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TITLE: AN ECONOMETRIC STRUCTURE FOR COST FUNCTIONS WITH APPLICATION TO MUNICIPAL WATER

Abstract approved: John A. Edwards

In an effort to determine empirical cost functions for municipal water supplies in the United States, the writer found it necessary to specify an acceptable mathematical form to represent the cost equation. A preliminary search yielded no theoretically consistent expression adaptable to the problem.

The primary concern in the study thus became one of identifying theoretically sound statistical estimation procedures for interrelated long- and short-run cost functions. Criteria were established to recognize a mathematical form qualified to function as a generalized cost equation.

The prescribed path wound its way through a traditional overview of economic production functions, cost theory literature, and curve estimation procedures; then it moved into a comprehensive review of empirical cost studies. This empirical section first identified studies of short-run cost relationships and gave examples of declining, constant, and increasing marginal costs. Theoretical literature, addressing possible reasons for these diverse shapes, was then cited.

The same format was maintained for long-run studies reporting diverse shapes. Theoretical explanations followed. The last part of the literature review inspected the explicit model structure of those studies combining long- and short-run cost curves.

When no econometric model was found in the literature which satisfied the pre-specified criteria for a generalized cost equation, the study assumed the task of developing such a framework. The resulting econometric structure exhibited the following properties: (1) The adopted equation generates both long- and short-run cost curves. (2) Two cost groups are retained in both the long- and short-run--costs which vary proportionately with output (i.e., operating costs), and costs which are independent of output (i.e., plant costs). (3) Plant capacity is strictly defined, and all short-run production of a plant is constrained to a quantity not to exceed that capacity. (4) Operating cost is a function of production and plant utilization while plant cost is a function of plant capacity.

Once the general econometric structure was developed it was then adapted to an empirical study of the cost for supplying water to municipalities. A survey of operating data for water utilities, collected by the American Water Works Association for the year 1965, was used as the principal data source for the application. Other independent variables, considered potentially important in determining cost, were evaluated and added to or omitted from the model. These characteristics included alternative treatments, types of customers, sources of water, city density, etc.. The resulting regression equations indicated the following industry structure: (1) Although the major portion of the industry is facing economies-to-scale, the long-run cost curve turns distinctly upward for large water suppliers. (2) Over 95 per cent of the plants face downward sloping short-run average cost curves. (3) With the available data no statistical evidence could be found to indicate a plant's operating cost is affected by the level of plant utilization.

The municipal water example was used to demonstrate the versatility of the generalized cost function in accommodating cost studies and hypothesis testing.

The author therefore asserts that the econometric structure developed in this study is qualified to fulfill the pre-selected requirements of a theoretically sound statistical estimation procedure for interrelating long- and short-run cost functions.

AN ECONOMETRIC STRUCTURE FOR COST FUNCTIONS WITH APPLICATION TO MUNICIPAL WATER

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INTRODUCTION

Need for the Study

Citizens of the United States, as well as many other world citizens, have in recent years become concerned with the quality of life which is potentially available to them. This awareness has taken many forms--one of which results in efforts to reduce all types of pollution whether they be in the environment around them or the substances that enter their bodies. Consequently, many bills have been passed; and more legislation will be enacted to implement these new values of society.

One such bill passed in the United States in recent years is specified "Title XIV--Safety of Public Water Systems" and signed by President Ford on December 17, 1974. The "Safe Drinking Water Act" (as it is more commonly known) places strict new standards on public drinking water systems. Construction maintenance and operation of improved systems capable of meeting these new standards will likely require sizable public and private investments. Society and its policy makers must be aware of the cost of imposing such restrictions since the welfare and production potential of a given society are constrained by the scarce resources it owns. Legislative decisions of this form will force resources--previously available to other areas--into use for water purification purposes. If the "cost" to society of such legislation can be determined while the legislation is being considered, this is desirable; but many times this information is not available in time to contribute to the decision process. Even after legislation is enacted, however, the cost impact information is useful since scarce resources must be channeled to implement the legislative requirements. If estimates are available as to the source and magnitude of these resource shifts, greater understanding will result which can often lead to a smoother transfer of the mobile resources; therefore, the need for ready access to systematic procedures of estimating industry and plant production costs demands recognition.

Methods of Cost Determination

Two methods of obtaining cost information are available: (1) a direct engineering survey to determine the specific needs of each production plant--an expensive and time-consuming method; and (2) a statistical survey which samples a part of the population and then generates a set of mathematical relationships (i.e., a model) to estimate and represent the costs relationships. Realizing that often estimates are needed and that without massive resources a complete engineering survey (method one) is prohibitive, an econometric model frequently becomes the only alternative feasible.

An econometric model has two aspects: (1) a set of expected behavioral characteristics which is predicted from a body of economic theory developed to explain the economic order, and

(2) a set of mathematical relationships which is general enough to permit all of the possible economic behavior patterns and specific enough to permit testing to determine if the economic relationships behave as the theory would predict.

An Overview of Cost Literature

In an overview of economic literature related to cost and cost modeling the following inadequacies surface: (1) Although there is an endless mass of cost studies, the literature does not seemingly hold together as a body of compatible and related knowledge.
(2) Very few of the studies attempt to address long-run costs and short-run costs in the same framework or even in the same study.
(3) An overview of the literature does not provide a consistent, logical, economic-statistical framework for a researcher to adopt as a foundation or cornerstone on which to build his model. As one explores the literature more critically he comes to realize why such a framework has not evolved.

Theoretical Shortcomings

A common practice in economic theory is to couch functional relationships in abstract symbolic forms and then prematurely terminate the exercise before specifying mathematical relationships consistent with the theory. Even when specific functional forms are suggested, due consideration is not given to possible statistical and econometric problems which might be encountered by the researcher when empirical data is integrated with the theory. As a result, the problems of specifying cost curves and functional forms in any particular research situation is left to the trial and error or *a priori* specification of the researcher. Trial and error routines can lead to much loss of time, money, and effort as well as in irretrievable forfeitures of coveted degrees of freedom. *A priori* specification, on the other hand, suggests *ex post facto* knowledge which can generate a fresh set of skeptical inquiries.

Theoretically Implied Shapes

Traditionally, economic theory has argued that short- and long-run average cost curves are U-shaped; (Ferguson, 1969, pp. 238-242; Leftwich, 1966, pp. 141-151; Stigler, 1966, pp. 153-158). Seeming discrepancies arise since empirical data reflects not only U-shaped but also constant and L-shaped short- and long-run average cost curves in various combinations. Although economic theory may not explicity exclude these other types of curves, it becomes the researcher's responsibility to explain why his study yields these unusual shapes. If, in fact, theory can be found to support these observations, then an education problem exists since the majority of empirical researchers appear unaware of the theory. If theory is not available then a conceptual conflict exists between theory and observed phenomena. In either case, a clarification of the theory is needed.

Statement of Problem

In light of the preceding discussion the predominant problem to be analyzed in this study is the specification of theoretically sound statistical estimation procedures for interrelated long- and short-run cost functions.

Formulation of the Study

If one accepts the premise that "clarification of the cost theory is needed," then a study must be formulated in such a way as to remove the nebulous nature of the concepts. In order to provide a central thrust to the study this author will specify a set of criteria he feels should characterize a generalized cost framework. The listed criteria will, therefore, become a standard for comparing the adequacy of reviewed literature and for evaluating any concepts developed within this study.

Criteria for a Generalized Cost Equation

The criteria necessary to qualify as a generalized functional form for long- and short-run cost equations may be stated as follows:

1. The function must incorporate both long- and short-run cost curves into a single framework.

2. The mathematical form must be able to generate an envelope of U-shaped short-run cost curves.

3. The equation must not restrict the curve forms to U-shaped curves.

4. The function must be able to accommodate a range of several magnitudes in plant size.

5. The formulation must be able to demonstrate economies-and/or diseconomies-to-scale.

6. The mathematical form must be adaptable to least squares linear regression techniques.

7. The framework must be conducive to hypothesis formulation and hypothesis testing.

Although some of the above criteria seem overly restrictive, the author feels they are necessary if a framework is to receive wide acceptance as an econometric tool for studying cost relationships.

Purpose of the Study

The purpose of the study is to test the following two hypotheses:

HO: A generalized functional form is not available from cost literature which meets the above criteria for a generalized cost equation.

HO: A functional framework can not be developed which will meet the pre-specified criteria for a generalized cost equation.

Procedure for the Analysis

In order to test the proposed hypotheses the study will be developed accordingly. Chapter II will review the existing theoretical and empirical literature. The discussion will begin with a review of the traditional cost model addressing the relationships between the production function and the cost functions. Long- and short-run cost curves will be reviewed in relation to their theoretically expected shapes. A slight diversion toward the middle of the chapter will briefly review methods of curve estimation. This digression is considered necessary in order to display the multiplicity of methods employed by various researchers in attempts to estimate cost curves. Although adaption to least squares regression is specified in the initial criteria for acceptance of a generalized functional form, it is by no means the only method available. This review will merely survey these alternative statistical procedures. Upon returning to the economies of cost, numerous empirical studies will be briefed. This section will be divided into the following three parts: short-run studies, long-run studies, and combined short- and long-run studies. After each group of empirical studies has been completed a discussion of relevant theoretical literature and its implications to the foregoing empirical studies will be capsuled. Chapter II is further designed to achieve three specific purposes: (1) to give the author and readers a better understanding of existing theoretical and empirical cost literature. (2) to identify areas of discrepancy between empirical evidence and economic theory and to cite selected economists' perceptions of cost relationships. (3) to search for mathematical forms which meet a part or all of the criteria specified earlier for the purpose of rejecting one or both of the principal hypotheses.

Chapter III will attempt a systematic development of a generalized functional form for cost equations using ideas extracted from the literature review along with innovative mathematical behavior patterns introduced in this section. Assumptions and definitions will be formulated consistent with the previously specified criteria. Mathematical forms will be chosen, evaluated, and improved. Statistical problems such as multicollinearity will be considered in relationship to these mathematical functions. The chapter will conclude with an evaluation of the developed function using the criteria given in Chapter I.

Theoretical constructs were criticized earlier for not testing the theoretical developments with empirical data. Chapter IV will, therefore, be devoted to empirical estimation. Since concern for legislation affecting the municipal water industry was responsible for identifying the overlying problem, it seems appropriate that this industry should be used as the example for the theoretical application. Although many of the problems addressed in Chapter IV are unique to municipal water, they serve as good examples of the types of problems encountered in moving from theory to empirical application. The exercise demonstrates how other variables are linked with the generalized cost function to create a completed mathematical expression for a system of short- and long-run cost curves. The chapter is intended to accomplish the following four purposes: (1) to survey the possible sources of data, (2) to incorporate the generalized functional form developed in Chapter III into a cost function for the municipal water industry, (3) to point out problems encountered and alternative solutions for selecting operational proxies for the theoretical components, (4) to demonstrate the process necessary to prepare data for the mathematical form and for linear regression estimation procedures.

Chapter V concludes the study. Results of the statistical regression equations developed in Chapter IV will be reported. Equations will be reviewed in the light of economic theory and the expected behavior of the cost functions. Because under some conditions the principles of economic theory are violated, Chapter V will establish a set of operating rules designed to avoid these violations. A short section will be devoted to using the knowledge gleaned from this study in an effort to understand more clearly cost curve behavior. The concluding remarks will re-examine the original cost function criteria, the principal hypotheses, and statement of problem in an effort to determine the degree to which the study accomplishes or fails to accomplish its objectives.

CHAPTER II

A REVIEW OF THEORETICAL AND EMPIRICAL STUDIES

> ". . . symbolism has been the soul of science ever since man began to organize his knowledge about actuality. Yet symbolism, if not supported by an operational interpretation of each symbol (or at least of each primary symbol), silently but unfailingly leads the student away from the most arduous and most important task of any special science, that of bringing the human mind in closer contact with actuality."
>
> --Nicholas Georgescu-Roegen, 1971

Perspective on the Literature Survey

Hundreds of studies have been conducted in an effort to determine cost equations for the estimation of production costs of various goods and services--both public and private. Economic literature abounds with examples of theoretical constructs designed to represent cost relationships and explain how cost curves "should" behave. Only limited efforts, however, have been directed at integrating these two bodies of knowledge. All too often the economic principles available are not used to their fullest value in developing empirical studies. When the theoretical constructs are used and expected behavior is not supported by empirical evidence the differences are usually discounted because of insufficient or inaccurate data sources. Untold attempts at cost estimation and well formulated studies have never reached the printed page because the findings were seemingly inconsistent with economic theory or with other studies conducted in related subject areas.

The following review of economic literature dealing with cost functions reveals a wide variation in approaches and problem formulation for seemingly basic problems of a kindred nature. As a result of these divergencies, complementary research on similar or even identical subject matters can seldom be systematically compared because the researchers did not use compatable models or test corresponding hypotheses.

Formulation of the Traditional Cost Model

Since cost curves are dependent upon production functions, they are not autonomous functions. Although it is not the intent of this paper to explore production functions, the author deems it beneficial to define the component parts of cost functions.

The Production Function

According to Boulding, "A production function is a feasible set of quantities of inputs and outputs which shows what quantities of inputs (factors) can be transformed into what quantities of output (product)." (Boulding, 1966, p. 423). Henderson and Quandt's (1958, p. 44, pp. 58-59) mathematical definition of a two factor input production function is

$$q = f(x_1, x_2, k) \tag{2.1}$$

where

q = quantity of output

x_1 and x_2 = the quantities of variable inputs

and k = level of the fixed input.

"The total cost of any quantity of output is simply the value of the inputs given up in its production." (Boulding, 1966, p. 423), or as Henderson and Quandt expresses it: (1958, p. 55, pp. 58-59)

$$C = r_1x_1 + r_2x_2 + h(k) \tag{2.2}$$

where new notations are

C = total cost of producing output q

r_1 and r_2 = prices of the input factors x_1 and x_2

and h(k) = cost associated with the fixed input k.

The total cost function assumes the quantities of x_1 and x_2 are chosen in such proportions so as to minimize the total cost of producing q. This is true when the ratio of the marginal inputs is equal to the inverse ratio of the input prices. (Ibid, p. 53).

$$\frac{\partial x_2}{\partial x_1} = \frac{r_1}{r_2} \tag{2.3}$$

From this expression evolves what economists call an expansion path

$$g(x_1, x_2) = 0 \tag{2.4}$$

which represents the optimum combinations of inputs x_1 and x_2 for alternative levels of output q. Optimum combination of inputs implies x_1 and x_2 are not restricted to fixed proportions because of a physical characteristic of production most often referred to as the law of diminishing returns. The law states that "if the input of one resource is increased by equal increments per unit of time while the inputs of other resources are held constant, total product output will increase, but beyond some point the resulting output increases will become smaller and smaller." (Leftwich, 1966, pp. 99-100). In a two variable input world, as described by Henderson and Quandt above, this means more q can be produced by increasing only x_1; or, conversely, the same amount of q can be produced by using different combinations of x_1 and x_2. The combinations of x_1 and x_2 capable of producing the same amount of q generate what are called isoquants or equal product curves (Stigler, 1966, p. 147). The expansion path described above, given a constant price for each variable input, is the locus of these optimum combinations.

Constant prices for the variable inputs imply each factor market is purely competitive (i.e., insufficient x_i is used in the production of q to influence the price of x_i). Throughout this study, factor or input prices will be assumed constant. For a rigorous presentation of what happens when these input prices are allowed to vary consult Boulding. (1966, pp. 438-441).

The Cost Function

Given the total cost Equation 2.2 as the equation representing the least cost combination of x_1 and x_2, and holding k constant, the equation can be segmented into two parts--the variable cost and the fixed cost. Ferguson (1969, p. 187) defines total variable cost as "the sum of the amounts spent for each of the variable inputs used." Fixed cost is defined in like manner as "the sum of the short-run explicit fixed costs and the implicit cost incurred by an entrepreneur." Henderson and Quandt (1958, p. 59) specifies fixed cost as an increasing function of plant size. Two new concepts are mentioned in the above definitions which thus far the writer has not clarified--short-run and plant size. The short-run, according to Stigler, is defined as "a period within which some inputs are variable, others fixed." (1966, p. 135). The long-run, on the other hand, is a period of time long enough for all inputs to vary. Plant size is a very illusive concept with few definitions available in economic literature. Stigler suggests two possibilities. Plant size can either be expressed as the output at which short-run average costs are at a minimum or as the output at which short- and long-run marginal costs are equal. (Ibid, p. 157). Plant size is a function of costs in both of the above definitions. This creates a situation in which plant size would change due to an increase in input prices while the physical characteristics of the plant would remain unaltered. Henderson and Quandt offers an alternative by defining plant size according to the physical limitations of the plant--"the levels of the entrepeneur's fixed inputs." In a later example the authors choose "square feet of selling space" to represent the plant size for a grocery store. (1958, p. 58).

Long-Run Cost Curves

Now that the principal terms and concepts have been defined, long- and short-run cost curves can be developed. Because of the conciseness of the procedure, the development as given by Henderson and Quandt will be used as the pattern for the cost curves used in this study. Let

$$q = f(x_1,x_2,k)$$

$$C = r_1x_1 + r_2x_2 + h(k)$$

$$0 = q(x_1,x_2,k)$$

be the production function, the total cost function, and the expansion path function respectively. By substitution, x_1 and x_2 can generally be removed leaving

$$C = f(q,k) + h(k) \tag{2.5}$$

where

$f(q,k)$ = variable cost

and $h(k)$ = fixed cost.

Equation 2.5 describes a family of total cost functions which can be generated by assigning different values of k. Once a specific value of k has been assigned h(k) becomes a constant, and Equation 2.5 becomes a short-run cost curve with h(k) fixed cost associated with plant size k. As a new k is chosen a new short-run cost curve results with a new fixed cost, etc.. The resulting curves can be represented by Figure 2.1.

If h(k) is a continuous function then a curve can be drawn on the lower side of the family of short-run cost curves touching each curve at its tangent point. This will show the minimum cost of producing each output q if the plant is allowed to adjust plant size k. By joining these tangent points the long-run cost curve is traced. As Henderson and Quandt points out: "The long-run cost curve is not something apart from the short-run cost curves. It is constructed from points on the short-run curves." (1958, p. 60).

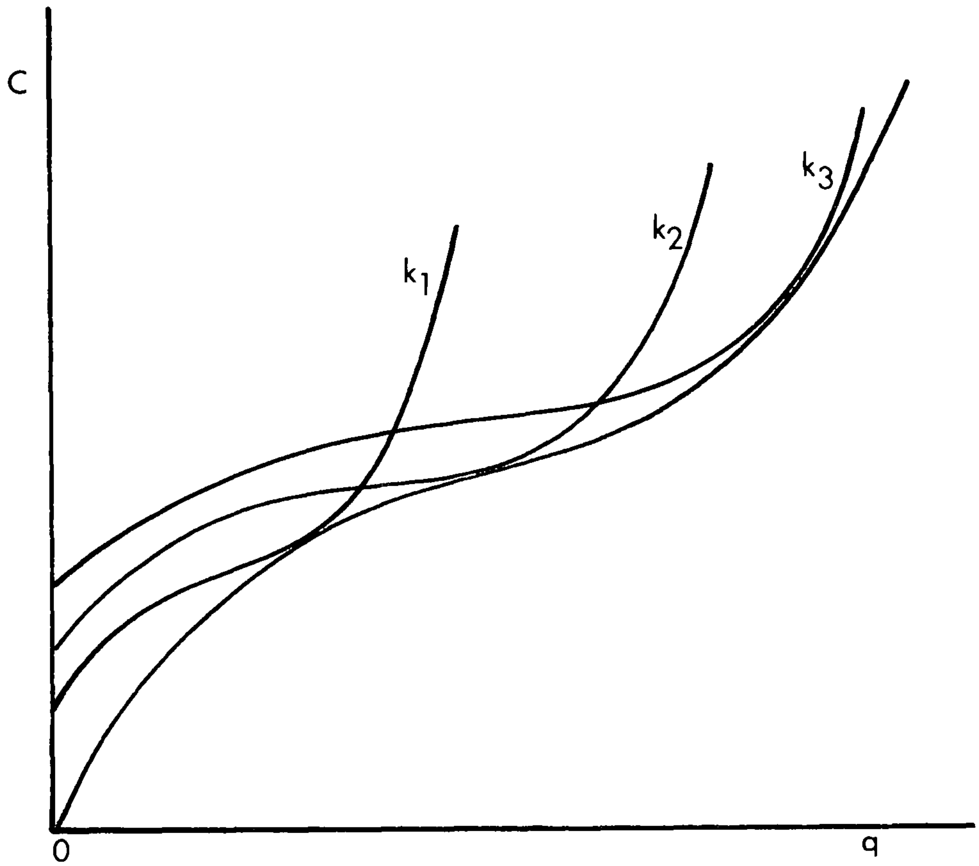

Figure 2.1. Long- and short-run total cost curves.

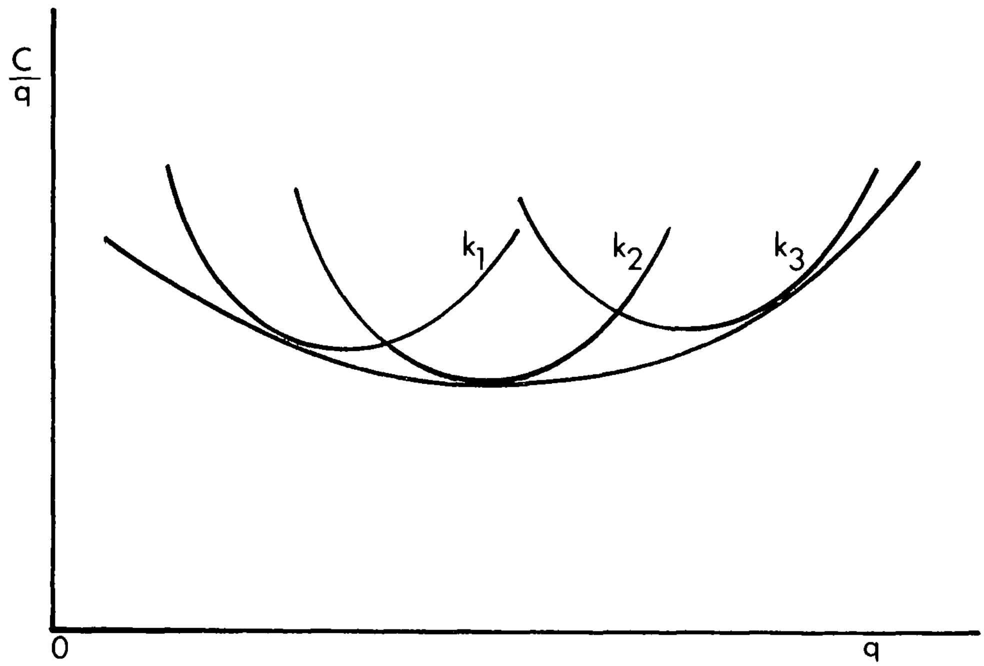

Figure 2.2. Long- and short-run average cost curves.

The average cost curves are simply the total cost curves divided by q. Figure 2.2 pictures the resulting curves from the long- and short-run average cost equation. Their U-shape gives them their common name, U-shaped cost curves.

The marginal cost, defined as the change in total cost associated with one additional unit of output, (Leftwich, 1966, p. 137) takes two forms also--short-run marginal cost and long-run marginal cost. Each marginal function is the first derivative of its respective total cost function. The long-run marginal cost, however, is not the envelope of the short-run marginal cost as is observed in the case with total cost. This condition holds because the long-run has a larger set of variable inputs than the short-run. The long-run marginal cost may be defined, however, as "the locus of those points on the short-run MC curve which corresponds to the optimum plant size for each output." (Henderson and Quandt, 1958, p. 60).

An example of a specific function is chosen by Henderson and Quandt of the form given in Equation 2.6.

$$C = a_2q^3 - a_1q^2 + a_0q - kq + a_3k^2 \tag{2.6}$$

Since fixed cost is zero in the long-run, by setting the partial derivative with respect to k equal to zero, Equation 2.7 results.

$$\frac{\partial C}{\partial k} = -q + 2a_3k = 0$$

$$k = \frac{q}{2a_3} \tag{2.7}$$

Inserting this value of k into Equation 2.6 yields the long-run cost function 2.8

$$LRC = a_2q^3 - a_1q^2 + a_0q - \frac{q^2}{2a_3} + \frac{1}{4a_3}q^2$$

$$LRC = a_2q^3 - \left(a_1 + \frac{1}{2a_3} - \frac{1}{4a_3}\right)q^2 + a_0q \qquad (2.8)$$

where new notation is

LRC = long-run cost.

Short-Run Cost Curves

The short-run total cost is represented by assigning a value for plant size to Equation 2.6.

$$SRC = a_2q^3 - a_1q^2 + a_0q - bq + a_3b^2 \qquad (2.9)$$

where new notation is

SRC = short-run cost

b = a constant assigned to the parameter k.

Likewise, short-run marginal cost can be derived from Equation 2.6

$$SRMC = 3a_2q^2 - 2a_1q + a_0 - b \qquad (2.10)$$

When evaluating the behavior of this equation (as will be the practice for each function encountered) one should realize that the short-run marginal cost, Equation 2.10, derived from the function, is a minimum when

$$\frac{\partial SRMC}{\partial Q} = 6a_2q - 2a_1 = 0$$

so that $q = \frac{2a_1}{6a_2}$. (2.11)

The value of q at the minimum point is a constant, implying the short-run marginal cost curve reaches a minimum at the same value of output regardless of the plant size (i.e., the marginal cost curves are equidistant curves differing only by the change in the b value). Following from this analysis the short-run average variable cost is also a minimum at a constant value. Appropriately, the minimum of the short-run average cost changes only as it is affected by the addition of the average fixed cost to the average variable cost. The resulting family of curves is not an envelope of curves but rather a family of curves with each member lying above the previous member at all points (i.e., the short-run curves do not cross). This shape is inconsistent with the expected long- and short-run cost relationships as shown in Figure 2.2. Due consideration will be given to this problem at a later point.

Expected Shape of the Cost Curves

The law of diminishing returns dictates that short-run total cost curves will be formulated as third degree polynomials of the shape represented by

$$y = a_0x - a_1x^2 + a_2x^3 \tag{2.12}$$

Figure 2.1 reflects this shape in its short-run cost curves.

Like shape for the long-run curve, however, does not evolve from the law of diminishing returns; it comes from what economists have labeled economies- and diseconomies-to-size.

Economies-to-Size

Many justifications are given for accepting the proposition of economies-to-size or what some economists call increasing returns-to-scale.[1] Stigler lists four reasons for his support of this theory: (1966, pp. 153-154).

> 1. There may be some unavoidable "excess capacity" of some inputs. A railroad has a tunnel which is essential for given traffic, but can handle twice as much traffic. The emphasis here is on "unavoidable." If the railroad has unused locomotives, in the long run they can be sold or worn out, and hence do not give rise to increasing returns.
>
> 2. Many inputs become cheaper when purchased on a larger scale. There are quantity discounts because of economies in larger transactions. Often equipment costs less per unit of capacity when larger sizes are ordered. . .
>
> 3. More specialized processes (whether performed by men or machines) are often possible as the scale of operations increases: the man can become more expert on a smaller range of tasks; the machine can be special purpose.
>
> 4. The statistical laws of large numbers give rise to certain economies of scale. For example, the inventory of a firm need not increase in proportion to its sales, because there is greater stability in the aggregate behavior of a larger number of customers.

On the other hand, Chamberlin (1965, pp. 235-236) gives only two principal causes: "(1) increased specialization made possible in general by the fact that the aggregate of resources is larger, and (2) qualitatively different and technologically more efficient units or factors, particularly machinery, made possible by a wise

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[1] This author is acutely aware of the distinction many economists make between economies-to-size and economies-to-scale. He contends that the distinction is irrelevant and, therefore, considers the terms synonymous. The forthcoming discussion will support this position.

selection from among the greater range of technical possibilities opened up by the greater resources." After giving this simple declarative statement as to why economies-of-scale exist he proceeds to a very lengthy and deliberate discussion thereby attempting to refute the existing arguments that economies-to-scale do not exist. This argument generates from a literal acceptance of Stigler's first cause listed above--that of excess capacity. The argument states that if factors were perfectly divisible then no economies-of-scale would exist. Chamberlin declares that "the common practice of treating proportions and size as separate problems has caused the current theory of the subject to go seriously astray, mainly through its becoming almost entirely a theory of proportions." (1965, p. 231).

A descriptive example of the path this "theory of proportions" follows is given by Henderson and Quandt in their discussion of homogeneous production functions. The conclusion reveals that the size of the firm is indeterminate (1958, pp. 62-66). This approach has been widely accepted because of the seemingly desirable properties summarized in Euler's theorem on homogeneous functions: ". . . if there is constant returns-to-scale, then the total product is equal to the sum of the marginal products of the various inputs, each multiplied by the quantity of its input." (Stigler, 1966, p. 152). The most widely acclaimed of this type function is the Cobb-Douglas production function (Shephard, 1970, pp. 35-36). Georgescu-Roegen (1972, p. 293) accents the magnitude of the adoption of homogeneous production functions by authors of empirical studies by stating: ". . . because of the computational facilities offered by the Cobb-Douglas type function, no effort worthy of mention seems to have been directed toward determining the shape properties of the basic function [non-homogeneous] on which most standard works often rely." Stigler (1966, p. 151) comments on the Cobb-Douglas production function:

> Its popularity is not due to its demonstrated validity as a description of actual production functions, however. Rather, it is used because (1) it yields diminishing returns to each productive factor separately, (2) it is simple to handle, being linear in logarithmic form, (3) in many investigations the precise nature of returns to scale is not very interesting, and constant returns is a convenient simplification, and (4) of a remarkable property of constant returns to scale. . . [Euler's Theorem]

Chamberlin (1965, p. 244) rejects all arguments for constant returns-to-scale and summarizes his views as follows:

> . . . it appears that indivisibilities play no part whatever in explaining economies of scale. Where all factors are perfectly divisible, efficiency remains nevertheless a function of size; so that the envelope curve, whether smooth or scalloped, descends to a minimum in its first phase. Where particular factors, or units of factors, remain fixed for substantial portions of the long-run average cost curve, and where this introduces scallops, the "trend" will be the same, and for the same reason.

Diseconomies-to-Size

The principal argument for decreasing returns-to-scale is stated by Stigler (1965, p. 155) in this manner:

> Decreasing returns to scale arises out of the difficulties of managing a large enterprise. The larger the enterprise, the more extensive and formal its administrative

> organization must be in order to provide the information necessary for central decisions and the sanctions necessary to enforce these decisions. A large organization must be less flexible--policies cannot be changed frequently and still be carefully controlled.

Methods of Curve Estimation

Before reviewing the shapes of cost curves observed in empirical data, the alternative methods of determining these shapes should be considered. Most studies can be divided into five categories: engineering curves, firm synthesis curves, average stratified curves, frontier cost curves, and statistical estimation curves.

Engineering Curves

Engineering estimates of cost are routinely calculated by production engineers. This approach, like cost accounting, divides the production into separate operations and assesses the cost of each operation. Walters (1963, p. 43) gives two difficulties with this approach: (1) The cost of processes may interact with one another and therefore are not additive. (2) Joint costs are often allocated in an arbitrary manner. Although these conditions do not pose large problems, economists have practiced very limited use of this approach.

One study by Chenery (1949) utilizes engineering data to study the cost of pipe line transportation. He estimates cost as a function of the diameter of the pipe, the thickness, the compression ratio, and the horse power of the pumps.

A second study by Ferguson (1950) attempts to estimate the marginal fuel costs of aircraft using the engineering production function.

Firm Synthesis Curves

The classic example of this type of curve estimation is presented in a book entitled, Marketing Efficiency in Puerto Rico by John Kenneth Galbraith and Richard Holton (1955). The study entails an estimation of the long-run cost curves for wholesale and retail food distribution stores in Puerto Rico. The curve determines the percentage markup from wholesale prices necessary to cover the cost of the retail outlet. Each owner of the sample stores studied was then requested to estimate the markup necessary to cover his cost at one-half, twice, and three times his present capacity. Each sample store with its four estimates was then plotted on a common graph. A curve drawn on the upper frontier of these line segments represented the upper bounds of a long-run cost function.

Average Stratified Curves

Average stratified curves are obtained by grouping the sample into production sizes. All cost observations within a given stratum are averaged, and this average per unit cost for that particular stratum represents its production size group. The resulting cost value can be plotted against the average production for that stratum, or the groups can be listed in ordinal sequence. The resultant curve is a cost per unit curve with m observed points where m equals the number of strata. This procedure was a common practice in the earlier days of empirical estimation. Wiles (1961, pp. 238-260) in an appendix to his book, Price Cost and Output, gives a brief description of approximately forty empirical studies which have used average stratified cost curves as a means of estimating long-run cost curves.

Frontier Cost Curves

Frontier cost curve estimation is a technique developed by M. J. Farrell (1957) for estimating cost curves using only firms that are on the efficiency frontier of the production and cost surfaces. The model assumes technology may change in time, and some firms will implement more efficient technology than others. Since cost curves are assumed to be derived from the most efficient combinations of resource inputs, then the firm which shows the lowest cost for a given size of output should be the firm chosen to represent all firms of that size. The curve generated by joining these most efficient firms will trace out the long-run cost curve. This procedure employs the same concept as is used in the formation of a production function which may be defined as the maximum amount (upper bound) of output that can be produced from any specified set of inputs when given the existing technology (Ferguson, 1969, pp. 116-118). By adopting the frontier cost curve technique one also assumes that the stochastic nature of the cost is a function of technology solely and that all other firms, or potential firms, by adjusting their technology and resource combinations can achieve this minimum cost. No consideration is given to quality of product or resource price differentials.

Wesley Seitz (1966) in a study designed to measure efficiency in steam-electric generating plants uses frontier cost curve techniques to estimate both efficiency and average cost curves. By using this procedure he estimates the change in technology between two time periods.

A second study on commercial banking conducted by Lionel Kalish III and R. Alton Gilbert (1973) uses Farrell's frontier cost technique to divide the average cost of banking into two parts: (1) the minimum average cost at which banks of a given size can operate, and (2) the excess cost over this minimum. The first component is defined as technological efficiency and the second as operational efficiency.

Frontier cost curves, although an interesting and useful concept, should be used with extreme caution because of the highly restrictive nature of the accompanying assumptions. This technique--in combination with other techniques like regression analysis (to be explained later)--could conceivably have value in separating management into average and superior management groups.

Statistical Estimation Curves

In more recent empirical economic studies statistical estimation is the most frequently used method of fitting cost curves. The advent of the electronic computer has freed man from the principal disadvantage of statistical analysis (i.e., the laborious calculations necessary to obtain the equation). The technique uses the dialectics of mathematics and statistics to express numerical data in systems of mathematical equations.

Least squares regression is the most commonly used of the statistical estimation procedures for equation building. This technique fits a mathematical equation to a set of numerical points in n dimensional space in such a way that no other equation of that particular form can be found which will give a smaller value for the sum of the squared distances from the observed points to the regression line (Li, 1964, pp. 289-294).

The mathematical forms of this technique are restricted to <u>intrinsically linear</u> relationships. Equations of this form are ". . . <u>nonlinear</u> with respect to the variables but <u>linear</u> with respect to the parameters to be estimated. The basic common characteristic of such models is that they can be converted into ordinary linear models by a suitable transformation of the variables." (Kmenta, 1971, pp. 451-452). Examples of intrinsically linear terms are power functions (x^2, $x^{.5}$), cross product terms (xy), division terms (x/y); or, in general, any combination of terms which can be associated with a single coefficient estimator.

Statistical estimation procedures other than least squares regression are available. Non-linear equations can be estimated with iterative techniques and algorithms. The linearization method or Taylor series is an iterative process which uses the results from successive stages of linear least squares to make increasingly better estimates of the coefficients for the non-linear expressions (Draper and Smith, 1966, pp. 267-270). It is possible for a Taylor series to cause the value of the beta coefficient to oscillate with no assurance that this progression will ever reach a stable beta.

A second method of non-linear estimation is the method of steepest descent (Ibid, pp. 270-272). This method, unlike the Taylor series, always converges on the solution, although it may necessitate many subsequent iterations to reach the optimum value of the coefficient.

Marquardt's compromising method (Ibid, pp. 272-273) combines the better characteristics of the previous two non-linear methods and is an algorithm which in most cases reaches a solution by way of a reasonably direct path.

Although these and other methods of statistical estimation are available to researchers, the majority are considerably more expensive than least squares regression techniques. In many cases the improved estimation equations resulting from the non-linear methods may not warrant the additional cost. Because of these and other reasons such as lack of familiarity with the alternative procedures and limited availability of computer program routines, the large majority of the empirical studies to date using statistical estimation procedures have adopted least squares regression as the method of obtaining statistical cost equations.

Shapes of Cost Curves

Individual empirical studies can now be reviewed to determine the observed shapes of the long- and short-run cost curves. As Waters points out: "There have been very few attempts to combine short period and long period observations to get estimates of both the short and long run cost curves." (1963, p. 45). Because of this fact the review of studies will be divided into three parts: (1) a sampling of short-run cost studies, (2) a sampling of long-run cost studies, and (3) a sampling of the limited number of combined short- and long-run studies. After each section space will be devoted to considering theoretical expositions which are specifically directed towards explaining empirical deviations from the traditional approach developed earlier in this chapter.

The writer's intent in this section is to provide a sampling of the various forms of observed results. Although this discussion does not constitute an inclusive review of available literature, the writer contends that its purpose is achieved since only one event is needed to disprove a theory. Because of the difficulty of interpreting empirical observations, however, several examples will be reviewed; furthermore, the author does not claim that all studies reviewed are of superior quality and logic. Some material was chosen simply because of its unique nature.

One final comment before the studies are reviewed. While surveying the studies the intent will be two-fold only: (1) to observe the reported shape of the curves, and (2) to search for a functional form generally adaptable to other empirical studies of cost relationships. A critical evaluation of the data set, the procedural techniques, and an extensive interpretation of the researcher's findings is beyond the scope of this thesis.

Empirical Short-Run Cost Curves

In order to render the review process more palatable the studies will be grouped into like curve shapes.

Studies Showing Declining Average Cost

A study by Joel Dean (1942) in which he collected data over a 60 months' period on large department stores reveals that the coat department showed constantly declining marginal and average cost curves of the nature shown in Figure 2.3.

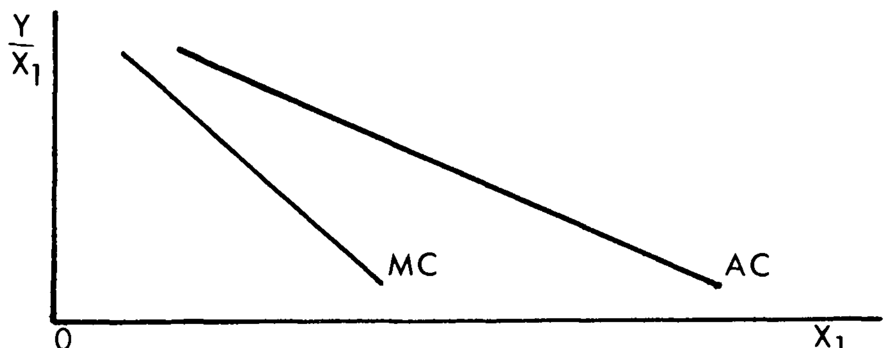

Figure 2.3. Constantly decreasing marginal and average cost.

The functional form used is given in Equation 2.12:

$$Y = -a_0 + a_1X_1 - a_2X_1^2 + a_3X_2 \qquad (2.12)$$

where

Y = total cost in the coat department

X_1 = number of transactions

X_2 = average value per transaction.

Two characteristics should be noted: (1) Dean uses total cost as his dependent variable. (2) The coerfficient a_3 on X_2 has a value of plus .787 implying for every one dollar increase in average value per transaction the total cost increases by \$.79. This would cause a vertical shift of \$.79 for the total cost function. The economic reasoning for including X_2 in the model is not apparent to this writer.

A second study by Wylie and Ezekiel (1940) of the U.S. Steel Corporation predicts the total cost of steel production by using an exponential cost function of the form in Equation 2.13.

$$Y = e^{a_1X_1 + a_2X_2} \qquad (2.13)$$

where

Y = production cost excluding depreciation and depletion

X_1 = percentage of capacity operated

X_2 = average hourly earnings.

This function is characterized by U-shaped average cost curves and positively sloped marginal cost curves (Figure 2.4). The value of X_1 at the minimum point on the average cost curve is equal to the inverse of a_1. In their empirical analysis Wylie and Ezekiel failed to observe any "indication of a tendency for cost per unit to rise with high output . . ." (1940, p. 790). This result implies that the observed value of a_1 is less than 1.0. The minimum point for the average cost curve, therefore, is in excess of 100 per cent of capacity resulting in a downward sloping average cost curve throughout the relevant range.

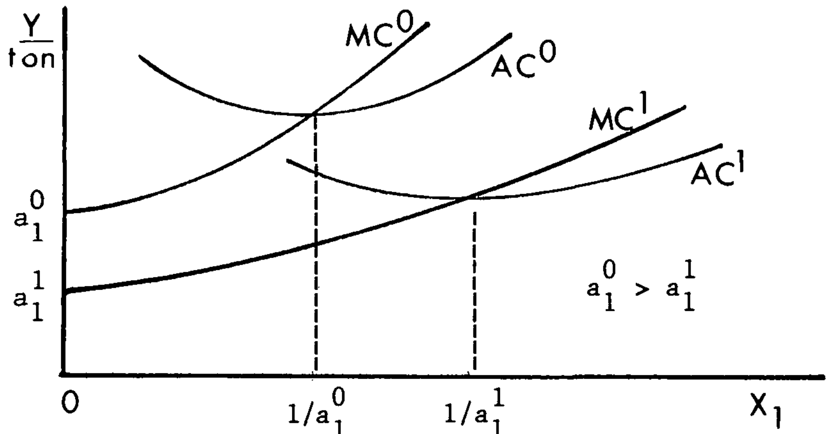

Figure 2.4. Cost curves of exponential function.

<u>Studies Showing Constant Marginal Cost</u>

A second study of U.S. Steel Corporation by T. O. Yntema (1940) based on annual data from 1927 to 1938 uses the functional form of Equation 2.13.

$$Y = a_0 + a_1X \tag{2.13}$$

where

Y = total cost

X = weighted output.

The results yield constant marginal and average variable costs--the only results permitted by the mathematical equation chosen.

Probably the first empirical study dealing with the shape of cost curves was conducted by Dean (1936) to determine short-run cost curves for a furniture factory. His results yield constant marginal cost and decreasing average total cost as shown in Figure 2.5:

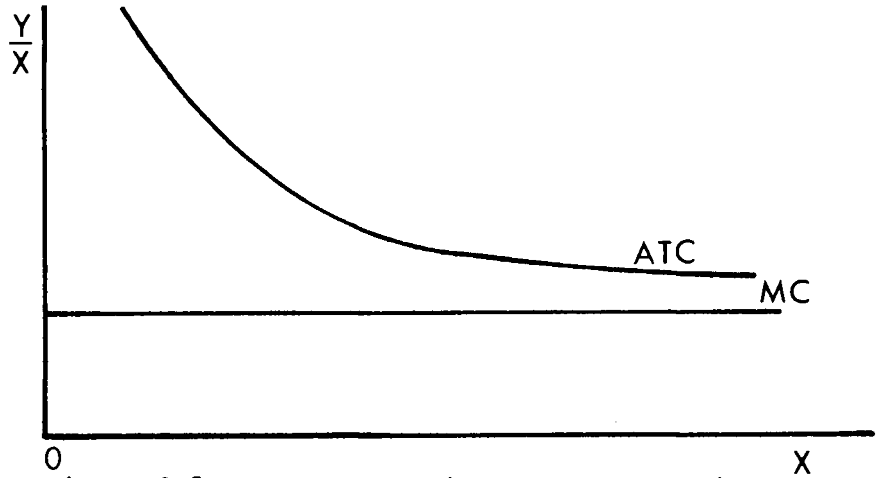

Figure 2.5. Cost curves with constant marginal cost.

If this curve is representative of the plant, the relationship between marginal cost and average cost would imply the decrease in average cost emanates entirely from the fixed cost sector.

A second short-run cost study was conducted by Dean (1941a) on a hosiery mill. The physical plant (composed of 81 identical knitting machines) and the plant technology did not change over the four year period of the study (1935-1939). The results indicate that both marginal cost and average cost remain constant as output increases. Second and third degree cost functions are fitted, but the higher terms are not significant. Whether or not depreciation and other fixed cost of the factory are included in the total cost is never clarified.

Studies Showing Increasing Marginal Cost

Nordin (1947) in his research on a light plant uses data from 541 eight-hour work shifts. Dealing only with the total fuel cost (a variable cost) and output as a percentage of capacity, he chooses a second degree polynomial for his functional form. The outcome discloses increasing marginal cost of the form pictured in Figure 2.6:

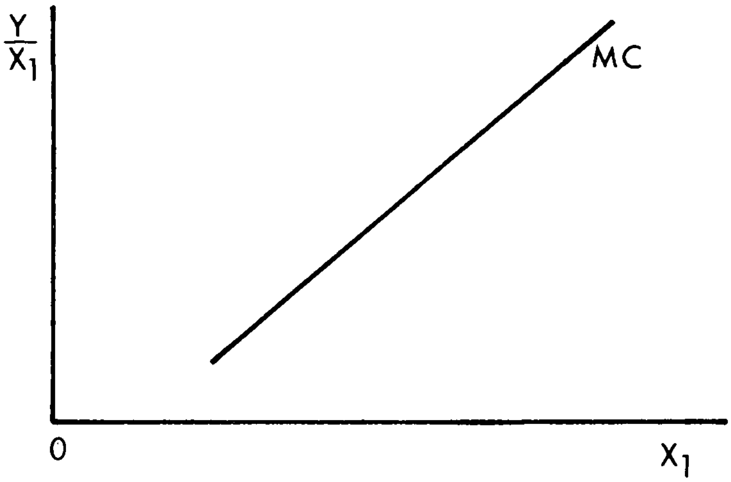

Figure 2.6. Light plant marginal cost curve.

He regresses the data using a third degree term, but the cubic term does not improve the fit.

Of Dean's many studies he has only one which shows increasing marginal cost--a study of a leather belt shop (1941b). Dean fits a cubic total cost function and finds all terms significant but rejects the model in favor of a constant marginal cost model. His reason for this rejection is two-fold: (1) an exceedingly high value for the correlation coefficient in the linear case already existed ($r = .998$), and (2) the possibility also existed that inferior factors of production would be employed at high levels of production thus violating the ceteris paribus assumption of cost curve construction. The model adopted is represented by Equation 2.14:

$$Y = a_0 + a_1X_1 + a_2X_2 \qquad (2.14)$$

where

Y = total cost

X_1 = output

X_2 = average weight of belting, pounds per square foot.

Several other short-run cost functions will be considered in the later sections devoted to the combined short- and long-run studies.

Survey of Business Managements' Opinions

A survey was conducted by Eiteman and Guthrie to determine what businessmen think about their cost curves. The assertion is made that "marginal price theory stands or falls depending upon what businessmen think, because their short-run decisions to expand or to contract are based upon what they believe rather than upon what is actually true." (1952, p. 832). A mail questionnaire was sent to 1,000 manufacturing companies and 366 replies were received. Of those reporting, 18 believed that average cost showed a substantial increase near capacity, 113 expressed the opinion that average cost might increase slightly towards capacity, and 203 stated that average cost would always decline. One respondent made a very acute observation that "even with the low efficiency and premium pay of overtime work, our unit cost will still decline with increased production since the absorption of fixed expenses would more than offset the added direct expenses incurred." (*Ibid*, p. 838).

Explanations of Marginal Cost Discrepancy

The evidence of the foregoing studies clearly shows that other short-run cost functions as well as the normally accepted U-shaped cost curves are present when empirical data are used to estimate the shapes of the short-run cost curves. Tintner (1952, pp. 49-50) gives three possible explanations why this discrepancy exists:

> (a) The range of the data is not great enough to cover the sections of the cost curve where increasing or decreasing marginal costs appear. . .
>
> (b) The assumptions of the economists are wrong, and we have actually in the economy constant marginal cost, at least over the relevant section of the cost curve. This would correspond to the assumption that within the relevant range of the data the factors of production are combined in more or less constant proportions. . .
>
> (c) The observed empirical cost curves are actually cost curves of enterprises functioning in a dynamic economy which is subject to the ups and downs of business fluctuations. The cost curves contemplated by the theoretical economists are static and hence not relevant in this situation. . .

He also points out the necessity for firms to build into their plants flexibility and adaptability to allow for fluctuating economic conditions. This, he says, would lead to near linear total cost curves in the middle section of the cost function. (*Ibid*, p. 51).

Stigler's Classification of Resources for Short-Run Analysis

According to Stigler (1939, pp. 308-311) there are certain technical conditions which can affect cost curve shapes. Some of them are alternatives available for choice; some of them are physical characteristics of the resources. He assigns two distinct properties to a resource which will determine the shape of the short-run cost curves. The first is divisibility, approximated by a large number of identical machines in a plant, or indivisibility, suggested by the roadbed for a railroad; the second is adaptability or unadaptability. Adaptability or flexibility is the property of combining resources in various proportions to achieve the same output. (i.e., It is associated with the shape of the isoquant curve described earlier.) Stigler then analyzes the possible combinations. Case I: A divisible plant that is completely adaptable results in the same marginal and fixed cost relationships for below and above optimum conditions. As the variable services are increased their marginal productivity will decline throughout the entire range while those of the fixed services will increase throughout. (Ibid, p. 309). Figure 2.7 represents this case.

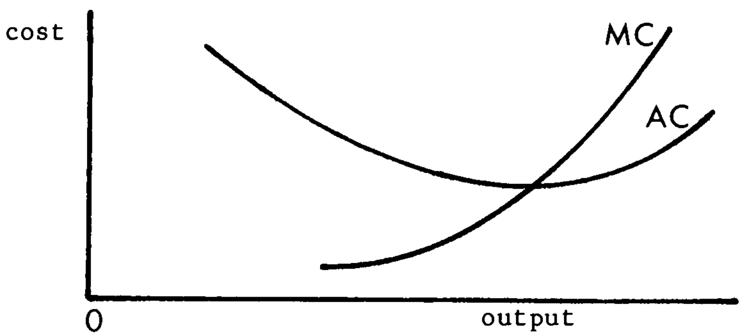

Figure 2.7: Stigler's Case I.

Case II: A plant which is divisible but unadaptable (e.g., ". . . numerous identical machines but each machine can be used only with a fixed amount of labor and materials.") (Ibid, p. 309). At less than optimum output the marginal product of the variable service (and, therefore, the marginal cost) will be constant. Outputs beyond optimum are not permitted; hence, marginal cost is infinity. Figure 2.8 represents the second case.

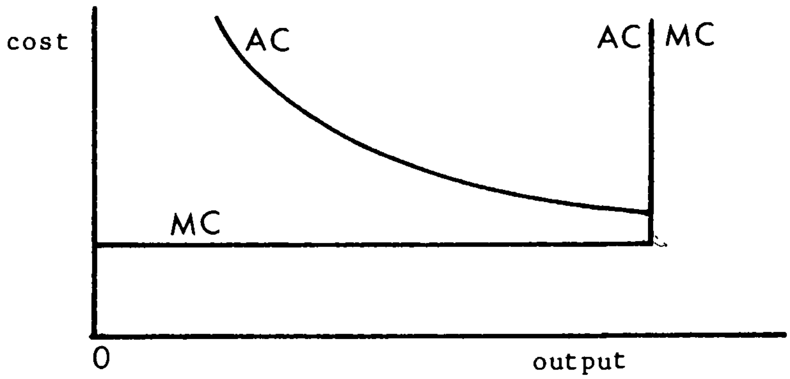

Figure 2.8. Stigler's Case II.

Case III: A fixed plant which is indivisible but completely adaptable to the variable inputs. This is the standard case of the law of diminishing returns which is applicable without qualification. Figure 2.9 pictures these traditional conditions.

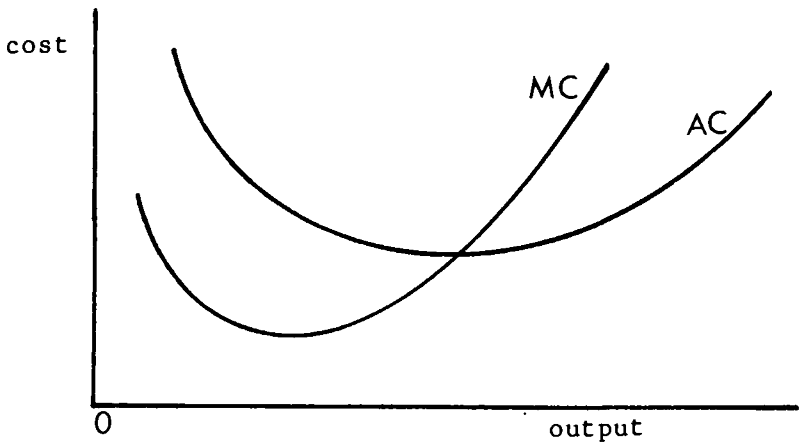

Figure 2.9. Stigler's Case III.

Case IV: A fixed plant which is indivisible and unadaptable. An example of a process which approaches this situation is a blast furnace. If there is only one fixed factor (and provided it is unadaptable) the plant can operate at only one output as pictured in Figure 2.10.

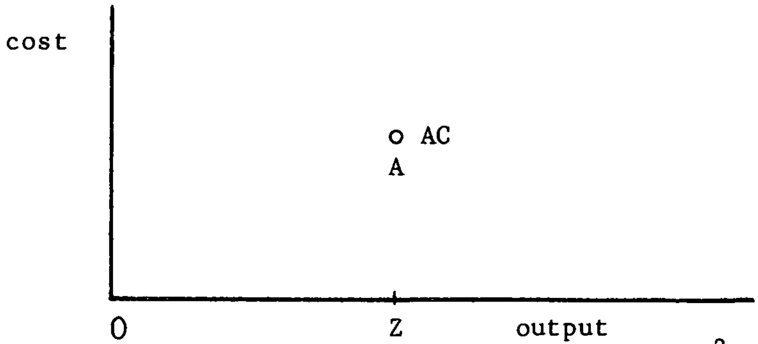

Figure 2.10. Stigler's Case IV.[2]

Stigler (Ibid, p. 310) summarizes:

> There is no need to labor the point that usually the fixed plant will be imperfectly divisible and partially adaptable, and, indeed, that the fixed plant will consist of numerous parts that differ greatly among themselves. Nevertheless, there is a possibility, at this stage of analysis, that the short-run marginal cost curve will be constant in the range of suboptimum outputs, if there are important divisible parts of plant. If there is also adaptability, the marginal cost curve will be rising in this range.

Stigler presented the above formulation in 1939, yet today Cases I and III are the only forms seriously recognized by economic theory; however, the review of empirical evidence gives support

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[2] Stigler draws the cost line perpendicular to the Q axis rather than plotting the single point A. This author feels an important distinction is made by allowing only point A since the line drawn to the axis implies Z could be produced at zero cost, and the line above A implies more than one combination of resources could be used to produce Z.

to the hypothesis that other forms of production and cost functions may exist.

Alchian's Reformulation of the Theoretical Cost Function

A major contribution to cost theory is presented in a paper by Armen Alchian in which he attempts to explain why most empirical studies may fail to show rising marginal cost. He formulates the quantity of output into two dimensions: (1) the rate of output, and (2) the scheduled volume of output. Traditional economic theory, he comments, has considered the rate as the crucial factor but "it is only one feature, and concentration on it alone has led to serious errors. . ." (Alchian, 1959, pp. 23-24). He specifies the functional relationship of Equation 2.15 by letting

$$C = F\ (V,X,T,m) \qquad (2.15)$$

where

C = cost (a change in equity)

V = scheduled volume of output

X = rate of output

T = time at which production begins

and m = length of production period.

According to Alchian's hypothesis marginal cost is always a rising function of the rate of output (X) when holding volume (V) constant and a falling function of volume of output (V) when holding rate (X) constant. (*Ibid*, pp. 25-26). Since economic theory has placed principal importance on rate, volume has been implicitly assumed constant (or infinite) while rate has changed to yield increasing marginal cost curves. On the other hand, businessmen seem to assume that the rate of production is held constant by the number of employees, plant facilities, etc.; and, therefore, to increase volume (V) they would change the length of run (m) thereby creating a decreasing marginal cost function.

A follow up paper by Jack Hirshleifer (1962) refines Alchian's contribution by challenging one of Alchian's conclusions: (Alchian, 1959, p. 34).

> ". . . there is not both a "long-run" and "short-run" cost for any given output program. For any given output program there is only <u>one</u> pertinent cost, <u>not</u> two. [Unambiguous] specification of the output or action to be costed makes the cost definition [unambiguous] and destroys the illusion that there are two costs to consider, a short- and a long-run cost for any given output. There is only one, and that is the <u>cheapest</u> cost of doing whatever the operation is specified to be. To produce a house in three months is one thing, to produce it in a year is something else. . ."

Specifically, Hirshleifer introduces uncertainty as a reason for retaining short- and long-run cost functions. If one assumes an increase in demand a given firm will increase volume by scheduling production and plant for long production runs (permanent increase in demand) or by scheduling more costly short production runs (temporary increase in demand). What each firm does depends on the expectations of its managers dealing in the context of uncertainty. As a result both short-run and long-run cost curves can exist. Hirshleifer (<u>Ibid</u>, p. 254) concludes that the Alchian model leads to a much weaker expectation of eventually rising marginal cost than does the classical cost model since the rate of output (X) and the volume of output (V) work in opposite directions when they are increased proportionately as output increases.

Empirical Long-Run Cost Curves

Studies of long-run cost curves are numerous and varied in economic literature. Only a very limited sampling will be given.

Studies Showing Declining Long-Run Average Cost

A study by Michel, Pelmoter, and Palange (1969) estimates the cost of operating and maintaining waste treatment plants with the use of a log function of the form given by Equation 2.16:

$$\log Y = \frac{1}{a + b \log X} \tag{2.16}$$

where

Y = annual cost per million gallons treated

X = plant average daily flow in million gallons.

Their results yield a definite decrease in unit cost as volume increased. The linear relationship developed by the log regression transforms into a cost function which is decreasing at a decreasing rate. This property is often referred to in the literature as an

L-shaped curve and is pictured in Figure 2.11.

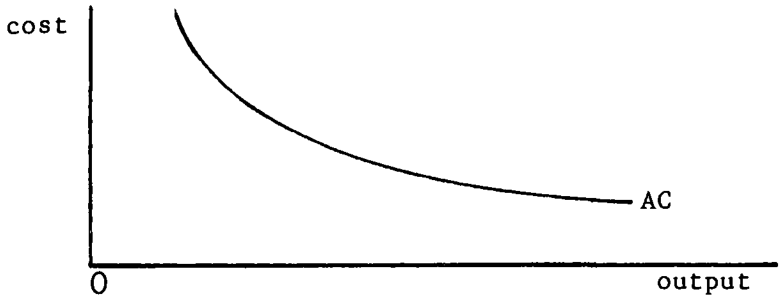

Figure 2.11. L-shaped curve.

A second study conducted by Michel one year later (1970), also pertaining to operating and maintenance costs of municipal waste water treatment, uses a different functional form as represented in Equation 2.17:

$$Y = aX^b \tag{2.17}$$

where

Y = total cost in dollars

X = plant load in million gallons per day.

This functional form denotes a total cost equation and transforms into the linear log Equation 2.18.

$$\log Y = \log a + b \log X \tag{2.18}$$

The power Equation 2.17 generates an average cost equation which has constant elasticity with respect to X.

$$AC = \frac{Y}{X} = aX^{b-1}$$

$$\frac{dAC}{dX} = (b-1)aX^{b-2}$$

$$= \frac{(b-1)AC}{X}$$

but elasticity $\left(\frac{dAC}{dX} * \frac{X}{AC} \right) = b-1 = \text{constant}.$

The shape of the average cost curve depends on the magnitude of b. If b=1 the average cost will be constant; if b<1 the curve will have an L shape as in Figure 2.11. A value of b>1 would result in an increasing cost function. Michel regresses more than 70 cost equations for various combinations of costs and treatments, as well as numerous labor requirement estimates, using this same functional form. All results reflect decreasing cost as volume increases (i.e., b<1.0).

An interesting study by Frederick Moore (1959) employs the six-tenths factor rule common to engineers as a functional form for testing economies of scale. The engineering principle is founded on the observation that "many pieces of capital equipment cost varies directly with surface area, while capacity is related to volume." (Johnston, 1960, p. 165). The functional form becomes Equation 2.19.

$$Y = aK^{b} \tag{2.19}$$

where

Y = capital expenditures

K = capacity.

When b is less than one economies-to-scale exist. Engineering and cost data suggest an average value of 0.6 for b. (p. 166).

Studies Showing U-Shaped Long-Run Average Cost

The pioneer study by Dean and James (1942) on long-run cost produces a U-shaped cost curve. Using the functional forms of

Equation 2.20, the researchers analyzed retail shoe stores.

$$\log Y = b_1 + b_2 \log X + b_3 \log X^2 \tag{2.20}$$

where

Y = total cost

X = volume of shoes sold.

The results show an increasing long-run marginal cost curve and a U-shaped average cost curve.

To determine cost functions for public high schools a study of the cost of public education was conducted by Cohn (1963). He chooses the functional form of Equation 2.21 for his cost curves.

$$Y = b_0 + b_1X_1 + b_2X_1^2 + b_3X_2 + \ldots \quad b_8X_7 \tag{2.21}$$

where

Y = cost per pupil

X_1 = average daily attendance

X_2 = change in composit score on Iowa Test of Educational Development from grade 10 to grade 12.

X_3 to X_7 = other variables considered significant.

Variable X_2 is used in an attempt to adjust for the quality of education as it might affect costs. The results of the regression indicate a U-shaped average long-run cost curve with minimum cost per student at a school size of 1,500 pupils.

A similar study by Riew (1966), utilizing the identical functional form of Equation 2.21, (except Riew omits class size as an independent variable), emits similar results. The U-shaped average cost curve reaches a minimum at 1,675 students--closely approximating Cohn's estimate. By specifying this functional form U-shaped long-run marginal cost curves are permitted and do, in fact, exist in these two studies.

Studies Showing Constant Long-Run Average Cost

"Expenditure Implications of Metropolitan Growth and Consolidation" by Werner Hirsch (1959) is evaluated as "one of the most important publications in the cost literature" by authors Alesch and Dougharty (1971, p. 31) in their report entitled, Economies-of-Scale Analysis in State and Local Government (prepared for the Council on Intergovernmental Relations, State of California). Hirsch's study on economies-to-size of local government services uses the functional form of Equation 2.22.

$$Y = b_0 + b_1X_1 + b_2X_1^2 + b_3X_2 + \ldots b_mX_n \tag{2.22}$$

where

Y = total cost per capita or per student

X_1 = night time population or average daily attendance

X_2 to X_n = other variables, designed to remove specification error problems, such as an index of quality for fire protection, average assessed value of real property, dwelling density per square mile, etc..

The results from Hirsch's study indicate police protection, refuse collection, and public education experience constant long-run average cost while fire protection and general local government administrative expenditures, such as city hall and board of education headquarters, show U-shaped long-run average cost curves.

The constant long-run average cost results in this functional form when the b_1 and b_2 coefficients are equal to zero.

A Single Study Showing Increasing Long-Run Average Cost

Tintner (1944) uses a Cobb-Douglas production function to draw implications about the shape of the long-run cost curves for Iowa farms. The sum of the elasticities for the studies add to less than one thereby implying the farms show decreasing returns to scale. Tintner surmises that this may be precipitated by the fact that management was excluded from the model. According to his opinion, the results are probably not typical of all Iowa farms but may reflect the conditions of the farms with higher management skills in as much as the sample was taken from above average management.

A Table Summarizing Other Long-Run Cost Studies

Wiles in his book entitled, Price Cost and Output (1961), summarizes 44 long-run studies, none of which are referenced above. In the cases reviewed 60 per cent show decreasing or L-shaped long-run average cost curves while 32 per cent reflect U-shaped average cost curves. Only 8 per cent show constant or increasing long-run average cost curves over the entire range. (Ibid, p. 261). Table 2.1 summarizes a large number of long-run studies not reviewed by Wiles. It serves as a compilation of summary tables by Walters (1963, pp. 48-49) and Hirsch (1970, p. 183) as well as several other studies reviewed by this author, and its intent is to demonstrate the wide variety of industries for which long-run studies have been conducted. The review is by no means all-inclusive.

A more detailed review of the majority of the studies referenced in the long-run and short-run surveys, along with many additional examples, can be located in Johnston (1960), Wiles (1961), Hirsch (1970), and Alesch-Dougharty (1971).

Theories Explaining Long-Run Cost Behavior

Two interesting factors should be cited at this point: the first is the concept of the regression fallacy, and the second is the discrepancy gap between Chamberlin and Stigler discussed in the first part of this chapter.

The regression fallacy is proposed by Milton Friedman in his comments on an article by C. A. Smith (1955). As Friedman illustrates: (1955, pp. 236-237)

> Suppose a firm produces a product the demand for which has a known two-year cycle, so that it plans to produce 100 units in year one, 200 in year two, 100 in year three, etc. Suppose also, that the best way to do this is by arrangement that involves identical outlays for hired factors in each year (no "variable" costs). If outlays are regarded as total costs, . . . average cost per unit will obviously be twice as large when output is 100 as when it is 200. If, instead of years one and two, we substitute firms one and two, a cross-section study would show sharply declining average costs. When firms are classified by actual output, essentially this kind of bias arises. The firms with the largest output are unlikely to be producing at an unusually low level; on the average,

> they are clearly likely to be producing at an unusually high level, and conversely for those which have the lowest output.

In essence, Friedman is saying that costs will be represented by outlays or operating expenditures of the firm; and at least some of these operating expenditures will be associated with the expected output rather than the actual output. An example of cost associated with expected output is the concept of user cost or depletion (i.e., that sum necessary for repairs and replacement in order to leave the value of capital assets intact). The normal charge for this user cost is expressed in accounting data as repair and maintenance. Friedman says that cross-sectional data may not reflect this cost since the true cost does not necessarily show up during the accounting period reviewed. By using volume solely there is no way of determining the magnitude of the difference between the planned output and realized output. The results are pictured in Figure 2.12.

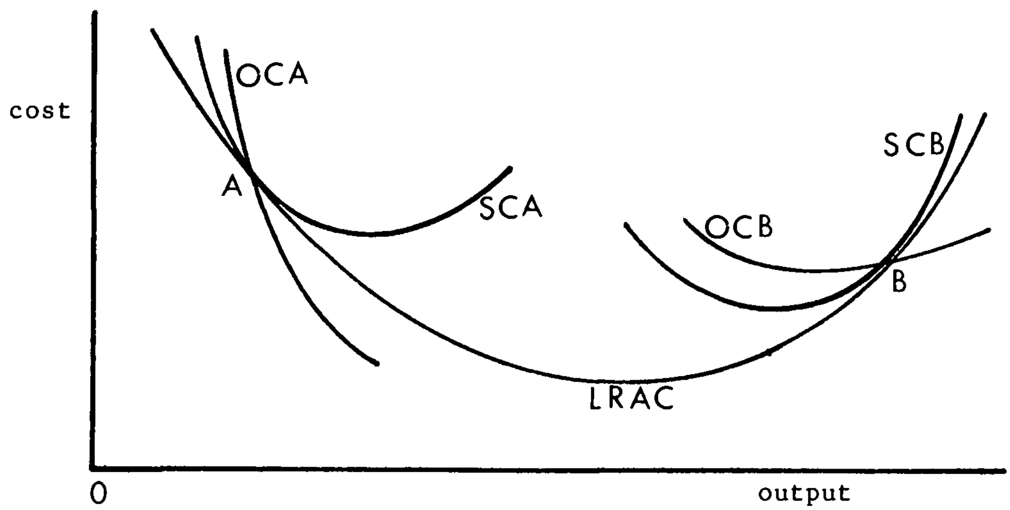

Figure 2.12. Results of regression fallacy.

TABLE 2.1. SUMMARY TABLE SHOWING SHAPE OF LONG-RUN AVERAGE COST.

AUTHOR:	YEAR:	INDUSTRY:	RESULTS:
Alpert	1959	Metal	Decreasing
Bain	1956	Manufacturing	Decreasing
Borts	1952	Railroad	Constant
Borts	1960	Eastern Railroads	Increasing
Borts	1960	South & West Railways	Decreasing
Bowers/Lovejoy	1965	Local Telephone Service	U-shaped
Cohen	1964	Hospitals	Constant
Cohn	1963	High Schools	U-shaped
Dean/James	1942	Shoe Stores	U-shaped
Gribbin	1953	Gas (U.K.)	Decreasing
Heady	1946	Iowa Farms	Constant
Heady/Shaw	1954	Farming	Decreasing
Hirsch	1959	Fire Protection	U-shaped
Hirsch	1959	School Administration	U-shaped
Hirsch	1959	Primary & Secondary Educ.	Constant
Hirsch	1960	Police Protection	Constant
Hirsch	1965	Refuse Collection	Constant
Holton	1956	Food Retailing (Puerto Rico)	Decreasing
Isard/Coughlin	1957	Sewage Plants	Decreasing
Johnston	1960	Electricity (U.K.)	Decreasing
Johnston	1960	Life Assurance	Decreasing
Kalish/Gilbert	1973	Commercial Banking	U-shaped
Kiesling	1966	Primary & Secondary Educ.	Constant
Lomax	1951	Gas	Decreasing
Lomax	1951	Gas (U.K.)	Decreasing
Lomax	1952	Electricity (U.K.)	Decreasing
Markham	1952	Rayon Industry	Decreasing
Michel, *et al.*	1969	Waste Treatment	Decreasing
Michel	1970	Waste treatment	Decreasing
Mohring	1972	Urban Bus Transportation	Decreasing
Moore	1959	Manufacturing	Decreasing
Nerlove	1961	Electricity	U-shaped
Riew	1966	High Schools	U-shaped
Ro	1963	Hospitals	Decreasing
Schmandt	1960	Police Protection	Constant
Seitz	1966	Steam-Electric Generating	Decreasing
Tentner	1944	Top Management Iowa Farms	Increasing
Tentner/Brownlee	1944	Iowa farms	Constant
Thomas	1973	Retailing	Decreasing
Will	1965	Fire Protection	Decreasing

The valid short-run cost curves are SCA and SCB, but the short-run cost curves observed from the regression process are OCA and OCB.

The second subject deals with the reason for economies-to-scale. This author believes the basic conflict between Stigler and Chamberlin pertaining to the causes for economies-and dis-economies-to-size is a result of the nesting of all costs into variable costs in the long-run. Stigler builds his argument around proportions and factor divisibility. In all likelihood, this approach would most accurately characterize that group of resource inputs labeled variable inputs in the short-run. Chamberlin, on the other hand, places primary importance on qualitative and technological differences in machinery. (1965, pp. 235-236). He discounts proportions and divisibility as unnecessary and irrelevant. The resource inputs characterized by Chamberlin appear to be factors comprising the group of fixed costs factors in the short-run equations. Although in the long-run these costs become variable costs, it does not change their basic nature which is different from short-run variable input.

In the light of these divergent viewpoints the writer proposes that greater understanding of proper relationships could be achieved if these two groups of variable inputs could remain separated in the long-run.

Combined Short- and Long-Run Cost Curves

A review of the literature which deals with short-run and long-run costs simultaneously is now deemed appropriate. One of the first such studies was conducted by Markham (1952) on the rayon industry. Short-run unit cost is expressed as a function of rate of capacity utilization. Markham's conclusions reveal that costs rise at an increasing rate when production is curtailed below 100 per cent of capacity (i.e., the curve is L-shaped). The shape is attributable to the high proportion of overhead cost and the relative inflexibility of the production process. The researcher terminates the short-run curve at 100 per cent of capacity since the cost curve, if extended, would be a vertical straight line. This situation would correspond to Case II of Stigler's analysis described earlier.

The long-run curve was also determined to be an L-shaped curve with minimal change in average cost associated with changes in plant size for the larger plants in the industry.

A study by Johnston (1960) on the electrical generating industries in England and the United States uses the functional form of Equation 2.23.

$$Y = a_0 + a_1X + a_2X^2 + a_3T \qquad (2.23)$$

where

Y = total variable cost

X = annual output

T = time period or year.

When X^2 is found to be insignificant Johnston eliminates it from the model. The final conclusion is that both average variable cost and marginal cost are constant in the short-run thereby rendering average cost L-shaped. In order to combine long-run and short-run relationship Johnston turns to United States data and uses the functional form of Equation 2.24.

$$Y = a + b_1X + b_2X^2 + b_3XZ + b_4Z + b_5Z^2 \tag{2.24}$$

where

Y = average variable cost

X = output in million units

Z = plant size in thousands of kilowatts.

Johnston does not apply significance tests to the squared terms in the above equation, nor does he address the problem of multi-collinearity between the variables X and Z (a problem to be discussed later). He determines that annual average fixed cost (note Y in Equation 2.24 is variable cost) is an L-shaped curve as output increases by the use of an independent equation. Thus he concludes that both short-run and long-run costs are L-shaped.

Borts (1960) uses a somewhat different approach in his study on railroad costs. He recognizes two important problems with respect to empirical regression analysis: (1) the existence of the regression fallacy, and (2) the problem of multicollinearity between output and plant size. One method of correcting for the regression fallacy is by introducing the plant size into the regression equation. If one assumes the plant size was originally determined by choosing the optimum plant size for producing a given output, it is reasonable to assume also this given output was the expected production of the management. The user cost will, therefore, exist as a function of this output and plant size.

Continuing his analysis, Borts chooses total freight car-miles (empty and loaded) as representative of plant size (X) and total freight carloads as the variable to depict output (Z). He observes, however, that the statistical correlation between X and Z is $+.85 \pm .08$. As two variables become highly correlated it becomes increasingly difficult to separate their effects on the dependent variable. The higher the intercorrelation, the lower the reliability for the individual regression coefficients. (Fox, 1968, p. 257). The foregoing factor creates the problem of <u>multicollinearity</u>. Borts uses a ratio of Z over X to circumvent this problem. This ratio becomes a measure of carload density, and its reciprocal can be considered a measure of the average length of haul. The mathematical form of the short-run cost curve thus be-becomes Equation 2.25:

$$Y = aX + b\frac{Z}{X} \qquad (2.25)$$

where

Y = total cost

X = total loaded and empty freight car-miles

Z = total freight carloads.

Implementing the above form Borts stratifies his sample into three sizes and three regions and applying covariance techniques estimates a series of short-run equations. Thereafter, he compares the coefficients (a) of output (X) and, observing no significant difference, concludes that short-run marginal cost is constant. When comparing regions he deduces that the Western and Southern regions show economies-to-size while the Eastern region shows diseconomies.

Two excellent examples of cost function specification are derived from studies conducted on the cost of providing hospital services. Ingbar and Taylor (1968) pools over 100 operating characteristics of a hospital into 14 variables and finds 11 to be useable. The authors specify Equation 2.26 as the mathematical form to be applied.

$$Y = a_0 + a_1X_1 + a_2X_1^2 + a_3X_2 + \ldots a_{12}X_{11} \tag{2.26}$$

where

Y = hospital service expenses per patient day

X_1 = number of beds

X_2 = occupancy rate

$X_3 \ldots X_{11}$ = other significant variables and indexes.

The regression results reveal constant long-run average cost and decreasing cost associated with utilization (occupancy rate).

Martin Feldstein (1967) uses the following modified functional form for his cost equation:

$$\frac{Y}{X} = -a_0Z + a_1Z^2 - a_2\frac{X}{Z} + a_3\left(\frac{X}{Z}\right)^2 + \ldots \tag{2.27}$$

where

Y = total operating cost

Z = number of hospital beds

X = number of hospital patients admitted per year.

One should also note that the utilization variable (X/Z) in the above equation is composed of a stock variable (Z), the number of beds; and a flow variable (X), the number of patients admitted per year. In both studies the plant capacity (i.e., the number of beds) and not the production is used as the primary indication of size. Feldstein's study reflects substantial economies-to-size for operating cost. The depreciation and investment cost for the plant and facilities are not included in the study.

Afer reviewing numerous short-run and long-run cost studies Alesch and Dougharty (1971) proposes the functional forms of Equations 2.28 and 2.29 as a reasonable representation for a study on school bus transportation.

$$AC = a_0 + a_1Q + a_2Q^2 + a_3U + a_4D + a_5L \tag{2.28}$$

and

$$U = b_0 + b_1D + b_2Q + b_3Q^2 \tag{2.29}$$

where

AC = cost of transportation per student per year

Q = number of students transported

U = utilization (number of students divided by bus seats)

D = miles driven annually per student (density variable)

L = proportion of bus fleet with more than 12 seats per bus.

By using two stage least squares the following conclusion is reached: "Since utilization is related to scale, it can be concluded that utilization is the mechanism by which economies to scale are being achieved." (Ibid, p. 50).

Summary of Literature Review

The survey of empirical cost curve estimation studies has yielded examples of short-run cost curves which exhibit either decreasing, constant, or increasing marginal costs. The shape of the curves and average cost/marginal cost relationships have frequently been determined by the functional form chosen rather than by the data utilized. Several theoretical explanations are available to help explain the diversity in cost curve shapes.

Long-run cost studies display a variety of shapes which have declining, U-shaped, and increasing properties. The majority of the research, however, has resulted in L-shaped long-run cost curves thereby implying economies-to-scale exist in many industries.

When models are constructed to incorporate both long- and short-run curves plant size must be introduced along with output. Most authors reviewed have chosen to introduce plant size into the model by creating a variable of utilization or plant idleness. This variable, however, is introduced in many different forms. The use of this ratio is desirable because it circumvents the problem of multicollinearity created when plant size is used as an independent variable within the same regression equation as output.

Some interesting functional forms emerged from the last section, but they are restrictive in nature and do not permit traditionally shaped long- and short-run relationships.

CHAPTER III

A GENERAL FUNCTIONAL FORM FOR COST EQUATIONS

Introduction

A review of the available literature has revealed the following recurring principles: (1) Many different shapes of cost curves are observable in empirical studies. (2) Some theoretical support exists for these shapes. (3) No general functional form is available to allow empirical determination of shape.

In the view of the author one of the most enlightening articles on the entire subject of the nature of cost relationships and production functions is an article in the Americal Journal of Agricultural Economics by Nicholas Georgescu-Roegen (1972). Although he deals primarily with the concepts of a production function, many deductions about the nature of cost are possible because cost curves are derived directly from production functions. His intent is to re-formulate the traditional concepts of a production function by integrating the concepts of time and plant size into the analysis. The ideas presented in this journal article will be used extensively in the development of Chapter III which will be concerned with the possibility of developing a versatile mathematical form for the cost equations thereby allowing empirical determination of shape.

Assumptions and Definitions

Before attempting to develop such a function some assumptions and definitions must be clarified. All definitions given with the traditional cost model will apply unless otherwise re-defined. Factor input prices will be held constant as in the earlier case.

Plant Capacity

Plant capacity will become a very important concept in this development; therefore, it must be clearly defined. One of the more functional definitions of plant capacity is expressed by employing absolute units. When possible it should be defined as the maximum amount of production which can be realized within a given set of fixed resources and an unrestricted set of variable resources in a given time period. This time period should be as long or longer than the maximum time period of potential plant operation. A 24 hour period is probably desirable. This would be the case for a water utility with fixed pumping and filtration capacity. An absolute production figure, however, may often be illusive. In this case, some level of production must be chosen to represent the absolute maximum beyond which the plant or firm cannot produce. For example, a grocery store might determine plant capacity as the square feet of floor space times a turnover rate of 70 times per year, providing no store being considered can exceed a turnover rate of 70 times within a given year. If necessary, this numerical value can be an arbitrary number provided it is standard between firms and production of a given firm is less than or equal to plant capacity at all times. Another alternative for the grocery store is to multiply the inventory value of a completely stocked store by 70. This definition, however, is inferior since value is not a physical characteristic.

Capacity Utilization

Capacity utilization is defined by Georgescu-Roegen (*Ibid*., p. 288) as:

$$\frac{tq}{k} = \frac{Q}{K} \tag{3.1}$$

where

t = working time (i.e., fraction of total time plant is in operation) $0 \leq t \leq 1$

q/k = capacity utilization at a point in time

Q = production in units

K = maximum daily production

q = output per unit time

k = plant capacity per unit time.

Total Cost Curves

In the analysis by Georgescu-Roegen total cost is divided into two parts:

t_v = the cost that varies proportionately with t

and t_f = the cost that is independent of t.

This distinction will be retained for both long- and short-run curves in this thesis as deemed desirable in the previous chapter.

In order to prevent confusing the traditional total variable cost and total fixed cost with the above terms, these two concepts will be named accordingly:

TOC = total operating cost

and TPC = total plant cost.

A justifiable point of departure is to let

$$TOC = f(Q) \tag{3.2}$$

$$TPC = f(K) \tag{3.3}$$

since the operating cost will be determined by the amount of output produced rather than the size of the plant producing it. Conversely, the plant cost is by definition independent of output.

Functional Forms

Choosing Third Degree Polynomial Functions

It now becomes necessary to choose specific functions to represent cost curves. Since the traditional curves are third degree polynomial functions this form appears to be a reasonable choice.

Total Cost

Let

$$TOC = a_0Q + a_1Q^2 + a_2Q^3 \tag{3.4}$$

$$TPC = b_0K + b_1K^2 + b_2K^3 \tag{3.5}$$

$$TC = TOC + TPC \tag{3.6}$$

then

$$TC = a_0Q + a_1Q^2 + a_2Q^3 + b_0K + b_1K^2 + b_2K^3 \tag{3.7}$$

For a fixed plant size the short-run total cost curve becomes

$$SRTC = C + a_0Q + a_1Q^2 + a_2Q^3 \tag{3.8}$$

where

$$C = b_0K + b_1K^2 + b_2K^3.$$

Under this situation C becomes the Y intercept, and the total cost curve initiates at C with the desired shape. On closer examination, however, difficulties are recognized. When one looks solely at TOC he realizes that the function reaches an inflection point with a constant value, i.e.,

$$\frac{\partial^2 TOC}{\partial Q^2} = 2a_1 + 6a_2Q$$

or $$Q = \frac{2a_1}{6a_2} \tag{3.9}$$

This, incidentally, is the same characteristic as exhibited by Henderson and Quandt's example given earlier in Equation 2.6. When Equation 3.7 is plotted on a graph it takes the form of Figure 3.1. Obviously, these are not the traditional cost curves as shown in Figure 2.1.

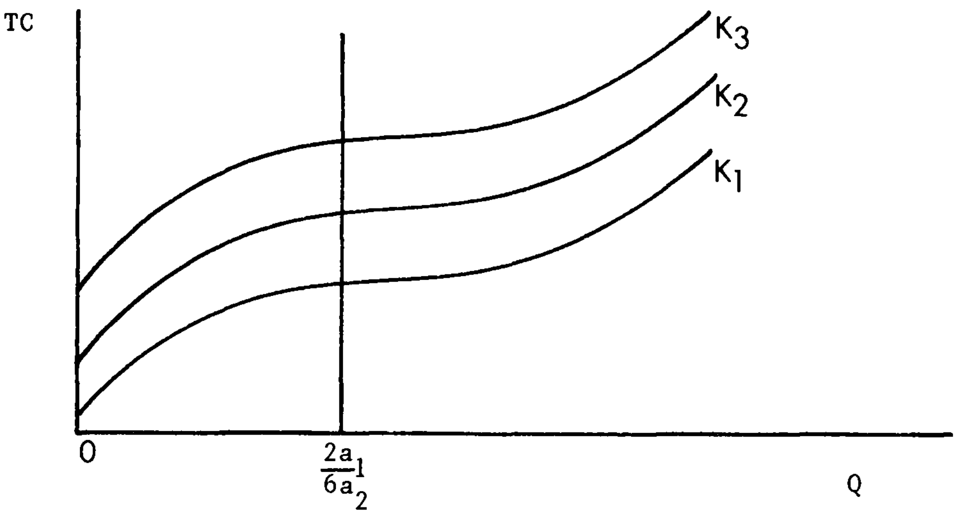

Figure 3.1. Total cost curves with a constant inflection point.

A curve of this nature has a TOC curve independent of plant size except for the upward shift caused by larger total plant cost.

Average Cost

An alternative analysis is provided by considering average cost curves. The average operating cost can be derived by dividing Equation 3.4 by Q thus yielding Equation 3.10:

$$AOC = a_0 + a_1 Q + a_2 Q^2 \qquad (3.10)$$

AOC is a minimum when

$$\frac{\partial AOC}{\partial Q} = a_1 + 2a_2 Q = 0$$

or $$Q = \frac{a_1}{2a_2} \qquad (3.11)$$

This minimum is also a constant.

The above equation indicates that short-run average cost curves do not form an envelope of curves but are a family of curves directly above each other being separated by the addition of average plant cost.

Introducing Time Considerations

Since the above function does not possess the desired characteristics, what new approaches could possibly aid in achieving an acceptable form? A further consideration of Georgescu-Roegen's re-formulation of production functions accents the importance he places on the time dimension and its implications to the above problem. He defines capacity utilization as

$$\frac{tq}{k} = \frac{Q}{K}$$

where output per unit time q and plant capacity per unit time k are time flow variables, and production Q and maximum production K are static unit variables. He emphasizes that this utilization characteristic is the link between a dynamic cost model and a static cost model since production Q can be produced from an infinite number of combinations of t and q (time and rate). To achieve his intended purpose Georgescu-Roegen holds q/k constant and varies t (i.e., he varies the hours that the plant operates). Using this basic concept, but allowing t and q both to vary, the ratio Q/K becomes a meaningful interpretation of the relative idleness of the total plant. The difference between this model and Georgescu-Roegen's model can be distinguished by noting that his model defines two planning cost curves: (<u>Ibid</u>, p. 291).

(1) T(Q; t,q/k) where t and q/k are fixed

(2) T(Q; q/k) where t is allowed to vary and q/k remains fixed.

This author proposes a model where working time t and working rate q/k are allowed to vary. When this is the case the time t and the output per unit time q values are absorbed into the Q value so that the proposed model would assume the form

T (Q;K) where K is fixed in the short-run and

T (Q,K) where K varies in the long-run with $Q \leq K$ in both cases.

One must pause at this point in the model development to relate and clarify the work which has been done by other economists. In the literature review of this paper it was pointed out that Alchian

(1959) also introduces the time factor in a rate-volume concept. Batch processing was his primary concern. He concludes that long- and short-run cost curves do not exist since there is only one least cost way of producing a given volume in a given time period; therefore, two different time periods require two different processes. Hirshleifer (1962) takes Alchian's model and adds uncertainty as the reason for not producing the cheapest way possible thereby allowing short- and long-run cost curves. The two cost curves are a result of the management's decisions made under uncertainty. Some managers make more costly temporary adjustments while others introduce more permanent changes. Neither Alchian nor Hirshleifer, however, uses plant capacity in his analysis. Georgescu-Roegen, on the other hand, introduces utilization (Q/K) as having two components: (1) the fraction of the total plant that operates during working hours (q/k), and (2) the number of working hours (t). He represents plant capacity as part of the utilization value.

The intent of this thesis is to combine the three concepts and explicitly specify

$$R = f(K,U) \tag{3.12}$$

where

R = rate

K = maximum daily production or maximum production possible in one production period (plant capacity)

U = utilization = Q/K = tq/k.

Simply stated, let

$$R = K * U. \tag{3.13}$$

Using Alchian's rate-volume concept then

$$Q = R * T \tag{3.14}$$

where

Q = volume

R = rate

T = time or length or a production run.

Realizing R is the traditional economic concept of units produced per time period since T = 1, then Q = R thereby allowing the most usual notation of output or Q. Using Equations 3.13 and 3.14, however, volume can now be expressed in the component parts as in Equation 3.15 or 3.16 given below:

$$V = K\left(\frac{Q}{K}\right)T \qquad Q \leq K \tag{3.15}$$

or $$V = K\left(\frac{tq}{k}\right)T \qquad Q \leq K \tag{3.16}$$

where

V = total volume produced

K = plant capacity in a 24 hour period

Q = output in a 24 hour period

t = fraction of a 24 hour period the plant is in operation $0 \leq t \leq 1$

q = output per hour

k = plant capacity per hour

T = length of production run in days.

If T = 1 as in traditional economics, then V = Q and output (Q) will be equal to plant capacity (K) times utilization (Q/K).

Modifying Third Degree Polynomial Functions

To give further clarification, the reason for the inclusion of K is to determine the relative idleness of the plant rather than the plant size itself. Since this is the primary intention for the introduction of K, the desirable relationship of utilization (idleness) should be determined. One reasonable approach is to specify plant relationships so that their relative shapes are all the same. This condition would be true if the average short-run operating cost reached a minimum at a given level of plant utilization.

If Equation 3.11 were modified to the form of Equation 3.17

$$\frac{Q}{K} = \frac{a_1}{2a_2} \qquad (3.17)$$

then the above condition would hold. Working in reverse through the equation sequence, Equation 3.10 would become Equation 3.18:

$$AOC = a_0 + a_1 Q + a_2 \frac{Q^2}{K} \qquad Q \leq K \qquad (3.18)$$

and the total operating cost Equation 3.4 would become Equation 3.19:

$$TOC = a_0 Q + a_1 Q^2 + a_2 \frac{Q^3}{K} \qquad Q \leq K \qquad (3.19)$$

Combining Equation 3.19 with total plant cost Equation 3.5 yields a new total cost Equation 3.20:

$$TC = a_0 Q + a_1 Q^2 + a_2 \frac{Q^3}{K} + b_0 K + b_1 K^2 + b_2 K^3 \qquad Q \leq K \qquad (3.20)$$

When Equation 3.20 is plotted the graph in Figure 3.2 will result. This equation has the desired properties consistent with U-shaped short- and long-run average cost curves of traditional economics.

Weighing Problems of Multicollinearity

When considering if Equation 3.20 is useable for empirical estimation one notices Q and K are both independent variables within the same estimation equation. Since one can expect Q and K to be highly correlated in almost all cases, the problem of multicollinearity returns. Two possibilities for correction exist: (1) The total cost equation can be subdivided into its component parts--operating cost and plant cost. These two equations can be estimated separately, or (2) Equation 3.20 can be modified in some manner expressing Q and K in a different form (e.g., Borts (1960) uses utilization as a means of circumventing the problem as discussed in Chapter II). The second possibility will be considered first.

If one proceeds with the analysis of Equation 3.20, however, rather than looking elsewhere for a new form to introduce, an interesting characteristic arises. When the average cost is derived from the total cost Equation 3.20, by dividing each term by Q the resultant is Equation 3.21.

$$AC = a_0 + a_1Q + a_2\frac{Q^2}{K} + b_0\frac{K}{Q} + b_1\frac{K^2}{Q} + b_2\frac{K^3}{Q} \qquad (3.21)$$

The above form transforms the K value into the reciprocal of utilization which has the same ability to reduce multicollinearity problems as does the utilization independent variable. The second possibility is to regress operating cost and plant cost; then add the equations to yield total cost. This approach is possible only if the data available allow for the separation of total cost into plant cost and operating cost. Following such a procedure implicitly assumes no interaction between operating cost and plant cost--a restrictive assumption which should be carefully evaluated before use. When cost separation is feasible Equation 3.18 can be used to regress average operating cost, or Equation 3.19 can be adopted to regress total operating cost. Plant cost can be derived by using Equation 3.5 to estimate total plant cost or by dividing Equation 3.5 by Q to yield average plant cost with the result of Equation 3.22:

$$APC = b_0\frac{K}{Q} + b_1\frac{K^2}{Q} + b_2\frac{K^3}{Q} \qquad Q \leq K \tag{3.22}$$

A third procedure is to divide Equation 3.5 by K thus yielding Equation 3.23:

$$TPC/K = b_0 + b_1K + b_2K^2 \tag{3.23}$$

One realizes that this is not a traditional average cost function since cost is divided by plant size (K) rather than output (Q), but once the regression process is complete the simple multiplication and division can be used to transform the equation into total plant cost or average plant cost equation forms.

Analyzing the Functional Form

Now that this functional form appears to be a reasonable choice a more detailed analysis of Equation 3.20 becomes necessary. The short-run total cost curve is obtained by assigning a value for K thus yielding Equation 3.24:

$$SRTC = C + a_0Q + a_1Q^2 + a_2\frac{Q^3}{K} \tag{3.24}$$

where

$C = b_0K + b_1K^2 + b_2K^3$ for a given value of K.

Referring to Figure 3.2 one observes that the short-run total cost curve for a plant size K = 3 begins at a Y intercept of C_3 and approaches but cannot exceed a value of three on the K axis. If the curve reaches a value of Q = 3, the function terminates at that point. The intercept C_3 is determined by the TPC function at a value of K = 3. The long-run total cost curve is generated by drawing a continuous line tangent to the lower edge of the family of short-run total cost curves. Each tangency point identifies the optimum output Q for each plant size K. These tangency points (LRTC) represent the long-run cost structure for a typical plant.

Many possible empirical combinations are permitted by this functional form. As an example, consider the four cases of Stigler's short-run analysis reviewed in the previous chapter. When coefficients a_0, a_2, b_0, and b_2 of Equation 3.20 are positive and coefficients a_1 and b_1 are negative, Cases I and III of Stigler's analysis are represented (i.e., the traditional cost curves). When a_1 and a_2 are equal to zero, short-run marginal cost

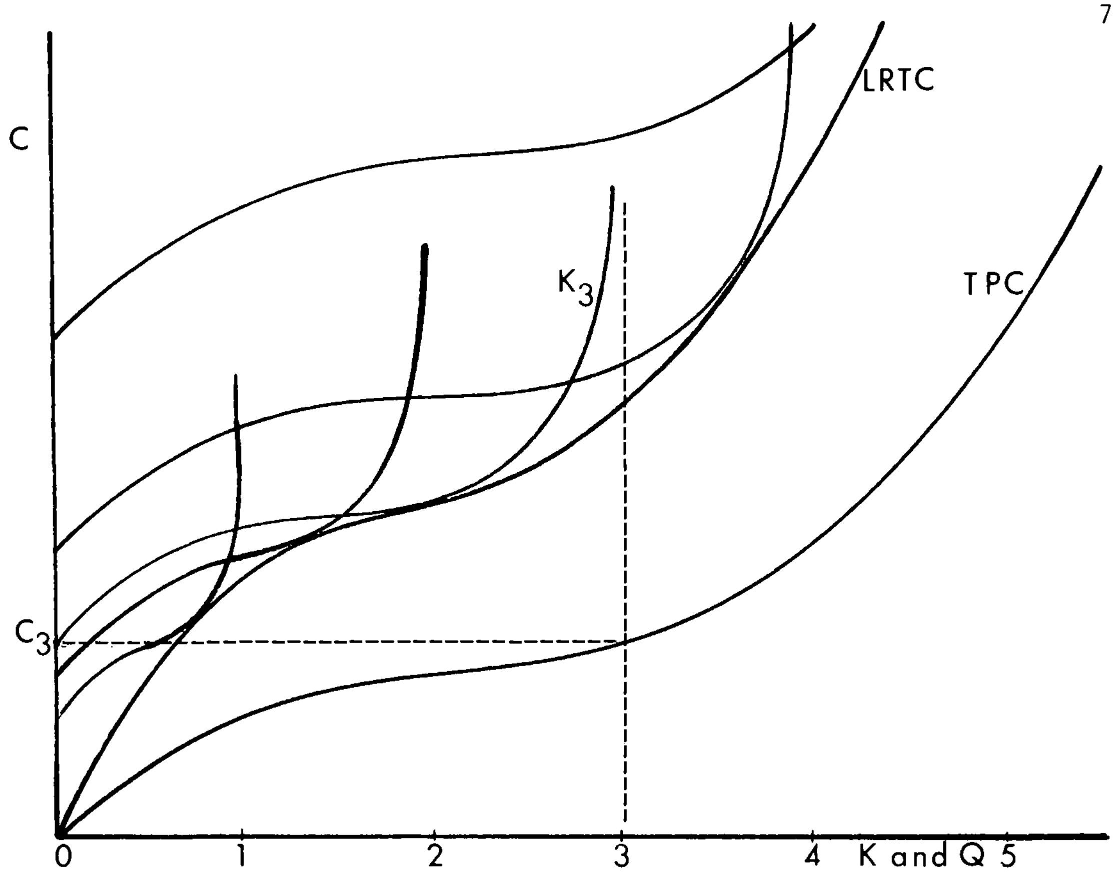

Figure 3.2. General cost function with U-shaped marginal cost.

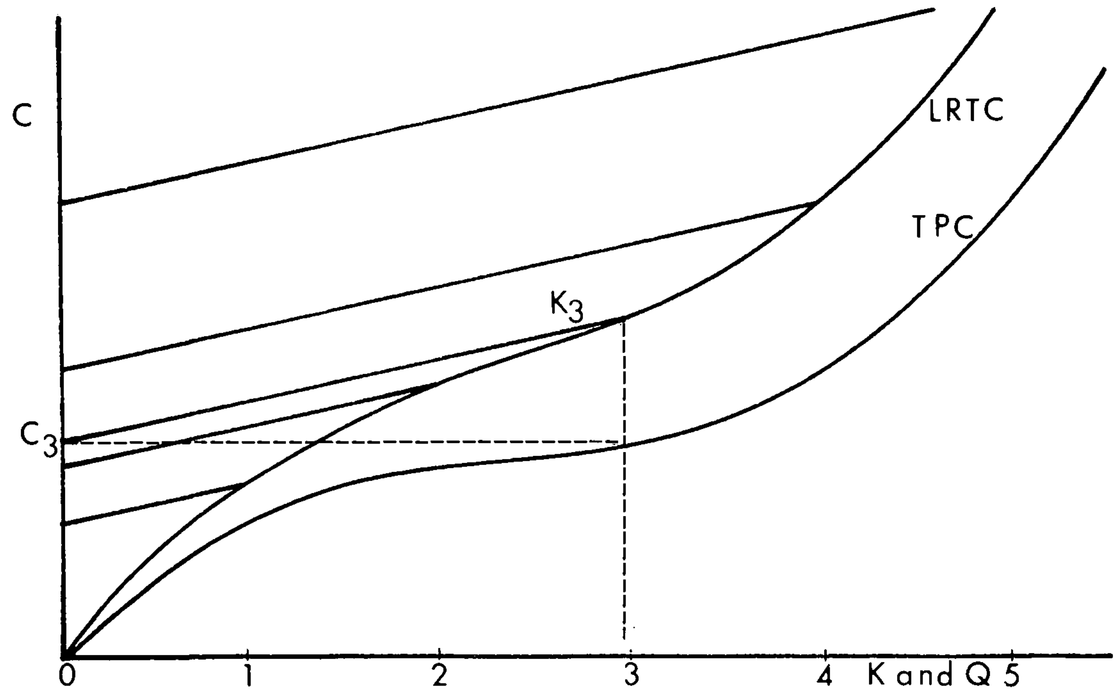

Figure 3.3. General cost function with constant marginal cost.

will be constant and average cost will be declining as in Case II of Stigler's analysis. In this case average operating cost is constant but average plant cost is declining with the net effect of average cost also declining. Resultingly, short-run marginal cost is constant while long-run average cost is U-shaped or L-shaped. Several examples of this relationship were observed in the empirical literature review section as represented in Figure 3.3. Case IV is represented when only one output is permitted for each plant size (i.e., $Q = K$). Under these conditions production can exist only when utilization is equal to 100 per cent; therefore, only the long-run cost curve is relevant.

One should realize that corner solutions are an integral part of this analysis since $Q \leq K$. Two possibilities exist for the formation of a short-run total cost curve: (1) It may become asymtotic to the K value, or (2) it may become equal to K and terminate (i.e., the function becomes discontinuous and cost attains infinity). Either likelihood is a desirable characteristic of the behavior of a general functional form since the industry structure itself will determine the resultant value of cost as output approaches capacity.

Chapter I listed a set of seven criteria deemed necessary to qualify a mathematical equation as a generalized functional form for cost equations. An evaluation of the functional form, using those criteria, is in order to determine if Equation 3.20 is acceptable as a generalized functional form.

1. The function incorporates both long- and short-run cost curves into a single framework.

2. The mathematical form is able to generate an envelope of U-shaped short-run cost curves.

3. The Stigler cases given above demonstrate that the equation is not restricted to U-shaped curves.

4. The function appears to be able to accommodate a range of several magnitudes in plant size.

5. The quadratic and cubic nature of the plant and operating cost functions permit the expressions of economies- and diseconomies-to-scale.

6. The mathematical form is adaptable to least squares linear regression techniques since the equation is intrinsically linear in nature.

7. The framework is amenable to hypothesis formulation and testing.

All of the pre-specified criteria have, therefore, been met. The general formulation appears to be sufficiently versatile to accommodate the majority of shapes found in the review of empirical literature while meeting the requirements of traditional cost theory. Chapter IV will adapt this general formulation to a specific empirical problem.

CHAPTER IV

DEVELOPMENT OF COST FUNCTION FOR MUNICIPAL WATER

Now that a general functional form has been developed which will allow the data to determine the curve shapes, it is desirable to apply the theory to an empirical study. The municipal drinking water industry will be chosen for this application. This chapter is intended to accomplish the following tasks: (1) to survey the possible sources of data, (2) to adapt the general functional form developed in Chapter III to a form suitable for a municipal water study, (3) to demonstrate the procedures necessary to prepare data for the mathematical form and linear regression equations.

Researching the Data on Municipal Water

Approximately 40,000 water utilities in the United States serve an estimated 160 million persons. Of this number 1,300 large systems serve 106 million people while the balance supply only 54 million people. Individual home supplies service the remaining 50 million population. (Senate Hearings, May, 1973, p. 186). Of this number 5,900 are investor-owned, serving about 30 million people. Through the years various studies have collected data on segments of these utilities. The most important ones with a brief description of the intent of each study and data collection are listed below:

(1) The Municipal Water Facilities Inventory of 1963 (USPHS, 1965) is probably the largest single survey ever conducted on municipal drinking water. The 1963 compilation is the most recent of federal inventories begun in 1939 by the United States Public Health Service, and it purposes to evaluate all municipal water facilities serving vicinities with a population of 100 or more. The inventory, intended to be taken every five years, was not taken in 1968 or 1973. The survey includes information on population served, year operation started, ownership, source of supply, maximum dependable draft, plant capacity, average plant output, treatment, distribution storage, and improvements needed. No cost or revenue information is included.

(2) Public Water Supplies of the 100 Largest Cities in the United States, 1962 (Durfor, 1964) by the U.S. Geological Survey deals with the chemical and physical properties of the raw and treated water of the larger cities. The second part conducts a city by city inventory on water supply, population, auxiliary sources, average daily use, mean discharge of streams used for public water supply, water treatment, plant capacity, and raw or finished water storage capacity. The study contains no cost data other than the cost of chemicals.

(3) Nationwide Study of High Municipal Water Rates by the Office of Saline Water was conducted in 1971. Its objectives may be stated as follows: ". . . to identify the largest possible number of public water supplies throughout the United States serving 1,000

people or more whose prices to their customers are so high that desalination technology might offer a competitive advantage, especially in considering additional sources of supply; to analyze the distribution of such high-priced water supplies in terms of states, size of population served, the age of their present rates, expectations of their officials as to future needs for additional supplies, and other variables." (USDI, 1971, p. 1). Over 11,000 questionnaires were mailed to municipal water agencies and 64 per cent were returned. Of those returned 1,174 (18 per cent) were found to have water rates of $7.50 or more for 10,000 gallons of water per month. The published data is given only for those utilities charging $7.50 or more per 10,000 gallons. The survey questionnaire asked only for information on population served, rate schedule, date rate became effective, and any anticipated change in rates. Using the rate information collected along with the 1963 inventory above, the study reveals that, on the average, the smaller cities have a higher rate structure than do the larger cities (<u>Ibid</u>, p. 31).

(4) <u>The Community Water Supply Survey</u> published by the United States Public Health Service Bureau of Water Hygiene in 1969 (USPHS, 1970) was undertaken to determine if the American consumer's drinking water met the 1962 Drinking Water Standards (USPHS, 1962). To accomplish this objective 969 water supply systems were investigated. Community water supply systems accounted for 885 systems, and 84 special water supply systems (mobile home parks, tourist accommodations) completed the total. Every water system within nine

selected regions was studied. The regions made up 5 per cent by number of all public water systems and 12 per cent (18.2 mil.) of the national population served by public systems. The study was primarily concerned with quality of the water but included some cost production data. The startling results of the report spurred Congressional Hearings in the first and second sessions of the Ninety-Second Congress (House, May 1971; Senate, March 1972; House, June 1972) and the first and second sessions of the Ninety-Third Congress (House, March 1973; Senate, May 1973; Senate, June 1973). Selected results were presented in the introduction of this paper.

(5) A Survey of Operating Data for Water Works is taken by the American Water Works Association every five years. The more detailed report was in 1964 for the 1960 data (AWWA, 1964a). This survey yields a massive amount of statistical information on the facilities reporting; however, the detailed questionnaire was sent only to cities serving populations of 10,000 persons or more. An abbreviated version was sent to a sample of water utilities serving less than 10,000 only in the 1960 survey (AWWA, 1964b). The surveys provide much production, cost, treatment, and distribution data but no information on the quality of the end product. The 1960 surveys have information on 870 utilities serving 10,000 or more population and 448 utilities serving less than 10,000 persons. These questionnaires, however, are not complete in a great number of cases. The 1965 and 1970 data contain less total information but are more complete for the reporting services (AWWA, 1974).

The 1965 data had 861 reporting water utilities while the 1970 data was limited to 769. In all cases the published reports reproduced the numerical data of the survey and a limited number of unrefined total tables for each survey. For example, the total tables ignored the missing data problem and assigned an arbitrary value of zero to the missing data. From the 1955 and 1960 surveys the Journal of American Water Works has published articles containing the very basic statistical summaries of the survey (Seidel, 1957; AWWA, 1966). State and local studies abound, but these are the only studies reporting on a national basis.

Adapting the General Cost Function to Municipal Water

Importance of Physical and Economic Characteristics

When determing an economic model the physical characteristics contributing to the economic cost must be determined. For municipal water the physical characteristics can be segmented into three parts: the source, the treatment, and the distribution. These three components should determine the primary contribution to cost of the physical characteristics.

The two primary sources of public drinking water are ground water and surface water. Although other sources are available--for example, the desalinization of salt water--these techniques are presently in limited use and will not be considered in this study. The cost of the ground water may be affected by the depth of the well, the pressure of the underground water, the cost of drilling the well, and many other contributing factors. The surface water, on the other hand, has less variation in potential cost associated directly with the source. Both ground and surface waters, however, are affected by the distance and elevation of the source from the city which it serves.

When considering the cost of treatment both type and level of treatment should be evaluated. Although many types and various methods of treatment are used in refining the water, most types can be divided into eight major categories: filtration, softening, disinfection, corrosion control, iron and magnesium removal, taste and odor control, fluoridation, and coagulation. The amount or level of treatment necessary will depend upon the quantity of dissolved and suspended materials in the water as well as the quality of water desired in the refined water.

After the water has been refined additional costs are then associated with the distribution. The distribution cost will depend upon the number, type, location, and volume usage of its customers; therefore, the proportionate use by residential, wholesale, commercial and industrial users will affect the resultant cost. Closely related with the volume of each user is the density of customers. Both volume and density will have significant bearing on the size and length of the distribution lines.

Social and economic characteristics can also affect cost. The behavior of any given firm varies greatly from city to city. In some cities municipal water is heavily subsidized from tax monies in order to supply adequate, reasonable water source to its customers. In other cities water income is used as a major source of revenue for financing of local city government. In still other cities the water companies are private companies which operate under a profit motive (given the constraints placed upon them by the respective governing boards); therefore, the price the consumer pays for public water may have very little correlation with the cost of producing that water. In the light of these facts the cost figures adopted by this study will be the cost of supplying, treating, and distributing water as reported by individual utility.

Addition of Treatment Variable

The basic model developed in Chapter III establishes the fundamental economic relationships; however, the above mentioned physical characteristics must now be considered as possible additions to the model. The characteristic of principal interest is that of treatment. In an effort to keep the discussion as concise as possible all treatments will be typified by a representative treatment T. Since the data available do not provide information on various levels of treatment, the only consideration relevant is the existence or non-existence of a particular treatment. Let treatment T be assigned a value of one when present in the plant and a value of zero when absent. In statistics variables of this nature are called dummy variables and are commonly used for non-quantifiable independent variables (Huang, 1970, pp. 163-167). Adding the treatment variable to Equation 3.21 will result in Equation 4.1:

$$AC = a_0 + a_1Q + a_2\frac{Q^2}{K} + b_0\frac{K}{Q} + b_1\frac{K^2}{Q} + b_2\frac{K^3}{Q} + c_0T + c_1\frac{TK}{Q} \qquad (4.1)$$

where $Q \leq K$

AC = average cost

Q = output

K = plant size

T = representative treatment.

When T = 0 Equation 4.1 becomes Equation 3.21; when T = 1 two changes result: (1) The intercept value a_0 changes by a value of c_0, and (2) the coefficient b_0 changes by a value of c_1. Equation 4.1 can be rearranged and represented by Equation 4.2 when treatment is present.

$$AC = (a_0 + c_0) + a_1Q + a_2\frac{Q^2}{K} + (b_0 + c_1)\frac{K}{Q} + b_1\frac{K^2}{Q} + b_2\frac{K^3}{Q} \qquad (4.2)$$

and $Q \leq K$.

The purpose of associating T with two variables can best be illustrated by observing the total cost equation. Equation 3.20 becomes Equation 4.3 when T is introduced.

$$TC = (a_0 + c_0T)\,Q + a_1Q^2 + a_2\frac{Q^3}{K} + (b_0 + c_1T)\,K + b_1K^2 + b_2K^3$$

and $Q \leq K$. (4.3)

In this equation the cost associated with treatment can affect the operating cost and/or plant cost by changing the slope of the total cost function. The end result is a new family of short-run cost curves and, therefore, a new long-run cost curve. The change in shape of the new curves, of course, will depend on the relative magnitudes and signs of c_0 and c_1. Adjustments in the square and cubic terms are possible using the same technique but will not be introduced in this study as the author feels their inclusion unduly complicates the model.

When the operating cost and plant cost equations are regressed independently, operating cost Equation 3.18 becomes Equation 4.5 and 4.6 respectively.

$$AOC = a_0 + c_0T + a_1Q + a_2\frac{Q^2}{K} \tag{4.4}$$

$$APC = b_0\frac{K}{Q} + c_1\frac{TK}{Q} + b_1\frac{K^2}{Q} + b_2\frac{K^3}{Q} \tag{4.5}$$

$$TPC/K = b_0 + c_1T + b_1K + b_2K^2 \tag{4.6}$$

The model now includes the three following principal independent variables: output, plant size, and treatment.

Evaluation of Selected Variables

Inventory of Specification Errors

Since the intent of the study is to ascertain the change in cost associated with improved water quality, caution must be exercised to prevent other variables from pre-empting the treatment variables. Consequently, all variables considered for inclusion in the model from this point on will be evaluated with respect to specification errors. The primary reason for their introduction into the cost equations will be to prevent misrepresenting the effects of the independent variables presently in the model. This misrepresentation would take the form of improper values for the statistical coefficients associated with the independent variables. For example, by omitting an important variable a treatment could show it adds more to total cost than it actually does. Kmenta (1971, p. 392) points out five kinds of possible specification errors:

1. Omission of a relevant explanatory variable.
2. Disregard of a qualitative change in one of the explanatory variables.
3. Inclusion of an irrelevant explanatory variable.
4. Incorrect mathematical form of the regression equation.
5. Incorrect specification of the way in which the disturbance enters the regression equation.

In this section the primary emphasis will be on specification errors one and three.

Source of Water Supply

Previous discussion has pointed out that the source of water may be important in determining water cost. Water source, however, has a great deal of effect on the quality of the raw water and, therefore, the treatments required. The introduction of water source into the model, consequently, could have a severe masking effect on treatment since the water source could become an aggregated variable representing several types of related water treatments. Because of the introduction of multicollinearity the significant effects of individual treatments would then dissipate.

Another important consideration in evaluating source is the recognition of the within variation as opposed to the between variation when determining cost. For example, much ground water is distributed without any processing, minus even disinfection, while other ground water requires the relatively high cost treatments of iron removal and/or softening. Likely, the variation in pumping cost within ground water sources is larger than any difference between ground and surface water. In light of the above circumstances, water source will not be included as an independent variable in the model.

Question of Purchased Water

A business structural characteristic, purchased water, will now be reviewed. Unlike the former comparison between ground and surface water where data information was more complete, this water enters the firm from an unknown origin and with unknown quality. The previous discussion intimates that the unknown origen will not cause any new difficulties; however, the same inference does not apply to treatment. The water purchased from wholesale water firms may be water ready for distribution to consumers, or it may be raw water requiring considerable treatment. In multiple cases the water will have already had extensive treatment at another treatment plant. Cost of the treatment, under these conditions, will be reflected in the purchase price of the water. The price of purchased water is included in operating cost.

Consequently, one would expect higher operating costs for purchased water. The types of treatment, however, are not known; thus any coefficients associated with an independent variable of "purchase" would reflect an average price and average treatment for the purchased water. The water entering the system would be no different than water entering from an owned surface or ground source. Two alternatives exist. Either all observations with purchased water could be removed from the sample, or a variable for purchased water should be introduced into the model. If the second alternative is chosen, one should realize that the extensiveness of the variation explained by the model will decline since the purchase variable will include unreported treatment types. This study will remove all observations containing purchased water.

Problem of Distribution Cost

In the beginning of this chapter the physical characteristics affecting cost were segmented into source, treatment and distribution. Previous discussion pointed out that the distribution cost would be affected by the number, location, density, type, and volume usage of the municipal water customers. The number of customers is already reflected in the production and plant size variable; therefore, the actual number of customers as a separate variable does not seemingly add anything to the analysis. Its usage in this model would likely cause severe multicollinearity problems. Given the available data the only possible approach to specifying a location variable would be through some type of proxy variable such as density.

Figures on city density are available through census data for the years 1960 and 1970. The census boundaries, however, are in many cases considerably different from the boundaries served by the municipal water supply. Internal to the data set some information might be gained on density by considering such combination variables as miles of transmission and distribution line per million gallons of water produced or per million gallons of plant size. A second alternative might be to consider miles of line per capita served. Although these are distinct possibilities, further consideration is necessary. What kind of cost-density relationship might be expected? The first apparent relationship is one of lower cost. As density increases usage per mile of line increases; therefore, per unit cost should decline. Although larger mains may be required, one would expect the net effect to be economies. Conversely, as density increases building height increases and, therefore, line pressure must increase. Also other diseconomies such as increased capacities for fire protection accelerate as density increases. Social structures such as labor unions are associated with increased density and population. External pressures of this nature may increase factor input prices thereby increasing cost although physical resources remain constant. A net effect of zero may result in opposing forces which cancel each other. The seemingly ever present problem of multicollinearity is active in these relationships and would require extremely careful monitoring. A density variable will not be included in the initial regression equations.

Type of Customer

The type of consumer is a related but somewhat different concept. If the pricing structure of municipal water is any indication of its true cost, one could expect higher cost per million gallons for water sold and delivered for residential use. Mace and Wicker (1968, p. 42) in a study, Do Single-Family Houses Pay Their Way?, evaluates the cost of public services to a single-family residence. The researchers conclude that water supply and sewage cost in the areas studied are self-supporting and revenues equal cost. Substantial discounts are given for volume sales to industry and manufacturing. Representation of this independent variable could presumably take the same form as a utilization variable (i.e., let the variable be represented by the following fraction--residential water sold divided by total water sold with resulting values between zero and one). No multicollinearity problems are apparent other than the possible tendency for smaller cities to sell a higher percentage of the water produced to residents. One would anticipate treatment to exist independently from the proportion of residential use. Some correlation between proportion of residential use and density, however, might be expected thereby enabling the residential variable to operate as a proxy variable for density adjustments. When the fraction of residential consumers is added to Equations 4.2, 4.4, 4.5, and 4.6 the following equations result:

$$AC = a_0 + c_0T + a_1Q + a_2\frac{Q^2}{K} + c_1\frac{TK}{Q} + b_0\frac{K}{Q} + b_1\frac{K^2}{Q} + b_2\frac{K^3}{Q} + dH$$

where $Q \leq K$ (4.7)

$$AOC = a_0 + c_0T + a_1Q + a_2\frac{Q^2}{K} + dH \qquad Q \leq K \tag{4.8}$$

$$APC = b_0\frac{K}{Q} + c_1\frac{TK}{Q} + b_1\frac{K^2}{Q} + b_2\frac{K^3}{Q} + dH \qquad Q \leq K \tag{4.9}$$

$$TPC/K = b_0 + c_1T + b_1K + b_2K^2 + dH \tag{4.10}$$

where

AC = average cost

AOC = average operating cost

APC = average plant cost

TPC/K = average plant cost per unit of plant size

Q = output

K = plant size

T = representative treatment

H = fraction of water sold to households.

Choice of Economic Model

When all factors mentioned above are considered the average cost Equation 4.7 along with the average operating and plant cost Equations 4.8 and 4.10 will be chosen as the mathematical form for the regression equations. Equation 4.10 is chosen in preference to Equation 4.9 for three reasons: (1) During the regression process Equation 4.10 does not have to be forced through the origin; and (2) Equation 4.10 has one less independent element (i.e., Q is missing); furthermore, (3) it is only a second degree equation while Equation 4.9 is a third degree equation. Cohen and Nagel (1943, pp. 312-315) points out that the simpler of alternative models should be chosen when it is capable of yielding the same results. This text defines simpler in these words: "One hypothesis is said to be simpler than another if the number of independent types of elements in the first is smaller than in the second." (*Ibid.*, 1934, p. 213). They further point out that this does not necessarily mean the least complex form mathematically. When the above factors are taken into consideration one would expect Equation 4.10 to provide the better regression estimates.

By utilizing the above three equations one can obtain two separate models. Equations 4.8 and 4.10 can be converted to their respective total cost equations. When they are added together a total cost equation will result containing both operating and plant costs. The combined equation can then be compared to the total cost equation derived from average cost Equation 4.7.

The chosen equations incorporate sound economic, statistical, and mathematical principles thereby arriving at short- and long-run cost functions which include the water treatment and distribution cost. Marginal cost and other useful economic relationships relevant to water cost can be derived from these three equations.

Choosing a Data Set for the Cost Function

The choice must now be made as to the data set to use. Of those available the American Water Works Association data are the only data sets with the cost information necessary for cost estimations. Three sets of data are presently available--1960, 1965, and 1970. The sets differ in the following aspects: The 1960 data reports plant capacity, maximum 24 hour demand, and original plant investment. The 1965 survey did not collect information on plant capacity but did collect data on the other two variables. In 1970 data on the original purchase price was not requested in the questionnaire. The most recent data set would logically be the preferred data; however, without the original capital investment the 1970 data is very limited since the original price is necessary to determine the plant cost variable. The 1965 data could conceivably be updated for those cities reporting in both 1965 and 1970 to attain an estimate of the original investment in the 1970 plant, but such a correction will not be attempted in this study.

The original model in Chapter III requires $Q \leq K$; however, this restriction is valid for both plant capacity and 24 hour maximum daily use since Q is the average production per day. At first glance the continuous utility capacity of the plant appears far superior to the others, but upon further deliberation some distinct problems arise. The majority of plants contain limiting factors--treatment plant, water supply distribution lines, pumping capacity, etc.. When continuous utility capacity is reported the most restrictive limiting factor will determine this volume. If, for example, the pumping capacity were reported--and this is a very likely way for an operator to report size since it is a figure which is readily available--the treatment plant portion and distribution lines might foreseeably accommodate more capacity. Another problem arises when determining the amount of water pressure used in the distribution lines. The other extreme is also a possibility. Pumping capacity, conceivably, could far exceed the capacity at which a filtration plant functions properly, although the water can be pumped through the filter. Since costs are expressed on a per unit basis, an overestimation of plant size would lead to an underestimation of per unit costs and visa versa.

The maximum amount of water used in a 24 hour period is a figure readily available to a plant operator; therefore, the reported value should be accurate. Also, the original determination of plant capacity is most likely a function of maximum needs. Linaweaver (1967, pp. 3-4) gives the standard procedure for determining plant capacity:

> The estimation of water demands in the design of water distribution systems has been accomplished for many years by estimating population, multiplying by an average daily per capita use to estimate total average use, and then applying peak to average ratios based on entire cities in order to estimate the peak demands. The peak to average ratios selected have been generally too low. The Minimum Design Standards acceptable to the Federal Housing Administration as revised in July 1965 include the following criteria for water

> distribution systems. Where experience data are available, the annual average demand should be ascertained on the basis of records of existing systems of similar nature in the area. In the absence of reliable experience records, an average demand of 100 gpcd and 4 persons per living unit should be used. This results in an average demand of 400 gpd per dwelling unit. A maximum daily demand of 200 per cent of the average demand is recommended. A maximum hourly demand of 500 per cent of the average demand is suggested except in areas where "extensive" lawn irrigation is commonly practiced, and then a rate of 700 per cent or more of the average daily demand is recommended. Thus, the Federal Housing Administration standards recommend an average demand of 400 gpd per dwelling unit, a maximum day of 800 gpd per dwelling unit, and a peak hour of 2,000 gpd per dwelling unit with extensive sprinkling. These criteria are an improvement, but there are situations where their use could lead to underdesign and in other situations to overdesign of the water distribution system.

This study will, therefore, designate the maximum amount of water used in a 24 hour period as the representative of plant size.

Considering the above factors the 1965 data set will be chosen as input into the regression model.

Preparing the Data for Regression Equations

Parts of the data must now be modified to provide the most usable form. One of the most important values which must be determined is the plant cost. A possible method is to use the reported dollar depreciation of the firm. A careful study of the data, however, suggests the rate of depreciation is more a concern of the cost of financing rather than the consideration of the useful life of the plant. A reasonable approach to correct the differing depreciation practices is to adjust plant cost to represent a uniform depreciation rate. In order to determine this adjusted depreciation cost the following four items of information are necessary: (1) the original cost of constructing the plant; (2) the age of the plant; (3) the uniform depreciation rate; and (4) an index of prices. A logical place to begin is with the age of the plant.

Plant Age

Determining plant age is no simple task even for a plant operator who is very familiar with a given plant and has access to many detailed records, but it becomes almost impossible for a study of this nature with its limited data and non-germane knowledge. A plant is most likely developed during several stages as modifications, additional capacity, new lines and new treatments are added. Given this data, a researcher can propose a pseudo or average age and assume that it will adequately represent the plant. The survey reported the following information: (1) original book value; (2) accumulated depreciation; (3) 1965 depreciation in dollars; and (4) reported depreciation rate. Using this information age can be determined by Equation 4.11.

$$A = \frac{AD}{OC} \frac{1}{R} \tag{4.11}$$

where

A = average age of plant

AD = accumulated depreciation

OC = original cost of the plant

R = depreciation rate

The reported depreciation rate R can be compared to the calculated depreciation rate as shown in Equation 4.12:

$$R = \frac{D}{OC} \tag{4.12}$$

where

R = calculated depreciation rate

D = dollar depreciation for 1965

OC = original cost of plant and facilities.

Using this method of age determination presupposes straight line depreciation on the original purchase price. All other forms of depreciation would give erroneous information. The comparison of the calculated depreciation rate to the reported depreciation rate should render some indication of which firms are using straight line depreciation. A somewhat arbitrary value of one percentage point difference between the calculated and reported rates will be chosen as the level beyond which observations used to determine fixed cost will be rejected. The observations which pass the test will be given a value for R equivalent to the average between the reported and calculated depreciation.

Inflation Index

The Index of Wholesale Prices from 1935-1965 will be chosen as an index of prices to update the original price (USDA, 1972, Table 621, p. 503). In order to make this correction the annual percentage increase in prices must be determined. Yearly increases of this type are logarithmic in nature rather than additive. Croxton (1967, p. 181), points out the procedures for determining average per cent of change. The general formula is identical to the compound interest formula. In this application the formula would take the form of Equation 4.13:

$$I_e = I_b\,(1 + CI)^n \tag{4.13}$$

where

I_e = index of wholesale prices for the ending period

I_b = index of wholesale prices for the beginning period

CI = relative increase or decrease per period expressed as a decimal

n = number of periods.

Solving Equation 4.13 for CI yields Equation 4.14:

$$\log\,(1 + CI) = \frac{\log I_e - \log I_b}{n}$$

or

$$CI = \left(\frac{I_e}{I_b}\right)^{\frac{1}{n}} - 1.0 \tag{4.14}$$

and inserting the values yields the solution

$$\log\,(1 + CI) = \frac{\log 96.6 - \log 41.3}{30}$$

$$CI = .0287$$

where

I_e = 96.6 = Index of Wholesale Prices 1965

I_b = 41.3 = Index of Wholesale Prices 1935

n = 30 = number of years

CI = change in Index of Wholesale Prices per year from 1935-65.

Replacement Value and Adjusted Depreciation

Once the age and inflation index have been determined the adjusted depreciation cost can be calculated using Equation 4.15:

$$TPC = [(A * CI * OC) + OC]\ R \tag{4.15}$$

where

TPC = adjusted annual depreciation cost

A = age of plant

CI = change in index of wholesale prices per year

OC = original cost of plant

R = rate of depreciation.

The quantities within the brackets combine to create the replacement value of the plant facility. Both sides of the equation can be divided by plant size to yield average fixed cost Equation 4.16.

$$TPC/K = [(A * CI * OC) + OC]\ R/K \tag{4.16}$$

Depreciation Rate

The choice of the uniform depreciation rate R is somewhat arbitrary. The value chosen will not affect the significance levels or the coefficient of determination resulting from the regression equations, but it will affect the values of the coefficients since it influences the value of the dependent variable. After the values for the statistical coefficients have been determined a different R value can be chosen, and the depreciation rate can be adjusted by dividing the right hand side of Equation 4.10 by the old R and multiplying by the new R. Therefore, the choice of R is not critical to the regression analysis but it is cost determining. One reasonable alternative is to choose the mean value of R as determined from the R in Equation 4.11. This value is determined to be .025 or an average useful life of 40 years. Choosing this technique should give a value representative of the industry depreciation practices.

Using these adjustments and preliminary calculations, Equations 4.7, 4.8, and 4.10 can be regressed using least squares regression techniques. The results are reported and discussed in Chapter V, the concluding chapter of the thesis.

CHAPTER V

CONCLUSIONS

The concluding chapter of this thesis will begin by reporting the preliminary results of the regression analysis. The empirical equations will be critically examined with reference to economic theory and the expected behavior of a cost function. A set of operating rules will be suggested for tuning the model so that economic principles will not be violated, and a final set of empirical equations will be reported using these operation rules. Selected comments will be directed toward using some of the principles established in the study to achieve a better understanding of cost curve behavior. Finally, a return to the original cost curve criteria, the hypotheses, and the statement of the problem will allow one to evaluate to what degree the original purposes were or were not achieved.

The Regression Results

The 1965 data source contains 159 observations useable in the regression process. Although Chapter IV considers treatment as a single independent variable, the multiple regression equation takes into account the following eight types of treatment for inclusion in the model: coagulation, filtration taste and odor control, disinfection, softening, corrosion control, iron removal, and fluoridation. K and Q are reported in million gallons of water throughout the chapter.

Reporting of Preliminary Regression Equations

Multiple regression techniques are used to fit statistically the data, and the resulting equations are reported below. Two average operating cost (AOC) equations are cited since neither can be rejected without additional evaluation.

$$AOC = 92.95 - .007419 \frac{Q^2}{K} + 124.52\ H + 48.57\ Soft \qquad (5.1)$$

Standard error	(.0003778)	(34.13)	(14.61)	$R^2 = .17$
t value: d.f.=155	(1.95)	(3.65)	(3.31)	F = 10.36

where

H = fraction of water sold to households

Soft = softening treatment

$$AOC = 92.31 - .00045\ Q + 125.72\ H + 48.82\ Soft \qquad (5.2)$$

Standard error	(.00023)	(34.07)	(14.61)	$R^2 = .17$
t value: d.f.=155	(1.94)	(3.69)	(3.34)	F = 10.32

$$TPC/K = 20.02 - .0001976\ K + .0000000006\ K^2 + 12.82\ Corr$$

Standard error	(.000117)	(.00000000038)	(4.39)
t value: d.f.=151	(1.67)	(1.55)	(2.92)

$$9.08\ Filt + 13.96\ Disin + 11.21\ Iron + 19.91\ H \qquad (5.3)$$

(4.61)	(5.75)	(4.76)	(11.44)	$R^2 = .27$
(1.97)	(2.43)	(2.35)	(1.73)	F = 7.89

where new notation is

Corr = corrosion control treatment

Filt = filtration treatment

Disin = disinfection treatment

Iron = iron removal treatment

When operating and plant costs are added and regressed as a single equation the resultant is Equation 5.4.

$$AC = 131.67 - .002582\, Q^2 + .000000001357\, \frac{K^3}{Q} + 195.78\, H$$

Standard error	(.0008541)	(.000000000596)	(43.85)
t value: d.f.=151	(3.02)	(2.28)	(4.46)

$$-207.71\, Soft + 153.69\, Soft*\frac{K}{Q} + 185.92\, Filt - 89.01\, Filt*\frac{K}{Q} \qquad (5.4)$$

(78.16)	(43.44)	(54.39)	(30.86)	$R^2 = .28$
(2.65)	(3.54)	(3.42)	(2.88)	F = 8.19

Examination of Preliminary Equations

The Total Cost Equation

Equation 5.4 indicates that only two types of treatment--softening and filtration--contribute significantly to total cost. When regressed separately, however, five of the eight treatments are significant. By combining the two costs a masking effect on treatment results which reduces the explanatory nature of the regression functions.

Plant Cost Equation

The Plant Cost Equation 5.3 displays expected signs for the K (a negative) and K^2 (a positive) terms. These signs represent the traditional U-shaped curve.

Operating Cost Equations

The operating cost equations, however, yield negative coefficients for both Q^2/K in Equation 5.1 and Q in Equation 5.2.

(i.e., A downward sloping average cost curve is implied throughout the relevant range.) This result is contrary to the expected behavior; therefore, further investigation, evaluation, and explanation are required. When the Q and Q^2/K terms are introduced into the same function both become insignificant. The alternate deletion of one of the terms yields Equations 5.1 and 5.2. The second of these equations is characterized by a linearly decreasing average cost function. The decreasing cost function, although not expected, is potentially stable and can react with marginal revenue to establish market equilibrium in a monopolistic or monopolistically competitive industry. The only requirement is that the marginal cost function must cross the marginal revenue function from below. (Henderson and Quandt, 1958, pp. 169-170). The constant slope of Equation 5.2 is a result of the mathematical form of the equation. Equation 5.2, therefore, is acceptable but restricted by the mathematical form.

Equation 5.1 is also a declining average operating cost function; however, it is non-linear in nature. Closer inspection reveals two characteristics even more disturbing than the downward slope. First, one observes that the non-linear nature of the short-run cost function (the function when K is held constant) is decreasing at an increasing rate. A total cost function concave to the X axis is thereby created. Economic theory would indicate that average cost functions when decreasing should decrease at a decreasing rate. Second, an assumption is violated by the nature of the equation when K is varied. As K increases each succeeding curve in the family of short-run cost curves lies above rather than below the previous curve. The purpose for introducing plant size (K) into the operating cost curve in Chapter III is to allow increasing costs that are associated with increasing utilization. All other operating costs are functions of Q. Consequently, succeeding operating cost curves should be below--not above--the previous member curve.[3] This unacceptable result can originate from either or both of two sources, namely--empirical or theoretical difficulties.

Before one attempts to analyze possible empirical causes he must be convinced that the theoretical framework is functioning properly. Therefore, the theoretical aspects will be considered first. One of the apparent lessons learned from Chapter II and its review of empirical literature is the fact that curves tend to be constrained by the mathematical nature of the function. The generalized functional form developed in Chaper III was an effort to relax this limitation. Since a term has been removed from this general form, however, a new evaluation of the mathematical behavior limitations is in order.

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[3] One should make careful note of this concept so as not to confuse the family of operating cost curves with the more familiar family of total cost curves. The family of total cost curves has incorporated into it the plant cost behaving as a curve shifter. Therefore, part or all of a specific total cost curve may lie above the next smaller plant size. Operating costs, on the other hand, differ only by cost associated with utilization.

When the term Q^2/K is analyzed one realizes that as Q increases the quotient changes at an increasing rate; in fact, this is one of the reasons it was originally chosen for the general functional form of the operating cost equation. When the Q^2/K term, however, has a minus coefficient associated with it in the absence of a Q term, a previously desirable characteristic becomes undesirable (i.e., the mathematical form without a second Q term will not permit a cost function which decreases at a decreasing rate). Likewise, one observes as K increases the quotient becomes smaller; and this negative coefficient causes each succeeding short-run operating cost curve to lie above the previous curve. Consequently, both problems can be explained by the mathematical nature of the function.

Additional insight into the empirical behavior of operating cost functions can also be gained by comparing Equations 5.1 and 5.2. Close inspection reveals that the two equations are very nearly identical with the exception of the a_1 coefficients on the quantity terms. Scrutinizing these two coefficients with their respective variables supplies some informative results. The second term from Equation 5.1 can be subdivided as follows:

$$.0007419 \frac{Q^2}{K} = .0007419\ Q * \frac{Q}{K} .$$

Substituting the mean value for utilization (Q/K) into the term gives

$$.0007419\ Q * .59 = .000438\ Q$$

which is very near the a_1 = .00045 coefficient for Equation 5.2. Since the explanatory power (R^2) of Equation 5.1 and Equation 5.2

are near equal, the collection of information implies that the level of utilization does not affect the operating cost of the industry (i.e., the average value serves equally as well as the actual value). Therefore, one can conclude that in this empirical study operating cost is a function of Q alone exclusive of K. This conclusion is also supported by the fact that when utilization is considered as a separate independent variable it adds no explanatory value to the model.

Reformulation of the Operating Cost Equation

The original form of the operating cost equation was chosen to allow increasing cost resulting from higher levels of plant utilization. Whether or not these increasing costs actually occur is more of an empirical question than a theoretical one. The model simply allows the increasing cost; it does not require it. One would expect this particular characteristic to vary from industry to industry . The data sample indicates that the municipal water industry in 1965 did not exhibit this characteristic. Assuming utilization is not important what adjustments are needed?

Equation 5.2, although theoretically sound, is unnecessarily limited to a linear relationship. Reformulating operating cost Equation 4.8 into Equation 5.5 removes the linear restriction thus allowing a non-linear relationship.

$$AOC = a_0 + c_0T + a_1Q + a_2Q^2 + dH \qquad Q \leq K \qquad (5.5)$$

The short-run operating cost curves generated by Equation 5.5 lie directly on top of each other with each curve terminating at Q=K. The family of short-run total cost curves still exists since the operating cost curves will be shifted up or down by the plant cost. One should note that Equation 5.5 exists in the identical form of Equation 3.4. Two things have changed, however: (1) Each individual curve terminates at plant capacity, and (2) the empirical evidence from the municipal water industry has specified this particular relationship from a more generalized form.

Ground Rules for Empirical Cost Studies

By expanding on the example above one can set up a list of "acceptable behavior characteristics" for testing regression results. In addition, rules designed to adjust the general form so that the mathematics will not prevent "acceptable behavior" seem useful.

Acceptable Behavior Characteristics

1. Average and marginal cost curves must be convex to the X axis. $\frac{d^2Q}{dx^2} \geq 1$

2. Each curve in the family of operating cost curves must lie below or coincident with the preceding curve.

3. The relative rate of change in the slope of the cost curve must remain constant or decline as plant size increases.

Operating Rules for Adjusting the General Form

1. One should substitute Q^2 for the Q^2/K term in the average operating cost equation as is done in Equation 5.5 under the two following conditions: (a) when Q is significant but Q^2/K is not significant, (b) when Q^2/K is significant with a negative coefficient and Q is insignificant. By removing the K the potential effect of increasing cost related to high plant utilization is eliminated.[4]

2. The K should be removed from the operating cost equation only if the regression process has shown the general operating cost Equation 3.18 to be unsatisfactory. This modification is necessary to prevent the mathematical nature of the function from creating a mathematical expression which behaves contrary to accepted economic behavior.

3. A Q^2/K term with a positive coefficient behaves like the general form in all ways except the average operating cost curve and marginal cost curve are positive over the entire range. No theoretical difficulties exist with this form. The Q term should be dropped if it is insignificant.

Using the above tests and rules to assure acceptable behavior, the operating cost data for the municipal water plants can be regressed engaging the form of Equation 5.5.

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[4] During the development of Equation 3.18 the following expressions were considered as alternatives to the Q^2/K term, but each was rejected as having undesirable or limiting properties: Q^3/K, $\sqrt{Q}/K$, QK, Q/K, and $Q/\sqrt{K}$. These are reported here only to prevent duplication of effort. The term Q^2/K^2 may warrant closer inspection as a possible alternative under close monitoring of resulting coefficients.

Digest of Final Regression Equations

Operating Costs Equation

Regressing the operating cost data in the form of Equation 5.5 results in Equation 5.6.

$$AOC = 89.53 - .00178\ Q + .0000000082\ Q^2$$

standard error (.00058) (.00000000332)

t value: df=154 (3.09) (2.47)

$$+ 135.81\ H + 42.33\ Soft + 16.58\ Coag \tag{5.6}$$

(34.40) (15.26) (11.97) $R^2 = .21$

(3.95) (2.77) (1.39) $F = 8.0$

where

H = fraction sold to households

Soft = softening treatment

Coag = coagulation treatment

Total Cost Equation

Adding Equation 5.3 to Equation 5.6 and converting to a total rather than average equation yields Equation 5.7.

$$TC = 89.53\ Q - .00178\ Q^2 + .0000000082\ Q^3 + 135.81\ H*Q$$

$$+ 42.33\ Soft*Q + 16.58\ Coag*Q + 20.02\ K - .0001976\ K^2$$

$$+ .0000000006\ K^3 + 12.82\ Corr*K + 9.08\ Filt*K + 13.96\ Disin*K$$

$$+ 11.21\ Iron*K + 19.91\ H*K \qquad Q \leq K \tag{5.7}$$

where

TC = total cost

Q = quantity produced in million gallons per year

K = plant capacity in million gallons per year

H = fraction of water sold to households

Soft = softening treatment

Coag = coagulation treatment

Corr = corrosion treatment

Filt = filtration treatment

Disin = disinfection treatment

Iron = iron removal treatment

Interpretation of Final Equations

Results of the regression of the average operating cost equation can be interpreted as meaning that throughout the observed municipal water suppliers, a significant number of plants did not exhibit increasing costs resulting from plant capacity limitations. This result is feasible considering the nature of the industry and the available data. The data are annual figures failing to reflect seasonal, daily, and hourly fluctuations. Also the highly varying demand requirements upon a water utility necessitate an average output considerably below the plant capacity; therefore, operating plants near capacity are rarely observed in the industry. Both the long-run average operating cost and long-run average plant cost are U-shaped curves.

Figure 5.1 shows the average cost curve for various plant size. The graph is derived from the information reported in Appendix B. Each curve represents an increase of 10 per cent of the observed range of plant sizes. The smallest plant capacity observed is 438 million gallons per year or 1.2 million gallons per day while the largest plant capacity is one billion gallons per day, an increase of more than 800 times. According to the graph the smaller the plant size the lower the cost will be for a given output. This results since the average plant cost declines as plant size decreases. Plants smaller than 108,537 million gallons of water per year will have decreasing average costs over the relevant range $Q \leq K$ while plants larger than this value will have U-shaped short-run average costs curves.

Decreasing Costs Analysis

The majority of the firms in this data set, however, fall in the declining costs' section. Appendix A reveals an average plant capacity of 17,536 million gallons per year with a standard deviation of 40,436. If the plant population is normally distributed 97.5 per cent of the plants would have capacities less than 98,398 and, therefore, show a declining cost throughout each plant's producing range.

A cost structure of this nature establishes an environment to encourage water consumption. A municipal water supplier capable of

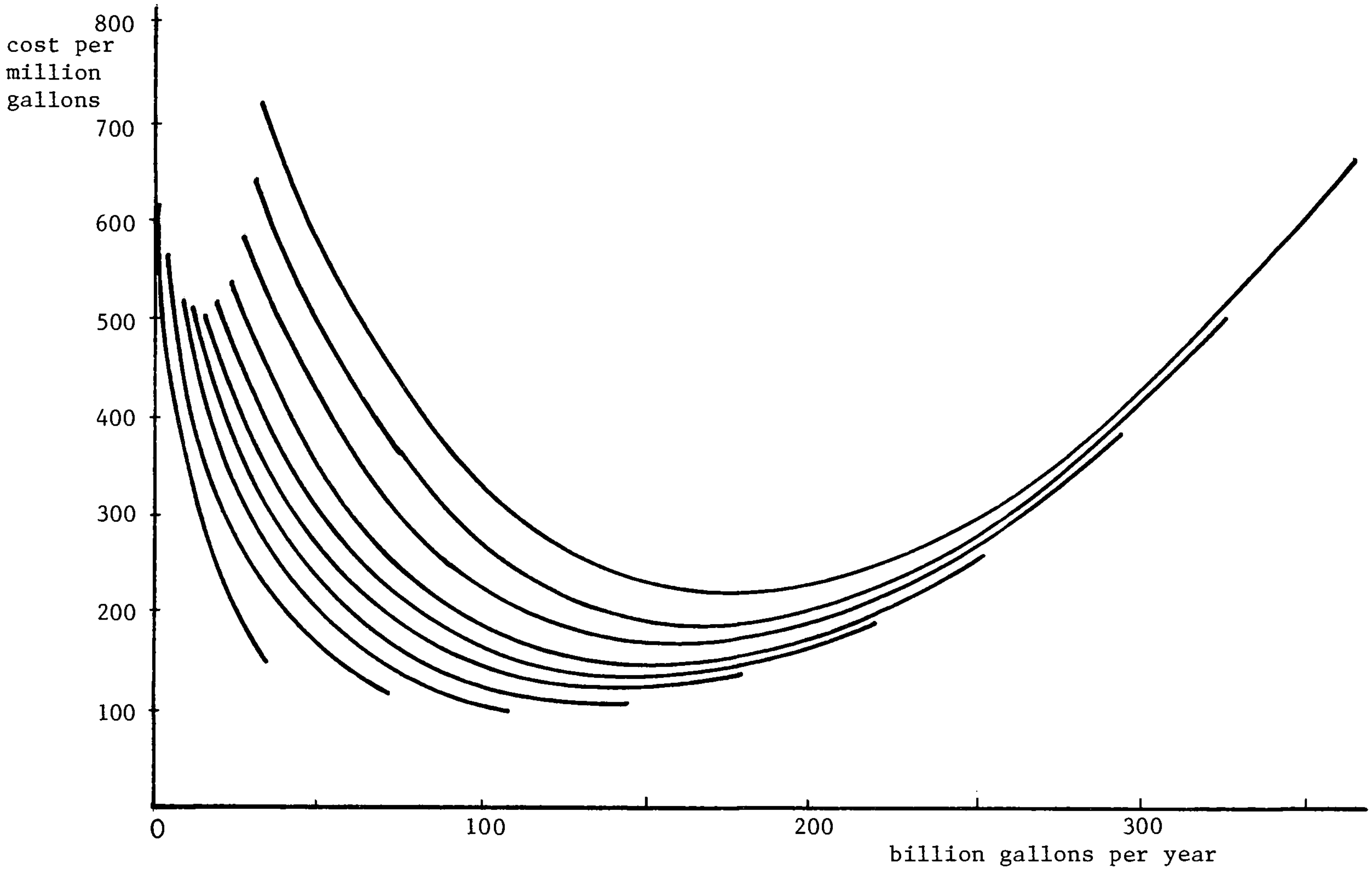

Figure 5.1. Short-run average cost curves of municipal water in the United States 1965.

influencing the demand curve by advertising or adding consumers can supply a larger quantity at a lower per unit cost. This reduction in cost can continue as long as the existing plant is capable of meeting peak demand needs. When the plant is no longer able to meet the needs of peak consumption, consumer pressure builds for a larger plant capable of supplying a larger amount of water at identical per unit costs. Then the cycle repeats itself. Figure 5.2 demonstrates why the process is considerably more explosive with decreasing cost firms. Assuming the equilibrium output of a profit maximizing firm is Q, a change in demand from D_1 to D_2 will result in a new equilibrium output of Q_i for an increasing cost firm and a much larger equilibrium output of Q_d for a decreasing cost firm.

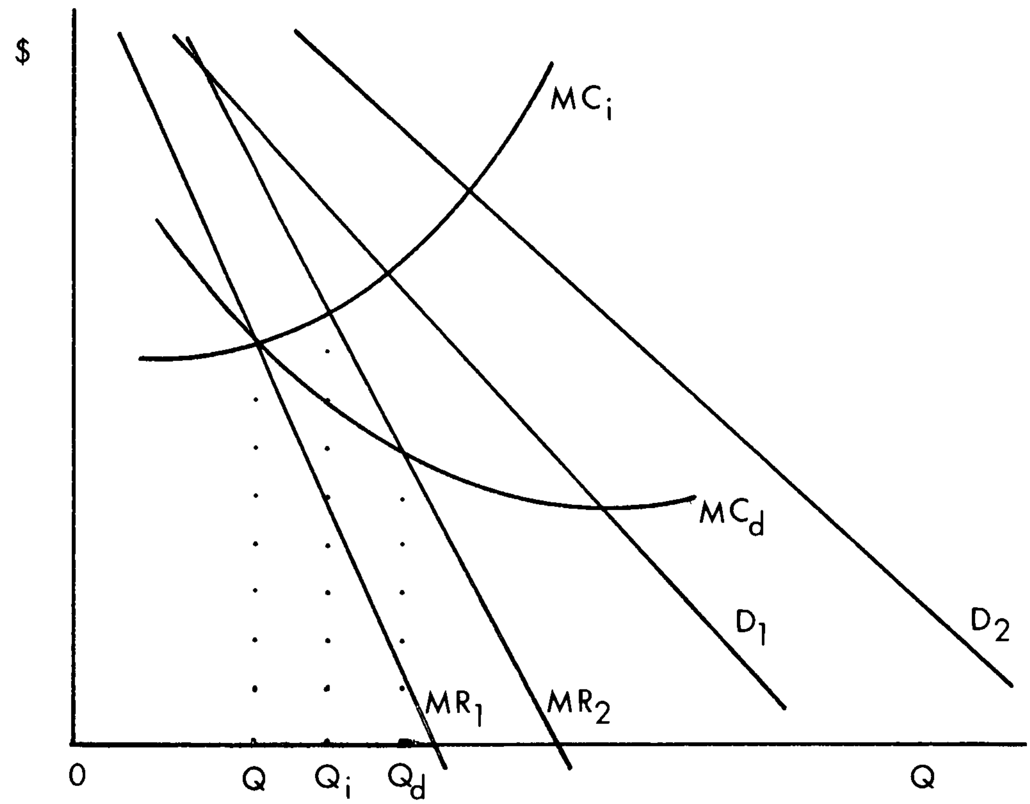

Figure 5.2. Effect of increasing demand.

The universal acceptance of the law of diminishing returns has caused economic theory to concentrate almost exclusively on the increasing cost function or the increasing cost portion of a U-shaped cost function. The increasing region is often called the region of rational economic activity (Heady, 1952, pp. 90-91). Ferguson's approach is typical of the rapid manner in which much economic literature discards the possibility of relevant decreasing marginal and average cost functions and decreasing regions of cost functions. Ferguson in discussing the existence of stage I (that part of a production function where average cost is declining) claims that "the distinction [existence of stage I]. . .is relevant only in theory because observed production relations are always those of stage II." (1969, p. 164). One should be alert to point out that these relationships hold only for pure competition, but this fact is all too often neglected and soon forgotten.

Realizing that declining cost functions do apparently exist in the municipal water industry and are observed frequently in the review of literature, the question of their relative importance and frequency within the cost structure is raised. Declining cost functions, when recognized, have traditionally been treated as an exception to the general rule or as a theoretical exercise in economic possibilities; that is to say, they have seldom been considered as real observable cost phenomena of frequent occurrence. With the function specification as described in the generalized functional form, however, declining cost functions become an acceptable and viable cost pattern. Therefore, the alternative types of declining cost functions and a description of industries which might be expected to exhibit these characteristics are in order.

Declining marginal cost firms can be grouped accordingly: (1) those firms with downward sloping marginal cost curves; (2) those firms with marginal cost curves which decline until the plant reaches 100 per cent of capacity and then becomes vertical. Although any regression equations developed for the industry will appear to have declining marginal costs, the second group does not theoretically exhibit declining costs at market equilibrium.

Group number one can be represented by the cost relationships of MC_d in Figure 5.2. Marginal cost declines throughout the relevant range since the excess capacity maintained by the firm is necessary to meet the varied demands of the market. Public utility industries and other types of service industries would likely exhibit this type relationship. The industry might be characterized by non-storable products or service and/or rapid fluctuations in consumer demands.

A third type of cost pattern, possibly constituting the largest group of declining cost curves, should now be cited--namely, the group of firms which have increasing marginal costs and perhaps average operating cost curves but reach plant capacity before the slope of the average total cost curve becomes positive. These plants may have a high ratio of plant capital to operating capital (i.e., the more plant capital intensive, the more L-shaped the short-run average cost curve).

Significant Concepts Overview

A better understanding of economic principles and their relationship to the mathematical and statistical tools used in empirical studies can be realized by summarizing several of the more important concepts used in this study.

Plant Size

The introduction and use of plant capacity in the model resulted in three direct benefits as well as numerous indirect benefits.

(1) The use of plant capacity established a mechanism for interlacing long- and short-run cost curves in a single mathematical relationship.

(2) A dynamic dimension was introduced into a static framework by specifying the hours of operation and the rate of plant output while in operation. Both of these characteristics were determined to be components in identifying plant utilization. This refinement will allow differentiation of plants even though they have identical outputs.

(3) By interlacing long- and short-run cost curves an improved environment was created whereby scale effects of plant size could be more systematically analyzed.

Restriction of $Q \leq K$

By restricting the output of a plant to a value always less than or equal to the plant capacity, two concepts emerged that have an impact on the relationship between economic theory and empirical findings.

(1) Corner solutions and their theoretical implications become an integral part of cost functions.

(2) Downward sloping average cost curves throughout the relevant range of cost functions become substantially more important.

Utilization

Plant utilization (Q/K) was specified as the sole linkage between plant size and operating cost. All other operating costs then become the products of output rather than plant size.

Plant and Operating Costs

This study "birthed" the concept of plant and operating costs as the component parts of total cost thereby replacing the more traditional fixed and variable costs concept. Unlike their predecessors plant and operating costs retain separate identities for both long- and short-run time periods. In so doing unique characteristics of the resource inputs are maintained.

Envelope of Short-Run Cost Curves

The mathematical form developed and used in this study was devised to allow an envelope of short-run average cost curves while also tracing out the long-run average cost curve. After an extensive review of cost literature was unable to identify a single mathematical expression whose behavior was consistent with the economic theory of cost the model was formulated. In its adopted form as expressed in Equation 3.20:

$$TC = a_oQ + a_1Q^2 + a_2\,\frac{Q^3}{K} + b_0K + b_1K^2 + b_2K^3 \qquad Q \leq K$$

there is a large amount of flexibility available for empirical estimation. This function can be expressed as a single average cost function (Equation 3.21), or it can be subdivided into an average operating cost function (Equation 3.18) and an average plant cost function (Equation 3.22 or 3.23) thereby allowing separate regression estimations whenever possible. The example in this study yielded the better results using separate regression equations; however, separate equations may not always be the better approach since they do not allow for interaction between plant costs and operating costs.

Multicollinearity

Because of the large probability of a high degree of correlation between output (Q) and plant size (K), empirical regression studies may encounter a problem with multicollinearity. By forming the total cost equation into an average cost equation (Equation 3.21) before regressing the data this empirical problem was avoided.

Summary Remarks

Review of Cost Curve Criteria

In the introduction to this study criteria were listed deemed necessary to qualify a mathematical expression as a generalized functional form for cost equations. A testing of the model in Chapter III validated the following criteria:

1. The function must incorporate both long- and short-run cost curves into a single framework.

The formulation does in fact meet this requirement as shown in Equation 3.20. Even when operating cost and plant costs are separated for regression purposes as in Equations 3.18, 3.22, and 3.23, they are formulated from the same framework. A simple addition of the two equations, after regression, yields a total cost equation for both the long- and short-run costs.

2. The mathematical form must be able to generate an envelope of U-shaped short-run average cost curves.

3. The equation must not restrict the curve forms to U-shapes.

The full cubic equation permits an envelope of U-shaped curves. When one or more of the independent variables are shown to be insignificant by the statistical testing, the U-shaped and/or the envelope of curves may disappear.

4. The function must be able to accommodate a range of several magnitudes in plant size.

In the municipal water example the function accommodated a plant size increase in magnitude of more than 800 times.

5. The formulation must be able to demonstrate economies- and/or diseconomies-to-scale.

The permitted cubic form of Equations 3.22 and 3.23 can accommodate economies- and/or diseconomies-to-scale. By maintaining the separate identity of plant cost these scale effects can be more easily distinguished.

6. The mathematical form must be adaptable to least squares linear regression techniques.

The form chosen is intrinsically linear in nature and, therefore, can utilize linear regression techniques. Average cost forms of the functions can be used to minimize problems of multicollinearity. Other independent variables can be added to the basic output and plant size variables without violating least squares linear regression criteria.

7. The framework must be conducive to hypothesis formulation and hypothesis testing.

Hypotheses can be formulated as to shape, magnitude, economies- or diseconomies-to-scale, and importance of independent variables on both the long-run and short-run cost of a firm or an industry. Standard statistical criteria can then be utilized in an effort to reject these hypotheses.

All the pre-determined criteria for a generalized cost equation, therefore, are met by the mathematical forms specified in Equations 3.20 and 3.21.

Results from Testing Hypotheses

Two hypotheses were proposed for this study:

HO: A generalized functional form is not available from cost literature which meets the above criteria for a generalized cost equation.

Hypothesis number one could not be rejected. The author, in a thorough review of the available cost literature, was unable to identify a mathematical formulation which met the criteria.

HO: A functional framework can not be developed which will meet the pre-specified criteria for a generalized cost equation.

Hypothesis number two is rejected. The foregoing study did, in fact, develop a framework which meets the criteria as was demonstrated in the previous section. The purpose of this study--the testing of these two stated hypotheses--has therefore been realized.

Reiteration of the Problem

The original problem as designated for this study was the specification of theoretically sound statistical estimation procedures for interrelating long- and short-run cost functions.

The author, believing that this problem has been properly addressed, feels that the cost framework developed is theoretically and empirically consistent with accepted economic and statistical principles. He is also of the opinion that the model is general enough to permit its adoption in a wide variety of cases and restrictive enough to avoid some of the common problems encountered in model building. Hopefully, this study has contributed to the understanding of cost relationships and their behavior thereby "bringing the human mind in closer contact with actuality." (Georgescu-Roegen, 1972, p. 279)

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APPENDICES

APPENDIX A

AVERAGE VALUES FROM 159 OBSERVATIONS OF CITIES OF 10,000 POPULATION OR GREATER IN 1965

Property	Mean	Standard deviation
Percentage of treatment:		
coagulation	46.6	
filtration	54.7	
taste and odor control	41.5	
disinfection	82.4	
softening	17.6	
corrosion control	38.3	
iron removal	22.0	
fluoridation	35.9	
Other percentages:		
ground water	40.7	
surface water	50.5	
purchased water	8.8	
privately owned utility	28.3	
amount sold to households	43.8	16.4
utilization	58.9	13.4
Costs:		
average operating	$151.31	76.01
average plant (TPC/Q)	84.94	38.67
average	236.25	99.66
average plant capacity (TPC/K)	50.29	26.35
original book value	25,065,026	60,160,461
replacement (1965)	35,392,799	
Other factors:		
age of plant (years)	12.61	5.90
yearly production (mil.gal/yr)	10,417	24,092
plant capacity (mil.gal/yr)	17,526	40,436
population	151,318	

COSTS OF PRODUCING MUNICIPAL WATER IN THE UNITED STATES 1965

K	Q	TOC	AOC	TPC	APC	TC	AC
500.	50.	8204.	164.08	26251.	525.01	34455.	689.09
500.	100.	16399.	163.99	26251.	262.51	42650.	426.50
500.	150.	24585.	163.90	26251.	175.00	50836.	338.91
500.	200.	32763.	163.81	26251.	131.25	59014.	295.07
500.	250.	40931.	163.73	26251.	105.00	67182.	268.73
500.	300.	49091.	163.64	26251.	87.50	75342.	251.14
500.	350.	57242.	163.55	26251.	75.00	83492.	238.55
500.	400.	65384.	163.46	26251.	65.63	91634.	229.09
500.	450.	73517.	163.37	26251.	58.33	99767.	221.71
500.	500.	81641.	163.28	26251.	52.50	107892.	215.78
36950.	3695.	582719.	157.70	1704054.	461.18	2286773.	618.88
36950.	7390.	1119316.	151.46	1704054.	230.59	2823370.	382.05
36950.	11085.	1612272.	145.45	1704054.	153.73	3316326.	299.17
36950.	14780.	2064069.	139.65	1704054.	115.29	3768123.	254.95
36950.	18475.	2477190.	134.08	1704054.	92.24	4181244.	226.32
36950.	22170.	2854116.	128.74	1704054.	76.86	4558170.	205.60
36950.	25865.	3197330.	123.62	1704054.	65.80	4901384.	189.50
36950.	29560.	3509313.	118.72	1704054.	57.65	5213367.	176.37
36950.	33255.	3792547.	114.04	1704054.	51.24	5496601.	165.29
36950.	36950.	4049515.	109.59	1704054.	46.12	5753569.	155.71
73400.	7340.	1112351.	151.55	3033527.	413.29	4145878.	564.83
73400.	14680.	2052362.	139.81	3033527.	206.64	5085889.	346.45
73400.	22020.	2839488.	128.95	3033527.	137.76	5873015.	266.71
73400.	29360.	3493184.	118.98	3033527.	103.32	6526711.	222.30
73400.	36700.	4032907.	109.89	3033527.	82.66	7066434.	192.55
73400.	44040.	4478113.	101.68	3033527.	68.88	7511640.	170.56
73400.	51380.	4848258.	94.36	3033527.	59.04	7881785.	153.40
73400.	58720.	5162798.	87.92	3033527.	51.66	8196325.	139.58
73400.	66060.	5441188.	82.37	3033527.	45.92	8474715.	128.29
73400.	73400.	5702885.	77.70	3033527.	41.33	8736412.	119.02
109850.	10985.	1599463.	145.61	4189004.	381.34	5788467.	526.94
109850.	21970.	2834599.	129.02	4189004.	190.67	7023603.	319.69
109850.	32955.	3770564.	114.42	4189004.	127.11	7959568.	241.53
109850.	43940.	4472596.	101.79	4189004.	95.33	8661600.	197.12
109850.	54925.	5005913.	91.14	4189004.	76.27	9194917.	167.41
109850.	65910.	5435731.	82.47	4189004.	63.56	9624735.	146.03
109850.	76895.	5827270.	75.78	4189004.	54.48	10016274.	130.26
109850.	87880.	6245748.	71.07	4189004.	47.67	10434752.	118.74
109850.	98865.	6756380.	68.34	4189004.	42.37	10945384.	110.71
109850.	109850.	7424305.	67.59	4189004.	38.13	11613309.	105.72

K	Q	TOC	AOC	TPC	APC	TC	AC
146300.	14630.	2046498.	139.88	5344827.	365.33	7391325.	505.22
146300.	29260.	3485088.	119.11	5344827.	182.67	8829915.	301.77
146300.	43890.	4469833.	101.84	5344827.	121.78	9814660.	223.62
146300.	58520.	5154795.	88.09	5344827.	91.33	10499622.	179.42
146300.	73150.	5694035.	77.84	5344827.	73.07	11038862.	150.91
146300.	87780.	6241621.	71.11	5344827.	60.89	11586448.	131.99
146300.	102410.	6951613.	67.66	5344827.	52.19	12296440.	120.07
146300.	117040.	7978073.	68.17	5344827.	45.67	13322900.	113.83
146300.	131670.	9475062.	71.96	5344827.	40.59	14819889.	112.55
146300.	146300.	11596677.	79.27	5344827.	36.53	16941504.	115.80
182750.	18275.	2455777.	134.38	6675336.	365.27	9131113.	499.65
182750.	36550.	4022890.	110.07	6675336.	182.64	10698226.	292.70
182750.	54825.	5001624.	91.23	6675336.	121.76	11676960.	212.99
182750.	73100.	5692267.	77.87	6675336.	91.32	12367603.	169.19
182750.	91375.	6395106.	69.99	6675336.	73.05	13070442.	143.04
182750.	109650.	7410431.	67.58	6675336.	60.88	14085767.	128.46
182750.	127925.	9038527.	70.65	6675336.	52.18	15713863.	122.84
182750.	146200.	11579708.	79.20	6675336.	45.66	18255040.	124.86
182750.	164475.	15334204.	93.23	6675336.	40.59	22009536.	133.82
182750.	182750.	20602304.	112.73	6675336.	36.53	27277632.	149.26
219200.	21920.	2829705.	129.09	8354865.	381.15	11184570.	510.24
219200.	43840.	4467065.	101.89	8354865.	190.58	12821930.	292.47
219200.	65760.	5430268.	82.58	8354865.	127.05	13785133.	209.63
219200.	87680.	6237501.	71.14	8354865.	95.29	14592366.	166.43
219200.	109600.	7406955.	67.58	8354865.	76.23	15761820.	143.81
219200.	131520.	9456806.	71.90	8354865.	63.53	17611664.	135.43
219200.	153440.	12905281.	84.11	8354865.	54.45	21260140.	138.56
219200.	175360.	18270512.	104.19	8354865.	47.64	26625376.	151.83
219200.	197280.	26070688.	132.15	8354865.	42.35	34425552.	174.50
219200.	219200.	36824064.	167.99	8354865.	38.12	45178928.	206.11
255650.	25565.	3170662.	124.02	10557760.	412.98	13728422.	537.00
255650.	51130.	4836675.	94.60	10557760.	206.49	15394435.	301.08
255650.	76695.	5820099.	75.89	10557760.	137.66	16377859.	213.55
255650.	102260.	6942989.	67.90	10557760.	103.24	17500736.	171.14
255650.	127825.	9027406.	70.62	10557760.	82.60	19585152.	153.22
255650.	153390.	12895411.	84.07	10557760.	68.83	23453108.	152.90
255650.	178955.	19369040.	108.23	10557760.	59.00	29926800.	167.23
255650.	204520.	29270384.	143.12	10557760.	51.62	39828144.	194.74
255650.	230085.	43421472.	188.72	10557760.	45.89	53979232.	234.61
255650.	255650.	62644480.	245.04	10557760.	41.30	73202240.	286.34

K	Q	TOC	AOC	TPC	APC	TC	AC
292100.	29210.	3481032.	119.17	13458367.	460.75	16939392.	579.92
292100.	58420.	5150782.	88.17	13458367.	230.37	18609136.	318.54
292100.	87630.	6235446.	71.16	13458367.	153.58	19693808.	224.74
292100.	116840.	7961217.	68.14	13458367.	115.19	21419584.	183.32
292100.	146050.	11554298.	79.11	13458367.	92.15	25012656.	171.26
292100.	175260.	18240896.	104.08	13458367.	76.79	31699248.	180.87
292100.	204470.	29247104.	143.04	13458367.	65.82	42705456.	208.86
292100.	233680.	45799344.	195.99	13458367.	57.59	59257696.	253.58
292100.	262890.	69123632.	262.94	13458367.	51.19	82581984.	314.13
292100.	292100.	100446096.	343.68	13458367.	46.07	113904448.	389.95
328550.	32855.	3763198.	114.54	17231008.	524.46	20994192.	639.00
328550.	65710.	5420447.	82.61	17231008.	262.23	22659440.	344.84
328550.	98565.	6740640.	68.39	17231008.	174.82	23971648.	243.21
328550.	131420.	9444679.	71.87	17231008.	131.11	26675660.	202.98
328550.	164275.	15285463.	93.05	17231008.	104.89	32516464.	197.94
328550.	197130.	26007840.	131.93	17231008.	87.41	43238848.	219.34
328550.	229985.	43356784.	188.52	17231008.	74.92	60587792.	263.44
328550.	262840.	69077248.	262.81	17231008.	65.56	86308256.	328.37
328550.	295695.	104913888.	354.80	17231008.	58.27	122144896.	413.08
328550.	328550.	152611792.	464.50	17231008.	52.45	169842800.	516.95
365000.	36500.	4019541.	110.12	22050016.	604.11	26069552.	714.23
365000.	73000.	5688727.	77.93	22050016.	302.05	27738736.	379.98
365000.	109500.	7400013.	67.58	22050016.	201.37	29450016.	268.95
365000.	146000.	11545871.	79.08	22050016.	151.03	33595872.	230.11
365000.	182500.	20518688.	112.43	22050016.	120.82	42568704.	233.25
365000.	219000.	36711008.	167.63	22050016.	100.68	58761024.	268.32
365000.	255500.	62515280.	244.68	22050016.	86.30	84565296.	330.98
365000.	292000.	100323872.	343.57	22050016.	75.51	122373888.	419.09
365000.	328500.	152529296.	464.32	22050016.	67.12	174579312.	531.44
365000.	365000.	221524064.	606.92	22050016.	60.41	243574080.	667.33