Zonal Momentum Balance at the Equator

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ABSTRACT

The conventional view of equatorial dynamics requires that the zonal equatorial wind stress be balanced, in the mean, by the vertical integral of "large-scale" terms, such as the zonal pressure gradient, mesoscale eddy flux, and mean advection, over the upper few hundred meters. It is usually presumed that the surface wind stress is communicated to the interior by turbulent processes. Turbulent kinetic energy dissipation rates measured at 140°W during the TROPIC HEAT I experiment and a production rate–dissipation rate balance argument have been used to calculate the zonal turbulent stress at 30 to 90 m depth. The calculated turbulent stress at 30 m depth amounts to only 20% of the wind stress, and decreases exponentially with depth below 30 m. Typical large-scale estimates of the zonal pressure gradient, mesoscale eddy flux, and advection have a depth scale larger than the turbulent stress, and are inconsistent with the vertical divergence of the stress as estimated from the dissipation rate measurements. It is concluded that either 1) the measured estimates of dissipation rate are too small, 2) the actual large-scale zonal pressure gradient, mesoscale eddy flux, and advection during our observation period were highly atypical and had a very shallow depth scale, 3) some process other than the simple diffusion of momentum through shear instabilities is transporting the momentum, or 4) the assumption of a production–dissipation balance in the turbulent kinetic energy budget is incorrect. The first two possibilities are unlikely.

1. Introduction

Two important features of the upper 100 m of the equatorial ocean are large vertical shear of zonal velocity and energetic turbulence. The Equatorial Undercurrent flows toward the east beneath the westward-flowing South Equatorial Current, and between these two currents, a vertical current shear as large as 0.02 s^{-1} is commonly observed. Recent measurements of equatorial turbulence were done as part of the TROPIC HEAT I experiment, during which one ship occupied an equatorial station from 19 November to 1 December 1984 (Moum and Caldwell 1985, hereafter MC), and another ship occupied a station a few tens of km distant from 25 to 30 November (Gregg et al. 1985, hereafter GPWOS). Both groups made vertical profiles 24 hours per day. Moum and Caldwell obtained 1749 good casts on station, and GPWOS collected 385.

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The large number of profiles collected in TROPIC HEAT I make possible the calculation of some elementary balances of the momentum budget in the upper 100 m. The most important contributions to the zonal momentum budget at the equator are the zonal convergence of zonal momentum, upwelling of eastward momentum, the zonal pressure gradient, the divergence of the mesoscale eddy flux, and the turbulent stress (Bryden and Brady 1985). Balancing these factors leads to a steady-state conservation equation

$$U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\partial}{\partial x}\overline{U'U'} - \frac{\partial}{\partial y}\overline{U'V'} - \frac{\partial \bar{F}_x}{\partial z}, \quad (1)$$

where U and V are mean zonal (positive eastward) and meridional (positive northward) velocities, P is mean pressure, ρ is density, x is the zonal direction (positive eastward), and F_x is the vertical turbulent flux of zonal momentum. By "mean", we intend an average over several months to years, and thousands of kilometers;

U' and V' are zonal and meridional velocity fluctuations in the mesoscale eddy band (i.e., fluctuations with time scales from a day to several weeks, length scales from tens of kilometers to hundreds of kilometers). Contributions of small-scale horizontal turbulent fluxes (i.e., turbulent fluxes with length scales less than tens of meters) are neglected on the grounds that horizontal velocity gradients are orders of magnitude less than vertical velocity gradients. The overbar indicates an average over several months to several years. It is assumed that the mean acceleration of fluid and the mean meridional velocity are small. The zonal balance of forces on a column of fluid of unit surface area extending from $z = a$ to $z = b$ is obtained by vertically integrating and rearranging (1):

$$\int_a^b \left\{ -\frac{1}{\rho}\frac{\partial P}{\partial x} - U\frac{\partial U}{\partial x} - W\frac{\partial U}{\partial z} - \frac{\partial}{\partial x}\overline{U'U'} - \frac{\partial}{\partial y}\overline{U'V'} \right\} \times dz - \bar{F}_x(b) + \bar{F}_x(a) = 0,$$

or

$$\mathrm{ZPG} + \mathrm{ZD} + \mathrm{UW} + \mathrm{EF} - \bar{F}_x(b) + \bar{F}_x(a) = 0 \quad (2)$$

where −ZPG is the integral of the zonal pressure gradient divided by density, −ZD is the integral of the zonal divergence of mean zonal momentum, −UW is the integral of the upwelling of zonal momentum, and −EF is the integral of the mesoscale eddy flux.

If the momentum flux in the fluid interior is due to turbulent processes, $-F_x(z)$ equals $\tau(z)/\rho$, where τ is the zonal turbulent stress; at the sea surface, $-F_x(0)$ equals τ_0/ρ, where τ_0 is the zonal surface wind stress. An eddy viscosity K_m is defined by

$$-F_x(z) = K_m\langle \partial u/\partial z\rangle, \quad z < 0, \quad (3)$$

where the angle brackets denote some averaging process: for example, an ensemble average, an areal average, or perhaps a Monte Carlo average. If a time average is used, it is an average over some arbitrary "short" time, longer than the time scale of turbulent fluctuations but shorter than the overbar average. For the measurements described below, hourly averages of turbulence quantities are used, although it is more correct to think of these as ensemble averages rather than time averages.

Bryden and Brady (1985, hereafter BB) made an order of magnitude estimate of K_m by assuming that the dominant term in (2) is ZPG, the zonal pressure gradient term. Using historical measurements of $\partial P/\partial x$ and wind stress, BB estimated a mean equatorial eddy viscosity at 75 m depth of $1.7 \times 10^{-3}\ \mathrm{m^2\ s^{-1}}$, an order of magnitude greater than has been estimated from dissipation measurements (GPWOS).

Our approach is to see if BB's results are consistent with MC's and GPWOS's measurements. We will use BB's estimates of the first four terms in (2), and use MC's estimates of $K_m(\partial u/\partial z)$ and the surface wind stress to evaluate the last two terms of (2). We seek to determine whether (2) can be balanced in this manner, and if not, which of the assumptions might be at fault.

In section 2, we review the dissipation method for estimating turbulent stresses. In section 3, we evaluate both sides of (2) and are unable to find equality. At first sight, this approach seems peculiar. BB's estimates are for the mean circulation over a wide longitude range, while MC's turbulence data is for a single 12-day period at one location several years after BB's estimates were made. We do expect, however, that the basic physical principles embodied in BB's estimates are not far from wrong, that the estimates can be viewed as "typical" estimates, and the difference between BB's estimates and the actual realization can be viewed as an "anomaly." In section 4, we ask whether or not the zonal momentum budget residual can be explained by a *reasonable* anomaly in large-scale terms, and examine the consequences if reasonable anomalies cannot be found.

2. The dissipation method

Estimates of eddy viscosities, diffusivities, and the turbulent fluxes of momentum and heat have been made using measurements of ϵ, the turbulent kinetic energy dissipation rate (e.g., Osborn 1980; Crawford 1982; GPWOS). The dissipation rate is calculated from measurements of high wavenumber vertical shear (vertical scale roughly between 0.01 and 1 m). Flux estimates are based on the turbulent kinetic energy equation (Wyngaard and Cote 1971; Osborn 1980; Dillon 1984, or any standard text):

$$\frac{\partial}{\partial t}\langle q^2\rangle + W\frac{\partial}{\partial z}\langle q^2\rangle = \mathcal{P} - \epsilon + B - \frac{\partial}{\partial z}\{\langle w'p'\rangle/\rho + \langle w'q^2\rangle\}. \quad (4)$$

Here, q^2 is the turbulent kinetic energy, W is the mean vertical velocity, and $\mathcal{P}$ is the rate of production of turbulent kinetic energy by shear instabilities,

$$\mathcal{P} = -\langle w'u'\rangle\langle \partial u/\partial z\rangle - \langle w'v'\rangle\langle \partial v/\partial z\rangle; \quad (5)$$

ϵ is the rate of kinetic energy dissipation, $B = -(g/\rho)\langle w'\rho'\rangle$ is the buoyancy flux, u', v' and w' are zonal, meridional, and vertical velocity fluctuations, and p' is a pressure fluctuation. In obtaining (4), it has been assumed that gradients of terms like q^2, $\langle p'u_i'\rangle$, and $\langle w'q^2\rangle$ are much larger in the vertical than in the horizontal because the forcing processes are, in an average sense, horizontally uniform over the region being considered.

The next step in using (4) to obtain eddy viscosities is to assume the time scale for changes in the shear production is long compared to the time scale for adjustment of the fluctuation field; i.e., that $\partial q^2/\partial t$ is small compared to $\mathcal{P}$. This assumption makes no statement about the state of the turbulence in any single observation. *In any single realization of a particular overturning event,* $|\partial q^2/\partial t|$ may be much greater than $\mathcal{P}$ and the fluctuation field may be rapidly growing or rapidly decaying, yet the steady-state assumption is reasonable, provided only that enough samples are included in the average. An analogy may be useful here; the ensemble average of sea level changes slowly, even though any instantaneous measurement at a point may show rapid variations.

The second term on the left of (4) is the advection of turbulent kinetic energy by a mean vertical velocity W. In many parts of the ocean this term may be safely neglected because W is small, but in equatorial waters, there is a net upwelling, with W in the range of 1 to 3 meters per day (BB). We can scale $W\langle\partial q^2/\partial z\rangle$ as Wq^2/h, where h is a typical equatorial mixing layer depth, say 50 m. Since ϵ scales as q^3/l, where l is the typical size of an overturn (say, 1 to 5 m), we estimate that $W\langle\partial q^2/\partial z\rangle/\epsilon \approx (Wl)/(hq) \approx 10^{-5}$. This term can be safely neglected.

Osborn (1980) argues that the last two terms on the right of (4) are divergences and hence can affect only the distribution of q^2 in space, not the total amount of turbulent energy, for stably stratified turbulent flows. This argument may be more nearly correct for the divergence of $\langle w'q^2\rangle$ than for $\langle w'p'\rangle/\rho$, because the former term is just the advection of turbulent kinetic energy by turbulent vertical velocity fluctuations, while the latter term is the internal wave energy flux. There is no a priori reason to believe that the amount of energy radiated away by internal waves is small, but we shall for the present assume that it is, and will discuss later the consequences of this assumption.

We are left with the common assumption that the rate of production of turbulent kinetic energy is balanced by its rate of dissipation plus the buoyancy flux. Experiments in the laboratory and in the atmospheric boundary layer (Osborn 1980 reviews some of these) indicate that the flux Richardson number, $R_f = B/\mathcal{P}$, is small, probably never exceeding 0.2. On these grounds, we neglect B in (4), and arrive at a production–dissipation balance:

$$\mathcal{P} = \epsilon. \tag{6}$$

If we define an eddy viscosity by

$$\langle u'w'\rangle = -K_m\langle\partial u/\partial z\rangle,\ \langle v'w'\rangle = -K_m\langle\partial v/\partial z\rangle, \tag{7}$$

we can use (5), (6), and (7) to obtain the eddy viscosity and zonal shear stress in terms of measured dissipation rates:

$$K_m = \frac{\epsilon}{\langle\partial u/\partial z\rangle^2 + \langle\partial v/\partial z\rangle^2}, \tag{8}$$

$$-F_x = -\langle u'w'\rangle = K_m\langle\partial u/\partial z\rangle = \frac{\epsilon\langle\partial u/\partial z\rangle}{\langle\partial u/\partial z\rangle^2 + \langle\partial v/\partial z\rangle^2}. \tag{9}$$

3. Equatorial turbulence estimates

Turbulent kinetic energy dissipation rates were determined from measurements made using the Rapid Sampling Vertical Profiler (RSVP; Caldwell et al. 1985) and were averaged over one-hour time intervals and 12 m depth intervals. The zonal shear was measured using a hull-mounted Acoustic Doppler Current Profiler (ADCP) and averaged over the same time and depth intervals (the depth averaging is an inherent limitation to the ADCP; our averaging interval was chosen to obtain independent shear estimates). An evaluation of the ADCP measurements made from the R/V *Wecoma* during the TROPIC HEAT I experiment may be found in Chereskin et al. (1986). Here K_m and $-F_x$ were calculated from the one-hour average dissipation rates and shears, and then averaged over the entire 12-day period. GPWOS calculated 4-day average values for ϵ and then used an estimate of the mean shear based on eight XCP casts.

Moum and Caldwell's dissipation rate measurements begin at 10 m depth and extend to 110 m. The 12-day average dissipation rate is nearly constant in the upper 30 m, but falls off exponentially in the 30 m to 70 m depth range (Fig. 1). The upper ocean is continuously stratified. The 12-day average buoyancy frequency is lowest near the surface and increases exponentially from 1 cph at 10 m depth to about 7 cph at 60 m depth; from 60 to 110 m, the buoyancy frequency is roughly constant at 7 to 8 cph. The zonal velocity measure-

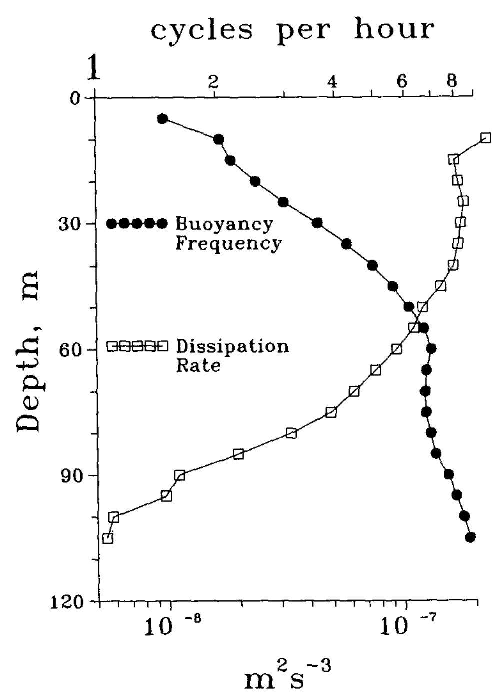

FIG. 1. Average buoyancy frequency (circles) and kinetic energy dissipation rate (squares) measured from R/V *Wecoma* during TROPIC HEAT I, 19 November to 1 December 1984, by MC.

ments from R/V *Wecoma* (Fig. 2) showed a consistent mean shear of about 0.017 s^{-1} over the upper 100 m.

The estimates of K_m and $-F_x$ using (8) and (9) begin at 33 m because that is the shallowest depth at which the shear could be independently determined from ADCP measurements. The eddy viscosity and momentum flux are largest nearest the surface, and monotonically decline with increasing depth (Figs. 3, 4). GPWOS dissipation rates are somewhat larger than those measured by MC (Fig. 1); the precise reason for the difference is unknown, but may be a result of different algorithms for calculating the dissipation rate. The shear estimates used by GPWOS are from eight XCP profiles made during their 4-day station and these do not agree in detail with the overlapping 4-day average from the MC dataset. The MC and GPWOS eddy viscosities are surprisingly small; BB's estimate at 75 m is 20 times larger than the eddy viscosity estimated from MC's data using (8).

Wind stress was calculated from R/V *Wecoma* observations. Wind speed and direction were recorded at 2-minute intervals, and first averaged over an hour. Atmospheric stability was determined from surface heat flux estimates using measured total downward solar radiation, the measured sea surface temperature, and humidity (measured with a dewpoint hygrometer). Large and Pond's (1981) iterative bulk-aerodynamic procedure, which provides drag coefficients corrected for atmospheric stability and sensor height, was used to calculate the zonal stress. The hourly stress was then

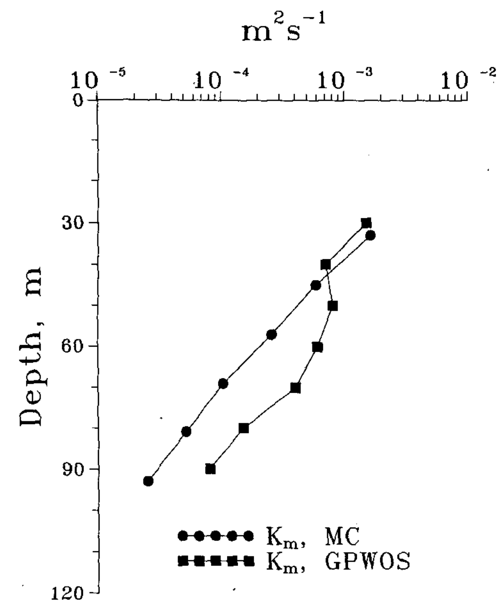

FIG. 3. Eddy viscosity estimates, K_m, vs depth, from MC (circles) and GPWOS (squares). MC's estimates were made from Eq. (8) using hourly averages of dissipation and velocity shears, then averaged over the 12-day station. Confidence limits for MC's estimates of ±1 standard deviation are no more than twice the size of the symbol; they were found from a "bootstrap" test using the parent population of the 288 hourly-averaged samples at each depth bin. Distributions of bootstrapped means are approximately Gaussian, and 68% of the values lie within the confidence limits.

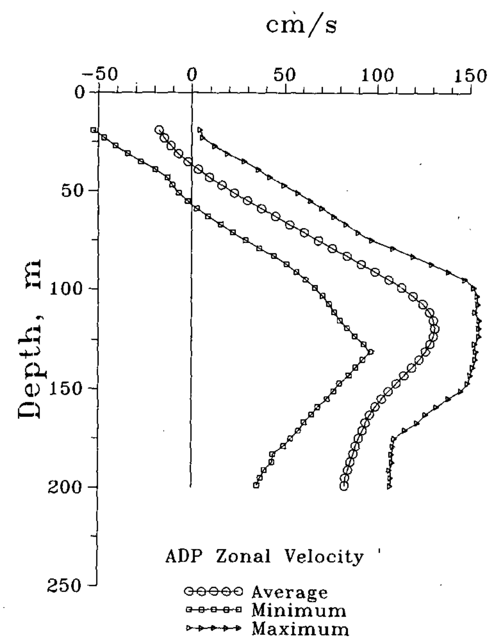

FIG. 2. The zonal velocity average (circles), minimum (squares) and maximum (triangles) measured from R/V *Wecoma* by Acoustic Doppler Current Profiler, 19 November–1 December 1984, by MC.

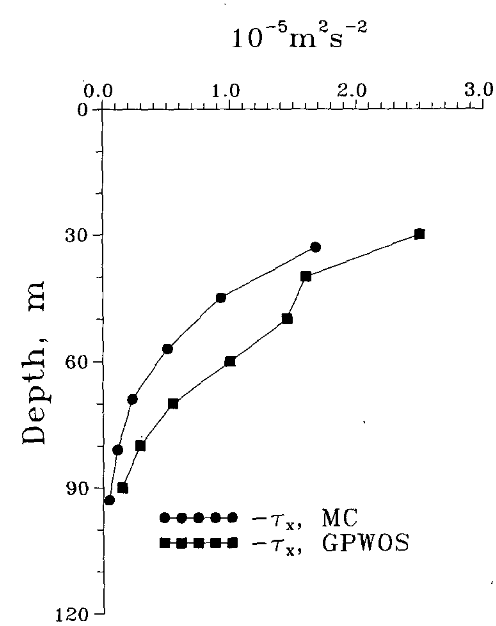

FIG. 4. Zonal component of the turbulent momentum flux estimated using Eq. (9) from MC (circles) and GPWOS (squares). For MC's data, the dissipation rate was first averaged over an hour, and combined with hourly-averaged ADCP shears; the hourly momentum flux was then averaged over the 12-day station.

used to calculate τ_0, the average zonal stress. For the 12-day station, $\tau_0 = 0.097$ N m^{-2}. The standard deviation was 0.030 N m^{-2}, and the sampling error in the mean is 0.002 N m^{-2}.

Large and Pond (1981) found good agreement between stability-corrected bulk stress estimates and direct eddy-correlation measurements. They list an uncertainty of 30% for daily average bulk estimates, with the largest discrepancies noted when winds were rapidly rising or falling; others have noted deficiencies in the bulk method when the relative wind/wave direction changes. During the TROPIC HEAT I experiment, the wind speed was nearly constant over the 12 days spent on station, ranging between 6 and 12 m s^{-1} (see Moum and Caldwell 1985 for a plot), with no rapid changes of direction, and an essentially infinite fetch. We therefore expect the *Wecoma* bulk estimates to have an uncertainty smaller than 30%; conservatively, we place the systematic uncertainty at 20%, and round the stress estimate to 0.10 N m^{-2}.

Surface stress must be continuous across a fluid boundary, assuming surface tension is negligible. Much of the surface stress can be taken up by the surface wave field in cases where the fetch is small, because waves can transport momentum away from the vicinity of the measurements. This is not the case, however, if the fetch is large. If surface wave momentum transport were an appreciable part of the momentum budget for infinite fetch, the growth of the wave field would be unbounded. When the fetch is large, as in the tropical Pacific, surface wave transport must play a small role in the average zonal momentum budget.

Surprisingly, the 12-day average momentum flux in the interior of the ocean, $F_x(z)$, is reasonably well described by

$$-F_x(z) = \tau_0/\rho e^{z/z_\tau} \qquad (10)$$

in the 30 to 90 m depth range. The shallowest estimate (33 m) of the turbulent momentum flux is much less than τ_0/ρ. The shear above 100 m does not vary much with depth, and the eddy viscosity and dissipation rate, as well as F_x, have an approximately exponential depth dependence. We performed a least-squares fit to MC's turbulent stress measurements, and found the scale depth for the stress divergence to be small: $z_\tau = 18 \pm 3$ m; the intercept of the fit at the surface was 0.11 N m^{-2}, about the same as the surface wind stress (Fig. 5). When the GPWOS momentum flux measurements were least-squares fitted to an exponential shape, we found essentially the same intercept 0.12 N m^{-2}, and a slightly larger depth scale, $z_\tau = 22$ m. We have no *dynamical* reason why ϵ, K_m, and F_x should be approximately exponential; rather, an exponential profile is simply a convenient way of summarizing the measurements. The fact that (10) intercepts the sea surface at the measured surface stress is probably circumstantial. The crucial point is that the measured profiles have a surprisingly small effective depth scale, and if the

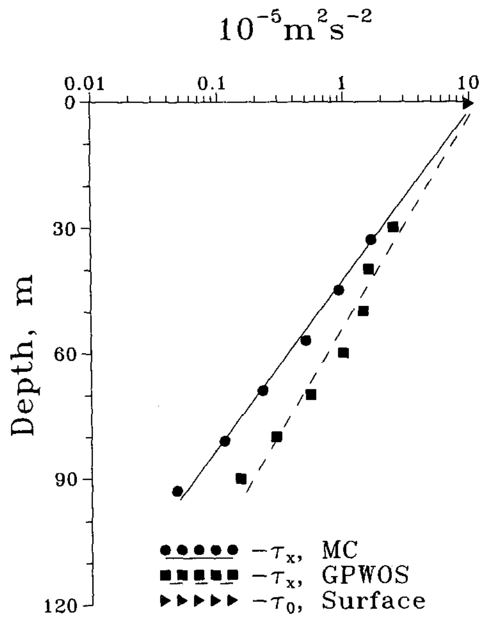

FIG. 5. Logarithmic plot of turbulent stress estimates from MC (circles) and GPWOS (squares). Lines are a least-squares exponential fit to MC (solid line) and GPWOS (dashed line) stress estimates. Intercept of fitted lines converge to the surface stress estimated from wind measurements (single triangle at the surface). The *e*-folding depth is 18 m for MC's stress and 22 m for GPWOS's stress.

surface stress is carried downward exclusively by turbulent processes, most of the surface stress does not penetrate very deeply.

4. A momentum imbalance?

Mean zonal pressure gradient and advective accelerations were calculated by BB from a diagnostic model of equatorial circulation. The horizontal eddy flux of zonal momentum was estimated by Bryden et al. (1986, hereafter BBH) using moored current meter arrays near 152°W from April 1979 to June 1980 and near 110°W from January 1979 to October 1981. Covariances were calculated using daily-average velocity measurements extending from the upper 15–20 to 250 m depth. The BB and BBH estimates were made based on measurements of more than a year duration, and we interpret them here as "typical" estimates of large-scale quantities. The BB and BBH estimates are presented with the turbulence estimates in Table 1.

The dominant term estimated by BB and BBH is ZPG, the vertical integral of the zonal pressure gradient. It is likely that the actual zonal pressure gradient during the experiment was different from the annual mean that we used in Table 1. However, the range of zonal pressure gradients measured by Mangum and Hayes (1984) was within a factor of 2 of the annual mean, and we therefore expect the actual ZPG during the experiment was no more than a factor of 2 larger or

TABLE 1. Momentum budget for the upper 90 m of the equatorial ocean in three 30 m segments. Units are 10^{-6} m^2 s^{-2}. ZPG, ZD, UW, and EF, taken from Bryden and Brady (1985) and Bryden et al. (1986), and "typical" estimates of large-scale terms. The stress estimates are from Moum and Caldwell's (1985) TROPIC HEAT I dissipation rate measurements. The uppermost 30 m segment has a large residual compared to the ZPG.

	Layer-depth range		
	0–30 m	30–60 m	60–90 m
$\text{ZPG} = \int -\frac{1}{\rho}\frac{\partial P}{\partial x}dz$	14	12	8
$\text{ZD} = \int -U\frac{\partial U}{\partial x}dz$	−0.9	−0.6	2
$\text{UW} = \int -W\frac{\partial U}{\partial z}dz$	5	6	1.5
$\text{EF} = -\int \frac{\partial}{\partial x}\overline{(U'U')} + \frac{\partial}{\partial y}\overline{(U'V')}\,dz$	−9	−6	−3
ZPG + ZD + UW + EF	9	11	9
τ/ρ, layer top	−100	−18	−4
τ/ρ, layer bottom	−18	−4	−0.6
$\delta\tau/\rho$, (top–bottom)	−82	−14	−3.4
Residual: R = ZPG + ZC + UW + EF + $\delta\tau/\rho$ (all units 10^{-6} m^2 s^{-2})	−72	−3	5.6
R/ZPG × 100%	−514%	−25%	70%

smaller than the mean listed in Table 1. As we shall see later, a factor of 2 uncertainty in the ZPG is irrelevant to the conclusions we draw.

A more important point concerns the depth scale of the zonal pressure gradient. The shape of the pressure gradient profiles found by Mangum and Hayes (1984) is more nearly gaussian than exponential, and can be modeled approximately as $P_0 = \exp[-(z/H)^2]$ (Fig. 6). The depth scale H is 100 to 140 m, while the depth scale for the turbulent momentum flux is about 20 m. No matter what else is assumed, we must conclude that in the upper 100 m, the force balance cannot be between the large-scale zonal pressure gradient and the divergence of the turbulent stress (Fig. 7).

Bryden and Brady found that the annual mean contribution to zonal momentum from upwelling (UW) is about half as large as the ZPG, and is positive in the upper 90 m. The contribution from large-scale eddies, EF, is about two-thirds that of the ZPG, and is negative, reinforcing the wind stress and opposing the ZPG. The only term small relative to the ZPG is ZD, the mean zonal divergence of zonal momentum.

Moum and Caldwell's turbulence and shear measurements extend from roughly 30 to 90 m, and we have divided the upper ocean into 3 sections: 0–30 m, 30–60 m, and 60–90 m (Table 1). We cannot expect to balance (2) exactly, and so rewrite it with a residual term, R, representing the imbalance:

$$\text{ZPG} + \text{ZD} + \text{UW} + \text{EF} + \tau(b)/\rho - \tau(a)/\rho = R. \tag{11}$$

Here, $\tau_0(z)/\rho = -F_x(z)$ represents wind stress at the surface $z = 0$ and is estimated as $K_m\langle \partial u/\partial z\rangle$ in the fluid interior; a represents the depth of the top of each layer, and b represents the bottom. The right side of (11) includes all of the terms we are unable to estimate, including the local acceleration $\partial u/\partial t$, as well as local anomalies in the zonal pressure gradient, advective accelerations, and, possibly, contributions to the stress which we are unable to measure (for example, internal wave horizontal-vertical velocity covariances in the frequency range of 1 day^{-1} to the local buoyancy frequency); by "anomalous" we mean "different from BB's 'typical' estimates."

The residual for the 60–90 m depth range is large, amounting to 70% of the ZPG for that depth range. This is unexpectedly large, but not alarming. One must expect any 12-day period to have substantial differences from long term means. In the 30–60 m depth range, the residual is only 25% of the ZPG.

The residual in the 0–30 m depth range is extraordinarily large, 5 times larger than the ZPG. We have no guarantee that the imbalance is confined to the 30

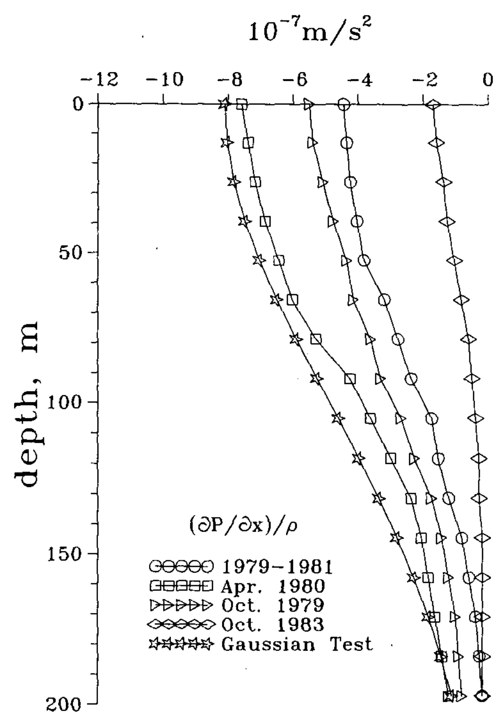

FIG. 6. Representative estimates of the zonal pressure gradient from Mangum and Hayes (1984). Measured profiles are for the 1979–81 mean (circles), from April, 1980 (squares), from October, 1979 (triangles), and from October, 1983 (diamonds). A gaussian profile (stars) is included for comparison; this profile has a depth scale of 135 m and is scaled so that the vertical integral of the pressure gradient balances a 0.1 N m^{-2} westward surface stress.

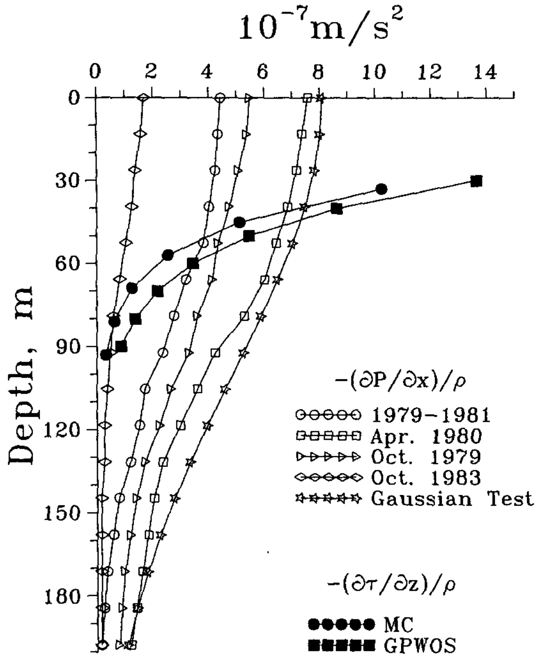

FIG. 7. Comparison of representative pressure gradients (open symbols as defined in Fig. 6) with estimates of derivatives of turbulent stress. Stress derivatives were estimated by differentiating an exponential curve least-squares fitted to MC (circles) and GPWOS (squares) data.

m depth range; it may originate at still shallower depths. The imbalance is so large that we must suspect the dynamics near the surface are different than we have assumed; i.e., our formulation of the problem may be fundamentally wrong. However, if our assumptions are so far off the mark in the upper depth range, we are not justified in making the same assumptions at deeper depths. We must consider the smaller residuals in deeper depth ranges as coincidental, possibly having no physical significance at all.

5. Discussion

A basic premise of equatorial dynamics is that the westward wind stress is transmitted from the sea surface to deeper waters by small-scale vertical transport, there to be balanced by larger-scale forces resulting from the mean zonal pressure gradient, the upwelling of eastward momentum, the convergence of zonal momentum, and the mesoscale eddy flux (e.g., Gill 1982). An equally basic premise of small-scale turbulence dynamics is that the local rate of production of turbulent kinetic energy is balanced by the rate at which it is locally dissipated, allowing the turbulent stress to be calculated from dissipation rate estimates. We have demonstrated that "typical" large-scale forces cannot balance the resultant of wind stress and turbulent stress forces in the upper 30 m at 140°W. The eastward turbulent stress at 30 m depth is too small to account for more than 20% of the westward surface wind stress, and typical large-scale forces can account for only 10% of the wind stress. Clearly, something is seriously wrong.

We have no assurance that the stress, as estimated by the dissipation method, is close to the wind stress above the 30 m level; on the contrary, the observations show the dissipation rate to be more nearly constant in the 10 m to 30 m depth range than below 30 m. If turbulent stress estimated using (9) is of the same order as the wind stress, the shear in the upper 30 m must necessarily be very small; this conclusion is not consistent with mooring measurements.

Two possibilities should be explored to resolve the apparent momentum imbalance: (a) the large-scale horizontal forces at the time of the TROPIC HEAT I experiment might have been much larger than our "typical" estimates; or (b) our estimate of the momentum flux may be unrealistically low. We examine each in turn.

a. Large-scale anomalies

The average wind stress during the 12-day turbulence experiment was 0.10 N m^{-2}, approximately a factor of 2 larger than the mean annual surface stress at 140°W (Weare and Strub 1981; Weare et al. 1981). The detailed response of the equatorial ocean to anomalous changes in the wind is unknown, and it may be overly optimistic to expect a rapid response in the zonal pressure gradient or other large-scale processes to fluctuations in the wind. Large-scale, annual average circulation terms can only balance the large-scale, annual average wind stress. If the mean annual surface stress (BB used 56×10^{-6} m^2 s^{-2}) is used instead of the measured wind stress, the residual R drops from -72×10^{-6} to -28×10^{-6} m^2 s^{-2}, still twice as large as the ZPG. Ideally we should use the same time scale to average the wind stress, the turbulent stresses, and the large-scale terms. Since such measurements are not available, we cast the problem in a different sense: If estimates of *typical* large-scale terms can explain only annually averaged surface stress, can the local wind stress be balanced by *anomalies* in the ZPG, the upwelling velocity, or the mesoscale eddy flux?

If our estimate of the turbulent momentum flux is correct, *momentum supplied by the local wind was not transmitted very deeply by turbulent stirring.* Can the disposition of the momentum supplied by the wind, but not transmitted below 30 m, be explained by a local, unmeasured anomaly with a depth scale of 30 m or less? Candidate anomalies which might explain the large residual are (i) the local acceleration of the top 30 m of the ocean; (ii) an anomalous zonal pressure gradient that is much larger than the mean pressure gradient; (iii) the local meridional advection of zonal momentum (i.e., an anomalous mesoscale eddy flux); (iv) a local anomaly in upwelling. Each of these is treated below.

(i) We have entirely neglected the local acceleration of the top 30 m of the ocean in (11); we now ask, "how large would the acceleration have to be to explain the residual?" If $R = \int \{\partial u/\partial t\} dz$, the average acceleration of the top 30 m would be 2.4×10^{-6} m s^{-2}, equivalent to a gain in westward velocity of 2.5 m s^{-1} over the 12-day period. No acceleration of this magnitude was observed, and we consider this explanation unlikely.

(ii) Suppose that the residual is in fact balanced by an anomalous pressure gradient. To evaluate $R = -\int \{\rho^{-1}\partial P_a/\partial x\} dz$, where P_a is the pressure anomaly, one must estimate the horizontal scale of the anomaly. Since "mean" zonal pressure gradients have the scale of thousands of kilometers (typically, the mean zonal pressure gradient is calculated from sections taken at 150° and 110°W, a distance of 4400 km), an "anomaly" should have a smaller horizontal scale, say, 500 km. The R could be explained by a difference in dynamic height of 0.12 m over the 500 km distance, and such a difference has been noted by Lemasson and Piton (1968) over the longitudes 135° to 145°W in the region of large thermocline slope. However, the required anomalous pressure gradient must be concentrated entirely in the upper 30 m. The mean zonal pressure gradient has a depth scale of 100 m or more (Mangum and Hayes 1984, Fig. 6); the anomalies shown by Lemasson and Piton (1968) are vertically variable but have depth scales comparable to the mean zonal pressure gradient. An anomalous pressure distribution with a 30 m depth scale can be caused by an anomalous temperature distribution, but in this case, the required expansion is 0.12 m/30 m, or 4 parts per thousand. A typical expansivity for sea water is 0.2 parts per thousand per degree latitude, so the pressure gradient anomaly would require a temperature change of about 20°C over 500 km. No temperature distribution of this magnitude has been observed on the equator, and we consider this explanation unlikely.

(iii) Suppose the residual is balanced by a meridional advection anomaly. If $R = \int \{v(\partial u/\partial y)\} dz$, one must estimate the meridional scale. ADCP measurements from the MC vessel show a 6-day time interval midway through the measurement period when the meridional velocity averaged about 0.4 m s^{-1} northward (Chereskin et al. 1986; this appears to be related to the surface-intensified 21-day waveform discussed by Philander et al. 1985). To balance the residual requires a 260 km meridional length scale. The change δu in zonal velocity over a distance of $\delta y = 260$ km would have to be 1 m s^{-1}. Changes in zonal velocity of this size (to within a factor of 2 or so) are sometimes observed near the equator (Moum et al. 1986), with one crucial difference: *such fluctuations have a vertical coherence scale of order 50 m,* not less than 30 m as demanded by the residual. The sign may also be wrong: BBH found that *the meridional advection of zonal momentum transports momentum westward, not eastward.* Halpern et al. (1988) were unable to support BBH's finding of westward momentum transport by meridional advection, but found that the 0.05 cpd equatorial current oscillations could not be driven by the local winds, but rather that the small variations in surface winds might be influenced by north–south advection of sea surface temperature variations. It therefore seems unlikely that meridional advection plays an important part in the local momentum budget.

(iv) If the residual is balanced by an upwelling anomaly, that is, $R = \int \{w(\partial u/\partial z)\} dz$, where w is an anomalous vertical velocity, one can estimate the size of w necessary for the balance. The vertical shear of zonal velocity during the 12 days of the turbulence experiment was of order 0.02 s^{-1} (Chereskin et al. 1986). Using this, we estimate that $w = 1.2 \times 10^{-4}$ m s^{-1} = 10 m day^{-1}. This is approximately 12 times as large as the mean upwelling velocity estimated by BB at 30 m depth, and we think it unlikely that upwelling anomalies could explain the residual.

We can find no evidence that the large zonal momentum flux residual can be explained by anomalies in the large-scale effects. It is unlikely that zonal pressure gradient, horizontal advection, or upwelling anomalies are large enough to balance the residual; (11) is so far out of balance that other explanations must be considered.

b. The dissipation method revisited

The turbulent stress at 30 m depth is 20% of the surface stress. If we use the dissipation method for calculating eddy viscosity, we find that most of the momentum supplied by the local wind was not transmitted below 30 m depth, yet we know that momentum put into the ocean by the annual wind stress must penetrate deeply in order to balance the ZPG. We can only conclude that *if the dissipation method is correct, local turbulent stirring in the upper 30 m does not respond rapidly to changes in the local surface stress.* But there is direct evidence which indicates that turbulent mixing is a strong function of surface conditions; both MC and GPWOS found a very strong diurnal signal in the strength of turbulence above the undercurrent. Perhaps the truth is that the momentum supplied by the local wind *is* penetrating deeply, but our estimate of the momentum flux is wrong.

There are three potentially large sources of error that should be questioned: (i) Was the sampling adequate to determine the mean kinetic energy dissipation rate? (ii) Is some process other than turbulent mixing responsible for vertically transporting most of the momentum at the equator? (iii) Is the *method* of estimating turbulent stresses from kinetic energy dissipation rates formally flawed? We address each of these questions in turn.

(i) Is it possible that we have underestimated the dissipation rate by a factor of 4 because of sampling error? MC and GPWOS together made over 2000 casts, each extending to 100 m depth or more. In all, well over 200 km of water was sampled; there is little hope of improving this record by another factor of 10 in the near future. We performed a "bootstrap" test of the significance of our mean turbulent stress calculation, using averages of the turbulent stress over one hour and 10 m vertically to define a parent probability distribution having 288 elements in each 10 m depth bin. One thousand example averages were calculated by randomly drawing 288 samples from the parent distribution, and a histogram of the example averages was formed. It was found that 90% of the example averages were within 20% of the average of the parent distribution. We conclude that sampling error is an unlikely source of a factor of 4 error.

It is worth noting that GPWOS and Peters et al. (1988) use the Cox number method (Osborn and Cox 1972) to estimate the mean vertical heat flux, finding about -80 W m^{-2} at 30 m depth; Moum et al. (1989) used an estimate of vertical heat flux based on ϵ and a flux Richardson number, and found the same result. This estimate seems reasonable (at least within a factor of 2 or so), because the net surface heat flux was of roughly the same size. If the kinetic energy dissipation rates were grossly undersampled and underestimated, the Cox number would also be undersampled and underestimated. Both the vertical heat flux and the momentum flux would be seen as much too small if the turbulence were severely undersampled. Instead, the *heat flux is roughly correct,* but the *momentum flux is much too small.* If sampling error is invoked to explain the large residual, one must explain how the momentum flux could be increased without making the interior heat flux larger than the surface heat flux. Estimates of the vertical heat flux from the MC data set and dissipation measurements (Osborn 1980) agree reasonably well (though perhaps only coincidentally) with the Cox number estimates of GPWOS and our own unpublished Cox number estimates.

(ii) We have not considered vertical momentum transport by internal wave interactions. Internal waves can be excited by turbulence (Townsend 1968; Wu 1969), and can sometimes transfer momentum vertically (Muller 1976; Booker and Bretherton 1967). Eriksen and Katz (1987) comment that critical layer absorption of internal waves might be important at the equator. The present measurements are inadequate to determine whether or not internal wave radiation is the major momentum transport process. The internal wave field in midlatitudes is horizontally homogeneous over long time scales, and so internal wave processes are not very effective in transporting momentum. On the equator, however, the presence of a large mean vertical shear breaks the symmetry, because eastward-propagating waves created at the surface can potentially be critically absorbed, while westward-going waves cannot be absorbed. In any event, wavelike processes transport kinetic energy easily, and we demonstrate below that *kinetic energy* transport by itself may be a major problem in the dissipation method.

(iii) It may not be possible to parameterize the vertical flux of horizontal momentum using the kinetic energy dissipation rate. The prescriptions for eddy viscosity and turbulent stress, (7), (8) and (9), all depend on (6), the assumption that the local rate of production of turbulent kinetic energy is balanced by the local dissipation rate. Third-order correlations and pressure–velocity correlations have both been neglected in deriving (6). Neither term has been measured in the ocean and there is no easy way to make such measurements directly. Wyngaard and Cote (1971) measured vertical velocity–turbulent kinetic energy correlations in the atmospheric boundary layer and found it to be a small quantity, sometimes positive and sometimes negative, when the stratification was stable. They estimated the divergence of pressure–vertical velocity fluctuations as a residual in the unstable atmospheric boundary layer, but did not estimate it when the stratification was stable (presumably because there were too few examples when the atmospheric boundary layer was stable). We thus have no a priori reason to suppose that a local production–dissipation balance holds when the ocean is stably stratified.

There is some qualitative evidence that the pressure–velocity correlation divergence may be a significant term in the turbulent kinetic energy budget. C. Paulson (personal communication) observed a significant increase in internal wave isotherm amplitudes during the night, when turbulence was most energetic and dissipation rates were largest. We do not intend to discuss diurnal variations in equatorial turbulence, but simply note that increases in the internal wave energy content is observed to be correlated with increases in turbulent kinetic energy.

It may be possible for turbulent overturns to drive the internal wave field, because the rate of increase of wave energy is controlled by pressure–velocity correlations (Lighthill 1978):

$$\frac{\partial E}{\partial t} = -\nabla \cdot (p\mathbf{u}), \tag{12}$$

where E is internal wave energy and $\mathbf{u}$ is the vector velocity. The mechanism described by (12) is the radiative loss of turbulent energy to an internal wave field.

If the pressure–velocity correlation is large and confined to a restricted region of the fluid, internal waves can be excited, and energy can be radiated away, eventually to be dissipated elsewhere. If the energy is radiated away from the region it is generated, a *local* production–dissipation balance cannot hold, and kinetic energy dissipation rates may be only loosely related to the eddy viscosity and turbulent stress. In view of our inability to explain the large momentum budget residual with any other mechanism, we must consider the pressure–velocity correlation divergence as a possibly significant source of error, and we must treat estimates of the turbulent stress based on dissipation rates with skepticism.

6. Summary and conclusion

We have shown that the momentum budget at 30 m depth and above on the equator at 140°W cannot be balanced using typical estimates of large-scale forces (zonal pressure gradients, upwelling of eastward momentum, convergence of westward momentum, and meridional eddy transport) and conventional estimates of the turbulent stress. One of two conclusions must be drawn from this study: Either the large-scale processes during the time of measurement were highly atypical, or estimates of the momentum flux based on a production–dissipation balance are seriously flawed.

A result similar to ours has been recently reported. McPhaden et al. (1988) studied the response of the upper equatorial Pacific to a westerly wind burst of several days duration and estimated that the eddy viscosity necessary to explain the observations would be order 100 cm^2 s^{-1}; this is 5 to 10 times larger than Gregg et al. (1985) estimated using microstructure measurements. For the present, we believe eddy viscosities and turbulent stresses estimated from dissipation rate and shear measurements may be too small and must be treated with caution.

Estimates of anomalies in the large-scale forces which could explain the apparent imbalance are unrealistically large, and do not have a realistic depth structure. Most of the problem may be attributable to the turbulent stress estimates because the stress estimate at 30 m depth is much smaller than the wind stress and decreases exponentially below 30 m. It is possible that internal wave interactions play a crucial role in near-surface equatorial dynamics, either by transporting momentum directly, or by radiating energy and invalidating the hypothesis of a local production–dissipation balance.

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