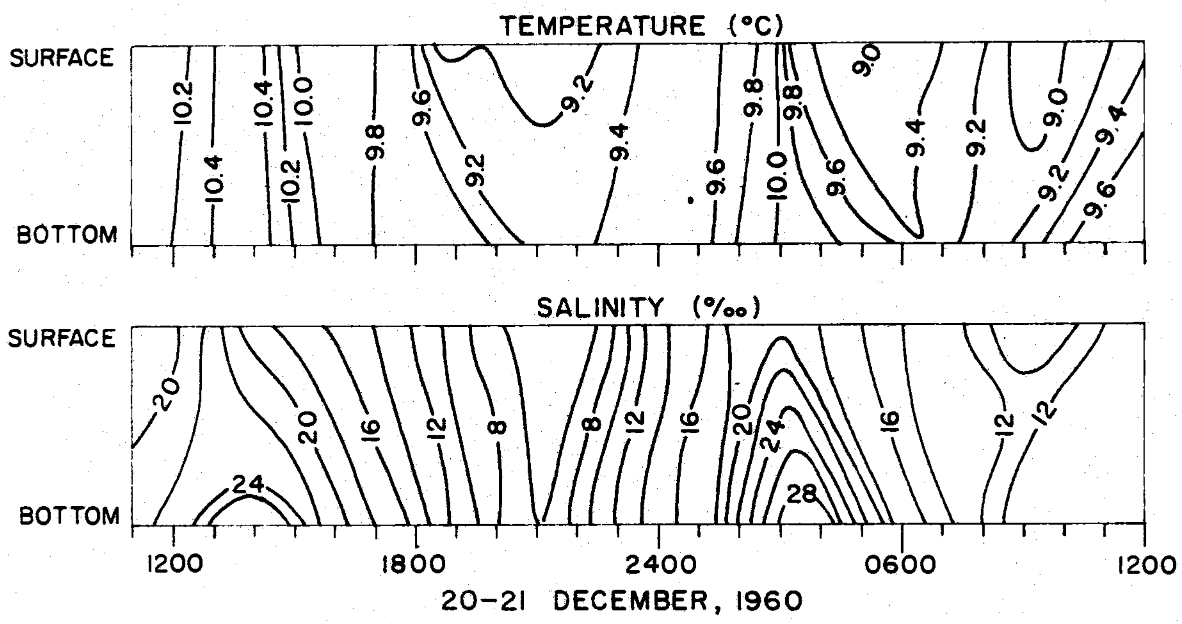


Figure 12. Partially mixed conditions at the U.S. 101 bridge.

five parts per thousand at low tide to 20-29 parts per thousand at high tide. The low salinity values at low tide illustrate the effect of fresh water dilution. At this time, Coos Bay was a partially mixed estuary.

At times intermediate between high and low runoff, the estuary may be only partially mixed at the head but well mixed at the mouth. During minimum runoff, however, Coos Bay is well mixed throughout.

A literature survey of Coos Bay was prepared by the University of Washington Department of Oceanography in 1955. In the Appendix of this survey (37, p. 115-135), the results of a tidal current survey made under the direction of the U.S. Army Corps of Engineers in February 1932 (32, p. 1-27) are reproduced. The tidal range data and the mean flood and ebb currents from this survey are listed in Table 1. The literature survey discloses no additional sources of useful information on currents and estuarine surveys.

In a tide survey performed by the Oregon State University Department of Oceanography, July 3, 1963, tidal ranges in four sections of Coos Bay were observed for an eight-hour period. The stations were evenly spread along

Table 1. Summary of Tidal Current Survey, Coos Bay, Oregon February 4, 1932.

(U.S. Army Corps of Engineers (32))

Miles from ocean	Higher high water (ft)	Lower high water (ft)	Higher low water (ft)	Lower low water (ft)	Mean flood velocity (ft/sec)	Mean ebb velocity (ft/sec)
2.1	7.4	5.0	2.3	-2.5	1.49	1.72
3.1	6.8	6.2	2.6	0.5	1.28	1.35
4.0	6.6	5.8	3.6	0.6	1.10	1.11
4.4	7.7	5.8	2.7	-1.7	1.54	1.70
4.5	7.0	5.8	2.4	0.1	1.48	1.60
5.2	7.5	4.8	2.5	-0.9	1.53	1.80
5.8	8.4	6.8	3.5	0.4	1.57	1.66
6.2	7.4	5.0	3.3	0.2	1.06	1.34
6.6	7.0	5.0	2.6	1.8	1.00	1.06
7.0	7.3	6.2	1.7	-1.0	1.60	1.72
7.3	6.8	4.8	2.5	1.8	0.97	0.87
7.8	7.0	4.7	1.5	-1.5	1.16	1.28
8.2	6.7	5.3	3.0	1.5	1.02	0.70
8.5	7.6	7.0	3.4	0.5	0.94	0.98
8.9	5.9	5.7	2.8	-0.3	1.38	1.13
9.2	7.8	7.1	2.2	0.4	1.46	1.33
9.7	8.8	6.8	3.2	-0.7	1.56	1.47
9.9	8.2	6.1	2.0	1.4	1.18	1.24
10.3	9.2	6.6	3.0	-0.2	1.38	1.41
10.9	7.7	5.7	1.1	1.0	1.10	1.07
11.5	7.6	6.8	2.5	1.3	1.11	0.77
12.5	7.3	6.5	3.5	-0.3	0.91	1.09
13.1	7.9	5.9	3.0	-1.0	1.19	1.39
13.8	7.5	5.7	2.3	-0.6	1.35	0.63
14.5	7.3	5.6	2.1	0.5	0.53	0.57
15.0	7.5	5.6	1.9	1.5	0.50	0.78

the estuary at Charleston, Empire, the railroad bridge, and the turning basin. Figure 13 presents a typical tide curve for Coos Bay. The results of the above survey are given in Figure 14. These results have been compared with the predicted tides in the U.S. Coast and Geodetic Survey Tide Tables and summarized in Table 2 below.

Table 2. Predicted Versus Observed Tides
July 3, 1963

	Entrance	Empire	Coos Bay
Predicted lag, minutes	0	39	88
Observed lag, minutes	0	21	52
Predicted range, feet	4.6'	4.3'	5.0'
Observed range, feet	5.15'	5.00'	6.35'

Streamflow data from the U.S. Geological Survey (35, p. 239) were examined in order to estimate the fresh water influx into Coos Bay. These estimates are rough at best since only one river in the entire Coos Bay drainage area, the West Fork of the Millicoma, is gaged. The gage is located just upstream from Allegany, Oregon.

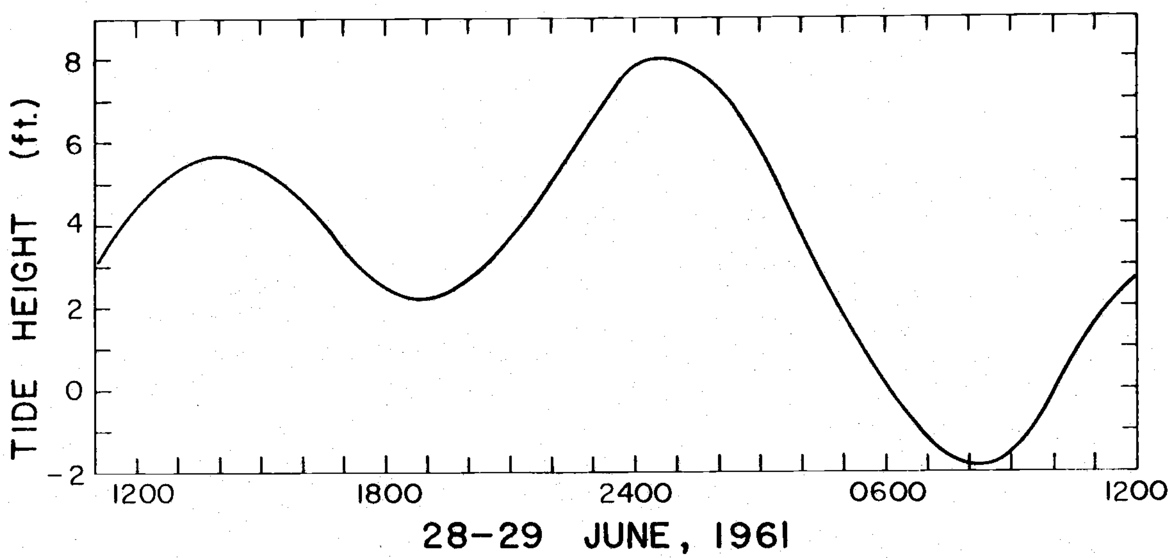


Figure 13. Tide curve at the U.S. 101 bridge.

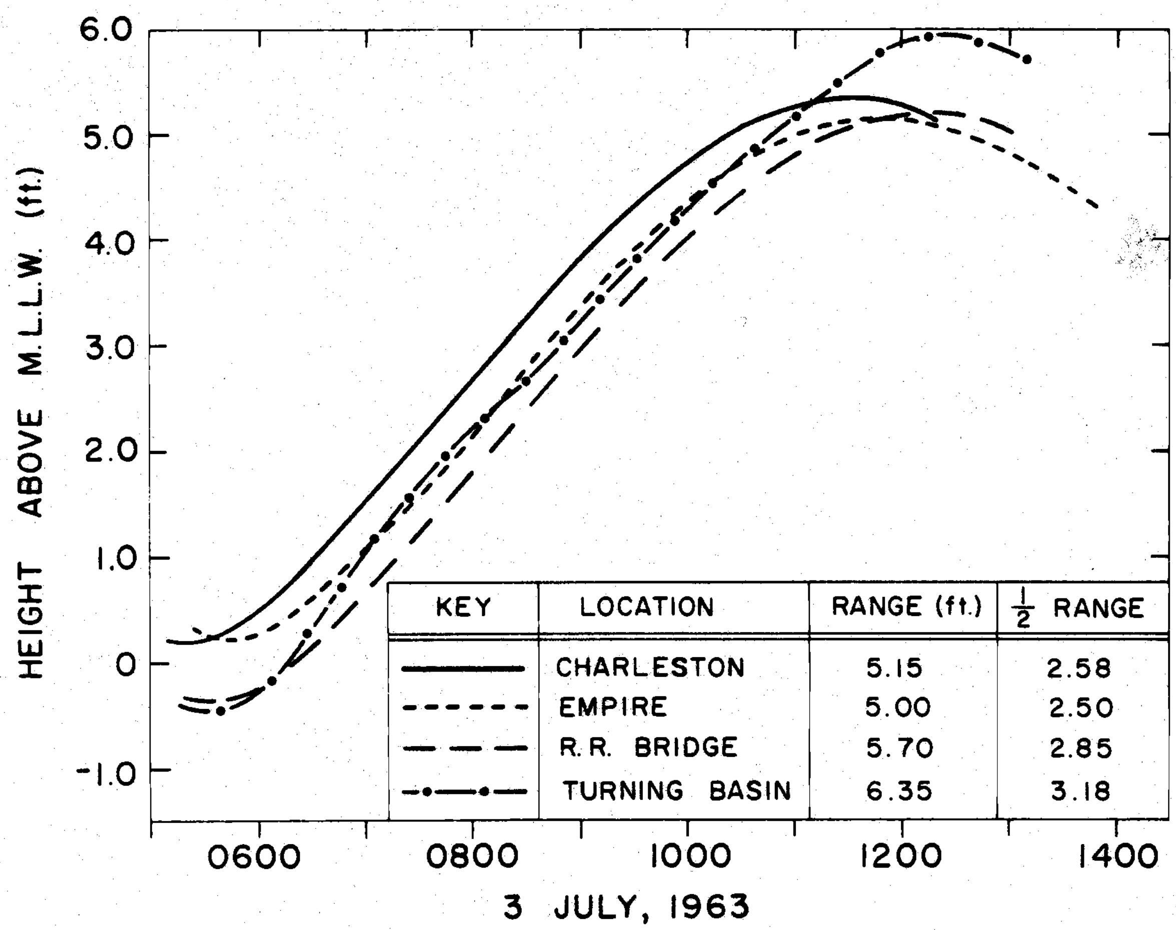


Figure 14. Tidal data for four locations in Coos Bay.

Runoff was expressed as a percentage of local rainfall:

$$\frac{\text{Runoff through gaging station}}{\text{Rainfall in Allegany region}} \times 100\%.$$

Percentages and averages were tabulated for the years 1955 to 1959. For this period, 83 percent of the rainfall became surface runoff.

From topographic maps, the drainage area into Coos Bay was determined. The area was divided into two sections. One was a lowlands area and runoff from this area was based on the rainfall at North Bend. The second was a highlands area with runoff based on the rainfall at Allegany. Ten inches were added to the recorded annual rainfall at Allegany, since it was thought that this figure, in part, would compensate for the mountainous region upstream from Allegany. The runoff (in acre-feet) was estimated as

$$\text{Runoff} = \text{rainfall (ft)} \times \text{surface area (acres)} \times 83\%.$$

Drainage into Coos Bay below the U.S. 101 bridge was neglected.

From mean flow rates in the river and in the bay, it was assumed that it would take 24 to 48 hours for the runoff at the Allegany gage to appear at the stations in Coos

Bay. An average was taken of the gage readings two days prior to the day being analyzed. Thus,

$$\text{Fresh water discharge} = (\text{Annual runoff}) \times \frac{\text{daily gage reading}}{\text{annual gage reading}}.$$

Table 3 summarizes the runoff figures obtained by this method.

Table 3. Fresh Water Runoff Under U.S. 101 Bridge

Date	Runoff feet^3/sec
21-22 June 1960	3950
12-13 July 1960	1450
16-17 Aug. 1960	525
20-21 Sept. 1960	635
22-23 Oct. 1960	1030
20-21 Dec. 1960	73500
28-29 Jan. 1961	9020
25-26 Mar. 1961	52250
20-21 May 1961	12000
28-29 June 1961	1630
20-21 July 1961	900
2-3 Aug. 1961	650

PROCEDURE AND RESULTS

In order to obtain numerical results concerning tidal energy and its dissipation in Coos Bay, tidal equations were needed that would describe the tidal elevation at any location in the estuary. These equations should include terms that will relate the damping of the tidal waves to its location in the bay. A method similar to that used by Redfield (28, p. 8-11) was used to derive these equations. A damping coefficient for Coos Bay was determined by this method using values of tidal ranges and high-water times observed in different sections of the tidal channel. In the equations that follow, one should refer again to the List of Symbols preceding the INTRODUCTION for an explanation of the terms used and to Figure 8 for an orientation of coordinates.

Derivation of Tidal Equations

The elevations of the primary and secondary waves are given by the equation,

$$\eta_1 = A_o e^{-\mu x} \cos(\sigma t - kx)$$

$$\eta_2 = A_o e^{\mu x} \cos(\sigma t + kx) \quad .$$

The actual tidal elevation is the sum of incident and reflected waves, or, adding (1) and (2),

$$\eta = \eta_1 + \eta_2 = A_o\left[e^{-\mu x}\cos(\sigma t - kx) + e^{\mu x}\cos(\sigma t + kx)\right]. \quad (3)$$

The time of high or low water at a particular location within an estuary occurs when $\frac{\partial\eta}{\partial t} = 0$; thus,

$$\frac{\partial\eta}{\partial t} = e^{-\mu x}\sin(\sigma t - kx) + e^{\mu x}\sin(\sigma t + kx) = 0 \ . \quad (4)$$

After re-arranging the above equation and noting that

$$\frac{e^{\mu x} - e^{-\mu x}}{e^{\mu x} + e^{-\mu x}} = \tanh \mu x,$$

the relative time of high water, σt_H, may be related to the phase change, <u>kx</u>, and the damping coefficient, <u>μ</u>:

$$\tan \sigma t_H = - \tan kx \tanh \mu x \ . \quad (5)$$

On substituting the time angle for high water, <u>σt_H</u>, from equation (5) into equation (3), the local high-water elevation (the elevation at any location in the estuary) can be calculated, or

$$\eta_H = 2A_o\sqrt{\tfrac{1}{2}(\cos 2kx + \cosh 2\mu x)} \ . \quad (6)$$

The elevation, <u>η_H</u>, in equation (6) is a function of the tidal amplitude at the estuary's head at x = 0, the phase change, kx, that the tidal wave experiences and the damping coefficient, μ, for the estuary. Since total reflection of the wave at x = 0 has been assumed, the elevation at the point of reflection is simply twice the incident amplitude, or $\eta_{OH} = 2A_o$ Thus, the ratio of local high water to the high water at the head is

$$\frac{\eta_H}{\eta_{OH}} = \sqrt{\tfrac{1}{2}(\cos 2kx + \cosh 2\mu x)} \ . \qquad (7)$$

It is assumed that both μ and k remain constant for a section that is uniform in cross-section and roughness. Therefore, these parameters will be in a constant ratio and can be related by a constant, ϕ, which will be called the dissipation constant. We may write

$$\mu = \left(\frac{\phi}{2\pi}\right)k \ . \qquad (8)$$

When equation (8) is substituted in both (5) and (7), the results are as follows:

$$\sigma t_H = \tan^{-1}\left[- \tan kx \tanh \left(\frac{\phi kx}{2\pi}\right)\right] \qquad (9)$$

$$\frac{\eta_H}{\eta_{OH}} = \sqrt{\tfrac{1}{2}\left\{\cos 2kx + \cosh \left(\frac{\phi kx}{2\pi}\right)\right\}} \ . \qquad (10)$$

From tidal data obtained in an estuary, it is a simple matter to calculate ratios of high-water elevation at several stations to the high-water elevation at the head, or $\frac{\eta_H}{\eta_{OH}}$. Also, the time angles of high water for the stations may be determined, σt_H. Since these same relationships are defined by equations (9) and (10), a nomograph may be constructed for various values of $\underline{\mu}$ and $\underline{k}$, with $\underline{\sigma t_H}$ as the abscissa and $\frac{\eta_H}{\eta_{OH}}$ as the ordinate (Fig. 15).

In an ideal situation, if $\frac{\eta_H}{\eta_{OH}}$ and σt_H values are plotted on the nomograph of equations (9) and (10) (Fig. 15), they should fall along a line of constant ϕ. Each of the points also will have a value of $\underline{kx}$ defining the phase change at that point. Since ϕ is related to μ, the dissipation constant is a parameter which describes the damping of the primary and secondary waves.

Energy Transport

By determining $\underline{\phi}$, $\underline{\mu}$ and $\underline{k}$ by the above method, one can describe the major properties of the primary and secondary wave. Since these govern the amplitude of the observed tidal wave, the parameters may be used to determine the energy transmitted in an estuary.

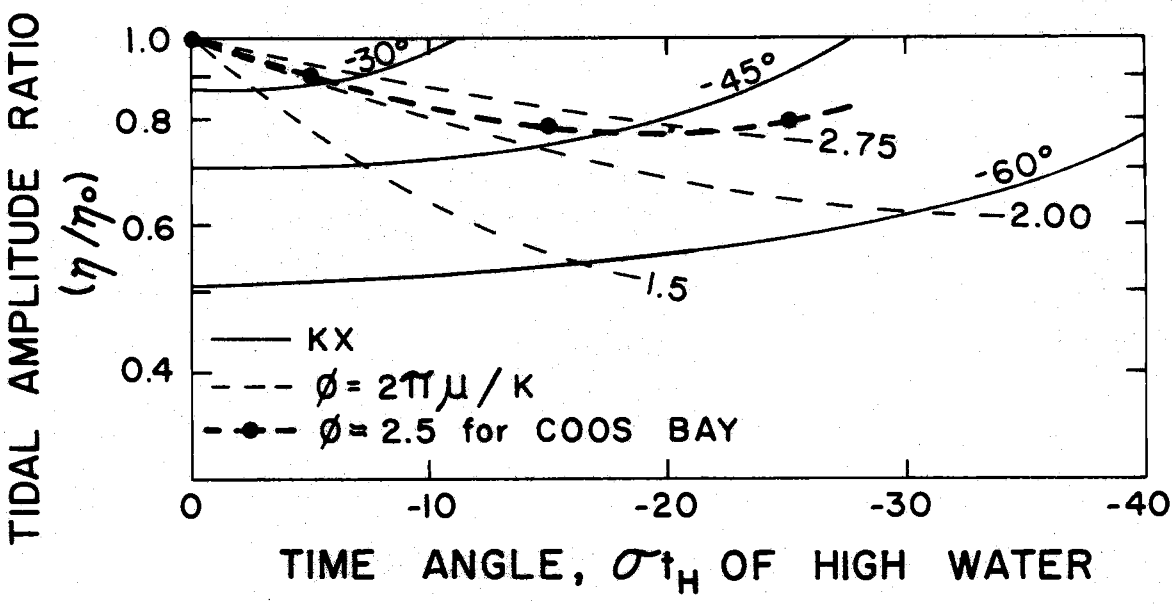


Figure 15. Nomograph for the determination of the dissipation constant, Φ.

It has already been said that the energy associated with a wave is proportional to the square of its height. Specifically, it can be stated that the energy per unit surface area, E_σ, is related to the amplitude, A, in the following manner:

$$E_\sigma = \tfrac{1}{2} \rho g A^2.$$

The rate at which energy is transported through a cross-section, P, is dependent upon the wave velocity, c, and the width of the channel, as well as the amplitude, A, of the wave; thus,

$$P = E_\sigma cb = \tfrac{1}{2} \rho g A^2 cb.$$

The amplitudes of tidal waves undergoing damping have already been defined by equations (1) and (2). Therefore, the mean rate of energy averaged over a tidal cycle transported by the incident and reflected waves, respectively, is

$$P_1 = \frac{cb\rho g}{2} A_o^2 e^{-2\mu x} \tag{11}$$

$$P_2 = \frac{-cb\rho g}{2} A_o^2 e^{2\mu x} \tag{12}$$

By adding equations (11) and (12) and noting that

$$\tfrac{1}{2} (e^{2\mu x} - e^{-2\mu x}) = \sinh 2\mu x,$$

the total rate of energy transport across any vertical section is

$$P = -\, cb\rho g A_o^2 \sinh(2\mu x) \qquad (13)$$

(11, p. 14). For the ocean entrance at $x = -L$,

$$P = \rho gcb A_o^2 \sinh(2\mu L). \qquad (14)$$

The difference in rate of energy transport between two sections will be equal to the energy dissipated or changed from simple wave energy in the interval between the sections. Therefore,

$$\text{Rate of energy dissipation} = P_{x_1} - P_{x_2}.$$

This rate of dissipation may conveniently be calculated on the basis of energy dissipation per unit mass, G_m, as

$$G_m = \frac{P_{x_1} - P_{x_2}}{\rho \bar{b}\bar{h}(x_1 - x_2)},$$

where $\bar{b}$ and $\bar{h}$ denote mean values of channel width and depth over the portion between the sections being considered.

Table 4 is a summary of the rate of energy transport across four sections in Coos Bay and the dissipation rate between the sections. Calculations were based on a ϕ of approximately 2.5 (Fig. 15).

Table 4. Results of the Energy Dissipation Study of Coos Bay

ϕ 2.5

Section	Charleston (Entrance)	Empire	Railroad Bridge	Turning Basin (Head)
Distance from head, meters	-22,000	-15,000	-9000	0
Width, meters	640	658	1006	158
Mean depth, meters	5.6	5.5	3.5	5.4
Wave velocity, meters/sec.	6.9	7.3	5.8	5.7
Phase change, kx	-50^{o}	-41^{o}	-25^{o}	0
k, rad/meter	$3.96 x 10^{-5}$	$4.77 x 10^{-5}$	$4.85 x 10^{-5}$	
μ, rad/meter	$1.57 x 10^{-5}$	$1.90 x 10^{-5}$	$1.93 x 10^{-5}$	
Energy transport rate P, in watts	$33 x 10^{6}$	$29 x 10^{6}$	$21 x 10^{6}$	$4.3 x 10^{6}$

Energy dissipation rate G, in watts	$4 x 10^{6}$	$8 x 10^{6}$	$17 x 10^{6}$
Energy dissipation rate G_m, in watts/kg	$1.6 x 10^{-4}$	$3.5 x 10^{-4}$	$7.1 x 10^{-4}$
Rate of potential energy gain, J, in watts/kg	$7.6 x 10^{-7}$		
Stratification number G_m/J	530		

As fresher water near the head of an estuary moves down toward the ocean, it gradually increases in salinity and, consequently, density. This increase of density shows up as an increase in potential energy. It has already been shown that this increase in potential energy reflects mixing of fresher and saltier water (Fig. 9). The change in potential energy may be illustrated by considering a section of the estuary with a length, _L_. A unit volume of fresh water enters the upstream end at the surface with a potential energy

$$\rho g\bar{h};$$

_h_ is measured from the bottom of the channel to the unit volume of fresh water. At a distance of _L_ downstream, the potential energy is increased to

$$(\rho + \Delta\rho)\, g\bar{h}\ .$$

It is assumed that the slope of mean sea level in the estuary is negligible over the distance _L_. The potential energy downstream minus the potential energy upstream equals

$$\Delta\rho g\bar{h}.$$

$\Delta\rho$ represents the additional salt that the unit volume of water has gained while moving down the estuary. The change of potential energy per unit mass is $\frac{\Delta\rho}{\rho} g\bar{h}$.

If one is interested in the rate at which the potential energy per unit mass increases, the above expression must be divided by the time necessary for the fresh water volume to move the distance, $\underline{L}$. The time for this is $L/\overline{U}_f$, where $\underline{\overline{U}_f}$ is the mean velocity of fresh water over the distance, $\underline{L}$. Thus, the rate of potential energy gain per unit mass of fluid, $\underline{J}$, is

$$J = g \frac{\Delta \rho}{\rho} \bar{h} \frac{\overline{U}_f}{L} .$$

This equation has also been used by Ippen and Harleman (11, p. 40).

$\underline{J}$ is a very important term and serves as an index of mixing. Even more important, it helps to describe the circulation pattern. If $\underline{\overline{U}_f}$ becomes large, $\underline{J}$ also increases. Since large amounts of fresh water will tend to stratify an estuary, high values of $\underline{J}$ may indicate a stratified estuary, other variables being constant. In a shallow and long estuary (high values of $\underline{L}$ and low values of $\bar{h}$), $\underline{J}$ becomes low. Estuaries with this characteristic are usually well mixed. Therefore, in a well-mixed estuary, there is relatively little potential energy change involved, whereas in an estuary that tends toward stratification, relatively large potential energy changes take place.

From the above, one can see that $\underline{J}$ is affected only by the fresh water runoff for any given estuary. Relative changes in density, $\Delta\rho$, are very small indeed when compared to changes in $\underline{\overline{U}_f}$. On the other hand, $\underline{G_m}$ is a function of tidal energy only for any one estuary. Any attenuation in the rate of energy transport appears as a dissipation of the tidal energy.

Ippen and Harleman (11, p. 40) have also defined a quantity called the stratification number. For an estuary this number is simply the ratio of the rate of energy dissipation per unit mass of water to the rate of potential energy gain per unit mass of water, or G_m/J. By considering how $\underline{G_m}$ and $\underline{J}$ change with respect to the factors discussed immediately above, one may see a relationship between G_m/J and the relative mixing in an estuary. Ippen and Harleman (11, p. 48) have shown the vertical salinity gradients for several stratification numbers. These relations are shown in Figure 16, and the curve for Coos Bay has been included.

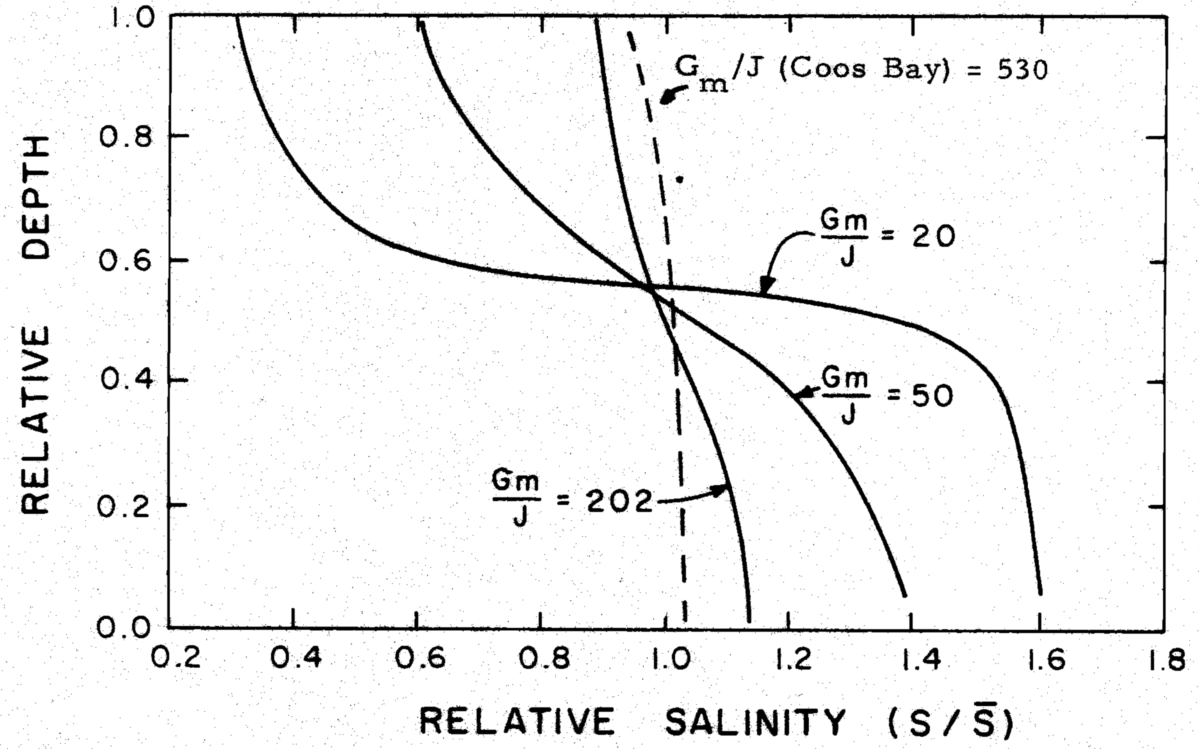


Figure 16. Vertical salinity gradients for several stratification numbers.

Source: Ippen and Harleman (11, p. 48)

Energy Spectra

In the section on Energy Flow, it was noted that the energy spectrum would be valuable in the investigation of the distribution of energy in the dissipative process, as well as in providing a simple explanation of turbulence. The following procedure was followed to obtain an energy spectrum for Coos Bay. The velocity data at one-minute intervals (17) show variations that can be considered as fluctuations due to interaction of a series of periods. The smallest period that could be determined from this data could be no smaller than the minimum time interval. These velocity fluctuations have been interpreted as lengths since eddy sizes are usually more meaningful than eddy periods. Energy spectra were obtained at times of high velocity and low velocity flows.

A base velocity of 10 cm/sec was chosen for the low velocity data whose mean velocity was in the region of 10 cm/sec. These data were presented to an IBM 1620 in digitized form as deviations from the mean tidal velocity which was determined from the "best fit" sine curve. When the mean tidal velocities were 10 cm/sec, the deviations were entered for each one-minute interval (the sampling interval). When the mean tidal velocities became greater than the base velocity, averages were entered for a time interval of less than one minute since an eddy of a given size (velocity fluctuation) would require proportionately less time to pass the velocity meter than during times of the 10 cm/sec current. The velocity base for the high velocity region was chosen at 55 cm/sec.

The IBM 1620 was programmed to provide a power spectrum analysis. This type of analysis is frequently used in the study of ocean waves, turbulence, and related subjects in the fields of oceanography, meteorology, and geophysics. Mr. Wilbur A. Rinehart of the Geophysics Group in the Department of Oceanography, Oregon State University, wrote the program used in this analysis and was kind enough to direct the procedures and interpret the power spectra.

Figure 17 shows energy density versus eddy size as wave number (units of k in 1/meter) for Coos Bay for both the high velocity and the low velocity bases. The results from a curve by Grant *et al*. (9, p. 253) may be compared with Figure 17. The data were collected from Discovery Passage on the east shore of Vancouver Island, British

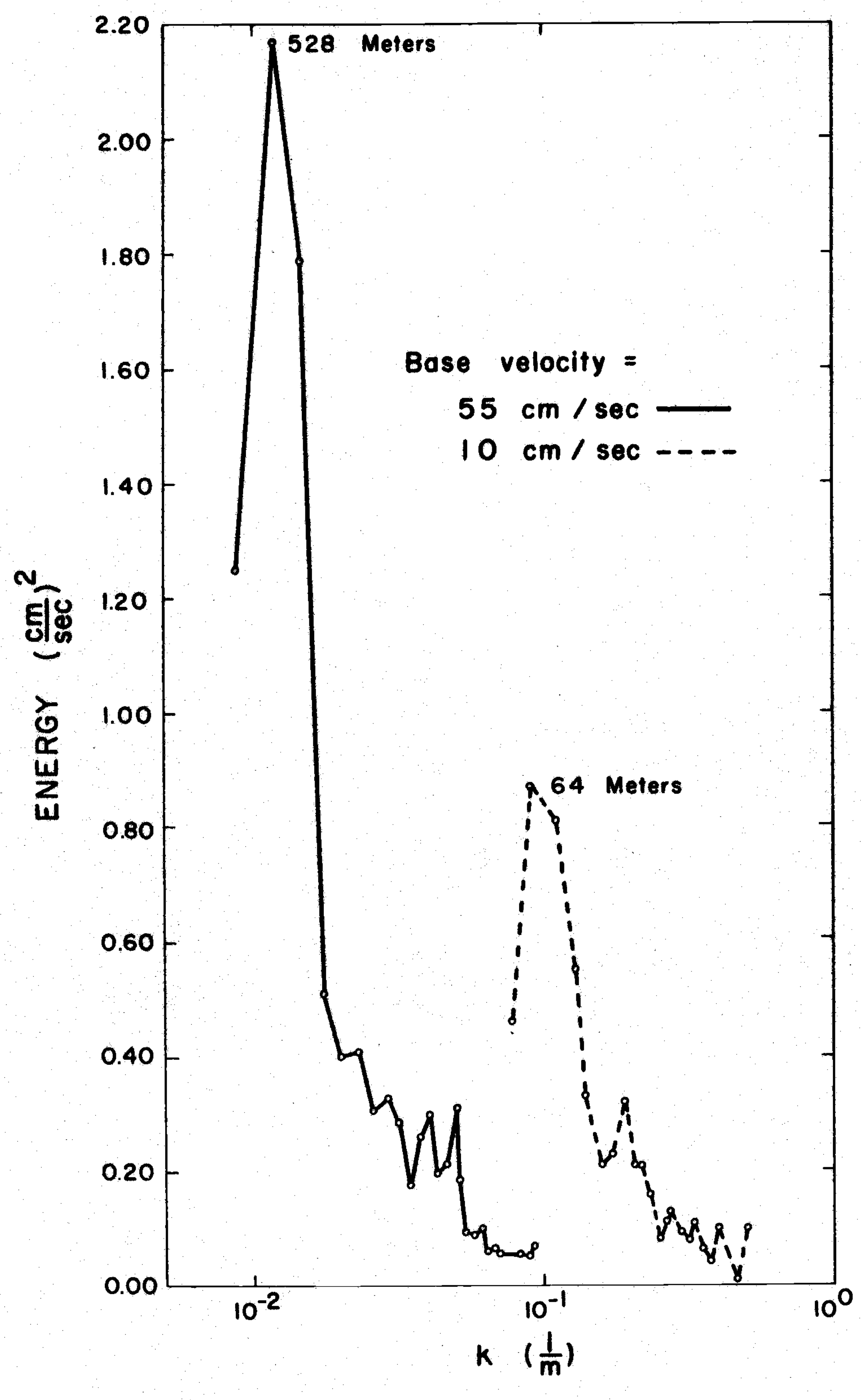


Figure 17. Energy spectra for high and low tidal velocities at the U.S. 101 bridge.

Columbia. The curve from Grant _et al_. shows energy mostly in the high wave numbers (small eddies), which is in the dissipative part of the spectrum. The obtaining of spectra in the dissipative range depends upon the capability of the current meter to measure extremely small velocity fluctuations over a period of a second or less. The meter used in Discovery Passage was a hot-film flow meter. The velocity meter used to obtain spectra in Coos Bay was an anemometer-type meter and was incapable of measuring these small fluctuations.

Discussion of Results

Table 4 presents a summary of the results of the energy dissipation study in Coos Bay. One can see that the energy transport rate decreases as the distance from the ocean increases. It is the difference in energy transported between sections that yields the energy that is dissipated between the stations listed in Table 4. This quantity, G_m, is seen to increase as one moves up the estuary. The greatest amount of dissipation occurs in the last section of the estuary between the railroad bridge and the turning basin. This energy is probably dissipated in the tidal flats (Fig. 1), and these flats predominate in the last section as compared to the other sections. This last section is also the one with the most phase change, 25 degrees.

The rate of potential energy gain calculated from the formula,

$$J = g \frac{\Delta \rho}{\rho} \bar{h} \frac{\bar{U}_f}{L}$$

is found to be quite small when compared to the dissipation of tidal energy. The low figure for J appears to be due to the great length of Coos Bay (about 15 miles) and the relatively shallow depth ($3\frac{1}{2}$ to $5\frac{1}{2}$ meters). Most homogeneous estuaries are shallow, and little energy is required to mix the ocean water with the fresh water. This is opposed to the fiords which are always relatively deep and are usually stratified.

The stratification number, G_m/J, was calculated to be 530 in Table 4. This number is characteristic of a well-mixed estuary (11, p. 48). Under partially mixed conditions, the fresh water velocity, $\bar{U}_f$, would increase the value of J. This in turn would decrease the value of G_m/J.

The energy spectra (Fig. 17) illustrate the eddy sizes where most of the turbulent energy was concentrated for the two sets of velocity data. The data gave higher energies for lower k numbers. In the case of the higher velocity data, the lowest k numbers represented eddies that were larger than the dimensions of the channel. These were neglected. For the low velocity data, the lowest k numbers were associated with energies that were thought to be too high to be representative of the data presented. When the data were entered into the computer as deviations from the best-fit sine curve, the sine curve acted as a band pass filter which filtered out the periods approaching those of a tidal period. However, this band pass filter is not perfect by any means, and any fluctuations with a period slightly more or less than this sine curve will not be filtered out. This, in effect, would tend to concentrate energy in the low frequencies surrounding a tidal cycle.

Most of the turbulent energy for the mean velocities around 55 cm/sec was contained in eddies whose dimensions were approximately 500 meters. In velocities close to 10 cm/sec, the energy appeared to be concentrated in eddies approximately 60 meters in size. Therefore, it may be that as the kinetic energy of the circulation increases, the turbulent energy increases, and this energy is contained in larger eddies.

As brought out in the section on Energy Flow, steady state conditions exist only if energy is added to the large eddies at the same rate that energy is dissipated in the smallest eddies. The spectra from Coos Bay offer evidence that steady state conditions do not exist. As the velocity decreases, the energy not only decreases, but the eddy size containing the most energy also decreases. This suggests that the energy from the smaller eddy probably comes from the larger eddy. If this were <u>not</u> the case, one would simply expect the energy to decrease, but the "hump" in the spectrum should remain over the same wave numbers. Therefore, as the velocity increases, energy is added to the medium size eddies faster than it can be dissipated, and the energy "hump" shifts to the lower wave numbers.

SUMMARY AND CONCLUSIONS

Coos Bay has been found to be well mixed during most of the year, but it may become partially mixed during periods of heavy runoff. Tides average around six feet in the estuary. The tide is the principal source of energy used to circulate and mix the waters in the estuary. Energy dissipation has been estimated from an analysis of the energy transported by the tidal wave at several sections in Coos Bay. The study of energy dissipation has shown a high rate of dissipation with much of the energy dissipated near the head of the estuary in a region with large areas of tide flats.

The energy spectra in Coos Bay cover much of the region of generation and most of the macro-scale turbulence, but they do not extend into the dissipative range. Both the high velocity and low velocity spectra tend to approach the same energy level in the higher wave numbers. The two curves (Fig. 17) represent regions of both high and low velocity. The approximate turbulent dissipation rate was calculated by computing the energy under both of the curves, subtracting the energies and dividing by the time difference between the two velocity records. The calculation of energy spectra by computer has one drawback. In an analysis of this kind, the longest period is limited by the length of the record; the shortest period is limited by the sampling interval. The computer must be told the periods in which to place the energy. One must resort to filters in order to define the upper period. The computer must be instructed to stop when the smallest period consistent with the sampling interval is reached.

The rate of energy dissipation calculated from the damping of the tidal wave was of the order of 10^{-4} watts per kilogram. The turbulent dissipation rate for Discovery Passage measured by Grant *et al*. (9, p. 253) was of the order of 10^{-3} watts per kilogram. The calculated dissipation rates in Coos Bay are smaller than those of Grant *et al*., as might be expected from the very turbulent regime measured by these authors. The rate of decay measured by the spectra at flood and at slack was of the order of 10^{-7} watts/kg. While this establishes a lower limit for the dissipation rate, it is much smaller than the mean rate. Since the mechanism for viscous dissipation is so effective in decaying turbulence, the slack water spectrum (10 cm/sec base) more likely represents the energy flow over a short period, say one hour, rather than representing the decay of the spectrum at maximum flood. This indicates that energy is withdrawn from the mean tidal cycle over most of the cycle and not exclusively during periods of highest velocities.

It appears that most of the energy extracted from the tides is dissipated into heat and horizontal mixing in the estuary. Apparently, only a relatively small amount of power is involved in vertical turbulent dissipation and mixing. In the one-dimensional analysis, no attempt was made to measure the extent of isotropy in any region of the spectrum. However, it would appear that there must be some means to introduce isotropy at small wave numbers (e.g., interfacial breaking waves (15, p. 489)) if a significant amount of the originally strongly anisotropic turbulent energy is to be diverted into vertical mixing.

The tidal wave in Coos Bay approximates a progressive wave, the tide occurring later along the bay, and slack water being intermediate rather than at high and low water. It is interesting to compare the tidal wave in Coos Bay with that of Yaquina Bay, an estuary located about 90 miles to the north of Coos Bay along the Oregon coast. Yaquina Bay has a substantially smaller area of tide flats. It is somewhat shallower, on the average, than Coos Bay, and the distance from the bar to the city of Toledo is about the same as from the bar to Coos Bay. In Yaquina Bay, the time difference for the arrival of high water over a 13-mile length of the bay is only 25 minutes. Slack water occurs within a few minutes of high or low water. This indicates a weakly damped standing wave and smaller dissipation in Yaquina Bay than in Coos Bay. Yaquina Bay is, however, generally more nearly well mixed than is Coos Bay. Realizing the inadequacy of one sample, it still seems pertinent to observe that while extensive tide flats can apparently result in high dissipation of the tidal energy, the withdrawal of mean kinetic turbulent energy into mixing and potential energy is apparently influenced importantly by considerations other than the sheer magnitude of the turbulent energy density.

It is hoped that this paper has been a contribution to the understanding of the dynamic processes acting in an estuary. An attempt has been made to describe these processes from the point of view of energy dissipation since any physical dynamic process must use energy.

Many doors have been open to the further study of the turbulent dissipation of energy in estuaries. A more rigorous measurement of the turbulent energy spectra at many parts of a tidal cycle and at many locations is needed. This study could best be undertaken with the use of a turbulence meter capable of measuring the spectra in the dissipative range. Future studies should deal with three dimensional analysis of the spectra in order to determine the wave lengths at which anisotropic turbulence decays into isotropic turbulence.

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