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The vertical and radial components of a seismic P wave can be decomposed by a Fourier transform into two sets of nonterminating sinusoidal waves with one set for each component. The tangent of the vertical transform divided by the radial transform gives by definition the apparent angle of emergence for that frequency. The actual angle of emergence can be calculated from the apparent angle. The change of the angle with frequency can be obtained by determining the angle over the entire frequency spectrum of the pulse.

The angle of emergence is only defined for a pure pulse. Just the short length of uncontaminated signal can be used to calculate the angle if the signal is interfered with by other signals. The actual angle of emergence was calculated as a function of frequency for stations near nuclear explosions. In all cases, the angle varied with frequency.

ANGLE OF EMERGENCE OF SEISMIC P WAVES AND ITS VARIATION WITH FREQUENCY

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ANGLE OF EMERGENCE OF SEISMIC P WAVES AND ITS VARIATION WITH FREQUENCY

I. INTRODUCTION

When an incident plane P wave of the form $A \cos(2\pi ft)$ strikes a free flat boundary of a semi infinite medium, in general, both a plane P wave $(B \cos[(2\pi ft) + \phi])$ and a plane SV wave $(C \cos[(2\pi ft) + \psi])$ are reflected. The direction determined by the outward horizontal component of the wave velocity will be called the radial direction. The ratio of particle motion, either displacement, velocity or acceleration, in the upward vertical direction (V) at a point on the free boundary to the radial motion (R) of the same type and at the same point on the free boundary gives a real number between zero and infinity.

The arc tangent of this ratio will be an angle between zero and ninety degrees, which is called the apparent angle of emergence $(\bar{e})$,

$$\bar{e} = \tan^{-1} (V/R). \tag{1}$$

If the vertical and radial particle motions are corrected for reflections from the free boundary occurring at the recording station, then the arc tangent of this corrected ratio is the actual angle of emergence. The actual angle (e) can be calculated from the apparent angle if the P and S wave velocities, α and β respectively, are known (Bullen, 1963, p. 130),

$$2 \cos^2 e = \frac{\alpha^2}{\beta^2} (1 - \sin \bar{e}). \tag{2}$$

Sometimes the incident angles $(\bar{i}, i)$ are used in place of the emergence angles.

Since the ratio of the particle motion is defined for a single frequency f, the angle determined by this ratio is for the same frequency. In order to find the angle for a different frequency, it would be necessary to use a plane wave source of that frequency.

Statement of the Problem

In the case just discussed, it is generally assumed that the angle is a constant for all frequencies. However, in a layered system there is both theoretical and experimental evidence that the angle may not be independent of frequency, even for the plane wave case. The purpose of this paper is to compute the actual angle of emergence for P waves recorded at seismological stations within thirty kilometers of a nuclear explosion, and determine the variations in the angle with changing frequency.

Previous Work

The simplest and most often used method for determining the angle of emergence of a transient pulse, as observed by Galitzin (Richter, 1958, p. 633) is to divide the peak vertical amplitude of a half cycle by the peak radial amplitude of the same half cycle. This method will obviously work for the single frequency case.

The slope of a hodograph (ground particle motion diagram) can be used to determine the angle of emergence. The vertical amplitude is given the y value on an x, y graph. The radial amplitude is assigned the x value, and the points (x, y) are plotted for successive increments of time. All points from the beginning of the first arrival to the beginning of the second arrival should, in theory, lie on a straight line. The angle this line makes with the x axis is the apparent angle of emergence. Many hodographs are not straight lines, making the apparent angle of emergence hard to to define.

The variation of the emergence angle with frequency or equivalently changes in the vertical to radial component amplitude ratios with frequency have been suspected for a long time. The Japanese seismologist, Suzuki (1932), generalized the peak amplitude method to include frequency dependence by calculating the angle for each pair of half cycles and assigning a frequency to each angle. The assigned frequencies are not frequencies in the mathematical sense,

$$F(t + 1/f) = F(t), \tag{7}$$

unless the pulse is periodic. Even if the pulse is periodic, the assigned frequencies may not be the frequencies of a sinusoidal function, $\sin(2\pi ft)$. Nuttli (1964) used a method similar to the one of Suzuki.

Most American seismologists have reported their results in terms of numerical ratios of the vertical and radial components instead of emergence angles. The investigators used relatively long period waves and large sections of the records. Some of the research involved finding the thickness of the crust.

These investigations of the crust began with the problem of using the seismic parameters of the crust or those of the mantle to calculate the reflection coefficients for long period waves reflected from the surface. The question was: "Could long period waves 'see' the crust?" Early research was done by Mie (1943) and continued by Byerly, Mie and Romney (1949). Nuttli and Whitman (1961) also worked on this problem.

A method that gives the exact transmission coefficient of a plane P wave through a layered system was developed by Thomson (1950). This method allows for frequency dependence in the angle of emergence. Haskell (1962) used this method with his improvements to calculate the ratio of the vertical and radial components at the surface as a function of frequency for a theoretical crust. Nuttli (1964) presented theoretical studies in terms of a frequency dependent incidence angle and compared these values with his observations of frequency dependent angles. Fernandez (1965) gives his theoretical and experimental results in terms of dimensionless parameters although these ratios imply a frequency dependent angle.

The Russian research in this field is more oriented toward exploration geophysics at shallow depths. Their results are usually given as an angle instead of a ratio. Gamburtsev (1954) proposed a seismic frequency sounding method; that is, using the frequency dependence of the angle of emergence to determine the geologic structure. Practical applications were attempted by Savarensky (1952) and Ivanova (1959 and 1960). Ivanova (1960) changed the instrument constants to pass different frequencies. He obtained angles for signals that had been filtered at different frequencies. Part of his research was at very shallow depths (2. 5 to 10. 5 meters) and at short distances (up to 35 meters from the shot).

II. METHOD OF ANALYSIS

The P wave from a nuclear explosion is a transient instead of being a steady state phenomenon. The transient nature of the pulse is actually quite useful. The angle of emergence can be found for a wide band of frequencies by taking the Fourier transform of the transient pulse. The response of the recording instruments, the seismometers, must be taken into account in order to obtain the correct angle of emergence.

Angles as Applied to More Complex Problems

Within thirty kilometers of a nuclear explosion, the P waves are not precisely plane; the terrain is usually not flat, and the subsurface geology is sometimes quite complex. These factors complicate studies of the emergence angle. The arc tangent of the ratio between the recorded vertical and radial components for the wave must be between zero and ninety degrees. Therefore, the apparent angle of emergence carries directly over into this more general case.

The actual angle of emergence will not exist if the apparent angle is less than a given number depending on α and β, the P and S wave velocities respectively. When

$$\alpha^2(1 - \sin \bar{e})/2\beta^2 > 1, \tag{3}$$

then

$$\cos (e) > 1, \tag{4}$$

and no real angle has a cosine greater than one. An imaginary angle,

$$\cos (ie) = \cosh (e) > 1 \tag{5}$$

would work, but could not be interpreted physically. That is, values for the apparent angle of emergence less than a certain minimum number are impossible in the theoretical case. For

$$\alpha^2/\beta^2 = 3 \tag{6}$$

or equivalently for a Poisson's ratio equal to 0. 25, this impossible result occurs when $\bar{e}$ is less than 19. 5 degrees. If the calculated apparent angles were random and evenly distributed, then over one-fifth of the actual angles would be impossible.

It was thought that the actual angle would be of more interest than the apparent angles because it takes the reflections into account, and can only exist for certain amplitude ratios. The actual angle of emergence is one of many ways of presenting the results of the analysis for this more complex problem. Some of the other forms of presentation were discussed in Chapter I.

A Poisson's ratio of 0. 25 was used for all stations rather than choosing a different ratio for each station. The ratio was probably not constant for all stations, but this fact was not considered in the calculations of the emergence angles as the exact values of Poisson's ratio were not known. However, low values of the emergence angle are extremely sensitive to small changes in Poisson's ratio. The minimum angle occurs in the zero to thirty kilometer epicentral distance range; so there can be a considerable difference between the actual and apparent angle because of the dependence on Poisson's ratio. All data from seismic stations for the explosions were treated in the same manner for computing the angle of emergence, and can therefore be directly compared with each other. It was thought that it would be better for comparison purposes to use the same approximation for all stations than to arbitrarily approximate each station separately.

Fourier Transform

In this research, the P pulse was transformed into the frequency domain by a Fourier transform, and the moduli of the transformed pulse were used to compute the angle of emergence as a function of frequency. The Fourier integral theorem states that most functions, $F(t)$, arising from physical problems can be represented as follows:

$$F(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(t') \exp\left[2\pi i f(t-t')\right] dt'\, df \quad . \qquad (8)$$

Substituting

$$g(f) = \int_{-\infty}^{\infty} F(t') \exp(-2\pi i f t')\, dt' \tag{9}$$

and t for t' leads to the Fourier transform pair:

$$g(f) = \int_{-\infty}^{\infty} F(t) \exp(-2\pi i f t)\, dt , \tag{10}$$

$$F(t) = \int_{-\infty}^{\infty} g(f) \exp(2\pi i f t)\, df . \tag{11}$$

The formula for $g(f)$ is called the transform, and that for $F(t)$ is called the inverse transform. The moduli obtained by this process are for sinusoidal waves going from minus infinity to plus infinity as required by the definition of the angle of emergence.

Since $F(t)$ is real and

$$\exp(-2\pi i f t) = \cos(2\pi f t) - i \sin(2\pi f t) , \tag{12}$$

$g(f)$ can be divided into a real part,

$$A(f) = \int_{-\infty}^{\infty} F(t) \cos(2\pi f t)\, dt , \tag{13}$$

and an imaginary part,

$$B(f) = -\int_{-\infty}^{\infty} F(t) \sin(2\pi f t)\, dt . \tag{14}$$

In this paper, the modulus,

$$G(f) = [A(f) + B(f)]^{1/2} , \tag{15}$$

and the phase,

$$P(f) = \cot^{-1} [A(f) / B(f)] , \quad (16)$$

were used. Only the modulus is needed to calculate the angle of emergence.

The integral over the infinite range can be replaced by an integral with a finite range,

$$\int_0^T F(t) \exp(-2\pi i f t) \, dt ,$$

where T is the length of the pulse because $F(t)$ is zero outside of this interval. The integral can be approximated with a sum by using some numerical integration procedure. In this case, the pulse amplitudes were digitized at $N+1$ equally spaced points with the time interval,

$$\Delta t = T/N , \quad (17)$$

between them, and the pulse was approximated by rectangles. The final formulas are

$$A(f) = \frac{1}{2\pi f} \{ \sum_{n=1}^{N} [(F[(n-1)\Delta t] - F[(n+1)\Delta t]) \sin(2\pi f n \Delta t)] \} + \frac{1}{2\pi f} \{F[(N-1)\Delta t] + F(N\Delta t)\} \sin(2Nf\pi \Delta t) \quad (18)$$

and

$$B(f) = \frac{1}{2\pi f} \{ \sum_{n=1}^{N} [(F[(n-1)\Delta t] - F[(n+1)\Delta t]) \cos(2\pi f n \Delta t)] \} - \frac{1}{2\pi f} \{F(0) + F(\Delta t) - (F[(N-1)\Delta t] + F(\Delta t)) \cos(2\pi f N \Delta t)\} . \quad (19)$$

Instrument Response

The modulus and phase of the transform are for the seismometer record. The true ground motion is distorted by the measuring instruments, in this case, the seismometers. The emergence angle is determined by the true ground motion so the seismograms must be corrected for the response of the seismometers. The seismometers at the stations used in this research had the response of an ideal pendulum, see Richter (1958).

Problem of Later Arrivals

Most disturbances in the earth which serve as sources for seismic waves are quite complex. Nuclear explosions do not seem to be any exception (Berg, Trembly and Laun, 1964). The earth modifies the signal by filtering, creates new signals through reflections, and even changes some of the P energy into SV energy. The seismogram is composed of many different signals.

The noise from the later arrivals creates an extremely severe problem in determining the emergence angle. Either the effect of later arrivals must be subtracted from the first signal, or only the time interval between the first and second signal can be used in the analysis. The exact arrival time and the exact shape of the later signals must be known if they are to be subtracted from the seismogram to give the first signal.

Using only the uncontaminated portion of the first signal means that the amplitude of the signal is arbitrarily set equal to zero for all time greater than the arrival time of the second signal. The seismogram is multiplied by a rectangular pulse of unit height going from minus infinity to the arrival time of the second signal. This process is called chopping or truncation, and is used in many fields. The effect that truncation in the time domain has on the frequency domain can be precisely calculated for a known signal. Unfortunately, most signals are truncated because they are unknown.

The convolution theorem gives the relation between the frequency spectrum of the signal and the frequency spectrum of the truncated signal. If $S(f)$ is the spectrum of the signal and $W(f)$ is the spectrum of the function, sometimes called a window, multiplying the signal, then the spectrum of the product is

$$\int W(s)\ S(f-s)\ ds$$

where s is a dummy variable of integration. The rectangular pulse is the window in the case of chopping. The difference between the spectrum of the signal and the spectrum of the truncated signal is a function of frequency, and may be quite small for some frequencies, depending on how and where the signal was truncated. The truncation is sometimes said to create or destroy certain frequencies. This statement is true, but it is better to look upon the truncation in the time domain as affecting all frequencies as clearly illustrated by the convolution theorem.

Much of the theory on truncating signals is stated in terms of power spectra where the windows are used to minimize the effect of the truncation on the power spectrum. The rectangular pulse is not a good window for this purpose, but it was used because part of the research was to see how the angle of emergence changed when part of the second arrival was included with the signal. The amplitude in the time domain of a good window is much smaller at the end of the time interval than in the middle of the time interval. When the seismogram is multiplied by this type of window, the amplitude of the last part of the time interval is small. Therefore, using a good window would hide the effect of the second arrival on the emergence angle because the last part of the time interval would not have much effect on the frequency spectrum.

The effect of truncation on the modulus of the zero frequency is quite clear. Its modulus is equal to the area under the truncated signal, which can vary over a fairly wide range as the length of the window is changed. This example shows that there may be very little relation between the shapes of the frequency-modulus curves for the signal and the truncated signal.

The problem is to see what effect the truncation has on the apparent angle of emergence when both components are truncated in the same way after the same length of time. The components can be either the true ground motion or the seismograms from identically matched instruments. It turns out that the truncation has absolutely no effect for a constant angle. The Fourier transform of the vertical component is related to the transform of the radial component as follows:

$$V(f) = R(f) \tan \bar{e} \, . \tag{20}$$

The convolutions, $V(f)*W(f)$ and $R(f)*W(f)$, of these transformed components with the same window are

$$V(f)*W(f) = \int W(f-s)\, R(s) \tan \bar{e} \, ds \tag{21}$$

and

$$R(f)*W(f) = \int W(f-s)\, R(s) \, ds \, . \tag{22}$$

Tan $\bar{e}$ can be taken out of the integral because it is not a function of frequency. The quotient of the two truncated components then becomes

$$\frac{V(f)*W(f)}{R(f)*W(f)} = \frac{\tan \bar{e} \int W(f-s)\, R(s)\, ds}{\int W(f-s)\, R(s)\, ds} = \tan \bar{e} \, . \tag{23}$$

The arc tangent of this ratio is exactly the apparent angle as defined for the original components in Equation (1). The actual angle depends on the apparent angle; so it, too, is exactly the same for both cases.

Since the truncation does not affect a constant angle, it would seem that the effect of truncation on an angle that had a small variation with frequency, say a few degrees, should be small. In this case,

$$V(f) = R(f) \tan \bar{e}(f) \tag{24}$$

where $\bar{e}(f)$ is the angle of emergence as a function of frequency. The convolutions are now:

$$V(f)*W(f) = \int W(f-s)\, R(s) \tan \bar{e}(s)\, ds \tag{25}$$

and

$$R(f)*W(f) = \int W(f-s)\, R(s)\, ds\,. \tag{26}$$

Now $\tan \bar{e}(f)$ is a function of frequency, and cannot be directly taken out of the integral. The quotient of the components is:

$$\frac{V(f)*W(f)}{R(f)*W(f)} \doteq \frac{\int W(f-s)\, R(s) \tan e(s)\, ds}{\int W(f-s)\, R(s)\, ds}\,. \tag{27}$$

The most important result of this discussion is that if the angle determined by the truncated components, either true ground motion or the records of matched instruments, is a function of frequency, then the angle for the first signal must be a function of frequency. That is, frequency dependence cannot be introduced by the truncation or by the response of matched recording instruments.

Summary of Calculating Procedure

The seismometer records were truncated to eliminate the later arrivals. These truncated records were transformed into the frequency domain by a Fourier transform. The moduli of the transformed seismometer records were corrected for the response of the seismometers. These corrected moduli were used to calculate the apparent angle of emergence as a function of frequency using Formula (1). This function was put into Formula (2) with α^2/β^2 equal to 3; that is, Poisson's ratio was 0.25, and the actual angle of emergence was computed as a function of frequency.

Comparison with Previous Work

The method of analysis used in this paper has several advantages over methods in the time domain. It has a better theoretical basis, since the formulas used to calculate the angle of emergence are for the frequency domain. The peak amplitude methods can only give one angle or one angle at one frequency per half cycle. The first arrival for stations near the source covers a very short interval of time and may not contain even one uncontaminated half cycle. Only the first few half cycles can be used in any case. Therefore, computing large numbers of angles to get a frequency dependent curve would require the records from many stations. Much scatter could be introduced by geologic conditions under these stations. The method used in this research gives the angle of the entire frequency spectrum by using a short length of the records from one station as input.

The previous American experimental results involving transform methods used long period waves (1 to 100 seconds) to obtain information about the crust. The crust was considered to be a plane layer on top of a semi-infinite half space. Deep distant earthquakes were used as sources. Thus, the waves were approximately plane, and a large part of the first arrivals could be analyzed. In most cases, multiple reflections from the crust-mantle boundary were included as part of the first arrival. The data were usually analyzed as ratios of the components. The theoretical results were obtained for plane waves, plane boundaries and multiple reflections.

Short period waves (0.05 to 2 seconds) were used for the calculations of this paper. The waves had some curvature. The layers were not flat, and the source was in the layers. The length of the uncontaminated pulse was very short, about 0.1 seconds. The purpose of this research was to calculate the angle of emergence for one arrival at the surface.

The depths and distances considered in this paper are between those of Ivanova (1960) and the American research on the thickness of the crust. The results are presented as an angle as was done in the Russian articles and Nuttli (1964). The analysis in this paper used transform methods similar to, but not as elaborate as those of Fernandez (1965). The Russian method of tuning the instruments or filtering can be made equivalent to transforming into the frequency domain.

Most paper considered the frequency dependence to be an effect of the layering and/or the free surface. A frequency dependent emergence angle does not have to imply a different ray path for each frequency. The velocity ratio α^2/β^2 in Equation (2) could be frequency dependent in the general case. The velocities could be considered as averages over depth or time instead of true velocities at a point. This comes back to the question of what velocity waves of different periods can "see." As far as the writer knows, results have never been presented in terms of this velocity ratio, although all practical applications involve finding velocities at depth.

III. SOURCES OF THE DATA

The yields and locations of the nuclear explosions used for this research are given in this section. Twenty-nine seismograms for various types of strong motion seismographs, whose setups are described below, were used in this research. The seismograms and calibration constants for these seismometers were supplied by the United States Coast and Geodetic Survey.

Data from four underground nuclear explosions were analyzed to check and improve the method of analysis. The first three of these explosions was at the Atomic Energy Commission's Nevada Test Site while Shoal was detonated in the Stillwater mountain range of central Nevada. The yield and geologic setting of the explosions were quite different for the various explosions. Table I contains the pertinent information about the explosions. It was condensed from VESIAC (1965).

Table I. General information about the nuclear explosions.

Event	Yield	Medium	Date
Antler	2.4 ± 0.25	tuff	September 15, 1961
Hardhat	5.9 ± 0.5	granite	February 15, 1962
Haymaker	56 ± 8	alluvium	June 27, 1962
Shoal	12 ± 2	granite	October 26, 1962

Six instruments, both a displacement meter and an accelerometer to record motions in each of the radial, transverse (horizontal and perpendicular to the radial) and the vertical directions, were generally used at each station. The instrument constants for the vertical and radial instruments used in this paper are given in Tables II, III and IV. The instrument constants for the vertical and radial accelerometers were about equal; that is, the instruments were matched.

The periods of the radial displacement meters were usually about twice as long as the periods of the vertical instruments for Shoal. The displacement meter constants were about equal for the other explosions, but only one component of each displacement meter group recorded the signal; so the records could not be used to calculate the angle of emergence. Therefore, emergence angles were computed from the records of ground displacement for Shoal only. The displacement meter constants for the other explosions are not given. All the displacement meters used for Shoal had a damping ratio of ten.

The recording stations for Shoal were laid out on four lines that diverged from the shotpoint. These lines will be referred to as the northwest, northeast, southwest or southeast line according to the direction they move from the shotpoint. There were not enough stations available for the other explosions to warrant

Table II. Displacement meter constants for Shoal.

Station Number	Vertical Period	Radial Period	Station Number	Vertical Period	Radial Period
2	1.60	4.04	10	2.10	4.00
3	1.40	4.05	12	1.70	3.90
4	4.04	3.97	13	2.60	3.91
5	1.31	3.82	18	1.60	3.82
6	2.01	4.02	19	1.46	4.10
7	1.60	4.01	21	1.85	3.50
8	1.80	3.76	25	4.10	4.02
9	2.60	3.91			

Table III. Accelerometer constants for Shoal.

Station Number	Vertical Period	Radial Period	Vertical Damping Ratio	Radial Damping Ratio
2	0.01435	0.014	10	10
3	0.01585	0.01719	9	5
4	0.0292	0.0306	9	10
5	0.0207	0.0203	12	11
6	0.0305	0.0303	10	11
7	0.0276	0.0284	12	10
8	0.0165	0.0204	7.6	10
9	0.01985	0.0179	6	8
10	0.04	0.0365	9	7
11	0.0356	0.0356	10	6
12	0.0361	0.0366	12	10
13	0.0382	0.0362	9	7
14	0.056	0.0571	8	9
15	0.0489	0.0518	10	10
16	0.052	0.0539	10	9
19	0.0576	0.06	10	8
21	0.0937	0.1035	10	11

Table IV. Accelerometer constants for the Antler, Hardhat and Haymaker events.

Event	Station Number	Vertical Period	Radial Period	Vertical Damping Ratio	Radial Damping Ratio
Antler	1	0.0795	0.0783	10	12
Antler	2	0.1140	0.1154	8	12
Hardhat	2	0.0295	0.0294	8	8
Hardhat	3	0.0357	0.0358	8	11
Hardhat	4	0.1595	0.1508	7	10
Hardhat	5	0.1487	0.1479	9	14
Haymaker	1	0.0278	0.0280	7	6
Haymaker	2	0.0392	0.0387	12	8
Haymaker	3	0.0557	0.0596	8	9
Haymaker	4	0.0834	0.0810	11	12

classifying them by their direction from the shot, and in most cases they were not on lines.

The vertical and radial accelerometer records from two stations were used to obtain the actual angles of emergence for Antler. Four sets of accelerometer records from four stations were analyzed for Hardhat. Four accelerometer stations were also used to calculate the angles of emergence for Haymaker. The angles of emergence were computed from both accelerometer and displacement meter seismograms for thirteen stations near Shoal. Two other stations with displacement meters only and four stations with accelerometers only were used for Shoal.

IV. PRESENTATION OF RESULTS

This section discusses in detail the emergence angles obtained using the data described in Chapter III. The locations of the stations and the types of instruments at each station are also included. The results of the analysis are presented in graphs for a few pertinent cases, in tables for all cases, and are discussed in the text. Since the description of the results is lengthy, it will be given separately for each explosion. All the cited distances from the shot are epicentral distances (the horizontal distance from surface zero to the station).

The following information is included in the tables:

(a) Type of instruments whose records were used to calculate the angle;

(b) Number of digitized points and their corresponding total time interval which were used to compute the Fourier transform;

(c) Angle obtained for the first half cycle by the peak of the half cycles method of analysis;

(d) Numerical values of all relative extrema of the frequency dependent angle and the frequencies they were attained at;

(e) Variation in the angle; that is, the absolute maximum angle minus the absolute minimum angle for the same station;

(f) Largest absolute value of the difference between angles at the same frequency as given by two pairs of accelerometers at the same location;

(h) Largest absolute value of the difference between angles at the same frequency as given by a pair of displacement meters and a pair of accelerometers at the same location.

Shoal

The data from Shoal offered many more ways of testing the method of analysis than the data from the other explosions. There were seismograms from nineteen stations available for Shoal, and both the displacement meters and the accelerometers recorded the signal at thirteen of these stations. Also two stations were put at exactly the same place in three cases. An angle could be calculated from the displacement meter and accelerometer records at each station, giving a total of four angles to be compared with each other. The results of the analysis for Shoal are summarized in Tables V, VI, VII and VIII.

Station two was the closest station to the shotpoint of Shoal that was used in this research. It was on the northeast line at a

Table V. Pertinent information and results for stations on the northeast line for Shoal.

Station Number	Distance in Kilometers	Type of Instruments	Number of Digitized Points	Total Time Interval	Angle (e) from Peaks of 1/2 Cycle	Minimum Angles (e)	Maximum Angles (e)	Frequency at which Attained	Variation of e	Largest Δ e from Accel.	Largest Δ e from Displ.
2	0.952	accel.	12	0.0600	63.4	62.0		0.5	3.1		
							65.1	20			
3	1.468	accel.	13	0.0598	50.5	53.7		0.5	1.5	2.6	1.9
							55.2	15.5			
						53.7		20			
4	1.468	accel.	13	0.0630	56.6	56.3		0.5	1.0	2.6	2.9
							57.3	14			
						54.9		20			
4	1.468	displ.	14	0.0679	36.7		55.8	0.5	2.1		
						53.7		20			
10	3.092	accel.	13	0.0699	52.5	42.6		0.5	2.7		
							44.6	17			
						41.9		20			
15	4.913	accel.	11	0.0700	32.7		36.4	0.5	11.2		
						25.2		20			
19	7.146	accel.	13	0.0746	75.6	73.7		0.5	5.4		
							79.1	17.5			
						75.7		20			
21	12.810	accel.	9	0.0436	81.2	76.9		0.5	2.6		
							77.0	4			
						75.8		12			
							78.4	20			

distance of 0.952 kilometers from the shot. The accelerometer records for this station give an angle of emergence that increases from 61.0 degrees at 0.5 cps to 65.1 degrees at 20 cps.

The angle calculated from the displacement meter seismograms for this station has a value of 45.6 degrees at 0.5 cps. The emergence angle then jumps down to 27.2 degrees at 1 cps and slowly increases to 31.0 degrees at 20 cps. The instrument constants for the accelerometers were nearly the same, while those of the displacement meters were quite different, as can be seen in Tables II and III. Apparently the method of analysis will only work when the recording instruments are matched. All the following angles are based on calculations made using records from instruments whose constants were about equal.

At station number three, the accelerometer sensitivity given for the vertical accelerometer was 2.34 cm/g ($g = 980\ cm/sec^2$), while the given value for the radial seismometer was 0.873 cm/g. The angle calculated by using these sensitivities is extremely low, about eight degrees. The magnifications of the accelerometers were also given. They were 115.2 for the vertical and 119.1 for the radial. The sensitivities (S) were compared to the magnifications (M) using the formula,

$$S = 980\ M\ T^2/4\pi^2 \tag{28}$$

where T is the period of the instrument. It was found that the vertical instrument's magnification had to be 385 or its sensitivity had to be 0.718. All the accelerometers had magnifications of about 115; so the sensitivity of 0.718 was used. An angle that varied from 53.7 degrees at 0.5 cps to a maximum of 55.2 degrees at 15.5 cps and then decreased to 53.7 degrees at 20 cps was obtained by using this corrected sensitivity. The periods for the displacement meters at station three were not about equal, and the emergence angle computed from these displacement records displays the same phenomenon as the angle calculated from displacement meter records of station two (p. 27).

Station <u>four</u> was at exactly the same location as station <u>three</u>, on the <u>northeast</u> line 1.468 km from the shot. Therefore, the angles determined from the seismograms for this pair of stations should agree. The graphs are given in Figure 1. The extent of the disagreement indicates the noise level and the accuracy of the method of analysis. The angle calculated from the accelerometer records using a total time interval of 0.0630 seconds (13 digitized points) changed from 56.3 degrees at 0.5 cps to a maximum of 57.3 degrees at 14 cps and then decreased to 54.9 degrees at 20 cps. The total time interval for the accelerometers of station three was 0.0598 seconds, and this interval also contained 13 points.

The numerical values of the angle computed from the

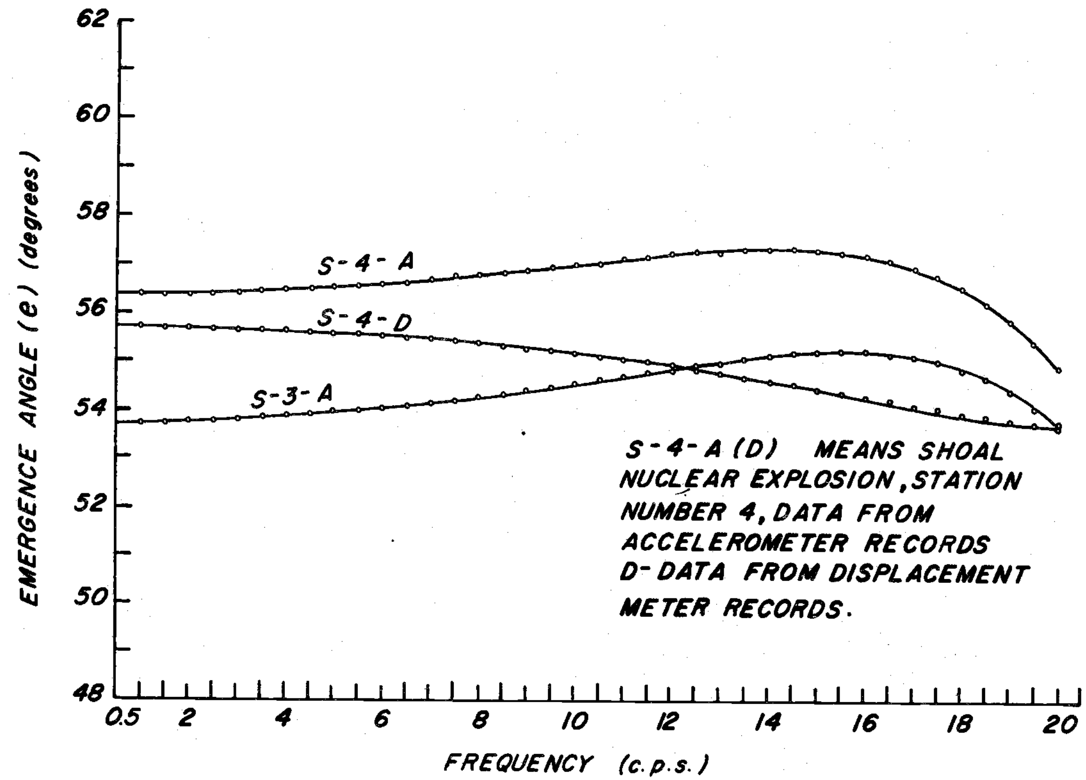

Figure 1. Actual angle of emergence as a function of frequency at stations three and four--Shoal.

displacement meter records at station four with a total time interval of 0. 0679 seconds (14 points) are quite close to the accelerometer values. However, the frequency dependence is different. The angle of emergence determined by the displacement meters decreases over the entire frequency range, going from 55. 8 degrees at 0. 5 cps to 53. 7 degrees at 20 cps. Part of this discrepancy could be due to the use of different time intervals.

The second pair of stations put at exactly the same location were numbers five and six, at a distance of 1. 483 km on the northwest line from the shot. The angle was calculated from the first 13 points on the accelerometer seismograms for both of these stations. The time intervals were 0. 0656 seconds for station five and 0. 0614 seconds for station six. An angle was also computed using the first 13 points (0. 0614 seconds) of the displacement meter records at station six. Figure 2 shows the results of all these calculations.

The accelerometers at station five gave an angle of 67. 7 degrees at 0. 5 cps. The angle increased with frequency and reached 77. 8 degrees at 20 cps. The results from the accelerometers of station six are similar. The angle increased from 66. 0 degrees at 0. 5 cps to 73. 5 degrees at 20 cps. The displacement meter records for station six indicated an angle of 71. 7 degrees at 0. 5 cps. The angle then jumped down to 67. 3 degrees at 1 cps. This effect could

Table VI. Pertinent information and results for stations on the northwest line for Shoal.

Station Number	Distance in Kilometers	Type of Instruments	Number of Digitized Points	Total Time Interval	Angle (e) from Peaks of 1/2 Cycle	Minimum Angles (e)	Maximum Angles (e)	Frequency at which Attained	Variation of e	Largest Δ e from Accel.	Largest Δ e from Displ.
5	1.483	accel.	13	0.0656	71.2	67.7		0.5	10.1	4.3	6.8
							77.8	20			
6	1.483	accel.	13	0.0614	65.0	66.0		0.5	7.5	4.3	2.5
							73.5	20			
6	1.483	displ.	13	0.0614	52.6	67.3		1	3.7		
							71.0	20			
13	4.72	accel.	11	0.0598	81.7	80.9		0.5	2.1		1.3
							83.0	20			
13	4.72	displ.	13	0.0707	78.8	80.7		0.5	2.0		1.3
							82.7	18			
						82.4		20			
18	6.921	displ.	13	0.1036	72.0		75.2	1			
						72.3		17.5	2.9		
							73.0	20			
25	25.428	displ.	13	0.1547	73.8	85.9		0.5	11.3		
							87.7	7			
						77.4		10.5			
							85.8	15.5			
						83.3		18.5			
							85.0	20			

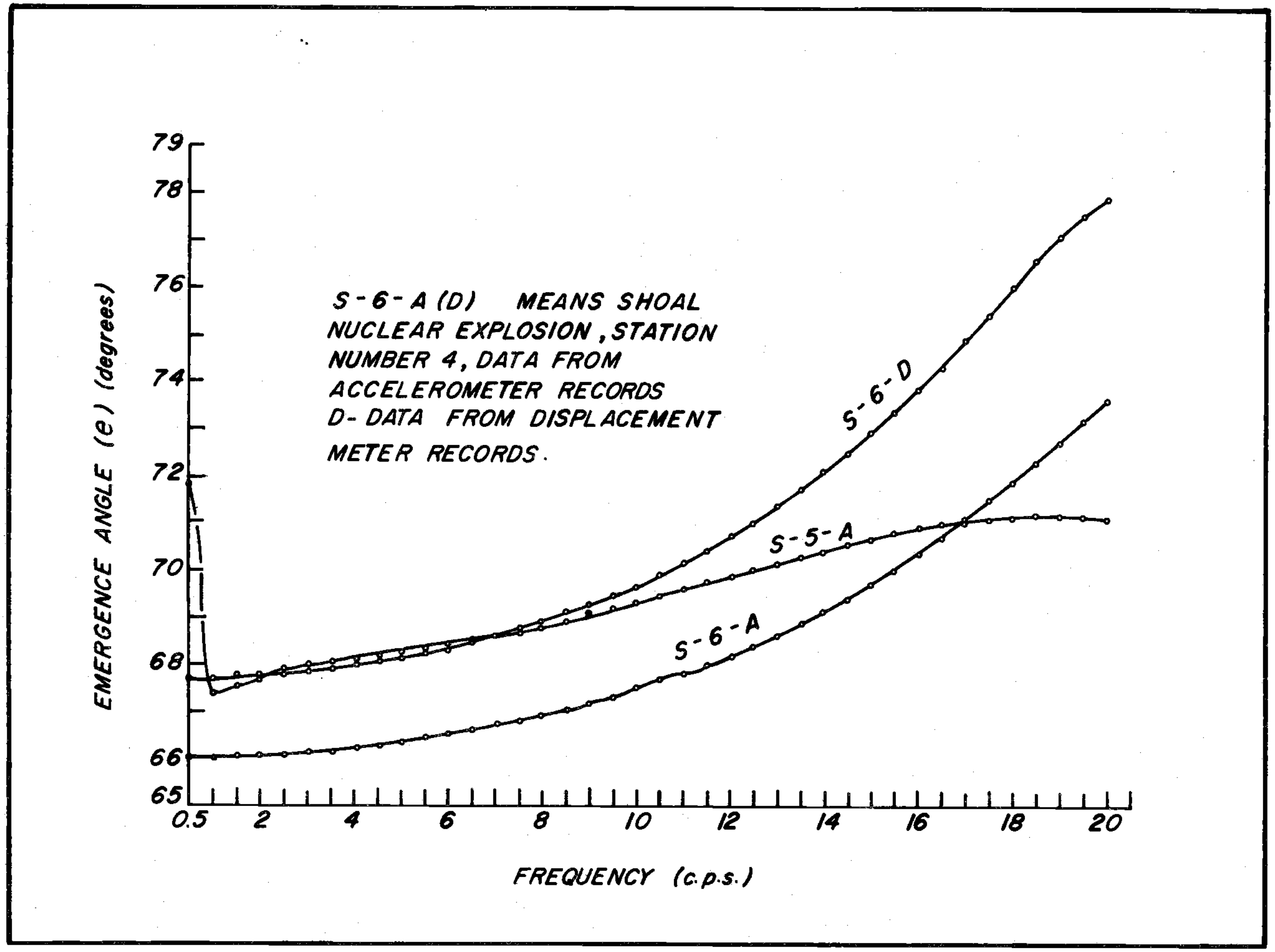

Figure 2. Actual angle of emergence as a function of frequency at stations five and six--Shoal.

be due to the slight mismatch of the seismometers. There was a steady increase in the angle as a function of frequency up to 17 cps, where a value of 71.0 degrees was reached. The angle was about constant from 17 cps to 20 cps.

Using the accelerometer values, the variation (with frequency) in the angle is 10.2 degrees for station five and 7.5 degrees for station six. However, the maximum difference between the angles at the same frequency as computed from the two accelerometers is only 4.3 degrees. The angle at both stations increases, and this increase is about twice the maximum difference between the angles at the same frequency. These facts indicate that this increase is real and is not an effect of noise.

Shoal was detonated in a granitic intrusion that outcropped on the surface. Some of the surface recording stations were on the granitic intrusion while others were on alluvium or metamorphic rock outside of the granite. The forementioned stations were just outside of the granite, where complex geologic structure could be expected. The complex geology might account for the large variation in the angle as a function of frequency.

Stations seven and eight, 1.733 km from the shot on the southwest line, were the last pair of stations put at the same place. The actual angle of emergence was computed using nine digitized points of the accelerometer records for both stations and ten points of the

Table VII. Pertinent information and results for stations on the southwest line for Shoal.

Station Number	Distance in Kilometers	Type of Instruments	Number of Digitized Points	Total Time Interval	Angle (e) from Peaks of 1/2 Cycle	Minimum Angles (e)	Maximum Angles (e)	Frequency at which Attained	Variation of e	Largest Δ e from Accel.	Largest Δ e from Displ.
7	1.733	accel.	9	0.0425	63.5	65.0		0.5	0.2	1.8	2.8
							65.2	16.5			
8	1.733	accel.	9	0.0448	63.6	63.2		0.5	1.2	1.8	1.1
							64.4	20			
8	1.733	displ.	10	0.0498	48.5	62.1		1	1.7		
							62.8	20			
12	3.375	accel.	8	0.0430	30.6	34.3		0.5	0.6		
							34.9	16.5			
						34.7		20			
16	5.691	accel.	9	0.0447	52.4	60.7		0.5	0.8		
							61.5	14			

Table VIII. Pertinent information and results for stations on the southeast line for Shoal.

Station Number	Distance in Kilometers	Type of Instruments	Number of Digitized Points	Total Time Interval	Angle (e) from Peaks of 1/2 Cycle	Minimum Angles (e)	Maximum Angles (e)	Frequency at which Attained	Variation of e
9	1.751	accel.	14	0.0664	65.7	62.9		0.5	9.9
							72.8	20	
11	3.372	accel.	9	0.0447	68.1	65.0		0.5	6.0
							71.0	20	
14	4.872	accel.	11	0.0503	64.1	55.9		0.5	9.1
							65.0	20	

displacement meter records for station eight. The points correspond to times of 0. 0448 and 0. 0498 seconds for the accelerometers and displacement meters of station eight and to 0. 0425 seconds for the accelerometers of station seven. The results are presented in Figure 3.

The accelerometers of station seven indicated an angle that increased from 65. 0 degrees at 0. 5 cps to a maximum of 65. 2 degrees at 16. 5 cps and then decreased to 65. 1 degrees at 20 cps. The angle calculated from the accelerometers of station eight increased from 63. 2 degrees at 0. 5 cps to 64. 4 degrees at 20 cps. The displacement meters for this station gave an angle that increased from 62. 1 degrees at 1 cps to 62. 8 degrees at 20 cps, except for the usual jump between 0. 5 and 1 cps.

The first station on the southeast line was station nine, 1. 751 km away from the shot. The angle computed from the accelerometer records increased from 62. 9 degrees at 0. 5 cps to 72. 8 degrees at 20 cps with most of the increase occurring at the higher frequencies.

The angle at station ten, 3. 092 km from the shot on the northeast line, was unusually small. It was 42. 6 degrees at 0. 5 cps, then attained a maximum of 44. 6 degrees at 17 cps and finally decreased to 41. 9 degrees at 20 cps. The variation with frequency was less than three degrees. Station eleven was on the southeast line, 3. 372 km from the shot. There was an increase of about six degrees, from

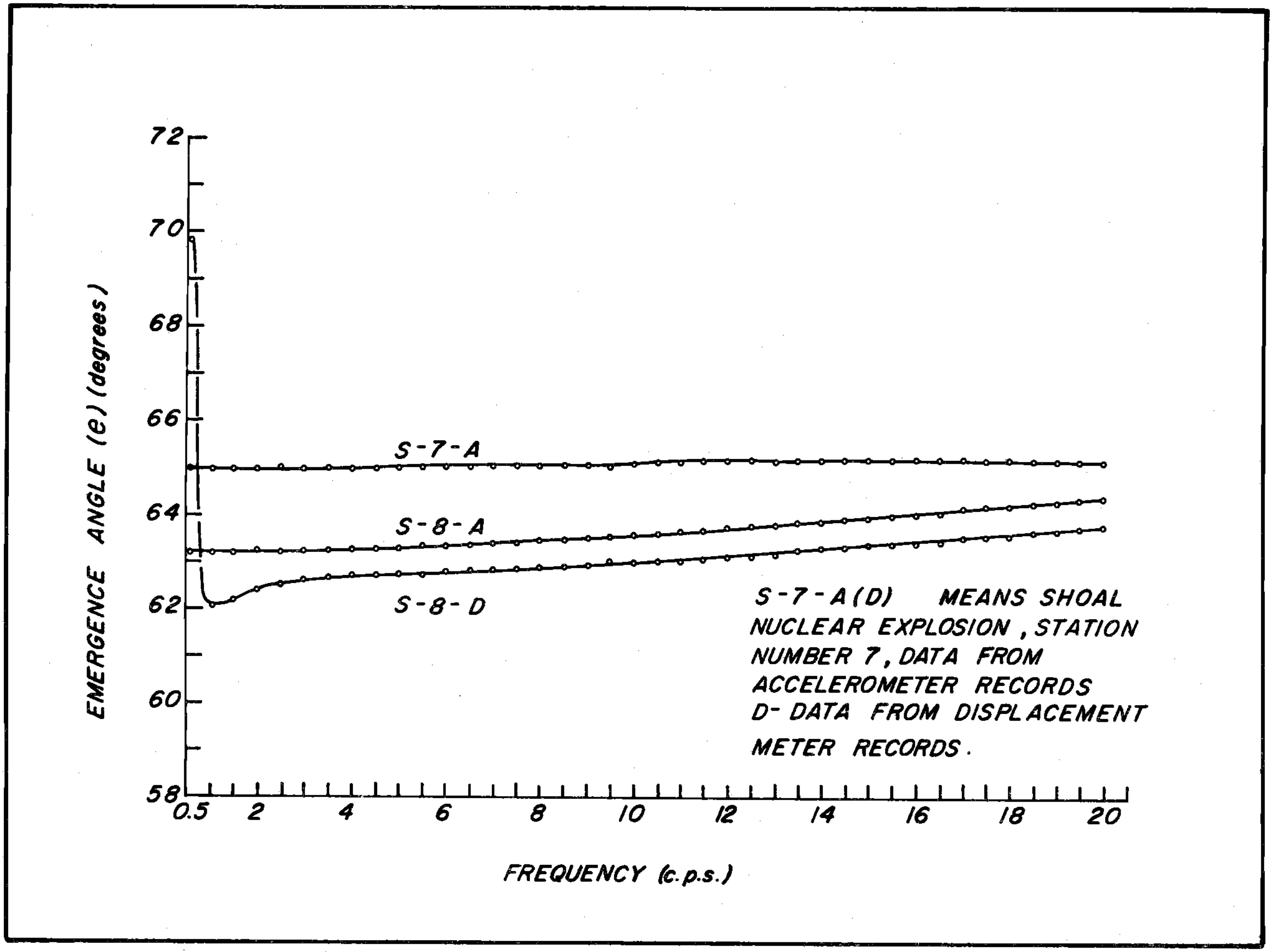

Figure 3. Actual angle of emergence as a function of frequency at stations seven and eight--Shoal.

65. 0 degrees at 0. 5 cps to 71. 0 degrees at 20 cps, for this station.

The results at station twelve, a distance of 3. 375 km away from the shot on the southwest line, are similar to the results at station ten. The numerical values of the angle were lower, and the change with frequency was smaller than those for station ten. The angle changed from 34. 3 degrees at 0. 5 cps to a maximum of 34. 9 degrees at 16. 5 cps and then decreased to 34. 7 degrees at 20 cps.

An extremely large angle of emergence was obtained at station thirteen, on the northwest line 4. 72 km from the shot. When computed from the transforms of 0. 0598 seconds (11 points) of the accelerometer seismograms, the angle of emergence increased from 80. 9 degrees at 0. 5 cps to 83. 0 degrees at 20 cps. The transforms of 0. 0707 seconds (13 points) on the displacement meter records gave an angle of 80. 7 degrees at 0. 5 cps, a maximum of 82. 7 degrees at 18 cps and 82. 4 degrees at 20 cps, in good agreement with the accelerometer values.

The angle at station fourteen, 4. 872 km southeast of the shot, increased from 55. 9 degrees at 0. 5 cps to 65. 0 degrees at 20 cps. Another usually small emergence angle was obtained at station fifteen, 4. 913 km away from the shot on the northeast line. The angle was about constant at 36. 4 degrees for frequencies between 0. 5 cps and 8 cps; then it decreased to 25. 2 degrees at 20 cps.

A small change from 60. 7 degrees at 0. 5 cps to a maximum of 61. 5 degrees at 14 cps, and then a decrease to 60. 8 degrees at 20 cps was noted for station sixteen on the southwest line 5. 691 km from the shot. Station nineteen, 7. 146 km from the shot in the northeast direction, gave an angle that increased from 73. 7 degrees at 0. 5 cps to 79. 1 degrees at 17. 5 cps and then decreased to 75. 7 degrees at 20 cps.

The results from the accelerometer seismograms at station twenty-one, the last station on the northeast line, 12. 810 km away from the shot, were more complex than the results at the nearer stations. The angle first showed a small increase from 76. 9 degrees at 0. 5 cps to 77. 0 degrees at 4 cps. A decrease to 75. 8 degrees at 12 cps was next noted. Then an increase to 78. 9 degrees at 20 cps was observed.

No accelerometer seismograms were available for stations eighteen and twenty-five on the northwest line; so displacement meter records were used. A decrease in the angle of emergence from 75. 2 degrees at 1 cps to a minimum of 72. 3 degrees at 17. 5 cps and then an increase to 73. 0 degrees at 20 cps was observed for station eighteen, 6. 921 km from the shot. The angle at station twenty-five, 25. 428 km from the shot, had a value of 85. 9 degrees at 0. 5 cps. It had maxima of 87. 7 degrees at 7 cps, 85. 5 degrees at 15. 5 cps and 85. 0 degrees at 20 cps. There were minima of 77. 4 degrees at 10. 5 cps and 83. 3 degrees at 18. 5 cps. The variation with frequency of over ten degrees in the angle of emergence is quite large when the size of the angle and the fact that it is bounded by ninety degrees are considered.

After obtaining a reasonable correlation between the angle of emergence and the distance from the shot, and getting fair reproducibility for the angle as a function of frequency, it was decided to use the same method of analysis on the other nuclear explosions to see if there was anything particular about Shoal or if the method of analysis was general. Only accelerometer records were available for these explosions.

Antler

The first station of the Antler event, 1.299 km from the shot, gave an angle that decreased from 37.2 degrees at 0.5 cps to a minimum of 37.0 degrees at 7.5 cps; then increased to a maximum of 38.6 degrees at 17 cps, and finally decreased to 37.8 degrees at 20 cps. An angle of 52.1 degrees at 0.5 cps was computed from the accelerometer records at Antler station two, 1.92 km from the shotpoint. A steady decrease with increasing frequency to a value of 34.9 degrees at 20 cps was noted for this station. The frequency dependence of the angle at station two does not compare very well with that of station one. There were only two stations for this explosion; so it is hard to say if this discrepancy is real, or if it is caused by the method of analysis. Table IX summarizes the information about Antler.

Table IX. Pertinent information and results for the Antler event.

Station Number	Distance in Kilometers	Number of Digital Points	Total Time Interval	Angle (e) from Peaks of 1/2 Cycle	Minimum Angles (e)	Maximum Angles (e)	Frequency at which Attained	Variation of e
1	1.299	14	0.0630	32.5		37.2	0.5	1.6
					37.0		7.5	
						38.6	17	
					37.8		20	
2	1.92	14	0.0644	45.5	52.1		0.5	17.2
						34.9	20	

Hardhat

Hardhat station two, 1.06 km from the shot, showed a small increase in the angle of emergence for increasing frequencies. The angle changed from 62.3 degrees at 0.5 cps to 64.0 degrees at 20 cps. An extremely small variation with frequency was found for Hardhat station three, 1.26 km from the source. The angle decreased from 34.0 degrees at 0.5 cps to a minimum of 33.7 degrees at 17 cps and then increased to 33.8 degrees at 20 cps. The angle at station four, 3.399 km from the shot, was 68.0 degrees at 0.5 cps and 71.2 degrees at 20 cps with a minimum of 66.6 degrees at 5 cps and a maximum of 71.6 degrees at 17 cps.

The last station for Hardhat, number five, 6.311 km away from the shot, indicated an abnormally large angle. It was 83.6 degrees at 0.5 cps; then it decreased to a minimum of 83.1 degrees at 6 cps. Finally it increased to 86.0 degrees at 20 cps. This station was on a thin layer of low velocity alluvium underlain by high velocity granite; so high angles would be expected. The change in the angle for increasing distance from the source is reasonable for Hardhat. Large angles should be obtained for stations near to the source. A minimum angle should be reached as the distance from the shot is increased; then an increase with increasing distance should be observed. This is exactly what happened. Table X gives pertinent information about Hardhat.

Table X. Pertinent information and results for the Hardhat event.

Station Number	Distance in Kilometers	Number of Digital Points	Total Time Interval	Angle (e) from Peaks of 1/2 Cycle	Minimum Angles (e)	Maximum Angles (e)	Frequency at which Attained	Variation of e
1	1.06	7	0.0420	63.9	62.3		0.5	1.7
						64.0	20	
3	1.26	10	0.0454	34.8		34.0	0.5	.3
					33.7		17	
						33.8	20	
4	3.399	10	0.0834	62.0		68.0	0.5	5.0
					66.6		5	
						71.6	17	
					71.2		20	
5	6.311	10	0.0432	85.5		83.6	0.5	2.9
					83.1		6	
						86.0	20	

Haymaker

Some remarkably consistent results were obtained from the four accelerometer stations of the Haymaker event (see Table XI and Figure 4). The low frequency values of the angle are fairly small for all four stations. The numerical values at 0. 5 cps are: 38. 0 degrees for station one, 47. 6 degrees for station two, 47. 3 degrees for station three and 48. 5 degrees for station four. The angle at all four stations increased with frequency to a maximum value. These maxima were: 61. 4 degrees at 12. 5 cps, 62. 4 degrees at 11. 5 cps, 64. 5 degrees at 12. 5 cps and 49. 5 degrees at 10 cps for stations one through four respectively. The size of the maximum and the frequency at which it was obtained are in very good agreement for the first three stations. The variation of the angle with frequency is considerably smaller at the fourth station than at the other three stations, and the absolute maximum value was obtained at the high end of the frequency range instead of in the middle of the range like the other stations.

The angle then decreased to a relative minimum for the first two stations and an absolute minimum for the last two stations. These minima were: 43. 2 degrees at 19 cps, 48. 7 degrees at 17 cps, 47. 3 degrees at 19. 5 cps and 46. 3 degrees at 15 cps in that order. Again, the frequency that the minimum occurs at is different for station four than for the other stations. An increase with frequency to 20 cps was finally noted where values of 44. 1 degrees for station one, 53. 2 degrees for station two, 42. 7 degrees for station three, and 50. 3 degrees for station four were obtained.

Table XI. Pertinent information and results for the Haymaker event.

Station Number	Distance in Kilometers	Number of Digital Points	Total Time Interval	Angle (e) from Peaks of 1/2 Cycle	Minimum Angles (e)	Maximum Angles (e)	Frequency at which Attained	Variation of e
1	1. 810	10	0. 0891	64. 9	38. 0		0. 5	23. 4
						61. 4	12. 5	
					43. 2		19	
						44. 1	20	
2	2. 709	10	0. 1008	73. 5	47. 6		0. 5	14. 8
						62. 4	11. 5	
					48. 7		17	
						53. 2	20	
3	3. 816	10	0. 0909	59. 9	47. 3		0. 5	21. 8
						64. 5	12. 5	
					42. 7		19. 5	
						42. 7	20	
4	5. 268	10	0. 0891	57. 6	48. 5		0. 5	4. 0
						49. 5	10	
					46. 3		15	
						50. 3	20	

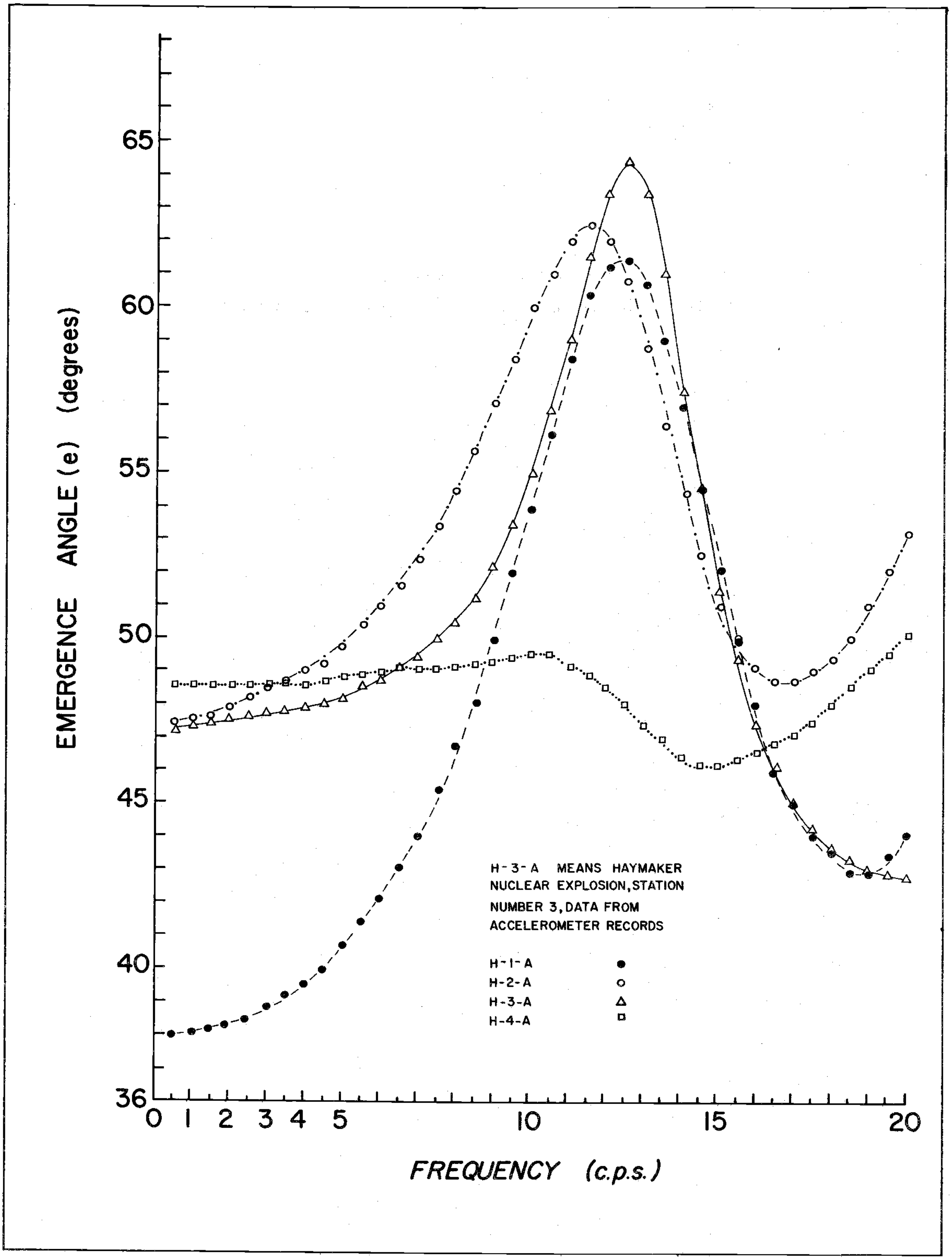

Figure 4. Actual angles of emergence as a function of frequency for stations of Haymaker.

V. DISCUSSION OF RESULTS

The numerical values of the emergence angle and the variations in the angle with changing frequency (the shape of the curve) will be discussed separately. The values of the angle can be in good agreement while the correlation between the changes with frequency is poor as the two curves for Shoal station four show (see Figure 1). One angle increases while the angle computed from the other set of records decreases, and the curves intersect. The two curves look like the letter "x," and the largest difference between angles at the same frequency is small.

Numerical Values of the Angle

Change with Distance

The angle at a given frequency changes for stations at various distances from the shot in a reasonable way. That is, the angle either continually increases with increasing distance or goes to a minimum and then increases with distance. The compatibility between the angle at one given frequency and the distance from the shot seems to be better at the low frequencies than at the high frequencies. For example, the far station for Antler, number two, has the larger angle of emergence at the low frequencies while the near station, number one, has the larger angle at the high frequencies. The first three stations of the Haymaker event also give a larger angle than that of the farthest station for frequencies between nine and sixteen cps. The angle at the high frequencies is more dependent on the geologic conditions under the station; so these results should be expected.

Varying local geology seems to hide much of the change with distance for Shoal. At four stations along the northeast line between points at 0.952 km and 4.913 km, the angles decreased with increasing distance. The minimum angle is attained at the outer edge of the granite, 4.913 km from the shot. This is an unusually long distance from the shot when the shallow depth of the source is considered. Large angles of around 77 degrees were found for the last two stations on this line which were outside of the granite.

On the northwest line, where all the stations were outside of the granite, the angle increased from about 70 degrees to about 80 degrees for an increase in distance from 1.483 km to 4.72 km. This result is the exact opposite of the behavior on the northeast line where the stations involved were inside of the granite. Very little change with distance was noted for the three stations on the southeast line.

Angles at three locations were available on the southwest line. The middle station, number twelve, 3.375 km from the shot, gave the minimum angle. Again, the station was on the outer edge of the granite; so this result is in very good agreement with the result on the northeast line.

The best comparison between the angle at any given frequency and distance was obtained from the four Hardhat stations. The angle decreased to a minimum value at the second station, 1.26 km from the shot, and then increased with increasing distance for the last two stations.

Compatibility with Geologic Structure

The size of the angle is compatible with the geologic structure under the station. The best example is Hardhat station five where extremely large angles were observed. The big values of the angle could be attributed to a large velocity contrast from low velocity alluvium at the surface to high velocity granite at depth. Abnormally large angles were also found for stations thirteen and twenty-five on the northeast line of Shoal. A large velocity contrast also existed under these stations.

Comparison with the Peak Amplitude Method

The angle computed from the ratio of the peak amplitudes of the seismograms lies inside the range of angles calculated from the frequency dependence method in nine cases. In seven other cases, the peak amplitude method gives results that are close to one end of the range of values determined by the frequency dependence method. The angle computed from the peak amplitudes is not close to the range of values of the frequency dependence method for six stations. The accuracy of the frequency dependence method is probably better than that of the peak amplitude method because more factors are taken into account by the frequency dependence method. These results then indicate the reliability of the peak amplitude method.

Precision of the Numerical Values

The angles calculated from two or more pairs of records of the ground motion at one location indicate the precision that can be obtained from the method of analysis. These angles would be exactly the same if the method of analysis were perfect. Records from two or three pairs of accelerometers and/or displacement meters with matched instrument constants were available at some locations for the Shoal event. The results at these stations were used to estimate the precision of the method of analysis.

The columns headed largest Δe from displacement meters or from accelerometers in Tables V, VI, VII and VIII show the results. Stations three and four gave values of 1.9, 2.6 and 2.9 degrees. These differences at stations five and six were 2.5, 4.3 and 6.8

degrees. Values of 1.1, 1.8 and 2.8 degrees were obtained for stations seven and eight. The largest difference between the angles calculated from accelerometer and displacement meter records at station thirteen was 1.3 degrees. The maximum over all stations of these largest differences was 6.8 degrees. The average difference, the sum of the differences at each frequency divided by the number of frequencies, would have been much smaller.

Variations with Frequency

The variation with frequency at Shoal stations five, six, nine, fifteen and twenty-five was greater than the maximum largest difference just discussed (6.8 degrees). The variation was more than ten percent of the total possible range, zero to ninety degrees, at all the fore-mentioned stations except number six. Antler station two and the first three Haymaker stations also vary with frequency by an amount greater than 6.8 degrees. Therefore, the frequency variation at these stations is too big to be attributed to scatter if the scatter for these stations is about the same as that for Shoal. Haymaker station one had the largest frequency variation. It was 23.4 degrees or more than a quarter of the total possible range.

Truncating the records of the ground motion cannot introduce frequency dependence as was proved in Chapter II. Small changes with frequency might be caused by noise, errors or the mismatch of the measuring instruments. Noise and errors are random and can be approximated by a white noise spectrum. The apparent angle of emergence for the sum of the white noise spectrum and the narrow band spectrum of the signal would approach 45 degrees outside of the frequency band of the signal. This effect was never observed. All accelerometers were quite well matched. No correlation between the instrument constants and the frequency dependence was found for the accelerometers.

The results at stations five and six put at the same location, where angles computed from all three pairs of instruments display an increase, indicate that the frequency dependence is real. The large variations with frequency at other Shoal stations (nine, fifteen and twenty-five) as well as Antler two and Haymaker one, two and three tend to verify the frequency dependence.

However, there is considerable doubt about the exact variation and shape of the frequency dependent angle. The fact that a frequency dependent angle cannot be taken out of the convolution integrals of Chapter II means that the length of the signal being analyzed and the mismatch of the instruments can affect the frequency dependence of the angle. The angles computed for different lengths and for different instruments will lie in a region that can be made to approach the true frequency dependent angle.

Onset of Later Arrivals

The analysis of the records for Shoal at Mould Bay indicate that the angle of emergence changes radically when a signal travelling along a different ray path arrives. This effect was observed by scanning the seismograms with windows of 0.3 seconds (15 points) and 0.16 seconds (8 points). The angle calculated using either of these windows changed radically for the 1.2 second section. Figure 5 shows the angle at 1 cps obtained using the 0.3 second window, and gives the seismogram for the station.

This same phenomenon is observed for angles in the zero to thirty kilometer range, but the time between the arrivals is much smaller. In many cases, the frequency dependence method finds a new arrival earlier than the effect of this arrival can be seen in the time domain. It takes time for the later arrival to build up enough amplitude to cause an interference pattern on the records.

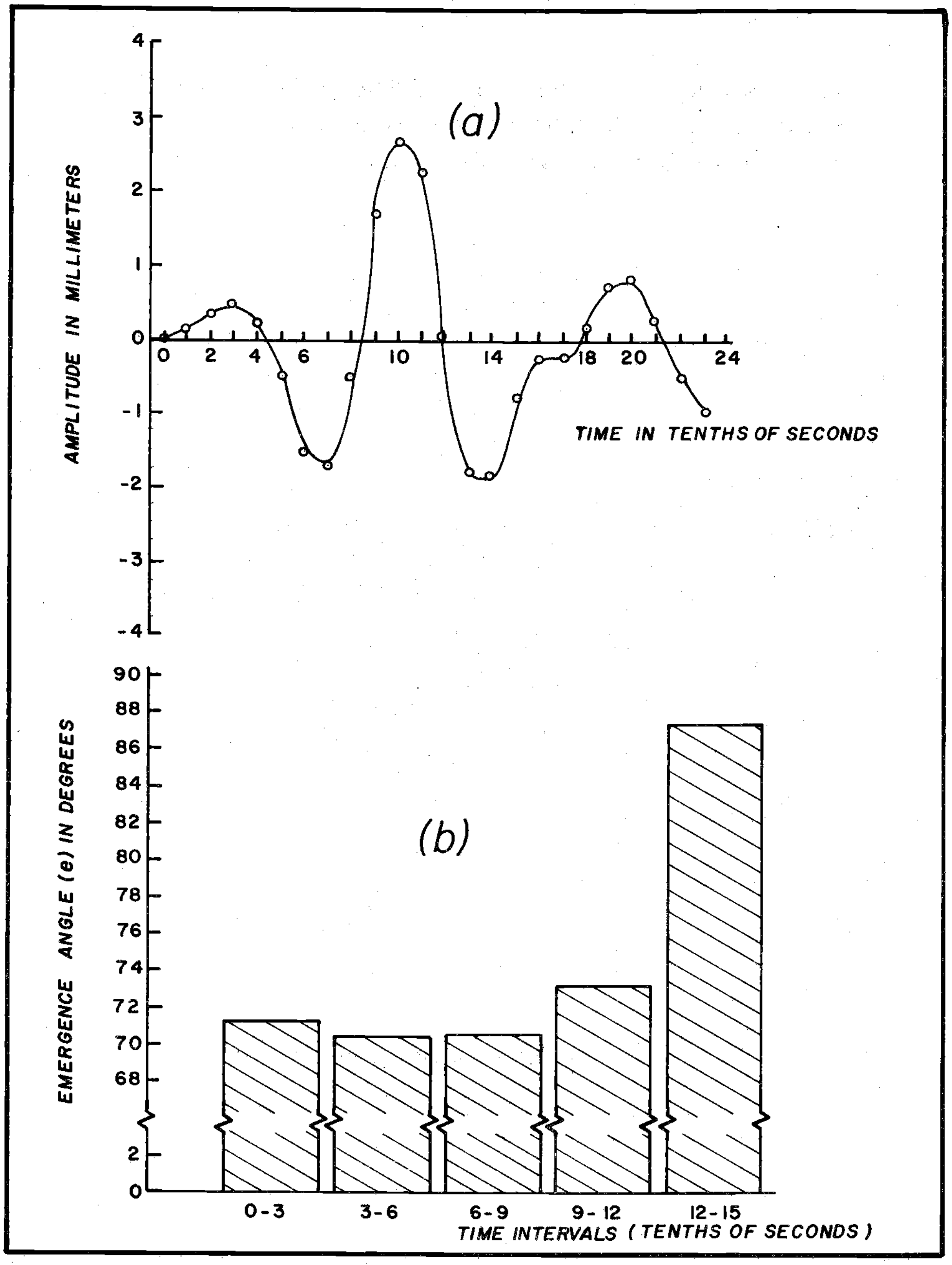

Figure 5. Comparison of recorded ground motion at Mould Bay with actual angle of emergence at 1 cps to show a later seismic arrival.

VI. CONCLUSIONS

The method of analysis used in this research requires that the data be transformed from the time domain to the frequency domain by numerical means; so any application of the method will require a large scale computer. However, this method of analysis permits emergence angles to be computed by using just the uncontaminated portion of the record for the first P arrival. The method as used in this paper gives reasonable and consistent results. The numerical values of the angles obtained seem to be very precise. More theoretical research into ways of minimizing the problems caused by later arrivals is needed to extend the general method of finding the frequency dependence of emergence angles.

The angles computed from this method of analysis can be used in formulas for the frequency domain involving the angle of emergence. When the frequency variation is small, an average of the frequency dependent angle can be used for formulas in the time domain.

A very exciting application is frequency sounding, using the frequency dependence of the angle to determine the geologic structure under the station. The average P to S velocity ratio of Chapter II is probably a better way of presenting these results than the angle of emergence. The changes in the angle as a function of frequency are needed for this application. That is, the derivatives must be known. These derivatives seem to have very large errors. The error in the derivatives of angles--as calculated by the method of this paper--is probably due to the convolution integrals. The velocities obtained by using the derivatives would be average values over varying distances. Interpretation of the results involves finding the true velocities at depth from the calculated average values. The results of this research indicate that much more work will have to be done before frequency sounding can be used with any assurance of reliable results.

Angles can also be calculated for small sections of the seismogram, and these angles can be compared with each other. Attempting to find the angle for a small section of the seismogram in the time domain cannot give reasonable results. The apparent angle will jump from zero to ninety degrees if the zero crossing times of the vertical and radial records are not the same. Therefore, the analysis must be done in the frequency domain.

The angles for small sections of the record computed by scanning the record with a window and transforming the results can be used to find a new arrival. The rectangular pulse is a good window for this purpose because the end of the time interval has as much effect on the transforms as the middle part of the time interval.

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