THE EFFECT OF ALTERNATING CURRENT ON THE RESISTANCE AND REACTANCE OF IRON AND STEEL WIRES.

THESIS

ON

THE EFFECT OF ALTERNATING CURRENT

ON THE RESISTANCE AND REACTANCE OF IRON AND STEEL

WIRES.

Submitted to the Faculty

of the

OREGON AGRICULTURAL COLLEGE

In Fulfillments of the Requirements

for the

Degree of Bachelor of Science in Electrical Engineering

by

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Dean of the School of Engineering.

PREFACE

During the past year the rapid increase in the cost of copper and aluminum has led operating companies to investigate the feasibility of using iron and steel conductors for trandmission and distribution purposes where small blocks of power are to be served. This thesis gives the results of tests on several sizes of iron and steel conductors suitable for transmission line purposes.

The author wishes to acknowledge the assistance given by Mr. Chas. E. Oakes, in conjunction with whom the tests were made, and to Messers Sinks, Stoppenbach, Kephart and Frost who rendered valuable assistance in the preliminary work.

W. Eckley.

INTRODUCTION.

When an alternating current is flowing thru a conductor the current thru out the cross section of the conductor will not be evenly distributed nor in phase. The current density will be greater near the surface of the conductor and is casued by the greater impedance at the center than at the surface. This phenomenon is known as skin effect and varies with the frequency of alternations.

The effective resistance ans inductance of the conductor as a whole are different for an alternating current than for a direct current, the effective resistance increasing, for non magnetic materials, and the inductance decreasing with an increase in frequency. When the material of the conductor has constant permeability and the conductor is of simple shape the effective resistance and inductance may be computed in a number of cases.[1] However when the permeability is not constant or the conductor is made up of ferromagnetic material the problem becomes so complex that a solution is apparently impossible. In such cases

[1]For refference see, Scientific paper of the Bureau of Standards, No.252, by John M. Miller.

the changes in effective resistance and inductance are very large and not only depend upon the frequency but also upon the current strength.

No attempt has been made in this article to treat the subject mathematically, but rather to give the actual test results of a number of samples of various kinds and qualities of iron and steel wires. The theory of skin effect has been treated with various degrees of refinement by numerous authorities and the reader is referred to the bibliography for a partial list of these references.

METHOD OF TEST.

The wires tested were strung on insulators and carefully spaced. The same kind and size of wire was used for the return conductor making it possible to compute the external reactance. The two wires were short-circuited at one end of the line with a heavy copper bar having a negligible resistance and reactance. At the other end of the line a voltmeter, ammeter and wattmeter were connected in the circuit. The ammeter and current coil of the wattmeter were connected in series with the line and the voltmeter and potential coil of the wattmeter were shunted across the line.

From the method of testing it will be seen that it was necessary to have a very accurate low reading voltmeter and a wattmeter with a low voltage potential coil as the resistance of each line was low for low current densities. In order to obtain low voltage readings the multiplier of a Weaton voltmeter, having a 10 volt scale, was shunted with a piece of copper wire and with this arrangement voltage readings as low as 0.5 volt, with an accuracy greater than 0.5 %, were obtained. The voltmeter was calibrated against a potentiometer, reverse readings being taken and the average of the two assumed to be the correct reading.

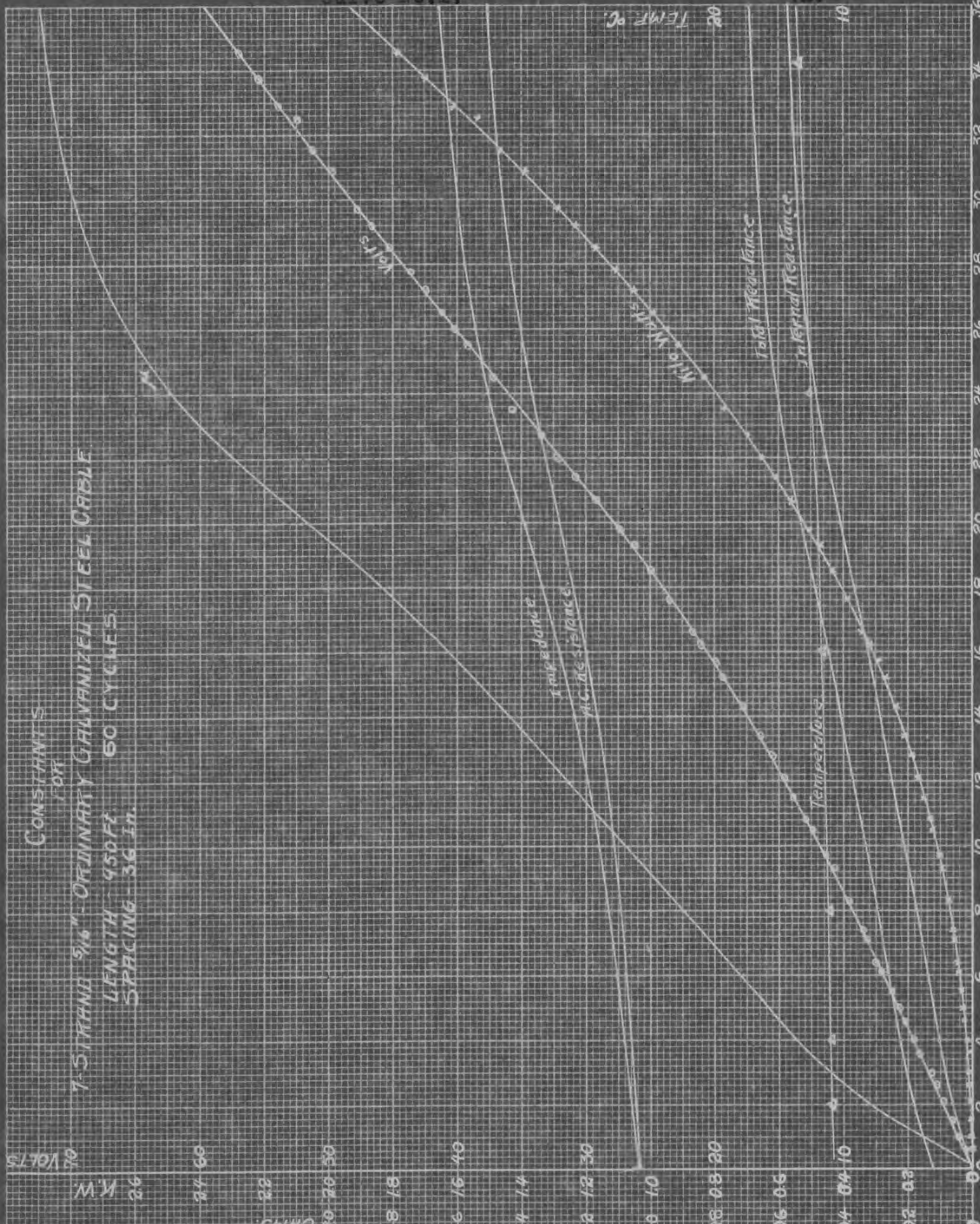
For the low reading wattmeter a Siemens Halske instrument was found having a 30 volt potential coil and a 2.5 amp. current coil. With this instrument power losses as low a 0.1 watt could be approximated as each scale division represented o.5 watt. For the larger current values precission instruments were used thru out.

The load was varied by the use of a lamp bank connected in series with the wire being tested. The power used was the commercial 60 cycle energy furnished by the Oregon Power Co.

Temperature measurements were obtained by a thermometer fastened to the wire.

The direct current resistance was determined by the drop of potential method, corrections being made for the current consumed by the voltmeter. Energy for these measurements was obtained from a low voltage storage battery.

The length of each conductor was carefully measured with a steel tape before stringing on the insulators. As an extra precaution the conductors were weighed to prevent any error being made in the size or length of wire.


CONSTANTS FOR
7-STRAND 5/16" - ORDINARY GALVANIZED STEEL CABLE
LENGTH 450 Ft. 60 CYCLES
SPACING 36 In.
AMPERES.
VOLTS
K.W.
OHMS
PERMEABILITY - μ
TEMP. °C.
Volts
Kilo Watts
Impedance
A.C. Resistance
Total Reactance
Internal Reactance
Temperature
μ
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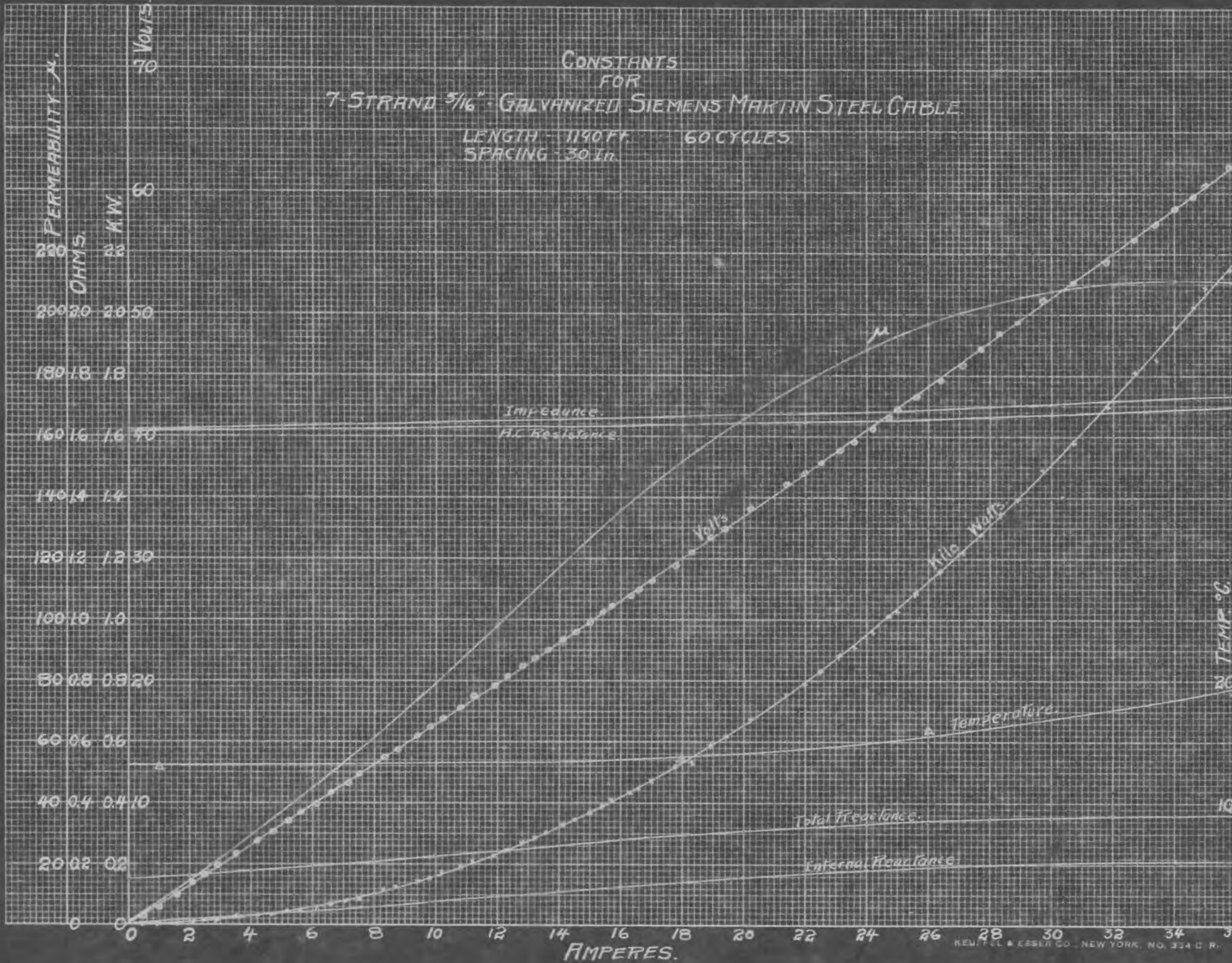
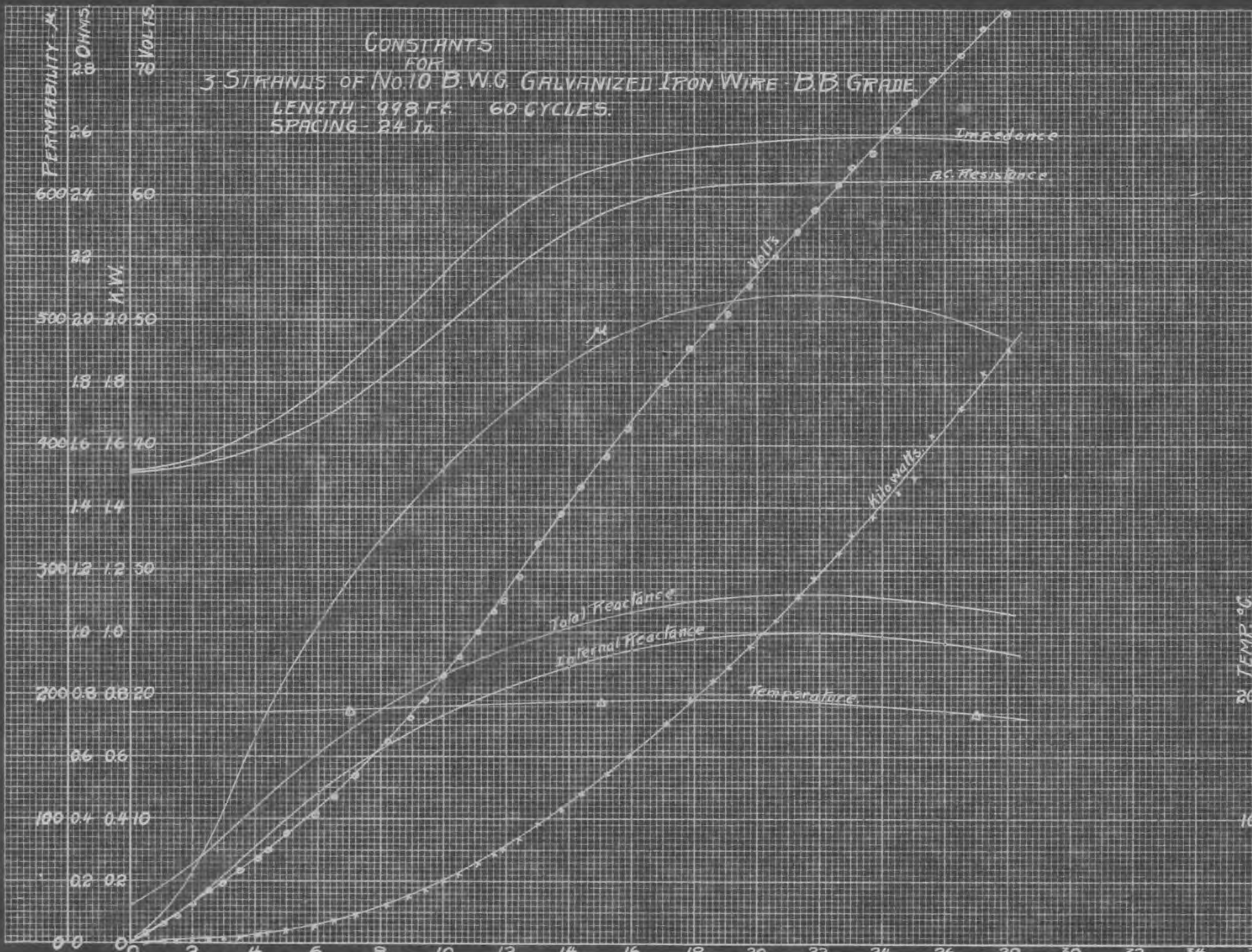
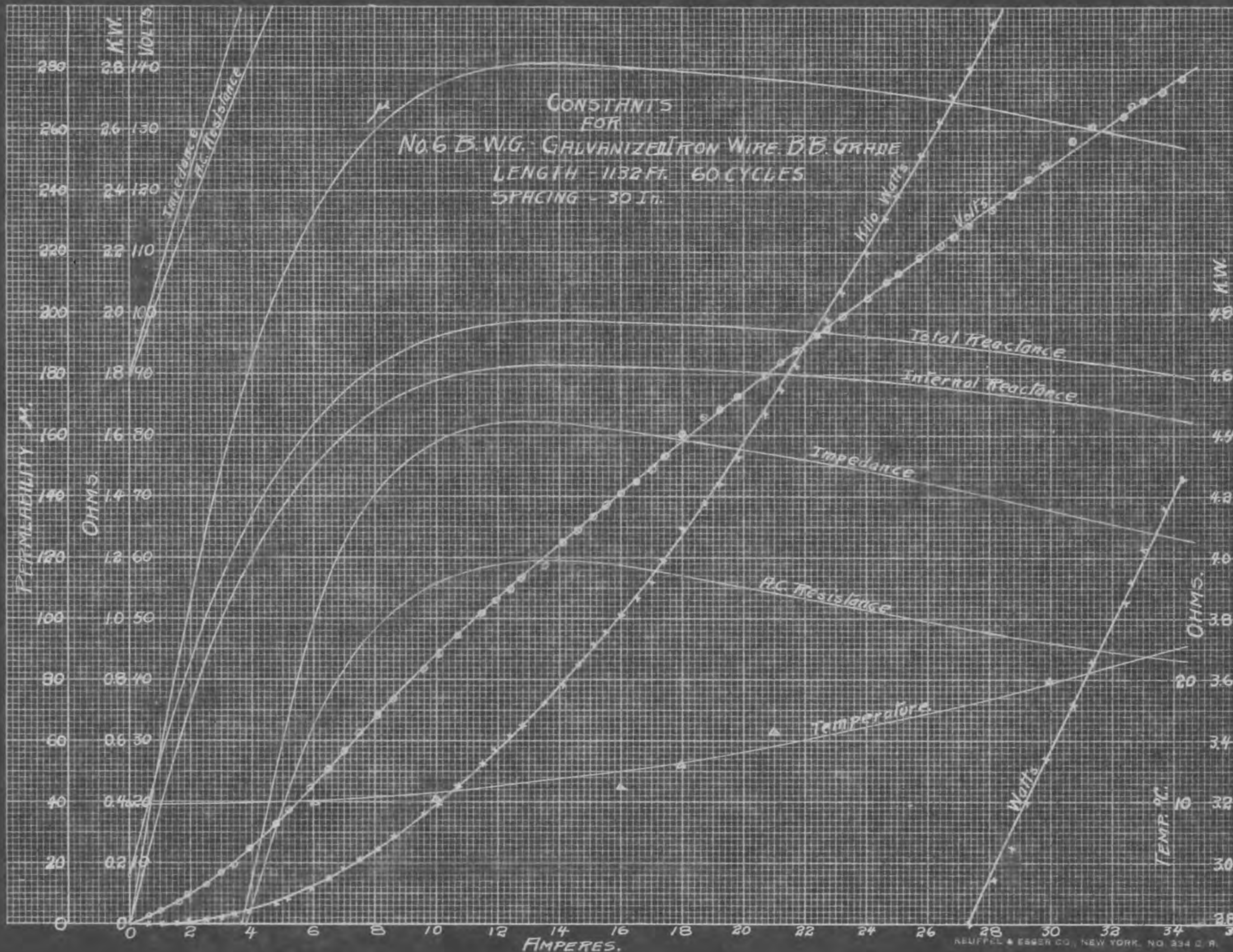

CONSTANTS
FOR
7-STRAND 5/16" - GALVANIZED SIEMENS MARTIN STEEL CABLE.
LENGTH - 1190 FT. 60 CYCLES.
SPACING - 30 In.
PERMEABILITY - μ.
OHMS.
K.W.
VOLTS.
TEMP. °C.
Impedance.
A.C. Resistance.
μ
Volts.
Kilo Watts.
Temperature.
Total Reactance.
Internal Reactance.
AMPERES.
KEUFFEL & ESSER CO., NEW YORK, NO. 334 C. R.


CONSTANTS
FOR
3 STRANDS OF No.10 B.W.G. GALVANIZED IRON WIRE - B.B. GRADE.
LENGTH - 998 Ft. 60 CYCLES.
SPACING - 24 In.
PERMEABILITY - μ.
OHMS.
K.W.
VOLTS.
TEMP. °C.
AMPERES.
Impedance
A.C. Resistance
Volts
μ
Kilowatts.
Total Reactance
Internal Reactance
Temperature.
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
KEUFFEL & ESSER CO., NEW YORK. NO. 334 C. B.


CONSTANTS
FOR
No. 6 B.W.G. GALVANIZED IRON WIRE B.B. GRADE
LENGTH - 1132 Ft. 60 CYCLES
SPACING - 30 In.
PERMEABILITY μ.
OHMS.
K.W.
VOLTS
AMPERES.
μ
Impedance
A.C. Resistance
Kilo Watts
Volts
Total Reactance
Internal Reactance
Impedance
A.C. Resistance
Temperature
Watts
TEMP. °C.
OHMS.
K.W.
KEUFFEL & ESSER CO., NEW YORK. NO. 334 C.R.

Test data were obtained on the following four sizes of wire:

7 Strand 5/16" Ordinary Grade Galvanized Steel

7 Strand 5/16" Siemens Martin Galvanized Steel

3 Strands of No.10 B.W.G., B.B.Grade Galvanized Iron

No.6 B.W.G., B.B.Grade, Solid, Galavnized Iron.

The data for these wires are shown in curves 1, 2, 3 & 4.

The following tables give the physical constants of the wires tested:

7 Strand 5/16" Ordinary Steel Cable,

Size of each strand - No. 12 B.W.G.

Diameter " " 0.106"

Max. Diameter of cable - 0.318 "

Area of each strand - 11280 cir. mils.

Area of cable - 78960 cir. mils

Spacing of conductor - 36"

Total length of wire tested - 950 ft.

Wt. of 950 feet of cable - 193 pounds

Wt. per mile " " - 1107 "

Resistance at 17.6 deg. C. - 0.9855 ohms

Res. per mile of conductor at 20 deg. C. - 5.56 ohms.

7 Strand 5/16" Siemens Martin Steel Cable,

Size of each strand - No.12 B.W.G.

Diameter of each strand - 0.109"

Max. dia. of cable - 0.327 "

Area of each strand - 11881 cir. mils

Area of cable - 83167 cir. mils

Spacing of conductor - 30"

Total length of conductor tested - 1190 ft.

Wt. of 1190 ft. of conductor - 263.5 lbs.

Wt per mile of " 1170 "

Res. at 13.7 deg.C. - 1.618 ohms

Res.per mile of conductor 20 deg. C. - 7.41 ohms

3 Strands of No. 10 B.W.G., B.B. Galv. Iron,

Size of each strand - No. 10 B.W.G.

Diameter of each strand - 0.134 "

Dia. of circumscribing circle - 0.288"

Spacing of conductor - 24"

Area of each strand - 17956 cir. mils.

Area of cable - 53867 cir. mils

Length of conductor tested - 998 ft.

Wt. of 998 ft. of conductor - 143.5 lbs.

Wt. per mile of conductor - 759 "

Resistance at 13.7 deg. C. - 1.3612 ohms

Res. per mile of cable 20 deg. C. - 7.45 ohms

No. 6 B.W.G., B.B. Iron, Solid, Galv.

Diameter of conductor - 0.2031"

Area of conductor - 41250 cir. mils.

Length of conductor tested - 1132 ft.

Wt. of 1132 ft of conductor - 123 lbs.

Wt. per mile of conductor - 375 lbs.

Spacing of conductor - 30"

Resistance at 16.6 deg. C. - 1.8482 ohms

Res. per mile of cond. at 20 deg. C. - 8.65 ohms

From the readings of volts, amperes and watts the values of internal reactance, effective resistance, or a.c. resistance, and permeability were computed. The derivation of the formulae for computing the reactance of stranded conductors as given by Dwight[1] is given:

Suppose that the current in each small wire has unit value. Then the flux density due to that current, at a distance x from the center of the wire, is 2/x.

If t is the distance between any two wires of the cable, the required voltage induced in the second by the first, per centimeter, is

$$2\pi f \int_t^S \frac{2}{x}\,dx = 2\pi f \times 2 \log_e S/t$$
$$= 2\pi fM$$

where f is the frequency and M is the inductance for the pair of wires.

[1]The Reactance of Stranded Conductors, by H.B.Dwight, Elec. World, Vol. 60, No. 16, Apr.19, 1913, page 828.

The inductance of a wire due to its own current is given by the usual formula

$$L_1 = 2 \log_e S/r + \tfrac{1}{2}$$

in which r is the radius of the wire and in which $\frac{1}{2}$ expresses the effect of the flux inside the wire.

The inductance per centimeter of cable of a single-phase circuit composed of seven-wire cables(Fig.1) is proportional to

$$L' = 7L_1 + 12M_1 + 30M_2$$

where L_1 is the inductance of each wire due to its own current, M_1 is the inductance for pairs consisting of an outer wire and the center wire, and M_2 is the average inductance for pairs consisting of two outer wires.

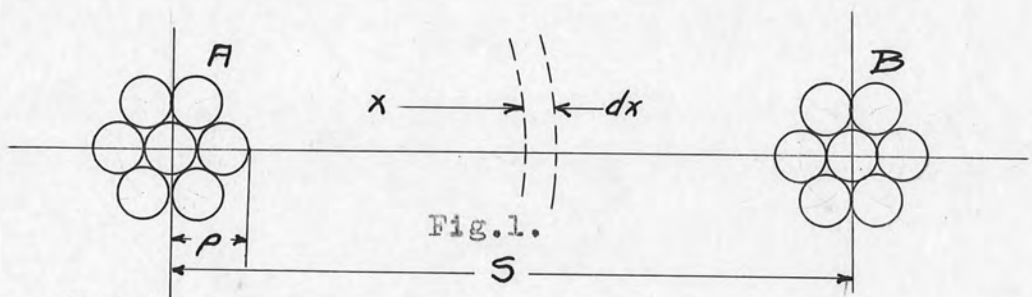


Fig.1.

Then,

$$L_1 = 2 \log_e S/r + \tfrac{1}{2}$$

and

$$M_1 = 2 \log_2 (S/2r)$$

For the third term it is convenient to use the following theorem:

If a circle of radius a (Fig.2) be divided into m equal parts at A, B, C, D, etc., then

AB, AC, AD, etc. (to m - 1 factors) = ma^{m-1} #

Thus the mean value of log t = $\log (1/am^{m-1})$

The average value of M_2 is therefore

$$2 \log_e \frac{S}{2 r 6^{1/5}}$$

Therefore,

$$L' = 98 \log_e S/r + 7/2 - 24 \log_e 2 - 60 \log_e 2 - 12 \log_e 6$$

In the above the voltages induced in the seven wires have been added together and unit current has been assumed for each wire. Considering the cable as a single conductor, its inductance will be

$$L = \frac{1}{49} L'$$

Changing to practicle units and referring to the maximum radius of the cable,

$$\rho = 3r$$

Then,

$$L = (103.3 + 741.13 \log_{10} S/\rho)10^{-6}$$

henrys per mile of conductor.

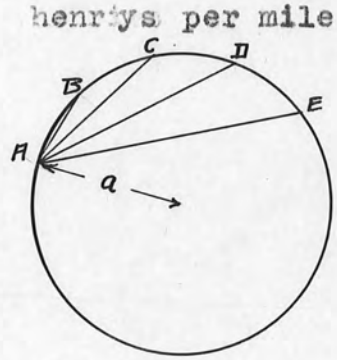


Fig.2.

Applying the above method to a strand of n wires composed of p layers around a center wire (Fig.3, b & c),

E.B.Rosa, Bulletin of the Bureau of Standards, 1907, Vol.IV, No.2, page 335.

$$n = 1 + 6 + 6 \times 2 + 6 \times 3 + \ldots + 6 \times p$$
$$= 3p^2 + 3p + 1$$

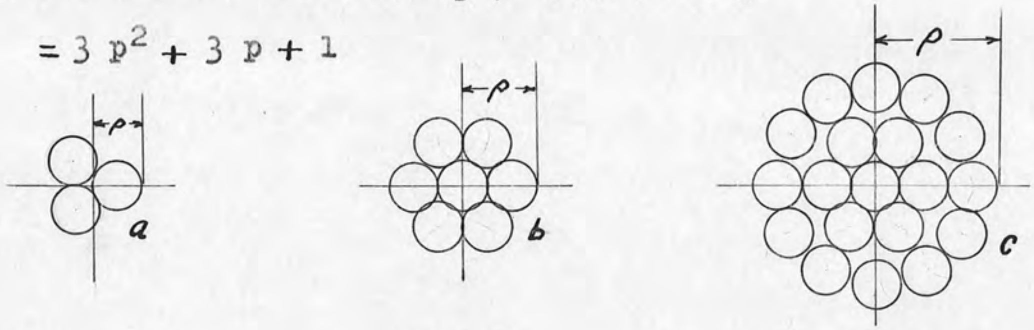


Fig.3.

and

$$L = 2\log_e S/\rho + 1/2n + 2\log_e(2p+1) - \frac{2(n-1)}{n^2}\log_e 3$$
$$- \frac{24}{n^2} + 4\log_e 2 + 2 \times 13\log_e 4 + 3 \times 28\log_e 6 + \ldots$$
$$\ldots + p(n-3p)\log_e 2p \qquad (1)$$

A three strand wire (Fig.7a) is a special case since there is no central wire. The inductance is

$$L = 2\log_e S/\rho + 1/6 + 2\log_e(2\sqrt{3}+3)/3 - 4/3\log_e 2$$
$$\text{per centimeter} \qquad (2)$$

Therefore $L = (125.2 + 741.13\log_e S/\rho)10^{-6}$ henrys per mile, where ρ is the radius of the circumscribing circle of the cable.

The above formulae were derived for non-magnetic material and the permeability is assumed as unity. For magnetic materials these formulae must be modified in that the permeability must be introduced into the term for internal reactance. Formula (1) thus modified is

$$L = 2\log_e S/\rho + \frac{\mu}{2n} + 2\log_e(2p+1) - \frac{2(n-1)}{n^2}\log_e 3$$
$$- \frac{24}{n^2}\Big\{4\log_e 2 + 2 \times 13\log_e 4 + 3 \times 28\log_e 6 + \ldots$$

$$\ldots + p(n-3p)\log_e 2p \Big\} \qquad (3)$$

and formula (2) is

$$L = 2\log_e S/\rho + \frac{\mu}{6} + 2\log_e(2\sqrt{3}+3)/3 - 4/3\log_e 2$$

per centimeter (4)

where, μ is the permeability for circular magnetization, the other factors having the same values as in equation (1)

The reactance per mile of single conductor is

$X = 2\pi f L \times 5280 \times 12 \times 2.54 \times 10^{-9}$ ohms per mile, where, L is the inductance per centimeter and f is the frequency in cycles per second.

The external reactance may be computed by leaving out the term for internal reactance. From the computed values of external reactance and the measured values of total reactance the internal reactance may be found and from these volues of internal reactance the permeability computed.

After determining the internal reactance from the experimental data the permeability was found by solving for μ. For the 7 Strand 5/16" ordinary steel cable equation (3) reduced to the following form applies, for a length of 950 ft. and 30 inch spacing,

$$\mu = \frac{x - 0.1243}{0.0078}$$

where x is the total reactance of the cable for 60 cycles.

In a like manner the permeability for the 7 Strand 5/16" Siemens Martin cable, having a length of 1190 ft.

and 30" spacing, was computed from the following formula;

$$\mu = \frac{x - 0.1505}{0.000975}$$

The formula for determining the permeability of the 3 Strands of No. 12 B.W.G., B.B. iron was obtained from equation (40) and is

$$\mu = \frac{x - 0.1244}{0.00191}$$

for a length of conductor of 998 ft. and 24 inch spacing.

The formula for computing the permeability of the solid No.6 B.W.G., B.B. iron wire was obtained from the Standard Handbook, 4th Edition, Sec.2, Art.77 and is

$$L = 0.1403 \log_{10} D/r + 0.01524\mu$$

millihenrys per 1000 ft.

where, L is the inductance, D is the spacing in inches and r is the radius of the conductor in inches. The reactance per 1000 feet is

$$X = 2\pi f L \qquad \text{ohms per 1000 feet}$$

where, f is the frequency in cycles per second and L is the total inductance per 1000 feet.

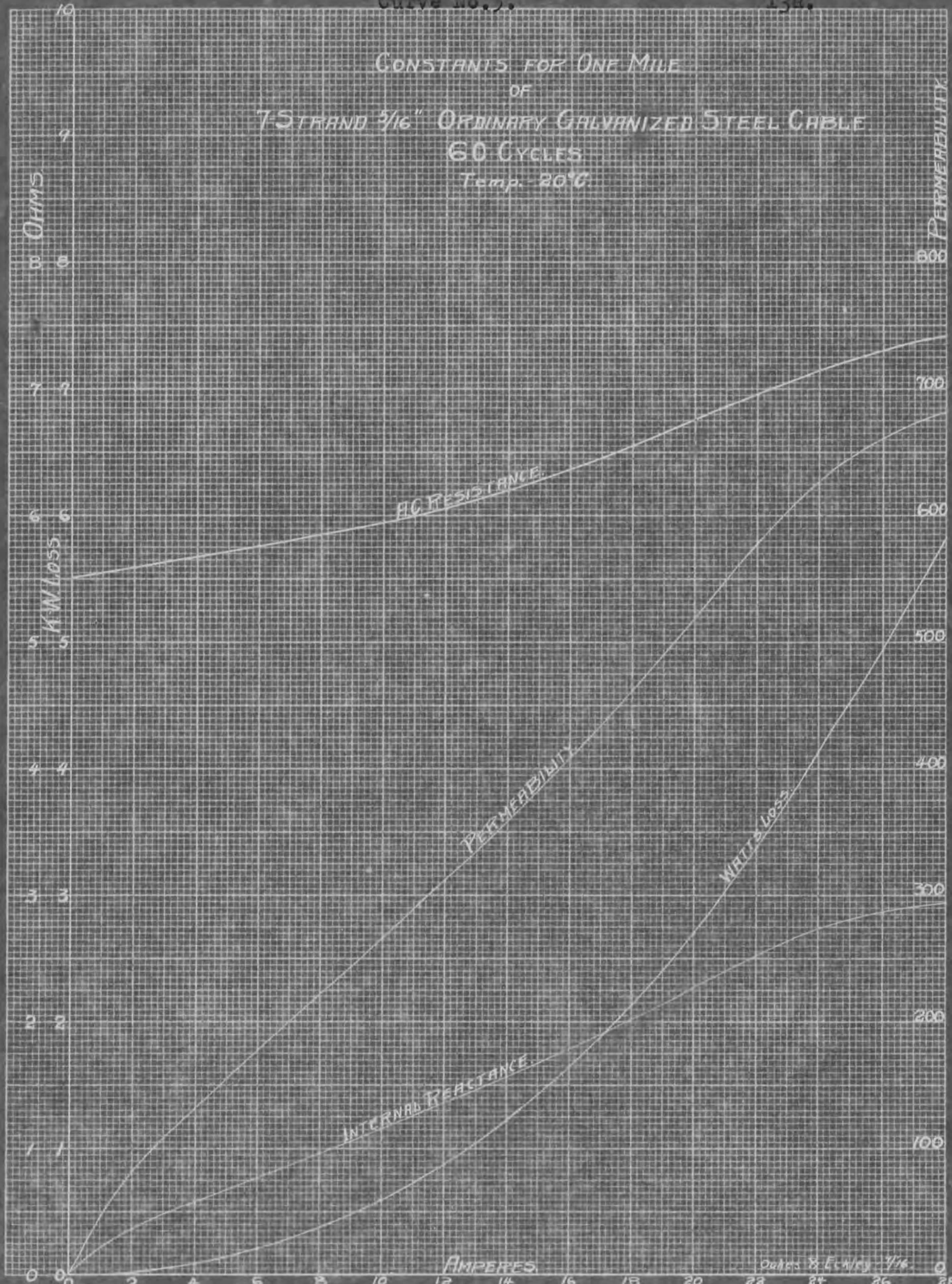
For a length of 1132 feet and 30 inch spacing the formula for permeability is

$$\mu = \frac{x - 0.1483}{0.065}$$

The values of permeability determined by these formulae are not strictly true as the current is assumed to be uniformly distributed through out the conductor and the magnetic path in the conductor is assumed to be composed of one material, i.e., not stranded nor having a layer of non-magnetic material perpendicular to the magnetic flux.

Referring to curves 5, 6, 7, & 8 comparisons can be made between the different conductors tested. The values given on these curves were computed from the actual test data and are given per mile of conductor. The values of permeability given on the curves show that the ordinary galvanized steel cable has the highest permeability while the Siemens Martin cable has the lowest. The Siemens Martin cable being of a harder material than ordinary steel should show a lower value of permeability, however, sinceboth cables are stranded any difference in thickness of the zinc coating would affect the permeability obtained by the formulae.

The softer materials, B.B. grade iron, show very low values of permeability. The 3 Strands of No.10 B.W.G. B.B.iron show a higher permeability than the No.6 B.B. iron. This difference is probably due to the greater concentration of current near the surface of the solid


CONSTANTS FOR ONE MILE
OF
7-STRAND 5/16" ORDINARY GALVANIZED STEEL CABLE
60 CYCLES
Temp. 20°C.
OHMS
K.W. LOSS
PERMEABILITY
AMPERES
A.C. RESISTANCE
PERMEABILITY
WATTS LOSS
INTERNAL REACTANCE
Oakes & Eckley 7/16.
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
100
200
300
400
500
600
700
800
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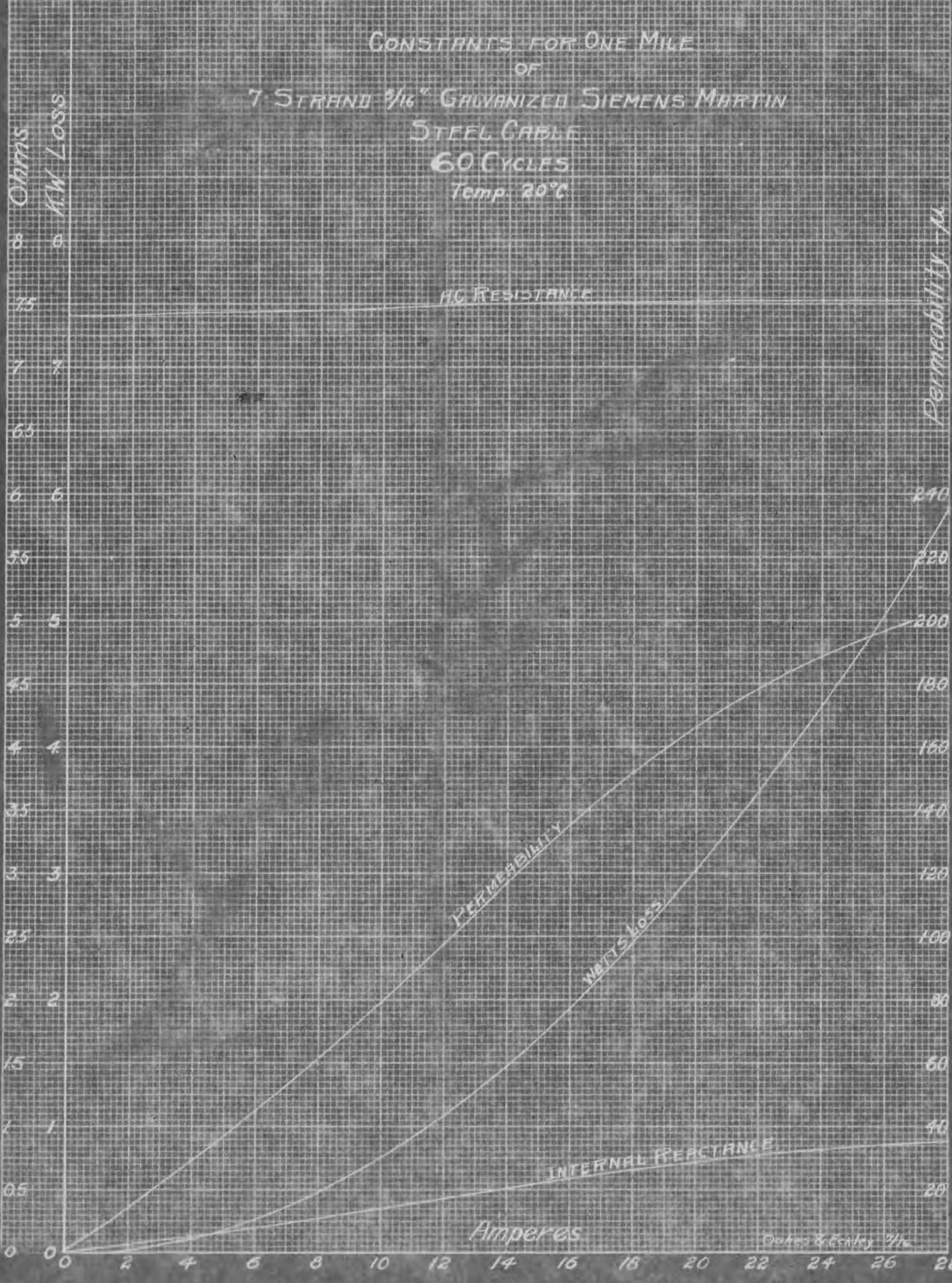

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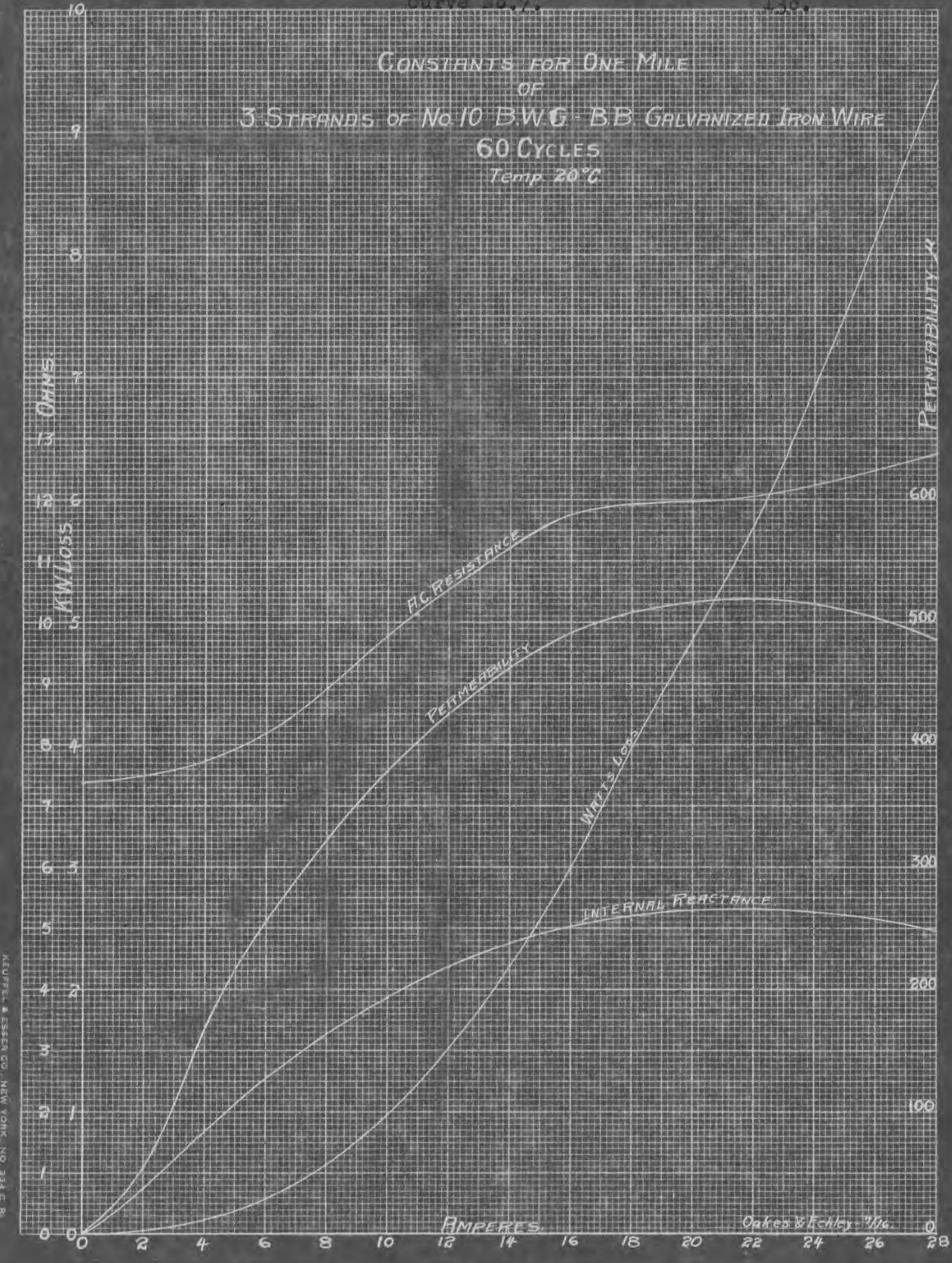
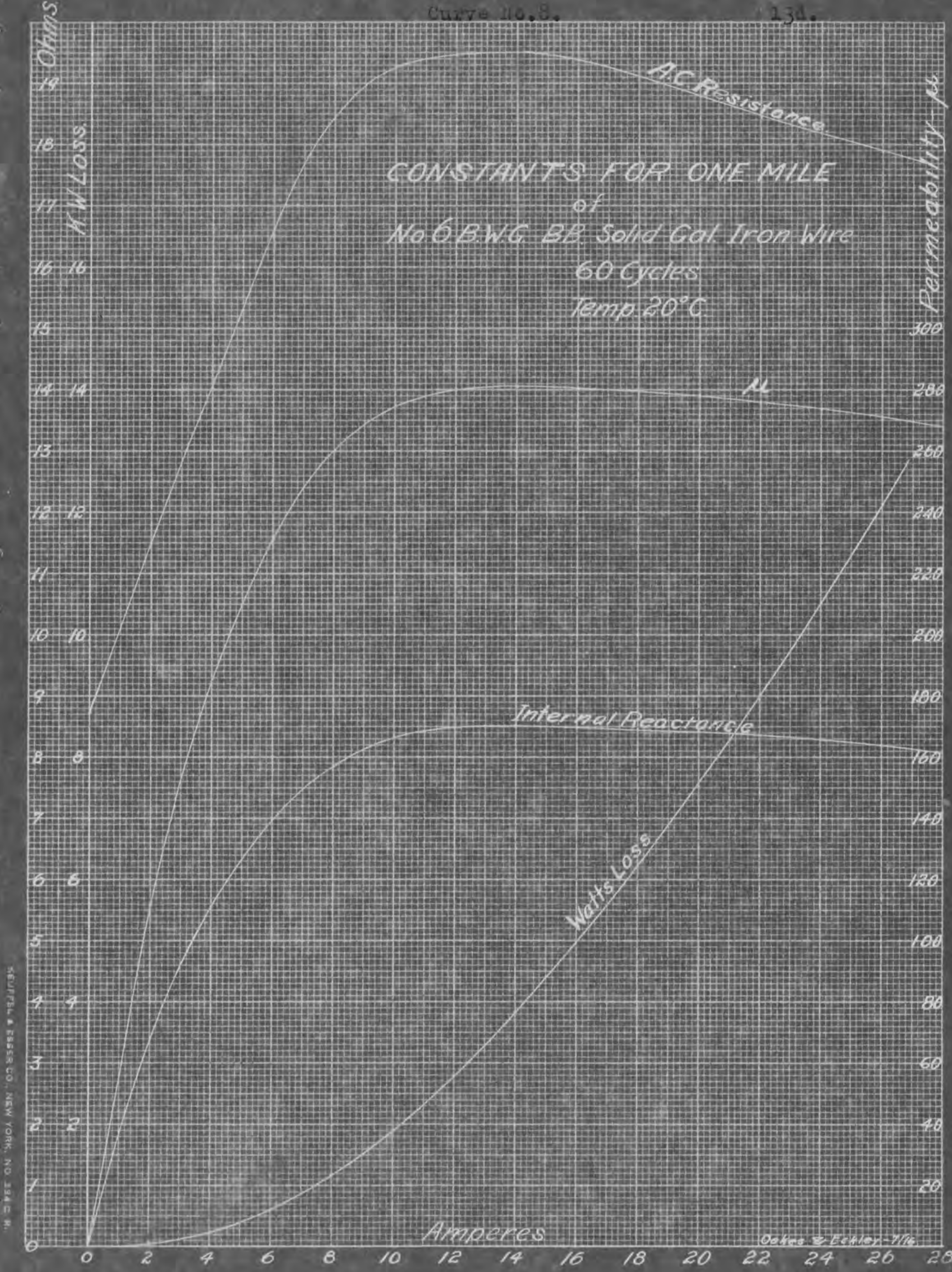

CONSTANTS FOR ONE MILE
OF
3 STRANDS OF No.10 B.W.G. B.B. GALVANIZED IRON WIRE
60 CYCLES
Temp. 20°C
A.C. RESISTANCE
PERMEABILITY
WATTS LOSS
INTERNAL REACTANCE
OHMS.
KW. LOSS
PERMEABILITY μ
AMPERES
Oakes & Echley - 7/16.
KEUFFEL & ESSER CO., NEW YORK. NO. 334 C RL


CONSTANTS FOR ONE MILE
of
No 6 B.W.G. BB Solid Gal. Iron Wire
60 Cycles
Temp. 20°C.
Ohms
K.W. Loss.
Permeability μ
A.C. Resistance.
μ
Internal Reactance
Watts Loss
Amperes
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
Oakes & Eckley - 7/16
KEUFFEL & ESSER CO. NEW YORK. NO. 354 C.R.

conductor than in the 3 Strand cable, thus there would be a greater flux concentration at the surface than near the center of the conductor.

Another factor that affects the skin effect, internal reactance and alternating-current resistance, in the stranded conductors, is the spiraling of the wires of the strand or cable.

A considerable decrease in the skin effect and internal reactance can be affected by properly stranding the conductor and reversing the direction of twist of each layer of strands.[1]

The method of manufacture makes it difficult to lay down any data that will apply to any particular grade of iron or steel. Since these materials are manufactured from a mechanical standpoint rather than from an electrical and a slight change in ingredients that would not change the mechanical properties might make a material difference in the magnetic and electrical characteristics. The number of times the material was drawn after annealing also affects the hardness and thus the conductivity and permeability are changed. Thus any test data obtained on a sample of wire would only give approximate results if used for a slightly different size of wire or wire of another manufacturer.

[1] Elektrotechnische Zeitschrift, Jan. 1915.

For convenience the following formulae were derived from formulae (3) and (4). These formulae give the reactance per mile of single conductor.

Reactance for 60 cycles.

No.6 B.W.G. Solid B.B. Iron.

$$X = 0.27941 \log_{10} \frac{D}{0.1055} + x'$$

5/16" 7 Strand Siemens Martin or ordinary grade steel cable.

$$X = 0.27941 \log_{10} \frac{D}{0.0545} - 0.09865 + x'$$

3 Strands of No.10 B.W.G., B.B. iron.

$$X = 0.27941 \log_{10} \frac{D}{0.144} - 0.0371 + x'$$

where, X is the total reactance in ohms per mile of wire, x' is the internal reactance from curve and D is the spacing in inches.

The following table gives the constants per mile length of conductor:

Material	Size	Ult. Strength Pounds	Res.per Mile 20 deg.C.	Wt. Lbs. per Mile	Actual Max.Dia. Inches.
Gal.Solid B.B.	No.6 B.W.G.	1652	8.65	573	0.203
Gal.Steel Ord.Grade	5/16"	3800	5.56	1107	0.327
Gal.Siemens Martin	5/16"	4800	7.41	1170	0.327
3 Strands Gal. B.B.	No.10 B.W.G.	2166	7.45	759	0.288

The effective a.c. resistance was computed from the watt curve, values being taken directly from the curve rather than using the readings. The a.c. resistance is a value such that when multiplied by the square of the current the power loss in the line is obtained. This resistance is composed of three factors; hysteresis loss, eddy currents, and skin effect, altho the last two named factors may be assumed as composed only of skin effect. The a.c. resistance is affected in the same manner as the reactance by the permeability.

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