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Title: CRITICAL BUCKLING LOAD FOR ROUND TAPERED

COLUMNS WITH VARIABLE END FIXITY

Abstract approved Redacted for Privacy

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The critical buckling load for round tapered columns with variable end fixity is presented. The theory used is presented together with the general solution for the resulting differential equation. This general solution is solved by use of a computer. The computer program written specifically for this thesis, the flow chart, the print out data and the input data file are included in Appendix A.

The data obtained substantiates published data and shows that the critical buckling load of a round tapered column, fixed at the small end and hinged and restrained against translation at the large end is increased approximately 100% when the large end of the column is fixed. Information is presented which shows how the critical buckling load for the column varies with relative fixity at the large end and with the column taper.

While this information applies to columns of any material, it is especially important to the timber industry since the data is useful
for timber piling design. Values for partial fixity are now available for use by the design engineer. In the past, engineers had only the values for the two extremes and the choice of full fixity or hinged was all that was available. A comparison of the increased buckling loads for a partially fixed column, using the curves in this thesis, with the buckling load for the hinged column, will show that increased economies can be realized.
Critical Buckling Load for Round Tapered Columns with Variable End Fixity

by

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INTRODUCTION

Round tapered columns in the form of timber piling are in common use today and have been used for many years to support heavy loads. In many cases the supported structure or pile cap is at or below the ground surface and the piling or tapered column is completely supported over its entire length. The maximum load which this column will support is mainly a function of the properties of the foundation material and of the soil-piling interaction and the maximum allowable load is determined by use of the principles of soil mechanics.

Docks, warehouses and other marine-type structures are many times supported by timber piling. The tops of the piles and the pile caps are located at a considerable distance above the soil into which the piles have been driven. The maximum load which the column will support becomes more a function of the unsupported length of the column than a function of the foundation properties as this unsupported distance is increased. Timber bracing has been used to decrease this unsupported length. This bracing is expensive and tends to collect floating debris. A solution which will permit an analysis of the long unsupported tapered column is developed and discussed in the
Many of the designs involving tapered piling have been approximations. One of these designs is the "rule of thumb" method (7). The equations for an exact analysis of the round tapered column with either fixed or pinned end conditions are presented in reference 3. This exact analysis is not developed for the variable end fixity condition. This method of analysis will be developed in later chapters and will satisfy all boundary conditions and tapers usually found in this type of installation. In most cases, higher buckling loads will be calculated using this method than that calculated using an approximate solution.

The solution for the critical buckling load of a round tapered column with variable end fixity at the large end and full fixity at the small end is developed from the theory of elastic stability. The general solution to the differential equation is first developed. This solution is in terms of the column taper, variable and fixity, and the term "k", equation (19). To solve this general solution by longhand methods would be a formidable task. The present-day computers can process large amounts of data quickly and economically with considerable accuracy. A computer program was written as part of the thesis to solve this general solution.
REVIEW OF PUBLISHED THEORY

Even though long tapered round columns in the form of timber piling have been used for many years as structural supporting members, a search of the literature reveals that very little accurate design information is available. No information was found which the design engineer could use to calculate the critical buckling load of a round tapered column with variable end fixity. A great deal of information, however, has been recorded in published articles concerning the soil resistive capacity and the soil-pile interaction. This soils information is vital to structures supported on long timber piling but the design of long slender piling is many times controlled by the structural behavior of the piling rather than the supporting value of the soil. It is this structural behavior that this thesis is concerned with. The tapered columns are considered restrained against translation at the top and are assumed to be long enough so that buckling will occur in the elastic range.

A typical isolated laterally supported unbraced piling is shown on Figure 1. Three properties affect the buckling strength of this column: the modulus of elasticity of the column material (E), the moment of inertia of the column (I), and the unsupported length of the column (L). The modulus of elasticity for any given material can usually be obtained from existing tables. The unsupported length for
Lateral displacement prevented by superstructure

Large end of round pile. The amount of rotational restraint dependent upon relative stiffness between superstructure and pile and the ability of connection to develop restraint against rotation

Round tapered pile
(Amount of taper varies)

Soil embedment determined from soil properties

Point of tip fixity developed by soil restraint

Small end of round piling

Figure 1. Typical tapered piling installation.
a given installation depends upon the characteristics of the soil to develop restraint. The method for determining this point of fixity is not within the scope of this thesis. Information can be obtained however from reference 3. The moment of inertia of a tapered piling varies along the length of the piling and the value used in the long column formulas needs careful consideration.

One common method of design for long tapered piling is the "rule of thumb" method (7). This method assumes that the critical section of piling is located at a point one-third the unsupported distance from the point of fixity. The moment of inertia at this point is used and the column is then designed as a uniform section without taper. This method has been proven to be adequate but the need to improve the economics of the timber piling industry has raised the question of just how conservative is this design method and will an exact analysis permit an increased allowable load.

An analytical method for determining the critical buckling loads for tapered columns has recently been presented by James M. Gere and Winfred O. Carter (2). This publication presents graphs for critical buckling loads for four different end conditions, pinned ends, fixed-free ends, fixed-pinned ends and fixed ends for seven different structural shapes. The shapes considered by Gere and Carter are wide flange, closed box, solid rectangular, open web, tower, solid circular and square sections. The article does not
include the variable fixity-fixed end case. A review of the results of Gere and Carter's work shows that the critical buckling load of a round tapered column can be increased by approximately 100% from a fixed-pinned case to a fixed-fixed case.

The fixed-fixed case is a more efficient use of the round piling as a column, however, a connection which will develop full fixity at the top of the round tapered wood column and still be economical, is difficult to construct. In most installations the top of the round piling-column is restrained to some degree varying between pinned and full fixity. No existing solutions were found which would give the critical buckling loads for this variable end condition. This thesis has developed the values for the variable fixity end condition.

The basic approach used in this theses is found in the theory of elastic stability by Timoshenko and Gere (6). A few elementary solutions will be included in this chapter to show the reasoning and theory used to develop the general solution in the case of partial fixity.

Figure 2. Beam Column: pinned-pinned ends (constant cross-section)
To solve this problem use the equation (1-3), referenced by Timoshenko and Gere (6).

\[ \frac{EId^2y}{dx^2} = -M \]  

(1)

The bending moment at any point along the deflected beam column = Py so that equation (1) becomes

\[ \frac{EId^2y}{dx^2} = -Py \]  

(2)

If \( k^2 \) is defined as \( k^2 = \frac{P}{EI} \) and then substituted into equation (2) the following basic differential equation is obtained.

\[ \frac{d^2y}{dx^2} + k^2 y = 0 \]  

(3)

The general solution to this differential equation from (2) is:

\[ y = A \cos kx + B \sin kx \]  

(4)

Using the boundary conditions:

1. \( y = 0 \) at \( x = 0 \)
2. \( y = 0 \) at \( x = L \)

the following solution is obtained:

\[ P_{\text{crit}} = \frac{\pi^2 EI}{L^2} \]  

(5)

which is the Euler column equation.
The basic equation for the elastic curve of a flexural member is:

\[
\frac{EId^2y}{dx^2} = -M
\]  

(1)

The moment at any point along the deflected beam column in Figure 3 is \(Py - Rx\); thus, the basic equation becomes:

\[
\frac{d^2y}{dx^2} + k^2 y = \frac{Rx}{EI}
\]  

(6)

The general solution to this differential equation from (6) is:

\[
y = A \cos kx + B \sin kx + \frac{Rx}{EIk^2}
\]

this becomes:

\[
y = A \cos kx + B \sin kx + \frac{Rx}{P}
\]  

(7)
\[
\frac{dy}{dx} = -k A \sin kx + B k \cos kx + \frac{R}{P} \tag{7A}
\]

using the following boundary conditions:

1. \(y = 0\) at \(x = 0\)
2. \(y = 0\) at \(x = L\)
3. \(\frac{dy}{dx} = 0\) at \(x = L\)

The following equations are obtained by substituting these boundary conditions into equations (7) and (7A):

1. \(A = 0\)
2. \(A \cos kL + B \sin kL + \frac{RL}{P} = 0\)
3. \(-Ak \sin kL + B k \cos kL + \frac{R}{P} = 0\)

Solving these equations,

\[
\tan kL = kL \tag{8}
\]

Equation (8) has been solved (6), page 53, and the equation for the critical buckling load is:

\[
P_{cr} = \frac{\pi^2 EI}{(0.699L)^2} \tag{9}
\]
Figure 4. Beam column: fixed-fixed ends (constant cross-section).

The moment at any point along the beam column is:

\[ M = Py + Rx - M_A \]

the basic equation (1) becomes

\[ \frac{EI d^2 y}{dx^2} = -M = -Py - Rx + M_A \]

or

\[ \frac{d^2 y}{dx^2} + k^2 y = \frac{MA}{EI} - \frac{Rx}{EI} \]  \hspace{1cm} (10) \]

The general solution to this differential equation from (6) is:

\[ y = A \cos kx + B \sin kx + \frac{M_A}{k^2 EI} - \frac{Rx}{k^2 EI} \]
and reduces to:

\[ y = A \cos kx + B \sin kx + \frac{M_A}{P} - \frac{R_x}{P} \]  

(11)

and

\[ \frac{dy}{dx} = -Ak \sin kx + Bk \cos kx - \frac{R}{P} \]  

(12)

The boundary conditions used to solve for the critical buckling load are:

1. \( y = 0 \) at \( x = 0 \)
2. \( y = 0 \) at \( x = L' \)
3. \( \frac{dy}{dx} = 0 \) at \( x = L \)
4. \( \frac{dy}{dx} = 0 \) at \( x = 0 \)

The following equations are obtained by substituting these boundary conditions into equations (11) and (12):

1. \( A + \frac{M_A}{P} = 0 \)
2. \( A \cos kL + B \sin kL + \frac{M_A}{P} - \frac{R_L}{P} = 0 \)
3. \( -Ak \sin kL + Bk \cos kL - \frac{R}{P} = 0 \)
4. \( Bk - \frac{R}{P} = 0 \)

This is solved by expanding the determinant:

\[
\begin{vmatrix}
1 & 0 & 1/P & 0 \\
\cos kL & \sin kL & 1/P & -L/P \\
-k \sin kL & k \cos kL & 0 & -1/P \\
0 & k & 0 & -1/P \\
\end{vmatrix}
\]
to obtain:

\[ 2(\cos kL - 1) + kL \sin kL = 0 \]  \hspace{1cm} (13)

Equation (13) has been solved (6), page 55, and the equation for the critical buckling load is:

\[ P_{\text{crit}} = \frac{4 \pi^2 EI}{L^2} \]  \hspace{1cm} (14)

Figure 5. Beam column: fixed-variable fixity ends (constant cross-section).

When \( C \) is very large a fixed-fixed case exists and when \( C \) approaches zero a pinned-fixed case is realized.

The moment in the beam column at any point is equal to

\[ M = Py + Rx - M_A \]
so from equation (1)

\[
\frac{EI}{dx^2} \frac{d^2 y}{dx^2} = -Py - Rx + MA
\]

is obtained. This becomes

\[
\frac{d^2 y}{dx^2} + k^2 y = \frac{MA}{EI} - \frac{Rx}{EI}
\]

which is the same as equation (10) and the general solution is the same as equations (11) and (12).

The boundary conditions to be used to solve this equation differ slightly and are listed below:

1. \(y = 0\) at \(x = 0\)
2. \(y = 0\) at \(x = L\)
3. \(\frac{dy}{dx} = 0\) at \(x = L\)
4. \(\frac{dy}{dx} = \frac{MA}{C}\) at \(x = 0\)

The following equations are obtained by substituting these boundary conditions into equations (11) and (12):

1. \(A + \frac{MA}{P} = 0\)
2. \(A \cos kL + B \sin kL + \frac{MA}{P} - RL/P = 0\)
3. \(-Ak \sin kL + Bk \cos kL - R/P = 0\)
4. \(Bk = \frac{MA}{C} - R/P = 0\)
solving this by expanding the determinant:

\[
\begin{vmatrix}
1 & 0 & \frac{1}{P} & 0 \\
\cos kL & \sin kL & \frac{1}{P} & -\frac{L}{P} \\
-k \sin kL & k \cos kL & 0 & -\frac{1}{P} \\
0 & k & -\frac{1}{C} & -\frac{1}{P}
\end{vmatrix}
\]

to obtain:

\[2k(\cos kL - 1) + \frac{1}{C}(PkL \cos kL - P \sin kL) + k^2L \sin kL = 0\]  
(16)

To check this solution let

\[C = \frac{M}{\theta} = 0\]  
for pinned end

equation (16) then reduces to

\[\tan kL = kL\]  
which checks with equation (8).

If we let \(C = \frac{M}{\theta} = \infty\) for fixed end

\[
\frac{1}{C} \to 0
\]

and equation (16) reduces to:

\[2(\cos kL - 1) + kL \sin kL = 0\]

which checks with equation (13).

The critical buckling load for values of end fixity between the pinned and fixed cases was not determined for the constant section. These values however can be easily obtained by solving equation (16) for the value of \(k\) to set the equation equal to zero for each
assumed value of end fixity. This example serves to show the
method to be used for the round tapered column which will be covered
in the next chapter.
DEVELOPMENT OF THEORY AND SOLUTION

Figure 6. Beam column: fixed-variable fixity case (variable cross-section).

Assumptions:

1. Beam column is fixed at point B.
2. Column is circular in cross-section.
3. Column is uniformly tapered with large end at point A.
4. Variable fixity at point A is shown by a spring constant
   \[ C = \frac{M_A}{\theta} = \frac{M_A}{\frac{dy}{dx}} \]
5. Basic differential equation is
   \[ EI \frac{d^2y}{dx^2} = -M \]
6. No translation is permitted at either end.

The moment at any point on deflected column is

\[ M = Py - R_Ax - M_A \]
Substituting this into the basic equation

\[
\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{RAx}{b} + \frac{MA}{b^2}
\]  

(17)

The moment of inertia of the beam column varies as follows:

\[
I_x = I_A [1 - \frac{x}{b}]^4
\]

Combining this with equation (17) we obtain

\[
EI_A [1 - \frac{x}{b}]^4 \frac{d^2y}{dx^2} + \frac{Py}{EI_A} = \frac{1}{EI_A} (RAx + MA)
\]

(18)

this becomes

\[
[1 - \frac{x}{b}]^4 \frac{d^2y}{dx^2} + \frac{Py}{EI_A} = \frac{1}{EI_A} (RAx + MA)
\]

defining

\[
k^2 = \frac{Pb^2}{EI_A}
\]

(19)

this equation becomes:

\[
[1 - \frac{x}{b}]^4 y'' + \frac{k^2}{b^2} y = \frac{1}{EI_A} (RAx + MA)
\]

(20)

by further rearranging the following equation is obtained.

\[
y'' + \frac{k^2y}{b^2 [1 - \frac{x}{b}]^4} = \frac{1}{EI_A} \left[ \frac{1}{[1 - \frac{x}{b}]^4} \right] (RAx + MA)
\]

(21)

This equation may be solved more easily by means of the following
The idea for these substitutions was obtained from reference 1, page 24.

\[
y' = \frac{dy}{dx} = \frac{dy}{dz} \frac{dz}{dx} = \frac{1}{b[1 - \frac{x}{b}]^2} \frac{dy}{dz} = \frac{z^2}{b} \frac{dy}{dz}
\]

\[
y'' = \frac{dy'}{dx} = \frac{dy'}{dz} \frac{dz}{dx} = \frac{z^2}{b} \frac{d}{dz} \left[ \frac{z^2}{b} \frac{dy}{dz} \right]
\]

\[
y''' = \frac{z^4}{b^2} \frac{d^2 y}{dz^2} + \frac{2z}{b^2} \frac{dy}{dz} = (22)
\]

Now substituting equation (22) into equation (21)

\[
\frac{z^4}{b^2} \frac{d^2 y}{dz^2} + \frac{2z}{b^2} \frac{dy}{dz} + \frac{k^2}{b^2} y z^4 = \frac{z^4}{EI_A} (R_A x + M_A) \quad (23)
\]

Multiplying this equation by \(b^2/z^3\)

\[
z \frac{d^2 y}{dz^2} + 2 \frac{dy}{dz} + k^2 z y = \frac{z^2}{EI_A} (R_A x + M_A) \quad (24)
\]

Now since

\[
z = \left[ \frac{1}{1 - \frac{x}{b}} \right]
\]
\[ x = b \left[ 1 - \frac{1}{z} \right] \] and substituting this into equation (24)

\[
\frac{d^2 y}{dz^2} + 2 \frac{dy}{dz} + k^2 y = \frac{R_A b^3 z}{E I_A} - \frac{R_A b^3}{E I_A} + \frac{M_A z b^2}{E I_A}
\] (25)

Now letting \( v = z y \)

\[
\frac{d^2 v}{dz^2} = v \frac{d^2 y}{dz^2} + 2 \frac{dy}{dz}
\]

Substituting this into equation (9) to obtain

\[
\frac{d^2 v}{dz^2} + k^2 v = \frac{R_A b^3 z}{E I_A} - \frac{R_A b^3}{E I_A} + \frac{M_A z b^2}{E I_A}
\]

Replacing \( k^2 \) by \( q \) this becomes:

\[
\frac{d^2 v}{dz^2} + q v = \frac{R_A b^3 z}{E I_A} - \frac{R_A b^3}{E I_A} + \frac{M_A z b^2}{E I_A}
\] (26)

The solution to this equation is:

\[
v = A \sin \sqrt{q} \; z + B \cos \sqrt{q} \; z + \frac{1}{q} \left[ \frac{R_A b^3 z}{E I_A} - \frac{R_A b^3}{E I_A} + \frac{M_A z b^2}{E I_A} \right]
\] (27)

If we now replace some of the terms using the following:

\[
v = z y
\]

\[
z = [1 \; 1 - x/b]
\]

\[k^2 = q\]
The general solution becomes:

\[ y = A[1 - \frac{x}{b}] \sin \frac{k}{[1 - \frac{x}{b}]} + B[1 - \frac{x}{b}] \cos \frac{k}{[1 - \frac{x}{b}]} + \frac{R_A b^3}{k^2EI} \]

\[-[1 - \frac{x}{b}] \frac{R_A b^3}{EI A^2} + \frac{M_A b^2}{k^2EI} \]  

(28)

Before proceeding farther this general solution is checked using equation (28). Differentiating equation (28)

\[
\frac{d^2y}{dx^2} = -\frac{A}{b} \left[ \frac{k}{b[1 - \frac{x}{b}]^2} \cos \frac{k}{[1 - \frac{x}{b}]} \right] + \frac{A}{b} \left[ \frac{k}{b[1 - \frac{x}{b}]^2} \cos \frac{k}{[1 - \frac{x}{b}]} \right]
\]

\[+ \frac{Ak}{b(1 - \frac{x}{b})} \left[ \frac{-k}{b(1 - \frac{x}{b})^2} \sin \frac{k}{[1 - \frac{x}{b}]} \right] - \frac{B}{b} \left[ \frac{-k}{b(1 - \frac{x}{b})^2} \sin \frac{k}{[1 - \frac{x}{b}]} \right]
\]

\[- \frac{B}{b} \left[ \frac{k}{b(1 - \frac{x}{b})^2} \sin \frac{k}{[1 - \frac{x}{b}]} \right] - \frac{Bk}{b(1 - \frac{x}{b})} \left[ \frac{k}{b(1 - \frac{x}{b})^2} \cos \frac{k}{[1 - \frac{x}{b}]} \right].
\]

Consolidating this equation

\[ y'' = \frac{d^2y}{dx^2} = -\frac{Ak^2}{b^2(1 - \frac{x}{b})^3} \sin \left[ \frac{k}{b(1 - \frac{x}{b})} \right] - \frac{Bk^2}{b^2(1 - \frac{x}{b})^3} \cos \left[ \frac{k}{b(1 - \frac{x}{b})} \right]
\]

(29)

The values for \( y \) from equation (28) and \( y'' \) from equation (29) are now substituted into equation (21) and consolidated to obtain:
\[ \frac{R_A x}{(1 - \frac{x}{b})^4} = \frac{R_A x}{(1 - \frac{x}{b})^4} \]  

so the general solution checks.

Next, the constants and unknowns in equation (28) are evaluated using the following boundary conditions:

1. \( y = 0 \) at \( x = 0 \)
2. \( y = 0 \) at \( x = L \)
3. \( \frac{dy}{dx} = 0 \) at \( x = L \)
4. \( \frac{dy}{dx} = \frac{M_A}{C} \) at \( x = 0 \)

Differentiating equation (28),

\[ \frac{dy}{dx} = A\left[ \frac{k}{b[1 - \frac{x}{b}]} \cos\left(\frac{k}{l - \frac{x}{b}}\right) - \frac{1}{b} \sin\left(\frac{k}{l - \frac{x}{b}}\right) \right] \]

\[ + B\left[ \frac{1}{b} \cos\left(\frac{k}{l - \frac{x}{b}}\right) - \frac{k}{b(1 - \frac{x}{b})} \sin\left(\frac{k}{l - \frac{x}{b}}\right) \right] + \frac{R_A}{EI_A} \left[ \frac{b^2}{k^2 \varepsilon} \right] \]  

Now substituting the boundary conditions into equation (28) and equation (31) the following equations are obtained:

\[ A \sin k + B \cos k + \frac{M_A b^2}{k^2 \varepsilon I_A} = 0 \]  

(32)
\[
A(1 - \frac{L}{b}) \sin \left(\frac{k}{1 - \frac{L}{b}}\right) + B(1 - \frac{L}{b}) \cos \left(\frac{k}{1 - \frac{L}{b}}\right)
\]
\[+ R_A \left(\frac{Lb^2}{k^2EI_A}\right) + \frac{M_A b^2}{k^2EI_A} = 0 \quad (33)\]

\[
A\left[\frac{k}{b(1 - \frac{L}{b})} \cos \left(\frac{k}{1 - \frac{L}{b}}\right) - \frac{1}{b} \sin \left(\frac{k}{1 - \frac{L}{b}}\right)\right]
\]
\[+ B\left[-\frac{1}{b} \cos \left(\frac{k}{1 - \frac{L}{b}}\right) - \frac{k}{b(1 - \frac{L}{b})} \sin \left(\frac{k}{1 - \frac{L}{b}}\right)\right] + R_A \left[\frac{b^2}{EI_A k^2}\right] = 0 \quad (34)\]

\[
A \left(\frac{1}{b} (k \cos k - \sin k)\right) + B \left(-\frac{1}{b} (\cos k + k \sin k)\right)
\]
\[+ \frac{R_A b^2}{EI_A k^2} - \frac{M_A}{C} = 0 \quad (35)\]

Placing equations (32), (33), (34), and 35) into determinant form,
<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( M_B )</th>
<th>( R_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin k )</td>
<td>( \cos k )</td>
<td>( \frac{b^2}{k^2EI_A} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( (1 - \frac{L}{b}) \sin \left( \frac{k}{1 - \frac{L}{b}} \right) )</td>
<td>( (1 - \frac{L}{b}) \cos \left( \frac{k}{1 - \frac{L}{b}} \right) )</td>
<td>( \frac{b^2}{k^2EI_A} )</td>
<td>( \frac{Lb^2}{k^2EI_A} )</td>
</tr>
<tr>
<td>( \frac{k}{b(1 - \frac{L}{b})} \cos \left( \frac{k}{1 - \frac{L}{b}} \right) )</td>
<td>( \frac{1}{b} \cos \left( \frac{k}{1 - \frac{L}{b}} \right) )</td>
<td>( 0 )</td>
<td>( \frac{b^2}{k^2EI_A} )</td>
</tr>
<tr>
<td>( -\frac{1}{b} \sin \left( \frac{k}{1 - \frac{L}{b}} \right) )</td>
<td>( -\frac{k}{b(1 - \frac{L}{b})} \sin \left( \frac{k}{1 - \frac{L}{b}} \right) )</td>
<td>( 0 )</td>
<td>( \frac{b^2}{k^2EI_A} )</td>
</tr>
<tr>
<td>( \frac{1}{b} (k \cos k - \sin k) )</td>
<td>( \frac{1}{b} (\cos k + k \sin k) )</td>
<td>( \frac{1}{C} )</td>
<td>( \frac{b^2}{k^2EI_A} )</td>
</tr>
</tbody>
</table>

Now assigning the following values:

\[ D = \sin k \]
\[ E = \cos k \]
\[ F = \frac{b^2}{k^2EI_A} \]
\[ G = (1 - \frac{L}{b}) \sin \left( \frac{k}{1 - \frac{L}{b}} \right) \]
\[ H = (1 - \frac{L}{b}) \cos \left( \frac{k}{1 - \frac{L}{b}} \right) \]
\[ T = \frac{k}{b(1 - \frac{L}{b})} \cos \left( \frac{k}{1 - \frac{L}{b}} \right) - \frac{1}{b} \sin \left( \frac{k}{1 - \frac{L}{b}} \right) \]

\[ U = -\frac{1}{b} \cos \left( \frac{k}{1 - \frac{L}{b}} \right) - \frac{k}{b(1 - \frac{L}{b})} \sin \left( \frac{k}{1 - \frac{L}{b}} \right) \]

\[ V = \frac{1}{b} (k \cos k - \sin k) \]

\[ W = -\frac{1}{b} (\cos k + k \sin k) \]

The previous determinant becomes:

\[
\begin{bmatrix}
D & E & F & O \\
G & H & F & LF \\
T & U & O & F \\
V & W & \frac{1}{C} & F
\end{bmatrix}
\]

Expanding this determinant the following equation is obtained:

\[
\frac{DHF}{C} - D F^2 U + D F^2 W - \frac{DLFU}{C} - \frac{E G F}{C} + E F^2 T - E F^2 V
\]

\[
+ \frac{E L F T}{C} + F^2 G U - G F^2 W - F^2 H T + F^2 H V + F^2 L T W
\]

\[- F^2 L U V = 0 \quad (36)\]

The expression, \[ k^2 = \frac{P b^2}{E I A} \] from equation (19)

can be rearranged as follows
\[ P = \frac{k^2 EI_A}{b^2} \]  
(37)

\( k^2 \) must be evaluated before the critical buckling load, \( P_{\text{crit}} \) can be determined.

The value of \( k^2 \) needed is that which will set equation (36) equal to zero.

Due to the magnitude of work involved to solve this equation for each tapered column and the fact that many different combinations of column taper will be considered, this problem can best be handled by use of a computer.

Before using the computer non-dimensionalize the terms involved and determine the form of the desired answer.

Referring to the article by Gere and Carter (2), the desired result should take the form of

\[ P_{\text{crit}} = (\text{constant}) \left( \frac{\pi^2 EI}{L^2} \right) \]  
(38)

multiplying equation (37) by

\[ \frac{L^2}{L^2} \times \frac{\pi^2}{\pi^2} \]

the following is obtained:
Referring back to the geometry of the beam-column

![Diagram of beam-column with annotations](image)

\[ P_{\text{crit}} = \left( \frac{k^2}{b} \frac{2}{\pi} \frac{L^2}{2} \right) \pi^2 \frac{E I_A}{L^2} \]  

(39)

Figure 7. Geometry of variable beam column.

and using \( \frac{D_A}{D_B} \) as a variable to define the geometry of the beam columns

\[ \frac{b}{b - L} = \frac{D_A}{D_B} = k_1 \]

(40)

Next, considering the variable spring constant end restraint at large end of beam column, this spring constant \( (k_2) \) varies from zero with pinned end to infinity at a fixed end condition. This spring
constant is defined in the following terms:

\[ k_2 = \text{spring constant} = \frac{C}{EI/L} \]

Assuming the following values

\[ L = 1 \]
\[ E = 1 \]
\[ I_A = 1 \]

then

\[ k_2 = C \] \hspace{1cm} (41) \]

\( k_1 \) from equation (40) becomes

\[ k_1 = \frac{b}{b - 1} \]

or rearranging

\[ b = \frac{-k_1}{1 - k_1} \]

Referring back to equation (36) and substituting the new values of \( L, E, I_A, C, \) and \( b \) into each term and multiplying each of these terms by

\[ \frac{C^2L^3}{L^4} \]

the following non-dimensionalized terms are obtained:
\[
\frac{\text{DHF}}{C} = \sin k \left(1 - \frac{L}{b}\right) \left(\cos \frac{k}{l - \frac{L}{b}}\right) \left(-\frac{b^2}{k^2 E_1 A}\right) \left(\frac{L}{C^2 L^3}\right) \\
= (\sin k) \left[\frac{k_1^2 + 1-k_1}{k_1}\right] \left[\frac{k_1}{k_1^2 + 1-k_1}\right] \left[\frac{-k_1}{1-k_1}\right] \left[\frac{-k_1}{k_1}\right] \left(\frac{1}{k^2}\right) (k_2) \\
= \frac{k_1 k_2}{k^2 (1 - k_1)^2} (\sin k) (\cos k k_1)
\]

\[
\text{DF}^2 U = \frac{k_2^2}{k^4} \left[\frac{k_1}{1-k_1}\right]^3 (\sin k \cos k k_1 + k k_1 \sin k \sin k k_1)
\]

\[
\text{DF}^2 W = \left(\frac{k_1}{1-k_1}\right)^3 \left(\frac{k_2}{4}\right) (\cos k \sin k + k \sin^2 k)
\]

\[
\frac{\text{DLFU}}{C} = \left(\frac{k_1}{1-k_1}\right) \left(\frac{k_2}{k}\right) (\sin k \cos k k_1 + k k_1 \sin k \sin k k_1)
\]

\[
\frac{\text{EGF}}{C} = \left(\frac{k_1}{1-k_1}\right)^2 \left(\frac{k_2}{k^2 k_1}\right) (\cos k \sin k k_1)
\]

\[
\text{EF}^2 T = -\left(\frac{k_1}{1-k_1}\right)^3 \left(\frac{k_2}{4}\right) (k k_1 \cos k \cos k k_1 - \cos k \sin k k_1)
\]

\[
\text{EF}^2 V = -\left(\frac{k_1}{1-k_1}\right)^3 \left(\frac{k_2}{4}\right) (k \cos^2 k - \cos k \sin k)
\]
\[
\frac{\text{ELFT}}{C} = -\left(\frac{k_1}{1-k_1}\right)\frac{k_2}{k} (k_1 \cos k \cos k_1 - \cos k \sin k_1)
\]

\[
F^2_{\text{GU}} = \left(\frac{k_1}{1-k_1}\right)^3 \frac{k_2^2}{4k k_1} (\sin k_1 \cos k_1 + k_1 \sin^2 k_1)
\]

\[
G F^2 W = \left(\frac{k_1}{1-k_1}\right)^3 \frac{k_2^2}{4k k_1} (\sin k_1 \cos k + k \sin k \sin k_1)
\]

\[
F^2_{\text{HT}} = -\left(\frac{k_1}{1-k_1}\right)^3 \frac{k_2^2}{k_1^4} (k_1 \cos^2 k_1 - \cos k_1 \sin k_1)
\]

\[
F^2_{\text{HV}} = -\left(\frac{k_1}{1-k_1}\right)^3 \frac{k_2^2}{k_1 k^4} (k_1 \cos k_1 \cos k - \cos k_1 \sin k)
\]

\[
F^2_{\text{LTW}} = -\left(\frac{k_1}{1-k_1}\right)^2 \frac{k_2^2}{k_1^4} (\cos k + k \sin k)(k_1 \cos k_1 - \sin k_1)
\]

\[
F^2_{\text{LUV}} = -\left(\frac{k_1}{1-k_1}\right)^2 \frac{k_2^2}{k^4} (k \cos k - \sin k)(\cos k k_1 + k \sin k_1)
\]

Designating each of these terms by a letter and number and factoring out

\[
\left[\frac{k_1 k_2}{k (1-k_1)}\right]
\]
from each term the following terms are obtained:

\[ A_1 = \frac{1}{(1-k_1)}(\sin k \cos k k_1) \]

\[ A_2 = \frac{k_1^2 k_2}{k^2(1-k_1)^2} (\sin k \cos k k_1 + k k_1 \sin k \sin k k_1) \]

\[ A_3 = \frac{k_1^2 k_2}{k^2(1-k_1)^2} (\cos k \sin k + k \sin^2 k) \]

\[ A_4 = (\sin k \cos k k_1 + k k_1 \sin k \sin k k_1) \]

\[ A_5 = \frac{1}{(1-k_1)}(\cos k \sin k k_1) \]

\[ A_6 = \frac{-k_1^2 k_2}{k^2(1-k_1)^2} (k k_1 \cos k \cos k k_1 - \cos k \sin k k_1) \]

\[ A_7 = \frac{-k_1^2 k_2}{k^2(1-k_1)^2} (k \cos^2 k - \cos k \sin k) \]

\[ A_8 = -(k k_1 \cos k \cos k k_1 - \cos k \sin k k_1) \]

\[ A_9 = \frac{k_1 k_2}{k^2(1-k_1)^2} (\sin k k_1 \cos k k_1 + k k_1 \sin^2 k k_1) \]
\[ A_{10} = \frac{k_1 k_2}{k^2 (1-k_1)^2} (\sin k k + k \sin k \sin k k_1) \]

\[ A_{11} = \frac{-k_1 k_2}{k^2 (1-k_1)^2} (k k_1 \cos k k_1 - \cos k k_1 \sin k k_1) \]

\[ A_{12} = \frac{-k_1 k_2}{k^2 (1-k_1)^2} (k \cos k k_1 \cos k - \cos k k_1 \sin k) \]

\[ A_{13} = \frac{-k_1 k_2}{k^2 (1-k_1)^2} (\cos k + k \sin k) (k k_1 \cos k k_1 - \sin k k_1) \]

\[ A_{14} = \frac{-k_1 k_2}{k^2 (1-k_1)^2} (k \cos k - \sin k) (\cos k k_1 + k k_1 \sin k k_1) \]

Rearranging terms to minimize computation by computer as follows:

\[ T_1 = 1/(1-k_1) \]

\[ T_2 = T_1 k_1 \left( \frac{k}{k_1} \right)^2 \]

\[ T_3 = T_2 (T_1) \]

\[ T_4 = T_3 (k_1) \]

\[ T_5 = \sin (k) \]

\[ T_6 = \sin (k k_1) \]

\[ T_7 = \cos (k) \]
\[ T8 = \cos (k \cdot k_1) \]
\[ T9 = k \cdot k_1 \]

The terms \(A_1\) to \(A_{14}\) can now be rewritten as follows:

\[
A_1 = (T1)(T5)(T8)
\]
\[
A_2 = T4 \left[ (T5)(T8) + (T9)(T5)(T6) \right]
\]
\[
A_3 = T4 \left[ (T7)(T5) + (k)(T5)^2 \right]
\]
\[
A_4 = (T5)(T8) + (T9)(T5)(T6)
\]
\[
A_5 = (T1)(T7)(T6)
\]
\[
A_6 = -T4 \left[ (T9)(T7)(T8) - (T7)(T6) \right]
\]
\[
A_7 = -T4 \left[ (k)(T7)^2 - (T7)(T5) \right]
\]
\[
A_8 = -(T9)(T7)(T8) + (T7)(T6)
\]
\[
A_9 = T3 \left[ (T6)(T8) + (T9)(T6)^2 \right]
\]
\[
A_{10} = T3 \left[ (T6)(T7) + (k)(T5)(T6) \right]
\]
\[
A_{11} = -T3 \left[ (T9)(T8)^2 - (T8)(T6) \right]
\]
\[
A_{12} = -T3 \left[ (k)(T8)(T7) - (T8)(T5) \right]
\]
\[
A_{13} = -T2 \left[ T7 + (k)(T5) \right] \left[ (T9)(T8) - T6 \right]
\]
\[
A_{14} = -T2 \left[ (k)(T7) - T5 \right] \left[ T8 + (T9)(T6) \right]
\]

Referring back now to equation (36) and substituting in the new values for each of the 14 terms we obtain:

\[
A_1 - A_2 + A_3 - A_4 - A_5 + A_6 - A_7 + A_8 + A_9 - A_{10} - A_{11}
\]
\[
+ A_{12} + A_{13} - A_{14} = 0
\]

(42)
This equation is in terms of three non-dimensional terms $k$, $k_1$, and $k_2$.

$k_1$ Defines the geometry of the column taper.

$k_2$ Defines the spring constant or relative amount of end fixity at the large end of the column.

and

$$k = \sqrt{\frac{P_{crit} b^2}{E I A}}$$

Values can be assigned to $k_1$ and $k_2$ and equation (42) solved to obtain the corresponding value of $k$.

This value of $k$ can then be used in equations (38) and (39) to obtain the value of the constant:

$$\frac{k^2}{b^2 \pi^2} = \frac{2}{L^2}$$

Using the values already assigned to $b$ and $L$ this constant reduces to

$$\frac{k^2(k_1 - 1)^2}{\pi^2 k_1^2} = C_{IA}$$

where

$$P_{crit} = C_{IA} \left( \frac{\pi^2 E I A}{L^2} \right)$$  \hspace{1cm} (43)
Equation (43) is in terms of the moment of inertia at the large end of the column but can be rewritten in terms of the moment of inertia at the small end $I_B$ as follows:

$$\frac{I_A}{I_B} \sim \frac{(D_A)^4}{(D_B)^4} k_1^4$$

$$C_{IB} = (k_1^4)(C_{IA})$$

Substituting this into equation (43) and using $I_B$ we obtain

$$P_{crit} = C_{IB} \left( \frac{\pi^2 EI_B}{L^2} \right)$$

Values have been obtained for $k$, $C_{IA}$ and $C_{IB}$ for each value of $k_1$ and $k_2$ by use of a computer. The computer program and flow chart are included in Appendix A together with the tabulated numerical output.

The data obtained from the computer has been plotted on Figures 8, 9, 10 and 11.

The values of $C_{IB}$ (to be used with the moment of inertia at the small end of the column) and the values of $C_{IA}$ (to be used with the moment of inertia, $I_A'$ at the large end of the column) are plotted with respect to $k_1$. Values of $k_1$ from 1.3 to 10.0 were used in the computations. The normal taper $D_A/D_B$ is less than 10 but
Figure 8 Critical Buckling Coefficients, $C_{1B}$
\[ P_{\text{crit}} = C_{IA} \left[ \frac{\pi^2 E I_A}{L^2} \right] \]

\[ K_2 = \frac{C}{EI_A} \]

**FIGURE 9 CRITICAL BUCKLING COEFFICIENTS**

- \( K_2 = 100.0 \)
- \( K_2 = 5.0 \)
- \( K_2 = 2.0 \)
- \( K_2 = 1.0 \)
- \( K_2 = 0.50 \)
- \( K_2 = 0.01 \)

- PINNED @ A
- FIXED @ A
- FIXED @ B

\[ \frac{DA}{DB} = K_1 \]
FIGURE 10 CRITICAL BUCKLING COEFFICIENTS, $C_{1B}$
FIGURE 11 CRITICAL BUCKLING COEFFICIENTS, $C_{IA}$

$P_{CRIT} = C_{IA} \left( \frac{N^2 E I_A}{L^2} \right)$

$K_2 = C / \frac{E I_A}{L}$

$K_2 = 100.0$
$K_2 = 10.0$
$K_2 = 5.0$
$K_2 = 3.0$
$K_2 = 2.0$
$K_2 = 1.00$
$K_2 = 0.50$
$K_2 = 0.10$
$K_2 = 0.01$

SEE REF. NO. 2
additional values can be obtained easily, if necessary, by using this computer program. The equations used in this thesis cannot be used as $k_1$ approaches 1.0. The values of $C_{IB}$ and $C_{IA}$ for $k_1 = 1.0$ for the two extreme cases of end fixity are shown on Figure 10. These values were obtained from reference 1.

Figures 10 and 11 represent the same data as Figures 8 and 9 except that the scales have been increased in order that the curves can be more useable in the lower range of values for $D_A/D_B$. 
CONCLUSIONS AND DISCUSSION

Coefficients to be used to calculate the critical elastic buckling loads for round uniformly tapered columns with a fixed end-variable fixed end condition have been determined in this thesis. The columns are considered fixed at the small end. The fixity at the large end of the columns varies from zero to 100 percent while translation at the large end is prevented. These values have been plotted in Figures 8, 9, 10 and 11. The actual computer printout data is shown in Appendix. These values apply to columns of any construction material, e.g., wood, steel, concrete, etc.

As discussed earlier in this thesis the results presented can be applied directly to the problems associated with the use of timber piling for the construction of waterfront or other similar structures. The small end of the piling is driven to a point of fixity while the large end is attached to a superstructure. The use of batter piles, anchor walls or other facilities to prevent translation of the tops of the piling is usually required to resist seismic forces and the lateral forces of ship docking and anchorage.

A review of the data in Appendix shows that when the stiffness, \( C \), of the deck is 100 times the stiffness of the piling (\( EI_A / L \)) with respect to the large end of the pile, the piling is essentially fixed at the top. The term \( k_2 \) is used to designate this ratio of...
relative fixity. These conclusions assume that the proper connections between the tapered column or piling and the superstructure is made efficiently. If these connections are not made properly some loss in relative stiffness will result which will affect the fixity at the top end of the column and the resulting critical buckling load. The type of connection to be used and the efficiency of such a connection is important to the design engineer but this is beyond the scope of this thesis. This should not be overlooked in practice because some connections may never develop the amount of fixity assumed and the actual critical buckling load my be considerably less than that obtained from the curves in Figures 8, 9, 10 and 11.

Previous articles (2) have given values for the critical buckling load for the fixed-fixed case and the fixed-pinned case. This thesis substantiates these values and also provides the values for intermediate partial fixity-fixed cases. The values of the constant $C_{IA}$ and $C_{IB}$ used to determine the critical buckling load can be obtained by first determining the ratio of $\frac{D_A}{D_B}$ then entering the curves for critical buckling coefficients and reading upward until the approximate curve for $k_2$ is reached and then horizontally to the scale for the coefficient.

The critical buckling load is then computed using one of the following equations:
The critical buckling load for the fixed-fixed case is approximately 200 percent of the fixed-pinned case. The critical buckling load for the case where the stiffness of the column based upon the moment of inertia at the larger end is equal to the stiffness of the deck, $k_2 = 1.0$, is approximately 168 percent of the fixed-pinned case. Therefore, where partial or full fixity can be obtained, a substantial increase in allowable column load can be realized over the pinned end case. In some structures it may be difficult and costly to obtain full fixity at the top of the piling. Partial fixity, however, may be obtained relatively easy and at lower cost. In the past, information has not been available for the partial fixity case and design engineers have been forced to design supporting columns for the fixed-pinned case even though some partial fixity was in fact obtained. The data presented in this thesis allows a much more economical use of timber piling than has been possible before. The formulas and data shown are not limited only to tapered wood columns and may be used for any material, provided all other design requirements are satisfied.
An example will be used to show how the curves in Figures 8, 9, 10 and 11 are to be used. Using the piling shown in Figure 1, assume the following:

Combined stiffness of Deck,

$$C_{\text{Deck}} = \left(\frac{EI}{L}\right)_{\text{Deck}} = 4 \times 10^7$$

Modulus of elasticity of piling = 1.6 x 10^6 psi

Unsupported length of piling, L = 70 feet

Diameter of piling at Top, $D_A = 20$ inches

Diameter of piling at point of fixity, $D_B = 10$ inches

$$\frac{D_A}{D_B} = k_1 = \frac{20}{10} = 2.0$$

$$I_A = \frac{\pi D_A^4}{64} = 7850 \text{ in}^4$$

$$I_B = \frac{\pi D_B^4}{64} = 492 \text{ in}^4$$

$$\frac{EI_A}{L} = \frac{1.6 \times 10^6 \text{ psi} \times 7850 \text{ in}^4}{840 \text{ in}} = 1.5 \times 10^7$$

$$k_2 = \frac{C/EI_A}{L} = \frac{4 \times 10^7}{1.5 \times 10^7} = 2.67$$
Entering the charts, Figures 10 and 11 at $k_1 = 2.0$ and moving up vertically to the intersection of $k_2 = 2.67$ on both sets of curves and then proceeding horizontally, we read:

\[
\begin{align*}
C_{IA} & = 0.75 \\
C_{IB} & = 12.0 \\
\text{or}
\end{align*}
\]

\[
\begin{align*}
P_{\text{crit}} &= C_{IA} \left( \frac{2EI_A}{L^2} \right) = 0.75 \left( \frac{2 \times 1.6 \times 10^6 \times 7850}{840 \times 840} \right) \\
&= 132 \text{ kips}
\end{align*}
\]

\[
\begin{align*}
P_{\text{crit}} &= C_{IB} \left( \frac{2EI_B}{L^2} \right) = 12.0 \left( \frac{2 \times 1.6 \times 10^6 \times 492}{840 \times 840} \right) \\
&= 132 \text{ kips}
\end{align*}
\]

If this same pile is assumed to be fixed at the top with $k_2 = 100$

\[
\begin{align*}
C_{IA} & = 1.00 \\
\text{or}
\end{align*}
\]

\[
\begin{align*}
P_{\text{crit}} &= 1.00 \times 132^k = 176 \text{ kips}
\end{align*}
\]

\[
\begin{align*}
C_{IB} & = 16.0 \\
\text{or}
\end{align*}
\]

\[
\begin{align*}
P_{\text{crit}} &= 16.0 \times 132^k = 176 \text{ kips}
\end{align*}
\]

If this same pile is assumed to be pinned at the top connection to the deck,
\[ k_2 \leq 0.001 \]

\[ C_{IA} = 0.51 \]

\[ P_{crit} = \frac{0.51}{0.75} \times 132^k = 89.6 \text{ kips} \]

\[ C_{IB} = 8.15 \]

\[ P_{crit} = \frac{8.15}{12.0} \times 132^k = 89.6 \text{ kips} \]

The following table will show how the critical buckling load varies with the amount of fixity at the large end.

<table>
<thead>
<tr>
<th>Amount of Fixity [k_2]</th>
<th>[C_{IA}]</th>
<th>[C_{IB}]</th>
<th>Critical Buckling Load [P_{crit}]</th>
<th>Design Load Assuming F. S. = 2.5 [F.L.]</th>
<th>Rule of Thumb Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001 (pinned)</td>
<td>0.51</td>
<td>8.15</td>
<td>89.6 [k]</td>
<td>35.8 [k]</td>
<td>26.0 [k]</td>
</tr>
<tr>
<td>0.50</td>
<td>0.58</td>
<td>9.3</td>
<td>102 [k]</td>
<td>40.8 [k]</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.64</td>
<td>10.2</td>
<td>112 [k]</td>
<td>44.8 [k]</td>
<td></td>
</tr>
<tr>
<td>2.67</td>
<td>0.75</td>
<td>12.0</td>
<td>132 [k]</td>
<td>51.8 [k]</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>0.84</td>
<td>13.4</td>
<td>147.5 [k]</td>
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BIBLIOGRAPHY


APPENDIX
COMPUTER PROGRAM

PROGRAM GFD

C THIS PROGRAM IS TO BE USED TO SOLVE AN EQUATION FOR THE VALUE OF K TO BE USED TO DETERMINE THE CRITICAL BUCKLING LOAD FOR A ROUND TAPERED COLUMN WITH VARIABLE END FIXITY.

REAL K, K1, K2
DIMENSION ANSW(1000)
IØ=61
IN=60
WRITE(IØ, 2)
   DØ 900 I=1, 28
   READ (IN, 13)K2
   DØ 900 J=1, 10
   K1=J
   IF(K1=1.)30, 25, 30
25 K1=1.3
30 CONTINUE
DELTA=. 1
START=. 3
DØ 500 M=1, 1000
K=START+DELTA*FLOAT(M)
T1=1./(1.-K1)
T2=T1*K1*K2/K**2
T3=T2*T1
T4=T3*K1
T5=SIN(K)
T6=SIN(K*K1)
T7=COs(K)
T8=COs(K*K1)
T9=K*K1
A1=T1*T5*T8
A2=T4*(T5*T8+T9*T5*T6)
A3=T4*(T7*T5+K*T5**2)
A4=T5*T8+T9*T5*T6
A5=T1*T7*T6
A6=-T4*(T9*T7*T8-T7*T6)
A7=-T4*(K*T7**2-T7*T5)
A8=-T9*T7*T8+T7*T6
A9=T3*(T6*T8+T9*T6**2)
A10=T3*(T6*T7+K*T5*T6)
A11=-T3*(T9*T8**2-T8*T6)
A12=-T3*(K*T8*T7-T8*T5)
A13=-T2*(K*T7-T5)(T9*T8-T6)
A14=-T2*(K*T7-T5)(T8*T9*T6)
IF(ABS(ANSW(M))-. 001)600, 600, 700
700 IF(M=1)800, 500, 800
IF((ABS(ANSW(M))+ABS(ANSW(M-1)))>ABS(ANSW(M) + ANSW(M-1)))S00, 500, 40
Q1=K-DELTA
QC=K
IF(ANSW(M)S, 5, 9
Q2=K-DELTA
Q1=K
B=Q1
A=Q2
X=(A+B)/2.
K=X
T1=1./(1.-K1)
T2=T1*K1*K2/(K**2)
T3=T2*T1
T4=T3*K1
T5=SIN(K)
T6=SIN(K*K1)
T7=CO(S(K)
T8=CO(S(K*K1)
T9=K*K1
A1=T1*T5*T8
A2=T4*(T5*T8+T9*T5*T6)
A3=T4*(T7*T5+K*T5**2)
A4=T5*T8+T9*T5*T6
A5=T1*T7*T6
A6=-T4*(T9*T7*T8-T7*T6)
A7=-T4*(K*T7**2-T7*T5)
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A10=T3*(T6*T7+K*T5*T6)
A11=-T3*(T9*T8**2-T8*T6)
A12=-T3*(K*T8*T7-T8*T5)
A13=-T2*(T7+K*T5)*(T9*T8-T6)
A14=-T2*(K*T7-T5)*(T8+T9+T6)
Y=ANSW(M)
IF((ABS(B-X)>0.0001))S3, 3, 4
G0 T0 600
3 G0 T0 6
4 IF(Y)?S, 3, 8
7 A=X
G0 T0 6
8 B=X
G0 T0 6
600 CIA=(K**2*(K1-1.))/(3.1416**2*K1**2)
CIB=K1**4*CIA
WRITE(I0, 1)K2, K1, K, CIA, CIB
G0 T0 900
500 CONTINUE
900 CONTINUE
1  FORMAT(1X, F15.6, 2F10.3, 2F18.3)
2  FORMAT(8X, K2=C/EIA/L, 2X, K1=DA/DB', 6X, 'K', 16X, 'CIA', 16X, 'CIB')
13 FORMAT(F6.3)
STOP
END

PROGRAM G DATA

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00003:.01
00004:.05
00005:.10
00006:.20
00007:.30
00008:.40
00009:.50
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00012:.80
00013:.90
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FLOW CHART

START GFD 22

PRINT C HEADING

DO 900 I=1, 28

READ 13 K2

DO 900 J = 1, 10
   K1 = J

30 -

IF
   K1 = 1.3

30 +

K1 = 1.3

25

30 Continue

G-DATA FILE
\[
\begin{align*}
\text{DELTA} &= 0.1 \\
\text{START} &= 0.3 \\
\text{DO} &\ 500 \ M = 1, 1000 \\
\text{END} \\
\text{ANSW}(M) &= A_1 - A_2 + A_3 \\
\text{IF} \quad \text{ABS(ANSW(M))} > 0.001 \\
\text{M} &= M - 1 \\
\text{IF} \quad \text{ABS(ANSW(M))} + \text{ABS(ANSW(M))} > 0.001 \\
\text{Q1} &= K - \text{DELTA} \\
\text{Q2} &= K
\end{align*}
\]
\[ Q_2 = K - \Delta \]
\[ Q_1 = K \]

**BISECTION METHOD**

\[ B = Q_1 \]
\[ A = Q_2 \]

\[ X = \frac{A + B}{2} \]
\[ K = X \]
\[ T_1 = \cdots \]
\[ Y = \text{ANSW}(M) \]

**IF**

\[ \text{ABS}(B - X) < 0.001 \]

\[ \text{GO TO} \ 600 \]

**IF**

\[ (Y) \]

\[ \text{GO TO} \ 600 \]

\[ \text{GO TO} \ 600 \]

\[ \text{GO TO} \ 600 \]
Bisection Method

As a matter of interest, a method of looping was first used in an attempt to converge on the root to the basic equation for the term "k". Each time the computer performed the loop, the interval used between each assumed value was reduced by a factor of ten. The intent was to converge quickly to a root. The first few computer runs proved that this method was not only inefficient but the limits of storage for the problem were also exceeded. The computer run time was in excess of 200 seconds.

The bisection method was next tried and although other methods may be more efficient it was found that this method reduced the initial computer run time from approximately 200 seconds for an incomplete run to approximately seven seconds for the computations of all values needed. This method is explained in detail on page 220 of reference 4.

This experience revealed that many methods have already been tried and tested and are available for use by the inexperienced programmer.