APPENDIX. Simplified Outline of Log and Carbon Market Model.

The model is an optimizing, intertemporal, spatial equilibrium market model. Projections are made over 100 years using 5-year time intervals. Solutions are found by maximizing the present value of the willingness-to-pay integrals under the log demand curves plus returns from carbon offset sales less costs of timber growing, log processing and handling (including capacity investment), and the carbon sales program over the projection period. Results reported here use a 6% discount rate. Logs flow from plots (subscript "PLOT" below) to processing facilities (subscript "MILLS"). Capacity acts both to shift the log demand (willingness to pay) function and to bound the level of log demand (termed log "Receipts") at each mill. The approach to timber inventory accounting is a combination of Johnson and Scheurman's (1977) model I and II forms. Stands can be managed using even-aged (^{EA}) systems of clearcutting and thinning, or "uneven-aged" (^{UA}) management with some form of partial cutting and no final clearcut. Plots may be either "in" the carbon offset sales program (COSP) or "out", an endogenous decision.

In western Oregon (USA), fiber for pulping is derived primarily as a by-product of sawlog/veneer log harvests and from the milling residues of lumber and plywood. Pulpwood prices have little impact on sawlog harvest decisions given the much higher values of saw and veneer logs. Thus, our model explicitly considers only the saw/veneer log market.

The general structure of the model is outlined below (the time subscript is suppressed, except where essential, to simplify the notation):

(A1) MAX Σ_{TIME} [Σ_{MILLS} Log demand willingness to pay(Receipts$_{MILLS}$, Capacity$_{MILLS}^{(i-1)}$)

- Capacity costs ($Maintenance^{(i)}_{MILLS}$, $Expansion^{(i)}_{MILLS}$)

- Transport costs (PLOT → MILLS)

- Harvest costs

- Planting and silvicultural costs

+ Carbon payments from CreditableCarbon on plots “in” the COSP

- Costs of COSP for “in” plots] $(1 + r)^{-TIME}$

+ Discounted Terminal Inventory Contribution (including COSP contributions)

Subject to:

(A2) All PLOTs must be allocated to some management regime in each period

(A3) ^{EA} planting ≤ area harvested in new and existing ^{EA} stands in a period – land area lost to development (for “out” PLOTs only)

(A4) $Harvest_{PLOT} = (Final\ Harvests_{PLOT} + Thinnings_{PLOT})^{EA} + (Partial\ Cuts_{PLOT})^{UA}$

(A5) $Harvest_{PLOT} \geq \Sigma_{MILLS}\ Shipments_{PLOT,\ MILLS} + Exports_{PLOT}$

(A6) $\Sigma_{PLOT}\ Shipments_{PLOT,\ MILLS} + Imports_{MILLS} + PublicHarvest_{MILLS} \geq Receipts_{MILLS}$

(A7) $Receipts_{MILLS} \leq Capacity_{MILLS}$

(A8) $Capacity^{(i)}_{MILLS,\ TIME} = Capacity^{(i)}_{MILLS,\ TIME-1}\ (1-\delta)^5 + Maintenance^{(i)}_{MILLS} + Expansion^{(i)}_{MILLS}$

(A9) CreditableCarbon(“in” PLOTs only) = Σ_{PLOT} Net Δ Carbon Stock + ω Harvested carbon in products - Leakage

(A10) Convexity Constraints (for piece-wise linearization of quadratic area under linear demand functions)

(A11) Non-negativity

where

$_{PLOT}$ subscript refers to the basic sample plot units that make up the inventory,

$_{MILLS}$ subscript refers to the individual mills or log processing units,

$^{(i)}$ and $^{(i-1)}$ superscripts refer to the capacity solution iteration number (see discussion below),

r is the discount rate,

δ is the capacity depreciation rate, and

ω is CAR's allowable fraction of harvested carbon going to product pools.

In the objective function (A1), willingness-to-pay, the integral of the area under the demand curves, is a function of log receipts and regional capacity (both endogenous). Constraints (A2) and (A3) are the standard accounting requirements for Johnson and Scheurman's (1976) models I and II. Harvests (A4) for each plot in each period are the sum of final harvests and thinnings from even-aged areas plus partial cuts from uneven-aged areas. Each plot in the sample represents a specific area in the inventory (determined by the plot's area expansion factor from the sample). In the solution, the area represented by each plot can be broken down into a number of even and uneven-aged treatments that may vary over time. The logs harvested from each plot may be shipped to mills within the region or exported (A5). Total exports are treated as exogenous. Receipts at mills (A6) comprise intra-regional log shipments from private lands, plus receipts from public lands, plus imports from outside the region. Imports are small and also treated as exogenous. Constraints (A7) and (A8) encompass the capacity model: receipts (mill output measured in units of log input) must be less than capacity (A7), capacity evolves over time according to the standard inventory identity (A8), with depreciation at a fixed rate (δ) and additions due either to maintenance activity or expansion.

Treatment of nonlinearities. Since the log demand functions are linear, the willingness-to-pay integral in (A1) is quadratic in mill receipts. This function also depends on mill capacity ($Capacity^{(i)}_{MILLS}$), which is endogenous and a further source of nonlinearity in the objective. The first nonlinearity was resolved by piece-wise linearization of the willingness-to-pay integral on receipts. Constraints (A10) insure convexity of these linearizations.

The potential nonlinear product of receipts and capacity was resolved by means of an iterative solution approach. In the first iteration, current capacity values in (A8) and in the objective function [$Capacity^{(i-1)}_{MILLS}$] are replaced by an estimate of equilibrium capacity in each period and the full problem (A1) – (A11) is solved. Revised values of capacity over the projection period are available from this solution [computed in constraint (A8)]. These are used, in turn, in the second iteration. This process continues until the changes in capacity between iterations fall below a small tolerance.

The model was coded in GAMS (Rosenthal, 2008) and used the CPLEX solver. Typical base case problems involved approximately 83,000 constraints and 2,810,000 activities.

The model explains log market behavior in only a single region, while products (such as lumber) are marketed at the national and international levels. Background assumptions/scenarios for prices and developments in these broader markets were derived from Haynes, et al. (2007). We used this older projection because it was based on an explicit macro-economic forecast and captures US housing adjustments during the recent recession.

Appendix references

Haynes R.W., Adams D.M., Alig R.J., Ince P.J., Mills J.R., Zhou X. 2007. The 2005 RPA timber assessment update. Gen. Tech. Rep. PNW-GTR-699. U.S. Department of Agriculture, Forest Service, Pacific Northwest Research Station, Portland, OR (212 p).

Rosenthal R.E. 2008. GAMS—A User's Guide. GAMS Development Corporation, Washington, DC (259 p).