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Oversampling analog-to-digital and digital-to-analog converters are gaining more popularity in many signal processing applications. Delta-sigma modulators are used in practical applications of oversampling systems because of their apparent practical advantage over other oversampling converters in terms of insensitivity to the inevitable imperfection of the analog circuitry.

In $\Delta\Sigma$ modulators, analog integrators are always very important components and are usually modeled as ideal in real applications. However, theoretical analysis shows that the integrator nonideality due to capacitor mismatching and finite op-amp gain cause large signal-to-noise ratio degradation. The primary disadvantages of the dual-quantization and cascade modulators are that they rely on the precise cancellation of terms derived from two separate circuits, one analog and one digital, and that there are added complexities on the digital sides.

This thesis describes digital adaptive correction of nonidealities in dual-quantization and cascade modulators. The sources and effects of nonidealities in a first-order delta-sigma loop are analyzed. Simple correction schemes are presented, and theoretical SNR improvements are calculated and compared with simulation results.

# Adaptive Correction Techniques
# For Delta-Sigma A/D Converters

by

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**To my parents Tijani and Hassiba Abdennadher**

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# Adaptive Correction Techniques For Delta-Sigma A/D Converters

## 1• Introduction

An analog-to-digital converter transform an analog input signal into a digital binary code representing a quantized value of the input signal. In the conventional *Nyquist rate A/D converters*, if the analog input signal is band limited to $f_B$, the minimum sampling rate required is $2f_B$. For *Oversampled A/D converters* the analog signal is sampled at a rate much higher than the Nyquist rate. In this case the signal is sampled at $OSR^* f_B$ where OSR is the oversampling ratio. One of the important advantage of oversampled A/D converters is that the requirement for analog pre-filtering is relaxed, where as Nyquist rate converters call for analog filters with sharp transition region to prevent aliasing.

Among oversampled A/D converters, delta-sigma modulators are gaining more popularity in many signal processing applications. $\Delta\Sigma$ modulators have several advantages over other oversampling converters in terms of insensitivity to the inevitable imperfection of the analog circuitry, and higher signal-to-noise ratio (SNR).

The basic $\Delta\Sigma$ modulator is the first-order lowpass modulator. In order to achieve a high SNR within the signal bandwidth, the order and the oversampling ratio (OSR) of the delta-sigma modulator must be high. However, the OSR is limited by the maximal speed of the circuit and for higher-order modulators stability is conditional. The first effective improvement to the basic modulator, without the stability issue, was the development of the Multistage Noise Shaping (MASH) Modulator. The basic idea of the MASH modulator is to realize higher-order modulators by using cascade of lower order $\Delta\Sigma$ modulators. If the individual loops are stable, then the whole modulator is stable. The primary disadvantage of this approach is that it relies on precise cancellation of terms derived from two separate circuits, one analog and one digital. Furthermore, there is added complexity on the digital side.

Another major improvement to the basic modulator, without the linearity issue, is the development of the dual-quantization modulator, introduced by Leslie and Singh. This modulator contains a single-bit $\Delta\Sigma$ loop with a multi-bit digital forward path. It was shown theoretically that this technique can result in a higher SNR compared to the first-order $\Delta\Sigma$ modulator. Similarly, this approach requires exact cancellation of terms from analog and digital circuitry. Theoretical results showed that the analog circuitry non-ideality can cause large SNR degradation and signal distortions

Presented in this thesis is a simple and effective self-calibrating scheme to estimate the non-ideal parameters of the analog components. These parameters are then used in the digital section to cancel the first-order quantization error. The emphasis of the thesis will be on the correction of the Leslie-Singh modulator and the MASH modulator, but the general concept is applicable to any other systems that suffers from sensitivity to the imperfection of the analog circuitry.

The circuit implementation of the adaptive scheme is discussed. The digital adaptive circuit requires no multiplication. The resulting converters' structures has much higher SNR over the conventional approaches.

The organization of this thesis is as follow.

Chapter 2 discusses the existing oversampling converter architecture. It also discusses the effect of analog components non-ideality on the converters' performance. In chapter 3, the new digital correction method for the Leslie-Singh modulator is described. The self-calibration scheme is also discussed in this chapter. The theoretical error analysis is presented and verified by simulation. In Chapter 4, the adaptive scheme is applied to the MASH modulator. The performance improvement is also presented. In the conclusion, the major results of the thesis, and possible future research directions are discussed.

# 2• OVERVIEW OF ΔΣ MODULATORS AND EFFECTS OF ANALOG COMPONENTS NON-IDEALITY

In this chapter the operation of the basic structure of ΔΣ modulator will be discussed, the effect of analog circuit non-ideality in the converter will be presented.

## 2.1 Basic ΔΣ Converter Structure

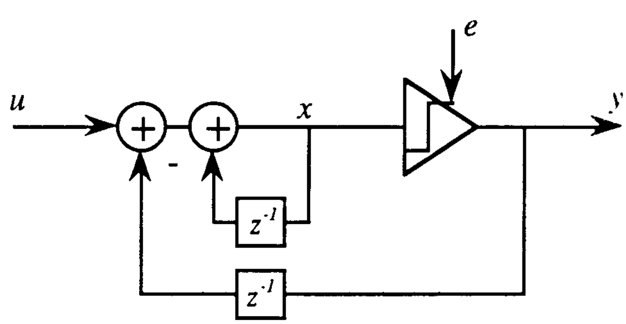


**Figure 1:** The first-order delta-sigma modulator.

The simplest ΔΣ modulator is the first-order lowpass modulator shown in Figure 1 [1]. The input signal $u$ is a discrete-time continuous-amplitude analog signal with a discrete-time digital signal output $y$. If the quantizer is modeled as an additive noise source, $y=x+e$ and the output can be expressed in the $z$-domain as

$$Y = U + H * E \tag{2.1}$$

where the noise transfer function is $H = 1 - z^{-1}$. In contrast, the signal transfer function is unity. Thus we see that the output is equal to the input signal plus an error term whose spectrum is shaped by $H$. H is clearly a high-pass filter function which tends to eliminate the quantization noise at low frequencies including the baseband. Therefore, the noise power is not present in the baseband avoiding any overlap with the signal. However, it has been shown that the error signal has a highly-colored, discrete spectrum which results in disturbing tones [2] [3].

A major shortcoming of this simple delta-sigma modulator is that it might generate low frequency tones, called pattern noise, for certain values of $u$. Methods for minimizing the tone includes: adding a small amount of dither to its input [4], using a finer quantizer with smaller levels, and a more successful solution is to uses a higher-order modulator [14].

An alternative strategy for the multi-bit modulator, without the non-linearity issue is Leslie-Singh modulator. Where as an alternative strategy for high-order noise shaping, without the stability issue of high-order feedback loop is the MASH modulator.

## 2.2 Leslie-Singh Modulator

The Leslie-Singh modulator, shown in Figure 2, consists of two stages: a first-order delta-sigma modulator with a one-bit quantizer and an $M$-bit A/D converter. The purpose of the $M$-bit A/D converter is to cancel the quantization error of the 1-bit ADC $e_1$ and replace it by the high-pass filtered quantization error $e_2$ of the $M$-bit ADC, which is $2^{M-1}$ times smaller. Analyzing the system shown in Figure 2 in the z-domain results in $Y_\epsilon = H_1 Y_1 + H_2 Y_2$ where

$$Y_1 = \frac{H_a U + E_1}{1 + z^{-1} H_a} \tag{2.2}$$

$$Y_2 = E_2 + Y_1 - E_1 = \frac{H_a U - z^{-1} H_a E_1}{1 + z^{-1} H_a} + E_2 \tag{2.3}$$

$$\Rightarrow Y_e = \frac{H_a (H_1 + H_2)}{1 + z^{-1} H_a} U + \frac{H_1 - z^{-1} H_2 H_a}{1 + z^{-1} H_a} E_1 + H_2 E_2 \tag{2.4}$$

The one-bit quantizer error $e_1$ can be eliminated from $y$ if $H_1 - z^{-1}H_2H_a = 0$ is chosen.

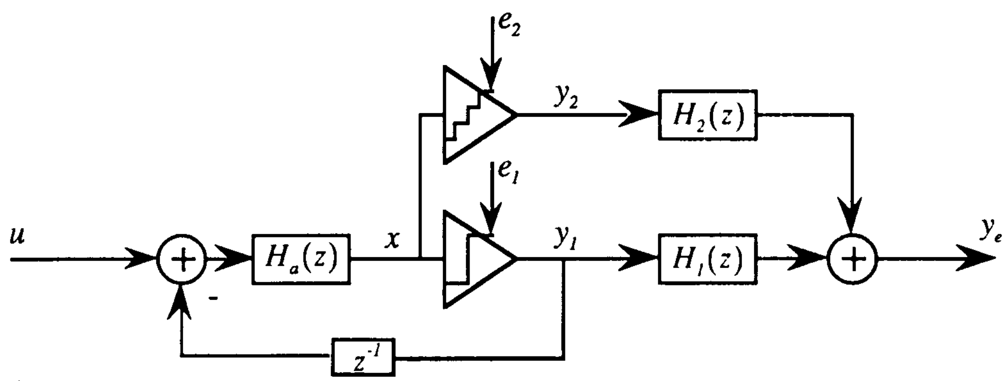


**Figure 2:** A General Leslie-Singh modulator.

For the first-order $\Delta\Sigma$ transfer function the analog loop filter is

$$H_a = \frac{z^{-1}}{1 - z^{-1}} \tag{2.5}$$

Noise cancellation of $e_1$ can be performed if $H_1 = z^{-1}$ and $H_2 = 1 - z^{-1}$. However, if the analog integrator is not ideal due to finite op-amp gain and capacitors mismatches, the cancellation of the first stage quantization noise will not be exact. Figure 3 shows the basic block diagram of the integrator in a switch-capacitor implementation. The actual first-stage analog transfer function $H_a$ of Figure 2 is

$$H_a = \frac{\alpha z^{-1}}{1 - \beta z^{-1}} \tag{2.6}$$

Where $\alpha$ is the integrator gain caused by the capacitor mismatches and $\beta$ is the integrator pole caused by the finite op-amp gain.

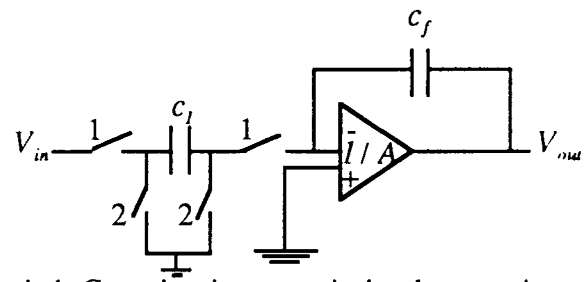


**Figure 3:** Switch-Capacitor integrator's implementation.

The output of the integrator can be described as

$$c_I(V_{in}z^{-1} + \frac{V_{out}}{A}) = -c_f(1+\frac{1}{A})(1-z^{-1})V_{out} \tag{2.7}$$

$$\frac{V_{out}}{V_{in}} = \frac{-\dfrac{c_I/c_f}{1+1/A+c_I/c_f}z^{-1}}{1-\dfrac{(1+1/A)}{(1+1/A+c_I/c_f)}z^{-1}} \tag{2.8}$$

From 2.7 and 2.8, $\alpha$ and $\beta$ could be expressed as a function of the op-amp gain and capacitor mismatch. Let $\Delta = c_I/c_f - 1$ and $\mu = 1/A$

$$\alpha = \frac{-(1+\Delta)}{(1+2\mu+\mu\Delta)} \approx \frac{c_I}{c_f} - \frac{2}{A} \tag{2.9}$$

$$\beta = \frac{(1+\mu)}{(1+2\mu+\mu\Delta)} \approx 1 - \frac{1}{A} \tag{2.10}$$

In CMOS circuit implementation the value of $|\Delta|$ is in the order of $10^{-2}$ to $10^{-3}$ and $\mu$ is in the order of $10^{-3}$ to $10^{-4}$. Hence, $0.9990 \le \beta \le 0.9999$ and $0.9880 \le \alpha \le 1.0098$. Ideally, $c_I = c_f$ and $1/A = 0$, so that $\alpha = \beta = 1$. However, due to the analog non-ideality the one-bit quantizer error $e_l$ would not be cancelled out and degrading the SNR performance of the modulator.

Figure 4 plots the simulated SNR of a first-order Leslie-Singh modulator as a function of the error gain $\alpha$ and the pole error $\beta$. The input signal is a sine wave with a peak amplitude of $A$ = 0.4472, -10dB, the oversampling ratio OSR=64, with an 8-bit ADC. The signal and in-band noise powers were determined using an FFT with a Hann-window. As can be seen from Figure 4, the SNR experiences a severe degradation if the analog integrator is not perfect. For an op-amp gain of 60 dB and capacitor tolerances of 1.5% , the SNR degrades is degraded by about 10 dB.

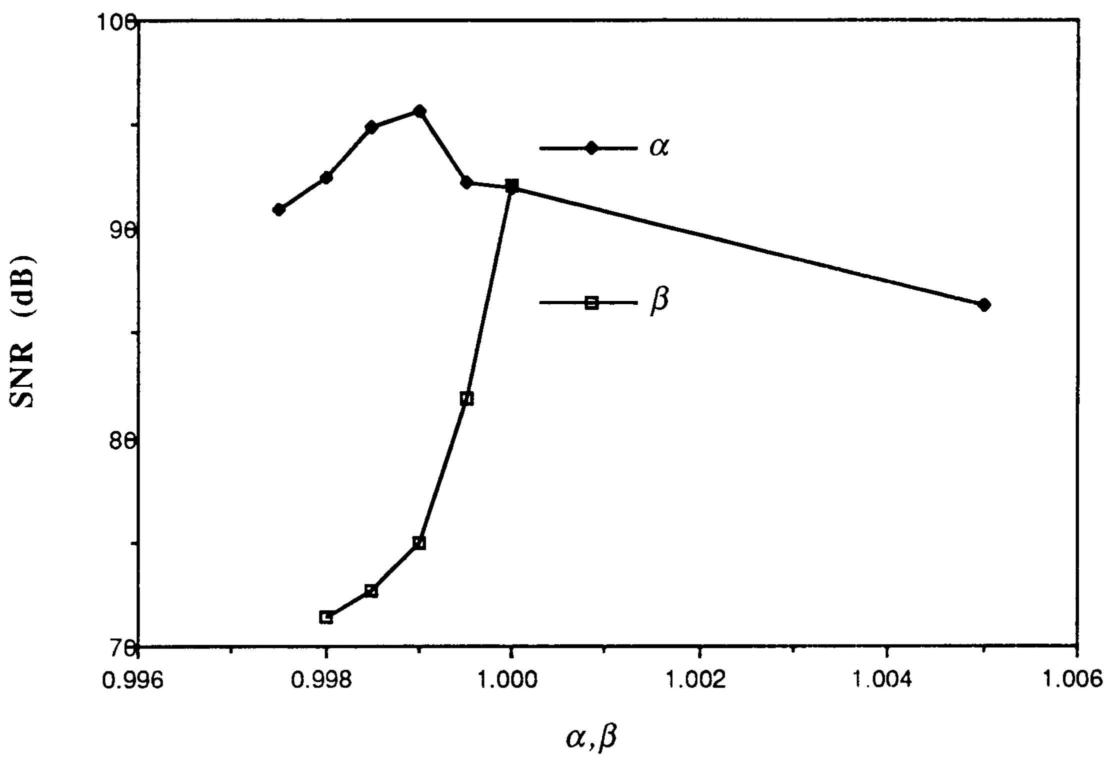


**Figure 4**: SNR degradation as a function of the pole and gain coefficients $\alpha,\beta$.

In order to improve the SNR performance of the modulator in the presence of non-ideal analog components we must improve the cancellation of $e_1$ by estimating the values of the integrator's pole $\beta$ and gain $\alpha$ and using them in the digital filter forward path. For the digital filters let $H_2 = 1-\hat{\beta}z^{-1}$ and $H_1 = \hat{\alpha}z^{-1}$. In this case, the output of the modulator is $Y_e = H_s U + H_{e_1}E_1 + H_{e_2}E_2$ where

$$H_s = \frac{\alpha\left(1+(\hat{\alpha}-\hat{\beta})z^{-1}\right)}{1+(\alpha-\beta)z^{-1}} \tag{2.11}$$

$$H_{e_1} = \frac{(\hat{\alpha}-\alpha)z^{-1}+(\alpha\hat{\beta}-\hat{\alpha}\beta)z^{-2}}{1+(\alpha-\beta)z^{-1}} \tag{2.12}$$

$$H_{e_2} = 1-\hat{\beta}z^{-1} \tag{2.13}$$

Ideally, in order to set $H_{e_1}$=0, we need $\hat{\alpha}=\alpha$ and $\hat{\beta}=\beta$. Figure 6 plots the simulated SNR of a first-order Leslie-Singh modulator as a function of $\hat{\beta}$ and $\hat{\alpha}$ where the analog parameters are $\beta$=0.9990 and $\alpha$=1.01. As shown in Figure 6, the SNR is reduced drastically for small error in the pole $\beta$ resulting from the finite op-amp gain, whereas the SNR degradation due to the capacitor mismatches is not severe. In order to achieve a high SNR, we need to find an accurate estimates of $\alpha$ and especially $\beta$.

Assuming both quantization noises $e_1$ and $e_2$ are zero-mean white noises and uncorrelated the theoretical in-band noise power $N_{e_1}$ and $N_{e_2}$ are

$$N_{e_1}^2 = \frac{\sigma_{e_1}^2(\beta-\hat{\beta})^2}{\hat{\alpha}(1+\alpha-\beta)^2}\Big[\frac{(\alpha(\hat{\beta}-\beta)(1-\alpha)+(\hat{\alpha}-\alpha)(1-\beta)(1-\alpha))^2}{OSR(1-\beta)(1-\alpha)} - \frac{\pi^2(\hat{\alpha}-\alpha)(\alpha(\hat{\beta}-\beta)-\beta(\hat{\alpha}-\alpha))}{3OSR^3}\Big] \tag{2.14}$$

$$N_{e_2}^2 = \left[\frac{(1-\beta)^2}{OSR} + \frac{\pi^2}{3OSR^3}\right] 4^{1-M} \sigma_{e_1}^2 \tag{2.15}$$

where ${\sigma_{e_1}}^2$ is the mean square value of $e_1$. $OSR$ is the oversampling ratio, and $M$ is the bit resolution of the multibit A/D. The first-stage error is proportional to $(\beta - \hat{\beta})^2$ which will be reduced drastically. The effect of the capacitor mismatches is different from the effect of the finite op-amp gain. The latter always causes a positive noise component, the former one may cause positive or negative noise component with a zero average. Figure 4 shows the SNR degradation due to the finite op-amp gain and the capacitors mismatches. If $\alpha$=0.9990 the SNR=95.69, where as ideally (i.e., $\alpha = \beta = 1$) the SNR=91.92. The objective is to estimate values of $\hat{\alpha}$ and $\hat{\beta}$ which will be described in chapter 3.

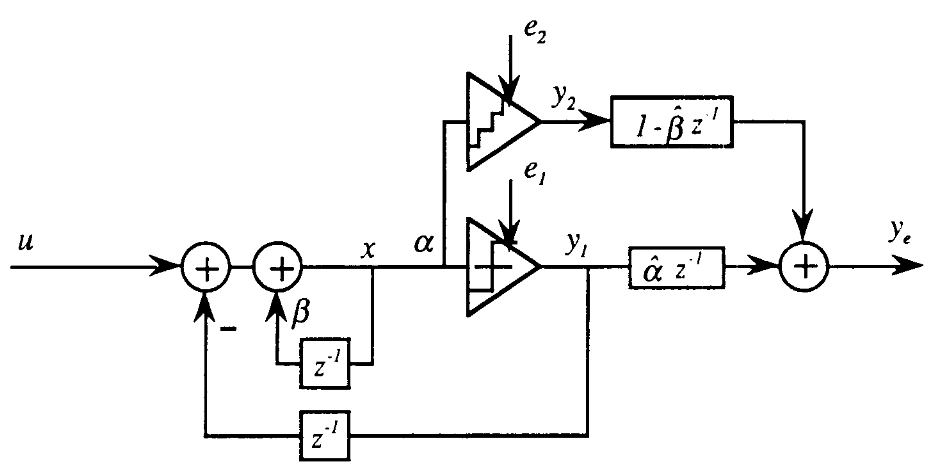


**Figure 5:** A first-order Leslie-Singh modulator with errors in the integrator transfer function and adjustable parameters in the digital transfer functions.

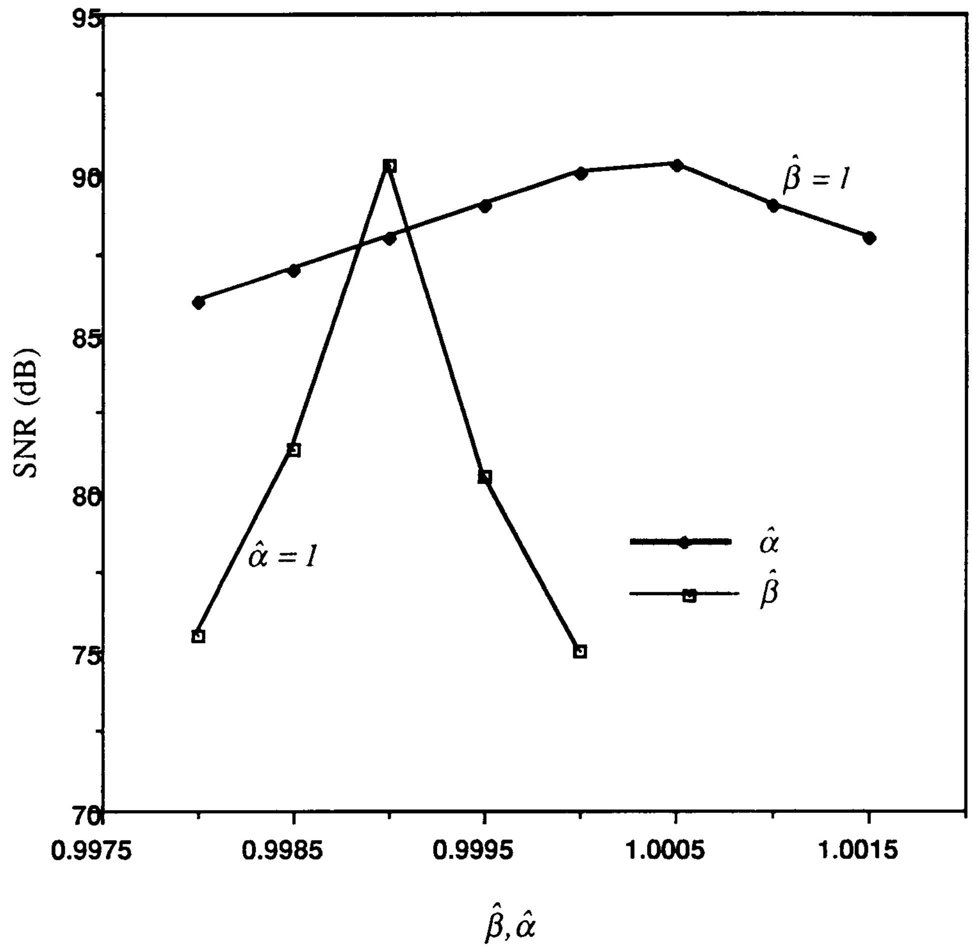


**Figure 6**: SNR degradation as a function of the estimated pole and gain coefficients $\hat{\beta}$, $\hat{\alpha}$.

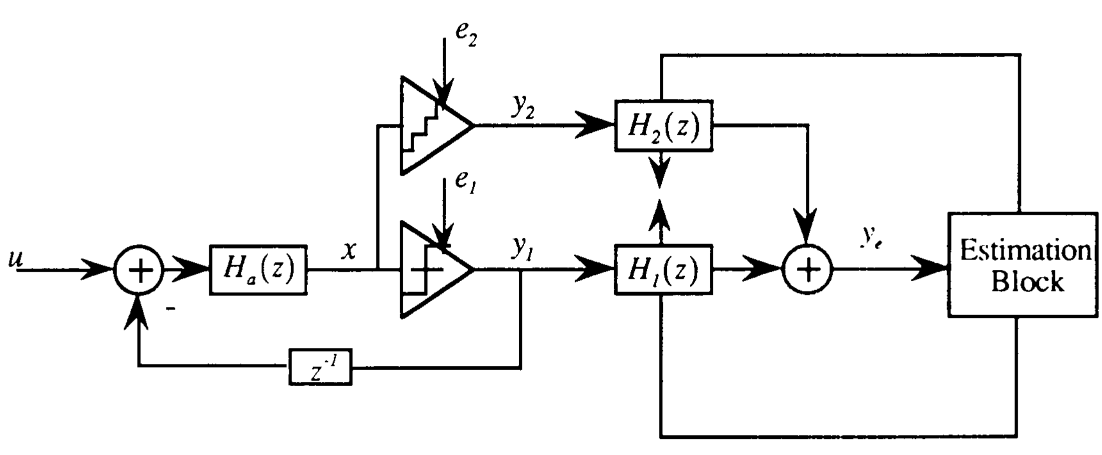


**Figure 7:** Leslie-Singh modulator with estimation block.

| Correction Residue in $\hat{\beta}_l$ | SNR |
|---|---|
| 80% | 77.09 |
| 40% | 82.48 |
| 20% | 86.66 |
| 10% | 88.94 |
| 5% | 89.79 |
| 0% | 90.14 |

**Table I:** Sensitivity of Leslie-Singh Modulator to the coefficient $\hat{\beta}_l$.

## 2.3 • MASH Modulators

Historically, the first improvement to the basic modulator was the development of the Multistage Noise Shaping (MASH) Modulator, shown in Figure 8 [5]. The basic idea of a two-stage MASH converter is to use a second $\Delta\Sigma$ modulator to digitize the error signal of the first stage. The output of the secondary modulator is digitally filtered and subtracted from the primary modulator's output to cancel the quantization noise $e_l$ leaving the quantization noise $e_2$ which has been shaped by the product of two noise-shaping functions. This approach, similar to Leslie-Singh method relies on the precise cancellation of terms derived from two separate circuits and coefficient mismatches can degrade the system performance.

Linear analysis of the system shows that the z-transform $Y_e(z)$ of the overall output signal is given by $Y_e = H_s U + H_{N_l} E_l + H_{N_2} E_2$ where the noise transfer function is

$$H_{N_1} = \frac{[(1 + z^{-1}I_2)H_1 - z^{-1}I_1I_2H_2]}{[(1 + z^{-1}I_1)(1 + z^{-1}I_2)]} \qquad (2.16)$$

Where $I_k$ and $H_m$ are the transfer functions of the various block as indicated in Figure 8. If $H_2 / H_1 = (1 + z^{-1}I_2)/(z^{-1}I_1I_2)$, then $H_{N1} = 0$ and $e_1$ will be cancelled from the output $y_e$ and $H_s = H_1$. For example, if we choose $I_1 = I_2 = 1/(1 - z^{-1})$, then $e_1$ is eliminated from the output if $H_1 = z^{-1}$ and $H_2 = 1 - z^{-1}$. In this case, the output contains only the delayed input $z^{-1}U$ and a filtered quantization noise of the second stage $(1 - z^{-1})^2 E_2$. Similarly to the first-order Leslie-Singh modulator, due to the unavoidable non-ideality in the analog integrators, the first stage quantization noise will not be perfectly cancelled. Let the actual transfer functions $I_1$ and $I_2$ in the MASH modulator be

$$I_1 = \frac{\alpha_1 z^{-1}}{1 - \beta_1 z^{-1}} \qquad (2.17)$$

$$I_2 = \frac{\alpha_2 z^{-1}}{1 - \beta_2 z^{-1}} \qquad (2.18)$$

and the digital filters with adaptive coefficients can be expressed as

$$H_2 = (1 + (1 - \hat{\beta}_2 - \hat{\beta}_1))z^{-1} + \hat{\beta}_1(\hat{\beta}_2 - 1)z^{-2}) \qquad (2.17)$$

where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the estimates of $\beta_1$ and $\beta_2$, respectively.

The first stage and the second stage output signals of the modulator shown in Figure 9 are $Y_1 = H_{s_1}U + H_{1_{e_1}}E_1$ and $Y_2 = H_{s_2}U + H_{2_{e_1}}E_1 + H_{1_{e_2}}E_2$

where

$$H_{1_{e1}} = \frac{1-\beta_1 z^{-1}}{1-(\beta_1 - 1)z^{-1}} \tag{2.18}$$

and

$$H_{2_{e1}} = \frac{-z^{-1}}{(1-(\beta_1 - 1)z^{-1})(1-(\beta_2 - 1)z^{-1})} \tag{2.19}$$

The overall output of the modulator $Y_e = H_s U + H_{e_1} E_1 + H_{e_2} E_2$ where $H_{e_1}$ is

$$H_{e_1} = [z^{-1} H_{1_{e1}} + (1 + (1 - \hat{\beta}_2 - \hat{\beta}_1)z^{-1}) + \hat{\beta}_1(\hat{\beta}_2 - 1)z^{-2})] H_{2_{e1}} \tag{2.20}$$

$$H_{e_1} = \frac{[(1-\beta_2-\beta_1)z^{-2} + \beta_1(\beta_2 - 1)z^{-3})] - [(1-\hat{\beta}_2-\hat{\beta}_1)z^{-2} + \hat{\beta}_1(\hat{\beta}_2 - 1)z^{-3})]}{(1-(\beta_1 - 1)z^{-1})(1-(\beta_2 - 1)z^{-1})}$$

Ideally, in order to set $H_{e_1}$=0 we need $\beta_1 = \hat{\beta}_1$ and $\beta_2 = \hat{\beta}_2$. Figure 10 plots the simulated SNR of a second-order MASH modulator as a function of the pole errors $\beta_1$ and $\beta_2$. The input signal is a sine wave with a peak of A=0.4472, -10dB amplitude, and the oversampling ratio OSR=64. For the MASH modulator the SNR experiences a more severe degradation from the first-stage pole error $\beta_1$ than from the second-stage pole error $\beta_2$ as shown in figure 10.

Table II shows the sensitivity of the MASH modulator to the first-stage estimate pole $\beta_1$ by examining the relative normalized error of $\hat{\beta}_1$ as $\frac{\hat{\beta}_1 - \beta_1}{1-\beta_1}$. As can been seen from the table an accurate estimate of $\beta_1$ is required to enhance the SNR performance of the MASH modulators.

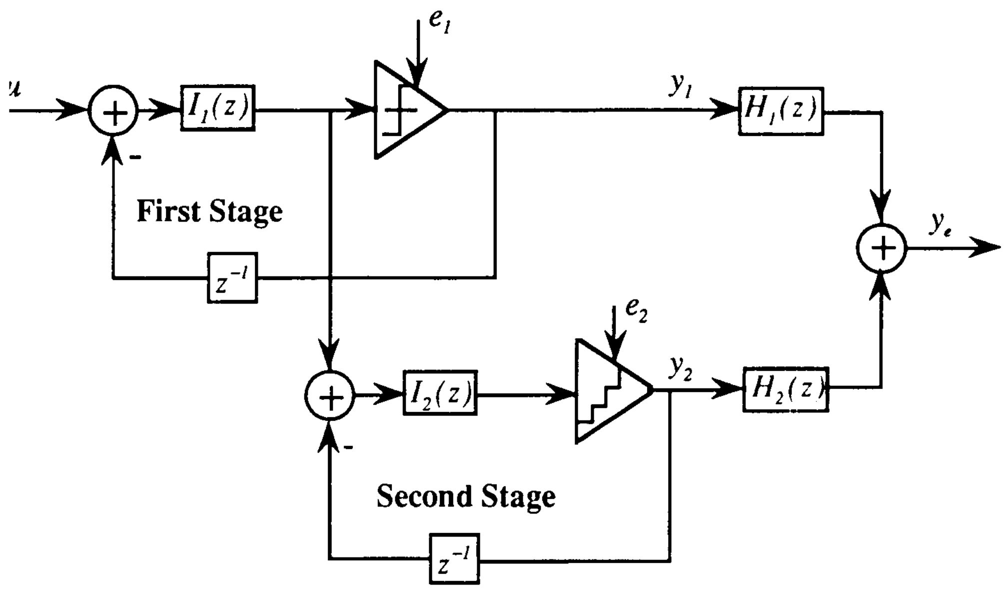


**Figure 8:** Second Order-Cascade Modulator.

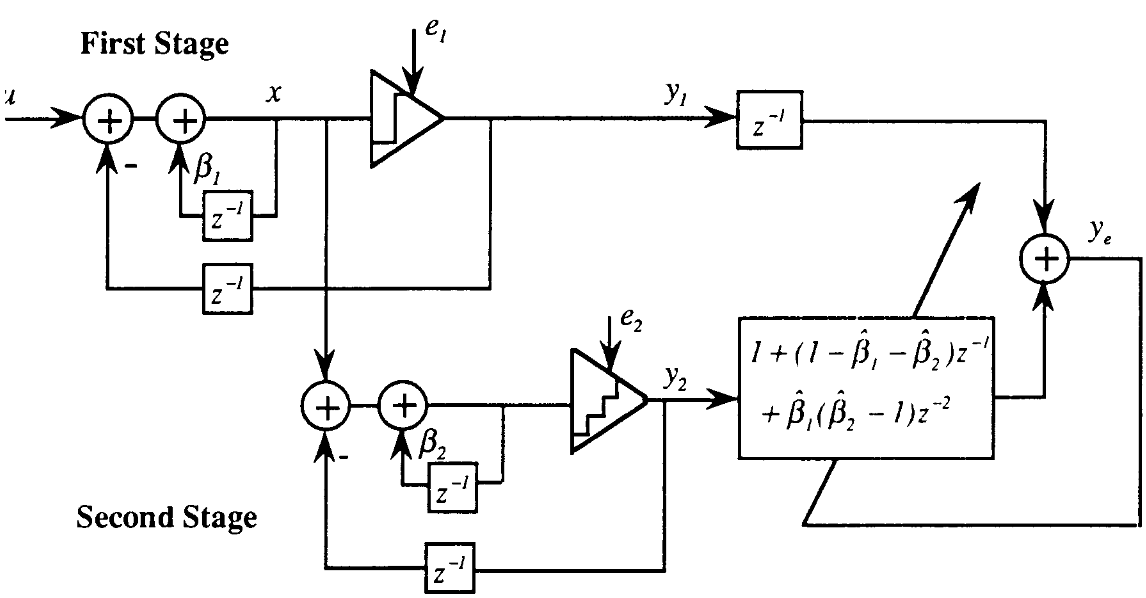


**Figure 9:** Second-order cascade Modulator with the Digital Adaptive Scheme.

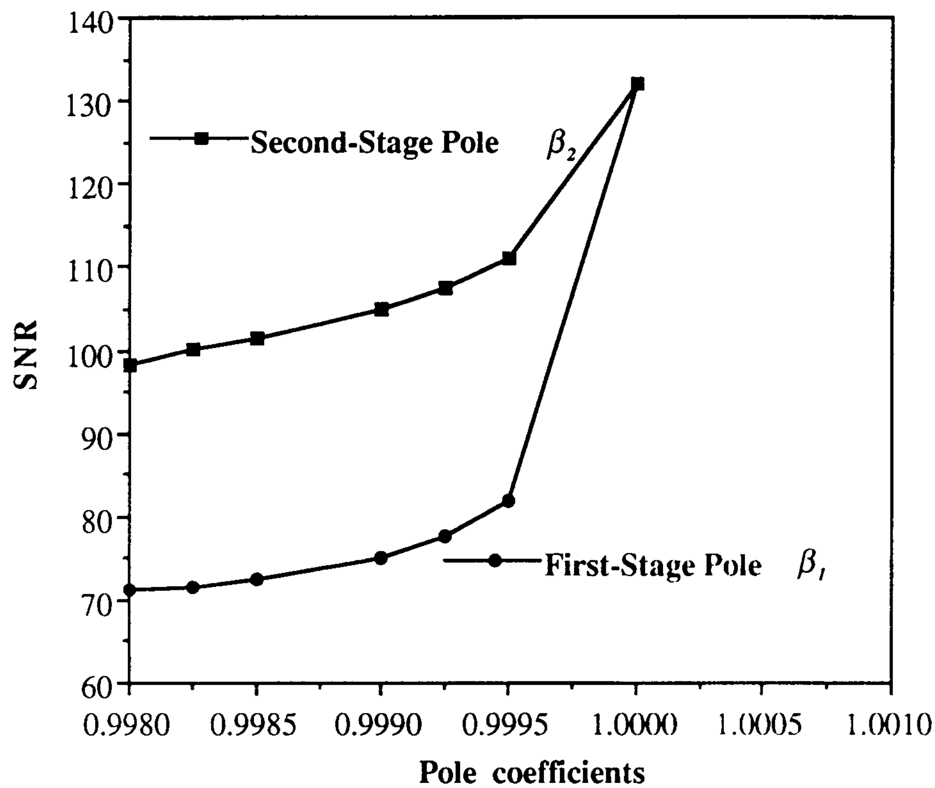


**Figure 10:** SNR degradation in the MASH modulator as a function of the first-stage and second stage poles

| Normalized error $\hat{\beta}_1$ | SNR |
|---|---|
| 10% | 95.22 |
| 5% | 101.24 |
| 2.5% | 107.26 |
| 0.3125% | 124.59 |
| 0.15625% | 128.66 |
| 0% | 131.28 |

**Table II:** Sensitivity of the MASH modulator to the op-amp coefficient $\hat{\beta}_1$

# 3• DIGITAL ADAPTIVE CORRECTION IN LESLIE-SINGH MODULATOR

In this chapter we will introduce adaptive digital processing techniques to estimate the non-ideal analog parameters of the integrator to enhance the performance of Leslie-Singh modulator.

## 3.1 Adaptive Estimation for The Leslie-Singh Modulator

### 3.1.1 Problem Formulation

In this section digital adaptive correction for the Leslie-Singh modulator will be analyzed. The objective is to use the output of the converter to estimate values of the analog components.

We can formulate our problem as a "model-matching problem" illustrated in Figure 11 where the unknown system, the *reference* system, is driven with a known input and generates an output which is corrupted by noise. The known input is passed through an adjustable system, the *model*, and the difference between its output and the corrupted output of the reference system is formed. The adaptive algorithm seeks to minimize the difference signal by adjusting the parameters of the model. For an accurate model, in the absence of noise, the difference signal can be reduced to zero and the model will match the reference.

In the context of our problem, the Leslie-Singh modulator is modified as represented in Figure 12, such that an estimate of the error of the one-bit quantizer could be determined by subtracting the output of the second stage from the output of the first stage of Leslie-Singh modulator. The unknown system is the actual noise transfer function, and the known input is an estimate of $e_1$, the error of the one-bit ADC. Both $e_2$, the error of the $M$-bit ADC and $u$, the arbitrary input, constitute disturbances. The adjustable system and the adaptive algorithm provide estimates of the $\alpha$ and $\beta$ parameters.

A method for estimating the parameters of the model is the least mean-square (LMS) method. The LMS method is a stochastic gradient algorithm that iteratevely finds the parameters $\hat{\beta}$ in the adjustable system in the direction of the negative gradient of the squared amplitude of an error signal $y_e^2$ where $y_e = (y_1' - y_2')$. The algorithm uses a gradient method where the gradient $\nabla_k$ is obtained by differentiating the mean-square error with respect to the variable parameter $\hat{\beta}$. To develop the LMS algorithm, we use $y_e^2$ itself as an estimate of $E(y_e^2)$ because the simplest choice of estimators of $E(y_e^2)$ is to use instantaneous estimates that are based on just sample values. At each iteration in the adaptive process, we have a gradient estimate $\nabla_k$. The instantaneous estimate of the gradient is determined as follow

$$\nabla_k = \frac{\delta y_e^2}{\delta \hat{\beta}} = 2 y_e \frac{\delta y_e}{\delta \hat{\beta}} \tag{3.1}$$

With the estimate of the gradient, the adaptive algorithm is

$$\hat{\beta}_{k+1} = \hat{\beta}_k - \mu \nabla_k \tag{3.2}$$

Where $\mu$ is the gain constant that regulates the speed and the stability of adaptation. It has been shown that minimizing the mean-square value of the difference signal will lead to perfect matching between $\beta$ and $\hat{\beta}$ if the disturbance is white noise and uncorrelated with the input signal [8].

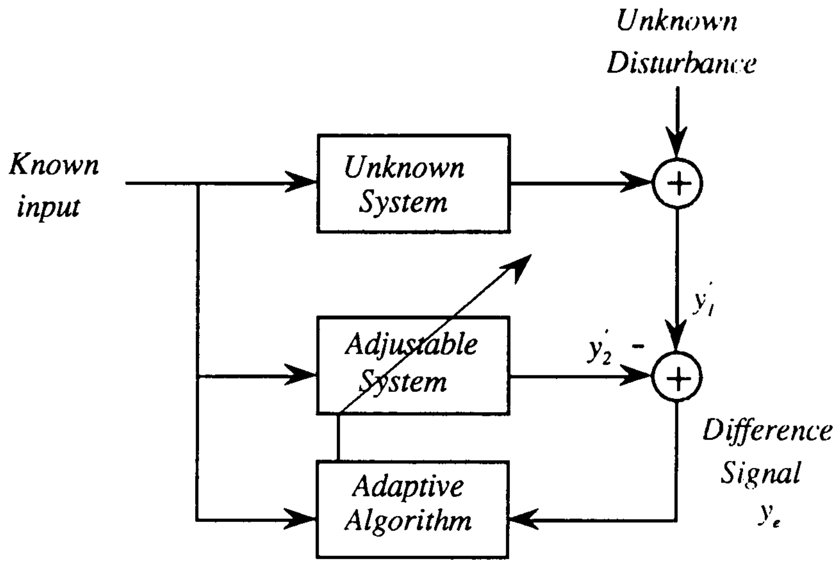


**Figure 11:** The setting of the model-matching problem of adaptive filtering.

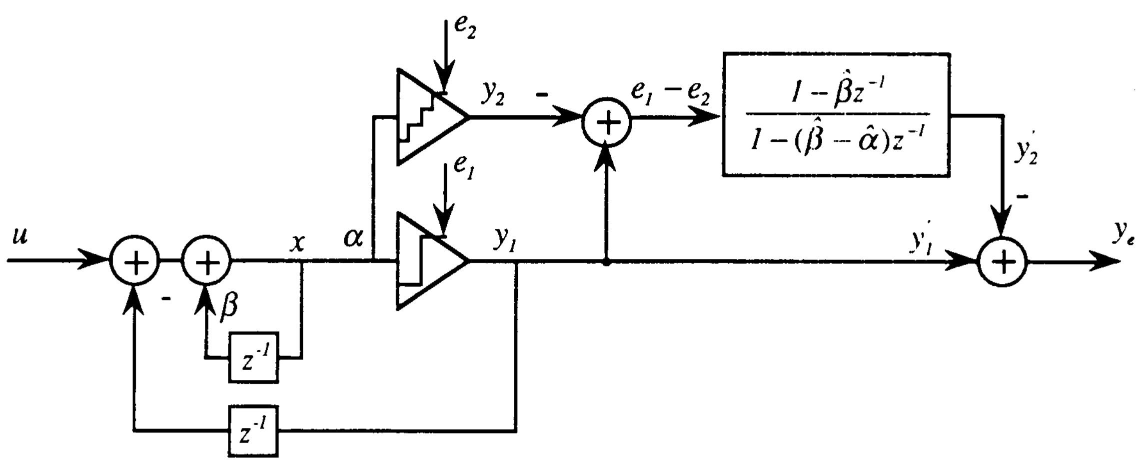


**Figure 12:** A modified dual-quantization modulator.

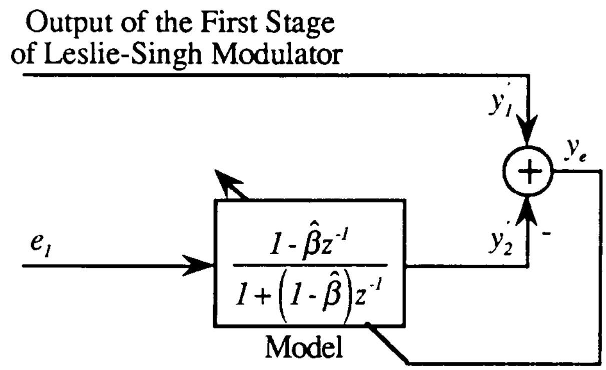


**Figure 13:** The model-matching problem for a first-order system corresponding to a first-order delta-sigma modulator with $\alpha = 1$.

From the foregoing discussion, it would appear that applying adaptive techniques to determine the noise transfer function of a delta-sigma modulator is fairly straightforward. However, it remains to be demonstrated that adequate convergence can be obtained in the face of noise. This will be shown next, for two important special cases.

### 3.1.2 LMS Adaptive Algorithm

As a first step toward verifying that adaptive algorithms are effective in the context of delta-sigma modulation, consider the system shown in Figure 13. This system corresponds to a first-order modulator wherein $\alpha = 1$. The equations for the LMS algorithm can be derived as follows. First, we examine the outputs $y_1'$ and $y_2'$ in the time domain to find an expression for the LMS error $y_e$.

The output $y_1'$ of the first stage modulator is

$$y_1'(k) = u(k) + e_1(k) - \beta e_1(k-1) + (\beta - 1)y_1'(k-1) \tag{3.3}$$

and the output of the adaptive digital section, using an infinite precision for the multibit quantizer, is

$$y_2'(k) = e_1(k) - \hat{\beta} e_1(k-1) + (\hat{\beta} - 1) y_2'(k-1) \tag{3.4}$$

defining the LMS error as $y_e(k) = y_1'(k) - y_2'(k)$.

$$\frac{\delta y_e(k)}{\delta \hat{\beta}} = e_1(k-1) - y_2'(k-1) \tag{3.5}$$

From the adaptive algorithm $\hat{\beta}_{k+1} = \hat{\beta}_k - \mu \nabla_k$ Substituting

$$\hat{\beta}_{k+1} = \hat{\beta}_k - \mu y_e(k)[e_1(k-1) - y_2'(k-1)] \tag{3.6}$$

Equation (3.4) is used to perform adaptation on the system shown in Figure 13 with the step-size parameter $\mu$ = 0.05 and with analog parameter $\beta$ = 0.999. For the simulations, three different test input signals were applied. The first test input is a white noise signal, the second test signal is set to zero (off-line method), and finally a sine wave signal was applied.

**Case I:** $u$ =white noise input signal

In the first case, where the input was white noise, $\hat{\beta}$ essentially converged to $\beta$, as predicted by theory. The small error is a result of not allowing $\mu$ approach zero sufficiently. Reducing $\mu$ by a factor of 10 would have reduced the error by a factor of 10.

**Case II:** $u$ =0 ( Off line method)

In the second case, $\hat{\beta}$ again converged to $\beta$ . This case and the prior one show that the error signal generated by Leslie-Singh modulator has sufficient spectral content to adequately excite the reference system.

**Case III:** $u$ =low frequency sine signal

In the third case, $\hat{\beta}$ again converged to $\beta$, although not with the same accuracy as before. The loss in accuracy is due to the extra "noise" present in $y_l$ : the output of $\Delta\Sigma$ modulator contain the input signal $u$ as well as the filtered error. Table III shows the value to which $\hat{\beta}$ converged and the improved SNR achieved for several situations.

| Situation | Estimated value of $\beta$ <br> $\hat{\beta}$ | Correction factor <br> $\frac{\hat{\beta}-\beta}{1-\beta}$ |
|---|---|---|
| $x$ is white noise | 0.9989995 | -.05% |
| $x$ chosen as the error generated by a $\Delta\Sigma$ modulator with $\beta$=0.9990 | 0.9989995 | -.05% |
| $y_1$ chosen as the output of a $\Delta\Sigma$ modulator with $\beta$=0.9990 | 0.99139991 | -.09% |

**Table III:** The estimates given by the LMS algorithm for the first-order system with $\alpha = 1$ and $\beta = 0.999$.

These results show that accurate knowledge of the error generated by the one-bit quantizer can be used to provide an accurate estimate of $\beta$.

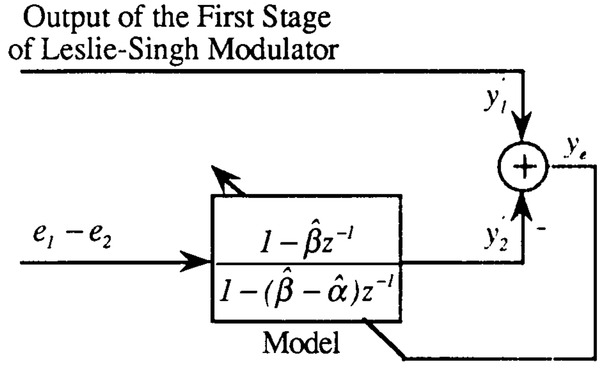


**Figure 14:** The model-matching problem for a first-order system corresponding to a Leslie-Singh modulator.

Next, we examined the effect of using a finite-precision quantizer as in Figure 2. We will consider the system shown in Figure 14 to verify that the Least Mean Square adaptive method is effective in an actual Leslie-Singh modulator.

Following the same procedure as before the update equations for $\hat{\beta}$ and $\hat{\alpha}$ are

$$\hat{\beta}_{k+1} = \hat{\beta}_k - \mu_\beta y_e(k)(e_1(k-1) - e_2(k-1) - y_2'(n-1)) \tag{3.7}$$

$$\hat{\alpha}_{k+1} = \hat{\alpha}_k - \mu_\alpha y_e(k) y_2'(k-1) \tag{3.8}$$

### 3.1.3 System Analysis and Simulation Results

Equations (3.7) and (3.8) were used to perform adaptation on the system shown in Figure 14 with different values for the step-size parameters, different quantizer word lengths, and with $\beta$ set to 0.999. It has been shown that one important criteria of LMS convergence is that the MSE should be bowl-shaped quadratic with a unique global optimum [10]. Examination of $E[y_e^2]$ as a function of $\hat{\beta}$ and $\hat{\alpha}$ shows that the error surface is not quadratic which explains the reason of the non-convergence of the update equations 3.4 and 3.8.

To get around this difficulty, we will look at a scheme, represented in Figure 15, that is suited to the filter synthesis problem. In this scheme, $1-\hat{\beta}z^{-1}$ and $(\hat{\beta}-\hat{\alpha})z^{-1}$ are adjusted separately as adaptive filters. The adaptive algorithm used to update $\hat{\beta}$ and $\hat{\alpha}$ can be derived as follow.

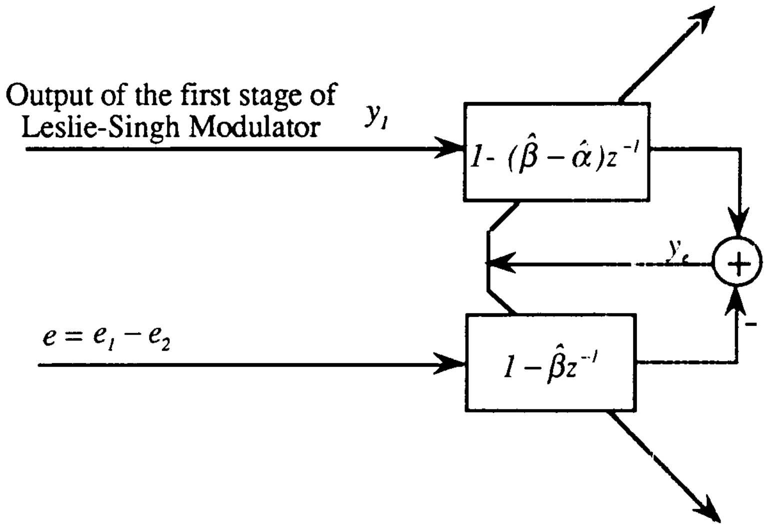


**Figure 15:** Filter synthesis by the equation error method.

$$Y_e = (1-(\hat{\beta}-\hat{\alpha})z^{-1})Y_1 - (1-\hat{\beta}z^{-1})E \tag{3.9}$$

$$\frac{\partial Y_e}{\partial \hat{\beta}} = z^{-1}(-Y_1 + E) \quad \Rightarrow \hat{\beta}_{k+1} = \hat{\beta}_k - \mu_\beta y_e(n)(e(n-1) - y_1(n-1)) \tag{3.10}$$

$$\frac{\partial Y_e}{\partial \hat{\alpha}} = z^{-1}Y_1 \qquad \Rightarrow \hat{\alpha}_{k+1} = \hat{\alpha}_k - \mu_\alpha y_e(n) y_1(n-1) \tag{3.11}$$

From equation (3.9) $Y_e$ could be expressed as follow:

$$Y_e = (1-\hat{\beta}z^{-1})(Y_1 - E) + \hat{\alpha}z^{-1}Y_1 \quad such\,that \quad Y_1 - E = Y_2 \tag{3.12}$$

So the topology represented in Figure 15 is equivalent to the digital part represented in the first-order Leslie-Singh Modulator Figure 2. The following simulation results is based on Leslie-Singh topology.

Equations (3.10) and (3.11) were used to perform adaptation on the system shown in Figure 5 with step-size parameters set to $\mu_\beta = 0.001$, $\mu_\alpha = 0.01$ and $\beta$ set to 0.999. Table

IV shows the value to which $\hat{\beta}$ converged and the correction factor achieved for several cases.

**Case I** : $u$ = White noise input signal

In the first case, $\hat{\beta}$ essentially converged to $\beta$. Appendix A shows the theoretical estimate of $\beta_e = \hat{\beta} - \beta$ . It has been shown that $\hat{\beta} - \beta$=0, if $u$ is white and uncorrelated with $e$. The small discrepancies between $\hat{\beta}$ and $\beta$ are due to the "noisy" gradient estimation.

**Case II** : $u = 0$ (Off line method)

In the second case, as predicted by theory, Appendix A, $\hat{\beta}$ again converged to $\beta$.

**Case III** : $u$ = Sine wave signal

In the third case, $\hat{\beta}$ did not converge to $\beta$ with the same accuracy as before. As predicted by theory, the loss of accuracy is due to the extra noise "$u$" and the noisy gradient estimate.

| Input Signal | pole estimates $\hat{\beta}$ | Correction factor $\frac{\hat{\beta}-\beta}{1-\beta}$ |
|---|---|---|
| u = white noise | 0.9991770 | 17.7% |
| u = 0 | 0.9991633 | 16.33% |
| u = sine wave signal | 0.9996210 | 62.1% |

**Table IV:** The estimates given by the LMS algorithm using different input signals.

### 3.1.4 SNR Performance

Simulated SNR Performance

The Signal to Noise Ratio performance of Leslie-Singh modulator is tabulated below. The multi-Bit quantizer used in the second stage of the modulator is an 8-bit quantizer , the input to the modulator is a -10 dB signal, and the Over-Sampling Ratio is 64.

| Parameters Values | Simulated SNR | Theoretical SNR | Comments |
|---|---|---|---|
| $\hat{\alpha}$=1.010 $\hat{\beta}$ =0.9990 | 90.14 | 93.76 | Perfect cancellation |
| $\hat{\alpha}$=1.000 $\hat{\beta}$=1.0000 | 74.81 | 78.93 | No Correction |
| $\hat{\alpha}$=1.015213 $\hat{\beta}$=0.999163 | 86.01 | 88.59 | Simulation Results |

Theoretical SNR Performance

If there is no correction for the op-amp phase and gain error (i.e. $H_2 = 1 - z^{-1}$ and $H_1 = z^{-1}$ in Figure 2) there will be quite large amount of uncancelled first stage error signal $e_1$ appearing in the output. Assuming both first and second stage noises, $e_1$ and $e_2$, are zero-mean white noises and uncorrelated the in-band noise power components are given by

$$N_{e_1}{}^2 = [\frac{(1-\beta)^2}{OSR(1+(\alpha-\beta))^2} - \frac{\pi^2(1-\alpha)(\alpha-\beta)}{3\ OSR^3(1+(\alpha-\beta))^2}]\sigma_{e_1}{}^2 \qquad (3.13)$$

$$N_{e_1}{}^2 = [\frac{4^{1-M}\pi^2}{3OSR^3}]\sigma_{e_1}{}^2 \qquad (3.14)$$

where $\sigma_{e_1}^{\ 2}$ is the mean square value of $e_1$. *OSR* is the oversampling ratio, and *M* is the bit resolution of the multibit A/D. Obviously, when M is small the overall SNR is dominated by the second stage order. Each bit added to the quantizer will be gained in the final SNR. But when M is larger, between 5 and 8 bits the SNR is dominated by the first stage error.

### 3.2 Implementation of the LMS Algorithm

In the digital implementation of the LMS algorithm, the adjustable filter coefficients as well as the signal levels are quantized to within a least significant digits. By doing so we are introducing a new source of error namely, *quantization* error.

Throughout, this section the update equation of $\hat{\beta}$ will be the typical example to be studied, same results applies to the update equation of $\hat{\alpha}$. Each data sample in the algorithm $y_e$ and $y_2$ is represented by $B_s$=8 bits. Similarly, we assume that the filter coefficient $\hat{\beta}$ is represented by $B_p$.

In the infinite precision form, a large gain constant $\mu_\beta$ is needed to accelerate convergence, while a small step size is needed to reduce the mean squared error. When, however, the LMS algorithm is implemented digitally a decrease in the gain constant $\mu_\beta$ can actually degrade the performance. In particular, if the product $\mu_{beta} y_e(n) y_2(n-1)$ is less in magnitude than half the parameter quantizing interval, the quantized value of the product is set equal to zero and the algorithm stops making any further adjustments. When the following condition is satisfied for all values of $k$ the LMS algorithm converges.

$$|\mu_\beta y_e(k) y_2(k-1)| < 2^{-Bp-1} \tag{3.15}$$

A possible solution for avoiding the arithmetic error arising when the algorithm is implemented digitally is by using more bits to represent the parameter $\hat{\beta}$ than the data samples. However, since the hardware complexity of digital signal processing systems is directly related to the digital wordlength, it is important to limit the numbers of bits in the various digital processing elements of the modulator. This requires an error analysis at each point in the system to assure that the system implementation does not degrade the modulator performance. The representation of $\hat{\beta}$ in 26 bits seems to be the first practical solution: $\mu_{beta}$ is represented in 10 bits, $y_e(n)y_2(n-1)$ is represented in 16 bits, hence the whole product could be represented in 26 bits.

In order to determine the minimum number of bits that $\hat{\beta}$ should be represented in without degrading the performance of the modulator we should analyze its internal representation:

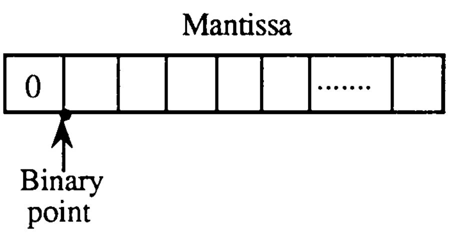


**Figure 16**: Finite arithmetic fractional wordlength

**Case 1:** If the mantissa is represented in 7 bits ($B_p$ = 8 bits), the best we can achieve for the accuracy of $\hat{\beta}$ is 0.99218750.

**Case 2:** If the mantissa is represented in 10 bits ($B_p$ = 11 bits), the best we can achieve for the accuracy of $\hat{\beta}$ is 0.999023437.

Table I shows the sensitivity of the modulator performance to the pole estimate $\hat{\beta}$. The relative normalized error of $\hat{\beta}$ should be less than 5% for an appropriate SNR enhancement. *11 bits* are the minimum number of bits that $\hat{\beta}$ should be represented in, in order to enhance the SNR performance of the modulator. In a typical example, simulation results proved that if $\hat{\beta}$ is represented in *11 bits*, the round-off errors will cause poor estimation for the analog non-ideal coefficients. Where as, if $\hat{\beta}$ is represented in *16 bits*, the update equation used to estimate $\hat{\beta}$ converges to $0.9990844$, when the analog filter coefficients are $\beta = 0.9990, \alpha = 1.01$.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

• Binary point

As mentioned in section 2.2, the range of the pole estimate is $0.9990 \le \beta \le 0.9999$ then the number of bits required for representing $\hat{\beta}$ is in the range $11 \le \#\,of\,bits \le 18$. This condition must be relaxed depending on the number of bits in the Multi-bit quantizer to reduce the introduced roundoff error.

If instead of storing values of $\hat{\beta}$, we'll store values of $1-\hat{\beta}$ during the updating of the parameters less bits will be needed to store these values and therefore a considerable hardware simplification can be achieved.

The LMS algorithm (3.10)-(3.11) requires multiplication of $y_e$ by $y_1$ and $y_2$. If 8-bit quantizer is used, an 8-bit multiply/accumulate is required to perform the update. To eliminate the multiplication involved in the adaptive algorithm a sign-data multiplier is used. Based on this, the update equations for $\hat{\beta}$ and $\hat{\alpha}$ will become

$$\hat{\beta}_{k+1} = \hat{\beta}_k + \mu_\beta y_e(k) sign(y_2(k-1)) \tag{3.16}$$

$$\hat{\alpha}_{k+1} = \hat{\alpha}_k - \mu_\alpha y_e(k) sign(y_1(k-1)) \tag{3.17}$$

| Method | $\hat{\beta}$ | $\hat{\alpha}$ | SNR | Number of Iteration | Number of Addition/ Subtraction | Number of Multiplication |
|---|---|---|---|---|---|---|
| Adaptive Sign-Data LMS | 0.9990625 | 1.015625 | 87.5 | 5420 | 5420 | None |
| Adaptive-LMS | 0.9990844 | 1.015625 | 87.5 | 5400 | 5400 | 5400 |
| No Correction | 1.0000000 | 1.000000 | 74.8 | ------------ | -------------- | ---------------- |

**Table V:** SNR performance and the number of operations involved in the new adaptive schemes.

The simulated SNR obtained using the sign-data is also plotted in Figure 17. Throughout the simulation, the iteration step sizes $\mu_\beta$ and $\mu_\alpha$ are of power 2 as $2^{-N}$ to replace the multiplication between $y_e$ and the iteration step sizes to only a shift operation(i.e. at each iteration shift the result of $y_2(k)$ by $N$ bits). Table V compares the sign-data results with the exact LMS algorithm with multipliers to the LMS algorithm. Both methods converged after approximately 5400 iterations. Using the Sign-Data method the number of operations is reduced drastically, without much reduction in the accuracy. Figure 18 shows the block diagram of the Sign-Data algorithm. Further hardware simplification can be achieved by performing post-decimation, reducing the data rate prior to the correction scheme. No high speed computation will be needed.

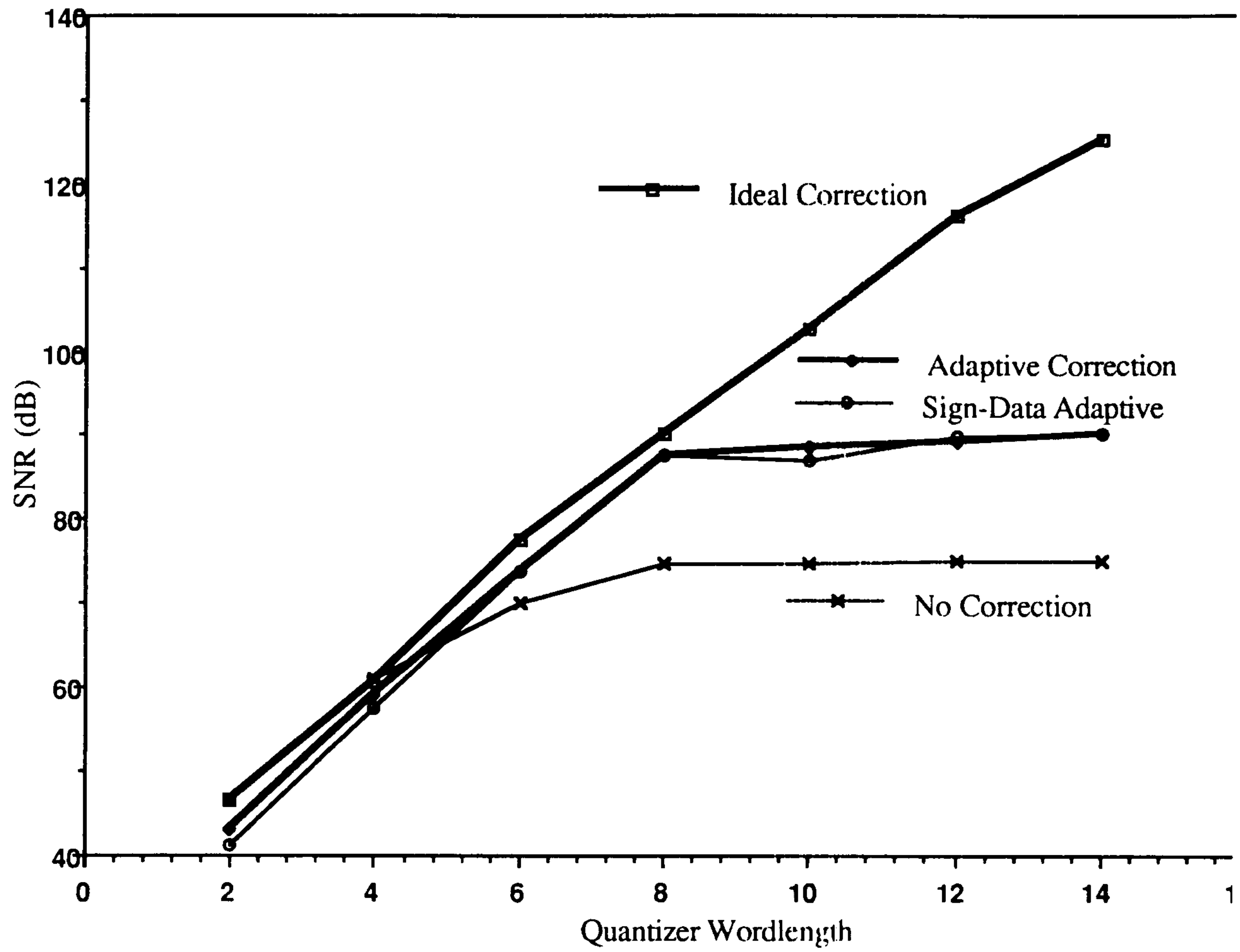


**Figure 17**: SNR plot vs. Quantizer bits of first-order Leslie-Singh modulator, -10 dB input, OSR =64, $\beta$=0.9990, $\alpha$=1.01.

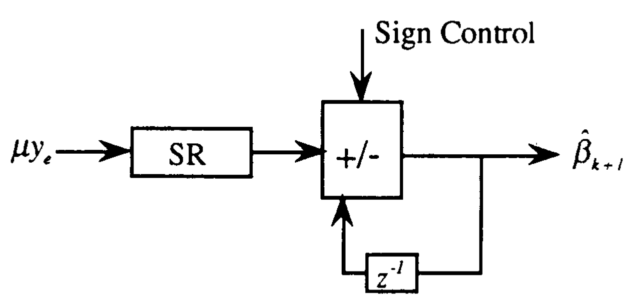


**Figure 18:** Block diagram of the digital adaptive counterpart for the op-amp gain coefficient $\hat{\beta}$.

# 4• APPLICATION OF THE ADAPTIVE ALGORITHM TO THE MASH MODULATORS

In this section, Application of the LMS algorithm to the MASH modulator is presented. Its applicability is discussed.

## 4.1 Adaptive MASH Modulator

The overall output of the MASH modulator, shown in Figure 9, in function of the first and second stage outputs can be written as

$$Y_e = z^{-1}Y_1 + (1 + (1 - \hat{\beta}_1 - \hat{\beta}_2)z^{-1} + \hat{\beta}_1(\hat{\beta}_2 - 1)z^{-2})Y_2 \tag{4.1}$$

Similar to the previous approach described in chapter 3, we use the LMS algorithm to estimate the values of the poles. The adaptive algorithm used to update $\hat{\beta}_1$ and $\hat{\beta}_2$ using the LMS method can be derived using the same procedure described in section 3.1. The gradient estimates $\nabla_{\beta_1}$ and $\nabla_{\beta_2}$ are determined by differentiating $y_e^2$ with respect to $\hat{\beta}_1$ and $\hat{\beta}_2$, respectively.

$$\frac{\delta Y_e}{\delta \hat{\beta}_1} = (\hat{\beta}_{2_k} - 1)z^{-2}Y_2 - z^{-1}Y_2 \Rightarrow \hat{\beta}_{1k+1} = \hat{\beta}_{1k} - \mu_{\beta_1} y_e(k)[(\hat{\beta}_{2_k} - 1)y_2(k-2) - y_2(k-1)] \tag{4.2}$$

$$\frac{\delta Y_e}{\delta \hat{\beta}_2} = \hat{\beta}_{1_k} z^{-2}Y_2 - z^{-1}Y_2 \quad \Rightarrow \hat{\beta}_{2k+1} = \hat{\beta}_{2k} - \mu_{\beta_2} y_e(k)[\hat{\beta}_{1_{k-1}} y_2(k-2) - y_2(k-1)] \tag{4.3}$$

## 4.2 System Analysis and Simulation Results

Equations (4.2) and (4.3) were used to perform adaptation on the system shown in Figure 9 with step-size parameters set to $\mu_{\beta_1} = 0.0001$, $\mu_{\beta_2} = 0.0001$, $\beta_1$ and $\beta_2$ set to 0.999. The multi-bit quantizer used in the second stage of the modulator is a 12-bit quantizer, the input is a -10 dB sine wave and the Over-Sampling Ratio is 64. Table VI shows the value to which $\hat{\beta}_1$ and $\hat{\beta}_2$ converged, the correction residue of $\hat{\beta}_1$ achieved, defined as $\frac{\hat{\beta}-\beta}{1-\beta}$, and the signal to noise ratio performance.

| $\hat{\beta}_1$ and $\hat{\beta}_2$ | **Correction residue** | **SNR** | **Comments** |
|---|---|---|---|
| $\hat{\beta}_1$=0.9990 $\hat{\beta}_2$=0.9990 | 0.00% | 131.82 | correction of $\hat{\beta}_1$ & $\hat{\beta}_2$ |
| $\hat{\beta}_1$=1.0000 $\hat{\beta}_2$=1.0000 | 100% | 75.20 | No correction |
| $\hat{\beta}_1$=0.9990262 $\hat{\beta}_2$=0.9984062 | 2.62% | 105.19 | Simulation Results |

**Table VI**: SNR performance of the second order MASH modulator.

The degradation in the SNR performance of a second order MASH modulator if no correction is performed on the op-amp gain coefficients, $\hat{\beta}_1$ and $\hat{\beta}_2$, is around 55 dB. If the adaptive scheme described in equations 4.2-4.3 was used 30 dB from the SNR degradation were recovered.

The LMS algorithm given by eqs. 4.5-4.6 requires the multiplication of $y_e$ by $[(\hat{\beta}_{2_k}-1)y_2(k-2)-y_2(k-1)]$ and $[\hat{\beta}_{1_{k+1}}y_2(k-2)-y_2(k-1)]$. To reduce these multiplications the update equation for $\hat{\beta}_1$ and $\hat{\beta}_2$ can be simplified by using sign-data multiplication.

$$\hat{\beta}_{1k+1} = \hat{\beta}_{1k} - \mu_{\beta_1} y_e(k) \mathrm{Sign}[(\hat{\beta}_{2_k} - 1) y_2(k-2) - y_2(k-1)] \tag{4.4}$$

$$\hat{\beta}_{2k+1} = \hat{\beta}_{2k} - \mu_{\beta_2} y_e(k) \mathrm{Sign}[\hat{\beta}_{1_{k+1}} y_2(k-2) - y_2(k-1)] \tag{4.5}$$

Further simplification can be done on the update equation of $\beta_1$ by approximating the sign of $[(\hat{\beta}_{2_k} - 1) y_2(k-2) - y_2(k-1)]$ to the sign of $[-y_2(k-1)]$. Simulation results proved that such approximation is valid and the update equation represented in eq. 4.4 is equivalent to

$$\hat{\beta}_{1k+1} = \hat{\beta}_{1k} + \mu_{\beta_1} y_e(k) \mathrm{Sign}[y_2(k-1)] \tag{4.6}$$

The simulated SNR obtained using eqs. 4.5-4.6, using different quantizer wordlength in the second stage of the MASH modulator, and representing $\hat{\beta}_1$ and $\hat{\beta}_2$ in 16 bits is plotted in Figure 19. "The saturation regions" in the adaptive correction and sign data correction graphs, when the quantizer wordlength is larger than 8, are due to the roundoff error introduced by representing the pole estimates in 16 bits. The SNR performance graphs of the adaptive and sign-data correction could easily follow the ideal correction graph by increasing the number of bits used to represent the pole estimates.

Further hardware simplification is achieved by choosing the gain constants $\mu_{\beta_1}$ and $\mu_{\beta_2}$ as power of 2 to reduce multiplication to a shift operation, performing decimation prior to the correction scheme will avoid the need for high speed multipliers.

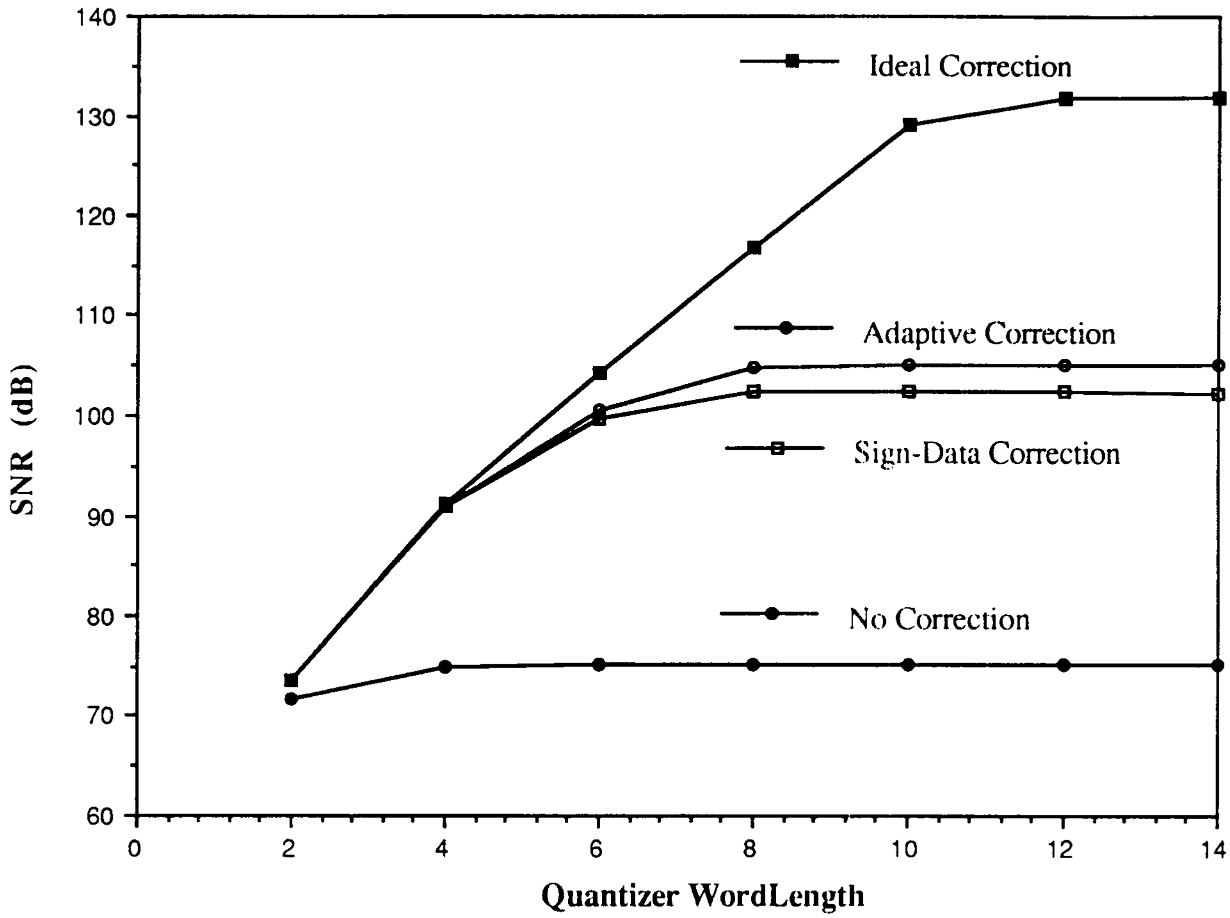


**Figure 19**: SNR plot vs. Quantizer bits of second-order cascade modulator, -10 dB input, OSR =64, $\beta_1$=0.9990, and $\beta_2$=0.9990.

Table VII shows the effect of correction and no correction in the op-amp gain coefficient $\hat{\beta}_2$. Between the simulation results and the total correction of $\hat{\beta}_2$ there is only a 1.5 dB degradation in the SNR performance of the MASH modulator.

| $\hat{\beta}_1$ and $\hat{\beta}_2$ | SNR | Comments |
|---|---|---|
| $\hat{\beta}_1$=0.9990262 $\hat{\beta}_2$=0.9984062 | 105.19 | Simulation Results |
| $\hat{\beta}_1$=0.9990262 $\hat{\beta}_2$=0.9990000 | 106.86 | Correction of $\hat{\beta}_2$ |
| $\hat{\beta}_1$=0.9990262 $\hat{\beta}_2$=1.0000000 | 102.77 | No Correction of $\hat{\beta}_2$ |

**Table VII:** Effect of correction of $\hat{\beta}_2$ on the SNR performance of the MASH modulator

The major degradation in the SNR performance of the two-stage MASH modulator is due to the error in the pole estimate $\hat{\beta}_1$. A major improvements results can be obtained if a third-order MASH modulator, realized as a cascade of a second-order and a first-order stage is used. It has been shown that a mismatch between the analog and digital transfer functions causes only first and second-order filtered quantization noise to appear in the output [17] [18]. Hence, the matching accuracy need not to be so extreme.

# 5• CONCLUSIONS AND FUTURE RESEARCH

Oversampled data converters use digital processing extensively taking advantage of the high operating speed offered by VLSI technology while being insensitive to the imprecise analog components. Certain topologies, such as Cascade and dual-quantization modulators, suffers from a severe degradation in terms of SNR performance due to these non-ideality.

An adaptive algorithm was proposed to enhance the performance of the Leslie-Singh modulator in the presence of nonideal analog components. The resulting modulator has a significantly higher signal-to-noise ratio. A simplified digital adaptive correction scheme was presented which required no multiplication. In a typical example, the SNR improvement may be about 18 dB for an op-amp with a 60 dB gain.

The self-calibrating method was applied to a second-order MASH modulators. In a typical example, the SNR improvement was about 30 dB for op-amp gains with 60 dB gains.

Although, in general, the correction procedure is quite straightforward, nevertheless, it present serious computational difficulties, especially when the filter contains a large number of parameters to be estimated and when the input data rate is high. An alternative procedure is to use decimation technique prior to the update algorithm to prevent high speed operations.

One of the important areas for future investigation is the implementation of the described self-calibration and correction method for the nonidealities of the analog components. Alternative adaptive techniques based on non-recursive algorithms have also been successfully used and will be described in a forthcoming publication.

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# APPENDIX

# APPENDIX : THEORETICAL ERROR ANALYSIS

### Convergence of the Least-Mean-Square (LMS) method

As with all adaptive algorithm, a primary concern with the LMS algorithm is its convergence to the optimum solution.

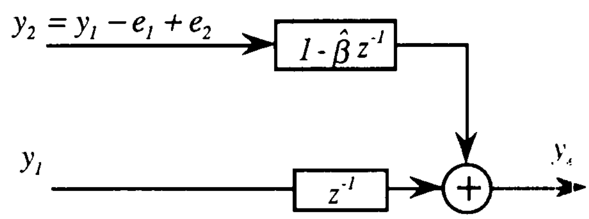


**Figure 20:** Digital adaptive block in Leslie-Singh modulator.

The adaptive process that we used is the LMS algorithm w41hich involves minimizing the mean-Square Error, $MSE = E(y_e^2)$.

The overall output signal of Leslie-Singh modulator, shown in Figure 20, can be expressed in the $z$-domain as follow

$$\begin{aligned} Y_e &= (1 + (1 - \hat{\beta})z^{-1})Y_1 - (1 - \hat{\beta}z^{-1})E_1 + (1 - \hat{\beta}z^{-1})E_2 \\ &= [(1 + z^{-1})Y_1 - E_1 + E_2] - \hat{\beta}[z^{-1}Y_1 - z^{-1}E_1 + z^{-1}E_2] \end{aligned} \qquad \text{(A.1)}$$

We now determine the mean-square error, and express it in terms of (A.1)

$$MSE = E(y_e^2)$$
$$= E([y_1(n) + y_1(n-1) - e_1(n)]^2) + \hat{\beta}E([y_1(n-1) - e_1(n-1)]^2) - \text{(A.2)}$$
$$2\hat{\beta}E([y_1(n) + y_1(n-1) - e_1(n)]*[y_1(n-1) - e_1(n-1)])$$

The LMS algorithm like many useful adaptive processes that cause the parameters to seek the minimum of the performance surface uses gradient methods. The gradient $\nabla$ can be obtained by differentiating (A.2) to obtain

$$\nabla = \frac{\delta MSE}{\delta\hat{\beta}} = 2\hat{\beta}E([y_1(n-1) - e_1(n-1)]^2) - 2E([y_1(n) + y_1(n-1) - e_1(n)]*[y_1(n-1) - e_1(n-1)])$$

To obtain the minimum mean-square error, the parameter $\hat{\beta}$ is set at its optimum value of $\hat{\beta}_{optimum}$ where the gradient is zero:

$$\frac{\delta MSE}{\delta\hat{\beta}} = 0 \quad \Rightarrow \quad \hat{\beta}_{Optimum} = \frac{E([y_1(n) + y_1(n-1) - e_1(n)]*[y_1(n-1) - e_1(n-1)])}{E([y_1(n-1) - e_1(n-1)]^2)} \tag{A.3}$$

**Estimate of $\hat{\beta}_e = \hat{\beta} - \beta$**

The output signal of the first stage in Leslie-Singh modulator, shown in Figure 5, can be expressed in the $z$- domain as follows

$$Y_1 = U' + \frac{1 - \beta z^{-1}}{1 + (1-\beta)z^{-1}}E_1 \quad \Leftrightarrow \quad Y_1 - E_1 = U' - \frac{z^{-1}}{1 + (1-\beta)z^{-1}}E_1 \tag{A.4}$$

Equation (A.4) can be expressed as

$$Y_1 - E_1 = U' - \frac{1}{z+b}E_1 \quad \text{such that } b = (1-\beta) << 1 \tag{A.5}$$

Note that $\frac{1}{z+b}E_1 \approx \frac{1}{z}E_1 - \frac{b}{z^2}E_1 + \frac{b^2}{z^3}E_1 - .....$ hence :

$$y_1(n) = u'(n) + e_1(n) - e_1(n-1) + be_1(n-2) - b^2 e_1(n-3) + \ldots.. \quad \text{(A.6)}$$

Substituting the above expression of the output signal of the first-stage in Leslie-Singh modulator in equation (A.3)

$$\hat{\beta}_{Optimum} = \frac{\begin{array}{c} E([u'(n) + be_1(n-2) - b^2 e_1(n-3) + \ldots + u'(n-1) - e_1(n-2) + \\ be_1(n-3) - b^2 e_1(n-4) + \ldots]*[u'(n-1) - e_1(n-2) + be_1(n-3) - \ldots]) \end{array}}{E([u'(n-1) - e_1(n-2) + be_1(n-3) - \ldots|^2)}$$

(A.7)

let $e'(n) = -e_1(n-2) + be_1(n-3) - b^2 e_1(n-4) + \ldots\ldots\ldots$ , then equation (A.9) is expressed as

$$\hat{\beta}_{Optimum} = \frac{E[(u'(n) + u'(n-1))(u'(n-1))] + E[(2u'(n-1) + u'(n))(e'(n))] + \beta E[e'(n)]^2}{E[u'(n-1)^2] + 2E[u'(n-1)e'(n)] + E|e'(n)|^2}$$

(A.8)

$U'$ is a mildly shaped version of $U$ that can be expressed as

$$\begin{aligned} U' &= \frac{1}{z+b} U \approx \frac{1}{z} U - \frac{b}{z^2} U + \frac{b^2}{z^3} U - \ldots\ldots. \\ U' + z^{-1} U' &\approx \frac{1}{z} U + \frac{\beta}{z^2} U - \frac{b\beta}{z^3} U + \ldots\ldots. \\ u'(n) + u'(n-1) &= u(n-1) + \beta(u(n-2) - bu(n-3) + \ldots.) \\ &= u(n-1) + \beta u'(n-1) \end{aligned} \quad \text{(A.9)}$$

Substituting the above equation into (A.8) yields

$$\hat{\beta}_{Optimum} = \frac{\beta E[(u'(n-1)^2] + E[u(n-1)u'(n-1)] + E[(2(u(n-1) + \beta u'(n-1)) - u'(n))e'(n)] + \beta E[e}{E[u'(n-1)^2] + 2E[u'(n-1)e'(n)] + E|e'(n)|^2}$$

(A.10)

$$\hat{\beta}_{Optimum} - \beta = \frac{E[u(n-1)(u(n-2) - bu(n-3)+...] + E[(u(n-1) + bu(n-2) - b^2u(n-3)+...)e'(n)]}{E[u'(n-1) + e'(n)]^2}$$

(A.11)

$\hat{\beta}_{Optimum} - \beta = 0$ if $u$ is white, and uncorrelated with $e$.

If the adaptive algorithm is performed in the off-line mode the $\hat{\beta}_{optimum}$ should converges exactly to $\beta$. However, in developing the LMS algorithm we used $y_e^2$ itself as an estimate of $MSE = E(y_e^2)$. Then at each iteration in the adaptive process, we have a gradient estimate $\nabla$.

$$\nabla = \frac{\delta y_e^2}{\delta \hat{\beta}} = 2y_e \frac{\delta y_e}{\delta \hat{\beta}} \qquad \text{(A.12)}$$

**Gradient Estimation and its Effect on Adaptation**

We now examine the effect of "noisy" gradient estimation on the parameter $\hat{\beta}$ during the adaptation process. It has been shown that the adaptation based on noisy gradient estimates results in noise in the parameter $\hat{\beta}$ and therefore a loss in performance [B. Widrow 1970].

Let us define $N_k$ as the noise in the gradient estimate at the $Kth$ iteration. Thus:

$$\tilde{\nabla}_k = \nabla_k + N_k \qquad \text{(A.13)}$$

If we assume that the LMS process, using a small step size $\mu$, has converged to a steady-state parameter solution near $\beta$ then $\nabla_k$ will be close to zero. then :

$$N_k = \tilde{\nabla}_k = -2y_e(n)y_2(n-1) \qquad \text{(A.14)}$$

Thus, due to the noisy estimate of $E(y_e^2)$ , the experimental optimum solutions are seen to be somewhat different from the theoretical solutions. Such a result is typical with the LMS algorithm.