Downwelling Circulation on the Oregon Continental Shelf: 
Part I: Response to Idealized Forcing

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ABSTRACT

Time-dependent downwelling on the Oregon continental shelf is studied with a two-dimensional approximation, that is, spatial variations across shelf and with depth, using the Blumberg–Mellor, finite-difference, stratified, hydrostatic, primitive equation model. The time-dependent response of a coastal ocean at rest to constant, downwelling-favorable wind stress is examined. Topography and stratification representative of the Oregon continental shelf are used for the basic case experiment. The wind stress forces onshore flow in a turbulent surface boundary layer. The compensating flow below the surface layer advects the density field downward and offshore and accelerates an alongshore current in the form of a vertically and horizontally sheared coastal jet. The dominant feature of the response flow field is a downwelling front that moves slowly offshore, leaving behind an inshore region where the density is well mixed. The downwelling front in the density field is concentrated near the bottom, while the front in alongshore velocity extends over the full depth and is nearly vertical, separating weak alongshore velocities inshore from the coastal jet offshore. The front contains strong vertical motion from the surface to the bottom and some recirculation. Much of the offshore flow from the base of the front is characterized by time- and space-dependent fluctuations involving spatially periodic separation and reattachment of the bottom boundary layer and accompanying recirculation cells. This flow has positive potential vorticity and appears to be finite-amplitude slantwise convection resulting from a hydrostatic symmetric instability. Additional experiments show the dependence of the response flow field on the magnitude of the wind stress, the initial stratification, and the shelf topography. Experiments with the vertical turbulent kinematic viscosity and diffusivity parameterized by a different turbulence closure scheme, as a function of a local Richardson number, or as constants show dependence of the response flow field on the choice of turbulence submodel. The occurrence of a well-mixed region inshore and the existence of time- and space-dependent fluctuations associated with slantwise convection in the near-bottom offshore flow appear to be robust features of the two-dimensional downwelling response.

1. Introduction

The time-dependent response of a stratified coastal ocean to a constant upwelling-favorable alongshore wind stress was studied in Allen et al. (1995). The Blumberg–Mellor (1987) hydrostatic, primitive equation, sigma coordinate, finite-difference model was used for numerical experiments in an idealized two-dimensional situation that includes spatial variations across-shelf and with depth, but assumes uniformity alongshore. Bottom topography for the continental shelf and slope from the Oregon coast was used. The Blumberg–Mellor (1987) model has the Mellor–Yamada (1982) level 2.5 turbulence closure submodel [with modifications described in Galperin et al. (1988)] embedded. In this paper, we pursue a parallel study for the response to downwelling-favorable wind stress. In contrast to the upwelling problem, the characteristics of coastal ocean flow fields under downwelling conditions have been largely unexplored in model studies. Results obtained here show interesting new behavior of downwelling flow fields, including the formation of downwelling fronts and the development of space- and time-dependent variability associated with slantwise convection in the near-bottom across-shelf circulation.

Specifically, using the same primitive equation model and the same bottom topography and initial stratification as in the upwelling experiments of Allen et al. (1995), we examine the model response, from a state of rest with a horizontally uniform density field, to constant downwelling-favorable alongshore wind stress. The general objective is to investigate the resulting time-dependent evolution of the velocity and density fields on timescales greater than an inertial period and, in particular, to examine the nature of the response as influenced by the turbulent surface and bottom boundary layers and turbulent frontal regions. To study parameter dependence we include, in addition, some experiments in which the magnitude of the wind...
stress, the initial stratification, and the shelf bottom topography are varied. For comparison with the results obtained using the Mellor–Yamada (1982) level 2.5 turbulence model, we also investigate the effects of representing the vertical turbulent viscosity and diffusivity with the Mellor–Yamada (1982) level 2 model, with a Richardson-number-dependent parameterization, and with constant values.

The outline of this paper is as follows. The model equations are described in section 2, with some details given in the appendices. The numerical experiments are defined in section 3. The results of the numerical experiments are presented for a basic case experiment in section 4 and for other experiments in section 5. Additional analyses of the experiments are discussed in section 6 and a summary is given in section 7.

2. Model formulation

The Blumberg and Mellor (1987) model is based on the hydrostatic primitive equations in sigma coordinates. We use potential density as a variable so that for two-dimensional flow the equations are

$$\frac{\partial \eta}{\partial t} + \frac{\partial (uD)}{\partial x} + \frac{\partial \omega}{\partial \sigma} = 0 \quad (2.1a)$$

$$\frac{\partial (uD)}{\partial t} + \frac{\partial (u^2D)}{\partial x} + \frac{\partial (u \omega)}{\partial \sigma} = -fvD$$

$$= -gD \frac{\partial \eta}{\partial x} - \frac{gD^2}{\rho_0} \int_0^\sigma \left( \frac{\partial \sigma_\theta}{\partial x} - \frac{\sigma D}{\partial \sigma} \frac{\partial \sigma_\theta}{\partial \sigma} \right) d\sigma$$

$$+ \frac{\partial}{\partial \sigma} \left[ \frac{K_M}{D} \frac{\partial u}{\partial \sigma} \right] + 2A_M \frac{\partial}{\partial x} \left( D \frac{\partial u}{\partial x} \right) \quad (2.1b)$$

$$\frac{\partial (q^2D)}{\partial t} + \frac{\partial (q^2uD)}{\partial x} + \frac{\partial (q^2 \omega)}{\partial \sigma} = \frac{\partial}{\partial \sigma} \left[ \frac{K_M}{D} \frac{q^2}{\partial \sigma} \right] + \frac{2K_M}{D} \left( \frac{\partial u}{\partial \sigma} \right)^2 + \frac{2g}{\rho_0} \frac{\partial}{\partial \sigma} \left( \frac{q^2}{B_i} \right) + A_H \frac{\partial}{\partial x} \left( D \frac{\partial q^2}{\partial x} \right) \quad (2.3a)$$

$$\frac{\partial (q^2uD)}{\partial t} + \frac{\partial (q^2u^2D)}{\partial x} + \frac{\partial (q^2u \omega)}{\partial \sigma} = \frac{\partial}{\partial \sigma} \left[ \frac{K_M}{D} \frac{q^2}{\partial \sigma} \right]$$

$$+ E_i \left\{ \frac{\partial}{\partial \sigma} \left( \frac{q^2}{B_i} \right) \right\} + \frac{g}{\rho_0} \frac{\partial}{\partial \sigma} \left( \frac{K_M}{D} \frac{\sigma_\theta}{\partial \sigma} \right)$$

$$- \frac{Dq^3}{B_i} \frac{\partial q}{\partial \sigma} + \frac{\partial}{\partial x} \left( D \frac{\partial q^2}{\partial x} \right) \quad (2.3b)$$

where

$$K_M = \tilde{K}_M + \nu_M, \quad \tilde{K}_M = q l S_M \quad (2.4a, b)$$

and the functions $S_M$, $\tilde{W}$ and the constants $B_1$, $E_i$ are defined in appendix A.

Since we find from the experiments that in general $|\eta_\sigma| \ll |(uD)_\sigma|, |\omega_\sigma|$, where the subscripts $i$, $x$ and $\sigma$

$$\psi(x, \sigma) = -\int_{\sigma_0}^\sigma \omega(x', \sigma) dx', \quad (2.5)$$

denote partial differentiation, it is useful to neglect $\eta_\sigma$ in (2.1a) and to calculate an approximate streamfunction for the $u$ and $\omega$ velocity components. We assume $(uD)_\sigma + \omega_\sigma \approx 0$, define $\psi_\sigma = uD$, $\psi_\sigma = -\omega$, and calculate
where \( x = x_0 \) is the location of the coast. The velocity component in the vertical \( \sigma \) (or \( z \)) direction (which may be calculated from \( u \) and \( \omega \)) is \( \omega \).

The model domain is an across-shelf section (Fig. 1) bounded by vertical walls. A right-handed coordinate system is used where \( x \) is positive onshore, \( y \) is positive northward, and \( \sigma \) varies from \( \sigma = 0 \) at the surface \([z = \eta(x, t)]\) to \( \sigma = -1 \) at the bottom \([z = -H(x)]\). The offshore boundary is at \( x = 0 \) and the coast is at \( x = x_0 \).

The boundary conditions at the surface are

\[
\omega = 0 \quad \text{at} \quad \sigma = 0, \quad (2.6a)
\]

\[
(K_M/D)(u_\sigma, v_\sigma) = (\tau^{(x)}, \tau^{(y)})/\rho_0, \quad (2.6b,c,d)
\]

\[
(K_M/D)\sigma_{\theta\sigma} = I_1, \quad \sigma \to 0 \quad (2.6b,c,d)
\]

\[
q^2 = B_I^{1/3} u_{\theta \sigma}^2, \quad q^2 I = 0, \quad \sigma = 0 \quad (2.6e,f)
\]

where \( \tau^{(x)}, \tau^{(y)} \) are the \( x, y \) components of the surface wind stress,

\[
u_{\theta \sigma}^2 = (\tau^{(x)2} + \tau^{(y)2})^{1/2}/\rho_0 \quad (2.6g)
\]

is the square of the surface friction velocity, \( I_1 = Q_\alpha/c_p \) is the longwave component of the surface insolation, \( \alpha \) is the coefficient of thermal expansion, and \( c_p \) is the specific heat at constant pressure.

The boundary conditions at the bottom are

\[
\omega = 0 \quad \text{at} \quad \sigma = -1 \quad (2.7a)
\]

\[
(K_M/D)(u_\sigma, v_\sigma) = (\tau^{(x)}, \tau^{(y)})/\rho_0, \quad (2.7b,c,d)
\]

\[
(K_M/D)\sigma_{\theta\sigma} = 0, \quad \sigma \to -1 \quad (2.7b,c,d)
\]

\[
q^2 = B_I^{1/3} u_{\theta \sigma}^2, \quad q^2 I = 0, \quad \sigma = -1 \quad (2.7e,f)
\]

\[
(\tau_b^{(x)}, \tau_b^{(y)}) = \rho_0 C_D (u_b^2 + v_b^2)^{1/2}(u_b, v_b), \quad (2.7g,h,i)
\]

\[
u_{\theta \sigma}^2 = (\tau_b^{(x)2} + \tau_b^{(y)2})^{1/2}/\rho_0. \quad (2.7g,h,i)
\]

The bottom velocity components \((u_b, v_b)\) are evaluated at the grid point next to the bottom, that is, at \( \sigma_b = -1 \) \( + \frac{1}{2\Delta\sigma_b} \), where \( \Delta\sigma_b \) is the \( \sigma \) finite-difference grid interval at the bottom. The drag coefficient

\[
C_D = \max \{ \kappa^2 [\ln (\Delta z_b/z_0)]^{-2}, 2.5 \times 10^{-3} \}, \quad (2.8)
\]

where \( \kappa = 0.4 \) is the von Kármán constant, \( z_0 \) is the bottom roughness parameter, and \( \Delta z_b = \frac{1}{2} \Delta\sigma_b H \).

The boundary conditions along the vertical sidewalls at the coast \((x = x_0)\) and offshore \((x = 0)\) are

\[
u = 0, \quad v_x = 0 \quad \text{at} \quad x = 0, x_0 \quad (2.9a,b)
\]

\[
\sigma_{\theta x} = q^2 = (q^2 I)_{x} = 0 \quad \text{at} \quad x = x_0, x_0. \quad (2.9c,d,e)
\]

Note: we use a free-slip condition (2.9b) for the tangential velocity component \( v \).

The shortwave solar insolation term \( \partial I/\partial \sigma \) in (2.1d) is found from

\[
I = I_1 \exp (\sigma D/\lambda_2), \quad (2.10a)
\]

where \( I_2 = Q_2 \alpha/c_p, Q_2 \) is the shortwave component of the surface insolation, and \( \lambda_2 \) is a constant extinction depth. The net incoming solar radiation is

\[
Q_0 = Q_1 + Q_2, \quad (2.10b)
\]

where the longwave radiation magnitude \( Q_1 \) appears in (2.6d).

The equations (2.1) and (2.3) are written in finite difference form on an Arakawa C grid as discussed in Blumberg and Mellor (1987). Explicit leapfrog time

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**Fig. 1.** The model computational domain. The shelf and slope topography represents the Oregon shelf at 45°15'N. The depth at the coast is 10 m. The grid spacing is uniform in \( \sigma \) and \( x \) with 60 sigma levels. (Note: the sigma resolution everywhere is that shown near the bottom.)
differencing combined with an implicit time differencing scheme for the vertical diffusion terms is used with a weak Asselin (1972) filter. A mode-splitting technique is employed so that the depth-independent barotropic component is calculated with a smaller time step than the depth-dependent baroclinic component.

3. Description of numerical experiments

The numerical experiments involve initial value problems for motion forced by a downwelling-favorable wind stress. The initial state corresponds to a stratified coastal ocean at rest with a horizontally uniform density field. The initial conditions are

\[ u = v = \omega = q^2 = q^2 l = 0, \quad \sigma_0 = \sigma_{00}(z) \quad \text{at} \quad t = 0. \]  
(3.1)

The wind forcing is by a spatially independent alongshore wind stress given by

\[ \gamma = \begin{cases} \tau_0 \sin((\pi t)/2T_R), & 0 \leq t \leq T_R, \\ \tau_0, & T_R \leq t. \end{cases} \]  
(3.2a)

where \( \tau_0 \) and \( T_R \) (=2 days) are constants. The other forcing terms in (2.1d) and (2.6) are zero; that is, \( \tau^{(x)} = I_1 = I_2 = 0. \)

For most of the experiments, the topography of the continental shelf and upper slope (Fig. 1) corresponds to that off Oregon at 45°15' N. Likewise, the initial density field for the basic case experiment \( \sigma_{00} = \sigma_{0BC}(z) \) with corresponding \( N_{0BC}(z) = -g \sigma_{0BC}/\rho_0 \). Fig. 2, is the horizontal average of an observed density field from 29 June 1973. This is the initial density field used in the upwelling experiments in Allen et al. (1995). We keep the initial density field the same here to allow direct comparison of the differences in the upwelling and the downwelling response. For all experiments, \( f = 1.0362 \times 10^{-4} \ \text{s}^{-1}. \)

For the basic case downwelling experiment, the Oregon topography (Fig. 1) and horizontally averaged observed density field \( \sigma_{0BC} \) are utilized with \( \tau_0 = \tau_{0BC} = 0.5 \ \text{dyn cm}^{-2}. \) Additional experiments, listed in Table 1, are conducted to study the dependence of the response on the topography, stratification, forcing, and the parameterization of the vertical turbulent viscosity and diffusivity. The dependence of the response on the stratification is investigated first by initializing with a constant \( N^2(t = 0) = N_0^2 = 1.5 \times 10^{-4} \ \text{s}^{-2}. \) The effects of changes in shelf topography are studied by using the same initial density field as the constant \( N^2(t = 0) = N_0^2 \) case with different geometries. Experiments are run with a shallow shelf and with a steep shelf topography (Fig. 3). Further investigations of the dependence of the response on the initial stratification are carried out in experiments using the basic case topography with \( N^2(t = 0) = \frac{1}{2} N_0^2 \) and \( N^2(t = 0) = 2N_0^2. \) The effect of wind stress magnitude is examined in an experiment with basic case topography, \( N^2(t = 0) = N_0^2, \) and with \( \tau_0 = 2\tau_{0BC} = 1 \ \text{dyn cm}^{-2}. \) For each experiment with \( N^2(t = 0) = \text{const}, \) the corresponding value of the nondimensional parameter \( S = H^2N^2(t = 0)/f^2 \) is also given in Table 1. The parameter \( S \) is a slope Burger number calculated using representative values of \( H \) over the shelf region where the depth \( H \leq 150 \ \text{m}. \) The dependence of the response on the parameterization used for the vertical turbulent kinematic viscosity and diffusivity is also examined to a limited extent with basic case conditions. A Mellor–Yamada (1982) level-2 turbulence closure submodel (see appendix A) is used in one experiment. Experiments are also run with \( K_M \) and \( K_H \) parameterized as a function of the Richardson number following Pacanowski and Philander (1981) and with constant values for \( K_M \) and \( K_H \) (see appendix B).

The basic case model domain is shown in Fig. 1. The depth at the coast is 10 m and the depth offshore is 500 m. The domain width for most experiments is \( x_0 = 100 \ \text{km}. \) For the finite-difference solutions, uniform grid spacing in \( x \) is used with 60 \( \sigma \) intervals. Consequently, the vertical grid size varies with \( x \) from about 0.17 m at the coast to 8.33 m in the deepest water. The horizontal grid size for most experiments is \( \Delta x = 0.25 \ \text{km}. \) A horizontal size of \( \Delta x = 0.5 \ \text{km} \) as used in Allen et al. (1995) appears to give adequate resolution here also. Because of the complex nature of the across-shelf circulation in the downwelling response flow field, however, we use \( \Delta x = 0.25 \ \text{km} \) in these experiments to ensure that the solutions are satisfactorily resolved. Additional experiments are run for the basic case conditions with \( \Delta x = 0.166 \ \text{km} \) to check the effects of horizontal grid resolution and with \( x_0 = 200 \ \text{km} \) (\( \Delta x = 0.5 \ \text{km} \)) to check the effects of domain size.

The horizontal kinematic eddy viscosity and diffusivity are chosen to be small; that is,

\[ A_M = A_H = 2 \ \text{m}^2 \ \text{s}^{-1} \]  
(3.3a, b)

so that horizontal diffusion processes play a negligible role in the resulting dynamical balances (section 6). We chose the background vertical viscosity \( \nu_M = 10^{-4} \ \text{m}^2 \ \text{s}^{-1} \) and diffusivity \( \nu_H = 5 \times 10^{-5} \ \text{m}^2 \ \text{s}^{-1} \) based on the values used in the Richardson number parameterization in Kantha and Clayson (1994). The bottom roughness parameter \( z_0 = 0.01 \ \text{m}. \) The reference density \( \rho_0 = 10^3 \ \text{kg m}^{-3}. \) Finite difference time steps are 60 s for the baroclinic and 2.5 s for the barotropic components.

4. The basic case experiment

The model response in the basic case experiment as a function of time is discussed below. We concentrate on a description of the flow field over the continental shelf. Dominant qualitative features of the response are noted here. Dynamical aspects are discussed further in section 6. All of the variables presented, unless otherwise noted, have been averaged over an inertial period to isolate the subinertial frequency response.
The $x$, $z$ fields of $\sigma_p$, $v$, $\psi$, $q^2$, and $K_M$ over the shelf for $t = 6$, $12$, and $18$ days are plotted in Fig. 4. The $\psi$ and $\sigma_p$ fields show that, as expected, the positive wind stress forces an onshore flow in a surface mixed layer. The depth of the mixed layer increases toward the coast to a region nearshore where the density field has been vertically mixed so that it is uniform from the surface to the bottom. A density front forms along the bottom at the offshore edge of the nearshore region of vertically uniform density. There is an associated front in alongshore velocity $v$ at the same location, which extends vertically over the full water depth and has a horizontal scale of $1-2$ km. That front sharply separates weak alongshore velocities in the nearshore region of vertically uniform density from an accelerating, horizontally and vertically sheared, alongshore coastal jet offshore. On the offshore side of the front in $v$, there is onshore flow in a surface layer, with a dominant dynamical balance of a linear Ekman layer (section 6). The depth of this surface layer, as indicated by the $K_M$ field, is $15-20$ m by day $18$. Inshore of the front, turbulent frictional effects extend over the full water depth, as indicated by the relatively high values of $K_M$, and a surface Ekman layer does not exist. The onshore flow in the surface Ekman layer, consequently, is turned abruptly at the front so that it flows vertically downward from the surface to near the bottom. This motion is evidently strong enough to induce consider-

\begin{table}
\centering
\begin{tabular}{ccc}
\hline
\textbf{Experiment} & \textbf{Description} & \textbf{Abbreviation} & \textbf{\textit{S}} \\
\hline
1 & Basic case & BC & \\
2 & $N^2(t = 0) = N_0^2$ & $N_0^2$ & 0.72 \\
3 & Shallow shelf, $N^2(t = 0) = N_0^2$ & Shallow $N_0^2$ & 0.16 \\
4 & Steep shelf, $N^2(t = 0) = N_0^2$ & Steep $N_0^2$ & 3.00 \\
5 & $N^2(t = 0) = \frac{1}{2}N_0^2$ & $\frac{1}{2}N_0^2$ & 0.36 \\
6 & $N^2(t = 0) = 2N_0^2$ & $2N_0^2$ & 1.45 \\
7 & $\tau_0 = 2\tau_{BC}$, $N^2(t = 0) = N_0^2$ & $2\tau_{BC}N_0^2$ & 0.72 \\
8 & Level 2 & Level 2 & \\
9 & $K_M$ and $K_H$ by Ri-dependent parameterization (P–P) & P–P & \\
10 & $K_M = K_H = 0.0005$ m$^2$ s$^{-1}$ & $K_M = K_H = 0.0005$ m$^2$ s$^{-1}$ & \\
11 & Basic case ($\Delta x = 0.166$ km) & \\
12 & Basic case ($x_0 = 200$ km) & \\
\hline
\end{tabular}
\caption{Summary of the numerical experiments denoted by a concise description of the difference from the basic case and the abbreviations used to designate the experiments in the figures. The nondimensional slope Burger number $S = H^2 N^2(t = 0)/f^2$, where $H$ is a representative value over the shelf, is also given for the experiments where $N^2(t = 0)$ is a constant.}
\end{table}
is taking place in an alternately separating and reattaching bottom boundary layer. This separation and reattachment process is time-dependent also, as will be shown later in this section, with the reattachment regions propagating onshore at about 2.5 km/day. The dynamics of this motion are discussed further in section 6.

The change with time of the vertical structure of the flow is illustrated in Fig. 5, where profiles at the 56-m isobath of $u$, $v$, $\sigma_\theta$, $q^2$, $K_M$, and $K_H$ are shown for days 6, 12, and 18. During this time period the downwelling front moves from a position inshore of the 56-m isobath on day 6 to a position offshore of it by day 18, passing the 56-m isobath on about day 12.

On day 6, the across-shelf velocities $u$ are onshore in a surface layer of depth about 12 m and offshore below. The surface layer is turbulent and mixed as indicated by the relatively large values of $q^2$, $K_M$, and $K_H$ and the vertical uniformity of $\sigma_\theta$. There is likewise a turbulent, well-mixed bottom boundary layer of thickness about 12 m. The offshore flow decreases slightly with depth through the inviscid interior region and has somewhat larger magnitudes in the bottom boundary layer. On day 12, the smooth transition of $u$ from onshore flow above 25-m depth to offshore flow below, the relatively large values of $v$ ($\leq 0.6$ m s$^{-1}$), the nearly uniform distribution of $\sigma_\theta$ and the large values of $q^2$ ($\leq 25 \times 10^{-4}$ m$^2$ s$^{-2}$), $K_M$ ($\leq 0.04$ m$^2$ s$^{-1}$), and $K_H$ ($\leq 0.05$ m$^2$ s$^{-1}$) indicate the presence of the offshore edge of the downwelling front. On day 18, the downwelling front has moved offshore of this isobath, and we find vertically uniform $\sigma_\theta$, very small across-shelf velocities $u$, and substantially weaker alongshore velocities $v$. The flow is turbulent throughout the depth as indicated by the relatively large, nearly uniform values of $q^2$ and the appreciable values of $K_M$ and $K_H$. As will be shown later, in this region the bottom stress approximately balances the surface stress so that the alongshore velocity is nearly steady and similar to a Couette flow with variable $K_M$.

The near-surface horizontal velocity vectors plotted as a function of $(x, t)$ in Fig. 6 show the development with time of the near-surface alongshore coastal jet and across-shelf velocities. The formation of relatively large gradients in $u$ and $v$ at the inshore edge of the coastal jet at the downwelling front and the movement offshore of the front as time increases are particularly evident. The velocity with which the front moves offshore between days 10 and 20 is about 0.4 km day$^{-1}$. The convergence of the across-shelf flow at the front is also clearly shown, for example, after day 6.

The net onshore transport, defined as

\[
U_{(+)} = \int_{-H}^{u} u_{(+)} dz, \quad u_{(+)} = u \quad \text{for} \quad u > 0, \\
U_{(-)} = 0 \quad \text{for} \quad u \leq 0, \quad (4.1)
\]

calculated as a function of $(x, t)$ is plotted in Fig. 7.
Fig. 4. Fields of the density $\sigma_\theta$, the alongshore velocity $v$, the streamfunction for the across-shelf flow $\psi$, twice the turbulent kinetic energy $q^2$, and the vertical kinematic viscosity $K_M$ from the basic case experiment at days 6, 12, and 18. All variables are averaged over an inertial period. The contour intervals are $\Delta \sigma_\theta = 0.166$ kg m$^{-3}$, where $\sigma_\theta = 24, 25,$ and $26$ kg m$^{-3}$ are marked by heavy lines; $\Delta v = 0.1$ m s$^{-1}$, where $v = 0.5$ m s$^{-1}$ is marked with a heavy line; $\Delta \psi = 0.1$ m$^2$ s$^{-1}$, where $\psi = 0$ is marked with a heavy line and $\psi > 0$ is dashed; $\Delta q^2 = 2 \times 10^{-4}$ m$^2$ s$^{-2}$, $\Delta K_M = 4 \times 10^{-3}$ m$^2$ s$^{-1}$. 
For reference, we also indicate the value of the Ekman transport $U_x = \tau^{(y)}/(\rho_0 f)$. Note that offshore, away from the coast, $|U_{(+)12}|$ is generally less than, but close to, $|U_{26}|$. This plot shows how the onshore transport $U_{(+)12}$ determined by the shelf circulation processes varies across the shelf and with time to attain a value near $U_x$ offshore. At day 2, $U_{(+)12}$ is small within 2 km of the coast but then increases smoothly to a value somewhat less than $U_x$ 26 km from the coast. Beginning on day 4, we find almost zero values of $U_{(+)12}$ inshore of the downwelling front, consistent with the absence of across-shelf circulation in that region indicated by the $\psi$ fields in Fig. 4. Relatively large values of $U_{(+)12}$ occur over a horizontal scale of 2–4 km around the downwelling front. The maximum values of $U_{(+)12}$ occur at the offshore edge of the front and reflect the recirculation in that region evident in the $\psi$ fields in Fig. 4.

The across-shelf velocities $u$ near the surface are plotted as function of $(x, t)$ in Fig. 8. Effects of the recirculation associated with this downwelling front are evident in the large values of $u$ (max $\approx 0.12$ m s$^{-1}$ on day 18) that occur over horizontal scales of 2–4 km on the offshore edge of the front. Also shown in Fig. 8 are plots of the vertical velocities $w$ at middepth as a function of $(x, t)$. The circulation in the front is characterized by large negative values of $w$ (max $\approx 0.0015$ m s$^{-1}$ on day 18) over a horizontal scale of about 1.5 km in the front, accompanied by large, but somewhat smaller magnitude, positive $w$ over a similar horizontal scale on the offshore side of the front. The behavior of both $u$ and $w$ shown in Fig. 8 is consistent, of course, with the circulation indicated by the $\psi$ fields in Fig. 4.

Scaling arguments for subinertial frequency coastal flow fields with alongshore spatial scales assumed to be greater than across-shelf scales imply that, away from turbulent boundary layers, the alongshore velocity is approximately in geostrophic balance, that is, $v \approx v_G$, where from (2.1b),

$$f v_G = g \frac{\partial \eta}{\partial x} + \frac{g D}{\rho_0} \int_0^\sigma \left( \frac{\partial \sigma}{\partial x} - \frac{\sigma}{D} \frac{\partial \sigma}{\partial x} - \frac{\partial \sigma}{\partial \sigma} \frac{\partial x}{\partial \sigma} \right) d \sigma. \tag{4.2}$$

It is useful to quantify the extent to which the alongshore velocity $v$ from the model solution (averaged over an inertial period) is geostrophically balanced. The $v_G$ and $|v - v_G|$ fields from the basic case experiment on days 6, 12, and 18 are plotted in Fig. 9. We see that $v$ is well approximated by $v_G$ except in the downwelling front, the surface layer offshore of the front, and on days 12 and 18 in some patchy regions near the bottom, which appear to be related to regions of large $q^2$.

Some of the characteristics of high-frequency fluctuations (not shown by the inertial-period-averaged variables) are examined. The spatial variability of the super-inertial frequency motion in $v$ is shown in plots of the inertial-period-averaged variance of $v$ about the inertial-period-averaged $v$, that is,
Instantaneous (i.e., not averaged over an inertial period) values of the gradient Richardson number,

$$\text{Ri} = -\frac{g}{\rho_0} \frac{\partial \sigma}{\partial z} \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right]^{-1}, \quad (4.4)$$

for days 6, 12, and 18 are also shown in Fig. 9. For clarity, only the values of Ri between 0 and 1.0 are contoured. In general, in the interior away from the surface and bottom boundary layers and offshore of the downwelling front Ri > 1. In most of the surface layer

\[ \langle v'^2 \rangle = \langle (v - \langle v \rangle)^2 \rangle, \quad (4.3a) \]

where

\[ \langle v \rangle(t_0) = \frac{1}{T_I} \int_{t_0-(1/2)T_I}^{t_0+(1/2)T_I} v \, dt \quad (4.3b) \]

for \( T_I \) the inertial period. Contour plots of \( \langle v'^2 \rangle \) fields for days 6, 12, and 18 are shown in Fig. 9. The downwelling front is seen to be a source of high-frequency \( \langle v'^2 \rangle \) variability on all of these days. In addition, high values of \( \langle v'^2 \rangle \) are found near the location of the largest alongshore currents \( v \) in the core of the jet \( v \) (Fig. 4) on days 6 and 12, and to a lesser extent on day 18. On days 12 and 18, patches of \( \langle v'^2 \rangle \) also occur 20–40 km above the bottom in regions where the spatially oscillating streamlines in the \( \psi \) field have their largest curvature.

\[ U_{(+)} \]

Fig. 7. Net onshore transport \( U_{(+)} \) (m² s⁻¹) defined in (4.1) as a function of \( (x, t) \) from the basic case experiment. The solid bar is the Ekman value \( \tau_{(x)} / (\rho_0 f) \).
and inshore of the downwelling front \( \text{Ri} < 0.25 \). In a region of variable height near the bottom, offshore of the front, \( \text{Ri} < 1 \). The regions near the bottom where \( \text{Ri} < 1 \) generally include the patches of large \( q^2 \) (Fig. 4), where the offshore flow is attached to the bottom in a bottom boundary layer. Some limited regions exist in the surface layer and inshore of the downwelling front where \( \text{Ri} < 0 \).

The near-bottom offshore flow associated with the periodic variation of streamline height and the recirculation regions shown in Fig. 4 on days 12 and 18 also involves time-dependent behavior. This is shown by time series of \( u, v, \) and \( q^2 \) from a location 7.5 m above the bottom over the 100-m isobath (Fig. 10). During the first four days, the alongshore current \( v \) increases rapidly to about 0.09 m s\(^{-1}\). Between days 4 and 9, the \( q^2 \) values increase and the rate of increase of \( v \) decreases, corresponding to the growth of the bottom boundary layer. After day 10, fluctuations in \( v, u, \) and \( q^2 \) develop with fairly regular periods of about two days. The across-shelf velocity \( u \) fluctuates positively and negatively about zero while \( q^2 \) fluctuates between positive and near-zero values. At this height above the bottom, \( u \) is roughly 180° out of phase with \( v \) and \( q^2 \) (i.e., minima in \( u \) correspond approximately to maxima in \( v \) and \( q^2 \)). A contour plot of the across-shelf velocity \( u \) near the bottom (\( \sigma = -0.958 \)) as a function of \( x \) and \( t \) (Fig. 11) shows that these fluctuations propagate toward the coast with velocities between 2 and 2.5 km/day. The quasi-regular spacing of these fluctuations in both space and time, indicated previously by the patterns in the \( \psi \) and \( q^2 \) fields in Fig. 4 and by the time series in Fig. 10, is particularly evident in this contour plot. The dynamics of this flow are discussed further in section 6 in connection with the potential vorticity \( \Pi \) fields in Fig. 9. The location of the downwelling front, marked by an inshore band of negative \( u \) about 1 km wide adjacent to an offshore band of positive \( u \) of similar scale, and the steady propagation of the front offshore at about 0.4 km/day are also clearly shown in Fig. 11.

5. Other experiments

In experiment 2, the initial stratification is uniform, with constant \( N^2(t = 0) = N^2_0 = 1.5 \times 10^{-4} \) s\(^{-2}\). The purpose of this experiment is to find the characteristics of the response flow field in the constant \( N^2 \) conditions frequently used in theoretical or conceptual models. The slope Burger number in this case is close to one, that is, \( S = 0.72 \). The day 18 fields (Fig. 12) are, in general, qualitatively similar to those of the basic case. Quantitative differences exist, however, consistent with the differences in the initial density field. In experiment 2, the magnitude and offshore decay scale of \( v \) and the horizontal gradients in \( v \) in the downwelling front are smaller than found in the basic case experiment. In addition, the downwelling front is located farther offshore compared to the basic case. These differences seem to be directly related to the lower initial stratification in the upper part of the water column in experiment 2 compared to the basic case. One of the most notable differences with constant initial \( N^2 \), however, is the reduced magnitude of the horizontal density gradients at the bottom at the offshore edge of the nearshore region of vertically mixed density. That difference makes it clear that in the basic case the large horizontal density gradients at the bottom of the downwelling front are a consequence of the relatively large vertical gradients in \( \sigma_\theta \) and large \( N^2 \) values between the surface and 40-m depth in the initial density field (Fig. 2). We also see in Fig. 12 that in water of depth greater than about 100 m the effects of larger initial \( N^2 \) in experiment 2 compared to the basic case leads to smaller heights of the bottom layer of mixed \( \sigma_\theta \) and to larger vertical gradients of \( \sigma_\theta \) in that layer. The structure of the potential vorticity \( \Pi \) field and its relation to the behavior of the flow near the bottom are discussed in section 6.

The change in the near-bottom behavior with constant initial \( N^2 \) compared to the basic case is further shown by contours of the across-shelf velocity \( u \) near the bottom as a function of \( (x, t) \) (Fig. 11). The time- and space-dependent oscillations develop earlier in time and, at these depths, are characterized by stronger fluctuations in \( u \). The onshore propagation velocities of the disturbances are somewhat smaller (\( \approx 1.5 \) km/day) in this case. Although the downwelling front is located farther offshore than in the basic case at the
Fig. 9. Fields of the alongshore geostrophic velocity $v_G$ (4.2), the difference $|v - v_G|$, the inertial-period averaged variance of $v$ about the inertial-period-averaged $v$ (4.3), the unaveraged Richardson number $Ri$ (4.4) and the potential vorticity $\Pi$ (6.2) from the basic case experiment at days 6, 12, and 18. The contour intervals are $\Delta v_G = 0.1$ m s$^{-1}$, where $v_G = 0.5$ m s$^{-1}$ is marked with a heavy line; $\Delta |v - v_G| = 0.01$ m s$^{-1}$, $\Delta \langle v^2 \rangle = 10^{-4}$ m$^2$ s$^{-2}$, and $\Delta \Pi = 10^{-6}$ kg m$^{-3}$ s$^{-1}$ with positive (negative) values solid (dashed) and 0 marked with a heavy line. For $Ri$, the values contoured are 1, 0.75, 0.50, 0.25, and 0 (dashed).
vertical gradients, and there is relatively little offshore transport in the bottom boundary layer. The acrossshelf circulation is still characterized by strong vertical and horizontal recirculation associated with the downwelling front. Some small fraction of the offshore flow near the bottom from the base of the front results in spatial oscillations in the streamlines, but the flow associated with these oscillations is weak and is found only

Fig. 10. Time series of unaveraged $u$, $v$, and $q^2$ at a location near the bottom at the 100-m isobath ($\sigma = -0.925, x = 86.375$ km) from the basic case experiment.

corresponding time, the offshore propagation speed is approximately the same at about 0.4 km/day (a similar result is found for experiment 5). Thus, the downwelling front evidently forms more quickly with reduced initial stratification near the surface.

The effects of shelf geometry are investigated in experiments 3 and 4 with a shallow shelf and with a steep shelf, respectively. In order to isolate effects of topography, the initial density field is the same as in the constant $N^2 (t = 0) = N_0^2$ experiment 2. The day 18 fields are plotted in Fig. 12. The nature of the shelf flow field differs considerably depending on the shelf geometry. Compared with experiment 2, over the shallow shelf the alongshore velocities $v$ are smaller, the downwelling front is located farther offshore, the bottom mixed layer in $\sigma_\theta$ is generally thicker ($\approx 55$ m), and a larger fraction of the offshore transport occurs in the bottom boundary layer. The streamlines for the offshore flow from the base of the downwelling front oscillate in space to heights above the bottom of about 55 m and extend offshore a horizontal distance of about 40 km, before becoming parallel to the bottom. In this experiment, the slope Burger number is less than one; that is, $S = 0.16$.

Over the steep shelf, the alongshore velocities are slightly larger, the downwelling front is located closer to the coast, the depth of the bottom mixed layer in $\sigma_\theta$ is considerably smaller ($\approx 10$ m) with generally larger

Fig. 11. Contours of unaveraged across-shelf velocity $u$ near the bottom ($\sigma = -0.958$) as a function of $x$ and $t$ from the basic case experiment (BC) and from experiment 4 with $N^2 (t = 0) = N_0^2 = 1.5 \times 10^{-4}$ s$^{-1}$. The contour interval is $\Delta u = 0.01$ m s$^{-1}$. Solid (dashed) contour lines correspond to positive (negative) values. The zero contour is omitted.
within 4 km of the front. The absence of appreciable values of $q^2$ near the bottom offshore of this region indicates the relative weakness of bottom boundary layer processes there. This reflects that, as found over the steep shelf in the upwelling experiments in Allen et al. (1995), the vertical gradients in $v$ near the bottom over the steep shelf are, to a large extent, geostrophically balanced through the thermal wind equation by horizontal gradients in $\sigma_\theta$. Time- and space-dependent fluctuations in the near-bottom offshore flow from the base of the downwelling front develop more strongly after day 24. In this experiment, the slope Burger number $S = 3$, considerably greater than one.

It is of interest to examine the behavior of the bottom stress $\tau_b^{(3)}$ in these experiments with different shelf topographies. The depth average of (2.1c) may be written

\begin{align*}
\frac{\langle \tau_b^{(3)}(z) \rangle}{\frac{g}{\rho} \left( \frac{\partial \rho}{\partial z} \right)} &= R + \frac{\nu}{\rho^*} \left( \frac{\partial^2 \rho}{\partial z^2} \right) \frac{g}{\rho} \left( \frac{\partial \rho}{\partial z} \right) \frac{\partial \rho}{\partial z} - \frac{\nabla \cdot \mathbf{u}}{\rho} \frac{g}{\rho} \left( \frac{\partial \rho}{\partial z} \right) \frac{\partial \rho}{\partial z}
\end{align*}
\[ V_t = (\tau^{(y)} - \tau_b^{(y)})/\rho_H - H^{-1} \int_{-1}^{0} (wD)_x d\sigma + H^{-1} \int_{-1}^{0} A_M(Dv_\sigma)_x d\sigma \quad (5.1a) \]

where

\[ V = H^{-1} \int_{-1}^{0} vDd\sigma. \quad (5.1b) \]

In a linear approximation with \( A_M = 0 \), spinup occurs; that is, \( V_t = 0 \) when the bottom stress has increased such that \( \tau_b^{(y)} = \tau^{(y)} \). In a barotropic linear problem with \( \tau_b^{(y)}/\rho_b = rv_b = rV \), the spinup time is \( D/r \) and varies with \( D \approx H(x) \). In Fig. 13, we plot \( \tau_b^{(y)} \) as a function of depth \( H(x) \) at day 18 from the \( N_0^2 \) wide shelf and shallow shelf experiments. Large positive \( \tau_b^{(y)} \) values, between 1.5 and 3 dyn cm\(^{-2}\), are found in the downwelling fronts that are located at depths between 75 and 90 m. Inshore of about 60-m depth, \( \tau_b^{(y)} \approx \tau^{(y)} = 0.5 \) dyn cm\(^{-2}\), reflecting a direct balance of the surface wind stress by the bottom stress through a nearly steady alongshore current. Offshore of the front, \( \tau_b^{(y)} \) oscillates around a somewhat lower value of about 0.2 dyn cm\(^2\). In the steep shelf experiment, offshore of 90-m depth \( \tau_b^{(y)} \) is small, consistent with the previous conclusion of weak bottom boundary layer processes there. It is evident from the \( \tau_b^{(y)} \) variations found in these experiments that the adjustment over shelf topography of stratified downwelling flow (where \( v_b \neq V \)) is considerably more complicated than that predicted by a linear, barotropic, depth-independent model.

The effects of reduced initial stratification with basic case topography are shown in Fig. 14 by the day 18 fields from experiment 5 with \( N^2(t = 0) = \frac{1}{2} N_0^2 \) and, thus, with \( S = 0.36 \). The spatial structure of the response flow field is qualitatively similar to that of experiment 2 with \( N^2(t = 0) = N_0^2 \). For decreased \( N^2 \), however, the alongshore velocities \( v \) are smaller and the offshore scale of the alongshore coastal jet in \( v \) is evidently also smaller. The horizontal gradients of \( v \) in the downwelling front are correspondingly weaker and the downwelling front has propagated farther offshore. The thickness of the density mixed layer on the bottom and the height (60–80 m) of the spatially oscillating streamlines for the offshore flow near the bottom are considerably larger than in experiment 2.

The effects of increased initial stratification are shown in Fig. 14 by the day 18 fields from experiment 6 with \( N^2(t = 0) = 2N_0^2 \) and \( S = 1.45 \). Compared with the day 18 fields from experiments 2 and 5, the alongshore velocities \( v \) are larger and the offshore scale of the coastal jet in \( v \) is also larger. The horizontal gradients of \( v \) in the downwelling front are stronger and the downwelling front is located closer to the coast. The thickness of the disturbed density layer on the bottom is smaller (20–40 m). The spatially oscillating streamlines occur over a shorter horizontal distance of about 8 km from the base of the downwelling front and have a correspondingly reduced height of about 30–40 m.

As in Allen et al. (1995), we examine the solutions obtained here for the scaling implied by balances from approximate linear models of the two-dimensional coastal response problem for constant wind stress \( \tau_0 \) and constant depth \( H_0 \). In that case, (4.2) holds and scaling estimates give \( u = \tau_0/\rho_0 f H_0 \) and \( v = t\rho f u \) or \( v = t\tau_0/\rho_0 H_0 \). This implies that, if the wind stress amplitude is doubled, the \( \psi \) field amplitude will be doubled and the same \( v \) and \( \sigma_\psi \) field will be attained in half the time. The fields at day 9 from experiment 7 with \( \tau_0 = 2\tau_{ac} \) and \( N^2(t = 0) = N_0^2 \) are shown in Fig. 14 for comparison to the fields from experiment 2 with \( N^2(t = 0) = N_0^2 \) at day 18 in Fig. 12. The \( v \) and \( \sigma_\psi \) fields are similar in structure in both experiments, as predicted by the scaling arguments, and the downwelling fronts are located in essentially the same position. The streamfunction field in experiment 7 is increased by a factor of \( \sim 2 \), as anticipated. The across-shelf circulation, shown by the \( \psi \) fields, is qualitatively similar in both cases with the exception that with \( \tau_0 = 2\tau_{ac} \) the downwelling circulation from the surface to the bottom occurs more in the well-mixed region inshore of
Fig. 14. Fields at day 18 from experiment 5 with $N^2 (t = 0) = \frac{1}{2} N^2_0$, at day 18 from experiment 6 with $N^2 (t = 0) = 2N^2_0$, and at day 9 from experiment 7 with $\tau_0 = 2\tau_{\text{app}}$ and $N^2 (t = 0) = N^2_0$. Contour intervals as in Fig. 4 with $\sigma_\theta = 27$ and $28$ kg m$^{-3}$ also marked with heavy lines in experiment 6. Contour interval for potential vorticity $\Pi$ as in Fig. 9.
the front. Intense downward vertical motion occurs at
the location of the front in $v$, but that only penetrates
to about one-half of the depth and is primarily involved
in the recirculation on the offshore side of the front. An
additional quantitative difference is that the structure
of the spatial fluctuations in $\psi$ for the near-bottom off-
shore flow appears somewhat less regular and exists
for a smaller horizontal distance with $\tau_0 = 2\tau_{nc}$.

The effects of different parameterizations for the ver-
tical turbulent kinematic viscosity and diffusivity are
investigated in experiments 8, 9, and 10 (Table 1) with
basic case stratification, topography, and forcing. In ex-
periment 8, a Mellor–Yamada (1982) level-2 turbu-
ence closure scheme (appendix A) is used. The Rich-
ardson-number-dependent parameterization of Paca-
nowski and Philander (1981) (appendix B) is utilized
in experiment 9, denoted by the abbreviation P–P.

Contour intervals as in Fig. 4.
(2.3a) and plotted for comparison with the previous results and for use as indicators of likely regions of small-scale turbulent motion.

The day 18 fields from experiment 8 (Fig. 15) with the level-2 closure scheme are very similar to those from the basic case (Fig. 4). The \( \sigma_\theta \) and \( \psi \) fields are almost indistinguishable. The structure of the downwelling front is also remarkably close, as evidenced by the similarity of the downwelling motion and the recirculation in the frontal region shown in the \( \psi \) fields and by the similarity of the \( q^2 \) and \( K_\psi \) fields in the front and in the region inshore of the front. Quantitative differences of the \( \psi \) and \( q^2 \) fields exist in the near-bottom space- and time-dependent offshore flow from the base of the front, but the qualitative agreement of these fields is strong. From comparison of these experiments, it appears that the considerably less complex level-2 turbulence submodel gives essentially the same results as the level 2.5 closure scheme in this shelf flow problem.

The \( \sigma_\psi \) and \( \psi \) fields in Fig. 15 from experiment 9 with \( K_M \) and \( K_H \) determined from the P–P Richardson-number-dependent parameterization (appendix B) show some agreement in structure and magnitude with those from the basic case. The \( \psi \) field is missing the large horizontal frontal gradients on the inshore edge of the coastal jet that are found in the basic case, but otherwise the coastal jet structure and magnitude of \( \psi \) are similar. The density at the base of the surface mixed layer is larger on day 18 in experiment 9, presumably the result of weaker vertical mixing with the P–P parameterization. The downwelling circulation is different, with vertical motion from near the surface to near the bottom found in the nearshore region of vertically well-mixed density. On the offshore side of this region, however, recirculation involving upward vertical motion and onshore flow occurs similar to that in the basic case. One significant qualitative point of agreement of the across-shelf circulation in experiment 8 and in the basic case is the nature of the offshore flow near the bottom that involves oscillations in the height of \( \psi \) contours and recirculation regions. The localized patches of \( q^2 \) along the bottom in experiment 9, evident in Fig. 15, indicate that the bottom boundary layer is characterized by alternate regions of separation and reattachment similar to the basic case. The contour plot in Fig. 16 of \( u(x, t) \) near the bottom shows that this behavior is time-dependent and qualitatively similar to that in the basic case (Fig. 11). The time- and space-dependent fluctuations start later, around day 14, and generally propagate onshore more slowly at about 1.2 km/day. The downwelling front moves offshore also with a lower velocity of about 0.3 km/day.

The fields with constant \( K_M = K_H = 0.0005 \text{ m}^2\text{s}^{-1} \) (Fig. 15) from experiment 10 also have several features in qualitative agreement with the basic case. The density field is vertically well mixed in an inshore region extending about 12 km from the coast. Offshore of that region, the structure and magnitude of the alongshore velocity field \( \psi \) and the spatial variability of the offshore flow near the bottom are similar to that in the basic case. Much of the downwelling from the surface layer to the bottom occurs in the outer part of the nearshore density well-mixed region as in the basic case, but there is no front in \( \psi \) at this location. In contrast to the basic case, appreciable values of \( \psi \) and strong across-shelf and vertical recirculation are found in the nearshore

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**Fig. 16.** Contours of unaveraged across-shelf velocity \( u \) near the bottom \( (\sigma = -0.925) \) as a function of \( x \) and \( t \) from experiment 9 with \( K_M \) and \( K_H \) parameterized as in Pacanowski and Philander (1981) (P–P) and from experiment 10 with \( K_M = K_H = 0.0005 \text{ m}^2\text{s}^{-1} \). The contour interval is \( \Delta u = 0.01 \text{ m s}^{-1} \). Solid (dashed) contour lines correspond to positive (negative) values. The zero contour is omitted.
region. The most noteworthy point of agreement of the flow in experiment 10 with the basic case is the space-
dependent fluctuations in the $\psi$ field for the offshore flow near the bottom. These fluctuations also have a
qualitatively similar time dependence characterized by onshore propagation, as shown by the contour plots of
$u(x,t)$ in Fig. 16. The propagation velocities, however, are considerably lower ($\approx 0.4$ km/day) here. These re-
sults indicate that inviscid processes are strongly involved in the space- and time-dependent fluctuations of the near-bottom offshore flow.

Finally, we note that a comparison of results of the basic case with those from experiment 11, where the $x$
grid size is reduced to $\Delta x = 0.166$ km, shows no qualitative differences in the solutions. Small quantitative
differences are found in the details of the time- and space-dependent near-bottom offshore flow, but these appear to be consistent with variations expected in sol-
lutions of unstable flow fields obtained under slightly different conditions. Also, a comparison of the results of the basic case with experiment 12, where the hori-
Zonal scale of the computational domain was increased to 200 km, shows no appreciable differences in the flow
fields over the shelf. This indicates that the results of these experiments are not dependent on the $x$ grid size
of 0.25 km or the domain width of $x_0 = 100$ km.

6. Additional analyses of the experiments

To gain additional information on the dynamical pro-
cesses acting over the shelf during the upwelling re-
sponse, we examine the balance of terms in the $v$
momentum equation (2.1c) and in the $\sigma_v$ potential density equation (2.1d). We choose constant $\sigma$
levels near the surface, at middepth, and near the bottom, as shown in Fig. 17, and plot the terms averaged over an inertial period at these locations on days 6, 12, and 18 for the $v$
equation in Fig. 18 and for the $\sigma_v$ equation in Fig. 19.

To simplify the discussion, we use the following shorthand notation for the terms in (2.1c,d):

$$v_t + [(uv)_x + (\omega v)_z] + fu - (K_{\theta}v_{\theta})_z - A_{\nu}v_{xx} = 0$$

(6.1a)

$$\sigma_{\theta_t} + [(u\sigma_{\theta})_x + (\omega_\theta)_z] - (K_{\theta}\sigma_{\theta\theta})_z - A_{\nu}\sigma_{\theta\theta} = 0,$$

(6.1b)

where the correspondence should be clear.

We examine the total balance in the $v$ momentum equation (Fig. 18) first. Near the surface, the locations of the upwelling front and the associated recirculation region offshore of the front are indicated by the large positive values of $fu$ as in Fig. 8. Offshore of the frontal region near the surface the negative vertical friction term, $-(K_{\theta}v_{\theta})_z$, is balanced primarily by a positive $fu$, corresponding to onshore flow, similar to a linear Ekman layer balance. The time derivative $v_t$ and the nonlinear advection term $[(uv)_x + (\omega v)_z]$ are both smaller in magnitude and positive and also help to bal-

nace the vertical friction term. As a consequence, when
the latter two terms are nonnegligible, as they are over the time-dependent coastal jet, the onshore transport in
the surface layer may be appreciably smaller than the linear Ekman value $\tau \sigma f/\nu f$. This relaxation of the linear Ekman transport constraint allows vertical mass flow out of the surface layer in the region over the
coastal jet as evident in the $\psi$ fields in Fig. 4. The behavior is also reflected by the variation of the net on-
shore transport $U_{(+)}$ (Fig. 7), which continuously decreases in magnitude as the downwelling front is ap-
proached from offshore. We note that this behavior is distinctly different than found in the upwelling-re-
sponse problem (Allen et al. 1995), where over the coastal jet $v_t$ and $[(uv)_x + (\omega v)_z]$ have opposite signs
and similar magnitudes. Thus, in the upwelling case, the transport in the Ekman layer offshore of the up-
welling front is generally closer to the linear Ekman value $\tau \sigma f/\nu f$ than found here.

In the vicinity of the downwelling front near the sur-
face, vertical friction $-(K_{\theta}v_{\theta})_z$ varies rapidly from large negative values offshore to a large positive value
over a narrow region about 1 km wide in the front. The positive $-(K_{\theta}v_{\theta})_z$ and positive $fu$ in the frontal region are balanced by large negative advection $[(uv)_x + (\omega v)_z]$. At middepth offshore of the front, an inviscid balance between a positive $v_t$ and a negative $fu$ is found as expected. In the frontal region, $fu$ is large and negative on the offshore side, while $-(K_{\theta}v_{\theta})_z$ is positive on the inshore side. These terms are balanced by advection $[(uv)_x + (\omega v)_z]$, which changes from large positive values offshore to negative values in the nar-
row region of downwelling in the front.

Near the bottom in the front, large positive advection $[(uv)_x + (\omega v)_z]$ balances large negative vertical fri-
Fig. 18. Terms in the $v$ momentum equation (in m s$^{-2}$; averaged over an inertial period and multiplied by 10$^6$) along the $\sigma$ levels shown in Fig. 17 from the basic case experiment at days 6, 12, and 18.
Fig. 19. Terms in the equation for potential density $\sigma_\theta$ (in kg m$^{-3}$ s$^{-1}$; averaged over an inertial period and multiplied by 10$^2$) along the $\sigma$ levels shown in Fig. 17 from the basic case experiment at days 6, 12, and 18.
tion $-(K_{p}v_2)_{x}$, which is clearly associated with the large positive values of the bottom stress found in the frontal region (Fig. 13). Offshore of the front near the bottom, a linear Ekman balance between negative $fu$ and positive $-(K_{p}v_2)_{x}$ exists at day 6. Later at days 12 and 18, offshore of the front we find that all of the terms (except horizontal friction) have appreciable magnitude and that they are characterized by spatial variations on scales of 2–4 km. These balances clearly are associated with the space- and time-dependent near-bottom flow. The relatively large negative fluctuations in $fu$, balanced largely by negative vertical friction $-(K_{p}v_2)_{z}$, correspond to regions where offshore flow is concentrated near the bottom in a turbulent bottom boundary layer. The $v_2$ and $[(u\sigma_{h})_{x} + (\omega\sigma_{h})_{z}]$ terms are roughly in antiphase with each other and out of phase with $fu$ and $-(K_{p}v_2)_{x}$ such that in the offshore direction positive $v_2$ and negative $[(u\sigma_{h})_{x} + (\omega\sigma_{h})_{z}]$ precede negative $fu$.

Considering the term balances in the $\sigma_{h}$ equation (2.1d) plotted in Fig. 19, we recall that, when $\sigma_{h}$ is balanced by advection $[(u\sigma_{h})_{x} + (\omega\sigma_{h})_{z}]$, the density is simply being advected by the velocity field so that $\sigma_{h}$ is conserved on fluid particles. This balance generally occurs at middepth where typically positive advection $[(u\sigma_{h})_{x} + (\omega\sigma_{h})_{z}]$ balances negative $\sigma_{h}$ as expected in a downwelling flow field. That balance is also found offshore of the front near the bottom on day 6. A nondiffusive balance between $\sigma_{h}$ and $[(u\sigma_{h})_{x} + (\omega\sigma_{h})_{z}]$ involving spatial fluctuation in sign of the terms on 2–4-km scales is found offshore of the front near the bottom on days 12 and 18 and is obviously associated with the time- and space-dependent near-bottom flow. In the downwelling front, negative vertical diffusion $-(K_{p}\sigma_{h})_{x}$, (mixing dense water upward) balances positive advection $[(u\sigma_{h})_{x} + (\omega\sigma_{h})_{z}]$ (advecting less dense water downward) near the surface, while positive diffusion balances either negative $\sigma_{h}$ or negative advection in the front near the bottom. The principal results here in the term balance in (6.1b) are that nonlinear density advection plays a significant role in the time- and space-dependent near-bottom flow that develops after day 10 and that vertical diffusion of density is important in the downwelling front near the surface and near the bottom.

Motivated by recent studies of Bishop and Chen (1995) and Haine and Marshall (1995) involving symmetric instability and finite amplitude slantwise convection in, respectively, baroclinic atmospheric boundary layers and oceanic mixed layers, we examine whether that process may be responsible for the space- and time-dependent near-bottom flow found here. Symmetric instability, also called slantwise convection (e.g., see Emanuel 1994, Chapter 12), can occur in two-dimensional, rotating, stratified fluid flow and involves a combination of the mechanisms responsible for inertial instability in a rotating nonstratified flow and for convective instability in a stratified nonrotating flow. An important point here is that symmetric instabilities can develop in flow situations stable to both pure inertial instability or pure convective instability. Thus, as the across-shelf circulation under downwelling conditions attempts to push lower density fluid under higher density fluid, symmetric instability can develop before the flow becomes convectively unstable, that is, before $\sigma_{h}$ becomes positive.

The condition for inviscid hydrostatic symmetric instability involves the sign of the potential vorticity. The potential vorticity $\Pi$ of the hydrostatic primitive equations for two-dimensional flow (2.1) (written in Cartesian coordinates for simplicity) is

$$\Pi = (f + u_2)\sigma_{h} - \nu\sigma_{h},$$  \hspace{1cm} (6.2)

Geostrophically balanced, steady, basic flows $v(x, z)$ in an unbounded region with $\sigma_{h} < 0$ are linearly unstable to small two-dimensional perturbations if $\Pi > 0$ (e.g., Bennetts and Hoskins 1979; Emanuel 1994). Hydrostatic symmetric instability is characterized by motion that is predominantly along density surfaces and typically develops into finite amplitude slantwise convection involving circulation in cells or eddies. Note that the initial potential vorticity is negative; that is,

$$\Pi (t = 0) = f\sigma_{hBC} < 0.$$  \hspace{1cm} (6.3)

Plots of the potential vorticity $\Pi$ fields for the basic case experiment on days 6, 12, and 18 are shown in Fig. 9. The potential vorticity $\Pi$ of the flow near the bottom and near the coast is positive, reflecting changes from the negative initial values due to frictional processes at the boundaries. Within a bottom layer of vertical extent that coincides with the height of the fluctuating streamlines and the modified density field, $\Pi$ is positive, relatively uniform, and near zero. From the $q^2$ and $K_{p}$ fields in Fig. 4, we see that a substantial fraction of the flow in this layer is inviscid. That conclusion is consistent also with the term balances in (6.1a) (Fig. 18). The potential vorticity is also positive in the nearshore region of well-mixed density, in the downwelling front, and in the recirculation region immediately offshore of the front.

The $\Pi$ fields on day 18 from experiments 2, 3, and 4 with different shelf topographies and with $N^2 (t = 0) = N_0^2$ are shown in Fig. 12. In the uniform $N_0^2$ experiment 2 and in the shallow shelf experiment 3, the time- and space-dependent fluctuating offshore flow and recirculation cells are likewise characterized by a layer above the bottom of relatively uniform, positive $\Pi$. Note from the $q^2$ fields that, as in the basic case experiment, frictional processes are only involved in a limited part of this layer in experiment 2. Similar structure is found in the day 9 $\Pi$ field from experiment 7 with $\tau_0 = 2\tau_{0BC}$ (Fig. 14) except that frictional processes appear to play a somewhat larger role. In the shallow shelf experiment 3, however, frictional effects are much stronger and the positive $\Pi$ values are smaller.
and more uniform. In the steep shelf experiment 4 where the offshore flow is more regular, the character of the \( \Pi \) field near the bottom is qualitatively different, involving large positive values and large gradients in a thin bottom layer. Similarly, the day 18 \( \Pi \) fields from experiments 5 and 6 (Fig. 14) with \( N^2 (t = 0) = \frac{1}{2} N_0^2 \) and \( N^2 (t = 0) = 2N_0^2 \), respectively, have positive values in the bottom layer of fluctuating across-shelf flow. The weak stratification experiment 5 has especially energetic fluctuations in a relatively thick bottom layer (80–90 m) and the \( \Pi \) values are small and uniform in that region. The strong stratification experiment 6 has weaker oscillations confined to a region offshore of the front with shorter horizontal scale and the \( \Pi \) field has larger positive values and larger gradients.

In all of the experiments, positive \( \Pi \) is found to characterize the near-bottom flow when it involves time- and space-dependent fluctuations in \( \psi \). This result, the fact that the streamlines in that layer are dominantly aligned along the direction of the \( \sigma_\theta \) surfaces in the bottom layer, and the major role played by inviscid dynamics give strong support to the identification of this process as finite amplitude slantwise convection resulting from a hydrostatic symmetric instability.

These results may be used to obtain approximate scales for the time- and space-dependent fluctuations in the near-bottom layer for the experiments 2, 3, and 5 where \( \Pi \) is small, positive, and relatively uniform. We assume that the finite amplitude slantwise convection maintains the flow in a condition near marginal stability, that is, that

\[ \Pi \approx 0, \]

that \( v \) is geostrophically balanced so

\[ v_z = -(g/f\rho_0)\sigma_{\theta x}, \]

and that

\[ |v_z| \ll f. \]

From (6.2) we obtain

\[ \left( \frac{\sigma_{\theta x}}{\sigma_{\theta z}} \right) = -\frac{f}{\sigma_\theta} \frac{\partial \sigma_{\theta x}}{\partial x} = f \frac{\rho_0}{g \sigma_{\theta x}}, \]

where we will use the notation

\[ \gamma = \left( \frac{\partial \sigma_{\theta z}}{\partial x} \right)_{\sigma_\theta}. \]

Note that (6.4b) and (6.5a) imply that in the bottom layer, the Richardson number formed from \( v_z \) is equal to one; that is,

\[ \hat{R}_i = -(g/\rho_0)\sigma_{\theta x}/v_z^2 = 1. \]

We assume also that the depth of the bottom layer with \( \Pi \approx 0 \) is uniform, that in the bottom layer \( \sigma_{\theta z} \approx 0 \), and that outside this layer \( \sigma_{\theta z} \) is close to its initial value; that is, \( \sigma_{\theta z} \approx \sigma_{\theta 0} \). If the density \( \sigma_\theta \) is vertically uniform in the bottom layer, that is, if \( \gamma^{-1} = 0 \), it follows that \( \sigma_{\theta x} \approx -H_x \sigma_{\theta 0} \) (recall \( H_x < 0 \)). For cases where \( \gamma^{-1} \neq 0 \), as observed in the experiments, we obtain

\[ \sigma_{\theta x} \approx \frac{H_x \sigma_{\theta 0}}{(1 + H_x \gamma^{-1})}. \]

Substitution of (6.7) in (6.5a) gives

\[ \gamma = \frac{f^2}{N^2 H_x} (1 + H_x \gamma^{-1}), \]

where \( N^2 = N^2 (t = 0) = -g \sigma_{\theta 0}/\rho_0 \), which is a quadratic equation for \( \gamma \). The relevant solution of (6.8) for \( \gamma \), assuming \( \gamma < 0 \) as observed, is

\[ \gamma = \frac{H_x}{2S} \left[ 1 + (1 + 4S)^{1/2} \right], \]

where

\[ S = H_x^2 N^2 f^2 \]

is the slope Burger number.

In addition, using (6.5a,b) and approximating \( v_z \approx \delta v/\delta z \), we obtain the estimate

\[ \delta z \approx -\delta \nu \gamma/f. \]

Expression (6.11) may be used to estimate the height \( \delta z \) of the bottom \( \Pi \approx 0 \) layer given \( \delta v \), the change of \( v \) across the layer. Further, if we assume, based on the structure of the calculated \( \psi \) fields, that the horizontal scale \( \delta x \) of the fluctuations is given by

\[ \delta x \approx -\delta \gamma^{-1}, \]

we obtain, using (6.11),

\[ \delta x = \delta v/f. \]

The scale (6.13) for \( \delta x \) is characteristic of estimates for horizontal scales in slantwise convection (e.g., Emanuel 1994, §12.3). The estimate (6.13) appears at first sight to contradict assumption (6.4c). However, the \( \delta v \) in (6.11) originates from \( v_z \) in (6.4b). Since \( v_z \approx H_x \sigma_{\theta x} \approx H_x \delta v/\delta z \approx -H_x \gamma/\gamma \) and generally \( H_x/\gamma \ll 1 \), it is not inconsistent to use (6.4c). The estimate (6.9) depends only on the initial \( N^2 \) plus known values of \( f \) and \( H_x \), whereas (6.11) and (6.13) depend on \( \delta v \), which in this situation is not known a priori and varies with time.

We apply the above scaling estimates to the day 18 fields from experiments 2, 3, and 5. For experiment 2 where \( f = 1.032 \times 10^{-4} \) s\(^{-1} \), \( N^2 = N_0^2 = 1.5 \times 10^{-4} \) s\(^{-2} \), \( H_x = -7.2 \times 10^{-3} \), and \( \delta v \approx 0.4 \) m s\(^{-1} \), we obtain \( \gamma \approx -1.48 \times 10^{-2} \), \( \delta z \approx 57 \) m, and \( \delta x \approx 4 \) km. For the shallow shelf experiment 3 where \( N^2 = N^2_0 \), \( H_x = -3.35 \times 10^{-3} \), and \( \delta v \approx 0.3 \) m s\(^{-1} \), we obtain \( \gamma \approx -2.43 \times 10^{-2} \), \( \delta z \approx 70 \) m, and \( \delta x \approx 3 \) km, while for the weak stratification experiment 5 where \( N^2 = 1/2 N_0^2 \), \( H_x = -7.2 \times 10^{-3} \), and \( \delta v \approx 0.3 \) m s\(^{-1} \), we
obtain $\gamma \approx -2.55 \times 10^{-2}$, $\delta z = 74$ m, and $\delta x \approx 3$ km. These estimates correctly predict the qualitative changes in the scales between the experiments. In addition, the estimates are in reasonable quantitative agreement with approximate scales obtained directly from the fields plotted in Figs. 12 and 14. For example, we find from experiment 2 at 150-m depth that $\gamma \approx -1 \times 10^{-2}$ to $-1.4 \times 10^{-2}$, $\delta z \approx 45$ m, $\delta x \approx 3.5$ km, from experiment 3 at 130-m depth that $\gamma \approx -3.2 \times 10^{-2}$, $\delta z \approx 55$ m, $\delta x \approx 3$ km, and from experiment 5 at 150-m depth, that $\gamma \approx -2.25 \times 10^{-2}$, $\delta z \approx 60$ m, $\delta x \approx 3$ km.

We observe that the finite amplitude slantwise convection develops energetic velocity fluctuations that extend over greater vertical scales in the experiments characterized by slope Burger numbers $S \ll 1$ compared with the experiments with $S > 1$. In this regard, the basic case experiment and experiments 8, 9, and 10 with $N^2(t = 0) = N_{bc}^2(z)$ have $S \approx 0.25 - 0.75$ based on $N_{bc}^2(z)$ values from depths between 30 and 150 m (Fig. 2). The day 18 near-bottom fluctuating motion is considerably less vigorous in experiment 6 with $S = 1.45$ and essentially absent in the steep slope experiment 4 with $S = 3$.

Although relatively short horizontal scales $\delta x$ develop in the time- and space-dependent near-bottom flow, that flow should remain consistently represented by the hydrostatic approximation in (2.1) since $\delta z/\delta x \approx -\gamma \ll 1$. It would be useful, nevertheless, to verify the validity of that approximation by comparison of these results to corresponding solutions obtained from the nonhydrostatic Boussinesq equations.

Finally, we note that, although there have been several recent interesting theoretical and model studies of bottom boundary layer behavior in stratified oceanic flows over sloping topography (e.g., Trowbridge and Lentz 1991; MacCreary and Rhines 1993; Garrett et al. 1993), the results of these studies do not seem to be directly applicable to a description of the finite amplitude slantwise convection and the associated spatially and temporally periodic separation and reattachment of the bottom boundary layer found to dominate the behavior of the near-bottom offshore flow here.

7. Summary

The numerical experiments illustrate the dynamical features of the two-dimensional response of a stratified coastal ocean at rest to downwelling-favorable wind stress over realistic topography from the Oregon continental shelf. The most notable results from the basic case experiment are the following: A downwelling front is formed. That front moves slowly offshore at about 0.4 km/day leaving behind an inshore region where the density is well mixed. The downwelling front in the density field is concentrated near the bottom, while the front in alongshore velocity extends over the full depth and is nearly vertical, separating relatively weak alongshore velocities nearshore from a coastal jet offshore. The front is the location of strong downward vertical motion from the surface to the bottom over a narrow region of horizontal scale about 1–2 km. Considerable recirculation, involving upward vertical and onshore motion, is present on the offshore side of the front. After 10 days, much of the near-bottom offshore flow from the base of the front is characterized by time- and space-dependent fluctuations that involve spatially periodic separation and reattachment of the bottom boundary layer and accompanying recirculation cells. The horizontal scale of these fluctuations varies from 2 to 4 km, while the vertical scale increases from about 30 m on day 12 to 50 m by day 18. The fluctuations in $u$ and in $q^2$ (corresponding to the regions of boundary-layer reattachment) propagate toward the coast, against the direction of the dominant offshore flow, at about 2.5 km/day. This time- and space-dependent flow has positive potential vorticity $\Pi$ and appears to be finite amplitude slantwise convection resulting from a hydrostatic symmetric instability.

Qualitatively similar features of the response flow field are found with increased wind stress in experiment 7 and with constant initial $N^2(t = 0) = N_0^2$ in experiment 2. In the experiments 3 and 4 with shallow and steep shelf topography, respectively, similar well-mixed regions inshore and downwelling fronts are formed. The near-bottom offshore flow over the shallow shelf has larger amplitude space- and time-dependent fluctuations in $\psi$ and stronger turbulence in the reattachment regions than in the basic case. Over the steep shelf, on the other hand, the offshore flow is considerably more regular, that is, distributed more uniformly with depth, and bottom boundary layer processes are relatively weak. Experiments 5 and 6 with reduced and increased initial stratification, respectively, also show generally similar behavior. Compared with experiment 2, the bottom layer is thicker and the velocity fluctuations are more energetic with weak stratification. Correspondingly, for strong stratification the bottom layer is thinner, the region of oscillating streamlines is more confined horizontally, and the velocity fluctuations are less energetic.

It is significant that several features of the downwelling response flow field that occur in the basic case experiment are also found in experiments 8, 9, and 10 with different parameterizations of the vertical turbulent viscosity and diffusivity. In all these experiments, a region where the density is well mixed vertically is found inshore. The characteristics of the downwelling vertical motion and the structure of the downwelling front differ depending on the exact parameterizations. Importantly, however, qualitatively similar time- and space-dependent fluctuations in the near-bottom offshore flow are found with the $P-P$ Richardson-number-dependent parameterization in experiment 9 and with constant eddy coefficients in experiment 10.
In experiment 8 under basic case conditions, but with the turbulence submodel replaced with the Mellor–Yamada (1982) level-2 closure scheme, the resulting fields are extremely close to those found in the basic case with the Mellor–Yamada (1982) level-2.5 closure scheme. From a comparison of these experiments, it appears that in this shelf flow problem the considerably less complex level-2 turbulence submodel gives essentially the same results as the level-2.5 closure scheme. Further investigations of the properties of turbulent parameterization schemes in numerical circulation models of coastal flow problems are needed to understand their implications and differences and the extent of their agreement with observations.

The inertial-period-averaged balance of terms in the \( v \) momentum equation (2.1c) and in the \( \sigma_{\theta} \) potential density equation (2.1d) show that nonlinear advection of both \( v \) and \( \sigma_{\theta} \) is important in the downwelling front and in the time- and space-dependent near-bottom offshore flow. In fact, all terms in the \( v \) momentum equation except horizontal diffusion, that is, \( v_{\nu}, [(\nu v), + (\omega_{\nu})], f u, \) and \( - (K_{c} p_{z}) \), play an appreciable role in the fluctuating near-bottom offshore flow.

The occurrence of the time- and space-dependent velocity fluctuations associated with slantwise convection of the near-bottom offshore flow appears to be a robust feature of the downwelling response. These fluctuations are found to extend over greater vertical scales in experiments characterized by slope Burger numbers \( S \leq 1 \) compared to the experiments with \( S > 1 \). Scaling arguments for the behavior of this flow based on assumptions (6.4a,b,c) and (6.7) give estimates for \( \gamma \) (6.8), \( \delta z \) (6.11), and \( \delta x \) (6.12) in reasonable agreement with values obtained from the solutions. The qualitative features of the process appear to be generally independent of the method of parameterization for the vertical turbulent mixing, as long as the eddy coefficients remain reasonably small. An obvious question arises as to the nature of the stability of these features to disturbances that include variations in the alongshore direction. Future work that addresses three-dimensional behavior is planned.

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**APPENDIX A**

**Turbulence Submodel**

The stability coefficients \( S_M \) and \( S_H \) in section 2 are given (Mellor and Yamada 1982; Galperin et al. 1988) by the solutions to

\[
S_M[1 - 9 A_1 A_2 G_H] - S_M G_H (18 A_1^2 + 9 A_1 A_2) = A_1 [1 - 3 C_1 - 6 A_1 B_1^{-1}],
\]

\[
S_H[1 - (3 A_2 B_2 + 18 A_1 A_2) G_H] = A_2 [1 - 6 A_1 B_1^{-1}]
\]

(2.1a)

(2.1b)

where

\[
G_H = \frac{l^2 g}{(\eta - z)^{-1} + (H + z)^{-1}} \partial \sigma / \partial \sigma,
\]

\[
G_H = |\partial \sigma / \partial \sigma|, 0.028.
\]

(A.1c,d)

In (2.4),

\[
S_M = 0.41 S_M \quad \text{so that} \quad \tilde{K}_Q = 0.41 \tilde{K}_M
\]

(A.2a,b)

and the wall proximity function

\[
\tilde{W} = 1 + \frac{E_2 (l / k L)^2}{2},
\]

\[
L^{-1} = (\eta - z)^{-1} + (H + z)^{-1}.
\]

(A.3a,b)

The constants \( A_1, A_2, B_1, B_2, C_1, E_1, E_2 = (0.92, 0.74, 16.6, 10.1, 0.08, 1.8, 1.33) \).

In the computations, when the solutions of (2.3) give either \( q^2 < \epsilon_1 \) or \( q^2 l < \epsilon_2 \), then both \( q^2 \) and \( q^2 l \) are set so that

\[
q^2 = \epsilon_1, \quad q^2 l = \epsilon_2,
\]

(A.4a,b)

where \( \epsilon_1 = 10^{-8} \text{ m}^2 \text{ s}^{-2}, \epsilon_2 = 10^{-8} \text{ m}^3 \text{ s}^{-2} \).

We impose an upper bound of

\[
l \leq l_{\text{max}} = 0.5 q / N,
\]

(A.5)

on the turbulence length scale following Galperin et al. (1988) (for additional discussion of this bound, see Kantha and Clayson 1994). The restriction on \( l \) (A.5) is imposed in the calculation of \( \tilde{K}_M, \tilde{K}_H \) (2.2c,d) and \( \tilde{K}_Q \) (2.4b). It is not used to directly modify the variable \( q^2 l \).

In experiment 7 we use what is essentially a Mellor–Yamada (1982) level-2 turbulence closure scheme. In that case, \( q^2 \) is obtained from the following equation, which replaces (2.3a):

\[
q^2 = B_1 l \left\{ \frac{\tilde{K}_M}{D^2} \left[ \left( \frac{\partial u}{\partial \sigma} \right)^2 + \left( \frac{\partial v}{\partial \sigma} \right)^2 \right] + \frac{g}{\rho_0} \frac{\tilde{K}_H}{D} \frac{\partial \sigma}{\partial \sigma} \right\}.
\]

(A.6a)

The length scale \( l \) is given by

\[
l = \kappa \left[ (\eta - z)^{-1} + (H + z)^{-1} \right],
\]

(A.6b)

which replaces (2.3b). The coefficients \( K_M, K_H \), and \( K_Q \) are obtained from (2.2) and (2.4) as before and the upper bound (A.5) is imposed on \( l \). In some applications of the level-2 model the length scale \( l \) may be determined in a different manner (Mellor and Yamada 1982).

**APPENDIX B**

**Other Parameterizations for \( K_M \) and \( K_H \)**

The use of formulations other than (2.2) for \( K_M \) and \( K_H \) is straightforward to implement in the Blumberg–
Mellor (1987) model. The Richardson-number-dependent parameterizations of Pacanowski and Philander (1981) are

\[
K_M = \nu_N (1 + 5 \text{Ri})^{-2} + \nu_B,
\]

\[
K_H = K_M (1 + 5 \text{Ri})^{-1} + \kappa_B, \quad (B.1a,b)
\]

where Ri is defined in (4.4), \(\nu_N = 5 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}\), \(\nu_B = 10^{-4} \text{ m}^2 \text{ s}^{-1}\), and \(\kappa_B = 10^{-5} \text{ m}^2 \text{ s}^{-1}\). These parameterizations are used in experiment 9 and are designated by the abbreviation P–P. Constant values \(K_M = K_H = 0.0005 \text{ m}^2 \text{ s}^{-1}\) are used in experiment 10.

In experiments 9 and 10, we eliminate any regions of unstable density stratification after each baroclinic mode time step by adjusting the density field in the vertical so that at points where \(\partial \sigma / \partial \sigma > 0\), \(\sigma_\theta\) is changed to give \(\partial \sigma / \partial \sigma = 0\) subject to the constraint that the depth integral of \(\sigma_\theta\) is conserved.

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