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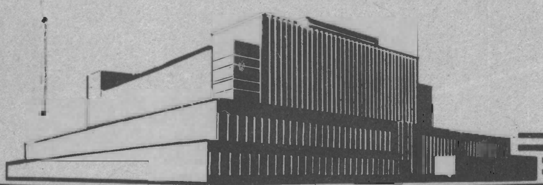

FOREST PRODUCTS LABORATORY
MADISON 5, WISCONSIN

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
In Cooperation with the University of Wisconsin

PLASTIC FLOW (CREEP) PROPERTIES OF TWO YELLOW BIRCH PLYWOOD PLATES UNDER CONSTANT SHEAR STRESS[1,2]

By

C. B. NORRIS, Engineer
and
W. J. KOMMERS, Engineer

Forest Products Laboratory,[3] Forest Service
U. S. Department of Agriculture

Summary

This report presents the results of tests made on two yellow birch plywood plates subjected to constant shear stresses to determine the plastic flow or creep properties of the material. The plates were of special 1:3:1 construction, in which the core consisted of three parallel-ply laminations. A mathematical analysis of a relationship between deflection, load, and time, satisfying the data obtained from tests, is also given. The tests were made by applying load (dead weights) perpendicular to the plane of a square plate at diagonally opposite corners, with supports at the other two corners. This loading has been demonstrated to subject the plate to shearing stress.[4] The plastic deflection-time curves obtained are parabolic. By testing a single specimen at various loads, a family of plastic deflection-time curves is obtained, one curve for each load. If the data are replotted in a family of plastic deflection-load curves, one curve for each time, the curves are again parabolic.

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[1] This report is one of a series of progress reports prepared by the Forest Products Laboratory relating to the use of wood in aircraft. Results here reported are preliminary and may be revised as additional data become available.

[2] Original report dated October 1943.

[3] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[4] Forest Products Laboratory Report No. 1301.

The equation is derived from the assumption that:

$$\sigma = K\epsilon^m \left(\frac{d\epsilon}{dt}\right)^n$$

and is

$$w = -\frac{a^2}{h}\left(\frac{m+n}{n}\right)^{\frac{n}{m+n}} \left[\frac{m+n+2}{2Kh^2}\right]^{\frac{1}{m+n}} P^{\left(\frac{1}{m+n}\right)} t^{\left(\frac{n}{m+n}\right)}$$

where $\underline{\sigma}$ is shear stress (pounds per square inch)

$\underline{\epsilon}$ is plastic shear strain

$\underline{w}$ is plastic deflection (inches) at gage points with respect to center of plate

$\underline{P}$ is load (pounds)

$\underline{t}$ is time (hours)

$\underline{a}$ is 1/2 the distance (inches) between adjacent gage points

$\underline{h}$ is thickness of plate (inches)

$\underline{K}$, $\underline{m}$, and $\underline{n}$ are constants of the material

(See Appendix for derivation).

Introduction

Creep or flow of materials deals with their plastic or inelastic behavior. Some solids, such as asphalt, flow appreciably at room temperatures under small loads; at sufficiently high temperatures nearly all metals will creep under stress. The subject of creep has been under investigation for many years for some of the widely used metals of construction. Wood is known to exhibit some plastic behavior, but very few data have been obtained on the subject. The work described in this report is part of the program in progress at the Forest Products Laboratory on the relation of the strength properties of wood and plywood to the duration of stress.

Material Tested

The two test specimens were cut from two panels of yellow birch plywood each consisting of five 1/16-inch plies arranged with the grain of the three central plies parallel to afford 1:3:1 construction. Each ply was a single piece of veneer. The thickness of the glue lines, the weight of the glue, and creep in the glue are assumed to be negligible throughout the calculations. The panels were received in a shipment of Tego-bonded aircraft plywood. The average specific gravity of the material tested was 0.64 based on the weight when oven dry and the volume at test. All tests were conducted under controlled conditions of 80° F. and 50 percent relative humidity resulting in approximately 9 percent moisture content in the wood. The specimens had dimensions of 11.0 by 11.0 by 0.300 inches.

Method of Test

Tests were made using the apparatus shown in figure 1. The specimens were placed so as to apply four equal loads at the corners of the square plate; two, at the extremity of one diagonal, acting downward (dead weights) and two, at the extremity of the other diagonal, acting upward (supports).[2] Small metal corner pieces were attached to the specimen so that the loads could be applied directly at the corners of the square. The deflection to be considered is the vertical displacement of a point on a diagonal with respect to the center of the specimen. With the arrangement used, as shown in figure 1, the mean upward displacement of two symmetrically located points on one diagonal is automatically added to the mean downward displacement of two similar points on the other diagonal. One half the dial gauge reading is thus the average of the vertical displacements of the four points with respect to the center of the specimen.

Weights were hung from the ends of one diagonal and deflection readings taken at approximately 1, 3, 6, 12, 18, 24, and 30 minutes and 1, 3, 6, 9, 12, 16, and 24 hours. The plates were alternated each 24-hour period and, in addition, each plate was loaded alternately on each diagonal. This report is based on 27 such loadings; 15 on one specimen and 12 on the other. One loading was with 10 pounds (5 pounds at each end of the diagonal, 11 with 14 pounds, 9 with 20 pounds, and 6 with 24 pounds. Loads less than 10 pounds were not used, as deflection increments due to

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[2]Forest Products Laboratory Report No. 1301.

such loads are too small to read with sufficient accuracy, and loads over 24 pounds were not used, as direct stresses due to bending of the plate become appreciable.

The recorded deflections are with respect to a zero established after the specimen was in place with the metal corners, hooks, and dial supports attached and the analysis assumes that all creep is due to the applied loads and none to the weight of the plate and the attached apparatus.

Analysis of Results

The time-deflection readings recorded during the tests were plotted on logarithmic graph paper. The resulting curves were of the shape shown in figure 2, curve A. The points of the middle curve are the same as those of Curve A with a correction term c added to each deflection reading which is the difference between the deflection at the observed zero time and the actual zero time. This difference is due to the difficulty of placing the weights on the specimen at any particular instant. By solving for this correction (see appendix), and replotting, the points are close to a straight line. The equation of this line is of the form $w = at^b$ where a and b are constants for the particular curve drawn. By testing the specimens several times at each load it was possible to determine average values of a and b for each load, in the equation $w = at^b$, which are more accurate than the values determined from any single test.

It was found that b did not vary appreciably with the load, so the values of b were averaged for all tests made, and arithmetically weighted according to the smoothness of the curve obtained from the test data. A value of a was chosen for each load which caused the plotted points of the equation to agree most closely with all of the test points for that load. These values of a were plotted against the corresponding loads on logarithmic graph paper (fig. 3). A straight line averaging these points was drawn and its equation was determined. The value of a in terms of the load, determined by this method, substituted in the equation $w = at^b$ yields equation (16) of the appendix,

$$w = 0.0007\, P^{0.54}\, t^{0.22}\ .$$

Figure 4 shows the comparison between the theoretical (equation (16) of the appendix) and average time-deflection curves plotted from data obtained from the two yellow birch plywood plates. The solid lines represent the theoretical curves. The plotted points result from using the average values of b from the equation $w = at^b$ for all the tests made and the value of $\underline{a}$ for each load determined as indicated. From this figure, it can be seen that for the four loads used, one test curve was coincident with the theoretical, two were slightly lower, and one was above.

It was realized that the plastic constants of one piece of plywood might be quite different from those of another. It was decided, therefore, to make a number of tests upon a single specimen in the preliminary study rather than to make a single test upon a number of specimens. It was assumed, because of the small loads used, that the plastic constants would not be materially changed by the treatment endured by the specimen during the several tests.

It has been found from other studies[6] that repeated and reversed stresses with loads greater in proportion to dimensions of specimens do have an effect on the modulus of elasticity of wood and plywood in bending and compression. It is also safe to assume that repeated and reversed stresses may affect the values of the plastic flow constants, even though 24 hours elapsed between tests of the plates. This effect may account in part for the discrepancy between the theoretical and experimental points shown in figure 4.

The discrepancy may also be due in part to the additional weight of the testing apparatus which could not be taken into account in the theory and to small inaccuracies in applying the loads exactly at the corners of the plates. The test points for the 10 pound load are for one test only and are not the averages of several tests as are the other load-deflection points of figure 4.

The middle curve of figure 2 shows the actual test points taken during a test of a plate under a 14-pound load. The recorded deflections have been corrected for the zero time error and show very good agreement with the theoretical curve of figure 2 which represents equation (16) of the appendix for a 14-pound load.

Table 1 lists the 27 tests with the equations of the curves as first plotted and the equations using a common exponent (the average of all tests). Also given are arithmetic weights used for each equation in calculating the average constant for each load.

The formulas in the appendix have been derived for solid wood plates with the surface in the LT plane (longitudinal-tangential). For rotary-cut veneer that is made into plywood, the surface of each ply is in the LT

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[6] Forest Products Laboratory Report No. 1320.

plane also, so that in this test (neglecting the effect of the glue) the same formulas apply in rotary-cut plywood plates as in flat-sawn wood plates. All data from these tests give the effect of flow in only the LT plane of shear, but it is expected that the flow in the LR (longitudinal-radial) plane will be of somewhat the same magnitude.

Conclusions

The data show that flat yellow birch plywood plates of 1:3:1 construction, when subjected to constant shear stress in the LT plane exhibit plastic flow at a rate which decreases with time. The plastic deformation-time curve is parabolic in form and concave downward. It is also shown that the plastic deformation at any given time is a parabolic function of the load such that the plastic deformation-load curve is concave downward.

From the limited number of loadings on only two plates of one material, it is believed that the tentative formula presented will give reasonable values of creep or plastic flow in shear, but it should not be assumed that other stresses on wood, such as compression or tension, will produce plastic flow according to the theory presented. Tests were not continued for more than 24 hours on any specimen, and while it is assumed that the theory applies to long-time tests, no such test data are yet available.

It appears that the assumption

$$\sigma = K\epsilon^{m} \left(\frac{d\epsilon}{dt}\right)^{n}$$

is a reasonable relation between the shear stress and the plastic shear strain.

Appendix

Notations used in calculations:

X, Y, and Z = choice of axes shown in figure 1B.

w = plastic deflection of a point on the surface of a square plate with respect to the center.

w^1 = deflection of a point on the surface of a square plate with respect to the center.

e = shear strain at a distance $\underline{z}$ from center plane of plate.

e_1 = shear strain at the surface of the plate.

h = plate thickness.

a = distance from center of plate to projection of gage point on the $\underline{x}$ or $\underline{y}$ axis.

P = total load on plate (pounds).

t = time (hours).

K, m, and n = constants of the wood.

σ = shear stress at central plane of the plate.

σ_1 = shear stress at surface of the plate.

ϵ = plastic deformation at a distance $\underline{z}$ from the central plane of the plate.

ϵ_1 = plastic deformation at the surface of the plate.

$u = \frac{d\epsilon}{dt}$ at central plane of the plate.

$u_1 = \frac{d\epsilon_1}{dt}$ at surface of the plate.

M_{xy} = twisting moment.

Calculations

From the theory of thin plates, it may be found[7,8] that for the case here considered the shear strain component is:

$$e = -2z \frac{\partial^2 w}{\partial x \, \partial y}$$

Assuming that *e* is independent of *x* and *y* and that $w^1 = 0$ when $x = 0$ and $y = 0$

$$w^1 = -\frac{e_1}{h} xy \quad \text{where } z = \frac{h}{2} \tag{1}$$

An elastic surface of this kind is obtained if a square plate is supported and loaded as shown in figure 1.[9]

In this case[8]

$$P = 4M_{xy} \tag{2}$$

Since the load *P* is a constant and the deflection w^1 is measured at various time intervals, it is assumed that the elastic deformation can be neglected, if the first deflection reading is taken just after the load is applied. It is also assumed that lines normal to the original surface of the plate will remain straight for plastic deformations as well as elastic. Equation (1) then applies to the plastic deformations[10] and

$$w = -\frac{\epsilon_1}{h} xy \tag{3}$$

and

$$\epsilon = 2\epsilon_1 \frac{z}{h} \tag{4}$$

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[7]Forest Products Laboratory Report No. 1301.

[8]Timoshenko, S. Theory of Plates and Shells. P. 91.

[9]Forest Products Laboratory Report No. 1301.

[10]MacCullough, G. H. An Experimental and Analytical Investigation of Creep in Bending. Applied Mechanics, April-June 1933.

$$\text{Let } u = \frac{d\epsilon}{dt} = 2\frac{z}{h}\frac{d\epsilon_1}{dt} = 2u_1\frac{z}{h} \tag{5}$$

where $\underline{u_1}$ = value of $\underline{u}$ at the surface of the plate.

$$\text{Assume } \sigma = K\epsilon^m u^n \quad \underline{11} \tag{6}$$

Substituting (4) and (5) in (6)

$$\sigma = K\left(2\epsilon_1 \frac{z}{h}\right)^m \left(2u_1 \frac{z}{h}\right)^n$$

$$\sigma = K\epsilon_1^{\,m} u_1^{\,n} \left(2\frac{z}{h}\right)^{m+n}$$

Then

$$\sigma = \sigma_1 \left(2\frac{z}{h}\right)^{m+n} \quad \text{where} \quad \sigma_1 = K\epsilon_1^{\,m} u_1^{\,n} \tag{7}$$

Now

$$M_{xy} = 2\int_0^{\frac{h}{2}} \sigma z \, dz \quad \underline{12} \tag{8}$$

Substituting (7) in (8)

$$M_{xy} = 2\sigma_1 \left(\frac{2}{h}\right)^{m+n} \int_0^{\frac{h}{2}} z^{m+n+1} \, dz$$

$$M_{xy} = \frac{1}{2}\frac{\sigma_1 h^2}{m+n+2}$$

$$\sigma_1 = \frac{2M_{xy}(m+n+2)}{h^2} \tag{9}$$

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[11] Davis, E. A. Creep of Metals at High Temperature in Bending. Journal of Applied Mechanics. March 1938.

[12] Forest Products Laboratory Report No. 1312.

Substituting (2) in (9)

$$\sigma_1 = \frac{P\ (m + n + 2)}{2h^2} \tag{10}$$

From equation (7)

$$\sigma_1 = K\epsilon_1^{\ m}\ u_1^{\ n} = K\epsilon_1^{\ m}\ \left(\frac{d\epsilon_1}{dt}\right)^n \tag{11}$$

Substituting (11) in (10)

$$K\epsilon_1^{\ m}\ \left(\frac{d\epsilon_1}{dt}\right)^n = \frac{P\ (m + n + 2)}{2h^2}$$

$$\epsilon_1^{\frac{m}{n}}\ \frac{d\epsilon_1}{dt} = \left[\frac{P\ (m + n + 2)}{2Kh^2}\right]^{\frac{1}{n}}$$

or

$$\epsilon_1^{\frac{m}{n}}\ d\epsilon_1 = \left[\frac{P\ (m + n + 2)}{2Kh^2}\right]^{\frac{1}{n}}\ dt$$

Since $\underline{P}$ does not vary with $\underline{t}$

$$\frac{\epsilon_1^{\frac{m}{n} + 1}}{\frac{m}{n} + 1} = \left[\frac{P\ (m + n + 2)}{2Kh^2}\right]^{\frac{1}{n}}\ t + C$$

If $\underline{\epsilon_1} = 0$ when $\underline{t} = 0$, then $\underline{C} = 0$.

Then

$$\epsilon_1 = \left(\frac{m}{n} + 1\right)^{\frac{n}{m + n}}\ \left[\frac{P\ (m + n + 2)}{2Kh^2}\right]^{\frac{1}{m + n}}\ t^{\left(\frac{n}{m + n}\right)} \tag{12}$$

Assuming that equation (3) applies to the plastic case, and substituting it in (12) where $\underline{x} = \underline{y} = \underline{a}$

$$-\frac{hw}{a^2} = \left(\frac{m}{n} + 1\right)^{\frac{n}{m+n}} \left[\frac{P\ (m + n + 2)}{2Kh^2}\right]^{\frac{1}{m+n}} t^{\left(\frac{n}{m+n}\right)}$$

or

$$w = -\frac{a^2}{h}\left(\frac{m + n}{n}\right)^{\frac{n}{m+n}} \left(\frac{m + n + 2}{2Kh^2}\right)^{\frac{1}{m+n}} P^{\frac{1}{m+n}}\, t^{\frac{n}{m+n}} \qquad (13)$$

This is the equation that has been used.

Sample calculation from data obtained during the test of a plywood plate under a load $\underline{P}$ of 14 pounds:

Curve A, figure 2, is a plotting of deflection readings taken during the test. This curve is of the form $w = at^b + c$ where $\underline{c}$ is the difference in deflections at the observed zero time and the actual zero time.

$t_1 = 0.2$ $\qquad$ $w_1 = 0.00099$

$t_2 = 2.0$ $\qquad$ $w_2 = .00227$

$t_3 = 20.0$ $\qquad$ $w_3 = .00440$

$$c = \frac{w_1 w_3 - w_2^2}{w_1 + w_3 - 2w_2} = -\ 0.00094$$

By adding this constant to all readings, and replotting (middle curve of fig. 2), a straight line can be drawn through the test points. Solving for the function:

$$w = 0.00275\ t^{.221} \qquad (14)$$

The average slope of the lines from all tests was found to be 0.22. The following equation results from drawing the best straight line of this slope through the test points of all the tests made with this load.

$$w = 0.00543\ t^{0.22} \tag{15}$$

From equation (13), since P does not vary with t, the value of $\frac{n}{m + n}$ = 0.22.

Plotting w against P when t = 1 on logarithmic paper for the various tests (fig. 3), it is possible to draw a straight line through the points, the slope of which yields the value of $\frac{1}{m + n} = 0.54$.

Thus $m = 1.44$

$n = 0.406$

Knowing these values, it is possible to solve for K which equals 1.433×10^{10}.

The final equation reduces to

$$w = 0.0007\ P^{0.54}\ t^{0.22} \tag{16}$$

Table 1.--Results of tests to determine plastic flow (creep) properties of yellow birch plywood of 1:3:1 construction under constant shear stress

Test No.	Speci-men	Load	Values of a and b in equation $w = at^b$: Individual curves as plotted: a	b	a for curves with: b = 0.22	Arith-metic weight	Weighted average of a for b = 0.22
		Lb.					
1	C	10	0.0029	0.197	0.0026	7	0.0026
2	C	14	.0023	.248	.0027	8	.0028
3	C	14	.0056	.118	.0029	8	.0028
4	C	14	.0021	.265	.0025	6	.0028
5	C	14	.0029	.208	.0027	7	.0028
6	C	14	.0023	.241	.0026	5	.0028
16	A	14	.0033	.196	.0028	7	.0028
17	A	14	.0031	.209	.0029	8	.0028
18	A	14	.0069	.130	.0039	2	.0028
19	A	14	.0049	.160	.0035	5	.0028
20	A	14	.0032	.174	.0025	7	.0028
21	A	14	.0021	.297	.0029	7	.0028
7	C	20	.0038	.201	.0035	5	.0036
8	C	20	.0037	.210	.0035	6	.0036
9	C	20	.0039	.201	.0037	7	.0036
10	C	20	.0025	.280	.0033	6	.0036
11	C	20	.0043	.207	.0040	5	.0036
12	C	20	.0039	.223	.0039	4	.0036
22	A	20	.0031	.280	.0040	5	.0036
23	A	20	.0049	.172	.0038	7	.0036
24	A	20	.0059	.120	.0031	8	.0036
13	C	24	.0041	.142	.0035	7	.0038
14	C	24	.0035	.244	.0039	8	.0038
15	C	24	.0043	.174	.0033	7	.0038
25	A	24	.0034	.220	.0034	7	.0038
26	A	24	.0037	.235	.0040	7	.0038
27	A	24	.0033	.285	.0044	7	.0038

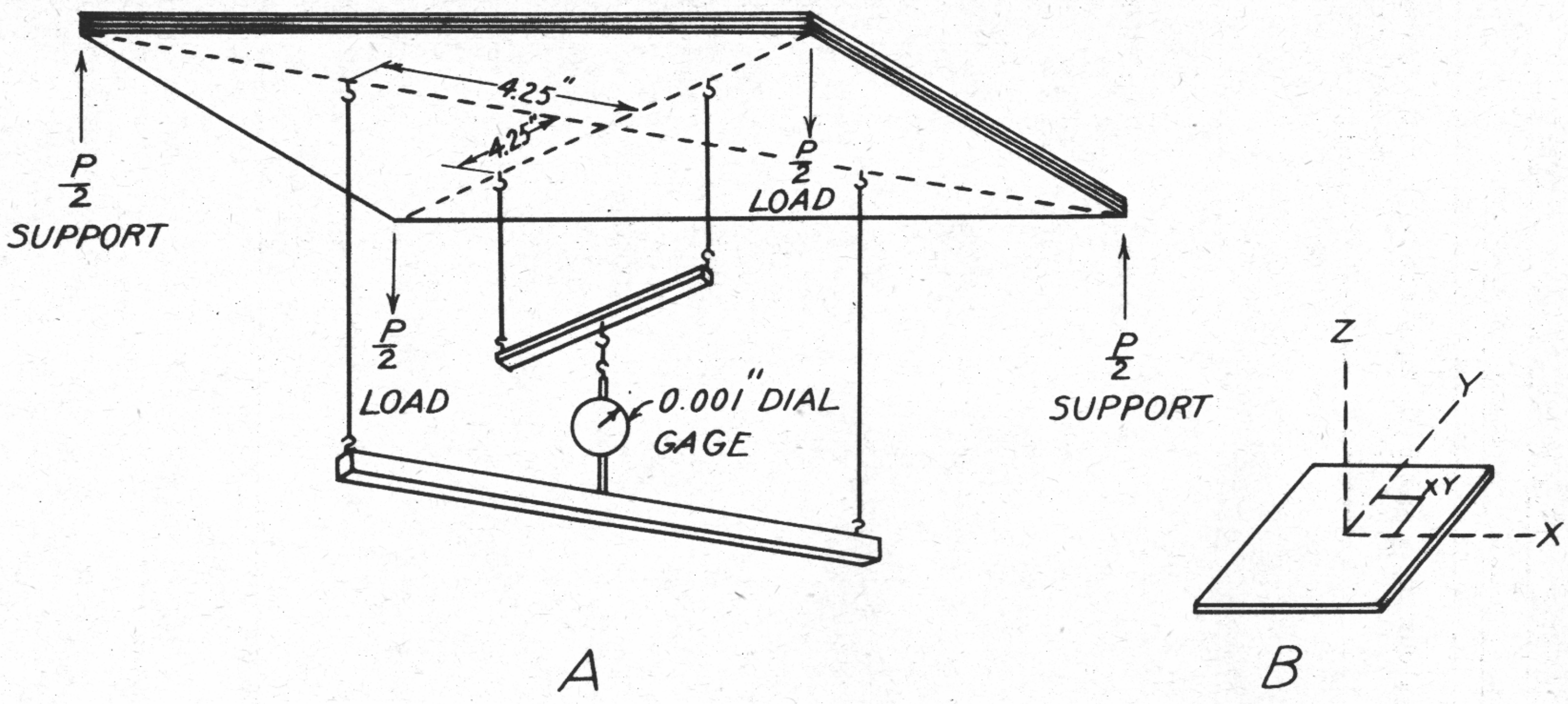

Figure 1.--Dead-load plate shear test: A, deflectometer apparatus attached to plate; B, choice of axes.

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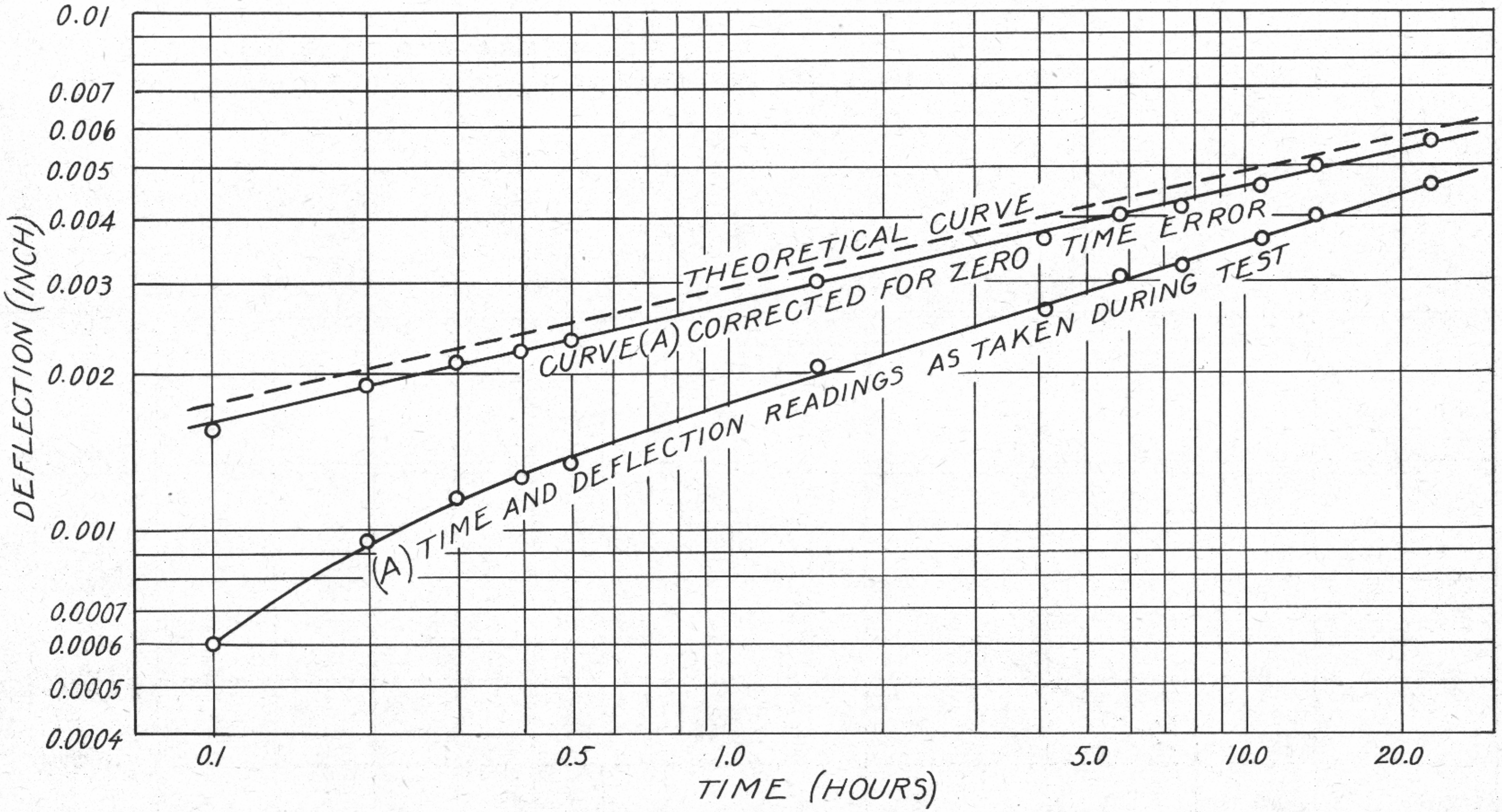

Figure 2.--Time-deflection curves for dead-load shear test on yellow birch plywood of 1:2:1 construction.

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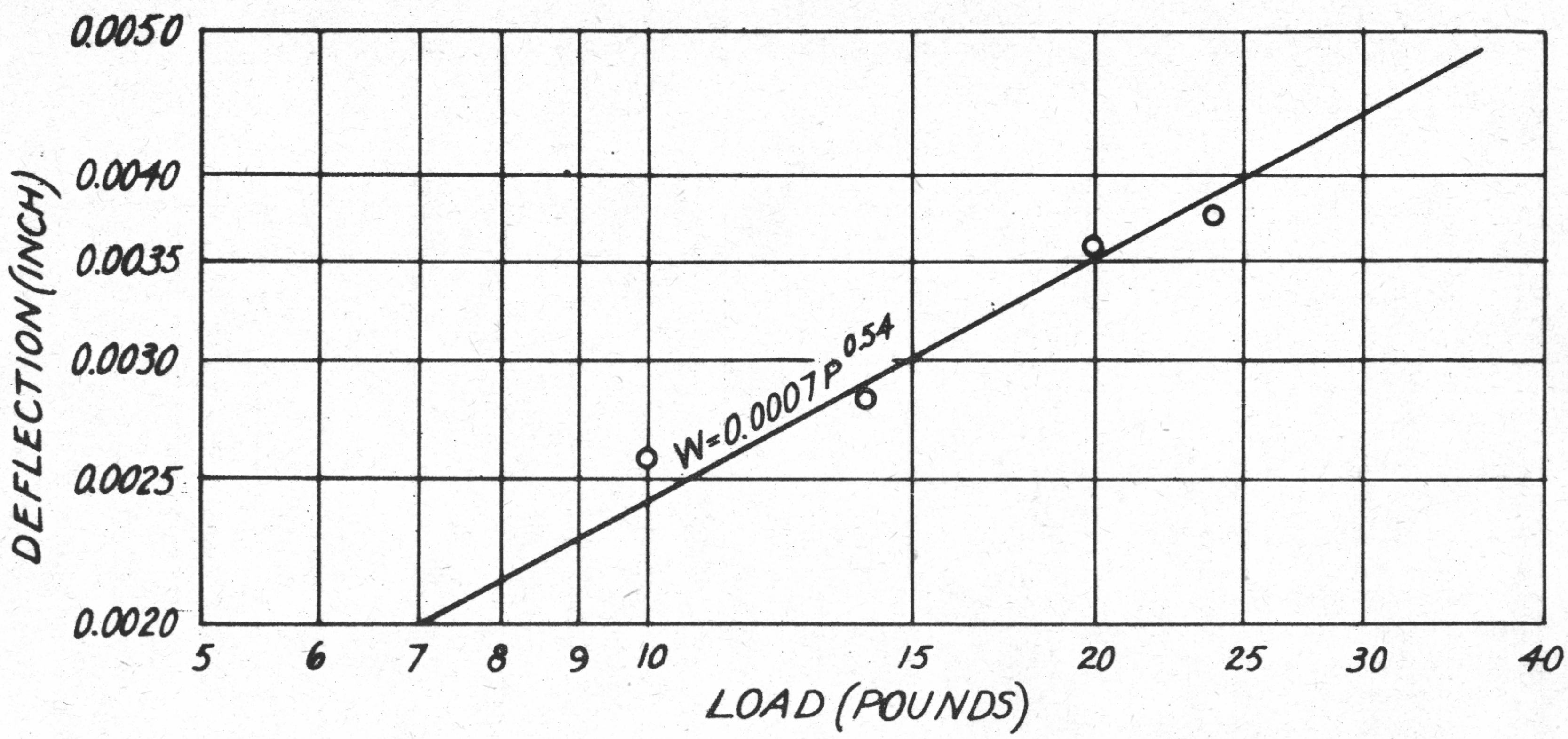

Figure 3.--Load-deflection curve of dead-load plate shear tests on yellow birch plywood of 1:3:1 construction when t=1.

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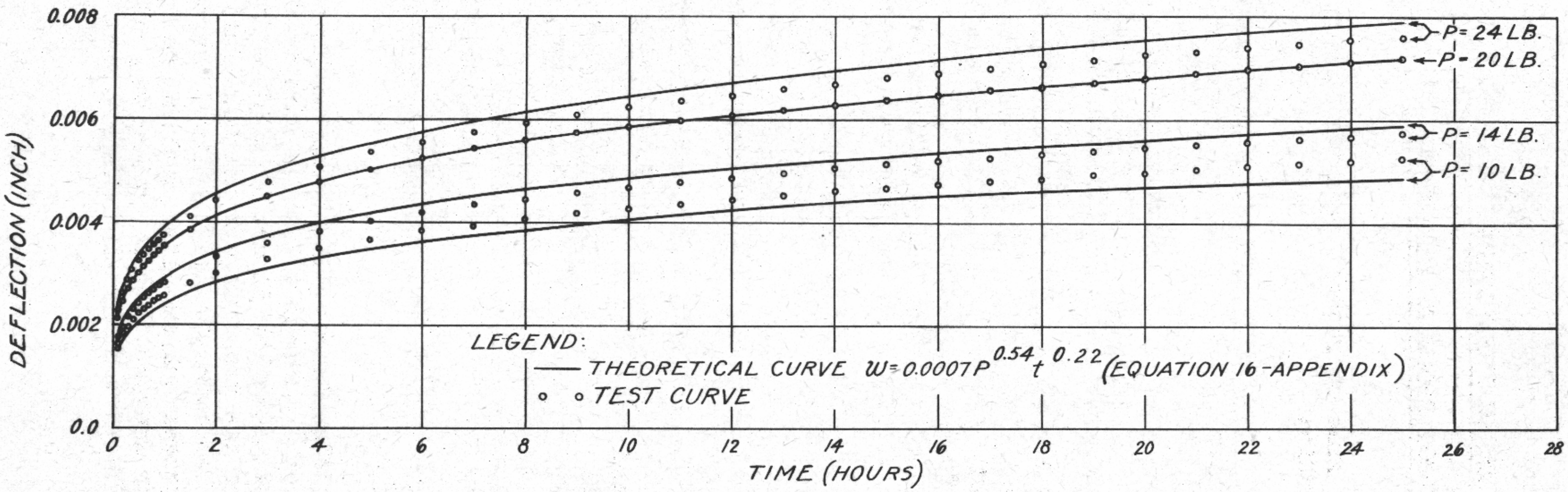

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Figure 4.--Time deflection curves for two yellow birch plywood plates under constant shear stress. Comparison of theoretical and test curves of plates of 1:3:1 construction.

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