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The purpose of this thesis is to represent and calculate unsymmetrical secondary faults on a grounded wye-to-delta transformer bank.

This thesis is specifically concerned with the secondary connections often encountered in industrial installations. Faults on these systems have not been adequately investigated.

Three types of faults are considered, namely:

1. A line-to-line fault between two phases.
2. A line-to-line fault on the center-tapped three-phase circuit.
3. A phase-to-ground fault with the opposite center-tap grounded.

Unsymmetrical faults on this system cause unsymmetrical currents and voltages. The method of symmetrical components is applied to determine these currents and voltages, and this can be considered a convenient approach to solve this kind of problem.

The above system calculating procedure was programmed in FORTRAN. The results are compared with the actual data taken in laboratory.

The calculated results and the actual system data, recorded in the laboratory, for currents, voltages and power show a close agreement; an error of $2 \%$ or less was obtained.

# Industrial Power System Faults 

by
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## DEDICATED TO MY FATHER

AND TO THE MEMORY OF MY MOTHER

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## I. INTRODUCTION

Power systems are subject to short-circuits. It is as important to determine the short-circuit currents in the system, as it is to determine the load currents when selecting circuit breakers and fuses to protect the system.

The system should be protected in such a way that a faulty or short-circuited section can be isolated from the rest of the system. This can be accomplished by the coordinated use of circuit breakers and fuses.

The proper size of circuit breakers or fuses is determined by the magnitudes of the short-circuit and load currents.

The magnitude of the short-circuit currents depends upon the energy sources, their impedances and upon the system impedance to the fault location.

Transformers are often referenced as a source of shortcircuit current, which is not true. Transformers change voltage and current magnitudes, but generate neither.

The short-circuit current delivered through a transformer depends upon its impedance and upon the impedances of the generator and the system to the terminals of the transformer.

The object of this thesis is to represent and calculate unsymmetrical faults on the delta side of a grounded wye-to-delta transformer bank. Three-phase and line-to-line type faults are adequately discussed by several authors. This thesis is specifically concerned with the transformer secondary connections often encountered in industrial installations. Faults on these systems for some transformer connections have not been adequately investigated with symmetrical components.

To accomplish this object, three types of faults are considered, namely:

1. A line-to-line fault between two phases.
2. A line-to-line fault on the center-tapped three-phase circuit.
3. A phase-to-ground fault with the opposite center-tap grounded.

Unsymmetrical faults in this system cause unsymmetrical currents to flow. To determine these currents, the method of symmetrical components is applied and it can be considered a convenient approach to solve such problems.

The behavior of the system can be determined by properly forming and interconnecting the positive and negative sequence equivalent networks of the complete system. If the unbalanced voltages and currents are resolved into two balanced sets of
components, the positive and negative sequence sets, the system then can be analyzed as a balanced circuit problem on a per-phase bases for each set. Zero sequence components are not included because there is no sequence path for current flow.

Each one of the above three cases is considered separately. The calculated results are compared with the actual system data as recorded in the laboratory.

## II. LINE-TO-LINE FAULT BETWEEN TWO PHASES

The circuit diag ram for a line-to-line fault on the secondary of a wye-delta connected bank of single-phase transformers, with the neutral point grounded, is shown in Figure l. The fault is indicated on lines $b$ and $c$ of the secondary side of the transformer.

This type of fault is an unsymmetrical fault, which causes unbalanced currents to flow in the system. The symmetrical component method is used herein, to determine the currents and voltages in all parts of the system after the occurrence of the fault. Thevenin's theorem allows us to find the current in the fault by representing the entire system as a single energy source and series impedance connected to the transformer.

The following equations, which describe the initial conditions for this kind of fault, are used to derive the circuit equations in terms of symmetrical component parameters.

$$
\left[\begin{array}{c}
\text { Vao }  \tag{1}\\
\text { Val } \\
\text { Va2 }
\end{array}\right]=\left[\begin{array}{l}
0 \\
\mathrm{EA} \\
0
\end{array}\right]-\left[\begin{array}{ccc}
\mathrm{Zo} & 0 & 0 \\
0 & \mathrm{Zl} & 0 \\
0 & 0 & \mathrm{ZZ}
\end{array}\right]\left[\begin{array}{c}
\text { Iao } \\
\text { Ial } \\
\text { Ia2 }
\end{array}\right]
$$

The initial conditions for this kind of fault can be represented by the following equations:

$$
\begin{aligned}
\mathrm{Vb} & =\mathrm{Vc} \\
\mathrm{Ia} & =0 \\
\mathrm{Ib} & =-\mathrm{Ic}
\end{aligned}
$$



Figure 1. Circuit diagram for a line-to-line fault on the secondary side of the transformer.

The symmetrical components for the voltages are given by:

$$
\left[\begin{array}{c}
\mathrm{Vao}  \tag{2}\\
\mathrm{Val} \\
\mathrm{Va2}
\end{array}\right]=\frac{1}{3}\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & \mathrm{a} & \mathrm{a}^{2} \\
1 & \mathrm{a}^{2} & \mathrm{a}
\end{array}\right]\left[\begin{array}{c}
\mathrm{Va} \\
\mathrm{Vb} \\
\mathrm{Vc}
\end{array}\right]
$$

With equation (2) we can resolve the three unsymmetrical voltage phasors into their symmetrical components. With the above initial conditions substituted into equation (2), the component voltages are:

$$
\left[\begin{array}{c}
\mathrm{Vao}  \tag{3}\\
\mathrm{Val} \\
\mathrm{Va2}
\end{array}\right]=\frac{1}{3}\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & \mathrm{a} & \mathrm{a}^{2} \\
1 & \mathrm{a}^{2} & \mathrm{a}
\end{array}\right]\left[\begin{array}{c}
\mathrm{Va} \\
\mathrm{Vb} \\
\mathrm{Vb}
\end{array}\right]
$$

Equation (3) can be written as three separate equations, as shown below:

$$
\begin{align*}
& \mathrm{Vao}=\frac{1}{3}(\mathrm{Va}+\mathrm{Vb}+\mathrm{Vb})  \tag{4}\\
& \mathrm{Val}=\frac{1}{3}\left(\mathrm{Va}+\mathrm{aVb}+\mathrm{a}^{2} \mathrm{Vb}\right)  \tag{5}\\
& \mathrm{Va} 2=\frac{1}{3}\left(\mathrm{Va}+\mathrm{a}^{2} \mathrm{Vb}+\mathrm{VVb}\right) \tag{6}
\end{align*}
$$

Equations (5) and (6) show that:

$$
\begin{equation*}
\mathrm{Va} 1=\mathrm{Va} 2 \tag{7}
\end{equation*}
$$

Equation (4) shows that no zero sequence components can exist if the sum of unbalanced phasors is zero. Since the sum of the line-to-line voltage phasors in a three-phase system is zero, zerosequence components are never present in the line voltages, regardless of the amount of unbalance. Therefore, during a line-to-line
fault, this system contains no zero sequence components and,

$$
\begin{equation*}
\mathrm{Vao}=0 \tag{8}
\end{equation*}
$$

The symmetrical components of the curreats are:

$$
\left[\begin{array}{c}
\text { Iao }  \tag{9}\\
\text { Ial } \\
\text { Ia2 }
\end{array}\right]=\frac{1}{3}\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a & a^{2} \\
1 & a^{2} & a
\end{array}\right]\left[\begin{array}{c}
\text { Ia } \\
\text { Ib } \\
I c
\end{array}\right]
$$

Substituting the initial condition, $I b=-1 c, I a=0$, into equation
(9), the symmetrical components for the currents then become:

$$
\left[\begin{array}{c}
\text { Iao }  \tag{10}\\
\text { Ial } \\
\text { Ia2 }
\end{array}\right]=\frac{1}{3}\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a & a^{2} \\
1 & a^{2} & a
\end{array}\right]\left[\begin{array}{c}
0 \\
-I c \\
\text { Ic }
\end{array}\right]
$$

Writing equation (10) in three separate equations gives:

$$
\begin{align*}
& I \mathrm{Ia}=\frac{1}{3} \quad(-\mathrm{Ic}+\mathrm{Ic})  \tag{ll}\\
& \text { Ial }=\frac{1}{3} \quad\left(-\mathrm{aIc}+\mathrm{a}^{2} \mathrm{Ic}\right)  \tag{12}\\
& \mathrm{Ia} 2=\frac{1}{3} \quad\left(-\mathrm{a}^{2} \mathrm{Ic}+\mathrm{aIc}\right) \tag{13}
\end{align*}
$$

The above equations clearly show that:

$$
\begin{align*}
& \text { Iao }=0  \tag{14}\\
& \text { Ia2 }=- \text { Ial } \tag{15}
\end{align*}
$$

and
Substituting the above equations, (7), (8), (14) and (15) into equation (l), we have:

$$
\left[\begin{array}{l}
0  \tag{16}\\
\mathrm{Val} \\
\mathrm{Val}
\end{array}\right]=\left[\begin{array}{l}
0 \\
\mathrm{EA} \\
0
\end{array}\right]-\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & \mathrm{Zl} & 0 \\
0 & 0 & \mathrm{Z2}
\end{array}\right]\left[\begin{array}{c}
0 \\
\text { Ial } \\
\text {-Ial }
\end{array}\right]
$$

Performing the matrix multiplication of equation (16), and substituting the results into the equality of equation (7), yields the following single equation:

$$
\begin{equation*}
0=\mathrm{EA}-\mathrm{Ial} \mathrm{Z} 1-\mathrm{Ial} \mathrm{Z} 2 \tag{17}
\end{equation*}
$$

Solving for Ial in equation (17) gives:

$$
\begin{equation*}
\mathrm{Ial}=\frac{\mathrm{EA}}{\mathrm{Z} 1+\mathrm{Z} 2} \tag{18}
\end{equation*}
$$

With this kind of procedure, we will be able to determine all the voltages and currents in the circuit due to the fault. From the above equations we can determine the sequence network connections that represent this fault on the system.

Since Zo does not enter into equation (18), the zero sequence network is not used in the network representation. The positive and negative sequence networks must be connected in parallel, as specified by equation (7); $\quad$ VA1 $=$ VA2.

Connecting the positive and negative sequence networks in parallel without the zero sequence network makes Ial $=-\mathrm{Ia} 2$ as specified in equation (15). The connection of the sequence networks for the line-to-line fault is shown in Figure 2.


Figure 2. Connection of the sequence networks for a line-to-line fault between lines $b$ and $c$.

The sequence diagrams of Figure 2, indicate the composition of the Thevenin's equivalent impedances for each network to be:

$$
\begin{aligned}
& \mathrm{Zl}=\mathrm{ZG1}+\mathrm{ZTAl} \\
& \mathrm{Z} 2=\mathrm{ZG} 2+\mathrm{ZTA} 2
\end{aligned}
$$

The sequence voltages equivalent to the transformer primary terminals, as indicated in Figure 2, can be calculated as follows:

$$
\begin{aligned}
& \text { Vat1 }=\mathrm{EA}-\text { Ial } * \mathrm{ZG} 1 \\
& \text { Vat2 }=\mathrm{Ia} 2 * \mathrm{ZG} 2 \\
& \text { Vat2 }=-\mathrm{Ial} * \mathrm{ZG} 2
\end{aligned}
$$

and
or

To calculate the true currents and voltages on the primary side of the transformer, the phase-shift of both currents and voltages between the primary and secondary side of the wye-delta connected transformer bank must be included. The phase-shift in a wye-delta connected transformer bank is discussed in Appendix C.

The voltages to neutral on the primary side of the transformer are:

$$
\begin{array}{ll} 
& \text { VAl }=-J(\text { Vatl }) \\
\text { and } & \text { VA2 }
\end{array}
$$

Also, the line currents on the primary side of the transformer are:

$$
\begin{aligned}
& \text { IA1 }=-J(\operatorname{Ia} 1) \\
& \text { IA2 }=+J(\operatorname{Ia} 2)
\end{aligned}
$$

The symmetrical components of the line-to-neutral voltages on the primary side of the transformer can be expressed as follows:

$$
\left[\begin{array}{c}
\mathrm{VA}  \tag{19}\\
\mathrm{VB} \\
\mathrm{VC}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & \mathrm{a}^{2} & \mathrm{a} \\
1 & \mathrm{a} & \mathrm{a}^{2}
\end{array}\right]\left[\begin{array}{c}
\mathrm{VAo} \\
\mathrm{VAl} \\
\mathrm{VA2}
\end{array}\right]
$$

Since VAo $=0$, equation (19) can be written as:

$$
\begin{aligned}
& \mathrm{VA}=\mathrm{VAl}+\mathrm{VA} 2 \\
& \mathrm{VB}=\mathrm{a}^{2} \mathrm{VA} 1+\mathrm{a} V A 2 \\
& \mathrm{VC}=\mathrm{a} V A 1+\mathrm{a}^{2} \mathrm{VA} 2
\end{aligned}
$$

The symmetrical components of the line currents are:

$$
\left[\begin{array}{l}
\text { IA }  \tag{20}\\
\text { IB } \\
I C
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a^{2} & a \\
1 & a & a^{2}
\end{array}\right]\left[\begin{array}{l}
\text { IAo } \\
\text { IA1 } \\
\text { IA2 }
\end{array}\right]
$$

Since $I A o=0$, equation (20) can be written as:

$$
\begin{aligned}
& I A=I A 1+I A 2 \\
& I B=a^{2} I A 1+a I A 2 \\
& I C=a I A 1+a^{2} I A 2
\end{aligned}
$$

The power in each phase of the primary side of the transformer can be calculated with the following equations:

$$
\begin{aligned}
& \mathrm{PA}=\mathrm{VA} * \mathrm{IA} * \cos \theta \mathrm{~A} \\
& \mathrm{~PB}=\mathrm{VB} * \mathrm{IB} * \cos \theta \mathrm{~B} \\
& \mathrm{PC}=\mathrm{VC} * \mathrm{IC} * \cos \theta \mathrm{C}
\end{aligned}
$$

Where $\theta$ is the angle between the voltage and the current in each phase. The total power for the three-phase system is:

$$
P_{\text {TOTAL }}=P A+P B+P C
$$

## Comparison of Experimental and Computer Results

In the preceding pages a method was developed to determine the values of voltage and current on the primary side of a wye-delta connected three-phase transformer bank, with the fault between lines $b$ and $c$.

A FORTRAN program was written to perform the above derived calculations for a secondary line-to-line fault.

The program listing and block diagram are shown in Appendix D. The computed results are shown on page 65. The system, as shown in Figure 12 of Appendix A, was connected in the laboratory with a zero impedance connection between terminals $b$ and $c$ of the transformer. A low-valued, three-phase voltage was applied from a wye connected laboratory generator, as indicated in Figure 12. Currents and voltages were then recorded from all parts of the circuit for various values of applied voltage. These data are shown in Table 1, on page 56.

The transformer impedances ZTAl and ZTA2, are determined in Appendix B.

The generator impedances ZGl and ZG 2 , are known from previous test data. These values are:

$$
\mathrm{ZG1}=1.0 \mathrm{ohms} \text { and } \mathrm{ZG} 2=0.3 \mathrm{ohms}
$$

The computer results on page 65 show close agreement with the
experimental values. A tabulated comparison for some of the values is shown in Table 2 on page 57. The error obtained is within $2 \%$ and can be attributed to reading laboratory data on the low end of the instrument scales.

The phasor sum of the voltages and currents on the primary side of the transformer are equal to zero.

A phasor diagram for the currents and voltages is shown in Figure 3. The relationship between the primary and secondary currents, as shown in Figure 1, are indicated below:

$$
\begin{aligned}
& \mathrm{Ibc}=\mathrm{IA} \\
& \mathrm{Iac}=\mathrm{IB} \\
& \mathrm{Iba}=\mathrm{IC}
\end{aligned}
$$

The fault current, IF, is:

$$
\mathrm{IF}=\mathrm{Ibc}+\mathrm{Iac}
$$

The magnitude of the currents were determined to have the following relationships: $\mathrm{Ibc}=2 \mathrm{Iac}=2 \mathrm{Iba}$

Then,

$$
\mathrm{IA}=2 \mathrm{Ib} \quad \text { and } \mathrm{IA}=2 \mathrm{IC}
$$

The fault current becomes:

$$
I F=3 I B \quad I F=3 I C \quad I F=1.5 I A
$$




Figure 3. Phasor diagrams, $a$ and $b$, for a line-to-line fault between two phases.

## III. A LINE-TO-LINE FAULT ON THE CENTER-TAPPED THREE-PHASE CIRCUIT

The circuit diagram for a line-to-line fault on the centertapped three-phase circuit, on the secondary side of a wye-delta connected transformer bank, with the neutral point grounded, is shown in Figure 4. The fault is indicated on lines $b^{\prime}$ and $c$.

This type of fault is an unsymmetrical fault which causes unbalanced currents to flow in the system.

The symmetrical components method is used herein, to determine the currents and voltages in all parts of the system after occurrence of the fault.

The initial conditions for this kind of fault can be represented by the following equations:

$$
\begin{aligned}
& \mathrm{Vb}^{\prime}=\mathrm{Vc}^{\prime} \\
& \mathrm{Ia}^{\prime}=0 \\
& \mathrm{Ib}^{\prime}=-\mathrm{Ic}
\end{aligned}
$$

The symmetrical components of the voltages are given in equation (2). With the above initial conditions substituted into equation (2), a solution will yield:

$$
\begin{equation*}
\mathrm{Val}^{\prime}=\mathrm{Va}^{\prime} \tag{21}
\end{equation*}
$$

With the same reasoning as applied to the first type of fault, the line-to-line fault on the center-tapped three-phase circuit contains no zero sequence components, therefore:


Figure 4. Circuit diagram for a line-to-line fault on the center-tapped three-phase circuit.

$$
\begin{equation*}
\operatorname{Vao}^{\prime}=0 \tag{22}
\end{equation*}
$$

The symmetrical components for the currents are given in equation (9). Substituting the initial conditions, $I b^{\prime}=-I c^{\prime}$ and $\mathrm{Ia}^{\prime}=0$, into equation (9) and solving, results in the following equations:

$$
\begin{align*}
& \mathrm{IaO}^{\prime}=0  \tag{23}\\
\text { and } \quad & \mathrm{Ia} 2^{\prime}=-\mathrm{Ial}^{\prime} \tag{24}
\end{align*}
$$

Substituting the above equations, (21), (22), (23) and (24), into equation (l), and substituting the solution into the equality of equation (21), yields the following single equation:

$$
\begin{equation*}
0=\mathrm{EA}-\mathrm{Ial}{ }^{\prime} * \mathrm{Zl}-\mathrm{Ia} 1^{\prime} * \mathrm{Z} 2 \tag{25}
\end{equation*}
$$

Solving for lal' in equation (25) gives:

$$
\begin{equation*}
\text { Ial' }=\frac{\mathrm{EA}}{\mathrm{Z} 1+\mathrm{Z2}} \tag{26}
\end{equation*}
$$

From the above equations we can determine the sequence network connections that represent this fault on the system.

Since Zo does not enter into equation (26), the zero sequence network is not used in the network representation. The positive and negative sequence networks must be connected in parallel to satisfy equation (21), i.e., $\operatorname{Val}^{\prime}=\operatorname{Vaz}^{\prime}$.

By connecting the positive sequence and negative sequence networks in parallel without the zero sequence network makes lal' = $-\mathrm{Ia}^{\prime}$ as specified in equation (24).

The sequence diagram of Figure 5, indicates the composition of the Thevenin's equivalent impedances for each network to be:

$$
\begin{aligned}
& \mathrm{Zl}=\mathrm{ZGl}+\mathrm{ZTBl} \\
& \mathrm{Z} 2=\mathrm{ZG} 2+\mathrm{ZTB} 2
\end{aligned}
$$

The sequence voltages equivalent to the transformer primary terminals, as indicated in Figure 5, can be calculated as follows:

$$
\begin{aligned}
& \text { Vatl }^{\prime}=\mathrm{EA}-\mathrm{Ia} \mathrm{l}^{\prime} * \mathrm{ZGl} \\
& \text { Vat2 }^{\prime}=\mathrm{Ia}^{\prime} * \mathrm{ZG} 2
\end{aligned}
$$

The line-to-neutral voltages, phase currents and power on the primary side of the transformer, can be calculated as in the first case.

## Impedance Derivation

The equivalent impedance for the wye-delta bank with a line-toline fault on the center-tapped circuit can be determined as follows:

The transformer bank connection can be represented as shown in Figure 6. A single-phase source is applied to one phase of the primary side of the transformer bank with the other two phases open.

Figure 6 shows that the transformers for phases $a, b$ and $c$ are represented by an impedance and an ideal transformer.

Assume that the transformer has Nl turns in the P winding and N2 turns in the $S$ winding. The voltages ep and es for the three transformers connected as shown in Figure 6, represent the voltages


Figure 5. Connection of the sequence networks for a line-to-line fault between lines $b^{\prime}$ and $c^{\prime}$.


Figure 6. Schematic diagram used in the impedance derivation for the second type of fault.
across the $P$ and $S$ windings of ideal transformers.
A single-phase source, EA, is applied to the primary terminals of phase a, with a line-to-line fault at the center-tap (the fault between lines $b^{\prime}$ and $c^{\prime}$ ).

The turns ratio between the $P$ and $S$ windings is $1: 1$. Assume that

$$
\begin{aligned}
& \mathrm{Zpa}=\mathrm{Zsa}=\mathrm{Z} \\
& \mathrm{Zpb}=\mathrm{Zsb} \\
& \mathrm{Zpc}=\mathrm{Zsc}
\end{aligned}
$$

Since the $S$ winding in phases $b$ and $c$ are each assumed to contain half of the total per unit impedance of a single phase transformer then:

$$
\begin{array}{lll}
\mathrm{Zpb}=\mathrm{Zsb}=\frac{1}{2} \mathrm{Z} & \text { p. u. } \\
\mathrm{Zpc}=\mathrm{Zsc}=\frac{1}{2} \mathrm{Z} & \text { p. u. }
\end{array}
$$

The following relations can be written by using the above assumptions:

For transformer b.

$$
\begin{aligned}
& \mathrm{N} 2 \mathrm{Ib}=\mathrm{N} 1 \mathrm{Ia} \\
& \mathrm{Ib}=\frac{\mathrm{N} 1}{\mathrm{~N} 2} \mathrm{Ia} \\
& \mathrm{esb}=\mathrm{Ib} \mathrm{Zsb} \\
& \mathrm{epb}=\frac{\mathrm{N} 1}{\mathrm{~N} 2} \text { esb }=\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right) \mathrm{Ib} \mathrm{Zsb}=\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Ia} \mathrm{Zsb} \\
& \mathrm{Vb}=\mathrm{epb}+\mathrm{Ia} \mathrm{Zpb}
\end{aligned}
$$

$$
\begin{aligned}
& =\left(\frac{\mathrm{Nl}}{\mathrm{~N} 2}\right)^{2} \mathrm{Ia} \mathrm{Zsb}+\mathrm{Ia} \mathrm{Zpb} \\
& =\mathrm{Ia}\left[\mathrm{Zpb}+\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Zsb}\right] \\
\mathrm{Zpsb} & =\frac{\mathrm{Vb}}{\mathrm{Ia}}=\mathrm{Zpb}+\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Zsb}
\end{aligned}
$$

Since the turns ratio is $1: 1$;

$$
Z p s b=Z p b+Z s b=\frac{1}{2} \quad Z+\frac{1}{2} \quad Z=Z
$$

where $Z p s b$ is the equivalent impedance for transformer $b$.

## Transformer c.

By following the same steps as for transformer $b$, we can determine that:

$$
\mathrm{Zpsc}=\frac{\mathrm{Vc}}{\mathrm{Ia}}=\mathrm{Zpc}+\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Zsc}
$$

Since the turns ratio is $1: l$;

$$
Z p s c=Z p c+Z s c=\frac{1}{2} Z+\frac{1}{2} \quad Z=Z
$$

where Zpsc is the equivalent impedance for transformer c .

## Transformer a:

$$
\begin{aligned}
& \mathrm{esa}=\mathrm{Ia} \mathrm{Zsa}+\mathrm{Vb}+\mathrm{Vc} \\
& \mathrm{epa}=\mathrm{Ia} \mathrm{Zpa} \\
& \mathrm{EA}=\mathrm{IA} \mathrm{Zpa}+\mathrm{esa}
\end{aligned}
$$

Since $I \mathrm{a}=\mathrm{Ib}=\mathrm{IA}$ then

$$
\begin{aligned}
& \mathrm{EA}=\mathrm{IA} \mathrm{Zpa}+\mathrm{IA}(\mathrm{Zpb}+\mathrm{Zsb})+\mathrm{IA}(\mathrm{Zpc}+\mathrm{Zsc}) \\
& \mathrm{EA}=\mathrm{IA}(\mathrm{Z} p \mathrm{pa}+\mathrm{Zsa}+\mathrm{Zpb}+\mathrm{Zsb}+\mathrm{Zpc}+\mathrm{Zsc})
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{EA}=\mathrm{IA}(\mathrm{Z}+\mathrm{Z}+\mathrm{Z}+\mathrm{Z}) \\
& \mathrm{EA}=\mathrm{IA}(4 Z) \\
& 4 Z=\frac{\mathrm{EA}}{\mathrm{IA}}
\end{aligned}
$$

Where $4 Z$ is the total equivalent impedance for the circuit. This impedance is equal to twice the impedance of a single-phase transformer.

Therefore, the equivalent impedance ZTBl and ZTB2 for the positive and negative sequence network representation for this type of fault is equal to twice the impedance of a single phase transformer.

## Impedance Calculation

The transformer impedance with a line-to-line fault at the center-tap (the fault between $b^{\prime}$ and $c^{\prime}$ ), can be determined from laboratory short-circuit data.

The recorded short-circuit data are:

$$
\begin{aligned}
& \text { VAsc }=8.7 \text { volts } \\
& \text { IAsc }=16.5 \mathrm{Amps} \\
& \text { PAsc }=94 \mathrm{Watts}
\end{aligned}
$$

From the above data, the transformer parameters can be determined.

$$
\mathrm{Ze}=\frac{8.7}{16.5}=0.528 \mathrm{ohms}
$$

$$
\begin{aligned}
& \mathrm{Re}=\frac{94}{(16.5)^{2}} 2=0.346 \mathrm{ohms} \\
& \mathrm{Xe}=\sqrt{\mathrm{Ze}^{2}-\mathrm{Re}^{2}}=0.3994 \mathrm{ohms}
\end{aligned}
$$

From which

$$
\mathrm{Ze}=0.528 \quad 49.15^{\circ}
$$

The value of the positive sequence impedance ZTBl and the negative sequence impedance $Z T B 2$ are each equal to Ze .

From the above calculations, the impedance of the trans former for this fault, is equal to twice the impedance of the single-phase transformer, as used for the phase-to-phase fault.

Comparison of Experimental and

## Computer Results

In the preceding pages a method was developed to determine the values of voltage and current on the primary side of a wye-delta connected three-phase transformer bank, with the fault between lines $b^{\prime}$ and $c^{\prime}$.

A FORTRAN program was written to perform the above calculations for a secondary line-to-line fault at the center-tap.

The program listing and block diagrams are shown in Appendix D. The computed results are shown on page 67. The system, as shown in Figure 12 of Appendix A, was connected in the laboratory with a zero impedance connection between terminals $b^{\prime}$ and $c^{\prime}$
of the transformer. A low-valued, three-phase voltage was applied from a wye-connected laboratory generator, as indicated in Figure 12. Currents and voltages were then recorded from all parts of the circuit for various values of applied voltage. These data are shown in Table 3, on page 58.

The symmetrical component sequence network connection for determining the currents and voltages associated with a system line-to-line fault are valid for either a phase-to-phase or center-tap-to-center-tap fault. The equivalent transformer impedance must be increased by a factor of two in the latter sequence representation.

The computer results on page 67 show close agreement with the experimental values. A tabulated comparison of some of the values is shown in Table 4 on page 59. The error obtained is within $1.5 \%$ and can be attributed to reading laboratory data on the low end of the instrument scales.

The phasor sum of the voltages and currents on the primary side of the transformer are equal to zero.

A phasor diagram of the currents and voltages is shown in Figure 7. The relationship between the primary and secondary currents, as shown in Figure 4, are indicated below:

$$
I^{\prime} \mathrm{b}^{2} \mathrm{cb}^{\prime}=\mathrm{IA}
$$

The fault current, IF, is:

$$
I F=I c^{\prime} b c b^{\prime}+I c^{\prime} a b^{\prime}
$$

The current

$$
\begin{aligned}
& \mathrm{Ic}^{\prime} \mathrm{ab}^{\prime}=2 \mathrm{Ic} \mathrm{c}^{\prime} \mathrm{b}^{\prime} \\
& \mathrm{IA}=2 \mathrm{IB} \\
& \mathrm{IA}=2 \mathrm{IC}
\end{aligned}
$$

The fault current magnitude in relationship to the primary currents are:

$$
\begin{aligned}
& \mathrm{IF}=3 \mathrm{IA} \\
& \mathrm{IF}=6 \mathrm{IB} \\
& \mathrm{IF}=6 \mathrm{IC}
\end{aligned}
$$


(a) Phasor diagram of the currents

(b) Phasor diagram of the voltages

Figure 7. Phasor diagrams, $a$ and $b$, for a line-to-line fault at the center-tap of a wye-delta connected transformer bank.

## IV. A PHASE-TO-GROUND FAULT WITH THE OPPOSITE CENTER-TAP GROUNDED

The circuit diagram for a phase-to-ground fault with the opposite center-tap grounded, on the secondary side of a wye-delta connected bank of single-phase transformers, with the neutral point grounded, is shown in Figure 8. The fault is indicated on lines a and $a^{\prime}$ of the secondary side of the transformer.

This type of fault is an unsymmetrical fault, which causes unbalanced currents to flow in the system. The symmetrical components method is used herein to determine the currents and voltages in all parts of the system after the occurrence of the fault. Thevenin's theorem allows us to find the current in the fault by representing the entire system as a single energy source and series impedance connected to the transformer.

The following equations which describe the initial conditions for this kind of fault are used to derive the circuit equations in terms of symmetrical component parameters.

The initial conditions for this kind of fault can be represented by the following equations:

$$
\begin{align*}
& \mathrm{Ib}=0 \\
& \mathrm{Ic}=0 \\
& \mathrm{Va}=0 \tag{27}
\end{align*}
$$



Figure 8. Circuit diagram for a phase-to-ground fault with the opposite center-tap grounded.

Substituting the above initial conditions into equation (9), the symmetrical components for the currents become:

$$
\left[\begin{array}{c}
\text { Iao }  \tag{28}\\
\text { Ial } \\
\text { Ia2 }
\end{array}\right]=\frac{1}{3}\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a & a^{2} \\
1 & a^{2} & a
\end{array}\right]\left[\begin{array}{c}
\text { Ia } \\
0 \\
0
\end{array}\right]
$$

Writing equation (28) in three separate equations gives:

$$
\begin{align*}
& \text { Iao }=\frac{1}{3} \quad \text { (Ia) }  \tag{29}\\
& \text { Ial }=\frac{1}{3} \quad \text { (Ia) }  \tag{30}\\
& \text { Ia } 2=\frac{1}{3} \quad \text { (Ia) } \tag{31}
\end{align*}
$$

The above equations clearly show that:

$$
\begin{equation*}
\mathrm{Ia} 1=\mathrm{Ia} 2=\mathrm{I} a \mathrm{o} \tag{32}
\end{equation*}
$$

Substituting the above equation (32) into equation (1), we have:

$$
\left[\begin{array}{c}
\text { Vao }  \tag{33}\\
\text { Val } \\
\text { Va2 }
\end{array}\right]=\left[\begin{array}{c}
0 \\
\text { EA } \\
0
\end{array}\right]-\left[\begin{array}{ccc}
\mathrm{Zo} & 0 & 0 \\
0 & \mathrm{Zl} & 0 \\
0 & 0 & \mathrm{Z} 2
\end{array}\right]\left[\begin{array}{c}
\text { Ial } \\
\text { Ial } \\
\text { Ial }
\end{array}\right]
$$

Performing the matrix multiplication of equation (33), and substituting the results into the equality of equation (27), yields the following single equation:

$$
\text { Vao }+ \text { Val }+\mathrm{Va} 2=- \text { Ial Zo }+\mathrm{EA}-\text { Ial Zl-Ial Z2 }
$$

Since

$$
\mathrm{Va}=\mathrm{Val}+\mathrm{Va} 2+\mathrm{Vao}=0
$$

Then

$$
\begin{equation*}
-\operatorname{Ial}(\mathrm{Zo}+\mathrm{Zl}+\mathrm{Z} 2)+\mathrm{EA}=0 \tag{34}
\end{equation*}
$$

Solving for Ial in equation (34) gives:

$$
\begin{equation*}
\text { Ial }=\frac{\mathrm{EA}}{\mathrm{Z} 1+\mathrm{Z} 2+\mathrm{Zo}} \tag{35}
\end{equation*}
$$

Zero sequence current can exist only when the circuit is arranged so that the phasor sum of the currents in the three phases is not forced to be zero. The zero sequence current can exist in the phases, on the delta side of a wye-delta connected transformer. Since the zero sequence components of the three currents are equal and in phase with each other, they will circulate around the delta, but they do not flow in the lines connected to the delta. Also, because the phasor sum of the three line-to-line voltages of a threephase system must always be zero, the existence of zero sequence components in the line-to-line voltages is not possible. Thus, even though zero sequence current can exist in the delta-connected secondaries of a transformer bank, they produce no zero sequence components in the line-to-line voltages. Therefore, during the fault, this system contains no zero sequence components.

The above equations (29), (30), (31) and (35) can be written as:

$$
\begin{align*}
& \text { Iao }=0  \tag{36}\\
& \text { Ia } 1=\mathrm{Ia} 2 \tag{37}
\end{align*}
$$

$$
\begin{equation*}
\text { Ia } 1=\frac{\mathrm{EA}}{\mathrm{Zl}+\mathrm{Z2}} \tag{38}
\end{equation*}
$$

With this kind of procedure, we will be able to determine all
the voltages and currents in the circuit due to the fault. From the above equations we can determine the sequence network connections that represent this fault on the system.

Since Zo does not enter into equation (38), the zero sequence network is not used in the network representation.

Connecting the positive sequence and negative sequence networks in series without the zero sequence network makes Ial = Ia2 as specified in equation (37). The connection of the sequence networks for a phase-to-ground fault with the opposite center-tap grounded is shown in Figure 9.

The sequence diagrams of Figure 9 indicate the composition of Thevenin's equivalent impedances for each network to be:

$$
\begin{aligned}
& \mathrm{Zl}=\mathrm{ZGl}+\mathrm{ZTCl} \\
& \mathrm{Z} 2=\mathrm{ZG} 2+\mathrm{ZTC} 2
\end{aligned}
$$

The sequence voltages equivalent to the transformer primary terminals, as indicated in Figure 9, can be calculated as follows:

$$
\begin{aligned}
& \text { Vatl }=\mathrm{EA}-(\mathrm{Ia} 1) *(\mathrm{ZGl}) \\
& \text { Vat2 }=-(\mathrm{Ia} 2) *(\mathrm{ZG} 2)
\end{aligned}
$$

To calculate the true currents and voltages on the primary side of the transformer, the phase-shift of both currents and voltages between the primary and secondary side of the wye-delta connected transformer must be included. The phase-shift in a wyedelta connected transformer is discussed in Appendix C.


Figure 9. Connection of the sequence networks for a phase-to-ground fault with the opposite center-tap grounded.

The line-to-neutral voltages, phase currents and power on the primary side of the transformer, can be calculated as in the first case.

## Impedance Derivation

The equivalent impedance for a wye-delta transformer bank with a phase-to-ground fault when the opposite center-tap is grounded can be determined. The transformer bank connection can be represented as shown in Figure 10, where the transformers in phases $b$ and a are represented by impedances and ideal transformers.

Assume that the transformers have Nl turns on the $P$ winding and N2 turns on the S winding. The voltages ep and es for the twotransformer connection, as shown in Figure l0, represent the voltages across the $P$ and $S$ windings of ideal transformers.

A single-phase source, EB, is applied to the primary terminals of phase b, with a short circuit on the secondary side of transformer a. The turns ratio between the $P$ and $S$ winding is $1: 1$.

Assume that

$$
\mathrm{Zpb}=\mathrm{Zsb}=\mathrm{Z}
$$

Since the $P$ and $S$ winding in phase a is half of the winding of a single phase transformer, it is assumed that the leakage reactance is half of the full winding.


Figure 10. Schematic diagram used in the impedance derivation for the third type of fault.

$$
\mathrm{Zpa}=\mathrm{Zsa}=\frac{1}{2} \mathrm{Z}
$$

The following relations can be written by using the above assumptions?

## Transformer a:

$$
\begin{aligned}
\mathrm{N} 2 \mathrm{Ia} & =\mathrm{Nl} \mathrm{Ib} \\
\mathrm{Ia} & =\frac{\mathrm{Nl}}{\mathrm{~N} 2} \mathrm{Ib} \\
\text { esa } & =\mathrm{Ia} \mathrm{Zsa} \\
\text { epa } & =\frac{\mathrm{Nl}}{\mathrm{~N} 2} \text { esa }=\frac{\mathrm{Nl}}{\mathrm{~N} 2} \mathrm{Ia} \mathrm{Zsa}=\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Ib} \mathrm{Zsa} \\
\mathrm{Va} & =\mathrm{epa}+\mathrm{Ib} \mathrm{Zpa} \\
\mathrm{Va} & =\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Ib} \mathrm{Zsa}+\mathrm{Ib} \mathrm{Zpa} \\
\mathrm{Va} & =\mathrm{Ib}\left(\mathrm{Zpa}+\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Zsa}\right) \\
\mathrm{Zpsa} & =\frac{\mathrm{Va}}{\mathrm{Ib}}=\mathrm{Zpa}+\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Zsa}
\end{aligned}
$$

since the turns ratio is $1: 1$, then

$$
\mathrm{Zpsa}=\mathrm{Zpa}+\mathrm{Zsa}=\mathrm{Z}
$$

where Zpsa is the equivalent impedance for transformer a.

Transformer b:

$$
\begin{aligned}
\text { esb } & =\mathrm{Ib} \mathrm{Zsb}+\mathrm{Va} \\
\text { epa } & =\mathrm{Ib} \mathrm{Zpb} \\
\mathrm{~EB} & =\mathrm{Ib} \mathrm{Zpb}+\mathrm{esb}
\end{aligned}
$$

since

$$
\mathrm{Ia}=\mathrm{Ib}=\mathrm{IB}
$$

then

$$
\begin{aligned}
& \mathrm{EB}=\mathrm{IB} \mathrm{Zpb}+\mathrm{IB} \mathrm{Zsb}+\mathrm{IB}(\mathrm{Zpa}+\mathrm{Zsa}) \\
& \mathrm{EB}=\mathrm{IB}(\mathrm{Zpb}+\mathrm{Zsb}+\mathrm{Zpa}+\mathrm{Zsa}) \\
& \mathrm{EB}=\mathrm{IB}(\mathrm{Z}+\mathrm{Z}+\mathrm{Z}) \\
& \mathrm{EB}=\mathrm{IB}(3 Z) \\
& 3 \mathrm{Z}=\frac{\mathrm{EB}}{\mathrm{IB}}=\mathrm{Zpsb}
\end{aligned}
$$

where Zpsb is the total equivalent impedance of the transformer connection as shown in Figure 10.

Figure 10 illustrates two transformers connected in cascade, one with a full primary and secondary winding and one with one-half the windings.

Therefore, the equivalent impedance, ZTCl and ZTC2, for the positive and negative sequence network representation for this type of fault is equal to 1.5 times the impedance of a single phase trans former.

## Comparison of Experimental and Computer Results

In the preceding pages, a method was developed to determine the values of voltages and currents on the primary side of a wyedelta connected three-phase transformer bank, with a phase-toground fault when the opposite center-tap is grounded.

A FORTRAN program was written to perform the above derived calculations for a phase-to-ground fault with the opposite
center-tap grounded.
The program listing and block diagrams are shown in Appendix D. The computed results are shown on page 69. The system, as shown in Figure 12 of Appendix A, was connected in the laboratory, with line a faulted to-ground, and with the opposite center-tap, $a^{\prime}$, also grounded. A low-valued, three-phase voltage was applied from a wye connected laboratory generator as indicated in Figure 12. Currents and voltages were then recorded from all parts of the circuit for various values of applied voltage. These data are shown in Table 5, on page 60.

The symmetrical component sequence network connection for determining the currents and voltages associated with a phase-toground fault with the opposite center-tap grounded, must be connected in series. The equivalent transformer impedance must be increased by a factor of 1.5 .

The computer results on page 69 show close agreement with the experimental values. A tabulated comparison for some of the values is shown in Table 6 on page 61. The error obtained is within $1.7 \%$ and can be attributed to reading laboratory data on the low end of the instrument scales.

It was determined from the laboratory data and the computer results, that the current IA is equal to zero. The currents IB and IC are equal and $180^{\circ}$ out of phase.

(b) Phasor diag ram of the voltage

Figure 11. Phasor diagrams, $a$ and $b$, for a phase-to-ground fault with the opposite center-tap grounded.

The phasor sum of the voltages and currents on the primary side of the transformer are equal to zero.

A phasor diagram for the currents and voltages is shown in Figure 11. The relationship between the primary and secondary currents, as shown in Figure 8, are indicated below:

$$
\begin{aligned}
\text { Iáca } & =\text { Iába } \\
I a^{\prime} b & =-I a^{\prime} c \\
I b a & =I c a \\
I A & =I a^{\prime} b+\left(-I a^{\prime} c\right)=0 \\
I B & =I c a \\
I C & =I b a \\
\text { then } & I B \\
& I F \\
& I I^{\prime} c a+\text { Iába }
\end{aligned}
$$

The fault current becomes:

$$
\begin{aligned}
& \mathrm{IF}=2 \mathrm{IB} \\
& \mathrm{IF}=2 \mathrm{IC}
\end{aligned}
$$

In this thesis an investigation has been made to determine the equivalent impedance representation for the calculation of faults in the secondary circuit of wye-to-delta connected transformers.

Secondary line-to-line fault calculations and their current representation have been adequately covered by other authors. This representation was verified.

Industrial systems often use a three-phase system whose supply is the center-tap positions on the delta connected secondary. The symmetrical component representation for a line-to-line fault on this system was derived and tested. The impedance representation in the positive and negative sequence networks for this transformer connection was found to be twice the value used for the phase-to-phase fault on the secondary of the standard wye-to-delta transformer bank.

The impedance representation in the positive and negative sequence networks for a fault connection from a phase conductor to ground when the opposite center-tap is grounded was derived and tested. This equivalent impedance value was determined to be 1.5 times the leakage impedance value of one of the transformers in the wye-to-delta bank.

The derived impedance values were verified by laboratory
testing. A wye-to-delta connected transformer bank was subjected to the three fault conditions. The measured currents and voltages agreed with the calculated values with an error of $2 \%$ or less.

The calculated values of voltage and current during the fault were obtained with a FORTRAN program specifically written for the three types of faults.

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APPENDICES

## APPENDIX A

## DESCRIPTION OF THE LABORATORY CIRCUIT

The circuit consists of three identical single-phase transformers connected to a three-phase alternating current generator.

The transformers are connected in a wye-delta configuration to perform as a three-phase transformer bank, with the common point on the primary side grounded. The generator was wye connected with the neutral grounded.

Each one of the three single-phase transformers has the following name-plate data except for the serial number:

Westinghouse Type E Transformer
Single phase Group 3 insulation
Serial No. 56D28535 Style 1629440-A
Voltage 240 x 480 to $120 / 240 \quad 60$ cycle 5 KVA
Continuous $150^{\circ} \mathrm{C}$ Tem rise. $\quad 1.8$ Impedance at $35^{\circ} \mathrm{C}$

Name-plate data for the AC generator:
G.E. AC Generator 3 phase

No. $307555 \quad 1200$ R PM
220/110 Volts $99 / 171$ AMPS 30 KW AT 0.8 P.F.
37.5 KVA CONT. AT $50^{\circ} \mathrm{C}$

The experiment was conducted at Oregon State University in the power system laboratory of Dearborn Hall and connected as shown in Figure 12.

A low voltage was applied from the three-phase ac generator to the primary side of the three-phase connected transformers.

Different kinds of faults were connected on the secondary terminals of the unloaded transformer and the fault data were recorded.


Figure 12. A three-phasc power system of a wye-delta transformer and an ac generator.

## APPENDIX B

## IMPEDANCE DERIVATION FOR WYE-DELTA BANK WITH A LINE-TO-LINE FAULT

Figure 13 shows a single-phase transformer represented by an impedance and an ideal transformer.

Assume that the transformer has Nl turns in the $P$ (primary winding) and $N 2$ turns in the $S$ (secondary winding). The voltages ep and es as shown in Figure 13 represent the voltages across the $P$ and $S$ windings of the ideal transformer.

Assume an ac voltage is applied to the $P$ winding, with the $S$ winding solidly short circuited.

The following relation can be written:

$$
\begin{gathered}
\mathrm{N} 2 \mathrm{Ia}=\mathrm{Nl} \mathrm{IA} \\
\mathrm{Ia}=\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right) \mathrm{IA} \\
\mathrm{es}=\mathrm{Ia} \mathrm{Zs} \\
\mathrm{ep}=\frac{\mathrm{N} 1}{\mathrm{~N} 2} \mathrm{es}=\frac{\mathrm{N} 1}{\mathrm{~N} 2} \mathrm{Ia} \mathrm{Zs}=\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{IA} \mathrm{Zs} \\
\mathrm{EA}=\mathrm{ep}+\mathrm{IA} \mathrm{Zp}=\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{IA} \mathrm{Zs}+\mathrm{IA} \mathrm{Zp} \\
\mathrm{EA}=\mathrm{IA}\left[\mathrm{Zp}+\left(\frac{\mathrm{N} 1}{\mathrm{~N} 2}\right)^{2} \mathrm{Zs}\right] \\
\mathrm{Zps}=\frac{\mathrm{EA}}{\mathrm{IA}}=\mathrm{Zp}+\left(\frac{\mathrm{Nl}}{\mathrm{~N} 2}\right)^{2} \mathrm{Zs}
\end{gathered}
$$

Where Zps is defined as the impedance between the primary and secondary circuits in ohms on the primary voltage base.


Figure 13. Circuit diagram for a short-circuit test on a single-phase transformer

Then $Z p s$ is the impedance between the $P$ and $S$ winding as measured by applying voltage to the $P$ winding with the $S$ winding short circuited.

## Impedance Calculation

To determine the impedance of the transformer, short-circuit test data were taken, and are as follows:
Vsc $=5.5$ volts
Isc $=20.8 \mathrm{Amps}$
Psc $=75 \mathrm{Watts}$

The turns ratio between the primary and the secondary windings of the transformer is $1: 1$.

From the above short circuit test data, the transformer parameters can be determined:

$$
\begin{aligned}
& \mathrm{Ze}=\frac{\mathrm{Vsc}}{\mathrm{Isc}}=\frac{5.5}{20.8}=0.264 \mathrm{ohms} \\
& \mathrm{Re}=\frac{\mathrm{Psc}}{(\mathrm{Isc})^{2}}=\frac{75}{432.64}=0.1733 \mathrm{ohms} \\
& \mathrm{Xe}=\sqrt{\mathrm{Ze}^{2}-\mathrm{Re}^{2}}=0.1997 \text { ohms } \\
& \mathrm{Zps}=\mathrm{Ze}=0.264 / 49.15^{\circ}
\end{aligned}
$$

The positive sequence impedance, ZTAl, and the negative sequence impedance, ZTA2, are used in the symmetrical component representation. They are equal to each other and have the value of Ze .

## APPENDIX C

## PHASE SHIFT IN WYE-DELTA CONNECTED TRANSFORMER

In this type of transformer connection, there is a phase shift, under balanced conditions, between the voltages on the primary side and the corresponding voltages on the secondary side of a wye-delta connected transformer. The line currents exhibit the same kind of phase shift between the two sides of the transformer.

Then, for unbalanced conditions when symmetrical components are used, a phase shift in the positive and negative sequence components of voltage and current can be expected, between the primary and secondary terminals of a wye-delta connected trans former bank.

The standard notation for the high tension terminals of a threephase transformer is marked H1, H2 and H3, and the low tension terminals are to be marked as $\mathrm{X} 1, \mathrm{X} 2$ and X 3 .

The wiring diagram for a wye-delta connected transformer is shown in Figure 14. The windings that are drawn in parallel directions are linked magnetically by being wound on the same core. The American standard for wye-delta transformers states that the voltage from Hl to neutral must lead the voltage Xl to neutral by 30 degrees, regardless of whether the wye or the delta winding is


Figure 14. Circuit diagram for a three-phase wye-delta connected transformer bank.
the high tension side. Also, the voltage at H 2 leads the voltage at X 2 by 30 degrees, and the voltage at H 3 leads the voltage at X 3 by 30 degrees. The phasor diagram for the positive sequence and negative sequence components of the voltages are shown in Figure 15, where VAl leads Vbl by 30 degrees. Inspection of the phasor diagram in Figure 15, shows that Val leads VAl by 90 degrees, and Va2 lags VA2 by 90 degrees. The phasor diagram for the current relations between the wye and delta are shown in Figure 16. Trans former theory for the current relationships show that IA and Ibc are 180 degrees out of phase if VA and Vbc are in phase. Also note that Ial leads IA1 by 90 degrees, and Ia 2 lags IA2 by 90 degrees. Then, the relationships between the symmetrical components of the line-to-neutral voltages between the primary and secondary terminals of the wye-delta transformer are:

$$
\begin{aligned}
& \text { Val }=+ \text { JVAl } \\
& \mathrm{Va} 2=-\mathrm{JVA2}
\end{aligned}
$$

And the symmetrical components of the line currents for the wyedelta transformer are:

$$
\begin{aligned}
& \text { Ial }=+ \text { JIAl } \\
& \text { Ia } 2=- \text { JIA2 }
\end{aligned}
$$



Positive sequence


Negative sequence

Figure 15. Voltage phasor diagram for a wye-delta three-phase transformer bank.


Figure 16. Current phasors of a three-phase wye-delta connected transformer bank.

Table 1. Laboratory Data for a Line-to-Line Fault Between Two Phases


APPENDIX D

Table 2. Comparison of Laboratory Data and Computer Results for a Line-to-Line Fault Between Two Phases

| Input voltage | Laboratory Data |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | $\begin{gathered} \text { VA } \\ \mathrm{V} \\ \hline \end{gathered}$ | $\begin{array}{r} \text { IA } \\ \mathrm{A} \\ \hline \end{array}$ | $\begin{aligned} & \mathrm{PA} \\ & \mathrm{~W} \\ & \hline \end{aligned}$ | $\begin{array}{r} \mathrm{VB} \\ \mathrm{~V} \\ \hline \end{array}$ | $\begin{array}{r} \text { IB } \\ \mathrm{A} \\ \hline \end{array}$ | $\begin{gathered} \mathrm{PB} \\ \mathrm{~W} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{VC} \\ \mathrm{~V} \end{gathered}$ | $\begin{array}{r} \text { IC } \\ \mathrm{A} \\ \hline \end{array}$ | $\begin{aligned} & \mathrm{PC} \\ & \mathrm{~W} \\ & \hline \end{aligned}$ |
| 6 | 1. 75 | 6.8 | 9 | 2.9 | 3.4 | 9 | 3.7 | 3.4 | 12 |
| 10 | 3 | 11.4 | 22 | 4.9 | 5.7 | 22 | 6 | 5.7 | 35 |
| 14 | 4.2 | 16.1 | 45 | 6.8 | 8.0 | 45 | 8.5 | 8 | 65 |
| 18 | 5.4 | 20.8 | 75 | 9.0 | 10.4 | 75 | 10.9 | 10.4 | 100 |

Computer Results

| 6 | 1.83 | 6.92 | 8.3 | 2.95 | 3.46 | 8.28 | 3.61 | 3.46 | 12.43 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 10 | 3.05 | 11.53 | 23.04 | 4.92 | 5.77 | 23.01 | 6.02 | 5.77 | 34.53 |
| 14 | 4.27 | 16.14 | 45.17 | 6.89 | 8.07 | 45.10 | 8.42 | 8.07 | 67.68 |
| 18 | 5.49 | 20.76 | 74.66 | 8.86 | 10.38 | 74.55 | 10.83 | 10.38 | 111.89 |

Table 3. Laboratory Data for a Line-to-Line Fault on the Center-Tapped Three-Phase Circuit

| Input voltage | Primary side |  |  |  |  |  |  |  |  | Secondary side |  |  | IN <br> A |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | $\begin{aligned} & \mathrm{VA} \\ & \mathrm{~V} \\ & \hline \end{aligned}$ | $\begin{array}{r} \text { IA } \\ \text { A } \\ \hline \end{array}$ | $\begin{gathered} \mathrm{PA} \\ \mathrm{~W} \\ \hline \end{gathered}$ | $\begin{aligned} & \text { VB } \\ & \mathrm{V} \\ & \hline \end{aligned}$ | $\begin{array}{r} \text { IB } \\ \text { A } \end{array}$ | $\begin{gathered} \mathrm{PB} \\ \mathrm{~W} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{VC} \\ \mathrm{~V} \\ \hline \end{gathered}$ | $\begin{array}{r} \text { IC } \\ \text { A } \end{array}$ | $\begin{gathered} \mathrm{PC} \\ \mathrm{~W} \end{gathered}$ | $\begin{gathered} \mathrm{Icbacb}^{\prime} \\ \mathrm{A} \end{gathered}$ | $\begin{gathered} \text { Ićáb' } \\ \text { A } \\ \hline \end{gathered}$ | $\begin{array}{r} \mathrm{IF} \\ \mathrm{~A} \\ \hline \end{array}$ |  |
| 4 | 2 | 3.5 | 4 | 2.4 | 1.8 | 3 | 2.9 | 1.8 | 5 | 3.5 | 7 | 10.5 | 0.0 |
| 6 | 2.8 | 5.4 | 10 | 3.5 | 2.7 | 6 | 4.3 | 2.8 | 12 | 5.4 | 10.8 | 16.2 | 0.0 |
| 8 | 3.9 | 7.2 | 18 | 4.9 | 3.6 | 12 | 5.8 | 3.7 | 20 | 7.2 | 14.4 | 21.6 | 0.0 |
| 10 | 4.9 | 9 | 30 | 6 | 4.5 | 17 | 7.2 | 4.5 | 31 | 9 | 18 | 27 | 0.0 |
| 12 | 5.8 | 10.8 | 40 | 7.2 | 5.5 | 25 | 8.6 | 5.5 | 45 | 10.8 | 21.6 | 32.4 | 0.0 |
| 14 | 6.8 | 12.6 | 55 | 8.4 | 6.4 | 35 | 10 | 6.4 | 62 | 12.6 | 23.2 | 35.8 | 0.0 |
| 16 | 7.8 | 14.5 | 72 | 9.6 | 7.3 | 45 | 11.5 | 7.3 | 80 | 14.5 | 29 | 43.5 | 0.0 |
| 18 | 8.7 | 16.5 | 94 | 10.8 | 8.2 | 57 | 12.9 | 8.2 | 102 | 16.4 | 32.8 | 49.2 | 0.0 |

Table 4. Comparison of Laboratory Data and Computer Results for a Line-to-Line Fault on the Center-Tapped Three-Phase Circuit

| Input voltage | Laboratory Data |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | $\begin{aligned} & \mathrm{VA} \\ & \mathrm{~V} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { IA } \\ & \mathrm{A} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { PA } \\ & \mathrm{W} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { VB } \\ & \text { V } \end{aligned}$ | $\begin{aligned} & \text { IB } \\ & \text { A } \\ & \hline \end{aligned}$ | $\begin{aligned} & \mathrm{PB} \\ & \mathrm{~W} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { VC } \\ & \text { V } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { IC } \\ & \mathrm{A} \\ & \hline \end{aligned}$ | $\begin{aligned} & \mathrm{PC} \\ & \mathrm{~W} \end{aligned}$ |
| 6 | 2.8 | 5.4 | 10 | 3.5 | 2.7 | 6 | 4.3 | 2.8 | 12 |
| 10 | 4.9 | 9 | 30 | 6 | 4.5 | 17 | 7.2 | 4.5 | 31 |
| 14 | 6.8 | 12.6 | 55 | 8.4 | 6.4 | 35 | 10 | 6.4 | 62 |
| 18 | 8.7 | 16.5 | 94 | 10.8 | 8.2 | 57 | 12.9 | 8.2 | 102 |

Computer Results

| 6 | 2.87 | 5.43 | 10.22 | 3.59 | 2.71 | 6.37 | 4.26 | 2.71 | 11.48 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 10 | 4.79 | 9.05 | 28.38 | 5.98 | 4.52 | 17.70 | 7.11 | 4.52 | 31.89 |
| 14 | 6.70 | 12.67 | 55.62 | 8.37 | 6.33 | 34.69 | 9.95 | 6.33 | 62.50 |
| 18 | 8.61 | 16.29 | 91.94 | 10.76 | 8.14 | 57.35 | 12.79 | 8.14 | 103.32 |

Table 5. Laboratory Data for a Phase-to-Ground Fault With the Opposite Center Tap Grounded

| Input voltage | Primary side |  |  |  |  |  |  |  |  | $\begin{gathered} \text { Secondary } \\ \text { side } \\ \hline \end{gathered}$ |  |  | $\begin{array}{r} \mathrm{IN} \\ \mathrm{~A} \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | $\begin{aligned} & \text { VA } \\ & \text { V } \end{aligned}$ | $\begin{array}{r} \text { IA } \\ \text { A } \\ \hline \end{array}$ | $\begin{gathered} \mathrm{PA} \\ \mathrm{~W} \\ \hline \end{gathered}$ | $\begin{gathered} \text { VB } \\ \text { V } \end{gathered}$ | $\begin{array}{r} \text { IB } \\ \mathrm{A} \end{array}$ | $\begin{gathered} \mathrm{PB} \\ \mathrm{~W} \end{gathered}$ | $\begin{gathered} \mathrm{VC} \\ \mathrm{~V} \end{gathered}$ | $\begin{array}{r} \mathrm{IC} \\ \mathrm{~A} \\ \hline \end{array}$ | $\begin{gathered} \mathrm{PC} \\ \mathrm{~W} \\ \hline \end{gathered}$ | $\begin{gathered} \text { Iáca } \\ \text { A } \\ \hline \end{gathered}$ | $\begin{gathered} \text { Iába } \\ \text { A } \\ \hline \end{gathered}$ | $\begin{array}{r} \mathrm{IF} \\ \mathrm{~A} \\ \hline \end{array}$ |  |
| 4 | 2.6 | 0.0 | 0.0 | 2.2 | 3.5 | 7 | 1.6 | 3.5 | 1 | 3.5 | 3.5 | 7 | 0.0 |
| 6 | 4 | 0.0 | 0.0 | 3.3 | 5.3 | 16 | 2.4 | 5.3 | 2.2 | 5.3 | 5.3 | 10.6 | 0.0 |
| 8 | 5.3 | 0.0 | 0.0 | 4.4 | 7 | 30 | 3.2 | 7 | 4 | 7 | 7 | 14 | 0.0 |
| 10 | 6.6 | 0.0 | 0.0 | 5.5 | 8.8 | 45 | 4.0 | 8.8 | 6.5 | 8.8 | 8.8 | 17.6 | 0.0 |
| 12 | 8 | 0.0 | 0.0 | 6.6 | 10.6 | 66 | 4.8 | 10.6 | 10 | 10.6 | 10.6 | 22.2 | 0.0 |
| 14 | 9.3 | 0.0 | 0.0 | 7.7 | 12.4 | 90 | 5.6 | 12.4 | 13 | 12.4 | 12.4 | 24.8 | 0.0 |
| 16 | 10.6 | 0.0 | 0.0 | 8.8 | 14 | 120 | 6.4 | 14 | 17 | 14 | 14 | 28 | 0.0 |
| 18 | 12 | 0.0 | 0.0 | 9.9 | 15.9 | 150 | 7.2 | 15.9 | 21 | 15.9 | 15.9 | 31.8 | 0.0 |

Table 6. Comparison of Laboratory Data and Computer Results for a Phase-to-Ground Fault With the Opposite Center-Tap Grounded

| Input voltage | Laboratory Data |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | $\begin{gathered} \mathrm{VA} \\ \mathrm{~V} \\ \hline \end{gathered}$ | $\begin{array}{r} \text { IA } \\ \mathbf{A} \\ \hline \end{array}$ | $\begin{gathered} \mathrm{PA} \\ \mathrm{~W} \\ \hline \end{gathered}$ | $\mathrm{VB}$ | $\begin{array}{r} \text { IB } \\ \text { A } \\ \hline \end{array}$ | $\begin{gathered} \mathrm{PB} \\ \mathrm{~W} \end{gathered}$ | $\begin{gathered} \mathrm{VC} \\ \mathrm{~V} \\ \hline \end{gathered}$ | $\begin{array}{r} \text { IC } \\ \mathrm{A} \\ \hline \end{array}$ | $\begin{gathered} \text { PW } \\ \mathrm{W} \end{gathered}$ |
| 6 | 4 | 0.0 | 0.0 | 3.3 | 5.3 | 16 | 2.4 | 5.3 | 2.2 |
| 10 | 6.6 | 0.0 | 0.0 | 5.5 | 8.8 | 45 | 4.0 | 8.8 | 6.5 |
| 14 | 9.3 | 0.0 | 0.0 | 7.7 | 12.4 | 90 | 5.6 | 12.4 | 13 |
| 18 | 12 | 0.0 | 0.0 | 9.9 | 15.9 | 150 | 7.2 | 15.9 | 21 |

Computer Results

| 6 | 3.98 | 0.0 | 0.0 | 3.79 | 5.28 | 16.88 | 2.42 | 5.28 | 2.4 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 6.64 | 0.0 | 0.0 | 5.49 | 8.8 | 46.90 | 4.03 | 8.8 | 6.67 |
| 14 | 9.29 | 0.0 | 0.0 | 7.69 | 12.32 | 91.92 | 5.64 | 12.32 | 13.08 |
| 18 | 11.95 | 0.0 | 0.0 | 9.88 | 15.83 | 151.95 | 7.25 | 15.83 | 21.62 |

## Program title

Declare the complex variables imaginary for the generator and transformer

Print the fault title

Print the outputs title

Initialize the input voltage

Calculate the current for each phase

Calculate the voltage for each phase


Calculate the power for each phase

Print the calculated currents, voltages and power

Increment the input voltage by one and repeat the above calculations

```
```

OS3 FOOTPNH VERSIO'I 3.13

```
```

```
OS3 FOOTPNH VERSIO'I 3.13
```

```
    pOOrRAM FA|LI
```

```
    pOOrRAM FA|LI
```










```
        FOOMAT(ロ⿱丷天心品)
```

        FOOMAT(ロ⿱丷天心品)
        A=「กイロ(-0.「,n.956)
        A=「กイロ(-0.「,n.956)
        AA=COMO(-1.5,-0,9E白)
        AA=COMO(-1.5,-0,9E白)
        75!=rnuc(7,1?.7C,1I)
        75!=rnuc(7,1?.7C,1I)
        7,?=C\capNO(7G?こ以,7F,2I
        7,?=C\capNO(7G?こ以,7F,2I
        ZT!=COMn(7T12,ZTII)
        ZT!=COMn(7T12,ZTII)
        \Delta口O,1=r`M(1.0)
        \Delta口O,1=r`M(1.0)
        Aวr.?=「OM(-1., )
        Aวr.?=「OM(-1., )
        71=7r,1+711
        71=7r,1+711
        7?=702+7T2
        7?=702+7T2
        7^TAL=Z1+72
        7^TAL=Z1+72
        W2TTF(51,20)
        W2TTF(51,20)
        FOOUATIFCX,#A LINE-TO-LINE FAULT उETWEEN TH? PHASES\not=/I
        FOOUATIFCX,#A LINE-TO-LINE FAULT उETWEEN TH? PHASES\not=/I
        WरITE(的,00)
    ```
        WरITE(的,00)
```








```
        CMOPENT CAIGULATIONS
```

        CMOPENT CAIGULATIONS
        EAD=0.?
        EAD=0.?
        ON 50 T= = , 20,1
        ON 50 T= = , 20,1
        EA?=FNQ+1.3
        EA?=FNQ+1.3
        FA=COMD(5A?,0.0)
        FA=COMD(5A?,0.0)
        | TAPUT=RE(EA)
        | TAPUT=RE(EA)
        COLL=FA/TVOTAL
        COLL=FA/TVOTAL
        CAPL=-CA1L
        CAPL=-CA1L
        CA1H=\trianglePC,OMCA1L
        CA1H=\trianglePC,OMCA1L
        C 12H=AFG1*CA2L
        C 12H=AFG1*CA2L
        CAU=CA1H+C.ARH
        CAU=CA1H+C.ARH
        C?1H=A\Delta* CA1H
        C?1H=A\Delta* CA1H
        CaつH=A*CA2H
        CaつH=A*CA2H
        CPH=C?1H+C132H
        CPH=C?1H+C132H
        CC1H=A* CA1H
        CC1H=A* CA1H
        CCPH=AA*CA>11
        CCPH=AA*CA>11
        CCH=CCLH+CCOH
        CCH=CCLH+CCOH
        CAHP= (VICPW!CAM)
        CAHP= (VICPW!CAM)
        उपH=XNORY(C,OH)
        उपH=XNORY(C,OH)
        COHH:= MNOマM(CCH)
        COHH:= MNOマM(CCH)
        ANCAH=SAO(C,1H)*(1190.0*7.7)/2?.0)
        ANCAH=SAO(C,1H)*(1190.0*7.7)/2?.0)
        ANCRH=?A\cap(r2H1)*((130.7*7.0))
        ANCRH=?A\cap(r2H1)*((130.7*7.0))
        voltarge caloulations
        voltarge caloulations
        VA!L= -A-(rA1L*ZG1)
        VA!L= -A-(rA1L*ZG1)
        VAOL=CA1L*TT,2
        VAOL=CA1L*TT,2
        VA1H=AFr, **V1L
        VA1H=AFr, **V1L
        V\triangle\H=A=r,1*VATL
        V\triangle\H=A=r,1*VATL
        VAH=VAIH+VIPH
        VAH=VAIH+VIPH
        VO1H=AA*VAIH
        VO1H=AA*VAIH
        V\つH=\triangle*V\DeltaつH
        V\つH=\triangle*V\DeltaつH
        V34=V?1H+V3?H
        V34=V?1H+V3?H
        VC1H=A*VA1H
        VC1H=A*VA1H
        VC?H=\Delta\Delta*VA?!
        VC?H=\Delta\Delta*VA?!
        VC4=VC1H+VC.OH
        VC4=VC1H+VC.OH
        VALN=XN\capOM(VAM)
        VALN=XN\capOM(VAM)
        VZHA:= YNOCN(y.aH)
        VZHA:= YNOCN(y.aH)
        VCHA:=YNORM(yrN)
        VCHA:=YNORM(yrN)
        ANVAH=3n#(VA4)*(1190.0*7.0)/22.0)
        ANVAH=3n#(VA4)*(1190.0*7.0)/22.0)
        ANVFH=30П(ソQH)*(119C.!N7.0)/22.0)
        ANVFH=30П(ソQH)*(119C.!N7.0)/22.0)
        AN!CH=PAN(VOH)* ((190.0*7.0)/22.01
        AN!CH=PAN(VOH)* ((190.0*7.0)/22.01
        FOHEG OALCULAIIONS
        FOHEG OALCULAIIONS
        AVGVA=ANVAH-ANI,AH
    ```
        AVGVA=ANVAH-ANI,AH
```










```
        PTCTAL=PA+ON+OC
```

        PTCTAL=PA+ON+OC
        YRITF(51,4\eta)V INPUT, CAHN, ANC,AH, C,OHN, ANC TH, CTHN, MNCCH,VATN, ANVAH, VB
        YRITF(51,4\eta)V INPUT, CAHN, ANC,AH, C,OHN, ANC TH, CTHN, MNCCH,VATN, ANVAH, VB
    1WN, ANVTH,VCHA!, ANVCH;QA,OG,FO,OTOTAL
    ```
    1WN, ANVTH,VCHA!, ANVCH;QA,OG,FO,OTOTAL
```




```
    1FG.?,?\,FG.:,?X,FG.?,2X,F5.2,?X,FG.?,2X,FG.?,2X,F5,?,?X,F5,?,?X,F-
```

    1FG.?,?\,FG.:,?X,FG.?,2X,F5.2,?X,FG.?,2X,FG.?,2X,F5,?,?X,F5,?,?X,F-
    ?F.3,?X,FR.?/1
    ?F.3,?X,FR.?/1
    \square
    \square
    FONTIMUF
    FONTIMUF
    FNT
    ```
    FNT
```

FORTRAN Program for a Line－to－Line Fault Between Two Phases


Computer Results of a Line-to-Line Fault Between Two Phases

```
OS3 FOQTRAN VERSION 3.13 04/30/76 1331
```

```
PPORRAM FAIILT
```



```
1 , CAIL,CA?L,CAIH,CA?H,CAH,CYIH,CQPH, CBH,CrIH, CC?H,CCH,VAIL,VA?L,
```



```
FEAD(FO,10)ZGIR,ZGII,ZG2R,ZGZI, TT1R,ZT1I,ZT2R,ZT2I
FORMAT (851~.5)
\(A=\operatorname{COMP}(-0.5,0.855)\)
\(\Delta A=\Gamma \cap М Р(-1.5,-0.96 \hbar)\)
7G1=COMR(7G1R,7G1I)
7r,2 = Сомр (Zr, ? , 2r,2I)
7T1 = COMF (TT1O, ZTII)
7 TR \(=C \cap M O(Z T 2 P\),
\(A R G 1=C O M(1.0)\)
\(A R T ?=\operatorname{COM}(-1 . n)\)
\(71=Z \square 1+7 T 1\)
\(2 T\) TAL \(=21+7\)
W2TTE(51,2?)
FOOMAT(57X, \(\mathcal{A}\) LINE-TO-LINE FAULT ON THE CENTER-TAPPEQ THREE-PHASE 1TIPCUIT \(\pm / 1\)
WPTTE 61,30\()\)
```





```
CUFRENT CALCULATIONS
\(E A R=0 \cdot ?\)
\(0050 \quad T=1,20,1\)
\(E A R=F A R+1, A^{B}\)
\(E A=\) GOMP (EAR, O, O)
\(\checkmark\) INPUT=PE(EA)
CALL=EA/ZTOTAL
C, \(A\) ?L \(=-\Gamma, \Delta 1 L\)
\(C A 1 H=A R G Z^{*} C A 1 L\)
\(C A D H=A P \Gamma 1 * G A D L\)
\(C A H=C A 1 H+C A ? H\)
CR1Y = AA C CAIH
\(\mathrm{C} \cdot \mathrm{BH}=\mathrm{C}, 5 \mathrm{H}+\mathrm{CB} \mathrm{B} \mathrm{H}\)
CCIH=A*CAIH
\(C C ? H=A \Delta * C A 2 H\)
\(\mathrm{C}, \mathrm{CH}=\mathrm{C} \mathrm{C}, 1 \mathrm{H}+\mathrm{C} \mathrm{C} 2 \mathrm{H}\)
CAHM=X:AOEM(CAH)
COH'J= YHOOM(C, तH)
CCMH=YNODM (CTH)
```



```
\(A N C B 4=2.1)(\mathrm{C} 94) *((190.0 * 7.7) / 22,0)\)
\(\Delta N C C H=R A C(C C H) *(180 . C * 7.0) / 22.0)\)
VOLTACE CALCILLATIONS
\(V \Delta!L=E A-(C A 1 L * Z G 1)\)
\(\checkmark A Z L=C A 1 L * Z G ?\)
\(V A 1 H=A R G 2 * V A 1 L\)
VA2H=ARG1*VA2L
\(V \Delta H=V A 1 H+V A 2 H\)
VB1H= \(\triangle A * V A 14\)
VB? \(\mathrm{H}=\mathrm{A}\) V V ? H
\(V R H=V B 1 H+V 82 H\)
VCIH=A*VA1H
\(V C \supset H=A \Delta * V A ? H\)
\(V\) CH=VC1H+VCつH
\(V A: J N=X M O R M(V A H)\)
\(V B H M=X N O P M(V B H)\)
VOHN =XNOEM (VCH)
ANVAH=OAП(YAH) F ( \(19 \mathrm{C} .0 * 7.0) / 22.0)\)
\(A N V B H=Q \Delta \cap(\mathrm{~V}!4) *((190.3 * 7.3) / 22.0)\)
\(\Delta \because V C H=O A)(V R . H) *((180.0 * 7.0) / 22.0)\)
FOWER CALCULATICNS
```



```
\(\triangle\) ACVB \(=A 11904-A\) NC 74
ANCUR=AHCOH-ANVOH
```





```
PTOTAL \(=P \Delta+\) PITPC
```



```
LHN, ANYRH,VCHI, ANVCH, IA, OB, PCOFTOTAL
FORMATI \(3 x, F, 1,2 x, F S, ?, 3 x, F S, ?, 2 x, F E, ?, 2 x, F 5, ?, ? X, F G, 2, ? x, F E, 2,2 x\),
1FG. 2,? \(\mathrm{X}, F \mathrm{~F}, ?, 2 X, F \in, ?, 2 X, F G, 2,2 X, F G, 2,2 X, F E, 2,2 X, F h, ?, 2 Y, F G, 2,2 X, F\)
2F. \(2 \times, \mathrm{F}_{6}\).21i
CONTINUE
```

FORTRAN Program for a Line-to-Line Fault on the CenterTapped Three-Phase Circuit


Computer Results of a Line-to-Line Fault on the Center-Tapped Three-
Phase Circuit

```
        TYFPRAUFAULT
```

```
        TYFPRAUFAULT
```



```
    2VAHH,VAOH,VAH,VEIH,VBPH,VBH,VCIH;VCPH,VCH;COM,COMP
```



```
    FORMGT(8F1%.5)
    A=CnN5:-0.5,0.955)
    AA=COMP(-i,5,-0.865)
    7G1=CクMP(zG1R,7\sigma1I)
    ZTS=CEMN(ZT2R,TROLI)
    7T?=T,OMP(ZT2R,ZT2I)
    APC,1=C刀M(1.0)
    APG?=COM(-1.0)
    Z1=76147T1
```



```
    ZTOTAL=71+72
    FOJMAT(SOX,&A PHASE-TO-GROUND FAULT WTTH THE OPROSITE CENTER-TAP G
    1ROMMOROF/1
    WOITE(F,,30)
```





```
    CURRENT CALCULATIONS
    FAP=0.0
    70 50 T=1,20,1
    EAR=EAR+1.7
    FA=rono(fio,0,0)
    VIHDUT=EE(EA)
    CAIL=EA/ZTCTAL
    CC?L=CA1L
    CA1H=GFr\rho=CA1L
    CAZH=ARC1*CARL
    CA4=CA1H+CA2H
    CB!L=\triangleA*CA1H
    CBZH=\triangle*CA?H
    CQH=C31H+CB2H
    CCiH=O*CA1H
    C^?H=AA*CA2H
    CCH=CC1H+CC2H
    C\triangleHI= YNORM(COH)
    r(3H:!=x'\CRM(CBH)
    CCHM= YHOOM(CO.H)
    &!NaH=FaO(CaH)*((18.0.0*7.0)/22.0)
    ANr.ロH==40(r,4)*((190.0*7.01/22.0)
    ANCC4=RAD(OCH)*(1180.0*7:0)/22.0)
    voltage calculations
    VA1L=EA-(CA1L*IF,1)
    VA?L=こ(CA?L*2r,2)
    VA1H=APro*VAIL
    VAOH=APSI*VA2L
    VAH=VA1H+VAZH
    VR1H=A *VA1H
    VR?H=A*VA?H
    VOH=V51H+VB2H
    VC!H=A*VA1H
    VC2H=AA*VAZH
    VCH=VC1H+Vr2H
    VA4:I=X.10OM(VAH)
    VBHH= VNOGM(VGH)
    VCHN = XHODM(VCH)
    LNYH=FAつ(VAH)*((180.0*7.0)/2?.n)
```



```
    ANVCH=RAO(YCH)*((18C:n*7:0)/2\grave{0.0)}
    POHER mALCJLATIOMS
    AN-VA=ANUAL-ANCAH
    AN:CVR-AM:VRH-ANCOH
    ANCVC=ANCTH-ANVCH
    OA=VAMA*CHH*COS(1ANCVA*?2.0)/1180.3*7.0)1
```




```
    PTOTAL=RA+PB+DC
    NRT:T-(O1,4OIV TIPUT,CAHN, ANCAH,CRHN, ANCBH,CCHN,ANCCH,VAHN, ANVAH,VI
    1HM,GNV3H,VEHN,AMVCH,PL,PQ,FR,FTOTAL
```




```
    CONTINUE
    ENO
```

FORTRAN Program for a Phase－to－Ground Fault with the Opposite Center－Tap Grounded


## Computer Results of a Phase-to-Ground Fault with the Opposite Center-Tap Grounded

## LIST OF NOMENCLATURE

## Variable

Description

IA Current in phase $A$ in amperes
IB Current in phase B in amperes
IC Current in phase $C$ in amperes
IF Fault current in amperes
IN $\quad$ Neutral current in amperes
VA Line to neutral voltage in phase $A$ in volts
VB Line to neutral voltage in phase $B$ in volts
VC Line to neutral voltage in phase $C$ in volts
PA Power in phase A in watts
PB Power in phase B in watts
PC Power in phase $C$ in watts
ZGl The generator positive impedance in ohms
ZG2 The generator negative impedance in ohms
Vatl The positive sequence voltage for phase a in volts
Vat2 The negative sequence voltage for phase a in volts

