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A cost function characterizes a firm's cost-minimizing behavior. It is defined as a function of the level of outputs produced and the prices of factors which enter the production process. Econometric estimation of a cost function allows one to test hypotheses regarding the structure of cost and the structure of the underlying technology. Cost function structure is indicative of production structure, namely, the relationships among factors and products involved in the production process.

In this study, the method of maximum likelihood is used to jointly estimate a cost function and labor share equation for a cooperative vegetable processing firm. The study concentrates on labor and energy inputs and on green beans, sweet corn, and an aggregate of other fruits and vegetables. Hypotheses of nonjointness in output prices (no factor substitutability) and nonjointness in inputs (no output complementarity), and a third hypothesis regarding regulation of raw product delivery quantities, are tested at the sample mean. Measures of conditional price elasticities of input demand, cost complementarity, and cost elasticity are derived from the estimated model.

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COST FUNCTION ANALYSIS OF FRUIT AND VEGETABLE PROCESSING IN AN OREGON COOPERATIVE

Chapter 1

Introduction

Analysis of the cost structure of a cooperative processing firm may help us understand the relationship between factors and products in the production process. This information can, in turn, assist us in drawing conclusions about the firm's operating goals as a farmer cooperative. The present thesis is an investigation of the cost structure of fruit and vegetable packing in an Oregon cooperative.

Oregon's share of frozen fruit and vegetable sales makes it an important center of the fruit and vegetable processing industry (Standard Industrial Classification--SIC--203). According to the *1982 Census of Manufactures*, Oregon's shipment value (total sales) of frozen vegetables (SIC 20372) was the highest in the country, and in frozen fruits (SIC 20371) was fifth highest (Table 6b, pp. 23-24). In the 1987 census, Oregon ranked second in SIC 20372 and third in SIC 20371 (*1987 Census of Manufactures*, table 6b, pp. 22-23). The firm chosen for analysis engages in activities related to fruit and vegetable canning (SIC 2033) and freezing (SIC 2037). Although the present study focuses on a stage of processing common to both canning and freezing activities, the firm's specialty is freezing; revenues from sale of frozen fruits and vegetables constitute 85% of its total revenues.

The present firm's 1989 annual average employment accounted for approximately 7.5% of total annual average employment in Oregon's fruit and vegetable processing industry, assuming the 1988 total level prevailed in 1989 (Kadera; *1989 Oregon Industrial Outlook*). The cooperative's membership of approximately 240 farmers, together with its contribution to employment and sales in the industry, clearly establish the importance of this firm for study.

Fluctuating factor and product prices affect a firm's financial viability. Changes in factor prices may change the firm's total cost level and changes in its product prices may change its total revenue. It is possible, by conducting an analysis of a firm's cost structure, to learn whether factors involved in production are substitutable, that is, whether the firm possesses the flexibility to change its factor mix in order to minimize cost. The analysis can also reveal whether the firm minimizes cost by producing various outputs in combination or whether specialization is a cheaper option.

In the present study, we wish to test the hypothesis that factors are not substitutable; this is the hypothesis of nonjointness in output prices. Rejecting the hypothesis would imply that the firm does indeed have flexibility to change its factor mix in order to minimize cost. Secondly, we would like to test the hypothesis that the marginal costs of particular outputs neither rise nor fall with changes in production levels of other outputs: the hypothesis of nonjointness in inputs. If marginal costs of particular outputs are unaffected by production levels of remaining outputs, production processes governing the outputs are in some sense separate. The firm is, therefore, able to minimize cost by producing these outputs separately. Rejecting this hypothesis would imply that marginal costs either rise or fall due to incremental increases in other outputs and production is in some sense joint.

Cooperatives may pursue a number of alternative goals. Some goals serve the interests of particular groups better than do others. We wish here to test the hypothesis that the cooperative chooses to regulate delivery quantities, possibly with the intention of maximizing per-unit returns to current members. Rejecting this hypothesis leaves open the possibility that the cooperative pursues other goals, such as maximizing total surplus revenue or choosing the scale of operation at which delivery quantities are at a financially viable maximum.

Econometric estimation of a cost function enables us to test these three hypotheses. Furthermore, certain important measures can be calculated from the estimated cost function. If inputs are substitutable, estimated conditional demand elasticities provide knowledge of the degree of substitutability. If production is joint, second-order derivatives of cost with respect to outputs indicate whether cost savings (complementarities) or losses (anti-complementarities) are present. A further measure, cost elasticity, provides information about increases in cost due to proportionate increases in output levels.

Econometric cost studies utilizing the translog functional form are found frequently in the literature. Denny and Pinto (1978), in a study of Canadian manufacturing, and Evans and Heckman (1983), in a study of the Bell system, use the translog form to test hypotheses of nonjointness in inputs. Both studies calculate demand elasticities as well. Kohli (1981), who developed the notion of nonjointness in output prices, uses a generalized-linear-generalized-Leontief functional form to test for this structure in U.S. private sector aggregate production. Brown et. al. (1979) use the translog form to estimate a cost function for U.S. freight and passenger railroads and to calculate cost elasticities at each observation point (that is, for each railroad). Sexton et. al. (1989) estimate a translog normalized profit function to test the hypothesis that cooperative firms in the California cotton ginning industry regulate delivery quantities. They consider this a test of the hypothesis that the cooperatives operate at a game-theoretic-stable equilibrium.

While the listed studies do not exhaust the body of empirical cost function literature, they are adequately representative of analyses whose objectives overlap our own. In this study, we attempt to add a new dimension to the literature. We claim that nonjointness in output prices is a necessary and sufficient condition for zero-valued own price demand elasticities for all inputs and zero-valued cross price demand elasticities for all input pairs. We restate Gorman's (1983) proposition that absence of both cost complementarities and anti-complementarities is necessary and sufficient for the absence of economies or diseconomies of scope. The latter proposition may be equivalently stated: absence of nonjointness in inputs is necessary and sufficient for the presence of either cost anti-complementarities or complementarities.

It is found useful here to calculate cost complementarity measures at each observation point (each corresponding to a separate plant). We also derive parameter restrictions to test for nonjointness in output prices for the translog cost specification. We show how an hypothesis regarding the cooperative's behavior may be tested by utilizing the expression for cost elasticity. Unlike most earlier studies, the cost structure analyzed corresponds to plants belonging to a single firm.

Since economic analysis must begin from a theoretical foundation, we begin by presenting the aspects of multiple output production theory necessary for specifying a cost function. A discussion of nonjointness in inputs, nonjointness in output prices, and a measure of cost elasticity are provided in the remaining sections of chapter two. Cost function analysis is then placed within the context of cooperative theory, permitting us to test a hypothesis regarding the cooperative's goal.

In chapter three, a sketch is provided of the firm's institutional framework and of the cost function used to model its production technology. Functional form, hypothesis tests, and estimation procedure are considered in chapter four. Chapter five contains the pertinent results. Analysis concludes with a summary in chapter six.

Chapter 2

Economic Theory

In this chapter, we define the cost function, state its properties and possible structural forms, and place it within the context of the theory of the marketing cooperative firm. We introduce first the notion of an input requirement set; that is, input bundles which can produce at most a given output bundle. Cost function analysis assumes that a firm is aware of the relationships between inputs and outputs defined by its input requirement set, and that it uses this knowledge to minimize production costs given input price levels. Only the minimal (boundary) points of the input requirement set, defined by the distance function, are important for our study. The set of minimal points of the input requirement set is the input isoquant for a given output bundle.

The firm's production technology may be characterized by certain restrictive structures. Structures we are interested in here are nonjointness in inputs and nonjointness in output prices. Nonjointness in inputs means that each output has its own input requirement set, or that a separate technological relationship governs the production of each output. Nonjointness in output prices means there are no substitution possibilities with respect to input use once a production plan has been chosen; that is, there is an input requirement set specific to each input.

Degree of cost elasticity, a measure derivable from cost function analysis, utilizes the notion of ray economies of scale. Cost elasticity is the inverse of ray economies of scale. Following the definition of cost elasticity, we summarize the theory of the marketing cooperative firm and discuss three possible objectives such a firm might pursue. In this final section, it is suggested that cost elasticity and hence a cost function may be used to isolate cooperative objectives.

2.1 Notation

A firm utilizes inputs, denoted by the n-dimensional vector $\mathbf{x}$, to produce outputs, denoted by the m-dimensional vector $\mathbf{y}$. The firm purchases its inputs at the input price levels given by the n-dimensional vector $\mathbf{w}$, and offers for sale outputs at output price levels given by the m-dimensional vector $\mathbf{p}$. The order of elements in a price vector is identical to the order of elements in the corresponding quantity vector. In the present chapter, i and j index inputs and k and l index outputs; exceptions to this rule will be indicated where appropriate.

2.2 Multiple Output Production

An input requirement set $X(\mathbf{y})$ is the set of all possible efficient (minimal) and inefficient input combinations that may at most produce $\mathbf{y}$, given the state of technology. Input requirement set $X(\mathbf{y})$ may be defined by

$$X(y) = \{x \mid F(y,x) \geq 1\}. \quad (2.1)$$

F(y,x) in (2.1) is known as a distance function. A distance function describes the relation between interior (inefficient) points of X(y) and the efficient points on its boundary.[1] From (2.1), it is evident that the distance function defines the input requirement set. Therefore, restrictions on F(y,x) determine restrictions on X(y). Part of our analysis is concerned with restrictions which might characterize a firm's input requirement set and, by definition, the distance function. Mathematical duality between the distance function F(y,x) and the cost function (defined below) permits us to gain a knowledge of production structure through an analysis of cost structure.

2.3 Cost Function

Input requirement set X(y) contains the information needed to gain a knowledge of the structure of production. The cost function can be defined in terms of X(y) as

$$C(y,w) = \min_x \{w \cdot x \mid x \in X(y)\} \quad (2.2)$$

(Chambers, p. 50). Definition (2.2) states that, given an input requirement set X(y), there exists a cost function C(y,w). Certain properties of X(y) determine properties of C(y,w). Results from duality theory inform us also that, given a cost function which satisfies these properties, there exists an input requirement set capable of defining that cost function (Chambers, pp. 86-84). Duality exists between F(y,x) and C(y,w); properties of F(y,x) with respect to **x** are identical to those of C(y,w) with respect to **w** (McFadden, p. 26). Here, we accept the validity of this duality result.

Relation (2.2) characterizes the cost minimizing behavior of a rational firm facing exogenous input prices **w**, that is, facing a factor market which is perfectly competitive. Minimization occurs with respect to input use; a firm takes advantage of its input substitution possibilities in the production of an output vector **y**. Cost function analysis places no restriction upon output markets; they may be perfectly or imperfectly competitive. The firm is assumed to have rationally decided upon its production plan **y** for a given period.

2.3.1 Regularity Conditions

To be valid in a cost function analysis, (2.2) must possess certain mathematical properties with respect to its arguments **y** and **w**. These properties are commonly known as the regularity conditions, denoted R1-R6.

R1: $C(\mathbf{y},\mathbf{w})$ is twice continuously differentiable in **y** and **w**.

R2: $C(\mathbf{y},\mathbf{w}) > 0$ for $\mathbf{y} > 0$ and $\mathbf{w} > 0$.

R3: $\partial C/\partial w_i = x_i \geq 0 \ \forall \ w_i$, where i indexes inputs.

R4: $C(\mathbf{y},\mathbf{w})$ is linearly homogeneous in **w**.

R5: $C(\mathbf{y},\mathbf{w})$ is concave in **w**.

R6: $\partial C/\partial y_k \geq 0 \ \forall \ y_k$, where k indexes outputs. (2.3)

R1 ensures that comparative static results are obtainable. Strictly speaking, R1 subsumes the necessary property of continuity in **w** (Chambers, pp. 52,56). Continuity in **y** and twice continuous differentiability are useful for applied analysis. R2 implies that outputs can be produced only at some cost. This is required by weak essentiality of inputs: a positive amount of input is necessary to produce positive output. If the input has positive price, cost must be positive. R3 holds by the cost function's property of being non-decreasing in **w** (the necessary property), which together with continuity ensures that the partial derivative $\partial C/\partial w_i$ exists and is nonnegative. By Shephard's lemma, $\partial C/\partial w_i$ is the cost minimizing conditional demand x_i for the ith input (pp.56-9). R4 implies that an equiproportionate change in factor prices leaves relative input use the same, so that costs change by that same proportion.

R5 is implied by the fact that inputs are generally substitutable and the firm is a cost minimizer. If there are two input vectors $\mathbf{w}^1$ and $\mathbf{w}^2$ and, corresponding to these price vectors, two cost-minimizing input bundles $\mathbf{x}^1$ and $\mathbf{x}^2$, then $\mathbf{w}^1 \bullet \mathbf{x}^1$ is the minimum cost associated with $\mathbf{w}^1$, and $\mathbf{w}^2 \bullet \mathbf{x}^2$ is the minimum cost associated with $\mathbf{w}^2$. Concavity in **w** means that the cost associated with a convex combination of $\mathbf{w}^1$ and $\mathbf{w}^2$, say $\mathbf{w}^*$, is not less than a convex combination of the minimum cost associated with $\mathbf{w}^1$ and the minimum cost associated with $\mathbf{w}^2$. If inputs were not substitutable and the firm utilized a fixed input vector, the minimum cost associated with input prices $\mathbf{w}^*$ would equal the corresponding convex combination of costs associated $\mathbf{w}^1$ and $\mathbf{w}^2$ (p. 53).

C(**y**,**w**) is required to be non-decreasing in **y**. Together with continuity, this ensures that the partial derivative $\partial C/\partial y_k$ exists and is nonnegative: property R6. (Chambers, pp. 52-59, 261-2). For econometric analysis, the set of conditions R1-R6 ensures that there exists a set X(**y**) which could have generated the estimated cost function.

2.4 Cost Structure

Function (2.2) may be characterized by certain restrictive structures, indicative of structures which characterize the input requirement set. Chambers (p.51) suggests that "the more *a priori* restrictions that are placed on the technology, the more constrained producers will be in solving the minimum-cost problem." In this section we concern ourselves with two structures: nonjointness in output prices (NJO) and nonjointness in inputs (NJI). Briefly, NJI and NJO result in an additively separable form of C(**y**,**w**).

2.4.1 Nonjointness in Output Prices

A production structure which exhibits nonjointness in output prices is one which is characterized by input nonsubstitutability once a production plan has been chosen (Kohli 1983, p. 213). Input usage is defined by a factor requirement function f^i(**y**), one for each input:

$$x_i \geq f^i(y_1,\ldots,y_k) \quad \forall\, i \tag{2.4}$$

where k indexes outputs and i indexes inputs. Input quantity x_i is the level of the ith input capable of producing the vector of outputs. The ith input requirement set is

$$X^i(\mathbf{y}) = \{x_i \mid x_i \geq f(\mathbf{y})\}. \quad \forall\, i$$

Thus, input requirement set X(**y**) for an NJO technology may be written as $\cap_{i \in n}$ X^i(**y**), that is as the intersection of all individual input sets (Chambers, p. 289). For a two-input technology, such input requirement sets produce right-angled isoquants, where the vertex would be the minimal vector of inputs capable of producing the chosen output vector. This vector of input levels is independent of relative input prices.

Chambers (p. 297) shows that a cost function of the form

$$C(y,w) = \Sigma_i w_i f^i(y). \tag{2.5}$$

is observationally equivalent to an NJO technology.[2] From (2.4), factor requirements are independent of factor prices. Using Shephard's lemma, we see also from (2.5) that the conditional demand for the ith input is independent of any input price, including its own price w_i.

Finally, the form of the cost function in (2.5) requires the hessian matrix of second-order derivatives of the cost function with respect to input prices be the null matrix:

$$\partial C/\partial w_i \partial w_j = 0 \ \forall \ i,j \tag{2.6}$$

(Chambers, p. 297). From (2.4) and (2.5), then, an NJO technology is characterized by nonsubstitutability of inputs and inelasticity of conditional demands.

2.4.1.a <u>Elasticities of Demand</u>

We noted above that an NJO technology is one in which each input demand is independent of input price, either its own or that of other inputs utilized in the production process. Thus, we may state that NJO is necessary and sufficient for zero-valued own price demand elasticities for all inputs and zero-valued cross price demand elasticities between all possible input pairs. These price demand elasticities are defined as

$$\epsilon_{ij} = (\partial x_i/\partial w_j)(w_j/x_i). \ \forall \, i,j \qquad (2.7)$$

For the two input cost function, $\epsilon_{ii} = - \epsilon_{ij}$ (Chambers, p. 65). Since ϵ_{ii} is required to be nonpositive, ϵ_{ij} will be nonnegative and inputs i and j either will be nonsubstitutable ($\epsilon_{ij} = 0$) or will be demand substitutes ($\epsilon_{ij} > 0$). The inputs cannot be related as demand complements.

2.4.2 Nonjointness in Inputs

A production structure which exhibits nonjointness in inputs is one which is characterized by the absence of complementarity amongst the technological processes which govern the k outputs. Each output, then, has its own production function

$$y_k \leq f^k(\mathbf{x}^k) \qquad (2.8)$$

(where $\mathbf{x}^k$ is the input vector specific to the kth output) subject to the restriction that total input usage, $\Sigma_k \mathbf{x}^k$, cannot exceed the total quantity of input $\mathbf{x}$ available. That is, $\Sigma_k \mathbf{x}^k \leq \mathbf{x}$ must hold.

There is an input requirement set for each output y^k, that is

$$X^k(y_k) = \{\mathbf{x}^k | \ y_k \leq f^k(\mathbf{x}^k)\} \quad \forall \, k$$

where $\mathbf{x}^k$ is the vector of inputs able to produce at most y_k. The input requirement set, X(y), for the entire technology would be written as $\Sigma_{k \epsilon m} X^k(y_k)$, the sum of sets corresponding to each output (Chambers, p. 287). Existence of NJI might serve as justification for decentralizing of the decision making authority which governs each of the k production processes; or production might, for example, be physically separated and occur at different plants within a firm.

The NJI production structure induces the cost function to have functional structure

$$C(\mathbf{y},\mathbf{w}) = \Sigma_k C^k(\mathbf{w},y_k). \tag{2.9}$$

With structure (2.9), $C(\mathbf{y},\mathbf{w})$ satisfies properties necessary to ensure that there exists an input set which could have generated the cost function (Chambers, p. 293). Finally, the form of (2.9) requires that off-diagonal elements of the hessian matrix of cost function second-order derivatives with respect to outputs be equal to zero:

$$\partial C(\mathbf{w},\mathbf{y})/\partial y_k \partial y_l = 0 \ \forall \ k,l, \ k \neq l. \tag{2.10}$$

2.4.2.a Cost Complementarities

If the off-diagonal terms of the cost function's output hessian matrix are non-zero, the technology exhibits either cost complementarities ($\partial^2 C/\partial y_k \partial y_l < 0$) or anti-complementarities ($\partial^2 C/\partial y_k \partial y_l > 0$). NJI therefore is a necessary and sufficient condition for the absence of both cost complementarities and anti-complementarities (Gorman, p. 435).

Economies of joint production (scope economies) have been put forth as a justification for the existence of a multiproduct firm (Bailey and Friedlander, pp. 1026-8). For a given binary partition of the firm's output vector, economies of scope exist when production costs are subadditive, that is, when

$$C(\mathbf{y}_1,\mathbf{y}_2,\mathbf{w}) < C(\mathbf{y}_1,0,\mathbf{w}) + C(0,\mathbf{y}_2,\mathbf{w}) \tag{2.11}$$

where $\mathbf{y}_1$ is a vector of outputs belonging to partition 1 and $\mathbf{y}_2$ is such a vector for second partition 2 (Gorman, p. 432). If k indexes outputs belonging to partition 1 and l indexes outputs belonging to partition 2, a sufficient condition for economies of scope is the existence of cost complementarity across partitions, which requires

$\partial^2 C/\partial y_k \partial y_l < 0 \; \forall \; k,l \; k \neq l$ (2.12)to hold.

A sufficient condition for diseconomies of scope--relation (2.11) with inequality reversed--would require relations (2.12) to hold with inequality reversed.

Thus, from (2.11) and (2.12), absence of NJI is necessary and sufficient for presence of cost complementarity or anti-complementarity, which, for a given partition of the firm's output vector, are, respectively, sufficient conditions for the presence of economies or diseconomies of scope. However, given an output vector partition, failure to establish cost complementarities and anti-complementarities as defined by (2.12) does not preclude the existence of economies or diseconomies of scope.

2.4.3 Nonjointness in Output Prices with Nonjointness in Inputs

If NJO and NJI hold simultaneously, the cost function takes the form

$$C(y,w) = \Sigma_k \Sigma_i \, w_i f^i_k(y_k) \qquad (2.13)$$

(Chambers, p.298). The condition of nonjointness in output prices requires relation (2.6) to hold so that concavity holds trivially. That is, from section 2.3.1, the cost associated with a convex combination w^* of input price vectors equals the convex combination of minimum costs associated with vectors w^1 and w^2. For an NJO and NJI technology, this is demonstrated as follows:

$$C(y,w^*) = \Sigma_k \Sigma_i \, [\Theta w_i^1 + (1-\Theta)w_i^2] f^i_k(y_k)$$

$$= \Sigma_k \Sigma_i [\theta w_i^1 f^i_k(y_k) + (1-\theta) w_i^2] f^i_k(y_k)]$$

$$= \theta C(y,w^1) + (1-\theta) C(y,w^2). \qquad 0 < \theta < 1$$

The demonstration above indicates that changing from an arbitrary input price vector $\mathbf{w}^*$--at which $\mathbf{x}^*$ ($x^*_i \geq f^i(y) = \Sigma_k f^i_k(y_k)$) minimizes cost of production--to price vector $\mathbf{w}^1$ or $\mathbf{w}^2$ does not involve input substitution. The firm continues to utilize the same input vector $\mathbf{x}^*$ at either the lower or higher price levels.

2.5 <u>Cost Elasticity</u>

Cost elasticity is defined as percentage change in total cost brought about by a unit percentage change in level of output. It is the inverse of the measure for ray scale economies. Ray scale economy R is defined as

$$R = C(ty^*) / \Sigma_k ty^*_k (\partial C(ty^*)/\partial y_k) = 1/[\Sigma_k (\partial \ln C(ty^*)/\partial \ln y_k)] \qquad (2.14)$$

where argument $\mathbf{w}$ is dropped for convenience (Bailey and Friedlander, pp. 1030-1). Here $\mathbf{y}^*$ is the output bundle which defines a composite commodity and t is the scale of that commodity.

As an intuitively appealing generalization of the single output case, the denominator of R in (2.14) exceeds, falls short of, or equals total costs as locally decreasing, increasing, or constant ray returns to scale hold. R is called ray economy since it is a measure of cost behavior as output proportions are held fixed along a ray $\mathbf{y}^*$ through the origin in the space of outputs. In (2.14), $\Sigma_k (\partial \ln C(ty^*)/\partial \ln y_k)$ is the degree of cost elasticity.[3]

2.6 Firm as Marketing Cooperative

In the situation analyzed here, members supply their raw product to the cooperative, which then processes it and brings it to market. A cooperative member might gain from the value added associated with converting his primary goods into secondary or processed ones.

2.6.1 Net Revenue Function

Helmberger and Hoos (1962) distinguish the marketing cooperative as a unique form of industrial organization, contending that processing or activity alone does not tell us about organization as such. Activity must be "coordinated toward the achievement of certain ends." To achieve these ends, there is some "'peak coordinator' consisting of a person or group of persons...that..engages in action to secure their achievement" (1962 pp. 277-79). The board of directors is an example of a peak coordinator. When only some factors are variable, the cooperative's profit function may be written

$$\pi = \mathbf{p}\bullet\mathbf{y} - \Sigma_i w_i x_i - \mathbf{w}^r\bullet\mathbf{r} - F \qquad (2.15)$$

where $\mathbf{p}$ = vector of output prices;

$\mathbf{y}$ = vector of output quantities;

w_i = price of variable factor i;

x_i = level of variable factor i;

$\mathbf{w}^r$ = vector of unit net returns to raw product;

$\mathbf{r}$ = vector of raw product quantities delivered per period;

F = level of fixed costs.

In (2.15), $\Sigma_i w_i x_i$ represents variable cost of production and $\mathbf{w^r} \cdot \mathbf{r}$ is total net return accruing to the raw product. Equivalently, $\mathbf{w^r} \cdot \mathbf{r}$ is the cost of raw product to the cooperative plus additional revenues called patronage refund. A cooperative is constrained to make zero profits and $\mathbf{w^r} \cdot \mathbf{r}$ is an unknown quantity to be determined. Hence the relationship of interest is the net return (NR) function

$$NR = \mathbf{w^r} \cdot \mathbf{r} = \mathbf{p} \cdot \mathbf{y} - \Sigma_i w_i x_i - F \tag{2.16}$$

(Helmberger 1964, p. 604).

The Helmberger and Hoos formulation of the cooperative model refers to the case of a homogeneous raw product. For the cooperative studied here, there are several different raw products and several different outputs. However, by choosing an output bundle $\mathbf{y}^*$, and hence a raw product bundle $\mathbf{r}^*$, we may reduce our multi-output analysis to that of a single output. Such a simplification enables us, in the following section, to appreciate the possible goals a cooperative may pursue.

In the single output setting where $\mathbf{w^r}$ and $\mathbf{r}$ are each of dimension one, the net average revenue product (NARP) function is defined as NR/r and the net marginal revenue product (NMRP) function is defined as $\partial(NR)/\partial r$. For an analysis involving multiple products, the analog to the NARP function would be a measure of NR divided by the scale t of the raw product bundle $\mathbf{r}^*$, NR/t. NMRP would be the derivative of NR with respect to increases in the scale t of the bundle $\mathbf{r}^*$, $\partial NR/\partial t$. Redefining (2.16) for an arbitrary $\mathbf{r}^*$ and $\mathbf{y}^*$, it is possible to utilize the conventional diagram relating NARP to NMRP to learn of possible cooperative firm objectives (demonstrated in section 2.6.3).

2.6.2 Three Cooperative Objectives

The major source of contention in the early literature on marketing cooperatives was over the equilibrium point at which a cooperative operates. There essentially are three possibilities (Sexton, p. 425). Each position in figure 1 coincides with a different goal set by the "peak coordinator."

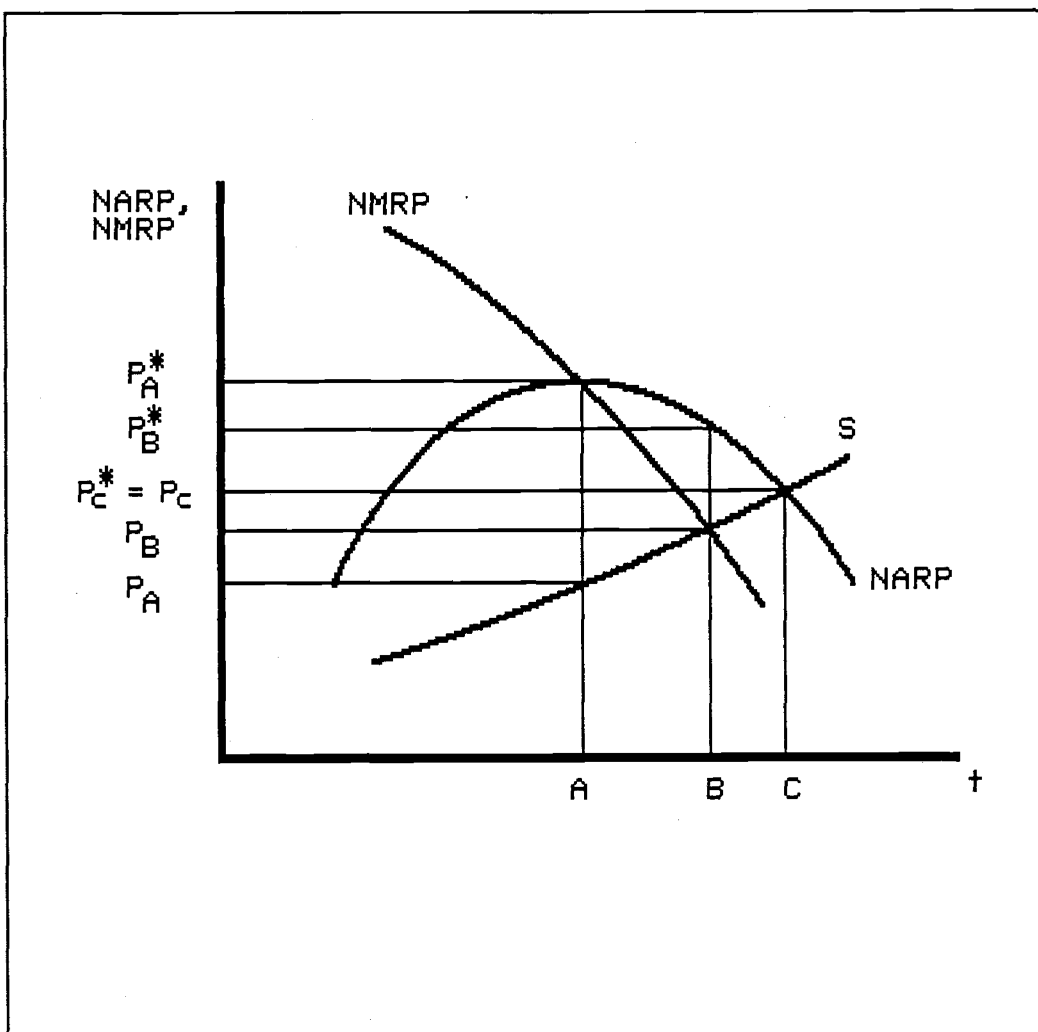
Point A, where NARP=NMRP, is the w^r- or NARP-maximizing solution. Operating where raw product deliveries are at the level corresponding to A requires delivery quantities to be regulated. It is referred to as the regulated-membership solution and assumes that delivery quota allotments per member are fixed. Since membership and quota sizes are not explicitly modelled in the present study, we concern ourselves only with changes in the level of delivery quantities and hence output levels.

Point B, where the member supply (on-farm marginal cost) curve intersects the NMRP curve, is the total-patronage refund-maximizing solution. Net return w^r is composed of price P_B paid to member suppliers of raw product r plus unit

Figure 1

Cooperative Objectives

Net Average and Marginal Revenue Product Curves



patronage refund $P_B^* - P_B$.# At delivery scale r, total patronage refund accruing to the cooperative is

$(P_B^* - P_B)r = NR - P_B r$. The variable quantity of patronage refund is maximized where the partial derivative of the patronage refund function with respect to r equals zero; that is, where NMRP equals the marginal farm production cost P_B of raw product r.

At point C, raw product deliveries are maximized because a cooperative firm accepting deliveries beyond this point would be operating at a loss. This is conventionally called the maximum-membership solution. At the level of operation corresponding to C, member supply curve intersects NARP.

In each of the cases A, B, and C, price to buyers per unit of raw product is derived from the NARP function. In the present exposition, $\mathbf{w}^r$ is referred to as a vector of buyer prices rather than unit net returns, because it is the vector of unit raw product prices competing processors would need to pay for purchases of raw product. If farmers sold their raw product in a competitive market, they could expect in the long run to receive a price equal to on-farm marginal cost of production, but as members of a cooperative they might obtain additional returns from the value added due to processing. This additional revenue would come from the total patronage refund (surplus) earned by the cooperative.

#The horizontal axis of figure 1 is labelled t, the scale of raw product bundle $\mathbf{r}^*$ (a vector of heterogeneous raw products). Discussion in section 2.6.2 is for the case of a single raw product, so that t of figure 1 should be interpreted as r, the quantity of a homogeneous raw product.

At A, buyer price P_A^* is highest. P_A^* - P_A is per unit patronage refund; thus, the farmer is paid P_A per unit of raw product delivered. At B, buyer price is P_B^*, and P_B^* - P_B is per unit patronage refund; the farmer member is paid P_B, the raw product's value marginal product. At C, buyer price equals the price paid to the farmer member per unit of raw product; buyer price is lowest of the three scenarios, while the farmer's unit raw product price is highest. There is no patronage refund at C.

The best scenario for farmer members as a group prevails at C. Members receive the highest possible price per unit of raw product, and absence of patronage refund means there will be no conflict over distribution of surplus revenues among members. If delivery quantities are regulated, as at A, members delivering this restricted quantity are potentially the best off (have the maximum unit net return to raw product). At B, total cooperative surplus (patronage refund) is maximized; the largest quantity of surplus revenues is available for distribution to members or retention by the cooperative.

Figure 1 has been drawn such that the total member supply curve intersects NARP beyond NARP's maximum. In the event member supply intersects NARP at point A, the goal of maximizing unit NR is indistinguishable from the goal of operating where the firm breaks even (delivery quantities are at a financially viable maximum), and from the goal of operating where total patronage refund is maximized.

Without a member supply curve, it is possible to know only whether raw product deliveries are consistent with operating at a point where NARP is rising, falling, or a maximum. If NARP intersects NMRP at the point of operation, any one of the goals consistent with A or B or C is possible, since the supply curve could intersect NARP at its maximum point. We might then conjecture that delivery quantities are regulated. If NARP exceeds NMRP, a goal consistent with B or C is possible; only the unit NR solution is ruled out. Recently it has been suggested that a cooperative's true price-output equilibrium would be at A even if in the intermediate run it operated at B or C. Beyond A, members would no longer have an incentive to contribute to the cooperative and would instead supply to another buyer; or they would form their own cooperative coalition that is smaller and more efficient than the original coalition (Sexton et al, p. 58).

2.6.3 Cost Function and Cooperative Objectives

In the absence of a member supply curve, we may utilize the measure of cost elasticity to decipher whether the cooperative operates where NARP exceeds, falls short of, or equals NMRP. Rewriting net return equation (2.16) in terms of raw product bundle $\mathbf{r}^*$ and output bundle $\mathbf{y}^*$ (each at the scale of operation t), and denoting variable costs of production as $C(t\mathbf{y}^*,\mathbf{w})$, we obtain

$$\mathbf{w}^r \cdot t\mathbf{r}^* = \mathbf{p} \cdot t\mathbf{y}^* - C(t\mathbf{y}^*,\mathbf{w}) - F \qquad (2.17)$$

as an expression for net returns along a ray defined by $\mathbf{r}^*$. Net average revenue product along the ray equals the quantity in (2.17) divided by the scale of operation t:

$$NARP = (\mathbf{w}^r \bullet t\mathbf{r}^*)/t = \mathbf{p} \bullet \mathbf{y}^* - C(t\mathbf{y}^*,\mathbf{w})/t - F/t. \quad (2.18)$$

Net marginal revenue product equals the derivative of equation (2.17) with respect to movements (changes in t) along the ray:

$$NMRP = \partial(\mathbf{w}^r \bullet t\mathbf{r}^*)/\partial t = \mathbf{p} \bullet \mathbf{y}^* - \partial C(t\mathbf{y}^*,\mathbf{w})/\partial t. \quad (2.19)$$

Setting NMRP equal to NARP results in the expression

$$\partial C(t\mathbf{y}^*,\mathbf{w})/\partial t = C(t\mathbf{y}^*,\mathbf{w})/t + F/t. \quad (2.20)$$

Multiplying both sides of (2.20) by $t/C(t\mathbf{y}^*,\mathbf{w})$, (2.20) reduces to a measure of cost elasticity

$$[\partial C(t\mathbf{y}^*,\mathbf{w})/\partial t][t/C(t\mathbf{y}^*,\mathbf{w})] = 1 + F/C(t\mathbf{y}^*,\mathbf{w}) \quad (2.21)$$

Degree of cost elasticity attains the value in (2.21) as NMRP equals NARP along the ray $\mathbf{r}^*$; this would be consistent with point A of figure 1. If NMRP exceeds NARP, cost elasticity exceeds $1 + F/C$. If NMRP falls short of NARP, cost elasticity falls short of $1 + F/C$ (consistent with points B and C of figure 1).

With the tools of the present chapter, we may proceed to the statistical estimation of (2.2). We then will conduct hypothesis tests to determine whether cost function structures (2.5) and (2.9), and a cost elasticity specified by (2.21), are consistent with the technology characterized by (2.2). If nonjointness in inputs, the structure consistent with (2.9), does not hold we will check for the presence of cost complementarities and anticomplementarities by using the expression for second order derivatives (2.10). If nonjointness in output prices, the structure consistent with (2.5), does not hold we can obtain estimates of conditional demand elasticities defined by (2.7), since the validity of (2.5) implies that inputs are nonsubstitutable. And, if cost elasticity does not take on the value specified by (2.21), we may obtain an estimate of its true value using the definition in (2.14).

Before proceeding to functional form, estimation, and hypothesis testing issues of chapter four, we diverge briefly in the next chapter to a discussion of the institutional framework of the present cooperative.

Chapter Endnotes

1. $F(y,x)$ is defined as $F(y,x) = \|x\| / \|\xi\| = 1 / \lambda$, where $\|(\cdot)\|$ designates the vector norm, $\xi = \min_\lambda \{\lambda x \mid \lambda x \in X(y)\}$ and λ is a scalar (Shephard, p. 67). x and ξ both lie along a ray from the origin. λ designates the smallest possible scalar such that when it is multiplied by x, a vector ξ, on the boundary of X(y), results. It is the minimal and hence efficient level of x such that output y can be produced. If $x \in X(y)$, then $x \geq \xi$ and $F(y,x) \geq 1$. If $x \notin X(y)$, then $x < \xi$ (since ξ is the smallest λx such that $\lambda x \in X(y)$, and in order for $x \notin X(y)$ x must be less than ξ; if ξ was smaller than x and $\xi \in X(y)$, required by definition, then $x \in X(y)$, which contradicts the assumption. Note that $\xi = x$ when $\lambda = 1$. When $F(y,x) \geq 1$, $x \in X(y)$. When $F(y,x) < 1$, $x \notin X(y)$.

2. In a perfectly competitive market, the price of output k would be reflected by its marginal cost, which for the case of NJO is $\partial C(y,w)/\partial y_k = \sum_i w_i \partial f^i(y)/\partial y_k$. Each $w_i \partial f^i(y)/\partial y_k$ represents the i^{th} factor's contribution to marginal cost or price of y_k, hence the term nonjoint in output prices.

3. The measure of ray scale economies is equivalently R = RAC/RMC. RAC = $C(ty^*)/t$, and RMC = $\partial C(ty^*)/\partial t$. To see that RMC = $\sum y^* \partial C/\partial y_k$ observe that

$$\partial C(ty^*)/\partial t = \sum_k (\partial C/\partial t y^*_k)(\partial t y^*_k/\partial t) = \sum_k y^*_k \partial C/\partial y_k.$$

Chapter 3

Institutional Context

The purpose of analyzing the cooperative's cost structure is to enable us to make statements about the structure of its technology. Here we review briefly the sort of production process in which the cooperative is engaged and derive a functional representation of its cost structure.

Following harvest, the cooperative members deliver their raw product (unprocessed fruits and vegetables) to the processing facility. During the first stage of processing (known as pack or wetpack), the raw product is weighed, graded, sorted, cleaned, and cut. The product is then blanched and either frozen or canned. Unofficial statistics indicate that 10% of the cooperative's pack is sold in frozen form to institutional buyers, 25% is canned and sold to retail and institutional outlets, and the remainder, approximately 65%, is frozen and placed in totes. Pack product placed in totes (large plastic bags within cardboard boxes) later undergoes the next stage of processing known as repack. Repack generally occurs after the harvest season, when the raw product delivery rate has slackened. During repack, some raw products may be blended with others. Blending may be of value to consumers and hence may affect price and payments to members (Stokstad 1989, p. 51).

Over the fiscal (FY) 1986-1990 period analyzed here, the firm's production activities were located at eight plants. Five of these plants were engaged in pack operations (plant#1 - plant#5), two were engaged in labelling and storage activities (plant#6 and #7), and one in repacking (#8). During the period FY86-FY90, variable costs incurred at plants 1 - 5 represented about 43% of the firm's total costs, with plants 6 and 7 accounting for 1% of total costs. Six percent of total costs were attributable to repack. The remainder, roughly 50%, were fixed costs, including the cost of capital, storage and transportation, insurance, administration, and management services.

In this study we focus on the pack processing stage and therefore on costs incurred at plants 1 - 5. These five plants are located in different geographic regions and handle approximately 25 types of raw farm products. Bean varieties (green, italian, and wax) and corn varieties (jubilee and sweet) account for the largest share (by weight) of pack output at most plants. The exception is at plant 5, which processes no corn and instead has beets as its second major product. Beans and corn each constitute roughly a third of total firm output volume. Other products are, in descending order of volume, carrots, broccoli, peas, cauliflower, strawberries, and beets.

Various inputs are utilized in the pack process and in other activities at plants 1 - 5. Examples are labor (direct and indirect), energy (electricity, gas, and fuel), containers, preserving ingredients, and trucking services. Over the period of our analysis, direct and indirect labor and energy costs constituted an average of 49% of annual total costs at plants 1 - 5.

3.1 Cost Function

In modelling the firm's cost structure, we regard labor and energy to be the relevant factors with respect to which the firm minimizes the cost of pack production at plants 1 - 5. C^O, the level of all other costs at plants 1 - 5 plus the cost at plants 6 - 8, is assumed to equal a quantity given by total labor and energy costs at plants 1 - 5 multiplied by a constant factor of proportionality α. Thus, C^O equals $\alpha C^P(\cdot)$ and the firm's cost equation takes the form

$$C^N = C^P(y^P,w) + C^O + F$$

where $C^P(\cdot)$, a function of input price and pack output levels, is the total labor and energy cost of pack production at plants 1 - 5; and F is the level of fixed non-plant-specific costs.

Pack production occurs separately at each of the five plants, so there essentially are five plant pack cost functions composing the pack total labor and energy cost function. Taking this into account, the firm's cost equation becomes

$$C^N = (1+\alpha)\Sigma_{i=1...5}\, C^P{}_i(y^{Pi},w_i) + F$$

where the sum $\Sigma_{i=1...5}\, C^P{}_i(y^{Pi},w_i)$ is the total labor and energy cost at plants 1 - 5.

It is impossible in this study to statistically estimate individual plant pack cost functions $C^P{}_i(\cdot)$. Estimating such plant-specific functions requires more data for each plant than we have. At present only five observations (five fiscal years) of data on each plant are available. However, we are able to estimate a pack cost function which is valid for a representative packing plant. Recall that our focus is on labor and energy costs. That is, we wish to estimate a labor and energy cost function

C^P_μ (y^P, w) valid for the representative packing plant. Hence, $5C^P_\mu(\cdot)$ is the level of total labor and energy cost at plants 1 - 5. Considering these additional matters, the firm- level cost equation takes the form

$$C^N = (1+\alpha)5C^P_\mu\,(y^P, w) + F.$$

3.2 Cooperative Objectives

In section 2.6.3, we showed that a measure of cost elasticity would help isolate cooperative firm goals. If $ty^{P\mu}$ is the mean level of pack output at plants 1 - 5, to which corresponds a raw product bundle tr^μ and final output bundle ty^μ, the net revenue function (2.17) is

$$NR = \mathbf{p} \bullet ty^\mu - (1+\alpha)5C^P_\mu(ty^{P\mu}, w) + F.$$

Along the ray, net average revenue NR/t is

$$\mathbf{p} \bullet y^\mu - (1+\alpha)5C^P_\mu(ty^{P\mu} w)/t + F/t$$

and net marginal revenue $\partial NR/\partial t$ along the ray is

$$\mathbf{p} \bullet y^\mu - (1+\alpha)5\partial C^P_\mu(ty^{P\mu}, w)/\partial t.$$

Thus, NMRP equals NARP along the ray if

$$(1+\alpha)5\partial C^P_\mu(ty^{P\mu}, w)/\partial t = (1+\alpha)5C^P_\mu(ty^{P\mu}, w)/t + F/t.$$

Multiplying both sides by $t/(1+\alpha)5C^P_\mu(ty^{P\mu} w)$ gives

$$[\partial C^P_\mu(ty^{P\mu}, w)/\partial t][t/C^P_\mu(ty^{P\mu}, w)] = 1 + F/(1+\alpha)5C^P_\mu(ty^{P\mu}, w). \quad (3.1)$$

The left-hand side of (3.1) is an expression for cost elasticity along a ray $y^{P\mu}$, and the right-hand side is the value of that elasticity where NMRP equals NARP. If the actual cost elasticity falls below the value in (3.1), NMRP falls below NARP.

NMRP exceeds NARP if actual elasticity exceeds the value in (3.1). The expression in (3.1) enables us to test the hypothesis that NMRP equals NARP at a firm's point of operation. F and α are treated for this purpose as exact non-sample information. F is the average level of fixed costs over the five-year period, and α is the factor of proportionality equal to the average value of (C^O/C^P) over the sample period.#

3.3 Outputs and Input Prices

In our study $\mathbf{y}^P$ is a 3-dimensional vector. The first component, y_1, is an aggregate of corn and beets. The second component, y_2, is an aggregate of beans, and the third component, y_3, is an aggregate of all other pack output. These aggregates are formed using the Laspeyres quantity index formula

$$L = \Sigma_i \, p^0_i y^1_i / \Sigma_i \, p^0_i y^0_i \qquad (3.2)$$

where i indexes the fruit or vegetable composing the aggregate (Varian, p. 127). For our purposes each p_i is the unit pack cost of the ith fruit or vegetable as given by the firm's Absorption Summary, and 0 and 1 index the base and current periods, respectively.

The input price vector **w** is 2-dimensional, the first component, w_1, being the aggregate labor price (composed of direct and indirect labor prices), and the second component, w_2, the aggregate energy price (composed of electricity price and fuel-gas

#The value of F is proprietary information and is therefore excluded from the text. α is approximately 2.21.

price index). Labor and energy price indexes are constructed using the Tornqvist formula

$$\ln P = 0.5\Sigma_i\, [(p^0{}_i x^0{}_i/\mathbf{p}^0\bullet\mathbf{x}^0) + (p^1{}_i x^1{}_i/\mathbf{p}^1\bullet\mathbf{x}^1)]\ln[p^1{}_i/p^0{}_i] \qquad (3.3)$$

where 0 and 1 index base and current periods and i indexes the input component forming the aggregate (Fuss, p. 96). We consider energy use to be held in fixed proportion to capital use so that energy price is, loosely speaking, a proxy for the price of capital. Hence, observing optimal behavior with respect to energy use is an indicator of optimal behavior with respect to capital use.

For convenience, $C^P{}_\mu(\cdot)$ is referred to below simply as C(y,w). The next chapter presents the functional form we will use for modelling C(y,w), the various algebraic restrictions needed to test the relevant hypotheses, and the statistical estimation procedure.

Chapter 4

Estimation

Econometric techniques are used to conduct the empirical analysis for this research. Estimating a cost function requires selecting a functional form, choosing an estimation method, and selecting a corresponding technique for conducting hypothesis tests. Choice of functional form is influenced by the hypotheses we wish to maintain and those we wish to test statistically. Here we utilize the method of maximum likelihood to estimate a translog approximation to a cost function and employ the likelihood ratio technique for hypothesis testing.

In section one we discuss how parameters of the translog form are restricted so that certain hypotheses may be maintained and others tested. The objective is to test statistically whether the data support a cost function consistent with the structures of nonjointness in output prices (NJO) and nonjointness in inputs (NJI), and whether it exhibits cost elasticity of a specified degree. If NJO does not hold, we may obtain measures of own and cross price elasticities of demand. If NJI does not hold, we may obtain measures of cost complementarity. A discussion of the method used to check regularity conditions completes the first section.

Estimation technique is considered in the chapter's second section. The chapter closes with a brief note on some limitations of the maximum likelihood method.

4.1 Translog Form

The translog form is a second-order numerical approximation to an arbitrary function; it approximates the slope and curvature of a function at the point of approximation, namely the vector of sample means for all variables. All regressor variables are scaled by the sample mean.

4.1.1 Parameter Restrictions

The translog is flexible in that it does not *a priori* restrict the cost structure to be consistent with nonjointness in output prices, nonjointness in inputs, or a specified constant cost elasticity. The latter structures can be tested for statistically by an appropriate restriction of parameters.

4.1.1.a Maintained Hypotheses

Twice-continuous differentiability of the cost function with respect to input prices, R1, is assumed if translog specification is interpreted as a second-order Taylor series approximation to the true cost function. The translog's parameters are first and second order derivatives of a monotonic (logarithmic) transformation of the approximate cost function with respect to logarithmic transforms of the variables (Chambers, p. 167). Condition R2, nonnegativity, holds because all costs, output levels, and input prices are positive. Symmetry of cross-partial derivatives holds by Young's Theorem (p. 162).

For a translog function with m outputs and n inputs, there are m + n first-order parameters, m(m+1)/2 + n(n+1)/2 + mn second-order parameters, and one scale parameter (Brown et al., p. 259). Assuming R1, R2, and symmetry of cross partial derivatives, the translog cost function for 2 inputs and 3 outputs is

$$\ln C(y_1, y_2, y_3, w_1, w_2) = \alpha 0 + \alpha_1 \ln w_1 + \alpha_2 \ln w_2 + \beta_1 \ln y_1 + \beta_2 \ln y_2 + \beta_3 \ln y_3 + 0.5(\alpha_{11}\ln^2 w_1 + \alpha_{22}\ln^2 w_2) + \alpha_{12}\ln w_1 \ln w_2 + 0.5(\beta_{11}\ln^2 y_1 + \beta_{22}\ln^2 y_2 + \beta_{33}\ln^2 y_3) + \beta_{12}\ln y_1 \ln y_2 + \beta_{13}\ln y_1 \ln y_3 + \beta_{23}\ln y_2 \ln y_3 + \delta_{11}\ln w_1 \ln y_1 + \delta_{12}\ln w_1 \ln y_2 + \delta_{13}\ln w_1 \ln y_3 + \delta_{21}\ln w_2 \ln y_1 + \delta_{22}\ln w_2 \ln y_2 + \delta_{23}\ln w_2 \ln y_3 \qquad (4.1)$$

where ln indicates the natural logarithm function and y_1, y_2, y_3, w_1, and w_2 are as defined in section 3.3.

Linear homogeneity in input prices (R4) reduces the number of free parameters in (4.1) to (m+n+1)(m+n)/2 by imposing the following m+n+1 linear restrictions

$$\alpha_1 + \alpha_2 = 1; \alpha_{11} + \alpha_{12} = 0 ; \alpha_{12} + \alpha_{22} = 0;$$
$$\delta_{11} + \delta_{21} = 0; \delta_{12} + \delta_{22} = 0; \delta_{13} + \delta_{23} = 0. \qquad (4.2)$$

An argument justifying (4.2) is as follows. Linear homogeneity of cost in input prices implies that an equiproportionate change in input prices results in a change in total cost by that same proportion, or

$$C(\lambda w_1, \lambda w_2, y) = \lambda C(w_1,w_2,y). \tag{4.3}$$

Letting $\lambda = 1/w_2$, (4.3) takes the form

$$C(w_1/w_2, y) = (1/w_2)C(w_1,w_2,y) \tag{4.4}$$

(Evans and Heckman, p.257). Taking the natural logarithm of both sides of (4.4) and using the translog form to represent the left-hand side, restrictions (4.2) are met with the specification

$$\begin{aligned} \ln C(w_1, w_2, y) - \ln w_2 &= \alpha_0 + \alpha_1(\ln w_1 - \ln w_2) + \beta_1 \ln y_1 + \beta_2 \ln y_2 + \beta_3 \ln y_3 + \\ &0.5\alpha_{11}(\ln w_1 - \ln w_2)^2 + 0.5(\beta_{11}\ln^2 y_1 + \beta_{22}\ln^2 y_2 + \beta_{33}\ln^2 y_3) + \beta_{12}\ln y_1 \ln y_2 + \\ &\beta_{13}\ln y_1 \ln y_3 + \beta_{23}\ln y_2 \ln y_3 + \delta_{11}(\ln w_1 - \ln w_2)\ln y_1 + \delta_{12}(\ln w_1 - \ln w_2)\ln y_2 + \\ &\delta_{13}(\ln w_1 - \ln w_2)\ln y_3 \end{aligned} \tag{4.5}$$

which is identical to (4.1) with restrictions (4.2) imposed (Denny and Pinto, p. 260).

By Shephard's lemma, $\partial C(y_1, y_2, y_3, w_1, w_2)/\partial w_1$ equals the cost-minimizing conditional demand for labor. We may use this relation to derive the labor share equation by noting that $[(\partial C/\partial w_1)w_1] / C$ is the expression for labor's share of total cost. But this expression is equivalent to $\partial \ln C/\partial \ln w_1$, which for the translog form in (4.5) is

$$\partial \ln C(y_1,y_2,y_3,w_1,w_2)/\partial \ln w_1 = \alpha_1 + \alpha_{11}(\ln w_1 - \ln w_2) + \delta_{11}\ln y_1 + \delta_{12}\ln y_2 + \delta_{13}\ln y_3 \tag{4.6}$$

Translog cost and labor share equations given by (4.5) and (4.6) are the unrestricted specifications for the purposes of this study.

4.1.1.b Tested Hypotheses

In order to conduct hypothesis tests, it is necessary to derive parameter restrictions corresponding to NJO, NJI, and the critical cost elasticity measure. We conduct individual likelihood ratio tests for each restriction and all possible combinations of restrictions. A likelihood ratio test compares the log-likelihood of the unrestricted model to the log-likelihood of the restricted model. The test statistic, namely the restricted log-likelihood value subtracted from the unrestricted log-likelihood value, multiplied by two, is distributed as a χ^2 random variable with degrees of freedom equal to the number of independent restrictions. There is no rule to guide the choice of significance level in such a test.

Cooperative Objective

From section 3.2, testing whether the cooperative operates where NMRP equals NARP can be reduced to a test of the form

$$\Sigma_k \, \partial \ln C(\cdot)/\partial \ln y_k = F/[(1+\alpha)5C(\cdot)] + 1 \tag{4.7}$$

where the left-hand side equals the cost elasticity along an output ray. The measure of cost elasticity from the translog is

$$\Sigma_k \, \partial \ln C(\cdot)/\partial \ln y_k = \Sigma_k \, [\beta_k + \beta_{kl} \ln y_l + \Sigma_i \, \delta_{ki} \ln w_i] \tag{4.8}$$

(Brown et al, p. 261-2). If we evaluate (4.8) at the sample mean for all variables, all terms involving regressor variables drop out. Recall that the model is specified with all variables scaled by their sample mean. From (4.8), cost elasticity κ therefore reduces to the sum of parameters

$$\kappa = \beta_1 + \beta_2 + \beta_3. \tag{4.9}$$

We may use (4.9) to conduct the cooperative objective test by setting κ equal to the right-hand side of (4.7):

$$F/[(1+\alpha)5C(\cdot)] + 1 = \beta_1 + \beta_2 + \beta_3. \tag{4.10}$$

$C(\cdot)$ in (4.10) is the estimated level of total cost. Taking the exponential of the translog form in (4.5), then evaluating all variables at the sample mean, reduces the expression for predicted cost to $\exp(\alpha_0)$. Substituting $\exp(\alpha_0)$ into (4.10), and rewriting in terms of parameter β_3, we obtain the parameter restriction

$$\beta_3 = F/[\exp(\alpha_0)(1+\alpha)5] + 1 - \beta_1 - \beta_2. \tag{4.11}$$

We restrict $\beta3$ in (4.5) to equal the expression in (4.11) and conduct a likelihood ratio test with one degree of freedom. As noted in section 3.2, F and α are considered for this purpose to be exact non-sample information.

Nonjointness Tests

Parameter restrictions needed to test for NJO and NJI are derived in a similar manner. NJI requires that all off-diagonal terms of the output hessian matrix be zero --relation (2.10)--and NJO requires the input price hessian matrix be the null matrix--

relation (2.6). Denny and Pinto (p.256-7) show that testing for nonjointness in inputs at the point of approximation requires restrictions of the form

$$\beta_k\beta_l = - \beta_{kl} \quad \forall \text{ k,l and } k \neq l. \tag{4.12}$$

For our analysis there are three such restrictions.

To derive the restrictions for NJO, first exponentiate the translog cost function in (4.5). This gives an expression for cost. Taking the derivative of this cost function first with respect to the ith input price and then with respect to the jth input price, one obtains an expression involving parameters and variables. However, if all variables are evaluated at the sample mean, the region in which the hypothesis test is valid, the resulting expression for the second-order derivative is

$$\exp(\alpha_0)[(\alpha_j/w_j)(\alpha_i/w_i) + \alpha_{ij}/w_iw_j] \quad \forall \text{ i,j}. \tag{4.13}$$

Each second order derivative from (4.13) equals zero when $\alpha_j\alpha_i + \alpha_{ij}$ equals zero, since $w_i,w_j > 0$ and $\exp(\alpha_0) > 0$. For the translog approximation in (4.5), this is met by the single parameter restriction

$$\alpha_{11} = \alpha_1(1-\alpha_1) \tag{4.14}$$

since (4.5) must be consistent with linear homogeneity restrictions (4.2).

4.1.1.c Economic Measures

The point of modelling a firm's cost structure is to gain insight into relationships between inputs and outputs in the production process. Measures of interest here are price elasticities of conditional demand and cost complementarity.

Own price elasticities of demand can be calculated from the translog cost function according to the formula

$$\epsilon_{ii} = [\alpha_{ii}/(\partial \ln C/\partial \ln w_i)] + \partial \ln C/\partial \ln w_i - 1 \qquad (4.15)$$

(Young et al, p. 19). In our analysis, (4.15) holds for i=1,2. Evaluating input shares (4.6) at the sample mean for all variables, (4.15) is expressible entirely in terms of the parameters as

$$\epsilon_{ii} = (\alpha_{ii}/\alpha_i) + \alpha_i - 1. \qquad (4.16)$$

It is useful to utilize (4.16) since we may obtain an estimate of its asymptotic standard error, the square root of the large-sample variance of a nonlinear function of parameters, using the approximate formula of Kmenta (p.486).

Parameter restriction (4.12) used for the test of NJI is a measure of the value of the second order derivative of cost with respect to outputs. The expression for the cross-partial derivative in terms of parameters is

$$\partial^2 C/\partial y_k \partial y_l = \beta_k \beta_l + \beta_{kl}. \qquad (4.17)$$

Checking for complementarities requires knowing the sign of these derivatives. From section 2.4.2.a, relation (2.12) needs to hold for k=1,2 and l=3. The relevant binary partition of the product set is into product set 1 = $\{y_1,y_2\}$, and product set 2 = $\{y_3\}$, a partition into major and minor products. We may use the approximate variance formula to obtain an estimate of the standard error of the expression in (4.17).

4.1.2 Checking Regularity Conditions

As noted in section 4.1.1.a, conditions R1, R2, and R4 are maintained in this study. However, for the translog cost function approximation to be a valid representation of the underlying technology, it is necessary to check whether R3, R5, and R6 are satisfied by our estimates. For monotonicity in input prices (R3) to hold, predicted input shares must be positive at each observation point.

Concavity in input prices (R5) is checked by noting whether the hessian matrix of second order derivatives of the cost function with respect to input prices is negative semidefinite (Young et al, p. 19, 21). For our study the hessian is two-dimensional, so the characteristic equation is a polynomial of degree two with possibly two distinct roots (Chiang, pp. 326-7, 330). These two roots can be calculated using the quadratic formula and must be nonpositive at each observation point. Monotonicity in outputs (R6) is checked by ensuring that predicted marginal costs for each output are nonnegative at each observation point.

4.2 Estimation Technique

The method of maximum likelihood is utilized to estimate the translog approximation to the cost function and share equation. It is necessary to explicitly impose an equality restriction upon those parameters which are common to the two equations. The likelihood function specifies the distribution from which our sample has been drawn. Concentrating the likelihood function enables use of the generalized

Gauss-Newton convergence criterion which is employed by the Time Series Processor (TSP version 4.1) estimation procedure.

4.2.1 Multiple Equation Model

A model with multiple equations may be written as

$$y_{ij} = x_{ij}^T \psi_j + \epsilon_{ij} \tag{4.18}$$

Cramer (p.100). x_{ij} in (4.18) denotes the ith vector of observations on the k_j regressor variables in equation j. Symbol ψ_j represents the k_j - dimensional coefficient vector for the jth equation. y_{ij} and ϵ_{ij} are, respectively, the ith observation of the dependent variable, and the error corresponding to observation i, both in the jth equation ($i=1...n$ and $j=1...g$). For the present analysis, $n=25$ and $g=2$. Equation 1 is the cost equation and equation 2 is the labor share equation.

Model (4.18) may be more usefully written as

$$y_i = X_i \psi + \epsilon_i \tag{4.19}$$

where $X_i = \text{diag}[x_{1i}^T, x_{2i}^T]$ and is of dimension 2xp; $\psi^T=[\psi_1,\psi_2]$ and is of dimension 1xp; and $y^T_i=[y_{1i},y_{2i}]$ and $\epsilon^T_i=[\epsilon_{1i},\epsilon_{2i}]$ are each of dimension 1x2 (Cramer, p.101). In (4.19), $\epsilon_i \sim N(0,\Sigma)$, where Σ is a 2x2 covariance matrix. That is, each equation has unique homoskedastic error variance (σ_{11} for equation 1 and σ_{22} for equation 2) and the two equations have a fixed disturbance covariance σ_{12}.

Regressor variables may be unique to a particular equation or appear in different equations; that is, $\Sigma_j k_j = p$, where p is the total number of not necessarily unique coefficients to be estimated. If there are r linear restrictions among the p coefficients of ψ, r elements of ψ may be expressed as linear combinations of the (p - r) unique elements as follows

$$\psi = F\psi_1 \tag{4.20}$$

(Cramer, p. 117). ψ_1 is the (p - r)-dimensional vector of unique parameters and F is a p x (p-r)-dimensional matrix expressing these restrictions. For example, the unrestricted translog model (4.5) with (4.6) would require components of ψ in (4.20) to be

$\psi_1=[\alpha_0, \alpha_1, \alpha_{11}, \beta_1, \beta_2, \beta_3, \beta_{11}, \beta_{22}, \beta_{33}, \beta_{12}, \beta_{13}, \beta_{23}, \delta_{11}, \delta_{12}, \delta_{13}]$ and

$\psi_2 = [\alpha_1^s, \alpha_{11}^s, \delta_{11}^s, \delta_{12}^s, \delta_{13}^s]$

where ψ_2 are the parameters of the labor share equation.

Restriction matrix F in (4.20) equates the elements of ψ_2 with their counterparts in ψ_1. Here there are a total of p=20 parameters, and r=5 linear equality restrictions, resulting in 15 unique parameters in the unrestricted model. As is evident from (4.5) and (4.6), it is necessary to impose these restrictions since the regressors corresponding to the share equation parameters are not the same regressors as those for the cost equation parameters.

4.2.2 Log-Likelihood Function

Model (4.19) with cross equation restrictions (4.20) may be written

$$y_i = X_i F \psi_1 + \epsilon_i \tag{4.21}$$

(Cramer, p.116). Assuming a normal distribution, the likelihood function for (4.21) is written

$$\begin{aligned} \text{LogL}(\psi_1,\Sigma) &= -(ng/2)\ln 2\pi - (1/2)\ln|\Sigma\otimes I_n| - (1/2)\mathbf{e}^T(\Sigma^{-1}\otimes I_n)\mathbf{e} \\ &= -(ng/2)\ln 2\pi - (n/2)\ln|\Sigma| - (1/2)\text{tr}[S\Sigma^{-1}] \end{aligned} \tag{4.22}$$

(Judge et al 1988, p. 553). **e** in (4.22) is the ng- or 50-dimensional vector of stacked residuals of the cost and labor share equations as defined by (4.21). S is a gxg (here 2x2) dimensional matrix with typical element $\mathbf{e}_m{}^T\mathbf{e}_q$ (m,q=1,2). For example, the residual vector for equation 1 would be $\mathbf{e}_1(\psi_1) = \mathbf{x}_1 F\psi_1 - \mathbf{y}_1$.

Since it is not possible to obtain estimates of ψ_1 with an unknown Σ, it is necessary to maximize the log-likelihood function with respect to Σ^{-1} to obtain a maximum likelihood estimator Σ^{ML} of Σ. The resulting Σ^{ML} is unique regardless of whether the log-likelihood is maximized with respect to Σ or Σ^{-1} (Harvey p. 99). For this purpose (4.22) may be more usefully written

$$\text{LogL}(\psi_1,\Sigma) = -(ng/2)\ln 2\pi + (n/2)\ln|\Sigma^{-1}| - (1/2)\text{tr}[S\Sigma^{-1}] \tag{4.23}$$

and then differentiated with respect to Σ^{-1} to obtain

$$\partial \text{LogL}(\psi_1,\Sigma)/\partial\Sigma^{-1} = (n/2)\Sigma - (1/2)S = \mathbf{0} \tag{4.24}$$

using certain differentiation results (Judge et al 1988, p. 553). Thus $\Sigma^{ML} = S/n$, which is substituted into (4.22) to obtain the concentrated log-likelihood function

$$\begin{aligned} \text{LogL}(\psi_1) &= -(ng/2)\ln 2\pi - (n/2)\ln|S/n| - (1/2)\text{tr}[I_g/n^{-1}] \\ &= -(ng/2)\ln 2\pi - ng/2 + (n/2)\ln(n) - (n/2)\ln|S| \end{aligned} \tag{4.25}$$

(Cramer p. 117).

4.2.3 TSP Procedure

In practice, estimation occurs by supplying an initial estimate ψ^0_1 of ψ_1. The initial estimate is used to calculate an initial residual vector $\mathbf{e}(\psi_1)$. This residual vector then is used to calculate S, which in turn can be employed to obtain a new estimate ψ^1_1 of ψ_1 using the formula for the generalized Gauss-Newton convergence criterion of the form

$$\psi^{t+1}_1 = \psi^t_1 + [\Sigma_{i\epsilon n}\, z_i(\Sigma^{ML})^{-1}z_i^T]^{-1}\Sigma_{i\epsilon n} z_i(\Sigma^{ML})^{-1}e_i \tag{4.26}$$

to determine whether updates of ψ_1 are approaching some limit (Harvey, p. 137; *TSP User's Guide*, p. 79). [z_i in (4.26) equals $-\partial e_i/\partial\psi_1$.] The TSP procedure is to employ an initial estimate of ψ_1 to determine e_i and z_i, then to regress e_i on z_i and obtain a new estimate of ψ_1 (Harvey p. 134, 137). A check is made to determine if the objective function, the negative of the log determinant of the residual covariance matrix S, is increasing, as this determines whether (4.25) is increasing also (*TSP Reference Manual*, p. 158). The validity of updating ψ_1 by means of a regression of e_i on z_i is clear from observing that the second term of (4.26) resembles the expression for a generalized least squares estimator (Harvey, pp. 70, 137).

4.3 Potential Sources of Error

Maximum likelihood estimation requires assuming that error terms ϵ_i of (4.21) are independent and multinormally distributed. Nonnormality of errors might be an important issue for the estimation of cost as a function of exogenous output levels and input prices. Observed costs must be greater than or equal to minimum costs, so that each disturbance term should be nonnegative ($\epsilon_i \geq 0$). The disturbance thus may be assumed to follow an exponential distribution. Or it may consist of two error components, one (exponentially distributed) component measuring the deviation from cost- minimizing behavior, and the other (normally distributed) component measuring the effects of all factors beyond a firm's control (Judge et al 1980, pp. 302-3). Evidence from frontier production function applications suggests that in the latter case the symmetric component may dominate the entire composite error term. If it does dominate, estimates may not be sensitive to the added nonnormal component. Because the estimator properties in this approach are unknown, normality is assumed in the present study.

The data set used in this study is a pooled one of 5 cross sections (plants) over 5 years. The 25 observations together are treated as one sample. This means that the 25 observations on the dependent variable, say cost, are 25 realizations of cost-minimizing behavior, that is, 25 sample values of the 25 cost random variables. Each cost random variable is identified by time and place, for example the cost at plant 1 in year 1. Each cost random variable has its own distribution with mean $x_{1i}^T\psi_1$ and variance σ_{11}^2. Associated with each realization there is an error; there are 25 such errors distributed $N(0,\sigma_{11}^2)$. Thus, each plant in each year has its own cost distribution, and if the experiment were to be repeated with identical initial conditions, i.e. identical x_{1i}, the result would be another set of 25 realizations of the dependent variables identified by a plant and year.

Note that plants and time periods are the same for the first and second realizations; in other words, time is assumed to be circular and can be repeated, enabling the second realization. For the data used here, increasing the sample size occurs by adding another five observations from a new year. The population from which this supposed random sample is drawn consists of all the 5 plants in all possible years that the firm could potentially operate. The population is finite; the firm has a finite life span. It is of no consequence that this fixed population size is unknown.

The assumption that errors ϵ_i are independent with variance-covariance matrix Σ may prove to be of greater concern as it implies an assumption of homoskedasticity and zero autoregression. Autoregressive errors might, for example, capture adjustments in cost minimizing behavior due to the learning process. It would seem more fruitful to challenge the homoskedasticity assumption since it is known *a priori* that our realizations are actually for 5 separate plants, each with a separate technological structure.

Unfortunately, taking account of variance inequality among five subsets of the 25 observations is not possible since the number of observations per subset, five, is far less than the number of parameters to be estimated. We might have wished to check whether product-diversified plants have had less cost variation than product-specialized plants. But even this is not possible with the quantity of data available.

Chapter 5

Results

A translog approximation to a cost function, valid for labor and energy costs at a representative packing plant, was estimated in a two equation model via the method of maximum likelihood. Hypothesis tests regarding nonjointness in output prices and nonjointness in inputs, and regarding a cost elasticity level consistent with regulation of delivery quantities, were conducted using the likelihood ratio technique. NJO and NJI individually were not rejected according to this test, but together they were rejected. The hypothesis that the cooperative regulates delivery quantities was rejected.

5.1 Estimated Model

The estimated translog approximation (4.5) to the firm's labor and energy pack cost function is found in table 1. An immediate observation is that parameters corresponding to second-order derivatives have high standard errors. However, the estimated model does satisfy regularity conditions R1-R6 (section 2.3.1) necessary for it to be a valid representation of technology. Recall, from section 4.1.1.a, that R1, R2, and R4 are maintained in the model specification given by (4.5) and (4.6). Predicted shares are positive at each observation point, so that R3 is satisfied (table 5). Characteristic roots at each observation point are less that 10^{-7}; thus, R5 is satisfied

Table 1

Parameter Estimates: Unrestricted Translog Model

Regressor Variable	Parameter	Parameter Estimate	Asymptotic Standard Error
	α_0	1.46	0.08
$\ln w_1 - \ln w_2$	α_1	0.81	0.01
$\ln y_1$	β_1	0.65	0.21
$\ln y_2$	β_2	0.45	0.19
$\ln y_3$	β_3	0.16	0.08
$0.5(\ln w_1 - \ln w_2)^2$	α_{11}	0.12	0.02
$0.5\ln^2 y_1$	β_{11}	-0.64	0.45
$0.5\ln^2 y_2$	β_{22}	0.23	0.19
$0.5\ln^2 y_3$	β_{33}	0.02	0.13
$\ln y_1 \ln y_2$	β_{12}	0.02	0.28
$\ln y_1 \ln y_3$	β_{13}	0.21	0.19
$\ln y_2 \ln y_3$	β_{23}	0.07	0.19
$(\ln w_1 - \ln w_2)\ln y_1$	δ_{11}	0.03	0.01
$(\ln w_1 - \ln w_2)\ln y_2$	δ_{12}	-0.001	0.01
$(\ln w_1 - \ln w_2)\ln y_3$	δ_{13}	-0.01	0.001

(table 6). Predicted marginal costs are positive at all observation points for y1 and y2, and at all but three observations for y3 (table 7). Hence, monotonicity in outputs, R6, is satisfactorily met. From section 2.3.1, meeting conditions R1-R6 ensures that there exists a technology underlying the cost function to which the estimated unrestricted translog model is an approximation.

5.2 Hypothesis Tests

We conducted likelihood ratio tests comparing the unrestricted model (4.5) and (4.6) to models restricted for NJO, NJI, and NJO with NJI. Additionally, we tested whether our data supported a cost function with a cost elasticity specified by (4.7); this null hypothesis is referred to as "coop" in table 2. As we were unable to take into consideration the nonnested nature of the hypothesis tests, we conducted tests of all possible combinations of the three hypotheses, in total seven tests. Table 2 presents the calculated test statistics and critical χ^2 values at the five percent level of significance.

A calculated test statistic which exceeds the χ^2 value causes us to reject the null hypothesis; that is, the data are judged not to suppport the null as well as they support the alternative (unrestricted) model. Choosing the 5% significance level means that we are willing to reject the null hypothesis 5% of the time when in fact it is true (commit a type one error).

The likelihood ratio test causes us not to reject NJO or NJI individually but to reject NJO and NJI structures taken together. A decision not to reject the

Table 2

Hypothesis Tests:

Calculated and Critical Chi-Square Values

Null Hypothesis[a]	Likelihood Ratio	Chi-Square (0.05)	Degrees of Freedom
NJO	2.97	3.84	1
NJI	5.73	7.82	3
NJO and NJI	10.44	9.49	4
Coop	907.89	3.84	1
NJO and Coop	106.35	5.99	2
NJI and Coop	106.03	9.49	4
NJI, NJO and Coop	112.26	11.07	5

[a/] NJI is nonjointness in inputs; NJO is nonjointness in output prices; Coop refers to the hypothesis of a cost elasticity value consistent with the point where NMRP=NARP.

structures individually might constitute a type two error, that is, failure to reject the null when in fact it is false, so that rejecting NJO and rejecting NJI would be the accurate decision. At the same time, rejecting NJO and NJI together might constitute a type one error, so that the accurate decision would have been the one reached by the individual jointness tests. In the absence of more data or a better hypothesis testing framework, one which would not lead to such conflict, we are uncertain which structure is truly consistent with the data.

The null hypothesis that the cooperative regulates delivery quantities requires the firm operate where NARP equals NMRP and thus to have cost elasticity specified by (4.7). This hypothesis is rejected with the likelihood ratio test (table 2). The extremely low log-likelihood value enables us to be somewhat confident about the result.

For the sake of completeness, we conducted individual tests of every possible combination of the three null hypotheses. That is, three additional tests of combined null hypotheses were conducted. All of these "combined" hypothesis tests were rejected with the likelihood ratio method (table 2).

5.3 Economic Measures

Table 3 presents the firm's conditional input demand elasticities and table 4 tabulates second order derivatives of cost with respect to outputs. Since we are uncertain whether the NJO and NJI structures are consistent with the data, we also calculated input demand elasticities for the NJI model and cost complementarity

Table 3

Price Elasticities of Input Demand at Sample Mean:
Unrestricted and Nonjoint-in-Inputs (NJI) Models

Maintained Model[a]	Own Price Elasticity	Standard Error
<u>Unrestricted</u>		
ϵ_{11}	-0.04	0.02
ϵ_{22}	-0.19	0.09
<u>NJI</u>		
ϵ_{11}	-0.05	0.02
ϵ_{22}	-0.22	0.09

a/ ϵ_{ij}=demand elasticity of ith input due to jth input price change; 1=labor; 2=energy; NJI=nonjoint in inputs.

Table 4

Cost Complementarities at Sample Mean:
Unrestricted and Nonjoint-in-Output Prices (NJO) Models

Maintained Model[a]	Cost Complementarity	Standard Error
Unrestricted		
C_{13}	0.32	0.23
C_{23}	0.001	0.22
NJO		
C_{13}	0.28	0.21
C_{23}	0.13	0.24

a/ $C_{kl} = \partial C/\partial y_k \partial y_l$; 1 = corn aggregate; 2 = bean aggregate; 3 = minor product aggregate; NJO = nonjoint in output prices.

measures for the NJO model.

Elasticity values in table 3 do not differ greatly across models and are significantly different from zero. Once again, the calculated standard error is the square root of the large-sample variance of a random variable (a function of the parameters). That is, the random variable (a nonlinear combination of normal random variables) is assumed to be normally distributed. The calculated test statistic is the value of the random variable divided by the value of its standard error; the critical test statistic value at the 5% level of significance is 1.96.

Estimated conditional demand elasticities indicating responses to changes in own price are greater for energy (and perhaps capital) than for labor. For the two-input cost function, we can infer labor demand responses to an energy price change and energy demand responses to a labor price change. From the unrestricted model, percentage change in labor demand due to a 1% energy price change is only 4%, while percentage change in energy demand due to a 1% labor price change is 19%.

From table 4, cost anti-complementarities prevail at the sample mean for both the unrestricted and NJO models but the estimates are not significantly different from zero. Recall that parameters corresponding to second-order output terms were not significant (table1); high standard errors for second-order output terms characterize the estimated NJO model as well.

At the sample mean, the value of cost elasticity κ is fairly constant across models. κ equals 1.27 for both the unrestricted and NJO models, 1.20 for the NJI model, and 1.19 for the NJO-NJI model. These values are not tabulated.

5.4 Interpretation

Calculating all three economic measures at each observation point, where an observation point corresponds to a plant in a particular year, permits us to make an intra-plant comparison of cost structure.

5.4.1 Cost Complementarities

Failure to establish cost complementarities does not imply that scope economies do not exist with respect to a binary partition into major and minor products. The data failed to satisfy what is merely a sufficient condition for scope economies (section 2.4.2.a). In addition, parameter estimates do not permit us to reach a reliable conclusion in this matter. The expression for cost complementarity (4.17) is an expression involving parameters corresponding to both first- and second-order output terms. Relatively high standard errors of estimated parameters β_{KL} cause the standard errors of (4.17) to be relatively high as well.

Calculating the cross partial derivatives C_{KL} at each observation point (table 8) reveals useful information. It is interesting that anti-complementarities prevail between major and minor products at plants 1 and 4; the sufficient condition for diseconomies of scope for the given partition is satisfied. These plants are the largest and most diversified in pack production. Plants 2, 3, and 5 seem to exhibit some cost complementarity between beans and minor products. At these plants, output of minor products is relatively insignificant. Presence of cost complementarity among beans and minor products may be due to the presence of a shared factor such as labor or energy input (a proxy for capital).

The minor products at plant 2 are zuchinni and yellow squash; at plant 3 they are squash and carrots; and at plant 5 they are winter squash, carrots, and plums. Discussion with plant production personnel revealed that, among the three aggregate outputs, a conveyor belt is the most common piece of shared equipment. Bean and corn processing requires highly specialized equipment. Some corn equipment (notably blanchers) is shared with one minor product (namely carrots), whereas no bean equipment is shared with minor products. Perhaps the source of complementarity between beans and minor products at plants 2, 3, and 5 is shared labor input.

5.4.2 Cost Elasticity

Cost elasticities at each observation point are presented in table 9. Cost elasticity is a measure of percentage change in cost due to a unit percentage change in output. Output volume is on average highest at plant 1, followed by plants 4, 3, 2, and 5. Cost elasticity is on average greatest for plant 4, followed by plants 1,3,5, and 2 in descending order. Plants 1 and 4, with the highest volume, have the highest cost elasticities, while plants 2 and 5, with the lowest volume, have the lowest cost elasticities.

Elasticities at plants 2 and 5 are, in most years, less than or close to one. From (2.14), this indicates that they may be operating at a point of increasing returns to scale. Otherwise, most cost elasticities are greater than one, implying locally decreasing returns to scale for the pack process. Decreasing returns to scale prevails at the sample mean, where cost elasticity equals 1.27 in the unrestricted model.

5.4.3 Demand Elasticities

Input demand elasticities at sample mean are useful indicators of a representative plant's input demand response to input price changes. Evaluating demand elasticities at each observation point (table 10) indicates that in almost all cases $|\epsilon_{11}| < |\epsilon_{22}|$, implying that labor demand is consistently less responsive than is energy demand to price moves. For the two-input cost function, labor's response to own-price change (ϵ_{11}) is the negative of labor's response to energy price change, that is, $\epsilon_{11} = -\epsilon_{12}$. The responsiveness of labor demand to own as well as cross-price changes can be known from ϵ_{11}, and the responsiveness of energy demand to own as well as cross-price changes can be known from the value of ϵ_{22}, since $\epsilon_{22} = -\epsilon_{21}$.

The ϵ_{11} value says that a one percent rise in the aggregate labor price induces a $|\epsilon_{11}|$ x 100 % decline in aggregate labor demand, and the ϵ_{22} ($\epsilon_{21} = -\epsilon_{22}$) value says that the rise in labor price will induce a $|\epsilon_{22}|$ x 100 % rise in aggregate energy demand. Aggregate labor price given by the tornqvist index, formula (3.3), is

$$W^L = (w^1_D/w^0_D)^{SD} \cdot (w^1_I/w^0_I)^{SI}$$

where SD and SI (SD + SI = 1) indicate, respectively, direct and indirect labor's share of total labor cost averaged over current and base (FY88 at plant1) periods. The corresponding tornqvist aggregate labor index would be

$$X^L = (x^1_D/x^0_D)^{SD} \cdot (x^1_I/x^0_I)^{SI}$$

where SD and SI are as defined above and x indicates quantity in hours of labor input.

The value of any elasticity ϵ_{ii} derived from the translog specification in (4.5) gives the percentage change in aggregate input demand due to a one percent change in aggregate input price. Values ϵ_{ii} provide no information about substitution behavior between direct and indirect labor components of the labor aggregate, nor about substitution behavior among electricity, gas, and fuel components of the energy aggregate. To be useful, elasticity measures would inform a firm how incremental changes of individual input prices affect input demands, and, therefore, costs. In order to determine the effect of individual input price changes on individual input demands in a cost function specified with aggregate input prices, one needs to know the effect of relative input price changes upon relative input demands. For our study, then, we may conclude only that labor demand is less responsive than energy demand to changes in input prices, and that both input demand responses are quite low.

5.4.4 Cooperative Objectives

Rejecting the null hypothesis of cost elasticity given by (4.7) is equivalent to rejecting the null hypothesis that the cooperative operates where NMRP equals NARP. This means that the cooperative might be operating where NARP is less than or greater than NMRP, that is, at a point such as B or C in figure 1. Operation at C is ruled out for the present cooperative since tabulated patronage refunds from the firm's Statement of Operations are positive in all five years of our analysis. Recall from figure 1 that if the member supply curve intersects NARP at the point of operation, there is no patronage refund.

To determine for the unrestricted model whether NMRP is less than or greater than NARP, we calculate at the sample mean the value of cost elasticity defined by (4.7). This critical value, approximately 1.62, is obtained by evaluating the expression $F/[(1+\alpha)5C(\cdot)] + 1$ at the sample mean, where $C(\cdot)$ equals $\exp(\alpha_0)$. The cost elasticity κ calculated from the unrestricted model was only 1.27. Since this falls short of the critical value, the firm does not appear on average to regulate delivery quantities so as to equate NMRP with NARP, that is to maximize per-unit net returns.

The firm therefore may operate at point B or at a point to the right or left of B in figure 1. If it is at B, unit net return and raw product price are neither the lowest nor the highest among the three possible goals discussed here. Patronage refunds earned at B may, upon negotiation between board and members, be returned to members or utilized to expand or diversify production.

Chapter 6

Conclusions

A cost function was estimated econometrically to assess the pack production structure at five plants in an Oregon fruit and vegetable processing cooperative. The cost function was specified for two input prices (aggregate labor price and aggregate energy price), and for three outputs (beans, corn, and a miscellaneous fruit and vegetable aggregate). Three hypotheses were tested at the sample mean.

Nonjointness in inputs (NJI) and nonjointness in output prices (NJO) were the two technology structure hypotheses. A test of the hypothesis regarding regulation of delivery quantities was conducted by determining whether cost elasticity was of a specified degree. The analysis required calculating conditional price elasticities of input demand, cost elasticity, and cost complementarities. These measures were evaluated at the sample mean and at each observation point.

Conclusions from hypothesis tests and values of economic measures at the sample mean provided an assessment of pack production at an "average" or representative plant. Our overall strategy was to test separately the validity of restricted models relative to an unrestricted model. Joint hypothesis tests of technology structure (NJO with NJI), and all tests (individual and joint) of cost elasticity consistent with regulating delivery quantities, were rejected using the likelihood ratio method.

Factor demand elasticities evaluated at the sample mean indicated labor demand is less responsive to input price change than is energy demand. Calculated conditional demand elasticities were low, indicating short-run responses to input price changes are not strong. No evidence of scope economies between major (beans and corn) and minor products was found at the sample mean. Our measure of cost elasticity at the unrestricted model's sample mean suggests the cooperative operates where net marginal revenue product is less than net average revenue product. That is, the cooperative does not regulate delivery quantities so as to maximize net average revenue product.

Cost elasticity and complementarity measures were strikingly different between the larger, diversified plants and the smaller, more specialized plants. In the larger plants (where diseconomies of size prevailed), anti-complementarities appeared between beans and minor products and between corn and minor products. At smaller, more specialized plants (where economies of size prevailed), there was some evidence of complementarity between beans and minor products. Since anti-complementarities are a sufficient condition for diseconomies of scope, some diseconomies of scope between major and minor products are in evidence at larger plants.

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APPENDIX

Table 5. Labor and Energy Cost Shares: Predicted Values from Unrestricted Model.

Plant	Labor Cost Share	Energy Cost Share
FY86		
1	0.82	0.18
2	0.81	0.19
3	0.78	0.22
4	0.78	0.22
5	0.78	0.22
FY87		
1	0.83	0.17
2	0.82	0.18
3	0.80	0.20
4	0.79	0.21
5	0.81	0.19
FY88		
1	0.83	0.17
2	0.83	0.17
3	0.80	0.20
4	0.80	0.20
5	0.80	0.20
FY89		
1	0.85	0.15
2	0.84	0.16
3	0.82	0.18
4	0.82	0.18
5	0.83	0.17
FY90		
1	0.85	0.15
2	0.84	0.16
3	0.81	0.19
4	0.83	0.17
5	0.81	0.19

Table 6. Characteristic Roots of Input Price Hessian Matrix: Predicted Values from Unrestricted Model

Plant	Characteristic Root 1 ($x10^{+8}$)[a]	Characteristic Root 2
FY86		
1	-0.47	-0.65
2	0.08	-0.08
3	-1.21	-0.13
4	-1.78	-0.59
5	0.52	-0.13
FY87		
1	-3.71	-0.56
2	-0.05	-0.07
3	0.03	-0.09
4	1.16	-0.46
5	0.07	-0.09
FY88		
1	0.00	-0.57
2	0.06	-0.06
3	0.79	-0.12
4	-0.52	-0.44
5	0.04	-0.08
FY89		
1	0.40	-0.19
2	-0.16	-0.04
3	-0.05	-0.09
4	-4.74	-0.32
5	-0.02	-0.06
FY90		
1	-0.62	-0.19
2	0.12	-0.04
3	-0.04	-0.08
4	-1.20	-0.35
5	0.12	-0.11

a/ For example, value of characteristic root corresponding to plant 4 in FY87 is 1.16 x 10^{-8}, the largest nonnegative root.

Table 7. Marginal Output Costs: Predicted Values from Unrestricted Model

Plant	Corn and Beets Products	Bean Products	Minor Products
FY86			
1	1.87	8.84	4.86
2	2.08	4.01	4.85
3	7.38	3.45	-3.02
4	9.02	5.97	1.10
5	4.25	3.89	1.10
FY87			
1	3.73	8.45	4.19
2	3.38	4.09	2.29
3	8.19	2.66	-0.70
4	8.79	5.30	1.07
5	4.62	3.64	0.04
FY88			
1	4.62	8.73	3.87
2	2.38	4.84	4.60
3	4.14	4.26	1.98
4	8.60	6.22	1.22
5	3.81	4.15	1.77
FY89			
1	4.80	8.54	3.29
2	1.86	5.01	5.69
3	5.35	3.71	0.98
4	9.23	6.14	1.18
5	3.53	4.36	1.04
FY90			
1	4.93	8.52	4.70
2	3.03	4.54	2.30
3	7.08	3.35	-2.67
4	6.48	7.65	2.15
5	7.25	3.92	0.53

Table 8. Cost Complementarities: Predicted Values from Unrestricted Model

Plant	C_{13}	C_{23}
FY86		
1	3.38	2.20
2	32.28	-28.71
3	49.27	-21.30
4	3.87	0.16
5	39.51	-15.00
FY87		
1	4.15	1.61
2	37.19	-18.02
3	17.58	-7.97
4	4.65	0.05
5	45.28	-21.01
FY88		
1	4.21	1.45
2	37.49	-19.12
3	27.76	-8.11
4	4.21	0.16
5	45.19	-22.47
FY89		
1	4.04	1.33
2	47.36	-20.95
3	21.15	-6.71
4	4.42	0.11
5	67.03	-27.62
FY90		
1	5.77	1.61
2	68.56	-35.86
3	45.83	-23.45
4	4.65	0.76
5	22.26	-5.63

Table 9. Cost Elasticity: Predicted Values from Unrestricted Model

Plant	Cost Elasticity
FY86	
1	1.12
2	0.77
3	1.21
4	1.38
5	1.05
FY87	
1	1.22
2	0.95
3	1.33
4	1.39
5	1.06
FY88	
1	1.25
2	0.84
3	1.05
4	1.35
5	0.97
FY89	
1	1.24
2	0.80
3	1.15
4	1.37
5	0.97
FY90	
1	1.26
2	0.89
3	1.19
4	1.26
5	1.25

Table 10. Own Price Input Demand Elasticities: Predicted Values from Unrestricted Model

Plant	Own Price Labor Elasticity ϵ_{11}	Own Price Energy Elasticity ϵ_{22}
FY86		
1	-0.03	-0.19
2	-0.05	-0.09
3	-0.07	-0.16
4	-0.07	-0.26
5	-0.07	-0.17
FY87		
1	-0.02	-0.16
2	-0.03	-0.02
3	-0.06	-0.16
4	-0.06	-0.24
5	-0.05	-0.07
FY88		
1	-0.02	-0.16
2	-0.03	0.01
3	-0.05	-0.12
4	-0.05	-0.24
5	-0.05	-0.09
FY89		
1	-0.01	-0.11
2	-0.02	0.14
3	-0.03	-0.08
4	-0.03	-0.19
5	-0.03	0.08
FY90		
1	-0.01	-0.09
2	-0.02	0.13
3	-0.04	-0.06
4	-0.03	-0.18
5	-0.04	-0.11