AN ABSTRACT OF THE THESIS OF

Gary Manford Buford for the M.S. in Civil Engineering (Name) (Degree) (Major)

Date thesis is presented 2/26/65

Title JET DIFFUSION AS RELATED TO OCEAN OUTFALL SYSTEMS

Abstract approved (Major professor)

This study discusses important aspects of the ocean which should be considered in the design of a sewage outfall system; conditions are restricted to those of the Oregon coastal area. They include predominant currents, upwelling tendencies and ocean bottom characteristics.

Major emphasis is placed upon investigation of diffusion characteristics of a horizontal fluid jet with particular interest in the cone-of-diffusion, the cone formed by mixing of the jet fluid with the surrounding fluid. The angle of expansion of the fluid mixture is generally referred to as the angle of the cone-of-diffusion. Past studies of the angle are briefly summarized while a rather extensive investigation is made of factors which affect the vertical position of the cone-of-diffusion axis.

Graphs are presented which show the effect of initial jet density and velocity, and surrounding fluid density and velocity upon
the position of the cone-of-diffusion axis. Curves of the corresponding horizontal and vertical positions of the axis are presented. A thorough discussion of equations used to obtain the curves is also included. Values of densities and velocities are limited to those which would apply to an ocean outfall system used for disposal of domestic sewage and kraft-mill effluent.
JET DIFFUSION AS RELATED TO OCEAN OUTFALL SYSTEMS

by

GARY MANFORD BUFORD

A THESIS
submitted to
OREGON STATE UNIVERSITY

in partial fulfillment of
the requirements for the
degree of

MASTER OF SCIENCE

August, 1963
APPROVED:

Redacted for Privacy

Professor of Civil Engineering

In Charge of Major

Redacted for Privacy

Head of Department of Civil Engineering

Redacted for Privacy

Dean of Graduate School

Date thesis is presented Oct 1963

Typed by Jolene Hunter Wuest
ACKNOWLEDGMENT

This study was accomplished under the sponsorship of the United States Public Health Service.

Gratitude is expressed to Charles E. Behlke, the writer's major professor, for his guidance and advice during the study; to Dr. Larry S. Slotta of the Oregon State University Civil Engineering Department for his suggestions and constructive criticisms; and to the United States Public Health Service for making the research grant available.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>FACTS OF THE OCEAN</td>
<td>4</td>
</tr>
<tr>
<td>Currents</td>
<td>4</td>
</tr>
<tr>
<td>Upwelling</td>
<td>5</td>
</tr>
<tr>
<td>Bottom Characteristics</td>
<td>6</td>
</tr>
<tr>
<td>SEWAGE AND KRAFT-MILL EFFLUENT DISPOSAL</td>
<td>8</td>
</tr>
<tr>
<td>Method of Disposal</td>
<td>8</td>
</tr>
<tr>
<td>Kraft-Mill Effluent</td>
<td>8</td>
</tr>
<tr>
<td>Ocean Outfall Problems and Considerations</td>
<td>11</td>
</tr>
<tr>
<td>JET DIFFUSION ANALYSIS</td>
<td>14</td>
</tr>
<tr>
<td>Angle of Cone-of-Diffusion</td>
<td>14</td>
</tr>
<tr>
<td>Position of Axis of Cone-of-Diffusion</td>
<td>15</td>
</tr>
<tr>
<td>Previous Studies</td>
<td>15</td>
</tr>
<tr>
<td>Dimensional Analysis</td>
<td>16</td>
</tr>
<tr>
<td>X/D - Y/D Curve Relationships</td>
<td>19</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>31</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>35</td>
</tr>
</tbody>
</table>
LIST OF FIGURES AND TABLES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cone-of-diffusion, vertical projection (equal density case)</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>Position of axis of cone-of-diffusion for variable $U_o^2/D_g$</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>Position of axis of cone-of-diffusion for variable density ratio</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>Position of axis of cone-of-diffusion for variable velocity ratio, $U_o^2/D_g = 0.10$</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>Position of axis of cone-of-diffusion for variable velocity ratio, $U_o^2/D_g = 0.40$</td>
<td>27</td>
</tr>
<tr>
<td>6</td>
<td>Position of axis of cone-of-diffusion for variable velocity ratio, $U_o^2/D_g = 0.80$</td>
<td>28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Values of the function $a^{1/2} Y$</td>
<td>21</td>
</tr>
</tbody>
</table>
JET DIFFUSION AS RELATED TO OCEAN OUTFALL SYSTEMS

INTRODUCTION

The timber products, paper products and other closely associated industries employ processes which involve the use of chemicals and water which must be disposed of at periodic intervals. These waste solutions generally have an objectionable odor in addition to being unsightly. They are quite similar, in many respects, to domestic sewage. A convenient solution to the problem of disposal is to discharge these objectionable solutions into a nearby river or large body of water. The ocean lends itself quite favorably as a receiving body for this waste because of its large extent and inherent diluting possibilities. It will be considered almost exclusively herein.

There are two rather important problems that are created when any body of water is used as a receiver of either industrial effluent or sewage. The first problem is that of the dissolved oxygen content of the receiving water. It is decreased as a result of reaction with the waste solutions. This decreases the capacity of the body of water for fish or other animals whose existences are dependent on the dissolved oxygen content of the water. The second problem which arises is the need for adequate dilution in order that no chemical or sewage is allowed to reach an inhabited area in a
sufficiently large concentration to create a threat to public health. Certainly, more than two problems are actually created if an objectionable waste solution is discharged into a body of water; however, they may usually be eliminated if the discharge process is adequately designed.

This study has chosen the ocean to be the receiving water. Disposal of waste solutions is therefore accomplished by pumping them through a pipe extending into the ocean and having a horizontal direction of discharge. This system shall be referred to as the ocean outfall method of waste disposal.

Adequate design of ocean outfall systems requires knowledge of both ocean and jet diffusion phenomena. No lack of information exists for very general ocean characteristics; however, magnitudes and definite statements are rather scarce at this time. It appears that it will be a number of years before oceanographers will be able to supply all of the information that is desired. There have been numerous studies of jet diffusion, however, few theoretical approaches have proved successful in the laboratory. Most investigators have chosen to fit diffusion equations to their laboratory curves.

Two jet diffusion characteristics of primary importance to horizontal fluid jets have been considered in this study. The first is the cone-of-diffusion angle which is actually the angle of expansion of the jet fluid after discharge from the nozzle into the ocean. This expansion is due to the mixing of the jet fluid with the surrounding fluid.
The angle obtained by projection of the cone-of-diffusion onto a vertical plane is discussed later. The second is the cone-of-diffusion axis. For the more general case of unequal fluid densities, it curves either upward or downward depending on the relative densities of the two fluids. The locus of the center of the cone-of-diffusion will be referred to as the cone axis. Curves are presented which predict the position of the axis for representative conditions that may be encountered by an ocean outfall system.

This study endeavors to collate and expand upon previous theoretical studies, relating the results to laboratory curves of other investigators. Dimensional analysis suggests the use of three dimensionless parameters to show the effect the ratio of ocean density to sewage density and the ratio of current velocity to nozzle discharge velocity have on the position of the cone-of-diffusion, a matter considered here to be of major interest.

Ocean conditions discussed which would affect the design or construction of an outfall system are almost exclusively restricted to those of Oregon coastal waters.
FACTS OF THE OCEAN

Currents

Hydrographic data from Oregon coastal waters (18, p. 26-27) indicates that the predominant offshore drift is northward during the period late October to early March, and it is south-westerly from mid-June to mid-October, the time when upwelling is most likely to occur. Upwelling, however, occurs only intermittently and should therefore not be considered a continual process. Drift bottle return data has failed to show existence of a definite north or south current during the period from March to June, and it appears that this may be the time of current reversal. Although the directions of coastal currents are fairly well known, there is relatively little available data at the present time covering their respective magnitudes or vertical velocity distributions. Direction and magnitude of currents are dependent upon the time of day (tide), time of year and weather; as a result, they are very difficult to predict. It is equally difficult to measure currents with any accuracy. Currents along the California coast appear, however, to be quite similar to those of Oregon; California current data is therefore considered to be representative of what could be expected in Oregon coastal waters.
There are practically no Oregon current data available; however, measurements of wind generated surface currents have been made off the California coast (10, p. 3.20). These measurements indicate that for a wind of 10 - 20 miles-per-hour, a surface current of approximately 0.4 feet-per-second will be created; for a wind of 50 - 60 miles-per-hour the surface current will be approximately 1.5 feet-per-second. Tidal current velocities can be considerably greater than those generated by wind. They often are as great as 7 feet-per-second. This study, however, has chosen to consider current velocities ranging up to 4.5 feet-per-second as representative of average conditions. Graphs which are included may be extended to cover conditions of unusually fast tidal currents.

Upwelling

This phenomenon occurs along the Oregon coast as a result of wind, blowing from the North, approximately parallel to the coastline, and the Coriolis deflecting force. The wind produces a movement of light surface water away from the coast and a resultant movement of water from greater depths toward the coast to replace the surface water. There has not yet been given a completely satisfactory explanation of upwelling. It is thought that the total process is made up of a number of sub-stages, each of which is controlled by other factors (4, p. 652).
Upwelling generally occurs during the period from mid-June to late October. This, however, is not the only time upwelling has been known to occur. The effects have been reported as late as January in the area of Cape Blanco along the southern coast of Oregon. Upwelling along the Oregon coast has been found to occur in depths of water seldom exceeding 600 feet and is intermittent in nature as opposed to a continual process. Although the process is generally understood there is very little data available at the present time covering the magnitude of the currents which it creates. An estimate of the extent of influence of upwelling may be made through comparison of salinity, oxygen content and temperature taken at surface stations along the coastline with those taken at various depths further out to sea. Surface salinity measurements taken at shore stations have been found to be quite useful in determining if upwelling is occurring. For example, a surface salinity greater than 33.00/o (parts per thousand) reported at a shore station on the Oregon coast has been found to be indicative of the upwelling process.

**Bottom Characteristics**

The ocean bottom adjacent to the Oregon coast is characterized by a continental shelf. The edge of the shelf occurs at
approximately five miles from shore at the south boundary of Oregon and twelve miles at the north. There is a nearly uniform increase in the width of the shelf from south to north except at a few points where it extends as far as twenty miles into the ocean. The depth at which the outer edge of the shelf occurs varies between 90 and 100 fathoms. Therefore, a representative bottom slope is approximately 95 fathoms per 8.5 miles or one foot of drop in 80 feet. This mild sloping section of the bottom is the area of major interest in this study since it is unlikely that a sewage outfall would need to be extended into water any deeper than 300 feet to obtain adequate diffusion.

Average conditions as stated above can be somewhat misleading if they are used in place of a thorough investigation. For instance, from Newport northward the existence of banks and reefs is intermittent; the assumption of a uniform beach with 80:1 slope would therefore be erroneous. Whenever an outfall system is contemplated the best policy is to obtain a reliable map of the ocean bottom and conduct an exploratory diving program to determine any peculiarities of the area.
SEWAGE AND KRAFT-MILL EFFLUENT DISPOSAL

Method of Disposal

The ocean outfall system has proved to be a convenient and efficient method of disposal of waste solutions in California and other areas, although there are few of these along the Oregon coast. An effective outfall is, naturally, desired in all cases; however, numerous installations have not been acceptable as originally constructed. Inadequate design has usually required that the pipe be modified in some way in order to obtain more adequate diffusion. In most cases, the outfall merely consists of a pipe, extending into the ocean, through which sewage or a waste solution is pumped. The typical waste solution considered in this study is kraft-mill effluent.

Kraft-Mill Effluent

It is difficult to accurately describe kraft-mill effluent because it may differ considerably from one mill to another depending upon the method of operation of each particular installation. The effluent, however, may be described as usually consisting of the following chemical compounds and approximate proportions:

Sodium Hydroxide, the prime constituent: 50 ppm

Aluminum Sulphate: 2 - 5 ppm

Sodium Sulphate, toxic: 0 - trace quantities
Black Liquor, mixture: 5 - 50 ppm

Fibre: 10 - 150 ppm

Others: 10 - 150 ppm

Presently there are no sanitary restrictions placed on ocean disposal of effluent other than the restriction of toxic chemicals to trace quantities or less. At any time, however, should undesirable conditions become apparent, the Public Health Service has the authority to demand that the responsible party correct the situation.

The temperature of kraft-mill effluent at time of disposal is generally 120°F; this is, however, not an absolute temperature for all mills. It is basically dependent upon two factors, the method of plant operation and temperature restrictions imposed by the outfall pipe itself. Plant management may elect to use the same solution in more than one operation or choose to operate at various temperatures, ultimately combining cold liquids with some which are much hotter to obtain only a warm solution. On the average, however, the temperature at time of plant discharge is, generally, no lower than approximately 100°F.

One phase of the operation of a kraft-mill near the Oregon coast illustrates the importance of effluent temperature. Engineering properties of the outfall pipe require that the temperature of its effluent not exceed 100°F. A cooling process is employed which
reduces the $120^\circ F$ temperature to $100^\circ F$. Temperature loss through approximately eight miles of its outfall pipe is thought to be very small, on the order of $10^\circ F$. Upon reaching the ocean, temperature of the effluent is approximately $90^\circ F$.

If an outfall discharge temperature of $90^\circ F$ is assumed, calculations may be made to show the similarity between kraft-mill effluent and domestic sewage. The specific gravity of kraft-mill effluent at $90^\circ F$ is 0.995. Using hydrographic data from Oregon coastal waters, an average value for the specific gravity of sea water near a point of discharge may be 1.026. Therefore, an average ratio of the density of sea water to that of effluent is 1.03. This value agrees favorably with work done by Rawn, Bowerman and Brooks (13, p. 65-105) on the disposal of sewage in the sea. From their work, the following data were observed.

\[
\text{Sewage: } \rho_o = 0.9987 \text{ @ } 70^\circ F \\
\text{Sea water: } \rho_s = 1.0259 \text{ @ } 50^\circ F
\]

\[
\text{Density Ratio } = \frac{\rho_s}{\rho_o} = \frac{1.0259}{0.9987} = 1.027.
\]

It may therefore be noted that the density ratio for kraft-mill effluent, as herein defined, is almost the same as that for domestic sewage.
Ocean Outfall Problems and Considerations

An ocean outfall, although a very convenient method of sewage and waste disposal, can be a very complicated system to analyze. The primary problem is that of jet diffusion. This study has chosen to investigate two basic aspects of the problem for a horizontal jet: prediction of the angle of the cone-of-diffusion and the vertical position of its axis when a density difference exists. Figure 1 shows a vertical view of the cone-of-diffusion if no density difference exists. Also shown is the cone-of-diffusion axis. If a density ratio were to exist, the cone would be deflected either upward or downward dependent on the relative values of the two densities.

Figure 1. Cone-of-diffusion, vertical projection (equal density case).
Also of importance are the velocities and concentrations along the axis of the cone and the distribution of the velocity and concentration at sections perpendicular to the axis. These aspects of the problem, however, are not considered in any detail in this study. The effectiveness of the mixing process can be accurately predicted only if all of the aspects of the problem of jet diffusion are understood. Various facets of this problem have been subjected to theoretical and laboratory studies for the past forty years.

Jet diffusion analysis is further complicated for an ocean outfall due to the statistical nature of the turbulence within the ocean and the lack of uniformity of the turbulence in the horizontal and vertical directions (15, p. 199-225). The jet must also be analyzed for the bouyant case due to the fact that the effluent is generally lighter than the salt water into which it is being diffused and therefore tends to rise.

Discharge velocity must satisfy two requirements. It should be low enough that there will not be an unnecessarily large head loss through the outfall pipe, yet it should be high enough that no debris is allowed to settle in the pipe. A study of existing domestic sewage outfalls (13, p. 65-105) has found that average pipe velocities vary between three and six feet-per-second.

Several factors determine the depth of discharge; dilution
requirements, safe diving depths and economics of construction.

Public health requirements in addition to preservation of the aesthetic value of the ocean require placing the pipe deep enough that adequate diffusion can be obtained before the solution reaches the surface where it may be carried shoreward. Outfall depths, however, should not be so great as to restrict Scuba divers, a convenient means of cleaning and checking the pipeline. A maximum depth for Scuba divers is 300 feet. Economic considerations are perhaps the most important. If adequate diffusion cannot be justified economically, another means of waste disposal should be investigated. An ocean outfall study of conditions in California waters (10, p. 420-3) has determined that a depth of 285 feet is ample for a domestic sewage outfall under the worst circumstances. This would apply to a discharge of effluent containing no sludge and having a specific gravity of 0.9987 at 70°F. Kraft-mill effluent, at time of discharge, is very comparable.
JET DIFFUSION ANALYSIS

Angle of Cone-of-Diffusion

Upon leaving the nozzle, a fluid jet expands and mixes with the surrounding fluid. The mixture of the two fluids forms a cone similar to that shown in Figure 1. The angle of the resultant cone will hereafter be referred to as the angle of the cone-of-diffusion.

There have been numerous studies made to determine the angle of the cone-of-diffusion. Each of these somewhat contradicts the others. Horn and Thring (7, p. 205) state that measured values of the half angle have been variously reported from 7 to 20 degrees. Tollmien's theoretical analysis (17, p. 468-78) found the half angle to be twelve degrees. A recent laboratory study by Horn and Thring (7) was conducted to determine, experimentally, the half angle and any influence of the density ratio upon the angle. Their measurements were made with nozzle fluids of specific gravities ranging from one to two. They found the half angle to be consistently eight and one-half degrees. This is in close agreement with the value reported by Binnie (2) for a water jet.

Results of the study by Horn and Thring (7) suggest that the spread functions of effluent concentration are independent of density ratio. Johnson (11) concluded from his study that the shape of a
vertical discharge cone, if the exit flow is in the turbulent flow region, is almost entirely a function of the mixing of the two liquids and essentially independent of the discharge pipe size, the specific gravities of the solutions, and the velocity of discharge. Unfortunately, he made no measurements of the cone-of-diffusion angle.

For purposes of predicting the cone-of-diffusion angle for an ocean outfall, it appears that the work of Horn and Thring (7) is the most applicable and the most reliable. Their jet fluid consisted of a magnetite slurry which was injected into a slowly moving mass of water in a transparent box. The jet was then photographed, thereby allowing measurements to be made of the angle of the cone-of-diffusion. A variation in density ratio from 1.0 to 2.0 was achieved by varying the density of the magnetite slurry.

Position of Axis of Cone-of-Diffusion

Previous Studies. There have been numerous studies of round, free turbulent jets but relatively few have investigated the effect of density differences on the jet path. The ability to predict the vertical position of the cone-of-diffusion can be quite useful. Diffusion is thought to be directly proportional to the length of time which the jet fluid remains submerged.

Groume - Grjimailo (4) made the first attempt (1923) to
predict a jet path under the conditions of initial density difference. Their formula, however, neglected all considerations of viscosity or entrainment of surrounding water, basically nothing more than the parabolic path of a projectile. Horn and Thring (7, p. 1081) (1955) and Bosanquet, Horn and Thring (3, p. 340-52) (1961) presented equations to be used for predicting the paths of jets under the condition of initial density difference. Their work included the effect of changing axial velocity and density. Their formulas are rearranged somewhat for this investigation and then used to show the effect of the variation of three dimensionless ratios upon the position of the axis of the cone-of-diffusion.

In the study of 1961, by Bosanquet, Horn and Thring, a jet of magnetite slurry was injected into a slowly moving mass of fluid, the fluid being contained in a large transparent box. The density ratio was varied by altering the mix proportions of the magnetite slurry. Photographs were then taken of the jet against an illuminated background. Slurry was forced through a 0.125 inch diameter nozzle to form the jet.

**Dimensional Analysis.** In this study it is desired to obtain a dimensionless curve of the horizontal and vertical positions of the axis of the cone-of-diffusion of kraft-mill waste in ocean water.

If one fluid is injected into another whose density differs
from that of the first, gravity forces exhibit a tendency to accelerate the jet either upward or downward dependent upon its relative density. Therefore, gravity forces should be taken into consideration in a dimensionless study. The following variables were considered for the dimensional analysis:

\[ g, \quad \text{gravity} \]
\[ X, \quad \text{horizontal distance from discharge end of outfall pipe} \]
\[ Y, \quad \text{vertical distance from discharge end of outfall pipe} \]
\[ D, \quad \text{nozzle diameter} \]
\[ U_o, \quad \text{jet velocity at origin} \]
\[ U_s, \quad \text{velocity of surrounding fluid} \]
\[ \rho_o, \quad \text{jet density at origin} \]
\[ \rho_s, \quad \text{density of surrounding fluid}. \]

A study of the variables indicates the need to adopt \( \rho_s/\rho_o \) and \( U_s/U_o \) as dimensionless parameters. Dimensional analysis suggests the use of \( U_o^2/Dg \) as an additional parameter to include the effect of gravity. Therefore, three dimensionless quantities may be varied to show their influences on the vertical position of the axis of the cone-of-diffusion. The following is a determination of the limits for the three quantities.
(1) \( \frac{\rho_s}{\rho_o} \);  
\[ \rho_s = 1.0259, \text{ relatively constant (seawater)} \]
\[ \rho_o = 0.9948, \text{ kraft-mill waste} \]

to

\[ \rho_o = 1.0259, \text{ thoroughly mixed with seawater} \]

and,

\[ \frac{\rho_s}{\rho_o} = \frac{1.0259}{1.0259} \text{ to } \frac{1.0259}{0.9948} \]

\[ \frac{\rho_s}{\rho_o} = 1.0 \text{ to } 1.032, \text{ approximate limits} \]

(2) \( \frac{U_s}{U_o} \);  
\[ U_s = 3 \text{ to } 6 \text{ fps, average velocities, jet} \]
\[ U_o = 0 \text{ to } 4.5 \text{ fps, current velocities} \]

and

\[ \frac{U_s}{U_o} = 0 \text{ to } \frac{4.5}{3} \]

\[ \frac{U_s}{U_o} = 0 \text{ to } 1.5 \]

(3) \( \frac{U_o^2}{Dg} \);  
\[ U_o = 3 \text{ to } 6 \text{ fps} \]
\[ D = 1.0 \text{ to } 5.0 \text{ ft.} \]

and

\[ \frac{U_o^2}{Dg} = \frac{(3)^2}{5(32.2)} \text{ to } \frac{(6)^2}{1(32.2)} \]

\[ = 0.056 \text{ to } 1.12 \]

\[ \frac{U_o^2}{Dg} = 0.05 \text{ to } 1.2 \]
Curves are presented later which show the influence of each of these variables on the vertical position of the axis of the cone-of-diffusion. The curves are plotted in the form of $X/D$ versus $Y/D$, where $X$ and $Y$ are the horizontal and vertical distances respectively from the origin; $D$ is the pipe diameter at the origin.

**$X/D - Y/D$ Curve Relationships.** The jet diffusion study by Bosanquet, Horn and Thring (3, p. 340-52) considers the effect of density differences on the path of a jet. They assume that horizontal momentum is conserved at any point along the jet. A term, $a$, is then introduced (Ibid, p. 347) which relates any $X$-coordinate on the jet axis to its respective $Y$-coordinate. The jet, however, must be treated as coming from a point source about $1.5 R$ behind the actual nozzle in order for the relationship to be completely satisfied (Ibid).

It is also assumed that the velocity of the surrounding fluid is zero. They give,

$$a = 0.139 \left( 1 - \frac{\rho_s}{\rho_o} \right) \left( \frac{\rho_s}{\rho_o} \right)^{1/2} \frac{\sec \theta_o}{RU_o^2}$$  \hspace{1cm} (1)

where $R =$ nozzle radius

$\theta = $ angle jet axis makes with horizontal
Suffixes

\( o \) relates to values at the nozzle

\( s \) relates to values in the surrounding fluid

and all other variables have been previously defined.

Table 1 shows values of \( \frac{d^2}{dX^2} Y \) as a function of \( \frac{d}{dX} X \) where \( X \) is the horizontal distance from the origin and \( Y = \int \tan \theta \, dX \). It is only part of a table presented in the paper by Bosanquet, Horn and Thring; the remainder was omitted because this study chose to consider only initially horizontal jets.

Table 1 was used to obtain values for two types of curves.

1. Position of Axis of Cone-of-Diffusion for variable \( U_o^2 / Dg \).

\[
\frac{U_s}{U_o} = \text{constant} = 0
\]

\[
\frac{\rho_s}{\rho_o} = \text{constant} \neq 0
\]

\[
\frac{U_o^2}{Dg} = \text{variable}
\]


\[
\frac{U_s}{U_o} = \text{constant} = 0
\]
Table 1. Values of the function $\frac{1}{\alpha^2} Y$

<table>
<thead>
<tr>
<th>$\alpha^2 X$</th>
<th>$\tan \theta = 0.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.0002</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.0013</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.0045</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.0107</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.0208</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.0360</td>
</tr>
<tr>
<td>0.7</td>
<td>-0.0572</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.0855</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.1219</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.1676</td>
</tr>
<tr>
<td>1.1</td>
<td>-0.2236</td>
</tr>
<tr>
<td>1.2</td>
<td>-0.2912</td>
</tr>
<tr>
<td>1.3</td>
<td>-0.3718</td>
</tr>
<tr>
<td>1.4</td>
<td>-0.4667</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.5777</td>
</tr>
<tr>
<td>1.6</td>
<td>-0.7065</td>
</tr>
<tr>
<td>1.7</td>
<td>-0.8553</td>
</tr>
<tr>
<td>1.8</td>
<td>-1.0264</td>
</tr>
<tr>
<td>1.9</td>
<td>-1.2227</td>
</tr>
<tr>
<td>2.0</td>
<td>-1.4474</td>
</tr>
<tr>
<td>2.1</td>
<td>-1.7044</td>
</tr>
<tr>
<td>2.2</td>
<td>-1.9983</td>
</tr>
</tbody>
</table>
\[ \frac{U_o^2}{Dg} = \text{constant} \neq 0 \]

\[ \frac{\rho_s}{\rho_o} = \text{variable} \]

Examples of the two types of curves, (1) and (2), are shown in Figures 2 and 3 respectively.

By rearranging the terms in \( \alpha \), multiplying by \( D^2 \) and taking the square root, the following expression can be obtained.

\[
\frac{1}{a^2} D = \sqrt{(0.278) \left( 1 - \frac{\rho_s}{\rho_o} \right) \left( \frac{\rho_s}{\rho_o} \right)^{\frac{1}{2}} \frac{1}{U_o^2/Dg}}
\]  

Equation (2) may be used directly with Table 1 to obtain \( X \) and \( Y \) coordinates.

A third type of curve considers the case in which the surrounding fluid has a velocity.

(3) Position of Axis of Cone-of-Diffusion for Variable Velocity Ratio.

\[ \frac{U_o^2}{Dg} = \text{constant} \neq 0 \]

\[ \frac{\rho_s}{\rho_o} = \text{constant} \neq 0 \]

\[ \frac{U_s}{U_o} = \text{variable} \]
\[
\frac{U_s}{U_o} = \text{const.} = 0
\]
\[
\frac{\rho_s}{\rho_o} = \text{const.} = 1.02
\]
\[
\frac{U_o^2}{Dg} = \text{variable}
\]

Figure 2. Position of axis of cone-of-diffusion for variable \(U_o^2/Dg\).
\[ \frac{U_s}{U_o} = \text{const.} = 0 \]
\[ \frac{U_o^2}{Dg} = \text{const.} = 0.20 \]
\[ \frac{\rho_s}{\rho_o} = \text{variable} \]

Figure 3. Position of axis of cone-of-diffusion for variable density ratio.
Examples of this type of curve are shown in Figures 4, 5 and 6.

In order to obtain a type (3) curve, some equations by Bosanquet, Horn and Thring (Ibid) must be rearranged. An equation is given for the horizontal component of velocity at the jet axis.

\[ U = U_0 B R \left( \frac{\rho_0}{\rho_s} \right)^{1/2} \left/ \int_0^\infty \sec \frac{3}{2} \theta \ dX' \right. \]  

(3)

where \( X' = \) distance traveled relative to the surrounding fluid

\[ B = 12.78, \] determined experimentally by Hinze and Zijnen (6, p. 435-61)

and all other variables have been previously defined.

The expression may be reduced to the following if a horizontal jet is considered and the region of the jet is restricted to that portion before it becomes very strongly curved.

\[ U = U_0 B R \left( \frac{\rho_0}{\rho_s} \right)^{1/2} \left/ X' \right. \]

and using \( R = D/2 \)

\[ U = \frac{U_0 B}{2} \left( \frac{X'}{D} \right)^{1/2} = \frac{U_0 B}{2} \left( \frac{X'}{D} \right)^{1/2} \left/ \left( \frac{\rho_s}{\rho_0} \right)^{1/2} \right. \]  

(4)
Figure 4. Position of axis of cone-of-diffusion for variable velocity ratio, $U_o^2/Dg = 0.10$. 
Figure 5. Position of axis of cone-of-diffusion for variable velocity ratio, $U_o^2 / Dg = 0.40$. 
Figure 6. Position of axis of cone-of-diffusion for variable velocity ratio, $U_o^2/Dg = 0.80$. 
Assuming that $U$ is equal to $\bar{U}$, the time mean value of the component of local jet velocity in the axial direction,

$$
\bar{U} = \frac{U^0 B}{2 \left( \frac{X'}{D} \right) \left( \frac{\rho_s}{\rho_o} \right)^{\frac{1}{2}}}
$$

(5)

If $X'$ is the distance traveled relative to the surrounding fluid and $X$ is measured from the point of discharge

$$
X = \int (1 + \frac{U_s}{\bar{U}}) \, dX'.
$$

(6)

If the region under investigation is again restricted to that portion before the jet becomes very strongly curved

$$
X = X' \left\{ 1 + \frac{1}{2} \left( \frac{U_s}{\bar{U}} \right) \right\}
$$

(7)

and by substitution of equation (5)

$$
X = X' \left\{ 1 + \frac{U_s}{\bar{U}} B \left( \frac{X'}{D} \right) \left( \frac{\rho_s}{\rho_o} \right)^{\frac{1}{2}} \right\}
$$

(8a)

and substituting the numerical value of $B$

$$
X = X' \left\{ 1 + 0.0783 \left( \frac{U_s}{\bar{U}} \right) \left( \frac{X'}{D} \right) \left( \frac{\rho_s}{\rho_o} \right)^{\frac{1}{2}} \right\}
$$

(8b)

Dividing both sides by the nozzle diameter

$$
\frac{X}{D} = \frac{X'}{D} \left\{ 1 + 0.0783 \left( \frac{U_s}{\bar{U}} \right) \left( \frac{X'}{D} \right) \left( \frac{\rho_s}{\rho_o} \right)^{\frac{1}{2}} \right\}
$$

(9)

For a particular case $U_s/\bar{U}$ and $\rho_s/\rho_o$ have constant values and therefore may be combined in a single constant

$$
K = 0.0783 \left( \frac{U_s}{\bar{U}} \right) \left( \frac{\rho_s}{\rho_o} \right)^{\frac{1}{2}}.
$$

(10)
A simplified version of equation (9) may be written

\[
\frac{X}{D} = \frac{X'}{D} \left\{ 1 + K\left(\frac{X'}{D}\right) \right\}
\]  \hspace{1cm} (11a)

and

\[
\frac{X}{D} = \frac{X'}{D} + K\left(\frac{X'}{D}\right)^2
\]  \hspace{1cm} (11b)

Therefore, any calculations for curves with \( \frac{U_s}{U_o} \) equal to zero, Figures 2 and 3, may now be modified by use of equation (11b) and used to obtain similar curves for variable velocity ratios as shown in Figures 4, 5 and 6.
CONCLUSIONS

There are three rather important ocean characteristics which will affect the design of an ocean outfall system.

1) Predominate surface and subsurface current direction
   Alignment of the direction of discharge with that of the predominate subsurface or tidal current will keep the effluent below the surface for the longest possible time and thereby allow maximum diffusion. The curves of Figures 4 through 6 show that a variation in the velocity ratio has a considerable effect on the position of the cone-of-diffusion axis.

2) Upwelling
   During the period of upwelling it might be desirable to dilute the effluent with seawater prior to the time it is discharged and perhaps eliminate the possibility of beach contamination due to upwelled effluent. Figure 3 shows that a variation in density ratio has a significant effect on the position of the axis of the cone-of-diffusion.
(3) **Bottom conditions**

The Oregon coastline, having an average bottom slope of 80:1, appears well suited for the ocean outfall method of waste disposal.

At the time of disposal the densities of domestic sewage and kraft-mill effluent are almost equal. Therefore, design criteria satisfactory for domestic sewage outfalls may also be used for those carrying kraft-mill effluent.

Ocean outfall design problems are numerous, however, the major problem is that of diffusion. Diffusion increases with the length of time that the jet remains submerged. It is therefore quite important that the designer knows the relative importance of a variation in density or velocity of the outfall effluent. It is anticipated that the curves which predict the vertical position of the axis of the cone-of-diffusion will allow a more theoretical and perhaps a more accurate method of ocean outfall design.

Important aspects of the three types of curves which have been presented may be summarized as follows:

1. If \( \frac{U_s}{U_o} \) and \( \frac{\rho_s}{\rho_o} \) remain constant, the length of time which the jet remains submerged is increased by increasing the discharge velocity (Figure 2).
(2) If \( \frac{U_s}{U_o} \) and \( \frac{U_o^2}{Dg} \) remain constant, the length of time which the jet remains submerged is increased by decreasing the density ratio (Figure 3). For Kraft-mill effluent, this may be accomplished most effectively by increasing the density of the jet fluid.

(3) If \( \frac{U_o^2}{Dg} \) and \( \frac{\rho_s}{\rho_o} \) remain constant, the length of time which the jet remains submerged is increased as the velocity ratio becomes larger. The velocity ratio is increased as a result of either increasing the surrounding fluid velocity or decreasing the jet discharge velocity.

It must be noted, however, that if the discharge velocity is to be changed, the pipe diameter must also be changed in order for \( \frac{U_o^2}{Dg} \) to remain constant.

This study has collated a large number of previous jet diffusion studies but has expanded the results of a relatively few of these which were felt to be of particular importance to the problem of jet diffusion as related to ocean outfall systems. The two original contributions which this thesis has made to the field of jet diffusion may be summarized as follows:

(1) Equations of previous investigators relating \( X/D \) to \( Y/D \) are revised to include three dimensionless parameters.
(2) Curves are presented which show the effect of a variation in any one of these parameters upon the position of the cone-of-diffusion axis.
BIBLIOGRAPHY


