A SLOTTED LINE FOR ULTRA-HIGH-FREQUENCY MEASUREMENTS

by

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A SLOTTED LINE FOR ULTRA-HIGH-FREQUENCY MEASUREMENTS

INTRODUCTION

Ultra-high-frequency waves are electromagnetic waves with wavelengths between 10 centimeters and 1 meter corresponding to frequencies between 3000 megacycles per second and 300 megacycles per second respectively (1, p.8). Within this frequency range, transmission of power by wire must be done with a coaxial transmission line. The losses through radiation from open wire lines increase with frequency and become very large in the ultra-high-frequency region. For wavelengths less than about 10 centimeters it becomes practical to transmit power through hollow pipes. These pipes must have a diameter of the order of a wavelength, however, which makes them extremely bulky when designed for use at ultra-high-frequencies.

A coaxial line consists of a rod supported concentrically within a hollow pipe by means of insulators. Currents flow in both the rod and the pipe, and an electromagnetic field exists between them. Since the system is completely enclosed, there is no radiation loss. A coaxial line will propagate an electromagnetic field in which the electric and magnetic fields are perpendicular to each other and to the axis of the line. In this mode, Maxwell's equations and the boundary conditions are satisfied independently of the frequency of the wave. That is, electromagnetic waves of any frequency can be propagated along a coaxial transmission line.

A coaxial line has a characteristic impedance, i.e., if a single wave is propagated along a coaxial line, the ratio of the voltage at a point to the current at that point is a constant which depends only on the geometrical properties of the line. When a coaxial line delivers power to an impedance which is different in magnitude from its characteristic impedance, some of the power transmitted down the line will be reflected back from the terminal impedance. This reflected power combines with the incident power to produce standing waves.

Standing waves increase the power losses in the transmission line, increase the peak voltage on the line, and send power back to the source thus decreasing the power delivered to the load.

In general, it is desirable to reduce standing waves to a minimum. In order to accomplish this it is necessary to determine to what extent they exist on a transmission line. The obvious method of detecting the presence of standing waves is to measure the electromagnetic field within the coaxial line. This may be done by slotting the outer conductor of the line axially and introducing a probe which may be moved along the coaxial line and which will measure the strength of either the electric or the magnetic field. A narrow axial slot has negligible effect on the properties of a coaxial line.

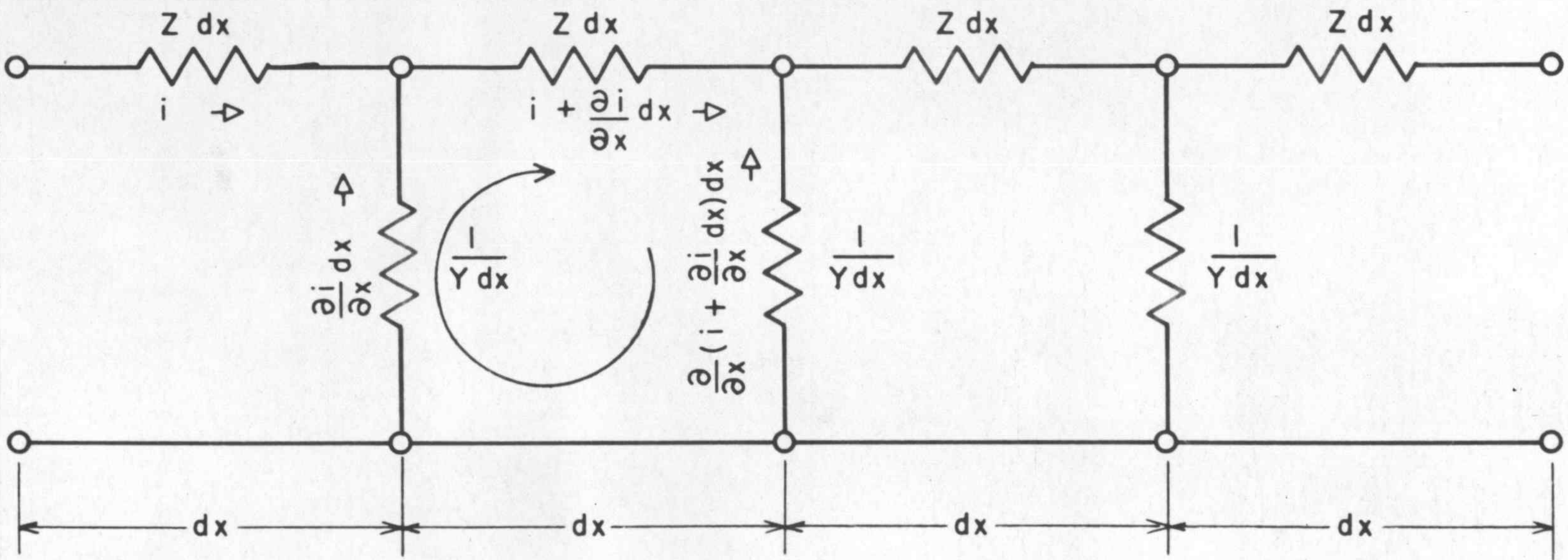
A section of coaxial transmission line provided with a slot and a travelling probe is called a slotted line. It is a most useful device for detecting standing waves, determining the frequency of waves transmitted through it and for measuring impedances, particularily those associated with transmission lines and antennas.

The problem of this thesis is the design, construction, testing and use of a slotted line. Enough transmission line theory is developed to permit a reasonably complete understanding of the use of the slotted line. This material is included since little is available in the literature relating to the theory of impedance measurements with a slotted line, although general transmission line theory is widely treated. Only those parts of transmission line theory are developed which will be directly useful in connection with slotted lines. Furthermore the development is such as to put results in terms of quantities which can be measured directly with a slotted line.

TRANSMISSION LINE THEORY

A coaxial transmission line propagating an electromagnetic wave in its normal mode is of primary interest. In this mode the coaxial line behaves like a two-wire transmission line in that a current flows axially along the center conductor and returns along the outer (2, P.160). It is convenient to think in terms of an open two-wire line.

The transmission line has a series impedance Z per unit length composed of series resistance and inductance, and a shunt admittance per unit length composed of shunt conductance and capacitance. Consider the line to be divided into sections of length dx and replace each section by a four terminal network as shown in Fig. 1. The impedance of each series element is then Z dx while the impedance of each shunt element is 1/Y dx. Let the current flowing in one series element be i. At a distance dx down the line, this current will have been changed by an amount $\frac{\partial i}{\partial x}$ dx, in first approximation. This is represented by adding a current through the shunt element of amount $\frac{\partial i}{\partial x}$ dx. The current flowing through the next series element is $i + \frac{\partial i}{\partial x}$ dx. This current will increase in the next distance by an amount $\frac{\partial}{\partial x}(i + \frac{\partial i}{\partial x})dx$, so that the current flowing in the third shunt element is



TRANSMISSION LINE

FIGURE 1

$\frac{\partial}{\partial x}(i + \frac{\partial i}{\partial x}dx)dx$. We may now write Kirchhoff's law for the closed current path indicated in Fig. 1. Thus,

$$(i + \frac{\partial i}{\partial x}dx)Z\,dx - (\frac{\partial i}{\partial x} + \frac{\partial^2 i}{\partial x^2}dx)dx\frac{1}{Ydx} + \frac{\partial i}{\partial x}dx\frac{1}{Ydx} = 0,$$

which reduces to

$$(i + \frac{\partial i}{\partial x}dx)\,Z - \frac{1}{Y}\,\frac{\partial^2 i}{\partial x^2} = 0.$$

Letting dx approach zero, this becomes:

$$\frac{\partial^2 i}{\partial x^2} - YZi = 0,$$

which has as a general solution

$$i = Ae^{-\gamma x} + Be^{\gamma x}, \text{ where } \gamma = \sqrt{YZ}.$$

CHARACTERISTIC IMPEDANCE. The general form of the current distribution on a transmission line is

$$i = Ae^{-\gamma x} + Be^{\gamma x}.$$

The relation existing between the current flowing at a point and the voltage at that point is required. Consider first a current given by

$$i = Ae^{-\gamma x}.$$

Referring to Fig. 1, it is seen that the current flowing in the shunt element is $\frac{\partial i}{\partial x}dx$ and that the element has an impedance $\frac{1}{Ydx}$. The current has been assumed to flow upward so that the voltage of the upper branch will be negative with respect to the lower line. Voltages will be expressed with respect to the lower line. Thus, the voltage at the point chosen is

$$V = -\frac{\partial i}{\partial x}dx\,\frac{1}{Ydx} = -\frac{1}{Y}\frac{\partial i}{\partial x} \quad .$$

For a current of the form

$$i = Ae^{-\gamma x}, \qquad \frac{\partial i}{\partial x} = -\gamma i,$$

so that the ratio of voltage to current at the point is

$$Z_o = \frac{V}{i} = \frac{\gamma}{Y} = \sqrt{\frac{Z}{Y}} \quad .$$

This ratio is the characteristic impedance of the transmission line and is independent of the constant A. If the second term of the general expression for the current had been chosen, the same result would have been obtained except for sign.

THE IMPEDANCE AT ANY POINT. In the general case,

$$\frac{\partial i}{\partial x} = -\gamma Ae^{-\gamma x} + \gamma Be^{\gamma x} \quad ,$$

so that the ratio of voltage to current is

$$Z_x = \frac{\gamma}{Y}\;\frac{Ae^{-\gamma x} - Be^{\gamma x}}{Ae^{-\gamma x} + Be^{\gamma x}} = Z_o\;\frac{Ae^{-\gamma x} - Be^{\gamma x}}{Ae^{-\gamma x} + Be^{\gamma x}} \; . \tag{1}$$

Evidently, in this case, the impedance is a function of distance along the line as well as of the ratio of the two components of the current.

INCIDENT AND REFLECTED WAVES. In general, $\gamma = \sqrt{ZY}$ is a complex quantity. Hence, the current will vary periodically as well as exponentially along the line. Furthermore, one deals usually with currents which are time dependent. With no loss of generality this time dependence can be represented by $e^{i\omega t}$. Particular currents of this type can be combined to produce any desired time dependence of current by using a Fourier series to represent the desired function.

The current in the line corresponding to a particular frequency is then,

$$i = (Ae^{-\gamma x} + Be^{\gamma x})e^{i\omega t} .$$

A particular value of the first term occurs at a point which moves toward plus x as time increases while, for the second term, the point will move toward minus x. That is to say, A is the coefficient of a wave, periodic in space and time, which is progressing in the direction of increasing x while B is the coefficient of a wave progressing in the direction of decreasing x. The space periodicity depends, of course, on the imaginary part of γ , while the real part of γ attenuates the wave.

A transmission line of characteristic impedance Z_o extending in the direction of increasing x and terminated in some impedance Z_r is now considered. Certainly, at the point of termination, the ratio of voltage to current must be Z_r. If Z_r is equal to Z_o, it will be possible for a single wave to travel toward the termination, because, for a single wave, the ratio of current to voltage is Z_o. If Z_r is different from Z_o there must be waves travelling in both directions corresponding to an incident wave and a reflected wave. The ratio of the amplitudes of these two waves may be expressed in terms of Z_r and Z_o using Eq. 1. Suppose the terminating impedance Z_r to be located at x = 0. Then,

$$Z_r = Z_o \frac{A - B}{A + B} \quad ,$$

or

$$\frac{B}{A} = \frac{Z_o - Z_r}{Z_o + Z_r} \quad .$$

The ratio $\frac{B}{A}$ is known as the reflection coefficient.

STANDING WAVE RATIO. In a coaxial line the series impedance per unit length consists of resistance and inductive reactance in series. The shunt admittance consists of conductance and capacitive susceptance in shunt. Thus,

$$Z = R + i\omega L,$$

where R is the series resistance per unit length and L is the series inductance per unit length, and

$$Y = G + i\omega C \ ,$$

where G is the shunt conductance per unit length and C is the shunt capacitance per unit length. Thus,

$$\gamma = \sqrt{YZ} = \sqrt{(G + i\omega C)(R + i\omega L)} \quad .$$

In a well designed coaxial line the shunt conductance and series resistance are usually small. In this case γ is almost purely imaginary, and the wave is propagated with little loss. Consider a lossless line extending in the direction of plus x and terminated in Z_r, with an incident wave propagating from minus x. By representing γ as ik, the voltage at any point in the line becomes

$$V = Z_o\,(Ae^{-ikx} - Be^{ikx})$$

where the time factor $e^{i\omega t}$ is omitted as unimportant in what follows. The current which will flow in a probe inserted into the line between the conductors is proportional to this voltage, and the power dissipated in a bolometer element connected to the probe is proportional to its square. It will be significant to compute the maximum and minimum values of V^2.

The quantities A and B are complex, in general. Let these be represented in polar form as

$$A = ae^{i\varphi} \quad , \quad B = be^{i\psi} \quad .$$

Then,

$$V = Z_o\,(\,ae^{i(\varphi - kx)} - be^{i(\psi + kx)} \quad ,$$

and

$$V^2 = VV^* = Z_oZ_o^*\,(a^2 + b^2 - abe^{-(\psi - \varphi + 2kx)} - abe^{i(\psi - \varphi + 2kx)}),$$

$$= Z_oZ_o^*\,(a^2 + b^2 - 2ab\cos 2(kx + \theta\,)\,).$$

where * means the complex conjugate of the quantity involved, and

$$\theta = \frac{\psi - \varphi}{2} \quad .$$

The values of x for which extreme values of V^2 will occur are those for which

$$\frac{\partial (V^2)}{\partial x} = 0 \ .$$

Differentiating,

$$\frac{\partial (V^2)}{\partial x} = 4kZ_oZ_o^* \, ab \sin 2(kx + \theta).$$

This expression is zero whenever

$$2(kx + \theta) = n\pi$$

where n is any integer. Hence, the extreme values of V^2 are given by

$$V^2 = Z_oZ_o^* \, (a^2 + b^2 - 2ab \cos n\pi)$$

$$= Z_oZ_o^* \, (a - (-1)^n b)^2 \ .$$

Thus the extreme values of the probe readings are proportional to

$$V^2max = ZoZo^* \, (a + b)^2$$

and

$$V^2 \, min = Z \, Z^* \, (a - b)^2 \ .$$

The standing wave ratio r is now defined by the relation

$$r = \frac{a + b}{a - b} \ .$$

Since a and b are magnitudes and hence essentially positive, and since, in order to conserve energy, $b < a$, r has been determined as the positive square root of

$$r^2 = \frac{V^2max}{V^2min} = \frac{\text{maximum bolometer output}}{\text{minimum bolometer output}} \ .$$

REFLECTION COEFFICIENT. If a wave of the form

$$Ae^{-i(kx - \omega t)}$$

is sent down a transmission line of characteristic impedance Z_o extending toward plus x and terminated with an impedance Z_r, it is seen that there must be a reflected wave, in general, of the form

$$Be^{i(kx + \omega t)} .$$

The reflection coefficient is defined as B/A. Solution of the defining equation for r gives as the magnitude of the reflection coefficient

$$\frac{B}{A} = \frac{r - 1}{r + 1} .$$

Thus measurement of the standing wave ratio permits calculation of the amplitude of the reflected wave. The square of the reflection coefficient gives the fraction of power reflected at Z_r.

THE R-X DIAGRAM. Given a coaxial line of characteristic impedance Z_o and a measured standing wave ratio of r, it is possible to determine some of the characteristics of the terminating impedance Z_r. It has been shown that

$$\frac{Z_r}{Z_o} = \frac{A - B}{A + B} = \frac{1 - B/A}{1 + B/A} .$$

Measurement of r gives

$$\frac{Z_r}{Z_o} = \frac{r - 1}{r + 1} e^{-2i\theta}$$

where θ is undetermined. Substituting this value of the reflection coefficient and separating the resulting equation into real and imaginary parts, essentially two equations are obtained since the real and imaginary parts must be satisfied separately. If θ is eliminated between these two equations, this very important result is obtained.

$$\left(\mathrm{Re}\,\frac{Z_r}{Z_o} - \frac{r + \frac{1}{r}}{2}\right)^2 + \left(\mathrm{Im}\,\frac{Z_r}{Z_o}\right)^2 = \left(\frac{r - \frac{1}{r}}{2}\right)^2$$

Let R represent the real part of Z_r and X represent the imaginary part of Z_r. Then, if coordinate axes are set up with R/Z_o as abcissa and X/Z_o as ordinates, the points Z_r/Z_o will lie on a circle for a given value of r.

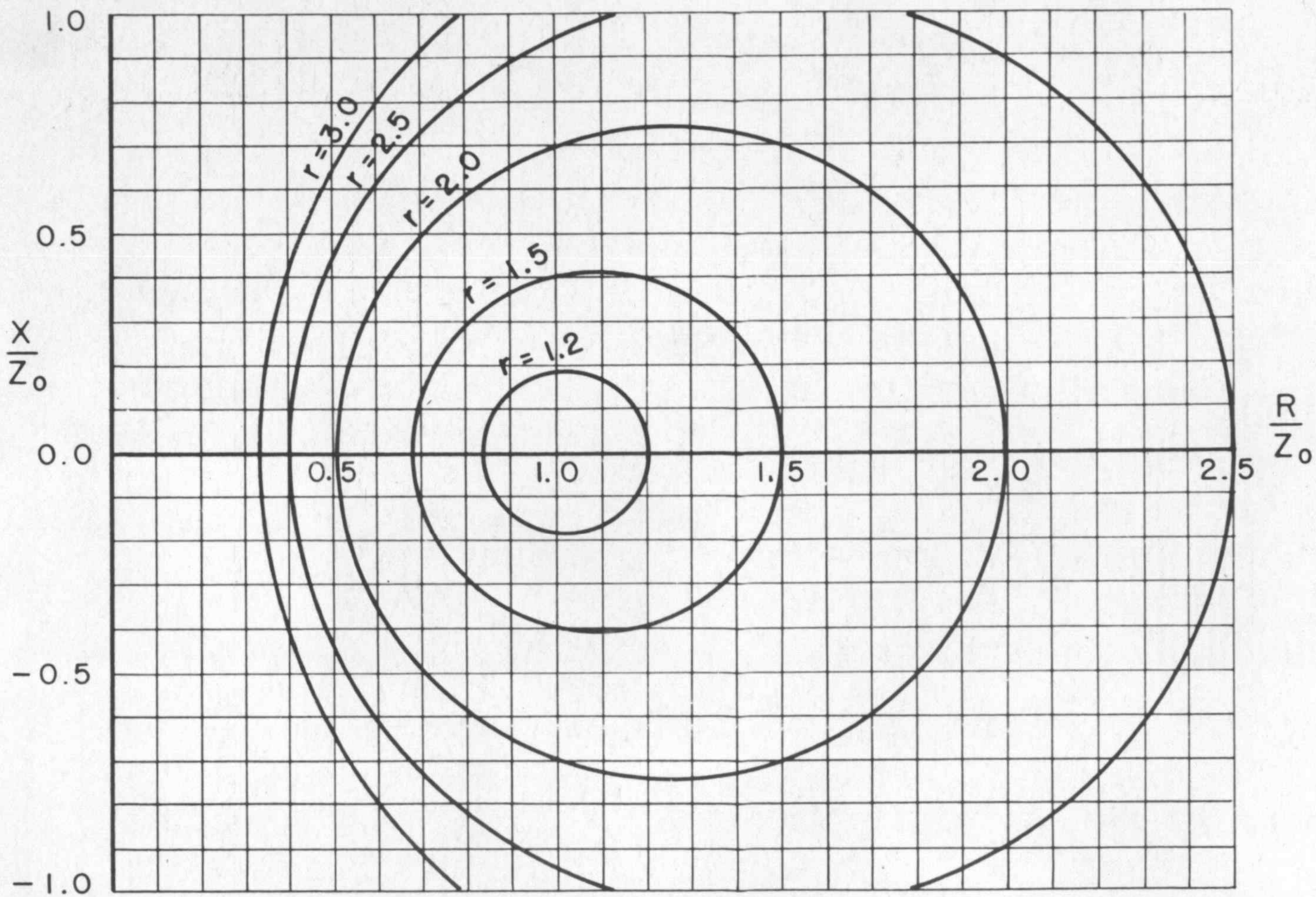
In general for a coaxial line

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + i\omega L}{G + i\omega C}} \quad .$$

For a lossless line this quantity is real. In this case R/Z_o is real and is proportional to the resistive component of Z_r. Similarly, X/Z_o is real and proportional to the reactive component of Z_r. A few of these circles, each for a different value of r, are shown in Fig. 2.

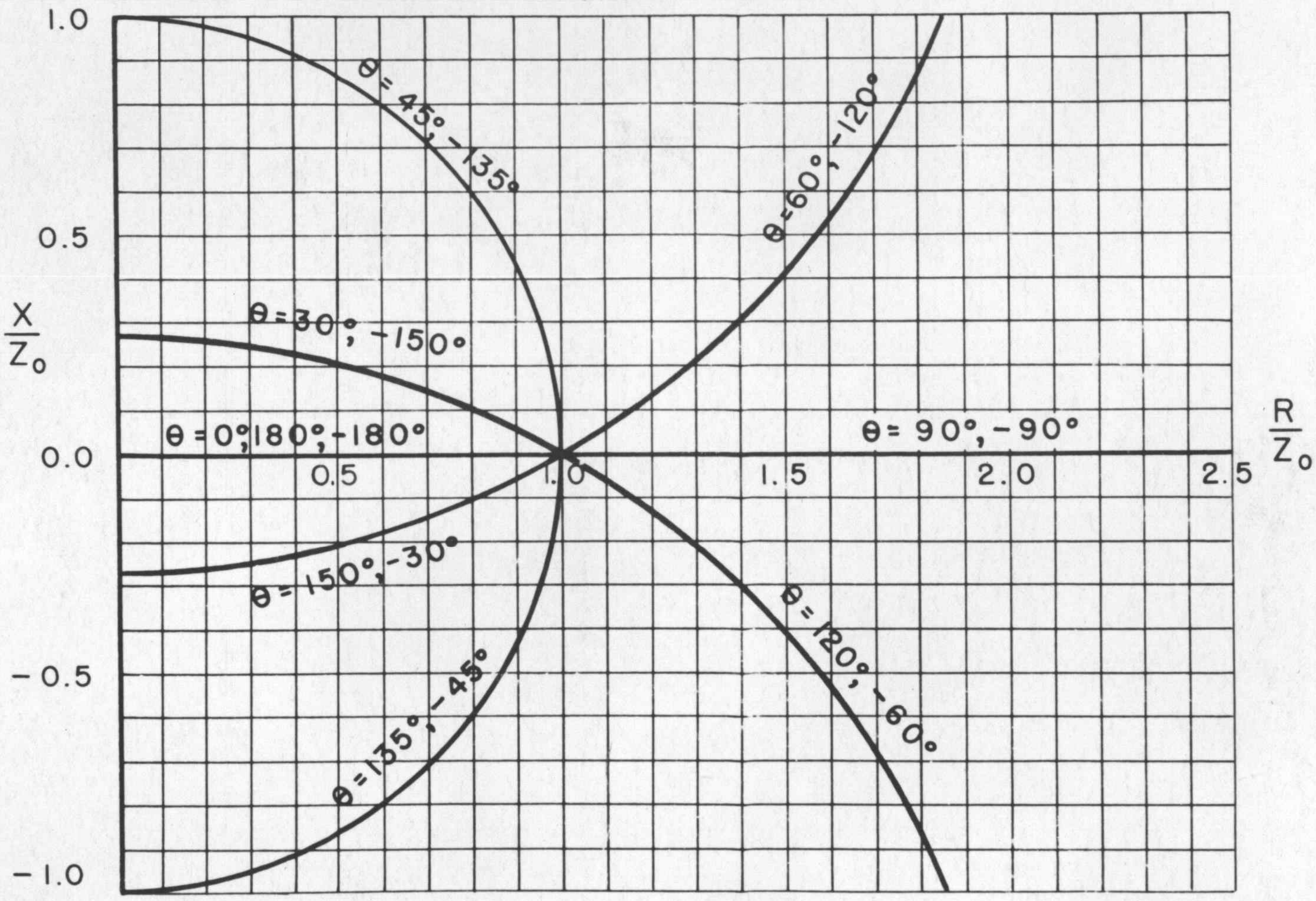
If r is eliminated between the two equations instead of θ and the lines for constant θ are plotted on the same set of coordinates, it is found that these lines are arcs of circles passing through the point $R/Z_o = 1$. Several of these lines are plotted in Fig. 3. Each arc is given by either of two values of θ. Thus as θ increases from zero to π the arcs move once around the point $R/Z_o = 1$ in a clockwise direction. As θ goes

FIGURE 2



CIRCLES OF CONSTANT STANDING WAVE RATIO

FIGURE 3



ARCS OF CONSTANT PHASE
OF REFLECTION COEFFICIENT

from zero to $-\pi$ the arcs move counterclockwise completely around the point. The line from the origin to $R/Z_0 = 1$ corresponds to $\theta = 0, \pi, -\pi$.

Of primary interest is the determination of the value of an unknown impedance terminating a transmission line in terms of quantities measurable at some other point of the line. It has been seen that measurements of the standing wave ratio, which can be made with a travelling probe, will locate the unknown impedance on a circle corresponding to r = constant in the R-X diagram. A method for determining θ is now needed.

From the foregoing discussion, it is seen that extreme values of V^2 occur at points for which

$$2(kx + \theta) = n\pi ,$$

or

$$x = \frac{n\pi}{2k} - \frac{\theta}{k} .$$

Since $k = 2\pi/\lambda$, this may be expressed as

$$x = n\frac{\lambda}{4} - \frac{\lambda\theta}{2\pi} .$$

Now if $Z_r = 0$ then θ is zero, because from

$$\frac{Z_r}{Z_0} = \frac{A - B}{A + B}$$

it is seen that this condition requires that $A = B$. This fact establishes a reference point for determining θ for an unknown Z_r.

Suppose a line is shorted and that the probe is set on a voltage minimum. Since in this case $Z_r = 0$, the probe must then be at a distance from the short given by

$$x_s = \frac{n\lambda}{4}$$

where n is even. Now let the short be removed and replaced with an unknown impedance Z_r. The reflection coefficient B/A will in general be complex and will have a phase 2θ where $-\pi \leq 2\theta \leq \pi$. The position of the minimum nearest x_s will be at

$$x_r = \frac{n\lambda}{4} + \lambda\frac{\theta}{2\pi} \quad .$$

Then

$$x_r - x_s = \frac{\lambda\theta}{2\pi} \quad ,$$

or

$$\theta = 2\pi\,\frac{\Delta x}{\lambda} \ , \quad \text{with} \quad \Delta x = x_r - x_s,$$

where it is seen from the restriction on θ that

$$\left|\frac{\Delta x}{\lambda}\right| \leq \frac{1}{4} \quad .$$

Thus, θ is measured by determining the shift of the minimas. Using a measured value of the standing wave ratio r, the reflection current is

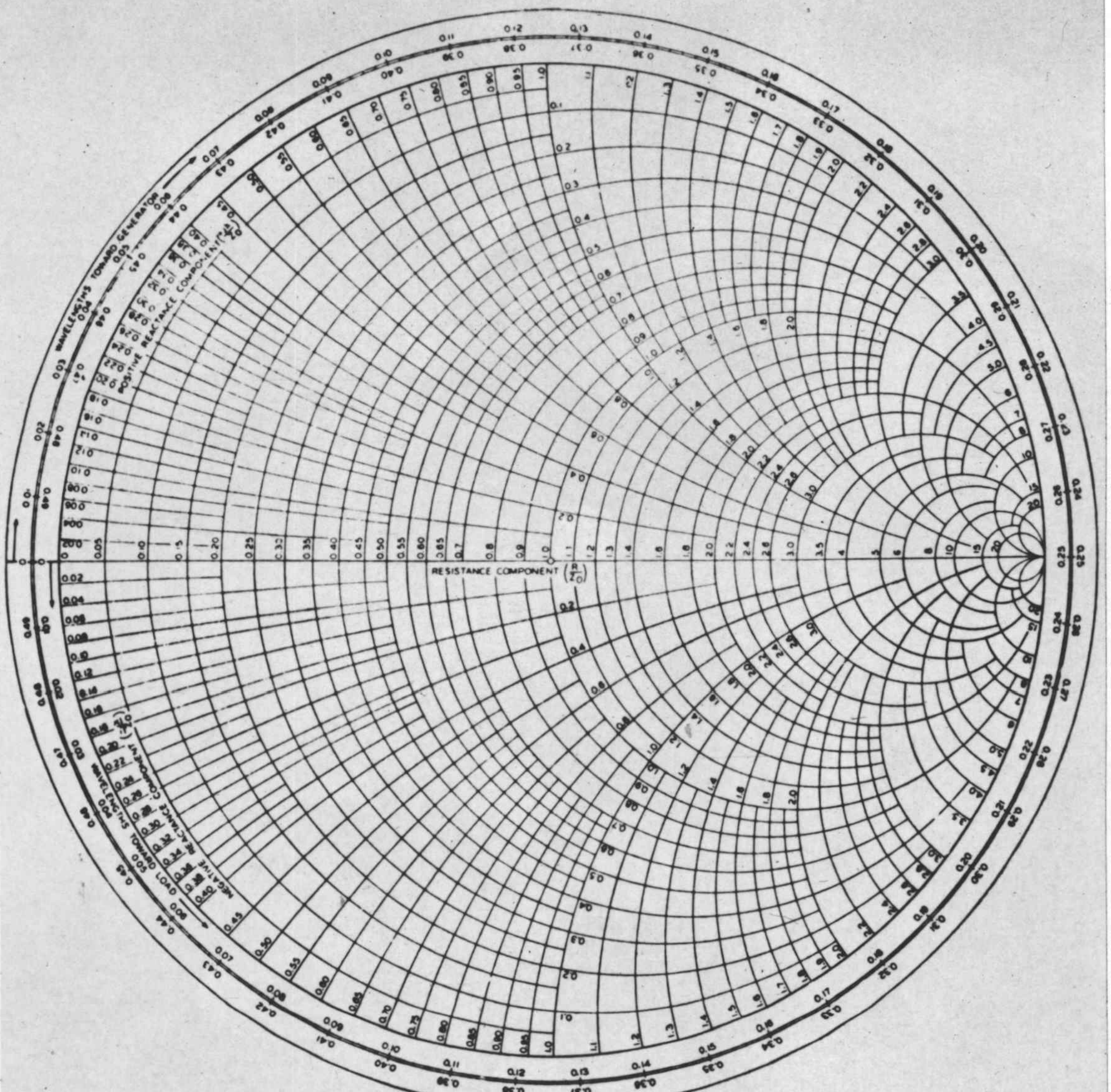
$$\frac{B}{A} = \frac{r-1}{r+1}\,e^{2i\theta} \quad ,$$

and the ratio of the unknown terminating impedance to the characteristic impedance of the line is known,

$$\frac{Z_r}{Z_o} = \frac{1 - B/A}{1 + B/A} \quad .$$

This last result is most readily obtained using the R-X diagram. By locating the intersection of the circle corresponding to the measured value of r with the circular arc corresponding to the measured value of θ , the value of $\frac{Z_r}{Z_o}$ can be read directly.

The R-X diagram may be put in a form more convenient for computation by means of a coordinate transformation which makes the circles of constant r concentric about the point $R/Z_o = 1$. With this transformation the arcs of constant θ become straight lines passing through the point $R/Z_o = 1$, and the lines X/Z_o and R/Z_o become circles. An R-X diagram transformed in this manner is known as a Smith chart (3, p.29). It may be fitted with an index which rotates about the point $R/Z_o = 1$ on which is marked the values of the standing wave ratio corresponding to the proper radius. Usually the scale corresponding to the phase of the reflection coefficient is marked off in terms of $\Delta x/\lambda$ rather than θ . A Smith chart is shown in Fig. 4.



SMITH CHART

FIGURE 4

DESIGN OF A SLOTTED LINE

Most high frequency electronic devices are designed to work into, or from, a 50 ohm transmission line. For this reason it is important that the slotted line itself have a characteristic impedance of 50 ohms. For a line with air dielectric this impedance is obtained if the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor is 2.67/1. The impedance is determined only by this ratio and the actual size of the conductors is optional. The size of one inch chosen in this case for the inner diameter of the outer conductor was a compromise based on convenience in machining.

The precision of a machining operation depends primarily on the accuracy of the machine being used and is more or less independent of the size of the work. Thus, the relative accuracy is increased as the size of the work is increased. In this case, the problem was to plane a semicircular groove into a solid bar of aluminum stock. Two such bars are placed together so that the semicircles meet to form an accurate hole which serves as the outer conductor. Thus, the larger the outer conductor the more accurately it can be made.

An upper limit is set on the diameter by the weight of the finished slotted line. As a piece of laboratory equipment it must be reasonably portable. An outer conductor diameter of one inch was chosen since it permits machining tolerances on the order of a few tenths of a percent while keeping the total weight of the slotted line down so that one man can easily handle it.

With this choice of outer conductor, the inner conductor then must have a diameter of 3/8 inch which is rather convenient. Accurately dimensioned tubing of this size is readily available.

Ideally, the inner conductor should be supported in such a fashion that there are no discontinuities along the axis of the line. Points of discontinuity will produce distortions of the field within the line and possibly cause misleading readings. This condition is achieved either by supporting the center conductor only at the ends or by a continuous strip of dielectric. The first possibility is impractical since the inner conductor would sag excessively thus changing the characteristic impedance from point to point. A continuous strip of dielectric would solve this problem. However, it is very difficult to machine such a strip so as to maintain the conductors coaxial to any degree of precision.

In spite of the possibility of introducing discontinuities it was decided to support the center conductor on a series of dielectric pegs small enough to introduce negligible distortion into the fields and to place them near enough together to avoid any appreciable sag. These pegs are made of 1/8 inch diameter polystyrene and are placed radially between the inner and outer conductors, in groups of three, spaced 120° apart at intervals of 6 inches.

For convenience in connecting the slotted line to other equipment, type 'N' connectors are attached to the ends of the line. Since the type 'N' connector has considerably smaller conductor diameters, it was necessary to construct tapered sections for connecting the slotted line to the connectors.

The outer conductor of the tapered sections was made by electroplating a heavy layer of copper on a mandril of aluminum which had been previously turned to the desired taper. The copper was removed from the mandril after plating by heating and then cooling the mandril. The coefficient of thermal expansion of aluminum is about twice that of electrolytic copper so that the plating was stretched slightly. The tapered copper tube could easily be slipped off the mandril after this treatment.

The inner conductor of the tapered section was turned on a lathe from an aluminum rod. This conductor is not supported in the tapered section but is attached directly to the inner conductor of the slotted line at one end, and to the center conductor of the type 'N' connector at the other.

A slotted line must be at least 1/2 wavelength long in order to insure that at least one standing wave maximum and one minimum will occur in the measuring section. This length is sufficient for impedance measurements. However, if wavelength is to be measured also, it is necessary that the section be at least a full wavelength long to insure that two minima will occur in the section. Wavelength measurements should be made at the minimum voltage points of the standing wave because these are quite sharp if the standing wave ratio is large while the maximums are broad at all times.

Conventional equipment, such as electronic devices with lumped parameters, can be built to operate at wavelengths as short as 100 centimeters. For laboratory demonstration purposes it is desirable to have a slotted line which can be used with this equipment as well as with the special equipment necessary to produce shorter wavelengths. Therefore a slotted line of at least one meter length is desirable.

Machine shop limitations placed an upper limit of five feet on the length. This length represents a satisfactory minimum value and was therefore chosen.

A bolometer is used with the slotted line as a detector. It was chosen because it requires no adjustments and because its operation is known. Crystal detectors are considerably more sensitive but need frequent calibration.

A bolometer is essentially a piece of very fine wire which is heated by an external current to a temperature near its melting point. When used as a detector for radio frequencies this wire is placed so as to dissipate some of the radio frequency power. This additional power raises the temperature of the wire, and, because of the temperature coefficient of resistance, increases its resistance. The voltage drop across the wire is thus increased. If the power obtained from the radio frequency source is small compared to that from the external source, the relation between radio frequency power dissipated and the change in bolometer resistance will be essentially linear.

It is important that the detector used with a slotted line draw negligible power from the line so that the voltage distribution along the line will be independent of the position of the detector. Thus, although the change in bolometer resistance could be measured with a bridge, it is more practical to modulate the radio frequency energy with a frequency whose period is approximately equal to the thermal time constant of the bolometer element. An alternating voltage will then be developed by the bolometer which will be proportional to the square of the voltage in the slotted line. This alternating voltage can be amplified to a value convenient for metering.

An amplifier having an effective input noise level of 10^{-7} volts can be constructed without serious difficulty. If this value is then taken as the minimum useful output from the bolometer, and if 10^{-2} volts is taken as the upper limit at which the bolometer output is small compared with the bolometer heating voltage (a few volts), then the bolometer will have a dynamic range of 50 db. which is adequate. Over this range the bolometer will produce an output voltage which is proportional to the square of the electric field in the slotted line and need not be calibrated.

The sensitivity of a bolometer can be increased considerably by mounting it a quarter wavelength from the shorted end of a coaxial line. An arrangement of this sort will insure that the maximum amount of the power taken from the slotted line is dissipated in the bolometer. However the length of the shorted tuning stub must be adjusted when the slotted line is used with a different frequency. It was thought that the inconvenience of adjusting such a tuning stub more than offset the advantage of increased sensitivity so that an untuned detector was used instead.

The detector used with the slotted line described in this thesis makes use of two 1/100 ampere Littelfuses as bolometer elements arranged so as to conduct the high-frequency current in parallel and the polarizing and modulation currents in series. The electrical circuit is shown in Fig. 5.

The complete slotted line is shown in Fig. 6.

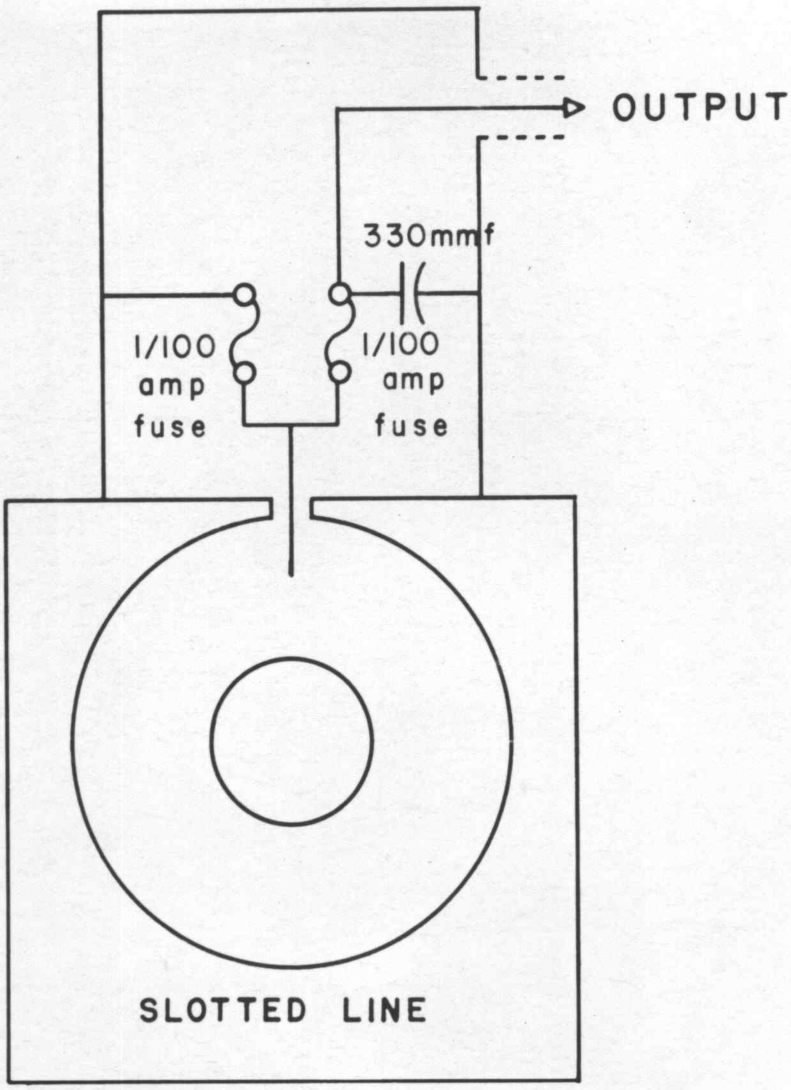
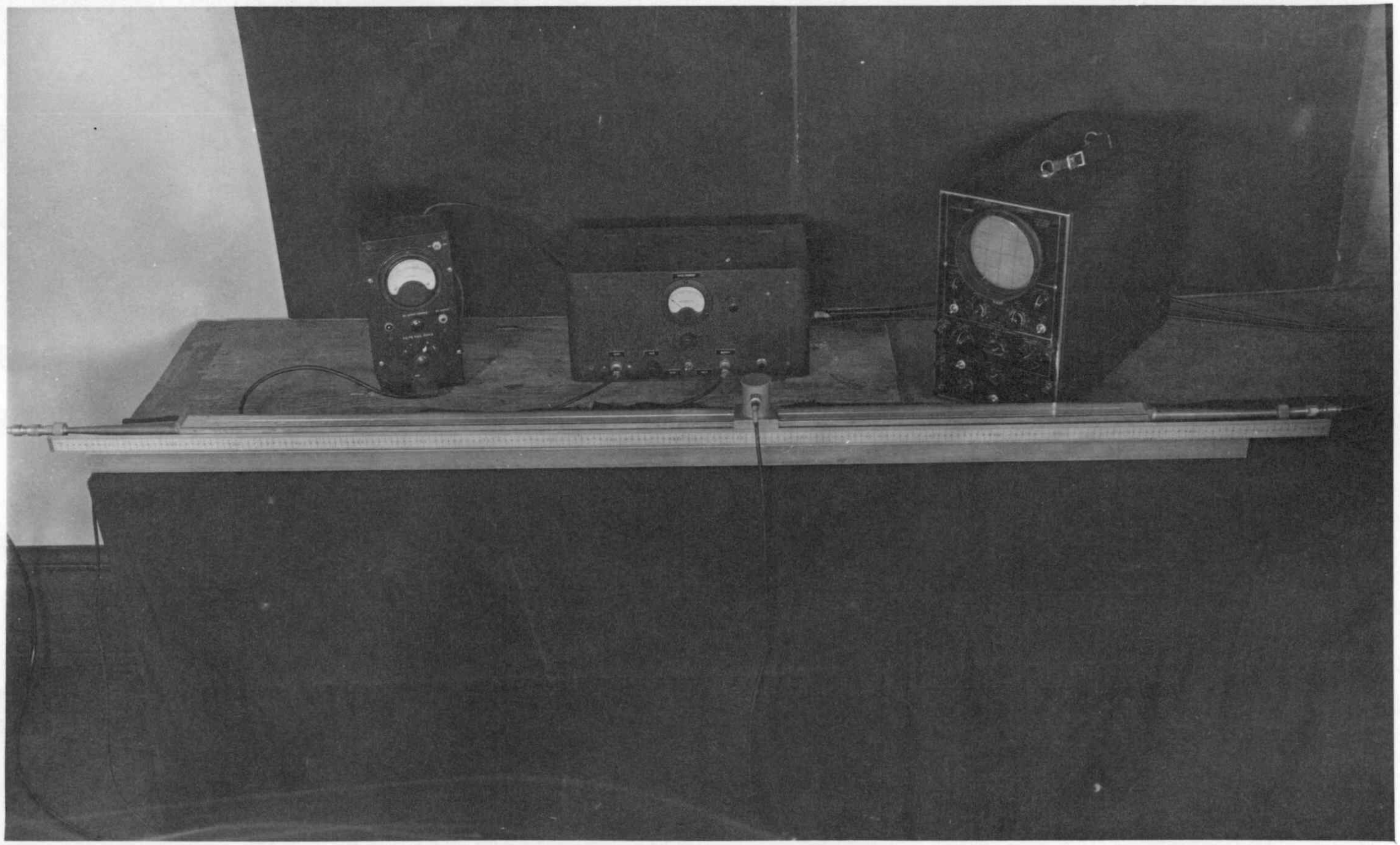


DIAGRAM OF DETECTOR

FIGURE 5

FIGURE 6



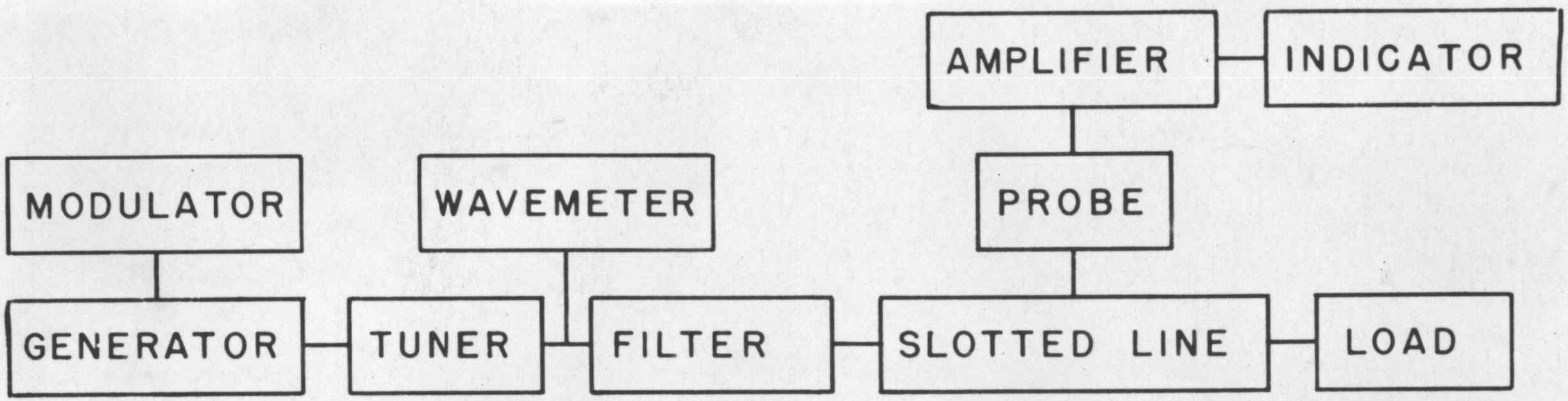
SLOTTED LINE

OPERATIONAL PROCEDURE AND PRACTICE

The essential components of a transmission-line measuring installation are shown in Fig. 7. It is necessary that an accurate means of measuring frequency be provided. The slotted line itself may be used if the standing wave ratio is high. However, the standing wave ratio is usually low with the unknown load connected to the measuring line and the minima are not sharply defined. Wavelength determinations will thus not be accurate. When the load is replaced with a short-circuit, a large amount of reactance will be reflected back into the line. Under these conditions the generator may shift frequency by a serious amount.

This difficulty may be overcome to a considerable extent by inserting a section of lossy line between the generator and the measuring section. If the attenuation in this connecting cable is sufficiently large the generator will see only the characteristic impedance of the line regardless of what occurs in the measuring section. At least 10 db. of attenuation is recommended. For frequencies above 300 megacycles per second this can be achieved with about 150 feet of RG-8/U cable.

The problem of frequency shift in the generator can also be solved by readjusting the frequency of the generator to the original value after the load is replaced with a short-circuit. A precise frequency meter



BLOCK DIAGRAM OF IMPEDANCE-MEASURING EQUIPMENT

FIGURE 7

is of course required.

The presence of harmonics of the generator frequency is usually harmful because the reflection coefficient of the load may be large for these harmonics. The use of filters to eliminate these harmonics is recommended. Again, a length of lossy line between the generator and the measuring section is helpful since the attenuation introduced by the lossy line increases with frequency. The use of a low-pass filter designed to cut off at a frequency below the second harmonic is better.

Often it is necessary to use a length of cable to connect the measuring line to the load. This introduces two problems. The attenuation of the line will reduce the standing wave ratio in the measuring section and lead to an erroneous value of load impedance. The formula for finding the load impedance when there is attenuation between the load and measuring section is

$$\frac{Z_r}{Z_o} = \frac{\frac{Z_m}{Z_o} \cosh(\gamma l) - \sinh(\gamma l)}{\cosh(\gamma l) - \frac{Z_m}{Z_o} \sinh(\gamma l)}$$

where Z_m/Z_o is the measured impedance ratio, γ is the propagation constant of the connecting line, and l is the length of line.

The second problem resulting from the use of a length of cable between the measuring section and the load is that of determining the position of the minimum when the line is shorted. If the generator changes frequency slightly when the short is introduced, the position of the minimum may be changed seriously if many wavelengths of cable are used. The use of a minimum length of connecting cable is recommended.

A typical data sheet is shown in Table 1. These data were obtained from measurements made on a piece of 50 ohm cable terminated by means of several carbon resistors selected to provide a direct current resistance of 50 ohms.

The output from the detector was amplified by a factor of 10,000, and measured with a Balantine voltmeter. The standing wave ratio was determined by taking the square root of the ratio of maximum voltmeter reading to the minimum voltmeter reading as the probe was moved along the slotted line. The distance Δx was obtained by subtracting the reading of the reference minimum position from that of the minimum position with the load connected to the line. This difference is positive when the reference minimum position is between the load and the minimum position with the load connected to the line. The pointer on a Smith chart is rotated a distance $\Delta x/\lambda$ from the point of zero impedance in a clockwise

Frequency Mcs.	Maximum Reading	Minimum Reading	Minimum Position	Reference Position	r	ΔX	$\frac{\Delta X}{\lambda}$	$\frac{Z_r}{Z_o}$	Z_r
300	0.100	0.051	127.0	130.9	1.40	− 3.9	−.039	.74 − j .11	38 − j 5.5
400	0.100	0.061	123.5	125.3	1.28	− 1.8	−.024	.80 − j .05	40 − j 2.5
500	0.091	0.049	119.2	119.2	1.36	0.0	.000	.75 + j 00	38 + j 0.0
600	0.100	0.050	108.2	107.9	1.41	0.3	.006	.71 + j .02	35 + j 0.1
700	0.087	0.051	114.6	112.3	1.31	2.3	.054	.80 + j .13	40 + j 6.5
800	0.072	0.043	121.4	118.6	1.30	2.8	.075	.84 + j .18	42 + j 9.0
900	0.060	0.039	115.3	112.2	1.25	3.1	.093	.90 + j .18	45 + j 9.0
1000	0.100	0.074	123.5	119.5	1.16	4.0	.133	1.00 + j .10	50 + j 5.0
1100	0.052	0.035	126.3	121.7	1.22	4.6	.169	1.09 + j .16	55 + j 8.0
1200	0.050	0.028	118.4	113.2	1.34	5.2	.208	1.26 + j .18	63 + j 9.0

TABLE I

direction when Δx is positive. The load impedance is then located on the pointer line at a distance from the center corresponding to the measured value of r. The impedance of the line is 50 ohms so that the values read from the Smith chart must be multiplied by 50 to obtain the load impedance.

CHARACTERISTICS OF THE SLOTTED LINE

It is important to know to what extent the slotted line may be depended upon to give accurate readings. Variations in probe depth and changes in the geometry of the line along its length will cause incorrect readings.

To measure the accuracy of the slotted line it is necessary to set up conditions for which results can be predicted. Inaccuracies in the slotted line can then be interpreted in terms of the departures from these results.

There are three conditions which are particularly useful for this purpose. If the line is terminated with either zero or infinite impedance, the standing wave pattern set up will vary as $\cos^2\theta$ with a sufficiently large standing wave ratio to permit accurate determinations of position and amplitude. Of these two conditions, the short-circuit was chosen. The curve was fitted to the experimental points at the origin, and at 75 centimeters. The results of this test are shown in Fig. 8. The noise level of the amplifier interfered with readings near the minima thus preventing the curve from touching the zero axis at these points.

A much more sensitive test of the slotted line can be made by studying the voltage distribution along the line when the line is terminated in its characteristic

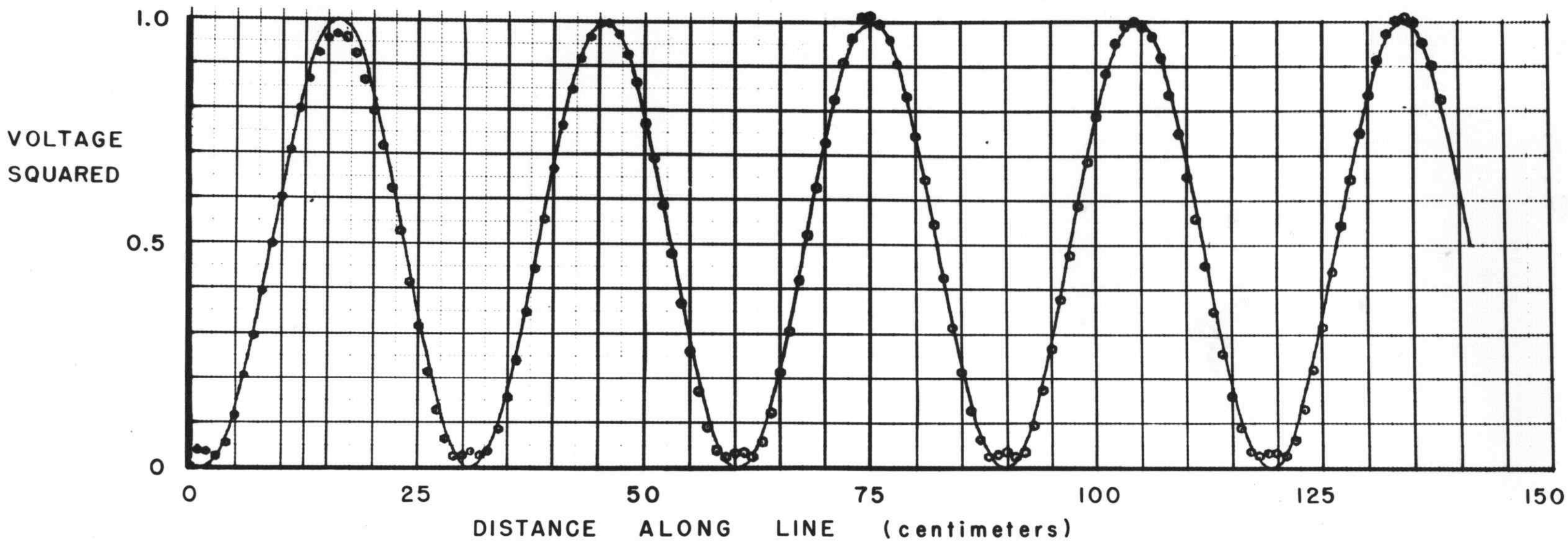


FIGURE 8

COMPARISON OF THEORETICAL AND EXPERIMENTAL VOLTAGE DISTRIBUTION ON THE SHORTED SLOTTED LINE AT 510 MC.

SOLID CURVE IS THEORETICAL -- POINTS ARE EXPERIMENTAL impedance. This condition is difficult to meet experimentally, however. An exactly matched load is strictly a theoretical quantity. For practical purposes, however, a load which is better than is normally met in practice will suffice. A piece of uniform coaxial cable long enough to attenuate the power 10 db. or more in each direction is a convenient form of "dummy load". In this case, an 800 foot length of RG-8/U cable terminated with a 50 ohm resistor was used. The standing wave ratio produced by this load was measured at several frequencies. The results are shown in Fig. 9. At 1200 megacycles per second a minimum in the standing wave ratio occurs corresponding to a standing wave ratio of 1.04/1. A study of the voltage distribution along the line was then made at this frequency. This is shown in Fig. 10. From this curve it is concluded that the residual standing wave ratio introduced by inaccuracies of the line when a perfectly matched load is connected will be about 1.03/1. This should be entirely adequate for impedance measurements. In the section from 40 to 110 centimeters, the inaccuracies of the line introduce a standing wave ratio of about 1.01/1. For very accurate work this section of the line should be used.

VOLTAGE STANDING WAVE RATIO PRODUCED BY 800 FEET OF RG-8-U CABLE CONNECTED DIRECTLY TO THE SLOTTED LINE

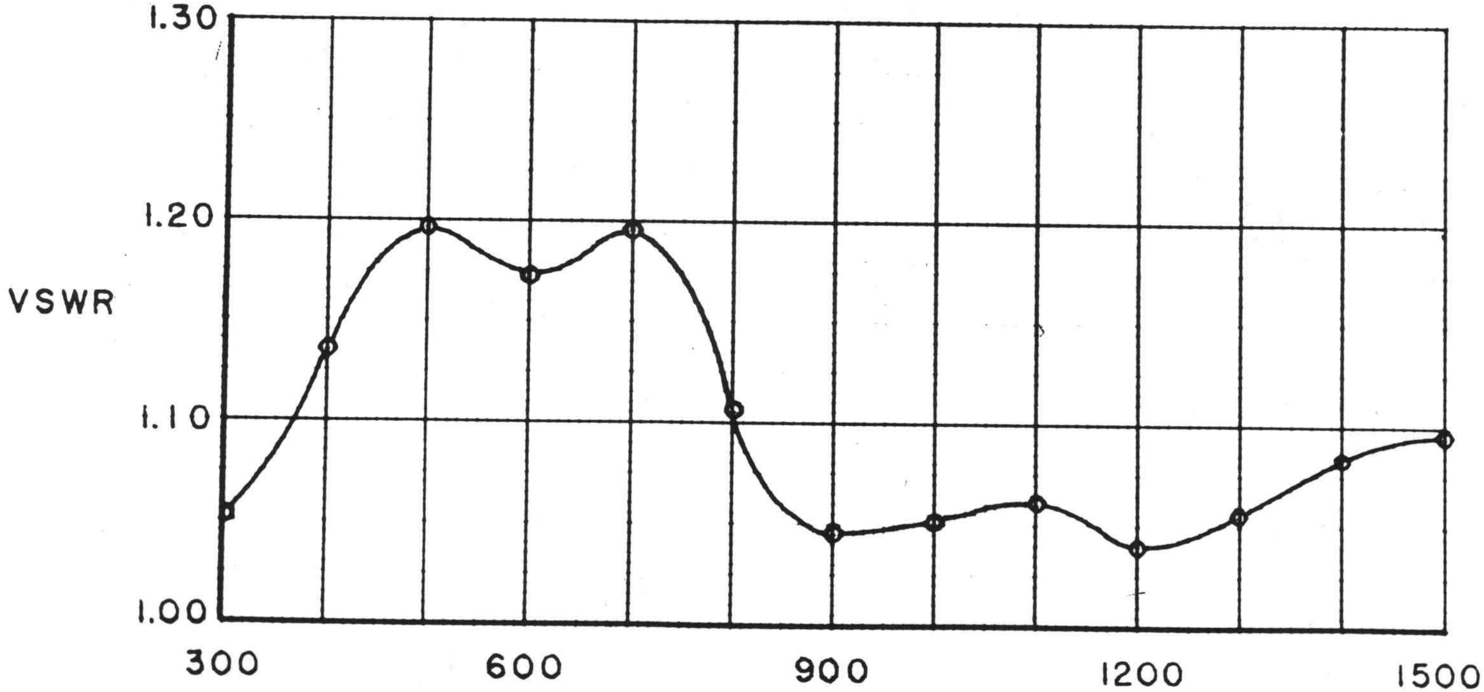


FIGURE 9 FREQUENCY (megacycles / second)

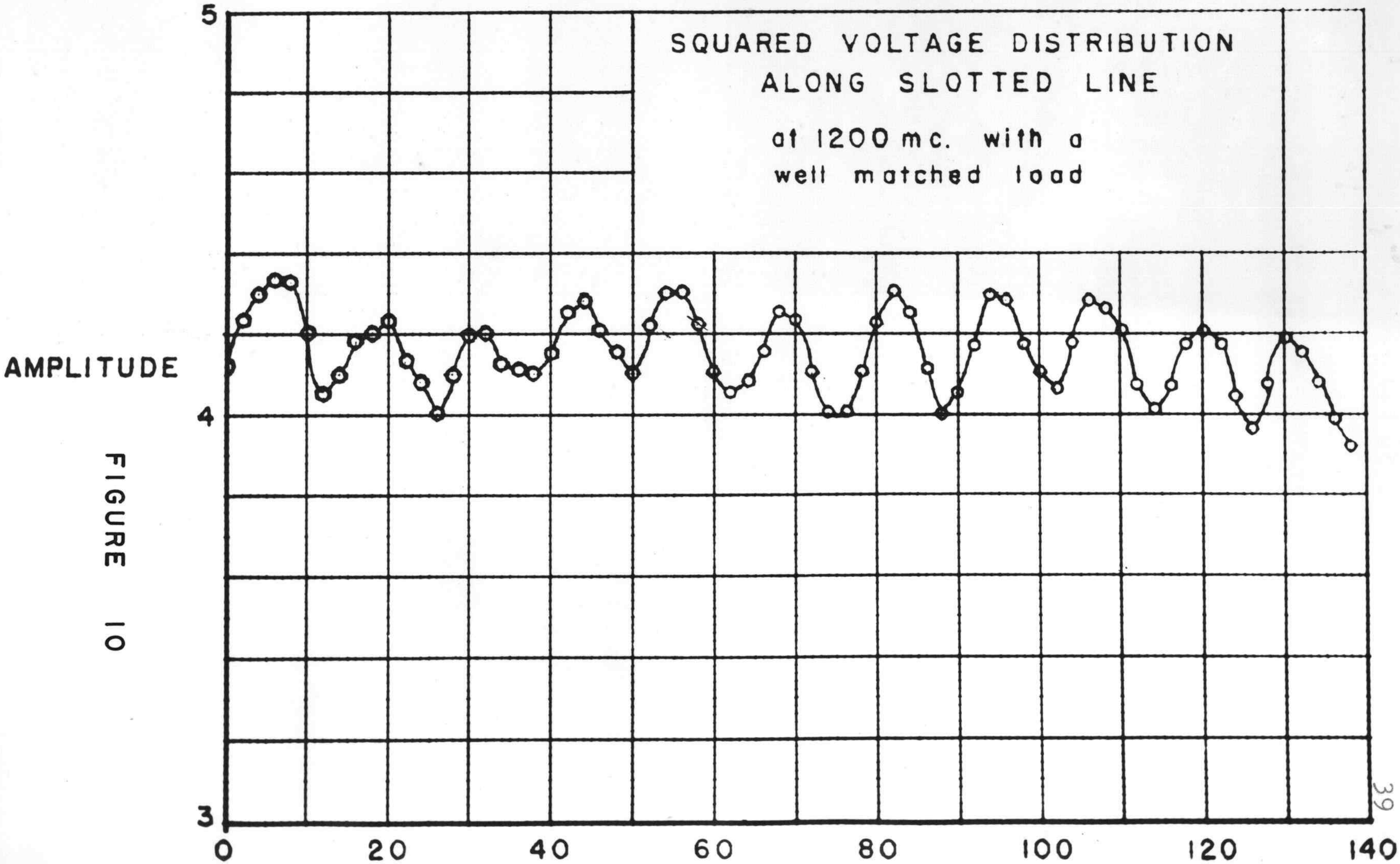

SQUARED VOLTAGE DISTRIBUTION
ALONG SLOTTED LINE
at 1200 mc. with a
well matched load
AMPLITUDE
5
4
3
0
20
40
60
80
100
120
140

FIGURE 10

ANTENNA IMPEDANCE

The following problem is a typical example of the use of a slotted line.

A dipole antenna 32.2 centimeters long was placed 35 centimeters from, and parallel to, a ground plane. The dipole was fed from a balun designed for a frequency of 420 megacycles per second. The problem was to determine the impedance of this antenna as a function of frequency.

The measuring equipment was set up and measurements taken by the methods previously described. The data resulting are given in Table II.

One additional problem is introduced in this set of measurements. It was necessary to use a length of transmission line between the measuring section and the antenna. Since the antenna was located on the roof above the room in which the measurements were taken, it was inconvenient to replace the antenna with a short-circuit at the point where the antenna was previously connected. Instead, the line feeding the antenna was removed and the short introduced at the measuring section. This procedure necessitates transferring the short-circuit, in effect, to the antenna terminals which was done by computing the number of wavelengths that existed between the actual position of the short-circuit and the antenna.

Frequency Mcs.	Maximum Reading	Minimum Reading	Minimum Position	Reference Position	r	ΔX	$\frac{\Delta X}{\lambda}$	$\frac{L}{\lambda}$	$\frac{\Delta X - L}{\lambda}$	$\frac{Z_r}{Z_o}$
329.3	0.88	0.045	79.10	67.95	4.41	11.65	.1279	5.2162	−.0883	.31 − j .57
374.8	0.97	0.183	116.30	124.20	2.30	−7.90	−.0987	5.9363	−.0350	.46 − j .18
405.9	1.00	0.570	122.10	130.30	1.32	−8.20	−.1110	6.4303	−.0413	.78 − j .10
428.3	0.90	0.620	113.80	99.10	1.20	14.70	.2098	6.7837	−.0739	.90 − j .11
447.8	0.94	0.330	106.10	103.60	1.69	2.50	.0373	7.0925	−.0552	.64 − j .22
461.3	1.48	0.615	89.90	74.15	1.55	15.75	.2422	7.3074	−.0652	.75 − j .21
463.8	1.63	0.500	92.40	74.78	1.63	17.62	.2723	7.3447	−.0724	.70 − j .27
487.8	0.93	0.270	91.50	81.10	1.86	10.40	.1691	7.7268	−.0577	.59 − j .24
510.2	0.95	0.160	119.10	115.80	2.44	3.30	.0561	8.0816	−.0255	.42 − j .13
531.9	0.95	0.113	87.70	91.20	2.67	−3.60	−.0638	8.4255	−.4893	.37 + j .05
561.8	0.92	0.086	94.50	97.25	3.27	−2.75	−.0515	8.8989	−.4504	.35 + j .29
589.4	0.98	0.062	99.70	102.45	3.97	−2.85	−.0560	9.3359	−.3919	.40 + j .73

TABLE II

The distance from the short to the antenna was found to be 317.5 centimeters. This corresponds to an electrical length of 475.2 centimeters since the wavelength in a coaxial line is less than the wavelength in free space by a factor equal to the reciprocal of the square root of the dielectric constant. The dielectric used in this line was polyethylene which has a dielectric constant of 2.26. A column is provided for recording the length of the connecting line in units of wavelength. Integer numbers of half-wavelength are dropped, and the resulting distance is subtracted from $\Delta x/\lambda$. This operation cancels out the effect of the connecting line. The attenuation of the length of cable used in this test is small at these frequencies and has been neglected.

The impedance of the antenna is plotted on the R-X diagram in Fig. 11.

The slotted line with which this thesis is concerned has been found to be satisfactory in performance and relatively easy to operate. It is simple in design, ruggedly yet precisely built, and will no doubt be a useful device for both instruction and research in the radio laboratories for which it was built.

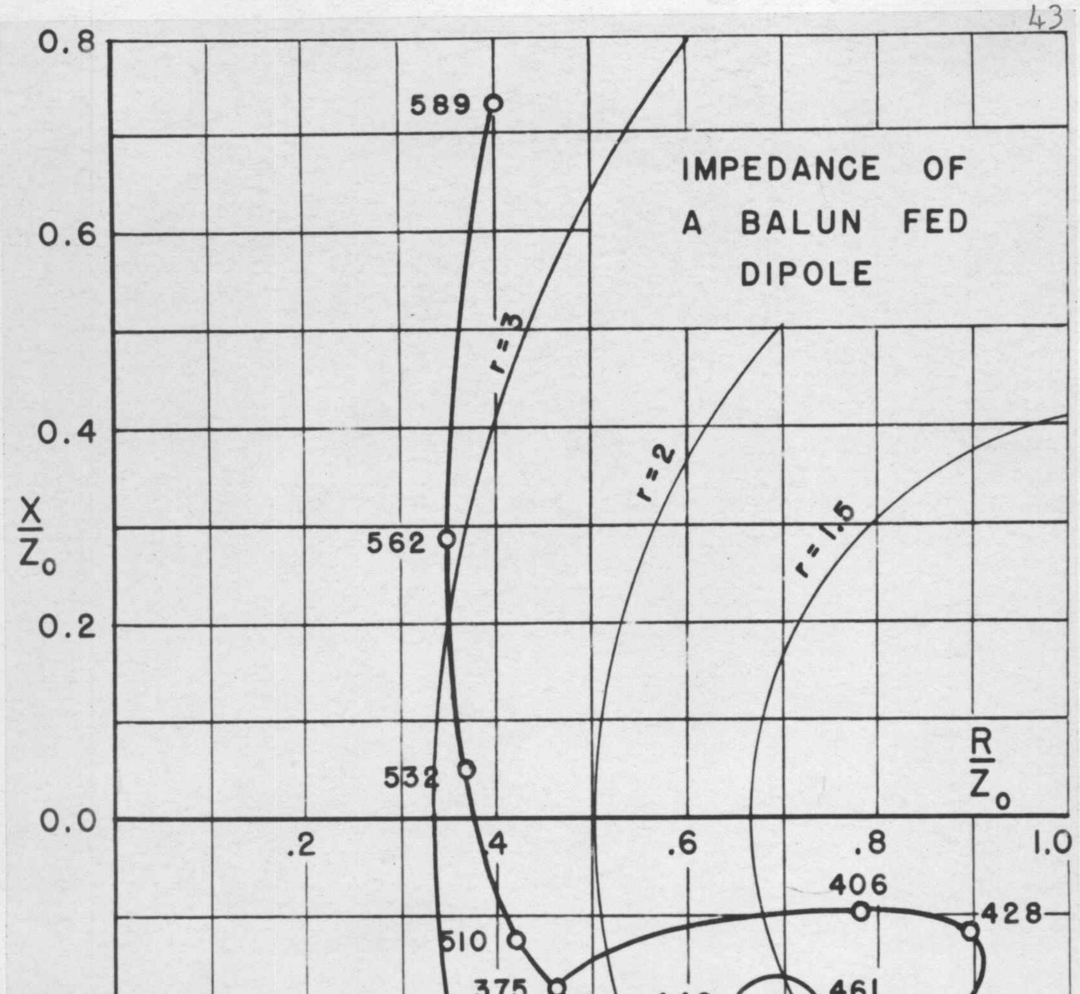

IMPEDANCE OF
A BALUN FED
DIPOLE
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
X/Z0
R/Z0
.2
.4
.6
.8
1.0
r = 3
r = 2
r = 1.5
589
562
532
510
375
329
406
428
448
461
489
464

FIGURE 11

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3. Smith, P. H. Transmission line calculator. Electronics 12:29-31. 1939.