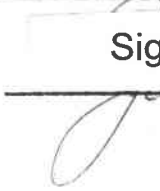AN ABSTRACT OF THE THESIS OF

Stephen Hosmer Smith   for the degree of   Master of Science

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Title: THE EVALUATION OF COMPETITIVE STRESS INDEX AS A MEASURE OF

STAND DENSITY FOR YOUNG GROWTH DOUGLAS-FIR

Abstract approved:

Signature redacted for privacy.

Dr. John F. Bell

The study evaluates the use of Competitive Stress Index (CSI) as a quantitative measure of stand density in a young-growth Douglas-fir stand in Western Oregon which had been subjected to various thinning regimes. Operational applications of the index are examined and new methodologies presented for the analysis of tree and stand growth response following thinning. Field procedures for sampling for competitive stress are outlined and the derivations of various sampling estimators are given. An index for predicting regular suppression mortality is derived and tested under natural stand density conditions. Results of the study indicate CSI is a viable expression of stand density and has tremendous potential as a management tool.

The Evaluation of Competitive Stress Index as a Measure of
Stand Density for Young Growth Douglas-Fir

by

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THE EVALUATION OF COMPETITIVE STRESS INDEX AS A MEASURE OF
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INTRODUCTION

The current trend of forest management research has focused on the
individual tree and its environment. Inter-tree competition is an im-
portant component in the analysis of tree and stand growth relationships.
New measures of stand density are needed to evaluate the relative level
of inter-tree competition in the stand. Competitive stress index (CSI)
is a measure of stand density which expresses the relative level of
competition between trees for growing space. It is a quantitative mea-
sure of the relative stress exerted on a tree by its neighbors. This
study examines the applications of the CSI model as a dynamic measure
of stand density.

The primary objectives of this study are two-fold. First, the
study attempts to evaluate the competitive stress model (CSI) as a
measure of stand density in young growth Douglas-fir stands. Second,
the study develops methodologies for operational application of the CSI
model. In accomplishing these objectives, the sensitivity of the index
in regulating thinning intensity, in predicting growth response of
trees and in determining a tree's competitive status will be examined.
In addition, procedures for deriving stocking and spacing guidelines,
sampling for competitive stress and predicting future mortality based
on the CSI model are also developed.

The scope of this study is limited to young growth Douglas-fir stands. Although the study results represent localized stand conditions, application of the methodologies employed are unrestricted. Hopefully, this study will provide new insights into the application of the CSI model to growth and yield forecasting for Douglas-fir in the Pacific Northwest.

THE STUDY AREA

The basic data used in the study were obtained from the Hoskins levels of growing stock study plots located approximately 22 miles West of Corvallis, near Hoskins, Oregon. The study area was established in a 20 year old, pure, even-aged stand of Douglas-fir. The stand was of natural origin. The study area had uniform soil and site conditions (approximately Site Class II). The Hoskins study plots are a part of a regional cooperative levels of growing stock study designed to examine the effects of various thinning regimes on stand growth and development. The Hoskins study consists of eight thinning treatments plus control replicated three times in a completely randomized design. Bell (1972) describes the Hoskins study and results in detail. Thinning treatments ranging from heavy to light in intensity were examined.

Since establishment, three thinning periods have occurred, at the end of 1966, 1970, and 1973 growing seasons, respectively. In this study, only data from treatments 1, 3, 5, 7, and control were analyzed. Table (1-1) gives the level of growing stock treatment schedule as the percent of the gross basal area increment of the control plot to be retained in growing stock.

Table (1-1): Percent of gross basal area
increment retained as growing
stock

Treatment	Thinning Period		
	1966	1970	1973
1	10	10	10
3	30	30	30
5	50	50	50
7	70	70	70
Control (unthinned)	100	100	100

Note the decrease in thinning intensity from treatment one to treatment seven. The thinning intensity by period depends upon the gross basal area increment of the control. Initial stand density for all treatments 1-7 were reduced to a prescribed level of about 50 sq. ft. BA/ACRE at the end of the 1963 growing season (Bell, 1972). Subsequent thinnings in 1966, 1970, and 1973 removed the prescribed percentage of the basal area for each treatment (see Chapter 3). Tables (1-2, 1-3, 1-4, and 1-5) give the stand data by treatment and measurement period.

DATA

The basic data for the study represented a ten year accumulation of individual tree measurements from 15 square, one-fifth acre, stem-mapped study plots. Detailed measurements of all trees were recorded

Table (1-2): Total basal area (TBA) by treatment 1963-1973
(square feet)

Treatment Number	1963	1966	1966 Thinning	1970	1970 Thinning	1973	1973 Thinning
1	49.4	85.5	55.1	99.5	60.4	85.1	64.4
3	49.0	85.0	64.2	111.5	81.6	111.3	85.9
5	49.2	86.0	74.9	126.5	103.6	137.6	121.1
7	50.1	85.9	84.9	139.4	124.5	161.8	150.2
Control (unthinned)	138.1	185.7	185.7	228.6	228.6	256.3	256.3

Table (1-3): Mean DBH by treatment 1963-1973 (inches)

Treatment Number	1963	1966	1966 Thinning	1970	1970 Thinning	1973	1973 Thinning
1	4.9	6.5	6.7	9.0	9.5	11.3	11.8
3	5.0	6.6	6.6	8.8	9.0	10.6	10.8
5	4.8	6.4	6.5	8.4	8.4	9.8	10.0
7	5.1	6.8	6.8	8.7	8.7	9.9	10.0
Control (unthinned)	3.6	4.3	4.3	5.5	5.5	6.3	6.3

Table (1-4): Average spacing by treatment 1963-1973 (feet)

Treatment Number	1963	1966	1966 Thinning	1970	1970 Thinning	1973	1973 Thinning
1	10.6	12.3	13.2	15.7	17.4	18.6	19.9
3	10.6	12.6	13.0	15.2	15.9	17.1	18.0
5	10.6	12.3	12.5	14.6	14.8	16.2	16.6
7	10.9	12.8	12.8	14.9	15.0	16.3	16.6
Control (unthinned)	8.2	9.0	9.0	10.4	10.4	11.5	11.5

Table (1-5): Number of trees per acre by treatment 1963-1973

Treatment Number	1963	1966	1966 Thinning	1970	1970 Thinning	1973	1973 Thinning
1	353	352	215	215	118	118	83
3	343	342	252	252	175	175	130
5	365	363	312	312	250	248	213
7	328	328	323	323	287	287	262
Control (unthinned)	1727	1640	1640	1272	1272	1087	1087

for each measurement period before and after thinning. These data include: plot number, tree number, x-y stem coordinates, diameter at breast height (nearest tenth inch), and tree status codes (dead, alive, cut). The basic computational procedures for determining a tree's competitive stress status using the CSI model are outlined below. Using the Hoskins Study data, individual plot records were sorted and stored on a permanent data file. Each fifth-acre plot served as the primary unit of interest. All trees contained within the plot were designated subject trees. Potential competitors were obtained by mirroring the original plot. Eight identical simulated plots were established by translating the original plot boundaries in cardinal directions (see Appendix E). The CSI model was fit to all subject trees by selecting simulated competitors from the nine nested plots (see Chapter 2).

The following data were generated and recorded for each subject tree: plot number, tree number, diameter, height, CSI, number of competitors, open grown crown area, average DBH of competitors, and average distance to competitors. These data were tabulated for each measurement period before and after thinning in 1963, 1966, 1970, and 1973. Collectively, these data comprised the basic data used in the study.

All analyses in the study were performed on the Oregon State University CDC 3300 computer facilities. Computer programs written in Fortran IV were developed to manipulate the data.

LITERATURE REVIEW

INTRODUCTION

Competition indices are measures of stand density that reflect the relative level of competition between trees for growing space. They are based on the hypothesis that the level of inter-tree competition is proportional to the size and spatial arrangements of trees in the stand. Relative competition for growing space can be expressed as a function of the area available to the tree relative to the area available to its neighbors.

The development of inter-tree competition indices has closely paralleled the development of stand simulation models. The higher degree of sophistication and improved methodologies associated with growth and yield forecasting has resulted in construction of more flexible and rigorous inter-tree competition models. The following sections describe six distinct types of competition models which have been developed to evaluate inter-tree competition. The format of presentation for each model focuses on how inter-tree competition is defined for individual trees in the stand. The order of presentation corresponds roughly to the level of sophistication of each competition model. Before the competition models are presented, however, it is essential that the terms inter-tree competition and stand density be fully understood.

The term competition is defined as the active demand by two or more organisms for a common vital resource (Wilson, 1971). The use of

the resource by one or more individuals reduces the amount available to other individuals in the population. The growth and survival of all individuals may be affected by a shortage of the resources (Whittaker, 1970).

Biological competition can be subdivided into interspecific and intraspecific competition. Interspecific competition is the competition which occurs between individuals belonging to different species. Intraspecific competition occurs between individuals which belong to the same species. Intraspecific competition in a single species forest stand is the most keen, since the niche requirements of each tree are identical to those of the other trees in the stand (Kormondy, 1969).

Inter-tree competition in a pure, even-aged Douglas-fir stand is a type of intraspecific competition. The growth of the individual tree is directly related to the proportion of the total resources available to it (Adlard, 1973). Inter-tree competition exists if these resources are reduced and subsequent growth of an individual tree is impaired by the presence of other trees in the stand. Inter-tree competition can be further subdivided into absolute and relative levels of biological interaction. Absolute competition refers to the amount and source of the interaction between two or more trees in the stand for the resources of the site, including: light, moisture, and nutrients. Trees compete aerially with their crowns and in the subsurface with their root systems. The absolute level of competitive interaction between two trees is difficult to assess. Relative competition expresses the interaction between trees for growing space. The growing space or influence zone

of a tree is the area in which it competes for the resources of the site necessary to sustain growth. Relative inter-tree competition does not describe the source or amount of the interaction between trees, but only quantifies the growing space requirement of the tree.

Measures of stand density or stocking are used to depict the degree to which the productive capacity of the site is being utilized by the stand (Husch, 1972). Although the two terms are often used interchangeably, a distinction between stand density and stocking is usually made. Standard definitions for stand density and stocking are given below. Stocking is a relative term used to indicate the number of trees in the stand as compared to the desirable number for best growth and management (Curtis, 1970). A stand may be under-stocked, over-stocked, or fully stocked, depending on what management objective is considered adequate for the given level of stand density (Gingrich, 1967). Stand density, on the other hand, is a quantitative expression of the number of trees, basal area, volume, or other criteria per unit area occupied by the stand (Bickford, 1957). Curtis (1970) states that "stocking is a comparison with current management objectives and stand density is almost any numerical quantity obtainable by measurement of the stand on an area basis."

In this paper the term stand density is defined as the degree of crowding of stems on an area occupied by trees without reference to any management objective. This definition is consistent with the common usage of the term (Curtis, 1970). Measures of stand density are quantitative variables which utilize the measurable characteristics of the

stand to describe the level of density or degree of inter-tree competition. They are generally independent of age and site index and directly interpretable in biologically meaningful terms (Curtis, 1970).

The following chapter reviews various measures of inter-tree competition including whole stand models, point density models, tree size and distance dependent models, growing space polygon models, miscellaneous competition models, and influence zone overlap models. The distinction between the general types of competition models and the classification of particular indices was subject to the discretion of the author. Unless a competition index is commonly referred to in the literature by a special acronym, the general acronym, CI (formed from competition index) was assigned to all indices.

WHOLE STAND MODELS

Measures of stand density which are based on the average area occupied or available area per tree relative to some standard density condition are collectively referred to as whole stand models. Such density measures provide the means of comparing actual stocking conditions in a stand with the stocking of similar stands grown under optimal or standard conditions. Two standard density conditions are commonly used as baseline references: the open-grown condition and the so-called "normal" or average maximum level of competition. These standard conditions represent the biological limits to growth in a natural stand. Whole stand models are relative measures of density which express an observed stand parameter as a percentage of the normal, fully stocked,

or open-grown stand density. Curtis (1970) has pointed out the similarities between a number of whole stand models commonly used to express relative stand density. Such measures of stand density are interpretable as expressions of the cumulative degree of crowding or competitive effects on the development of the "average" trees in closed stands.

Basal area is frequently referred to as a measure of stand density (Bickford, 1957). The ratio of the stand's basal area per acre to the normal basal area per acre for stands of the same age and site index or average stand diameter is a relative measure of stand density (Husch, 1972). The ratio of observed basal area to the normal basal area expresses the total area available in a normal stand to the trees in the observed stand per unit area. Similarly, stand volume can be used as a relative expression of stand density (Husch, 1972). Density is determined by comparing the volume of the observed stand with the volume of the standard stand of the same age and site quality.

The number of individuals per unit area is typically used in ecological studies to describe population density. In forest stands, however, the number of stems per unit area is an imperfect expression of stand density because of the wide variation in tree sizes (Husch, 1972; Curtis, 1970). A fully stocked stand may contain fewer stems per unit area than an understocked stand with many smaller individuals (Loetsch, 1973). For this reason, the number of trees per unit area conveys a measure of stand density only if it is qualified by some measure of tree size (Husch, 1972). A measure of stand density based

on the number of trees per unit area and average stand diameter is Reineke's (1933) stand density index.

Stand Density Index (SDI) expresses the number of trees per unit area as a percentage of the number of trees per unit area for a fully stocked stand of the same average diameter.

$$SDI = \frac{N_o}{N_e} \qquad (2\text{-}1)$$

Where:

N_o = the observed number of trees per unit area in the stand

N_e = the normal number of trees per unit area

The expected or normal number of trees per unit area for a specified average stand diameter is given by:

$$N_e = a\overline{D}^b \qquad (2\text{-}2)$$

Where:

$\overline{D}$ = the quadratic mean diameter of the stand (i.e., the diameter of the tree of mean basal area)

a, b = constants

A least squares solution to Equation (2-2) would fit:

$$\log N_e = a + b \log \overline{D} \qquad (2\text{-}3)$$

Reineke (1933) found that the constant b was -1.605 for several species. It is assumed that SDI is not strongly correlated with age or site index but according to some investigators this is not always the

case (Bickford, 1957; Loetsch, 1973; Paine, 1976). Curtis (1970) showed that the ratio of N_o to N_e in Equation (2-1) can be interpreted as the ratio of the total area occupied by the observed number of trees in the normal stand to the unit area available to trees in the observed stand with the same average diameter.

S.F. Gingrich (1967) used the basal area and number of trees per acre to derive a relative density measure, the stocking percent. He derived a stocking chart for hardwood stands in the central U.S. which gives the stocking percent as a function of the number of trees and basal area in relation to a normal stand condition. A stocking chart indicates whether or not the stand is fully stocked, overstocked, or understocked. Gingrich's stocking chart is based on the premise that all tree heights have the same stocking percent for a given number of trees and basal area (Loetsch, 1973).

The Tree Area Ratio (TAR) proposed by Chisman and Schumacher (1940) is a relative measure of stand density which is independent of age and site quality. It is based on the hypothesis that the growing space area of a tree is a function of tree diameter.

$$TA_j = b_0 + b_1 D_j + b_2 D_j^2 \tag{2-4}$$

Where:

   TA_j = the tree area occupied by the jth tree in the stand (j = 1,N)

   D_j = the diameter at breast height of the jth tree

   b_0, b_1, b_2 = constants

The total area of growing space occupied by the N trees in the stand is represented by summing over all the j tree areas on the unit area occupied by the stand.

$$TTA = \sum_{j=1}^{N} TA_j \qquad (2\text{-}5)$$

Where:

TTA = the total tree area of the stand

Substituting Equation (2-4) for the tree area of the jth tree into Equation (2-5) and simplifying gives:

$$TTA = \sum_{j=1}^{N} (b_0 + b_1 D_j + b_2 D_j^2) = b_0 N + b_1 \sum_j D_j + b_2 \sum_j D_j^2 \qquad (2\text{-}6)$$

The tree area ratio for the stand is the ratio of the total tree area to the area occupied by the stand:

$$TAR = \frac{TTA}{A_c} = \frac{(b_0 N + b_1 \sum_j D_j + b_2 \sum_j D_j^2)}{A_c} \qquad (2\text{-}7)$$

Where:

TAR = tree area ratio

A_c = ground area occupied by the stand

For a normal stand of unit area the TAR in Equation (2-7) is equal to 1.0. The constants b_0, b_1, and b_2 can be estimated from a sample of normal stands adjusted to a unit area basis, using least squares estimators. Once these constants have been determined for normal stands, Equation (2-7) can be applied to non-normal stand conditions. The TAR

for a stand expresses the proportion of the unit area which would be occupied by the stand if each tree has the area occupied by a tree of the same diameter in a normal stand (Curtis, 1970). If the constants in Equation (2-7) were derived from data collected in normal or fully stocked stands, the TAR for an observed stand will reflect the proportion of full stocking (i.e., TAR = 1.0) represented by the stand (Husch, 1972).

Crown Competition Factor (CCF) developed by Krajicek et al. (1961) is a measure of stand density based on open-grown crown areas. CCF estimates the average growing space area available to trees in the stand relative to the maximum growing space available to open growing trees of similar diameter. The CCF for a stand occupying a unit area is:

$$CCF = (100) \frac{\sum_{j=1}^{N} A_j}{A_c} \tag{2-8}$$

Where:

A_j = the maximum crown area of the jth tree in the stand

N = the number of trees in the unit area

A_c = the ground area occupied by the stand

The maximum growing space area (A_j) for a tree of specified diameter is represented by the vertical projection of the crown area of an open-grown tree of the same diameter. This maximum growing space area is assumed to be a circular area with radius, CR. The relationship between open-grown crown width or crown radius is often expressed as a function of tree diameter (Arney, 1973):

$$CW = b_0 + b_1 D + b_2 D^2 \qquad (2\text{-}9)$$

Where:

CW = open grown crown width of the tree

D = the diameter of the tree at breast height

b_0, b_1, b_2 = coefficients

The coefficients in Equation (2-9) are estimated using the method of least squares from measurement of a satisfactory number of open-grown trees.

The maximum crown area A_j is expressed using Equation (2-9) to define open-grown crown width as:

$$A_j = \frac{\pi(CW_j)^2}{4} \qquad (2\text{-}10)$$

The percentage of an acre occupied by the open-grown crown area is obtained by dividing A_j by the area of an acre in square feet.

$$Q_j = \frac{100\ A_j}{43560} \qquad (2\text{-}11)$$

Summing over all Q_j values for the stand gives an alternative expression of CCF per acre as:

$$CCF = \sum_{j=1}^{N} \frac{Q_j}{A_c} \qquad (2\text{-}12)$$

CCF differs from the Tree Area Ratio in that the growing space areas for open grown trees are used as the baseline reference rather than the

average areas occupied by trees in a normal stand. Krajicek (1961) states, "CCF estimates the area available to the average tree in the stand in relation to the maximum area it could use if it were open-grown." CCF values less than 100 indicate that the sum of the open grown crown areas in Equation (2-12) are less than the ground area occupied by the stand. In this case, the growing space available is not being fully utilized. CCF values greater than 100 indicate that the trees in the stand more than fully cover the ground area occupied by the stand. For example, a CCF = 200 represents the condition where the sum of the maximum crown areas are 2 times the ground area occupied by the stand. A CCF = 100 represents full occupancy of the site for a uniformly spaced stand. Note that in this case, not all the trees in the stand are necessarily open-grown. The growing space of maximum crown areas of an individual tree is assumed to be circular. An open-grown tree's growing space area is not overlapped by another tree's growing space circle. In a uniformly spaced stand, all of the ground area cannot be fully occupied by non-overlapping circles. The truly open-grown condition has a CCF value of less than 100. Krajicek (1961) states that CCF is not a measure of crown closure, as complete crown closure can occur from CCF = 100 to the maximum for the species.

CCF has proved to be a useful measure of stand density that is independent of age and site quality. Several investigators have fit the CCF model to a variety of species (Vezina, 1962; Dahms, 1966; Alexander 1967; Paine, 1976).

The whole stand models described previously offer only crude estimates of inter-tree competition in the stand. The degree of competition

experienced by individual trees in the stand are based on the average tree. Whole stand models are insensitive to the competitive relationships between individual trees. Spatial relationships in the stand are ignored in estimating stand density. Conventional measures of stand density are expressions of cumulative crowding or competition effects in the development of average trees only when the stand is reasonably uniform and fully closed (Curtis, 1970). Aggregate competitive effects are derived on a whole stand basis, relative to some standard density condition. Such standard density conditions as normal or fully stocked are not easily defined and application of these standard density conditions to managed stands is questionable. A major limitation of whole stand models is that stand density is used to describe individual tree competition. The use of average tree conditions obscures the causal relationship between inter-tree competition and individual tree growth (Bella, 1970).

POINT DENSITY MODELS

Point density models use measures of stand density around individual trees to express the intensity of inter-tree competition. Point density models are based on sample plot, angle count or angle summation measurement techniques. Inter-tree competition is assumed to be proportional to the basal area per acre observed at a sample point in the stand. If the point is centered on an individual tree, the observed density is an expression of the relative amount of inter-tree competition the tree experiences. In general, the higher the point density,

the greater the degree of competition the tree receives. Point density models only indirectly utilize the size and spatial relationships between trees in the stand. The growing space for an individual tree is poorly defined by point density models. The growing space of a tree is typically arbitrarily assigned proportional to tree DBH. The greatest limitation of point density models is that stand density is used to express individual tree competition. Basal area per acre is an absolute measure of stand density that has no direct interpretation in terms of the competitive influence exerted by competing neighbors. The generalization that the higher the stand density the less the expected growth of the trees tends to obscure the cause and effect relationship between competition and tree growth (Bella, 1970).

Steneker and Jarvis (1963) centered a sample plot of fixed radius around individual subject trees in the stand. The competition index or point density for a tree was defined as the sum of the basal areas of all the trees on the plot expressed on a per acre basis. In general:

$$CI_j = \sum_{i=1}^{n} BA_i \qquad (2-13)$$

Where:

CI_j = the point density observed for the jth sample tree's plot

BA_i = the basal area of the ith competitor tree in the plot at breast height

n = the number of competitors in the jth tree's plot

The growing space of a tree was assumed to be equal to the area of a circular plot of fixed radius centered around the tree. The competitors of the subject tree were those trees which were contained within the sample plot's boundaries.

Spurr (1962) and Opie (1968) criticized the sample plot method of evaluating point density. The size and shape of the sample plot effect the point density estimates. Different basal area estimates will be obtained for different sized plots centered on the same tree. The index in Equation (2-13) does not distinguish between degrees of competitive influence. Trees located near the sample point should exert a greater competitive influence than competing trees further away. In the sample plot method, all competitors contribute equally to the point density index without regard to the spatial distribution around the subject tree.

Lemmon and Schumacher (1962) developed a point density index based on angle count summation for individual trees in the stand. They assumed that basal area measurements with an appropriate angle gauge reflected the stocking density around an individual tree. A sample point was centered on a subject tree. The number of overlapping circles or "in" trees at the point were used to estimate the stocking density for the tree. It was assumed that the basal area per acre density observed at the point indicated the relative level of competition experienced by the tree.

$$CI_j = BAF\ t_j \qquad\qquad (2\text{-}14)$$

Where:

CI_j = the point density index for the jth tree (expressed as basal
   area per acre)

BAF = basal area factor used in sampling

t_j = the tree count or angle count for the jth tree

The potential competitors of a subject tree are defined as those trees which subtended the critical angle of the BAF used to measure point density. One of the weaknesses of the angle count model is its definition of growing space. The model assumes that the growing space of a tree is a function of the BAF selected. The larger the BAF, the smaller the growing space of a given size tree. Lemmon and Schumacher (1962) discuss the problems of spatial distribution and irregularities associated with selecting a BAF relative to defining a tree's growing space. However, their best results were obtained using a 10 BAF angle gauge.

Although the model in Equation (2-14) eliminated some of the questions raised by fixed radius model in Equation (2-13), several limitations are apparent. Spurr (1962) criticized the angle count summation model because only the total number of trees subtending the critical angle are counted at a given point. The precision of a single point density estimate is therefore only as small as the contribution of each tree. For example, using 10 BAF, each tree tallied contributes 10 square feet per acre, while using 30 BAF angle gauge, each "in" tree contributes 30 square feet of basal area. The angle count model only indirectly uses the spatial relationships of the surrounding trees in

measuring the point density of the subject tree. Any tallied tree contributes equally to the estimate regardless of its size of proximity to the subject tree (Opie, 1968).

Spurr (1962) developed a measure of point density based on angle summation similar to Bitterlich's angle count method. He proposed an index which involved the measurement of the actual angles subtended by trees around a given point by stems of surrounding trees in sequence, rather than using a fixed angle gauge which was arbitrarily chosen. Spurr's index is defined as:

$$CI_j = \frac{76.625}{n} \sum_{k=1}^{n} (k - 1/2) \left(\frac{D_k}{L_k}\right)^2 \qquad (2\text{-}15)$$

Where:

CI_j = the point density estimate for the jth tree expressed in terms of basal area per acre

n = the number of trees selected

D_k = the diameter at breast height of the kth competitor (in inches)

L_k = the distance from the kth competitor to the sampling point (in feet)

The angle summation method is based upon the same principles used to derive the Angle Count Model in Equation (2-14). The Angle Summation Model is more precise in that tree size and relative position with respect to the sample point determines the contribution to the point density estimate (Spurr, 1962). Competitor trees are sequentially selected

and ranked by their $(D/L)^2$ ratio. A limiting angle is chosen so as to
cause the kth competitor to be exactly subtended. The progressive sum-
mation of the basal area estimates for all n competitors gives the mean
area for the point, adjusted for the spatial distribution of the sur-
rounding trees.

A serious limitation of Spurr's Point Density Index is its failure
to identify which trees qualify as competitors. In addition, Spurr's
index in Equation (2-15) is tedious to apply, although it is an improve-
ment over the Angle Count Method discussed earlier.

TREE SIZE AND DISTANCE DEPENDENT MODELS

Competition indices which express the interdependence of individual
trees in the stand only as a function of their physical size and spatial
location relative to the size and location of their competitors are
collectively referred to as tree size and distance dependent models
(Ek, 1975). This type of competition model is a direct outgrowth of
the distance measuring techniques used in ecological sampling. A number
of such methods are available which describe the distance to the nearest
neighbor as a function of the spatial relationships of the population
(Clark, 1954; Laylock, 1975). These methods provide estimates of
population density based on the spatial distribution of individuals.
Tree size and distance dependent models combine the spatial relation-
ships between individual trees in the stand and tree size in assessing
relative competitive status.

The amount of competition a tree experiences is assumed to be pro-
portional to the relative size, location, and number of its competitors.

Tree size is usually some measurable characteristic of the tree (e.g., DBH, crown dimensions, height). Tree size and distance dependent models are one of the simplest forms of expressing inter-tree competition in the stand. They played an integral part in the development of early individual tree simulation models (Ek, 1975). The available growing space of the tree is not clearly defined by the model. Usually the growing space of a tree is assumed to be a circular plot of fixed radius centered on the tree. The size of the plot is proportional to the size of the tree. Newnham (1966) calculated the competition radius of a tree using open grown crown width as a function of tree diameter. He also used various BAF (40 and 80) to arbitrarily define the growing space radius of a tree as the limiting distance for a given critical angle.

All trees within the growing space of a subject tree can potentially contribute to the level of stress exerted on the subject tree. A competitor's contribution is usually assumed to be directly related to its size and inversely related to the distance from the subject tree.

Tree size and distance dependent models offer a mathematical description of stand structure. Their chief advantage is their simplicity in assessing inter-tree competition in the stand. The most serious limitation in application of the models is their relative nature. Tree size and spatial location are only empirically related to tree growth and are not biologically interpretable as measures of inter-tree competition. A large number of competition indices based on tree size and distance relationships can potentially be formulated. Their validity as true measures of inter-tree competition is questionable.

Hegyi (1974) defined a relative measure of inter-tree competition
for each tree as:

$$CI_j = \sum_{i=1}^{n} \frac{D_i}{D_{sj}} \frac{1}{L_{ij}}$$ (2-16)

Where:

CI_j = the competition index of the jth subject tree

D_{sj} = the diameter of the subject tree at breast height

D_i = the diameter of the ith competitor at breast height

L_{ij} = the distance between the jth subject tree and the ith competitor

n = the number of competitors within a circle of fixed radius

Hegyi states, "the rationale for the CI lies in the assumption that
a tree may be subjected to competition even in the absence of crown
overlaps." Species which naturally exhibit wide spacings or irregular
crown relationships can be evaluated using this relative measure of
competition.

The index in Equation (2-16) uses a weighting factor based on the
ratio of the diameter of the competitor to the diameter of the subject
tree to adjust the contribution of the ith competitor; the larger the
size of the subject tree, the smaller the contribution. In general,
the higher the index, the greater the relative level of competition
the tree experiences. If the growing space radius of the tree is ex-
pressed as a function of maximum crown expansion, a CI = 0 indicates
an open grown tree (i.e., no competition). The expression in Equation

(2-16) is not directly interpretable in terms of the amount of competition a tree experiences, but only as a relative ranking of the tree's competitive status.

Quenet (1976) defines a competition index using only the size of the competition and its distance from a subject tree (or sample point:

$$CI_j = \sum_{i=1}^{n} \frac{D_i^2}{L_{ij}} \tag{2-17}$$

Inter-tree competition in Equation (2-17) is assumed to be directly proportional to the diameter squared (basal area) of the ith competitor and inversely to the distance from the subject tree. Unless a growing space area is defined for the subject tree, all trees in the stand are potential competitors since L_{ij} can be defined for each tree. Quenet's index does not account for a differential weighting between the ith competitor and the subject tree. In fact, the index in Equation (2-17) is completely independent of the size of the subject tree.

Newnham (1966) tested various expressions of tree size as indicators of growth in red spruce stands. He found certain crown and crown related measures can be used to estimate growth increment over short periods. Functions of tree diameter and distance, similar to those discussed earlier, provide the highest correlation with growth, although not as effectively as the more sophisticated competition indices tested.

Mendiboure (1974) analyzed the growth increment in three red spruce stands which were geometrically thinned. Inter-tree competition was evaluated using functions of tree height, diameter, and basal area.

A general competition index was formulated as:

$$CI_j = \sum_{i=1}^{n} \frac{G_{sj} - G_i}{L_{ij}} \qquad (2\text{-}18)$$

Where:

G_{sj} = the size of the jth subject tree (e.g., height, DBH, basal area)

G_i = the size of the ith competitor

The competition experienced by the jth subject tree is expressed as the difference in the size of the subject tree and the ith competitor divided by the distance between the tree. Equation (2-18) had the highest correlation when basal area was used as the measure of tree size to predict annual growth increment.

GROWING SPACE POLYGON MODELS

Growing space polygon models describe the primary influence zone of an individual tree as an irregular closed polygonal area surrounding the tree. The boundaries of the polygon are formed by intersecting lines between and perpendicular to the tree of interest and each of its competitors (Moore, 1973). The area of the polygon reflects the competitive status of the tree relative to its neighbors. The larger the area of the polygon, the greater the influence zone of the tree. As the competitive stresses exerted by neighboring trees increase, the influence zone or area potentially available to the tree is reduced. Competition for growing space is a fucntion of tree size; the larger the tree,

the greater the competitive influence it exerts on its neighbors. Polygon based models are geometric models that take into account the irregularity in the spatial and competitive relationships in the stand (Adlard, 1973). Aerial and root competition effects are expressed by the non-symmetrical development of a tree's growing space area with respect to time. Each tree is assigned a mutually exclusive growing space area. Competing neighbors are defined automatically in the construction of the growing space polygon for each tree.

Brown (1965) advocated that the growing space area of a tree could be quantified by constructing a polygon around each tree in the stand. The area of each polygon would approximate the area potentially available to each tree. The growing space area for the stand could then be partitioned into a closed network of interlocking polygons. The area of each polygon would reflect the competitive status of the tree. Brown assumed that the boundary of the area was located half way between two competing trees and perpendicular to the line between their centers. By connecting these boundary lines for all pairs of neighbors in the stand, a closed network of non-overlapping polygons is formed. The area potentially available to an individual tree in the stand is approximated by the area of the polygon which surrounds it.

Using Brown's concepts, Moore (1973) developed a modified competition index called Area Potentially Available (APA). The index uses the area of the polygon enclosing each tree as the relative measure of its competitive status:

$$APA_j = A_{pj} \qquad (2\text{-}19)$$

Where:

APA$_j$ = the Area Potentially Available competition index value for the jth tree in the stand

A$_{pj}$ = the polygon area enclosing the jth tree in the stand

The APA index is quantified by constructing an irregular polygon around each tree in the stand. Brown's method of defining polygon boundaries was independent of tree size. Moore (1973) assumed that the competitive influence of a tree is proportional to tree size: the area between two competing trees is divided in proportion to the relative size of the competitors. The position of the boundary line is:

$$\text{DIST}_B = \frac{D_{sj}^2}{D_{sj}^2 + D_i^2} \, L_{ij} \tag{2-20}$$

Where:

DIST$_B$ = the distance from the subject tree to the boundary line for the jth competitor

D$_{sj}$ = the diameter at breast height of the subject tree

D$_i$ = the diameter at breast height of the ith competitor

L$_{ij}$ = the distance between the centers of subject tree's bole and the ith competitor's bole

The polygon boundaries are formed perpendicular to the lines connecting the subject tree to each of its neighbors. The individual polygons do not overlap. Moore (1973) states that approximately seven percent of the total stand area is not assigned to any tree's growing space and represent areas where no single tree dominates.

Adlard (1973) also used growing space polygons to quantify the potential resource available to individual trees. He used a more sophisticated version of the growing space polygons by defining twice as many polygon sides as Moore's APA index.

Primary limitations in the use of growing space polygon models are the excessive search and computation time required to define individual polygons. Each polygon is defined from a set of intersected points. All potential neighbors must be examined with respect to their size, distance, and location, and appropriate computations made to determine their contribution to the polygon's construction. In addition, some sort of graphical or analytical method of determining the area of the polygons must be employed. A second limitation in the use of polygon models is their application to managed stand conditions. Wide or irregular spacings may cause the polygons to assume unreasonable sizes and shapes.

MISCELLANEOUS COMPETITION MODELS

Not all competition indices can be neatly classified into a general type of inter-tree competition model. The following competition indices are unique with respect to their evaluation of a tree's competitive status.

Hatch (1975) developed a mathematical index which reflected an individual tree's growth potential based on its exposed crown surface area (ECSA). The index directly incorporates the size and spatial position of the crowns of surrounding trees in determining the level of competition for a tree.

$$ESCA_j = \sum_{i=1}^{n} \frac{CSA_{ij}}{H_{sj}} \frac{BA_{sj}}{BA_i} \qquad (2\text{-}21)$$

Where:

$ECSA_j$ = the competition index for the jth tree in the stand

CSA_{ij} = the exposed crown surface area of the jth subject tree when restricted by the ith competitor

H_{sj} = the length of the bole from the crown to breast height for the jth subject tree

BA_{sj} = the basal area of the jth subject tree at breast height

BA_i = the basal area of the ith competitor

n = the number of competing neighbors

The index in Equation (2-21) utilizes a tree's effective crown size and the distance with respect to DBH to express its growth potential. Hatch (1975) notes that the greater the distance between the crown and the breast height point the less the diameter growth. The weighting factor BA_{sj}/BA_i adjusts for a non-linear competitive interaction based on relative tree size. The weighting factor assumes that the larger the tree the greater its competitive influence relative to its neighbors. The interaction between two trees is assumed to be proportional to their respective basal areas.

In general, the larger the magnitude of the competition index in Equation (2-21), the greater a tree's relative growth potential. An implicit assumption of the model is that competition for light is more limiting than competition for nutrients or soil moisture (Hatch, 1975).

The potential competitors for a subject tree are defined by dividing the area surrounding the tree into 16 equal sections. The tree nearest the subject tree in each section is selected as a potential competitor. The growing space area of a subject tree is not defined since the index is an absolute rather than a relative measure of the interaction.

ESCA is unique in that a vertical component is recognized in the determination of a tree's competitive status. Application of the index on an operational basis is limited by the complexity of the computations involved. The use of bole lengths require additional data not commonly measured on an individual tree basis. The identification of 16 competitors systematically around each subject tree involves considerable computation time. Hatch (1975) also notes that the use of the ESCA index may be limited to species which are intolerant or moderately intolerant to light.

Mitchel (1975) also used measurement of tree crowns to develop a competition index. He expressed the level of competition as the ratio of a tree's actual volume of foliage to the maximum foliar volume obtained under an open-grown condition.

$$CI_j = \ln \left(\frac{FV_j}{FV_{max}}\right) \qquad (2-22)$$

Where:

FV_j = the foliar volume of the jth subject tree

FV_{max} = the maximum volume of foliage achieved in an isolated condition

The index in Equation (2-22) is biologically meaningful in that it expresses the proportion of the utilizable growing space of a tree not relinquished to its competitors (Mitchel, 1975). Maximum volume of foliage for a tree is defined as a function of tree height, distance from leader and age of the leaves when the horizontal expansion of the crown is not restricted by competing neighbors. Grier (1974) presents empirical relationships for determining foliar mass as a function of sapwood basal area.

Bella (1970) points out that competition indices based on integrated expressions of actual crown dimensions reflect only past cumulative effects of competition and not a tree's current competitive status. Two trees of the same size may have equal volume of foliage, but not the same competitive status due to differences in the size and proximity of their respective competitors (Ek, 1975). Mitchel's index in Equation (2-22), therefore, is not applicable in a practical sense as a measure of inter-tree competition.

Lin (1973) developed a competition index which evaluated the relative space or resources available to a tree for growth. It is a quantitative expression of the growing space available to a tree relative to the growing space of open grown and extremely suppressed trees of the same diameter. Linear relationships between crown width and stem diameter were determined for the two extreme growing space conditions. Competitive angles corresponding to these limiting conditions determine the maximum and minimum distances to potential competitors as a function of tree size. If a tree subtends a critical angle which is less than

the mininum competition angle when viewed from the subject tree, it is
not in competition with the subject tree. If a competitor subtends an
angle which is greater than the maximum competition angle, its competi-
tive influence is so great that no growing space is available to the
subject tree. Lin's growing space index (GSI) is defined as:

$$GSI_j = \sum_{i=1}^{n} 25 - (\frac{25}{\Theta_{max} - \Theta_{min}}) (\Theta_i - \Theta_{min}) (\frac{D_{sj} + D_i}{2D_{sj}}) \qquad (2-23)$$

Where:

GSI_j = the growing space index for the jth tree

Θ_i = the observed subtended angle of the competition in the ith
quadrant

Θ_{max} = maximum competition angle indicating complete suppression

Θ_{min} = minimum competition angle indicating an open grown condition

D_i = the DBH of the competition selected in the ith quadrant

D_{sj} = the DBH of the jth subject tree

A subject tree's hypothetical growing space is divided into four
quadrants. Each quadrant is assigned 25 GSI units. An open grown tree
has a GSI = 100, since no competitors subtend an angle larger than the
minimum competition angle. A completely suppressed tree has a GSI = 0,
since all of the available growing space is occupied by competing neigh-
bors in close proximity to the tree. In this case, the angle subtended
by the competitors in each quadrant is larger than maximum competition
angel. Trees with intermediate available growing space conditions have
a GSI between 0 and 100, depending on the size and distance of their

competitors. The growing space index for a subject tree is the sum of the GSI for each quadrant. Within each quadrant, the tree which subtends the largest angle (Θ_i) is selected as the competitor for the quadrant. The weighting factor $(D_i + D_{sj})/2\, D_{sj}$ is used to weigh the contribution of the competitor in each quadrant proportional to tree size.

Lin's GSI incorporates the spatial arrangements and size of competition into a single expression of a tree's available growing space. The index uses the empirical relationships for expressing growing space for open-grown and suppressed individuals to establish the total range of growing space available. The index selects representative competitors based on their size and proximity to the subject tree in each quadrant. The index assumes that the space available between the selected competitor and the subject tree is occupied proportional to tree size. Since only one competitor is identified in each quadrant, the GSI may under or overestimate the actual competition, depending on the uniformity of spacing in the stand.

INFLUENCE ZONE OVERLAP MODEL

The influence zone overlap model is based on the hypothesis that the level of competitive stress sustained by a tree is proportional to the area of overlap of its growing space by the growing space areas of its neighbors. A tree's growing space or influence zone is that area in which it competes for the resources of the site (Bella, 1971). Each tree in the stand interacts with the environment that surrounds it. The area that a tree occupies when unrestricted by competition is

termed its influence zone. Influence zone models assume that optimal
growing space area can be approximated as a circular area surrounding
the tree. Competition occurs between two trees when their growing space
areas overlap. The degree of competition for a tree is assumed to be
proportional to the area of overlap or the amount of intrusion on its
growing space by the influence zones of competing neighbors. The con-
tribution of each competitor is separately identifiable and additive.
The amount of overlap depends upon the size and proximity of the com-
peting trees. The number of competitors and their competitive influence
is clearly defined.

Figure (2-1) shows the overlap and influence zones of a hypothetical
stand. Several investigators have proposed competition indices which
can be classified as influence zone models. The differences between
the indices are mainly due to their definition of growing space area
and the level of complexity in determining the actual area of overlap.

The determination of the radius of the competition circle is an
important component in the influence zone model. The radius of growing
space area is usually assumed to be proportional to tree size. Compe-
tition radius can be expressed as a simple function of tree diameter.
Newnham (1966) and Opie (1968) used various basal area factors to derive
the competition radius for a tree. Bella (1971), Arney (1973), and
Newnham (1966) used the relationship between tree diameters and open
grown crown width to define the maximum radius of influence under
no competition. Bella (1971) also incorporated a radius expansion
factor to adjust for competition beyond the open grown crown width.
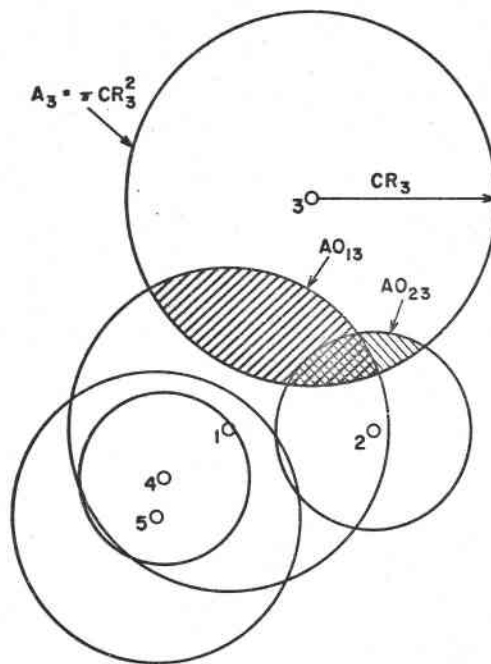
Figure (2-1): The overlap areas and influence zones for a hypothetical stand

Keister (1971) used actual tree measurements to determine the radius of influence as a function of tree height and crown dimensions.

The competitive radius will vary with species and tree size. The area of growing space is defined as the circular area with a competition radius proportional to tree size.

$$A_j = \pi(CR_j)^2 \tag{2-24}$$

Where:

A_j = the area of the influence zone or growing space for the jth tree

CR = the competition radius for the jth tree

Various expressions of inter-tree competition have been proposed which reflect the overlap of the growing space of a tree.

Staebler (1951) was one of the first to describe inter-tree competition with an influence zone model. He reasoned that the actual area of overlap of a tree's competitive circle indicated the intensity of competition experienced by the tree. The competition index is defined as the sum of the radial width of the overlap region for all competitors:

$$CI_j = \sum_{i=1}^{n} LO_{ij} \tag{2-25}$$

Where:

LO_{ij} = the linear overlap or radial width of the ith competitor's growing space circle with the growing space of the jth subject tree

Staebler recognized that the actual area of overlap provided a direct measure of competition. Lacking modern computer facilities necessary for repeatedly determining this area mathematically, he assumed the linear overlap of the competitor's competition circle would approximate the actual competition relationship.

Newnham (1966) used an angular measure to define inter-tree competition. He assumed that the proportion of the circumference of the subject tree's growing space circle overlapped by those of adjacent trees reflected the intensity of competition exerted on the tree.

$$CI_j = \frac{1}{2\pi} \sum_{i=1}^{n} \Theta_{ij} \qquad (2\text{-}26)$$

Where:

Θ_{ij} = the interior angle subtended for jth tree's circle by the ith competitor's overlap zone (in radians)

The interior angle is formed by the chords connecting the center of the jth subject tree and the points of intersection of the ith competitor's overlap region. Newnham's index in Equation (2-26) assumes that the greater the interior angle subtended the greater the influence of the competitor on the subject tree. The actual area of overlap is only indirectly related to the size of the interior angle.

Opie (1968) attempted to quantify the overlap area using angle gauges. He delineated the overlap zones of the tree and expressed the total area of overlap of the relative growing space area of the tree as a measure of density.

$$CI_j = BAF \frac{\sum\limits_{i=1}^{k} OA_{ij} \cdot i}{\sum\limits_{i=1}^{k} OA_{ij}} \qquad (2\text{-}27)$$

Where:

OA_{ij} = the area occupied by the ith overlap zone

i = the number of overlaps

Opie's model in Equation (2-27) actually is a sophisticated point density model. The level of inter-tree competition is assumed to be proportional to the point density observed at the sample point. The CI in Equation (2-27) is expressed in terms of basal area per acre. The model is classified as an influence zone model because the actual area of overlap by competing neighbors is considered.

The models proposed by Staebler, Newnham, and Opie in Equations (2-25, 2-26, and 2-27) are crude forms of the influence zone model. Although each of these competition indices recognizes the importance of the relative area of overlap of growing spaces, they do not mathematically calculate it directly.

Gerrard (1969) was one of the first to propose an index which mathematically quantified the area of overlap between two competing trees. Using geometrical relationships based on the tree size and spatial location, Gerrard formulated a modified competition index as:

$$CI_j = \frac{\sum AO_{ij}}{A_j} \qquad (2\text{-}28)$$

Where:

AO_{ij} = the area of overlap of the ith competitor's growing space
with the area of the jth subject tree

A_j = the area of the growing space of the jth subject tree

Equation (2-28) states that the competition stress sustained by a
tree is directly proportional to the sum of the overlaps of its growing
space by competition circles of its neighbors, and inversely propor-
tional to the area of its growing space. Notice the similarity between
Opie's competition index in Equation (2-27) and Gerrard's competition
index in Equation (2-28). In both cases, the competitive effect is
evaluated directly as the sum of the areas of overlap relative to the
area of the growing space or influence zone of the tree. Opie used
angle gauges and dot counts to determine the area of overlaps, while
Gerrard used mathematical expressions based on geometrical relation-
ships. Gerrard also assumed that the radius of the growing space circle
was proportional to the DBH of the tree.

Keister (1971) developed an index identical to Gerrard's CI with
the exception of how a tree's growing space area is defined. Instead
of assuming the maximum radius of a tree's influence zone was propor-
tional to tree diameter, Keister used a function of tree height, crown
width, and dead limb length.

Bella (1970) developed a competition index which incorporated an
exponential type of competitive effect based on tree size. He hypothe-
sized that the larger the size of the tree, the greater the degree of

its competitive influence on neighboring trees. Larger trees are able to more fully exploit the resources of the site than smaller trees. Instead of assuming that the area of overlap of a tree's growing space is linearly additive, Bella's index gives greater competitive weight to larger trees than smaller ones.

$$CIO_j = \frac{\sum\limits_{i=1}^{n} AO_{ij}}{A_j} \left(\frac{D_i}{D_{sj}}\right)^a \qquad (2\text{-}29)$$

Where:

CIO_j = the competitive influence-zone overlap index for the jth tree

D_{sj} = the DBH of the jth subject tree

D_i = the DBH of the ith competitor

a = competition parameter

The weighting factor (D_i/D_{sj}) allows for a differential weighting of the competitive influence of the ith competitor based on its size relative to the size of the subject tree. Bella's index requires calibration by species to determine the most appropriate exponent.

Ek (1974) also used a differential weighting factor to account for non-linear competitive influence based on relative tree size. A tree's competitive status is calculated as:

$$CI_j = \sum_{i=1}^{n} \frac{AO_{ij}}{A_j} \left(\frac{S_i}{S_j}\right) \qquad (2\text{-}30)$$

Where:

S_i = the size of the ith competitor (height x crown radius)

S_j = the size of the jth subject tree

In Equation (2-30) the influence zone overlap of the ith competitor is weighted by the relative ratio of tree size.

Both Bella's and Ek's competition indices use relative weighting factors to adjust for tree size. These models allow for great flexibility in describing inter-tree competition. However, with their level of sophistication, computational and calibration time becomes excessive. It is not clear whether or not the use of these arbitrary weighting factors is justified. Influence zone models are relative measures of inter-tree competition. The gains made by the addition of such complexity to an abstract measure is questionable.

Arney (1973) developed a competition index for Douglas-fir which evaluated the relative level of inter-tree competition as the percent overlap of growing space.

$$CSI_j = 100 \frac{(\sum_{i=1}^{n} AO_{ij} + A_j)}{A_j} \qquad (2\text{-}31)$$

Where:

CSI_j = the competitive stress index of the jth subject tree

AO_{ij} = the area of overlap of the ith competitor's growing space with the jth subject tree's growing space

A_j = the growing space area of the jth tree

If a tree has no competitors (i.e., it is open grown) then it has a CSI = 100, since $\sum AO_{ij} = 0$. As the sum of the area of overlaps increase, the level of competitive stress also increases. Arney's CSI model eliminates tree size by computing the percent overlap of growing space. Although larger trees have larger growing space areas than smaller trees, their competitive stress index values are comparable on a relative basis. Table (2-1) gives the ranges of CSI typically encountered in forest stands.

Arney used the relationship between open-grown crown width and DBH to determine the maximum growing space area required by a tree. The model in Equation (2-31) assumes that the open grown tree of a given size exhibits the maximum expansion of crown dimensions. For Douglas-fir, the width of the crown approximates the extent of the root system (Smith, J.H.G., 1964). The maximum growing space area for a tree is assumed to be the vertical projection of the open-grown crown area into the x-y plane. Figure (2-2) shows the relationship between open grown crown radius and diameter. Two trees compete if their growing space circles overlap. Using Figure (2-2) the radius of competition can be defined as a function of tree size.

The primary limitations of the model are the requirements of detailed stem coordinates and diameter measurements on an individual tree basis. In addition, some of the model assumptions may be restrictive. The competitive effect of neighboring trees is assumed to be linearly additive. No differential weighting between the relative size of competitors is included, which may not be realistic. Another assumption

OPEN GROWN CROWN RADIUS

$CW = 4.7071 + 2.0168 \, DBH - 0.0186 \, DBH^2$
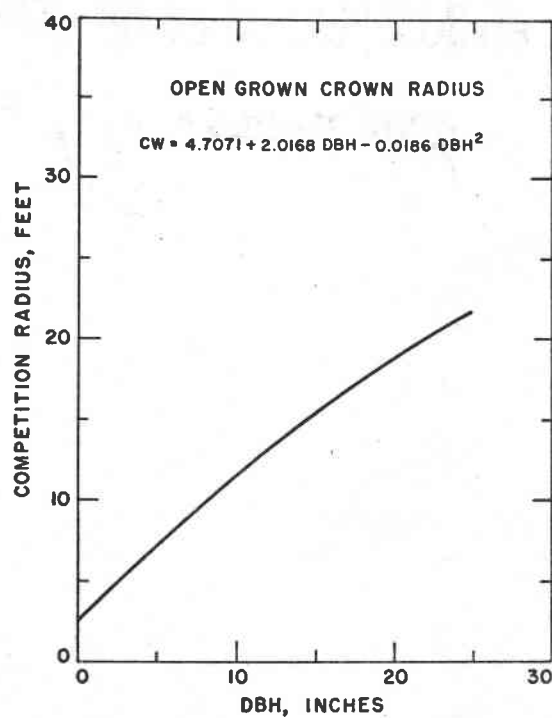
Figure (2-2): The relationship between open grown crown radius and diameter for Douglas-fir

is that growing space is approximated by the open-grown crown width.
This does not allow for certain biological phenomena such as root
grafting, and does not include a vertical component of competition.
The model also assumes that competition is uniformly distributed over
the zone of overlap. These limitations may be minimized by the rela-
tive nature of the index.

CSI is a quantitative expression of inter-tree competition. Using
tree diameter and x-y stem coordinate data, CSI can be routinely calcu-
lated (see Appendix B).

Table (2-1): Range in CSI values normally
found in forest stands

CSI Level	Condition
100	Open grown
250-350	Average value for managed stands
350-450	Average value for normal stands
200-300	Dominant crown class
300-450	Co-dominant crown class
450-550	Intermediate crown class
550+	Suppressed crown class

STAND DENSITY

INTRODUCTION

Measures of stand density are used to depict the degree to which the resources of the site are being utilized by trees in a stand (Husch, 1972). As defined previously, stand density is a measure of the degree of crowding of trees on an area without reference to a specific management objective.

The following chapter evaluates CSI as a dynamic measure of stand density and inter-tree competition. CSI is used to quantitatively describe the degree of thinning intensity. The index is tested as an independent variable in describing tree growth response following thinning. Spacing and stocking guidelines based on CSI are also developed. The results of these analyses indicated CSI is an effective measure of stand density.

THINNING INTENSITY

The usefulness of stand density in expressing the effect of inter-tree competition is important in the analysis of tree and stand growth relationships. Whole stand measures of density conceal the causal relationships between stand density and tree growth by averaging out the competitive effect on the individual tree. Basal area per acre is commonly used to reflect stand density. The degree of thinning intensity can be measured by the amount of reduction in initial stand basal area due to thinning. Basal area measures assume that the greater the

amount of basal area removed, the heavier the thinning intensity and the greater the reduction in the average competitive stress levels for trees in the stand. However, basal area per acre only indirectly reflects the competitive relationships between individual trees. It is an aggregate measure of stand density which expresses the relative degree of competition exerted on the average tree, independent of the spatial relationships in the stand. Reduction in stand basal area by thinning does not necessarily reduce the level of competitive stress uniformly for all trees in the stand. Basal area per acre is also dependent upon the age and site quality of the stand.

CSI is a quantitative measure of the inter-tree competition. It can be objectively applied to individual trees in the stand. Average CSI for the stand reflects the average level of competition for all trees. The degree of thinning can be objectively measured as a reduction in the competitive stress due to thinning. Average CSI for the stand is biologically interpretable as a measure of stand density. It is independent of age and site conditions. Reduction in CSI by thinning reflects the reduction in the level of inter-tree competition in the stand, relative to tree size and spatial arrangements. The following section compares CSI and basal area per acre as measures of stand density.

CSI data for each of the four thinning treatments and control for the Hoskins study plots were tabulated by measurement period from 1963-1973. Table (3-1) gives the average CSI by treatment for each period. In 1963 all study plots except the control plots were thinned. The

Table (3-1): Average competitive stress index by treatment 1963-
1973 (CSI units)

Treatment Number	1963	1966	1966 Thinning	1970	1970 Thinning	1973	1973 Thinning
1	178	233	162	218	149	176	144
3	169	222	177	237	189	222	185
5	177	233	210	282	237	277	249
7	173	227	225	298	271	316	295
Control (unthinned)	532	591	591	612	612	614	614

average CSI for all treatments in 1963 reflects the uniformity in the
calibration thinning. Subsequent thinnings occurred at the end of the
1966, 1970, and 1973 growing seasons. Figure (3-1) depicts the change
in CSI with respect to period for each treatment. Reductions in CSI
due to thinning are indicated by the dashed lines. Notice that the
intensity of thinning decreases from Treatment 1 to 7.

The 1966 thinning did not markedly affect the average CSI for
Treatments 5 and 7. Even though a portion of the stand basal area was
removed, the competitive status of trees in these plots were not af-
fected. The 1970 and 1973 thinnings were more affective in reducing
the level of competition across the range of treatments. Comparison of
the unthinned control with the thinned treatments indicates the dif-
ferences between managed and natural stand conditions.

Table (3-2) compares the reduction in average CSI with reduction in
stand basal area by treatment and thinning period. The change in CSI is

Table (3-2):   Change in CSI and BA by treatment and
              thinning period

		1966 Thinning	1970 Thinning	1973 Thinning
Treatment 1	ΔCSI	71	69	32
	%	30.5	31.7	18.2
	ΔBA	30.4	39.1	20.7
	%	35.6	39.3	24.3
Treatment 3	ΔCSI	45	48	37
	%	20.3	20.3	16.7
	ΔBA	20.8	29.9	25.4
	%	24.5	26.8	22.8
Treatment 5	ΔCSI	23	45	28
	%	9.9	16.0	10.1
	ΔBA	11.1	22.9	16.5
	%	12.9	18.1	12.0
Treatment 7	ΔCSI	2	27	21
	%	.9	9.1	6.6
	ΔBA	1.0	14.9	11.6
	%	1.2	10.7	7.2
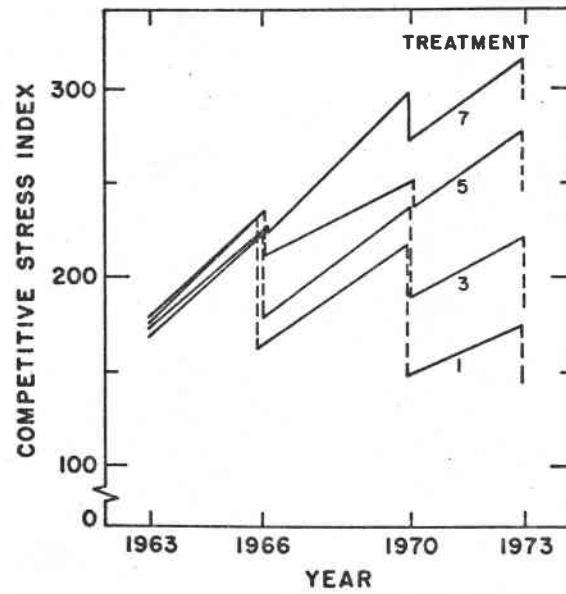
Figure (3-1):   Change in CSI by period and
treatment

closely related to change in basal area in all cases. The change in CSI can be interpreted as a reduction in the level of inter-tree competition, while change in basal area per acre cannot be directly related to the competitive status of the stand.

A direct comparison of CSI and basal area per acre as measures of stand density were made using 1963-1973 thinning data. Using basal area per acre as a measure of stand density, each treatment was thinned to a specified level. Reduction in stand density was assumed to be uniformly representative of the degree of release experience by all trees within a treatment. The coefficient of variation for stand basal area estimates per acre ranged from 2-5 percent for all treatments, indicating a highly uniform stand condition. Table (3-3) gives the coefficient of variation for the CSI by treatment. The coefficient of variation ranges from 16-20 percent, indicating moderately variable stand density conditions exist. The basal area per acre density indicates all of the treatments are uniform with respect to stand density. CSI density for the same treatments indicate that inter-tree competition is variable within each treatment. Clearly, CSI is a more sensitive measure of inter-tree competition within a treatment. High coefficients of variation for CSI density indicate that there is a lack of uniformity in competitive stress levels between trees within a treatment. Although stand basal area estimates indicate the treatment densities are uniform, individual trees within the treatments are subject to different degrees of competition. The variation in the competitive status of trees within a treatment confounds thinning study

Table (3-3): Coefficient of variation of CSI in % by treatment 1963-1973

Treatment Number	1963	1966	1966 Thinning	1970	1970 Thinning	1973	1973 Thinning
1	.205	.176	.201	.165	.172	.161	.171
3	.197	.182	.179	.148	.207	.195	.182
5	.191	.178	.192	.174	.243	.148	.169
7	.203	.183	.186	.159	.176	.160	.156
Control (unthinned)	.207	.181	.181	.183	.183	.166	.166

Table (3-4): Range in CSI by treatment 1963-1973

Treatment	1963	1966	1966 Thinning	1970	1970 Thinning	1973	1973 Thinning
1	224	252	151	182	127	146	95
3	174	242	141	194	195	228	155
5	174	220	220	271	200	221	207
7	212	258	258	248	259	272	258
Control	568	539	539	566	566	448	448

results. Treatments with a heavy thinning intensity may have trees with CSI levels comparable to trees in treatments with lighter thinning intensities.

Table (3-4) gives the range of CSI for each treatment as the difference between the maximum CSI and the minimum CSI observed. The uniformity of density within a treatment is reflected by the range of CSI observed. The range of CSI should be reduced following thinning. The data in Table (3-4) indicate that the use of basal area per acre density does not result in uniform levels of inter-tree competition.

Future thinning studies should adopt CSI as a measure of a stand density. If CSI is used to control stand density, more uniformity between trees within a treatment can be achieved.

GROWTH RESPONSE EQUATIONS

A measure of stand density is dependent upon its ability to describe the growth and development of trees in the stand. Measures of stand density are useful only to the extent that they are related to tree growth. Gingrich (1969) states, "the forester must be able to predict how future growth might be redistributed by management action." The following procedures were adopted to examine the influence of the level of inter-tree competition for growing space on individual tree growth. It was hypothesized that if CSI is a biologically meaningful expression of inter-tree competition in the stand, then the growth response of individual trees should be related to their level of CSI before thinning and to their reduction in CSI following thinning. The amount

of release from competition is reflected by the amount of reduction of their CSI levels due to thinning.

$$\Delta CSI = CSI_B - CSI_T \tag{3-1}$$

Where:

ΔCSI = the reduction in CSI due to removal of competing neighbors by thinning

CSI_B = the level of competitive stress before thinning

CSI_T = the level of competitive stress after thinning

Three independent variables were used to predict periodic diameter increment for a tree including the tree's DBH, its initial level of CSI before thinning, and its change in CSI due to thinning.

Data from 59 sample trees systematically selected from all four thinning treatments described earlier were used to evaluate the predictive ability of the independent variables in accounting for the observed variation in tree DBH growth response following thinning. Border effect was eliminated by careful selection of subject trees from the 12 study plots using a 30 foot edge buffer zone (see Appendix E). Two growth periods were used in the study: a four year growth period from 1966-1970 and a three year growth period from 1970-1973. Table (3-5) gives a description of the average subject tree data by measurement period.

Periodic diameter growth was expressed as a function of initial diameter, initial CSI, and change in CSI due to thinning as follows:

$$\Delta G = f(DBH_o) \tag{3-2}$$

$$\Delta G = f(\Delta CSI_o) \tag{3-3}$$

$$\Delta G = f(CSI_o) \tag{3-4}$$

$$\Delta G = f(CSI_o, \Delta CSI_o) \tag{3-5}$$

$$\Delta G = f(DBH_o, \Delta CSI_o, CSI_o) \tag{3-6}$$

Where:

ΔG = the dependent variable, periodic diameter growth

DBH_o = initial tree DBH at the beginning of the period

CSI_o = initial CSI at the beginning of the period

ΔCSI_o = the change in CSI due to thinning

Table (3-5): Subject tree data by measurement period

	1963	1966	1966 Thin	1970	1970 Thin	1973
N	59	59	59	59	59	59
DBH	5.15	6.78	6.78	8.86	8.86	10.29
CSI	166	219	194	260	215	254
XN	4.4	6.8	5.2	8.3	6.2	7.7
S	10.8	12.7	13.0	15.2	15.8	17.1

Where:

XN = the average number of competitors

S = average spacing

Equation (3-2) expresses periodic diameter increment as a function of initial diameter alone. Equations (3-3 and 3-4) express periodic diameter increment as a function of ΔCSI_o and CSI_o, respectively. Equation (3-5) combines CSI_o and ΔCSI_o into a single form which expresses the contribution of the competitive stress variable alone. Equation (3-6) adds tree size and expresses the total contribution of all three independent variables combined.

Two models were used to describe the growth relationships above. Model I assumed ΔG was a linear function of the independent variables. Model II assumed that ΔG was a non-linear function of the independent variables.

Model I:

$$\Delta G = b_0 + b_1 X + b_2 X_2 + \ldots + b_n X_n + \varepsilon_I \qquad (3\text{-}7)$$

Model II:

$$\Delta G = e^{(b_0 + b_1 X + b_2 X_2 + \ldots b_n X_n)} + \varepsilon_{II} \qquad (3\text{-}8)$$

where

ε_I, ε_{II} = random error terms

In Equation (3-7) $\Delta G = \Delta DBH$, where ΔDBH is the periodic diameter increment for the tree. In Equation (3-8), ΔG is transformed by taking the natural logarithm and becomes $\Delta G = \ln (\Delta DBH)$. Model I and Model II were used to fit the data for each growth period and for each functional form of the independent variables given in Equations (5-2 through 5-6).

Four transformations of each independent variable were tested, including linear and quadratic terms and their respective inverses.

$$f(I) = (I, I^2, I^{-1}, I^{-2})$$ (3-9)

Where:

I = the independent variable tested (i.e., DBH_o, CSI_o, ΔCSI_o)

Stepwise regression procedures were used in selection of significant independent variables for each equation. Standard t-tests of significance and minimum residual mean square criteria were used to determine which independent variable entered the model. A 10 percent probability level was used to test the significance of each variable in the model.

Table (3-6) gives the multiple coefficient of determination (R^2) for various combinations of DBH_o, CSI_o, and ΔCSI_o for Models I and II using 4 year and 3 year growth data. The R^2 for functions of DBH alone (Column 1) accounted for 51.6 to 73.7 percent of the variation in periodic diameter growth. Functions of CSI_o and ΔCSI_o (Column 4) accounted for 36.3 to 55.1 percent of the variation. Functions of ΔCSI_o or CSI_o alone (Columns 2 and 3) did not explain much of the variation associated with periodic diameter growth. Tree size, level of competitive stress and change of competitive stress variables accounted for the most variation in diameter growth. Functions of DBH_o, CSI_o, and ΔCSI_o explained from 63.9 to 86.8 percent of the variation in periodic ΔDBH growth columns (column 5). The contribution of the CSI variables can be

expressed on a relative basis as:

$$\Delta R = \frac{R^2_{(5)} - R^2_{(1)}}{R^2_{(1)}}$$

(3-10)

Where:

ΔR = the contribution of the CSI variables expressed as a percent change in R^2

$R^2_{(1)}$ = multiple coefficient of determination using DBH alone in reduced model (Column 1)

$R^2_{(5)}$ = multiple coefficient of determination using full model (Column 5)

The ΔR values in Column 6 indicate that the addition of CSI variables (ΔCSI_o, CSI_o) cause an increase in R^2 of approximately 18 to 29 percent.

Table (3-7) gives the mean square error for each Equation tested. Column 6 gives a relative measure of the percent reduction in MSE when the CSI variables are added to the model.

$$\Delta MSE = \frac{MSE_{(1)} - MSE_{(5)}}{MSE_{(1)}}$$

(3-11)

Where:

ΔMSE = the percent reduction in MSE for full model relative to DBH alone

The addition of ΔCSI_o and CSI_o as independent variables cause a reduction in MSE of approximately 23 to 40 percent for the full model.

Table (3-6): Multiple coefficient of determination for various combinations of diameter, CSI, and ΔCSI

	DBH (1)	ΔCSI (2)	CSI (3)	ΔCSI, CSI (4)	DBH, ΔCSI, CSI (5)	Δ_R (6)
ΔDBH 1966-1970	.524	.274	.091	.398	.674	+.286
ℓn (ΔDBH) 1966-1970	.516	.226	.112	.363	.639	+.238
ΔDBH 1970-1973	.666	.127	.367	.551	.808	+.213
ℓn (ΔCBH) 1970-1973	.737	.098	.339	.489	.868	+.179

Table (3-7): Mean square error for various combinations of diameter, CSI, and ΔCSI

	DBH (1)	ΔCSI (2)	CSI (3)	ΔCSI, CSI (4)	DBH, ΔCSI, CSI (5)	Δ_{MSE} (6)
ΔDBH 1966-1970	.130	.202	.248	.167	.094	-.277
ℓn (ΔDBH) 1966-1970	.044	.072	.080	.059	.034	-.227
ΔDBH 1970-1973	.069	.181	.131	.096	.042	-.391
ℓn (ΔDBH) 1970-1973	.052	.197	.144	.115	.031	-.414

These results indicate that functions of CSI_o and ΔCSI_o are important variables in explaining periodic diameter growth. Tree size alone accounts for a large portion of the variation in diameter growth. Addition of CSI variables increase the predictive power of the model and also reduce the variability of the estimate. These results imply that functions of CSI are important variables in the assessment of tree growth and development and should be considered by the forester in managing forest stands.

STOCKING AND SPACING GUIDES

Spacing and stocking guidelines are extremely useful to the forest manager in the manipulation of the stand. Thinning can be viewed as a temporary reduction of stand density in order to achieve a specific management objective. Spacing and stocking guidelines provide some means of controlling how the thinning is to be performed relative to given stand conditions. For useful field guides it is convenient to express spacing and stocking in terms of tree size and residual stand density. The following section presents theoretical and empirical stocking and spacing guidelines for a managed stand. Stand density is expressed as the average CSI for the stand. A reduction in stand density following thinning reflects a corresponding reduction in the level of inter-tree competition in the stand. Theoretical stocking and spacing guidelines can be developed for a uniform stand with regular spacing as follows. Suppose the stand consists of trees with uniform DBH = k. The area available to the open grown tree of DBH = k is given as a

function of its open grown crown width:

$$A_k = \frac{\pi (CW_k)^2}{4}$$ (3-12)

Where:

A_k = the open grown crown area of a tree with DBH = k

CW_k = the open grown crown width of a tree with DBH = k (see

Equation (3-22)

The number of open grown trees per acre of uniform DBH = k is cal-culated as:

$$TPA_{(k,100)} = \frac{43560}{A_k}$$ (3-13)

Where:

$TPA_{(k,100)}$ = the number of uniformly spaced open grown trees

(i.e., CSI = 100) per acre with DBH = k

Equation (3-13) gives the expected number of open grown trees per acre spaced at the corners of squares of equal size. The number of trees per acre corresponding to an average stand density of CSI_o is equal to:

$$TPA_{(k,CSI_o)} = TPA_{(k,100)} \frac{CSI_o}{100} \quad (3-14)$$ (3-14)

Equation (3-14) states that the number of trees per acre of DBH = k with stand density equal to CSI_o is the product of the number of open

grown trees per acre of same diameter and the ratio $CSI_o/100$. Theoretically, a stand with $CSI_o = 200$ has twice the area of overlap as a stand with density $CSI_o = 100$ and hence has twice as many uniformly spaced trees per acre.

The average area available per tree assuming uniformly square spacing of the trees in the stand is:

$$A_{(k,CSI_o)} = \frac{43560}{TPA_{(k,CSI_o)}} \tag{3-15}$$

The distance between trees in the stand of uniform DBH = k, and stand density = CSI, is equal to the square root of $A_{(k,CSI_o)}$.

$$SP_{(k,CSI_o)} = \sqrt{A_{(k,CSI_o)}} = \sqrt{A_k/(\frac{CSI_o}{100})} \tag{3-16}$$

Where:

$SP_{(k,CSI_o)}$ = average square spacing for trees with DBH = k and

stand density = CSI_o

Equation (3-16) states that the trees in the stand are distributed at the corners of equal sized squares with sides of length $SP_{(k,CSI_o)}$ and area $A_{(k,CSI_o)}$. (Note that there is a slight bias since the area of a square with sides X only approximately equals the area of a circle with radius X).

In Equation (3-16), the average open grown area per tree is reduced by a factor of $100/CSI_o$ as CSI_o increases from $CSI_o = 100$. A stand with $CSI_o = 200$ has only one-half the area per tree as a stand with $CSI_o = 100$.

Tables (3-8) and (3-9) give the theoretical stocking and square spacing guides for a uniform stand of DBH = k and density = CSI_o. Given the desired stand density level and stand diameter the expected spacing and stocking guidelines can be determined (see Figure 3-2). Suppose a stand had a uniform DBH = 10 and a thinning is proposed which will cause the residual stand density to have an average CSI_o = 250. The theoretical spacing required between trees is 12.9 by 12.9 and the corresponding number of trees per acre is 262 (see Tables 3-8 and 3-9).

Solving Equation (3-16) for CSI gives the expected stand density as a function of spacing and stand DBH (see Table 3-9b).

$$CSI_{(k,SP_o)} = A_k(100)/SP_o \qquad\qquad (3-17)$$

Equation (3-17) expresses the expected stand density of CSI when a square spacing of SP_o is used as a field guideline to thin a stand with DBH = k. For example, if DBH = 10 and SP_o = 12, thinning all trees to a 12x12 square spacing will result in a residual stand with an expected stand density of $CSI_{(10,12)}$ = 289.

Tables (3-8 and 3-9) are only useful for hypothetical stands of uniform DBH and square spacing. Except in plantations, these special conditions are not generally applicable to forest stands.

The previous discussion has focused on the development of theoretical stocking and spacing guidelines based on hypothetical stand conditions. A study was performed to develop empirical relationships between spacing and stand density in a managed stand. The average distance between a subject tree and its competitors is an estimate of its spacing.
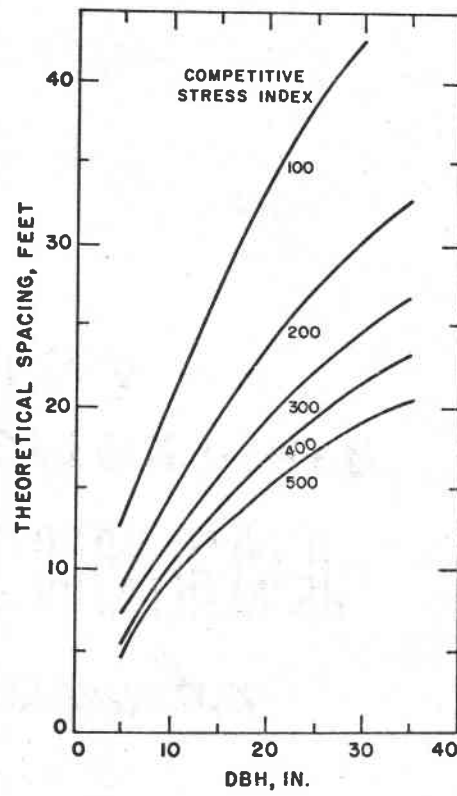
Figure (3-2): Theoretical spacing by stand
CSI and DBH

Table (3-8): Theoretical stocking levels based on stand DBH
and CSI (number of trees per acre)

Stand CSI

DBH	100	150	200	250	300	350	400	450	500	550	600
1	1234	1850	2467	3084	3701	4317	4934	5551	6168	6785	7401
2	738	1108	1477	1846	2215	2585	2954	3323	3692	4062	4431
3	495	742	989	1236	1484	1731	1978	2225	2473	2720	2967
4	356	534	713	891	1069	1247	1425	1603	1781	1960	2138
5	270	405	540	676	811	946	1081	1216	1351	1486	1621
6	213	319	426	532	639	745	852	958	1065	1171	1278
7	173	259	346	432	519	605	691	778	864	951	1037
8	144	215	287	359	431	503	574	646	718	790	862
9	122	182	243	304	365	426	487	547	608	669	730
10	105	157	209	262	314	366	419	471	524	576	628
11	91	137	183	228	274	320	365	411	457	502	548
12	81	121	161	202	242	282	322	363	403	443	484
13	72	108	144	180	216	251	287	323	359	395	431
14	65	97	129	162	194	226	258	291	323	355	388
15	59	88	117	146	176	205	234	264	293	322	351
16	53	80	107	134	160	187	214	240	267	294	321
17	49	74	98	123	147	172	196	221	245	270	294
18	45	68	91	113	136	159	181	204	227	249	272
19	42	63	84	105	126	147	168	189	210	231	252
20	39	59	78	98	118	137	157	177	196	216	235
21	37	55	73	92	110	129	147	165	184	202	220
22	35	52	69	86	104	121	138	155	173	190	207
23	33	49	65	81	98	114	130	147	163	179	196
24	31	46	62	77	93	108	123	139	154	170	185
25	29	44	59	73	88	103	117	132	147	161	176
26	28	42	56	70	84	98	112	126	140	154	168
27	27	40	53	67	80	93	107	120	133	147	160
28	26	38	51	64	77	89	102	115	128	141	153
29	25	37	49	61	74	86	98	110	123	135	147
30	24	35	47	59	71	83	94	106	118	130	142
31	23	34	46	57	68	80	91	102	114	125	137
32	22	33	44	55	66	77	88	99	110	121	132
33	21	32	43	53	64	75	85	96	107	117	128
34	21	31	41	52	62	72	83	93	103	114	124
35	20	30	40	50	60	70	80	91	101	111	121

Table (3-9): Theoretical square spacing based on stand DBH and
CSI (in feet)

Stand CSI

DBH	100	150	200	250	300	350	400	450	500	550	600
1	5.9	4.9	4.2	3.8	3.4	3.2	3.0	2.8	2.7	2.5	2.4
2	7.7	6.3	5.4	4.9	4.4	4.1	3.8	3.6	3.4	3.3	3.1
3	9.4	7.7	6.6	5.9	5.4	5.0	4.7	4.4	4.2	4.0	3.8
4	11.1	9.0	7.8	7.0	6.4	5.9	5.5	5.2	4.9	4.7	4.5
5	12.7	10.4	9.0	8.0	7.3	6.8	6.3	6.0	5.7	5.4	5.2
6	14.3	11.7	10.1	9.0	8.3	7.6	7.2	6.7	6.4	6.1	5.8
7	15.9	13.0	11.2	10.0	9.2	8.5	7.9	7.5	7.1	6.8	6.5
8	17.4	14.2	12.3	11.0	10.1	9.3	8.7	8.2	7.8	7.4	7.1
9	18.9	15.5	13.4	12.0	10.9	10.1	9.5	8.9	8.5	8.1	7.7
10	20.4	16.7	14.4	12.9	11.8	10.9	10.2	9.6	9.1	8.7	8.3
11	21.8	17.8	15.4	13.8	12.6	11.7	10.9	10.3	9.8	9.3	8.9
12	23.2	19.0	16.4	14.7	13.4	12.4	11.6	11.0	10.4	9.9	9.5
13	24.6	20.1	17.4	15.6	14.2	13.2	12.3	11.6	11.0	10.5	10.1
14	26.0	21.2	18.4	16.4	15.0	13.9	13.0	12.2	11.6	11.1	10.6
15	27.3	22.3	19.3	17.2	15.7	14.6	13.6	12.9	12.2	11.6	11.1
16	28.5	23.3	20.2	18.1	16.5	15.3	14.3	13.5	12.8	12.2	11.7
17	29.8	24.3	21.1	18.8	17.2	15.9	14.9	14.0	13.3	12.7	12.2
18	31.0	25.3	21.9	19.6	17.9	16.6	15.5	14.6	13.9	13.2	12.7
19	32.2	26.3	22.8	20.4	18.6	17.2	16.1	15.2	14.4	13.7	13.1
20	33.3	27.2	23.6	21.1	19.2	17.8	16.7	15.7	14.9	14.2	13.6
21	34.4	28.1	24.4	21.8	19.9	18.4	17.2	16.2	15.4	14.7	14.1
22	35.5	29.0	25.1	22.5	20.5	19.0	17.8	16.7	15.9	15.1	14.5
23	36.6	29.9	25.9	23.1	21.1	19.5	18.3	17.2	16.4	15.6	14.9
24	37.6	30.7	26.6	23.8	21.7	20.1	18.8	17.7	16.8	16.0	15.3
25	38.6	31.5	27.3	24.4	22.3	20.6	19.3	18.2	17.2	16.4	15.7
26	39.5	32.3	27.9	25.0	22.8	21.1	19.7	18.6	17.7	16.8	16.1
27	40.4	33.0	28.6	25.6	23.3	21.6	20.2	19.1	18.1	17.2	16.5
28	41.3	33.7	29.2	26.1	23.8	22.1	20.6	19.5	18.5	17.6	16.9
29	42.1	34.4	29.8	26.7	24.3	22.5	21.1	19.9	18.8	18.0	17.2
30	43.0	35.1	30.4	27.2	24.8	23.0	21.5	20.2	19.2	18.3	17.5
31	43.7	35.7	30.9	27.7	25.3	23.4	21.9	20.6	19.6	18.7	17.9
32	44.5	36.3	31.5	28.1	25.7	23.8	22.2	21.0	19.9	19.0	18.2
33	45.2	36.9	32.0	28.6	26.1	24.2	22.6	21.3	20.2	19.3	18.5
34	45.9	37.5	32.4	29.0	26.5	24.5	22.9	21.6	20.5	19.6	18.7
35	46.5	38.0	32.9	29.4	26.9	24.9	23.3	21.9	20.8	19.8	19.0

Table (3-9b): The expected stand density as a function of
density and stand DBH

	Spacing (in feet)									
DBH	3	6	9	12	15	18	21	24	27	30
1	392	100	100	100	100	100	100	100	100	100
2	655	164	100	100	100	100	100	100	100	100
3	979	245	109	100	100	100	100	100	100	100
4	0	340	151	100	100	100	100	100	100	100
5	0	448	199	112	100	100	100	100	100	100
6	0	568	253	142	100	100	100	100	100	100
7	0	700	311	175	112	100	100	100	100	100
8	0	842	374	211	135	100	100	100	100	100
9	0	995	442	249	159	111	100	100	100	100
10	0	0	514	289	185	128	100	100	100	100
11	0	0	589	331	212	147	108	100	100	100
12	0	0	667	375	240	167	123	100	100	100
13	0	0	748	421	269	187	137	105	100	100
14	0	0	832	468	300	208	153	117	100	100
15	0	0	918	517	331	230	169	129	102	100
16	0	0	1006	566	362	252	185	142	112	100
17	0	0	1096	616	394	274	201	154	122	100
18	0	0	0	667	427	297	218	167	132	107
19	0	0	0	719	460	320	235	180	142	115
20	0	0	0	771	494	343	252	193	152	123
21	0	0	0	824	527	366	269	206	163	132
22	0	0	0	876	561	389	286	219	173	140
23	0	0	0	928	594	413	303	232	183	149
24	0	0	0	980	627	436	320	245	194	157
25	0	0	0	1032	661	459	337	258	204	165
26	0	0	0	1083	693	482	354	271	214	173
27	0	0	0	0	726	504	370	284	224	181
28	0	0	0	0	758	526	387	296	234	189
29	0	0	0	0	789	548	403	308	244	197
30	0	0	0	0	820	570	418	320	253	205
31	0	0	0	0	850	590	434	332	262	213
32	0	0	0	0	880	611	449	344	271	220
33	0	0	0	0	908	631	463	355	280	227
34	0	0	0	0	936	650	477	366	289	234
35	0	0	0	0	962	668	491	376	297	241

Average spacing was expressed as a function of tree size and level of CSI for 177 subject trees. The functional form of the relationship used is:

$$S_e = b_0 + b_1 D + b_2 CSI + [b_3 D^2 + b_4 CSI^2 + b_5 (D)(CSI)] \qquad (3\text{-}18)$$

where

S_e = the empirical spacing in feet

A first order equation was fit using D and CSI as independent variables. Second order terms were then added to the model and found to be significant (at the 1% significance level). The resulting equation was:

$$S_e = 9.8473 + .92849D - .011585CSI \qquad (3\text{-}19)$$

Table (3-10) expresses spacing as a function of tree diameter and CSI level. Thinning the stand using these spacing guidelines will reduce the stand density to a specified level for each diameter class. Figure (3-3) depicts the relationship between spacing and the diameter for various levels of CSI. Note as the residual stand density increases, the average distance between trees decreases proportionally.

Table (3-10) expresses spacing guidelines in terms of residual stand density independent of the stand condition prior to thinning. The reduction in CSI due to thinning can be measured as the difference between the initial stand density before thinning and the residual stand density following thinning. An empirical relationship between the average distance between trees following thinning and the stand condition prior to thinning was developed as follows. Spacing following
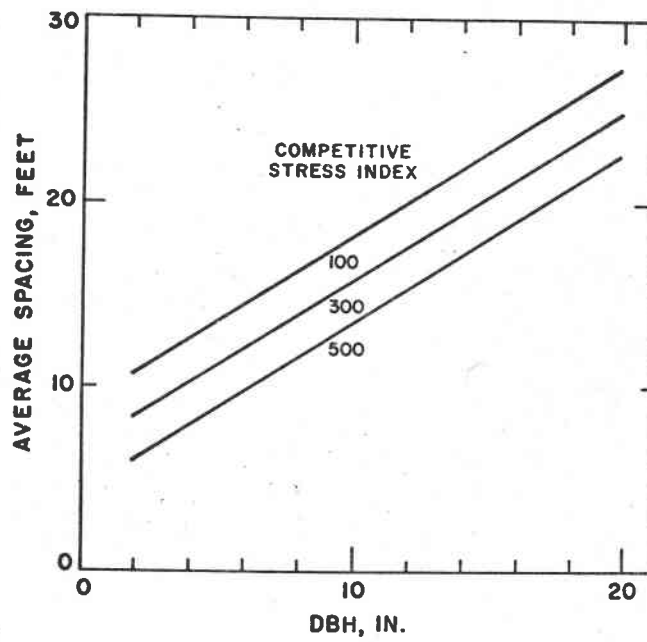
Figure (3-3): Average spacing as a function
of CSI and DBH

thinning was expressed as a function of tree size, initial CSI before thinning and change in CSI due to thinning.

$$S_{T_k} = b_0 + b_1 D_k + b_2 CSI + b_3 \Delta CSI \qquad (3\text{-}20)$$

Where:

S_{T_k} = the average spacing for a tree of DBH = k based on initial stand density and degree of thinning

The coefficients for Equation (3-19) were estimated as:

$$S_{T_k} = 10.602 + .8977D - 0.10684 CSI + .02379 \Delta CSI \qquad (3\text{-}21)$$

Specifying the initial CSI of the stand or tree and the degree of thinning proposed, Equation (3-21) gives empirical spacing for each DBH class. Thinning the stand using these guidelines will result in the reduction of initial CSI by ΔCSI units to the desired residual level.

Equations (3-19) and (3-21) give the forest manager the means of controlling stand density relative to the specific management objectives. The spacing guides developed regulate the level of stand density as a function of tree size. Arney (1973) suggests that the range of CSI are generally applicable to a managed stand is 250-350 CSI units. Fully stocked natural stands typically have stand densities ranging from 400-500 CSI units.

A field guide for determining when two trees in the area are competing based on their respective diameter can be derived from the open grown crown width relationship (Arney, 1973):

Table (3-10): Average spacing as a function of tree diameter
and CSI level (in feet)

					Tree CSI						
DBH	100	150	200	250	300	350	400	450	500	550	600
1	9.6	9.0	8.5	7.9	7.3	6.7	6.1	5.6	5.0	4.4	3.8
2	10.5	10.0	9.4	8.8	8.2	7.7	7.1	6.5	5.9	5.3	4.8
3	11.5	10.9	10.3	9.7	9.2	8.6	8.0	7.4	6.8	6.3	5.7
4	12.4	11.8	11.2	10.7	10.1	9.5	8.9	8.3	7.8	7.2	6.6
5	13.3	12.8	12.2	11.6	11.0	10.4	9.9	9.3	8.7	8.1	7.5
6	14.3	13.7	13.1	12.5	11.9	11.4	10.8	10.2	9.6	9.0	8.5
7	15.2	14.6	14.0	13.5	12.9	12.3	11.7	11.1	10.6	10.0	9.4
8	16.1	15.5	15.0	14.4	13.8	13.2	12.6	12.1	11.5	10.9	10.3
9	17.0	16.5	15.9	15.3	14.7	14.1	13.6	13.0	12.4	11.8	11.3
10	18.0	17.4	16.8	16.2	15.7	15.1	14.5	13.9	13.3	12.8	12.2
11	18.9	18.3	17.7	17.2	16.6	16.0	15.4	14.8	14.3	13.7	13.1
12	19.8	19.3	18.7	18.1	17.5	16.9	16.4	15.8	15.2	14.6	14.0
13	20.8	20.2	19.6	19.0	18.4	17.9	17.3	16.7	16.1	15.5	15.0
14	21.7	21.1	20.5	19.9	19.4	18.8	18.2	17.6	17.1	16.5	15.9
15	22.6	22.0	21.5	20.9	20.3	19.7	19.1	18.6	18.0	17.4	16.8
16	23.5	23.0	22.4	21.8	21.2	20.6	20.1	19.5	18.9	18.3	17.8
17	24.5	23.9	23.3	22.7	22.2	21.6	21.0	20.4	19.8	19.3	18.7
18	25.4	24.8	24.2	23.7	23.1	22.5	21.9	21.3	20.8	20.2	19.6
19	26.3	25.7	25.2	24.6	24.0	23.4	22.9	22.3	21.7	21.1	20.5
20	27.3	26.7	26.1	25.5	24.9	24.4	23.8	23.2	22.6	22.0	21.5

$$CW_i = 4.7071 + 2.01680 \, D_i - .0186 \, D_i^2 \qquad (3-22)$$

Where:

CW_i = the open grown crown width of the ith tree

D_i = the DBH of the ith tree

The open grown crown radius is equal to:

$$CR_i = \frac{CW_i}{2} = 2.3535 + 1.008 \, D_i - .0096 \, D_i^2 \qquad (3-23)$$

Dropping the D_i^2 term from Equation (3-23) gives:

$$CR_i = 2.3535 + 1.008 \, D_i \qquad\qquad (3\text{-}24)$$

Two trees compete (i.e., their competition circles overlap) if the sum of their open grown crown radii is greater than the distance between their centers. Using Equation (3-24):

$$R = CR_1 + CR_2 = 4.7070 + 1.008 \, (D_1 + D_2) \qquad\qquad (3\text{-}25)$$

Where:

R = the sum of the open grown crown radii for trees 1 and 2

Equation (3-25) can be used as a convenient "diameter plus" rule of thumb. The two trees are competing if the sum of their diameters at breast height plus 4.7 is greater than the distance between them. One major limitation to the diameter plus rule of thumb as a field guide is that the degree of competition between the two trees is not specified. To solve this problem, an alternative spacing guide can be defined as follows. Using the data presented in Table (3-10), a field guide based on diameter and level of CSI can be derived for individual trees. The average spacing for a tree is expressed as a function of its diameter and CSI as:

$$S_e = 9.84730 + .92849 \, D - .011585 \, CSI \qquad\qquad (3\text{-}26)$$

For a uniform level of stand density (i.e., $CSI = \overline{CSI}$), S_e in Equation (3-26) becomes a function of tree diameter alone.

$$S_e = (9.84730 - .011585 \overline{CSI}) + .92849 D \qquad (3-27)$$
$$\text{CONSTANT}$$

Solving the above Equation (3-27) for S_e under various density conditions (assuming .92849 $\sim$ 1.0) gives an approximate diameter plus spacing guide. Table (3-11) gives the appropriate diameter plus rule of thumb for various levels of CSI.

The appropriate diameter plus constant for a given stand density specifies the average spacing required based on tree size. Thinning to these guidelines will reduce the average CSI in the stand to the desired level. Since these guides are based on empirical results, caution should be exercised in their application to other stands.

Table (3-11): D-plus rule of thumb for various stand densities

Stand Density CSI	D Plus Spacing Rule
100	8.69
150	8.11
200	7.53
250	6.95
300	6.37
350	5.79

SAMPLING FOR COMPETITIVE STRESS

INTRODUCTION

Inter-tree competition is an important component in the analysis of tree and stand growth relationships. Competition indices are measures of stand density which express the relative level of competition between trees for growing space. Several indices have been proposed as suitable measures of inter-tree competition. The primary emphasis of this thesis has been on the evaluation of competitive stress index as a measure of inter-tree competition. It is obvious that inter-tree competition indices are primarily research tools. Application of the indices require specialized data sets including: tree identification codes, stem coordinates, and accurate diameter measurements. Such data sets are expensive to obtain and are not generally available on a large scale. In addition, the complexity of the computations associated with using the various competition indices require that some sort of computer processing procedures be adopted. These requirements seem very restrictive in the application of the indices on an operational basis. In the future, one would expect that the ability to estimate the level of competition in a stand would be as necessary as obtaining estimates of basal area per acre, volume per acre, or average stand diameter. Hopefully, this will provide some insight into assessing inter-tree competition on an operational level using the CSI model.

The objectives of this chapter are to develop the theory and procedures of sampling for CSI in a forest stand. A sampling scheme is proposed which will provide estimates of CSI for a stand or for individual trees within the stand. Results of simulated field samples are also presented.

Before the theory of sampling for CSI is presented, it is informative to briefly reexamine the CSI model in the whole stand and in single tree cases.

The CSI for a single tree is given by (Arney, 1973):

$$CSI_j = 100 \frac{(\sum_i AO_{ij} + A_j)}{A_j} \tag{4-1}$$

$$= \frac{100\ TAO_j}{A_j}$$

$$= PO_j$$

Where:

CSI_j = the competitive stress level for the jth tree in the stand

AO_{ij} = the area of overlap of growing space of the ith competitor and the jth tree

A_j = the growing space area of the jth tree defined as the open grown crown area of the tree

TAO_j = the total area of overlap of growing space of the jth tree (including the area of overlap of the jth tree itself)

PO_j = the percent overlap of growing space for the jth tree.

Equation (4-1) states that the CSI for a single tree can be expressed as the percent overlap of its growing space by the growing space circles of competing neighbors.

The average CSI level for the stand can be expressed in two forms: as a ratio of the means or as the mean of the ratios of the total area of overlap to the area of growing space.

Mean of the ratio form:

$$\overline{CSI} = \sum_{j=1}^{N} \frac{CSI_j}{N} = 100 \sum_{j=1}^{N} \frac{TAO_j/A_j}{N} = \sum_{j=1}^{N} \frac{PO_j}{N} = \overline{P} \tag{4-2}$$

Ratio of the means form:

$$\overline{CSI}_R = 100 \frac{TAO.}{A.} = \frac{100 \sum\limits_{j=1}^{N} TAO_j}{\sum\limits_{j=1}^{N} A_j} \tag{4-3}$$

Where:

$\overline{CSI}$ = the mean of the ratio form of expressing the average CSI of the stand

N = the number of trees in the stand

$\overline{P}$ = the average percent overlap of growing space per tree in the stand

$\overline{CSI}_R$ = the ratio of the means form of expressing the average CSI of the stand

TAO. = the total area of overlap of growing space for all N trees in the stand

A. = the total area of growing space for all N trees in the stand

$\overline{P}_R$ = the average percent overlap of growing space for the N trees in the stand based on the ratio of the means estimator

Using either form, the average CSI level for the stand can be expressed as the average percent overlap of growing space.

It will be shown that when sampling for the average CSI for the stand, an unbiased estimate of crown competition factor (CCF) can also be obtained. In fact, the CCF and CSI models are closely related on a whole stand basis. CCF is a measure of stand density based on the potential open grown crown areas or growing space of the stand relative to the ground area occupied by the stand. The CCF for a closed stand occupying one acre of ground area is given as:

$$CCF = 100 \sum_{j=1}^{N} \frac{A_j}{A_c} = 100 \frac{\sum_{j=1}^{N} A_j}{43650} = \sum_{j=1}^{N} Q_j \qquad (4\text{-}4)$$

where:

CCF = the crown competition factor for the stand

A_j = the open grown crown area or growing space of the jth tree in the stand

A_c = the ground area occupied by the stand (in this case A_c = 93560 sq. ft.)

Q_j = the open grown crown area or growing space of the jth tree expressed as a percent of an acre

$$\sum_{j=1}^{N} Q_j = \text{the percent of an acre occupied by the total crown areas}$$
or total growing space of the stand.

EVALUATION OF ARNEY'S SAMPLING ESTIMATOR

Arney (1973) developed a sampling estimator for estimating average CSI or CCF in a closed stand based on Bitterlich's point sample design. The sampling procedure is as follows: n sample points are systematically established in the stand. At each sample point a tally of "in" trees is made by one-inch diameter classes, using an appropriate basal area factor (BAF). The BAF is selected so as to give an average tree count of 7-10 trees per sample point (Arney, 1973). The CSI or CCF estimator for the jth sample point is:

$$U_{A_j} = BAF \sum_{i=1}^{k} \frac{y_{ij}}{x_{ij}} TC_{ij} = BAF \sum_{i=1}^{k} R_{ij} \qquad (4-5)$$

Where:

U_{A_j} = Arney's estimator of average CSI or CCF for the jth sample point

BAF = the basal area factor used in sampling

y_{ij} = the open grown crown area or growing space of the ith DBH class expressed as a percent of an acre at the jth sample point

x_{ij} = the basal area in square feet of the ith DBH class at the jth sample point

k   = the number of DBH classes tallied

R_{ij} = the sum of the y_{ij}/x_{ij} ratio for all trees in the ith DBH class at the jth point

TC_{ij} = the tree count observed for the ith DBH class at the jth point

The estimator for a sample of n points in the stand is:

$$U_A = \sum_{j=1}^{n} \frac{U_{Aj}}{n} = \frac{BAF}{n} \sum_{j=1}^{n} \sum_{i=1}^{k} \frac{y_{ij}\, TC_{ij}}{x_{ij}}$$

$$= BAF \frac{\sum_{j=1}^{n} \sum_{i=1}^{k} R_{ij}}{n} = BAF \frac{\sum_{j=1}^{n} R_{\cdot j}}{n} = BAF\ \overline{R} \qquad (4\text{-}6)$$

Where:

U_A = Arney's estimator of average CSI or CCF for a closed stand based on n sample points

$\overline{R}$ = the average ratio weighting factor for all DBH classes

The sample variance of the estimator in equation (4-6) is:

$$V(U_A) = V(BAF \cdot \overline{R}) = BAF^2 V(\overline{R})$$

But:

$$V(\overline{R}) = \sum_{j=1}^{n} \frac{(R_{\cdot j} - \overline{R})^2}{(n)(n-1)}$$

Therefore:

$$V(U_A) = BAF^2 \sum_{j=1}^{n} \frac{(R_{\cdot j} - \overline{R})^2}{(n)(n-1)} \qquad (4\text{-}7)$$

Where:

$V(U_A)$ = the sample variance of Arney's estimator U_A

$V(\bar{R})$ = the sample variance of the average ratio weighting factor $\bar{R}$

$R_{.j}$ = the weighting factor for the jth sample point summed over all DBH classes

Arney (1973) states that U_A is a valid estimator of average CSI or CCF in a closed stand. It is assumed that a "closed stand" is one that fully occupies the available growing space of the site. In a closed stand, the average CSI and CCF values are highly correlated, but are not necessarily the same value. The average CSI for a stand is always greater than the corresponding CCF value. Arney's estimator in Equation (4-6) does not distinguish between the average CSI and the CCF for a stand.

Using the terminology presented by Husch (1972) for polyareal plot sampling, Arney's estimator for CSI or CCF in a closed stand can be viewed from a different perspective. Husch uses the term "factor" to specify the number of units per acre that are represented by each tree tallied in the sampling process for any tree characteristic. A "tree factor" is the per acre conversion factor which specifies the number of trees per acre represented by each tree tallied. A "volume factor" represents the number of units per acre represented by each tree tallied. In general, a "tree characteristic factor" specifies the number of units of the particular tree characteristic per acre which are represented

by each tree tallied. In horizontal point sampling the "tree factor" is:

$$STF_i = \frac{BAF}{BA_i} \tag{4-8}$$

Where:

    STF_i = the tree factor or stand table factor for the ith DBH class

    BAF = the basal area factor used in the sampling process

    BA_i = the basal area of the ith diameter class

In general, for any tree characteristic Y_i, corresponding to a tree in the ith DBH class, the "characteristic factor" can be expressed as (Husch, 1972):

$$F_{yi} = (y_i)\ (\frac{BAF}{BA_i}) = (y_i)\ (STF_i) \tag{4-9}$$

Where:

    F_{yi} = the "tree characteristic factor" for the ith DBH class

    y_i = the tree characteristic of interest (e.g., volume, diameter, percent growing space)

Equation (4-9) expressed the "tree characteristic factor," F_{yi}, for the ith DBH class, as the product of the characteristic of interest and the tree factor for the ith DBH class.

An estimate of the per acre characteristic y_{ij} at the jth sample point is the product of the tree characteristic factor and the tree count of the ith DBH class observed at the jth sample point.

$$y_{ij} = (F_{yi})\ (TC_{ij})$$

Where:

    TC_{ij} = the observed tree count of the ith DBH class at the jth

          sample point

The estimate of the characteristic per acre for all DBH classes at the jth sample point is:

$$y_{\cdot j} = \sum_{i=1}^{k} F_{yi} TC_{ij} \tag{4-10}$$

Where:

    $y_{\cdot j}$ = the per acre estimate of the tree characteristic y_i at the

          jth sample point summed over all DBH classes

    k   = the number of DBH classes tallied

An average per acre estimate for the tree characteristic y_i obtained from n sample points in the stand is calculated by dividing the sum of the n per acre estimates $y_{\cdot j}$ by n, using Equations (4-9) and (4-10).

$$\bar{y} = \frac{\sum_{j=1}^{n} y_{\cdot j}}{n} = \sum_{j=1}^{n} \sum_{i=1}^{k} \frac{F_{yi} TC_{ij}}{n} \tag{4-11}$$

Substituting for F_{yi} in Equation (4-11) using Equation (4-8) gives:

$$\bar{y} = \frac{\sum_{j=1}^{n} \sum_{i=1}^{k} y_{ij} \frac{BAF}{BA_i} TC_{ij}}{n} = \frac{BAF \sum_{j=1}^{n} \sum_{i=1}^{k} \frac{y_{ij}}{BA_i} TC_{ij}}{n} \tag{4-12}$$

Let y_i equal the growing space area of the ith DBH class as a percent of one acre. Then equation (4-12) states that the average per acre estimate of the tree characteristic of interest is given by $\bar{y}$. Here, $\bar{y}$ is the average growing space area per acre expressed as percent of an acre. In other words, $\bar{y}$ is the percent of an acre occupied by the growing space area of the stand or the CCF value for the stand (see Equation 4-4). Substituting $x_{ij} = BA_{ij}$ in Equation (4-6) and comparing this result with Equation (4-12) shows that Arney's estimator is equal to $\bar{y}$. Thus, Arney's estimator for CSI or CCF in a closed stand is, in fact, an estimator of CCF in a closed stand and not average CSI, although as stated previously, the average CSI and CCF values for a stand are generally highly correlated.

Arney's estimator for the average CSI or CCF for a stand was evaluated under a variety of stand conditions using natural and hypothetical populations. The results of these simulated field samples confirm the theoretical analysis presented above. Arney's estimator consistently underestimated the CSI value of the stand but was an excellent estimator of the CCF value of the stand.

Arney's estimator can be decomposed into its basic components under the following special conditions. Suppose the stand consists of N trees of a uniform diameter k. The y_i/x_i ratio for all the trees in the stand is therefore equal to a constant, C, since all the trees have the same diameter. Arney's estimator U_A in Equation (4-5) can be simplified to:

$$U_A = BAF \sum_{j=1}^{n} \frac{R_{.j}}{n} = BAF \sum_{j=1}^{n} \frac{C\ TC_j}{n} = BAF\ C\ \overline{TC} = BAF\ \overline{R} \qquad (4\text{-}13)$$

Where:

C = the constant ratio y_i/x_i or the ratio of the growing space of the ith DBH class as a percent of an acre to its basal area

TC_j = the tree count at the jth point

$\overline{TC}$ = the average tree count per point for the n sample points

Equation (4-13) states that the estimator of competitive stress in a uniform stand of DBH = k can be expressed as the product of the constant weighting factor C corresponding to the stand DBH = k and the average tree count per point. If C and $\overline{TC}$ are known, and an approximate CCF level is estimated for the stand, the appropriate BAF can be determined by solving Equation (4-13) for BAF.

$$BAF = \frac{\hat{U}_A}{\overline{TC}\ C} \qquad (4\text{-}14)$$

Where:

BAF = the appropriate BAF for the stand

$\hat{U}_A$ = the estimated CCF for the stand (e.g., low = 150, medium = 300, high = 450)

Table (4-1) gives the appropriate BAF for a uniform stand with a constant DBH = k corresponding to an estimated CCF level. An average tree count of 5 trees per point is assumed.

Table (4-1): The appropriate BAF corresponding to
a uniform stand DBH = k and various
CCF values using Equation (4-14)

Stand Diameter	Stand CCF				
	100	200	300	400	500
5	7.6	15.2	22.8	30.4	38.0
10	11.3	22.6	33.9	45.2	56.5
15	14.3	28.6	42.9	57.2	71.5
20	17.1	34.2	51.3	68.4	85.5

When sampling in a uniform stand of diameter k, the critical com-
ponent of Arney's estimator is the average tree count per point. When
sampling under conditions in which the stand is not closed, Arney's
estimator underestimates the CCF for the stand because the average TC
falls below a minimum level. Table (4-2) gives the minimum tree count
required when using a given BAF for various uniform stand diameters.
The CCF value for an open grown stand that fully occupies the site is
100. CCF values less than 100 indicate that the stand is not fully
utilizing all the growing space available to it.

DERIVATION OF CSI ESTIMATORS

The following sections presents the theory used to develop esti-
mators of CSI for a whole stand or for individual trees within the
stand. Derivations of the expected values and variances of the esti-
mators are presented in detail in Appendix A. Since the estimator

developed for the single tree case is a special form of a whole stand
estimator, the single tree case will be presented first.

Figures (4-1) and (4-2) illustrate a hypothetical stand consist-
ing of three trees. Figure (4-1) depicts the overlap zones of the
trees relative to their growing space areas. The variable i indicates
the number of times that a particular area has been overlapped by
competing neighbors. Figure (4-1) is representative of the overlap
zones which might typically be encountered in a natural stand. Figure
(4-2) depicts each of the trees and their corresponding overlap zones
individually.

Table (4-2): Minimum tree count required for
a given basal area factor

Stand CCF = 100

	Average Stand DBH			
BAF	5"	10"	15"	20"
15	2.54	3.79	4.76	5.70
20	1.91	2.84	3.57	4.27
25	1.53	2.27	2.86	3.42
30	1.27	1.89	2.38	2.85
40	0.95	1.42	1.79	2.14
60	0.64	0.95	1.19	1.42
80	0.48	0.71	0.89	1.07
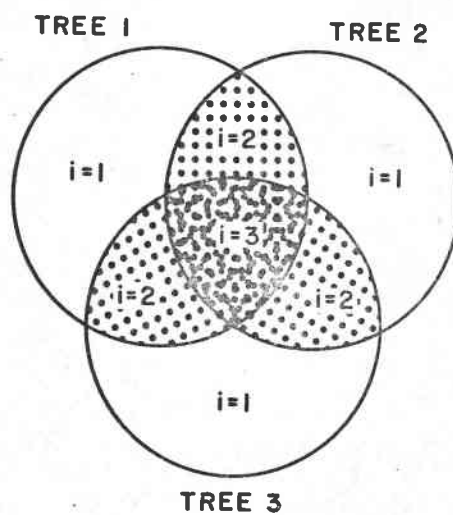
Figure (4-1): Overlap conditions in a
hypothetical stand
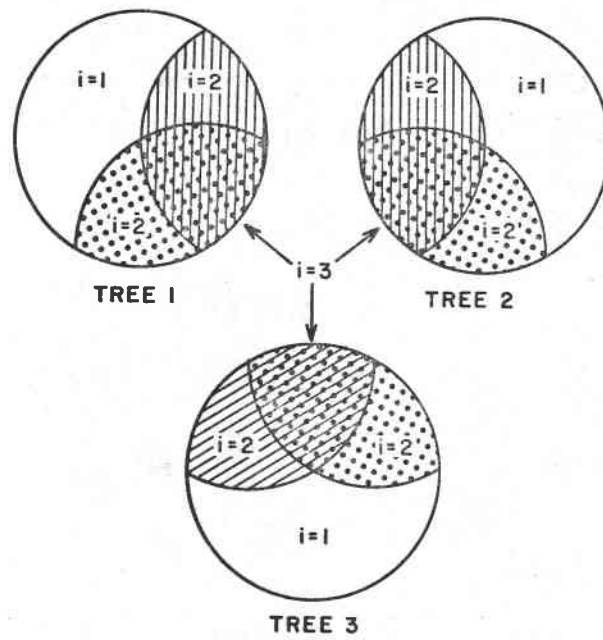
Figure (4-2): Individual overlap zones
by tree

It was shown earlier that the CSI for a tree is equivalent to its percent overlap of growing space (see Equation 4-1). An open grown tree has a CSI of 100 units or has 100 percent of its growing space occupied. A tree with a CSI of 200 units has 200 percent of its growing space occupied or overlapped by competitors' growing space circles. Sampling for CSI for a single tree is equivalent to sampling for its percent overlap of growing space.

An expression of the CSI for a single tree was given in Equation (4-1). An alternative expression of a tree's CSI is derived from the notation presented in Figure (4-3). The growing space area for the jth tree equals the sum of the areas occupied by the i overlap zones of the tree. The total area of overlap for the jth tree equals the area occupied by the ith tree count zone times the number of times the zone has been overlapped, summed over all i overlap zones. The CSI level for the jth tree can be expressed as (see Figure 4-3):

$$CSI_i = 100 \; \frac{TAO_j}{A_j} = \frac{\sum_i OA_{ij} \cdot i}{A_j} = \frac{\sum_i OA_{ij} \cdot i}{\sum_i OA_{ij}} \tag{4-15}$$

Where:

A_j = the growing space for the jth tree

TAO_j = the total area of overlap of the jth tree

i = the overlap condition of tree count value (i = 1, 2, 3)

OA_{ij} = the area occupied by the ith tree count or overlap zone in the jth tree's growing space
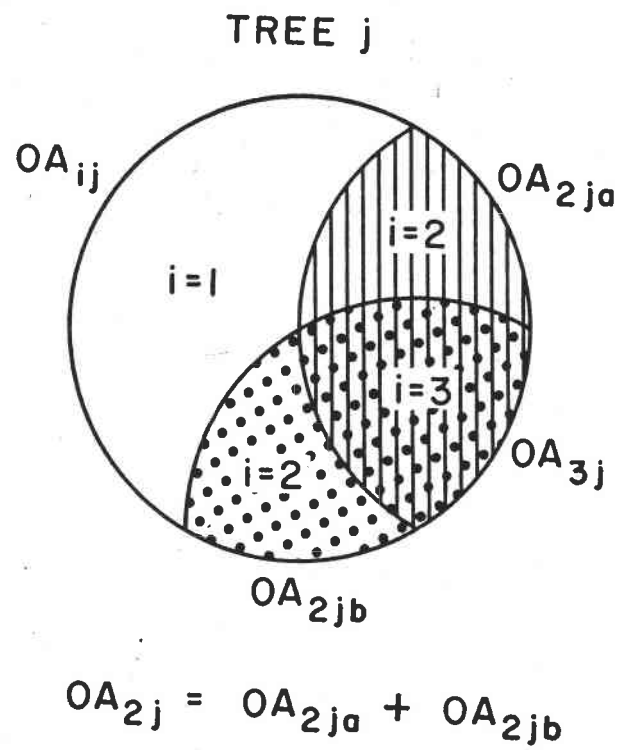
$$OA_{2j} = OA_{2ja} + OA_{2jb}$$

Figure (4-3): Area occupied by the ith overlap zone.

For example, the CSI for tree j in Figure (4-3) is computed as follows:

$$CSI_j = (100) \frac{(TAO_j)}{A_j} = \frac{100 \ (OA_{1j} \ (1) + OA_{2j} \ (2) + OA_{3j} \ (3))}{OA_{1j} + OA_{2j} + OA_{3j}} \qquad (4\text{-}16)$$

Where:

$$OA_{2j} = OA_{2j_a} + OA_{2j_b}$$

This derivative of CSI for a single tree is equal to the expression given in Equation (4-1). The CSI for tree j in Figure (4-4) using Equation (4-1) is:

$$CSI_j = \frac{100 \ TAO_j}{A_j} = \frac{100 \ (AO_{2j} + AO_{3j} + A_j)}{A_j} \qquad (4\text{-}17)$$

But:

$$AO_{2j} = OA_{2j_a} + OA_{3j}$$
$$AO_{3j} = OA_{2j_b} + OA_{3j}$$
$$A_j \quad = OA_{1j} + OA_{2j_a} + OA_{2j_b} + OA_{3j}$$
$$TAO_j = OA_{1j} + 2(OA_{2j_a} + OA_{2j_b}) + 3(OA_{3j})$$

Substituting these expressions into (4-17) gives:

$$CSI_j = \frac{100 \ (OA_{1j} + 2 \ (OA_{2j}) + 3 \ (OA_{3j}))}{OA_{1j} + OA_{2j} + OA_{3j}} \qquad (4\text{-}18)$$

، which is identical to the CSI computed in Equation (4-16).
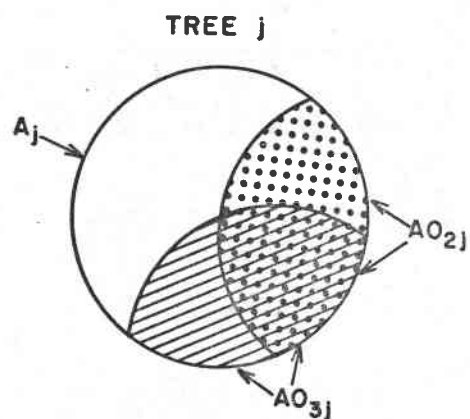
Figure (4-4): Overlap of tree j by its
              competitors

The percent overlap value for a single tree can be estimated from a sample of points located within the tree's hypothetical growing space area. The systematic grid of sample points in Figure (4-3) illustrates how the percent overlap value for the ith tree can be estimated. Consider the set of m sample points which fall into the jth tree's growing space circle. The probability that the observed tree count at the kth sample point will fall into the ith overlap zone equals the area occupied by the ith overlap zone divided by the area of the growing space for the jth tree. Symbolically:

$$P_t(i) = \frac{OA_{ij}}{A_j} \tag{4-19}$$

Where:

$P_t(i)$ = the actual probability that the observed tree count at a given sample point will fall into the ith tree count zone for a given tree

Referring to Figure (4-3), note as the area occupied by the ith tree count zone increases (OA_{ij}), the greater the probability a sample point will fall into that zone (see Equation 4-19). A sample point falling into a tree count zone equal to 1 is equivalent to a percent overlap condition of 100. An observed tree count equal to 3 corresponds to a percent overlap of 300. The average tree count or overlap condition of the m sample points for the jth tree reflect the average overlap percentage of growing space or PO value for the tree.

The percent overlap value for a single tree can be estimated using the following estimator:

$$T_j = \frac{100 \sum\limits_{k=1}^{m} t_{jk}}{m} = \bar{t}_j (100) \qquad (4\text{-}20)$$

Where:

T_j = an unbiased estimator of the percent overlap of growing space for the jth tree

t_{jk} = the number of trees which overlap the kth sample point or the overlap condition observed at the kth sample point

$(k = 1,m)$

$\bar{t}_j$ = the average tree count or overlap condition for the jth tree

m = the number of sample points taken in the jth tree's growing space area

It can be shown that T_j is an unbiased estimator of PO_j or the CSI for the jth tree (see Appendix A). The corresponding sample variance of T_j is:

$$V(T_j) = 100^2 \frac{\sum\limits_{k=1}^{m} (t_{jk} - \bar{t}_j)^2}{m-1} \qquad (4\text{-}21)$$

Where:

$V(T_j)$ = the sample variance of the T_j estimator for the jth tree

Two estimators are proposed for estimating the average CSI level for the whole stand. The first whole stand estimator can be derived using the single tree estimator presented above. Since T_j is an unbiased estimator of the CSI for the jth tree in the stand, it is intuitive that the mean of a sample of n individual trees would be an

unbiased estimator of the average CSI of the stand.

$$T_s = \sum_{j=1}^{n} \frac{T_j}{n} = \sum_{j=1}^{n} \frac{100 \left(\sum_{k=1}^{m_j} \frac{t_{jk}}{m_j} \right)}{n} \qquad (4\text{-}22)$$

Where:

T_s = the estimator for the average CSI for the whole stand

n = the number of trees in the sample

m_j = the number of sample points observed for the jth tree

If the number of sample points observed per tree is the same for all n trees in the sample, Equation (4-22) can be simplified by substituting $m = m_j$ to:

$$T_s = 100 \frac{\sum_{j=1}^{n} \sum_{k=1}^{m} t_{jk}}{mn} = \overline{\overline{t}} \qquad (4\text{-}23)$$

Where:

$\overline{\overline{t}}$ = the average percent overlap of growing space of average tree count of all sample points

T_s is the mean of the ratio estimator for the average CSI given in Equation (4-2). It is based on the mean of the individual tree CSI estimates for the n sample trees selected in the stand. The sample variance associated with the estimator given in Equation (4-23) is:

$$V(T_s) = \frac{V\left(\sum_{j=1}^{n} T_j \right)}{n} = \sum_{j=1}^{n} \frac{V(T_j)}{n^2} \qquad (4\text{-}24)$$

$$= 100^2 \sum_{j=1}^{n} \sum_{k=1}^{m} \frac{(t_{jk} - \overline{t}_j)^2}{(n)^2 (m-1)}$$

Where:

$V(T_s)$ = the sample variance of the whole stand estimator T_s

The whole stand estimator T_s estimates the average percent overlap of growing space per tree in the stand. The primary limitation in using the estimator on an operational basis is the total number of sample points required. In order to adequately sample for average CSI in the stand, a reasonable sample size of individual trees is necessary. In addition, the critical factor in estimating a single tree's CSI is the number of sample points selected per tree. As a result, as n becomes large enough to reasonably estimate the average CSI in the stand, the total number of sample points becomes excessive. Let M equal the total number of sample points required. Then:

$$M = (m)(n) \tag{4-25}$$

For n = 30 sample trees, and m = 4 sample points per tree, M = 120 total sample points required. An excessive number of total sample points causes the estimator in Equation (4-23) to be less than desirable on a whole stand basis.

A second and more realistic approach to estimating the average CSI level in the stand is to develop an estimator based on a sample of points rather than on a sample of individual trees within the stand. The second proposed whole stand estimator is based on a random or systematic sample of points within the stand without regard to individual trees. A weighted average tree count or percent overlap estimator can be derived to estimate the average CSI for the stand. Figure (4-5)

shows the overlap zones of a hypothetical stand of three trees. Super-
imposed is a systematic grid of points. The circled points indicate
the sample points randomly selected for observation. Notice that a
sample point may be overlapped by more than one tree's growing space
circle. In deriving the whole stand estimator based on a sample of
points rather than individual trees within the stand, a weighting
factor must be employed to adjust for multiple overlaps. Figure (4-6)
shows the individual tree case described earlier. The selected sample
points for the whole stand estimator are indicated for each tree. A
whole stand estimator based on individual trees can be computed from
the overlap conditions indicated by Figure (4-5). A single sample
point falling into an overlap zone of 2 in Figure (4-5) is represented
by two sample points in Figure (4-6), one for each sample tree that
overlaps it.

The weighting factor for the second whole stand estimator must
account for the multiple overlap of a given sample point. A weighted
average tree count or percent overlap for the stand depicted in Figure
(4-5) can be constructed as:

$$S = 100 \ \frac{\sum\limits_{k=1}^{\ell} t_k \, w_k}{\sum\limits_{k=1}^{\ell} w_k} \tag{4-25}$$

Where:

S = the weighted average tree count adjusted for multiple overlaps

t_k = the tree count or overlap of the kth sample point (k = 1,1)
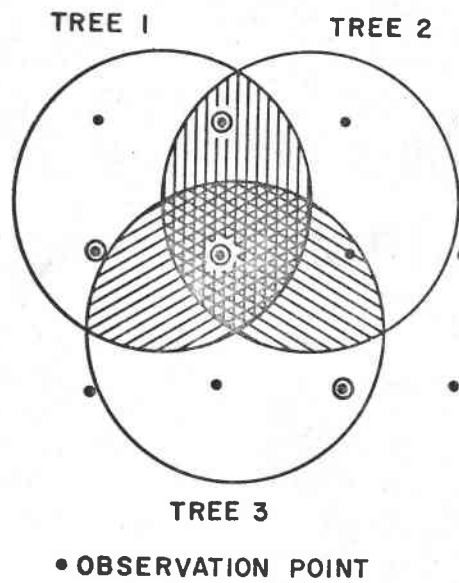
Figure (4-5): Overlap zones of a hypothetical stand
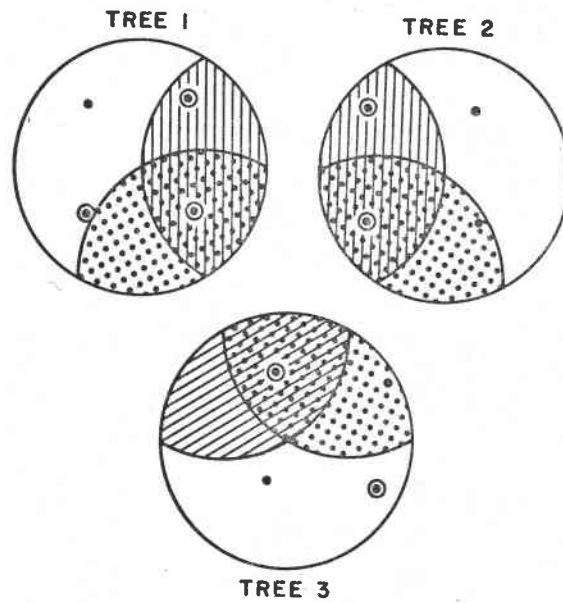
Figure (4-6):   Overlap zones for each tree
                in the stand

w_k = the weighting factor used at the kth sample point

k = the sample point number

ℓ = the sample size

The weighting factor w_k depends upon the tree count t_k observed at the kth sample point. If the tree count at the kth point equals 1, then the point is overlapped by one tree and w_k = 1. If t_k = 2, then the kth sample point is contained within two trees' growing space circles and the weighting factor w_k = 2. In general, if the tree count at the kth point t_k = i, then the weighting factor w_k = i or w_k = t_k. Substituting this relationship between t_k and w_k into Equation (4-25) yields:

$$S = \frac{100 \sum_{k=1}^{\ell} t_k \cdot t_k}{\sum_{k=1}^{\ell} t_k} = \frac{100 \sum_{k=1}^{\ell} t_k^2}{\sum_{k=1}^{\ell} t_k} \tag{4-26}$$

The whole stand estimator S represents the weighted average tree count or percent overlap of growing space for the stand, based on a sample of ℓ points. The sample variance of S is approximately (Cochran, 1963):

$$V(S) = 100^2 \sum_{c=1}^{\ell} \frac{(t_k^2 - St_k)^2}{\ell-1} \cdot \frac{(1 - \frac{\ell}{N})}{\ell\bar{t}} \tag{4-27}$$

Where:

V(S) = the approximate standard error of the estimator S.

The probability that a sample point falls into the ith overlap zone is equal to the area occupied by the ith overlap zone divided by

the area occupied by the stand. Symbolically:

$$P_s(i) = \frac{OA_i}{A_c} = \frac{OA_i}{\sum\limits_i OA_i} \tag{4-28}$$

Where:

$P_s(i)$ = the probability that the kth sample point falls into the ith overlap zone of the stand

OA_i = the area occupied by the ith overlap zone in the stand

A_c = the ground area occupied by the stand

It can be shown that for a reasonable size $\ell(\ell \geq 10)$, the expected value of S is equal to the average CSI for the stand. The whole stand estimator S in Equation (4-26) corresponds to the ratio of the means estimator of the average CSI given in Equation (4-3).

Equation (4-26) states that a whole stand estimator of the average CSI level in the stand can be expressed as the weighted average tree count of ℓ sample points in the stand. If no weighting factor is used in Equation (4-25), S becomes the unweighted average tree count for the ℓ sample points. The expected value of this unweighted estimator can be shown to be equal to the CCF value for the stand (see Appendix A). An estimator for average CSI or for CCF for the stand can be obtained from the sample of ℓ points depending upon the weighting factor used to derive the estimator S. If the weighting factor in Equation (4-25) is unity, S becomes:

$$S = 100 \sum_{k=1}^{\ell} \frac{t_k \cdot 1}{\displaystyle\sum_{k=1}^{\ell} 1} = 100 \sum_{k=1}^{\ell} \frac{t_k}{\ell} = 100 \, \bar{t}_u \qquad (4\text{-}29)$$

Where:

$\bar{t}_u$ = the unweighted average tree count or overlap condition of the ℓ sample points

The whole stand estimator T_s derived from individual tree estimates of CSI is identical to the whole stand estimator S if the number of sample points per tree is constant for all trees sampled.

In order to fully understand the differences between the two whole stand estimators S and T_s the following example is given. Figure (4-5) depicts the whole stand case used to derive the estimator S. The figure shown ℓ = 7 sample points randomly selected in the stand. At each point the overlap or tree count condition is tallied. Substituting the ob-served tree count or overlap values into Equation (4-26) gives S as:

$$S = \left(\frac{1^2 + 2^2 + 1^2 + 3^2 + 2^2 + 2^2 + 1^2}{1 + 2 + 1 + 3 + 2 + 2 + 1}\right) (100) = \frac{24}{12} (100) = 200$$

Figure (4-6) depicts the individual trees represented in the stand. The sample points used to estimate S are given for each sample tree. The whole stand estimator T_s can be derived for the M = (n)(m) = (3)(4) = 12 total sample points. By substituting the actual tree counts for each tree into Equation (4-23):

$$\begin{array}{ccc} \text{Tree 1} & \text{Tree 2} & \text{Tree 3} \end{array}$$

$$T_s = \left[\frac{(1+2+3+2) + (2+1+3+2) + (2+3+2+1)}{3(4)}\right] 100 = \frac{24 \, (100)}{12} = 200$$

In the example, notice that the whole stand estimator S requires 7 sample points to estimate the average CSI of the stand while the whole stand estimator T_s requires 12 total sample points.

SAMPLING PROCEDURES FOR THE SINGLE TREE CASE

The sampling procedures for estimating the competitive stress index for an individual tree within a stand are described as follows:

First, identify the tree of interest and establish m sample points within its growing circle. At each sample point, a tally of "in" trees is made, using an appropriate basal area factor, from the surrounding trees in the stand. Each potential competitor is observed at breast height (BH). The tally of "in" trees represents the number of times the sample point has been overlapped by the hypothetical growing space circles of the trees in the stand. The CSI estimate and sample variance of the CSI for the subject tree are given as:

$$T = 100 \sum_{k=1}^{m} \frac{t_k}{m} = \bar{t} \, (100) \tag{4-30}$$

$$V(T) = \frac{\sum_{k=1}^{m} (t_k - \bar{t})^2}{m-1} \, (100)^2 \tag{4-31}$$

The establishment of the sample points within the subject tree's hypothetical growing space circle can theoretically be randomly or systematically located. However, in order to maintain uniformity from tree to tree, it is recommended that the m sample points be located at an equal distance from the tree's bole. If a tree's growing space circle is divided into circular sections, the centroid or geometric

center of each section can be used to locate the m sample points. The
centroid radius or the distance from the center of the tree to the
centroid of each section can be mathematically derived (see Appendix
C). The centroid radius SR, for any circular section, can be expressed
as a function of the circular radius. In the case of a tree, the
centroid radius is a function of the competition radius of the tree's
growing space area. Figure (4-7) shows the geometrical relationships
involved between the centroid radius, SR, and the competition radius,
CR, for a circular section with an interior angle of 2 Θ.

The desired sample size m determines the size of the circular sec-
tion and the functional relationship between the centroid radius and the
competition radius for the tree. Table (4-3) gives the relationship
between SR and CR for various sample sizes m. A detailed examination
of the geometrical relationships presented in Figure (4-7) is given in
Appendix C.

The establishment of the sample points within the tree's growing
space circle is critical. The location of the sample points at the
geometrical center of each circular section assumes a uniform distri-
bution of sample points within the tree's growing space circle. Random
or other systematic locations of the m sample points are also possible,
but were not field tested.

Figure (4-8) depicts a hypothetical stand consisting of five trees.
Suppose an estimate of CSI is desired for tree 3. Four sample points
are located at the centroid of each quadrant. At each sample point,
the number of trees overlapping the point are tallied. Figure (4-9)

GROWING SPACE CIRCLE
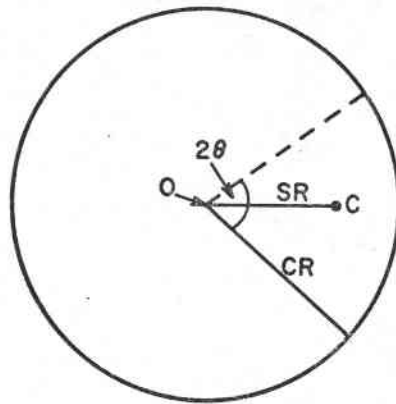
O = TREE CENTER
CR = COMPETITION RADIUS
SR = CENTROID RADIUS
C = CENTROID OF SECTION
2θ = INTERIOR ANGLE

Figure (4-7): Determination of the centroid
radius of a circular section

SINGLE TREE

OBSERVATION POINT
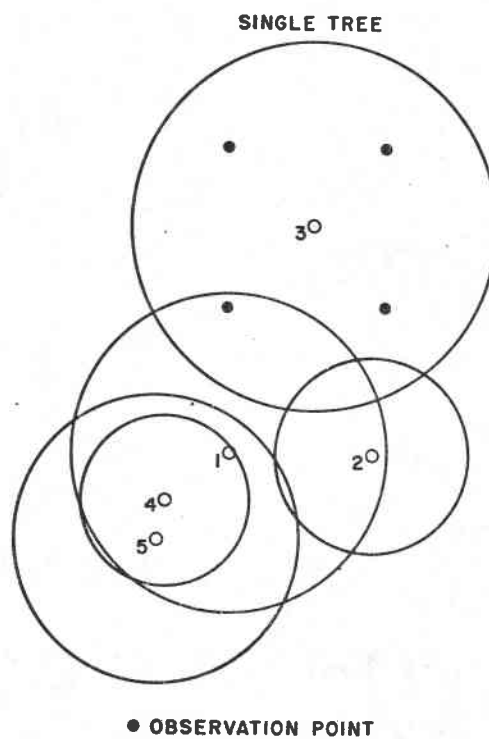
Figure (4-8): Sampling for CSI in the single tree case
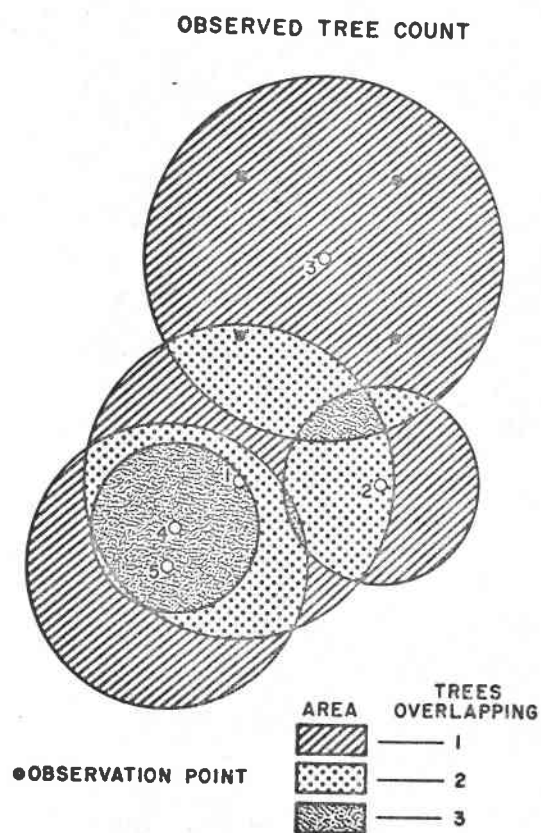
Figure (4-9): Tree count zones for tree 3

Table (4-3): Relationship between centroid
radius and competition radius
for a given sample size

Sample Size	Centroid Radius
m	
2	(.42441)(CR)
4	(.60021)(CR)
6	(.63662)(CR)
8	(.64966)(CR)
10	(.65575)(CR)

shows the tree count or overlap condition observed for each sample
point. Three of the points fall into an area with only one tree over-
lapping (e.g., the subject tree). The fourth point falls into an
overlap area where two trees overlap the point. The estimate of CSI
for tree 3 can be computed as:

$$T_3 = 100 \sum_{k=1}^{4} \frac{t_k}{4} = 100 \frac{(1 + 1 + 1 + 2)}{4} = 100 \left(\frac{5}{4}\right) = 125$$

$$V(T_3) = 100^2 \frac{\Sigma(t_k - \bar{t})^2}{3} = (100)^2 (.25) = 2500$$

The estimated CSI level for tree 3 is 125. The actual CSI level for
tree 3 is 121.

SAMPLING PROCEDURES FOR THE WHOLE STAND CASE

If the sampling objective is to estimate the average competitive
stress level for the whole stand either of the whole stand estimators
described earlier can be used. If the whole stand estimator T_s is used,

individual tree estimates will be obtained for the n sample trees as discussed above. If the whole stand estimators is used, the sampling procedure would be as follows:

Using either a random or systematic sampling scheme, ℓ sample points are selected in the stand. Each of the sample points is located independently of individual trees in the stand. At each sample point, a tally of "in" trees is made using an appropriate BAF, from the surrounding trees in the stand. Each potential competitor is observed at breast height. The tally of "in" trees represents the number of times the point has been overlapped by the hypothetical growing space circles of trees in the stand. The average CSI for the stand can be estimated as weighted average of observed tree counts using Equation (4-25) as:

$$S = \frac{100 \sum_{k=1}^{\ell} t_k^2}{\sum_{k=1}^{\ell} t_k} \qquad (4-32)$$

Figure (4-10) depicts a hypothetical stand consisting of five trees. The growing space circle for each tree is represented. Superimposed is a systematic grid of 16 sample points. At each sample point, tree count is observed using an appropriate BAF. Sample points with tree counts equal to zero are not contained within any trees growing space circle. A tree count of zero is not defined since the CSI value for an open grown tree of 100 corresponds to tree count of overlap equal to one. Also, CSI is a characteristic associated with a given tree. A tree count of zero does not provide any information
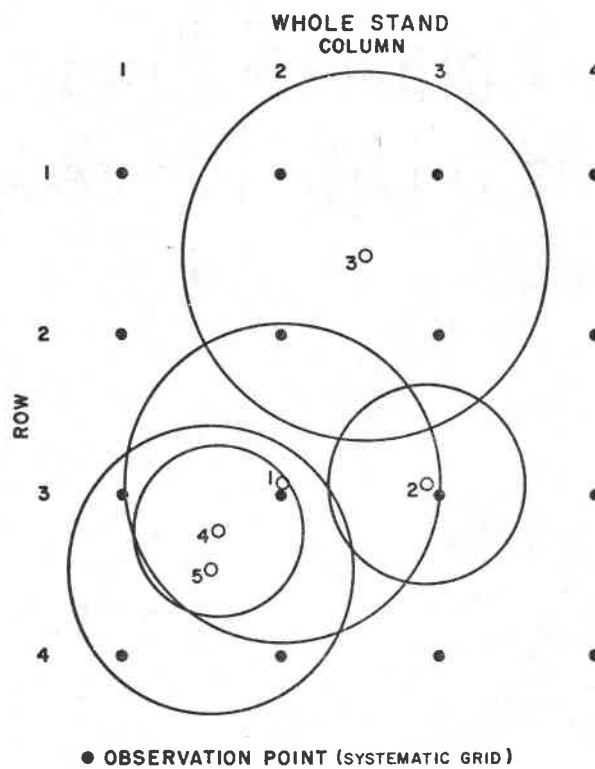
Figure (4-10): Sampling for CSI in the whole stand case

about the CSI or percent overlap for a tree since it falls outside of any growing space circle. Using the weighted whole stand estimator S, a tree count of zero has no effect on the estimator. A tree count of zero is not possible in estimating the CSI of a single tree since all m sample points are located within the sample tree's growing space. The minimum tree count for the single tree case is therefore equal to 1. In this case, only the subject tree overlaps the sample point. To maintain consistency in sampling for CSI all sample points with a tree count or overlap condition equal to zero should be ignored. Figure (4-11) shows the overlap condition observed at each sample point. Sample point (1,2) falls into a tree count zone = 1. Point (3,2) lies in a tree count zone = 3. Point (3,4) has a tree count = 0 and is discarded from the sample. The average CSI estimate for the stand can be computed as:

$$S = \frac{100 \sum_{i=1}^{\ell} t_k^2}{\sum_{k=1}^{\ell} t_k} = 100 \left(\frac{23}{13}\right) = 177$$

The whole stand estimate for the stand is 177 units. The actual CSI for the hypothetical stand is 198 units.

The number of sample points selected in the stand is dependent upon the objectives of the sampler. A rule of thumb suggested by Beers (1964) is 40 points per 40 acres or one point per acre. The simulated samples drawn from field plots used 50 points per acre or 10 points per 1/5 acre plot.
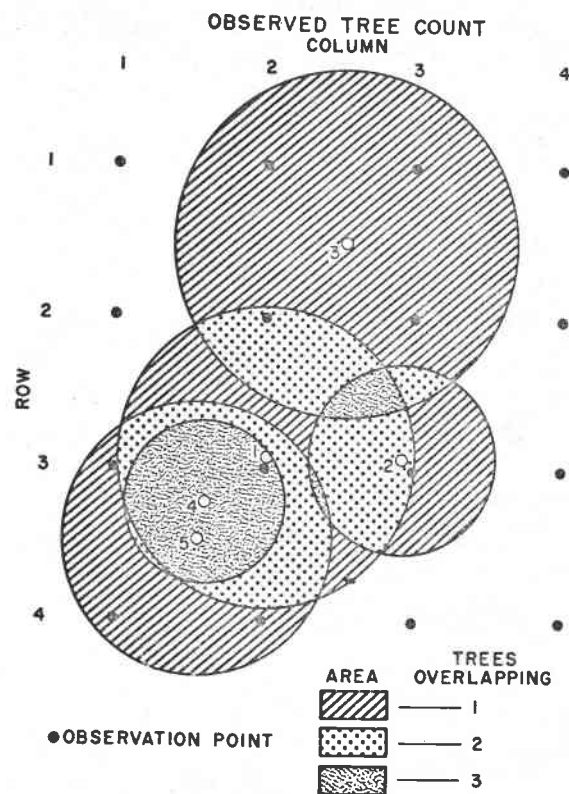
Figure (4-11): Overlap zones for the stand

SELECTING THE BASAL AREA FACTOR

The primary objective of selecting the appropriate BAF is to approximate the growing space circle with the variable plot circle for a given size tree. If each tree's growing space area is blown up proportionately, then tree size and spatial arrangement relationships in the stand can be used to estimate the average CSI.

Suppose the stand to be sampled consists of N trees of some uniform diameter k. Under this assumption, all the trees in the stand have the same size "growing space," since growing space is a function of diameter in the CSI model. The criteria used to select the basal area factor (BAF) to be used in the sampling process is as follows. Select a BAF such that the variable plot area equals the growing space area for a tree of diameter k. An equivalent expression is to select a BAF such that:

$$\text{Plot radius} = \text{Competition radius} \qquad (4\text{-}33)$$
$$R \qquad\qquad\qquad CR$$

for a tree of diameter k. From variable plot theory, BAF can be expressed as a function of tree diameter and plot radius (Beers, 1964), where

$$BAF = \frac{75.625\ (DBH)^2}{R^2} \qquad\qquad PRF = \frac{8.696264}{\sqrt{BAF}} \qquad (4\text{-}34)$$

Let CR = R and k = DBH. Substituting into the above equation gives

$$BAF_k = \frac{75.625\ (k)^2}{CR^2} \qquad\qquad PRF_k = \frac{8.696264}{\sqrt{BAF_k}} \qquad (4\text{-}35)$$

Where:

BAF_k = the basal area factor which has a plot radius R equal to the competition radius CR for a tree of diameter k

PRF_k = the plot radius factor corresponding to BAF_k

Thus for any tree diameter k, the basal area factor which has a variable plot area equal to the growing space area can be determined.

Figure (4-12) compares the variable plot area for three basal area factors with the actual growing space area for a five inch tree. A 20 BAF has a plot radius which overestimates the actual competition radius of the tree. A 50 BAF has a plot radius which is smaller than the competition radius for the tree. A 36 BAF has a plot radius equal to the competition radius for a five inch DBH tree.

Figure (4-13) shows the relationship between competition radius and various BAFs over a range of diameters. The appropriate BAF corresponding to a given tree diameter can readily be interpolated. As tree diameter increases, the required BAF also increased. For trees less than 5 inches DBH, most of the BAFs underestimate the competition radius of the tree. This underestimation of competition radius for smaller diameter trees is a source of bias in sampling for CSI. Similarly, the use of small BAFs in stands with a large DBH will radically overestimate the competition radius and result in an inflated CSI estimate. Table (4-4) gives the appropriate BAF corresponding to a given diameter tree. For operational use, these BAFs should be rounded to the nearest 5 units.
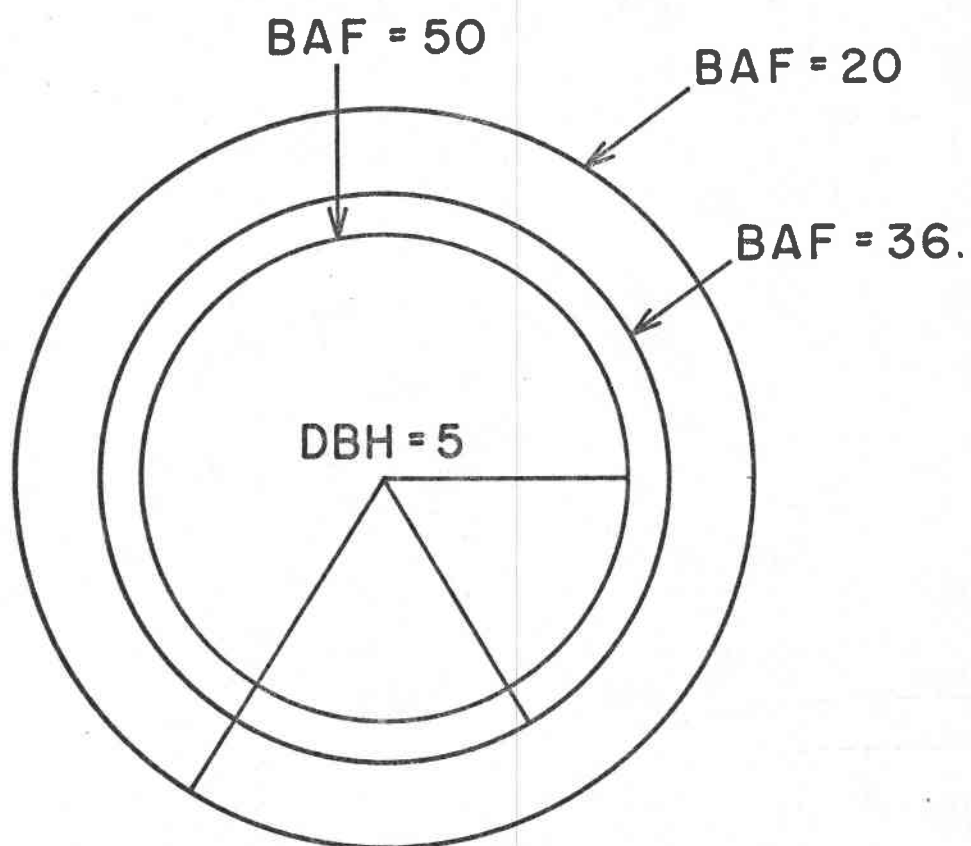
Figure (4-12): Comparison of plot radii for various BAFs
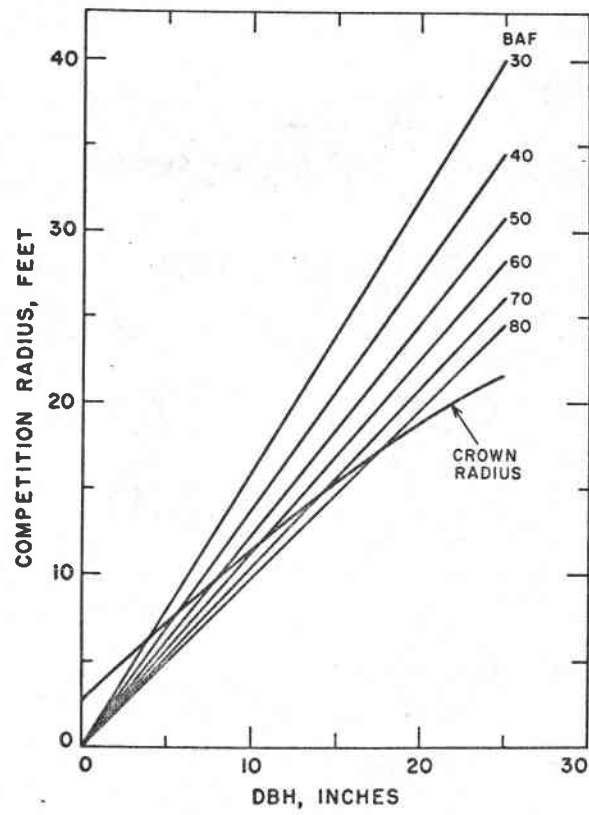
Figure (4-13):   Relationship between competition
radius and BAF for various DBH
trees

Table (4-4): Appropriate basal area
              factors for given diameters

DBH	BAF	PRF
1	6.73	3.3526
2	16.11	2.1666
3	24.28	1.7650
4	31.09	1.5596
5	36.85	1.4326
6	41.81	1.3449
7	46.19	1.2795
8	50.13	1.2282
9	53.75	1.1862
10	57.11	1.1508
11	60.28	1.1201
12	63.31	1.0929
13	66.23	1.0685
14	69.08	1.0463
15	71.87	1.0258
16	74.62	1.0067
17	77.36	.9887
18	80.09	.9718
19	82.82	.9556
20	85.57	.9401
21	88.35	.9252
22	91.17	.8968
23	94.03	.8968
24	96.94	.8833
25	99.90	.8700

It has been assumed that the stand consists of N trees of uniform diameter k. In general, $DBH_i \neq DBH \neq k$. The implication of this more general case considerably alters the selection of an appropriate BAF based on tree size. A different BAF must be selected for each size class in the stand in order to preserve the relationship between plot radius and competition radius for a given size tree. This requirement is restrictive in an operational setting for the following reasons:

(1) It involves a potentially large number of BAFs (one for each class)

(2) The availability of appropriate BAFs not commonly used is questionable

(3) It introduces additional personal bias in matching BAF to tree size

Three alternatives are proposed for selecting a BAF to be used in sampling for CSI. The first alternative is to partition the N trees in the stand into DBH classes and use the BAF corresponding to the midpoint of each DBH class in sampling for CSI. Within a DBH class the selected BAF will approximate the plot radius-competition radius relationship for the trees in the stand. Using BAFs corresponding to various DBH classes reduces the number of different BAFs required. However, the DBH of every potential competitor must be estimated prior to sampling with an appropriate BAF. If the DBH classes are large enough, the variation associated with estimating individual tree diameter classes is reduced. The personal error associated with selecting potential competitors based on tree size is difficult to assess. The decision to examine an individual tree prior to selection of the BAF is an example of a potential personal error. Borderline trees may not be included in the sample if their diameters are incorrectly estimated or if the wrong BAF is used. Examination of a large number of trees to determine their status is tedious and time consuming, lending itself to additional personal error.

The second alternative in selecting an appropriate BAF is to use the BAF which corresponds to the average DBH of the stand to be sampled.

This alternatiave requires the use of only one BAF in the sampling process. The need to estimate tree diameters is also eliminated. The tree counts at each sample point are the only data needed to derive the average CSI estimate for the stand. When using average stand DBH to select the appropriate BAF, the relationship between plot and radius and competition radius associated with each tree size is no longer preserved. For trees with diameters smaller than the average diameter of the stand, the BAF will underestimate their competition radius and fewer smaller trees will be included in the sample. Trees larger than the average stand DBH will have a plot radius which overestimates their actual competition radius and more larger trees will be included in the sample than if a larger BAF had been used. A compensation in tree count between larger and smaller size classes will occur. The distribution of trees by DBH classes in even-aged stands of young growth Douglas-fir is generally normally distributed (Assmann, 1970). Consequently, the compensation in tree counts will probably not significantly alter the CSI estimate for the stand. The inclusion of more trees of larger DBH will be offset by the exclusion of smaller trees. One major problem arises when using average stand DBH to select the BAF. Estimates of average stand DBH are required prior to sampling to determine the appropriate BAF. Sampling bias will occur if the estimated stand DBH used to select the BAF is significantly different than the actual stand DBH.

The third alternative proposed in selecting an appropriate BAF is the most feasible on an operational basis. A BAF is selected which is most convenient to the sampler. In traditional variable plot cruising,

a BAF which gives an average of 4-7 trees per sample point is recommended to minimize sampling error and personal error (Dilworth, 1974). The selected BAF is used to obtain an uncorrected estimate of average CSI in the stand. This uncorrected estimate is then adjusted using a correction factor based on stand DBH. This alternative in selecting the BAF offers the greatest flexibility in sampling for CSI. No prior estimates of stand characteristics are required. Only one BAF is used in the sampling process, and this BAF is commonly available. In addition, there exists the possibility of combining sampling for CSI with sampling for volume of basal area per acre.

In the three proposed alternatives, after selection of the BAF has been made, any measuring device can be used in the sampling process. Wedge prisms or relascopes or other devices are available with a wide variety of BAFs. Wedge prisms ground to the selected BAF might be difficult to obtain, especially if the selected BAF is not commonly used. The relascopes offer a variety of BAF on both the American and international scale (see Appendix D).

DERIVATION OF THE STAND CORRECTION FACTOR

It can be shown that the estimate of CSI obtained when using the BAF corresponding to the average diameter of the stand correctly predicts the actual stand CSI level. A stand correction factor has been derived to adjust the CSI estimate obtained when using any BAF as a function of stand DBH. The correction factor uses the ratio of the BAF used in sampling to the BAF corresponding to the average DBH of

the stand to adjust the uncorrected estimate, S_u. The adjusted estimate S_A is calculated as:

$$S_A = (C_k)(S_{u_k}) \qquad (4\text{-}36)$$

Where:

S_A = the adjusted estimate of CSI for the stand

C_k = the correction factor corresponding to the estimated average stand diameter k

S_{u_k} = the unadjusted estimate of CSI for the stand when using a BAF corresponding to an estimated average stand BDH = k

k = the estimated stand diameter

a = the actual stand diameter

Note that if $C_k \geq 1$ then the adjusted estimate is greater than the unadjusted estimate. If $C_k \leq 1$ then the adjusted estimate is less than the unadjusted estimate. The stand correction factor standardizes the unadjusted estimate for any BAF used in sampling for CSI.

The basis for deriving the stand correction factor from variable plot theory is as follows:

Let:

P_{ik} = the probability of observing a tree of diameter i when using a BAF corresponding to some estimated stand diameter k to determine tree overlap status

P_{ia} = the probability of observing a tree of diameter i when using a BAF corresponding to the actual stand diameter to determine tree overlap status

Then:

$$\Delta k = \frac{P_{ik}}{P_{ia}} \tag{4-37}$$

Where:

> Δ_{ik} = the percent of the correct probability of observing a tree of diameter i when a BAF corresponding to an estimated stand diameter k is used to determine tree count, instead of the BAF represented by the actual stand diameter.

From variable plot theory, the probability of observing a tree of diameter i can be expressed as (Beers, 1964):

$$P_{ij} = \frac{1}{STF_{ij}} = \frac{1}{BAF_j/BA_i} = \frac{BA_i}{BAF_j} \tag{4-38}$$

Where:

> P_{ij} = the probability of observing a tree of diameter i when using the jth BAF

> STF_{ij} = the stand table factor or per acre conversion factor for a tree of diameter i using the jth BAF

> BAF_j = the jth basal area factor used in sampling

> BA_i = the basal area in square feet for a tree of diameter i at breast height

Equation (4-38) states that the probability of observing a tree of diameter i, when using the jth basal area factor, is the ratio of the basal area factor to the basal area of the ith diameter class. Substituting this result into Equation (4-37) using the previous notation gives:

$$\Delta_{ik} = \frac{P_{ik}}{P_{ia}} = \frac{BA_i/BAF_k}{BA_i/BAF_a} = \frac{BAF_a}{BAF_k} \qquad (4\text{-}39)$$

The expression given in the above equation is independent of tree size.
Let C_k equal:

$$C_k = \frac{1}{\Delta_{ik}} = \frac{BAF_k}{BAF_a} \qquad (4\text{-}40)$$

Where:

C_k = the stand correction factor for BAF_k

Equation (4-40) states that the stand correction factor for the
BAF_k is the ratio of the BAF used in sampling to the BAF corresponding
to the average stand diameter. C_k is also independent of tree size.
The following relationships between the estimated stand DBH = k and the
actual stand DBH are evident. If the estimated stand diameter k is
greater than the actual stand diameter implies:

$$BAF_k > BAF_a \quad \text{and} \quad C_k > 1.0 \qquad (4\text{-}41)$$

Conversely, if $k \le a$, then:

$$BAF_k \le BAF_a \quad \text{and} \quad C_k \le 1.0 \qquad (4\text{-}42)$$

Equation (4-41) states that if the estimated stand diameter k used
to select BAF_k is greater than the actual stand DBH, then C_k is greater
than 1.0. If the estimated stand diameter used to select BAF_k is less
than or equal to the stand DBH, then the stand correction factor is
less than or equal to one (Equation (4-42).

In general, the stand correction factor can be applied to any estimate obtained when using the jth BAF.

$$C_j = \frac{BAF_j}{BAF_a} \qquad (4\text{-}43)$$

Where:

C_j = the stand correction factor corresponding to the jth basal area factor

BAF_j = any desired basal area factor used in sampling for CSI

BAF_a = the BAF corresponding to the actual average stand DBH = a

Equation (4-43) states that the stand correction factor can be applied to any basal area factor BAF_j used in sampling for CSI in the stand. The BAF_j does not necessarily have to be selected with respect to average stand diameter as discussed earlier. BAF_j can be any BAF desired. The BAF most commonly used in volume sampling is probably the best choice.

The stand correction factor for the generalized case in Equation (4-43) is the ratio of the BAF corresponding to the average stand DBH and the basal area factor used to estimate average CSI in the stand. The basal area factor corresponding to an average stand DBH = a is given as:

$$BAF_a = 75.625 \, (a)^2 CR_a \qquad (4\text{-}44)$$

$$= \frac{75.625 \, (a)^2}{2.35355 + 1.60930 \, (a) - .00930 \, a^2}$$

Where:

a = average stand diameter

CR_a = the competition radius for a tree of DBH = a

Substituting various values of a into Equation (4-44) gives the basal area factor corresponding to average diameter of the stand. If the average stand DBH is not known prior to sampling, the following modified sampling procedure can be used. Select any desired BAF for sampling for CSI in the stand. At each sample point, observe tree count as usual. Tally "in" trees by appropriate DBH classes (e.g., 1", 2", 4" classes) using the selected BAF. Construct a stand table from the average tree count for each diameter class as:

$$f_i = STF_i \ \overline{TC_i} = (\frac{BAF}{BA_i}) \ (\overline{TC_i}) \qquad (4-45)$$

Where:

f_i = the frequency of trees per acre in the ith DBH class

STF_i = the stand table factor for the ith DBH class

$\overline{TC_i}$ = the average tree count per point for the ith DBH class

BAF = the selected BAF used in sampling for CSI

BA_i = the basal area in square feet for the ith DBH class

i = the ith diameter class (i = 1,k)

Using the frequency of trees by diameter class per acre given in Equation (4-45), estimate the average tree stand diameter as:

$$ADBH = \frac{\sum\limits_{i=1}^{k} f_i \cdot DBH_i}{\sum\limits_{i=1}^{k} f_i} \qquad (4\text{-}46)$$

Where:

ADBH = the weighted average DBH of the stand

DBH_i = the midpoint of the ith DBH class

Substituting the average stand diameter estimated in Equation (4-46) into Equation (4-44) gives the appropriate BAF corresponding to the average diameter of the stand. The appropriate correction factor is the ratio of the BAF used in sampling to the basal area factor calculated in Equation (4-44). The adjusted estimate of CSI is equal to the product of the correction factor and the unadjusted estimate where:

$$S_A = C_j S_{u_j} \qquad (4\text{-}47)$$

MORTALITY ESTIMATION

INTRODUCTION

Two types of mortality typically occur in even-aged stands: regular mortality and irregular mortality (Staebler, 1955). Regular mortality is characterized by the overtopping and suppression of trees in the smallest diameter classes (Lin, 1973). Irregular mortality in an even-aged stand is generally associated with some catastrophic or random event such as windthrow, fire, insect or disease epidemic, or snowbreak. Consequently, irregular mortality is difficult to predict (Lin, 1973). In this chapter, the regular mortality which has occurred in an even-aged stand of young growth Douglas-fir over a ten year period will be examined.

When two trees of different sizes compete, they do not affect each other equally (Keister, 1972). The larger tree will exploit the resources of the site to a greater degree than will the smaller tree (Bella, 1970). Environmental stresses induced by severe competition will result in a general reduction of growth of the smaller tree (Bormann, 1960). Eventually, the reduction in photosynthetic rate below a threshold level will cause the tree to experience a net loss of photosynthate and growth will cease. At this level, the photosynthetic processes of the tree are incapable of supporting growth and unable to meet the demands of respiration (Mitchel, 1972). If these conditions exist for an extended period, the tree will die. Regular mortality is a dynamic process which occurs continually in the natural development of the stand. It is a steady, progressive selection of

those individual trees which are most vigorous or best adapted to the environmental stresses (Smith, D.M., 1962).

Regular mortality in an even-aged stand generally occurs in the intermediate or suppressed crown classes (Smith, D.M., 1962). Assman (1970) states that:

> The typical social differences which are particularly prominent in regular, closed stands without any human interference are mainly the results of a merciless struggle for light and growing space.

These "social differences" have given rise to a ranking of trees into natural classes, according to their appropriate standing in the community. Dominant, co-dominant, intermediate, and suppressed individuals in the stand can be identified. Individuals are placed into an appropriate "social class" using such criteria as crown development, vigor, or tree size. All natural classification schemes are based on the premise that the observable characteristics used to establish a tree's "social ranking" indicate its future status as well (Assman, 1970). Future regular mortality in a stand, using natural classification schemes, would be expected to occur primarily in the present intermediate and suppressed classes which contain the weakest, least vigorous individuals. Such natural classification schemes for predicting future mortality are qualitative in nature and subject to personal judgment. It is desirable to develop a more objective means of predicting future regular mortality in the stand. The following sections present a detailed examination of the quantitative prediction of future mortality in a natural stand. A derivation of a mortality index based on Arney's competitive stress index is presented. Comparisons of

those individual trees which are most vigorous or best adapted to the environmental stresses (Smith, D.M., 1962).

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on Arney's competitive stress index is presented. Comparisons of three variables for predicting future regular mortality are also made.

DERIVATION OF A MORTALITY INDEX BASED ON CSI

Keister (1972) developed a method for mathematically identifying suppression mortality in southern pine plantations. The risk of future suppression mortality was predicted using a mortality index based on Keister's inter-tree competition index (Keister, 1971). The mortality index value for a tree was defined as the ratio of its competitive stress level and number of competitors. Using the index, Keister determined high and low risk classes for predicting which trees have a high probability of becoming future suppression mortality. Keister's mortality index is a dynamic variable in predicting regular mortality in simulated southern pine plantations. The objective of this chapter is to extend Keister's concept to the Competitive Stress Index model for Douglas-fir developed by Arney (1973).

Keister's mortality index for a single tree is defined as:

$$MI_k = \frac{I}{xn_k} \tag{5-1}$$

Where:

MI_k = the mortality index for the tree

I = Keister's inter-tree competition index

xn_k = the number of competitors for the tree using Keister's index

A mortality index based on Arney's CSI model can similarly be derived for a tree as:

$$MI_A = \frac{CSI}{xn_A} \left(\frac{1}{100}\right) \qquad (5\text{-}2)$$

Where:

MI_A = the mortality index based on Arney's competitive stress index for the tree

CSI = the competitive stress index value for the tree

xn_A = the number of competitors based on Arney's CSI model for the tree

Equation (5-2) states that the mortality index for a tree is equal to the ratio of its CSI level and the number of competing neighbors. The mortality index value indicates the average proportion of the relative competition contributed by a competitor. High mortality index values are associated with trees with a high risk of future mortality, while low index values are associated with trees of higher vigor. Notice the expression given in Equation (5-2) is independent of tree size.

Assuming that the definition of tree growing space is the same for both Keister's index I and Arney's CSI model, the indices are linearly related as follows:

$$I = \frac{CSI}{100} - 1 \qquad (5\text{-}3)$$

Keister's mortality index given in Equation (5-1) is therefore related to the mortality index based on Arney's CSI model given in Equation (5-2) as:

$$MI_k = \frac{I}{xn_k} = \frac{\left(\frac{CSI}{100} - 1.\right)}{xn_k} = \frac{CSI}{(100)\,(xn_k)} - \frac{1}{xn_k} \qquad (5\text{-}4)$$

Substituting the expression for MI_A in Equation (5-2), and assuming $xn_k = xn_A$ gives:

$$MI_k = MI_A - \frac{1}{xn_A} \qquad\qquad (5-5)$$

Keister's mortality index MI_k in Equation (5-5) is expressed as a function of the mortality index MI_A and the number of competitors xn_A, based on Arney's CSI model. Keister (1972) found that an $MI_k = .5$ was the critical value in defining the "high risk" mortality class in southern pine. Trees with MI_k values greater than .5 exhibited a high probability of suppression mortality over a 5 year period.

Figure (5-1) shows the critical value for MI_A, corresponding to Keister's $MI_k = .5$ as a function of the number of competitors. A tree falls into a "high" risk category if the tree's MI_A value and number of competitors xn_A are greater than the critical value given in Figure (5-1). If a tree's mortality index and number of competitors plot below the critical MI_A value, the tree is in a "low" risk mortality class.

Two important points need to be clarified. First, the definition of risk classes given in Figure (5-1) is based on the critical mortality index value developed for southern pine. The critical mortality risk value for Douglas-fir is undoubtedly different. The critical mortality risk value is assumed to be species dependent and should be expected to vary with species tolerance and geographical region. Second, the defi-nition of growing space or influence zone has been assumed to be the same for both Keister's index and Arney's CSI model. In general, this
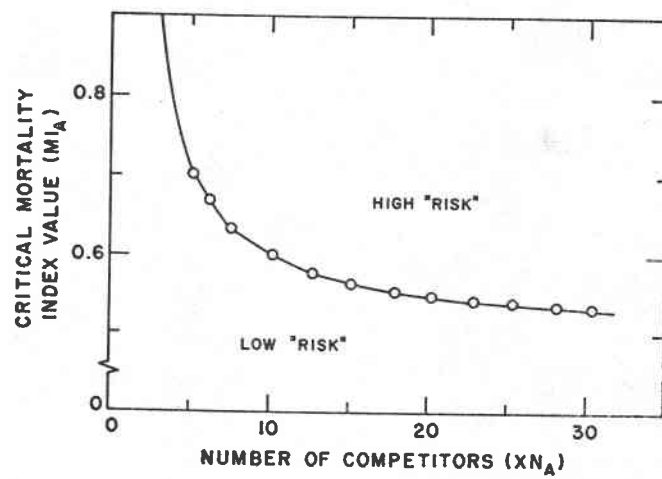
Figure (5-1): The relationship between
Keister's critical value for
MI_k and MI_A

assumption is not valid. Keister (1971) defines the growing space radius of a tree as a function of tree height, actual crown width, and live crown length using the measured dimensions of the tree. Arney (1973) defines the competition radius of a tree as a function of tree size, using the crown radius of open grown trees as a measure of maximum crown extension. These two approaches in defining the growing space of a tree are fundamentally different. Arney's definition of growing space for Douglas-fir is larger than Keister's growing space for southern pine for the same sized tree. As a result, the competition indices and corresponding mortality indices are not comparable. In general, the larger the growing space the greater the number of competitors and the smaller the mortality index for a given level of competition. Clearly, the critical value for Keister's mortality index is not necessarily the critical value for an even-aged Douglas-fir stand.

The above discussion suggests that there is a need to define the critical mortality index value for MI_A. A preliminary study was undertaken to determine the critical value of the mortality index for dichotomizing a stand into high and low mortality risk classes. A sample of 133 trees was selected from the interior of three one-fifth acre, unthinned control plots established in a 20 year old Douglas-fir stand. Using the 1963 stem data, a mortality index was computed for each tree based on the ratio of its 1963 CSI level and the number of its competitors (see Equation 5-2). In addition, the status of each tree at the end of the 1973 growing season was tablulated as dead or alive. All

of the 133 sample trees were alive in 1963. All mortality occurring in the stand during the 1963-1973 study period was assumed to be regular suppression mortality.

Table (5-1) gives the retrospective cross-classification of the 133 sample trees by tree status in 1973 and by mortality index class in 1963. Of the 133 sample trees, 52 died during the 1963-1973 period (39.1%), while 81 of the trees (60.9%) survived. The mortality index classification indicates that only 13 of the 52 trees (25.0%) that died had a 1963 mortality index value of $MI_A \leq .35$, while 68 of the 81 survivors in 1973 (84.0%) had a 1963 mortality index of $MI_A \leq .35$. The data in Table (5-1) were used to test the independence of tree status in 1973 and mortality index class in 1963, using the standard chi-square test statistic (Snedecor, 1967). The null hypothesis is that future tree status is independent of mortality risk class (i.e., there is no association between a tree's status in 1973 and its mortality risk index in 1963). The chi-square test statistic is:

$$V = \sum_{i=1}^{2} \sum_{j=1}^{7} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \qquad (5-6)$$

Where:

V = the chi-square test statistic with 6 d.f.

O_{ij} = the observed frequency in the ith row and jth column of Table (5-1)

E_{ij} = the expected frequency in the ith row and jth column of Table (5-1)

Table (5-1):   Retrospective classification of sample trees by 1963
               mortality index and 1973 tree status

1963 Mortality Index Class

1973 Status[2]	1	2	3	4	5	6	7	Total
Mortality	0	0	2	11	12	11	16	52
	.000	.000	.015	.083	.090	.083	.120	.391
Survivor	8	24	22	14	5	6	2	81
	.060	.180	.165	.105	.038	.045	.015	.609
Total	8	24	24	25	17	17	18	133
	.060	.180	.180	.188	.129	.128	.135	
Class boundary[1]	.20	.25	.30	.35	.40	.45	.50	

[1] The upper boundary for each class is indicated (i.e., $MI_i \leq UB < MI_{i+1}$).

[2] The sample proportions are indicated below the observed frequency in each class.

Table (5-2):   Expected frequencies under the null hypothesis of
               independence[1]

Expected Frequencies Under H_0

1973 Status	1	2	3	4	5	6	7	Total
Mortality	3.128	9.383	5.810	9.774	6.647	6.647	7.038	52
Survivor	4.872	14.617	14.617	15.226	10.353	10.353	10.962	81
Total	8.000	24.000	24.000	25.000	17.000	17.000	18.000	133
Class boundary	.20	.25	.30	.35	.40	.45	.50	

$V = 60.838^2$ $P < .001$

[1] The expected frequency in cell (i,j) is:

$$E_{ij} = \frac{(\text{Total i Row}) \ (\text{Total j Column})}{\text{Grand Total}}$$

[2] Chi-square test statistic value (see Equation 5-6).

Table (5-2) gives the expected frequencies under the null hypo-
thesis of independence. Note the high expected frequencies in the
smaller 1963 mortality index classes for the mortality trees compared
with the observed frequencies given in Table (5-1). The null hypothesis
was rejected with a P-value < .001, indicating a highly significant
relationship between future tree status and 1963 mortality index
classification. The future status of a tree at the end of a 10 year
period is dependent upon its mortality index value at the beginning of
the period. Trees that died during the 1963-1973 period tended to have
higher 1963 mortality index values than survivor trees. This result
supports Keister's conclusion: a tree's mortality index at the be-
ginning of a period is useful in identifying individuals with a high
risk of becoming future suppression mortality.

The primary objective of this study is to determine the critical
mortality index value most sensitive in predicting the future status of
individual trees. This critical value can be used to define "high" and
"low" mortality risk classes based on the mortality index values of the
trees in a stand. Future tree status at the end of a period can be pre-
dicted at the beginning of the period with a specified level of confi-
dence.

Six mortality index values were tested as possible critical levels.
The 133 sample trees discussed earlier were cross-classified into one
of six 2x2 contingency tables using future tree status in 1973 (dead
or alive) and 1963 mortality risk class (low, high). Figure (5-2)
gives the generalized 2x2 contingency tables for observed frequency

and sample proportions and their respective marginal totals for a sample of n individuals.

In Figure (5-2), the 1973 tree status is dichotomized as dead (B) or alive ($\overline{B}$). Similarly, the 1963 mortality risk class is tabulated as low ($\overline{A}$) or high (A). The risk classification indicates the likelihood that suppression mortality will occur during the 1963-1973 period. The mortality risk classes are defined using a critical value for the mortality index denoted by CMI_A. The cell frequencies (n_{ij}) represent the number of individuals which belong to the ith status class (B or $\overline{B}$) and the jth risk class ($\overline{A}$ or A). The corresponding sample proportions (p_{ij}) indicate the proportion of the total sample n which have the characteristics (A or $\overline{A}$) and (B or $\overline{B}$). The marginal totals for each row and column represent the total number or total proportion of individuals in a given row or column. For example, $p_1.$ denotes the proportion of the 133 sample trees which died during the 1963-1973 period. $p_2.$ represents the proportion of the 133 sample trees which survived the 1963-1973 period. The proportion of the 133 trees which are in the low risk of mortality class in 1963 are denoted by $p._1$, while $p._2$ indicates the proportion of the total sample which have a high risk of mortality based on their 1963 mortality index value. Notice that in a 2x2 table the following relationship exists:

$$p_1. = 1 - p_2. \tag{5-7}$$

$$p._1 = 1 - p._2$$

1973 Status	1973 Risk Class			1973 Status	1963 Risk Class		
	Low $\overline{A}$	High A	Total		Low $\overline{A}$	High A	Total
Dead B	n_{11}	n_{12}	$n_1.$	Dead B	p_{11}	p_{12}	$p_1.$
Alive $\overline{B}$	n_{21}	n_{22}	$n_2.$	Alive $\overline{B}$	p_{21}	p_{22}	p_{21}
Total	$n._1$	$n._2$	$n..$	Total	$p._1$	$p._2$	1.0

Figure (5-2): The generalized 2x2 contingency tables for observed frequency and sample proportions

Where:

n_{ij} = the observed frequency of the cell in the ith row and jth column of the 2x2 table (i = 1,2; j = 1,3)

$n_i.$ = the marginal total of the ith row i = 1,2

$n._j$ = the marginal total of the jth column j = 1,2

$n..$ = the total sample size

p_{ij} = the sample proportion in the ith row and jth column of the table ($p_{ij} = n_{ij}/n..$)

$p_i.$ = the marginal proportion in the ith row ($p_i. = n_i./n..$)

$p._j$ = the marginal proportion in the jth column ($p._j = n._j/n..$)

A different critical CMI_A value was assigned to each of the six 2x2 tables and used to define the high and low risk classes. If a sample tree's MI_A value was greater than the assigned critical value, the tree was classified into the "high" risk category. The critical values tested in the study were mortality index values equal to $CMI_A = .25$, $CMI_A = .30$, $CMI_A = .35$, $CMI_A = .40$, $CMI_A = .45$, $CMI_A = .50$.

Table (5-3) gives the six 2x2 contingency tables developed. Each table specifies the critical mortality index value used to define the high and low risk classes. The 133 sample trees were classified into one of four cells depending on their MI_A index in 1963 (low, high) and their status at the end of the 1973 growing season (mortality, survivor) for each 2x2 table.

The independence of future status and risk class was tested for each 2x2 contingency table. The null hypothesis for the test was that future tree status and initial mortality risk classification are independent (i.e., the probability that a tree dies over the 10 year period being independent of the risk class it belonged to at the beginning of the period). The significance of the association between future status and mortality risk class can be assessed using the following standard chi-square test statistics (Fleiss, 1973) (see Figure 5-2):

$$V = \frac{n_{..}(|n_{11}n_{22} - n_{12}n_{21}| - \frac{1}{2}n_{..})^2}{(n_{.1})(n_{.2})(n_{1.})(n_{2.})}$$

Where:

V = the chi-square test statistic with 1 d.f.

Table (5-3):   2x2 contingency tables for each critical value

Critical MI_A = .25

1973 Status	1963 Risk		
	Low $\overline{A}$	High A	Total
Dead B	0	52	52
Alive $\overline{B}$	32	49	81
Total	32	101	133

Critical MI_A = .40

1973 Status	1963 Risk		
	Low $\overline{A}$	High A	Total
Dead B	25	27	52
Alive $\overline{B}$	73	8	81
Total	98	35	133

Critical MI_A = .30

1973 Status	1963 Risk		
	Low $\overline{A}$	High A	Total
Dead B	2	50	52
Alive $\overline{B}$	54	27	81
Total	56	77	133

Critical MI_A = .45

1973 Status	1963 Risk		
	Low $\overline{A}$	High A	Total
Dead B	36	16	52
Alive $\overline{B}$	79	2	81
Total	115	18	133

Critical MI_A = .35

1973 Status	1963 Risk		
	Low $\overline{A}$	High A	Total
Dead B	13	39	52
Alive $\overline{B}$	68	13	81
Total	81	52	133

Critical MI_A = .50

1973 Status	1963 Risk		
	Low $\overline{A}$	High A	Total
Dead B	48	4	52
Alive $\overline{B}$	80	1	81
Total	128	5	133

The chi-square test of independence is an excellent measure of the significance of the association between future status and mortality risk, but is not a useful measure of the degree of association between the two characteristics (Fliess, 1973). To estimate the degree of association between future tree status and mortality risk class the following notation is presented. There are a variety of measures of association for 2x2 tables which estimate the degree of association between two characteristics A and B (Fliess, 1973; Snedecor, 1967). One such measure is the odds ratio. Fliess (1973) suggests that, in general, when one characteristic is antecedent to another, the "risk" of experiencing outcome B when antecedent factor A is present is given by the odds (see Figure 5-2):

$$O_A = \frac{P(B\backslash A)}{P(\overline{B}\backslash A)} = \frac{P_{12}/P_{\cdot 2}}{P_{22}/P_{\cdot 2}} = \frac{P_{12}}{P_{22}} = \frac{n_{12}}{n_{22}} \tag{5-8}$$

Where:

O_A = the odds outcome B will occur when A is present

$P(B\backslash A)$ = the conditional probability that outcome B will occur given that A is present

$P(\overline{B}\backslash A)$ = the conditional probability that outcome $\overline{B}$ (not B) will occur given that A is present

Similarly, when A is absent, the odds of factor B occurring are:

$$O_{\overline{A}} = \frac{P(B\backslash \overline{A})}{P(\overline{B}\backslash \overline{A})} = \frac{P_{11}/P_{\cdot 1}}{P_{21}/P_{\cdot 1}} = \frac{P_{11}}{P_{21}} = \frac{n_{11}}{n_{21}} \tag{5-9}$$

Where:

$O_{\overline{A}}$ = the odds of outcome B occuring when factor $\overline{A}$ (not A) is present

$P(B\backslash\overline{A})$ = the conditional probability of outcome B occurring given $\overline{A}$ has occurred

$P(\overline{B}\backslash\overline{A})$ = the conditional probability of outcome $\overline{B}$ (not B) occurring given $\overline{A}$ has occurred

The odds O_A and $O_{\overline{A}}$ given in Equations (5-8) and (5-9) can be combined into a single expression of relative odds. The odds ratio O, is a measure of association between factors A and B.

$$O = \frac{O_A}{O_{\overline{A}}} = \frac{n_{12}/n_{22}}{n_{11}/n_{21}} = \frac{n_{12}n_{21}}{n_{22}n_{11}} \tag{5-10}$$

If the conditional probabilities $P(B\backslash A)$ and $P(B\backslash\overline{A})$ are equal then the two odds O_A and $O_{\overline{A}}$ are also equal and the odds ratio O equals 1.0. This result indicates the independence or lack of association between A and B. Note that in Equation (5-10), if either n_{11} or n_{22} is equal to zero, O is not defined. An improvement using a modified estimate is (Fliess, 1973):

$$O_c = \frac{(n_{12} + .5)(n_{21} + .5)}{(n_{22} + .5)(n_{11} + .5)} \tag{5-11}$$

Where:

O_c = the odds ratio corrected for n_{11} or n_{22} equal to zero.

The standard error of O_c is given as (Fliess, 1973):

$$S.E.(O_c) = O_c \sqrt{\frac{1}{n_{11}+.5} + \frac{1}{n_{22}+.5} + \frac{1}{n_{12}+.5} + \frac{1}{n_{21}+.5}} \qquad (5\text{-}12)$$

A tree's mortality index value in 1963 is antecedent to its future status in 1973. Large MI_A values in 1963 identify individuals with a high probability of future suppression mortality. Using the generalized notation presented above, let:

A = the event that a tree's 1963 mortality index falls into a

high risk class (i.e., $MI_A \geq CMI_A$)

$\overline{A}$ = the event that a tree's 1963 mortality index falls into a low

risk class (i.e., $MI_A < CMI_A$)

B = the event that a tree dies during the 1963-1973 period (i.e.,

1963 status = dead)

$\overline{B}$ = the event that a tree survives the 1963-1973 period (i.e.,

1973 status = alive)

The odds of future mortality (B) occurring given that a tree has a high risk mortality index value $(\overline{A})$ from Equations (5-11) are:

$$O_{A_c} = \frac{P(B\backslash A)}{P(\overline{B}\backslash A)} = \frac{(n_{12}+.5)}{(n_{22}+.5)} \qquad (5\text{-}13)$$

Where:

O_{A_c} = the odds that suppression mortality will occur over the

period 1963-1973 given that the tree is in the high risk

mortality index class in 1963 (Adjusted for continuity,

see Equation 5-8)

$P(B\backslash A)$ = the conditional probability that suppression mortality

will occur given that the tree's 1963 risk class is "high"

$P(\overline{B}\backslash A)$ = the conditional probability that the tree will survive the

1963-1973 period given that its risk class in 1963 is

"high"

Similarly, when the mortality risk class is "low", the odds that suppression mortality will occur are (see Equation 5-9):

$$O_{\overline{A}_c} = \frac{P(B\backslash\overline{A})}{P(B\backslash A)} = \frac{n_{11}+.5}{n_{21}+.5}$$

(5-14)

Where:

$O_{\overline{A}_c}$ = the odds that suppression mortality will occur during the

1963-1973 period, given that the tree's 1963 mortality

risk is low (Adjusted for continuity, see Equation 5-11)

$P(B\backslash\overline{A})$ = the conditional probability that suppression mortality

will occur given that the 1963 risk class is low

$P(\overline{B}\backslash\overline{A})$ = the conditional probability that suppression mortality

will not occur (i.e., the tree survives the 1963-1973

period) given that the mortality risk class is low

The odds ratio O_c gives the relative odds or association between future status in 1963 and initial mortality risk class in 1973.

$$O_c = \frac{O_A}{O_{\overline{A}}} = \frac{(n_{12}+.5)(n_{21}+.5)}{(n_{22}+.5)(n_{11}+.5)}$$

(5-15)

Where:

O_c = the odds of suppression mortality occurring over the 1963-1973 period when the 1963 mortality risk class is high relative to the odds of suppression mortality occurring when the risk is low (Adjusted for continuity, see Equation 5-11)

The odds ratio in Equation (5-15) can be interpreted as: the odds suppression mortality occurs when the risk is high are O_c times those for the low risk class. The odds ratio provides a quantitative measure of the sensitivity of a given critical value in partitioning the stand into mortality risk classes. The ideal critical value would have the following properties. The probability of future suppression mortality (B) would be small when the risk class was low ($\overline{A}$):

$$P(B\backslash\overline{A}) \rightarrow 0 \tag{5-16}$$

The probability of survival ($\overline{B}$) when the initial risk class is low ($\overline{A}$) would be large:

$$P(\overline{B}\backslash\overline{A}) \rightarrow 1.0 \tag{5-17}$$

The resulting odds of future mortality occurring when the initial risk was low would be small since $P(B \overline{A})$ in Equation 5-16 is small:

$$O_{\overline{A}} = \frac{P(B\backslash\overline{A})}{P(\overline{B}\backslash\overline{A})} \rightarrow 0 \tag{5-18}$$

Similarly, the probability that suppression mortality (B) occurs when the initial risk is high (A) ideally is large, since the initial risk classification is antecedent to future mortality:

$$P(B\ A) \rightarrow 1.0 \tag{5-19}$$

The resulting odds of future mortality occurring when the mortality risk class is high would also be large since the probability of survival when the risk is high goes to zero.

$$P(\overline{B}\ A) \rightarrow 0 \quad \text{implies} \quad 0_A = \frac{P(B\ A)}{P(\overline{B}\ A)} \rightarrow \infty \tag{5-20}$$

The ideal odds ratios 0_C would tend to be large since 0_A in Equation (5-20) is large and $0_{\overline{A}}$ in Equation (5-18) is small.

$$0_C = \frac{0_A}{0_{\overline{A}}} \rightarrow \infty \tag{5-21}$$

These ideal properties for a theoretical critical value provide an objective means of empirically evaluating the six critical values tested. The sensitivity of a given critical value can be judged by how well it corresponds to the theoretical properties presented above in Equations 5-16 through 5-21.

Table (5-4) presents the data from the analysis of the six 2x2 contingency tables given in Table (5-3). Columns 1-11 give the chi-square test statistic, the conditional probabilities of mortality and survival for a given risk class, the odds of future mortality occurring for a given risk class, the odds ratio and its standard errors, and the proportion of trees in the high risk class in 1963 for each critical value.

All of the critical values except CMI = .50 had a highly significant chi-square test statistic (Column 1), indicating a significant

Table (5-4): Summary of the 2x2 contingency analysis

High Risk Class	V	$P(B\backslash\overline{A})$	$P(\overline{B}\backslash\overline{A})$	$O_{\overline{A}_c}$	$P(B\backslash A)$	$P(\overline{B}\backslash A)$	O_{A_c}	$O_c = \dfrac{OA_c}{O\overline{A}_c}$	$SE(O_c)$	$SE(O_c)\%$	$P(A)$
$CMI_A = .25$	24.93	.015	.985	.015	.517	.483	1.061	68.90	99.22	143.87	.759
$CMI_A = .30$	48.72	.044	.956	.046	.652	.348	1.836	40.00	27.57	68.88	.574
$CMI_A = .35$	43.77	.166	.834	.197	.752	.248	2.926	14.84	6.438	43.36	.391
$CMI_A = .40$	26.74	.259	.741	.347	.775	.225	3.235	9.32	4.241	45.47	.263
$CMI_A = .45$	19.31	.316	.684	.459	.892	.108	6.600	14.37	10.171	70.75	.135
$CMI_A = .50$	2.080	.377	.623	.602	.818	.182	3.000	4.979	4.781	96.01	.038
Column	1	2	3	4	5	6	7	8	9	10	11

relationship exists between future status and 1963 mortality risk classifications. The degree of association as measured by the odds ratio is given in Column 8.

Comparing the data in Table (5-4) with the ideal properties of a critical value discussed previously reveals the following generalizations. As the critical value is increased, the $P(B\ \overline{A})$ in Column 2 increases, the $P(\overline{B}\ \overline{A})$ decreases, (Column 3) and $O_{\overline{A}}$ increases (Column 4), contrary to the properties of the hypothetical critical value given in Equation (5-20). The odds that future suppression mortality will occur when the 1963 risk is low increases significantly as the critical value increases. The higher the 1963 critical value the less sensitive it is in distinguishing between which trees will live and which trees will die during the 1963-1973 period.

An opposite trend occurs in Columns 5-7. As the critical value increases from $CMI_A = .25$ to $CMI_A = .50$ the probability of suppression mortality occurring when the risk is high (Column 5) increases, while the probability of survival when the risk is high (Column 6) decreases. The odds that suppression mortality will occur (Column 7) increase slightly as the critical value increases. These results compare favorably with the properties outlined for the optimal critical value described earlier (see Equation 5-18). The larger the critical value CMI_A, the greater the differentiation between the probabilities of future mortality and future survival when the 1963 risk is high. For example, when $CMI_A = .45$, the odds of suppression mortality occurring (Column 7) when the risk is high are six times those of surviving. Only

10.8 percent of the survivor trees that ultimately die are in the 1963 high risk class, while 89.2 percent of the trees that ultimately die are in the 1963 high risk class. When the critical value CMI_A = .25 is used to define the risk classes, only 51.7 percent of the future mortality trees fall into the high risk class in 1963, while 48.3 percent of the 1973 survivors have a high risk in 1963. The odds that a tree dies when the risk is high are only 1.06 times the odds of surviving when CMI = .25 is used to define the high and low mortality risk classes.

The odds ratio in Column 8 combines the odds of mortality occurring in the high and low risk classes into a single expression of relative odds for each critical value. As stated previously, the ideal critical value would have a large odds ratio. As the critical value increases, the odds ratio in Column 8 decreases significantly. The odds that suppression mortality will occur during the 1963-1973 period when the 1963 mortality index risk class is high are 68.9 times as large as those when the risk class is low, when CMI_A = .25 is used to define the risk classes. Clearly, the larger the odds ratio in Column 8 the more sensitive the critical value is in predicting the future status of individual trees based on their 1963 mortality index. Figure (5-3) depicts the odds ratio in Column 8 of Table (5-4) for each of the critical values tested. Note that as the critical value increases, the odds ratio falls significantly.

The standard error of the odds ratio and the standard error of the odds ratio in percent are given in Columns 9-10, respectively, in Table (5-4). Figure (5-4) shows the relationship between the standard error
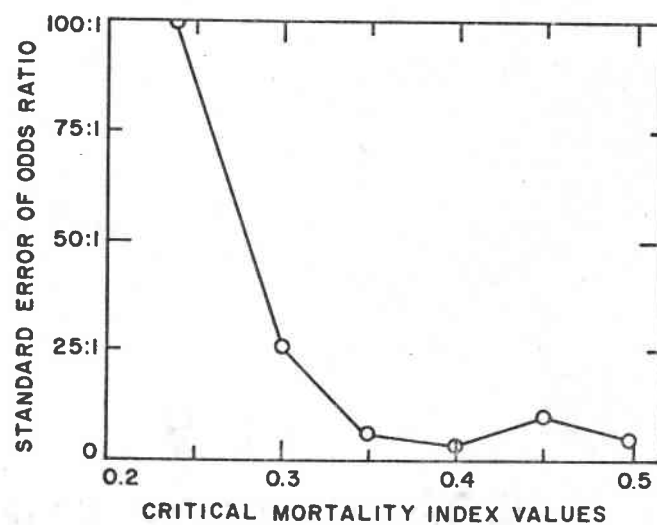
Figure (5-3):   Odds ratio for various
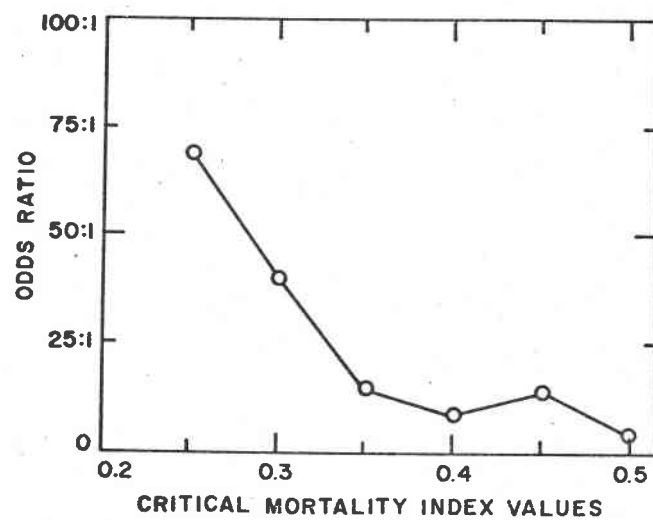critical values

Figure (5-4):   Standard error of the odds
ratio by critical value

of the odds ratio as a function of the critical values used to define the risk classes. The high variability of the odds ratio estimate for small critical values is caused by small observed cell frequencies (see Equation 5-12). Figure (5-5) shows the standard error of the odds ratio in percent. The minimum S.E.% occurs at a critical value CMI_A = .35.

Based on the discussion of the data presented in Table (5-4) and Figures (5-3 through 5-5), the critical value of CMI_A = .35 was selected as the most sensitive critical value for identifying high and low mortality risk individuals. The odds that a tree will die when the risk is high (i.e., $MI_A \geq .35$) are roughly 15 times those when the risk is low (i.e., $MI_A < .35$). Over 75 percent of the trees in the high 1963 mortality risk class died during the following 10 year period, while only 17 percent of the trees that died had a low mortality index in 1963. Using the critical value $CMI_A \geq .35$, individuals with a high probability of becoming future suppression mortality can be identified. Approximately 40 percent of the 133 sample trees were in the high risk class in 1963.

The previous discussion has shown that 1963 mortality risk classification is a useful prediction of future tree status. Expanding upon this conclusion, a test was performed to examine the effectiveness of the 1963 risk classification in predicting which period a tree would die.

Using the 1963-1973 stem data, the 52 mortality trees were re-classified according to the period in which they died. The status of
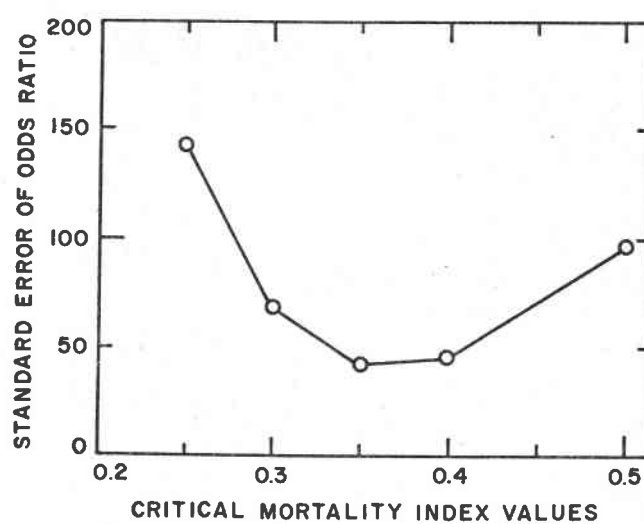
Figure (5-5): Standard error of the odds
ratio in percent for various
critical values

each tree was retrospectively classified as dead or alive at the end of 1966, 1970, and 1973 growing seasons and grouped by mortality period. Using the critical value CMI_A = .35, the 52 sample trees were cross-classified into low and high 1963 mortality risk classes. The data in Table (5-5) show the tabulation of the 52 mortality trees by period and risk class. The significance of the association between the two criteria was tested using the standard chi-square test statistic given in Equation (5-6). The null hypothesis is that the proportion of individuals in the high risk class is the same for each mortality period. Symbolically:

$$H_o : P_1 = P_2 = P_3 = p$$

Where:

p_k = the true portion of individuals in the high risk class for the ith mortality period k = (1,2,3)

p = the true proportion of individuals in the high risk class for the population

Table (5-6) gives the expected frequencies under the null hypothesis of independence (i.e., H_o : p = .750). The null hypothesis was not rejected (p-value = .50), indicating that the 1963 mortality risk classification was independent of the period in which the mortality occurred. Although the 1963 mortality risk classification of the 133 sample trees was strongly associated with future suppression mortality, it apparently is unable to distinguish in which period the mortality is to occur.

Table (5-5): Tabulation of mortality trees
by risk class and period

1963 Risk Class

	Low MI < .35	High MI ≥ .35	Total
Period 1	1	8	9
	.111	.889	.173
Period 2	6	22	28
	.214	.786	.538
Period 3	6	9	15
	.400	.600	.288
Total	13	39	52
	.250	.750	1.00

Table (5-6): Expected frequencies under the
null hypothesis

1963 Risk Class

	Low MI < .35	High MI ≥ .35	Total
Period 1	2.255	6.750	9
Period 2	7.000	21.000	28
Period 3	3.750	11.250	15
Total	13	39	52

$V = 2.916$

COMPARISONS OF MORTALITY PREDICTORS

A previous section introduced the processes associated with regular mortality in an even-aged Douglas-fir stand. Natural classification schemes can be used to qualitatively determine a social ranking of trees in the stand. Future status can also be predicted using quantitative variables. Individuals with a high risk of becoming future suppression mortality can be identified prior to their actual mortality.

Three quantitative variables were tested as predictors of future mortality: tree DBH, CSI, and mortality risk index. Each of these variables are susceptible to measurement at the beginning of a growth period. They are dynamic variables which are biologically oriented to the growth and development of the tree. To test the ability of each variable to differentiate between trees with high and low probability of becoming future mortality, the following strategy was adopted.

The 133 sample trees described earlier were retrospectively classified as dead or alive at the end of the 1966, 1970, and 1973 growing seasons. Trees were grouped into four status classes as follows: dead 1963-1966, dead 1966-1970, dead 1970-1973, alive 1973. Three cumulative mortality classes representing 3, 7, and 10 year periods were developed from the status classes above as: cumulative mortality 1963-1966, cumulative mortality 1963-1970, cumulative mortality 1963-1973. In addition, each tree was crossclassified into their respective DBH (1"), CSI (50 unit), and MI (.05 unit) classes. Using these data, average DBH, CSI, and MI were computed in 1963 by mortality, cumulative mortality and survivor classes, and for all trees combined. Comparison

of the three variables by future status classification demonstrates their relative predictive abilities based on measurement at the beginning of the period.

Table (5-7) gives the 1963 data for each mortality prediction by future status classification. Columns 1-3 give the 1963 mean values of DBH, CSI, and MI for trees that died during 1963-1966, 1963-1970, and 1963-1973 periods, respectively. Column 4 gives the mean values for trees which were alive in 1973. Column 5 gives the combined data for all trees in 1963. The difference between the average DBH of survivors and the average DBH of mortality trees is highly significant. Future survivors had a 1963 DBH of 4.47 inches compared to an average DBH of 2.37 inches for all mortality trees. CSI in 1963 does not differentiate strongly between future survivors and mortality trees. Mortality index in 1963 appears to be significantly different for survivors and mortality trees. Mortality trees had 1963 MI values greater than .40 while 1973 survivors had MI values less than .30.

Figures (5-6, 5-7, and 5-8) compare the frequency of trees by DBH class, CSI class, and MI class, respectively. Note how 1963 DBH and 1963 MI partition the total stand into distinct classes. Future survivors tend to have a much larger DBH and smaller MI than mortality trees. CSI in Figure (5-7) does not sharply distinguish between future status classes. The distribution of trees by 1963 class is roughly identical for each status group.

Tables (5-8, 5-9, and 5-10) give stand tables based on diameter class, CSI class, and MI class. Note that in Table (5-9), tree DBH and tree MI in 1963 are independent of CSI class.

Table (5-7): Average DBH, CSI, and MI in 1963 by future status classes

| 1963 Data	1 Cumulative Mortality 1963-1966	2 Cumulative Mortality 1963-1970	3 Cumulative Mortality 1963-1973	4 Survivors 1973	5 All Trees
N	9	37	52	81	133
DBH average	2.00	2.28	2.37	4.47	3.65
median	1.80	2.20	2.30	4.20	3.20
SD	.636	.612	.590	1.631	1.67
CSI average	593.7	554.7	548.0	522.6	532.50
median	557.0	543.0	548.5	514.0	531.00
SD	109.3	101.6	109.6	110.4	110.38
MI_A average	.426	.417	.406	.281	.329
median	.414	.425	.407	.272	.324
SD	.075	.066	.068	.078	.096

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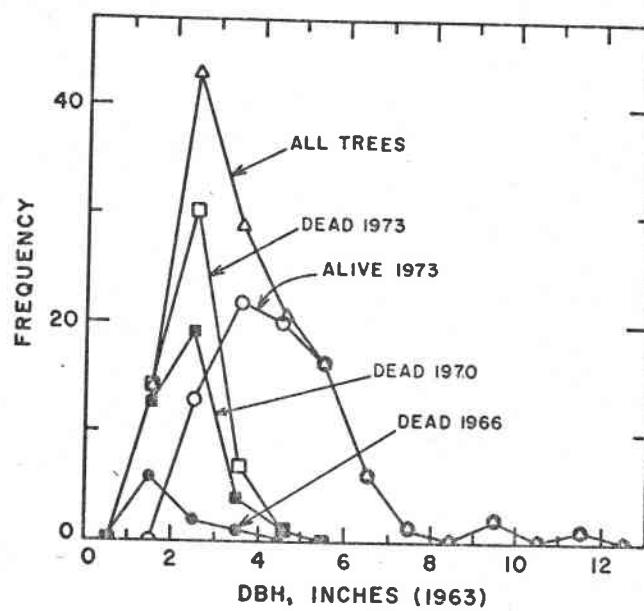
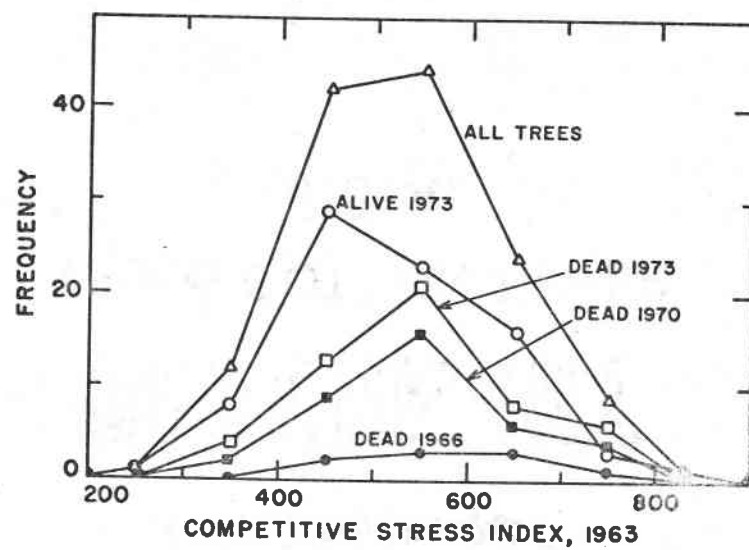Figure (5-6):   Frequency of trees by 1963
DBH class

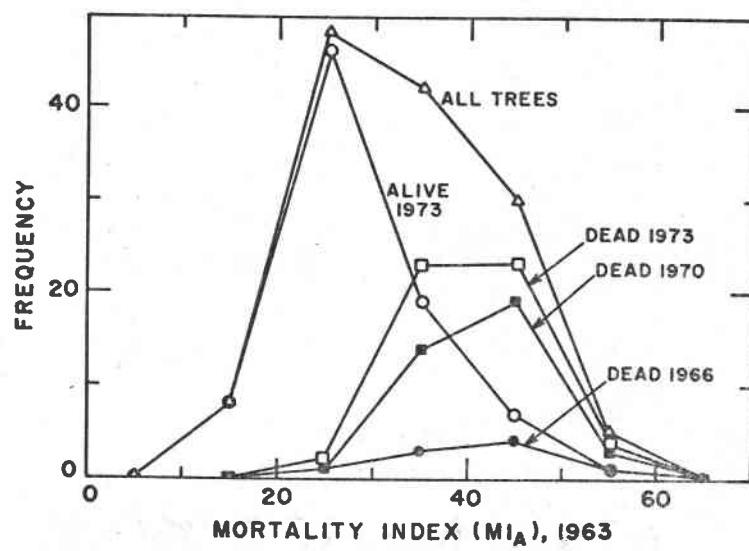Figure (5-7): Frequency of trees by 1963
CSI class

Figure (5-8): Frequency of trees by 1963
MI class

Table (5-8): Frequency of trees by future status and DBH class in 1963 (in inches)

	2	4	6	8	10	12	Average
Mortality							
CSI	560.3	546.5	432.0	--	--	--	548.0
MI	.4387	.3960	.3323	--	--	--	.4060
N	14	37	1	0	0	0	52
Survivor							
CSI	--	535.5	532.0	444.0	433.0	463.0	522.6
MI	--	.3366	.2537	.2001	.1515	.1129	.281
N	0	35	36	7	2	1	81
Total							
CSI	560.3	541.1	529.3	444.0	433.0	463.0	532.5
MI	.4387	.3671	.2559	.2001	.1515	.1129	.3290
N	14	72	37	7	2	1	133

Table (5-9): Frequency of trees by future status and CSI
class in 1963

	300	400	500	600	700	800	Average
Mortality							
DBH	--	2.27	2.33	2.48	2.16	2.43	2.37
MI	--	.4077	.4123	.4031	.3967	.4168	.4060
N	0	4	13	21	8	6	(52)
Survivor							
DBH	5.00	4.61	4.84	4.38	3.92	4.27	4.47
MI	.2938	.3001	.2467	.2990	.2903	.3128	.2810
N	1	8	29	23	16	3	(81)
Total							
DBH	5.00	3.83	4.07	3.47	3.33	3.04	3.65
MI	.2938	.3360	.2979	.3487	.3258	.3821	.3290
N	1	12	42	44	24	9	(133)

Table (5-10): Frequency of trees by future status and mortality index class in 1963

	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	Avg
Mortality											
DBH	--	--	--	2.60	2.75	2.33	2.32	2.19	2.00	2.05	2.37
CSI	--	--	--	459.0	544.0	581.8	544.6	519.0	525.5	671.0	548.0
XN	0	0	0	2	11	12	11	12	2	2	52
Survivors											
DBH	10.55	6.73	5.10	4.01	3.47	3.20	3.25	2.70	3.00	--	4.47
CSI	455.5	432.7	513.0	504.3	581.4	616.2	549.5	531.0	372.0	--	522.6
XN	2	6	24	22	14	5	6	1	1	0	81
Total											
DBH	10.55	6.73	5.10	3.90	3.16	2.59	2.65	2.23	2.33	2.05	3.65
CSI	455.5	432.7	513.0	500.5	564.9	591.9	546.3	519.9	474.3	671.0	532.5
XN	2	6	24	24	25	17	17	13	3	2	13

SUMMARY

Competitive stress index (CSI) is a quantitative measure of inter-tree competition developed by Arney (1973). It is an expression of the relative stress placed on an individual tree by its neighbors. The index does not attempt to describe the source of competition (i.e., light, nutrients, moisture) but only to quantify relative levels of competition for growing space based on the overlap of open crowns. The underlying assumption of the index is that the open grown tree, which developed under conditions of no competitive stress, represents the maximum development in stem dimensions and growth response for a tree of given size.

The degree of competition exerted on a tree is assumed to be directly proportional to the sum of the area of overlap of its growing space by neighboring trees and inversely proportional to the size of its growing space area. The computational requirements of the index are diameter measurements and x-y stem coordinates of all trees in the stand. The CSI model was fitted to 10 years of thinning data from the Hoskins study plots. The basic data were summarized by measurement period before and after thinning.

A review of the literature was undertaken to provide a background of the historical development of inter-tree competition models. Applications of CSI to specific aspects of the study were then considered. Average CSI and basal area per acre were compared as measures of thinning intensity. Based on results from this study, while both basal area per acre and CSI are effective measures of density, CSI is

more sensitive in evaluating the uniformity of stand density. This is evidentially due to the ability of CSI to accurately measure competitive status of individual trees, in comparison with the general averaging of competition involved in basal area per acre measures.

CSI was also found to be related to periodic diameter growth. Functions of CSI and ΔCSI explained up to 40 percent of variation found in periodic DBH growth. Functions of DBH, CSI, and ΔCSI explained 80 percent of the variation of periodic DBH growth. In the full model, CSI variables were found to increase the predictive power and decrease the standard mean square error.

Spacing and stocking guidelines were developed as functions of CSI. A diameter plus spacing rule was derived as a field guide to control stand density for achieving specific CSI levels.

Sampling procedures were developed to estimate CSI on a whole stand or single tree basis. Procedures for sampling for CSI in the field were outlined in detail. Sampling estimators were based on functions of observed tree count using an appropriate basal area factor. Special consideration for selecting an appropriate basal factor were discussed. A correction factor was derived as a function of stand diameter to improve the accuracy of the stand estimates. These techniques can be retrospectively applied to conventional cruise data without additional modification or data collection. The proposed sampling scheme makes CSI available to the forest manager on an operational basis. Examples of simulated samples for the single tree and whole stand estimators are also given.

In the final section of the study a mortality index was developed using a ratio of a tree's CSI level and the number of its competitors. This mortality index was found to be highly associated with future tree status. An evaluation of mortality index revealed that MI = .35 was the critical level in determining the high and low risk classification of trees. Seventy-five percent of the 1963 high risk mortality trees examined died during the subsequent 10 year period. Tree diameter, CSI, and mortality index (MI) were compared as possible predictors of future suppression mortality. Study results indicated that a tree's initial DBH and MI could be used to predict its future status, while CSI did not distinguish between future mortality and survivor trees.

CSI appears to have tremendous potential for application in research and operational areas. This study indicates that CSI is an effective measure of stand density and can be used as a predictive tool.

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APPENDICES

APPENDIX A

The expected value of a discrete random variable X is defined as (Mood, 1974):

$$E(X) = \sum X\, p(X) = \mu_X \tag{A-1}$$

Where:

$E(X)$ = the expected value of X

μ_X = the first moment of X

$p(X)$ = the probability of observing X

Using the definition of expected value in Equation (A-1), the expected value of the sampling estimators for CSI can be derived. The expected value of T_j in Equation (4-20) is:

$$E[T_j] = E[100 \sum_{}^{m} \frac{t_j k}{m}] = \frac{100}{m} \sum_{}^{m} E\,(t_j k) \tag{A-2}$$

The expected value of $t_j k$ is:

$$E(t_j k) = \sum_{}^{m} i\, p_t(i) = \sum_{}^{m} \frac{iOA_{ij}}{A_j} = \frac{TAO_j}{A_j} = \frac{CSI_j}{100} \tag{A-3}$$

Substituting the results in (A-3) into (A-2) gives:

$$E(T_j) = \frac{100}{m} \sum_{}^{m} CSI_j = \frac{m}{m} CSI_j = CSI_j \tag{A-4}$$

Equation (A-4) states that the expected value of T_j is an unbiased estimator of CSI_j for the jth tree.

The expected value for T_s can be routinely derived using Equation (A-4) as:

$$E[T_s] = E \frac{[\sum T_j]}{n} = \frac{1}{n} \sum E(T_j) = CSI \tag{A-5}$$

Equation (A-5) states that T_s is an unbiased estimate of the stand's CSI level. The expected value of S is more difficult to derive.

Let $y - \sum t_k^2$ and $X = \sum t_k$.

Then $S = y/X$.

Mood (1974) gives that the approximated expected value of y/X as:

$$E[y/X] \approx \frac{\mu_y}{\mu_X} - \frac{1}{\mu_X^2} \ cov(X,y) + \frac{\mu_X}{\mu_X^3} \ VAR \ [X[\tag{A-6}$$

Where:

$\mu_X = \mu_1$ = first moment of t

$\mu_X = \mu_2$ = second moment of t.

The expected value of y/X is approximately equal to (μ_2/μ_1) - [bias term]. But:

$\mu_2 = \xi \ [t^2]$ and $\mu_1 = E[t]$.

Using Equation (A-1) and (4-28):

$$E[t] = \sum iP_s(i) = \sum \frac{iOA_i}{A_i} = \frac{A.}{A_c} = \mu_1 \tag{A-7}$$

$$E[t^2] = \sum i^2 P_s(i) = \sum \frac{i^2 OA_i}{A_i} = \frac{TAO.}{A_c} = \mu_2 \tag{A-8}$$

Using these results and Equations (A-6) and (4-3):

$$E[S] = E[y/X] \approx \frac{E[t^2]}{E[t]} = \frac{\frac{TAO.}{A_i}}{\frac{A.}{A_i}} = \frac{TAO.}{A.} = CSI_R \tag{A-9}$$

Equation (A-9) states that the expected value of S, ignoring the bias term which is a function of ℓ, is CSI_R.

APPENDIX B

Arney (1973) presents the formulas for mathematically determining the areas of overlap between two competing trees. Five distinct conditions are recognized based on the size and spatial relationships of the competitors. The formulas below can be routinely derived using calculus.

CASE (1) $R_1 + R_2 \geq L$ (B-1)

AO = 0   (no overlap between trees)

CASE (2) $R_1 \geq L$ (B-2)

$AO = \pi R_2^2$ (complete overlap of smaller tree)

CASE (3) $X_1 < L$ (B-3)

$$AO = R_1^2 \sin^{-1} \left(\frac{C}{R_1}\right) - (L)(C)$$

CASE (4) $X_1 = L$ (B-4)

$$AO = \pi R_2^2 + R_1^2 \sin^{-1} \left(\frac{R_2}{R_1}\right) - (D)(R_2)$$

CASE (5) $X_1 > L$ (B-5)

$$AO = \pi R_2^2 - R_2^2 \sin^{-1} \left(\frac{C}{R_2}\right) + X_2(C) + R_1^2 \sin^{-1}$$

$$\left(\frac{C}{R_1}\right) - (X_1)(C)$$

Where:

AO = area of overlap between larger tree and smaller tree

L = distance between tree centers in feet

R_1 = competition radius for larger tree (open grown crown radius)

R_2 = competition radius for smaller tree (open grown crown radius)

$S = (R_1 + R_2 + L)/2$

$C = \dfrac{2}{L} \sqrt{S(S-R_1)(S - R_2)(S-L)}$

$X_1 = \sqrt{R_1^2 - C^2}$

$X_2 = L - X_1$

APPENDIX C

The objective of the following appendix is to derive the relationship between the centroid radius and the competition radius for various sample sizes when sampling for CSI in the single tree case. The desired sample sized per tree determines the number of circular sections into which the sample tree's hypothetical growing space circle must be partitioned. The centroid of a circular section is derived in general for any desired sample size m.

The centroid of a homogeneous area is located at its geometric center or center of mass. The coordinates of the centroid of a plane area are:

$$\bar{x} = \frac{M_y}{A} \qquad\qquad \bar{y} = \frac{M_x}{A} \qquad\qquad (C-1)$$

Where:

$\bar{x}$ = the x coordinate of the centroid of the plane area

$\bar{y}$ = the y coordinate of the centroid of the plane area

M_y = the first moment of the plane area with respect to the y axis

M_x = the first moment of the plane area with respect to the x axis

A = the area of the section

The centroid coordinates of a circular section of radius r and interior angle 2Θ can be derived as follows (see Figure C-1). Center the circular section so that the centroid lies on the x axis. The x coordinate of the centroid can be calculated as the quotient of the first moment with respect to the y axis (M_y) and the area of the section
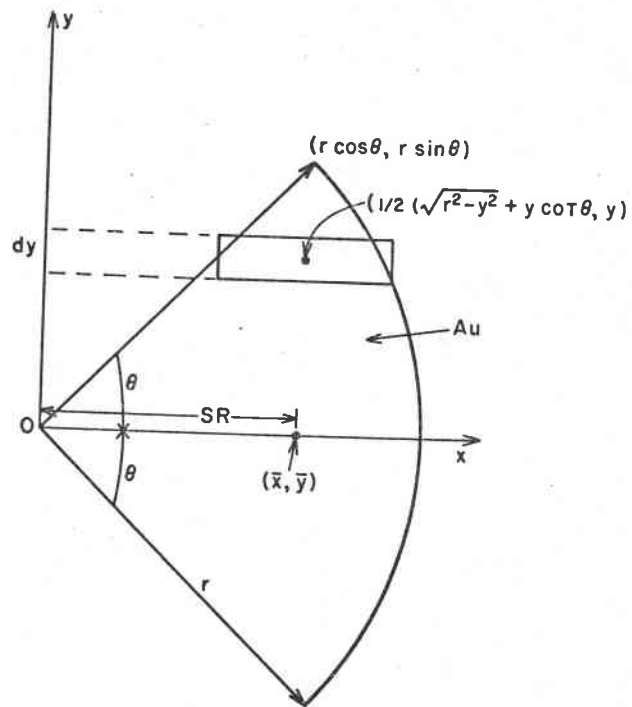
Figure (C-1): Geometry of determining the centroid of a circular section

lying above the x axis (A_u) using Equation (C-1) where:

$$A_u = \int_0^{r\sin\Theta} (\sqrt{r^2 - y^2} - y\cot\Theta)dy$$

$$= \frac{1}{2} y \sqrt{r^2 - y^2} + \frac{1}{2} r^2\sin^{-1}(\frac{y}{r}) - \frac{1}{2} y^2\cot\Theta \Big|_0^{r\sin\Theta} = \frac{1}{2} r^2\Theta$$

$$M_y = \int_0^{r\sin\Theta} \frac{1}{2} (\sqrt{r^2 - y^2} + y\cot\Theta) (\sqrt{r^2 - y^2} - y\cot\Theta)dy$$

$$= \int_0^{r\sin\Theta} \frac{1}{2} (r^2 - y^2 - y^2\cot^2\Theta)dy$$

$$= \frac{1}{2} (r^2 y - \frac{1}{3} y^3 - \frac{1}{3} y^3\cot^2\Theta \Big|_0^{r\sin\Theta} = \frac{1}{3} r^3\sin\Theta$$

Therefore:

$$A_u = \frac{1}{2} r^2\Theta \qquad\qquad M_y = 1.3 r^3\sin\Theta \qquad\qquad\qquad (C-2)$$

The x coordinate of the upper area A_u using Equation (C-2) is:

$$\bar{x} = \frac{M_y}{A_u} = \frac{\frac{1}{3} r^2\sin\Theta}{\frac{1}{2} r^2\Theta} = \frac{2r\sin\Theta}{3\Theta}$$

By symmetry, the coordinates of the centroid of the circular section with interior angle 2Θ are:

$$\bar{x} = \frac{2r\sin\Theta}{3\Theta} \qquad\qquad \bar{y} = 0 \qquad\qquad\qquad (C-3)$$

Where:

r = radius of the circular section

Θ = half of the interior angle of the circular section (in radians)

The radius of the centroid SR is related to the radius of the circle r as follows. From Equation (C-3):

$$SR = \frac{2\sin\Theta}{3\Theta} r \qquad\qquad (C-4)$$

Where:

SR = the centroid radius or the distance from the center of the circular section to the centroid of the section

r = the radius of the circle.

The centroid radius for any circular section with interior angle 2Θ can be calculated by evaluating the expression in Equation (C-4) for various values of Θ.

When sampling for CSI for a single tree, the number of sample points m desired determine the appropriate 2Θ value to use in calculating the centroid radius SR. The interior angle 2Θ corresponding to a given m is:

$$2\Theta_m = \frac{2\pi}{m} \qquad\qquad (C-5)$$

Solving Equation (C-5) for Θ gives

$$\Theta_m = \frac{\pi}{m}$$

Where:

Θ_m = the critical angle corresponding to a desired number of
      sample points m

m = the desired number of sample points per tree

Table (C-1) expresses the centroid radius SR as a function of the
competition radius CR for various sample sizes m for sampling for CSI
in the single tree case.

Figure (C-2) shows the sample point location for various sample
sizes and corresponding interior angles in degrees. For a given sample
size m, the sample points are systematically located at the centroid of
each circular section.

Table (C-1): Centroid radius for various sample sizes

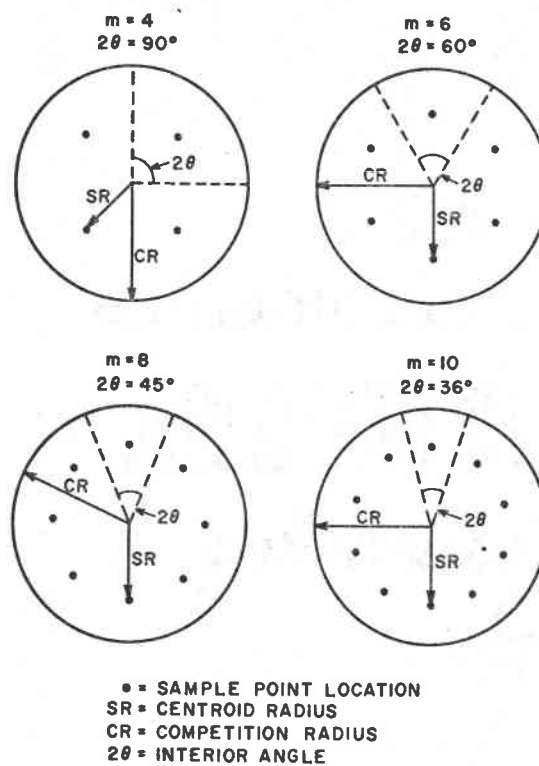| Sample Size	Interior Angle (Degrees)	Interior Angle (Radians)	(Radians)	Centroid Radius
m	2Θ	2Θ	Θ	SR
2	180	π	$\pi/2$	(.42441)(CR)
4	90	$\pi/2$	$\pi/4$	(.60021)(CR)
6	60	$\pi/3$	$\pi/6$	(.63662)(CR)
8	45	$\pi/4$	$\pi/8$	(.64966)(CR)
10	36	$\pi/5$	$\pi/10$	(.65575)(CR)

Figure (C-2):   Sample point locations for
various sample sizes

APPENDIX D

Various basal area factors can be derived using the wide scale relascope. The number of full and quarter bars can be converted into appropriate basal area factors.

$$BAF = 4.3561 \ (U^2) \qquad\qquad (D-1)$$

Where:

BAF = basal area factor

U   = number of full and quarter bars used

Table (D-1) gives the basal area factor associated with a given U value.

Table (D-1): Relationship
between
U and BAF

U	BAF
2.00	17.42
2.25	22.05
2.50	27.23
2.75	32.94
3.00	39.20
3.50	53.56
4.00	69.70

APPENDIX E

Each of the Hoskins study plots had an x-y coordinate axis. Individual tree stem coordinates were located with respect to this reference axis. To generate CSI data for trees on the edge of the plot a simulated stand condition was used. Each tree's stem coordinates were mirrored into eight identical plots by tranlation of their x-y stem coordinates. Potential competitors were then selected from all nine nested plots. The difference in tree characteristics due to an edge effect was assumed to be negligible. Figure (E-1) depicts the simulated coordinates of a tree on the original study plot.
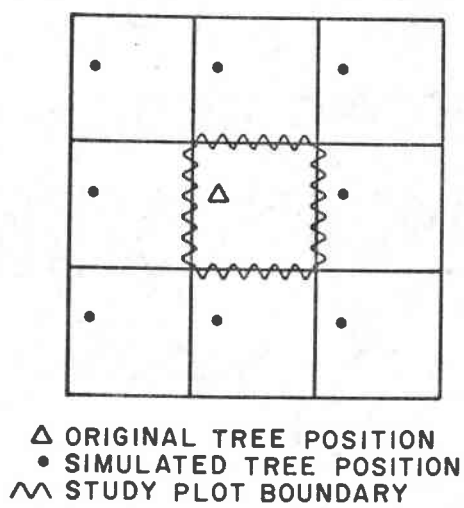
△ ORIGINAL TREE POSITION
● SIMULATED TREE POSITION
ᴧᴧ STUDY PLOT BOUNDARY

Figure (E-1):   Nested arrangement of trans-
                lated plots