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Title: CRITICAL BUCKLING LOAD FOR ROUND TAPERED COLUMNS WITH VARIABLE END FIXITY

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The critical buckling load for round tapered columns with variable end fixity is presented. The theory used is presented together with the general solution for the resulting differential equation. This general solution is solved by use of a computer. The computer program written specifically for this thesis, the flow chart, the print out data and the input data file are included in Appendix A.

The data obtained substantiates published data and shows that the critical buckling load of a round tapered column, fixed at the small end and hinged and restrained against translation at the large end is increased approximately 100% when the large end of the column is fixed. Information is presented which shows how the critical buckling load for the column varies with relative fixity at the large end and with the column taper.

While this information applies to columns of any material, it is especially important to the timber industry since the data is useful

for timber piling design. Values for partial fixity are now available for use by the design engineer. In the past, engineers had only the values for the two extremes and the choice of full fixity or hinged was all that was available. A comparison of the increased buckling loads for a partially fixed column, using the curves in this thesis, with the buckling load for the hinged column, will show that increased economies can be realized.

Critical Buckling Load for Round Tapered Columns with Variable End Fixity

by

George Franklin Dotson

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CRITICAL BUCKLING LOAD FOR ROUND TAPERED COLUMNS WITH VARIABLE END FIXITY

INTRODUCTION

Round tapered columns in the form of timber piling are in common use today and have been used for many years to support heavy loads. In many cases the supported structure or pile cap is at or below the ground surface and the piling or tapered column is completely supported over its entire length. The maximum load which this column will support is mainly a function of the properties of the foundation material and of the soil-piling interaction and the maximum allowable load is determined by use of the principles of soil mechanics.

Docks, warehouses and other marine-type structures are many times supported by timber piling. The tops of the piles and the pile caps are located at a considerable distance above the soil into which the piles have been driven. The maximum load which the column will support becomes more a function of the unsupported length of the column than a function of the foundation properties as this unsupported distance is increased. Timber bracing has been used to decrease this unsupported length. This bracing is expensive and tends to collect floating debris. A solution which will permit an analysis of the long unsupported tapered column is developed and discussed in the following chapters.

Many of the designs involving tapered piling have been approximations. One of these designs is the "rule of thumb" method (7). The equations for an exact analysis of the round tapered column with either fixed or pinned end conditions are presented in reference 3. This exact analysis is not developed for the variable end fixity condition. This method of analysis will be developed in later chapters and will satisfy all boundary conditions and tapers usually found in this type of installation. In most cases, higher buckling loads will be calculated using this method than that calculated using an approximate solution.

The solution for the critical buckling load of a round tapered column with variable end fixity at the large end and full fixity at the small end is developed from the theory of elastic stability. The general solution to the differential equation is first developed. This solution is in terms of the column taper, variable and fixity, and the term "k", equation (19). To solve this general solution by longhand methods would be a formidable task. The present-day computers can process large amounts of data quickly and economically with considerable accuracy. A computer program was written as part of the thesis to solve this general solution.

REVIEW OF PUBLISHED THEORY

Even though long tapered round columns in the form of timber piling have been used for many years as structural supporting members, a search of the literature reveals that very little accurate design information is available. No information was found which the design engineer could use to calculate the critical buckling load of a round tapered column with variable end fixity. A great deal of information, however, has been recorded in published articles concerning the soil resistive capacity and the soil-pile interaction. This soils information is vital to structures supported on long timber piling but the design of long slender piling is many times controlled by the structural behavior of the piling rather than the supporting value of the soil. It is this structural behavior that this thesis is concerned with. The tapered columns are considered restrained against translation at the top and are assumed to be long enough so that buckling will occur in the elastic range.

A typical isolated laterally supported unbraced piling is shown on Figure 1. Three properties affect the buckling strength of this column: the modulus of elasticity of the column material (E), the moment of inertia of the column (I), and the unsupported length of the column (L). The modulus of elasticity for any given material can usually be obtained from existing tables. The unsupported length for

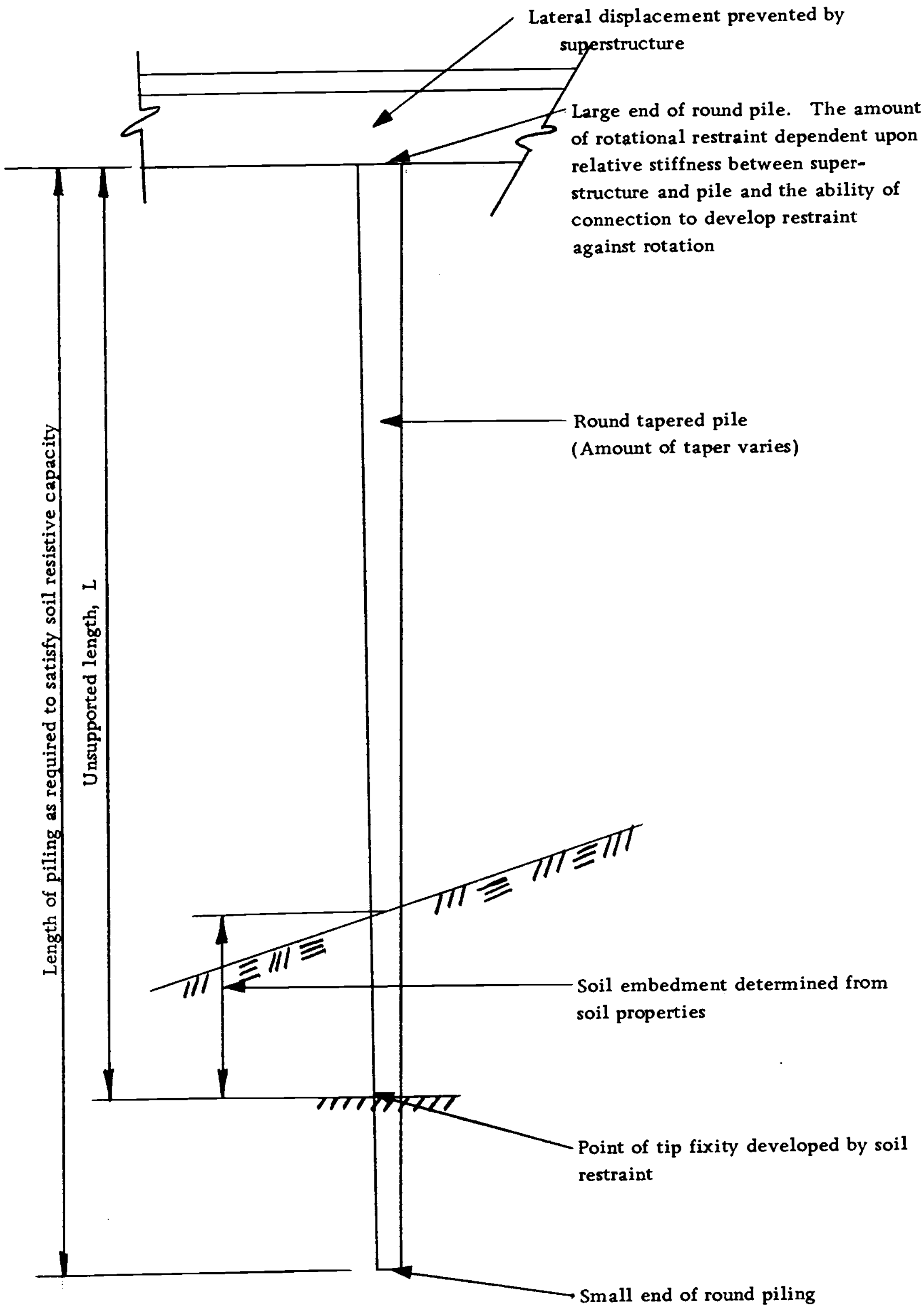


Figure 1. Typical tapered piling installation.

a given installation depends upon the characteristics of the soil to develop restraint. The method for determining this point of fixity is not within the scope of this thesis. Information can be obtained however from reference 3. The moment of inertia of a tapered piling varies along the length of the piling and the value used in the long column formulas needs careful consideration.

One common method of design for long tapered piling is the "rule of thumb" method (7). This method assumes that the critical section of piling is located at a point one-third the unsupported distance from the point of fixity. The moment of inertia at this point is used and the column is then designed as a uniform section without taper. This method has been proven to be adequate but the need to improve the economics of the timber piling industry has raised the question of just how conservative is this design method and will an exact analysis permit an increased allowable load.

An analytical method for determining the critical buckling loads for tapered columns has recently been presented by James M. Gere and Winfred O. Carter (2). This publication presents graphs for critical buckling loads for four different end conditions, pinned ends, fixed-free ends, fixed-pinned ends and fixed ends for seven different structural shapes. The shapes considered by Gere and Carter are wide flange, closed box, solid rectangular, open web, tower, solid circular and square sections. The article does not include the variable fixity-fixed end case. A review of the results of Gere and Carter's work shows that the critical buckling load of a round tapered column can be increased by approximately 100% from a fixed-pinned case to a fixed-fixed case.

The fixed-fixed case is a more efficient use of the round piling as a column, however, a connection which will develop full fixity at the top of the round tapered wood column and still be economical, is difficult to construct. In most installations the top of the round piling-column is restrained to some degree varying between pinned and full fixity. No existing solutions were found which would give the critical buckling loads for this variable end condition. This thesis has developed the values for the variable fixity end condition.

The basic approach used in this theses is found in the theory of elastic stability by Timoshenko and Gere (6). A few elementary solutions will be included in this chapter to show the reasoning and theory used to develop the general solution in the case of partial fixity.

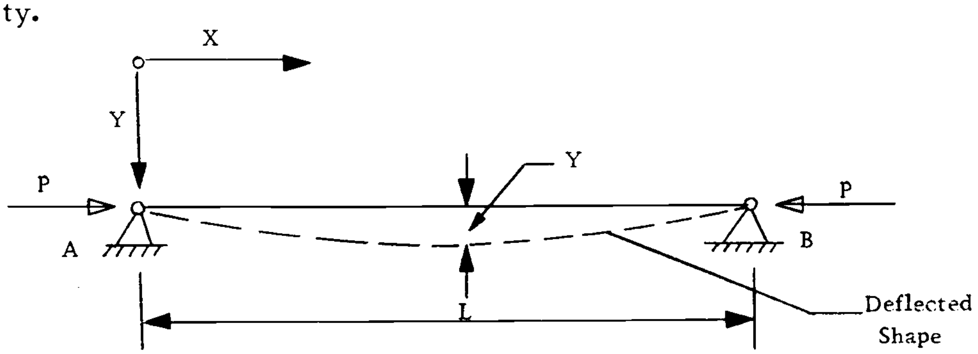


Figure 2. Beam Column: pinned-pinned ends (constant cross-section.

To solve this problem use the equation (1-3), referenced by Timoshenko and Gere (6).

$$\frac{EId^2y}{dx^2} = -M \tag{1}$$

The bending moment at any point along the deflected beam column = Py so that equation (1) becomes

$$\frac{EId^2y}{dx^2} = -Py \tag{2}$$

If k^2 is defined as $k^2 = P/EI$ and then substituted into equation (2) the following basic differential equation is obtained.

$$\frac{d^2y}{dx^2} + k^2y = 0 \tag{3}$$

The general solution to this differential equation from (2) is:

$$y = A\cos kx + B\sin kx \tag{4}$$

Using the boundary conditions:

1. $y = 0$ at $x = 0$
2. $y = 0$ at $x = L$

the following solution is obtained:

$$P_{crit} = \pi^2 EI/L^2 \tag{5}$$

which is the Euler column equation.

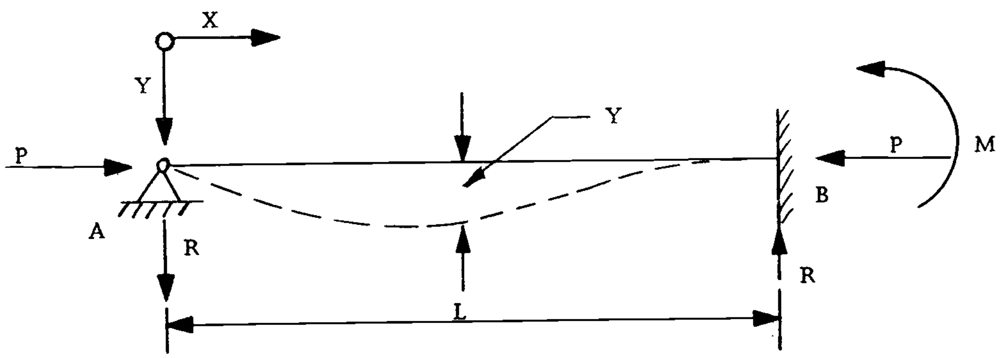


Figure 3. Beam column:pinned-fixed ends (constant cross-section).

The basic equation for the elastic curve of a flexural member is:

$$\frac{EId^2y}{dx^2} = -M \tag{1}$$

The moment at any point along the deflected beam column in Figure 3 is Py - Rx; thus, the basic equation becomes:

$$\frac{d^2y}{dx^2} + k^2y = \frac{Rx}{EI} \tag{6}$$

The general solution to this differential equation from (6) is:

$$y = A\cos kx + B\sin kx + \frac{Rx}{EIk^2}$$

this becomes:

$$y = A\cos kx + B\sin kx + \frac{Rx}{P} \tag{7}$$

and

$$dy/dx = -k A \sin kx + B k \cos kx + R/P \tag{7A}$$

using the following boundary conditions:

1. $y = 0$ at $x = 0$
2. $y = 0$ at $x = L$
3. $dy/dx = 0$ at $x = L$

The following equations are obtained by substituting these boundary conditions into equations (7) and (7A);

1. $A = 0$
2. $A \cos kL + B \sin kL + RL/P = 0$
3. $-Ak \sin kL + B k \cos kL + R/P = 0$

Solving these equations,

$$\tan kL = kL \tag{8}$$

Equation (8) has been solved (6), page 53, and the equation for the critical buckling load is:

$$P_{cr} = \frac{\pi^2 EI}{(0.699L)^2} \tag{9}$$

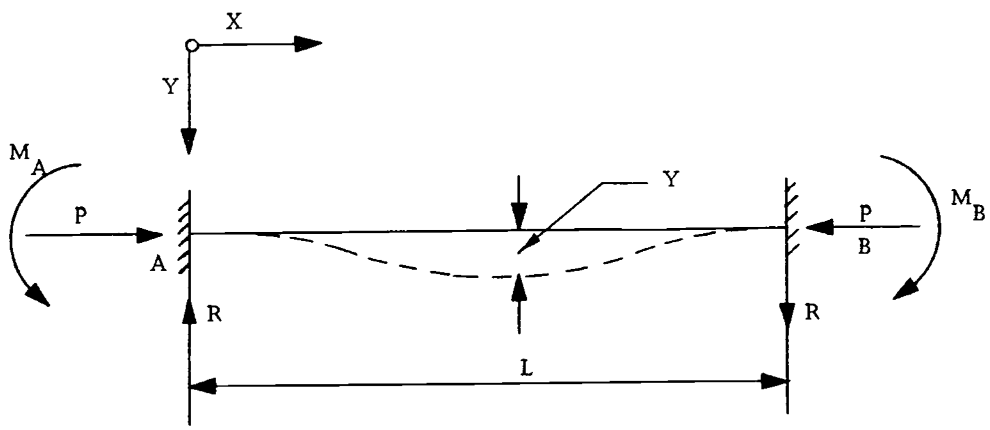


Figure 4. Beam column:fixed-fixed ends (constant cross-section).

The moment at any point along the beam column is:

$$M = Py + Rx - M_A$$

the basic equation (1) becomes

$$\frac{EId^2y}{dx^2} = -M = -Py - Rx + M_A$$

or

$$\frac{d^2y}{dx^2} + k^2y = \frac{M_A}{EI} - \frac{Rx}{EI} \qquad (10)$$

The general solution to this differential equation from (6) is:

$$y = A\cos kx + B\sin kx + \frac{M_A}{k^2EI} - \frac{Rx}{k^2EI}$$

and reduces to:

$$y = A \cos kx + B \sin kx + M_A/P - Rx/P \tag{11}$$

and

$$\frac{dy}{dx} = -Ak \sin kx + Bk \cos kx - R/P \tag{12}$$

The boundary conditions used to solve for the critical buckling load are:

1. $y = 0$ at $x = 0$
2. $y = 0$ at $x = L$
3. $dy/dx = 0$ at $x = L$
4. $dy/dx = 0$ at $x = 0$

The following equations are obtained by substituting these boundary conditions into equations (11) and (12);

1. $A + M_A/P = 0$
2. $A \cos kL + B \sin kL + M_A/P - RL/P = 0$
3. $-Ak \sin kL + Bk \cos kL - R/P = 0$
4. $Bk - R/P = 0$

This is solved by expanding the determinant:

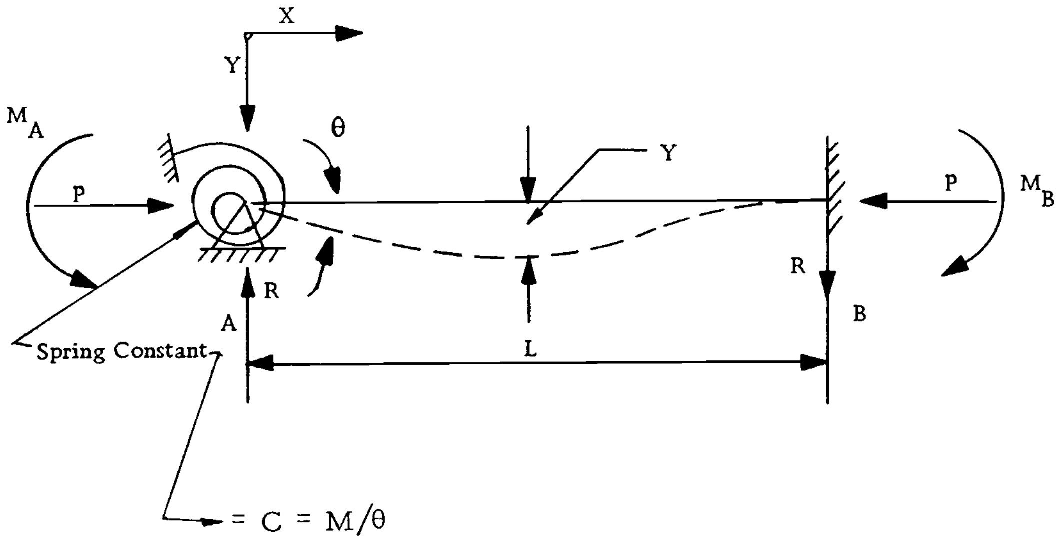
$$\begin{vmatrix} 1 & 0 & 1/P & 0 \\ \cos kL & \sin kL & 1/P & -L/P \\ -k \sin kL & k \cos kL & 0 & -1/P \\ 0 & k & 0 & -1/P \end{vmatrix}$$

to obtain:

$$2(\cos kL-1) + kL \sin kL = 0 \tag{13}$$

Equation (13) has been solved (6), page 55, and the equation for the critical buckling load is:

$$P_{crit} = \frac{4\pi^2 EI}{L^2} \tag{14}$$



$$= M/dy/dx \ \text{or} \ \frac{dy}{dx} = \frac{M_A}{C}$$

Figure 5. Beam column:fixed-variable fixity ends (constant cross-section).

When C is very large a fixed-fixed case exists and when C approaches zero a pinned-fixed case is realized.

The moment in the beam column at any point is equal to

$$M = Py + Rx - M_A$$

so from equation (1)

$$\frac{EId^2y}{dx^2} = -Py - Rx + M_A$$

is obtained. This becomes

$$\frac{d^2y}{dx^2} + k^2y = \frac{M_A}{EI} - \frac{Rx}{EI} \qquad (15)$$

which is the same as equation (10) and the general solution is the same as equations (11) and (12).

The boundary conditions to be used to solve this equation differ slightly and are listed below:

1. $y = 0$ at $x = 0$
2. $y = 0$ at $x = L$
3. $dy/dx = 0$ at $x = L$
4. $dy/dx = M_A/C$ at $x = 0$

The following equations are obtained by substituting these boundary conditions into equations (11) and (12):

1. $A + M_A/P = 0$
2. $A \cos kL + B \sin kL + M_A/P - RL/P = 0$
3. $-Ak \sin kL + Bk \cos kL - R/P = 0$
4. $Bk - M_A/C - R/P = 0$

solving this by expanding the determinant:

$$\begin{vmatrix} 1 & 0 & 1/P & 0 \\ \cos kL & \sin kL & 1/P & -L/P \\ -k \sin kL & k \cos kL & 0 & -1/P \\ 0 & k & -1/C & -1/P \end{vmatrix}$$

to obtain:

$$2k(\cos kL - 1) + \frac{1}{C}(PkL \cos kL - P \sin kL) + k^2 L \sin kL = 0 \quad (16)$$

To check this solution let

$$C = M/\theta = 0 \quad \text{for pinned end}$$

equation (16) then reduces to

$\tan kL = kL$ which checks with equation (8).

If we let $C = M/\theta = \infty$ for fixed end

$$1/C \rightarrow 0$$

and equation (16) reduces to:

$$2(\cos kL - 1) + kL \sin kL = 0$$

which checks with equation (13).

The critical buckling load for values of end fixity between the pinned and fixed cases was not determined for the constant section. These values however can be easily obtained by solving equation (16) for the value of k to set the equation equal to zero for each assumed value of end fixity. This example serves to show the method to be used for the round tapered column which will be covered in the next chapter.

DEVELOPMENT OF THEORY AND SOLUTION

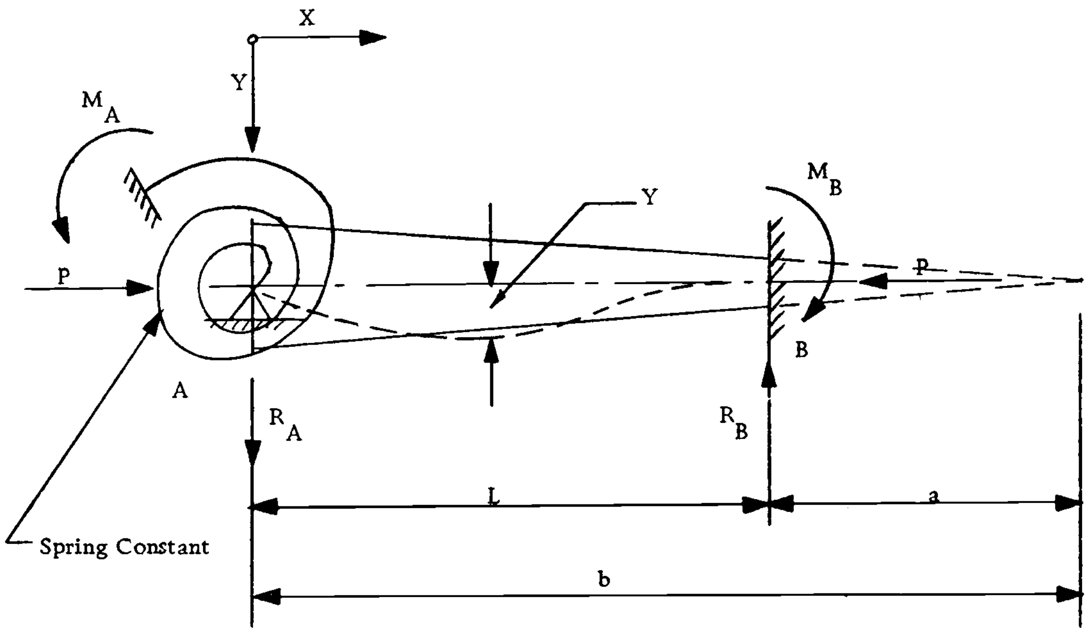


Figure 6. Beam column:fixed-variable fixity case (variable cross-section).

Assumptions:

1. Beam column is fixed at point B.

2. Column is circular in cross-section.

3. Column is uniformly tapered with large end at point A.

4. Variable fixity at point A is shown by a spring

   constant $= C = M_A/\theta = M_A/\frac{dy}{dx}$

5. Basic differential equation is $EI_x \, d^2y/dx^2 = -M$

6. No translation is permitted at either end.

The moment at any point on deflected column is

$$M = Py - R_A x - M_A$$

Substituting this into the basic equation

$$EI_x \frac{d^2y}{dx^2} + Py = R_A x + M_A \qquad (17)$$

The moment of inertia of the beam column varies as follows:

$$I_x = I_A[1 - \frac{x}{b}]^4$$

Combining this with equation (17) we obtain

$$EI_A[1 - \frac{x}{b}]^4 \frac{d^2y}{dx^2} + Py = R_A x + M_A \qquad (18)$$

this becomes

$$[1 - \frac{x}{b}]^4 \frac{d^2y}{dx^2} + \frac{Py}{EI_A} = \frac{1}{EI_A}(R_A x + M_A)$$

defining

$$k^2 = \frac{Pb^2}{EI_A} \qquad (19)$$

this equation becomes:

$$[1 - \frac{x}{b}]^4 y'' + \frac{k^2}{b^2} y = \frac{1}{EI_A}(R_A x + M_A) \qquad (20)$$

by further rearranging the following equation is obtained.

$$y'' + \frac{k^2 y}{b^2[1 - \frac{x}{b}]^4} = \frac{1}{EI_A}\left[\frac{1}{[1 - \frac{x}{b}]^4}\right](R_A x + M_A) \qquad (21)$$

This equation may be solved more easily by means of the following substitutions:

$$z = \left[\frac{1}{1 - \frac{x}{b}}\right]$$

The idea for these substitutions was obtained from reference 1, page 24.

$$y' = \frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} = \frac{1}{b\left[1 - \frac{x}{b}\right]^2}\frac{dy}{dz} = \frac{z^2}{b}\frac{dy}{dz}$$

$$y'' = \frac{dy'}{dx} = \frac{dy'}{dz}\frac{dz}{dx} = \frac{z^2}{b}\frac{d}{dz}\left[\frac{z^2}{b}\frac{dy}{dz}\right]$$

$$y'' = \frac{z^4}{b^2}\frac{d^2y}{dz^2} + \frac{2z^3}{b^2}\frac{dy}{dz} \qquad (22)$$

Now substituting equation (22) into equation (21)

$$\frac{z^4}{b^2}\frac{d^2y}{dz^2} + \frac{2z^3}{b^2}\frac{dy}{dz} + \frac{k^2}{b^2}yz^4 = \frac{z^4}{EI_A}(R_Ax + M_A) \qquad (23)$$

Multiplying this equation by b^2/z^3

$$z\frac{d^2y}{dz^2} + 2\frac{dy}{dz} + k^2zy = \frac{zb^2}{EI_A}(R_Ax + M_A) \qquad (24)$$

Now since

$$z = \left[\frac{1}{1 - \frac{x}{b}}\right]$$

$x = b[1 - \frac{1}{z}]$ and substituting this into equation (24)

$$z\frac{d^2y}{dz^2} + 2\frac{dy}{dz} + k^2zy = \frac{R_A b^3 z}{EI_A} - \frac{R_A b^3}{EI_A} + \frac{M_A zb^2}{EI_A} \tag{25}$$

Now letting $v = zy$

$$\frac{d^2v}{dz^2} = z\frac{d^2y}{dz^2} + \frac{2dy}{dz}$$

Substituting this into equation (9) to obtain

$$\frac{d^2v}{dz^2} + k^2v = \frac{R_A b^3 z}{EI_A} - \frac{R_A b^3}{EI_A} + \frac{M_A zb^2}{EI_A}$$

Replacing k^2 by q this becomes:

$$\frac{d^2v}{dz^2} + qv = \frac{R_A b^3 z}{EI_A} - \frac{R_A b^3}{EI_A} + \frac{M_A zb^2}{EI_A} \tag{26}$$

The solution to this equation is:

$$v = A \sin \sqrt{q}\; z + B \cos \sqrt{q}\; z + \frac{1}{q}[\frac{R_A b^3 z}{EI_A} - \frac{R_A b^3}{EI_A} + \frac{M_A zb^2}{EI_A}] \tag{27}$$

If we now replace some of the terms using the following:

$v = zy$

$z = [1/ 1 - x/b]$

$k^2 = q$

The general solution becomes:

$$y = A[1-\frac{x}{b}] \sin\frac{k}{[1-\frac{x}{b}]} + B[1-\frac{x}{b}] \cos\frac{k}{[1-\frac{x}{b}]} + \frac{R_A b^3}{k^2 EI_A}$$

$$- [1-\frac{x}{b}] \frac{R_A b^3}{EI_A k^2} + \frac{M_A b^2}{k^2 EI_A} \qquad (28)$$

Before proceeding farther this general solution is checked using equation (28). Differentiating equation (28)

$$\frac{d^2y}{dx^2} = -\frac{A}{b}[\frac{k}{b[1-\frac{x}{b}]^2} \cos\frac{k}{(1-\frac{x}{b})}] + \frac{A}{b}[\frac{k}{b[1-\frac{x}{b}]^2} \cos\frac{k}{(1-\frac{x}{b})}]$$

$$+ \frac{Ak}{b(1-\frac{x}{b})}[\frac{-k}{b(1-\frac{x}{b})^2} \sin\frac{k}{(1-\frac{x}{b})}] - \frac{B}{b}[\frac{-k}{b(1-\frac{x}{b})^2} \sin\frac{k}{1-\frac{x}{b}}]$$

$$- \frac{B}{b}[\frac{k}{b(1-\frac{x}{b})^2} \sin\frac{k}{(1-\frac{x}{b})}] - \frac{Bk}{b(1-\frac{x}{b})}[\frac{k}{b(1-\frac{x}{b})^2} \cos\frac{k}{(1-\frac{x}{b})}].$$

Consolidating this equation

$$y'' = \frac{d^2y}{dx^2} = \frac{-Ak^2}{b^2(1-\frac{x}{b})^3} \sin[\frac{k}{1-\frac{x}{b}}] - \frac{Bk^2}{b^2(1-\frac{x}{b})^3} \cos[\frac{k}{1-\frac{x}{b}}] \qquad (29)$$

The values for y from equation (28) and y'' from equation (29) are now substituted into equation (21) and consolidated to obtain:

$$\frac{R_A x}{(1-\frac{x}{b})^4} = \frac{R_A x}{(1-\frac{x}{b})^4} \qquad (30)$$

so the general solution checks.

Next, the constants and unknowns in equation (28) are evaluated using the following boundary conditions:

1. $y = 0$ at $x = 0$
2. $y = 0$ at $x = L$
3. $dy/dx = 0$ at $x = L$
4. $dy/dx = M_A/C$ at $x = 0$

Differentiating equation (28),

$$\frac{dy}{dx} = A\left[\frac{k}{b[1-\frac{x}{b}]}\cos\left(\frac{k}{1-\frac{x}{b}}\right) - \frac{1}{b}\sin\left(\frac{k}{1-\frac{x}{b}}\right)\right]$$

$$+ B\left[-\frac{1}{b}\cos\left(\frac{k}{1-\frac{x}{b}}\right) - \frac{k}{b(1-\frac{x}{b})}\sin\left(\frac{k}{1-\frac{x}{b}}\right)\right] + R_A\left[\frac{b^2}{EI_A k^2}\right] \qquad (31)$$

Now substituting the boundary conditions into equation (28) and equation (31) the following equations are obtained:

$$A \sin k + B \cos k + \frac{M_A b^2}{k^2 EI_A} = 0 \qquad (32)$$

$$A(1 - \frac{L}{b}) \sin (\frac{k}{1 - \frac{L}{b}}) + B(1 - \frac{L}{b}) \cos (\frac{k}{1 - \frac{L}{b}})$$

$$+ R_A(\frac{Lb^2}{k^2 EI_A}) + \frac{M_A b^2}{k^2 EI_A} = 0 \qquad (33)$$

$$A[\frac{k}{b(1 - \frac{L}{b})} \cos (\frac{k}{1 - \frac{L}{b}}) - \frac{1}{b} \sin (\frac{k}{1 - \frac{L}{b}})]$$

$$+ B[-\frac{1}{b} \cos (\frac{k}{1 - \frac{L}{b}}) - \frac{k}{b(1 - \frac{L}{b})} \sin (\frac{k}{1 - \frac{L}{b}})] + R_A[\frac{b^2}{EI_A k^2}] = 0 \qquad (34)$$

$$A (\frac{1}{b} (k \cos k - \sin k)) + B (-\frac{1}{b} (\cos k + k \sin k))$$

$$+ \frac{R_A b^2}{EI_A k^2} - \frac{M_A}{C} = 0 \qquad (35)$$

Placing equations (32), (33), (34), and 35) into determinant form,

A	B	M_B	R_B
$\sin k$	$\cos k$	$\frac{b^2}{k^2 EI_A}$	0
$(1 - \frac{L}{b}) \sin (\frac{k}{1 - \frac{L}{b}})$	$(1 - \frac{L}{b}) \cos (\frac{k}{1 - \frac{L}{b}})$	$\frac{b^2}{k^2 EI_A}$	$\frac{Lb^2}{k^2 EI_A}$
$\frac{k}{b(1 - \frac{L}{b})} \cos (\frac{k}{1 - \frac{L}{b}})$	$-\frac{1}{b} \cos (\frac{k}{1 - \frac{L}{b}})$	0	$\frac{b^2}{k^2 EI_A}$
$-\frac{1}{b} \sin (\frac{k}{1 - \frac{L}{b}})$	$- \frac{k}{b(1 - \frac{L}{b})} \sin(\frac{k}{1 - \frac{L}{b}})$		
$\frac{1}{b} (k \cos k - \sin k)$	$- \frac{1}{b}(\cos k + k \sin k)$	$- \frac{1}{C}$	$\frac{b^2}{k^2 EI_A}$

Now assigning the following values:

$$D = \sin k$$

$$E = \cos k$$

$$F = b^2/k^2 EI_A$$

$$G = (1 - \frac{L}{b}) \sin (\frac{k}{1 - \frac{L}{b}})$$

$$H = (1 - \frac{L}{b}) \cos (\frac{k}{1 - \frac{L}{b}})$$

$$T = \frac{k}{b(1 - \frac{L}{b})} \cos\left(\frac{k}{1 - \frac{L}{b}}\right) - \frac{1}{b}\sin\left(\frac{k}{1 - \frac{L}{b}}\right)$$

$$U = -\frac{1}{b}\cos\left(\frac{k}{1 - \frac{L}{b}}\right) - \frac{k}{b(1 - \frac{L}{b})}\sin\left(\frac{k}{1 - \frac{L}{b}}\right)$$

$$V = \frac{1}{b}(k \cos k - \sin k)$$

$$W = -\frac{1}{b}(\cos k + k \sin k)$$

The previous determinant becomes:

$$\begin{vmatrix} D & E & F & O \\ G & H & F & LF \\ T & U & O & F \\ V & W & -\frac{1}{C} & F \end{vmatrix}$$

Expanding this determinant the following equation is obtained:

$$\frac{DHF}{C} - DF^2U + DF^2W - \frac{DLFU}{C} - \frac{EGF}{C} + EF^2T - EF^2V$$

$$+ \frac{ELFT}{C} + F^2GU - GF^2W - F^2HT + F^2HV + F^2LTW$$

$$- F^2LUV = 0 \qquad (36)$$

The expression, $k^2 = \frac{Pb^2}{EI_A}$ from equation (19)

can be rearranged as follows

$$P = \frac{k^2 EI_A}{b^2} \tag{37}$$

k^2 must be evaluated before the critical buckling load, P_{crit} can be determined.

The value of k^2 needed is that which will set equation (36) equal to zero.

Due to the magnitude of work involved to solve this equation for each tapered column and the fact that many different combinations of column taper will be considered, this problem can best be handled by use of a computer.

Before using the computer non-dimensionalize the terms involved and determine the form of the desired answer.

Referring to the article by Gere and Carter (2), the desired result should take the form of

$$P_{crit} = (\text{constant}) \left(\frac{\pi^2 EI}{L^2}\right) \tag{38}$$

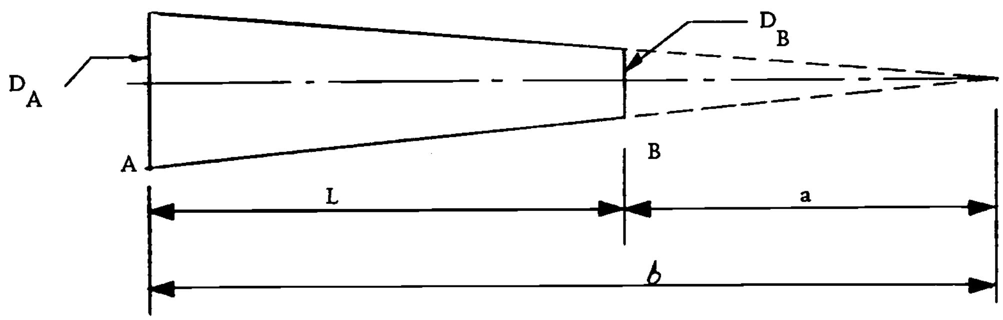
multiplying equation (37) by

$$\frac{L^2}{L^2} \times \frac{\pi^2}{\pi^2}$$

the following is obtained:

$$P_{crit} = \left(\frac{k^2}{\frac{b^2\pi^2}{L^2}}\right)\left(\frac{\pi^2 EI_A}{L^2}\right) \qquad (39)$$

Referring back to the geometry of the beam-column



$$\frac{D_A}{D_B} = \frac{b}{b-L}$$

Figure 7. Geometry of variable beam column.

and using D_A/D_B as a variable to define the geometry of the beam columns

$$\frac{b}{b - L} = \frac{D_A}{D_B} = k_1 \qquad (40)$$

Next, considering the variable spring constant end restraint at large end of beam column, this spring constant (k_2) varies from zero with pinned end to infinity at a fixed end condition. This spring constant is defined in the following terms:

$$k_2 = \text{spring constant} = \frac{C}{EI/L}$$

Assuming the following values

$$L = 1$$

$$E = 1$$

$$I_A = 1$$

then

$$k_2 = C \qquad (41)$$

k_1 from equation (40) becomes

$$k_1 = \frac{b}{b - 1}$$

or rearranging

$$b = \frac{-k_1}{1 - k_1}$$

Referring back to equation (36) and substituting the new values of L, E, I_A, C, and b into each term and multiplying each of these terms by

$$\frac{C^2 L^3}{L^4}$$

the following non-dimensionalized terms are obtained:

$$\frac{DHF}{C} = \sin k\,(1 - \frac{L}{b})(\cos \frac{k}{1 - \frac{L}{b}})(\frac{b^2}{k^2 EI_A})(\frac{1}{C})(\frac{C^2L^3}{L^4})$$

$$= (\sin k)[\frac{k_1 + 1 - k_1}{k_1}]\,[\frac{\cos k}{\frac{k_1 + 1 - k_1}{k_1}}]\,[\frac{-k_1}{1 - k_1}]\,[\frac{-k_1}{1 - k_1}]\,(\frac{1}{k^2})(k_2)$$

$$= \frac{k_1 k_2}{k^2(1 - k_1)^2}\,(\sin k)(\cos k\,k_1)$$

$$DF^2U = \frac{K_2^2}{k^4}\,[\frac{k_1}{1-k_1}]^3(\sin k \cos k\,k_1 + k\,k_1 \sin k \sin k\,k_1)$$

$$DF^2W = (\frac{k_1}{1-k_1})^3(\frac{k_2^2}{k^4})\,(\cos k \sin k + k \sin^2 k)$$

$$\frac{DLFU}{C} = (\frac{k_1}{1-k_1})(\frac{k_2}{k^2})\,(\sin k \cos k\,k_1 + k\,k_1 \sin k \sin k\,k_1)$$

$$\frac{EGF}{C} = (\frac{k_1}{1-k_1})^2(\frac{k_2}{k^2 k_1})\,(\cos k \sin k\,k_1)$$

$$EF^2T = -(\frac{k_1}{1-k_1})^3\,(\frac{k_2^2}{k^4})\,(k\,k_1 \cos k \cos k\,k_1 - \cos k \sin k\,k_1)$$

$$EF^2V = -(\frac{k_1}{1-k_1})^3(\frac{k_2^2}{k^4})\,(k \cos^2 k - \cos k \sin k)$$

$$\frac{ELFT}{C} = -\left(\frac{k_1}{1-k_1}\right)\left(\frac{k_2}{k^2}\right)(k\,k_1 \cos k \cos k\,k_1 - \cos k \sin k\,k_1)$$

$$F^2GU = \left(\frac{k_1}{1-k_1}\right)^3\left(\frac{k_2^2}{k^4 k_1}\right)(\sin k\,k_1 \cos k\,k_1 + k\,k_1 \sin^2 k\,k_1)$$

$$GF^2W = \left(\frac{k_1}{1-k_1}\right)^3\left(\frac{k_2^2}{k^4 k_1}\right)(\sin k\,k_1 \cos k + k \sin k \sin k\,k_1)$$

$$F^2HT = -\left(\frac{k_1}{1-k_1}\right)^3\left(\frac{k_2^2}{k^4 k_1}\right)(k\,k_1 \cos^2 k\,k_1 - \cos k\,k_1 \sin k\,k_1)$$

$$F^2HV = -\left(\frac{k_1}{1-k_1}\right)^3\left(\frac{k_2^2}{k_1 k^4}\right)(k \cos k\,k_1 \cos k - \cos k\,k_1 \sin k)$$

$$F^2LTW = -\left(\frac{k_1}{1-k_1}\right)^2\left(\frac{k_2^2}{k^4}\right)(\cos k + k \sin k)(k\,k_1 \cos k\,k_1 - \sin k\,k_1)$$

$$F^2LUV = -\left(\frac{k_1}{1-k_1}\right)^2\left(\frac{k_2^2}{k^4}\right)(k \cos k - \sin k)(\cos k\,k_1 + k\,k_1 \sin k\,k_1)$$

Designating each of these terms by a letter and number and factoring out

$$\left[\frac{k_1 k_2}{k^2(1-k_1)}\right]$$

from each term the following terms are obtained:

$$A_1 = \frac{1}{(1-k_1)}(\sin k \cos k\, k_1)$$

$$A_2 = \frac{k_1^{\,2} k_2}{k^2(1-k_1)^2}(\sin k \cos k\, k_1 + k\, k_1 \sin k \sin k\, k_1)$$

$$A_3 = \frac{k_1^{\,2} k_2}{k^2(1-k_1)^2}(\cos k \sin k + k \sin^2 k)$$

$$A_4 = (\sin k \cos k\, k_1 + k\, k_1 \sin k \sin k\, k_1)$$

$$A_5 = \frac{1}{(1-k_1)}(\cos k \sin k\, k_1)$$

$$A_6 = \frac{-k_1^{\,2} k_2}{k^2(1-k_1)^2}(k\, k_1 \cos k \cos k\, k_1 - \cos k \sin k\, k_1)$$

$$A_7 = \frac{-k_1^{\,2} k_2}{k^2(1-k_1)^2}(k \cos^2 k - \cos k \sin k)$$

$$A_8 = -(k\, k_1 \cos k \cos k\, k_1 - \cos k \sin k\, k_1)$$

$$A_9 = \frac{k_1 k_2}{k^2(1-k_1)^2}(\sin k\, k_1 \cos k\, k_1 + k\, k_1 \sin^2 k\, k_1)$$

$$A_{10} = \frac{k_1 k_2}{k^2(1-k_1)^2} (\sin k k_1 \cos k + k \sin k \sin k k_1)$$

$$A_{11} = \frac{-k_1 k_2}{k^2(1-k_1)^2} (k k_1 \cos^2 k k_1 - \cos k k_1 \sin k k_1)$$

$$A_{12} = \frac{-k_1 k_2}{k^2(1-k_1)^2} (k \cos k k_1 \cos k - \cos k k_1 \sin k)$$

$$A_{13} = \frac{-k_1 k_2}{k^2(1-k_1)} (\cos k + k \sin k)(k k_1 \cos k k_1 - \sin k k_1)$$

$$A_{14} = \frac{-k_1 k_2}{k^2(1-k_1)} (k \cos k - \sin k)(\cos k k_1 + k k_1 \sin k k_1)$$

Rearranging terms to minimize computation by computer as follows:

$$T1 = 1/(1-k_1)$$

$$T2 = T1(k_1)\left(\frac{k_2}{k}\right)^2$$

$$T3 = T2\,(T1)$$

$$T4 = T3\,(k_1)$$

$$T5 = \sin(k)$$

$$T6 = \sin(k k_1)$$

$$T7 = \cos(k)$$

$$T8 = \cos(k\, k_1)$$

$$T9 = k\, k_1$$

The terms A_1 to A_{14} can now be rewritten as follows:

$$A1 = (T1)(T5)(T8)$$

$$A2 = T4\,[(T5)(T8) + (T9)(T5)(T6)]$$

$$A3 = T4\,[(T7)(T5) + (k)(T5)^2]$$

$$A4 = (T5)(T8) + (T9)(T5)(T6)$$

$$A5 = (T1)(T7)(T6)$$

$$A6 = -T4\,[(T9)(T7)(T8) - (T7)(T6)]$$

$$A7 = -\,T4\,[(k)(T7)^2 - (T7)(T5)]$$

$$A8 = -(T9)(T7)(T8) + (T7)(T6)$$

$$A9 = T3\,[(T6)(T8) + (T9)(T6)^2]$$

$$A10 = T3\,[(T6)(T7) + (k)(T5)(T6)]$$

$$A11 = -T3\,[(T9)(T8)^2 - (T8)(T6)]$$

$$A12 = -T3\,[(k)(T8)(T7) - (T8)(T5)]$$

$$A13 = -T2\,[T7 + (k)(T5)]\,[(T9)(T8) - T6]$$

$$A14 = -T2\,[(k)(T7) - T5]\,[T8 + (T9)(T6)]$$

Referring back now to equation (36) and substituting in the new values for each of the 14 terms we obtain:

$$A1 - A2 + A3 - A4 - A5 + A6 - A7 + A8 + A9 - A10 - A11 + A12 + A13 - A14 = 0 \qquad (42)$$

This equation is in terms of three non-dimensional terms k, k_1 and k_2.

k_1 Defines the geometry of the column taper.

k_2 Defines the spring constant or relative amount of end fixity at the large end of the column

and

$$k = \sqrt{\frac{P_{crit} b^2}{EI_A}}$$

Values can be assigned to k_1 and k_2 and equation (42) solved to obtain the corresponding value of k.

This value of k can then be used in equations (38) and (39) to obtain the value of the constant:

$$\frac{k^2}{\frac{b^2 \pi^2}{L^2}}$$

Using the values already assigned to b and L this constant reduces to

$$\frac{k^2 (k_1 - 1)^2}{\pi^2 k_1^2} = C_{IA}$$

where

$$P_{crit} = C_{IA} \left(\frac{\pi^2 EI_A}{L^2}\right) \tag{43}$$

Equation (43) is in terms of the moment of inertia at the large end of the column but can be rewritten in terms of the moment of inertia at the small end I_B as follows:

$$\frac{I_A}{I_B} \sim \frac{(D_A)^4}{(D_B)^4} \sim k_1{}^4$$

$$C_{IB} = (k_1{}^4)(C_{IA})$$

Substituting this into equation (43) and using I_B we obtain

$$P_{crit} = C_{IB}\left(\frac{\pi^2 EI_B}{L^2}\right)$$

Values have been obtained for k, C_{IA} and C_{IB} for each value of k_1 and k_2 by use of a computer. The computer program and flow chart are included in Appendix A together with the tabulated numerical output.

The data obtained from the computer has been plotted on Figures 8, 9, 10 and 11.

The values of C_{IB} (to be used with the moment of inertia at the small end of the column) and the values of C_{IA} (to be used with the moment of inertia, I_A, at the large end of the column) are plotted with respect to k_1. Values of k_1 from 1.3 to 10.0 were used in the computations. The normal taper D_A/D_B is less than 10 but

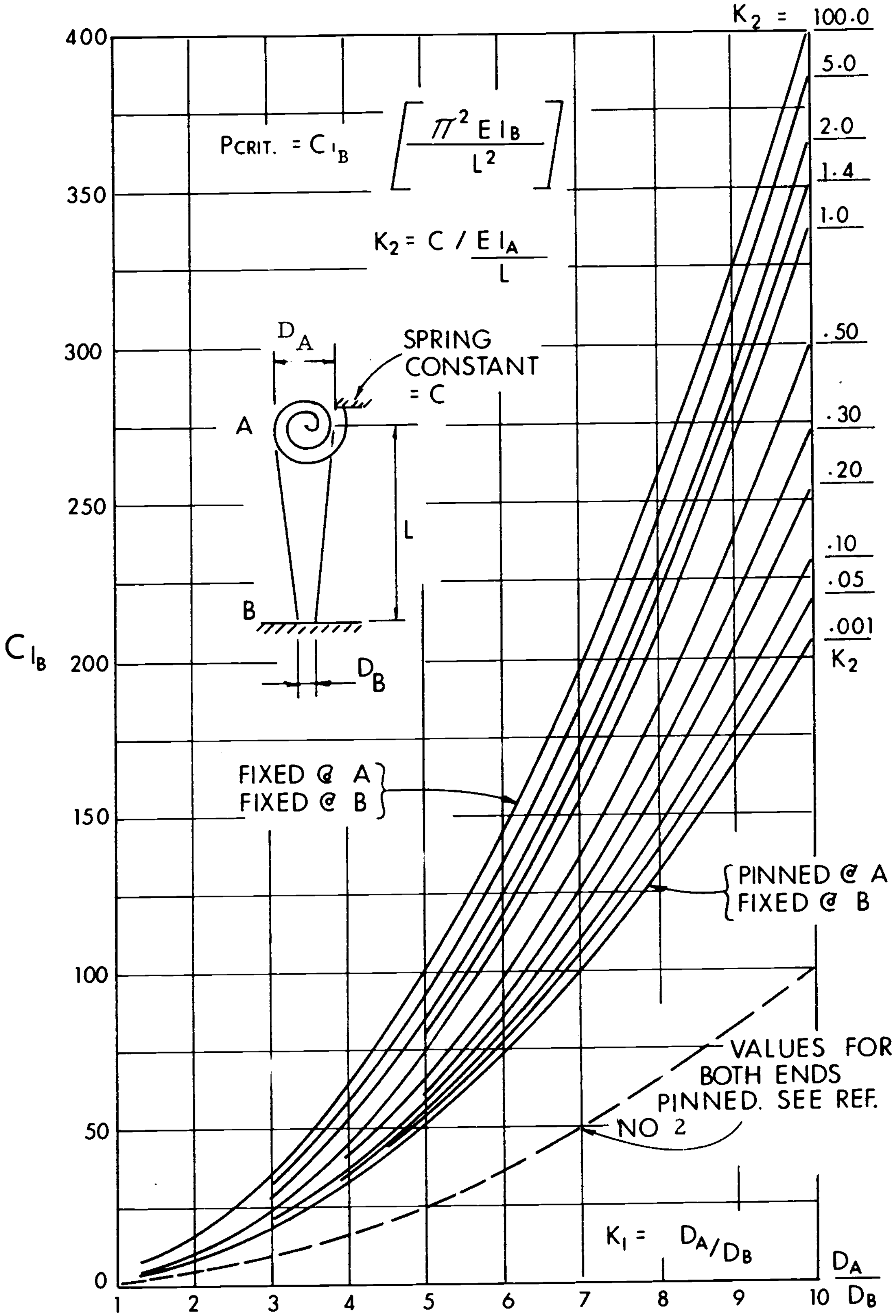


FIGURE 8 CRITICAL BUCKLING COEFFICIENTS, C_{I_B}

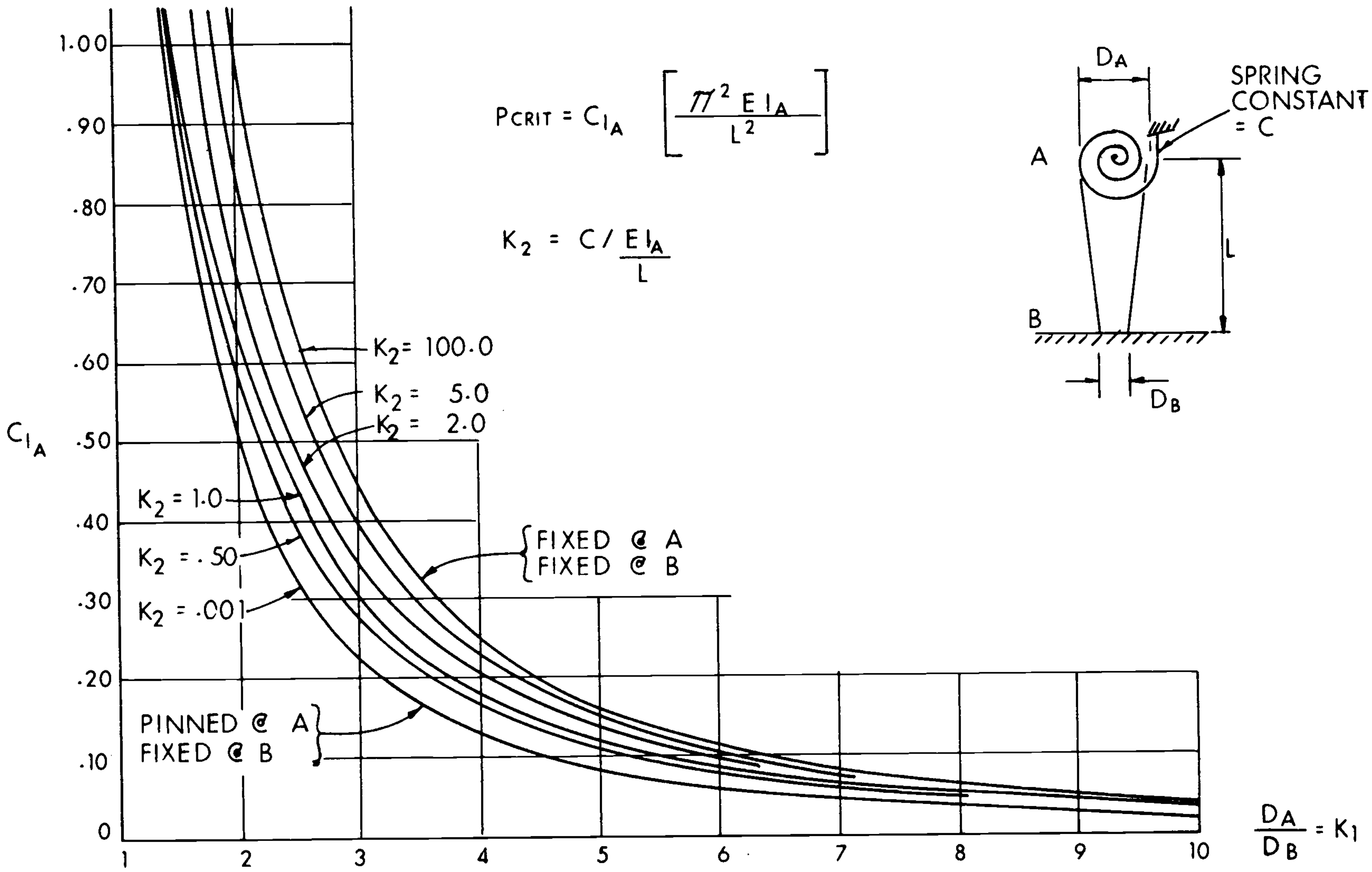


FIGURE 9 CRITICAL BUCKLING COEFFICIENTS C_{1_A}

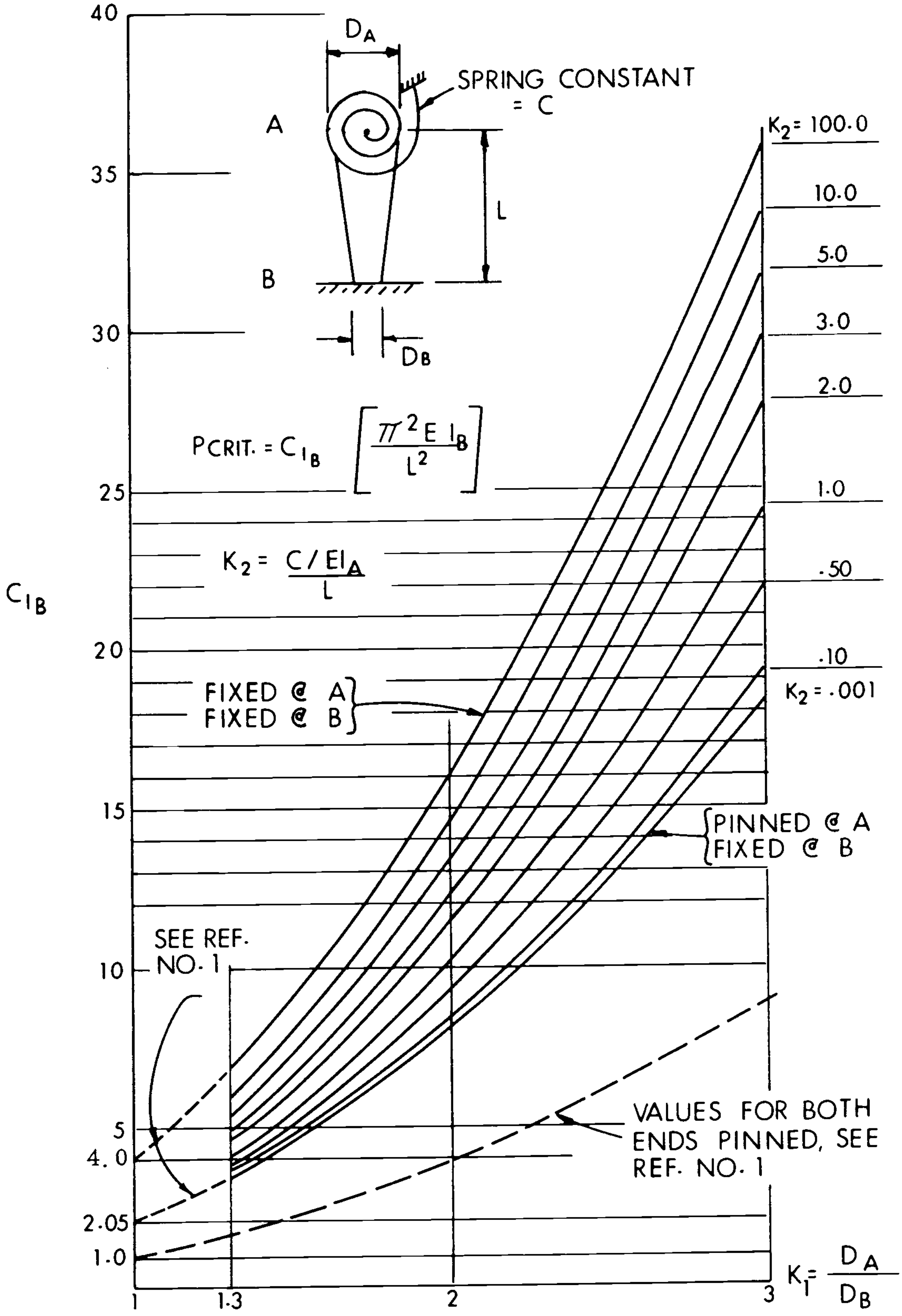


FIGURE 10 CRITICAL BUCKLING COEFFICIENTS, C_{1_B}

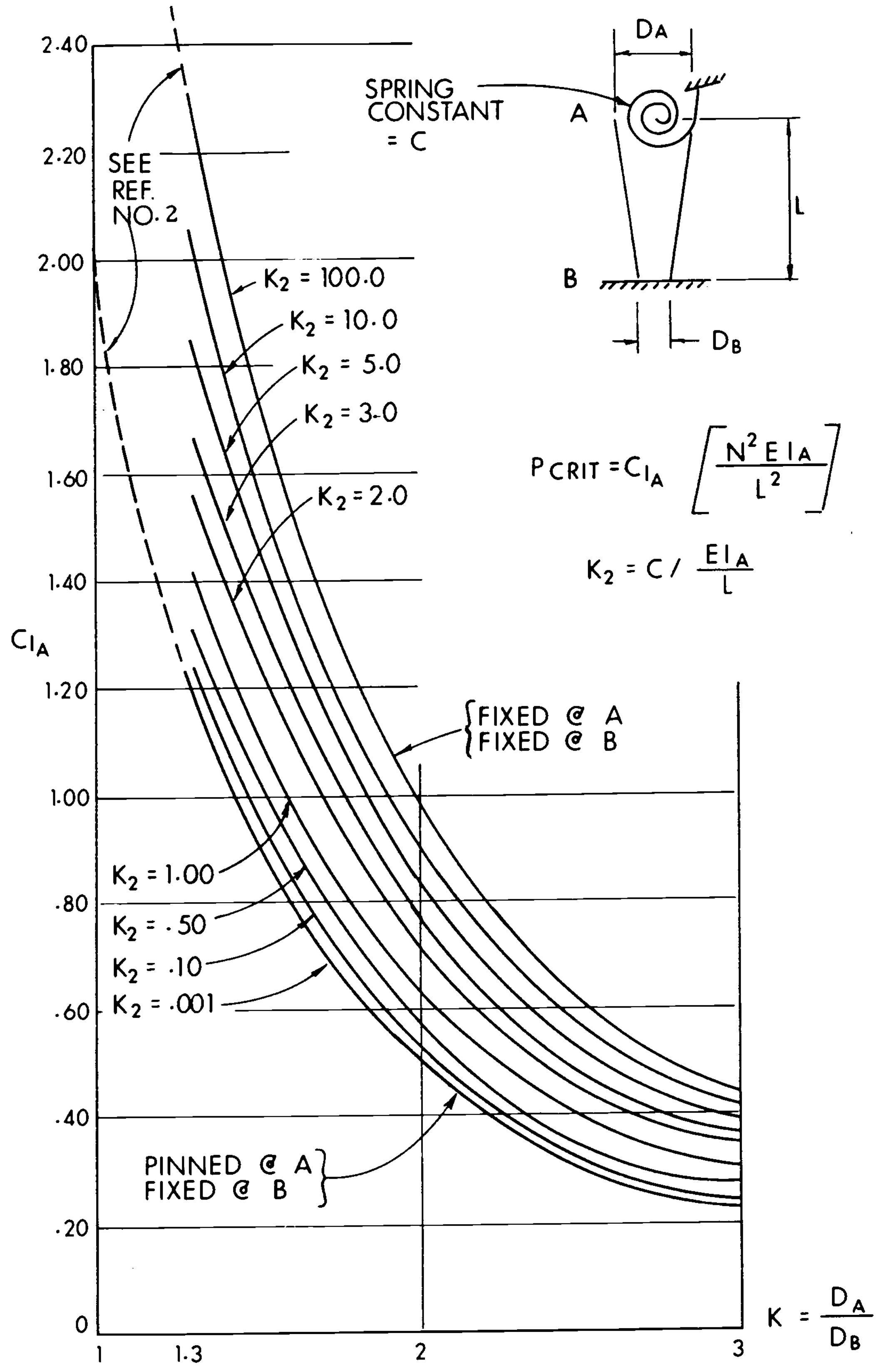


FIGURE 11 CRITICAL BUCKLING COEFFICENTS, C_{I_A}

additional values can be obtained easily, if necessary, by using this computer program. The equations used in this thesis cannot be used as k_1 approaches 1.0. The values of C_{IB} and C_{IA} for $k_1 = 1.0$ for the two extreme cases of end fixity are shown on Figure 10. These values were obtained from reference 1.

Figures 10 and 11 represent the same data as Figures 8 and 9 except that the scales have been increased in order that the curves can be more useable in the lower range of values for D_A/D_B.

CONCLUSIONS AND DISCUSSION

Coefficients to be used to calculate the critical elastic buckling loads for round uniformly tapered columns with a fixed end-variable fixed end condition have been determined in this thesis. The columns are considered fixed at the small end. The fixity at the large end of the columns varies from zero to 100 percent while translation at the large end is prevented. These values have been plotted in Figures 8, 9, 10 and 11. The actual computer printout data is shown in Appendix. These values apply to columns of any construction material, e.g., wood, steel, concrete, etc.

As discussed earlier in this thesis the results presented can be applied directly to the problems associated with the use of timber piling for the construction of waterfront or other similar structures. The small end of the piling is driven to a point of fixity while the large end is attached to a superstructure. The use of batter piles, anchor walls or other facilities to prevent translation of the tops of the piling is usually required to resist seismic forces and the lateral forces of ship docking and anchorage.

A review of the data in Appendix shows that when the stiffness, C, of the deck is 100 times the stiffness of the piling (EI_A/L) with respect to the large end of the pile, the piling is essentially fixed at the top. The term k_2 is used to designate this ratio of relative fixity. These conclusions assume that the proper connections between the tapered column or piling and the superstructure is made efficiently. If these connections are not made properly some loss in relative stiffness will result which will affect the fixity at the top end of the column and the resulting critical buckling load. The type of connection to be used and the efficiency of such a connection is important to the design engineer but this is beyond the scope of this thesis. This should not be overlooked in practice because some connections may never develop the amount of fixity assumed and the actual critical buckling load my be considerably less than that obtained from the curves in Figures 8, 9, 10 and 11.

Previous articles (2) have given values for the critical buckling load for the fixed-fixed case and the fixed-pinned case. This thesis substantiates these values and also provides the values for intermediate partial fixity-fixed cases. The values of the constant C_{IA} and C_{IB} used to determine the critical buckling load can be obtained by first determining the ratio of D_A/D_B then entering the curves for critical buckling coefficients and reading upward until the approximate curve for k_2 is reached and then horizontally to the scale for the coefficient.

The critical buckling load is then computed using one of the following equations:

$$P_{crit} = C_{IA}\left(\frac{\pi^2 EI_A}{L^2}\right)$$

or

$$P_{crit} = C_{IB}\left(\frac{\pi^2 EI_B}{L^2}\right)$$

The critical buckling load for the fixed-fixed case is approximately 200 percent of the fixed-pinned case. The critical buckling load for the case where the stiffness of the column based upon the moment of inertia at the larger end is equal to the stiffness of the deck, $k_2 = 1.0$, is approximately 168 percent of the fixed-pinned case. Therefore, where partial or full fixity can be obtained, a substantial increase in allowable column load can be realized over the pinned end case. In some structures it may be difficult and costly to obtain full fixity at the top of the piling. Partial fixity, however, may be obtained relatively easy and at lower cost. In the past, information has not been available for the partial fixity case and design engineers have been forced to design supporting columns for the fixed-pinned case even though some partial fixity was in fact obtained. The data presented in this thesis allows a much more economical use of timber piling than has been possible before. The formulas and data shown are not limited only to tapered wood columns and may be used for any material, provided all other design requirements are satisfied.

An example will be used to show how the curves in Figures 8, 9, 10 and 11 are to be used. Using the piling shown in Figure 1, assume the following:

Combined stiffness of Deck,

$$C_{Deck} = \left(\frac{EI}{L}\right)_{Deck} = 4 \times 10^7$$

Modulus of elasticity of piling $= 1.6 \times 10^6$ psi

Unsupported length of piling, L = 70 feet

Diameter of piling at Top, D_A = 20 inches

Diameter of piling at point of fixity D_B = 10 inches

$$\frac{D_A}{D_B} = k_1 = 20/10 = 2.0$$

$$I_A = \frac{\pi D_A^{\ 4}}{64} = 7850 \text{ in}^4$$

$$I_B = \frac{\pi D_B^{\ 4}}{64} = 492 \text{ in}^4$$

$$\frac{EI_A}{L} = \frac{1.6 \times 10^6 \text{ psi} \times 7850 \text{ in}^4}{840 \text{ in}} = 1.5 \times 10^7$$

$$k_2 = \frac{C/EI_A}{L} = \frac{4 \times 10^7}{1.5 \times 10^7} = 2.67$$

Entering the charts, Figures 10 and 11 at $k_1 = 2.0$ and moving up vertically to the intersection of $k_2 = 2.67$ on both sets of curves and then proceeding horizontally, we read:

$C_{IA} = 0.75$

$C_{IB} = 12.0$

$$P_{crit} = C_{IA}\left[\frac{\pi^2 EI_A}{L^2}\right] = 0.75\left[\frac{\pi^2 \times 1.6 \times 10^6 \times 7850}{840 \times 840}\right]$$

$$= \underline{\underline{132 \text{ kips}}}$$

or

$$P_{crit} = C_{IB}\left[\frac{\pi^2 EI_B}{L^2}\right] = 12.0\left[\frac{\pi^2 \times 1.6 \times 10^6 \times 492}{840 \times 840}\right]$$

$$= \underline{\underline{132 \text{ kips}}}$$

If this same pile is assumed to be fixed at the top with $k_2 \geq 100$

$C_{IA} = 1.00$

$$P_{crit} = \frac{1.00}{.75} \times 132^k = \underline{\underline{176 \text{ kips}}}$$

$C_{IB} = 16.0$

$$P_{crit} = \frac{16.0}{12.0} \times 132^k = \underline{\underline{176 \text{ kips}}}$$

If this same pile is assumed to be pinned at the top connection to the deck,

$k_2 \leq .001$

$C_{IA} = .51$

$P_{crit} = \frac{.51}{.75} \times 132^k = \underline{\underline{89.6 \text{ kips}}}$

$C_{IB} = 8.15$

$P_{crit} = \frac{8.15}{12.0} \times 132^k = \underline{\underline{89.6 \text{ kips}}}$

The following table will show how the critical buckling load varies with the amount of fixity at the large end.

Amount of Fixity k_2	C_{IA}	C_{IB}	Critical Buckling Load	Design Load Assuming F. S. = 2.5	Rule of Thumb Method
.001 (pinned)	.51	8.15	89.6^k	35.8^k	26.0^k
.50	.58	9.3	102^k	40.8^k	
1.0	.64	10.2	112^k	44.8^k	
2.67	.75	12.0	132^k	51.8^k	
5.0	.84	13.4	147.5^k	59.0^k	
10.0	.91	14.55	160^k	64.0^k	
100 fixed	1.00	16.0	176^k	70.5^k	50.7^k

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APPENDIX

COMPUTER PROGRAM

```
      PROGRAM GFD
C     THIS PROGRAM IS TØ BE USED TØ SØLVE AN EQUATIØN FØR THE VALUE ØF K
      TØ BE USED TØ BE USED TØ DETERMINE THE CRITICAL BUCKLING LØAD FØR A
      RØUND TAPERED CØLUMN WITH VARIABLE END FIXITY.
      REAL K, K1, K2
      DIMENSIØN ANSW(1000)
      IØ=61
      IN=60
      WRITE(IØ, 2)
         DØ 900 I=1, 28
         READ (IN, 13)K2
      DØ 900 J=1, 10
      K1=J
      IF(K1-1.)30, 25, 30
25    K1=1.3
30    CØNTINUE
      DELTA=.1
      START=0.3
      DØ 500 M=1, 1000
      K=START+DELTA*FLØAT(M)
      T1=1./(1.-K1)
      T2=T1*K1*K2/K**2
      T3=T2*T1
      T4=T3*K1
      T5=SIN(K)
      T6=SIN(K*K1)
      T7=CØS(K)
      T8=CØS(K*K1)
      T9=K*K1
      A1=T1*T5*T8
      A2=T4*(T5*T8+T9*T5*T6)
      A3=T4*(T7*T5+K*T5**2)
      A4=T5*T8+T9*T5*T6
      A5=T1*T7*T6
      A6=-T4*(T9*T7*T8-T7*T6)
      A7=-T4*(K*T7**2-T7*T5)
      A8=-T9*T7*T8+T7*T6
      A9=T3*(T6*T8+T9*T6**2)
      A10=T3*(T6*T7+K*T5*T6)
      A11=-T3*(T9*T8**2-T8*T6)
      A12=-T3*(K*T8*T7-T8*T5)
      A13=-T2*(T7+K*T5)*(T9*T8-T6)
      A14=-T2*(K*T7-T5)*(T8+T9*T6)
      ANSW(M)=A1-A2+A3-A4-A5+A6-A7+A8+A9=A10-A11+A12+A13-A14
      IF(ABS(ANSW(M))-.001)600, 600, 700
700   IF(M-1)800, 500, 800
```

```
800   IF((ABS(ANSW(M))+ABS(ANSW(M-1)))-ABS(ANSW(M)+ANSW(M-1)))500, 500, 40
 40   Q1=K-DELTA
      Q2=K
      IF(ANSW(M))5, 5, 9
  9   Q2=K-DELTA
      Q1=K
  5   B=Q1
      A=Q2
  6   X=(A+B)/2.
      K=X
      T1=1./(1.-K1)
      T2=T1*K1*K2/K**2
      T3=T2*T1
      T4=T3*K1
      T5=SIN(K)
      T6=SIN(K*K1)
      T7=CØS(K)
      T8=CØS(K*K1)
      T9=K*K1
      A1=T1*T5*T8
      A2=T4*(T5*T8+T9*T5*T6)
      A3=T4*(T7*T5+K*T5**2)
      A4=T5*T8+T9*T5*T6
      A5=T1*T7*T6
      A6=-T4*(T9*T7*T8-T7*T6)
      A7=-T4*(K*T7**2-T7*T5)
      A8=-T9*T7*T8+T7*T6
      A9=T3*(T6*T8+T9*T6**2)
      A10=T3*(T6*T7+K*T5*T6)
      A11=-T3*(T9*T8**2-T8*T6)
      A12=-T3*(K*T8*T7-T8*T5)
      A13=-T2*(T7+K*T5)*(T9*T8-T6)
      A14=-T2*(K*T7-T5)*(T8+T9*T6)
      ANSW(M)=A1-A2+A3-A4-A5+A6-A7+A8+A9-A10-A11+A12+A13-A14
      Y=ANSW(M)
      IF(ABS(B-X)-.0001)3, 3, 4
  3   GØ TØ 600
  4   IF(Y)7, 3, 8
  7   A=X
      GØ TØ 6
  8   B=X
      GØ TØ 6
600   CIA=(K**2*(K1-1.)**2)/(3.1416**2*K1**2)
      CIB=K1**4*CIA
      WRITE(IØ, 1)K2, K1, K, CIA, CIB
      GØ TØ 900
500   CØNTINUE
900   CØNTINUE
```

```
1      FØRMAT(1X, F15.6, 2F10.3, 2F18.3)
2      FØRMAT(8X, K2=C/EIA/L, 2X, K1=DA/DB', 6X, 'K', 16X, 'CIA', 16X, CIB')
13     FØRMAT(F6.3)
       STØP
       END
```

PRØGRAM G DATA

```
00001:.0001
00002:.001
00003:.01
00004:.05
00005:.10
00006:.20
00007:.30
00008:.40
00009:.50
00010:.60
00011:.70
00012:.80
00013:.90
00014:1.0
00015:1.1
00016:1.2
00017:1.3
00018:1.4
00019:2.0
00020:3.0
00021:4.0
00022:5.0
00023:6.0
00024:8.0
00025:9.0
00026:10.0
00027:100.0
00028:1000.0
1
#UNEQUIP, 1
#UNEQUIP, 60
#EQUIP, 1=GFD22
#EQUIP, 60-GDATA
#FØRTRAN, I=1, R
```

$K2=C/EI_A/L$	K1=DA/DB	K	CIA	CIB
.000100	1.300	14.978	1.211	3.457
.000100	2.000	4.493	.511	8.183
.000100	3.000	2.247	.227	18.413
.000100	4.000	1.498	.128	32.730
.000100	5.000	1.123	.082	51.142
.000100	6.000	.899	.057	73.655
.000100	7.000	.749	.042	100.248
.000100	8.000	.642	.032	130.919
.000100	9.000	.562	.025	165.672
.000100	10.000	.499	.020	204.614
.001000	1.300	14.980	1.211	3.453
.001000	2.000	4.494	.512	8.185
.001000	3.000	2.247	.227	18.419
.001000	4.000	1.498	.128	32.755
.001000	5.000	1.124	.082	51.178
.001000	6.000	.899	.057	73.719
.001000	7.000	.749	.042	100.352
.001000	8.000	.642	.032	131.078
.001000	9.000	.562	.025	165.903
.001000	10.000	.500	.020	204.934
.010000	1.300	14.993	1.213	3.464
.010000	2.000	4.500	.513	8.208
.010000	3.000	2.252	.228	18.496
.010000	4.000	1.502	.129	32.935
.010000	5.000	1.128	.082	51.534
.010000	6.000	.903	.057	74.328
.010000	7.000	.753	.042	101.296
.010000	8.000	.646	.032	132.517
.010000	9.000	.566	.026	167.985
.010000	10.000	.503	.021	207.666
.050000	1.300	15.052	1.223	3.492
.050000	2.000	4.527	.519	8.307
.050000	3.000	2.272	.232	18.831
.050000	4.000	1.520	.132	33.719
.050000	5.000	1.144	.085	53.063
.050000	6.000	.918	.059	76.923
.050000	7.000	.768	.044	105.437
.050000	8.000	.660	.034	138.597
.050000	9.000	.580	.027	176.562
.050000	10.000	.517	.022	219.440
.100000	1.300	15.125	1.234	3.526
.100000	2.000	4.561	.527	8.430
.100000	3.000	2.296	.237	19.235
.100000	4.000	1.541	.135	34.661
.100000	5.000	1.164	.088	54.871
.100000	6.000	.937	.062	79.996
.100000	7.000	.785	.046	110.209
.100000	8.000	.677	.036	145.652
.100000	9.000	.596	.028	186.451
.100000	10.000	.533	.023	232.731

$K2=C/EI_A/L$	K1=DA/DB	K	CIA	CIB
.200000	1.300	15.266	1.258	3.592
.200000	2.000	4.624	.542	8.664
.200000	3.000	2.342	.247	20.002
.200000	4.000	1.580	.142	36.413
.200000	5.000	1.198	.093	58.180
.200000	6.000	.968	.066	85.527
.200000	7.000	.815	.049	118.700
.200000	8.000	.705	.039	157.917
.200000	9.000	.622	.031	203.319
.200000	10.000	.558	.026	255.096
.300000	1.300	15.402	1.280	3.656
.300000	2.000	4.683	.556	8.889
.300000	3.000	2.383	.256	20.719
.300000	4.000	1.614	.148	38.015
.300000	5.000	1.228	.098	61.138
.300000	6.000	.996	.070	90.389
.300000	7.000	.840	.052	125.977
.300000	8.000	.728	.041	168.230
.300000	9.000	.643	.033	217.208
.300000	10.000	.577	.027	273.097
.400000	1.300	15.533	1.302	3.718
.400000	2.000	4.740	.569	9.104
.400000	3.000	2.421	.264	21.386
.400000	4.000	1.645	.154	39.482
.400000	5.000	1.255	.102	63.791
.400000	6.000	1.019	.073	94.622
.400000	7.000	.860	.055	132.207
.400000	8.000	.746	.043	176.827
.400000	9.000	.660	.035	228.567
.400000	10.000	.592	.029	287.712
.500000	1.300	15.659	1.323	3.779
.500000	2.000	4.792	.582	9.308
.500000	3.000	2.457	.272	22.011
.500000	4.000	1.673	.159	40.825
.500000	5.000	1.278	.106	66.157
.500000	6.000	1.038	.076	98.322
.500000	7.000	.877	.057	137.605
.500000	8.000	.761	.045	184.122
.500000	9.000	.673	.036	238.140
.500000	10.000	.604	.030	299.601
.600000	1.300	15.780	1.344	3.838
.600000	2.000	4.843	.594	9.504
.600000	3.000	2.489	.279	22.596
.600000	4.000	1.698	.164	42.054
.600000	5.000	1.298	.109	68.277
.600000	6.000	1.056	.078	101.604
.600000	7.000	.892	.059	142.237
.600000	8.000	.774	.046	190.411
.600000	9.000	.684	.038	246.079
.600000	10.000	.614	.031	309.561

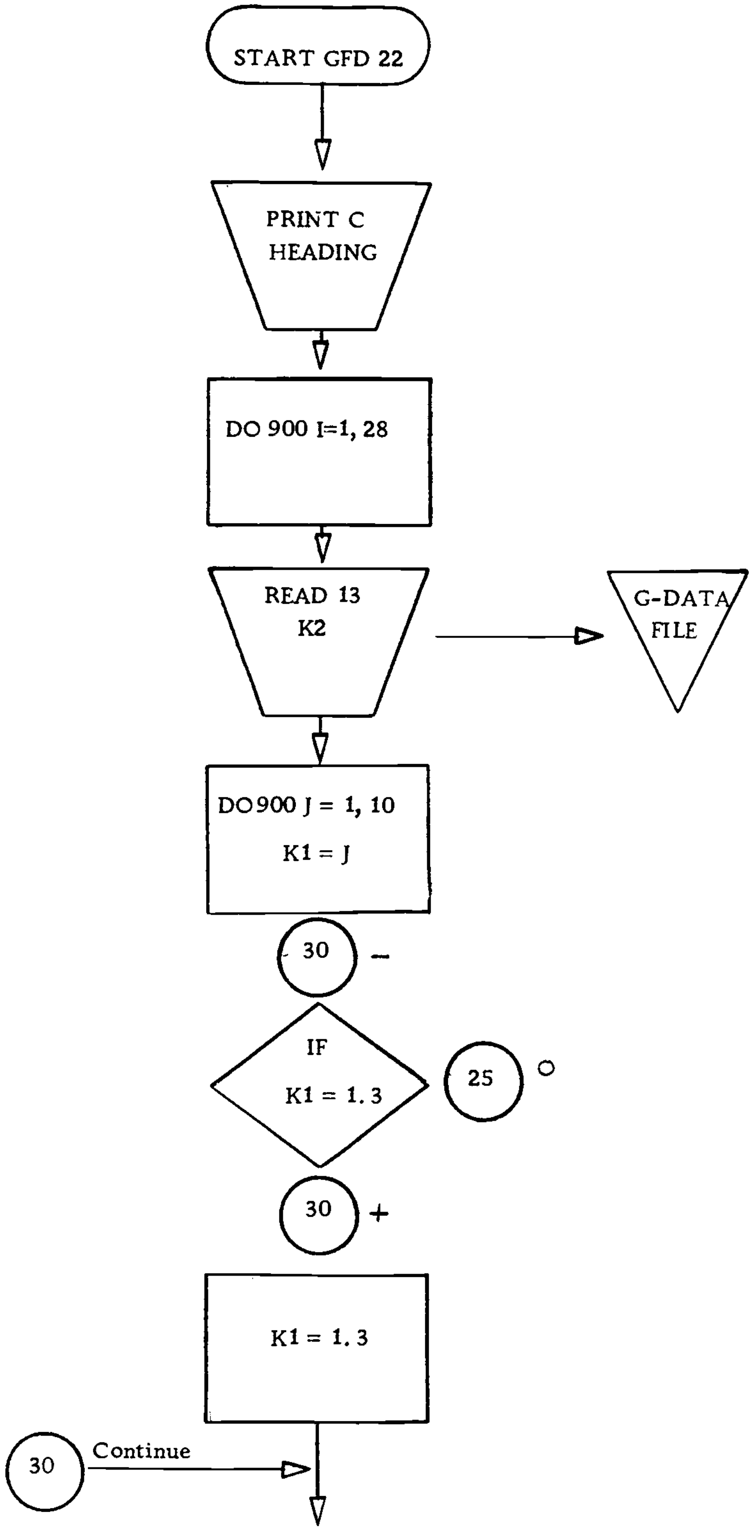
$K2=C/EI_A/L$	K1=DA/DB	K	CIA	CIB
.700000	1.300	15.897	1.364	3.895
.700000	2.000	4.890	.606	9.692
.700000	3.000	2.519	.286	23.142
.700000	4.000	1.720	.169	43.174
.700000	5.000	1.316	.112	70.201
.700000	6.000	1.071	.081	104.520
.700000	7.000	.905	.061	146.315
.700000	8.000	.785	.048	195.732
.700000	9.000	.694	.039	252.367
.700000	10.000	.622	.032	317.886
.800000	1.300	16.009	1.383	3.950
.800000	2.000	4.935	.617	9.871
.800000	3.000	2.547	.292	23.655
.800000	4.000	1.741	.173	44.210
.800000	5.000	1.332	.115	71.920
.800000	6.000	1.084	.083	107.091
.800000	7.000	.916	.062	149.810
.800000	8.000	.794	.049	200.338
.800000	9.000	.702	.039	258.593
.800000	10.000	.629	.032	324.706
.900000	1.300	16.118	1.402	4.004
.900000	2.000	4.978	.628	10.042
.900000	3.000	2.572	.298	24.132
.900000	4.000	1.759	.176	45.157
.900000	5.000	1.347	.118	73.489
.900000	6.000	1.095	.084	109.380
.900000	7.000	.925	.064	152.958
.900000	8.000	.802	.050	204.299
.900000	9.000	.708	.040	263.656
.900000	10.000	.635	.033	330.783
1.000000	1.300	16.223	1.420	4.056
1.000000	2.000	5.018	.638	10.206
1.000000	3.000	2.596	.303	24.582
1.000000	4.000	1.776	.180	46.034
1.000000	5.000	1.359	.120	74.903
1.000000	6.000	1.105	.086	111.418
1.000000	7.000	.933	.065	155.748
1.000000	8.000	.809	.051	207.898
1.000000	9.000	.714	.041	267.889
1.000000	10.000	.640	.034	335.891
1.100000	1.300	16.324	1.438	4.106
1.100000	2.000	5.057	.648	10.363
1.100000	3.000	2.618	.309	25.005
1.100000	4.000	1.792	.183	46.847
1.100000	5.000	1.371	.122	76.200
1.100000	6.000	1.115	.087	113.277
1.100000	7.000	.941	.066	158.169
1.100000	8.000	.815	.051	210.921
1.100000	9.000	.719	.041	271.712
1.100000	10.000	.644	.034	340.418

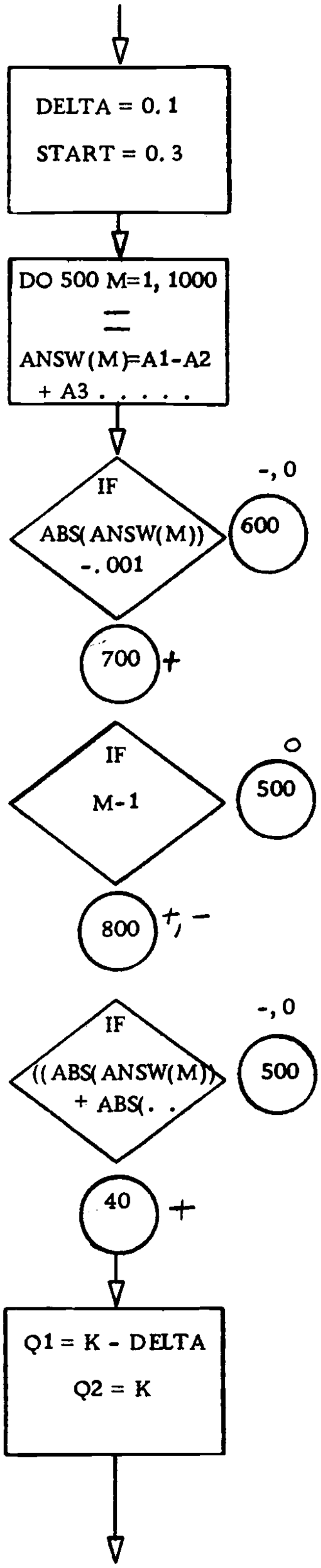
$K2=C/EI_A/L$	K1=DA/DB	K	CIA	CIB
1.200000	1.300	16.421	1.455	4.156
1.200000	2.000	5.093	.657	10.513
1.200000	3.000	2.639	.314	25.402
1.200000	4.000	1.806	.186	47.596
1.200000	5.000	1.382	.124	77.377
1.200000	6.000	1.123	.089	114.950
1.200000	7.000	.947	.067	160.343
1.200000	8.000	.820	.052	213.762
1.200000	9.000	.724	.042	274.968
1.200000	10.000	.648	.034	344.353
1.300000	1.300	16.515	1.472	4.203
1.300000	2.000	5.128	.666	10.656
1.300000	3.000	2.658	.318	25.776
1.300000	4.000	1.819	.189	48.288
1.300000	5.000	1.391	.126	78.452
1.300000	6.000	1.130	.090	116.435
1.300000	7.000	.953	.068	162.333
1.300000	8.000	.825	.053	216.212
1.300000	9.000	.727	.042	277.945
1.300000	10.000	.651	.035	347.683
1.400000	1.300	16.606	1.488	4.250
1.400000	2.000	5.161	.675	10.794
1.400000	3.000	2.676	.323	26.125
1.400000	4.000	1.831	.191	48.923
1.400000	5.000	1.400	.127	79.447
1.400000	6.000	1.137	.091	117.807
1.400000	7.000	.958	.068	164.135
1.400000	8.000	.829	.053	218.367
1.400000	9.000	.731	.043	280.638
1.400000	10.000	.654	.035	350.820
2.000000	1.300	17.092	1.576	4.502
2.000000	2.000	5.329	.719	11.508
2.000000	3.000	2.764	.344	27.858
2.000000	4.000	1.887	.203	51.964
2.000000	5.000	1.440	.134	83.987
2.000000	6.000	1.166	.096	123.959
2.000000	7.000	.981	.072	171.919
2.000000	8.000	.847	.056	227.831
2.000000	9.000	.745	.044	291.846
2.000000	10.000	.666	.036	363.722
3.000000	1.300	17.722	1.695	4.840
3.000000	2.000	5.527	.774	12.382
3.000000	3.000	2.857	.367	29.767
3.000000	4.000	1.943	.215	55.085
3.000000	5.000	1.477	.141	88.396
3.000000	6.000	1.192	.100	129.671
3.000000	7.000	1.001	.075	178.974
3.000000	8.000	.862	.058	236.317
3.000000	9.000	.758	.046	301.561
3.000000	10.000	.676	.037	374.905

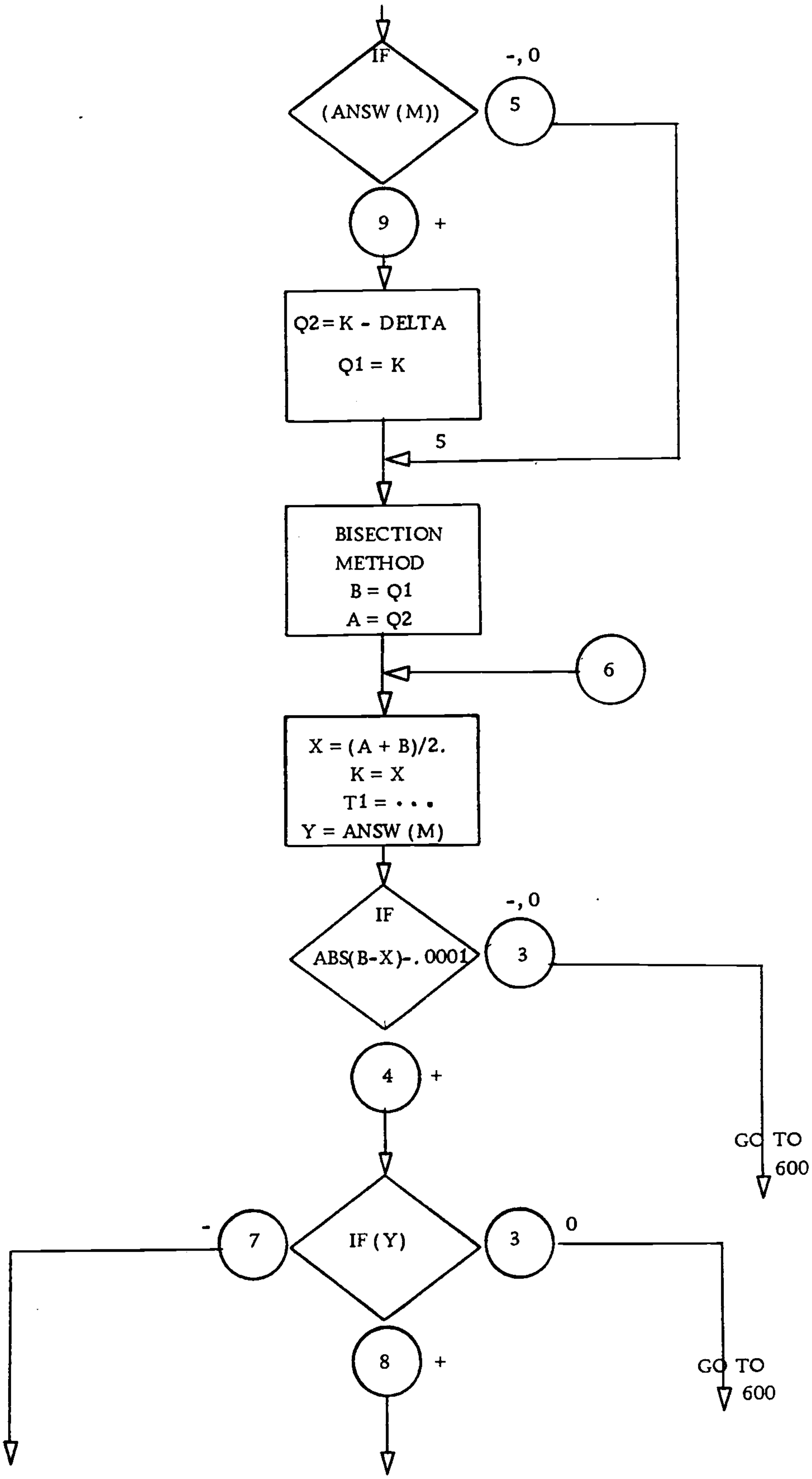
$K2=C/EI_A/L$	K1=DA/DB	K	CIA	CIB
4.000000	1.300	18.192	1.786	5.100
4.000000	2.000	5.662	.812	12.992
4.000000	3.000	2.915	.383	30.989
4.000000	4.000	1.976	.223	56.961
4.000000	5.000	1.498	.146	90.939
4.000000	6.000	1.207	.103	132.919
4.000000	7.000	1.012	.076	182.908
4.000000	8.000	.871	.059	240.942
4.000000	9.000	.764	.047	306.870
4.000000	10.000	.681	.038	380.778
5.000000	1.300	18.552	1.857	5.304
5.000000	2.000	5.758	.840	13.437
5.000000	3.000	2.954	.393	31.821
5.000000	4.000	1.997	.227	58.207
5.000000	5.000	1.511	.148	92.583
5.000000	6.000	1.217	.104	134.992
5.000000	7.000	1.018	.077	185.388
5.000000	8.000	.876	.060	243.760
5.000000	9.000	.768	.047	310.172
5.000000	10.000	.684	.038	384.499
6.000000	1.300	18.834	1.914	5.467
6.000000	2.000	5.829	.861	13.772
6.000000	3.000	2.981	.400	32.421
6.000000	4.000	2.012	.231	59.075
6.000000	5.000	1.521	.150	93.735
6.000000	6.000	1.223	.105	136.382
6.000000	7.000	1.023	.078	187.027
6.000000	8.000	.879	.060	245.721
6.000000	9.000	.771	.048	312.383
6.000000	10.000	.687	.039	387.137
8.000000	1.300	19.246	1.999	5.708
8.000000	2.000	5.928	.890	14.241
8.000000	3.000	3.018	.410	33.229
8.000000	4.000	2.032	.235	60.228
8.000000	5.000	1.533	.152	95.210
8.000000	6.000	1.231	.107	138.218
8.000000	7.000	1.029	.079	189.176
8.000000	8.000	.884	.061	248.238
8.000000	9.000	.775	.048	315.238
8.000000	10.000	.690	.039	390.226
9.000000	1.300	19.400	2.031	5.800
9.000000	2.000	5.963	.901	14.410
9.000000	3.000	3.031	.414	33.513
9.000000	4.000	2.038	.237	60.622
9.000000	5.000	1.537	.153	95.720
9.000000	6.000	1.234	.107	138.833
9.000000	7.000	1.031	.079	189.967
9.000000	8.000	.885	.061	249.006
9.000000	9.000	.776	.048	316.192
9.000000	10.000	.691	.039	391.332

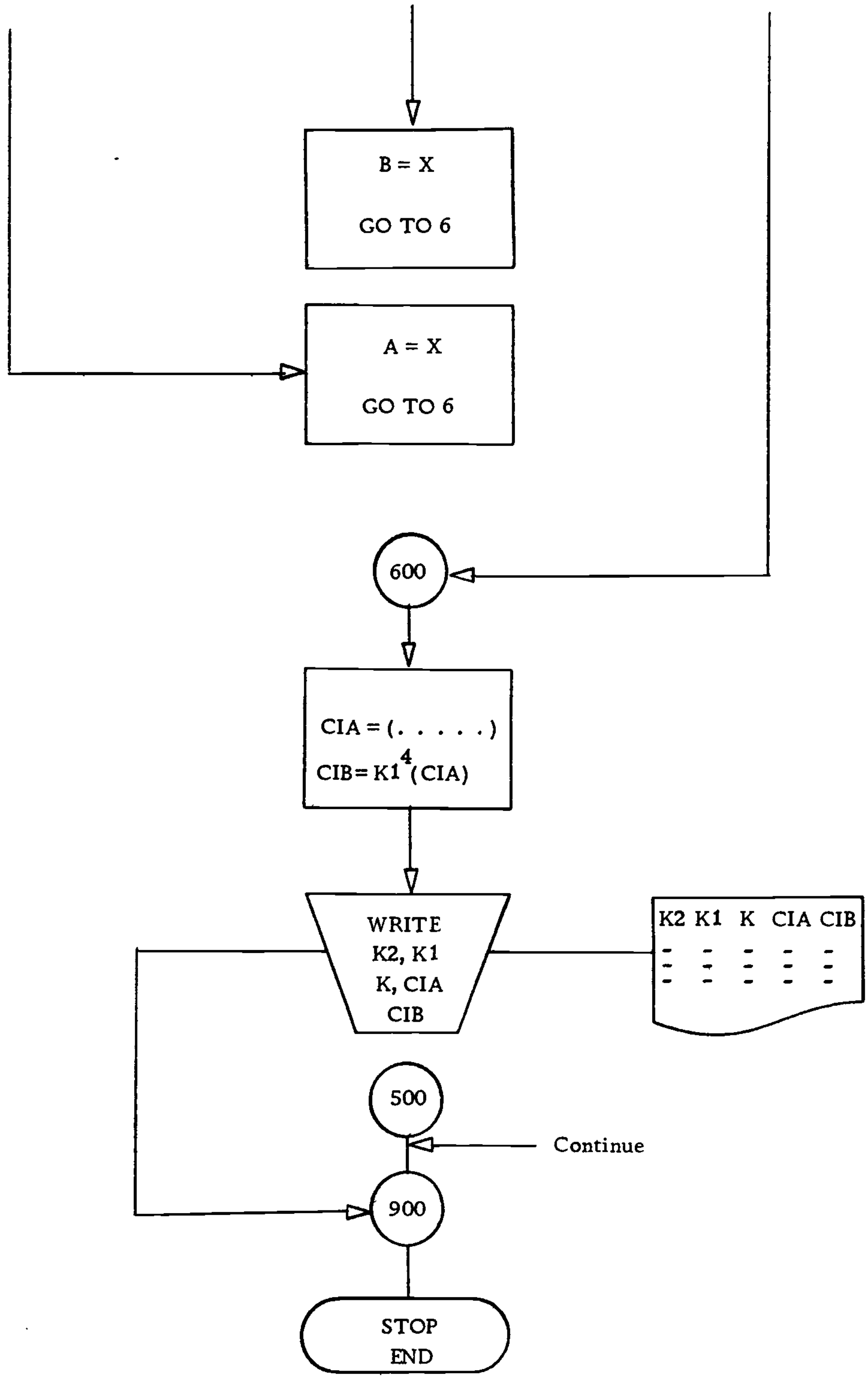
$K2=C/EI_A/L$	K1=DA/DB	K	CIA	CIB
10.000000	1.300	19.528	2.058	5.877
10.000000	2.000	5.992	.909	14.551
10.000000	3.000	3.042	.417	33.743
10.000000	4.000	2.044	.238	60.936
10.000000	5.000	1.540	.154	96.134
10.000000	6.000	1.236	.107	139.316
10.000000	7.000	1.033	.079	190.543
10.000000	8.000	.887	.061	249.776
10.000000	9.000	.777	.048	316.989
10.000000	10.000	.691	.039	392.218
100.000000	1.300	20.784	2.331	6.657
100.000000	2.000	6.252	.990	15.841
100.000000	3.000	3.131	.441	35.761
100.000000	4.000	2.089	.249	63.680
100.000000	5.000	1.568	.159	99.603
100.000000	6.000	1.255	.111	143.531
100.000000	7.000	1.046	.081	195.477
100.000000	8.000	.896	.062	255.308
100.000000	9.000	.784	.049	323.235
100.000000	10.000	.697	.040	399.118
1000.000000	1.300	20.928	2.363	6.750
1000.000000	2.000	6.280	.999	15.984
1000.000000	3.000	3.141	.444	35.975
1000.000000	4.000	2.094	.250	63.966
1000.000000	5.000	1.570	.160	99.950
1000.000000	6.000	1.256	.111	143.933
1000.000000	7.000	1.047	.082	195.915
1000.000000	8.000	.898	.062	255.976
1000.000000	9.000	.785	.049	323.879
1000.000000	10.000	.698	.040	400.013

FLOW CHART


START GFD 22
PRINT C
HEADING
DO 900 I=1, 28
READ 13
K2
G-DATA
FILE
DO900 J = 1, 10
K1 = J
30
-
IF
K1 = 1. 3
25
0
30
+
K1 = 1. 3
30
Continue


DELTA = 0. 1
START = 0. 3
DO 500 M=1, 1000
ANSW(M)=A1-A2
+ A3 . . . . .
IF
ABS(ANSW(M))
-. 001
-, 0
600
700
+
IF
M-1
0
500
800
+, -
IF
((ABS(ANSW(M))
+ ABS(. .
-, 0
500
40
+
Q1 = K - DELTA
Q2 = K


IF
(ANSW (M))
-, 0
5
9
+
Q2 = K - DELTA
Q1 = K
5
BISECTION
METHOD
B = Q1
A = Q2
6
X = (A + B)/2.
K = X
T1 = . . .
Y = ANSW (M)
IF
ABS(B-X)-. 0001
-, 0
3
4
+
GO TO
600
-
7
IF (Y)
3
0
8
+
GO TO
600


B = X
GO TO 6
A = X
GO TO 6
600
CIA = (. . . . . .)
CIB= K1⁴(CIA)
WRITE
K2, K1
K, CIA
CIB
K2 K1 K CIA CIB
500
Continue
900
STOP
END

BISECTION METHOD

As a matter of interest, a method of looping was first used in an attempt to converge on the root to the basic equation for the term "k". Each time the computer performed the loop, the interval used between each assumed value was reduced by a factor of ten. The intent was to converge quickly to a root. The first few computer runs proved that this method was not only inefficient but the limits of storage for the problem were also exceeded. The computer run time was in excess of 200 seconds.

The bisection method was next tried and although other methods may be more efficient it was found that this method reduced the initial computer run time from approximately 200 seconds for an incomplete run to approximately seven seconds for the computations of all values needed. This method is explained in detail on page 220 of reference 4.

This experience revealed that many methods have already been tried and tested and are available for use by the inexperienced programmer.