AN ALTERNATING CURRENT, RESISTANCE ANALOG NETWORK SIMULATOR FOR A-C POWER SYSTEM STABILITY STUDIES

by

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TABLE OF SYMBOLS

ABCD - Generalized circuit constants.

E - Potential.

e - Base of Naperian logarithms.

F - Force.

f - Frequency.

G - Synchronous machine rating.

g - A subscript denoting generator.

H - Inertia Constant.

I - Current.

I - Moment of inertia.

j - Unit phasor in the Y direction.

k - A constant equal to $(\Delta t)^2/M$.

KE - Kinetic energy.

L-L - Line-to-line voltage.

L-N - Line-to-neutral voltage.

M - Angular momentum.

M - Inertia constant.

M - Network simulator constant.

m - A subscript denoting motor.

m - Mass.

N - Network simulator constant.

n - A subscript denoting the nth interval.

n - Synchronous machine speed.

P - A subscript denoting an in-phase quantity.

P - Real power, watts.

TABLE OF SYMBOLS (CONT.)

P_s - Shaft or mechanical power.

pu - Per unit.

Q - A subscript denoting a quadrature component.

Q - Reactive power, vars.

R - Resistance.

r - Radius.

Re - Real part of.

S - Complex power, volt-amperes.

T - Torque.

t - Time.

V - Potential difference.

v - Velocity.

W_s - Stored kinetic energy at synchronous speed.

WR^2 - Moment of inertia of an electric machine.

X - Reactance.

X_a - Inductive reactance of aerial lines.

X_d - Inductive reactance spacing factor for aerial lines.

X'_d - Transient reactance.

Z - Impedance.

α - Angular acceleration.

$\alpha\beta\gamma\rho$ - Angles associated with the generalized circuit constants.

Δ - Indicates a change in value.

δ - Angle of machine internal voltage with respect to a synchronously rotating reference axis.

δ_c - Critical angle of system oscillation.

TABLE OF SYMBOLS (CONT.)

θ - Angular position with respect to a stationary reference axis.

θ - Impedance angle.

ω - Angular velocity.

ω_s - Synchronously rotating reference axis.

° - Placed above a symbol to indicate a phasor quantity.

* - Placed above a symbol to indicate a conjugate quantity.

AN ALTERNATING CURRENT, RESISTANCE
ANALOG NETWORK SIMULATOR FOR A-C
POWER SYSTEM STABILITY STUDIES

INTRODUCTION

An electric power system may be considered an electrical coupling between mechanical devices. To assure continuous operation of these devices, the power system should be maintained free of disturbances since these may cause instability of the system, and in some cases initiate a complete system shut down. Since all electric power systems are subject to disturbances of a transient nature, some method should be available for studying power systems in order that protection against such disturbances may be employed. Several methods have been developed, the solutions of which may be carried out by hand or machine calculation. Machines in the form of analog computers are commonly used to study large power systems since hand analysis is extremely difficult and tedious. These machines are generally known as network simulators, analyzers or calculators, the terms being used interchangeably. Most simulators contain resistive, inductive and capacitive elements which are used to represent power system impedances. This thesis considers a network simulator which uses only resistive elements to represent the impedances of power systems. The simulator was designed and constructed for studying transient behavior of power systems, but may also be used to study the steady-state condition.

The basic theory of power system stability has been studied in detail, and from it have come the equations necessary for solving stability problems either by hand or machine. Mathematical solution of these equations is generally not possible, and a step-by-step solution, which is ideally suited to computer techniques, is used instead.

The network simulator which has been constructed has been used to study transient stability problems. One of these problems has been included as an example in this thesis. This example illustrates both the use of the network simulator, and the general approach to the solution of all power system stability problems.

THE STABILITY PROBLEM

History Of Stability

On commercial alternating-current electric power systems, the generators, rotory condensers and some motors are of the synchronous type. It is absolutely necessary that synchronism be maintained between these machines if electrical energy is to be transferred. Maintaining synchronism becomes increasingly difficult as the power systems and the interconnections between these systems continue to grow. The tendency of the components of an alternating-current electric power system to develop forces to maintain synchronism and equilibrium is known as stability. The problem of stability was first encountered when synchronous machines were operated in parallel.

The amount of power that can be transferred between two synchronous machines is limited by operating conditions and transfer impedance. The load corresponding to the maximum power value is known as the stability limit. The machines lose synchronism or "fall out of step" when the stability limit is exceeded. With the advent of the modern-day high reactance transmission lines, the difficulty of maintaining power system stability has increased. In the event of a fault, it was often necessary to entirely separate interconnected systems in order to maintain stability or restore synchronism.

The principle developments in system stability came from the study of long distance transmission lines. Initially the problems involved determining the capability of proposed systems for transmitting a desired amount of power. A more important problem, and one which is now a better measure of stability, is whether or not a proposed system will be able to withstand transient disturbances likely to be encountered in operation. These transient disturbances may be classified into three general categories as follows:

a. Disturbances resulting from a sudden increase in load.

b. Disturbances from switching operations which open one or more transmission lines.

c. Disturbances resulting from system faults.

Definitions

Stability and stability limit refer to both steady-state and transient conditions. The American Standard Definitions of Electrical Terms defines stability and stability limit as follows (1, p. 123):

> "Stability, when used with reference to a power system, is that attribute of the system, or part of the system, which enables it to develop restoring forces between the elements thereof, equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements.
>
> "A stability limit is the maximum power flow possible through some particular point in the system when the entire system or part of the system to which the stability limit refers is operating with stability."

Steady-state stability refers to the maximum possible flow of power through a given point in a system without loss of stability when the power being transferred is very gradually increased (7, p. 321). Transient stability, on the other hand, is concerned with maximum possible power flow through a given point without loss of stability when the system is subjected to an aperiodic disturbance (4, p.436). An aperiodic disturbance occurs irregularly, and only at intervals such that the system regains stability between disturbances. The two types of stability differ in that transient stability depends on the nature and severity of the disturbance whereas steady-state stability does not. Transient stability is of greater importance because its limiting value is below the steady-state stability limit.

Stability problems then, deal with the transmission of electric power between groups of synchronous machines when the systems are subjected to disturbances. Actual power systems consist of many groups of machines, but for purposes of analysis each group may be replaced by a single equivalent machine since the machines of any group tend to "swing" as a complete unit. Using this method often reduces a multi-machine system to a two machine or one machine and an infinite bus equivalent system. The factors affecting the equivalent system are essentially the same as those affecting the actual system, but the analysis is far less complicated.

The Synchronous Power System

As an example of a synchronous power system, consider Figure 1 which shows the seven essential parameters in the stability problem (4, p. 434).

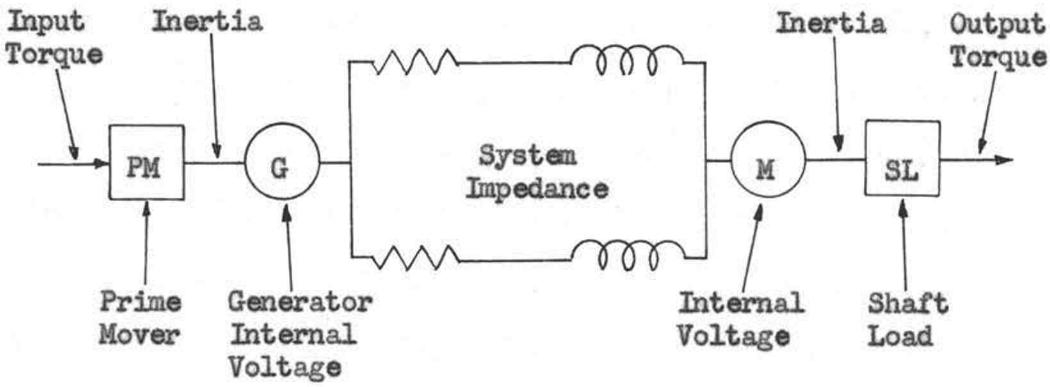


Figure 1. Basic diagram for a two-machine stability problem.

The four mechanical and three electrical parameters are as follows:

a. Prime mover input torque.

b. Inertia of the prime mover and generator.

c. Generator internal voltage.

d. System impedance.

1. Generator.

2. Line.

3. Motor.

e. Motor internal voltage.

f. Inertia of motor and shaft load.

g. Shaft load output torque.

The power that can be transferred between the machines of Figure 1 is a function of the internal voltages of the two machines, the angle between these voltages and the total equivalent series impedance of the system. Current flowing in the field winding of a synchronous machine creates a sinusoidally varying mmf wave. This mmf wave in turn, establishes the rotor flux wave, stationary with respect to the rotor, but travelling at synchronous speed with respect to the armature, which links the conductors of the armature circuit. It is this flux linkage with the armature that generates the internal voltage of the machines. The internal voltage has a direct angular relation to the rotor flux wave, and depends on the power factor of the machine. Thus the angles between internal voltages of the machines of a power system are also the angles between the rotors of those machines. Figure 2 shows the relation between some of the above mentioned electrical quantities.

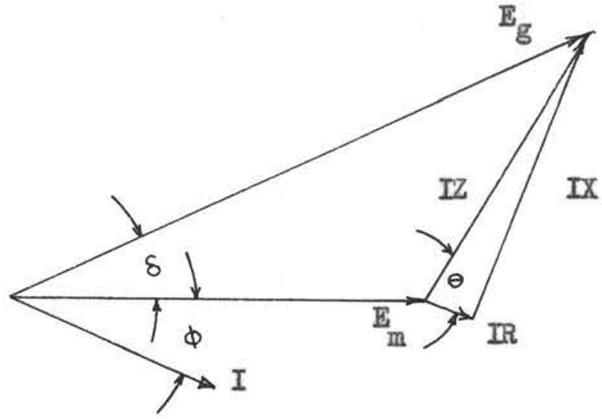


Figure 2. Phasor diagram for the system of Figure 1.

Often, it is convenient to represent the electrical portion of Figure 1 as the four terminal network shown in Figure 3.

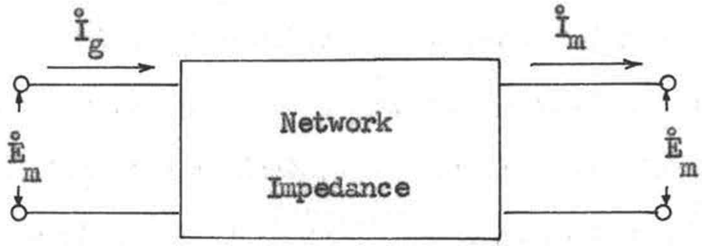


Figure 3. Four terminal equivalent network for the system of Figure 1.

The currents and voltages of Figure 3 may be written as functions of generalized circuit constants as follows:

$$\mathring{E}_g = \mathring{A}\mathring{E}_m + \mathring{B}\mathring{I}_m \qquad \text{Eq. (1)}$$

$$\mathring{I}_g = \mathring{C}\mathring{E}_m + \mathring{D}\mathring{I}_m \qquad \text{Eq. (2)}$$

where

$$\left.\begin{aligned} \mathring{A} &= Ae^{j\alpha} \\ \mathring{B} &= Be^{j\beta} \\ \mathring{C} &= Ce^{j\gamma} \\ \mathring{D} &= De^{j\rho} \end{aligned}\right\} \qquad \text{Eq. (3)}$$

From Equations (1) and (2), the generator current in terms of machine internal voltages is

$$\mathring{I}_g = \frac{\mathring{D}\mathring{E}_g}{\mathring{B}} - \frac{\mathring{E}_m}{\mathring{B}} \qquad \text{Eq. (4)}$$

The above equations can be solved for the motor voltage and current as follows:

$$\mathring{E}_m = \mathring{D}\mathring{E}_g - \mathring{B}\mathring{I}_g \qquad \text{Eq. (5)}$$

$$\mathring{I}_m = -\mathring{C}\mathring{E}_g + \mathring{A}\mathring{I}_g \qquad \text{Eq. (6)}$$

and

$$\mathring{I}_m = \frac{\mathring{E}_g}{\mathring{B}} - \frac{\mathring{A}\mathring{E}_m}{\mathring{B}} \qquad \text{Eq. (7)}$$

The electrical power transferred between machines is

$$\mathring{S} = \mathring{E}\overset{*}{I} \qquad \text{Eq. (8)}$$

For the generator

$$\mathring{S}_g = \mathring{E}_g\overset{*}{I}_g$$

$$= \mathring{E}_g \frac{\overset{*}{E}\overset{*}{D}_g}{\overset{*}{B}} - \frac{\overset{*}{E}_m}{\overset{*}{B}}$$

$$= \frac{|E_g^2|\overset{*}{D}}{\overset{*}{B}} - \frac{|E_g E_m|e^{j\delta}}{\overset{*}{B}} \qquad \text{Eq. (9)}$$

and

$$\mathring{S}_m = \mathring{E}_m\mathring{I}_m$$

$$= \mathring{E}_m \frac{\overset{*}{E}_g}{\overset{*}{B}} - \frac{\overset{*}{A}\overset{*}{E}_m}{\overset{*}{B}}$$

$$= -|E_m^2|\frac{\overset{*}{A}}{\overset{*}{B}} + \frac{|E_g E_m|e^{-j\delta}}{\overset{*}{B}} \qquad \text{Eq. (10)}$$

In stability calculations, the item of primary interest is the ability of the power system to transfer real power during a system disturbance. The reactive power is of little interest and Equations (9) and (10) may be reduced to their real parts.

$$P_g = \mathrm{Re}\left[\mathring{S}_g\right]$$

$$= \mathrm{Re}\left[\left|E_g{}^2\right|\frac{\overset{*}{D}}{\overset{*}{B}} - \frac{\left|E_g E_m\right|}{\overset{*}{B}}\, e^{j\delta}\right] \qquad \text{Eq. (11)}$$

$$= \left|E_g{}^2\right|\frac{D}{B}\cos(\beta-\rho) - \frac{\left|E_g E_m\right|}{B}\cos(\beta+\delta)$$

$$P_m = \mathrm{Re}\left[\mathring{S}_m\right]$$

$$= \mathrm{Re}\left[-\left|E_m{}^2\right|\frac{\overset{*}{A}}{\overset{*}{B}} + \frac{\left|E_g E_m\right|}{\overset{*}{B}}\, e^{-j\delta}\right] \qquad \text{Eq. (12)}$$

$$= -\left|E_m{}^2\right|\frac{A}{B}\cos(\beta-\alpha) + \frac{\left|E_g E_m\right|}{B}\cos(\beta-\delta)$$

Equivalent ABCD constants for most commonly encountered circuit configurations are available in many references. As an example, see those given by Harder (6, p. 327).

EQUAL AREA CRITERIA FOR TRANSIENT DISTURBANCES

Criterion Of Stability

According to Evans and Muller (4, p. 435), the criterion of stability for electric power systems is as follows:

> "A power-transmission system operating under specified circuit and transmitted load conditions is said to be stable if, when displaced from these conditions by any small arbitrary forces, the system upon removal of these forces develops restoring forces tending to return it to its original conditions."

The criterion of stability deals with systems in which disturbing forces are applied and subsequently removed. The engineer studying stability is just as concerned with the system in which the disturbing forces are not removed. Such disturbances are load increases, switching operations and system faults, all previously mentioned. Each of these transient disturbances may be studied with the aid of power-angle diagrams which are obtained by plotting power, in Equations (11) and (12), versus the angle δ between machine internal voltages. Power-angle diagrams are shown in Figure 4 for the transient disturbances mentioned above. These diagrams are drawn for disturbances on the two-machine power system of Figure 1.

Equal Area Criteria

The changing conditions of the synchronous machines of a power system subjected to a transient disturbance can be understood by a thorough study of Tables 1 and 2. Rotor positions of Tables 1 and 2 refer to corresponding points of Figure 4 (a). These tables indicate that after the application of the disturbance, the machines of the system will oscillate about the point corresponding to the new system operating conditions. Resistance of the system will damp out these oscillations resulting in stable operation. Both tables apply to disturbances resulting from switching operations and system faults. Only Table 2, for synchronous motors applies to the load increase type of disturbance. On an unregulated power system, generator input power will remain constant during transient disturbances. The transmission system between motor and generator is unaffected by a sudden load increase, with the result that the transferred power of the system remains constant during the disturbance. Under the conditions of constant power and increased load torque, the only alternative is for the entire rotating portion of the system to decrease its angular velocity. Thus both motor and generator will decelerate from synchronous speed. If the power system is regulated, the regulation will function to change the generator input torque to supply the increased demand for power without deceleration of the generator.

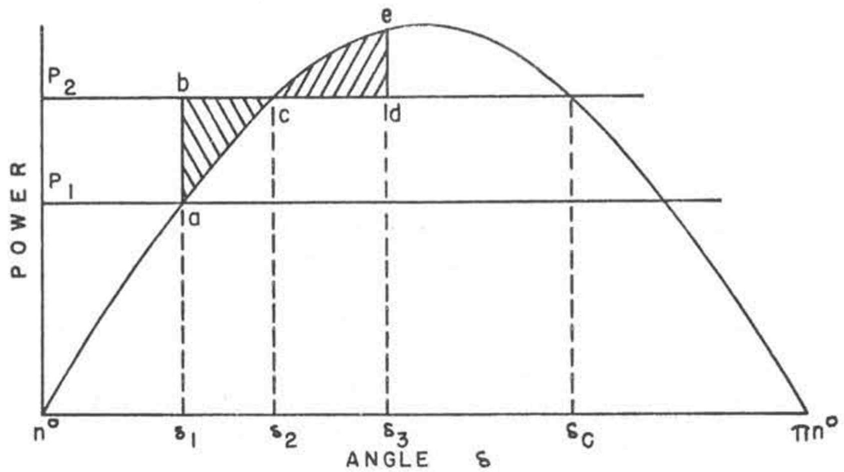
Considering Figure 4 (a), if during the system oscillations, angle δ becomes greater than δ_c, the system will become unstable. Once this condition occurs, the transferred electric power becomes less than the power required by the motor load. This condition causes the angle δ to increase, but in this case, the electric power decreases causing δ to further increase. This process continues

Table 1. Changing conditions for a synchronous generator of a two machine system undergoing a transient disturbance.

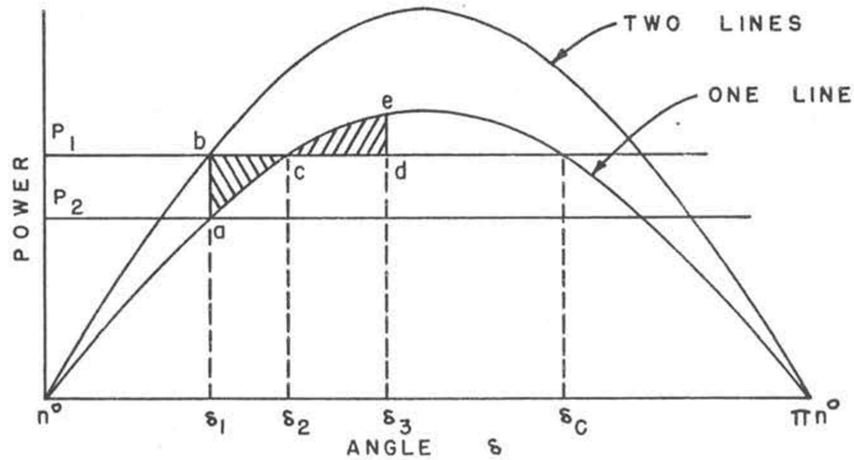
Rotor Position	Generator Speed ω	Angle δ	Electric Power P_e	Stored Energy W	Machine Undergoing
at a	$\omega = \omega_s$ increasing	$\delta = \delta_1$ minimum	$P_e < P_s$ minimum	$W = W_s$ increasing	acceleration
a to c	$\omega > \omega_s$ increasing	increasing	$P_e < P_s$ increasing	$W > W_s$ increasing	acceleration
at c	$\omega > \omega_s$ maximum	$\delta = \delta_2$ increasing	$P_e = P_s$ increasing	$W > W_s$ maximum	———
c to e	$\omega > \omega_s$ decreasing	increasing	$P_e > P_s$ increasing	$W > W_s$ decreasing	deceleration
at e	$\omega = \omega_s$ decreasing	$\delta = \delta_3$ maximum	$P_e > P_s$ maximum	$W = W_s$ decreasing	deceleration
e to c	$\omega < \omega_s$ decreasing	decreasing	$P_e > P_s$ decreasing	$W < W_s$ decreasing	deceleration
at c	$\omega < \omega_s$ minimum	$\delta = \delta_2$ decreasing	$P_e = P_s$ decreasing	$W < W_s$ minimum	———
c to a	$\omega < \omega_s$ increasing	decreasing	$P_e < P_s$ decreasing	$W < W_s$ increasing	acceleration
a to c	cycle repeats				

Table 2. Changing conditions for a synchronous motor of a two machine system undergoing a transient disturbance.

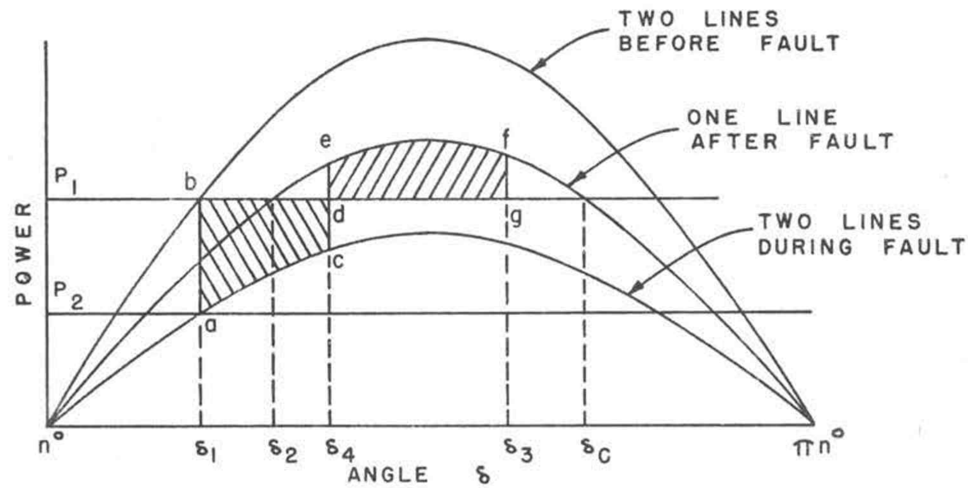
Rotor Position	Motor Speed ω	Angle δ	Electric Power P_e	Stored Energy W	Machine Undergoing
at a	$\omega = \omega_s$ decreasing	$\delta = \delta_1$ minimum	$P_e < P_s$ minimum	$W = W_s$ decreasing	deceleration
a to c	$\omega < \omega_s$ decreasing	increasing	$P_e < P_s$ increasing	$W < W_s$ decreasing	deceleration
at c	$\omega < \omega_s$ minimum	$\delta = \delta_2$ increasing	$P_e = P_s$ increasing	$W < W_s$ minimum	————
c to e	$\omega < \omega_s$ increasing	increasing	$P_e > P_s$ increasing	$W < W_s$ increasing	acceleration
at e	$\omega = \omega_s$ increasing	$\delta = \delta_3$ maximum	$P_e > P_s$ maximum	$W = W_s$ increasing	acceleration
e to c	$\omega > \omega_s$ increasing	decreasing	$P_e > P_s$ decreasing	$W > W_s$ increasing	acceleration
at c	$\omega > \omega_s$ maximum	$\delta = \delta_2$ decreasing	$P_e = P_s$ decreasing	$W > W_s$ maximum	————
c to a	$\omega > \omega_s$ decreasing	decreasing	$P_e < P_s$ decreasing	$W > W_s$ decreasing	deceleration
a to c	cycle repeats				



(a) SUDDEN LOAD INCREASE.



(b) SWITCHING OF PARALLEL LINES.



(c) SYSTEM FAULT WITH CLEARING.

FIGURE 4. POWER-ANGLE DIAGRAMS FOR TRANSIENT DISTURBANCES.

until no power is transferred and the system loses synchronism entirely. Angle δ_c is the critical angle of system oscillation. To maintain stability, a disturbance must not cause δ to exceed this critical angle.

In stability studies, it is helpful to know whether or not the critical angle will be exceeded for a given disturbance. Assuming that the power system under study contains no losses, during a disturbance the kinetic energy released by the system must equal that absorbed by the system. Also during a disturbance, if the system is stable the speed of the machines will not vary more than a few percent from synchronous speed, and may be considered as being constant. From the laws of the mechanics of rotation, which apply to electric machines, the power produced is directly proportional to the torque on the machine when the speed of the machine remains constant. Figure 4(a) could thus be in terms of torque rather than power. Again, from the mechanics of rotation, the kinetic energy of a machine, in joules, is the integral of the product of torque expressed in joules per radian and the time rate of change of the angle in radians through which the torque acts. The area under the power-angle diagram of Figure 4(a) is now directly proportional to the kinetic energy of the system. Specifically, the shaded area of Figure 4(a) between δ_1 and δ_2 represents the kinetic energy dissipated by the system during the disturbance. Similarly, the shaded area between δ_2 and δ_3 represents the kinetic energy absorbed by the system. From the assumption that the kinetic energy released must equal the kinetic energy absorbed, the areas abc and cde of Figure 4(a) must be equal. This method of analysis is known as the equal area criteria of stability. Using the equal area criteria, the power system of Figures 1 and 4(a) is stable if the area cde can be located above the new power level P_2, equal to area abc. Similar considerations apply to Figures 4(b) and 4(c).

The power-angle diagrams of transient disturbances given above indicate that the system fault condition is the most serious type encountered. Generally speaking this is true, and most systems on which stability may become a problem, are designed to withstand the most serious type of fault for which protection is economically feasible. In the order of increasing severity, the types of faults and their frequency of occurance are (7, p. 11):

a.	Single line-to-ground fault.	70%
b.	Line-to-line fault.	15%
c.	Double line-to-ground fault.	10%
d.	Three-phase fault.	5%

For maximum reliability, the power system would be designed to withstand a three-phase fault in the worst possible location, but this is very often economically impractical. Instead, systems are generally designed to sustain a less severe fault, usually the double line-to-ground type.

Equal area criteria is a fast and convenient method of handling stability analysis. The only information required is the power-angle diagrams and the system operating conditions before, during and after the disturbance. Unfortunately, this information often is not readily available. A second disadvantage to this method is that no means are easily available for determining the angle at which a fault must be cleared in order to retain system stability. From a practical standpoint, faults are not cleared as a function of angular difference between machine rotors, but as a function of time and current. Relays and circuit breakers used to clear and isolate faulty sections of a system have definite operating times and are independent of angular displacement. The solution to stability problems thus resolves into finding a relation between the angular displacement of machine rotors and time.

THE SWING EQUATION AND ITS SOLUTION

Development Of The Swing Equation

When torque T is applied to a particle, the particle experiences angular acceleration. If the particle has a differential mass dm, the tangential force required to accelerate it is

$$dF = r\alpha dm \qquad \text{Eq. (13)}$$

This force acts with a lever arm r to produce torque.

$$dT = r^2\alpha dm \qquad \text{Eq. (14)}$$

The torque for the entire mass is obtained by integration of Equation (14).

$$\begin{aligned} T &= \alpha\int r^2 dm \\ &= \alpha I \qquad \text{Eq. (15)} \end{aligned}$$

The laws of rotation apply to synchronous machines and Equation (15) may be rewritten as

$$I\alpha = \frac{I d^2\theta}{dt^2} = T_a \qquad \text{Eq. (16)}$$

where T_a is the net torque acting on the machine including shaft torque due to the generator prime mover or motor load, torque from mechanical and electrical losses, and electromagnetic torque. Letting T_i be the shaft torque and T_o the electromagnetic torque, each corrected for losses, the net torque will be the difference of the two.[1] The torques are positive for generator action, mechanical input and electrical output, and negative for motor action, electrical input and mechanical output. For steady-state conditions, the torques are equal and the net torque is zero. Under transient conditions, the net torque is not zero and is available to produce acceleration or deceleration of the machines. The net torque is either a positive or negative accelerating torque and Equation (16) may be written as

$$I\frac{d^2\theta}{dt^2} = T_a = T_i - T_o \qquad \text{Eq. (17)}$$

The problem now is to solve Equation (17) for machine rotor angles as a function of time. However, the torques of electric machines are not easily measured. A more convenient quantity to use is power, which is related to torque by the angular velocity of the machine. Multiplying Equation (17) through by angular velocity yields

$$I\omega\frac{d^2\theta}{dt^2} = T_a\omega = T_i\omega - T_o\omega \qquad \text{Eq. (18)}$$

By definition, $I\omega$ is angular momentum, and torque times angular velocity is power. Substitution of these quantities into Equation (18) results in

$$M\frac{d^2\theta}{dt^2} = P_a = P_i - P_o \qquad \text{Eq. (19)}$$

---

1. These losses are inherent in the system and include bearing and friction losses, windage losses and various core and copper losses of the equipment.

The net power P_a is the difference between shaft power and electric power, and is available for accelerating the machine rotor. P_a will subsequently be referred to as accelerating power.

For any machine in the system, angle θ is the angular position of the rotor at any given time measured from a fixed space reference and is continuously changing quantity. It is more convenient to measure the angular position with respect to a synchronously rotating reference axis. For this purpose, let

$$\theta = \omega_s t + \delta \qquad \text{Eq. (20)}$$

where $\omega_s t$ is the synchronously rotating reference axis and δ is the angular displacement from that axis.

Taking derivatives of both sides of Equation (20) with respect to time yields

$$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt} \qquad \text{Eq. (21)}$$

and

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \qquad \text{Eq. (22)}$$

Substitution of Equation (22) into Equation (19) results in a relation between the internal voltage angle of a machine and time known as the swing equation in which M now is known as the inertia constant (6, p. 22).

$$M\frac{d^2\delta}{dt^2} = P_a = P_i - P_o \qquad \text{Eq. (23)}$$

Thus far, no units have been specified for any of the equations leading up to the swing equation. In most power system work, power is expressed in megawatts. It is also common practice to express power in per unit notation. Angles are normally in electrical degrees, and seconds are most frequently used as time units. The solution of Equation (23) for M is

$$M = \frac{P_a}{\frac{d^2\delta}{dt^2}} \qquad \text{Eq. (24)}$$

which in units becomes

$$M = \frac{\text{megawatt-seconds}^2}{\text{electrical degree}}$$

$$= \frac{\text{megajoule-second}}{\text{electrical degree}} \qquad \text{Eq. (25)}$$

When the per unit system is used the units of M are

$$M = \frac{\text{per unit power-seconds}^2}{\text{electrical degree}} \qquad \text{Eq. (26)}$$

In rotational mechanics, M is the angular momentum of a rotating body. When used in power system calculations, M is known as the inertia constant. This constant is not usually known or given for a machine, and hence must be computed. Often, a manufacturer will specify the moment of inertia WR^2, and the speed n, of the machine. The stored kinetic energy of the machine is

$$KE = \frac{1}{2}I\omega^2 \qquad \text{Eq. (27)}$$

In terms of the given machine constants, the kinetic energy in megajoules is given by (6, p. 23)

$$KE = \frac{746}{550} \times 10^{-6} \times \frac{1}{2} \times \frac{WR^2}{32.2} \times \left[\frac{2\pi n}{60}\right]^2 \qquad \text{Eq. (28)}$$

The kinetic energy of a rotating machine may also be written as

$$KE = \frac{1}{2} M\omega \qquad \text{Eq. (29)}$$

where

ω = 360f electrical degrees per second.

Equation (29) reduces to

$$KE = 180\ Mf \qquad \text{Eq. (30)}$$

Substituting Equation (30) into Equation (29) and solving for M results in

$$M = \frac{746}{550} \times 10^{-6} \times \frac{1}{2} \times \frac{WR^2}{32.2} \times \left[\frac{2\pi n}{60}\right]^2 \times \frac{1}{180f} \qquad \text{Eq. (31)}$$

$$= 1.28 \times 10^{-6} \times \frac{WR^2 n^2}{f}$$

Equation (31) is useful when the machine constants are known. Frequently they are not and the evaluation of M is impossible with the equation as given. The moment of inertia WR^2 and consequently M vary over a wide range and depend on the particular machine being considered, so that no average or mean value of M can be assigned. It is now convenient to introduce another inertia constant H, such that it is equal to the stored kinetic energy of the machine in megajoules divided by the machine rating G in megavolt-amperes.

$$H = \frac{\text{kinetic energy of machine}}{\text{megavolt-ampere rating G}} \qquad \text{Eq. (32)}$$

or

$$GH = KE \qquad \text{Eq. (33)}$$

Substituting the kinetic energy from Equation (30) into Equation (33) and solving for M gives

$$M = \frac{GH}{180f} \qquad \text{Eq. (34)}$$

Machine rating and frequency will most always be known. The inertia constant H has the advantage of being nearly constant for a particular type of machine throughout the range of power ratings in which the machine is manufactured. Figure A-1 (8, p. 189) of the appendix shows some typical values of inertia constants for generators that may be used if the specific constants required in Equation (31) are not known. For synchronous motors, a value of 2.00 may be used for the inertia constant H (7, p. 327).

Solution Of The Swing Equation

Having computed the value of M by either of the methods just discussed, the swing equation can be solved. The mathematical solution involves elliptical integrals so that generally the only feasible method of solving the swing equation is by making a point-by-point solution. This method is carried out by assuming one or more variables of the swing equation to be constant during a small interval of time $\triangle t$ in order that the change in angular position of the machine rotor can be computed during the interval. New values of the assumed constants are then computed and used with the next interval of time.

A first assumption is that if the power system is stable, the prime mover input power to the generator remains constant throughout the transient disturbance. This is permissible since the prime mover is governor controlled, and governors are inherently slow acting. During a transient disturbance, a governor will hold the generator input constant for approximately three fourths of a second (6, p. 5) or about 45 cycles, which is roughly ten times as long as a circuit breaker requires to clear a fault. In addition, governors are not sensitive to speed changes of less than one or two percent. Unless a power system loses synchronism entirely, its machines will not change much more than this amount under a transient disturbance. Constant shaft torque may, however, necessitate a change in system frequency.

The point-by-point solution of the swing equation gives values of δ as a function of time. It is customary to plot a curve of δ versus time, such a curve being called the "swing curve." If the swing curve indicates that after reaching a maximum, the angle δ starts to decrease, the system is considered to be stable and that the oscillations around the equilibrium point will eventually damp out. Not only does the swing curve indicate whether or not the system is stable, but it provides a basis for estimating the margin of stability.

The point-by-point solution of the swing equation consists of two processes that are alternately calculated. First is the computation of the angular positions of the machine rotors at the end of a given time interval from a knowledge of the rotor positions at the beginning of that interval. The angular velocity is assumed constant throughout any interval at a value computed for the middle of the interval. Secondly, the accelerating power of each machine is computed for each interval from the angular position of the rotor during that interval. The assumption is made that the accelerating power remains constant from the middle of the preceding interval to the middle of the interval concerned. This assumption is a closer approximation to the true accelerating power than would be obtained if the accelerating power was assumed constant throughout the interval. Figure 5 will illustrate these relationships (7, p. 343).

None of the assumptions made are exactly true, but if the time interval is made small enough, the assumptions approach the actual values. If the nth interval of Figure 5 between $t = (n-1)\Delta t$ and $t = n\Delta t$ is considered, the angular position at the beginning of the interval is δ_{n-1} and the accelerating power $P_{a(n-1)}$ is constant from $t = (n-3/2)\Delta t$ to $t = (n-1/2)\Delta t$. The change in angular velocity, which is constant throughout the nth interval, will be given by

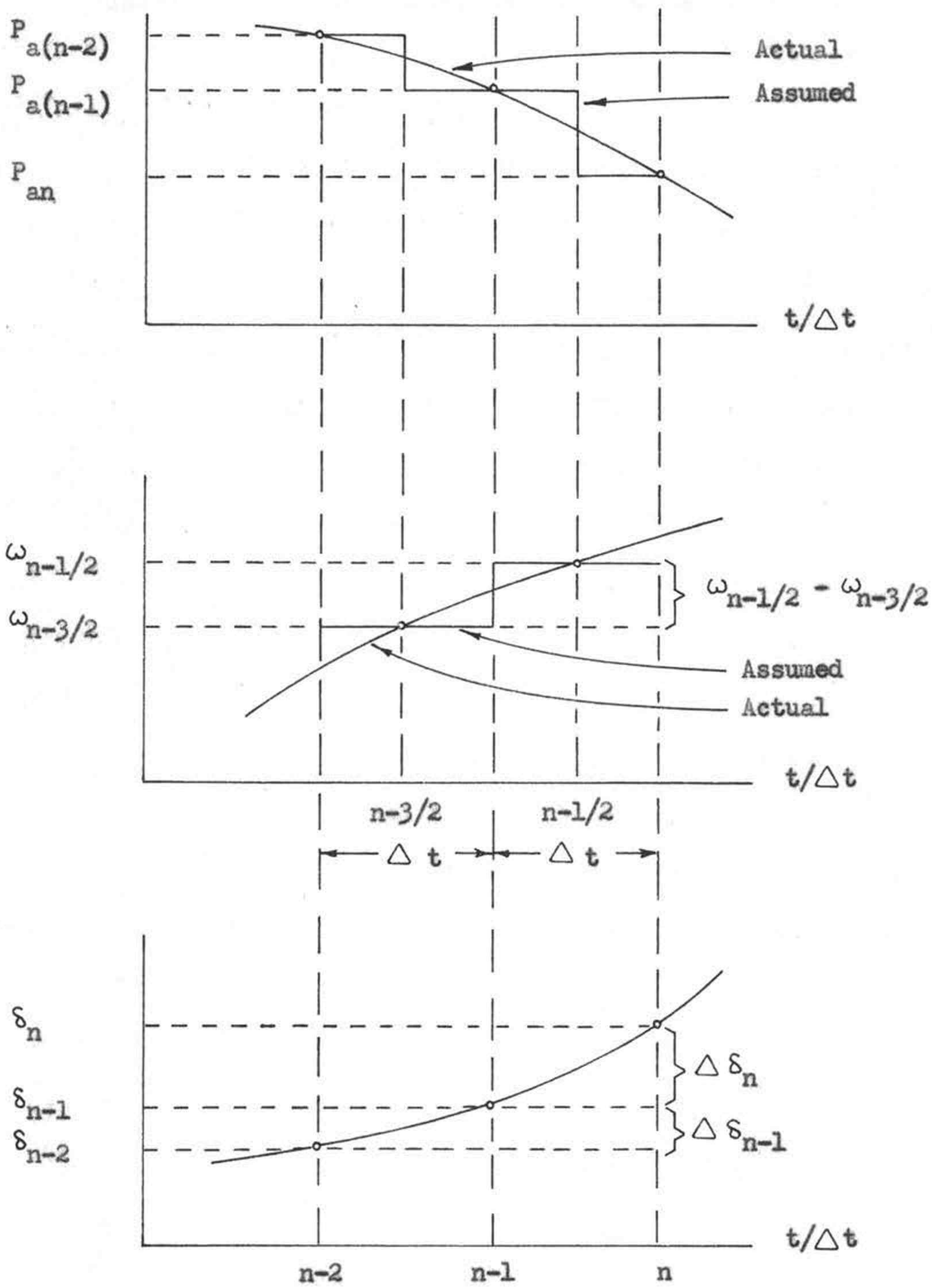


Figure 5. Actual and assumed values of P_a, ω and δ as a function of time; point-by-point calculation.

$$\Delta\omega_{n-1/2} = \omega_{n-1/2} - \omega_{n-3/2}$$

$$= \alpha_{n-1}\Delta t$$

$$= \frac{d^2\delta}{dt^2}\Delta t$$

$$= \frac{P_{a(n-1)}}{M}\Delta t \qquad \text{Eq. (35)}$$

The angular velocity for the nth interval then becomes

$$\omega_{n-1/2} = \omega_{n-3/2} + \Delta\omega_{n-1/2} \qquad \text{Eq. (36)}$$

The change in rotor angular position which is the product of velocity and time is

$$\Delta\delta_n = \omega_{n-1/2}\Delta t \qquad \text{Eq. (37)}$$

and the angular position at the end of the nth interval will be

$$\delta_n = \delta_{n-1} + \Delta\delta_n$$

$$= \delta_{n-1} + \omega_{n-1/2}\Delta t$$

$$= \delta_{n-1} + (\omega_{n-3/2} + \Delta\omega_{n-1/2})\Delta t$$

$$= \delta_{n-1} + \omega_{n-3/2}\Delta t + \frac{P_{a(n-1)}}{M}(\Delta t)^2 \qquad \text{Eq. (38)}$$

By analogy with Equation (37)

$$\Delta\delta_{n-1} = \omega_{n-3/2}\Delta t \qquad \text{Eq. (39)}$$

Equation (38) now reduces to

$$\delta_n = \delta_{n-1} + \Delta\delta_{n-1} + \frac{P_{a(n-1)}}{M}(\Delta t)^2 \qquad \text{Eq. (40)}$$

Substitution of Equation (38) into Equation (40) and solving for the change in rotor position during the nth interval results in

$$\begin{aligned} \Delta\delta_n &= \delta_n - \delta_{n-1} \\ &= \Delta\delta_{n-1} + \frac{P_{a(n-1)}}{M}(\Delta t)^2 \end{aligned} \qquad \text{Eq. (41)}$$

Equation (41) is the basic equation in the solution of the swing equation, and shows how to obtain the change in rotor position for a given time interval when the value for the previous interval is known. If the angular velocity of the machine for the nth interval is desired, it may be obtained from the following relation:

$$\omega_{n-1/2} = \frac{\Delta\delta_n}{\Delta t} \qquad \text{Eq. (42)}$$

Evaluation of Equation (41) is best done in tabular form. Table 3 is typical of the type that might be used in a stability study. It should be kept in mind that Table 3 is for a system containing one machine and an infinite bus, or for one machine of a multi-machine system. If more than one machine must be analyzed, a separate table of calculations must be made for each. Also, the change in rotor position during a particular time interval must be computed for all machines before the next interval is considered.

When a transient disturbance occurs, there is a discontinuity in the accelerating power. Immediately before the disturbance, the accelerating power is zero, while immediately after, it has a finite value. The accelerating power now has two values at the beginning of the first time interval. The assumption that the accelerating power remains constant from the middle of the previous interval to the middle of the interval being considered, requires that one value of accelerating power be used. Therefore, an average accelerating power is used as shown by the computation in Table 3. When the discontinuity occurs in the middle of a time interval, no special procedure is required since the accelerating power is assumed to change at the middle of the interval. Should a discontinuity occur other than at the beginning or middle of an interval, a weighted average of the accelerating power should be used. This will seldom be necessary as the time interval is usually taken small enough that the discontinuity may be assumed to occur at either the beginning or middle of the interval without introducing appreciable error. Usual values for the time interval are 0.05 and 0.10 seconds. In general, a time interval is chosen so that the angular change in rotor position does not exceed twenty or thirty degrees in any one time interval. If the angular change is greater than this amount, a smaller time interval should be chosen. To illustrate the point-by-point method of solution of transient stability problems, the following example is presented.

Table 3. Computation sheet for power system stability studies.

Machine No. ______ Total Mva ______ Shaft Power P_s ______ Mw

H ______ Mw-sec/Mva $M = GH/180f$ ______ Mw-sec^2/elect. degree

Δt ______ Seconds $k = (\Delta t)^2/M$ ______ Mw/ elect. degree

(1) Time Seconds	(2) Initial Angle δ_{n-1}	(3) $\pm C'$	(4) $\cos(\beta \pm \delta_n)$	(5) $\mp C'' \cos(\beta \pm \delta_n)$	(6) Electric Power P_e	(7) Accelerating Power P_a	(8) $k \times P_a$	(9) Angular Change $\Delta\delta_n$	(10) Final Angle δ_n	(11) Initial Angle Of Other Machine δ'_{n-1}	(12) $\delta_{n-1} - \delta'_{n-1}$
—	—	—	—	—	3 + 5	P_s - 6	7 x k	$8 + 9_{n-1}$	$10_{n-1} + 9$	—	2 - 11
t_o −	δ_o	C'_o	$\cos X_o$	$C''\cos X_o$	P_{eo} −						
t_o +	δ_o	C'_1	$\cos X_o$	$C''\cos X_o$	P_{eo} +	P_{ao}					
$t_{o\ avg}$	δ_o	C'_1	$\cos X_o$	$C''\cos X_o$	P_{eoavg}	$P_{ao}/2$	$k \times P_{ao}$	$\Delta\delta_o$	δ_1	δ'_o	$\delta_o - \delta'_o$
t_1	δ_1	C'_1	$\cos X_1$	$C''\cos X_1$	P_{e1}	P_{a1}	$k \times P_{a1}$	$\Delta\delta_1$	δ_2	δ'_1	$\delta_1 - \delta'_1$
t_2	δ_2	C'_1	$\cos X_2$	$C''\cos X_2$	P_{e2}	P_{a2}	$k \times P_{a2}$	$\Delta\delta_2$	δ_3	δ'_2	$\delta_2 - \delta'_2$

Example One

The single machine and infinite bus system shown in Figure 6 has the following characteristics.

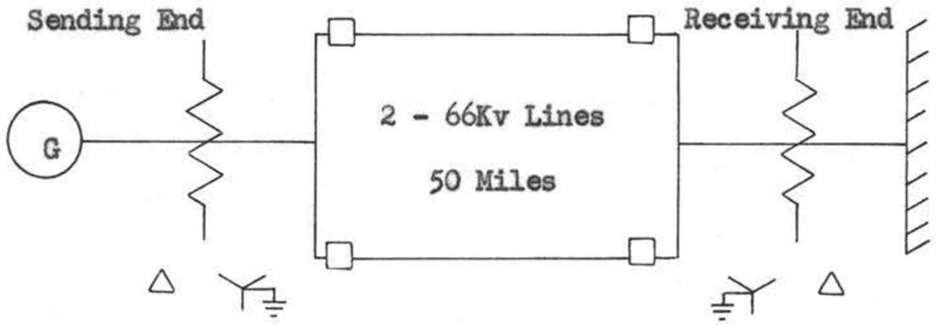


Figure 6. Single-machine system of example one.

Transmission Lines:

Two circuits in parallel, 50 miles long, 10 foot flat spacing, 12.6 foot equivalent delta spacing. Conductors are 250,000 circular mil, copper. Distance between lines is 40 feet and the conductors are transposed. Fifty megawatts unity power factor are delivered to the infinite receiver system. Normal voltage is 66Kv.

Sending End:

One 60 Mva, three-phase, 60 cycle waterwheel generator

Unsaturated synchronous reactance:	X_d =63.8%
Rated current transient reactance:	X_d' =25.4%
Inertia constant (Mw-sec/Mva):	H =3

Transformers:

One 60 Mva, three-phase, 60 cycle bank connected as shown in Figure 6, at each end of the transmission lines.

Reactance = 8%.

Receiving End:

Low-voltage side of receiver transformer connected to an infinite inertia system. Receiver low-voltage bus fixed at 95% of normal voltage.

Transient Disturbance:

A three-phase fault at the midpoint of one line. Circuit breakers at each end of the faulted line isolate the fault simultaneously. The circuit breakers do not reclose. Neglecting system resistance, calculate and plot the swing curves for a sustained fault and for fault clearing in 0.15 seconds.

The problem will be worked using the per unit system with base values of 60,000 Kva and 66 Kv for the three-phase system. The solution of the problem is given in the appendix, with the results of that solution shown in Figure 7.

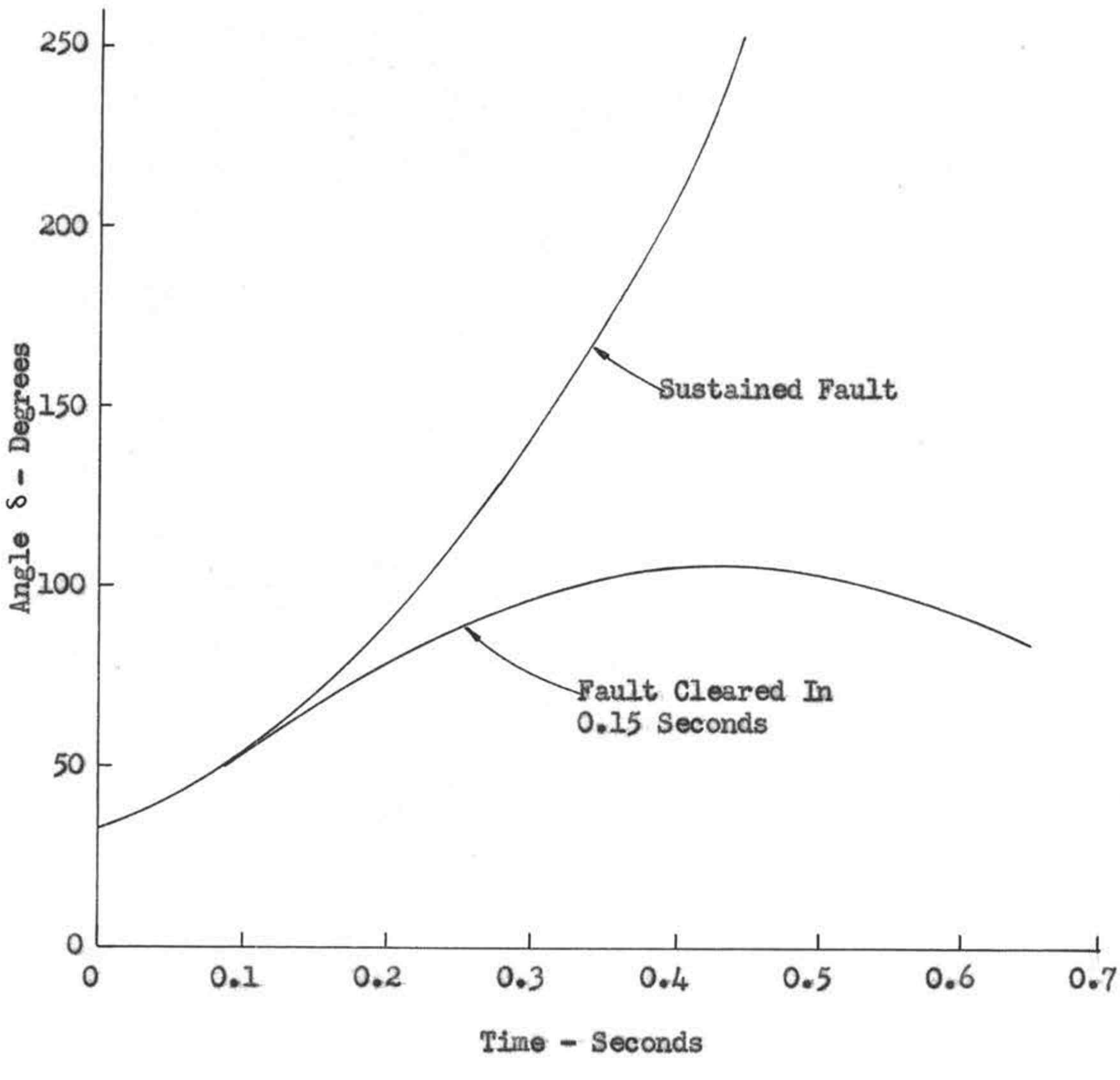


Figure 7. Swing curves for the single-machine system of Example One.

THE A-C NETWORK SIMULATOR

General Design Features

The solution of the single machine example presented here involves a certain amount of network reduction and a large share of hand calculations in the point-by-point computations. Two machine problems may be handled without much more effort, but if more than two machines are to be analyzed, the amount of work becomes exceedingly great. The use of an alternating current network simulator will greatly reduce the labor involved in the solution of power system stability problems. The design of the network calculator is such that all essential elements of a power system can be reproduced in a miniature replica. Using multipliers and conversion factors, the various parts of the system are represented accurately in proportions suitable for observation and measurement. The majority of the network calculators now in use have resistive, inductive and capacitive elements that are used to represent transmission lines and other system impedances. The network simulator presented in this thesis uses resistance units to represent power system impedances.

The resistance network simulator was originally developed by Mr. Waldo E. Enns to solve system power flow and voltage problems (3, p. 96).[2] This simulator was somewhat limited in its operation.

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2. Mr. Enns is presently Chief Systems Engineer of the Portland General Electric Company, Portland, Oregon.

Network impedances between voltage buses had to be reduced to equivalent circuits for use on the simulator since it was not possible to connect impedance units in series. Because the simulator was designed specifically to study the steady-state power system, the phase angles of all bus voltages were limited to approximately 20 degrees lead or lag. Transient stability problems involve large angular swings of machine internal voltages and thus could not be represented on the original network simulator. Investigation of power system transients requires voltage sources with phase angles which are adjustable between zero and 180 degrees lead or lag.

The network simulator presented in this thesis, and based on the principle of Mr. Enns' original simulator, is designed to study power systems under transient conditions. The limitations discussed above have been eliminated in this design, with the result that the network simulator may be used for investigating steady-state power systems as well as those involving transients. Little or no network reduction is necessary if the power system does not exceed the capacity of the simulator. Resistance units are so arranged that they may be connected in series to represent the various portions of the impedance network of the power system. Bus voltage units are connected to either positive or negative potential sources, measured with respect to a neutral bus. In this manner bus voltages may be adjusted to any leading or lagging phase angle from zero to 180 degrees. This feature is discussed in more detail in the section entitled Bus Units.

In recent years, digital computers have played an increasingly more prominent role in the analysis of electric power systems. These computers have enough capacity to analyze most power systems with high speed and accuracy, and therefore have become a valuable tool to the power system engineer. Often in power system design and analysis, it is necessary to make many changes in the system in order to obtain the desired power transfer or proper handling of transient disturbances. System changes, when using digital computers, involves extensive reprogramming and a large consumption of time. Analog network simulators have the advantage that changes in the system under study can be easily made. These changes involve simply a change of impedance connections on the simulator or a change of bus voltage, and are accomplished in a few minutes. Because all system changes can be made directly on the analog simulator, it is much more convenient to use than the digital computer, although accuracy of measurement is somewhat sacrificed. The resistance analog network simulator presented in this thesis is based on the following theory (2, p. 1-4).

Theory Of The Network Simulator

Current and voltages of the power system of Figure 8 may be represented by the phasor diagram of Figure 9.

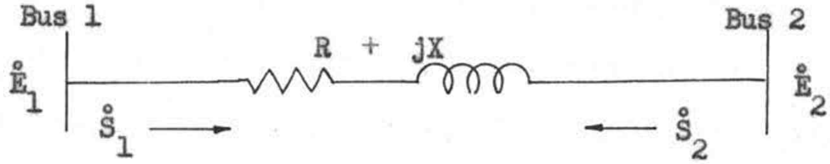


Figure 8. Two-bus system for illustrating network simulator theory.

The positive direction for power and vars will be taken when these quantities flow from a bus into a line.

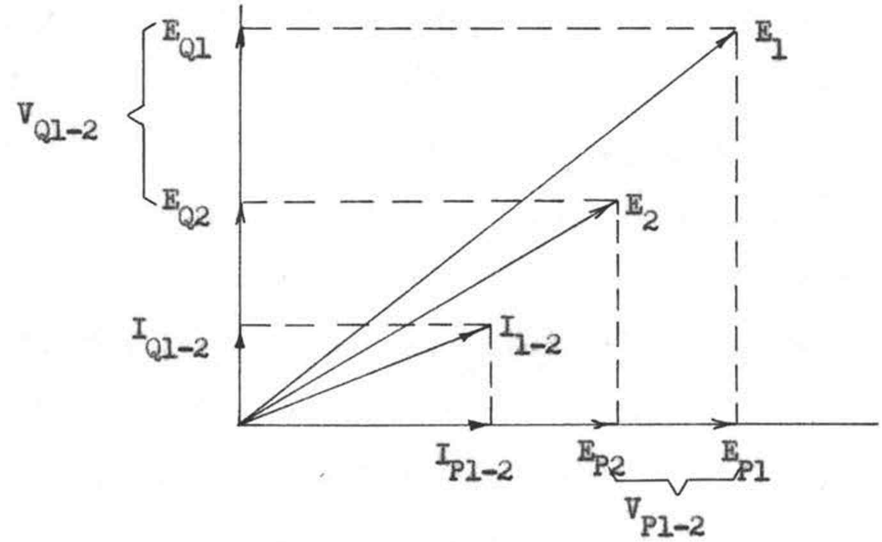


Figure 9. Phasor diagram for the system of Figure 8.

Power flow at each bus of Figure 8 is given by

$$\mathring{S}_1 = \mathring{E}_1 \overset{*}{I}_{1-2} \qquad \text{Eq. (43)}$$

$$\mathring{S}_2 = \mathring{E}_2 \overset{*}{I}_{2-1} \qquad \text{Eq. (44)}$$

The current in the system is

$$\mathring{I}_{1-2} = \frac{\mathring{E}_1 - \mathring{E}_2}{\mathring{Z}}$$

$$= \frac{V_{P1-2} + jV_{Q1-2}}{R + jX}$$

$$= \frac{(V_{P1-2}R + V_{Q1-2}X) + j(V_{Q1-2}R - V_{P1-2}X)}{R^2 + X^2}$$

$$= (V_{P1-2}G + V_{Q1-2}B) + j(V_{Q1-2}G - V_{P1-2}B) \qquad \text{Eq. (45)}$$

For convenience in operating the network simulator, two new quantities, M and N are introduced such that

$$M = \frac{1}{G} = \frac{Z^2}{R} = \frac{Z}{\cos \theta_Z} \qquad \text{Eq. (46)}$$

$$N = \frac{1}{B} = \frac{Z^2}{X} = \frac{Z}{\sin \theta_Z} \qquad \text{Eq. (47)}$$

Substituting these values into Equation (45) results in

$$\mathring{I}_{1-2} = \left(\frac{V_{P1-2}}{M} + \frac{V_{Q1-2}}{N}\right) + j\left(\frac{V_{Q1-2}}{M} - \frac{V_{P1-2}}{N}\right) \qquad \text{Eq. (48)}$$

which has as its real and reactive components of current

$$I_{P1-2} = \frac{V_{P1-2}}{M} + \frac{V_{Q1-2}}{N} \qquad \text{Eq. (49)}$$

$$I_{Q1-2} = \frac{V_{Q1-2}}{M} - \frac{V_{P1-2}}{N} \qquad \text{Eq. (50)}$$

Positive power at bus 1 is given by

$$\mathring{S}_1 = \mathring{E}_1 \overset{*}{I}_{1-2}$$

$$= (E_{P1} + jE_{Q1})(I_{P1-2} - jI_{Q1-2})$$

$$= \left[E_{P1}\left(\frac{V_{P1-2}}{M} + \frac{V_{Q1-2}}{N}\right) + E_{Q1}\left(\frac{V_{Q1-2}}{M} - \frac{V_{P1-2}}{N}\right)\right]$$

$$+ j\left[E_{Q1}\left(\frac{V_{P1-2}}{M} + \frac{V_{Q1-2}}{N}\right) - E_{P1}\left(\frac{V_{Q1-2}}{M} - \frac{V_{P1-2}}{N}\right)\right] \qquad \text{Eq. (51)}$$

The real and reactive components of power at bus 1 are

$$P_1 = E_{P1}\left(\frac{V_{P1-2}}{M} + \frac{V_{Q1-2}}{N}\right) + E_{Q1}\left(\frac{V_{Q1-2}}{M} - \frac{V_{P1-2}}{N}\right) \qquad \text{Eq. (52)}$$

$$Q_1 = E_{Q1}\left(\frac{V_{P1-2}}{M} + \frac{V_{Q1-2}}{N}\right) + E_{Q1}\left(\frac{V_{Q1-2}}{M} - \frac{V_{P1-2}}{N}\right) \qquad \text{Eq. (53)}$$

The positive power and vars at bus 2 are given by the same type of equation as those used at bus 1, but with the signs reversed since $\mathring{I}_{2-1} = -\mathring{I}_{1-2}$. The voltages are the rectangular coordinates of the bus voltage E_2.

$$P_2 = -E_{P2}\left(\frac{V_{P1-2}}{M} + \frac{V_{Q1-2}}{N}\right) - E_{Q2}\left(\frac{V_{Q1-2}}{M} - \frac{V_{P1-2}}{N}\right) \qquad \text{Eq.(54)}$$

$$Q_2 = -E_{Q2}\left(\frac{V_{P1-2}}{M} + \frac{V_{Q1-2}}{N}\right) + E_{P2}\left(\frac{V_{Q1-2}}{M} - \frac{V_{P1-2}}{N}\right) \qquad \text{Eq.(55)}$$

All of the quantities that appear in the above equations for power and vars are scaler numbers. Electrically they could be represented by direct voltages and resistances, but for convenience they are represented on this network simulator by single-phase alternating voltages and resistors. Four resistors and four voltage units are required to represent a single series impedance with a bus at each end. Figure 10 is the network simulator equivalent circuit for such an impedance.

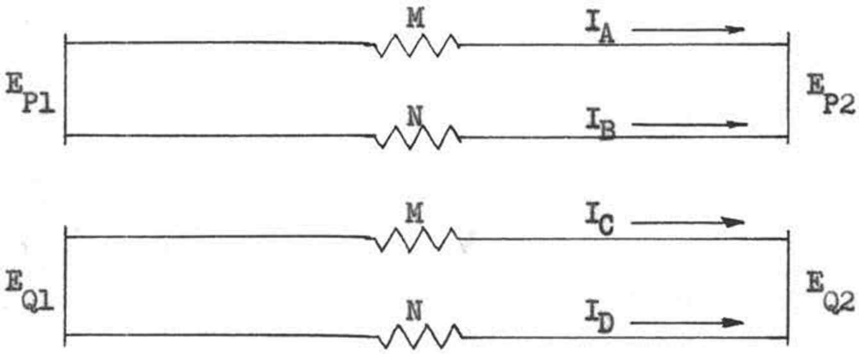


Figure 10. Network simulator equivalent circuit for power system impedances.

The line currents of Figure 10 are

$$I_A = \frac{E_{P1} - E_{P2}}{M} = \frac{V_{P1-2}}{M} \qquad \text{Eq. (56)}$$

$$I_B = \frac{E_{P1} - E_{P2}}{N} = \frac{V_{P1-2}}{N} \qquad \text{Eq. (57)}$$

$$I_C = \frac{E_{Q1} - E_{Q2}}{N} = \frac{V_{Q1-2}}{M} \qquad \text{Eq. (58)}$$

$$I_D = \frac{E_{Q1} - E_{Q2}}{N} = \frac{V_{Q1-2}}{N} \qquad \text{Eq. (59)}$$

Substituting Equations (56) to (59) into Equations (52) to (55), the expressions for power and vars become

$$P_1 = E_{P1}(I_A + I_D) + E_{Q1}(I_C - I_B) \qquad \text{Eq. (60)}$$

$$Q_1 = E_{Q1}(I_A + I_D) - E_{P1}(I_C - I_B) \qquad \text{Eq. (61)}$$

$$P_2 = -E_{P2}(I_A + I_D) - E_{Q2}(I_C - I_B) \qquad \text{Eq. (62)}$$

$$Q_2 = -E_{Q2}(I_A + I_D) + E_{P2}(I_C - I_B) \qquad \text{Eq. (63)}$$

The current combinations are made with three winding current transformers and the multiplications and sums are made with a two element wattmeter.

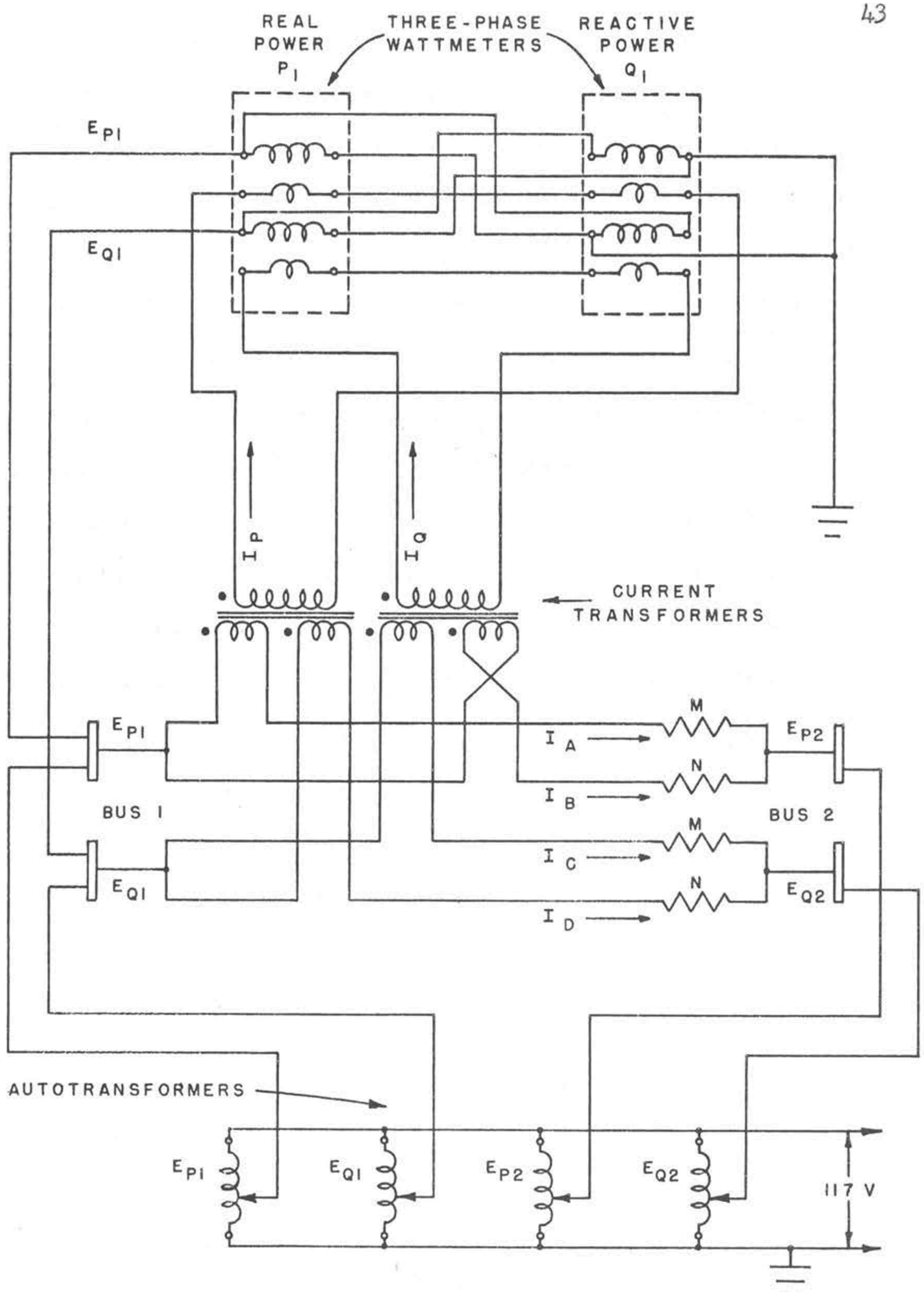
The network calculator is designed to use the per unit system, thus values of M and N must be expressed in per unit of the base impedance of the computer. Base values of voltage and impedance for the calculator have been selected as 75 volts and 2500 ohms. Base current is then 30 milliamperes, and base power becomes 225 volt-amperes.

Description of Network Simulator

As the mathematical development has shown, the operation of the analyzer is based on the principle that an impedance can be accurately represented by four resistors (3, p. 96) properly connected to direct and quadrature components of terminal voltages. When these voltage components are connected to the buses of the system under study, the need for actual generators and loads is eliminated because the power flow in each circuit is defined by the circuit constants and terminal voltages. The power load or generation at any bus is the algebraic sum of the power flowing in all the circuits connected to that bus. Var flow is determined in a similar manner. Figure 11 is a simplified schematic diagram of the network simulator showing one circuit connected between two buses. This illustrates the measurement of power and vars at bus 1.

Two element, three-phase wattmeters connected as shown in Figure 11 will read separately the real and reactive power flow at bus 1. The calculator discussed here, however, uses a single wattmeter and a double-throw switch labeled "P-Q" that changes the potential circuits from the watt (P) to the var (Q) connection. A second double-throw switch reverses the polarity of the wattmeter potential circuits and is used to indicate the direction of power and var flow with respect to the bus being metered. This switch is labeled "Out-In." The "Out" position indicates the positive direction of power or var flow, while the "In" position indicates negative flow. Thus, both real and reactive power flow are measured with the same instrumentation.

<u>Bus Units</u> The solution of Example One indicates that in order to solve stability problems on a network analyzer, it is necessary to have continuous phase adjustment of bus voltages from zero to 360 degrees. Each voltage unit of this analyzer consists of two 0.165 Kva, 120/0-132 volt, 1.25 ampere continuously variable autotransformers which represent the in-phase and quadrature components of bus voltage. Power is supplied to all voltage units by a 1.5 Kva,



FIGURE·11. SIMPLIFIED DIAGRAM OF NETWORK SIMULATOR ILLUSTRATING POWER MEASUREMENT AT BUS ONE.

120/240 volt, single-phase power transformer. The secondary of this transformer has a grounded center-tap which is used as the reference bus throughout the calculator. The two remaining secondary terminals have opposite polarities with respect to the grounded center-tap, and are used as relative positive and negative voltages sources for the analyzer. The common connection of each autotransformer is grounded. The other side of the primary winding is fused and may be connected through a three position switch to either the positive or negative voltage bus. The autotransformer secondaries are independently adjustable in magnitude, from zero to 1.75 per unit volts. Therefore, the voltage at any bus may be adjusted within the specified limits, to any desired magnitude and phase angle.

Because each bus voltage is represented by direct and quadrature components, the question may arise as to why a two-phase voltage supply is not used. First, the voltage components only represent direct and quadrature voltages and are not 90 degrees displaced but actually in phase. Each of the component voltages is initially impressed on a separate circuit and later combined with the proper calculator currents to represent power and var flow. A two-phase voltage supply would introduce an unwanted 90 degree phase shift in the calculator resulting in erroneous power measurements. This is evident if one examines Equations (60) to (63) for power and vars in which all quantities are scalers without associated phase angles.

Having adjusted the bus voltages of the network simulator to the proper magnitudes and phase angles, it is now necessary to measure the power and var flow and the voltage at any desired point in the system being studied. The primary requirement when inserting instruments into the calculator circuits is that they must draw negligible power from the circuits.

Instrumentation Circuits The direct and quadrature components of bus voltage are measured using two identical measuring schemes. The voltage measuring circuits may be inserted into the power system at any desired bus, or into any circuit connected to a bus. The bus selection is relay operated and controlled by a single on-off toggle switch. One switch and the associated relays are provided for each bus in the calculator. The individual circuits to be measured are push button selected, and another push button is provided to measure the entire bus. When any particular bus in the system is being measured, a 15 megohm resistor in series with a 75 thousand ohm resistor is connected between each of the component buses and ground. Voltages proportional to each component voltage are taken from the 75 thousand ohm resistors to ground and conducted through shielded cables to voltage amplifiers. The voltage amplifiers have gains of approximately 23 when used with the 16 ohm termination. The output voltage of the amplifiers is fed to impedance matching step-up transformers with impedance ratios of 16 to 5000 ohms. The 5000 ohm terminals of these transformers are connected to either the direct or quadrature voltmeters and to the watt-var selector switches. The two voltmeters are direct-current instruments and are equipped with external, selenium, full wave bridge rectifiers. The voltmeters and the watt and var potential circuits each have approximately 5000 ohms resistance to match the secondaries of the step-up transformers. In order to maintain a good impedance match between components, the voltmeters and the watt-var potential circuits cannot be connected to the step-up transformers at the same time. The voltmeters indicate only when the watt-var meter is switched off. The meter switching is done with the watt-var selector switch.

If the portion of Figure 11, between the E_{P1} and E_{P2} buses is redrawn as the equvalent circuit of Figure 12, the power loss due to the voltage measuring circuit may be investigated.

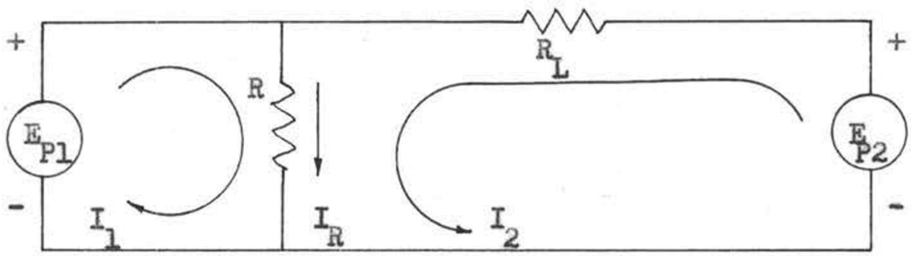


Figure 12. Equivalent circuit illustrating method of voltage measurement.

Resistance R_L is the equivalent resistance between buses, denoted as M and N on Figure 11. The resistance of each current transformer is 3.2 ohms and is small enough to be neglected. R is the 15 megohm resistor for the voltage metering system. The 75 thousand ohms resistor in series with the shunt branch is small compared to 15 megohms and can also be neglected. The loop equations for the voltages of Figure 12 are:

$$E_{P1} = RI_1 + RI_2 \qquad \text{Eq. (64)}$$

$$E_{P2} = RI_1 + RI_2 + R_L I_2 \qquad \text{Eq. (65)}$$

Equations (64) and (65) can be solved for the loop currents I_1 and I_2, the sum of which is the current I_R through the shunt branch.

$$\begin{aligned} I_R &= I_1 + I_2 \\ &= \frac{E_{P1}}{R} + \frac{E_{P1}}{R_L} + \frac{E_{P2}}{R} - \frac{E_{P1}}{R_L} - \frac{E_{P2}}{R} \\ &= \frac{E_{P1}}{R} \end{aligned} \qquad \text{Eq. (66)}$$

The current flowing in the shunt branch is a function of the applied voltage at the bus being measured, and is independent of power system impedance parameters since R is constant. The voltage at any particular bus will commonly be in the range of 0.80 to 1.20 per unit. If the voltage E_{P1} of Figure 12 is 1.10 per unit, the shunt branch current is

$$\begin{aligned} I_R &= \frac{1.10 \text{ pu} \times 75 \text{ volts/pu}}{15 \times 10^6 \text{ ohms}} \\ &= 5.5 \text{ microamperes} \end{aligned} \qquad \text{Eq. (67)}$$

The power drawn by the shunt branch is

$$P = I_R^2 R$$

$$= (5.5 \times 10^{-6})^2 \times 15 \times 10^6$$

$$= 0.454 \text{ milliwatts} \qquad \text{Eq. (68)}$$

A typical value of E_{P2} is 1.00 per unit volts while R_L will vary throughout the range of 0.10 to 2.00 per unit impedance. The power transferred in the series branch of Figure 12 for the assumed limits of line impedance is

$$P_{max} = \left[\frac{(1.10 - 1.00)\text{pu} \times 75 \text{ volts /pu}}{0.10 \text{ pu} \times 2500 \text{ ohms/pu}}\right]^2$$

$$= \frac{7.5^2}{250}$$

$$= 225 \text{ milliwatts} \qquad \text{Eq. (69)}$$

$$P_{min} = \left[\frac{(1.10 - 1.00)\text{pu} \times 75 \text{ volts/pu}}{2.0 \text{ pu} \times 2500 \text{ ohms/pu}}\right]^2$$

$$= \frac{7.5^2}{5000}$$

$$= 13.25 \text{ milliwatts} \qquad \text{Eq. (70)}$$

Throughout the range of R_L, the transferred power is much larger than the power drawn by the shunt branch of Figure 12. Hence, the voltage measuring circuits are a very small burden on the circuits of the calculator.

The current measuring circuits of Figure 11 are over simplifications of the actual circuits. The secondary of each three-winding current transformer is connected to a current amplifier which has both gain and phase adjustments. The amplifier outputs are each fed into other current transformers having turns ratios of 40 to 1. In turn, each of these transformer secondaries is connected to one of the watt-var meter current coils. The over-all gain of the current instrumentation circuits is approximately 90, and is adjustable from the control panel of the calculator by 50,000 ohm and 2,000 ohm variable resistors connected in series across the secondary winding of each three-winding current transformer. The primary windings of the three-winding current transformers are rated at 3.2 ohms or 0.00128 per unit impedance each and are negligible compared with any circuit impedance which might be used. The burden of the current measuring circuits is thus very small and does not influence the power transfer of the system.

Resistance Units The network simulator is designed to have a capacity of 60 buses and 92 circuits. Fifty of the buses have a capacity of four circuits, the remaining ten buses have eight circuit capacity. Each circuit represents a power system impedance such as a transmission line or generator reactance and is represented on the calculator by four resistor units, two of which are M units and two are N units. Each unit is a 0-5000 ohm continuously variable resistor of the wire-wound type and has a power rating of three watts. Wire-wound resistors were chosen rather than composition type because of better stability characteristics. Preliminary investigation showed that the wire-wound resistors had a resistance variation of one to four percent depending upon the initial setting. Composition resistors were found to vary between two and one half and four percent of their initial values. The temperature-resistance characteristics of the samples were determined by connecting a one and one half volt battery to each for a period of 30 minutes and noting the change in resistance during that period. Resistance was measured before and after each test with a resistance bridge, and the cumulative resistance drift was displayed on a dual-trace oscilloscope. After a period of 30 minutes, the wire-wound resistors appeared to have stabilized, whereas the composition type had not. The circuit used and the data taken are given in Appendix C.

On a per unit basis, each of the resistance units is adjustable between zero and two per unit. In series with each unit is an open circuit which must be jumpered with a wire having banana plugs on each end. The purpose of this jumper connection is three-fold. First, it provides a quick and accurate check on which resistance units are in use. Secondly, in some studies only reactance components are used. These components correspond to the N resistance units. By leaving the M units unjumpered in such a study, there is no chance that they can be accidently connected into the circuit and result in false readings of power or var flow. Finally, it may be found necessary to occasionally use values of M and N greater than the two per unit available in each resistor unit. The jumpers provide a means of connecting extra resistors in series to obtain the proper values of the circuit constants.

To provide maximum speed and convenience in setting up problems on the analyzer, two modified Strowger step-by-step connector switches are used to connect any of the 92 circuits to an ohmmeter for setting the M and N constants. The switches, operated by a modified telephone dial, are connected one on either side of the circuit being measured. Each switch has two banks containing 100 sets of two contacts. The banks are semicylindrical in form and are mounted one above the other with the sets of contacts arranged in ten horizontal rows. The two contacts of each set are placed vertically with a thin insulator between them. Each bank has a pair of spring wipers which may connect to any of the 100 sets of contacts. The two pair of wipers of each switch are arranged on a vertical shaft so that when the upper pair is raised to a given level on the upper bank of contacts, the lower pair is raised to the corresponding level on the lower bank. The four wipers of each switch have 100 possible positions and at each position they are connected to four contacts. Ninety-two of these positions are occupied by the analyzer circuits. The four contacts of each position correspond to the two M and two N resistance units of each circuit. The circuits have been assigned numbers from 01 through 92 and each may be selected by dialing the appropriate number.

The spring wipers of each switch are connected to an auxiliary four-position switch which allows independent measurement of the four resistance units of the circuit. The auxiliary switches are operated with a single push button. Another push button returns both Strowger switches and the auxiliary switches to their normal unoperated positions.

System Fault Panel System fault studies on the network simulator require that the system can be short circuited to the neutral bus at any desired point. A separate shorting panel provided for this purpose connects the fault point to the neutral bus through a four-pole single-throw switch. The panel is designed to be used with two circuits connected in series. It is inserted in the circuit to be faulted, with one end of the shorting switch between the resistor units of the two series circuits. The resistors are then adjusted to place the fault in the desired location in the circuit. Terminals are provided on the shorting panel for connection of external resistors when it is desired to study faults through impedances.

Complete circuit diagrams of the network simulator and its component parts are found in Appendix D.

Simulator Solution Of The Swing Equation

Stability problems are solved on the network simulator by considering them as a set of simultaneous equations under successive steady-state conditions. The system under study is first reduced to a common base and normally set up on the basis of the positive sequence network. Each machine internal voltage is connected in series with its transient reactance. The system is connected on the network simulator for conditions prior to the disturbance. Values of machine internal voltages, power flow and machine rotor angles are recorded at each bus in the study. Real power at each bus of the system prior to the disturbance is assumed to be the mechanical power at that bus. Internal voltage magnitudes of the machines are held constant throughout the study, with only the phase angles of these voltages allowed to change. Initial phase angles of the machine voltages are the normal power system phase angles, and departures from these angles are calculated from the real power and dynamic characteristics of the machines.

When the operating conditions of the system prior to the disturbance have been properly connected on the network simulator, the transient disturbance is applied at the desired point in the system. Machine voltages and phase angles are adjusted to their pre-disturbance values and the new power at each bus is read. The difference in the power before and after the disturbance is applied is the accelerating power available for changing the angular position of the machine internal voltages. From the relations between machine inertia, change in power and time interval, the angular change of each machine rotor is calculated. The simulator bus voltage angles are then shifted to the new values. The procedure is repeated for the next interval, and continued until the system is proven either stable or unstable for the conditions being studied.

To illustrate the network simulator solution of transient stability problems, the following example is offered.

Example Two

The two machine power system of Figure 13 has the following characteristics.

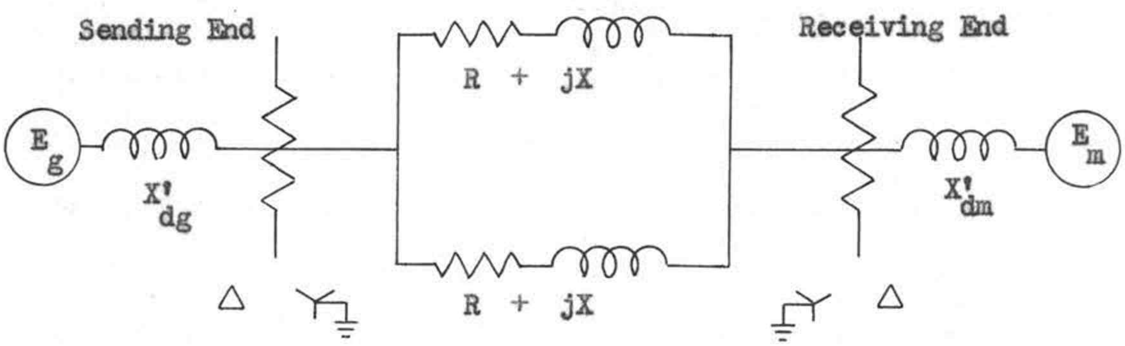


Figure 13. Two-machine power system of Example Two.

All values of impedance are expressed on a 50 Mva, 33 Kv base.

Transmission Lines:

Two circuits in parallel. The power delivered at the receiving end is 0.75 per unit at unity power factor and one per unit voltage. Normal voltage is 33 Kv. Each line is assumed to have the following impedances:

a. $0 + j0.75$ pu.

b. $0.53 + j0.53$ pu.

c. $0.75 + j0$ pu.

Sending End:

One 100 Mva, 13.8 Kv, three-phase, 60 cycle waterwheel generator.

Rated current transient reactance: $X'_d = 15\%$

Inertia constant (Mw-sec/Mva): $H = 3$

Transformers:

One 75 Mva, three-phase, 60 cycle bank at the sending end, connected as shown in Figure 13.

Reactance: $X = 4.66\%$

One 50 Mva, three-phase, 60 cycle bank at the receiving end connected as shown in Figure 13.

Reactance: $X = 7\%$

Receiving End:

One 50 Mva, 13.8 Kv, three-phase, 60 cycle synchronous motor.

Rated current transient reactance: $X'_d = 25\%$

Inertia constant (Mw-sec/Mva): $H = 2$

Transient Disturbance:

The system is initially operating with both transmission lines in service. One line is then opened by circuit breakers. For each of the transmission line impedance conditions, calculate and plot the swing curves of the power system.

The data obtained from the simulator and the associated calculations are given in Appendix E. Swing curves of Example Two are shown in Figure 14.

Synchronous Condensers

Synchronous condensers are over-excited synchronous motors used primarily for power factor correction. These condensers operate without mechanical shaft torque, and under steady-state conditions, do not draw power from the system except to overcome losses. Being over excited machines, they draw a leading current and thus act as capacitors. In power system studies, the synchronous condenser is often represented by a static capacitor. Most commercial network analyzers make this replacement since the synchronous condenser is generally small compared with the motors and generators of the system. Because the condenser is small, its inertia will also be small and will not influence the stability of a power system to any great degree. If the condenser is large enough to be of importance in stability studies, it is treated as any other synchronous machine. That is, it is represented as an internal voltage in series with its transient reactance. Initially, only reactive power flows in the condenser, but under transient conditions, real power will also flow.

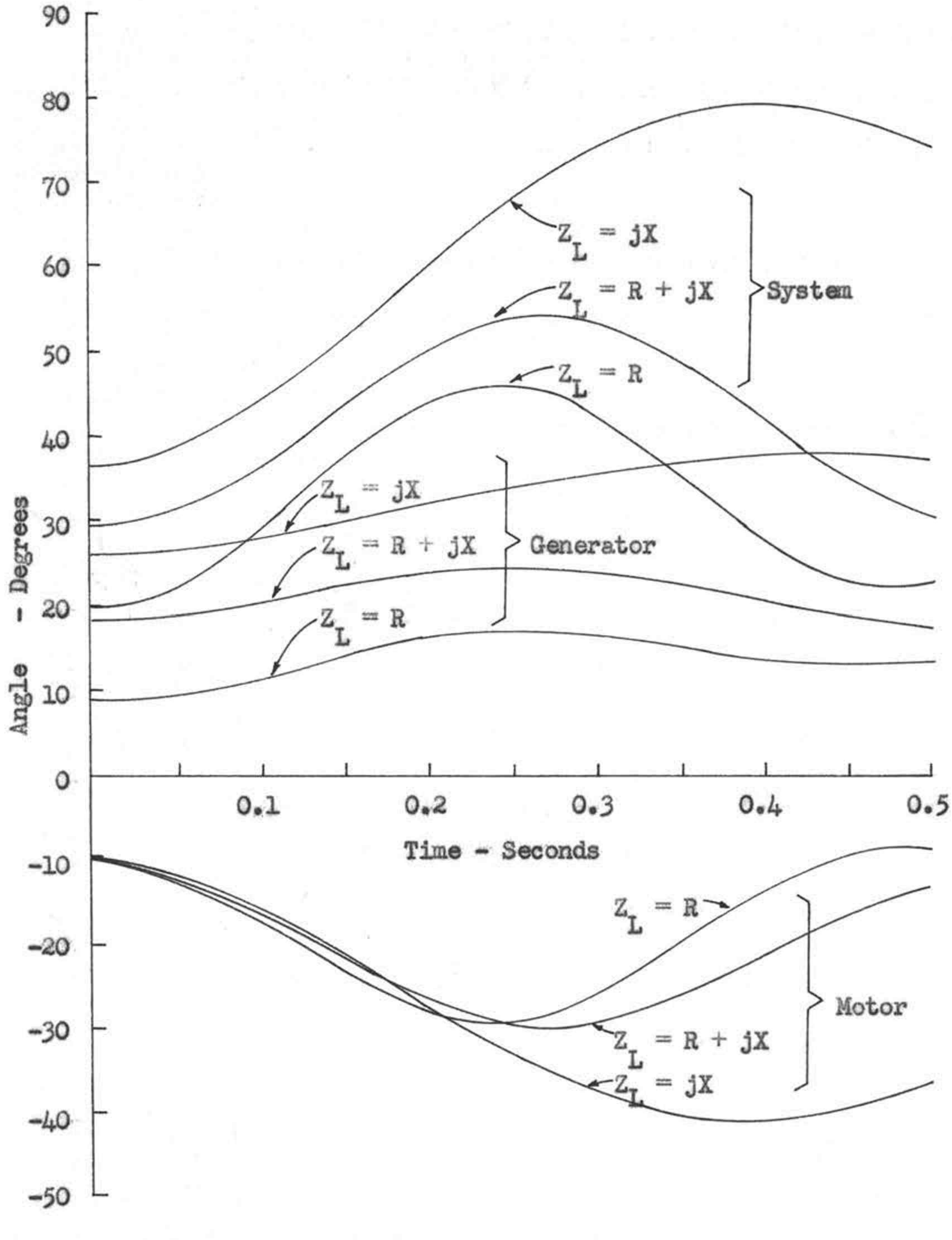


Figure 14. Swing curves for the two-machine system of Example Two.

The network simulator presented here has no provision for representing capacitance. Therefore, condensers must be represented as synchronous machines, and analyzed in the same manner.

Simulator Limitations

Voltage instrumentation circuits of the network simulator require half an hour or more to thoroughly warm up and stabilize. During this warm up period, the input voltage to the amplifiers decreases about ten percent indicating a positive temperature coefficient of resistance of the voltage measuring resistors. These resistors are one watt carbon resistors with plus or minus five percent tolerance. Precision resistors of a higher power rating would help to eliminate voltage drift. If, during the warm up period, the bus voltage is increased to compensate for the drift in the measuring circuit, the current flow through the simulator circuits will increase. This results in too much current at the terminals of the wattmeter and an incorrect power measurement. The present procedure to counteract voltage drift during warm up is to change the measuring resistors. Fixed resistors of the type previously mentioned are provided for this purpose in a range 12 to 18 megohms. Clip leads are used to make the connections between the resistors and the input terminals of the voltage amplifiers.

The instruments used in the network simulator have scale divisions of 0.05 per unit, and can be read only to two places with accuracy. The over-all accuracy of measurement is within ten percent of actual values. Example Two presented here was checked by hand calculation. The value of electric power obtained from the simulator in no instance varied more than ten percent from hand calculations, and in most cases the error was about five percent.

CONCLUSIONS

On electric power systems consisting of more than two machines, stability analysis without the aid of a network simulator is very difficult and tedious. Some of the advantages of a network simulator include:

a. Little or no network reduction is necessary, provided the power system to be simulated does not exceed the capacity of the simulator.

b. Initial system operating conditions need not be computed, but may be obtained by a steady-state study on the simulator.

c. Load changes, switching operations and symmetrical three-phase transient disturbances may be applied on the simulator just as they are on the actual power system. These disturbances employ only the positive sequence impedance network. To study unbalanced system faults, the negative and zero sequence impedance networks must also be used.

d. The number of steps in the point-by-point solution of stability problems is greatly reduced. The simulator reads directly the electric power, from which are computed the accelerating power and angular change of the machines of the system.

e. The simulator can be used as a design tool for electric power systems. In this respect, it has a decided advantage over high speed digital computers.

The network simulator has been designed to have a capacity of 60 buses and 92 circuits. Half of the 92 circuits connect directly through relays to the metering bus of the simulator and are used when a single impedance is connected between buses. The other 46 circuits have Cinch-Jones plugs on both ends and may be connected in series to represent the various portions of a power system impedance. These circuits have no connection to the metering bus, and when used, must have as the terminating unit, a circuit which does connect to the metering bus.

Fifty of the buses are designed to have a maximum of four circuits connected to them. The remaining ten buses have a capacity of eight circuits each. The simulator is presently in the construction stage, and contains only enough buses and circuits to study two-machine power systems.

The network simulator is based on the principle that an impedance can be represented by four resistors if they are properly connected to in phase and quadrature magnitudes of the voltages at the terminals of the impedance. Connecting these voltage components to the buses of the system eliminates the need for actual generators and loads because the power flow in each circuit is determined by its constants and terminal voltages.

Wire-wound resistors were chosen for the simulator circuits because of their resistance stability under temperature changes. The units selected are continuously variable from 0-5000 ohms and have a power rating of three watts.

The per unit system of notation is used on the network simulator with base values of voltage and impedance of 75 volts and 2500 ohms. The simulator is instrumented by in phase and quadrature voltmeters and a two-element, three-phase wattmeter which measures both real and reactive power. Current and voltage amplifiers are provided in the metering system, thus reducing the power loss from the circuits of the simulator.

Transient stability problems are solved by employing a point-by-point analysis of the swing equation. On the network simulator, this is done by solving a set of simultaneous equations under successive steady-state conditions. The analysis is done in tabular form and continued until the system is proven either stable or unstable. A special short circuit panel is provided for system fault studies, while for load changes, the bus voltages are changed, and for switching operations, circuit connections are changed.

Often in transient stability studies, transmission line resistance is neglected. Example Two shows the effect on stability of including this resistance. Resistance is seen to cause damping of system oscillations around the equilibrium point. The amount of damping increases with increasing resistance.

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APPENDIX

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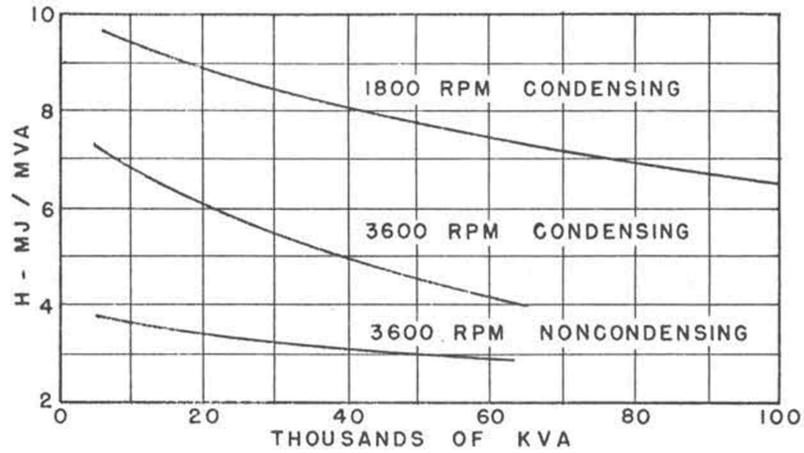
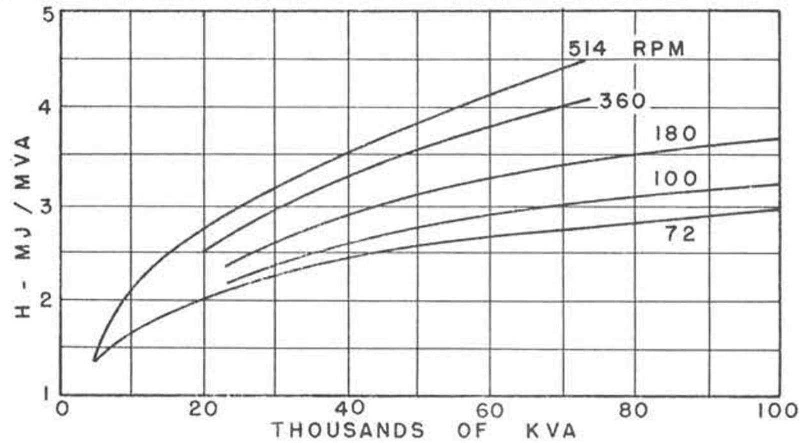
APPENDIX A

GENERATOR CHARACTERISTICS



(a) LARGE TURBINE GENERATORS, TURBINE INCLUDED.



(b) LARGE VERTICAL TYPE WATERWHEEL GENERATORS, INCLUDING ALLOWANCE OF 15 PERCENT FOR WATERWHEELS.

FIGURE A-1. INERTIA CONSTANTS OF GENERATORS (8, P. 189).

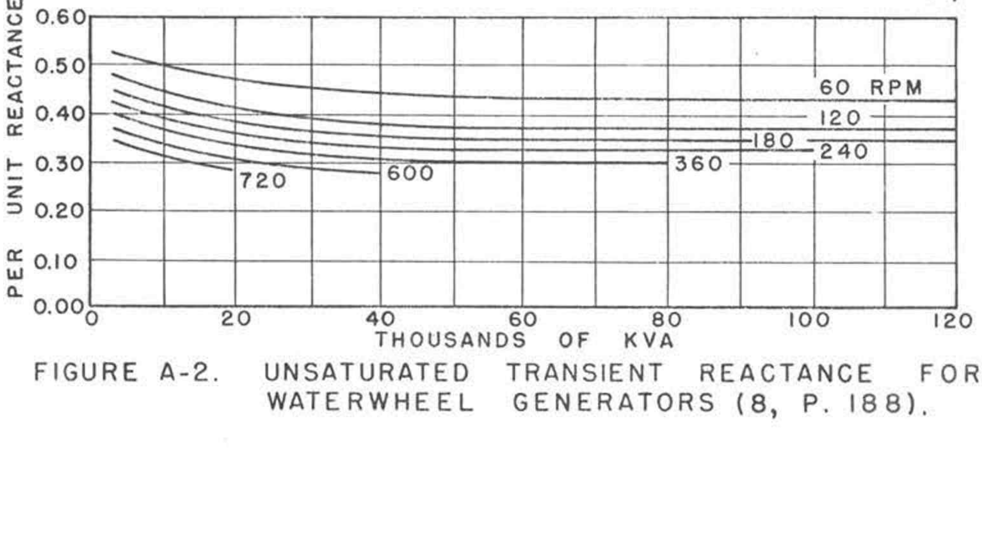


FIGURE A-2. UNSATURATED TRANSIENT REACTANCE FOR WATERWHEEL GENERATORS (8, P. 188).

APPENDIX B

SOLUTION OF EXAMPLE ONE

On a single-phase basis, the base values of voltage, current and impedance are

$$\text{Base voltage} = \frac{\text{L-L Kv}}{3}$$

$$= \frac{66\ \text{Kv}}{3}$$

$$= 38.1\ \text{Kv L-N} \qquad \text{Eq. (B-1)}$$

$$\text{Base current} = \frac{3\phi\ \text{Kva}/3}{\text{L-L Kv}/3}$$

$$= \frac{60{,}000\ \text{Kva}}{66\ \text{Kv}/3}$$

$$= 525\ \text{amperes} \qquad \text{Eq. (B-2)}$$

$$\text{Base impedance} = \frac{\text{Base voltage}}{\text{Base current}}$$

$$= \frac{38.1\ \text{Kv}}{525\ \text{amps}}$$

$$= 72.6\ \text{ohms} \qquad \text{Eq. (B-3)}$$

Resistance of the system has been neglected, reducing all impedances to pure reactances. The transmission line reactance is calculated with the aid of Tables 1 and 6 of reference 9 (9, p. 49-54).

$$X = X_a + X_d$$
$$= 50 \text{ miles } (0.487 + 0.3073) \text{ ohms/mile}$$
$$= 39.7 \text{ ohms} \qquad \text{Eq. (B-4)}$$

In per unit notation, the reactance of each line becomes

$$X_{pu} = \frac{X \text{ ohms}}{\text{Base } Z}$$
$$= \frac{39.7}{72.6}$$
$$= 0.547 \text{ pu} \qquad \text{Eq. (B-5)}$$

The fault on the system is the three-phase type, and only the positive sequence reactance diagram of Figure B-1 need be considered.

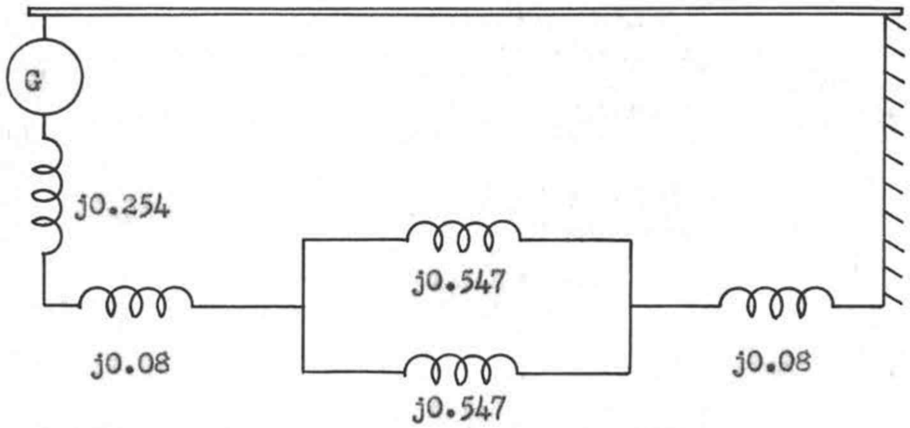


Figure B-1. Positive sequence reactance diagram of Example One prior to the fault.

The equivalent reactance between the generator terminals and the infinite bus prior to the fault is

$$X_e = j0.08 + j\frac{0.547}{2} + j0.08$$
$$= j0.434 \text{ pu} \qquad \text{Eq. (B-6)}$$

Receiver load is 50,000 Kw or 0.833 pu with the voltage constant at 0.95 pu. The current flowing in the system is

$$\mathring{I} = \frac{0.833}{0.95}$$
$$= 0.877\ e^{j0^\circ}\ \text{pu} \qquad \text{Eq. (B-7)}$$

The generator terminal voltage is then

$$\mathring{E}_t = \mathring{E}_m + \mathring{I}X_e$$
$$= 0.95 + 0.877 \times j0.434$$
$$= 0.95 + j0.381$$
$$= 1.023\ e^{j22^\circ}\ \text{pu} \qquad \text{Eq. (B-8)}$$

and the voltage behind the generator transient reactance is

$$\mathring{E}_g = \mathring{E}_t + \mathring{I}X'_d$$
$$= 0.95 + j0.381 + 0.877 \times j0.254$$
$$= 0.95 + j0.604$$
$$= 1.12\ e^{j32.4^\circ}\ \text{pu} \qquad \text{Eq. (B-9)}$$

The sending end power is given by Equation (11) of the text, which is the power equation for any system. When system resistance is neglected, Equation (11) reduces to the following form.

$$P_g = \frac{E_g\ E_m}{X_e + X'_d} \sin \delta$$
$$= \frac{1.12 \times 0.95}{0.434 + 0.254} \sin \delta$$
$$= 1.55 \sin \delta\ \text{pu} \qquad \text{Eq. (B-10)}$$

When the fault is applied, the positive sequence reactance diagram becomes that of Figure B-2.

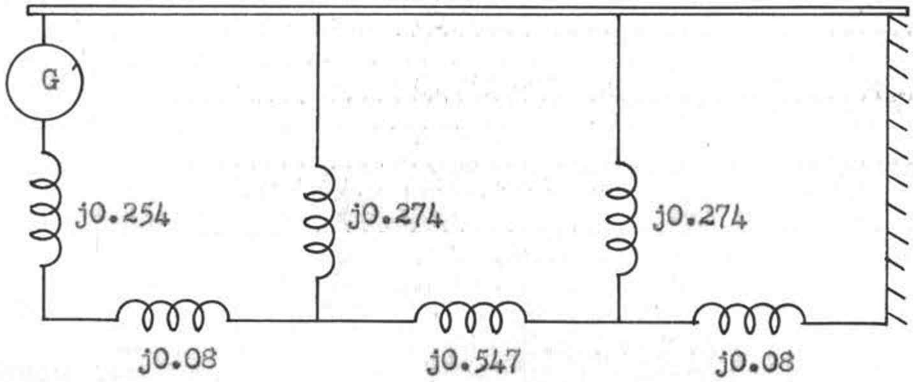


Figure B-2. Positive sequence reactance diagram of Example One during fault.

By successive wye-delta conversions, Figure B-2 may be converted to an equivalent delta circuit as shown in Figure B-3.

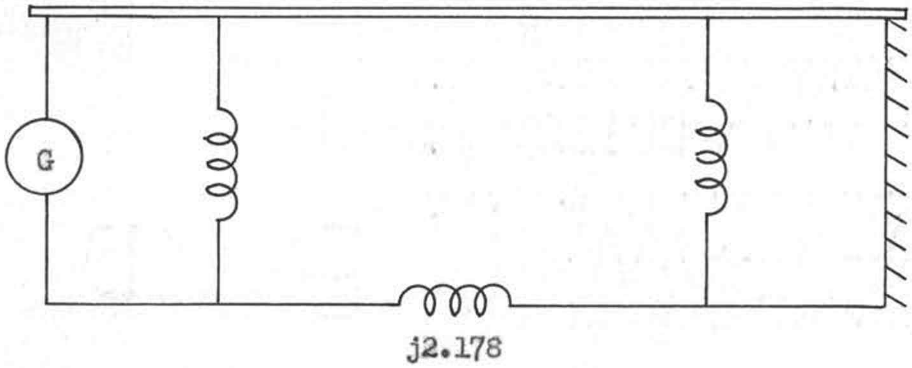


Figure B-3. Equivalent reactance diagram of Figure B-2.

The values of the shunt branches need not be determined since they are pure reactances and have no effect on the power transfer of the system, although they do increase the var flow. Sending end power during the fault is given by

$$P_g' = \frac{1.12 \times 0.95}{2.178} \sin \delta$$

$$= 0.488 \sin \delta \text{ pu} \qquad \text{Eq. (B-11)}$$

When the fault is cleared, the equivalent reactance of the system reduces to the generator transient reactance plus the transformer reactances in series with the reactance of the remaining line, or 0.957 pu. The power transferred through one line is

$$P_g'' = \frac{1.12 \times 0.95}{0.957} \sin \delta$$

$$= 1.11 \sin \delta \text{ pu} \qquad \text{Eq. (B-12)}$$

All necessary information for calculation of the swing curve is now available, and the tabular solution follows. A time interval of 0.05 seconds will be used.

The critical angle of system oscillation can be determined from Equation (B-12) for the transferred power over one line when the fault has been cleared. The amount of power to be transferred is 0.833 pu. The critical angle is given by

$$\delta_c = 180^\circ - \sin^{-1} \frac{P_s''}{1.11}$$

$$= 180^\circ - 48.6^\circ$$

$$= 131.4^\circ \qquad \text{Eq. (B-13)}$$

Table B-1. Computation sheet for power system stability studies.

Machine No. 1   Total Mva 60   Shaft Power P_s 50 Mw

H 3 Mw-sec/Mva $M = GH/180f$ 2.78×10^{-4} Mw-sec^2/elect. degree

Δt 0.05 Seconds $k = (\Delta t)^2/M$ 9 Mw/ elect. degree

(1) Time Seconds	(2) Initial Angle δ_{n-1}	(3) $\pm$ C'	(4) $\cos(\beta \pm \delta_n)$	(5) $\mp$ C'' $\cos(\beta \pm \delta_n)$	(6) Electric Power P_e	(7) Accelerating Power P_a	(8) k x P_a	(9) Angular Change $\Delta\delta_n$	(10) Final Angle δ_n	(11) Initial Angle Of Other Machine δ'_{n-1}	(12) $\delta_{n-1} - \delta'_{n-1}$
—	—	—	—	—	3 + 5	P_s - 6	7 x k	8 + 9_{n-1}	10_{n-1} + 9	—	2 - 11
0 -	32.4	0	-0.537	0.833	0.833				32.4		
0 +	32.4	0	-0.537	0.262	0.262	0.571			32.4		
0 avg						0.285	2.66	2.66	35.06		
0.05	35.06	0	-0.575	0.281	0.281	0.552	4.97	7.63	42.69		
0.10	42.69	0	-0.678	0.331	0.331	0.502	4.52	12.15	54.74		
0.15	54.74	0	-0.816	0.390	0.390	0.443	3.98	16.13	70.87		
0.20	70.87	0	-0.945	0.461	0.461	0.372	3.35	19.48	90.35		
0.25	90.35	0	-1.000	0.488	0.488	0.345	3.10	22.58	112.93		
0.30	112.93	0	-0.92	0.449	0.449	0.384	3.45	26.03	138.96		
0.35	138.96	0	-0.656	0.320	0.320	0.513	4.62	30.03	169.61		
0.40	169.61	0	-0.180	0.088	0.088	0.745	6.71	37.36	206.97		
0.45	206.97	0	0.453	-0.221	-0.221	1.054	9.50	46.86	253.85		

Table B-2. Computation sheet for power system stability studies.

Machine No. 1 Total Mva 60 Shaft Power P_s 50 Mw

H 3 Mw-sec/Mva M = GH/180f 2.78×10^{-4} Mw-sec^2/elect. degree

Δt 0.05 Seconds k = $(\Delta t)^2/M$ 9 Mw/ elect. degree

(1) Time Seconds	(2) Initial Angle δ_{n-1}	(3) $\pm$ C'	(4) $\cos(\beta \pm \delta_n)$	(5) $\mp$ C" $\cos(\beta \pm \delta_n)$	(6) Electric Power P_e	(7) Accelerating Power P_a	(8) k x P_a	(9) Angular Change $\Delta\delta_n$	(10) Final Angle δ_n	(11) Initial Angle Of Other Machine δ'_{n-1}	(12) $\delta_{n-1} - \delta'_{n-1}$
–	–	–	–	–	3 + 5	P_s - 6	7 x k	8 + 9_{n-1}	10_{n-1} + 9	–	2 - 11
0.15 -	54.74	0	-0.816	0.390	0.390	0.443					
0.15 +	54.74	0	-0.816	0.906	0.906	-0.073					
0.15 avg						0.185	1.67	13.82	68.56		
0.20	68.56	0	-0.930	1.033	1.033	-0.200	-1.80	12.02	80.58		
0.25	80.58	0	-0.985	1.094	1.094	-0.261	-2.35	9.67	90.25		
0.30	90.25	0	-1.000	1.110	1.110	-0.277	-2.49	7.18	97.43		
0.35	97.43	0	-0.990	1.100	1.100	-0.267	-2.40	4.78	102.21		
0.40	102.21	0	-0.977	1.085	1.085	-0.252	-2.27	2.51	104.27		
0.45	104.27	0	-0.967	1.075	1.075	-0.242	-2.18	0.33	105.05		
0.50	105.05	0	-0.966	1.073	1.073	-0.240	-2.16	-1.83	104.22		
0.55	104.22	0	-0.969	1.077	1.077	-0.244	-2.20	-4.03	100.19		
0.60	100.19	0	-0.984	1.093	1.093	-0.260	-2.34	-6.37	93.82		

APPENDIX C

RESISTOR STABILITY CHARACTERISTICS

The stability characteristics of wire-wound and composition type resistors were desired before resistor units were purchased for the network simulator circuits. Samples of each type of resistor were connected to the circuit of Figure C-1 to determine how current flow through the units affected the resistance.

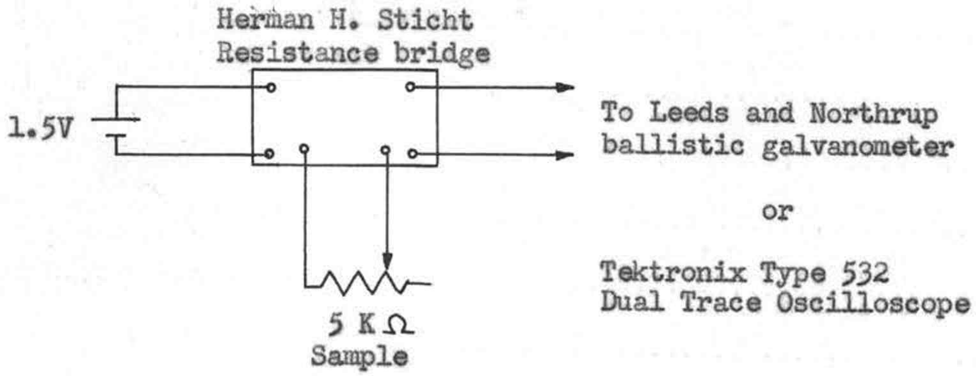


Figure C-1. Circuit for measuring resistor stability characteristics.

The wire-wound sample resistor was set at 5000 ohms using the resistance bridge and ballistic galvanometer. The galvanometer was then replaced with the oscilloscope which had been allowed to warm up and stabilize. The B channel of the oscilloscope was used as a reference, with the output terminals of the resistance bridge connected to the A channel. The traces of the two channels were made to coincide. As the resistor temperature increased due to current flow from the battery, the A trace drifted away from the

B trace indicating a resistance change. This procedure was repeated with other settings of the sample and with other samples. After being connected to the circuit for 30 minutes, the wire-wound resistors appeared to have stabilized, whereas the composition type had not. One hour was allowed between tests on any one sample resistor. Results of the tests are given in Table C-1.

Table C-1. Resistance stability characteristics of wire-wound and composition resistors.

Type Of Resistor	Initial Setting	Resistance Increase	Percent Change
Wire-wound	5000	50	1.0
	1000	12	1.2
	500	20	4.0
Composition	5000	120	2.4
	1000	24	2.4
	500	20	4.0

APPENDIX D

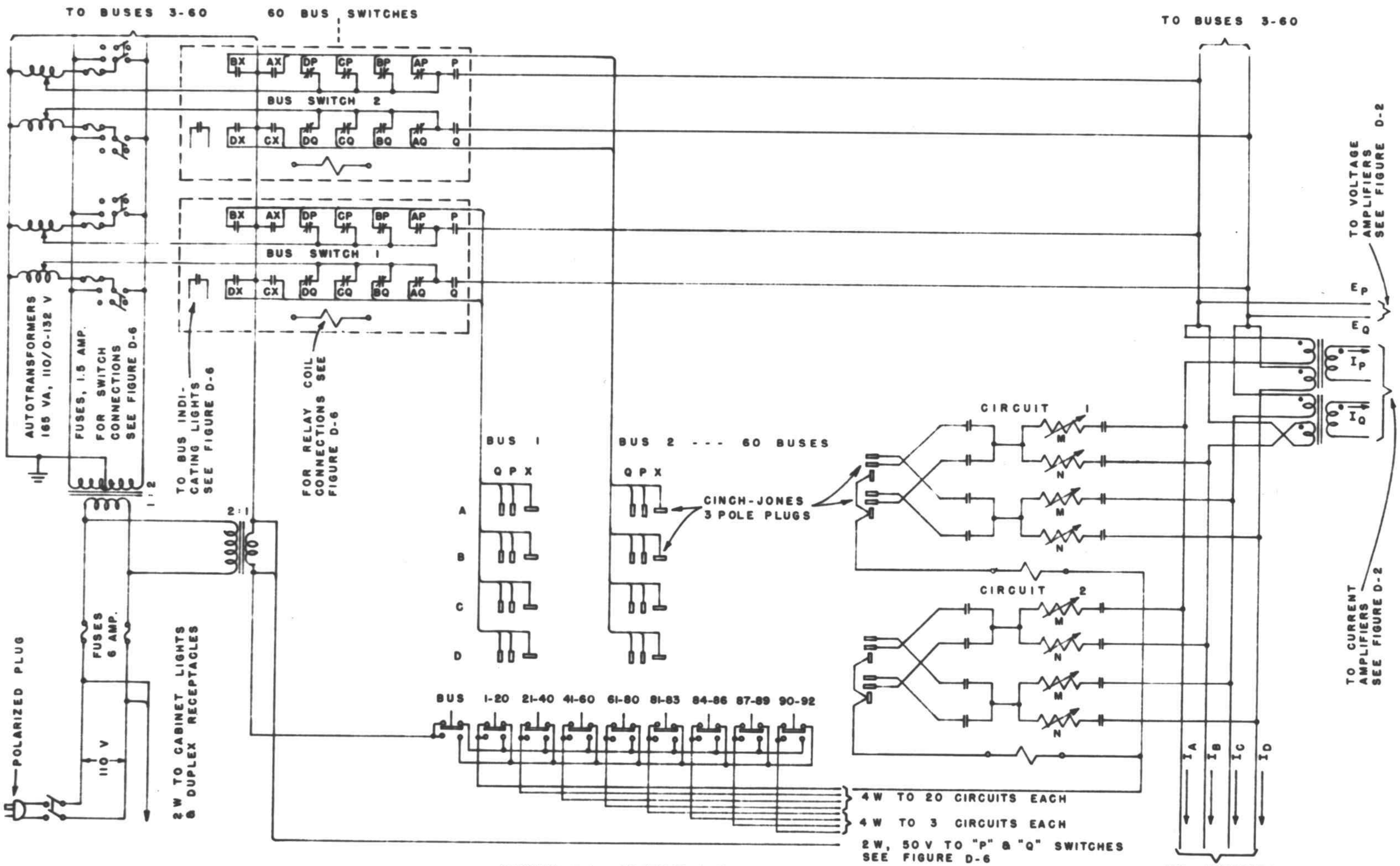


FIGURE D-1. SCHEMATIC DIAGRAM OF NETWORK SIMULATOR BUSES AND CIRCUITS

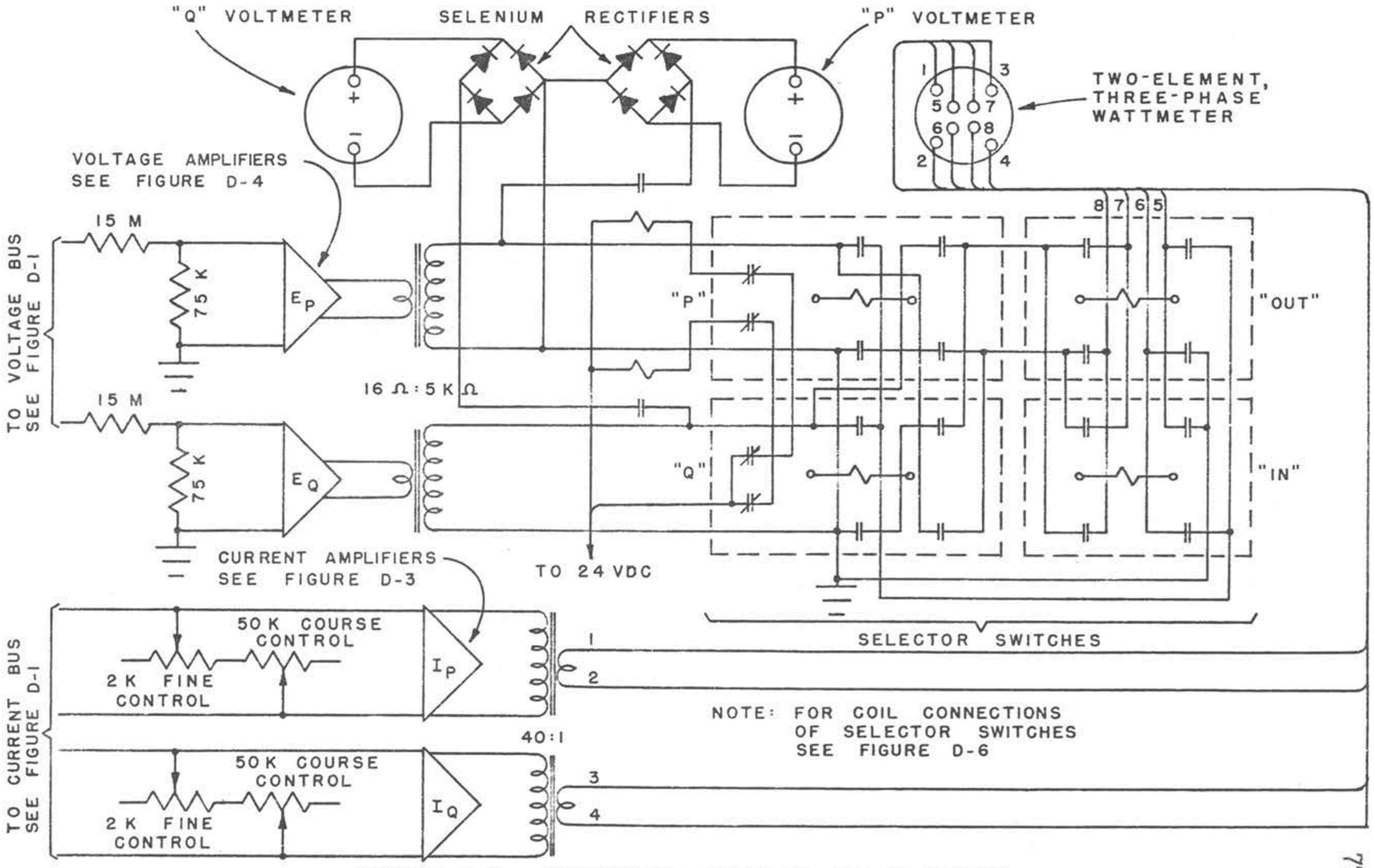


FIGURE D-2. SCHEMATIC DIAGRAM OF NETWORK SIMULATOR METERING CIRCUITS

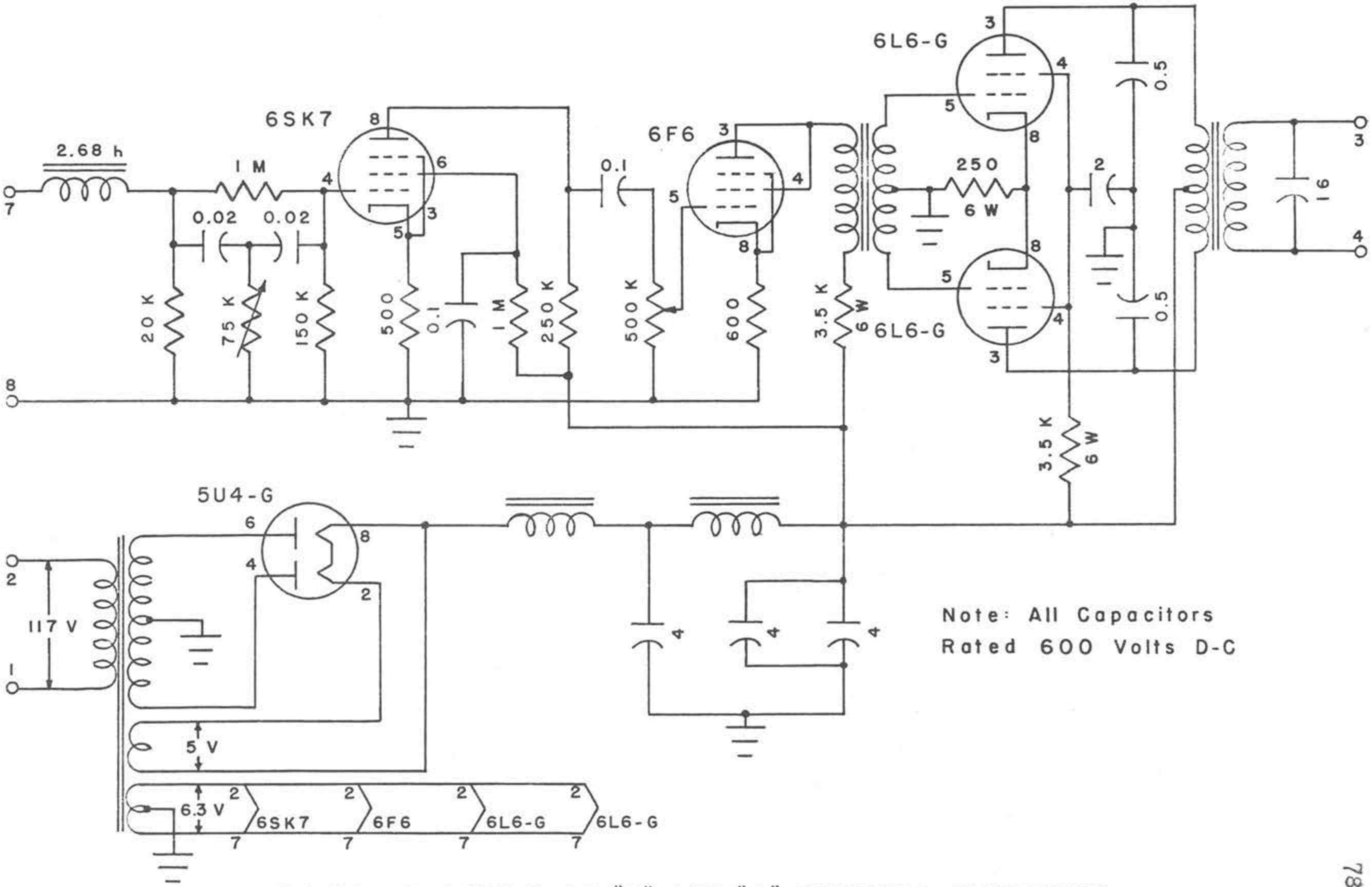


FIGURE D-3. CIRCUIT OF "P" AND "Q" CURRENT AMPLIFIERS

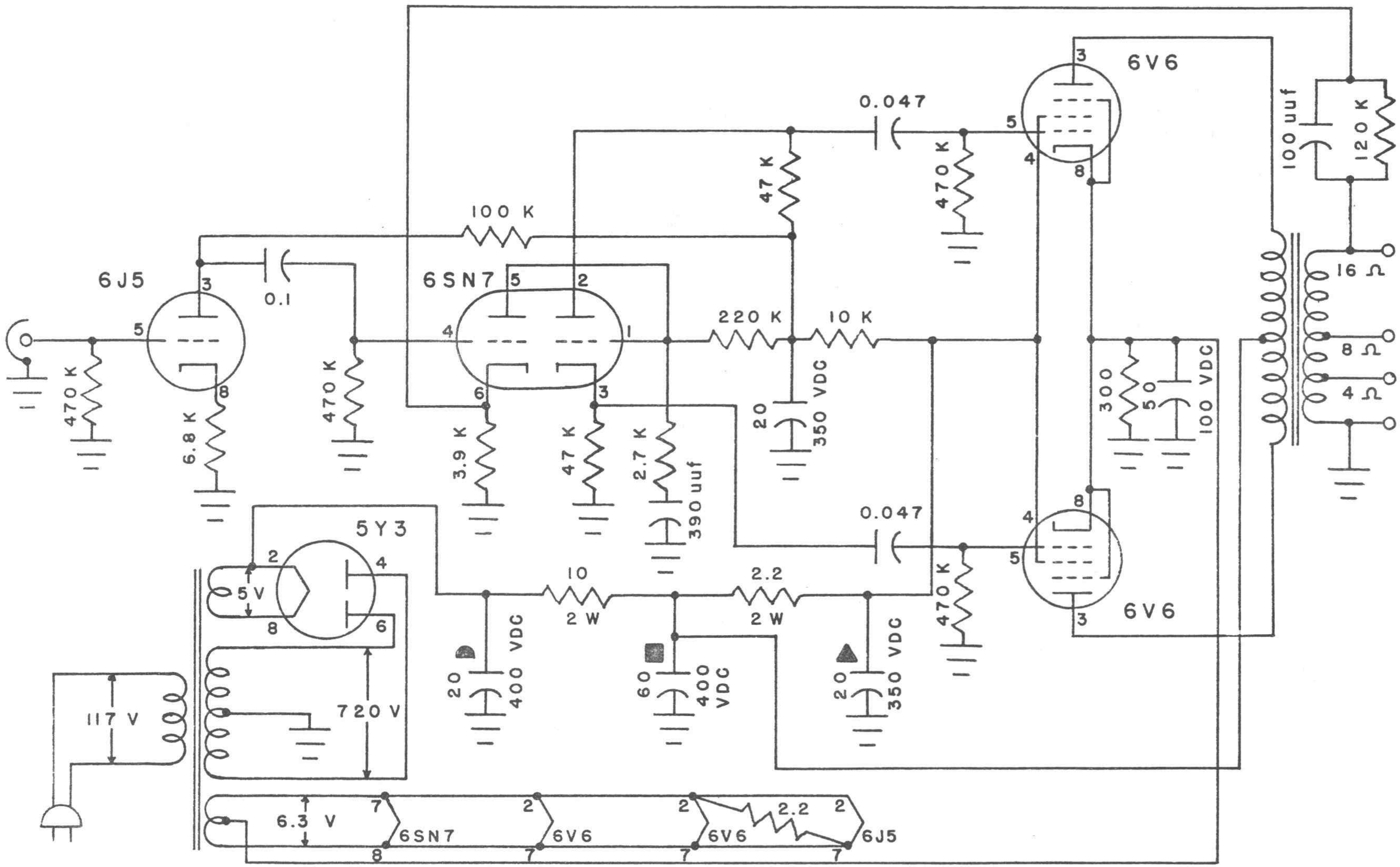


FIGURE D-4. CIRCUIT OF "P" AND "Q" VOLTAGE AMPLIFIERS

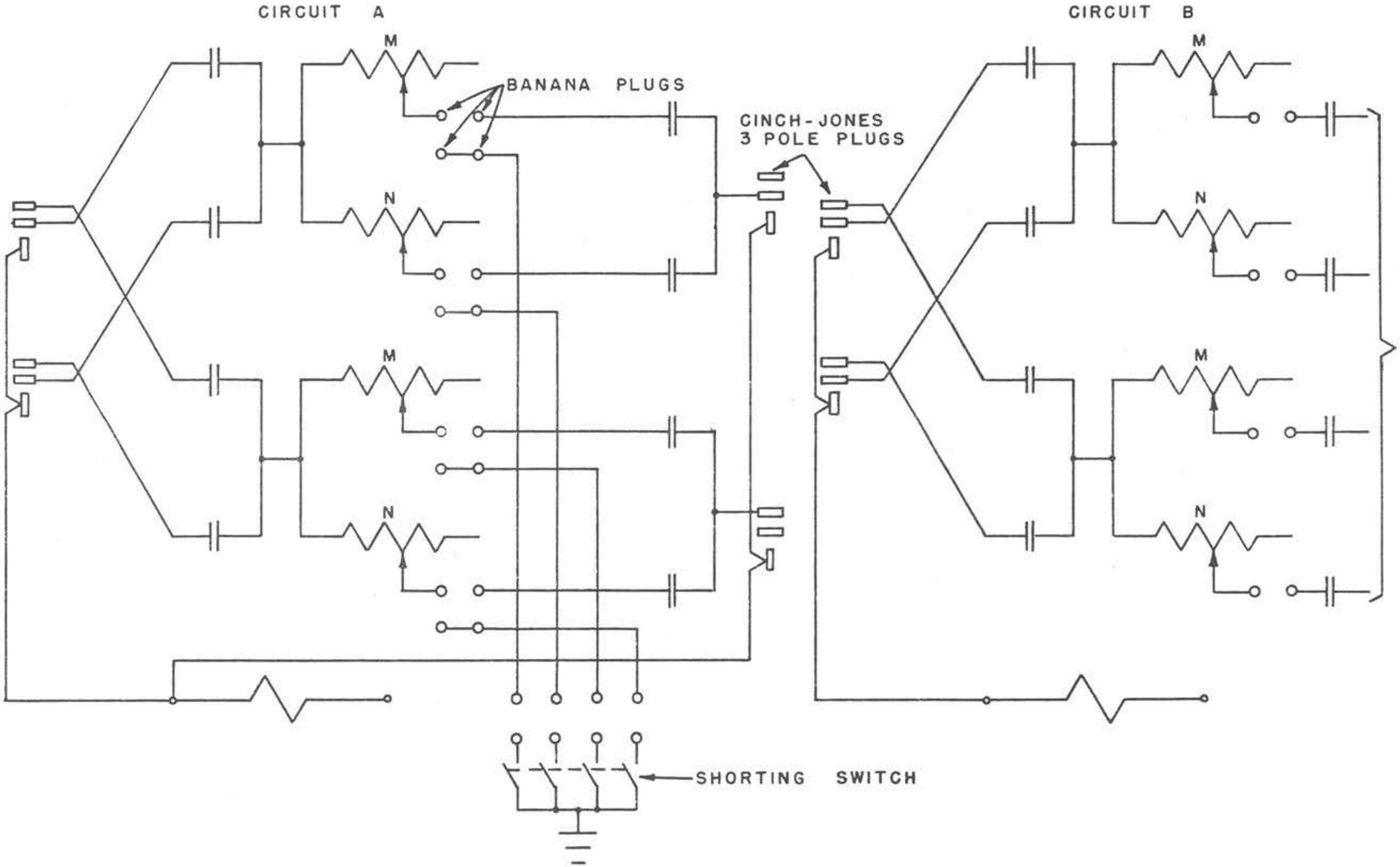


FIGURE D-5. SCHEMATIC DIAGRAM OF SYSTEM FAULT CIRCUIT SHOWN CONNECTED IN A SERIES CIRCUIT.

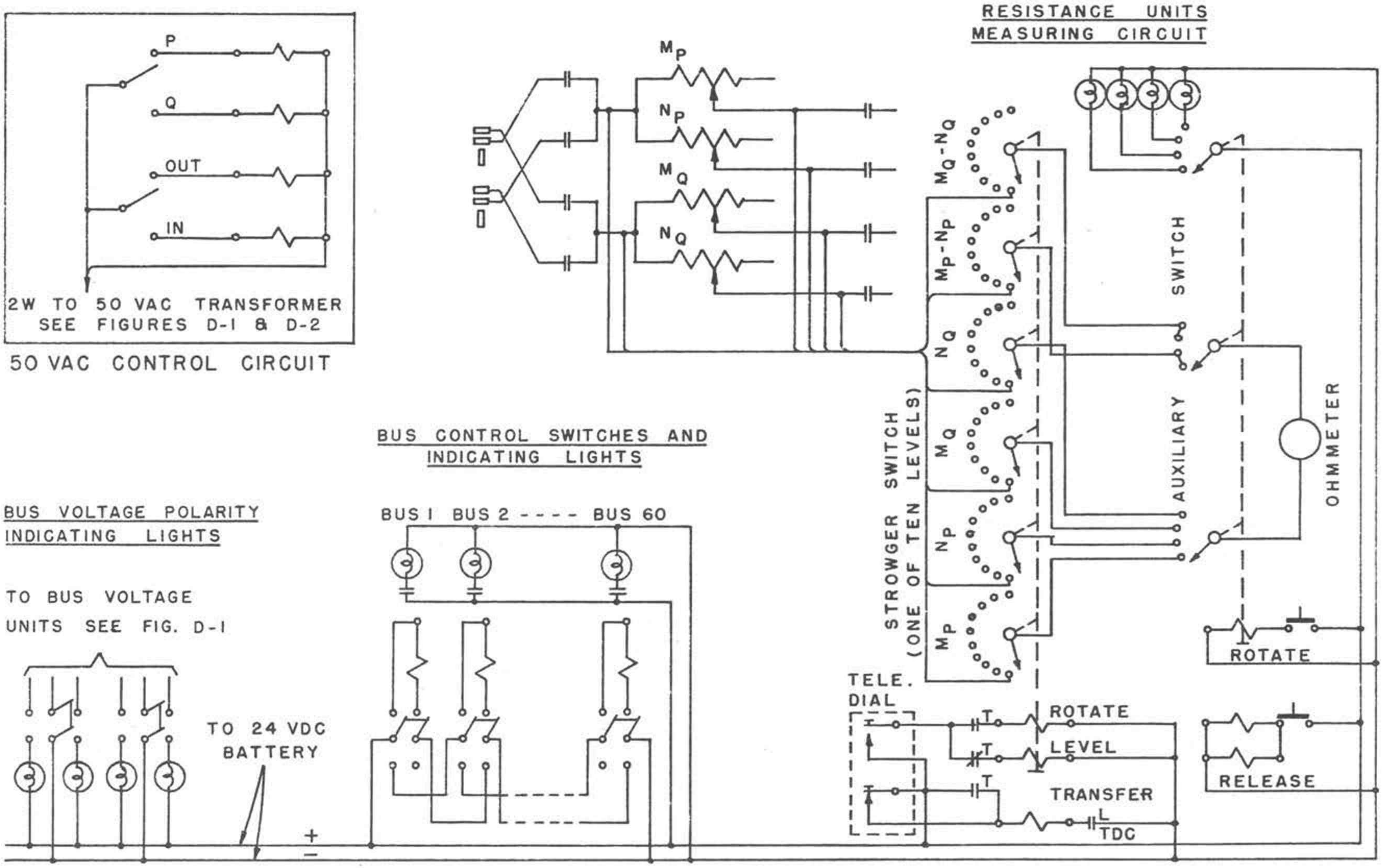


FIGURE D-6. SCHEMATIC DIAGRAM OF 24 VDC AND 50 VAC CONTROL CIRCUITS.

APPENDIX E

Table E-1. Computation sheet for power system stability studies.

Machine No. Gen. pt. (a) Total Mva 100 Shaft Power P_s 375 Mw

H 3 Mw-sec/Mva $M = GH/180f$ .000555 Mw-sec^2/elect. degree

Δt 0.05 Seconds $k = (\Delta t)^2/M$ 4.5 Mw/ elect. degree

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Time Seconds	Initial Angle δ_{n-1}	$\pm$ C'	$\cos(\beta \pm \delta_n)$	$\mp$ C" $\cos(\beta \pm \delta_n)$	Electric Power P_e	Accelerating Power P_a	k x P_a	Angular Change $\Delta\delta_n$	Final Angle δ_n	Initial Angle Of Other Machine δ'_{n-1}	$\delta_{n-1} - \delta'_{n-1}$
—	—	—	—	—	3 + 5	P_s - 6	7 x k	8 + 9_{n-1}	10_{n-1} + 9	—	2 - 11
0 -					0.75				25.7		
0 +					0.53	0.22			25.7		
0 avg						0.11	0.50	0.50	26.2		
0.05					0.55	0.20	0.902	1.402	27.6		
0.10					0.62	0.13	0.586	1.988	29.6		
0.15					0.70	0.05	0.225	2.213	31.8		
0.20					0.77	-0.02	-0.09	2.123	33.9		
0.25					0.83	-0.08	-0.361	1.762	35.7		
0.30					0.86	-0.11	-0.50	1.262	37.0		
0.35					0.87	-0.12	-0.541	0.721	37.7		
0.40					0.88	-0.13	-0.586	0.135	37.8		
0.45					0.88	-0.13	-0.586	-0.451	37.3		

Table E-2. Computation sheet for power system stability studies.

Machine No. **Mtr. pt. (a)** Total Mva 50 Shaft Power P_s 37.5 Mw

H 2 Mw-sec/Mva M = GH/180f .000185 Mw-sec^2/elect. degree

Δt 0.05 Seconds k = $(\Delta t)^2/M$ 13.5 Mw/ elect. degree

(1) Time Seconds	(2) Initial Angle δ_{n-1}	(3) $\pm$ C'	(4) $\cos(\beta \pm \delta_n)$	(5) $\mp$ C" $\cos(\beta \pm \delta_n)$	(6) Electric Power P_e	(7) Accelerating Power P_a	(8) k x P_a	(9) Angular Change $\Delta\delta_n$	(10) Final Angle δ_n	(11) Initial Angle Of Other Machine δ'_{n-1}	(12) $\delta_{n-1} - \delta'_{n-1}$
—	—	—	—	—	3 + 5	P_s - 6	7 x k	8 + 9_{n-1}	10_{n-1} + 9	—	2 - 11
0 -					-0.75				-10.6		
0 +					-0.53	-0.22			-10.6		
0 avg						-0.11	-1.49	-1.49	-12.1		
0.05					-0.56	-0.19	-2.57	-4.06	-16.2		
0.10					-0.63	-0.12	-1.62	-5.68	-21.9		
0.15					-0.71	-0.04	-0.541	-6.22	-28.1		
0.20					-0.79	0.04	0.541	-5.68	-33.8		
0.25					-0.85	0.10	1.35	-4.33	-38.1		
0.30					-0.88	0.13	1.76	-2.57	-40.7		
0.35					-0.89	0.14	1.89	-0.68	-41.4		
0.40					-0.90	0.15	2.03	1.35	-40.0		
0.45					-0.90	0.15	2.03	3.38	-36.6		

Table E-3. Computation sheet for power system stability studies.

Machine No. Gen. pt. (b) Total Mva 100 Shaft Power P_s 44.5 Mw

H 3 Mw-sec/Mva $M = GH/180f$ .000555 Mw-sec^2/elect. degree

Δt 0.05 Seconds $k = (\Delta t)^2/M$ 4.5 Mw/ elect. degree

(1) Time Seconds	(2) Initial Angle δ_{n-1}	(3) $\pm C'$	(4) $\cos(\beta \pm \delta_n)$	(5) $\mp C'' \cos(\beta \pm \delta_n)$	(6) Electric Power P_e	(7) Accelerating Power P_a	(8) $k \times P_a$	(9) Angular Change $\Delta\delta_n$	(10) Final Angle δ_n	(11) Initial Angle Of Other Machine δ'_{n-1}	(12) $\delta_{n-1} - \delta'_{n-1}$
—	—	—	—	—	3 + 5	P_s - 6	7 x k	8 + 9_{n-1}	10_{n-1} + 9	—	2 - 11
0 -					0.89				18.4		
0 +					0.65	0.24			18.4		
0 avg						0.12	0.54	0.54	18.9		
0.05					0.70	0.19	0.857	1.357	20.3		
0.10					0.81	0.08	0.361	1.718	22.0		
0.15					0.95	-0.06	-0.271	1.447	23.4		
0.20					1.07	-0.18	-0.822	0.652	24.0		
0.25					1.12	-0.23	-1.04	-0.415	23.6		
0.30					1.10	-0.21	-0.947	-1.362	22.2		
0.35					1.02	-0.13	-0.586	-1.948	20.3		
0.40					0.90	+0.01	0.045	-1.903	18.4		
0.45					0.77	0.12	0.54	-1.363	17.0		

Table E-4. Computation sheet for power system stability studies.

Machine No. Mtr. pt. (b)   Total Mva 50   Shaft Power P_s 34 Mw

H 2 Mw-sec/Mva $M = GH/180f$ .000185 $Mw\text{-}sec^2$/elect. degree

Δt 0.05 Seconds $k = (\Delta t)^2/M$ 13.5 Mw/ elect. degree

(1) Time Seconds	(2) Initial Angle δ_{n-1}	(3) $\pm C'$	(4) $\cos(\beta \pm \delta_n)$	(5) $\mp C'' \cos(\beta \pm \delta_n)$	(6) Electric Power P_e	(7) Accelerating Power P_a	(8) $k \times P_a$	(9) Angular Change $\Delta\delta_n$	(10) Final Angle δ_n	(11) Initial Angle Of Other Machine δ'_{n-1}	(12) $\delta_{n-1} - \delta'_{n-1}$
–	–	–	–	–	3 + 5	P_s - 6	7 x k	8 + 9_{n-1}	10_{n-1} + 9	–	2 - 11
0 –					–0.68				–10.6		
0 +					–0.44	–0.24			–10.6		
0 avg						–0.12	–1.62	–1.62	–12.2		
0.05					–0.48	–0.20	–2.20	–4.32	–16.5		
0.10					–0.59	–0.09	–1.22	–5.54	–22.0		
0.15					–0.73	0.05	0.656	–4.884	–26.9		
0.20					–0.84	0.16	2.16	–2.724	–29.6		
0.25					–0.90	0.22	2.97	0.246	–29.4		
0.30					–0.89	0.21	2.84	3.086	–26.3		
0.35					–0.81	0.13	1.76	4.846	–21.5		
0.40					–0.69	0.01	0.135	4.981	–16.5		
0.45					–0.57	–0.11	–1.49	3.491	–13.0		

Table E-5. Computation sheet for power system stability studies.

Machine No. Gen. pt. (c) Total Mva 100 Shaft Power P_s 47 Mw

H 3 Mw-sec/Mva $M = GH/180f$ .000555 Mw-sec^2/elect. degree

Δt 0.05 Seconds $k = (\Delta t)^2/M$ 4.5 Mw/ elect. degree

(1) Time Seconds	(2) Initial Angle δ_{n-1}	(3) $\pm C'$	(4) $\cos(\beta \pm \delta_n)$	(5) $\mp C'' \cos(\beta \pm \delta_n)$	(6) Electric Power P_e	(7) Accelerating Power P_a	(8) $k \times P_a$	(9) Angular Change $\Delta\delta_n$	(10) Final Angle δ_n	(11) Initial Angle Of Other Machine δ'_{n-1}	(12) $\delta_{n-1} - \delta'_{n-1}$
—	—	—	—	—	3 + 5	P_s - 6	7 x k	8 + 9_{n-1}	10_{n-1} + 9	—	2 - 11
0 -					0.94				8.9		
0 +					0.66	0.28			8.9		
0 avg						0.14	0.631	0.631	9.5		
0.05					0.70	0.24	1.08	1.711	11.2		
0.10					0.85	0.19	0.856	2.567	13.8		
0.15					1.04	-0.10	-0.451	2.116	15.9		
0.20					1.20	-0.26	-1.17	0.946	16.8		
0.25					1.25	-0.31	-1.40	-0.454	16.3		
0.30					1.15	-0.21	-0.946	-1.40	14.9		
0.35					0.97	-0.03	-0.014	-1.414	13.5		
0.40					0.78	0.16	0.722	-0.692	12.8		
0.45					0.67	0.27	1.22	0.528	13.3		

Table E-6. Computation sheet for power system stability studies.

Machine No. Mtr. pt. (c) Total Mva 50 Shaft Power P_s 18.5 Mw

H 2 Mw-sec/Mva $M = GH/180f$ .000185 Mw-sec^2/elect. degree

Δt 0.05 Seconds $k = (\Delta t)^2/M$ 13.5 Mw/ elect. degree

(1) Time Seconds	(2) Initial Angle δ_{n-1}	(3) $\pm C'$	(4) $\cos(\beta \pm \delta_n)$	(5) $\mp C'' \cos(\beta \pm \delta_n)$	(6) Electric Power P_e	(7) Accelerating Power P_a	(8) $k \times P_a$	(9) Angular Change $\Delta\delta_n$	(10) Final Angle δ_n	(11) Initial Angle Of Other Machine δ'_{n-1}	(12) $\delta_{n-1} - \delta'_{n-1}$
—	—	—	—	—	3 + 5	P_s - 6	7 x k	8 + 9_{n-1}	10_{n-1} + 9	—	2 - 11
0 -					-0.37				-10.6		
0 +					-0.09	-0.28			-10.6		
0 avg						-0.14	-1.89	-1.89	-12.5		
0.05					-0.14	-0.23	-3.11	-5.00	-17.5		
0.10					-0.29	-0.08	-1.08	-6.08	-23.6		
0.15					-0.48	0.11	1.49	-4.59	-28.2		
0.20					-0.64	0.27	3.65	-0.94	-29.1		
0.25					-0.68	0.31	4.19	3.25	-25.8		
0.30					-0.58	0.21	2.84	6.09	-19.7		
0.35					-0.40	0.03	0.405	6.13	-13.6		
0.40					-0.22	-0.15	-2.03	4.10	- 9.5		
0.45					-0.10	-0.27	-3.65	0.45	- 9.0		