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Title THE DEVELOPMENT OF THE THEORY OF METAL ROLLING AND ITS APPLICATION TO ROLLING MILL CONTROL.

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The relationships of the variables of a process to be controlled are expressed in a mathematical model of the process. In the case of the rolling of metal the process is described by the physics of the deformation of the metal in the roll gap. In developing the theory of rolling it is necessary to survey the field for existing theories and decide upon the theory that most nearly describes the process. The final model is in a form which shows the relationship of manipulated variables to measurable variables.

The control of rolling is done by a machine, or rolling mill. From a control point of view it is necessary to study the combined behavior of machine and metal deformation to find the variables that may be used to most effectively regulate the process.

A typical problem in this area is to control the output thickness to a given deviation from nominal. The machine-process equations show that output thickness is controlled readily by strip

tensions and rolling speed. The mathematical model also shows which variables most effectively control the thickness and what the relationships are between changes in controlled variables and correction in output sheet thickness.

Since the control of the process is the object of the investigation, the controller must be capable of working into conventional rolling mill drive equipment. This interrelationship is discussed in light of the equipment and the mathematical model.

THE DEVELOPMENT OF THE THEORY OF METAL ROLLING AND ITS APPLICATION TO ROLLING MILL CONTROL

by

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LIST OF SYMBOLS

C - Hitchcocks Constant for roll flattening (1.67×10^{-4} in/ton)

f - Frictional force

h_o - Thickness of strip at entry to mill stand

h_1 - Thickness of strip at exit of mill stand

h_n - Thickness of strip at neutral plane

k_o - Yield stress in shear at entry plane

k_1 - Yield stress in shear at exit plane

P - Roll force

$\bar{p}$ - Mean roll pressure

$\bar{p}_o$ - Mean pressure between entry and neutral planes

$\bar{p}_1$ - Mean pressure between neutral and exit planes

R - Roll radius

R' - Flattened roll radius

S - Forward slip

t - Time

v_o - Velocity of strip entering mill stand

v_1 - Velocity of strip leaving mill stand

w - Width of strip

μ - Coefficient of friction

ν - Poisson ratio

σ_b - Back tension stress $\frac{T_b}{w h_o}$

LIST OF SYMBOLS (continued)

σ_f - Front tension stress $\frac{T_f}{wh_1}$

ϕ_x - Angle of contact measured between plane of exit and reference plane

ϕ - Angle of contact between plane of exit and plane of entry

ϕ_n - Neutral angle measured from entry plane

ω - Angular velocity of roll

Increments in quantities such as ϕ are noted by $\Delta\phi$, differentials are noted as $d\phi$; i.e. $\lim_{\Delta \to o} \Delta\phi \to d\phi$

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THE DEVELOPMENT OF THE THEORY OF METAL ROLLING AND ITS APPLICATION TO ROLLING MILL CONTROL

INTRODUCTION

In the realm of process control it is the rigidness of specification upon the finished product which dictates the refinement necessary for adequate control. As the end product becomes more and more critical the solution to the control problem can no longer be found in the intuitive or historical approach, but must come from a more theoretical expression of the problem and a rigorous analytical treatment of the process. By analysis of the theory of the process, in light of its limitations, it is possible to define the problem in terms of the relationships between the variables. It is then possible to determine which of the variables are controllable and which are dependent or uncontrollable and to arrive at a method of process control. It must be remembered that the control and instrumentation problems are inseparable in that the adequacy of control depends upon the type and quality of measurements made upon the process. Similarly, the overriding factor of engineering judgment and cost control must dictate the proper economic solution to process control.

In this treatise a common process, that of the rolling of metal, is analyzed from the point of view of determining the theoretical expression of the process, defining the method by which the machine rolls the metal to the desired thickness and the combination of machine and process to get an overall mathematical model of the metal - machine process. From the above investigation a method of control will be developed with attention to the effectiveness of the controlled variables and availability of measurements made on the process variables. Finally, a theoretical controller will be discussed for controlling the process by analog methods and it will be compared with a theoretical digital controller for the same process.

Throughout this treatise it will be noticed that current state of the theory of rolling fails to define certain things that happen in the process. Since this work is not aimed at definition of the metallurgical processes involved, these different points of view are presented here without editorial comment or attempt to rationalize them except where conflicting opinion makes further analysis impossible.

The method of developing a mathematical model is the same either way, so the basis of further analysis will be stated and the investigation continued. Further treatment of conflicting theory will be found in the referenced material.

I. THE ROLLING PROCESS

The theory of rolling expresses the relationships between the various variables of the rolling process. In general, the theory aims at (1) predicting the manner of deformation and the forces involved in the process; and, (2) provide a basis of appraisal for the operation of existing facilities and planning for new facilities. In particular the theory will enable prediction of the process variables which are to be controlled and establish limits upon which the engineering design of mill and control must be based.

Rolling theory can be divided into two broad and general categories, that which applies to hot-rolling and that which applies to cold rolling. In hot-rolling the yield stress characteristic of the metal is strain-rate dependent and frictional force between the roll and stock is high. In cold-rolling the yield stress characteristic of the metal is essentially independent on the rate of deformation and the frictional force between roll and stock is low (1).

The theory of rolling is complicated by a number of factors to the extent that a solution to even the relatively simple case of rolling flat sheet is not possible. As it will be shown, of primary influence upon the rolling process are the following parameters: the work roll diameter, the reduction in one pass, the initial thickness of the work piece, the speed of rolling (rate of strain), the front and back tensions, the nature of friction between the rolls and the material being rolled, the temperature field in the material, the physical properties of the material being rolled, the shape of the roll contour and the mill behavior under load. Most of these are properties or characteristics of the process under consideration rather than the machine (mill) doing the work.

In addition to the above considerations are such things as the memory of the material, or the effect of previous treatment; the elastic deformation of the rolls under load; and, the state of anistropy. It is obvious that the pure mathematical expression of the work process will be difficult to explain even for the case of flat rolling and that various assumptions have to be made to simplify the problem. The most complete and theoretically correct work in this field was done by Orowan (1943) (7). His work was based upon the assumptions set down by Von Karman (1925) (14) in his pioneer paper on rolling mill theory. Bland and Ford (1948) (1), further assumed away some of the problems involved in Orowan's theory, and it is upon the former that the bulk of the following is based.

To limit the scope of the problem and adequately illustrate the principles involved in creating a mathematical model this thesis shall be limited to the cold-rolling of strip. In order to obtain a solution the theory will be further restricted to those assumptions set down by Von Karman and Orowan and detailed later in this work.

Figure 1 shows in general terms what happens when a strip of material of initial thickness h_o enters the rolls. The material is compressed elastically until it yields, is then subjected to plastic deformation, and upon leaving the roll gap there is elastic recovery to final thickness h_1. Except for very hard, thin material (stainless steel or razor blade stock) the elastic recovery is quite small and is usually ignored so that for analytical purposes the exit thickness h_1 is the same as the separation between the work rolls.

Figure 1 shows the material in the roll gap, and the external forces acting upon the plane strip of height 2y and incremental thickness dx. Before attempting to write the basic differential equation for what takes place in the roll gap it is necessary to make some further simplifying assumptions. Although Orowan used more basic assumptions the generally accepted conditions are those advanced by Von Karman and accepted by Siebel (10), Nadai (6) and Tselikov (13) in their simplifying works. Those assumptions are as follows:

1. The metal being formed is a continuous isotropic medium.
2. The stock in the gap consists of thin vertical strips of incremental thickness with no shear stress and only a normal pressure between adjacent segments; the plastic deformation is assumed to be homogeneous compression. The accompanying assumption here is that segments of an initially plane cross section remain plane.

3. The yield stress of the material as it passes through the rolls remains the same. As pointed out previously this is not strictly true since work hardening takes place in cold-rolling to change the yield stress along the arc of contact; in hot-rolling the yield stress is dependent upon the rate of deformation which varies along the arc of contact.
4. The arc of contact of metal with roll is circular.
5. The coefficient of friction is constant along the arc of contact.
6. The problem is considered to be two-dimensional. Thus, any lateral spread is neglected and the problem is considered to be one of plane strain.
7. Elastic compression of the strip is neglected.
8. The Huber - Von Mises condition of plasticity holds. See Appendix A.
9. Slipping takes place along the arc of contact.
10. The contact angle is small.
11. Constant speed of rolling exists.

While the above assumptions appear to be rather formidable, comment will be made upon the effect of these assumptions in the body of this discussion. The most important of these assumptions is that which stipulates that plane sections perpendicular to the direction of rolling remain plane; that is, deformation is homogeneous compression. This requires that there is a slipping friction over the whole arc of contact.

In passing through the roll gap a strip of constant width w, undergoes a thickness change from h_o to h_1, or Δh, and because the deformation has been assumed to be one of plane strain on an incompressible material the following relationship holds:

$$w\, h_o\, v_o = w\, h\, v = w\, h_1\, v_1\ .$$

Thus the velocity of the strip increases steadily from entry to exit and the velocity of the rolls must have a value somewhere v_o and v_1. On the entry side the strip moves slower than the roll and the frictional force acts to pull the metal into the roll; on the exit side the metal strip exceeds the speed of the roll so that the frictional force opposes the delivery of the strip. It is apparent, then, that somewhere between the entry plane AA and the exit plane BB there exists a point where the surface of the strip and the peripheral velocity of the rolls are the same. This point (or plane NN) is called the neutral point and its position depends upon the equilibrium of external forces such as front and back tensions and roll load.

Considering the strip shown in Figure 1 and summing horizontal forces on the incremental strip of height 2y it can be shown (Appendix B) that the basic differential equation for pressure distribution in the roll gap is

$$\frac{p_x - \sigma_x}{y} \frac{dy}{dx} - \frac{d\sigma_x}{dx} \pm \frac{f_x}{y} = 0 \qquad (1)$$

where the sign of the frictional force f_x is determined by the relative velocity of the strip with respect to the roll surface. The positive sign is for an elemental strip between entry and neutral planes, the negative sign for an element between neutral and exit planes.

Because of the Von Mises condition of plasticity, substitution of p_x for σ_1, and σ_x for σ_3 yields

$$\frac{d(p_x - 2k)}{dx} - \frac{2k}{y} \frac{dy}{dx} \pm \frac{f_x}{y} = 0 \qquad (2)$$

which is the basic differential equation for the specific pressure. The important question in the theory of rolling is the question of the distribution of this pressure along the arc of contact and its correlation between separating force, and torque and power consumption. Obviously an exact solution to the differential equation is dependent upon a valid correlation between the pressure and the tangential stress at the contact surface. It is in the nature of this relationship that the various basic theories differ.

Complete solutions are obtained by treating the above expression as two separate distributions; that between entry and neutral planes and that between neutral and exit planes. The general solution is then the superposition of the two. This is necessary because the integral of the equilibrium differential equation has only one arbitrary constant whereas there are two different boundry conditions given at entry and exit. The discontinuity of the derivative of the pressure curve and discontinuity of friction at the netural point, where there is no relative slip, appear to be the largest single drawback in the formulation of a universal rolling theory.

Although it was stated earlier that a number of theories are existent and that no attempt would be made to rationalize the differences between them, the following summary of the more popular theories is presented in order that this record might be complete and the effect of vast differences upon the control problem might be appreciated.

A. Von Karman's Solution to Pressure Distribution Equation

On the premise that the frictional force is dry slipping friction, which is proportional to the value of local normal force the relationship between f_x and p_x is $f_x = \mu p_x$

With that assumption Von Karman's solution to the basic differential equation is:

$$\frac{d(p_x - 2k)}{dx} - \frac{2k}{y}\frac{dy}{dx} \pm \mu p_x = 0 \qquad (3)$$

and:

$$p_x = e^{\mp \int \frac{\mu}{y} dx} \left[C + \int \frac{2k}{y} e^{\pm \int \frac{\mu}{y} dx} dy \right] \qquad (4)$$

Obviously this equation is not of a form which lends itself readily to computation, automatic or otherwise. Mr. T. L. Smith (5) performed the following simplications, assuming that the arc of contact was parabolic rather than circular.

$$p_x = k\left\{\left[\zeta_o + \frac{2}{m^2}(1 - mu_o)\right] e^{m(u_o - u)} - \frac{2}{m^2}(1 - mu)\right\} \quad (5)$$

for the region between entry and neutral plane and:

$$p_x = k\left\{\left(\zeta_1 + \frac{2}{m^2}\right) e^{mu} - \frac{2}{m^2}(1 + mu)\right\} \quad (6)$$

for the region between neutral and exit planes.

Here:

$$m = \frac{2\mu}{h_1 \Delta h}$$

$$u_o = \arctan \frac{\Delta h}{h_1}$$

$$u = \sqrt{\frac{\Delta h}{h_1} \cdot \frac{x}{\ell}}$$

$$\zeta_o = 1 - \frac{\sigma_b}{2k}$$

$$\zeta_1 = 1 - \frac{\sigma_f}{2k}$$

Similarly, if the arc of contact is approximated by a chord, the specific pressure becomes:

$$p_x = \frac{2k}{\delta}\left[\left(\zeta_o \delta - 1\right)\left(\frac{h_o}{h_x}\right)^{\delta} + 1\right] \quad (7)$$

between entry and neutral point, and

$$p_x = \frac{2k}{\delta}\left[\left(\zeta_1 \delta + 1\right)\left(\frac{h_x}{h_1}\right)^{\delta} - 1\right] \quad (8)$$

for the zone between neutral and exit planes, where

$$\delta = \frac{2\mu}{\Delta h} .$$

B. Siebel's Solution to the Pressure Distribution Equation

Siebel's solution to the basic differential equation stipulates that the frictional stress is constant along the arc of contact. Here:

$$f_x = \text{constant} = f \simeq \mu K$$

If the parabolic arc of contact is again assumed, the pressure distribution becomes

$$p_x = K\left\{\left[\zeta_o - \log\left(\frac{Z_o^2 + 1}{Z^2 + 1}\right)\right] + 2f\sqrt{\frac{r}{h_1}}\arctan\left(\frac{Z_o - Z}{1 + Z_o Z}\right)\right\} \quad (9)$$

for the zone between entry and neutral and

$$p_x = K[\zeta_1 + \log(Z^2 + 1) + 2f\sqrt{\frac{r}{h_1}}\arctan Z] \quad (10)$$

between neutral and exit where

$$Z = \sqrt{\frac{\Delta h}{h_1}}\ \frac{x}{\ell}$$

$$Z_o = \sqrt{\frac{\Delta h}{h_1}}$$

and ζ_o and ζ_1 are as previously defined.

C. Nadai's Solution to the Pressure Distribution Equation

Nadai was the first to investigate the occurrence of viscous friction between the roll and the strip. His solution to the Equation (2) is based upon the relationship

$$f_x = \mu \frac{dv}{dy}$$

where f_x = the frictional stress

μ = the dynamic viscosity of the lubricant

$\frac{dv}{dy} = \frac{v_x - v_r}{\epsilon}$

where v_x = velocity at section "x"

v_r = peripheral velocity of roll

ϵ = mean oil thickness

$$v_x = \frac{v_1 y_1}{y}$$

where v_1 = velocity at exit

y_1 = half thickness at exit

$$v_r = \frac{v_1 y_1}{y_H}$$

where y_H = half the thickness at the neutral plane

Under these conditions if τ is the specific frictional force when the velocity of slippage equals the velocity of the metal it can be shown that $f_x = \mp \tau y_1 \left(\frac{1}{y_1} - \frac{1}{y_H}\right)$

and the basic Equation (3) becomes

$$\frac{d(p_x - 2k)}{dx} - \frac{2K}{y}\frac{dy}{dx} \pm \tau\left(\frac{1}{y} - \frac{1}{y_H}\right)\frac{y_1}{y} = 0 \qquad (11)$$

Again using the parabolic approximation for the arc of contact evaluating the constants of integration from the boundry conditions this equation has the solution

$$p_x = 2k\,[\zeta_1 + \log(1 + Z^2)] + \frac{A\tau}{2}\left[\frac{Z}{1+Z^2} - B\arctan Z\right] \qquad (12)$$

Where

$$B = \frac{1 - Z_H^2}{1 + Z_H{}^2}\left\{\frac{\frac{2k}{A\tau}[\zeta_1 - \zeta_o + \log(1 + Z_o^2)] + \frac{Z_o}{1 + Z_o^2}}{\arctan Z_o}\right\}$$

$$Z_H = \sqrt{\frac{1 - B}{1 + B}}$$

D. Tselikov's Approximation and Solution to the Distribution Equation

Tselikov based his solution on the assertion that the material does not necessarily slip at all but one point on the arc of contact, rather that there was a region of no-slip condition. This point of view has much substantiation in experimental work, but represents a significant departure from the fundamental theory of Von Karman. The resulting expression for distribution of roll force is largely empirical and is not included in this discussion.

E. Orowan's Theory

Orowan used an entirely analytic approach in which he includes the elastic deformation of the rolls. It does not assume slippage between roll and material but includes criteria for determining where sticking occurs and where slipping occurs. Thus many of the restrictions of the Von Karman, Siebel, Nadai theories are circumvented. The Orowan theory is very complex in its nature and application and as such does not lend itself readily to computational methods.

The Orowan theory was the basis for further work done by Bland and Ford upon which the following mathematical model is based.

F. Mathematical Model Based upon Work of Bland and Ford

1. Roll Pressure

The theory advanced by Bland and Ford recognized the necessity of reducing the calculation for specific roll pressure to an expression of known or measurable quantities. Reasoning that the calculation for roll pressure would be used primarily for determining either load design for a new mill or maximum reduction for an existing one, Bland and Ford reduced the equation for roll pressure to a function of known or assumed variables:

$$p = F\,(h_o,\ h_1,\ h_n,\ k_o,\ k_1,\ k_n,\ \sigma_f,\ \sigma_b,\ R').$$

As derived in the Appendix C,

$$p^+ = \frac{2\,k_n\,h_n}{h_1}\,(1 - \frac{\sigma_f}{2k_1})\ e^{\mu H} \tag{13}$$

for the exit side, and

$$p^- = \frac{2\,k_n\,h_n}{h_o}\,(1 - \frac{\sigma_b}{2k_o})\ e^{\mu(H_o - H)} \tag{14}$$

for the entry side. The new quantity H is defined as

$$H = 2\sqrt{\frac{R'}{h_1}}\,\tan^{-1}\left(\frac{R'}{h_1}\cdot\phi\right) \tag{15}$$

and H_o is the value of H at the entry; k_o and k_1 are yield shear stress at entry and exit.

The total roll force P is now available from Equations (13) and (14) as

$$P = \int_o^{\phi} p\,R'\,d\phi \tag{16}$$

or

$$P = \int_{o}^{\phi n} p^{+} R' \, d\phi \; + \int_{\phi n}^{\phi} p^{-} R' \, d\phi \qquad (17)$$

2. The Neutral Angle

The calculation for position of the neutral angle in the roll gap frequently enters into the calculations for tensions and rolling speeds. The expression for the neutral angle is found from Equations (13) and (14) by recognizing that at the neutral point $p^{+} = p^{-}$. Equating Equations (13) and (14) and solving for Hn

$$H_n = \frac{H_o}{2} - \frac{1}{2\mu} \quad \ell n \left[\frac{h_o}{h_1} \quad \frac{(1 - \sigma_f / 2 k_1)}{(1 - \sigma_b / 2 k_o)} \right] \qquad (18)$$

from which

$$\phi_n = \sqrt{\frac{h_1}{R'}} \quad \tan \frac{H_n}{2} \quad \sqrt{\frac{h_1}{R'}} \qquad (19)$$

where R' is from Hitchcock's equation

$$R' = R \, \frac{(1 + cP)}{(h_o - h_1)} \qquad (20)$$

As with Von Karman's equation for pressure distribution, the Equation (19) is too cumbersome for calculation purposes so it is proposed to use an approximation by Fajnberg (3), a derivation of which is given in the Appendix D.

$$\phi_n = \frac{\phi_o}{2} \left(1 - \frac{\phi_o}{2\mu}\right) + \frac{1}{4\mu \, R' \bar{p}} \left(\sigma_f h_1 - \sigma_b h_o\right) \qquad (21)$$

where ϕ_o is calculated from

$$\phi_o = \sqrt{\frac{h_o - h_1}{R'}} \qquad (22)$$

This completes the definition of the basic process in terms of known or measurable process variables. The reason for definition of the pressure and neutral point will become apparent. At this point it is sufficient to note that the mathematical model for the deformation of the metal in the roll-gap gives the distribution of pressure necessary to reduce the thickness from h_o to h_1. This model is predicated upon the assumption that the coefficient of friction is constant and that slipping occurs all along the arc of contact except at one point, called the neutral point. To complete the expression of the process the location of the neutral plane is expressed in terms of the same variables that were used to define the pressure distribution.

3. Rolling Torque

Since the control of the process is done primarily by the motor driving the work rolls, it is of interest to express the torque supplied by the motor in terms of the rolling variables used to define the pressure distribution. From Figure 1 it is apparent that the torque G per roll is as follows:

$$\text{maximum torque per roll per unit width} = G_R = R \int_{\phi_n}^{\phi} \mu R' p^- d\phi - \int_o^{\phi_n} \mu p^+ R' d\phi$$

$$= \mu R R' \int_{\phi_n}^{\phi} p^- d\phi - \int_o^{\phi_n} p^+ d\phi$$

The above equation is not in a form for ready calculation. In their work in 1948 Bland and Ford (1) assumed μ constant and simplified the above to

$$G_r = RR' \left[\int_0^{\phi} p\phi \, d\phi + \frac{(p_1 h_1 - p_2 h_2)}{2R'} \right]$$

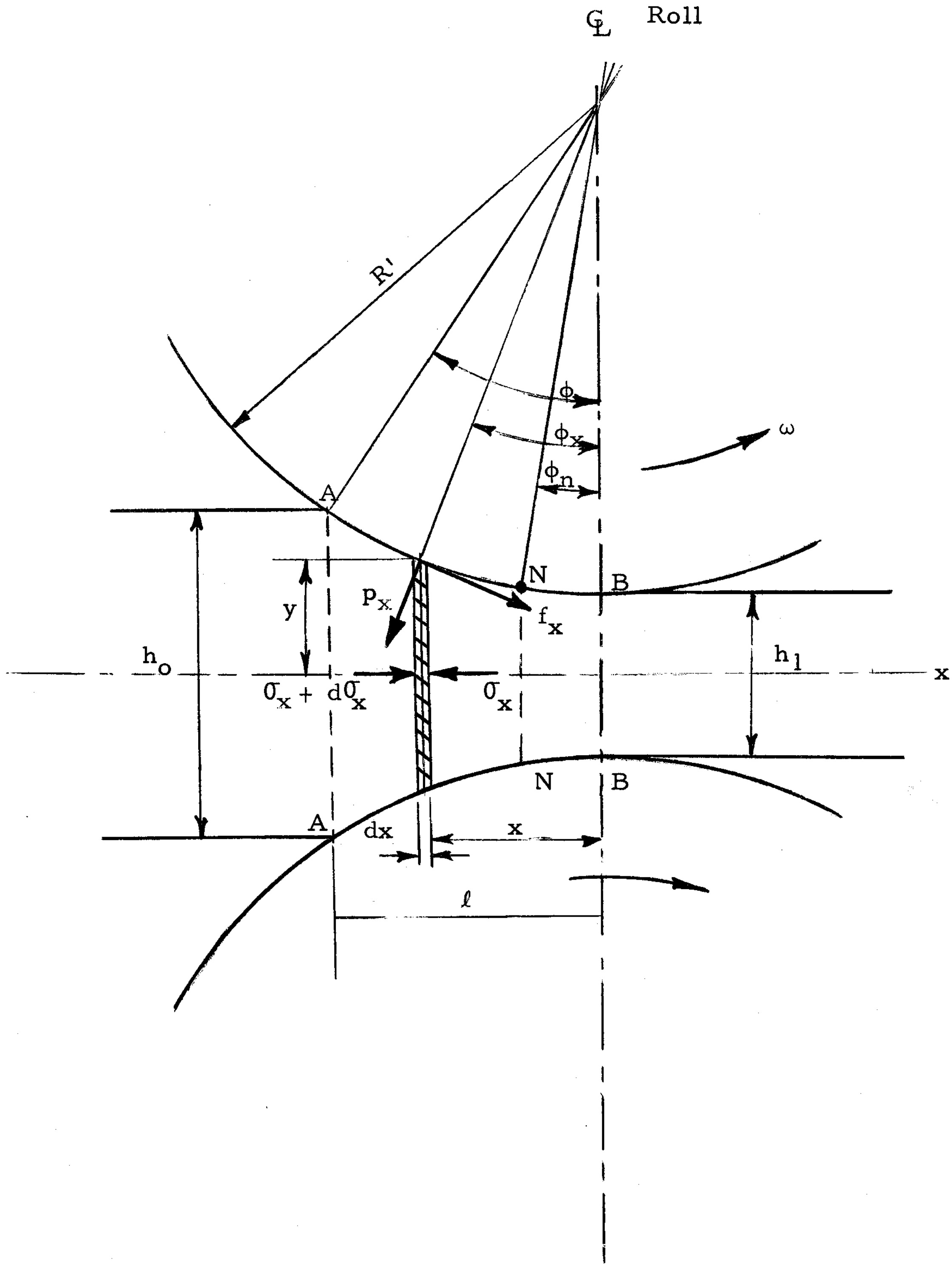


FIGURE 1

A-A - Plane of entry
N-N - Neutral plane
B-B - Exit plane

II. THE MACHINE

A single stand rolling mill appears physically as shown in Figure 2. The mill shown there is called four-high because of the stack of two upper and two lower work rolls. The auxiliary equipment has been omitted for simplicity, since it effects neither the control nor the mechanical problem. It is worthy of mention that on cold strip mills the upper roll is counter balanced by a hydraulic system so that the roll opening may be adjusted and will remain even though there is not metal in the gap. This also means that the force exerted by the roll upon the metal does not include the weight of the upper roll.

For cold strip mills the work roll is generally driven while the upper and lower rolls, called back-up rolls, are used to add rigidity to the roll (i. e., reduce bending of the work roll).

A typical mill for rolling cold aluminum in the range of 0.125 inches thick up to 100 inches wide would have the following physical characteristics:

Work roll	- 18" dia. x 105" long
Back-up	- 42" dia. x 105" wide
Drive motor	- 3000 HP, 600 V, 3920 A 225/440 rpm
Winding reel motors	- 2-200 HP, 230 V, 740 A 300/1200 rpm (G. R. 8.58:1)
Screwdown motors	- 2-75 HP, 230 V, 273 A

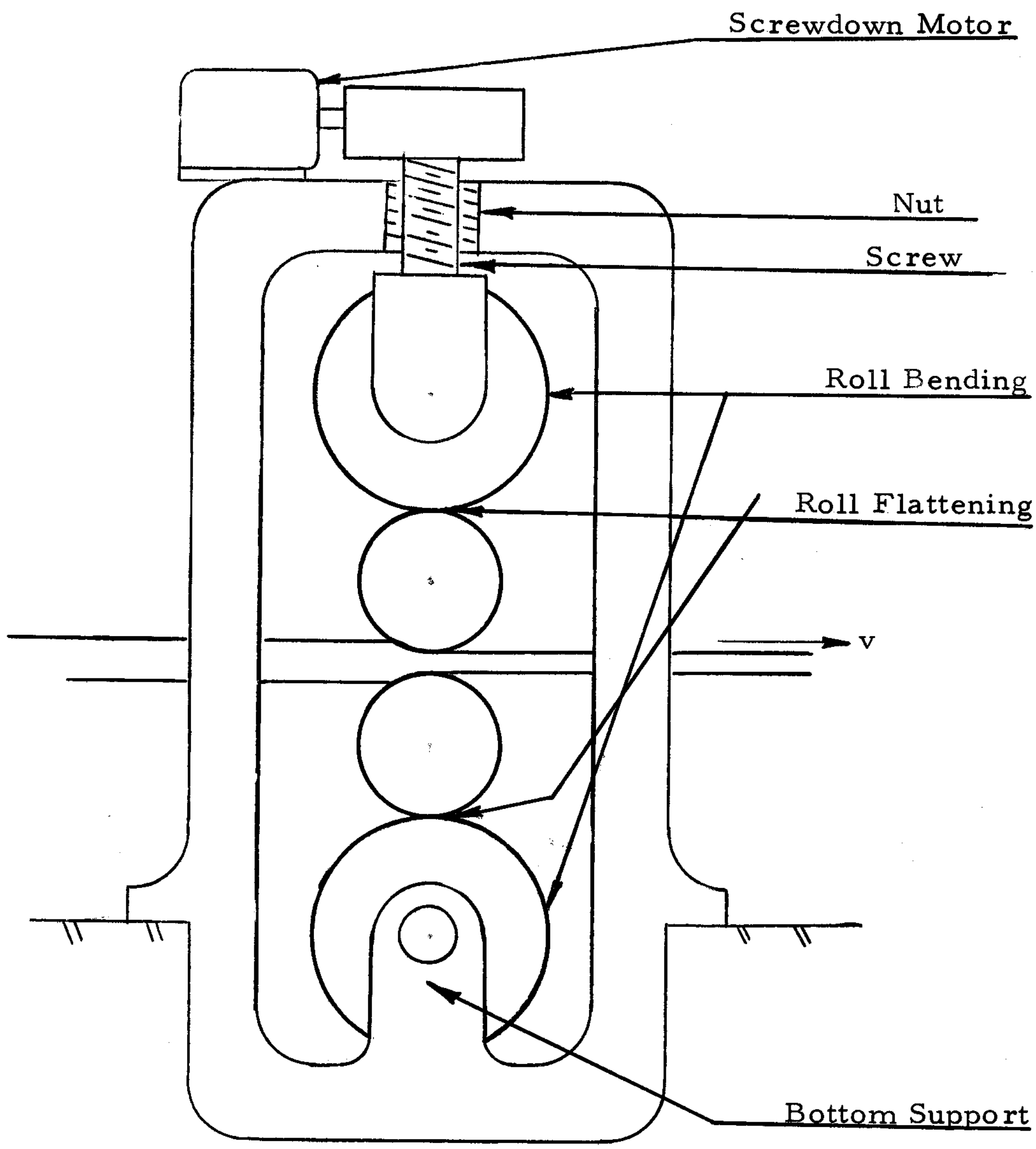
The roll gap is adjusted by running the screwdown motors.

This turns the screw in the nut which is held rigid by the top of the mill housing. In this manner the roll gap is adjusted to any preset value when the mill is empty or, as we shall see, the screwdown may be used to adjust the gage (thickness) of the metal. One of the screw-down motors is on each side of the housing. They may be run either one at a time to equalize the roll gap or together to change the whole opening. For this study it will be assumed that the screws move together and it will be understood that the rolls remain level.

Ideally the roll gap should be set to an opening of h_1 to give a delivered gage of h_1. For the practical case, however, there are three major factors which cause the gage to differ from the original roll opening. They are:

1. roll flattening
2. roll deflection
3. housing stretch

Figure 2 indicates where these phenomena take place. To get a clearer understanding of the forces involved, consider Figure 3. This figure shows a piece of metal of width w being rolled between two work rolls. In the previous discussion of the process it was shown that the force exerted by the rolls on the work piece caused deformation of the metal. Considering now the external forces acting on the roll, the metal exerts a uniformly distributed force P against the roll. Assuming that the strip is centered in the roll gap the opposing (or equilibrium) forces would be of magnitude P/2



MILL HOUSING

FIGURE 2

at each roll chock. From the strength of materials the deflection of the roll at the center point is

$$\delta = \frac{P\ell}{AE} \tag{23}$$

where P = total roll force (or separating force)

ℓ = distance between roll supports

A = cross sectional area of the work roll and backup roll

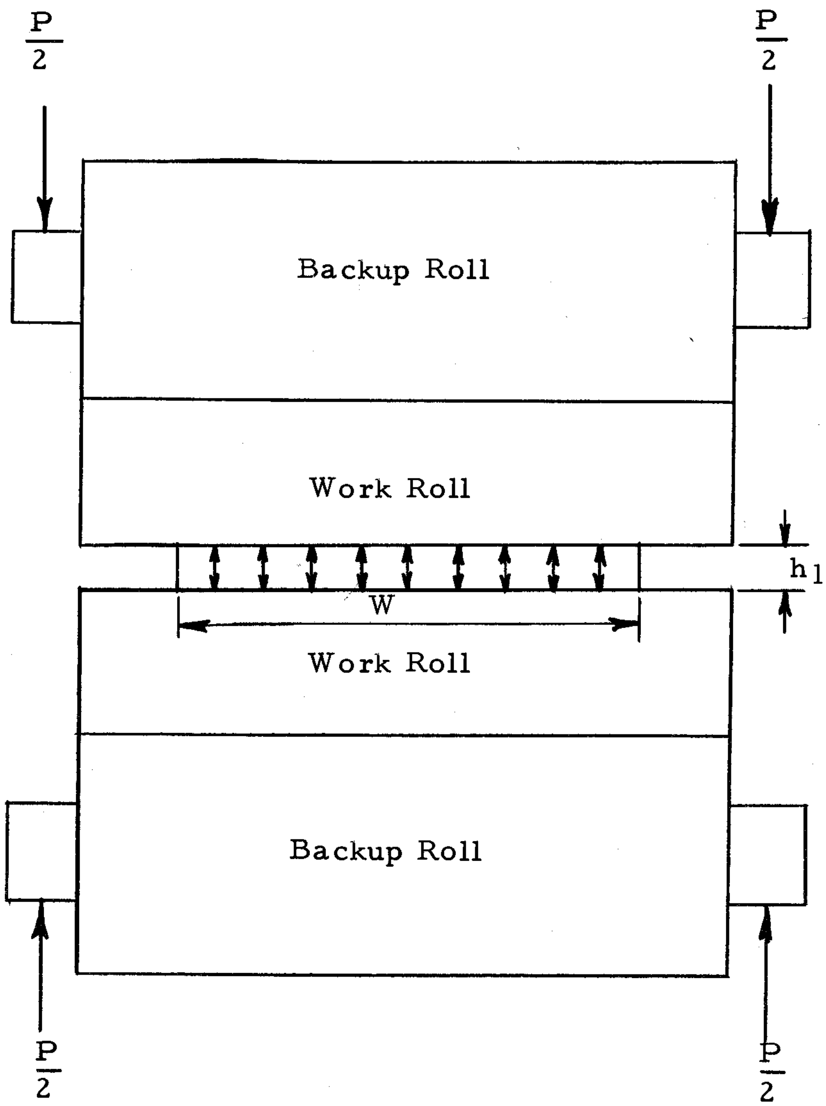
E = modulus of elasticity of the roll

In a mill such as the sample one the roll deflection would be:

$$\delta = \frac{(5 \times 10^6)(120'')}{T_1[(9)^2 + (21)^2]\ 30 \times 10^6} = .012 \text{ inches}$$

for top and bottom roll or a total departure from original gage setting of .024 inches. Normally the work roll will be ground with a crown to compensate for the roll deflection for the products normally rolled on that mill, so that the computation for roll deflection will not generally be used as an equation of a machine variable.

Reference again to Figure 3 shows the origin of the second machine-related variable, that of roll flattening. The standard work in this area is that of Hitchcock (5) in which the roll flattening is equated to a new or deformed roll radius. Flattening of the roll occurs in two areas, that of contact between the work and back-up rolls and that of contact between work piece and work roll. As shown in Part I the effect of flattening in the area of contact with the strip is to change the effective roll radius R to



LOADED ROLLS

FIGURE 3

$$R' = R\frac{(1 + 2\,cP)}{(h_o - h_1)}\ . \tag{24}$$

The amount of flattening between the rolls is entirely load dependent, and is generally included in the third machine-variable, housing stretch.

From Figures 2 and 3 it is apparent that the separating force exerted upon the rolls is borne by the roll supports or chocks. The effect of this load is to elastically compress the mill screws and nuts and bottom support and to elastically elongate the mill housing. Except for a small amount of backlash (or taking up the slack) at light loads, the combined effect is a spring relationship in which the amount of separation of rolls is a linear function of the separating force. Since roll flattening is also an elastic phenomena it is usually included in an overall constant called the Mill Spring. This relationship is essential to accurate control of thickness and is usually determined experimentally by placing hydraulic jacks between the roll chocks and measuring the separation of the chocks as a function of applied pressure. Figure 4 shows a Mill Spring line for a hypothetical mill. It is apparent that the Mill Spring constant is a function of the mill's mechanical design; it is sometimes referred to as the "stiffness" of the mill.

The auxiliary equipment of the mill, the unwind and rewind equipment are important in that they must be able to apply tensions to the strip in magnitudes high enough to serve as an effective controller of gage. Actual tension control is accomplished by use of electric motors, which will be discussed under electrical characteristics.

The electrical characteristics of the mill are the controls which are available to control the machine in such a way that the process is also controlled. In a single stand mill the electrical circuits must be able to control the unwind and rewind speeds and tensions. The main mill motors speed and torque, and the position of the mill screws must be similarly controlled.

For purposes of continued analysis it is necessary to decide upon the types of electrical control systems which will accomplish the described control functions. Let us assume the following:

1. The Mill Screw Control - The mill screw control will be a DC shunt motor, which will be pulsed by the control when a gage correction is desired.
2. The Main Mill Drive - The main mill drive will be a D-C shunt motor with an amplidyne regulating system for speed control. Primary control for accelerating will be controlled by a motor operated rheostat which starts the motor at a maximum field, minimum armature voltage. The armature voltage increases to base speed, whereupon the field begins to weaken. Field weakening

continues to a preset value for a given running speed. Thereafter speed control is accomplished by continuous adjustment of the armature voltage by the amplidyne regulator.

3. The Unwind Regulator - The unwind tension and speed will be controlled by an amplidyne regulator which controls the motor armature voltage. During acceleration and while running the speed of the unwind drag generator will be determined by the speed of the main motor since the sheet provides the pull to turn the drag generator.

4. The Rewind Regulator - The rewind motor speed will also be amplidyne-regulated armature voltage controlled. Speed during acceleration and deceleration will be controlled by the voltage output of a pilot generator which is driven by the main mill drive motor.

This section has outlined in general terms the characteristics of the machine which we shall use to control the process described in Part I. In the following section the combination of machine and process will be treated from a more analytical point of view. This section, then, lays the groundwork or describes the specification for the following one.

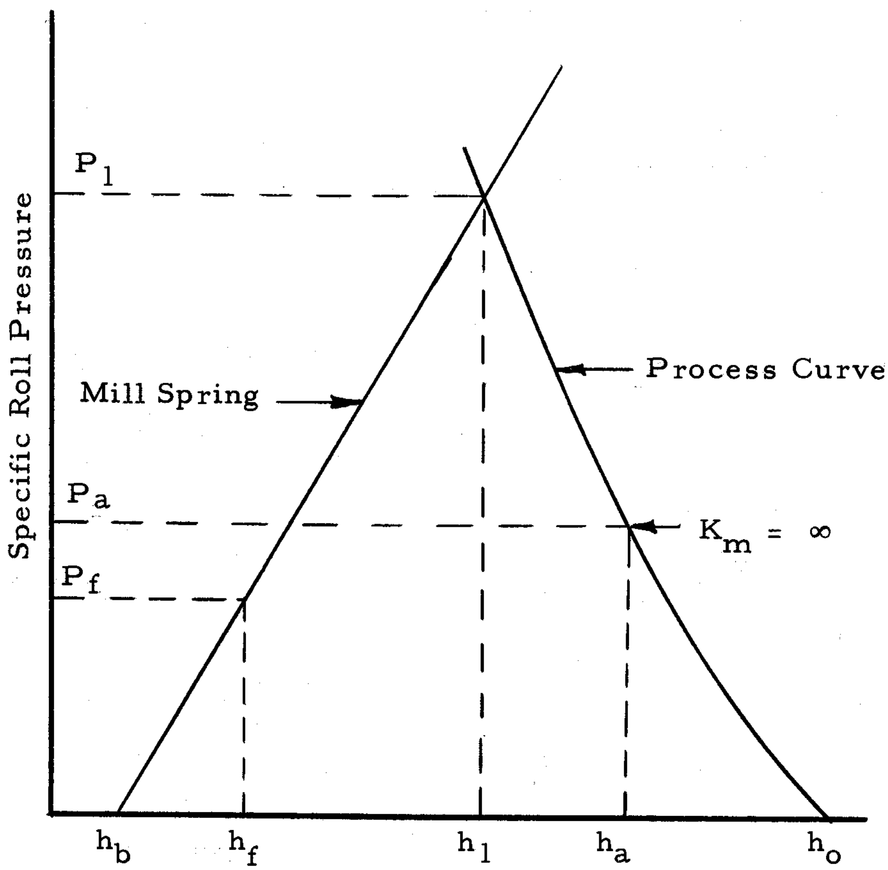
III. THE COMBINATIONAL PROBLEM

Consider the graph shown as Figure 4. This graph shows how the thickness (gage) of the product is determined by the process as well as the machine. The curve labeled "process curve" is the relationship between thickness of the strip and the roll load considering a perfectly rigid mill (i. e., Mill Spring Constant $k_m = \infty$). Interpretation of the process curve is that if a strip enters the mill at initial thickness h_o and if the mill is perfectly rigid an initial roll opening of h_a would produce a separating force of P_a. The resulting exit gage would then be h_a.

The Mill Spring curve says that if the original roll opening is h_b, a separating force of P_f will produce a resultant opening of h_f. Or viewed another way, a separating force of P_f will result in a stretch of h_f-h_b inches.

Combining the above two curves gives an expression of the overall problem: when a strip of initial thickness h_o, having a process curve as shown in Figure 4 is rolled by a mill with initial roll opening h_b having a mill spring curve as shown in Figure 4, what will be the resultant output gage? The answer is apparently the point of simultaneous solution of the two curves. This will produce an output gage of h_1 and a resulting roll force P_1.

In order to control the process it is necessary to know the



OUTPUT GAGE

FIGURE 4

response of the system to changes in the rolling variables. Since the bulk of the coil is rolled under "steady state" conditions the behavior of the process about the operating point (intersection of the two curves in Figure 4) is of primary importance and will be considered first. This study has been approached in a variety of ways: by differential equations in the manner of Sekulic and Alexander (9); with finite difference equations as by Courcoulas and Ham (2); and in terms of per unit perturbations by Phillips (8). The availability of a computer to solve for the constants makes the method of Phillips both more practical and easier to apply to the control of the process.

As shown in Part I the equations which relate the variables are non-linear and extremely complex. Fortunately, for the small changes which occur about the operating point, the coefficients of linearized equations may be obtained numerically using the non-linear equations. The principal causes of changes in roll separating force F, rolling torque, and neutral point thickness are changes in incoming and outgoing tensions, changes in incoming and outgoing thickness, and changes in the coefficient of friction.

Assume that for a particular stand equations of this form can be written

$$\frac{\Delta P}{P} = K_1 \frac{\Delta h_1}{h_1} + K_2 \frac{\Delta T_b}{A_o} + K_3 \frac{\Delta T_f}{A_1} + K_4 \frac{\Delta h_o}{h_o} + K_{13} \frac{\Delta \mu}{\mu} \qquad (25)$$

$$\frac{\Delta G_r}{G_r} = K_5 \frac{\Delta h_1}{h_1} + K_6 \frac{\Delta T_b}{A_o} + K_7 \frac{\Delta T_f}{A_1} + K_8 \frac{\Delta h_o}{h_o} + K_{14} \frac{\Delta \mu}{\mu} \qquad (26)$$

$$\frac{\Delta h_n}{h_n} = K_9 \frac{\Delta h_1}{h_1} + K_{10} \frac{\Delta T_b}{A_o} + K_{11} \frac{\Delta T_f}{A_1} + K_{12} \frac{\Delta h_o}{h_o} + K_{15} \frac{\Delta \mu}{\mu} \qquad (27)$$

These equations are essentially the same as those suggested by R. A. Phillips (8) except for the addition of K_{13}, K_{14} and K_{15} to include changes in the coefficient of friction. As previously noted from the work of Nadai, this term is to include the well-known speed effect which says that for equal tensions, entry gage, and screw opening, the exit gage will be thinner at a higher speed.

The constants K_1 through K_{15} are calculated from the equations for roll force P, torque G_r and height at the neutral point as given by the mathematical model derived in Part I. The force F, torque G_r, and neutral point thickness h_n are calculated at the initial operating point. Then one of the conditions, say h_1, is changed a few percent. A new roll force, torque and neutral point thickness are calculated and from them K_1, K_5, and K_9 are determined. In a like manner the other K's are computed.

Combining the process Equations (25) through (27) with the machine equation for the rolling mill will express the machine-process system in terms of relationships which are measurable and controllable. From an economic point of view the objective function is to produce a controlled product with the least out-of-tolerance.

Generally, sheet or rolled products are specified by their thickness so that the obvious variable to control is that of finished gage. For this reason this control model is derived for controlling the exit thickness of the metal at the least deviation from nominal. There exist other conditions which limit the ranges of some of the variables. Some of these are: (1) surface quality, (2) available speed from the motors, (3) available torque from the motors, (4) limits of load on mill housing and roll, (5) limits of tension to stay within the elastic range of the sheet, and (6) the neutral angle ϕ_n must be less than the angle of contact or the roll will slip on the work piece. These and other limitations are discussed in the literature and would only serve here to cloud the issue. This model makes use of the simplifying assumption that the operating point described by Equations (25) through (27) is well within the design limits of the mill and that any corrective action will not exceed those limits.

The magnitude of the change in roll separating force, P_1 is dependent upon the combined spring of mill, as defined in Part II, and the change in outgoing gage and screw setting change, if any.

Mathematically, a change in P about the operating point will be, by Hooke's law

$$\Delta P = M\ (\Delta h_1 - \Delta S) \tag{28}$$

where ΔP is change in roll force

M is combined spring constant, millions of pounds per inch

ΔS is change in screw position, positive upward.

It is of interest to note here that the relationship in Equation (28) is usually defined by a second order differential equation, but since the natural frequency of the mill can be assumed very high compared to the regulating systems of the mill, the inertia and damping terms are neglected.

Substituting Equation (28) into Equation (25) and solving for $h_1/\Delta h_1$, the result may be substituted into Equation (26) and (27) giving the following set of equations

$$\frac{\Delta h_1}{h_1} = \frac{K_2}{C}\frac{\Delta T_b}{A_o} + \frac{K_3}{C}\frac{\Delta T_f}{A_1} + \frac{K_4}{C}\frac{\Delta h_o}{h_o} + \frac{K_{13}}{C}\frac{\Delta \mu}{\mu} + \frac{M/P}{C}\Delta S \qquad (29)$$

$$\frac{\Delta G_r}{G_r} = \left[\frac{K_5K_2}{C} + K_6\right]\frac{\Delta T_b}{A_o} + \left[\frac{K_5K_3}{C} + K_7\right]\frac{\Delta T_f}{A_1} + \left[\frac{K_5K_4}{C} + K_8\right]\frac{\Delta h_o}{h_o}$$
$$+ \left[\frac{K_5K_{13}}{C} + K_{14}\right]\frac{\Delta\mu}{\mu} + \left[\frac{K_5\, M/P}{C}\right]\Delta S \qquad (30)$$

$$\frac{\Delta h_n}{h_n} = \left[\frac{K_9K_2}{C} + K_{10}\right]\frac{\Delta T_b}{A_o} + \left[\frac{K_9K_3}{C} + K_{11}\right]\frac{\Delta T_f}{A_1}$$
$$+ \left[\frac{K_9K_4}{C} + K_{12}\right]\frac{\Delta h_o}{h_o} + \left[\frac{K_9K_{13}}{C} + K_{15}\right]\frac{\Delta\mu}{\mu} + \left[\frac{K_9\, M/P}{C}\right]\Delta S \qquad (31)$$

where $C = \frac{Mh_1}{P} + K_{12}$

Equations (29), (30) and (31) are useful from a control point of view in that they enable one to define certain "transfer functions" of the mill-process system.

One frequently used transfer function is called the "screw down transfer function," which is defined as $\frac{\Delta h_1}{\Delta S}$, the change in output gage caused by a change in screw position. From Equation (29) with constant tensions, constant coefficient of friction and no change in input thickness the screwdown transfer function is

$$\frac{\Delta h_1}{\Delta S} = \frac{mh_1}{P} \left[\frac{1}{\frac{mh_1}{P} + K_1} \right] \tag{32}$$

In a similar fashion it is possible to define the transfer function for various other rolling variables; however, these are valid only for small changes and assume that other variables remain constant. As we shall see in a multivariable problem of this type, these assumptions may not be valid. Transfer functions so derived, although in common usage, must be used with discretion if they are to have any value at all. Since other transfer functions based upon these same equations will be subject to the same limitations they will not be derived here.

Equations (29), (30) and (31) are useful in defining the various transfer functions and interrelationships between the process variables. Before the process is to be controlled, however, it is necessary to define the response characteristics of the controller in terms of the process. For example, if the unwind generator is to control the input tension, it is desirable that it react quickly enough to changes in the system to hold input tension constant. The desired relationship is that which expresses a change in tension, in this case, to other changes in the system. The relationship governing changes in tension has its basis in the elastic property of the strip between unwind and roll.

Appendix E derives the equation for tensions in the strip as

$$\frac{T_b}{A_o} = \frac{E}{L_u} \int_o^t (v_o - v_u)\, dt \qquad (33)$$

$$\frac{T_f}{A_1} = \frac{E}{L_r} \int_o^t (v_r - v_1)\, dt \qquad (34)$$

where E = modulus of elasticity

L_u = distance to unwind

L_R = distance to reel

v_u = velocity of strip at unwind

v_R = velocity of strip at reel

At the roll gap, as previously noted, the mass flow relationship may be written as

$$h_o\, v_o = h_n\, v_n = h_1\, v_1 \qquad (35)$$

This relationship assumes no change in density of the metal from plastic deformation and no change in width of the strip.

Again restricting the derivation to small changes in speed, it is possible to develop a set of linear equations to analyze the process. Replacing each of the variables in Equation (35) with an initial value plus a small change or perturbation,

$$h_o = h_o + \Delta h_o$$

$$h_n = h_n + \Delta h_n$$

$$h_1 = h_1 + \Delta h_1$$

$$v_o = v_o + \Delta v_o$$

$$v_n = v_n + \Delta v_n$$

$$v_1 = v_1 + \Delta v_1 \tag{36}$$

with the reference point such that the relationship in (35) still holds. Substituting Equation (36) in (35) and dividing by (35) gives

$$\left[1 + \frac{\Delta h_o}{h_o}\right]\left[1 + \frac{\Delta v_o}{v_o}\right] = \left[1 + \frac{\Delta h_n}{h_n}\right]\left[1 + \frac{\Delta v_n}{v_n}\right] \tag{37}$$

$$\left[1 + \frac{\Delta h_1}{h_1}\right]\left[1 + \frac{\Delta v_1}{v_1}\right] = \left[1 + \frac{\Delta h_n}{h_n}\right]\left[1 + \frac{\Delta v_n}{v_n}\right] \tag{38}$$

Since the delta quantities are small compared to unity, the products of delta quantities can be ignored and

$$\frac{\Delta v_o}{v_o} = \frac{\Delta v_n}{v_n} + \frac{\Delta h_n}{h_n} - \frac{\Delta h_o}{h_o} \tag{39}$$

$$\frac{\Delta v_1}{v_1} = \frac{\Delta v_n}{v_n} + \frac{\Delta h_n}{h_n} - \frac{\Delta h_1}{h_1} \tag{40}$$

If Equations (29) and (30) are substituted into Equations (39) and (40) the result is

$$\frac{\Delta v_o}{v_o} = \frac{\Delta v_n}{v_n} + \left[\frac{K_9 K_2}{C} + K_{10}\right]\frac{\Delta T_b}{A_o} + \left[\frac{K_9 K_3}{C} + K_{11}\right]\frac{\Delta T_f}{A_1}$$
$$+ \left[\frac{K_9 K_4}{C} + K_{12} - 1\right]\frac{\Delta h_o}{h_o} + \left[\frac{K_9 K_{13}}{C} + K_{15}\right]\frac{\Delta \mu}{\mu} + \left[\frac{K_9 M/P}{C}\right]\Delta S \tag{41}$$

$$\frac{\Delta v_1}{v_1} = \frac{\Delta v_n}{v_n} + \left[\frac{K_2\,(K_9-1)}{C} + K_{10}\right] \frac{\Delta T_b}{A_o} + \left[\frac{K_3\,(K_9-1)}{C} + K_{11}\right] \frac{\Delta T_f}{A_1}$$

$$+ \left[\frac{K_4\,(K_9-1)}{C} + K_{12}\right] \frac{\Delta h_o}{h_o} + \left[\frac{K_{13}\,(K_9-1)}{C} + K_{15}\right] \frac{\Delta \mu}{\mu} \qquad (42)$$

$$+ \left[\frac{(K_9-1)\,M/P}{C}\right] \Delta S$$

Rewriting Equations (33) and (34) into small difference form to express plastic flow in the roll gap gives

$$s\,\frac{\Delta T_b}{A_o} = \frac{E\,v_u}{L_u}\left[\frac{\Delta v_o}{v_o} - \frac{\Delta v_u}{v_u}\right] \qquad (43)$$

$$s\,\frac{\Delta T_f}{A_1} = \frac{E\,v_1}{L_R}\left[\frac{\Delta v_R}{v_R} - \frac{\Delta v_1}{v_1}\right] \qquad (44)$$

Substitution of Equations (41) and (42) into (43) and (44) and solving for the tensions puts the equation in time constant form.

$$\frac{\Delta T_b}{A_o} = \frac{1/A}{1 + \frac{1}{AE}\,\frac{L_u}{v_u}\,s}\left[-\frac{\Delta v_n}{v_n} + \frac{\Delta v_u}{v_u} - B\frac{\Delta T_f}{A_1}\right.$$

$$\left. - D\,\frac{\Delta h_o}{h_o} - F\,\frac{\Delta \mu}{\mu} - H\,\Delta S\right] \qquad (45)$$

where

$$A = \frac{K_9 K_2}{C} + K_{10}$$

$$B = \frac{K_9 K_3}{C} + K_{11}$$

$$D = \frac{K_9 K_4}{C} + K_{12} - 1$$

$$F = \frac{K_9 K_{13}}{C} + K_{15}$$

$$H = \frac{K_9 \, M/P}{C}$$

$$\frac{\Delta T_f}{A_1} = \frac{1/N}{1 + \frac{1}{NE} \frac{LR}{v_1} s} \left[\frac{\Delta v_R}{v_R} - \frac{\Delta v_n}{v_n} - J \frac{\Delta T_b}{A_o} - W \frac{\Delta h_o}{h_o} - Q \frac{\Delta \mu}{\mu} - X \, \Delta S \right] \qquad (46)$$

where

$$J = \frac{K_2 \, (K_9 - 1)}{C} K_{10}$$

$$N = \frac{K_3 \, (K_9 - 1)}{C} K_{11}$$

$$W = \frac{K_4 \, (K_9 - 1)}{C} K_{12}$$

$$Q = \frac{K_{13} \, (K_9 - 1)}{C} K_{15}$$

$$X = \frac{(K_9 - 1) \, M/P}{C}$$

The tension control equations lead directly to the expressions for changes in torque delivered by the motor of the mill stand. The stand motor must supply the rolling torque plus the torque due to incoming tension minus the torque due to outgoing tension. In small oscillation form

$$\Delta G_m = \Delta G_r + A_o R \times 10^6 \frac{\Delta T_b}{A_o} - A_1 R \times 10^6 \frac{\Delta T_b}{A_1} \qquad (47)$$

Dividing by G_r for per unit form

$$\frac{\Delta G_m}{G_r} = \frac{\Delta G_r}{G_r} + \frac{A_o\ Rx10^6}{G_r}\ \frac{\Delta T_b}{A_o} - \frac{A_1\ Rx10^6}{G_r}\ \frac{\Delta T_1}{A_1} \qquad (48)$$

This completes the derivation of the machine-process equations. To summarize at this point, the above equations express the relationships which exist between the process and machine variables; or, to be more specific, it defines changes in key variables in terms of constants and other variables which are measurable. Since Equations (45) and (46) also include the transient response to changes in the variables, these equations contain all the information for complete transient and steady state analysis of the system. The tasks remaining are to determine the characteristics of the controller and to determine the method of control.

IV. THE SERVO PROBLEM

In the earlier discussion of the machine some reference was made to the electrical characteristics. It should be appreciated that the entire mill control is a complex combination of servomechanisms, protective relaying and various interconnecting devices. Although all the electrical properties of the control certainly have an effect upon the total machine-process performance, there are certain portions of the control system which are most directly concerned with the process under discussion. In general, the systems to be discussed are:

(1) The main mill motor speed control

(2) The unwind tension control

(3) The reel tension control

The purpose of the main mill motor speed control is to regulate the speed of rolling in a manner which produces the desired control of the process. From the standpoint of machine operation it is obvious that the speed should stay as high as possible to get the maximum utilization from the equipment. Similarly, the process equations show that the speed of rolling is an effective means of controlling the output thickness of the metal.

The basic servomechanism for speed control such as described in Part II is in common use in the industry (11; 12). The basic multi-stage rotating amplifier is shown in Figure 5, along with its block diagram. In this figure the reference speed is preset, the input T_L is actual motor load which is fed back through the flux in the machine.

The rotating amplifier here is the combination of amplidyne and the drive generator. The gain of this equivalent amplifier is

$$G_A = \frac{K_A K_G}{(S T_f + 1)(S T_q + 1)} \tag{49}$$

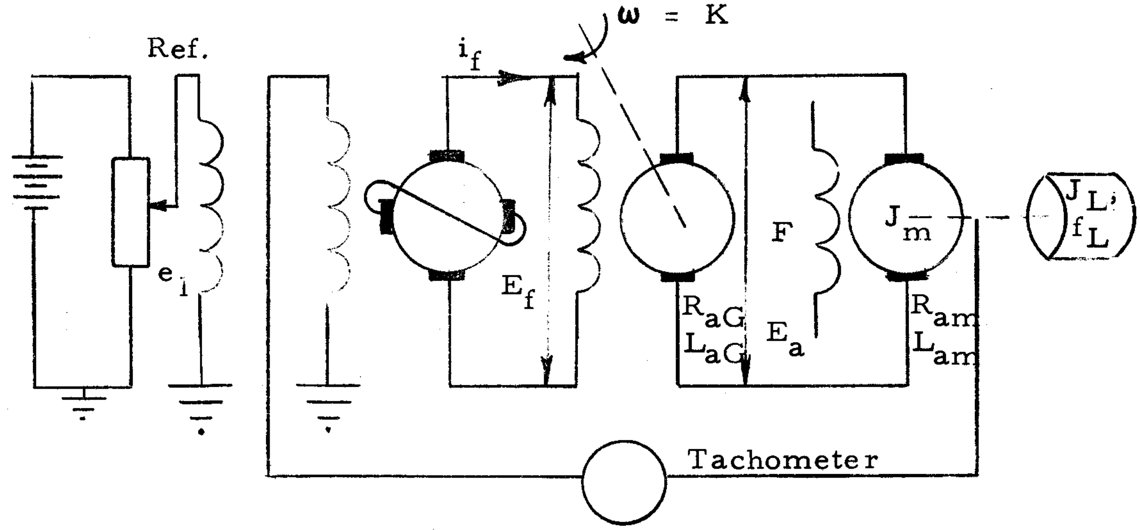
where K_A = amplifier gain constant

K_G = generator gain constant

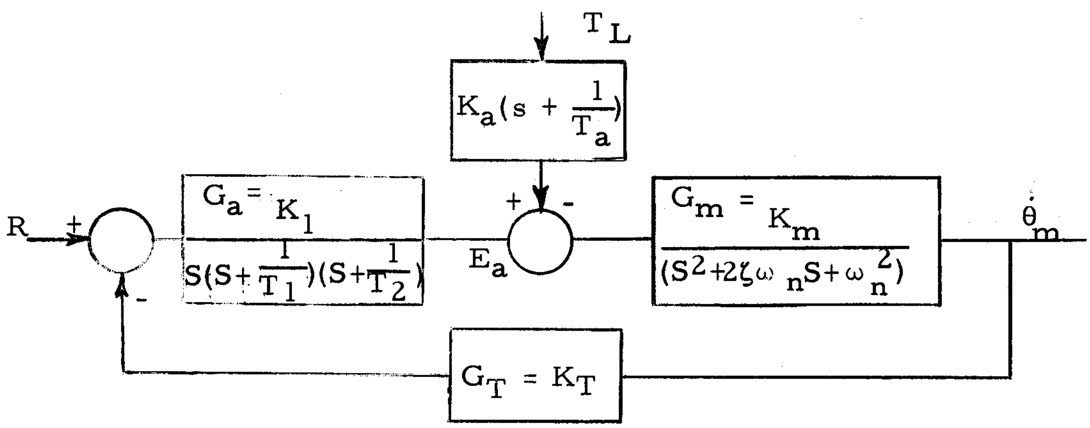
T_f = control field time constant

T_q = quadrature field time constant

The time constants are lumped, that is, if multiple control fields are used the total time constant is T_f. Likewise, multiple fields may effect the values of the gain constants K_A and K_G. As shown in Figure 5 the reference signal is fed into a control field on the amplidyne. In order to control the process adequately, there must be control fields available to accept the signals from the changing process variables. If we wish to control the output thickness we will supply a control field current proportional to the deviation of h_1 from its nominal amount. The effectiveness of that signal, i.e., how much speed change is necessary to correct the thickness deviation comes from the evaluation of the coefficients of the linear



SCHEMATIC DIAGRAM



BLOCK DIAGRAM

Equations of Motion

$$E_f = Ae_1 = i_f\,(R_f + L_f \frac{d}{dt})$$

$$E_a = K^1_{VG} i_f = (R_{aG} + R_{am})\, i_a + (L_{aG} + L_{am}) \frac{di_a}{dt} + K_{VM} i_a \dot{\theta}_m$$

$$K^1_{VG} = K_{VG}\omega_G$$

For the amplifier with second order time constant

$$K_1 v_1 = \left[\frac{d^2}{dt^2} + \left(\frac{1}{T_1} + \frac{1}{T_2}\right)\frac{dt}{dt} + \frac{1}{T_1 T_2}\right] E_a$$

FIGURE 5

difference equations in Part III. By now it should become apparent how the process equations are used to control the process.

In a similar manner the servo controls of the reel motor are used to track the speed of the main drive motor. Since the process equations indicate that the exit thickness can be controlled by the front tension, it might be anticipated that a thickness deviation could be a signal for changing the exit tension $\frac{\Delta T_f}{A_1}$. As the equations show [Equation (40)] a tension change is accomplished by changing the relative velocities of the sheet at reel and mill, which, in turn, is by motor speed control.

Finally, the unwind motor controls are used to control the back tension on the sheet. One conventional way of controlling the unwind tension is to let the unwind machine operate as a drag generator; that is, be pulled along by the sheet. The back tension is then controlled by adjusting the load on the generator. As before, this may be done by preset signal which is modified by a process signal, say thickness deviation, to control back tension. The effectiveness of this control is that calculated from the process equations.

Of primary importance in consideration of the servomechanism problem is that of the investigation of the time constants involved. Equations (45) and (46) show the time constants involved before a change in variable yields a change in tension. In a single stand mill it is seen from the equations that the time constants are very small, particularly compared to the servomechanism time constants given in Figure 5. Therefore, for this process the process may be assumed to be linear and the response of the machine-process system is dependent only upon the response of the mill's regulating system.

The remaining consideration for time response of the system has to do with the problem of measuring the thickness of the metal. Despite some attempts to measure thickness of the metal by direct measurement of the roll gap, the most common way of determining the exit thickness is by means of a contact gage or x-ray gage located between the mill stand and the take-up reel. This introduces a transport delay, or distance-velocity delay which is probably the dominant time constant in the system. The transport delay produces a time lag in the form of $e^{-T_T s}$ which causes some difficulty in analytical treatment. Truxal (12) suggests treatment of a transportation plot by Bode or Nyquist plots in the conventional manner. Practically the transport delay is handled by damping the response of the gage with a RC filter of time constant about equal to the transport time. In this manner the input to the controller reads the average gage rather than instantaneous values.

The discussion thus far has defined the process and the characteristics of the machine. This section has described the servomechanism and the considerations that govern the control of the mill. To explain how the instrumentation of mill variables are used by the controller to control the mill, the following section deals with a typical problem; that of exit gage control.

V. THE CONTROLLER

As noted in the introduction to this treatise, the object of detailed analysis of the process is to obtain a better controlled product. The details of the design or synthesis of the controller is subject for another paper, but it is difficult to show the importance of the detailed analysis without showing how the machine-process equations are used to control the process. As with any engineering problem, the refinement of the control will be dependent upon the price the management is willing to pay. Economic considerations will determine which variables may be monitored by the instrumentation and how precise the control is to be. This, in turn will determine which of the variables will be controlled directly and which by approximation. The first step, therefore, is to determine what signals are available from the instrumentation and based upon those signals which variables can best be manipulated to control the process. Since most rolled products are specified by their thickness (or gage), the specific control problem will be that of maintaining a nominal output gage. Further, since the discussion has been limited thus far to the "steady state" condition of rolling, the control problem will be limited to that condition also. This does not imply that problems with threading the mill and control as the tail end leaves the mill are not important, it simply states that those problems are beyond the limits of the present discussion. If the control is to be based upon the process equations developed earlier it is subject to the same limitations as the equations.

The use of an amplidyne, or multi-field rotating amplifier will be continued, although it should be realized that control is not limited to this type of device. In common use now are magnetic amplifier and silicon controlled rectifiers in addition to various other power amplifying devices. Any can be employed in the same way that the amplidyne is; the amplidyne has been chosen arbitrarily to provide a clearer picture of the control function.

The variables which are of interest in controlling the exit gage h_1, according to Equation (29) are: front tension $\frac{T_f}{A_1}$, back tension $\frac{T_b}{A_o}$, input thickness h_o, the coefficient of friction μ, the screw position S, and the roll force P. Immediately we may eliminate a couple of the variables from the measurable category. One of these is the entry thickness h_o because an input gage would be prohibitively expensive and because the deviation should not be great if the previous rolling was done on a well controlled mill. The other that may be eliminated is the roll force P, mainly on an economic basis. If it were possible to obtain this measurement at reasonable expense the control problem would be somewhat less troublesome. The roll load P is, however, only the result of having to do work upon the metal within the gap and as such is a resultant variable rather than a manipulated one. For this reason we are justified in letting the roll force assume any level it will so long as the physical limitations of the mill are not exceeded. The process equations have shown that the roll force is dependent upon the tensions as well as the coefficient of friction so that for a given reduction these variables may be adjusted to keep the roll force within its limits.

Table 1 shows the instrumentation which we can make fairly easily. Front and back tensions are functions of the armature currents of reel and unwind machines. As such they are easily instrumented. The tensions are conventionally set by operator or punched card, by adjustment of the amplidyne reference field. The armature current signal is then fed into another control field to serve as the feedback signal. Changes in desired tension may come from resetting of the manual control, or by changing the control field current as a result of some variable changing.

Another measurement that is readily available is that of the main mill motor drive speed. This comes from a tachometer generator geared directly to the motor shaft and can be calibrated directly in the peripheral velocity of the work roll. This, by definition, is v_n, or the velocity of the metal in the roll gap at the neutral point. This signal is used by the amplidyne to regulate the mill motor speed to that preset by the operator. One other comment must

TABLE I

Variable	Instrumentation	Controller		Reference
		Amplifier	Machine	
Tension, back	Unwind generator Armature current	Amplidyne Control field	Unwind drag generator field current	Preset by operator
Tension, forward	Reel motor Armature current	Amplidyne Control field	Reel motor Armature voltage	Preset by operator
Coefficient of friction	Mill motor drive speed. Tachometer generator	Amplidyne Control field	Main motor Armature voltage	Preset by operator
Screw position	Direct measurement or implicit in h_1	none	Screw drive motor	Initially set by operator for given h_1
Roll force	Load cells under mill screw			
Output thickness h_1	X-ray or contact gage			Initially set by the operator

be made here about motor speed. It is well known by rolling experts that the output thickness, particularly in cold strip rolling, is very much dependent upon the speed of rolling. Of the classic works in rolling theory only Nadai and Orowan attempt to explain this phenomenon. The most plausible explanation is that the coefficient of friction, μ, and hence the frictional force, f_x, is the result of viscous friction and is therefore speed dependent. Thus the so called "speed effect" is taken into account by the term in the general process Equation (29). Thus a change in the roll speed and μ will change the output gage if all other variables remain fixed.

The screw position, which is a measure of the unloaded roll opening is not an essential measurement as far as the "steady state" of the rolling process is concerned, since the roll gap will be adjusted to whatever value is necessary to give the proper exit thickness. This measurement is rather important for mill setup, however, because the initial roll opening determines the operating line for the "Mill Spring" curve in Figure 4. The screw position is generally monitored at least as a visual indication of the initial opening.

The last bit of essential instrumentation is that of output gage measurement. This conventionally is done by a contact gage (or micrometer) which rides on the moving strip, or by a noncontacting x-ray gage. Each has its advantages and disadvantages but it would serve no purpose to argue their relative merits here. It is sufficient for the present purpose to note that the signal is continuous and is proportional to the deviation of the actual thickness from a desired nominal thickness.

The next step is to determine a method of control which will base its corrective action upon the signals available and work into the individual controllers of the variables that are to be manipulated. The variables most easily controlled are the tensions, the mill motor speed and the screw position. The screw motor and resultant screw position are by their very nature slow and controllable only in discrete steps. That is, the screw motor turns on, runs at a fixed speed until it turns off. It can control the gage over a very wide range and is, therefore, the logical choice for the coarse control. (Recent schemes of making the screw motors turn at different speeds by adjusting its armature voltage open the way to finer control by manipulation of screw positions. For the present the use of the screwdown is limited to coarse control of exit gage.)

The choice of using the tensions or the mill motor speed for vernier control becomes pretty much a matter of personal preference, depending primarily upon the type of mill and the range of product rolled. For the single stand mill under discussion the choice will depend upon the effectiveness of the controls as determined by the constants of Equations (25), (26) and (27). The constants for steel sheet have been worked out for certain assumed conditions (8) which confirm operating experience in showing that back tension is more effective than forward tension in controlling the exit gage, so long as the neutral point stays near the middle or somewhat forward of the center of the arc of contact. Since the position of the neutral point is very much dependent upon both forward and back tensions [Equation (21)] it would seem simpler to let the tension controllers attempt to maintain a constant tension and use the speed of the mill stand drive motor as a vernier control for output gage. Such will be the case. (For apparent reasons the speed control is not always the best. In a tandem mill for example, the problem of keeping the stand speeds syncronized overshadows the problem of tension control so that tension control is used to hold the gage near nominal.)

The deviation signal comes from the thickness gage located somewhere between the mill stand and the take-up reel. The previously discussed transport delay means that transient response of the mill to changes in exit gage must be damped out if control is to stay smooth. From an engineering point of view, this is reasonable since the customer usually is interested in the average thickness over a reasonable length rather than the strict adherence of each individual measurement to nominal thickness. To accumulate the deviation the signal will be fed into an integrator whose output will be the time integral of the deviation of the thickness from nominal. The integrator output will drive a power amplifier, which in turn, drives an amplidyne control field in the mill drive motor speed control circuit. The polarity is such that the speed increases as the gage tends to heavy and decreases as the gage tends to be light. This method of control continues until the speed reaches its limit, or the integrator saturates. At that time the screw motors are pulsed to make a change in roll opening and simultaneously the integrator is reset and the mill drive motor returned to its base speed. At this point the description becomes redundant.

It should be appreciated that this method is only one means of control. All the conventional techniques of proportional rate, and derivative action can be used as required to solve the specific problem. The object here was to hold the exit gage as near as possible to the desired value. Perhaps a maximum rolling rate in tons per hour or optimum loading of the mill drive equipment could be the object of the control. The case of gage control was used here because it is a common one and because it illustrates the way the machine-process equations developed earlier are used to determine the method of process control.

The glamour area today is in the area of digital computer process control. Any work which deals with processes would be conspicuous if it omitted some reference to this area of endeavor.

The rolling process has been an area of intense activity in the computer field, from the standpoint of pacing the mill and production data logging as well as computer and direct digital control. The direct digital control for mill set up, i. e., setting screw openings to their initial positions, mill motors to their base speeds, setpoints on tensions to nominal values, has been a fruitful field. Data logging and mill pacing have received a great deal of attention mainly from an accounting and cost analysis point of view.

On-line computer control of the actual process of rolling metal is still in the formative stages. The single stand mill hardly lends itself to digital computer control because the process is well controlled by analog methods. The small number of measurable and manipulated variables does not justify the expense of the computer and associated converting equipment. Present attention in the on line computer area is being directed toward multi-stand mills where repetitive computations make the economic considerations more attrative. The major problem still lies in the inability to define the physics of the rolling process well enough to make a mathematical model which is precise enough to control the process adequately and still simple enough to permit rapid iterative computations of process variables.

The development of control equations gives the control engineer a systematic presentation of the problems which are readily adaptable to simulation either on analog or digital computers. These simulations, along with newer methods of analysis of existing data from operating mills will lead to better methods of control whether they be analog or digital.

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APPENDICES

APPENDIX A

THE HUBER-VON MISES CONDITIONS FOR PLASTICITY

The Von Mises theory of plasticity relates the stresses on a given volume to the known yield stress of the metal to determine the stresses necessary to make the metal flow.

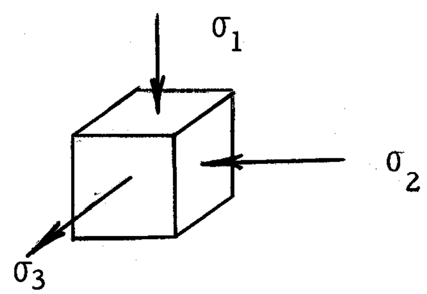

Figure A-1

σ_1 is vertical compression
σ_2 is lateral compression
σ_3 is longitudinal spread (tension)

The Huber-Von Mises equation relates these forces in the following fashion

$$\sigma_1 + \sigma_2 - \sigma_3 = 2k = \frac{2}{\sqrt{3}} Y \qquad \text{(A-1)}$$

where k = the yield stress in pure shear

Y = yield in uniaxial compression

Since rolling supplies no lateral forces $\sigma_2 = 0$ and

$$\sigma_1 - \sigma_3 = 2k = \frac{2}{\sqrt{3}} Y \qquad \text{(A-2)}$$

For the rolling problem this means that for the metal to flow the algebraic sum of specific roll pressure (compressive) and tension must be equal to $\frac{2}{\sqrt{3}}$ Y. This is sometimes called the constrained yield stress, S_o.

APPENDIX B

DERIVATION OF BASIC DIFFERENTIAL EQUATION FOR PRESSURE DISTRIBUTION IN THE ROLL GAP

Figure 1 shows the metal in the roll gap between the work rolls. With the assumptions as detailed in Part I the derivation of the differential equation for pressure distribution in the roll gap is as follows:

Consider the forces acting upon the plane of height 2y at a distance x from the center line of the work rolls. "x" is assumed to be on the entry side of the neutral plane.

Resolving the forces f_x and p_x into horizontal and vertical components and writing the equilibrium equation gives

$$[2(y + dy)][-(\sigma_x + d\sigma_x)] + (\sigma_x)2y - 2f_x\, dx \cos\phi_x + 2p_x\, dx \sin\phi_x = 0 \qquad \text{(B-1)}$$

which reduces to

$$-y(d\sigma_x) - (dy)\,\sigma_4 - (dy)(d\sigma_x) - f_x\, dx \cos\phi_x + p_x\, dx \sin\phi_x = 0$$

$$\text{if } \phi \text{ is small } \cos\phi_x \simeq 1, \quad \sin\phi_x \simeq \tan\phi_x = \frac{dy}{dx} \qquad \text{(B-2)}$$

$$(p_x\, dx \frac{dy}{dx} - \sigma_x\, dy) - (y + dy)\, d\sigma_x - f_x\, dx = 0 \qquad \text{(B-3)}$$

$$(p_x - \sigma_x)\, dy - y\, d\sigma_x - f_x\, dx = 0 \qquad \text{(B-4)}$$

$$\frac{(p_x - \sigma_x)}{y}\frac{dy}{dx} - \frac{d\sigma_x}{dx} - \frac{f_x}{y} = 0$$

Putting in the conditions for the other side of the neutral plane gives

$$\frac{p_x - \sigma_x}{y} \frac{dy}{dx} - \frac{d\sigma_x}{dx} \mp \frac{f_x}{y} = 0 \qquad \text{(B-5)}$$

where the minus sign is for the area between entry and neutral plane, and the plus sign is for the area between neutral and exit planes.

APPENDIX C

DERIVATION OF BLAND AND FORD'S SOLUTION FOR PRESSURE DISTRIBUTION IN THE ROLL GAP

Referring to Figure 1 Bland and Ford derived the equation for the specific roll pressure as a function of known or calculable rolling variables. Specifically those variables are angle of contact, shear yield stress, tensions, roll radius (flattened per Hitchchocks approximation), thickness at entry, exit and neutral plane, and coefficient of friction. Mathematically

$$p_x = f\,(\phi_x,\ k,\ \sigma_f,\ \sigma_b,\ R',\ h_o,\ h_1,\ h_n,\ \mu) \qquad \text{(C-1)}$$

Summing horizontal forces in Figure 1, letting h be the height of the unknown plane, and σ_3 the horizontal stress:

$$\sigma_3 h + 2p_x R' d\phi_x \sin\phi_x \pm 2\mu p_x R' d\phi_x \cos\phi_x = \sigma_3 h + d(\sigma_3 h) \qquad \text{(C-2)}$$

rearranging

$$\frac{d(\sigma_3 h)}{d\phi_x} = 2p_x R' (\sin\phi_x \pm \mu \cos\phi_x) \qquad \text{(C-3)}$$

where the positive sign is for the region between neutral and exit, positive between entry and neutral planes.

Likewise for vertical forces on the referenced plane

$$\sigma_1 R' d\phi_x \cos\phi_x = p_x R' d\phi_x \cos\phi \mp \mu p_x R' d\phi \sin\phi \qquad \text{(C-4)}$$

$$\sigma_1 = p_x (1 \mp \mu \tan\phi_x) \qquad \text{(C-5)}$$

Now assuming:

(1) The contact angle is small and μ is small Equation (C-5) becomes

$$\sigma_1 \simeq p_x \tag{C-6}$$

(2) Sin $\phi_x \simeq \phi_x$, Cos $\phi_x \simeq 1$ Equation (C-3) becomes

$$\frac{d(\sigma_3 h)}{d\phi_x} = 2\, p_x\, R' \,(\phi \pm \mu) \tag{C-7}$$

From the Huber - Von Mises condition of plasticity

$$\sigma_1 - 2k = \sigma_3 \tag{C-8}$$

Combining Equations (C-8) and (C-7)

$$\frac{d[\,h(p_x - 2k)\,]}{d\,\phi_x} = 2\, p_x\, R' \,(\phi_x \pm \mu) \tag{C-9}$$

or

$$\frac{d}{d\phi_x}\left[\left(2kh\,\frac{p_x}{2k}\right) - 1\right] = 2\, p_x\, R' \,(\phi_x \pm \mu) \tag{C-10}$$

differentiating

$$2kh\,\frac{d}{d\phi_x}\left(\frac{p_x}{2k}\right) + \left(\frac{p_x}{2k} - 1\right)\frac{d}{d\phi_x}(2kh) = 2\, p_x\, R' \,(\phi_x \pm \mu) \tag{C-11}$$

Since $\left(\frac{p_x}{2k} - 1\right)$ is very small and 2kh is nearly a constant, the second term can be dropped. Omitting that term Equation (C-11) becomes

$$\frac{\frac{d}{d\phi}\left(\frac{p_x}{2k}\right)}{\left(\frac{p_x}{2k}\right)} = \frac{2R'}{h}\,(\phi_x \pm \mu) \tag{C-12}$$

The variation in strip thickness is

$$h = h_1 + 2\,R'\,(1 - \cos\phi_x) \tag{C-13}$$

$$h = h_1 + R'\,\phi_x^{\;2} \tag{C-14}$$

Substituting Equation (C-14) into (C-12)

$$\frac{\frac{d}{d\phi}\left(\frac{p_x}{2k}\right)}{\left(\frac{p_x}{2k}\right)} = \frac{2\,R'\,(\phi_x - \mu)}{h_1 + R'\,\phi_x^2} \tag{C-15}$$

Integrating

$$\ln\left(\frac{p_x}{2k}\right) = \ln\left(\frac{h}{R'}\right) \pm 2\mu \sqrt{\frac{R'}{h_1}} \cdot \tan^{-1} \sqrt{\frac{R'}{h_1}} \cdot \phi_x + \ln c \tag{C-16}$$

where C is a constant of integration.

Solving for p_x

$$p_x = C(2k \frac{h}{R'} \quad e^{\pm \mu H}) \tag{C-17}$$

where

$$H = 2 \sqrt{\frac{R'}{h_2}} \tan^{-1} \sqrt{\frac{R'}{h_2}} \cdot \phi_x \tag{C-18}$$

At the exit $\sigma_x = -\sigma_f$, the front tension. The roll pressure p_1, at exit is $p_1 = 2k_1 - \sigma_f$ where k_1 is the shear stress at exit.

Making these substitutions in Equation (C-17) and evaluating the constant

$$C = \frac{R'}{h'} \left(1 - \frac{\sigma_f}{2k_1}\right) \tag{C-19}$$

The roll force on the exit side (C-17) becomes

$$p_x^+ = \frac{2kh}{h_1} \left(1 - \frac{\sigma_f}{2k_1}\right) e^{\mu H}$$

$$p_x^- = \frac{2kh}{h_o} \left(1 - \frac{\sigma_b}{2k_o}\right) e^{\mu H} \tag{C-20}$$

At the neutral plane $p_n^+ = p_n^-$, equating the two parts of Equation (C-20) at that point and solving for ϕ_n, the neutral angle

$$\phi_n = \sqrt{\frac{h_1}{R'}} \tan \sqrt{\frac{h_1}{R'}} \cdot \frac{H_n}{2} \tag{C-21}$$

APPENDIX D

FAJNBERG'S APPROXIMATION AND SOLUTION FOR LOCATION OF THE NEUTRAL POINT

In Figure D-1 and D-2 the sum of horizontal forces, with positive to the right gives

$$- \int_{\phi_n}^{\phi} p_o^- w R' \sin a_1 \, da_1 + \int_{\phi_n}^{\phi} \mu \bar{p}_o w R' \cos a_1 \, da_1$$

$$- \int_{o}^{\phi_n} \bar{p}_1 w R' \sin a_2 \, da_2 - \int_{o}^{\phi_n} \mu \bar{p}_1 w R' \cos a_2 \, da_2$$

$$+ \sigma_f w \frac{h_1}{2} - \sigma_b w \frac{h_o}{2} - \frac{M}{2} v_1 + \frac{M}{2} v_o = 0 \qquad \text{(D-1)}$$

Assuming that the mean pressures $\bar{p}_o$ and $\bar{p}_1$ are equal to the total mean pressure $\bar{p}$, and the width of the strip W, the coefficinet of friction, μ, and the deformed roll radius, R', are constant, integration of Equation (D-1) gives

$$\bar{p} w R' \left[\cos a_1 \right]_{\phi_n}^{\phi} + \mu \bar{p} w R' \left[\sin a_1 \right]_{\phi_n}^{\phi}$$

$$+ \bar{p} w R' \left[\cos a_2 \right]_{o}^{\phi_n} - \mu \bar{p} w R' \left[\sin a_2 \right]_{o}^{\phi_n}$$

$$+ \frac{w}{2} (\sigma_f h_1 - \sigma_b h_o) - \frac{M}{2} (v_1 - v_o) = 0 \qquad \text{(D-2)}$$

Using the approximation that

$$\sin \phi \simeq \phi, \quad \cos \phi = 1 - \frac{1}{2} \phi^2$$

Equation (D-2) becomes

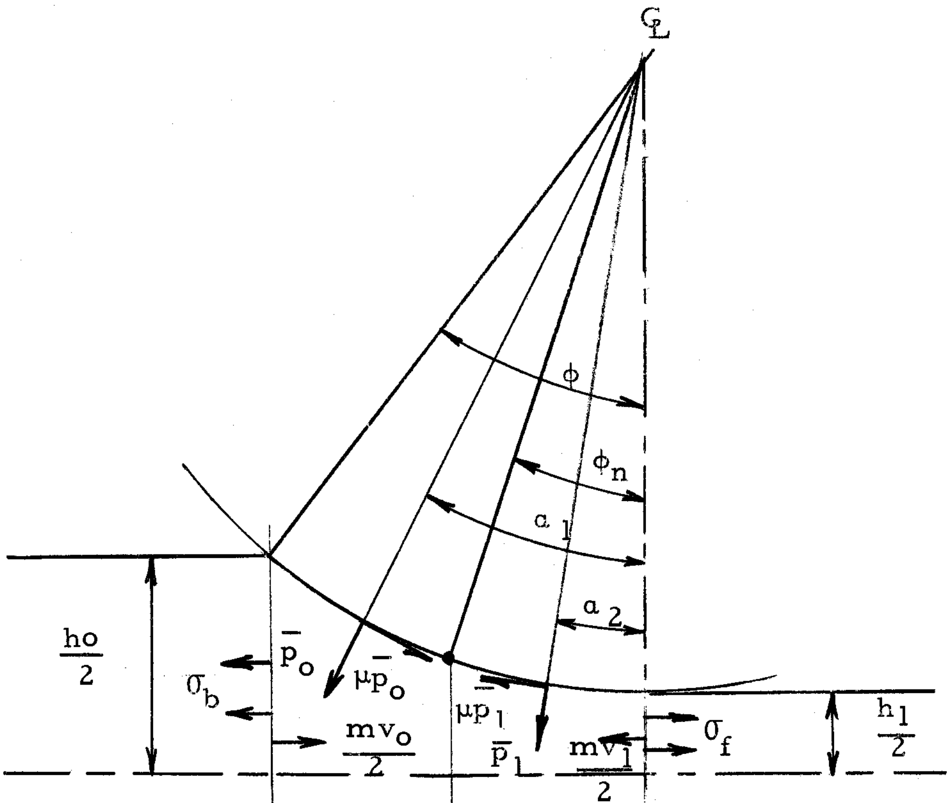

FIGURE D-1

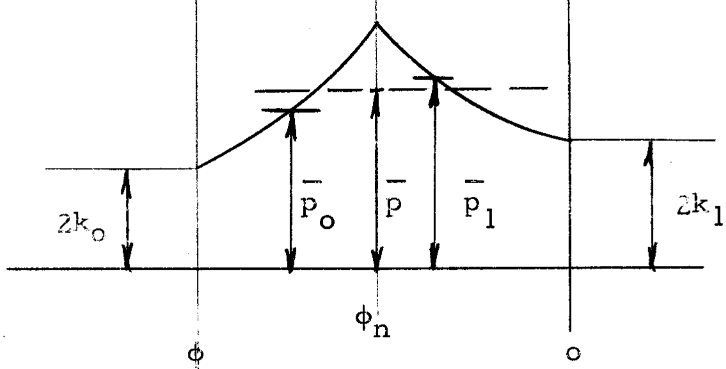

FIGURE D-2

DISTRIBUTION OF PRESSURE IN ROLL GAP

$$\phi_n = \frac{\phi}{2}\left(1 - \frac{\phi}{2\mu}\right) + \frac{1}{4\mu\bar{p}R'}(h_1\sigma_f - h_o\sigma_b) - \frac{M}{4\mu\bar{p}wR'}(v_1 - v_o)$$

and with the further approximation that

$$\frac{M}{4\mu\bar{p}\ R'}(v_1 - v_o) = 0$$

Equation (D-2) becomes

$$\phi_n = \frac{\phi}{2}\left(1 - \frac{\phi}{2\mu}\right) + \frac{1}{4\mu\bar{p}R'}(h_1\sigma_f - h_o\sigma_b) \qquad \text{(D-3)}$$

APPENDIX E

DERIVATION OF EQUATIONS FOR TENSION IN THE STRIP

Elongation of the strip between the reel and the mill stand occurs when the section is subjected to stress. The cross sectional area of the strip will not generally be absolutely uniform so that the total tension may not be the same everywhere in the sheet. Usually the well controlled mill will inherently roll strip to close tolerance, however, so we are justified in assuming uniform cross section. The stress in the sheet is, therefore, the total pounds tension divided by the nominal cross sectional area.

$$\sigma = \frac{T}{A} \qquad \text{(E-1)}$$

Since the strip is elastic between the rolls and reels it obeys Hooke's Law and

$$e = \frac{T\ell}{AE} \qquad \text{(E-2)}$$

where e = elongation

T = pounds tension force

L = distance from reel to center line of the rolls

A = nominal cross-sectional area

E = modulus of elasticity

ℓ = unstretched length of sheet

It therefore follows that

$$L = e + \ell \qquad \text{(E-3)}$$

Within the normal ranges of rolling the elongation is very small compared to the length between reel and center line of the rolls so that Equation (E-2) is very nearly (within about 0.2 percent)

$$e = \frac{TL}{AE} \qquad \text{(E-4)}$$

The change in elongation between roll and reel is the consequence of a difference in the velocity of the strip leaving the reel and arriving at the roll or leaving the roll and arriving at the reel, or

$$e = \int_o^t (v_o - v_u)\, dt \quad . \qquad \text{(E-5)}$$

From which

$$\frac{T_o}{A_o} = \frac{E}{L_u} \int_o^t (v_o - v_u)\, dt \; . \qquad \text{(E-6)}$$