


AN ABSTRACT OF THE THESIS OF

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Title DETERMINATION OF THE MAXIMUM LOADING OF
PARALLEL CONNECTED THREE-PHASE TRANSFORMERS WITH
UNEQUAL IMPEDANCE

Abstract approved


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The comparative values of impedance and resistance to reactance ratios are important parameters in determining the load capacity of an interconnection of power transformer, called a bank.

The full design capabilities of parallel or three-phase connected transformers cannot be utilized unless the following constraints are applied.

1. All transformer voltage ratios must be identical.
2. All transformer percent impedances must be equal.
3. The resistance to reactance ratios of all transformers must be equal.

Departure from these conditions involves either an uneconomical division of current, or a circulating current, both of which will lower the efficiency and decrease the load that the bank can carry without overheating.

A transformer bank whose individual transformers do not have equal percent impedances will have a load distribution that is unbalanced. The transformer with the lowest percent impedance will supply its full-rated MVA capacity while the other transformers are underloaded. This condition represents a loss in the capacity of the transformer bank.

Transformers having widely different impedance values can be made to divide their load in proportion to their rating by placing the proper impedance in series with those transformers that have low impedances.

The replacement of a damaged transformer in a bank will produce the best transformer load distribution if the following constraints are applied:

1. The transformer should have impedance and voltage ratio equal to those of the existing transformers in the bank.
2. The voltage ratios and the percent complex impedances should be equal to those of the existing transformers even though the KVA ratings are not equal.
3. When the resistance and/or the reactance values are different, they should be larger than those values of the existing transformers.

A difference in the voltage ratios of paralleled or banked transformers will produce a much greater reduction in load capacity than mismatched impedances. This condition should be avoided if at all possible.

DETERMINATION OF THE MAXIMUM LOADING
OF PARALLEL CONNECTED THREE-PHASE
TRANSFORMERS WITH UNEQUAL IMPEDANCE

by

SAWASDI PUIPUNTHAVONGTH

A THESIS

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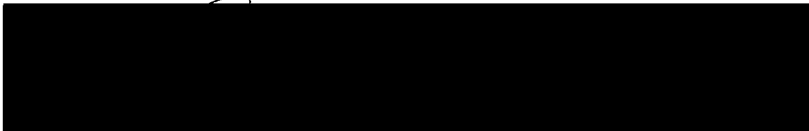
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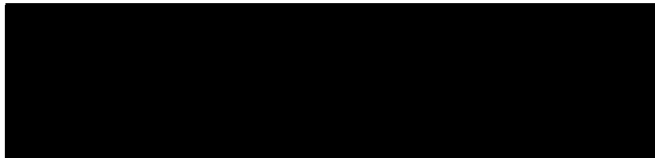
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DETERMINATION OF THE MAXIMUM LOADING OF PARALLEL CONNECTED THREE-PHASE TRANSFORMERS WITH UNEQUAL IMPEDANCE

INTRODUCTION

This thesis presents an analysis of the maximum loading of three-phase transformer banks whose impedances and resistance-reactance ratios are not equal. Digital computer programs were written and used for the calculation of the required data. The division of load between the transformers is also discussed.

A transformer bank whose individual transformers do not have equal impedance will have a load distribution that is unbalanced. The transformer with the lowest impedance will carry its full-rated MVA capacity when the bank is properly loaded. This condition represents a loss in the capacity of the transformer bank.

Transformers having widely different impedance values can be made to divide their load in proportion to their rating by placing the proper impedance in series with those transformers that have low impedances or by changing the transformation ratio. The ratio of transformation is adjusted to suit the working conditions by means of tap-changing. In this way the unequal voltage drops in the transformers of the bank can be made equivalently equal. However, due to the relatively large steps in voltage that are normally provided with transformer taps, this method is not recommended.

If either or both the voltage ratios or the impedances of a transformer bank are different, the circulating current due to the difference in ratio or impedance should be combined with each unit's share of the load current to obtain the actual total current in each winding.

In general a 10% mismatch in impedance between two banks of transformers arranged in parallel is considered acceptable (2,p. 1791).

American Standard No. ASA C57. 12a for transformer requirements specify that, "The impedance of a two-winding transformer shall have a tolerance of plus or minus 7.5% of the established value. Differences of impedance between two duplicate two-winding transformers, when two or more units of a given rating are produced by one manufacturer at the same time, shall not exceed 7.5% of the established value" (2, p. 1791). The requirement for three-winding and autotransformers differ from these requirements by permitting a tolerance of $\pm 10\%$.

This thesis will discuss two types of transformer banking when the impedances are not equal. A procedure for computing the maximum capacity of the following transformer bank connections will be presented:

- (1) Three single-phase dissimilar transformer in a three-phase connection.
- (2) Three dissimilar three-phase transformer banks in parallel.

FUNDAMENTAL EQUATIONS

In discussing transformers under load conditions, it is customary to neglect the existing current and the core loss power. These quantities will be assumed negligible in this thesis.

It will be assumed throughout this discussion that the load is connected to the secondary side of the transformer banks.

Since the investigation covers two main points, the equations used for each digital program are presented separately.

1. Equations for a delta-delta connected transformer bank.

The following equations are used to determine the maximum load that can be supplied by the transformer bank without exceeding the rating of any unit. The unbalance in current of the transformer bank is also obtained. It is assumed that the primary voltages and the secondary line currents are balanced. Let

$$E_A', E_B', E_C' = \text{primary voltages.}$$

$$E_A, E_B, E_C = \text{secondary voltages.}$$

$$I_a', I_b', I_c' = \text{primary line currents.}$$

$$I_a, I_b, I_c = \text{secondary line currents.}$$

$$I_A', I_B', I_C' = \text{primary winding currents.}$$

$$I_A, I_B, I_C = \text{secondary winding currents.}$$

$$N_A, N_B, N_C = \text{turns ratio, secondary to primary}$$

R_A, R_B, R_C = resistance in Ohms.

X_A, X_B, X_C = reactance in Ohms.

The resistance and reactance values are referred to the secondary side. The following are assumed conditions:

$$|E'_A| = |E'_B| = |E'_C|$$

$$\bar{E}'_B = \alpha^2 \bar{E}'_A$$

$$\bar{E}'_C = \alpha \bar{E}'_A$$

where α and α^2 are defined as unit complex operators.

$$\alpha = -0.5 + j0.866$$

$$\alpha^2 = -0.5 - j0.866$$

The symbol $\bar{}$ is used to indicate a phasor quantity.

$$|I'_a| = |I'_b| = |I'_c|, \text{ by assumption.}$$

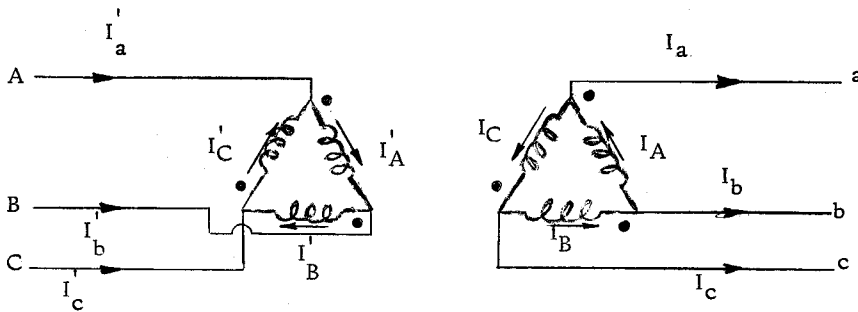


Figure 1. Delta-Delta Connection.

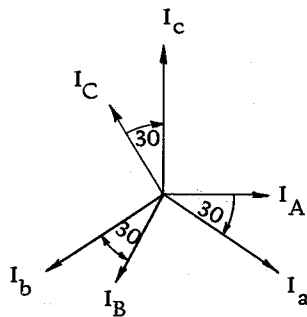


Figure 2. Phasor diagram for the secondary currents of a delta-delta bank under balanced conditions.

By Kirchoff's law, the secondary line currents from Figure

1 are:

$$\bar{I}_a = \bar{I}_A - \bar{I}_C \dots\dots\dots (1)$$

$$\bar{I}_b = \bar{I}_B - \bar{I}_A \dots\dots\dots (2)$$

$$\bar{I}_c = \bar{I}_C - \bar{I}_B \dots\dots\dots (3)$$

$$\bar{I}_A = \bar{I}_B - \bar{I}_b = \bar{I}_B - a^2 \bar{I}_a \dots\dots\dots (4)$$

$$\bar{I}_C = \bar{I}_B + \bar{I}_c = \bar{I}_B + a \bar{I}_a \dots\dots\dots (5)$$

Solving the above equations for \bar{I}_B in terms of \bar{I}_a ,

$$\bar{E}_A = N_A \bar{E}'_A - \bar{I}_A (R_A + jX_A) \dots\dots\dots (6)$$

$$= N_A \bar{E}'_A - (\bar{I}_B - a^2 \bar{I}_a) (R_A + jX_A) \dots\dots\dots (7)$$

$$\bar{E}_B = N_B \bar{E}'_B - \bar{I}_B (R_B + jX_B) \dots\dots\dots (8)$$

$$\bar{E}_C = N_C \bar{E}'_C - \bar{I}_C (R_C + jX_C) \dots\dots\dots (9)$$

$$= N_C \bar{E}'_C - (\bar{I}_B + a \bar{I}_a) (R_C + jX_C) \dots\dots\dots (10)$$

Now;
$$\bar{E}_A + \bar{E}_B + \bar{E}_C = 0 \dots\dots\dots (11)$$

By substituting equations (7), (8) and (10) into equation (11), the equation for \bar{I}_B becomes:

$$\bar{I}_B = \frac{(N_A + \alpha^2 N_B + \alpha N_C) E_A' + [\alpha^2 (R_A + jX_A) - \alpha (R_C + jX_C)] \bar{I}_a}{R_A + R_B + R_C + j(X_A + X_B + X_C)}$$

\bar{I}_B can be determined for any assumed value of \bar{I}_a by the substitution of \bar{I}_a and all known quantities into equation (11). Then \bar{I}_A can be found from equation (4) and \bar{I}_C from equation (5). By substituting the currents \bar{I}_A , \bar{I}_B , and \bar{I}_C into equations (6), (8) and (9) the transformer terminal potentials can be determined. Their phasor sum should equal zero.

The secondary transformer currents obtained by the above process are those winding current that will flow as a result of the assumed total load used for determining \bar{I}_a . It may be found that one or more transformers are overloaded; in which case the assumed load can be reduced by the greatest ratio of rated current to calculated current.

2. Equations for determining the parallel operation of three, three-phase transformer banks.

The equations derived for studying three, three-phase transformer banks in parallel are not the same as those equations for the three single-phase transformers. Thus it is necessary to derive a new equation so that the load supplied by each bank can be determined.

It will be assumed that the transformer banks differ not only in impedance but also in capacity.

The fundamental principles that control the behavior of three transformers operating in parallel are as follows:

(1). The voltage impressed on the primary winding of one transformer is equal to and in phase with that impressed on the primary winding of the other transformers.

(2). The voltage appearing at the secondary terminals of one transformer is equal to and in phase with that appearing at the secondary terminals of the other transformers.

From the first principle, it follows that the secondary open-circuit voltage of the three transformers are in phase with each other. The three secondary open-circuit voltages are not equal in magnitude unless the ratios of transformation are equal.

When the load is applied to the secondary side of the parallel transformer banks, the impedance drop in each transformer must be of such a magnitude and direction that the second consideration is satisfied. This is shown by the phasor diagram of Figure 3.

Let

\bar{I}_1 = primary current of transformer 1

\bar{I}_2 = primary current of transformer 2

\bar{I}_3 = primary current of transformer 3

\bar{I}_0 = total current

\bar{Z}_1 = equivalent impedance of transformer 1

\bar{Z}_2 = equivalent impedance of transformer 2

\bar{Z}_3 = equivalent impedance of transformer 3

All impedances are referred to the primary sides of the transformers.

\bar{V}_1 = primary equivalent of secondary voltage

\bar{E}_1 = primary terminal voltage

\bar{Y}_1 = admittance of transformer 1

\bar{Y}_2 = admittance of transformer 2

\bar{Y}_3 = admittance of transformer 3

\bar{Y}_0 = sum of the admittances

\bar{Z}_0 = equivalent impedance of banks.

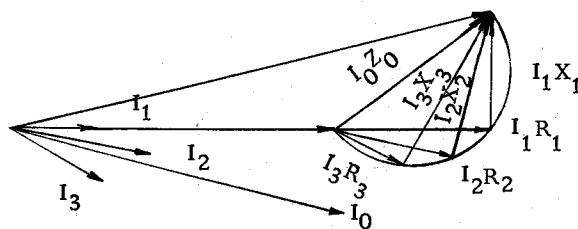


Figure 3. Phasor diagram for one phase of three transformer banks in parallel.

The solution of the general equations for parallel operation when the ratios of the transformation are equal, are as follows:

$$\bar{E}_1 - \bar{I}_1 \bar{Z}_1 = \bar{V}_1$$

$$\bar{E}_1 - \bar{I}_2 \bar{Z}_2 = \bar{V}_2$$

$$\bar{E}_1 - \bar{I}_3 \bar{Z}_3 = \bar{V}_3$$

$$\bar{I}_0 = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

$$= (\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3)(\bar{E}_1 - \bar{V}_1)$$

It is assumed that the total current will be known from an assumed load; thus the above equations can be solved for the common impedance drop.

$$(\bar{E}_1 - \bar{V}_1) = \bar{I}_0 / \bar{Y}_0$$

The current in each transformer is given by:

$$\bar{I}_1 = \bar{Y}_1 \bar{I}_0 / \bar{Y}_0$$

$$\bar{I}_2 = \bar{Y}_2 \bar{I}_0 / \bar{Y}_0$$

$$\bar{I}_3 = \bar{Y}_3 \bar{I}_0 / \bar{Y}_0$$

The above determined currents are the primary currents for each transformer bank relative to the assumed load. If an overload condition is indicated, the assumed load can be reduced by the greatest ratio of rated current to the calculated over load current for the transformer system.

METHOD OF ANALYSIS

The percent impedance of "identical" transformers may not be the same even though they have been made by the same manufacturer from the same specifications. This is because the impedance values do not depend entirely upon the voltage and current ratings.

The intent of this thesis is to illustrate and to discuss the division of load between these "identical" transformers when they are used in three-phase combinations. The load distribution data were calculated with an IBM 1620 digital computer. Actual transformer name plate ratings and impedances were used as the basic information to the computer program. The impedance data were then varied from the actual values to obtain the results for discussion. The limits of variation are beyond the $\pm 7.5\%$ as indicated in (2) only to illustrate concretely the effect of unmatched transformers. The variation in impedance values for the computer program was made in the following manner:

(1) The resistance component of the impedance of one transformer was varied above and below the given value. The inductive reactance value was held constant. The impedance values for all other transformers were also held constant.

(2) The inductive reactance component of impedance was varied above and below the given value. All other impedance values were

maintained constant at their given values.

(3) The value of impedance of one transformer was varied above and below the given value while holding the ratio of resistance to reactance constant. The impedance values of all other transformers were held constant as given.

a. Three single-phase transformers connected delta-delta in a three-phase bank

The variation of applied voltage or the effect of unequal turns ratios because of unequal transformer tap settings will produce results similar to those due to unbalanced impedances. This effect of unequal internal voltages is illustrated and discussed for the single three-phase transformer bank only.

The following transformer data were assumed as the initial conditions for determining the distribution of load in the transformer bank (1, p. 1597-1598).

Transformer	A	B	C
capacity	1000 KVA	1000 KVA	1000 KVA
primary voltage	33 KV	33 KV	33 KV
secondary voltage	2.3 KV	2.3 KV	2.3 KV
resistance on the			
low side	0.029 ohms	0.029 ohms	0.029 ohms
reactance on the			
low side	0.0328 ohms	0.0328 ohms	0.0328 ohms

Assumed load power factor was 0.8 lagging.

Results and Discussion

(1) The internal voltage was changed through the range of $\pm 2\%$ from the rated value to give sufficient data for discussion. Through this range the useful transformer loading decreases to about 60% of the bank rating. These results are shown in Figures 4 and 5.

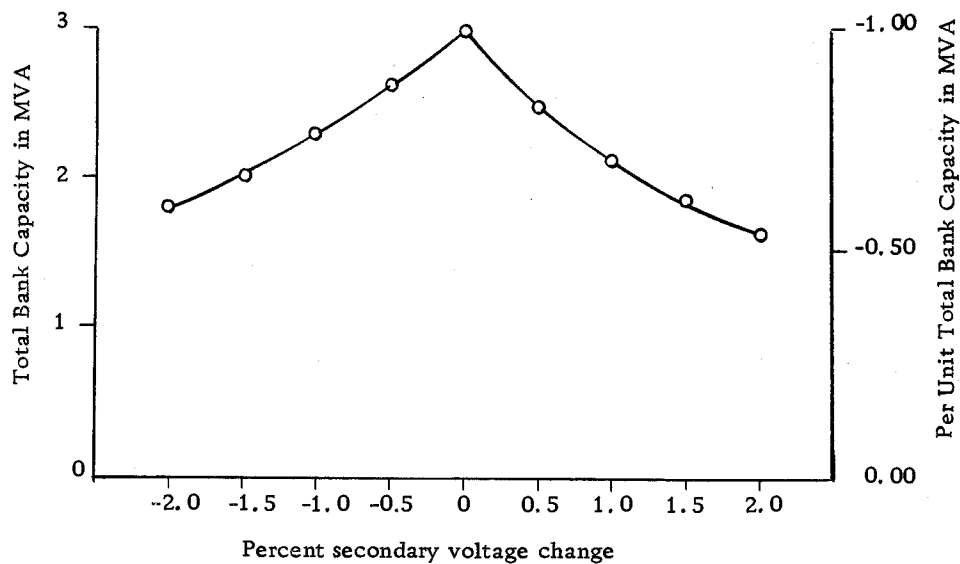


Figure 4. Transformer bank load capacity as a function of the change in the secondary voltage of one transformer.

Figure 4 shows the maximum total load in MVA that the transformer bank will supply without overloading any one transformer. For a given value of voltage change on one transformer (plus or minus), the condition of lower internal voltage will allow a higher maximum bank loading than for the same magnitude of voltage

increase. In either case, the amount of load supplied is approximately equal for the two transformers with normal voltage. Hence when these two transformers are near full load and supply the limiting condition, the total bank load will be larger than the total bank load when only one transformer reaches full load.

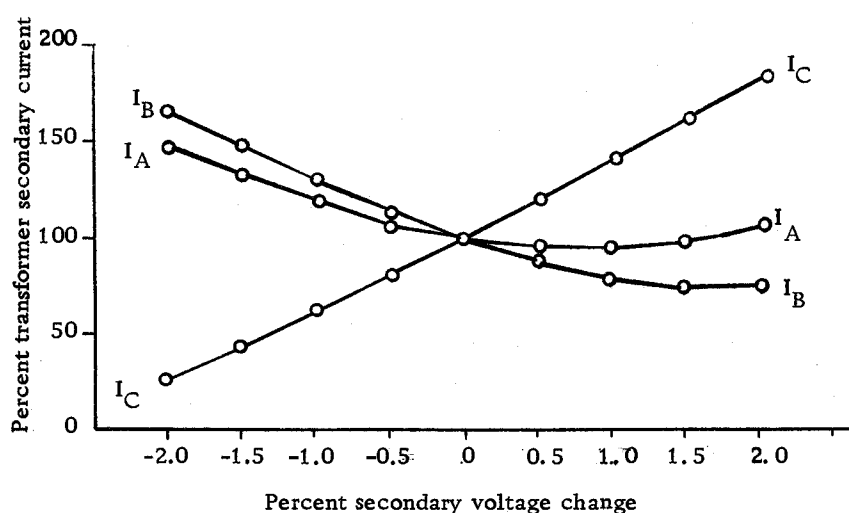


Figure 5. Transformer secondary current in percent as a function of the change in secondary voltage of one transformer with a constant load on the bank.

Figure 5 shows the secondary current in percent of rated current for each transformer if the bank supplied rated load. This figure indicates which transformer is supplying the greatest share of the total load and the approximate percent of the overload for that transformer. The total bank load must then be reduced by the same ratio as the maximum secondary current must be reduced to limit this

current to 100 percent. The unbalance of the currents in Figure 5 is due to the zero sequence current in the delta connected winding. This zero sequence current is equal to the sum of the transformer impedances divided into the phasor sum of the induced voltages in the transformer secondary winding.

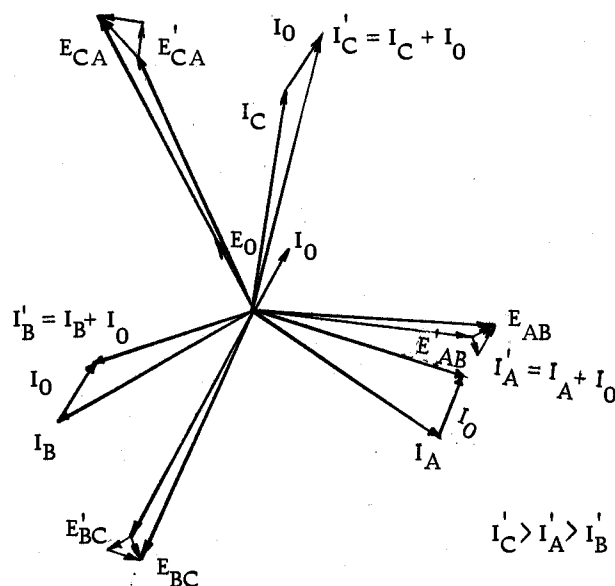


Figure 6. Phasor diagram of the secondary voltages and currents for unbalance applied voltages.

The phasor diagram, Figure 6, shows E_0 (zero sequence voltage) to be inphase with E_{CA} when the internal voltage of transformer C is increased, I_0 (zero sequence current) will lag E_0 by the impedance angle. When I_0 is added to the three symmetrical load currents, the resulting currents, I'_A , I'_B , and I'_C indicate the same relative magnitudes is shown on the right side of Figure 5.

(2) The effect of changing the resistance of one transformer in the three-phase bank while holding the inductive reactance value constant. The results are shown in Figures 7 and 8.

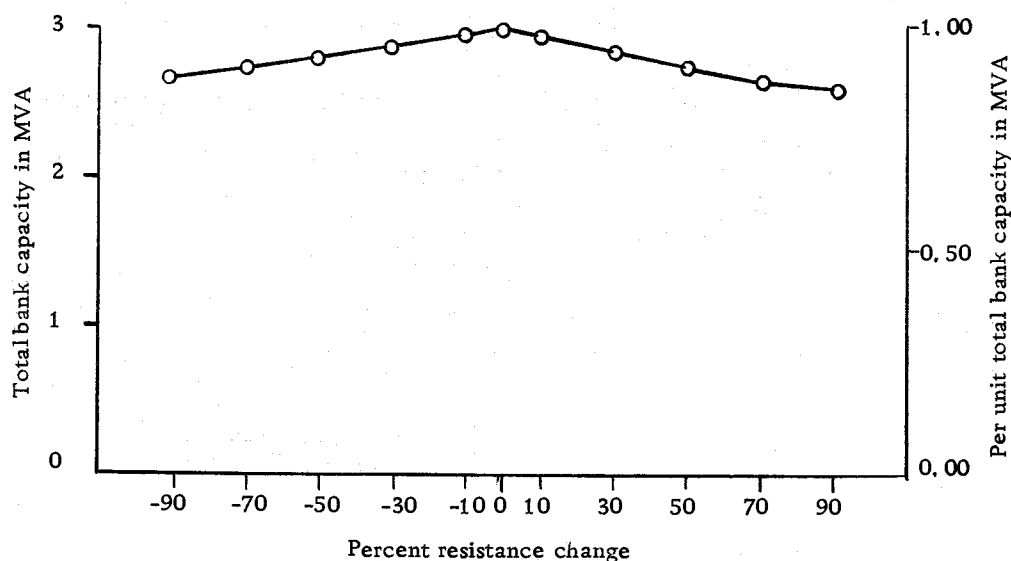


Figure 7. Transformer bank load capacity as a function of the change in resistance of one transformer.

Figure 7 shows the maximum total load in MVA that the transformer bank will supply without overloading any one transformer, as a function of the percent change in the resistance. This figure appears to have the same general shape as Figure 4 although the magnitude of change is not comparable. A 57% increase in resistance is required to produce the same reduction in load capacity (10%) as an increase of 0.25% in the induced voltage of one transformer of the bank. The transformer bank will supply a greater load when the

resistance is decreased than when the resistance is increased by the same amount above the normal value. This follows the same reason as given for Figure 4.

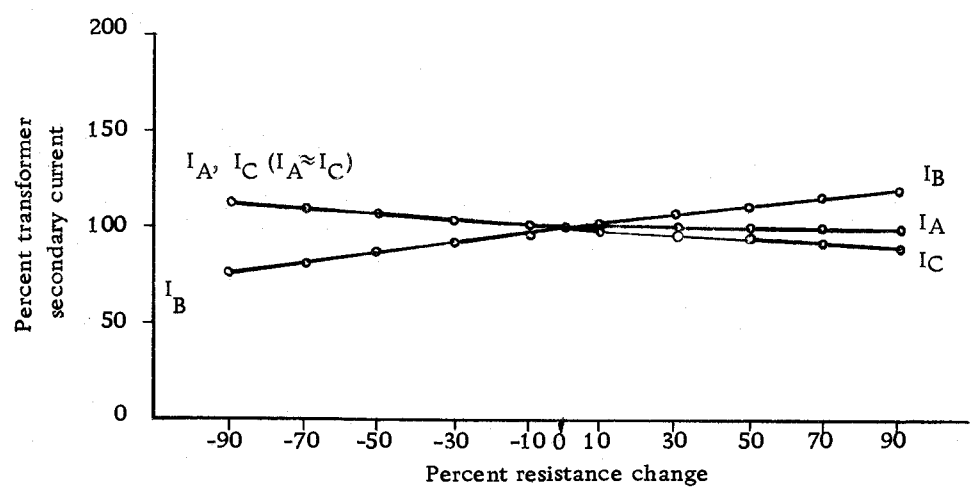


Figure 8. Transformer secondary current in percent as a function of the change in resistance of one transformer with a constant load on the bank.

Figure 8 shows the secondary current in percent of rated current for each transformer if the bank supplied rated load. This figure indicates that the bank capacity is limited by the load of transformer B for an increase in the resistance component of transformer C. However, for a decrease in resistance, the bank capacity is limited by transformer C. A comparison with the curves in Figure 5 shows that the above characteristics of Figure 8 are in reverse to those shown in Figure 5. This is due to the "reversal" of zero sequence current when the resistance is changed as compared to the

change in induced voltage. The "reversal" is not 180 degree, but the general direction is opposite. The actual magnitude and direction is controlled by the magnitude of the resistance change.

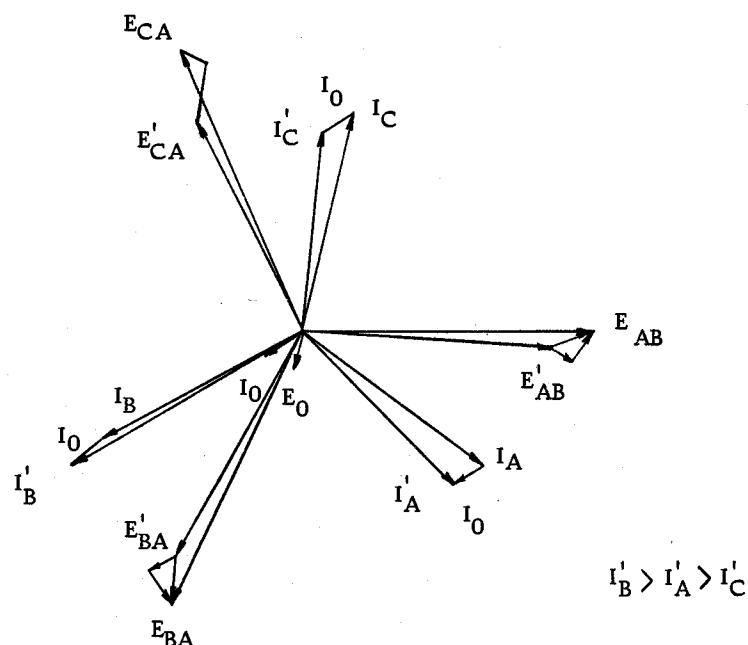


Figure 9. Phasor diagram of the secondary voltage and current for an increase in resistance of transformer C.

Figure 9 illustrates the resultant direction of I_0 when the resistance component of transformer C is increased. The magnitude and direction of E_0 is equal to the phasor sum of the terminal voltages E'_{AB} , E'_{BC} , and E'_{CA} .

(3) The effect of changing the inductive reactance of one transformer in the three-phase bank while holding the resistance value constant. The results are shown in Figure 10 and 11.

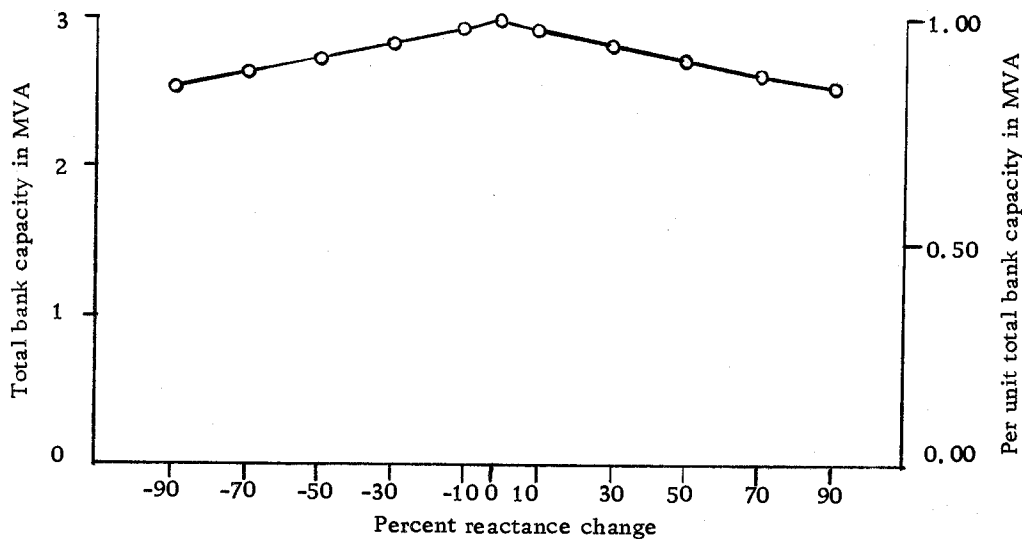


Figure 10. Transformer bank load capacity as a function of the change in inductive reactance of one transformer.

Figure 10 shows the maximum total load in MVA that the transformer bank will supply without overloading any one transformer. This figure appears to have the same shape as Figure 7 although the change in the magnitude is smaller. A 55% decrease in the inductive reactance is required to produce the same reduction in load capacity (10%) as a decrease of 73% in the resistance of one transformer of the bank. For the interval, plus and minus 60%, the transformer bank will supply a greater load when the inductive reactance is decreased than when it is increased by the same amount. Beyond this range the transformer bank will supply a greater load for an increase in the inductive reactance. This behavior follows the same

reason as given for Figure 4.

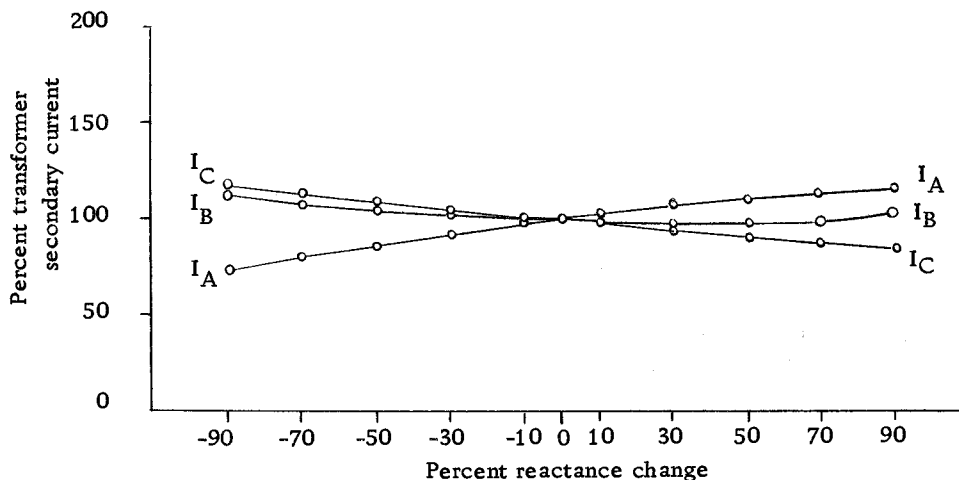


Figure 11. Transformer secondary current in percent as a function of the change in reactance of one transformer with a constant load on the bank.

Figure 11 shows the secondary current in percent of rated current for each transformer if the bank supplied rated load. This figure indicates that the bank capacity is limited by the load of transformer A for an increase in the inductive reactance component of transformer C. Although, for a decrease in the inductive reactance, the bank capacity is limited by transformer C. A comparison of the curves in this figure with those in Figure 8 show that the characteristics of the secondary currents of transformers A and B are reversed. This is due to the relative phasor directions of the zero sequence currents for the two conditions. This can be shown by a comparison of Figures 9 and 12. The actual magnitude and

direction of I_0 is controlled by the amount of the inductive reactance change.

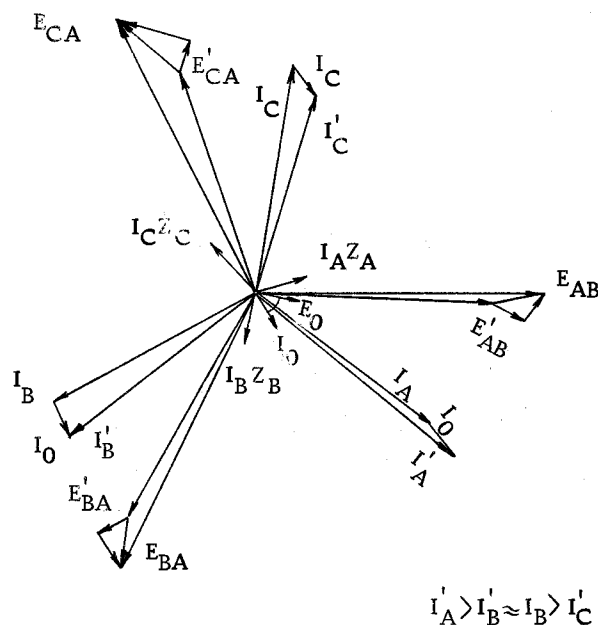


Figure 12. Phasor diagram of the secondary voltage and current for an increase in reactance of transformer C.

Figure 12 illustrates the resultant direction of I_0 when the inductive reactance component of transformer C is increased. The magnitude and direction of E_0 is equal to the phasor sum of the terminal voltages E'_{AB} , E'_{BC} , and E'_{CA} .

(4) The effect of changing the impedance of one transformer in the three-phase bank while holding the ratio of resistance to reactance constant. The results are shown in Figures 13 and 14.

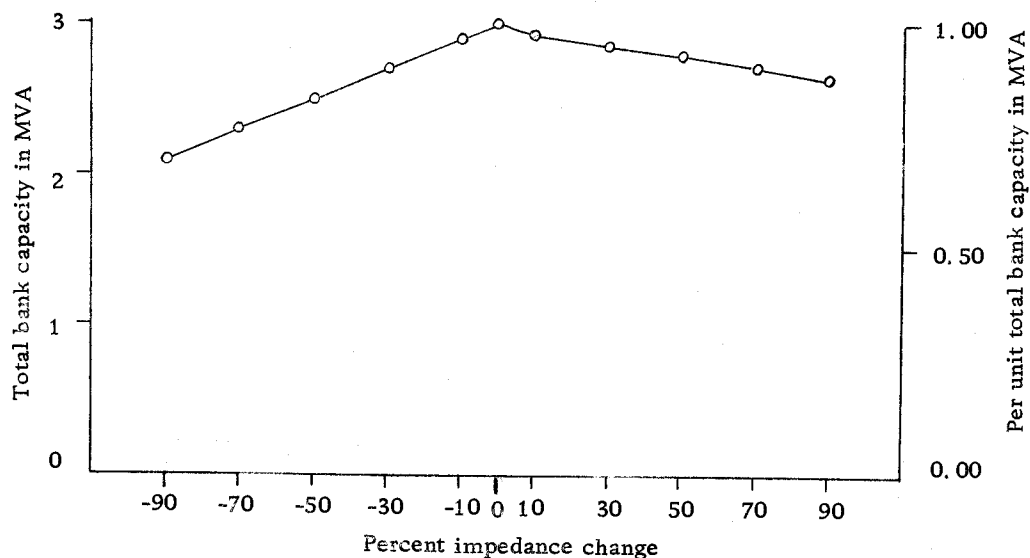


Figure 13. Transformer bank load capacity as a function of the change in the impedance of one transformer.

Figure 13 appears to have a different shape when compared to Figures 4, 7, and 10. A 74% increase in the impedance of transformer C is required to produce the same reduction in load capacity (10%) as a decrease of 30% in the impedance; or for a 55% change in the inductive reactance component only. The transformer bank will supply a greater load when the impedance is increased. This reason is the same as given for Figure 4.

Figure 14 shows that the bank capacity is limited by the load of transformer C when the impedance is decreased and by the load of transformers A and B for an increase in the impedance. This is due to the "reversal" of the zero sequence current phasor relative

to the direction of the impedance change.

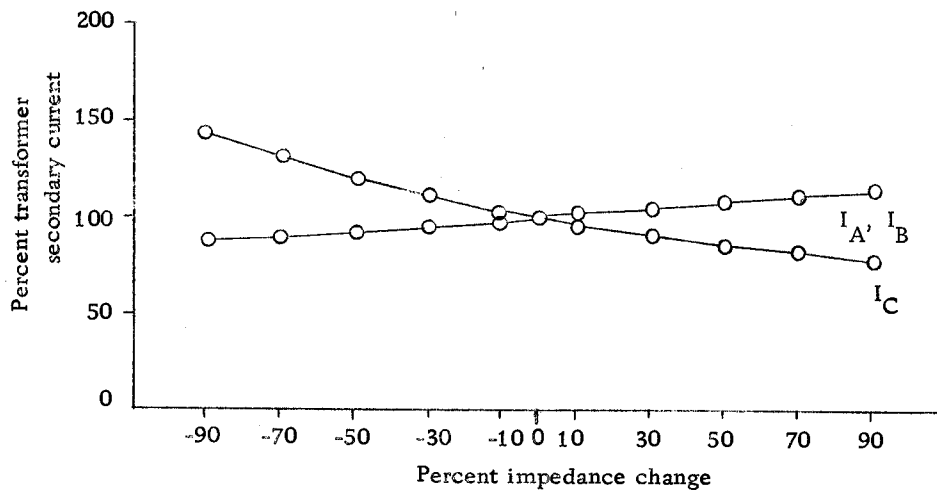


Figure 14. Transformer secondary current in percent as a function change in the impedance of one transformer with a constant load on the bank.

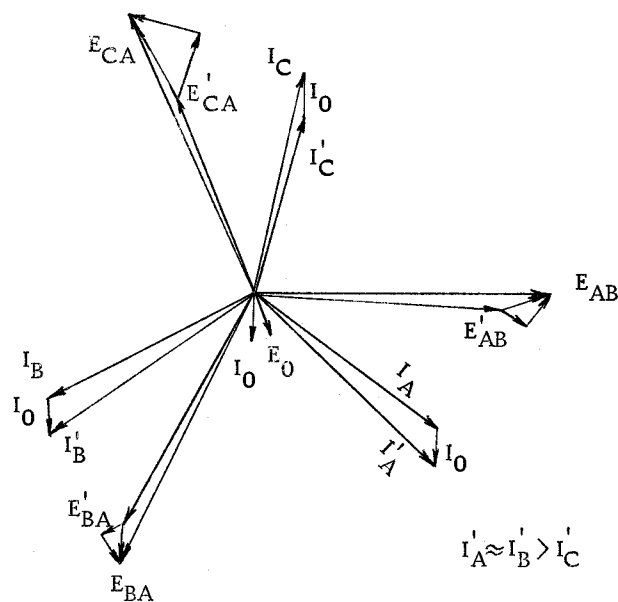


Figure 15. Phasor diagram of the secondary voltage and current for an increase in the impedance of transformer C.

b. Three three-phase transformer banks connected in parallel

The effect of unequal transformer impedance on transformer load division is illustrated and discussed in the following section.

The variation of transformer impedance is confined to all transformers of one bank of three paralleled three-phase banks. All transformer banks are assumed to be wye-wye connected with a common neutral bus. It is assumed that all loads are balanced.

The following transformer bank data were assumed as the initial conditions for determining the transformer load distribution (4, p. 370-372).

Part 1

Transformer Bank	No. 1	No. 2	No. 3
capacity	3, 500 KVA	3, 500 KVA	3, 1000 KVA
primary voltage	11 KV	11 KV	11 KV
secondary voltage	2.3 KV	2.3 KV	2.3 KV
resistance on the high side	2.2489 ohms	2.2489 ohms	0.9050 ohms
reactance on the high side	5.8994 ohms	5.8994 ohms	5.8555 ohms

Part 2

Transformer Bank	No. 1, No. 2, and No. 3
capacity	3, 500 KVA
primary voltage	11 KV
secondary voltage	2.3 KV
resistance on the high side	2.2489 ohms
reactance on the high side	5.8994 ohms

Assumed load power factor was 0.8 lagging.

Results and Discussion

Part 1. The rated capacity of transformer bank No. 3 is twice that of transformer banks No. 1 and No. 2.

(1) The effect of changing the resistance of transformer bank no. 3 while holding the inductive reactance value constant. The results are shown in Figures 16 and 17.

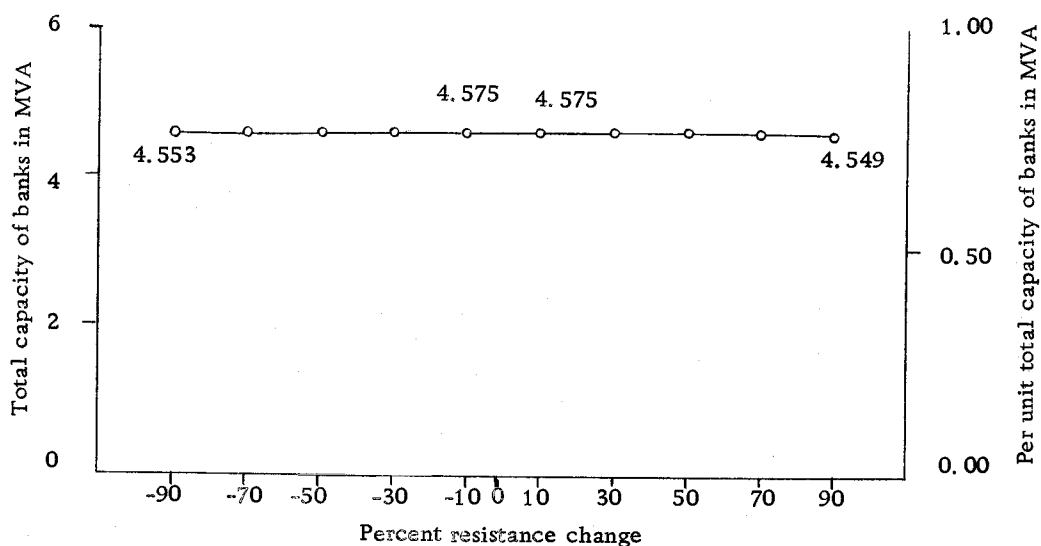


Figure 16. Transformer bank load capacity as a function of the change in the resistance of transformer bank No. 3.

Figure 16 shows that the increase in the percent resistance reduces the total load of the transformer banks. A decrease in resistance reduces the impedance of transformer bank No. 3, thus requiring additional current and load for this bank. This requirement

is to equalize the impedance drop in all banks since the turns ratios of all transformers are equal.

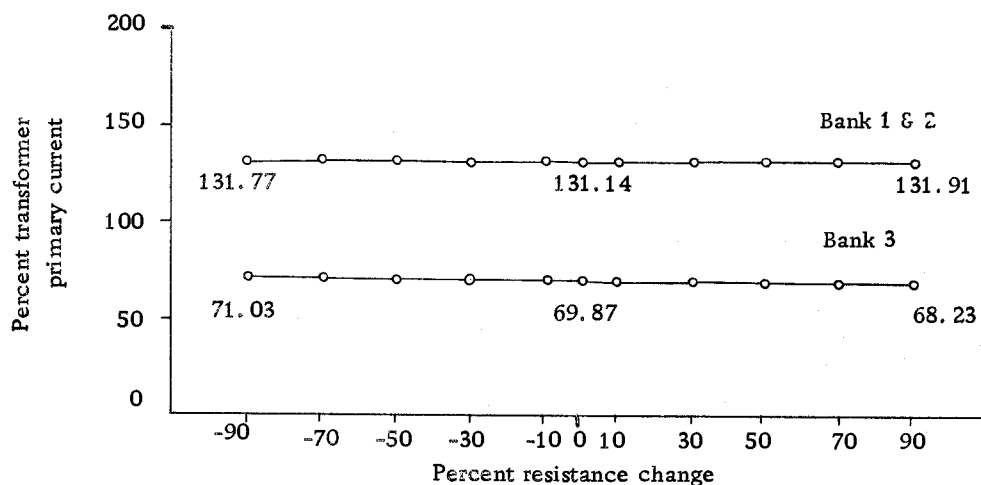


Figure 17. Transformer primary current in percent of rated current as a function of the change in the resistance bank No. 3

Figure 17 shows the primary current in percent of rated current for each bank when the transformer banks supply a load equal to the total MVA of the transformer system. This figure indicates that banks No. 1 and 2 are equally overloaded (above 130% load) for all values of resistance used in bank No. 3. For the conditions of these data the total load supplied by all banks together is 6 MVA. A reduction in MVA load to such a value that banks No. 1 and 2 are supplying their rated load value will reduce all currents by the same proportion. This is assuming the change in terminal voltage is negligibly small. The magnitude of this reduced load is shown in

Figure 16. The value of resistance in the transformers of bank No. 3 is approximately 15% of the reactance value; therefore, the change in this resistance over the range of $\pm 90\%$ of its value will not appreciably change the impedance of bank No. 3. Since the MVA rating of bank No. 3 is equal to the sum of the rating of banks No. 1 and 2, and the reactance values of all three banks are equal, bank No. 3 will be approximately 50% loaded and supply only one-third of the total load.

As shown in Figure 16, the maximum balanced load that this transformer system will supply is approximately 75% of the total rating of all transformers.

(2) The effect of changing the inductive reactance of transformer bank No. 3 while holding the resistance value constant. The results are shown in Figures 18 and 19.

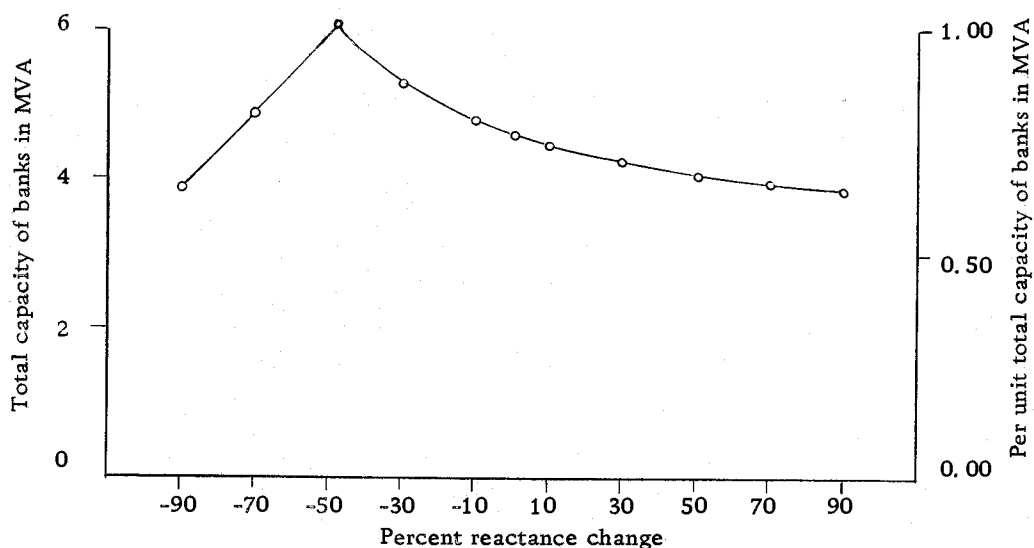


Figure 18. Transformer bank load capacity as a function of the change in the reactance of transformer bank No. 3.

Figure 18 indicates that the transformer banks will supply rated capacity when the inductive reactance of the third bank is decreased by 48% of the normal value. A 48% decrease of the inductive reactance will cause the impedance drop of transformer bank No. 3 to become equal to the impedance drops of the other two banks at its rated current.

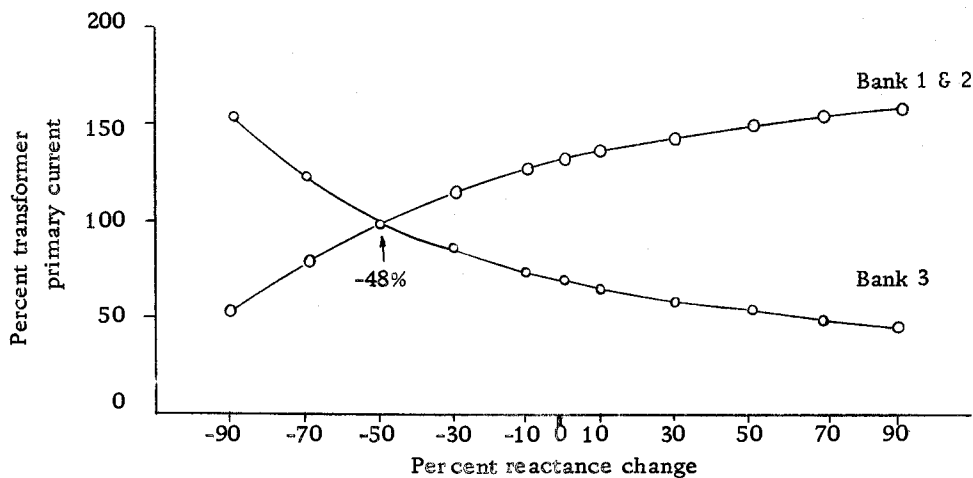


Figure 19. Transformer primary current in percent as a function of the change in the reactance of transformer bank No. 3.

Figure 19 shows that the magnitudes of the currents in each bank become equal to their rated value when the reactance of the third bank is decreased by 48%. The effect of the small comparative resistance of the third bank prevents the reduction of inductive reactance to 50% as might be indicated by the relative values of

MVA capacity and reactance.

(3) The effect of changing the impedance of transformer bank No. 3 while holding the ratio of the resistance to reactance constant. The results are shown in Figures 20 and 21.

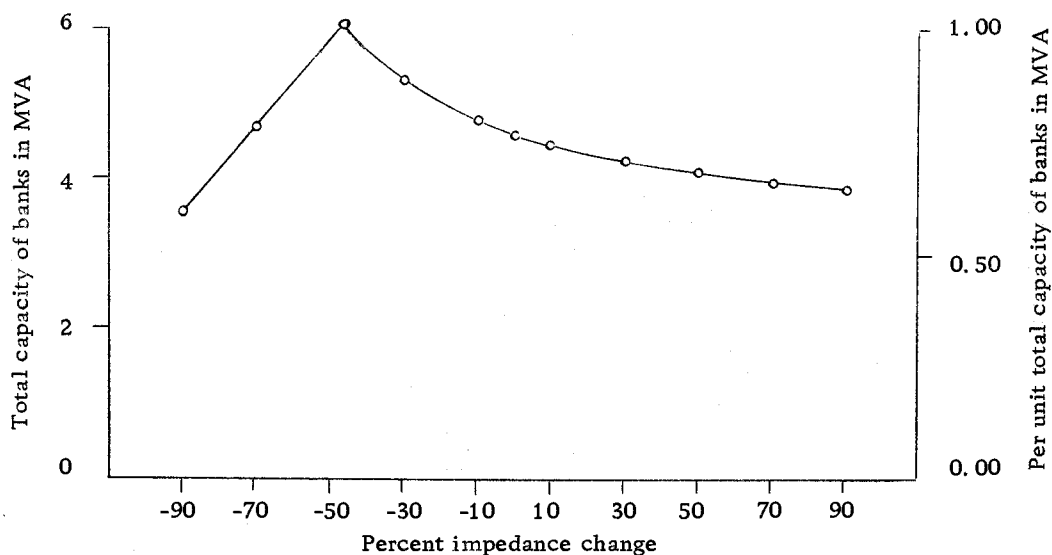


Figure 20. Transformer bank load capacity as a function of the change in the impedance of transformer bank No. 3.

Figure 20 shows that the transformer banks will supply their rated capacity when the impedance is decreased by 47% of the normal value. This figure has a shape similar to Figure 18 relative to MVA magnitude. A 67.5% decrease in the impedance of transformer bank No. 3 is required to produce the same reduction in load capacity (20%) as a decrease of 70% change in the inductive reactance component only.

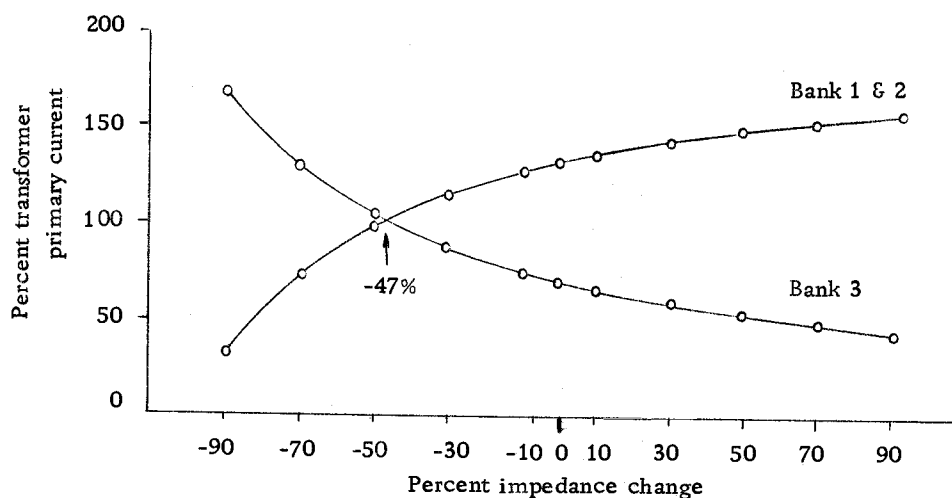


Figure 21. Transformer primary current in percent as a function of the change in the impedance of transformer bank No. 3.

Essentially the same information is conveyed by Figure 21 as for Figure 19. This is because the influence of the resistance component of the third bank is very small.

Part II. Each of the three transformer banks has the same rated capacity. The initial impedance of all transformers are equal. Figure 22, 24, and 26 show the maximum total load in MVA that the transformer banks will supply without overloading any one bank as a function of the percent change in the complex impedance of one bank. Respectfully, the figures represent the load characteristics for changes in resistance, reactance, and absolute value of impedance.

Figures 23, 25, and 27 show the corresponding load currents for each bank for the respective changes in complex impedance.

In each case, the load supplied by the transformer-system is equal in MVA to the sum of the MVA ratings of all transformers in the system. Thus, Figures 23, 25 and 27 indicate the percent load current of each bank with the same load for all illustrated conditions of the independent variable.

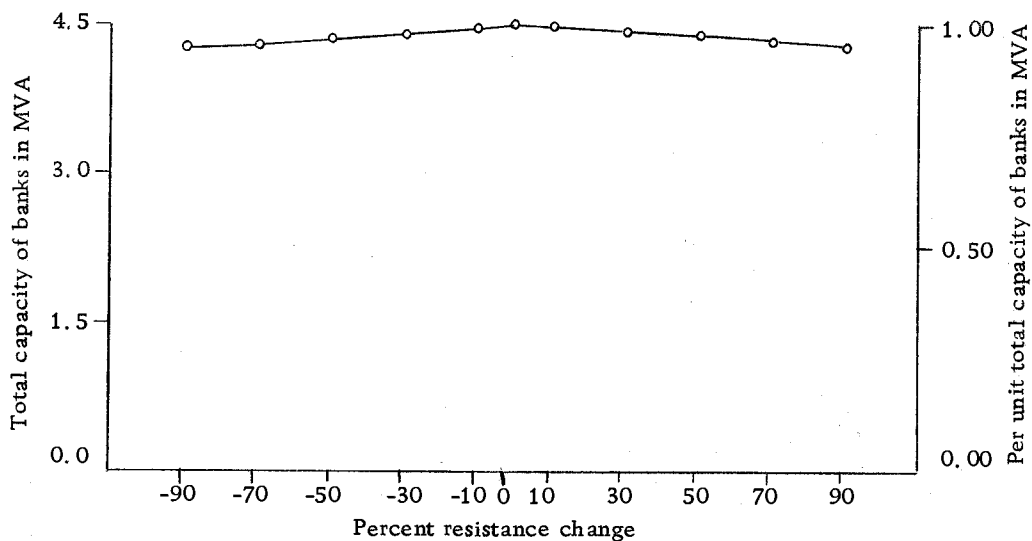


Figure 22. Transformer bank load capacity as a function of the percent change in the resistance of one transformer bank.

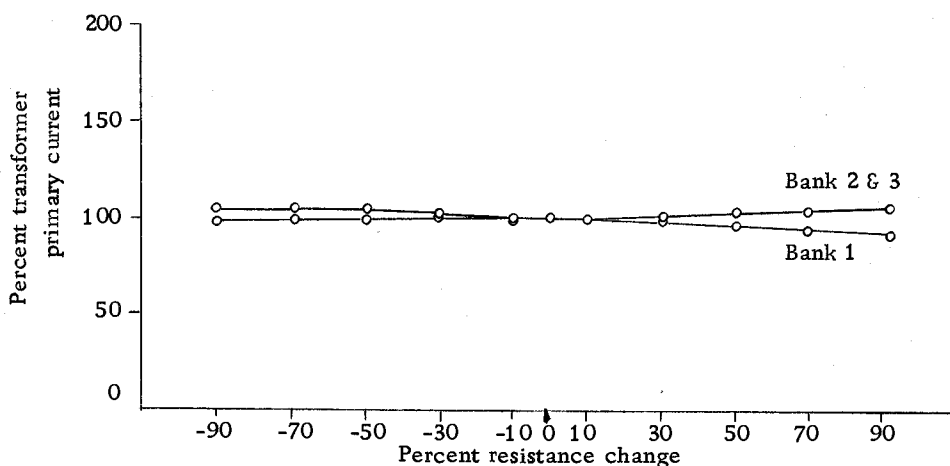


Figure 23. Transformer primary current in percent as a function of the percent change in the resistance of one transformer bank.

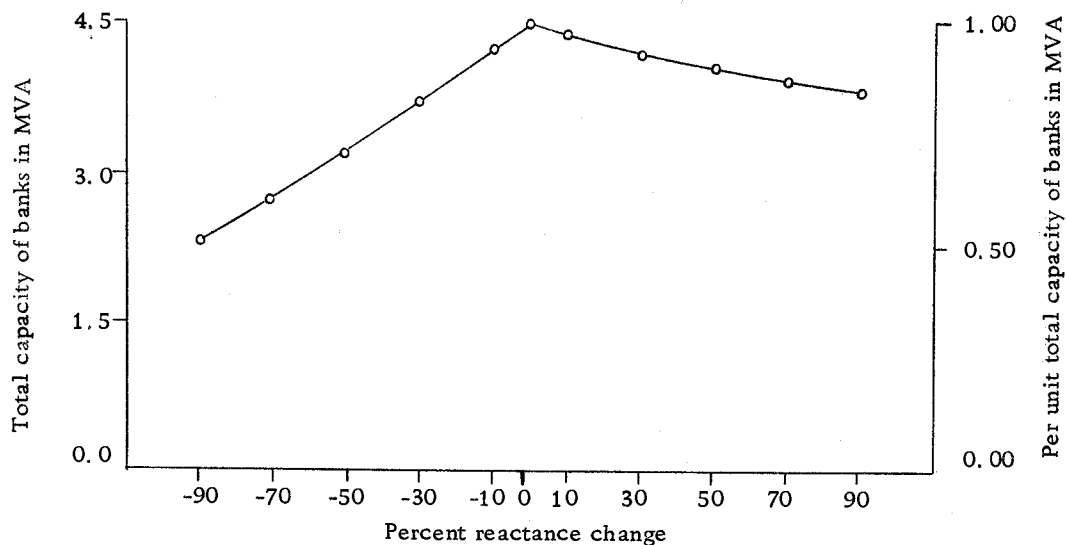


Figure 24. Transformer bank load capacity as a function of the percent change in the reactance of one transformer bank.

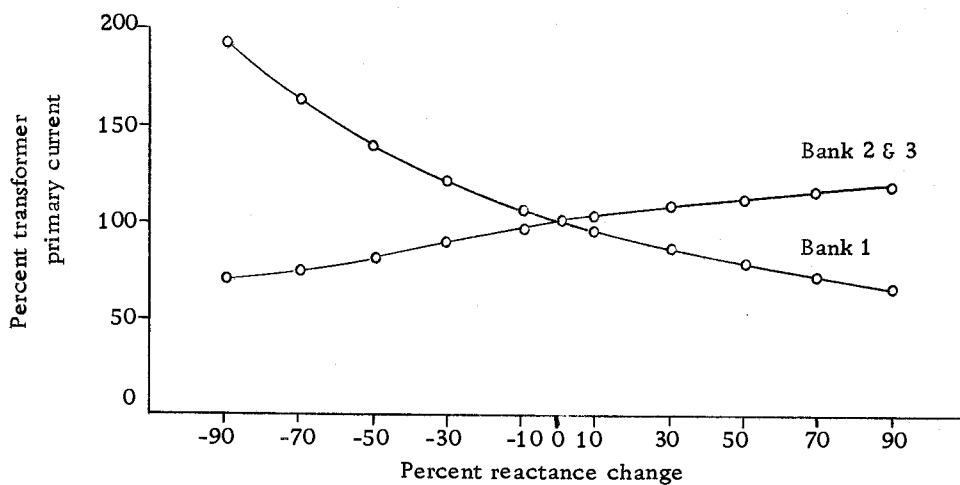


Figure 25. Transformer primary current in percent as a function of the percent change in the reactance of one transformer bank.

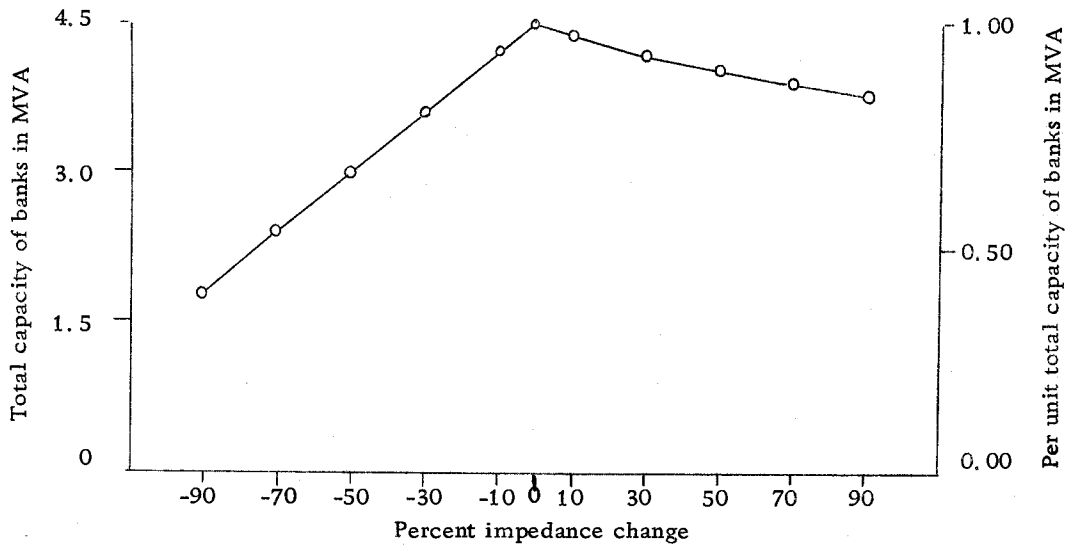


Figure 26. Transformer bank load capacity as a function of the percent change in the impedance of one transformer bank.

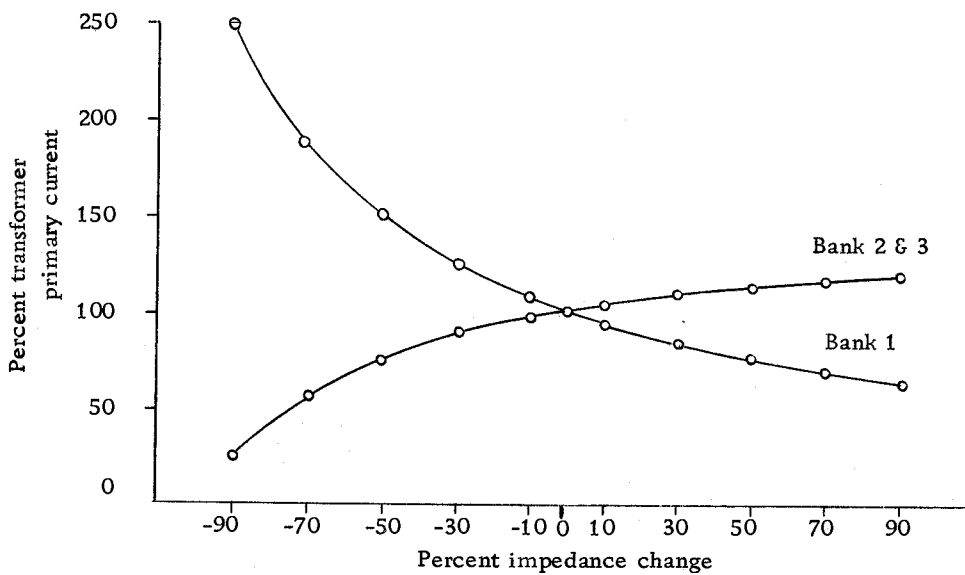


Figure 27. Transformer primary current in percent as a function of the percent change in the impedance of one transformer bank.

CONCLUSION

The comparative values of impedance and resistance to reactance ratios are very important parameters in determining the compatibility of power transformers when they are interconnected.

The full design capabilities of parallel or three-phase connected transformers can be realized only if the following constraints are applied.

- a. Their voltage ratios are identical .
- b. Their percent impedances must be equal.
- c. The resistance to reactance ratios must be equal.

Departure from these conditions involves either an uneconomical division of current, or a circulating current, both of which will lower the efficiency and decrease the load that the bank can carry without overheating.

The results indicate that the replacement of a damaged transformer in a bank should follow the preference list given below:

1. The transformer should have impedance and voltage ratio equal those of the existing transformers in the bank.
2. The voltage ratios and the percent complex impedances should be equal to those of the existing transformers even though the KVA ratings are not equal.
3. When the resistance and/or the reactance values are

different, they should be larger than those values of the existing transformers. The voltage ratios must be equal.

In either of the latter two recommendations, calculations should be made to determine the maximum load that can be supplied by the transformer banks without overloading any one transformer.

A difference in the voltage ratios of paralleled or banked transformers will produce a much greater reduction in load capacity than mismatched impedances. This condition should be avoided if at all possible.

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APPENDIX

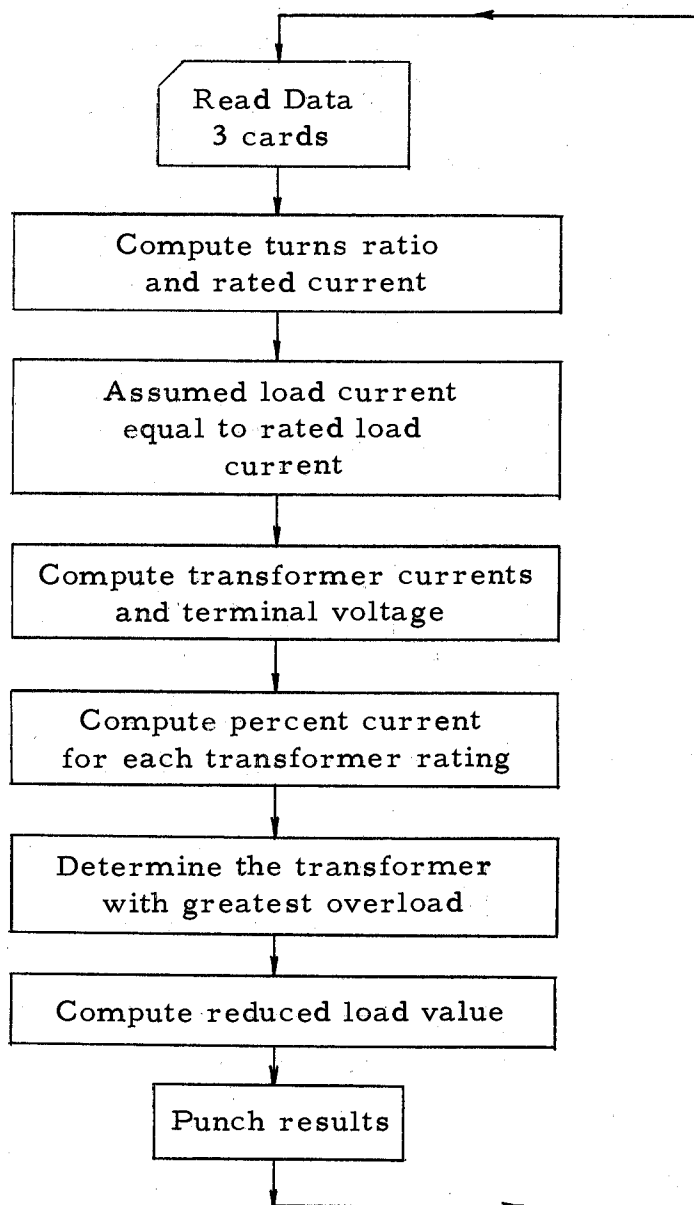


Figure 1A. Flow chart for computing the maximum transformer load for a three-phase Delta to Delta connected transformer bank.

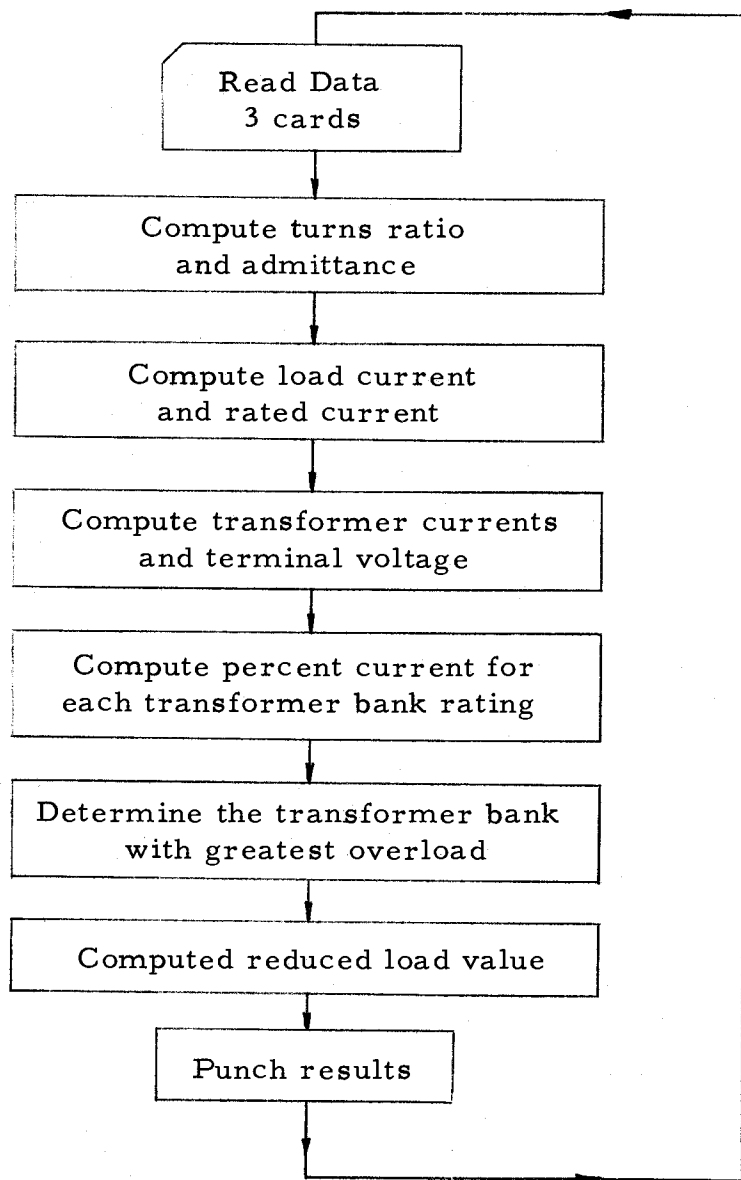


Figure 2A. Flow chart for computing the maximum transformer bank load when three, three-phase transformer banks are connected in parallel.