Semidiurnal Internal Tide in JASIN: Observations and Simulation

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Temperature was observed in the upper 80 m by moored thermistor chains at three locations in Rockall Channel west of Scotland. Isotherms were interpolated, and a 1-week period of exceptionally energetic tidal oscillations was analyzed. The moored array (horizontal separations ranging from 6 to 20 km) was used as an antenna to determine the dominant horizontal wavelength and direction of propagation of the internal tide within the array. Rockall Bank, 100 km to the southeast, was identified as the source of the internal tide. The semidiurnal internal tide generated by the interaction of the surface tide with Rockall Bank was simulated by use of a model due to Prinsenberg and Rattray. The model predicts generation of the internal tide at the shelf break and propagation seaward as energetic beams which lie along internal wave characteristics. Some energy is trapped near the surface in association with the pycnocline. There is substantial structure in the velocity and vertical displacement fields. Most of the total energy of the internal tide is in the first vertical mode. However, at particular depths, vertical modes as high as 4 (horizontal wavelengths of 25 km) dominate. The high degree of spatial variability in the modeled internal tide illustrates the potential for error when basing a description of the tide on sparse observations. There is good agreement between the modeled vertical displacements and the 1-week period of energetic oscillations of isotherm depth observed at the moorings. In addition, there is good agreement between the model and the tidal velocity variance measured during the same week at depths ranging from 10 to 1000 m and distances ranging from 50 to 130 km from Rockall Bank. Richardson numbers associated with the vertical shear of the modeled internal tide range down to values less than 2.

1. INTRODUCTION

It is now generally accepted that the internal tide is generated by the interaction of the surface tide with bottom topography [e.g., Hendershott, 1981]. However, observations of the internal tide in deep water have seldom been unequivocally identified with generation at a particular topographic feature. Regal and Wunsch [1973] found that the internal tide south of Cape Cod was intensified and coherent near the surface. They attributed the intensification to generation on the continental shelf about 60 km away and subsequent propagation of energy along internal wave characteristics which approached the surface at the point of observation. Hendry [1977] analyzed moored measurements of temperature and velocity in the western North Atlantic and concluded that the Blake Escarpment 700 km away was a major generation area for the observed internal tide. There have also been measurements on the continental slope [Torgrimson and Hickey, 1979] which demonstrate generation on the slope and propagation along characteristics.

Theoretical models of the generation of the internal tide over topography have grown in sophistication over the past 2½ decades. The earliest models describe generation in a two-layer fluid [e.g., Rattray, 1960]. The limitations of a two-layer fluid were overcome by Rattray et al. [1969], who investigated generation on a step continental shelf in an ocean having constant buoyancy frequency N. This work was continued by Prinsenberg et al. [1974] for the case of a sloping continental shelf and further extended for variable N by Prinsenberg and Rattray [1975]. Baines [1973, 1974] has developed a model for generation of the internal tide in an ocean with nearly arbitrary bottom topography and density stratification. The model predicts strong generation of the internal tide at the shelf break where the bottom slope is greater than or equal to the slope of the local internal wave characteristic. The internal tide generated at the shelf break propagates seaward along characteristics as energetic beams. Comprehensive reviews of the theories and observations of the internal tide have been given by Wunsch [1975] and Hendershott [1981].

In this paper we analyze observations of the internal tide and compare them to a model. The analysis is concentrated on a 1-week period of exceptionally energetic oscillations observed west of Scotland during the Joint Air-Sea Interaction (JASIN) experiment. The topographic source of the observed internal tide is identified, and the generation and propagation from this source are simulated with a model due to Prinsenberg and Rattray [1975]. The model has a step shelf and depth-dependent buoyancy frequency. Predicted vertical displacements and tidal velocity variances are compared with observations. The spatial variability of the internal tide is described, and its contribution to shear-induced mixing is evaluated.

2. OBSERVATIONS

As part of the JASIN experiment [Pollard et al., 1983], observations of temperature were taken from late July to early September 1978 by use of thermistor chains moored in Rockall Channel about 300 km west of Scotland. Thermistor chains, manufactured by Aanderaa, were attached to surface moorings at four locations, B1, B2, B3, and B4, shown in Figure 1. Moorings B1, B2, and B3 were part of the Fixed Intensive Array (FIA), a cluster of many moorings within a 6-km square (Figure 1). Measurements at B3 were unreliable and have been excluded. Thermistors were distributed throughout the upper 81.5 m of the water column at each mooring and sampled at 10-min intervals. The observations analyzed in this paper come primarily from thermistors 5 m apart at depths ranging from 36.5 to 81.5 m. This range of depths encompasses the maximum in the buoyancy frequency (Figure 2). Isotherms were linearly interpolated between adjacent temperature measurements.

Temperature observations were also taken during JASIN with a towed thermistor chain [Baumann et al., 1980]. The chain was towed at a speed of 3 m/s around a 15-km square which contained the FIA moorings (Figure 1). The chain had thermistors installed at 2-m intervals from approximately 20
to 70 m depth. Pressure was measured at three locations on the chain. The temperature observations were averaged over sequential 30-s intervals to filter the effects of ship motion. Isotherm depths were linearly interpolated.

The time series of isotherm depths from the B moorings were divided into segments of 1024 points (nearly 1 week; see Table 1), and spectra were computed. Spectra of isotherm displacement at a mean depth of 50 m are shown in Figure 3. The spectral level at mooring B4 is higher on average than the level at B1 or B2, but levels are similar at the frequency of the semidiurnal tide (0.0805 cph). The tidal peak is the most prominent feature in the spectra and is larger than the spectral estimate at the inertial frequency (0.0714–0.0716 cph) by a factor of 4 or more at all three moorings. At frequencies higher than the semidiurnal tide the spectra decrease with a slope of about $\omega^{-1}$ except for a small peak at twice the semidiurnal frequency. A general discussion of internal waves observed during JASIN is given by Levine et al. [1983].

The spectral levels and coherences in the semidiurnal tidal band calculated over nearly week-long segments during the experiment are shown in Figure 4. For each segment, three raw spectral estimates were averaged in the frequency band extending from 0.0732 to 0.0908 cph. The temporal variation of variance in the tidal band (Figure 4a) was similar at moorings B1 and B2. The variance at all moorings was the greatest during segment 4. High horizontal coherence was observed between moorings B1 and B2 during segment 4 at all depths, and coherence decreased as the variance fell during segment 5 (Figure 4b). The maximum variance at mooring B4 was larger than that at the other two moorings, but coherence at the larger horizontal separations (B1-B4 and B2-B4) fell slightly during segment 4. The high variance in the tidal band during segment 4 was also accompanied by very high vertical coherence for all vertical separations at all three moorings. Figure 4c illustrates this for mooring B2. Note that the low coherence during segment 3 is not necessarily representative of moorings B1 and B4 but that the high coherences during segment 4 occurred to the same extent at all three moorings.

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**TABLE 1.** Time Periods Used for Spectral Analysis

<table>
<thead>
<tr>
<th>Segment</th>
<th>Start Date</th>
<th>Start Time, UT</th>
<th>End Date</th>
<th>End Time, UT</th>
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<td>Aug. 1</td>
<td>1220</td>
<td>Aug. 8</td>
<td>1450</td>
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<td>Aug. 29</td>
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<td>Sept. 6</td>
<td>0130</td>
</tr>
</tbody>
</table>

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**Fig. 3.** Isotherm displacement spectra for isotherms with a mean depth of 50 m. The ordinate scale applies to the spectrum from mooring B2. Spectra from moorings B1 and B4 are offset by one decade upward and downward, respectively. The spectra are ensemble averaged over four or five segments. No band averaging was performed at frequencies below 0.115 cph; smoothing was performed above 0.115 cph by band averaging into 15 nonoverlapping bands per decade. Light lines are 95% confidence intervals.

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**Fig. 1.** Location of the B moorings in relation to topography and other moorings. The cluster of moorings centered on 59°N, 12°30'W (see inset) is called the Fixed Intensive Array (FIA). The dotted line in the inset is the track of the towed thermistor chain. Bottom slope along the dashed lines labeled A, B, and C is shown in Figure 8.

**Fig. 2.** Mean buoyancy frequency profile. The profile is a smoothed representation of a profile calculated by Pennington and Briscoe [1979] from conductivity, temperature, and depth casts during the experiment.
In the section that follows we analyze segment 4, the 1-week period during which the tidal variance observed at the B moorings was exceptionally high.

3. DIRECTIONAL WAVE SPECTRA

The horizontal array of moorings, B1, B2, and B4 (Figure 1), was used as an antenna to determine the dominant horizontal wavelength and direction of propagation of the internal tide during segment 4. The technique used to estimate the wave number spectrum is called the beam-forming method [Capon, 1969]. This method is easy to implement, and the resolution in wave number space is as good as in more sophisticated methods, provided the observed wavelengths are of the order of the largest sensor separation [Davis and Regier, 1977]. The applicability of the beam-forming method to the scales encountered in this study is demonstrated by examination of Figure 5. Time series of 2-hour moving averages of isotherm depth at about 50 m calculated once an hour during segment 4 are shown for all three moorings. The internal tide at mooring B4 is about 180° out of phase with observations at mooring B1, and the phase difference between observations at moorings B1 and B2 is always small (Table 2). These phase differences suggest that the internal tide during segment 4 was traveling in a direction close to north to south, with a wavelength approximately twice the largest sensor separation, or about 40 km.

The estimate of the wave number spectrum by the beam-forming method at horizontal wave number k and frequency \( \omega_0 \) is given by [Capon, 1969]

\[
S(k, \omega_0) = \frac{1}{M^2} \sum_{p,q=1}^{M} \left( \frac{f_{pq}(\omega_0)}{f_{pq}(\omega_0) f_{pq}(\omega_0)} \right)^{1/2} \exp \left\{ i2\pi k \cdot (x_p - x_q) \right\}
\]

(1)

where \( f_{pq} \) is the cross spectrum at frequency \( \omega_0 \) between observations at sensors p and q, M is the number of sensors in the array, and \( x_p \) is the location of the pth sensor. The wave number window for this estimator, called the beam pattern (Figure 6) of the array, depends only on sensor separations and is given by

\[
B(k) = \frac{1}{M^2} \sum_{p,q=1}^{M} \exp \left\{ i2\pi k \cdot (x_p - x_q) \right\}
\]

(2)

Instead of using a conventional estimator of the cross spectrum \( f_{pq} \), we follow the example of Hendry [1977] and use the "raw" Fourier coefficients at tidal frequency, that is

\[
f_{pq} = \hat{\xi}(x_p) \hat{\xi}^*(x_q)
\]

where \( \hat{\xi}(x_p) \) is the Fourier transform of vertical displacement at location \( x_p \). This is a modification of the beam-forming method in that one assumes that the internal tide can be modeled as a deterministic rather than a random process. It can then be shown that for any noncolinear array of three points the wave number spectrum (1) will be exactly the beam pattern (2) reproduced with its center at \( k_0 \), where \( k_0 \) is the average wave number of the tide.

The calculated wave number spectrum at tidal frequency for segment 4 is shown in Figure 6. The most probable wave solutions occur at peaks in the wave number spectrum; a peak at wave number \( (k_x, k_y) \) represents a wave of wavelength \([1/(k_x^2 + k_y^2)]^{1/2}\) traveling in a direction defined by a vector drawn from the peak toward the origin. For example, the peak closest to the origin in the southwest quadrant of the wave number spectrum (Figure 6) is interpreted as a wave with an average wavelength of 36 km traveling in the direction of the arrow, about 25° to the east of north. The uncertainty in this estimate is indicated by the broadness of the peak in the beam pattern. The peak in the northwest quadrant that is closest to

<table>
<thead>
<tr>
<th>Mooring Pair</th>
<th>Separation, km</th>
<th>Observed Phase Difference, deg</th>
<th>Modeled Phase Difference, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1-B2</td>
<td>5.8</td>
<td>-25</td>
<td>-22</td>
</tr>
<tr>
<td>B1-B4</td>
<td>19.2</td>
<td>-166</td>
<td>-167</td>
</tr>
<tr>
<td>B2-B4</td>
<td>19.7</td>
<td>-141</td>
<td>-145</td>
</tr>
</tbody>
</table>

Modeled phase differences are from Prinsenberg and Rattray [1975].

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**Fig. 4.** (a) Temporal variation in the semidiurnal tidal band of the variance of isotherm displacement at about 50 m depth. (b) Horizontal coherence between isotherms at each mooring. (c) Vertical coherence at mooring B2 for isotherms which have various vertical separations, regardless of actual mean depth. All coherences in Figure 4c are significantly different than zero at the 90% level. Time segments are defined in Table 1.

**Fig. 5.** Time series of 2-hour moving averages of isotherm depth with a mean of 50 m during segment 4. The modeled plane wave (equation (3)) is shown as a dashed curve. The isotherms are 10.8°C at B1, 10.6°C at B2, and 10.0°C at B4.

**Table 2.** Observed and Modeled Phase Differences at Semidiurnal Frequency at About 50 m Depth During Segment 4
the origin also represents a wave with a wavelength near 36 km but traveling toward the southeast, about 155° to the east of north. These two peaks closest to the origin are the most likely solutions. All peaks farther from the origin are interpreted as aliases of the main peaks. These aliases occur because there may be integral multiples of waves between the moorings in addition to the fractional waves associated with the peaks closest to the origin. The wavelengths associated with the aliased peaks are short, 12 km or less. The true spectral density at these wavelengths is likely to be less than the density at longer wavelengths, because we expect that the longer waves will be more energetic at the point of generation and that they will be less easily dissipated. Hence, locally at a depth of 50 m, the vertical displacement \( \zeta \) during this time can be reasonably represented as a plane wave of the form

\[ \zeta = A \cos \left[ 2\pi(k_x x + k_y y + \omega_0 t) + \phi \right] \]  

(3)

where \((k_x, k_y)\) are determined from the beam-forming analysis and \(\phi\) is the absolute phase. Values of \((k_x, k_y)\) at all peaks in the wave number spectrum (Figure 6) give the same solution for \(\zeta\) at the three moorings. Comparison of this representation with the data for \(A = 10\) m, \(\omega_0 = 1/12.2\) cph (average of \(M_2\) and \(S_2\)) is shown in Figure 5. The agreement is good in both amplitude and phase except for the last few days at B2 and B4.

One may determine which of the two peaks closest to the origin (Figure 6) is the most likely solution from the location of possible topographic internal wave generators. If the arrow drawn from the peak in the southeast quadrant of the wave number spectrum is superimposed on the topography in Figure 1, it points directly from Rockall Bank toward the FIA. Weiland et al. [1969] showed that internal waves propagate normal to bathymetric contours regardless of the angle of incidence of the barotropic wave. This is consistent with generation at Rockall Bank. An arrow drawn in the direction indicated by the peak in the northwest quadrant of the wave number spectrum does not lead directly from any topographic feature.

The local spatial variation of isotherm depth provides another means of distinguishing between the two most likely solutions given by the wave number spectrum (Figure 6). Isotherms were constructed from observations by a thermistor chain towed around the moorings (dotted line, Figure 1) for two time periods during segment 4 [see Baumann et al., 1980]. Isotherm depths from a pair of repeated east-to-west transects on August 25 are shown in Figures 7a and 7b; a similar pair of isotherm profiles from a north-to-south track on August 27 are shown in Figures 7c and 7d. The isotherms were smoothed to eliminate small-wavelength oscillations. The plane wave solution (3) derived from the wave number spectrum is compared with the observed horizontal profiles of isotherm depth (Figure 7). The predicted displacements for the two plane waves with about 36-km wavelengths, one propagating from the northwest and one from the southwest, are plotted. For the east-to-west tracks the observations agree with a wave propagating from Rockall Bank better than with a wave from the northwest. The time interval between repeated tracks is about 6 hours or about one-half the tidal period, and the tracks nearly coincide with the times of maximum (Figure 7a) and minimum (Figure 7b) tidal displacement. The agreement of observed displacement with the plane wave propagating from Rockall Bank is also better for the north-to-south tracks, although for the first track (Figure 7c) there is not much to distinguish between the two solutions. Most of the 16 transects that were compared in this manner indicated either a

![Fig. 7. Comparison between isotherm depth from the towed thermistor chain (see Figure 1 for tow track) and plane wave solutions (3) from the two directions indicated by the two peaks in the wave number spectrum (Figure 6). Solid curves are the observed horizontal profiles of isotherm depth for (a and b) two time periods (August 25, 1620-1740 and 2145-2245 UT) of east-to-west tracks and (c and d) two time periods (August 27, 1332-1452 and 1902-2022 UT) of north-to-south tracks. Plus signs represent the model plane wave solutions for an internal wave propagating from Rockall Bank. Dashed curves represent the plane wave solutions from a direction 25° to the west of north. Start times for the model are the same as for Figure 5.](image)
better fit to the wave from Rockall Bank or not enough difference in the two solutions to choose a better fit. Hence the data from the towed thermistor chain support the conclusion that the tide can be described locally as a plane wave propagating from the southwest.

4. Generation Model and Comparison with Observations

While the internal tide may appear as a plane wave on small spatial scales, on larger scales we expect that the correlated modes generated by interaction of the barotropic tide with topography will produce a modulated, beamlike pattern. Topographic features were examined to determine their effectiveness as generators. The internal tide propagates along characteristics with slope given by

$$ \frac{dz}{dx} = \frac{\alpha^2 - f^2}{N^2 - \omega^2} \left( \frac{1}{\alpha} \right) $$

The most effective generator is one with slope equal to or greater than the slope of the characteristic [Baines, 1974]. Slopes of three topographic features near the mooring array are shown in Figure 8, and the corresponding section lines are drawn in Figure 1. Also shown is the slope of the tidal characteristic calculated from (4) by use of the JASIN mean buoyancy profile (Figure 2). The slope of Rockall Bank (profile A) is approximately equal to or steeper than the slope of the characteristic over depths ranging from 350 m to 1400 m, which indicates that it may be an effective generator. Lousy Bank to the north of the array (profile C) has one of the least steep slopes in the area and is therefore not an effective generator. George Bligh Bank, located directly to the west (profile B), and the remaining topographic features, located to the east of the moored array (Figure 1), have slopes favorable for the generation of an internal tide, but the wave number spectrum indicates a low probability that the observed internal tide propagated from the west or east during segment 4.

The internal tide generated by Rockall Bank was calculated by use of a model due to Prinsenberg and Rattray [1975]. The model consists of a steplike shelf (region I) of depth $H_1$ and width $L$ and a deep ocean (region II) of depth $H_2$ (Figure 9). The solution in each region is the sum of a surface standing wave (barotropic tide) and a set of progressive internal waves propagating away from the shelf break. Internal waves traveling up the shelf can be reflected at the coast but are not expected to return as far as the shelf break [Prinsenberg et al., 1974]. The $N$ profile is taken to be a function of depth only, and the internal waves in each region are represented as a sum of horizontally propagating, vertically standing modes. The vertical wave functions $\Phi_n$ and horizontal wave numbers $\kappa_n$ corresponding to mode $n$ are determined by solving the boundary value problem

$$ \frac{d^2 \Phi}{dz^2} + k^2 \left( \frac{N^2 - \omega^2}{\alpha^2 - f^2} \right) \phi = 0 $$
with $\phi = 0$ at the surface and bottom. The first five modes for vertical displacement and velocity in region II are shown in Figure 10, and wavelengths, wave numbers, and phase speeds are given in Table 3. The vertical displacement in regions I and II is then given by [Prinsenberg and Rattray, 1975, equations 12 and 13]

$$\eta^I = \frac{1}{H_1} \cos \kappa_o (x + L)$$

$$\eta^{II} = \frac{1}{H_2} \cos \kappa_o (x + x_0)$$

$$+ \sum_{n=1}^{\infty} A_n^I \sin \left( -i \kappa_n x \right) e^{-i \omega t}$$

$$+ \sum_{n=1}^{\infty} A_n^{II} \sin \left( i \kappa_n x \right) e^{i \omega t}$$

where $A_0$ and $\kappa_0$ are the amplitudes and wave numbers of the barotropic tide, $A_n$ is the amplitude of mode $n$, and $\kappa_n$ is the wave number of mode $n$. The parameter $x_0$ shifts the phase of the barotropic wave in region II to match the horizontal and vertical velocities of the barotropic wave in region I at the shelf break. Relationships between the barotropic and baroclinic amplitudes are obtained by matching the total vertical displacement and horizontal velocity above the shelf break. These relationships are coupled matrix equations containing sums over an infinite number of modes but can be solved approximately for $A_n^I$ and $A_n^{II}$ in terms of $A_0^I$ by using a finite number of modes. The higher modes mainly contribute fine structure to the solutions and in general have smaller amplitudes than the lower modes. In particular, for the case of constant $N$ the contribution of the $n$th mode is proportional to $(1/n) \sin (n \pi H_2 / H_1)$ [Rattray et al., 1969]. The amplitude thus decreases roughly proportionally to mode number but has zeros at integer values of $nH_2 / H_1$. For realistic stratification the mode shapes are no longer sinusoidal, and the relative contribution of each mode is more complicated than but qualitatively similar to the case of constant $N$. The model of the internal tide generated at Rockall Bank was obtained by increasing the number of modes in the sums (6a) and (6b) until the solution for the amplitudes of the first 10 modes was not significantly changed by the addition of more modes. Thirty modes were used. This approach was validated by comparing the simulation for constant $N$ with the exact solution.

One expects internal waves generated at Rockall Bank to propagate away from the shelf break along internal wave characteristics, forming beams of internal wave energy. The position of the beams and the amplitude of the internal tide are determined by the stratification, the frequency and amplitude of the barotropic tide, and the dimensions of the topography. If the shelf depth is small, reflection from the surface occurs near the shelf break, and the two beams emanating from the break combine into a single beam [e.g., Rattray et al., 1969]. In the present analysis, Rockall Bank is approximated by a steplike shelf with constant depth on each side. The actual topography is compared to this approximation in Figure 9. A sharp discontinuity in depth eliminates the generation from internal waves along the slope between the shelf break and the ocean floor. The effect of the weak generation along the slope is a broadening of the beams [Prinsenberg and Rattray, 1975]. The depth at which Rockall Bank becomes steeper than the slope of the internal wave characteristics, about 350 m, was used as the shelf depth $H_2$. The depth of the shelf determines the tangent point of the characteristics to the topography, affecting the horizontal position of the beams. The shelf width controls the phase of the barotropic standing wave in region I over the shelf break and sets the amplitude of the baroclinic tide relative to the barotropic, while leaving the details of the solution relatively unaffected. A standing wave over a shelf may not be the best representation of the barotropic tide, because there is no true coast, although Rockall Island surfaces about 50 km to the south of the shelf break. Rather than a detailed modeling of the local modification of the barotropic tide by the surrounding topography, the model shelf width was treated as a free parameter. The need to adjust the shelf width is an artifact of approximating the topography of Rockall Bank by a step shelf. Given a surface tide with an amplitude of about 1 m in deep water [Cartwright et al.,

**TABLE 3. Wave Numbers, Wavelengths, and Phase Speeds of the First Five Internal Wave Modes at 1/12.2 cph**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Wave Number, rad/m</th>
<th>Wavelength, km</th>
<th>Phase Speed, cm/s</th>
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<tr>
<td>1</td>
<td>$7.45 \times 10^{-5}$</td>
<td>84.4</td>
<td>192.</td>
</tr>
<tr>
<td>2</td>
<td>$1.24 \times 10^{-4}$</td>
<td>50.6</td>
<td>115.</td>
</tr>
<tr>
<td>3</td>
<td>$1.86 \times 10^{-4}$</td>
<td>33.7</td>
<td>76.7</td>
</tr>
<tr>
<td>4</td>
<td>$2.53 \times 10^{-4}$</td>
<td>24.8</td>
<td>56.5</td>
</tr>
<tr>
<td>5</td>
<td>$3.14 \times 10^{-4}$</td>
<td>20.0</td>
<td>45.6</td>
</tr>
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</table>
in the beams. The internal tide at the depth of the B moorings (50 m) has wavelengths of about 40 km. However, at 1000 m energy. Maximum values of vertical displacement exceed 25 m zero to 25 cm/s. Velocity contours tend to be parallel to the characteristics to form beams of internal wave regions of high vertical displacement tend to follow the characteristics. The spatial variability in vertical displacement associated with the modeled internal tide is illustrated in Figure 11.

The stratification and the frequency of the barotropic tide affect the width and horizontal position of the beam by controlling the slope of the characteristics given by (4). In the present analysis we choose a frequency of 1/12.2 cph, a shelf depth of 350 m, and a shelf width of 350 km. Superimposed are internal wave characteristics calculated from (4). The envelope of vertical displacement (dashed curves) is shown at depths of 50, 200, 600, 1000, and 1400 m together with vertical displacement at four times separated by one-eighth the tidal period. A vertical displacement of 1 m is represented by 5 m on the ordinate.

The modeled horizontal velocity field is shown in Figure 12. The stratification and the frequency of the barotropic tide are overcome by the modeled internal tide is further illustrated in Figure 13, which shows the horizontal wave number composition of vertical displacement (Figure 13b) and horizontal velocity (Figure 13a) at various depths. The contribution from each of the modes is a strong function of depth, varying over an order of magnitude or more at wavelengths corresponding to each of the first 10 modes. At a given depth, mode 1 is not always dominant. At depths of 200 and 1400 m the dominant contributions to the variance of vertical displacement are from mode 4. The dominant contribution to the tidal velocity variance at a depth of 1000 m comes from mode 3. Even when the first mode is dominant, there are often significant contributions from mode 2, 3, or 4.

The modeled vertical displacement associated with the internal tide is compared with observed isotherm displacement at the B moorings in Figure 14. The start time for the model, which fixes the absolute phase of the tide, was chosen to fit the data. Even with the large horizontal and vertical variability predicted by the model, the relative phases at the three moorings are in good agreement for all except two locations. The measured amplitudes are at locations where the model predicts strong gradients in amplitude (Figure 12a). Relatively minor shifts in position of strong gradients by processes neglected in the model would cause differences between the measurements and the model. Given the uncertainties in the measurements and the complexity of the velocity field generated by an idealized model, the agreement between observations and theory is good.
5. DISCUSSION

The analysis (section 3) of the energetic oscillations of isotherm displacement observed at the B moorings showed that the internal tide had a horizontal wavelength of 36 km at a depth of 50 m within the moored array. This wavelength may be compared with the wavelengths associated with the vertical mode solutions (6) for horizontally propagating free waves. The wave numbers, wavelengths, and phase speeds of the first five vertical modes at tidal frequency are given in Table 3, and circles have been drawn on the wave number spectrum (Figure 6) to represent the wave number of each mode. The mode 3 circle nearly intersects the center of each of the two peaks in the wave number spectrum which are closest to the origin. Hence within the FIA at a depth of 50 m the vertical displacement appears to have the wavelength of a mode 3 plane wave. This result may not be valid in other regions because of the beamlike nature of the internal tide (Figure 11).

The modeled spatial variability of semidiurnal tidal amplitude and wavelength (Figures 11 and 12) illustrates the potential for error when describing the internal tide based on sparse measurements. We observed a wavelength characteristic of mode 3 at a depth of 50 m within the B moored array. This wavelength is consistent with the model at the B moorings (Figure 14 and Table 2), but it is impossible to generalize to other depths and locations because of the spatial variability expected on the basis of the model (Figures 11 and 12). At 50 m depth, mode 2 is dominant in the model (Figure 13), but the dominant wavelength measured in a particular region may not be characteristic of mode 2, e.g., within the B array. If the B moorings had been located 50 km closer to Rockall Bank, the model predicts that the measured horizontal wavelength would have been much longer than 36 km (Figure 11). Although mode 1 is dominant in the aggregate, modes as high as 4 may dominate at a particular depth (Figure 13).

Independent estimates of the horizontal wavelength of the
internal tide were made during JASIN by Pollard [1983b]. These observations were made 1 week after segment 4, 10–40 km west of the FIA at depths of 14–42 m from a drifting spar buoy. Pollard reported that the internal tide had a horizontal wavelength of 25–30 km, corresponding to mode 3 or 4, consistent with analysis of the B moorings.

The inconsistencies in previous descriptions of the internal tide may have been caused by the attempt to generalize from sparse measurements. In the deep ocean far from large topographic features, such as continental shelves, the internal tide is traditionally thought to be composed of a few surviving low modes which have become uncorrelated. Using this assumption in the analysis of data from Mid-Ocean Dynamics Experiment (MODE), Hendry [1977] found most energy in mode 1. He identified Blake Plateau 700 km to the west as the source of the 160-km wavelength tide. In contrast, mid-ocean observations in the Pacific from the Pole experiment (35°N, 155°W) [Simpson and Paulson, 1979] and the Mixed Layer Experiment (MILE) (50°N, 145°W) [Davis et al., 1981] indicate that modes higher than 1 were probably present, because 10-m vertical displacements were found in the upper 100 m at semi-diurnal frequency. If the vertical structure were entirely mode 1, this would imply a maximum amplitude at middepth of at least 40 m, which may be unrealistically large. From measurements of horizontal coherence in the mid-Pacific by Barnett and Bernstein [1975], Simpson and Paulson [1979] estimate a horizontal wavelength of 35 km, corresponding to a wave of order mode 3. One explanation for the differences among these midocean determinations of the structure of the internal tide may be differences in sampling. Hendry [1977] may not have found significant energy in modes higher than 1, because he lacked data in the upper 400 m where higher modes may dominate. The spatial complexity of the modeled internal tide (Figures 11 and 12) illustrates the difficulty of determining the characteristics of the tide from sparse measurements.

The sampling problem is further compounded by temporal variability. Large temporal variation in the internal tide was observed at the B moorings (Figure 4). Ketzler [1983] also found significant variation in the amplitude of the semidiurnal tide on time scales of a week during JASIN. The observations consisted of horizontal velocity measured from July 18 to August 28 at six depths at the K1 mooring (Figure 1) in the FIA. The depths ranged from 70 to 980 m. The variation observed at 70 m depth is qualitatively similar to the variation in the variance of vertical displacement shown in Figure 4. In particular, there was an increase by a factor of 3 in the ampli-
Fig. 13. Modal composition of (a) horizontal velocity and (b) vertical displacement for the first 10 internal wave modes in the deep ocean from the model of Prinsenberg and Rattray [1975] (see Figure 11 for model parameters).

Fig. 14. Time series of 2-hour moving averages of isotherm depth with a mean of 50 m during segment 4 (solid curve). The modeled plane wave [Prinsenberg and Rattray, 1975] is shown as a dashed curve. (See Figure 11 for model parameters.) The isotherms are 10.8°C at B1, 10.6°C at B2, and 10.0°C at B4.

tude of the semidiurnal tide at 70 m during the week (August 22–29) when the variance of vertical displacement was also high (Figure 4). The vertical variation of semidiurnal tidal amplitude during this week was qualitatively similar to the modeled vertical variation (Figure 12) at the location of the K1 mooring (Figure 1). The observed amplitude was a maximum near the surface (10 cm/s at 70 m) and decreased to low values (2–4 cm/s) at depths of 480 m and below. These amplitudes and their variation compare favorably with the measured and modeled amplitudes (Table 4 and Figure 12) at the W1 and W2 moorings, which were less than 5 km away from K1 (Figure 1). The results of Ketzler’s analysis are consistent with ours and provide additional support for the model.

The 1-week period of tidal oscillations which we analyzed appears to be unusually energetic (Figure 4). In his review paper, Wunsch [1975] remarks "... the most common observation is that the internal tides tend to come and go, i.e., they appear to be intermittent." There are several possible causes of intermittency. The generation process may be affected by variations in the magnitude of the surface tide and fluctuations of density and velocity in the region of generation [Baines, 1982]. Once generated, the high-energy beams may be refracted, reflected, or trapped by fluctuations of density and velocity along the paths of propagation [e.g., Mooers, 1975]. In JASIN the 1-week energetic period nearly coincides with a maximum in the surface tide [Cartwright et al., 1980]. However, the amplitude of the surface tide changes slowly and by no more than 30% during the 5 weeks of observations. It is more likely that the intermittency is caused by shifting of the beams of internal wave energy due to fluctuations in density and veloc-
model solution for an internal wave generated at Rockall Bank and propagating toward 25° to the east of north. Start times for the model are the same as for Figure 14.

...the two time periods (August 25, 1620–1740 and 2145–2245 UT) of north-to-south tracks. Plus signs represent the model solution for an internal wave generated at Rockall Bank and propagating toward 25° to the east of north. Start times for the model are the same as for Figure 14.

The temporal variation of the tidal velocity amplitude at the K1 mooring (Ketzler [1983], described above) is consistent with a horizontal shifting of beams of internal wave energy. During the 1-week period analyzed here the tidal velocity amplitude at 70, 170, and 270 m depth at K1 was high, a factor of 2–3 times higher than during the first month of observation. The amplitude at 980 m during the same week, however, was low, 4–8 times lower than during the previous weeks. From Figure 12a, if the beam reached the surface after its first reflection just slightly closer to Rockall Bank, then the pattern of high and low velocities measured with depth at the FIA would be entirely different. The shallow sensors would be located in a region of low velocity, and the deeper ones in a region of higher velocities. This may have been the case during the last few weeks in July at K1.

There is indirect evidence that changes in the magnitude of the internal tide during JASIN may have been associated with changes in the eddy field. The velocity near the thermistor chain moorings changed direction (northwest to southwest) from August 20 to 22 [Weller and Halpern, 1983], just before the onset of the large-amplitude tidal oscillations. This nearly simultaneous occurrence may have been fortuitous. There is probably not enough information to determine a cause and effect relationship. In any case, an investigation of the effects of the eddy field on the internal tide is beyond the scope of this paper.

Little is known about the role of the internal tide in mixing the interior [Hendershott, 1981], although Baines [1974] suggests that mixing driven by tidal shear is easily achieved. The model predicts large velocity shear in the beams and near the surface (Figure 12a) which may contribute to mixing. A measure of the potential for overturning is the Richardson number $N^2/(f\omega^2)$, where $\omega$ is the horizontal velocity of the internal tide. Contours of a minimum Richardson number over a tidal cycle are shown in Figure 16. The contours tend to parallel internal wave characteristics, with lower Richardson numbers occurring along the beams which emanate from the shelf break. The greatest potential for the internal tide to contribute to mixing exists near the surface where Richardson numbers less than 2 are found. Even though a Richardson number of 2 is not subcritical, the addition of shear from other sources or fine structure in the vertical density profile could cause mixing. The model neglects nonlinear processes which could also contribute to mixing.

6. SUMMARY

We have presented the results of an investigation of the internal tide. The investigation included an analysis of observations in the Rockall Channel during the JASIN experiment, simulation of the generation and propagation of the internal tide by use of a model [Prinsenberg and Rattray, 1975], and comparison of the model with observations.

The internal tide was exceptionally energetic in the upper 80 m during the 1-week period chosen for analysis (Figure 4). Analysis of vertical displacement of isotherms observed by an array of three moored thermistor chains (Figure 1) and a towed thermistor chain showed that the internal tide at a depth of 50 m within the array propagated from the direction of Rockall Bank, about 100 km away. Because of its location and favorable topography (Figure 8), Rockall Bank was identified as the source of the observed internal tide.

The internal tide generated by the interaction of the barotropic tide with Rockall Bank was simulated by use of a model due to Prinsenberg and Rattray [1975], which has a step shelf and a depth-dependent buoyancy frequency. The modeled internal tide exhibits a large degree of spatial variability (Figures 11, 12, and 13) which is associated with beams propagating from the shelf break along internal wave characteristics (Figure 9). Tidal energy is also concentrated near the surface, in association with the seasonal pycnocline. In the aggregate most of the modeled tidal energy is in the first mode, but at particular depths, modes as high as 4 (wave-lengths of 25 km) contain most of the energy (Figure 13). The spatial variability in the modeled amplitudes and wavelengths illustrates the potential for error when drawing conclusions about the structure of the internal tide from sparse observa-

<table>
<thead>
<tr>
<th>Mooring</th>
<th>Distance From Rockall Bank, km</th>
<th>Depth, m</th>
<th>10</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>600</th>
<th>1000</th>
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<tbody>
<tr>
<td>I3</td>
<td>52</td>
<td>4.2 (2.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W1</td>
<td>107</td>
<td>7.5 (6.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.5 (3.2)</td>
</tr>
<tr>
<td>W2</td>
<td>109</td>
<td>17.8 (16.5)</td>
<td></td>
<td>7.2 (4.6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I4</td>
<td>130</td>
<td>6.5 (4.8)</td>
<td>6.1 (4.9)</td>
<td>5.7 (7.6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Amplitudes are in centimeters per second. Data from moorings I3, W1, and W2 are averaged over a 7-day period from August 22 to 29 (segment 4) and from mooring I4 over a 33-day period starting from August 22. Amplitudes in parentheses are from the model (Figure 11).


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