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Title SPATIAL VISUALIZATION ABILITIES OF CENTRAL WASHINGTON STATE COLLEGE PROSPECTIVE ELEMENTARY AND SECONDARY TEACHERS OF MATHEMATICS

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This investigation was designed to determine the extent to which students who are completing planned curricula in mathematics education are proficient in spatial visualization abilities and possess mathematical understandings. The effects of mathematics curricula upon the development of these abilities and understandings and the relationships between these abilities and understandings were examined by comparing group mean test scores of prospective elementary and secondary mathematics teachers with those of (a) prospective social science teachers, (b) prospective English teachers, (c) prospective science teachers, (d) prospective art/industrial arts teachers, (e) freshman mathematics students, (f) experienced elementary mathematics teachers, and (g) experienced secondary mathematics teachers.

Criterion tests were the _Differential Aptitude Test of Space Relations, Form A_, (DATSR), the _Revised Minnesota Paper Form Board Test, Series MB_, (MPFB), and the _Sequential Test of Educational_

Progress-Mathematics, Form 1B, (STEPM). The study being of a post-test-only design, the criterion instruments were administered to the freshman students at the beginning of Fall Term 1964. Prospective teachers completed the tests during the term in which they were enrolled in their respective special teaching methods courses.

Single classification of analysis of covariance using DATSR, MPFB, and STEPM group means was employed to statistically test the null hypotheses. Means of combined verbal and mathematical subtest scores on the Washington Pre-College Tests were applied as covariance controls of scholastic aptitude, and group mean cumulative grade-point-averages were similarly used to control group differences in academic ability. F ratios were computed and evaluated to determine whether differences in group means on the criterion instruments were significant. The data were further analyzed to determine correlations between the variables and other curricula data. Reliability coefficients for the criterion instruments were computed for each group involved in the study.

FINDINGS

The following conclusions relative to students at Central Washington State College were drawn from the data obtained and analyzed in this investigation:

1. The spatial visualization abilities of prospective elementary mathematics teachers are significantly different from similar abilities of experienced elementary mathematics teachers, while the spatial visualization abilities of prospective secondary mathematics teachers do not differ significantly.
2. Spatial visualization abilities of prospective secondary mathematics teachers and experienced elementary mathematics teachers are significantly different, as are the similar abilities of prospective elementary mathematics teachers and experienced secondary mathematics teachers.
3. The spatial visualization abilities of prospective elementary mathematics teachers are not significantly different from similar abilities of prospective teachers in other academic fields of endeavor involved in the study, except in the field of science.
4. Spatial visualization abilities of prospective secondary mathematics teachers are significantly different from similar abilities of prospective teachers of social sciences and prospective teachers of English, but are not significantly different from spatial visualization abilities of prospective teachers of art/industrial arts or prospective teachers of science.
5. Prospective secondary mathematics teachers have a significantly different degree of development of spatial visualization

abilities than do prospective elementary mathematics teachers. It would appear, based on the means of the collected data, that the former group has a higher degree of development.

6. The spatial visualization abilities of first-quarter freshman mathematics students are significantly different from similar abilities of prospective elementary mathematics teachers but are no different from these same abilities of prospective secondary mathematics teachers.
7. Mathematics achievement can be used, at significant levels of confidence, as a valid index of spatial visualization abilities of prospective teachers in several academic fields, freshman mathematics students, and experienced elementary and secondary mathematics teachers.
8. There is no significant dependence of a teacher's spatial visualization abilities on his scholastic ability. This is also true for the first-quarter freshman mathematics students.

SPATIAL VISUALIZATION ABILITIES OF CENTRAL WASHINGTON STATE COLLEGE PROSPECTIVE ELEMENTARY AND SECONDARY TEACHERS OF MATHEMATICS

by

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CHAPTER I

INTRODUCTION

General Remarks

In recent years there has been a great upsurge of interest in the school mathematics program. One major result has been a realization of the importance of a fact long known but largely neglected. This is that the mathematics which students study in school is a continuous stream rather than a collection of separate units. Realization of this fact has created a demand for information about the mathematical processes and methods involved. Such information, if sufficiently detailed, could make it possible for teachers to improve their effectiveness in the classroom and enhance their students' realization of their own potentials in the present culture.

Education is of great concern to all Americans. Without education our young people could not get jobs, would be unable to participate intelligently and responsibly in civic and social activities, and would fail to achieve a measure of individual realization. Education is increasingly being recognized as the servant of all aspects of a full and rewarding way of life.

The interest in, and demand for, change in the mathematical programs of the schools came from the above. Expressions of concern over the mathematical inadequacies of high school graduates have been heard for many years, and many of the expressions of concern have been substantiated by reliable experimentation (9, p. 87). Concern over the mathematical program in our schools was both widespread and acute. It was found among laymen as well as among professional people. The significance of this educational problem was emphasized by the work of various committees representing the thinking of professional organizations and by extensive experimentation being carried on in the schools and colleges.

One aspect of the concern relative to the mathematics programs was directly related to the mathematical preparation and background of the teachers of mathematics. The Mathematical Association of America, and other professional groups, have made recommendations in this area (13, p. 13). The Cambridge Conference on School Mathematics ignored the whole problem of teacher training and acted on the assumption that if a teachable mathematics program were developed, teachers would be trained to handle it (10, p. 3). It was recognized, however, that curricular development and teacher preparation must keep pace with each other in order to have an effective mathematics program in our schools.

Possibly the most critical element in the composite structure of an effective mathematics program in the schools is the teacher himself. A poorly prepared teacher could destroy the effectiveness of any carefully selected and well-organized curriculum with inadequate and unenthusiastic instruction, inaccurate and uninformed interpretation, in addition to indifferent and negative attitudes.

It has been shown that a teacher who has a solid understanding of mathematics himself is more likely than not to develop a similar understanding in his pupils. It has been found that this "solid understanding" is frequently absent in teachers (17, p. 16).

Teachers on all levels, in elementary and secondary schools as well as in colleges, must be competent to teach mathematics with an understanding of traditional mathematics and an appreciation of the modern point of view, and they must be able to convey to the students a new insight into the nature of mathematical thought and reasoning and its role in our culture. The teacher's mastery of mathematical ideas must substantially exceed that represented by his textbook, if he is to teach with a spirit of enthusiasm and inquiry which will stimulate his students.

Several of the recent mathematics curriculum proposals have incorporated many of the recommendations of the College Entrance Examination Board (12, p. 26), one of which was to eliminate from the secondary mathematics curriculum a separate course in solid geometry. The recommendations further stipulated that the solid geometry, or space, concepts should be incorporated into what was formerly referred to as a "plane" geometry course. The effective teaching of such concepts by the mathematics teachers is of concern to mathematics educators.

Statement of Problem

There is a need, then, for careful investigation of the spatial visualization abilities of mathematics teachers at the elementary and secondary levels. To provide data relevant to this need, the present study investigated the spatial visualization abilities of prospective elementary and secondary mathematics teachers as measured by appropriate testing devices.

One of the major problems which has confronted mathematics teachers for a long time has been the development of the student's ability to visualize spatial relationships (26, p. 5). Minnick has stated:

> Perhaps the student's greatest difficulty in beginning the study of geometry is seeing a three-dimensional figure. The three-dimensional figure is drawn in a two-dimensional space, but he is asked to see it in a three-dimensional sense. Such a transition is not easy. . . . Then the first new instructional problem which the mathematics teacher meets is that of teaching students to read three-dimensional figures (40, pp. 237-239).

Kinney and Purdy also supported this view by stating that "without question, the most important teaching problem in solid geometry is that of developing ability to visualize" (32, p. 174).

The assumption has been made that no assistance in the visualization of three-dimensional figures is needed by students (26, p. 5). There is very little data from research studies upon which to base an evaluation, however. The presence of the aptitude to predict, or visualize, how an object will look when its shape and position are changed is a major factor in selecting persons from the general population for certain endeavors requiring this aptitude. The presence of sufficient perceptual acuity with both two- and three-dimensional objects is a critical factor for successful performance of construction projects (38, p. 2).

People live in a three-dimensional world. Hence, the major aim of early instruction in geometry should be to help the student acquire an understanding of the spatial relationships existing in the world around him; to train him in spatial imagination; to acquaint him with spatial forms; and to give him an analytic insight into his surroundings. Mere intuition does not do this for him. His imagination must be cultivated through intelligent guidance (5, p. 311).

Here the mathematics teacher enters the picture. Teachers exert both a personal and an intellectual influence on their students. Hence, the teacher's state of mind tends, for better or worse, to propagate itself. Thurstone has raised the question which forms the basis for this study:

> . . . whether much more of this kind of thinking (spatial visualization) could be introduced into teaching in many fields if teachers had more ability or training in this area. Again, we are referring to the fact that teachers, being largely selected on the basis of their own ability in the verbal factor, are likely to restrict their handling of any subject to the abilities in which they themselves excel (66, p. 571).

It should be mentioned here that at Central Washington State College, where this investigation was conducted, there were no mathematics content courses required in the elementary education program. There were four courses, two offered by the mathematics department and two offered by the education department, available periodically as electives to prospective elementary teachers. The only required course related to mathematics was a course, Education 323, in the methods of teaching arithmetic. Individuals certificated to teach mathematics at the secondary level were required to complete a minimum of 22 quarter hours of mathematics, including a methods course in the teaching of secondary mathematics. It was entirely possible, and has happened, for individuals to graduate from Central Washington State College and be certified to teach elementary school mathematics without having completed a mathematics content course since the high school mathematics completed at the ninth or tenth grade levels.

Statement of Purposes

Within the limitations imposed, and under the assumptions listed in Chapter II, the purposes of this study were:

1. To determine the extent to which college mathematics experiences developed spatial visualization ability in prospective elementary and secondary mathematics teachers.
2. To determine and compare the degree of spatial visualization ability in several groups of prospective elementary and secondary mathematics teachers.
3. To measure the mathematical achievement of several groups of prospective elementary and secondary school teachers.
4. To determine the relationships which existed between the spatial visualization ability, mathematical achievement, and scholastic aptitude.
5. To determine the relationship between two spatial visualization measuring instruments.
6. To determine whether revisions in the mathematics curriculum at Central Washington State College should be recommended to provide for instruction in subject matter designed to develop the spatial visualization ability in prospective elementary and secondary mathematics teachers.
7. To make recommendations to mathematics teachers relative to their spatial visualization ability.

CHAPTER II

AREA OF STUDY AND METHODOLOGY

Importance of the Study

Thurstone, in his research relative to primary mental abilities, recognized a space factor, S, a mental ability, which is associated with the visualization of objects in space. Most intelligent people are fairly well endowed with this factor, but there are many highly intelligent people who are conspicuously poor in this ability (64, p. 585). There are wide individual differences in the ability to think of objects in space (65, p. 527). Paterson and others reported that individuals were frequently baffled in their attempts to solve mechanical puzzles where the third dimension was involved (49, p. 290). He further stated:

> It is possible that the prevailing educational systems may be responsible in part for the fact that most people habitually think in terms of two dimensions represented by the plane of the blackboard and the maps and diagrams commonly used as educational devices. . . . In the early stages of reading a child recognizes a word upside down as readily as right side up. By the time adulthood is reached, this capacity is lost, but we do not know how or why. We do know, however, that it may be acquired by adults; that so short a practice period as ten hours will give considerable facility in reading upside down. . . . Considerations of this sort suggest that such abilities as that of dealing with three-dimensional space may be determined not so largely by particular native capacities as by practice and experience (49, pp. 290-1).

The problem of whether this spatial visualization ability is something which can be learned or is an inherited characteristic was a fact that seemed to be left unanswered in the literature. Piaget found that, in his experiments with young children, measurement in three dimensions, like that in two dimensions, showed a continuous development from level I on (51, p. 171). He also stated:

> The forms of spatial knowledge, as a consequence of sensori-motor operations, are the resultants of actual manipulations by the organism within his environment. In this sense the organization of space is relatively egocentric, i. e., the intuition of space is not differentiated from the actions involved in its apprehension (33, p. 209).

Wohlwill placed the emphasis in teaching scientific and mathematical subjects to young children on an enrichment of their experiences; for example, getting them actively engaged in measuring shapes and sizes (72, p. 266).

Paterson (49, p. 291), Kinney (32, p. 174), Thorndike (63, p. 20), and Meserve (37, pp. 326-9) have indicated that such abilities as that of dealing with three-dimensional space may be determined not so much by particular inherited traits as through experience and practice.

On the other hand, Vernon wrote "... from early childhood we are aware of a three-dimensional spatial continuum of objects, extending away in all directions from ourselves as the centre" (69, p. 81). The factors of experience and familiarity undoubtedly exist and improve accuracy of judgments of distance, although it may be, as claimed by the Gestalt psychologists, that we have an innate tendency to perceive space three-dimensionally (69, p. 110). In this same vein, Flavell wrote:

> Our adult representation of space is thus said to result from active manipulations of the spatial environment rather than from any "reading off" of this environment by the perceptual apparatus. . . . It just seems natural to assume that we see space as it is and have always seen it that way (20, p. 328).

Under most conditions a person's intelligence remains fairly constant, although, as Stephens said, "we do know that there is some growth in mental abilities through the twentieth year at least" (60, pp. 136-8). This would tend to indicate that the spatial visualization ability, a mental ability according to Thurstone (64, p. 585), could possibly be developed further, at least through the twentieth year. However, Super found that changes in spatial visualization ability took place up to about age 14, after which it appeared that this ability was rather stable (62, p. 295).

Butler specified that geometric content of junior high school geometry should include developing the ability to draw and see solids in their proper perspective (9, pp. 470-1). A study of the outlines and materials of several of the recent mathematics curriculum proposals revealed that space geometry is not neglected in the over-all picture. The School Mathematics Study Group materials, for example, contained two strong units on space geometry in grade eight, and a unified treatment of plane and solid geometry in grade ten. Three-dimensional rectangular coordinate geometry and three-dimensional vectors are studied in grade eleven (47, p. 69).

The Cambridge Conference on School Mathematics suggested that geometry should be studied along with arithmetic and algebra from the kindergarten level on including experiences in visualization, such as constructing two-dimensional patterns and forming them into three-dimensional figures (10, p. 33).

The Fifteenth Yearbook of the National Council of Teachers of Mathematics listed as a major goal in tenth-grade geometry:

> IV Development of elementary spatial insight, and the habit of noticing geometric relations in three dimensions, and . . . . Such a development can be secured by discussion and generalization of certain propositions or problems of plane geometry (46, p. 93).

The College Entrance Examination Board listed the "development of concepts of spatial relations, and of spatial perception" as an objective of mathematics teaching and suggested that a solid geometry course should be included in the curriculum (12, pp. 26-7). A survey of literature made by Breslich disclosed that, among the important objectives of intuitive geometry, training in space perception and imagination was to be emphasized. He defined "intuitive geometry" as "the geometry all people should know" (5, p. 235). The Committee on the Reorganization of Mathematics in Secondary Education listed as major goals in teaching mathematics:

> . . . . familiarity with the geometric forms common in nature, industry and life; the elementary properties and relations of these forms, including their mensuration; the development of space perception; and the exercise of spatial imagination; . . . . (5, p. 236).

Relative to recommendations for the training of teachers of mathematics in spatial concepts the Mathematical Association of America has recommended courses in informal geometry for elementary teachers and more formalized courses in geometry for secondary teachers of mathematics (13, pp. 11-12). Meserve included in his list of topics, which should be considered in the preparation of teachers of mathematics, a course which would extend geometry to three-space and higher dimensions (36, p. 439). Mathematics programs, such as the School Mathematics Study Group (55, p. 341), the Report of the Commission on Mathematics of the College Entrance Examination Board (12, p. 26) and the University of Illinois Committee on School Mathematics (29, pp. 1-2) have prepared materials concerned with spatial visualization and teaching techniques for the use of mathematics teachers and have included such materials and experiences in their student texts.

Davis suggested that prospective secondary mathematics teachers should take a course in solid analytic geometry which, among other things, develops better space perception, and, further, that a prospective teacher should gain a sufficient mastery and perception of the subject matter so that he not only "understands it himself but is able to explain it clearly to others" (16, pp. 358-9). Fehr agreed with this suggestion as he claimed that "To develop space perception, we must teach a type of geometry different from the traditional Euclidean course" (19, p. 16). He advocated a space coordinate geometry combined with descriptive and projective geometries to fill the need.

Young made a statement which suggested the goal in developing space perception, in addition to all of mathematics, namely:

> . . . Whether we regard mathematics from the utilitarian point of view, according to which the pupil is to gain facility in using a powerful tool, or from the purely logical aspect, according to which he is to gain the power of logical thinking, it is clear that the chief end of mathematical study must be to make the pupil think. If mathematical teaching fails to do this, it fails altogether. . . . (73, p. 4).

Structure of the Study

This study would be considered of a "true" experimental design, a recommended experimental procedure where the sources of internal and external invalidity are controlled to a great extent, and where the use of pretests in the ordinary sense are not convenient (21, pp. 195-6). Within limits of confidence specified by the tests of significance, randomization can suffice without the administration of a pretest. Design of the study is designated as:

$$\begin{array}{lll} R_i & X & O_1, O_2, O_3 \\ R_j & & O_1, O_2, O_3 \end{array}$$

where R_i represents the groups of prospective elementary and secondary mathematics teachers, R_j represents the remaining groups of prospective teachers, experienced teachers, and freshman mathematics students; X, the experimental variable, represents the mathematical preparation of prospective elementary and secondary teachers of mathematics; and O_1, O_2, O_3 represent the dependent variables used in this study.

The dependent variable or criterion used in this study was defined in terms of the score each individual attained on each of two standardized spatial visualization tests and on a standardized mathematics achievement test.

The independent variables or control variables used in this study were the cumulative grade-point-average and a measure of scholastic aptitude. Using these criteria for each group tested, an attempt was then made to determine whether or not there was any significant difference in the spatial visualization abilities and mathematics achievement among the various groups of individuals being investigated.

Analysis of covariance techniques were used to provide the investigator with a means of attaining some measure of control of individual differences in scholastic aptitude and academic ability. Analysis of covariance incorporates elements of the analysis of variance and of regression (70, p. 343).

Population for the Study

The population for this study consisted of undergraduate and graduate students at Central Washington State College, Ellensburg, Washington. Central Washington State College was a tax-supported coeducational institution with a 1964-65 enrollment of approximately 3600 students, of which approximately 75 percent were enrolled in a teacher-education curriculum. An extensive summer session program was organized for experienced teachers to complete work on a fifth-year program or a master's degree. The sample consisted of junior and senior students who, prior to their student-teaching experiences, were enrolled in special methods classes in certain subject-matter areas during the 1964-65 academic year, first-quarter freshman mathematics students in the fall quarter 1964, and experienced elementary and secondary mathematics teachers enrolled during the summer session 1965. A total of 530 students was involved in the study. Randomization of the sample was accomplished at registration.

Assumptions

Major assumptions related to this study were:

1. Spatial visualization ability is important to the development of our culture. Super supported this assumption by his statement:

> It is an aptitude which has long been considered important in such clearly similar activities as machine-shop work, carpentry, mechanical drawing, in which the worker must judge shape and size and translate two-dimensional drawings into three-dimensional objects, and which has been considered likely to be important in certain other occupations, the principal activities of which were not quite so clearly similar, such as engineering and art (62, p. 283).

He also stated that presence of the trait in a high degree cannot be considered a good prognosticator of success but that its relative absence in an individual can be considered a deterrent to success (62, p. 307). Fehr stated that one of the most frequently mentioned desired outcomes of the study of geometry has been the development of space visualization and that this perception is considered a great need in architectural and engineering study (19, p. 16).

2. Spatial visualization ability in teachers of mathematics is necessary in order to develop this ability in their students. Thurstone stated that teachers are likely to restrict their handling of any subject to the abilities they themselves have (66, p. 371). The Cambridge Conference on School Mathematics also stated that if the elementary teachers in the next generation do not understand what they are teaching then the results may be worse than we are getting now (10, p. 27). Efficient and successful teaching of demonstrative geometry in the secondary school requires on the part of the teacher much more than a knowledge of subject matter (59, p. 99). Kinney inferred that a student cannot be expected to possess any great competence in spatial

insight when he enters a geometry class, and as a result, specific instruction must be given (32, p. 174).

3. Spatial visualization abilities can be developed through instruction. Corle suggested that discussion of space is a good beginning for the elementary teacher to introduce geometric space concepts to children (15, p. 284). Breslich stated that it required training in order for an individual to understand diagrams (5, p. 312). The 15th Yearbook of the National Council of Mathematics recommended that much attention should be given to the visualization of spatial figures and relations (46, p. 97). Kinney also supported this assumption by stating that "specific instruction must be given" (32, p. 174). Paterson stated that:

> . . . such abilities as that of dealing with three-dimensional space may be determined not so largely by particular native capacities as by practice and experience (49, p. 291).

4. Spatial visualization abilities can be measured objectively. Thurstone supported this assumption when he indicated two space factors S_1 and S_2 existed as primary abilities (65, p. 527). Piaget also measured the development of spatial relationships (51, p. 171). Lawshe stated that "some authorities contend that tests of this sort are really non-language intelligence tests" (34, p. 124).

5. Valid and reliable testing instruments were available for measuring spatial visualization abilities and mathematics achievement. Validity and reliability of testing instruments will be discussed in another section of this chapter.

Hypotheses

The study was planned to test the hypothesis that Central Washington State College prospective elementary and secondary school teachers with college mathematics backgrounds have developed a higher degree of spatial visualization ability than those prospective elementary and secondary teachers who have not had college mathematics backgrounds. It was predicted that prospective elementary and secondary teachers with college mathematics backgrounds would have greater proficiency in spatial visualization than would those prospective teachers without college mathematics backgrounds.

To test the hypothesis, the following null hypotheses were established to be treated statistically.

1. There is no difference between the spatial visualization abilities of prospective elementary teachers of mathematics and the spatial visualization abilities of prospective secondary teachers of mathematics.

2. There is no difference between the spatial visualization abilities of prospective elementary teachers of mathematics and the spatial visualization abilities of experienced elementary teachers of mathematics.

3. There is no difference between the spatial visualization abilities of prospective elementary teachers of mathematics and the spatial

visualization abilities of experienced secondary teachers of mathematics.

4. There is no difference between the spatial visualization abilities of prospective secondary teachers of mathematics and the spatial visualization abilities of experienced elementary teachers of mathematics.

5. There is no difference between the spatial visualization abilities of prospective secondary teachers of mathematics and the spatial visualization abilities of experienced secondary teachers of mathematics.

6. There is no difference between the spatial visualization abilities of prospective elementary teachers of mathematics and the spatial visualization abilities of prospective elementary and secondary teachers in other academic fields, such as science, English, social sciencies, fine arts and industrial arts.

7. There is no difference between the spatial visualization abilities of prospective secondary teachers of mathematics and the spatial visualization abilities of prospective elementary and secondary teachers in other academic fields, such as science, English, social sciences, fine arts and industrial arts.

8. There is no difference between the spatial visualization abilities of prospective teachers of mathematics and the spatial visualization abilities of first-quarter freshman students enrolled in college

level mathematics classes.

9. There is no relationship between achievement in mathematics and spatial visualization abilities.

10. There is no relationship between scholastic ability and spatial visualization abilities.

Limitations of the Study

The investigation reported in this study was subject to the following limitations:

1. Spatial visualization abilities will be the only factors of concern.

2. The spatial visualization abilities of the individuals involved in this study may be influenced by past and present learning situations, both academic and non-academic, which were not considered.

3. The study was limited to experienced teachers, prospective elementary and secondary teachers, and first-quarter freshman mathematics students enrolled at Central Washington State College during the academic year 1964-65 and the summer session 1965.

4. The study was limited to relatively small groups of students at a single educational institution, thus limiting the extent to which the results can be generalized.

5. Individuals involved in the study were aware of the fact that they were subjects of an experiment, and this fact may have had some

effect on the results of the study, although efforts were made to control this possible influence (the Hawthorne effect).

6. The use of random sampling techniques was limited by the necessity to use the best statistical procedures for testing hypotheses and the availability of classes of individuals to be used in this study.

Definitions of Terms and Abbreviations Used

For the purposes of this study, the following terms were used:

1. Prospective teacher: any college student enrolled in the education program leading to a bachelor's degree and certification to teach in the public schools of the State of Washington.

2. Elementary mathematics teachers: elementary school teachers who teach mathematics in addition to other subjects.

3. Visual space perception: ability depending on combinations of the following eight factors: distinctness, shadow, position, relative size, relative motion, sensations of accomodation in the eye, binocular differences in the eye, and sensations of convergence in the eye (45, p. 103).

4. Spatial visualization, or spatial imagination: visual relations of objects in space as they are visually manipulated (23, p. 601); the ability to judge the relations of objects in space, to judge shapes and sizes, to manipulate them mentally, to visualize the effects of putting them together or of turning them around (62, p. 283).

5. Experienced teacher: a teacher who has taught in the public schools at least one year.

6. Coefficient of correlation (r): a pure number, ranging between positive one and negative one, which denotes the tendency for corresponding observations in two or more sets of data to vary together from the averages of their respective sets of data. The deviation-score method of computation of the correlation was used in this study to facilitate computations involved:

$$r = \frac{N\Sigma XY - \Sigma X \Sigma Y}{\sqrt{(N\Sigma X^2 - (\Sigma X)^2)(N\Sigma Y^2 - (\Sigma Y)^2)}}$$

where	r	is the coefficient of correlation between the two criteria,
	N	is the number of scores,
	ΣX	is the sum of the scores on a first criterion,
	ΣY	is the sum of the scores on a second criterion,
	ΣX^2	is the sum of the squares of the scores on the first criterion,
	ΣY^2	is the sum of the squares of the scores on the second criterion,
	ΣXY	is the sum of the products of scores on the first criterion and second criterion for each member of a sample.

7. Analysis of covariance: an extension of the methods of analysis of variance in which adjustments are made in the data for the

criterion variable on the basis of data collected on one or more other variables which vary concomitantly with the criterion variable. Assumptions underlying the use of the analysis of covariance are similar to those of analysis of variance and regression, namely:

1. Observations in each category must be random samples.
2. The variances within the subgroups are from a normally distributed population.
3. Linearity of the data is also assumed.

In the actual research situation it may be difficult to satisfy all the assumptions; however, it is doubtful whether this failure is sufficiently great to invalidate the application of the technique. Evidence suggested that the limits of tolerance within which the assumptions must be approximated are wider than it was originally thought (70, p. 183, 343).

8. Reliability coefficient: A measure of reliability of a test, derived from the Kuder-Richardson formula (25, p. 225) and based on the assumption that all test items are of the same difficulty. While this is not perfectly true of the tests administered in this study, it is approximately true. The computation is relatively simple and uses only the mean, variance and number of items on the test, as follows:

$$r_{xx} = \frac{N}{N - 1}\left(1 - \frac{\overline{X} - \frac{\overline{X}^2}{N}}{s^2}\right)$$

where r_{xx} is the reliability of the test,

N is the number of items on the test,

$\overline{X}$ is the mean of the test scores,

s^2 is the variance of the test scores.

9. Validity coefficient: "Validity is a highly relative concept", according to Guilford (24, p. 461). This same viewpoint will be followed in this study. Guilford adds that "A test is valid for (some activity) if its scores correlate highly with (some criterion)" (24, p. 463). Using both these concepts, all obtained correlations will be considered valid only on relative bases. Since a particular test may have different correlations with some other single test obtained from two different groups of individuals, there can be no single "validity" for any given test. Each test is only relatively valid, and for different criteria yields a different validity index or coefficient of validity.

The following abbreviations were used throughout the study and were used in reporting the data obtained:

WPCVM Washington Pre-College Testing Program combined verbal and quantitative test scores administered through the University of Washington to all high school graduates entering colleges in the State of Washington.

DATSR	Spatial Relations Test - This is part of the DAT, Differential Aptitude Tests battery, published by the Psychological Corporation.
MPFB	Revised Minnesota Paper Form Board Test - This test is also published by the Psychological Corporation.
STEPM	Sequential Tests of Educational Progress, Mathematics-Level 1, Form B - A mathematics achievement test published by Cooperative Test Division, Educational Testing Service.
GPA	Grade-point-average - In the case of prospective teachers and experienced teachers this referred to college grade-point-average; for the freshman mathematics students this referred to eighth-semester high school grade-point-average.
PEM	Prospective elementary mathematics teachers - There were 57 individuals in this group for this study.
PSM	Prospective secondary mathematics teachers - There were 75 individuals in this group for this study.
PE	Prospective English teachers - There were 46 individuals in this group.
PSS	Prospective social science teachers - There were 45 individuals in this group.
PAI	Prospective art and/or industrial arts teachers - There were 41 individuals in this group.
PS	Prospective science teachers - There were 49 individuals in this group.
FM	Freshman mathematics students - There were 157 individuals in this group.
EEM	Experienced elementary mathematics teachers - There were 35 individuals in this group.

ESM	Experienced secondary mathematics teachers - There were 25 individuals in this group.

Instruments Used in the Study

A brief, easily-answered questionnaire, designed by the investigator, to aid in assessing the mathematical background, college background, teaching experience, and other personal data was completed by each individual in the study. (See Appendix.)

The Washington Pre-College Verbal and Quantitative Test scores were obtained from records available in the Counseling and Testing Office at Central Washington State College for each undergraduate student in the study. The verbal test score was obtained from a 25-minute, 72-item vocabulary test aimed at evaluating the individual's ability to abstract or generalize and to think constructively, rather than at simple fluency or vocabulary recognition. The antonym form of test items was appropriate for the measurement of reasoning ability. The quantitative test score was obtained from a 23-minute, 80-item quantitative skill test measuring the individual's ability to do quantitative reasoning as measured by arithmetic problems, number series, and figure analogies. It was a test to measure the student's ability to reason with numbers, to manipulate numerical relationships and to deal intelligently with quantitative materials. The total of the verbal test score and the quantitative test score was used as a measure of general learning ability or scholastic ability. On the basis for which these tests were designed the validity and reliability coefficients, respectively, were 0. 64 and 0. 57.

Differential Aptitude Space Relations Test (DATSR).

This test employed items of the type called "surface development". A pattern for a geometric surface was provided for each item. Each item was provided with five isometric drawings of various figures. The task was to indicate all the solid figures which could be made from the given pattern. The solid figures were rotated into various positions in space. All the items involved three-dimensional space. The maximum possible score was 100. Scoring was based on the number correct less the number incorrect. This test was designed to evaluate the ability to deal with concrete materials through visualization and to create structure in an individual's mind from a given pattern. In the data furnished by the publisher, the coefficients of reliability ranged from 0. 86 to 0. 94.

Revised Minnesota Paper Form Board Test (MPFB)

This test contained 64 items to be completed in 20 minutes. Each item contained a given box containing several separate plane figures. There were five additional plane drawings provided, which showed a single plane figure consisting of joined parts. The problem was to select the single joined figure which represented the assembly of the given separate figures. This test was complicated by the fact that some items had similar parts, but of different dimensions. Scoring was based on the number correct. This test was designed to measure the ability to visualize the assembly of geometric shapes into a whole design.

Sequential Tests of Educational Progress - Mathematics (STEPM)

This was a 70-minute test, consisting of two 35-minute units, consisting of 50 multiple-choice items from which the correct conclusion based on the given information was to be selected. The test items were constructed in such a way as to measure broad understanding of general mathematical concepts rather than specific course work in mathematics. Reliability coefficients, reported by the publisher, ranged from 0.83 to 0.89. One major deficiency in terms of content of the tests was the highly verbal character of the problems.

Procedures and Methods of Research

In the fall of 1964 the investigator discussed the study with the Dean of Instruction at Central Washington State College and received his permission to proceed in implementing the testing program necessary for the study. Colleagues of the investigator in the mathematics department volunteered to assist in administering tests to the freshman mathematics classes. Other members of the faculty who were involved in teaching special methods classes in the areas of English, social science, fine arts, industrial arts, elementary mathematics, science, and secondary mathematics during the academic year 1964-65 were approached by the investigator and were asked to permit the testing program to be conducted with their students in these special methods classes. All members of the faculty who were approached, except for one professor teaching elementary mathematics methods, graciously granted permission to use their students and the class time in which to administer the tests and questionnaire. Each quarter the investigator made specific arrangements to administer the tests.

On the second day of classes in the fall quarter all freshman students enrolled in mathematics classes were asked to complete the brief personal data questionnaire and the Differential Aptitude Spatial Relations Test (DATSR) and the Revised Minnesota Paper Form Board Test (MPFB) were administered. Special arrangements were made with the students involved to take the Sequential Test of Educational Progress - Mathematics (STEPM).

On previously arranged days during the first four weeks of each quarter of the academic year students in the special methods classes, which had been selected, completed the personal data questionnaire, and the DATSR, the MPFB, and the Sequential Test of Educational

Progress - Mathematics (STEPM) were administered.

During the summer session 1965 similar procedures were followed with special classes in elementary and secondary mathematics in which there were a number of experienced teachers enrolled.

In all instances instructors were asked not to alert their students relative to participating in the study. The investigator administered the questionnaire and the tests in all cases, except for some of the freshman mathematics sections. A brief explanation of the purposes of the testing was offered in each class. All tests were hand-scored by the investigator. Test scores were made available for the information of both students involved in the study and interested instructors.

For other relevant data needed in the study the investigator found the transcripts of students and test scores readily available in the offices of the Director of Testing and Counseling and the Registrar of the college.

Each student participating in the study was given a code number by the investigator specifying his category, i. e., freshman, prospective English teacher, etc.; his college status, i. e., undergraduate or graduate; his level of teaching preparation, i. e., elementary or secondary; and his standing in his category. Some of the pertinent personal data was also coded by the investigator for easier reference and use in analysis.

The student number; the coded personal data; the WPCVM scores; the scores on the DATSR, MPFB, and STEPM tests; total credits, and grade-point-average (GPA) for each student were entered on IBM cards. Statistical programs were written and debugged by the investigator for the IBM 1620 computer which was available at Central Washington State College for research use. After the data and the statistical programs were accumulated, they were processed on the computer by the investigator. The first phase calculated means, standard deviations, and various correlation coefficients for each of the categories of individuals involved; the second phase calculated data needed for the following phase, which used this data and applied it in the analysis of covariance computations.

The analysis of covariance computations were carried out considering two control variables. Since scholastic aptitude and academic ability could conceivably influence each individual's response to a spatial visualization test, DATSR or MPFB, or the mathematics achievement test, STEPM, these influences were controlled by obtaining the WPCVM scores as a measure of scholastic aptitude and the college grade-point-averages as a measure of academic ability for each individual in the sample. By using these scores as control variables in the analysis of covariance, the possible bias introduced by individual differences was removed insofar as these factors adequately represent the differences in question. High school grade-point-averages were used as a measure of academic ability for control purposes for the freshman mathematics students involved in the study, as no college grade-point-averages were yet available.

The analysis of covariance computations involving the two groups of experienced teachers, EEM and ESM, were carried out considering only one control variable, college grade-point-average, as a measure of academic ability, as the WPCVM scores were not available for the majority of these individuals, and there were no other common criteria available for control purposes with the prospective teacher groups.

The data were then summarized and an attempt was made to analyze them.

Overview of Thesis

Significant writings related to spatial visualization abilities will be reviewed in Chapter III. In Chapter IV the educational background, teaching experience, and other personal data obtained through the use of the brief questionnaire will be summarized and discussed, while in Chapter V the data obtained in the investigation will be organized, analyzed, and interpreted.

Chapter VI will be devoted to summarizing the findings and to presenting a number of recommendations and implications based on the data obtained in the study.

CHAPTER III

REVIEW OF RELATED LITERATURE

No research studies were found which related directly to the relationship of spatial visualization ability of a mathematics teacher and his educational background. Neither was there found any literature relating directly to the problem of a mathematics teacher's success in the classroom and his spatial visualization ability. Tending to confirm the absence of studies directly related to the present study, Watson and Blade reported,

> . . . so far as we know, there have been no previous experimental studies which undertook to relate an individual student's background to his spatial visualization score. . . . (2, p. 2).

Literature did exist which related spatial visualization ability to success in such fields as architecture, drafting, engineering, and industrial arts teaching.

Perception of Space

Piaget defines schemata as repeatable behavior (39, p. 201). Thus, a representation of an observable behavior pattern exists in the cognitive structure and forms the basis for future cognitive and behavioral functioning in subsequent situational contexts. In some ways one could describe a schema as a response in the sense that it is repeatable and reliable in a specific situation. Now it may be argued that these responses will always be intuitively available in spatial, numerical, or causal form. In discussing the implications of Piaget's work, Langer stated:

> These categorical forms of intuition provide the continuity of cognitive organization even though it is constantly changing in the course of developmental growth. . . . The forms of spatial knowledge as a consequence of sensori-motor operations, are the resultants of actual manipulations by the organism within his environment. In this sense the organization of space is relatively egocentric, i. e., the intuition of space is not differentiated from the actions involved in its apprehension (33, p. 209).

The positive, constructive something which we inherit, Piaget stated, is a mode of intellectual function. We do not inherit cognitive structures as such--these come into being only in the course of development. The mode of functioning which Piaget said, ". . . constitutes our biological heritage", remains essentially constant throughout life (20, p. 43). Just as with object development, the development of space perception is primarily one of progressive "externalization and desubjectification" (20, p. 135). Wohlwill supported this view:

> In other words, I think one has to look at the role of experience in relation to the level of the development of the child. . . . More particularly, it seems to me that the experience should ideally have a built-in feedback possibility for the child (72, p. 225).

Piaget found that in young children the perception of space is an early developmental product, while the representation of space is quite different and appears much later (20, p. 232). Vernon found that,

> The factors of experience and familiarity undoubtedly exist and improve accuracy of judgments of distance, although it may be, as claimed by the Gestalt psychologists, that we have an <u>innate</u> <u>tendency</u> to perceive space three-dimensionally (69, p. 110).

Flavell indicated that his findings

> . . . show clearly that it would be a mistake to imagine that human beings have some innate or psychologically precocious knowledge of the spatial surroundings organized in a two- or three-dimensional reference frame (20, p. 333).

Wohwill recommended providing children with a broad base of experience which would assure them of developing their cognitive skills (72, p. 226). Peel even went so far as to state that,

> . . . it looks as if concept formation in adolescence and adult life differs from concept formation in childhood only in that the heirarchial and multiplicative structures of the logic of classes and relations are available. But new criteria of action (mental and physical) and perception may still have to be constructed at all ages (50, p. 229).

Jerome Bruner has provided a psychological rationale for the major ideas of the new mathematics curricula (8). As much of his interpretation of new concepts and new methods in education was made in the light of Piaget's discussion of intellectual development, Bruner has stimulated much interest among mathematics educators in Piaget's work. For example, Piaget's evidence that the child of nine or ten can handle many of the basic concepts of Euclidean spatial representation and measurement is mirrored in the placement of such topics in the elementary curriculum by the School Mathematics Study Group (31, p. 248).

Boehm reported that a few mathematicians have exceptional spatial visualization and computational skills (3, p. 22). He further concluded that "mathematicians probably cannot visualize ordinary shapes as clearly as engineers and mechanics. . ." (3, p. 23). Minnick found that incomplete drawings made by individuals in various professions were due largely to their inability to visualize the figure from a verbal description (40, p. 192).

The Case for Solid Geometry

Whether the inclusion of solid geometry concepts in the elementary and secondary school curricula would lead to extensive development of spatial visualization abilities has been a matter for concern for many of the curricula study groups. Relative to this Johler commented that,

> "Seeing solid", that is looking at a drawing or a figure in one plane and comprehending it in three dimensions, is one essential concept of solid geometry. The converse, to think of, comprehend, or see a three-dimensional object. . . is of equal importance (30, p. 253).

Breslich pointed out that one of the difficulties experienced by students in high school courses in solid geometry was that they were unable to understand geometric figures all of whose points did not lie in one plane (5, p. 311). He blamed the intensive study of plane geometry for this problem and suggested that three-dimensional geometry instruction and experiences should begin in the seventh and eighth grades (5, p. 312).

The College Entrance Examination Board in 1959 recommended the inclusion of solid geometry concepts as a part of the secondary geometry course (12, p. 26), a recommendation that the School Mathematics Study Group followed in their courses of study (56, p. 20). The University of Illinois Committee on School Mathematics, for instance, went further by developing a programmed text in solid geometry to be used as a three or four week supplement to their course of study (29, p. 2). The recommendation for inclusion of solid geometry concepts in the high school geometry course was not of recent vintage as the Fifteenth Yearbook of the National Council of Teachers of Mathematics in 1940 stated that the development of spatial insight "can be secured by discussion and generalization of certain propositions or problems of plane geometry" (46, p. 93).

The National Council of Teachers of Mathematics warned, in 1960, that

> The fact that none of the improved programs contains a course called solid geometry should not lead us to conclude that solid geometry is regarded as unimportant. There has been a change in the way space geometry is presented (47, pp. 68-69).

Fehr reported a study of over 1,000 students in which it was found that the traditional course in solid geometry, or excerpts of this course imbedded in the plane geometry course, did not develop space perception significantly. Students who did not study solid geometry made more gain in space perception, as measured by space relations tests, than those students who did study the traditional course (19, p. 16). Brown reported a similar study which indicated that little, if any, improvement in space perception resulted from the study of solid geometry (7, p. 11). Gibney reported from her study that high school geometry students scored lowest on primary mental ability tests in spatial relationships (22, p. 182).

Holcomb conducted an investigation relative to teaching solid geometry concepts to different classes by the traditional Euclidean unit method and an "analytic" unit method and pre- and post-testing the students with different forms of the Differential Aptitude Space Relations Test. He found the greatest gain in spatial visualization ability was made by the class taught by the "analytic" unit method (28).

Relationships With Spatial Visualization Ability

Turner reported the results of her study on the measurement of spatial visualization concepts in grades one through nine. She found that test items which contained factors other than shape alone tended to discriminate between students at different levels of performance to a greater degree than test items which did not. A two-way analysis of variance was used to establish the relationship of age, sex, and mental ability to spatial visualization ability. She found that there was a highly significant relationship between chronological age and spatial visualization ability and, also, a highly significant relationship between mental age and spatial visualization ability. She found no significant relationship existed between sex and spatial visualization ability (67).

Shaw reported a study among entering ninth-grade students of the relationship between primary mental abilities and high school achievement. He found that spatial visualization ability had little relationship to measures of achievement (58).

Super found that spatial visualization ability did not correlate well with other tests of mental ability and, in fact, gave poor predictions of school achievement, increasingly so with increasing age. He suggested that, as a result,

> . . . the degree of spatial judgment possessed in middle adolescence or adulthood is not a good indicator of the amount of any other mental ability possessed by the individual (62, p. 283).

He further concluded that the spatial visualization testing resulted in measuring a "trait which is related to mental ability in childhood but relatively independent in adulthood" (62, p. 283). Super found, also, that a moderately high spatial visualization score in a very able person did not mean special aptitude for professional or technical work and that a high spatial visualization score for a person of low average intelligence might well indicate promise for the skilled trades (62, p. 295).

Thurstone, in a study of high school students, found a correlation of 0.304 between the Primary Mental Abilities (Space) Test and the Iowa Test of Ability to Do Quantitative Thinking and a correlation of 0.283 between the Primary Mental Abilities (Space) Test and the Iowa Test of Background in the Natural Sciences (66, p. 576). This again seemed to indicate that there was little relationship between spatial visualization ability and achievement.

Mendocino, in 1958, administered the Differential Aptitude Mechanical Reasoning Test and the Differential Aptitude Spatial Relations Test (DATSR) to tenth-grade vocational machine shop students and tenth-grade non-vocational students and found that machine shop experiences in grade ten resulted in no greater growth in mechanical reasoning or space perception than did a year of a non-vocational curriculum. He concluded that the increase in scores of the control group was due to growth or development and in no way attributable to training (35).

Schmidt and Rothney reported that tenth-grade students selecting future occupations as machinists, mechanics, doctors, art and engineering did not make significantly higher scores on spatial visualization tests, which were said to measure abilities necessary for these occupations, than students who selected other future occupations. Students who selected drafting as a future occupation indicated superiority on the spatial visualization tests (54).

Bowman, in a study of the basic mathematical skills needed to teach industrial arts in the public schools, recommended that, based on study evidence, prospective industrial arts teachers be required to complete a course in, among others, solid geometry. He also reported that inservice teachers of industrial arts, at the public school and college levels, and individuals involved in the fields of industrial arts activities rated algebra and solid geometry backgrounds as necessary for success in the field (4).

The Blade-Watson Study of Engineering Students (2)

Blade and Watson conducted a study investigating the effect of practice and/or training on scores on a spatial relationship test at Cooper Union School of Engineering. The test was given as a part of an entrance examination and consisted of two parts, Intersections and Blocks, with a one-hour time limit. After one year of engineering study, including mechanical drawing and mathematics, scores on the spatial visualization test, after a re-test, increased by almost one standard deviation over the first set of scores. They concluded that spatial visualization was subject to noteworthy changes due to the kind and extent of training, and that low scores did not necessarily indicate lack of spatial visualization ability. Although no correlations were published, Blade and Watson indicated that those students who scored highest on the spatial visualization test also indicated higher achievement in mathematics and engineering drawing.

The Roe Study of Eminent Scientists (53, pp. 21-25)

A special spatial relations test was constructed by the Educational Testing Service for the use of Roe in the study of 64 physicists, biologists, and social scientists. The test consisted of 24 items to be completed in 20 minutes. Each item consisted of four isometric drawings in various positions. The problem was to select the unique pair of blocks which were congruent. A mathematics achievement test of 35 minutes duration was also given to each of the participants in the study. The age range of the participants was 40 to 50 years.

Roe reported a correlation of 0.21 between spatial visualization score and mathematical achievement, a correlation of 0.40 between spatial visualization and age, and a correlation of 0.00 between age and mathematical achievement. The only significant correlation was that between age and spatial visualization ability--this at the 0.01 level of significance.

The Lorge Study of Doctoral Candidates (57, pp. 20-21)

Lorge administered the same spatial relations test as Roe used to 174 doctoral candidates in Education and Philosophy at Columbia University. Other tests were also administered in this study. The age range of the participants was from 20 to 35 years. Lorge reported a correlation of 0.38 between spatial visualization and mathematics achievement, a correlation of 0.24 between spatial visualization and age, and a correlation of 0.23 between mathematics achievement and age. All of these correlations were found to be significant, the spatial visualization-mathematics achievement correlation at the 0.01 level of significance and the others at the 0.05 level of significance.

The Sharo Study of 60 High School Students (57)

Sharo selected 60 students at Stuyvesant High School and administered four different spatial relations tests, the spatial test used by Roe, the spatial test used by Blade and Watson at Cooper Union School of Engineering, the Minnesota Paper Form Board Test (MPFB), and the Abstract Reasoning Test of the Differential Aptitude Battery (57, p. 15). Only those students who had previously taken a Physics Regents Examination were involved in the study. Several mathematics tests and physics tests were also administered to the participants in the study. The age range of the participants was from 16 to 19 years.

Sharo reported a correlation of 0.34 between spatial visualization and solid geometry, a correlation of 0.40 between spatial visualization and algebra, a correlation of 0.09 between spatial visualization and age (57, p. 36). All of these correlations, except the last one, were significant at the 0.01 level of significance. The last correlation was not significant.

Among the conclusions Sharo drew from his study were the following:

> 1. Some Visual Spatial Relations tests can be used, at significant levels of confidence, as valid indices of academic achievement in Physics and Mathematics. They are somewhat less valid as indices of Chemistry achievement, and of negligible or negative validity as indices of English achievement.
>
> 2. Mathematics tests are more valid indices of Physics than are either Visual Spatial Relations tests or English tests (57, p. 45).

He also reported that solid geometry seemed to be a good index of academic achievement in the study of Physics (57, p. 39).

Summary

The review of the related literature supplied background information and indicated the significance and timeliness of the study.

A broader perspective of the factors involved in the ability to visualize spatially was derived from textbooks and journal articles.

The data obtained from the research in cognitive development and the uncertainty of psychologists and sociologists as to the development of this spatial visualization ability were valuable. The debate relative to the inclusion and placement of solid geometry concepts in the school curricula was of current interest as the many curricula reforms and studies are being tested and implemented. Studies reporting indications that solid geometry study in the schools had no greater value in helping develop the spatial visualization ability were of particular interest, if not disturbing.

Several reports emphasized the small relationship between spatial visualization and mathematics achievement, while other studies indicated that there was a significant relationship.

The problems cited and the importance attached to the spatial visualization ability indicated the importance of gathering further data relative to the development of this ability, particularly with respect to prospective teachers of elementary and secondary mathematics.

CHAPTER IV

PERSONAL DATA OF PARTICIPANTS IN THE STUDY

Status of Prospective Teachers

One hundred nine of the 313 prospective teachers at Central Washington State College, who participated in the study, were in their third year of college study, 185 were in their fourth year of college study, and 19 had received a bachelor's degree in some academic field and were completing the requirements for a teaching credential.

The average age of the prospective teachers who participated in the study was 23 years with a range from 19 years to 51 years, a span of 32 years.

One hundred eighty-eight of the prospective teachers were preparing to teach at the secondary school level, while 125 were preparing to teach at the elementary school level. There were 175 male prospective teachers and 138 female prospective teachers involved in the study. Of the male prospective teachers, 124 were preparing to teach at the secondary level, and 51 were preparing to teach at the elementary level. Sixty-four female prospective teachers were preparing to teach at the secondary level, while 74 were preparing to teach at the elementary level. These data are presented in Table I together with the number of prospective teachers in each of the groups

Table I. Summary of Personal Information of Prospective Teachers.

Criteria		Groups* PSM	PEM	PSS	PAI	PE	PS	All Groups
Number of Participants		57	75	45	41	46	49	313
Sex	Male	44	28	23	26	16	38	175
	Female	13	47	22	15	30	11	138
Age	Range	19-43	19-51	19-44	19-37	20-45	19-30	19-51
	Mean	23.1	23.7	24.8	22.6	23.2	22.7	23.4
Year in College	3rd	22	19	10	15	29	14	109
	4th	30	51	31	24	15	34	185
	5th	5	5	4	2	2	1	19
Teaching Level Preference								
	Elementary	0	75	20	10	11	6	122
	Secondary	57	0	25	31	35	43	191
Degree Status	None	52	70	41	39	44	48	294
	B. A.	5	5	4	2	2	1	19
	M. A.	0	0	0	0	0	0	0
Student Teaching Experience	Completed	11	18	5	5	7	8	54
	Not Completed	46	57	40	36	39	41	259

*PSM - Prospective Secondary Mathematics Teachers
PEM - Prospective Elementary Mathematics Teachers
PSS - Prospective Social Science Teachers
PAI - Prospective Art /Industrial Arts Teachers
PE - Prospective English Teachers
PS - Prospective Science Teachers

involved in the study.

Fifty-four of the participants had completed their student-teaching requirements prior to participating in the study, while 257 had not entered the student-teaching phase of their program of study.

Mathematics Preparation of Prospective Teachers

A summary was made of the number of credits in mathematics completed by the participants in the study at the high school level and the college level. The average number of semester credits completed at the high school level was 6. 2, while the average number of quarter hours completed in mathematics at the college level was 9. 1. These data are not valid indications, however, as they were biased by either the lack of mathematics completed by, say, the group of prospective English teachers or by the number of mathematics credits completed by prospective secondary mathematics teachers. These data are presented in Table II.

Attention should be given to the data relative to the number of college mathematics credits attained by the prospective teachers in each category, particularly the prospective elementary and secondary mathematics teachers. On the basis of 22 quarter hours in college-level mathematics required for minimum certification endorsement to teach mathematics at the secondary level, as specified by Central Washington State College, 38 of the 57 prospective secondary

Table II. Mathematics Preparation, College Credits, and Grade-Point-Averages of Prospective Teachers.

Criteria		Groups						All Groups
		PSM	PEM	PSS	PAI	PE	PS	
High School Mathematics Preparation*	Range	4-9	1-9	1-9	2-9	1-9	0-9	0-9
	Mean	7.8	5.8	6.4	5.3	5.5	6.2	6.2
College Mathematics Preparation**	Range	0-57	0-21	0-20	0-41	0-10	0-26	0-57
	Mean	29.9	4.6	2.9	6.1	0.9	7.7	9.1
College Credits**	Range	121-228	113-303	113-124	115-228	115-207	126-217	113-303
	Mean	136.9	146.4	126.2	132.2	129.0	148.4	137.7
Grade Point Average	Maximum	3.95	3.74	3.71	3.74	3.78	3.29	3.95
	Minimum	1.77	1.61	1.96	1.91	1.98	1.87	1.61
	Mean	2.55	2.56	2.58	2.44	2.60	2.53	2.53

* These data are in terms of semester credits.

** These data are in terms of quarter hours.

mathematics teachers had achieved, or exceeded, this number. On the basis of 12 quarter hours in college level mathematics, as recommended by the Mathematical Association of America as minimum preparation for elementary school teachers (13, p. 11), it was found that only seven of the 75 prospective elementary mathematics teachers had achieved, or exceeded, this amount. In fact, it was found that 34 of the prospective elementary mathematics teachers had completed no mathematics at the college level.

Credits and Grade-Point-Averages Attained by Prospective Teachers

The number of college credits and the college grade-point-averages attained at the time of the testing were also compiled. The range of college credits for all prospective teachers was from 113 quarter hours to 303 quarter hours with an average of 137. 7 quarter hours. The average college grade-point-average was 2. 55 with a range from 1. 61 to 3. 95. These data are summarized in Table II for all groups involved in the study.

Major and Minor Academic Fields of Prospective Teachers

A compilation was made of the areas of concentrated study of the prospective teachers, particularly those who indicated mathematics as a major or a minor area of emphasis. These data are presented in Table III.

Table III. Major and Minor Areas of Emphasis of Prospective Teachers.

		Groups						
Academic Area		PSM	PEM	PSS	PAI	PE	PS	Totals
Mathematics	Major	45	2	0	2	0	0	49
	Minor	13	5	4	4	0	9	35
English	Major	2	0	0	1	37	0	40
	Minor	1	7	6	3	8	1	26
Biology	Major	1	1	0	0	0	20	22
	Minor	1	0	0	0	0	8	9
Social Science	Major	1	5	44	0	9	0	59
	Minor	6	11	4	5	12	2	40
Art/Ind. Arts.	Major	1	0	0	31	0	0	32
	Minor	1	3	1	6	3	2	16
General Science	Major	4	0	0	0	0	28	32
	Minor	6	4	0	3	0	2	15
Chemistry	Major	0	0	0	0	0	0	0
	Minor	5	1	2	1	0	10	19
Physics	Major	0	0	0	0	0	0	0
	Minor	6	1	0	0	0	2	9
Psychology/ Philosophy	Major	0	0	0	0	0	0	0
	Minor	2	3	7	3	6	2	23
Home Economics	Major	0	0	0	0	0	0	0
	Minor	0	2	1	2	0	1	6
Professional Subjects	Major	3	7	1	7	0	1	19
	Minor	3	67	17	3	4	3	97
Other Areas	Major	0	0	0	0	0	0	0
	Minor	13	22	3	11	13	7	69

It was noted that two prospective elementary mathematics teachers indicated mathematics as a major field, while five indicated mathematics as a minor field of study.

Status of Freshman Mathematics Students

One hundred fifty-seven freshman mathematics students were tested in this study at the beginning of their first quarter of residence at Central Washington State College. The average age of these students was 18.4 years with a range from 17 years to 26 years. None of these students was asked to make a commitment as to his future plans, i.e., whether to enter the teaching profession, or to declare major and minor areas of study.

There were 38 female students and 119 male students included in this group.

Mathematics Preparation of Freshman Mathematics Students

The average number of semester credits completed in mathematics at the high school level was found to be 4.4 with a range from two credits to ten credits. This average was affected by the fact that a minimum of one year of high school mathematics was required for high school graduation in the State of Washington.

Grade-Point-Averages Attained by Freshman Mathematics Students

The grade-point-averages of the freshman mathematics students were obtained from their eighth-semester high school grade reports. The average grade-point-average for the 157 students was 2.67 with a range from 1.74 to 3.87 on a four point scale.

Status of Experienced Teachers

Fifty-five of the 60 experienced elementary and secondary mathematics teachers who participated in the study had completed a bachelor's degree. Three teachers had obtained a master's degree in education. Five teachers did not have a bachelor's degree but completed the requirements for the degree at the end of the 1965 summer session.

Twenty-five teachers were teaching at the secondary level, while 35 were teaching at the elementary level. The average number of years of teaching experience for the entire group of experienced teachers was 5.7 years with a range from one year to 26 years. The average number of years of teaching experience for the group of secondary mathematics teachers was 5.0 years with a range from one year to 23 years, and the average number of years of teaching experience for the elementary teachers was 6.2 years with a range from one year to 26 years. These data are presented in Table IV.

Table IV. Summary of Personal Information of Experienced Teachers.

Criteria		Groups*		Total
		EEM	ESM	
Number of Participants		35	25	60
Sex	Male	12	19	31
	Female	23	6	29
Age	Range	23-53	22-51	22-53
	Mean	34.7	30.9	33.1
Degree Status	None	5	0	5
	B. A.	30	22	52
	M. A.	0	3	3
Level of Teaching	Elementary	35	0	35
	Secondary	0	25	25
Years of Teaching Experience	Maximum	26	23	26
	Minimum	1	1	1
	Mean	6.2	5.0	5.7

* EEM-Experienced elementary mathematics teachers.
ESM-Experienced secondary mathematics teachers.

The average age of the group of experienced teachers was 33. 1 years with a range from 22 years to 53 years. Included in this group were 31 male teachers and 29 female teachers. Twelve of the 35 elementary teachers were male, and 19 of the 25 secondary teachers were male. These data are also presented in Table IV.

Mathematics Preparation of Experienced Teachers

A summary was made of the number of credits in mathematics completed by the experienced teachers involved in the study. The average number of semester hours of mathematics completed at the high school level was 5. 3, while the average number of quarter hours of mathematics completed at the college level was 21. 2.

These averages had little meaning, as bias existed due to the lack of mathematics preparation of the elementary teachers, for example. The averages for the individual groups of experienced teachers had more meaning, however. For high school mathematics, in semester hours, and for college mathematics, in quarter hours, the averages were, respectively, 4. 4 hours and 4. 4 hours for experienced elementary teachers, and 6. 5 hours and 44. 8 hours for the experienced secondary teachers. Fifteen of the 35 experienced elementary teachers had completed no mathematics at the college level. These data are summarized in Table V.

Table V. Mathematics Preparation, College Credits, and Grade-Point-Averages of Experienced Teachers.

Criteria		Groups		Total
		EEM	ESM	
High School Mathematics Preparation*	Range	1-8	2-10	1-10
	Mean	4.4	6.5	5.3
College Mathematics Preparation**	Range	0-25	3-103	0-103
	Mean	4.4	44.8	21.2
College Credits**	Range	0-177	3-169	0-169
	Mean	97.5	103.7	100.1
Grade Point Average	Maximum	4.00	4.00	4.00
	Minimum	1.61	2.00	1.61
	Mean	2.85	3.07	2.94

* These data are in terms of semester credits.

** These data are in terms of quarter hours.

Credits and Grade-Point-Averages Attained by Experienced Teachers

The number of college credits and the college grade-point-averages attained by the experienced teachers was compiled. The range of graduate college credits for all experienced teachers was from three quarter hours to 169 quarter hours with an average of 21.2 quarter hours. The average college grade-point-average was 2.94 with a range from 1.61 to 4.00. These data are summarized in Table V.

Major and Minor Academic Fields of Experienced Teachers

A summary of the areas of concentrated study of the experienced teachers are presented in Table VI. It was interesting to note that one elementary teacher listed mathematics as a major, that only two elementary teachers listed mathematics as a minor, and that two of the secondary mathematics teachers did not list mathematics as either a major or minor field of study. The latter two individuals had 20 hours of mathematics credit between them, one having completed 17 hours and the other having completed three hours.

Summary

This chapter has summarized some of the personal data and background and experience of the participants of the study. These

Table VI. Major and Minor Areas of Emphasis of Experienced Teachers.

Academic Area		Groups		Total
		EEM	ESM	
Mathematics	Major	1	18	19
	Minor	2	5	7
English	Major	4	0	4
	Minor	4	1	5
Biology	Major	1	1	2
	Minor	0	3	3
Social Science	Major	5	1	6
	Minor	4	3	7
Art/Ind. Arts	Major	1	3	4
	Minor	0	0	0
General Science	Major	0	2	2
	Minor	1	4	5
Chemistry	Major	0	0	0
	Minor	0	3	3
Physics	Major	0	0	0
	Minor	0	2	2
Psychology/ Philosophy	Major	0	0	0
	Minor	1	0	1
Home Economics	Major	0	0	0
	Minor	2	0	2
Professional Subjects	Major	23	0	23
	Minor	16	0	16
Other Areas	Major	0	0	0
	Minor	5	4	9

data will be utilized in Chapter V relative to their effect on spatial visualization abilities.

The ages of the experienced teachers were greater than those of the prospective teachers, as would be expected, and the ranges of ages differed considerably.

The mathematics preparation of the various groups of individuals varied a great deal, although grade-point-averages were compatible. Nineteen prospective teachers already had received a bachelor's degree and were completing requirements for a teaching credential, while there were five experienced teachers, with a total of 51 years of teaching experience who were completing the requirements for a bachelor's degree.

It was noted that 34 of the 75 prospective elementary mathematics teachers and 15 of the 35 experienced elementary teachers in the study had completed no college mathematics courses.

The findings from this compilation and summary of personal data, training, and teaching experiences of the participants in the study provided a basis for recommendations offered later.

CHAPTER V

TREATMENT, ANALYSIS, AND INTERPRETATION OF FINDINGS

Treatment of Data

Data for this study were obtained from various groups of prospective teachers, freshman mathematics students, and experienced teachers enrolled at Central Washington State College. Stratification of the samples used in this study was made on the bases of area of teacher preparation and level of college progress. Minimum sample sizes were determined in discussion with the Statistics Department at Oregon State University, and, in each sample selected, this minimum was attained or exceeded.

Each of the groups of prospective teachers, experienced teachers, and freshman mathematics students was given the Differential Aptitude Space Relations Test (DATSR), the Revised Minnesota Paper Form Board Test (MPFB), and the mathematics test of the Sequential Tests of Educational Progress (STEPM), in that order, with the DATSR and MPFB tests given during the same class period, and the STEPM test given during a subsequent class period. This method of administration of the testing instruments was followed each quarter.

As it was felt that spatial visualization ability of the participants in the study could have been related in some manner to both academic ability and scholastic achievement, analysis of covariance procedures were indicated and, hence, were used in analyzing the data obtained and testing the previously-stated hypotheses.

Since scholastic aptitude and academic ability could conceivably influence each individual's response to the criterion measuring instruments, these individual differences were controlled by utilizing the combined scores on the Washington Pre-College Verbal and Quantitative tests as a measure of scholastic aptitude and the college grade-point-averages (eighth-semester high school grade-point-averages in the case of the freshman mathematics students) as a measure of academic ability for each individual in the samples. As a result, possible bias introduced by individual differences was removed insofar as these factors represented the differences in question. This does not preclude the possibility of other factors affecting each individual's response to the criteria being used. Analysis of covariance techniques have been discussed in the literature (70, pp. 343-62).

To test the null hypotheses relative to differences in spatial visualization abilities of various groups of individuals involved in the study, single classification analysis of covariance procedures were used where two groups of individuals were of concern. Single classification analysis of covariance procedures were also used to test for a significant difference in spatial visualization abilities among all groups involved in the study.

Pearson product-moment correlations were also computed for the data for each group of individuals in order to test the null hypotheses relative to the relationship between spatial visualization ability and mathematics achievement and spatial visualization ability and intelligence. Other correlations were also calculated as part of the development and were reported.

The Pearson product-moment correlations provided the basis for significant validity indices and relative validity indices between the different measures and criteria treated.

The reliability coefficient, using the Kuder-Richardson formula defined and discussed in Chapter II, was computed for each of the testing instruments used in this study.

Considerations in the Analysis of Data

The following were major aspects anticipated in analyzing the findings of this study.

1. Levels of significance for the "r" validity indices between criteria were calculated on the basis of the t-test, for the 0.05 and 0.01 levels of significance. Using the formula suggested by Guilford (24, p. 219),

$$t = r\sqrt{\frac{N - 2}{1 - r^2}}$$

the investigator computed the lower limits for "r", which would indicate significance levels of 0.05 and 0.01 for the various N involved in the study.

2. The reliability indices obtained and other validity indices below the 0.05 level of significance, will be discussed and compared on a purely relative basis.
3. Results significant at the 0.05 and 0.01 levels of confidence in the analysis of covariance computations will be indicated as they are reported.

DATSR, MPFB, and STEPM Results

The data collected with regard to two spatial visualization tests and the mathematics achievement test given to each participant in the study are presented in Tables VII and VIII. These data indicate a considerable range in attained scores in each group of participants. Table VII indicates that the prospective secondary mathematics teacher group attained the highest average score on the DATSR and STEPM tests and had the second highest average on the MPFB test, followed by the prospective science teachers group or the prospective art/industrial arts teacher group.

Table VIII shows that the freshman mathematics group attained a higher average score on the DATSR than either of the experienced teachers groups and attained an average score on the MPFB test just slightly lower than the experienced secondary mathematics teachers

Table VII. Maximums, Minimums, Means, and Standard Deviations of Spatial Visualization and Mathematics Achievement Scores of Prospective Teachers.

Criteria		Groups*						
		PSM	PEM	PSS	PAI	PE	PS	All Groups
Sample Size		57	75	45	41	46	49	313
DATSR	Maximum	95	80	77	95	72	96	96
	Minimum	12	5	14	13	12	5	5
	Mean	59.18	47.04	44.73	52.34	45.41	55.47	50.69
	STDEV**	15.79	14.75	14.40	17.27	13.29	19.69	16.69
MPFB	Maximum	57	57	57	60	53	62	62
	Minimum	20	18	9	23	21	1	1
	Mean	43.70	40.84	39.53	43.63	39.22	43.84	41.77
	STDEV**	7.23	8.27	9.21	8.20	6.65	10.24	8.51
STEPM	Maximum	316	307	312	312	305	315	316
	Minimum	270	230	262	232	252	232	230
	Mean	301.38	284.39	288.84	289.46	287.07	284.31	289.17
	STDEV**	9.54	14.71	12.61	16.87	11.66	20.18	15.69

* PSM-Prospective Secondary Mathematics Teachers.
PEM-Prospective Elementary Mathematics Teachers.
PSS-Prospective Social Science Teachers.
PAI-Prospective Art/Industrial Arts Teachers.
PE-Prospective English Teachers.
PS-Prospective Science Teachers.
**Standard Deviation.

Table VIII. Maximums, Minimums, Means, and Standard Deviations of Spatial Visualization and Mathematics Achievement Scores of Freshman Mathematics Students and Experienced Teachers.

Criteria		Groups*			
		FM	EEM	ESM	EEM and ESM
Sample Size		157	35	25	60
DATSR	Maximum	96	70	84	84
	Minimum	20	5	27	5
	Mean	59.89	41.17	58.60	48.43
	STDEV**	15.69	15.42	14.25	17.17
MPFB	Maximum	61	57	58	58
	Minimum	28	10	33	10
	Mean	44.28	38.86	46.76	42.15
	STDEV**	7.00	8.82	7.06	8.97
STEPM	Maximum	312	302	315	315
	Minimum	230	265	274	265
	Mean	286.74	282.17	300.08	289.63
	STDEV**	17.17	12.56	9.36	14.35

* FM-First-Quarter Freshman Mathematics Students.
EEM-Experienced Elementary Mathematics Teachers.
ESM-Experienced Secondary Mathematics Teachers.

**Standard Deviation.

group. The latter group achieved an average STEPM score somewhat higher than either the freshman mathematics group or the experienced elementary teachers group.

Comparison of Tables VII and VIII indicates that the prospective secondary mathematics teachers group, the freshman mathematics students group, and the experienced secondary mathematics teachers group attained approximately equivalent average scores on the DATSR, while the latter group had a higher average MPFB score than the other two mentioned groups, which were almost the same on this test. Both the experienced and prospective secondary mathematics teachers groups achieved a higher average STEPM score.

No DATSR or MPFB norms were available for comparing the scores of the various groups at the college level, but norms for college students were available for comparing STEPM scores (18, p. 23). The average STEPM results in Table VII for prospective teachers and in Table VIII for freshman mathematics students compared with these norms indicated that the mean STEPM score for each group exceeded the mean score given by the publishers of the test. Data from 202 freshman students from 99 colleges were furnished by the publishers, as were data from 200 sophomore students from 97 colleges. Because of the limited number of students involved from each college, these comparisons may not have too much value.

Analysis of Covariance Results

Analysis of covariance with a single classification was made first with all groups included in the computations; then with all prospective and experienced teachers groups; and then with only the prospective teacher groups included in the computations. WPCVM scores were utilized to control scholastic aptitude and cumulative grade-point-averages were utilized to control academic ability in each case. The first three rows of Table IX present these results for each of the criteria involved. These data indicate, to the degree that scholastic aptitude is controlled by the WPCVM, and to the degree that grade-point-average is a measure of control of academic ability, and to the degree that all other pertinent factors related to spatial visualization ability and mathematics achievement have not introduced a bias in this study, there is little doubt that the spatial visualization abilities of the groups of individuals involved in the study are different. This is, also, true relative to the mathematics achievement of these groups. Presumably these differences can be attributed to the type of educational training and/or experience the various groups have encountered. Insufficient evidence has been found from these data to indicate that the difference between various combinations of two or more means is significant.

As a consequence of these data, analysis of covariance was computed based on mean scores for pairs of various groups involved in the study. Table IX contains analysis of covariance results for several groups of individuals, other than the original grouping which was established for the testing procedures. These results indicate that, subject to the control factors, female prospective teachers and male prospective teachers differ significantly in their spatial visualization abilities and their mathematics achievement. There were 175 males and 138 females in these two groups. Similar results are indicated for the two experienced teachers groups, ESM and EEM.

Prospective secondary teachers and experienced secondary teachers were found to differ significantly at the 0.05 level in mathematics achievement, but did not differ significantly in spatial visualization abilities, while prospective elementary teachers and experienced elementary teachers were found to differ significantly only in spatial visualization ability, as measured by DATSR. This difference in mathematics achievement between the secondary teachers might be due to the little mathematics background of the prospective English and social science teachers included in this group of prospective secondary teachers. The significant difference in spatial visualization ability of the elementary teachers is not so easily explained.

A significant difference in spatial visualization abilities and mathematics achievement is also indicated in Table IX between the prospective elementary teachers and the prospective secondary teachers. These results follow the trend indicated in Table IX

Table IX. Analysis of Covariance of Spatial Visualization and Mathematics Achievement Scores of Various Groups of Individuals Involved in the Study.

	F-Ratios		
Comparison Groups	DATSR	MPFB	STEPM
All Groups Involved in the Study@	10. 8887**	5. 0785**	9. 3418**
All Prospective Teachers (PSM, PEM, PSS, PAI, PE, PS)	6. 3341**	3. 7704**	9. 6550**
All Prospective and Experienced Teachers (PSM, PEM, PSS, PAI, PE, PS, EEM, ESM)@	8. 5921**	4. 5371**	11. 3405**
Prospective Elementary Teachers and Prospective Secondary Teachers	9. 0710**	4. 3417*	6. 7012*
Prospective Elementary Teachers and Experienced Elementary Teachers@	5. 8789*	2. 9324	3. 1701
Prospective Secondary Teachers and Experienced Secondary Teachers@	1. 2729	2. 0769	4. 6844*
Experienced Elementary Mathematics Teachers (EEM) and Experienced Secondary Mathematics Teachers (ESM)@	16. 4967**	11. 4450**	32. 0680**
Female Prospective Teachers and Male Prospective Teachers@	16. 5075**	4. 8180*	21. 2061**

* Significant at 0. 05 level of significance.

** Significant at 0. 01 level of significance.

@ These values were computed with one control variable, while all others were computed with two control variables.

comparing these same criteria for experienced elementary and experienced secondary teachers.

Combining the results on the DATSR, MPFB, and STEPM tests for the prospective elementary mathematics teachers and the prospective secondary mathematics teachers and computing analysis of covariance for this combined group and each of the remaining groups involved in the study resulted in the data reported in Table X. These data indicate that significant differences in spatial visualization abilities exist between prospective elementary and secondary mathematics teachers and the groups of prospective social science teachers, prospective English teachers, freshman mathematics students, and experienced elementary mathematics teachers, as measured by either DATSR or MPFB.

Mathematics achievement did not differ significantly between this group of elementary and secondary mathematics teachers and the prospective social science teachers or the group of prospective art/industrial arts teachers, but a significant difference was indicated in all other instances.

Table X also shows that the combined group of prospective elementary and secondary mathematics teachers differ significantly in spatial visualization ability, as measured by DATSR, when compared with the group of all experienced teachers involved in the study. No significant difference was indicated in spatial visualization ability,

Table X. Analysis of Covariance of Spatial Visualization and Mathematics Achievement Scores of All Prospective Mathematics Teachers.

	F-Ratios		
Comparison Groups	DATSR	MPFB	STEPM
Prospective Social Science Teachers (PSS)	9.2724**	4.2176*	1.5038
Prospective Art/Industrial Arts Teachers (PAI)	0.0784	2.4913	0.4532
Prospective English Teachers (PE)	9.5158**	7.0156**	4.6021*
Prospective Science Teachers (PS)	1.2768	1.8149	7.3355**
Freshman Mathematics Students (FM)	11.8449**	3.2180	8.5484**
Experienced Elementary Mathematics Teachers (EEM) @	17.7215**	9.2935**	12.6862**
Experienced Secondary Mathematics Teachers (ESM) @	1.5706	0.9758	4.8195*
All Experienced Teachers (EEM, ESM) @	5.7601*	2.2850	2.1034

* Significant at 0.05 level of significance.

** Significant at 0.01 level of significance.

@ These values were computed using one control variable, while all other values were computed using two control variables.

as measured by MPFB, or in mathematics achievement between these two groups.

In order to investigate the differences in spatial visualization abilities and mathematics achievement between groups involved in the study, analysis of covariance was made comparing the mean scores on the DATSR, MPFB, and STEPM tests for prospective elementary teachers with those mean scores of the other groups involved in the study. WPCVM and cumulative grade-point-averages were used as control variables in all cases, except those involving the experienced teachers groups whose WPCVM scores were not available. These data are reported in Table XI.

These data indicate that prospective elementary mathematics teachers differ significantly in spatial visualization abilities when compared with prospective secondary mathematics teachers, prospective science teachers, freshman mathematics students, and both groups of experienced mathematics teachers involved in the study as measured by DATSR. They differ significantly in this ability from the prospective secondary mathematics teachers, the freshman mathematics students, and the group of experienced elementary mathematics teachers, as measured by the MPFB test. It was also found that prospective elementary mathematics teachers differ significantly in mathematics achievement, as measured by STEPM, with only the two groups of secondary mathematics teachers.

Table XI. Analysis of Covariance of Spatial Visualization and Mathematics Achievement Scores of Prospective Elementary Mathematics Teachers.

	F-Ratios		
Comparison Groups	DATSR	MPFB	STEPM
Prospective Secondary Mathematics Teachers (PSM)	10.2372**	3.9574*	47.1983**
Prospective Social Science Teachers (PSS)	1.6134	1.4453	2.4166
Prospective Art/Industrial Arts Teachers (PAI)	2.5617	3.8558	3.2796
Prospective English Teachers (PE)	1.1348	2.2907	1.0923
Prospective Science Teachers (PS)	5.2044*	2.6138	0.0039
Freshman Mathematics Students (FM)	24.9241**	6.1705*	0.7557
Experienced Elementary Mathematics Teachers (EEM)@	6.8997*	3.8889*	1.2368
Experienced Secondary Mathematics Teachers (ESM)@	7.6805**	2.0893	16.9084**

* Significant at 0.05 level of significance.

** Significant at 0.01 level of significance.

@ These values were computed using one control variable, while all other values were computed using two control variables.

Data for analysis of covariance for the mean scores on DATSR, MPFB, and STEPM tests for prospective secondary mathematics teachers and other groups involved in the study are presented in Table XII. These data indicate that the prospective secondary mathematics teachers differed significantly in spatial visualization abilities, when compared with the prospective elementary mathematics teachers, the prospective social science teachers, the prospective English teachers, and the group of experienced elementary teachers, as measured by both DATSR and MPFB. Thus, prospective secondary mathematics teachers differed significantly in spatial visualization ability with four of the eight comparison groups.

Table XII also indicates a highly significant difference in mathematics achievement between prospective secondary mathematics teachers and each of the other groups involved in the study, with the exception of the group of experienced secondary mathematics teachers, where no significant difference was apparent as based on STEPM results.

Coefficients of Correlation

Using the formula suggested by Guilford (24, p. 219) the investigator computed the minimum values of "r", which would indicate significance levels of 0.05 and 0.01 for the various sample sizes involved in the study. These computed "r" values are indicated in

Table XII. Analysis of Covariance of Spatial Visualization and Mathematics Achievement Scores of Prospective Secondary Mathematics Teachers.

	F-Ratios		
Comparison Groups	DATSR	MPFB	STEPM
Prospective Elementary Mathematics Teachers (PEM)	10.2372**	3.9574*	47.1983**
Prospective Social Science Teachers (PSS)	17.7608**	5.7880*	26.8563**
Prospective Art/Industrial Arts Teachers (PAI)	1.2714	0.1654	15.0826**
Prospective English Teachers (PE)	20.2226**	10.9475**	45.4982**
Prospective Science Teachers (PS)	0.1430	0.2429	27.4340**
Freshman Mathematics Students (FM)	0.3703	0.1860	35.4098**
Experienced Elementary Mathematics Teachers (EEM)@	37.7939**	11.9349**	69.7973**
Experienced Secondary Mathematics Teachers (ESM)@	0.2850	0.0898	0.6315

* Significant at 0.05 level of significance.

** Significant at 0.01 level of significance.

@ These values were computed using one control variable, while all other values were computed using two control variables.

Table XIII.

Results of computations of Pearson product-moment coefficients of correlation between scores on the DATSR, MPFB, and STEPM tests and data from WPCVM and mathematics preparation are presented in Tables XIV and XV. WPCVM data were not available for the groups of experienced teachers, as the WPC tests were not administered on a state-wide basis to students until January 1960, after most of these individuals had finished high school.

The spatial visualization tests, DATSR and MPFB, did not correlate very highly with WPCVM for any of the groups, with the range from .00 to .39. DATSR and MPFB correlations with mathematics preparation varied considerably among each group but were quite low, .15 to .11, for the entire group of 530 participants in the study. The coefficients of correlation between the DATSR and MPFB results ranged from .40 to .80 for the various groups, with a coefficient of .57 for the entire group.

STEPM, as indicated in Table XV, did not correlate well with WPCVM in any of the groups, with the largest coefficient, .21, indicated for the group of prospective social science teachers. STEPM correlated significantly with the mathematics preparation of the PEM, PSS, PAI, and PS groups but quite low in all the other groups. The STEPM-Mathematics Background coefficient of correlation for the entire group was .35. This coefficient of correlation for the group

Table XIII. Computed Minimum Significance Levels of Correlations. *

N	Minimum Significant Values of R	
	0.05 Level	0.01 Level
25	0.396	0.505
35	0.334	0.430
41	0.308	0.398
45	0.294	0.380
46	0.291	0.376
49	0.281	0.364
57	0.260	0.339
75	0.227	0.296
157	0.155	0.205

*See Guilford (24, p. 219)

Table XIV. Coefficients of Correlation between Spatial Visualization Scores and Other Criteria for Groups Involved in Study.

Criteria	Groups									All Groups
	PSM	PEM	PSS	PAI	PE	PS	FM	EEM	ESM	
WPCVM-DATSR	.39**	.21	.21	.24	.09	.20	.10	@	@	.21**
WPCVM-MPFB	.26*	.25*	.10	.00	-.10	.18	.06	@	@	.09*
Mth Prep-DATSR	-.13	.20	.41**	.39*	-.12	.40**	.05	.21	.00	.15**
Mth Prep-MPFB	-.28**	.12	.36*	.13	.10	.25	.07	.22	.09	.11**
DATSR-MPFB	.51**	.61**	.71**	.43**	.52**	.51**	.40**	.70**	.80**	.57**
DATSR-STEPM	-.34**	-.29*	-.55**	-.63**	-.61**	-.39**	-.68**	-.70**	-.69**	-.18**
MPFB-STEPM	-.18	-.25*	-.47**	-.32*	-.31*	-.29*	-.31**	-.59**	-.52**	-.06

@ These data were not available.
* Significant at 0.05 level of significance.
** Significant at 0.01 level of significance.

Table XV. Coefficients of Correlation Between Mathematics Achievement Scores and Other Criteria for Groups Involved in Study.

	Groups									
Criteria	PSM	PEM	PSS	PAI	PE	PS	FM	EEM	ESM	All Groups
WPCVM-STEPM	.20	-.03	.21	.11	.04	.11	.05	@	@	.04
Mth Prep-STEPM	.00	.31**	.53**	.39*	.18	.46**	.06	.16	.17	.35**
DATSR-STEPM	-.34**	-.29*	-.55**	-.63**	-.61**	-.39**	-.68**	-.70**	-.69**	-.18**
MPFB-STEPM	-.18	-.25*	-.47**	-.32*	-.31*	-.29*	-.31**	-.59**	-.52**	-.06

@These data were not available.
*Significant at 0.05 level of significance.
**Significant at 0.01 level of significance.

of prospective secondary mathematics teachers was found to be .001, while for the experienced secondary mathematics teachers, this coefficient of correlation was found to be only .17.

STEPM coefficients of correlation with the spatial visualization tests, DATSR and MPFB, ranged from a negative .18 (STEPM-MPFB for the PSM group) to a negative .70 (STEPM-DATSR for the EEM group). For the entire group of participants the STEPM-DATSR coefficient of correlation was negative .18 and the STEPM-MPFB coefficient of correlation was negative .06.

Coefficients of Reliability

Reliability coefficients were computed for each test, separately for each group of individuals involved in the study. These coefficients are presented in Table XVI. This table indicates that the goal of adequate test reliability has, in general, been achieved for each group involved in the study.

The DATSR test had higher coefficients of reliability for every group than either the MPFB or the STEPM tests. The STEPM coefficients of reliability compared favorably with the coefficients of reliability reported by North (48, pp. 47-56). He reported Spearman-Brown reliability coefficients from .42 to .87 for STEPM given to college freshman and sophomores.

It may be noted that the relative order of reliability for the tests,

Table XVI. Coefficients of Reliability for Spatial Visualization and Mathematics Achievement Tests for Groups Involved in Study. *

	Groups									
Criteria	PSM	PEM	PSS	PAI	PE	PS	FM	EEM	ESM	All Groups
DATSR	.91	.89	.89	.93	.87	.95	.91	.91	.89	.92
MPFB	.75	.80	.83	.81	.76	.88	.73	.82	.76	.80
STEPM	.78	.86	.81	.90	.77	.92	.89	.78	.76	.99

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*Computed by Kuder-Richardson Formula.

as applied to all the groups with the exception of PSS and EEM, was the same, namely, DATSR, STEPM, MPFB. For the PSS and EEM groups the relative order was DATSR, MPFB, STEPM. All coefficients of reliability were greater than, or equal to, .73.

Summary

Data have been tabulated and reported in this chapter relative to ranges, means, and standard deviations of scores attained by each group of individuals on the DATSR, MPFB, and STEPM tests. Analysis of covariance results were presented relative to all groups, several specialized groups, and the groups of prospective elementary mathematics teachers and the prospective secondary mathematics teachers when compared, with controls for academic ability and scholastic aptitude, with each of the other groups of individuals involved in the study. Significant results were duly noted and discussed.

Coefficients of correlation between scores attained on DATSR, MPFB, and STEPM tests, WPCVM data, and mathematics preparation information were tabulated and discussed. The DATSR, MPFB, and STEPM test data appeared to correlate quite well among themselves, but did not correlate well with other criteria, except in isolated instances.

Coefficients of reliability for each of the DATSR, MPFB, and STEPM tests were presented for each group of individuals. These coefficients were all greater than .70, which would indicate a fairly good degree of reliability.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

Expressed in terms of the original null hypotheses formulated in Chapter II, a summary of this study is:

1. Prospective secondary mathematics teachers at Central Washington State College have a significantly different degree of development of spatial visualization abilities than do prospective elementary mathematics teachers at the same institution. It would appear, based on the means of the collected data, that the former group has a higher degree of development of these abilities. Consequently, null hypothesis one is rejected.

2. The spatial visualization abilities of prospective elementary mathematics teachers at Central Washington State College are significantly different from the spatial visualization abilities of experienced elementary mathematics teachers. As a consequence, null hypothesis two is rejected.

3. The spatial visualization abilities of prospective elementary mathematics teachers at Central Washington State College and experienced secondary mathematics teachers are significantly different. Therefore, null hypothesis three is rejected.

4. Spatial visualization abilities of prospective secondary mathematics teachers at Central Washington State College and experienced elementary mathematics teachers are significantly different. Consequently, null hypothesis four is rejected.

5. The spatial visualization abilities of prospective secondary mathematics teachers at Central Washington State College and experienced secondary mathematics teachers are not significantly different. Therefore, null hypothesis five is accepted.

6. The spatial visualization abilities of prospective elementary mathematics teachers at Central Washington State College are not significantly different from similar abilities of prospective teachers in other academic fields of endeavor at Central Washington State College, except in the field of science. Hence, null hypothesis six is accepted.

7. Prospective secondary mathematics teachers' spatial visualization abilities are significantly different from the spatial visualization abilities of both prospective teachers of social sciences and prospective teachers of English at Central Washington State College, but are not significantly different from the spatial visualization abilities of prospective teachers of science, or prospective teachers of art/industrial arts. Therefore, null hypothesis seven is accepted.

8. The spatial visualization abilities of first-quarter freshman mathematics students at Central Washington State College are

significantly different from similar abilities of prospective elementary mathematics teachers at the same institution, but these same abilities of freshman mathematics students are not significantly different from those of prospective secondary mathematics teachers at Central Washington State College. On this basis, null hypothesis eight is accepted.

9. There is a significant relationship between achievement in mathematics and spatial visualization abilities of prospective teachers in several academic fields, freshman mathematics students, and experienced elementary and secondary mathematics teachers. It would appear, based on correlation data, that mathematics achievement can be used, at significant levels of confidence, as a valid index of spatial visualization ability. As a result, null hypothesis nine is rejected.

10. There is no significant relationship between scholastic ability and spatial visualization abilities of prospective teachers in several academic fields and freshman mathematics students at Central Washington State College. Therefore, null hypothesis ten is accepted at significant levels of confidence.

Conclusions

This study has indicated, to the degree that the limitations, assumptions, and control factors of the study have been accepted and used, that there is little doubt that Central Washington State College prospective elementary and secondary school teachers with mathematics preparation have developed a higher degree of spatial visualization ability than those prospective elementary and secondary school teachers who have had little or no mathematics preparation. No significant difference in spatial visualization abilities was indicated between groups of prospective teachers in fields where several courses in mathematics were required, or strongly recommended, such as science or art/industrial arts, and the groups of prospective elementary and secondary mathematics teachers.

It has also been shown that, at Central Washington State College, students preparing to teach mathematics at the secondary level, and, hence, following the prescribed mathematics curriculum for certification and graduation in the field of education, have developed, in addition to a greater degree of mathematics achievement, a higher degree of spatial visualization ability than have the prospective elementary mathematics teachers at the same institution of higher learning.

The results of this study have also indicated that there is, in general, no relationship between scholastic aptitude and spatial visualization ability, at least for the individuals involved in the study at Central Washington State College. This seemed to verify what Super (62, p. 283-4), Thorndike (63, p. 20), and Thurstone (66,p. 576) found relative to the relationship between scholastic aptitude and spatial visualization ability.

There appears to be a difference in the degree of spatial visualization abilities of the prospective elementary school teachers and the experienced elementary school teachers. This might be attributed to the fact that there have been the four elective mathematics courses available for the former group of individuals at Central Washington State College, while there were no such specific mathematics courses available when the experienced teachers were undergraduates.

Implications

With respect to the prescribed mathematics curricula for prospective elementary and/or secondary mathematics teachers at Central Washington State College, the following may be considered:

1. In order to develop spatial visualization abilities, mathematics content courses for the elementary school teacher should be required in the elementary curriculum for teacher certification.

2. The prescribed mathematics curriculum for prospective secondary school mathematics teachers appears to develop the spatial visualization ability, and any changes in said curriculum should be considered in this light.

Recommendations for Further Study

Investigations suggested by this study include:

1. Consideration of a similar study with prospective teachers from all colleges and universities in the State of Washington.

2. A follow-up study relative to spatial visualization abilities of the prospective elementary and secondary mathematics teachers involved in the present study after several years of teaching experience to investigate gain or loss in this ability.

3. Consideration of a similar study with various prospective and experienced teacher groups in future years at Central Washington State College to add further information to the present study. It is also suggested that, if this same study were to be repeated, that better spatial visualization tests might be available, or devised, and that other control factors might be considered in the analysis of covariance computations.

4. A yearly follow-up study of the spatial visualization abilities of the individuals, who were included in the freshman mathematics students' group in the present study, in order to relate their gain or loss in spatial visualization ability to their mathematics and/or college preparation, maturation, and other factors which might affect development of this ability.

5. A further study relative to the relationship between spatial visualization ability and scholastic aptitude among students enrolled at Central Washington State College.

6. A study designed to measure the ability of prospective and experienced teachers of mathematics to draw three-dimensional objects adequately and meaningfully. This ability would seem to be important in mathematics teaching, in addition to other fields of endeavor.

7. Consider a similar spatial visualization study with mathematics students at the elementary and/or secondary school levels. The teachers could also be included in such a study, where an attempt might be made to relate the students' spatial visualization development to the teachers' success in teaching, their spatial visualization abilities, and other criteria which might seem of importance.

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APPENDIX

APPENDIX
PERSONAL DATA QUESTIONNAIRE

This part for examiner's use only.

WPCV________________
WPCM________________
WPCVM_______________
STEPM_______________
DATSR_______________
MPFB________________
GPA_________________
TCRED_______________

(Do not write above this line)

Personal Data Sheet

Instructions: The purpose of the information requested below is to obtain personal data about the individuals involved in this study. Please complete all the items which are appropriate for you.

1. NAME____
Last First Middle Initial

2. AGE:__________

3. SEX (Check one):
______Male ______Female

4. YEAR OF HIGH SCHOOL GRADUATION:____________

5. CLASS (Check one):
______Freshman
______Sophomore
______Junior
______Senior
______Graduate
______Other (specify)______________

6. TEACHING MAJOR (OR FIRST MINOR) (Check one):
______Mathematics
______English
______Industrial Arts, Fine Arts
______Biology
______General Science
______Social Science
______Elementary
______Other (specify)______________

7. TEACHING MINOR (Check appropriate subjects):

____Art
____Biology
____Botany
____Chemistry
____English
____Foreign Languages
____General Science
____Home Economics
____Industrial Arts
____Mathematics
____Music
____Physical Education/Health
____Physical Science
____Physics
____Professional Subjects
____Science Education
____Social Sciences
____Zoology
____Other (specify)____________

8. TEACHING EXPERIENCE: (Complete either Part A or Part B, but not both):

A. Prospective Teachers Only:

Student Teaching Completed: ___Yes ___No

B. Experienced Teachers Only:

Years of Teaching Experience: ______________

9. LEVEL OF LATEST TEACHING EXPERIENCE, OR TEACHING PREPARATION (Check one):

______Elementary	______Secondary	______College
(Grades 1-6)	(Grades 7-12)	(Grades 13-?)

10. HIGH SCHOOL MATHEMATICS BACKGROUND (Encircle the number of semesters completed where applicable):

1	2	3	4	General Mathematics
1	2	3	4	Algebra I
1	2	3	4	Geometry (including Solid Geometry)
1	2	3	4	Trigonometry
1	2	3	4	4th year mathematics (College Algebra, Analytic Geometry, Math Analysis, Calculus, Statistics, Matrix Algebra, et cetera.)

11. COLLEGE MATHEMATICS BACKGROUND (quarter hours):____

12. DEGREE STATUS (Check highest degree attained):

______None
______BA, BS
______MA, MS, MEd
______PhD, DEd
______Other (specify)______________