

TRANSISTOR DC AMPLIFIER DESIGN THEORY

by

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# ABBREVIATED LIST OF SYMBOLS

$h_{ie}$	Common-emitter input impedance with the output short-circuited. Also referred to as $h_{11e}$ .
$h_{re}$	Common-emitter backward voltage transfer ratio with the input open. Also referred to as $h_{12e}$ .
$h_{fe}$	Common-emitter forward current transfer ratio with the output short-circuited. Also referred to as $h_{21e}$ .
$h_{oe}$	Common-emitter output admittance with the input open. Also referred to as $h_{22e}$ .
$\alpha_n$	Ratio of collector current to emitter current, normal connection.
$\alpha_i$	Ratio of collector current to emitter current, inverted connection (collector and emitter terminals interchanged).
$f_\alpha$	Frequency at which alpha becomes 0.707 of its low-frequency value, common-base configuration.
$I_{co}$	Collector saturation current with the emitter open.
$I_{eo}$	Emitter saturation current with the collector open.
$I_e$	Emitter current.
$I_b$	Base current.
$I_c$	Collector current.
$V_{eb}$	Emitter-to-base voltage.
$V_{ce}$	Collector-to-emitter voltage.
$V_{cb}$	Collector-to-base voltage.
$K$	Boltzmann's constant, $1.37 \times 10^{-19}$ erg/degree.
$T$	Absolute temperature, $^{\circ}\text{K}$ .
$q$	Magnitude of charge on an electron, $1.6 \times 10^{-19}$ coulomb.
$\mathcal{J}$	Algebraically equal to $KT/q$ , or 0.026 volts at $300^{\circ}\text{K}$ .

$\Delta$  Difference operator.

$\rightarrow$  An amplifier, usually understood to be a dc amplifier unless otherwise indicated.

$E_s$  Signal voltage source.

$\mu$  micro,  $10^{-6}$ .

$m$  milli,  $10^{-3}$ .

$k$  kilo,  $10^3$ .

# TRANSISTOR DC AMPLIFIER DESIGN THEORY

## INTRODUCTION

This paper was prepared to present a practical and realizable solution to the problems of transistor dc amplification. Thus it consists not only of theoretical dc amplifier requirements, problems, and possible solutions for both dc and modulated dc systems, but also the design considerations and experimental results leading to the construction of an acceptable dc amplifier.

Inherent to the subject of general dc amplification is the problem of thermal drift associated with direct-coupled circuitry. In addition, the transistor functions according to well-defined thermal laws thus making temperature considerations even more critical in this application. Offsetting this disadvantage, however, is the tremendous ruggedness, reliability, and low power consumption of the transistor. At the present state of the art, the transistor has been proven to be as reliable as the best vacuum tubes, able to withstand shocks of several thousand g's capable of operating on milliwatts of power at temperatures up to  $150^{\circ}\text{C}$ , and available at about the same cost as a vacuum tube.

From the preceding description it is seen that the transistor is very desirable for many applications. In particular this report emphasizes the application of the dc amplifier to an operational voltage amplifier;



however, most of the information is equally relevant to general dc amplifier applications. Pertinent to all applications are considerations of stability, bandwidth, impedance levels, amplification noise, transistor interchangeability, and cost. These are some of the more general problems treated in this report.

A rather specific analysis is given to the chopper-type dc voltage amplifier which was experimentally investigated in detail. This amplifier consisted of a series arrangement of a transistor chopper, ac amplifier, demodulator, and dc amplifier.



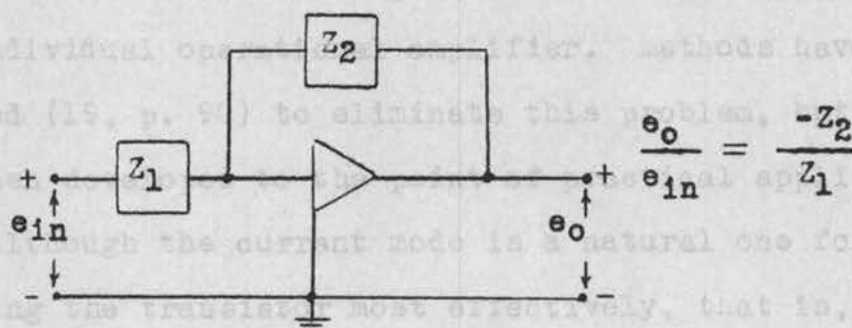
## DERIVATION OF DC AMPLIFIER REQUIREMENTS

Before an intelligent choice of a dc amplification system can be made, the requirements of such a system must be specified. Thus this phase of the report will attempt to develop the theoretical requirements of an operational amplifier\* along with the practical considerations involved in such a project.

### Operational Amplifier Analysis

The operational amplifier is commonly understood to be the basic building block of analog computers. In the intended application to an analog computer, a natural choice exists as to the mode of operation, voltage or current. The network configurations and idealized responses for each mode are indicated below (15, p. 410-13).

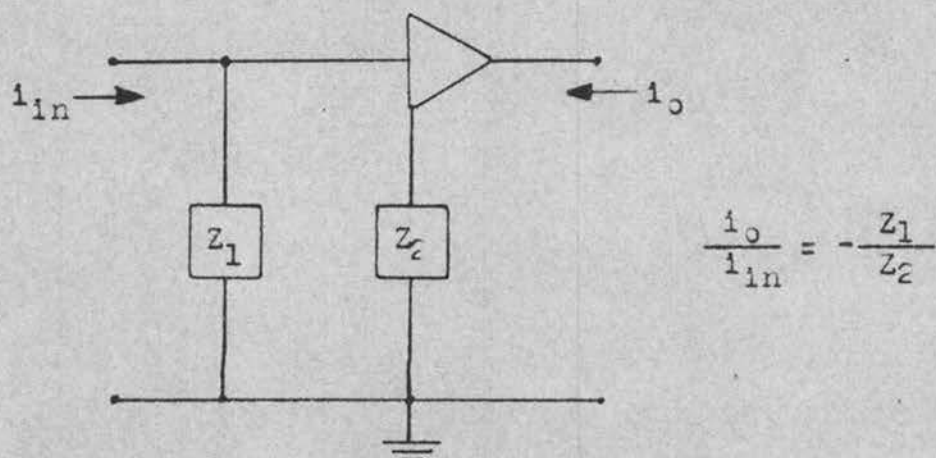
Figure 1. An Operational Voltage Amplifier




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\* An operational amplifier can be defined as a dc amplifier used with various feedback networks in analog computations.

Figure 2. An Operational Current Amplifier



The choice of the operational mode is now seen to be imposed by the proposed application. Since an analog computer requires at least ten operational amplifiers to handle a sufficiently large collection of problems, it is imperative that each amplifier be capable of operating from common power supplies. This condition eliminates the operational current amplifier from further consideration since the feedback impedance,  $Z_2$ , isolates each amplifier thus necessitating isolated power supplies for each individual operational amplifier. Methods have been proposed (19, p. 90) to eliminate this problem, but none have been developed to the point of practical applications. Thus, although the current mode is a natural one for utilizing the transistor most effectively, that is, the transistor is a current operated device, has low input impedance, and a high output impedance, the prohibitively

large bulk and cost of such an analog computer eliminates this choice.

At this point our problem is now confined to the operational voltage amplifier, figure 1. Since all such amplifiers have a common ground, they may utilize common power supplies and are thus readily adaptable to large analog computer operations.

Ideally, a voltage operated device such as an operational amplifier\* has infinite input impedance and zero output impedance. If in addition, the dc amplifier has infinite voltage amplification, an identity is established such that the response of the network shown in figure 1 becomes  $\frac{e_o}{e_{in}} = \frac{-Z_2}{Z_1}$ . This equation is the basis for all analog computer operations. Thus the utility of the operational amplifier lies in the accuracy with which this relationship may be obtained and the flexibility of operations which may be performed by choosing different impedances for  $Z_1$  and  $Z_2$ .

Since a transistor dc amplifier will by necessity have a finite input impedance, finite voltage amplification, and a non-zero output impedance, it is desirable to know what error is produced by virtue of these non-ideal conditions. The complete expression for the response of

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\* Throughout the rest of the report, the term "operational amplifier" will be understood to mean a voltage operated dc amplifier used with various feedback networks in analog computations.



the network shown in figure 1 can be shown (Appendix II) to be:

$$(1) \quad \frac{e_o}{e_{in}} = -\frac{Z_2}{Z_1} \left[ 1 + \frac{1}{A} \left( 1 + \frac{R_o}{r} + \frac{Z_2}{r} + \frac{R_o}{Z_1} + \frac{Z_2}{Z_1} + \frac{R_o}{Z_2} \right) \right]$$

Where:  $R_o$  = output impedance of dc amplifier  
 $r$  = input impedance of dc amplifier  
 $A$  = voltage amplification of dc amplifier

The error term is easily recognized as:

$$(2) \quad \text{error} = \epsilon = \frac{1}{A} \left( 1 + \frac{R_o}{r} + \frac{Z_2}{r} + \frac{R_o}{Z_1} + \frac{Z_2}{Z_1} + \frac{R_o}{Z_2} \right)$$

The limiting values of  $Z_1$  and  $Z_2$  for a given magnitude of error are easily found from this expression. For the specific case of  $A = 25,000$ ,  $R_o = 1,000$  ohms, and  $r = 2,000$  ohms the limiting values are plotted in figure 44. This figure illustrates the error of the operational amplifier which results from a given choice of  $Z_1$  and  $Z_2$  and an amplifier with the above mentioned specifications.

Certain conclusions are obvious from this analysis. First, it illustrates that an analog computer based on a dc amplifier with  $A = 25,000$ ,  $R_o = 1,000$  ohms, and  $r = 2,000$  ohms is not as limited by the choice of  $Z_1$  and  $Z_2$  as is commonly supposed, a maximum dc gain of 200 being possible with an error of only 1%. This indicates

that a transistorized operational amplifier, although restricted, has many applications.

The complete operational amplifier network responses for an integrator and differentiator operation with the non-ideal dc amplifier are developed in Appendix III and IV, respectively.

Equation 1 also implies many more requirements other than amplification,  $Z_1$ ,  $Z_2$ , input impedance and output impedance. These are:

1. The output drift due to temperature and aging are negligible.
2. The amplification  $A$ , is large at all frequencies for which the computation accuracy is to be good.
3. The amplifier including the feedback loop  $Z_1$  and  $Z_2$  is stable.
4. The effects of noise and non-linearity are negligible.

These requirements then impose conditions of thermal drift, aging, amplification, bandwidth, stability, phase-shift, noise, and non-linearity upon the operational amplifier.

### Practical Consideration

An inherent implication in any practical project is that it be economically feasible. This provides another requirement which will have to be considered since, practically speaking, it is always desirable to obtain the most

acceptable performance for the least cost.

Included indirectly in the cost requirement is the added condition that component tolerance be as large as possible. Thus it is desired to eliminate as many matched or hand-picked components as possible.

Another factor to be considered in a satisfactory dc amplifier design is the effects of interchanging transistors. It is desirable to be able to replace transistors of the same type with as little adjustment as possible and still obtain satisfactory results.

A further consideration is the stability of a series of cascaded operational amplifier networks with an overall feedback loop. In this configuration, which is commonly encountered, the phase-shift is additive thus imposing more stringent requirements on the phase-shift per amplifier.

Another requirement is apparent from an investigation of multiple network operations. Namely, that to prevent saturation in succeeding amplifiers each amplifier must be designed so that for the zero input condition, there is also a zero output. This condition is commonly known as an automatic zero-set, AZS arrangement.

#### Summary of Compromise Specifications

With all the previously mentioned considerations

in mind, a set of specifications may now be drawn up. It will be noted that the following specifications are quite lenient --the operational amplifier as a whole falling in about the five per-cent error category. Of course, it is desirable to refine the specifications so that the overall error would be less than one per-cent; however, these requirements provide a measure of usefulness as well as a suitable starting point which is practical.

A limitation which has not been mentioned previously is that for the presently available transistors, the maximum output voltage swing is about  $\pm 15$  volts. This figure appears below.

From the above conditions the following specifications may be derived for the proposed dc amplifier which is to be used in an operational amplifier network.

1. Voltage amplification to be equal to or greater than 20,000.
2. Input impedance to be equal to or greater than 2,000 ohms.
3. Output impedance to be equal to or less than 1,000 ohms.
4. Maximum output voltage swing to be  $\pm 15$  volts.
5. Thermal drift at the output to be equal to or less than 0.25 volt in the temperature range of  $70^{\circ}\text{F} \pm 15^{\circ}\text{F}$ .
6. Long term output drift to be less than 0.10 volt per hour.
7. Bandwidth to be dc to 1,000 cps, or greater.



8. Phase-shift at output to be less than  $10^\circ$  deviation from the net  $180^\circ$  phase-shift.
9. Noise at output to be less than 0.10 volt peak-to-peak.
10. Non-linear distortion to be less than 2%.
11. Transistors and components to be interchangeable with minimum of custom adjustments or hand-picking.
12. Cost to be minimum for acceptable performance.

## DISCUSSION OF TRANSISTOR OPERATIONAL CHARACTERISTICS

With the desired specifications in mind, the transistor characteristics may now be examined to isolate and analyze the particular aspects which will be important in this application. Consequently, this section of the report is included to provide the background information which is necessary before the significance of particular transistor dc amplification systems can be appreciated.

### Physical Characteristics

First to be considered are those characteristics in which the transistor exceeds its vacuum tube counterpart. These qualities are reliability, ruggedness, power dissipation, size, and weight. It is for these reasons that the transistor has been so rapidly developed and applied.

The reliability of transistors is usually considered from the standpoint of short-term and long-term effects. Short-term effects are those contributed by temperature and shock. Electrically the effects of temperature are well known, an increase in the saturation current of reverse-biased junctions being the primary effects of a rise in temperature. Thus it produces no failures in well designed circuits.

As far as mechanical ruggedness is concerned, it has experimentally been found that a shock of 500 g's will

damage less than 1% of the transistors tested. Even at 8,000 g's, a 60% to 75% survival rate was obtained. Vibration and acceleration characteristics are equally impressive, thus making the mechanical ruggedness of the transistor greater than that of many passive circuit elements (20, p. 95-97).

Long term effects generally include the changes produced by continuous operation, namely, aging effects and failure rates. Generally, it has been found that aging produces the same effects as noted from an increase in temperature. These effects are a reduction in the value of current gain and an increase in the collector saturation current,  $I_{co}$ . Most of the total change occurs in the first few hundred hours; silicon transistors usually showing considerably less change than germanium transistors. These changes are primarily due to changes in the transistor surface conditions (2, p. 376-86).

According to a recent Bell System study, (2, p. 388-91) grown-junction transistors in their P-carrier system exhibited a failure rate of 0.67% per 1,000 hours. In less complex circuitry where transistor requirements were not so stringent, failure rates ranged from  $10^{-4}$  % to 0.20% per 1,000 hours. As a basis for comparison, tube failure rates in IBM computer service were found to range from 0.17% to 2.0% per 1,000 hours. In the more

conservatively designed Bell systems the tube failure rate ranges from 0.019% to 0.29% per 1,000 hours. Also it is to be noted that in a submarine cable system, long considered the ultimate in reliability, the required failure rate was just under 0.03% per 1,000 hours. Thus the analysis of transistor failures in field equipment shows failure rates that are comparable with well-designed, conservatively used vacuum tubes. Also, since these results were obtained from transistors of 1956 or earlier vintage, it is probable that presently developed transistors have a higher reliability than these figures indicate.

The other more obvious characteristics which make the transistor a necessity for many applications are its extremely small size, weight and power dissipation.

The above mentioned characteristics are the primary reasons for adapting the transistor to a dc amplifier system.

### Inherent Limitations

Since a transistor exhibits different characteristics in each of its three configurations --common-emitter, common-base, and common-collector, a specific configuration must be specified with any discussion of its particular characteristics. The only configuration which is capable of an iterative voltage gain, however, is the

common-emitter stage (24, p. 73-88); thus this stage is the only one which deserves much consideration as a voltage amplifier. The common-base and common-collector stages, may be very useful as input and output stages where either a small or large, input or output resistance is desired. For these reasons the remainder of this discussion will be primarily directed at the characteristics of the common-emitter configuration.

A fundamental property of any junction transistor is the dependence of the collector saturation current,  $I_{co}$ , upon the temperature of the device. Since  $I_{co}$  varies exponentially with temperature, doubling about every  $10^{\circ}\text{C}$ , it is very temperature sensitive and considerable care must be exercised to account for its variation. However, the effects of  $I_{co}$  are well defined (24, p. 52-54), and consequently there is no mystery involved in designing a suitable temperature stable amplifier. The full significance of this limitation will not be developed until a later section.

Other factors which contribute to the thermal dependence of the transistors are the linear decay of  $V_{be}$  ( $\approx 2 \frac{\text{mv}}{^{\circ}\text{C}}$ ) a slow exponential rise in  $h_{oe}$ , and a small increase in  $h_{fe}$  at lower temperatures and a decrease at the higher temperatures (13, p. 25-30). These changes are, however, of secondary importance and are not usually considered



directly. In high temperature applications these factors may become appreciable.

Another transistor characteristic which is of paramount importance is the relationship between the magnitude and phase of the common-emitter current amplification,  $h_{fe}$ , and frequency. A valid expression or experimental result of this relationship is most difficult to obtain since  $h_{fe}$  is dependent to a great extent on the external circuit and on the extrinsic parameters, besides the intrinsic transistor. For our purposes, however, it is sufficient to note that feedback loops around two common-emitter stages are nearly always stable, while feedback loops around three stages are generally unstable. Also it should be noted that the upper cutoff frequency in the common-emitter configuration is usually slightly less than the quotient of the alpha cutoff frequency in the common-base configuration and the low frequency  $h_{fe}$ .

Like all other electronic devices the transistor exhibits definite noise properties. Generally the noise encountered in transistors is considered to include three different types; the thermal noise of an ohmic conductor, shot noise, and excess or semiconductor noise. The first two types are white noise while the last type is characterized by a  $\frac{1}{f}$  behavior and is usually found below 1,000 cps. Thus in dc applications it is quite likely that the

semiconductor noise would be most troublesome.

Outside of choosing a low noise-figure transistor to begin with, there are certain general procedures which tend to minimize the noise tendency. Driving the transistor from a source impedance of about 1,000 ohms seems to reduce the noise (23, p. 14-20) as does reducing the collector-to-emitter voltage and emitter current. Thus the operating point on the critical input stages should be made as low as possible.

Another condition to be considered is the non-linearity of  $h_{fe}$ . Generally  $h_{fe}$ , the small-signal parameter, decreases with increasing collector current. Thus for large-signal amplification the output will be appreciably distorted. The amount of distortion can be anticipated from a visual display of the characteristic curves of the transistor in question. To reduce this non-linear tendency it is feasible to use as large a load impedance as possible, and if further improvement is desired, to apply negative feedback around individual or multiple stages.

Other important parameters to be considered in a satisfactory design are the maximum collector power dissipation, and the maximum collector-to-emitter voltage. Needless to say, the ratings of the transistor should never be exceeded. These parameters are usually



determined by the type of transistor and its construction.

If interchangeability is a requirement to be considered then some thought must be given to the deviations which are likely to be found in transistors of the same type. Generally the manufacturer will state a maximum, minimum, or average  $h_{fe}$ , or a combination of the above. Other parameters are quite likely to vary as much as 100% from average. Thus the design must be able to accommodate very large deviations with a minimum of custom adjustments. Usually one must specify a minimum and maximum  $h_{fe}$  to insure that the transistor will be biased in a linear region. The other parameters are generally not so critical.

### Bias Stabilization

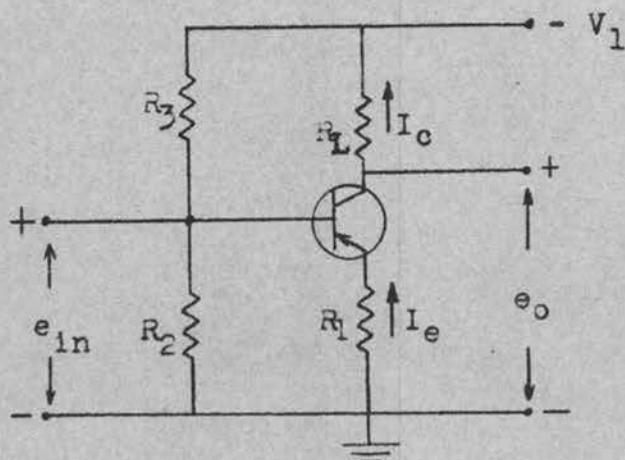
The temperature dependence of  $I_{co}$  has been previously mentioned as an inherent limitation of the transistor. In this section we shall discuss the stabilization, with regard to  $I_{co}$ , of the single-stage common-emitter biasing network and its effect on the dc voltage amplification of the stage.

It is easily found that besides providing a suitable biasing arrangement for the transistor, the biasing network determines the sensitivity of the external circuit to temperature. Thus a well-designed biasing network is mandatory. Several suitable biasing schemes exist (24 p. 68-71), however, we shall examine just one of them

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which is very general and useful. Other biasing schemes would yield the same general conclusions.

Figure 3. A Common-Emitter Single Battery Biasing Circuit



Certain stability factors may be derived for this circuit on the basis of the variation of  $I_{co}$  and its effect on the external circuit. According to Shea's analysis (24, p. 54-71), the current stability factor is defined as the quotient of the change in emitter current and the change in  $I_{co}$ , or algebraically:

$$(3) \quad S_I = \frac{\Delta I_e}{\Delta I_{co}} = \frac{G_1}{G_2 + G_3}, \text{ for average conditions, } (G_n = \frac{1}{R_n})$$

Likewise, the voltage stability factor is defined as the quotient of the change in collector-to-base voltage and the change in  $I_{co}$ .

$$(4) \quad S_V = \frac{\Delta V_{cb}}{\Delta I_{co}} = S_I R_1 + (1 + S_I) R_L, \text{ for average conditions.}$$

To meet the small thermal drift requirements, therefore, ideally  $S_V$  and  $S_I$  should equal zero. To approach this result, equation 3 states that  $R_1$  should be large and  $R_2$  and  $R_3$  should be small. The voltage stability equation states that the above mentioned conditions should be met plus the condition that  $R_L$  should be small.

One other result is needed to fully appreciate the significance of the above statements. This result is the expression for the input impedance of the transistor.

$$(5) \quad R_{in} = h_{ie} + h_{fe} R_1, \text{ for average conditions.}$$

From these equations the full significance of the gain versus bias stability conflict may be understood. By increasing  $R_1$  and decreasing  $R_2$  and  $R_3$ , thereby increasing the thermal stability, the input impedance of the transistor is raised geometrically while the input impedance of the complete stage is decreased. Thus if this stage is driven by a current source, most of the signal current will be lost through  $R_2$  and  $R_3$  and very little current will enter the transistor. If the stage is driven by a voltage source, the shunting effect of  $R_2$  and  $R_3$  will not be so noticeable; however, the high input impedance of the transistor will definitely limit the current which can enter the transistor. In either case,

the signal current which can enter the transistor is comparatively small. It naturally follows then that the voltage amplification of the stage will be very small since the transistor is a current operated device. Furthermore, the condition that  $R_L$  shall be small for good thermal stability limits the possible voltage amplification even more. For these reasons it is very difficult to achieve appreciable dc voltage amplification per stage with suitable temperature stability.

From the definitions of the voltage and current stability factors, it is apparent that the stability is directly dependent upon  $\Delta I_{co}$ . Thus if  $\Delta I_{co}$  could be decreased for the same change in temperature, the gain versus stability conflict would be alleviated somewhat. This is the reason for the introduction of the silicon transistor. The  $I_{co}$  for both germanium and silicon increase at about the same rate with temperature, however, at room temperature an average silicon transistor has an  $I_{co}$  of the order of  $10^{-8}$  amps while an average germanium transistor has an  $I_{co}$  of the order of  $10^{-6}$  amps. Thus silicon transistors are much better suited to high temperatures applications and dc amplification. It seems that nature always works against the engineer, for accompanying the beneficial temperature characteristics of



silicon is a cost of about ten times that of germanium.

So, a conflict still exists.



## ANALYSIS OF DC AMPLIFIER SYSTEMS

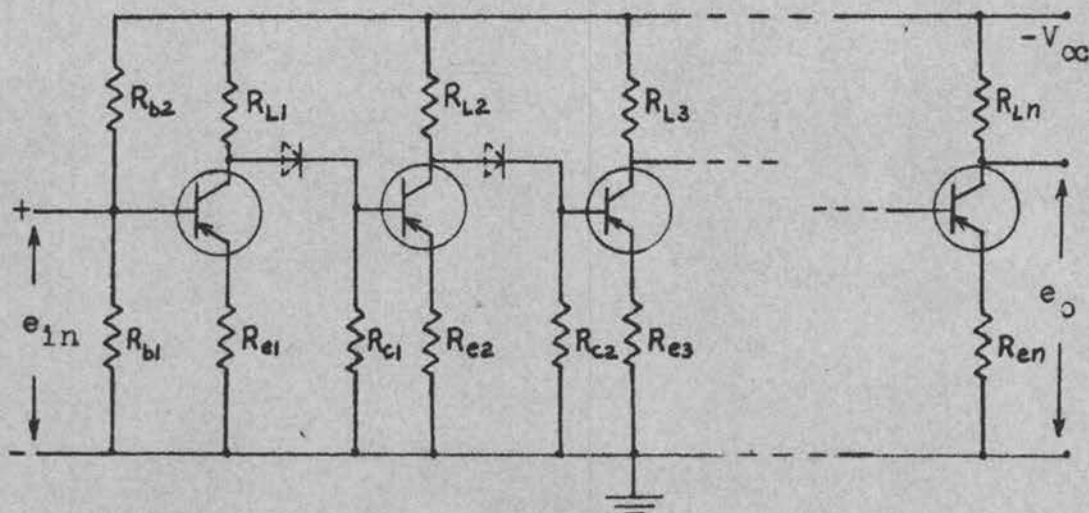
To obtain the optimum system for a given application, all applicable systems must be evaluated. It is for this reason that a brief analysis must be given to all the systems which could possibly satisfy the previously set forth requirements. From this analysis, then, a system can be chosen which will best satisfy the requirements for the application.

Generally a dc amplifier system will be either a direct-coupled configuration or a modulated dc system. These are the classifications which will be discussed in this section.

### Direct Coupled Configuration

The characteristics of the individual transistor common-emitter stage have already been examined in the previous section. From this discussion a rather obvious possibility is apparent, that of a cascade of several common-emitter stages. A conventional arrangement of such a circuit is illustrated below.

Figure 4. A Direct-Coupled Single-Sided DC Amplifier



Since this type of circuit is merely a series of common-emitter stages, all the limitations of the single stage still apply individually. However, it should be noted that if all stages are identical, each iterative stage has perfect  $I_{CO}$  stability, and the temperature characteristics of the complete circuit become approximately those of only the first stages. Even with considerable difference between stages, as long as the transistors are all of the same polarity --either PNP or NPN, some cancellation of temperature characteristics from stage to stage will be effected. On the other hand, if the transistors alternate polarity; that is, PNP-NPN-PNP --etc., the temperature characteristics from stage to stage combine additively so that the complete circuit would have very poor thermal stability characteristics.

Thus in a real application where it is certainly not desirable to have all stages identical from the standpoint of their quiescent operating points, a certain amount of cancellation of temperature characteristics may be effected from stage to stage by using similar polarity transistors. However, a compromise is still in order between the voltage amplification per stage and thermal stability considerations. This compromise means accepting a low amplification per stage, which indicates a large number of stages, in an attempt to preserve some thermal stability.

This compromise, however, with its large number of stages introduces other problems, namely the voltage accumulation from stage to stage. If a large collector supply voltage is used it further aggravates the gain versus bias stability conflict because of the necessity for larger resistors in the front stages of the circuit. A smaller collector voltage supply may be used if breakdown diodes are used between stages to compensate for the voltage accumulation. If the diodes are used, as indicated in figure 4, the resistors  $R_{c1}$ ,  $R_{c2}$ , ...,  $R_{cn}$ , are usually necessary to provide sufficient current to fully saturate or break down the diode. Otherwise these resistors may not be necessary. Their insertion provides an additional shunt path thus lowering the amplification.

The breakdown diode in saturation exhibits an

intrinsic incremental resistance of:

$$(6) \quad R_{inc} = \frac{\delta}{I} \quad \text{where} \quad \delta = \frac{KT}{q}$$

$\delta = 0.026 \text{ volts at } 300^{\circ}\text{K}$

$I = \text{Current through the diode}$

Additionally, the breakdown diode has a positive temperature coefficient of from 0.001 to 8.0% which must be considered.

The voltage accumulation presents an additional problem in that an output device must be constructed to satisfy the automatic-zero-set requirement.

It is easily recognized that the voltage accumulation problem is eliminated if the transistors are of alternate polarity; however, as previously indicated this combination would exhibit very poor thermal characteristics.

Several non-linear compensating techniques have been developed so that an amplifier may exhibit appreciable gain per stage and yet have satisfactory thermal stability. Among these techniques are the utilization of the positive voltage temperature coefficient of reverse biased diodes, the negative voltage temperature coefficient of forward biased diodes, the negative resistance coefficient of both forward and reverse biased diodes, other temperature sensitive resistors, and extra transistors of the same type



used as compensating current generators (24, p. 20-23). All these techniques, however, are dependent upon the matching of the temperature characteristics of the compensated and compensating devices. This requires special hand-picked and matched components which are hard to obtain, costly, and definitely undesirable from the standpoint of interchangeability.

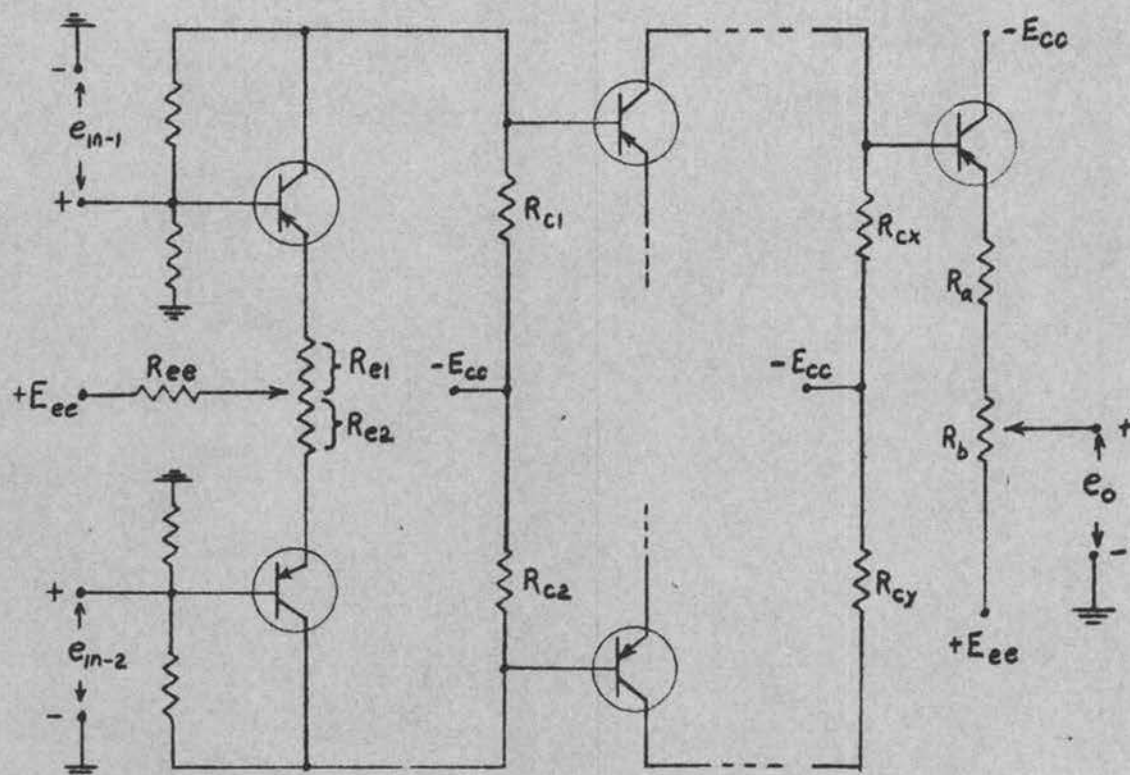
From this brief analysis, the single-sided cascaded dc amplifier is seen to have definite shortcomings. To achieve appreciable amplification per stage either the temperature stability must be sacrificed, the increased cost of silicon transistors must be absorbed, or the increased cost and complexity of matched compensating devices must be tolerated. Additionally, the voltage accumulation necessitates an output device to provide an AZS arrangement.

A typical figure for the drift of such a dc amplifier is 100 millivolts referred to the input (8, p. 249) for a temperature change of 20°C to 50°C.

Another direct coupled configuration which has considerable merit as a dc amplifier is the differential amplifier (12, p. 157-60). The differential amplifier is a form of double-sided or double-ended circuit which amplifies the difference between its inputs. A conventional arrangement is shown below:



Figure 5. A Differential DC Amplifier



The principle advantages attributed to this type of circuit (18, p. 194-98) over the single-sided circuit are that the temperature induced variations tend to oppose each other, and that the emitter resistor  $R_{ee}$  can be made as large as necessary, thus making the sum of emitter currents as stable as desired, without adversely affecting the gain per stage. Other desirable characteristics are that the gain and input impedance are twice that of its single-sided counterpart. The differential amplifier also has a voltage accumulation from stage to stage which

requires the same consideration as discussed for the single-sided amplifier. An AZS arrangement is also shown in the figure.

The primary problem with this circuit is matching each pair of transistors with respect to temperature induced  $I_{co}$  variations, emitter current,  $h_{fe}$ , and base-to-emitter voltage variations with temperature. The degree of matching naturally determines the extent of idealization which may be expected from the amplifier.

A typical figure for the thermal drift of such a dc amplifier in the temperature range of 20°C to 50°C is about 2 millivolts referred to the input (8, p. 249).

Another matter of some practical concern in applying this circuit to an operational amplifier is that a device must be provided to eliminate the dc bias at the input.

From this brief discussion, it is seen that the differential amplifier is much to be preferred over the single-sided amplifier; however, it is still not very desirable for this application because of its necessity for matched transistors.

At this point it is generally concluded that the application of direct-coupled dc amplifier configurations is definitely limited, primarily because of the gain versus bias stability conflict. At best the situation is alleviated somewhat by matched transistors and components,

complex circuitry, and custom adjustments; all being prohibitive from the standpoint of cost, time and interchangeability. The next section deals with a method which practically eliminates the previously noted conflict.

### Modulated DC Systems

According to IRE standards, modulate means to vary some characteristic of one wave in accordance with another wave. In this discussion a modulated dc system refers to a system which utilizes the advantages gained by changing the dc signal to an ac signal in some aspect of its operation, the advantage being that it is much easier to amplify ac signals with acceptable thermal stability than dc signals. This advantage is attributable to the use of reactive bypass and coupling components.

An illustration of possible systems configurations is shown on the following pages. It will be noted that a discrimination is made between dc chopper amplifiers and dc chopper stabilized amplifiers. A chopper amplifier is one in which the chopper and its associated circuitry are the sole mode of amplification; whereas a chopper stabilized amplifier is one in which the chopper amplifier is used in conjunction with another mode of amplification.

Figure 6-A illustrates a rather novel, although conventional, approach to eliminating thermal drift. In this system the output of the conventional dc amplifier is divided by its amplification factor and compared to the input. The



Figure 6, CHOPPER STABILIZED DC AMPLIFIER SYSTEMS. 30

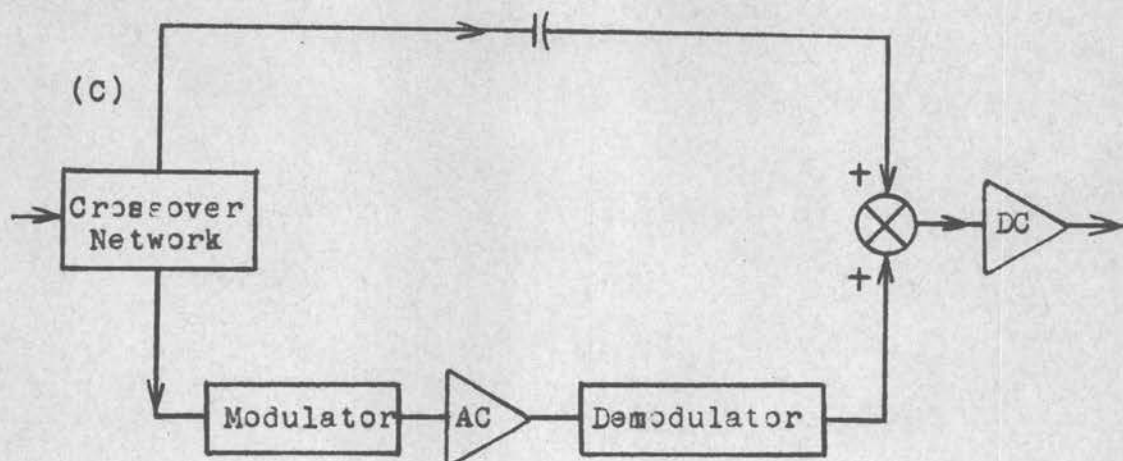
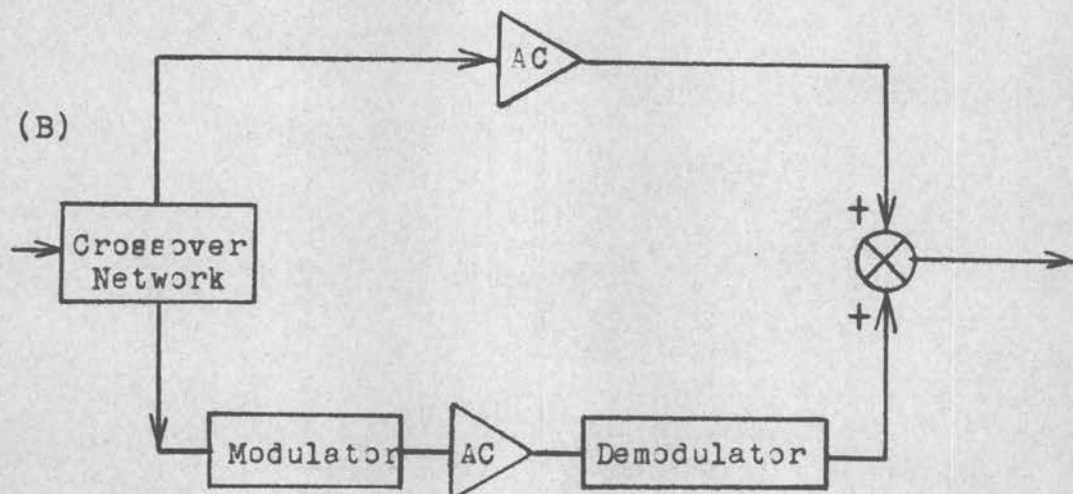
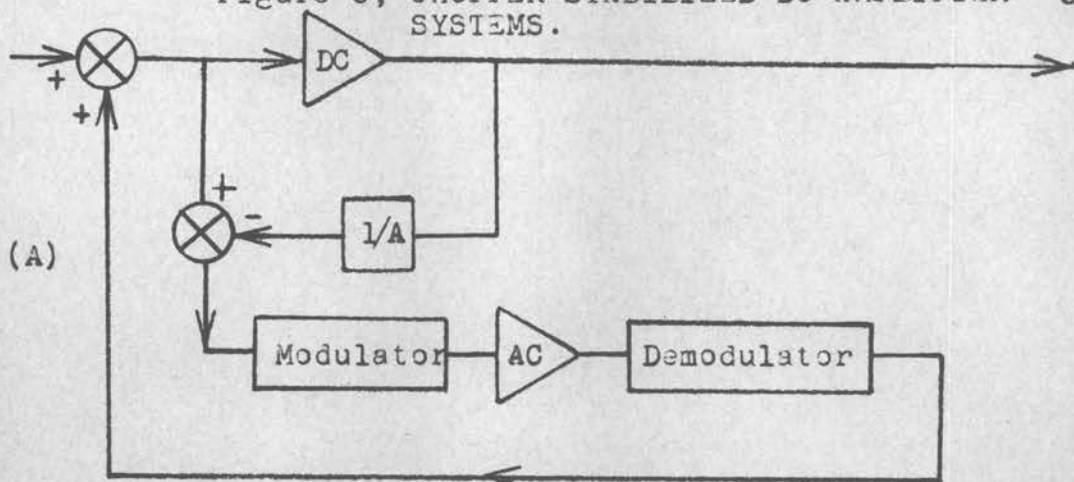
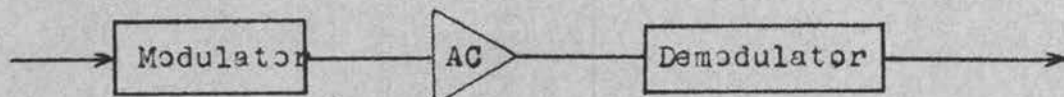
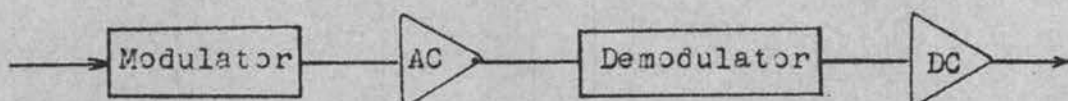


Figure 7, DC Chopper Amplifier Configurations

(A)



(B)



difference then activates the chopper-amplifier which produces an input to counteract this difference. Thus the drift of the main dc amplifier is reduced by the amplification of the chopper amplifier. The basic disadvantage of this system is that it appears to be too complex and refined for this application. Undoubtedly, it could meet the desired requirements. However, a simpler technique is more desirable.

The system shown in figure 6-B provides a different amplifier for the ac and dc signal components. Although



this system could provide high gain with a minimum of drift and an extremely large bandwidth, it again suffers because of its complexity.

The system of figure 6-C is merely a simplification of figure 6-B. Instead of providing a separate ac amplifier the high frequency signals are fed directly into the high-level output dc amplifier. Because of this simplification the bandwidth is considerably reduced over that of figure 6-B. The stabilization of this system could logically provide a practical problem, and the system again appears to be too complex for this application.

Of all the modulated dc systems, the chopper amplifier shown in figure 7-A is seen to be the ultimate in simplicity. However, since the demodulator is usually simply a low-pass RC filter, the system could be unstable with certain types of feedback encountered in operational amplifier operations (5, p. 10). The system of figure 7-B remedies this situation by isolating the demodulator from the output by a high-level dc amplifier. Thus if the modulating techniques will allow a sufficient bandwidth and the ac and dc amplifiers provide a sufficient voltage amplification, this system appears to be the most desirable.

From this analysis the system shown in figure 7-B was chosen as the one to be experimentally evaluated.

Consequently, most of the rest of this report is an investigation of this system.

### Optimum System Analysis (Figure 7-B)

Theoretically the modulation technique could utilize the phase, frequency, or amplitude of the carrier signal. However, the simplest technique is merely that of chopping the input signal. The chopped signal is then amplified in the ac amplifier, the amplified dc input level is recovered in the demodulator, and the output high-level dc amplifier further amplifies the signal and isolates the output from the demodulator.

By virtue of its simplicity the chopper is a natural choice of the modulation technique. However, here a choice exists as to full-wave or half-wave chopping. With full-wave chopping there is no inherent phase-shift; however, it requires a double-sided chopper, ac amplifier, and demodulator. This indicates twice as many components as those required for a half-wave chopping system. The half-wave chopping system, though, introduces an effective time lag, or phase shift, when the chopped signal is demodulated, equal to half the period between samples.

Algebraically:

$$(7) \quad \phi = \frac{f_s}{f_c} 180^\circ, \quad \text{where} \quad \begin{array}{l} f_s = \text{signal frequency} \\ f_c = \text{chopping frequency} \\ \phi = \text{phase-shift} \end{array}$$

Thus to restrict the phase-shift to  $10^0$  or less with a maximum frequency of 1,000 cps, the chopping frequency must be at least 18,000 cps. This is perfectly feasible with a transistor chopper.

Additionally, since the chopping technique is merely a form of sampling, the sampling theorems and principles (3, p. 37-41) do apply. The basic information which is to be obtained from this analysis, however, is that the sampled wave must be sampled at a rate at least twice that of the highest significant signal frequency to obtain all of the original information which is contained in the sampled wave. Practically, the sampling rate must be considerably faster than that predicted theoretically. Also, the theoretical considerations show that the demodulator should consist of a low pass filter with a cutoff frequency of one-half the chopping frequency. Under these idealized conditions no information in the sampled wave will be lost.

A practical analysis indicates that other frequency sensitive networks, usually low-pass RC filters, must be used to preserve stability. By necessity these shaping networks must be placed in the dc sections of the amplifier; that is, at the input, after the demodulator, and after or within the dc amplifier (7, p. 10-13).

To minimize the thermal drift it is also seen that the dc amplifier gain should be as low as possible. This

necessitates a high gain ac amplifier.



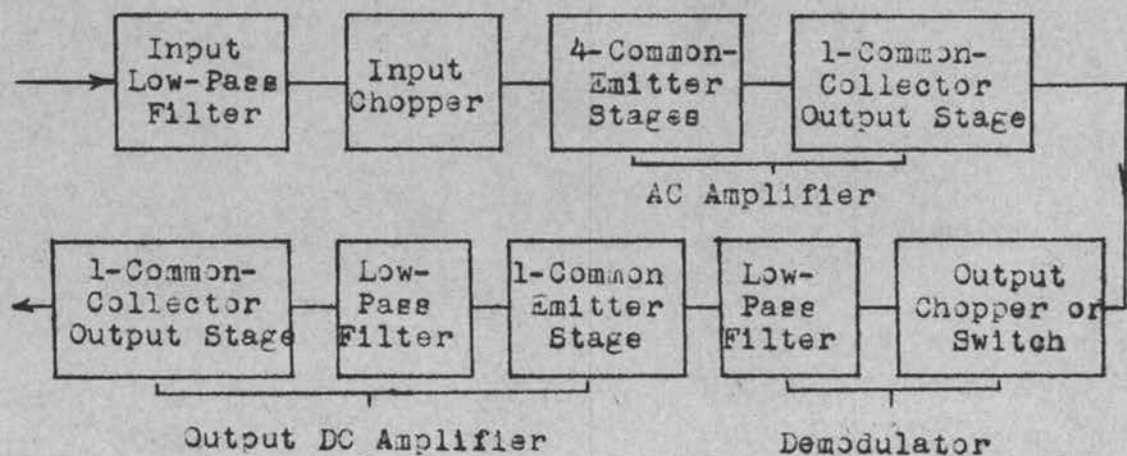
## CIRCUIT DESIGN ANALYSIS OF THE OPTIMUM SYSTEM

With the desired system and specifications now in mind, the individual components may be developed. An attempt is made to make each component a complete discussion within itself. This development will follow the natural sequential order; that is, chopper, ac amplifier, demodulator, and dc amplifier.

Preliminary Design Layout

Before entering into the development of each component, it is advantageous to have a more specific idea of the overall system which is being designed. Consequently, the system block diagram is shown below in order that one may better understand the development to follow. This block or flow diagram is derived in part from the previous discussion and the remainder from the development which follows:

Figure 8. A Specific Chopper-Amplifier Block Diagram





At this point certain conclusions can be drawn from the preliminary layout (17, p. 34-39). First, it is noted that the input filter must have an input resistance of 2,000 ohms to meet the previously set forth requirements. Next, it is apparent that the input chopper must have a very small bias error, or in other words, for a zero input signal it must have a very near zero output since the rest of the circuit cannot tell the difference between a bias error produced by the chopper or an external signal. Thus the chopper must be a very high quality chopper and capable of switching very rapidly. It is obvious that the sensitivity of the chopper determines the sensitivity of the system. Consequently, the success of the system, to a large extent, is determined by the performance of the chopper.

To be discussed in more detail later in the report, are the common-collector output stages on the end of the ac amplifier and dc amplifier. These stages, it will be noted, produce no phase-shift and a low output resistance.

Overall there is seen to be a  $180^\circ$  phase-shift which is necessary in order that the feedback loop and amplifier be stable. Also, the low-pass filters help maintain stability since they are inserted to attenuate the higher frequencies where the total phase-shift may approach  $0^\circ$  and oscillations would otherwise occur.

The single-stage dc amplifier must obviously be quite stable with temperature if the system is to be stable with temperature variations. However, if most of the amplification can be obtained in the ac amplifier, the dc amplifier need not produce very much gain.

At this point it should also be mentioned that a good choice of power supply voltages is best made if one starts at the output which must be at 0 dc level and work back toward the front. Naturally, the transistors and other circuitry must be well in mind before this choice is made.

#### Chopper Design Consideration

A search for a suitable chopper or switch to convert dc to ac will yield many different methods. Among these are the mechanical chopper, vacuum and gas tube switching circuits, diode switching circuits, cryatron, transistor switches utilizing the Hall effect, and the conventional transistor switching circuits. At this point we shall confine our attention to the latter method --conventional transistor switching circuits. This choice is dictated by the requirements of availability, simplicity, high quality, and high switching frequency.

The basic principle of conventional transistor switching circuits is suggested by the well-defined dependence of the emitter current, and hence the collector

current, upon the emitter-to-base voltage. Algebraically, the relationship can be shown (18, p. 344) to be:

$$(8) \quad I_e = I_{eo} \left( e^{\frac{V_{eb}}{\mathcal{J}}} - 1 \right) - \alpha_i I_c$$

$$\mathcal{J} = \frac{KT}{q}$$

$$(9) \quad I_c = I_{co} \left( e^{\frac{V_{eb}}{\mathcal{J}}} - 1 \right) - \alpha_n I_e, \quad \text{where: } \alpha_i = \text{Inverted alpha}$$

$$\alpha_n = \text{Normal alpha}$$

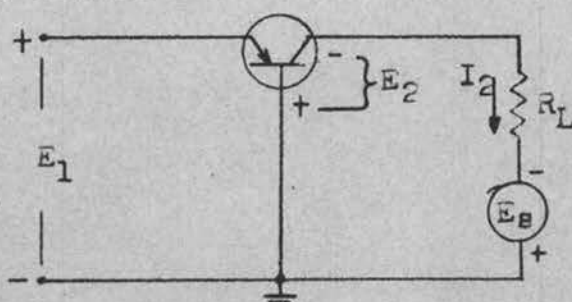
It will be noted that these equations as well as the following discussion apply to polarities associated with a PNP transistor. The same information applies to NPN transistors with polarities reversed.

These equations show that if the emitter junction is made positive with respect to the base, a large emitter current of exponential order will flow. (Since  $I_c$  is normally negative, the two components of  $I_e$  combine additively.) On the other hand, if the emitter is made negative with respect to the base, a very small emitter current will flow. Thus the state of the switch is determined by the polarity of the emitter-to-base voltage.

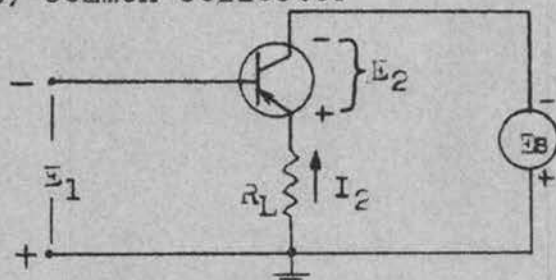
Extremely different switching characteristics are obtained in each of the different switching configurations. Thus an analysis of each configuration is in order. These configurations are illustrated on the following page.

Figure 9. The Basic Switch Configurations

## (A) Common-Base



## (B) Common-Collector

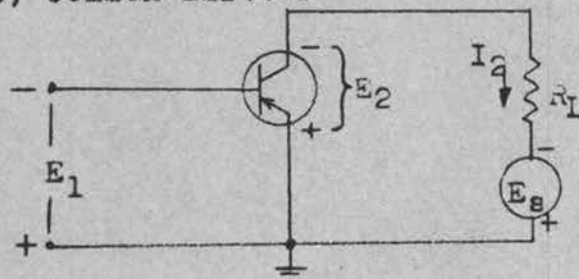
Ideal CharacteristicsSaturated (ON)  $E_2 = 0$ 

$$I_2 = \frac{E_s}{R_L}$$

Cutoff (OFF)  $I_2 = 0$ 

$$E_2 = E_s$$

## (C) Common-Emitter



In the preceding figures it is understood that  $E_1$  is the voltage which determines the state of the switch, and that  $E_s$  is the voltage which is being switched into the load resistance,  $R_L$ .  $E_2$  and  $I_2$  are the quantities which are dependent upon the transistor and determine the quality of the switch.

Under the assumption that these switches are switched

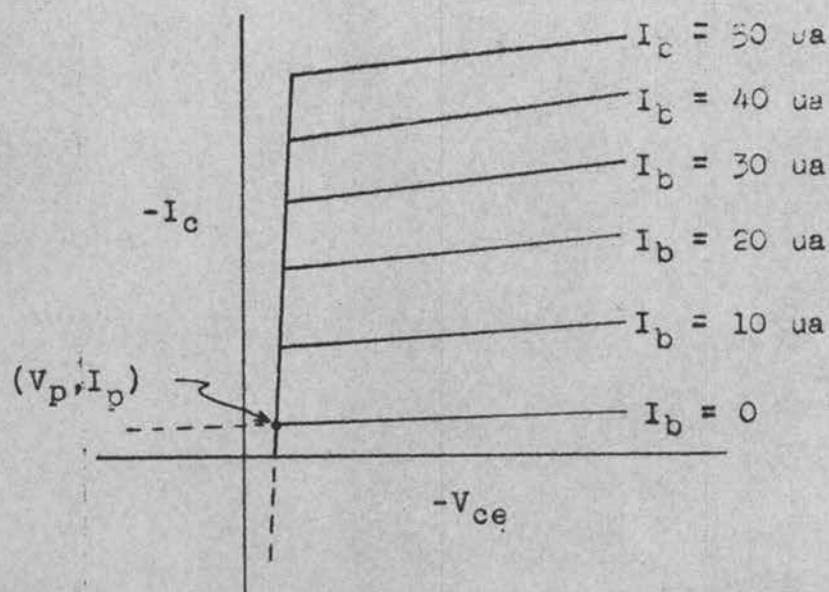


from the saturation region to the cut-off region, the characteristics of each configuration may be obtained (18, p. 339-352). Basically, the common base configuration is undesirable because the saturation voltage,  $E_2$ , is negative and by no means negligible, being in the order of a tenth of a volt. The common collector switch exhibits a small saturation voltage, but the cut-off current,  $I_2$ , is negative, which is also undesirable. The common-emitter switch is the best behaved configuration as both the cut-off current,  $I_2$ , and saturation voltage,  $E_2$ , are very small and always positive. Thus the common-emitter configuration offers the most ideal characteristics.

Now that the basic configuration has been determined, a more detailed analysis of its operating characteristics should be made in order to find the causes, effects, and solutions of the non-ideal characteristics.

Of primary importance is the magnitude of the saturation voltage and cut-off current. These parameters can easily be recognized from the common-emitter characteristic curves. An idealized and exaggerated set of these curves is illustrated on the following page to emphasize the point.

Figure 10. Idealized PNP Common-Emitter Characteristic Curves



During the saturation the voltage  $V_p$  exists between the collector and emitter thus yielding the saturation voltage, and during cutoff, a cutoff current  $I_p$  flows through the collector leg thus yielding the cutoff current. The above discussion then suggests that the transistor switch can be thought of as an ideal switch shunted by a current source with finite impedance during cutoff, or in series with a voltage source with internal resistance during saturation.

In terms of the transistor parameters, the non-ideal saturation voltage and cutoff current can be expressed as (16, p. 16-21):

$$(10) \quad I_p^n = \frac{(1 - \alpha_i)}{(1 - \alpha_n \alpha_i)} I_{co}$$

$$(11) \quad V_p^n = \frac{KT}{q} \ln \frac{1}{\alpha_i}$$

For ordinary germanium transistors, these parameters usually are in the order of several microamps and several millivolts.

Now if the transistor is inverted; viz, the collector and emitter terminals interchanged, the above parameters are reduced by approximately one order of magnitude (16, p. 16-21).

$$(12) \quad I_p^1 = \frac{(1 - \alpha_n) I_{eo}}{1 - \alpha_n \alpha_i}$$

$$(13) \quad V_p^1 = \frac{KT}{q} \ln \frac{1}{\alpha_n} \quad \text{Where } I_{eo} \alpha_n I_{eo} \alpha_i$$

This inversion is possible since both the collector and emitter terminals are of the same polarity, N-type or P-type. Thus either one may act as the emitter or collector. However, in the normal configuration the collector is made larger than the emitter so as to have a high collector efficiency. Thus when these functions are reversed the collector efficiency is considerably reduced. For this reason, generally the  $\alpha_i$  ranges from 0.3 to 0.9 and  $\alpha_n$  ranges from 0.9 to 1.0. Accompanying the reduced collector efficiency is also a decrease



in the maximum allowable power dissipation.

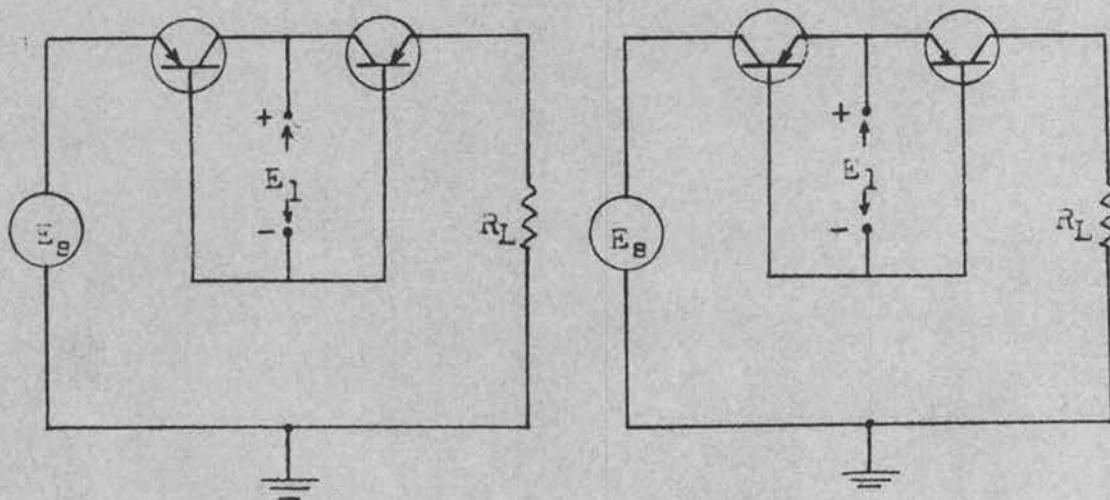
A closer examination of the single stage common-emitter switch will also show that it is incapable of turning off appreciable negative signals.

After understanding the characteristics of the single-stage common-emitter switch, we are now ready to proceed with the next logical step in a refinement of the transistor switch, namely, the differential type, or series-pair switch.

Figure 11. The Series-Pair Switch

(a) Inverted Connection

(b) Normal Connection



A rather obvious advantage of this switch is that at every interval of time, the saturation voltage or cutoff current, depending upon the state of the switch,



of the two transistors combine differentially. This greatly improves the quality of the switching characteristics.

In analyzing this and other more complex transistor switches a useful rule to keep in mind is as follows: The transistor may conduct in either direction if either junction is forward biased, and the transistor will be cutoff only if both junctions are reverse biased. From this information it is seen that with  $E_1$  positive as indicated, both transistors are turned on; and with  $E_1$  negative, one transistor always is turned off regardless of the signal polarity,  $E_s$ . Thus this switch may be used to switch signals of either polarity up to the breakdown voltage of the transistors.

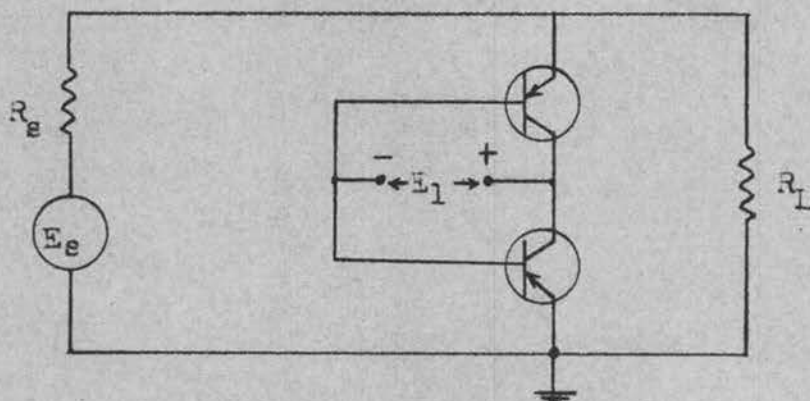
As far as current limitations are concerned, it is advisable to restrict the current through the switch in the inverted connection to about  $2/3$  of the maximum rated current. This restriction is primarily due to the smaller inverted alpha. In the normal connection, the full rated  $I_c$  may be recognized.

Generally speaking, the normal configuration is preferable in high level applications, while the inverted configuration provides better low level performance. Also, the series switch, meaning that the switch is a series with the load, is usually preferable in low impedance

(less than a thousand ohms) circuits; while the shunt switch, meaning that the switch is in parallel with the load, is used more advantageously in the high impedance circuit (11, p. 28-35).

At this point then, we are ready to choose a configuration to be used as the input chopper in the proposed dc chopper amplifier. Since this application requires a high quality low-level switch to be used in a fairly high impedance circuit, the natural choice is the shunt, inverted connection, series-pair configuration.

Figure 12. The Shunt Series-Pair Switch, Inverted Connection



With this configuration in mind, we are now in a position to study the more specific characteristics of the switch in actual operation.

As to the choice of transistors, theoretical considerations indicate that a high  $\alpha_n$ , low  $\alpha_i$ , and low  $I_{co}$  are desirable. Also to be considered are the

collector saturation resistance,  $R_{cs}$ , and transient switching characteristics.

Of the available transistors, the Philco 2N346 surface barrier transistor seemed to be best suited to this application. Only germanium transistors were considered, although silicon transistors are more desirable from the standpoint of their lower  $I_{co}$ . Also it was found that grown junction transistors are definitely unsuitable for low-level applications because of their large ohmic resistance which makes the collector saturation resistance very large, several hundred ohms.

The 2N346 was found to have an  $\alpha_n$  of about 0.975,  $\alpha_i$  of about 0.8 and, an  $I_{co}$  of about 1 to 2  $\mu a$ . Also, its alpha cutoff frequency of 75 Mc was found to be indicative of excellent high speed switching characteristics.

Of basic importance in the "on" or conducting state are the collector saturation resistance and collector saturation voltage. The emitter-to-collector saturation resistance,  $R_{cs}$ , for a transistor in the inverted connection has been shown (5, p. 5) to be:

$$(14) \quad R_{cs} = \frac{kT}{q} \frac{1}{I_b} (1 - \alpha_n \alpha_i) \alpha_i + R_x \quad \text{where: } I_b = \text{Base current} \\ R_x = \text{Ohmic resistance}$$



For the specific case of the 2N346, this equation becomes:

$$(15) \quad R_{cs} = 7.15 \times \frac{10^{-3}}{I_b} + 10, \text{ ohms}$$

(The ohmic resistance of 10 ohms is estimated)

And at  $I_b = 400 \text{ ua}$ ,  $R_{cs} = 28 \text{ ohms}^*$ .

As a verification of this theoretical result, the experimental values of  $R_{cs}$  were obtained. The experimental value is easily obtained from a visual display of the characteristic curves. A typical set of common-emitter, inverted connection, characteristic curves with saturated resistances indicated are shown on the following page.

It is easily proven that the experimental and theoretical values compare favorably indicating the validity of the equation. The experimental results also show that fairly small saturation resistances can be obtained by driving the transistor with sufficient base current.

The collector saturation voltage is a slightly more complex function since it depends on both the effective load resistance and base current. This relationship is illustrated in figure 14. From this set of curves it is seen that if the effective load resistance as seen by

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\* Power transistors have much lower  $R_{cs}$  values but are not suited for low-level or high-frequency switching.



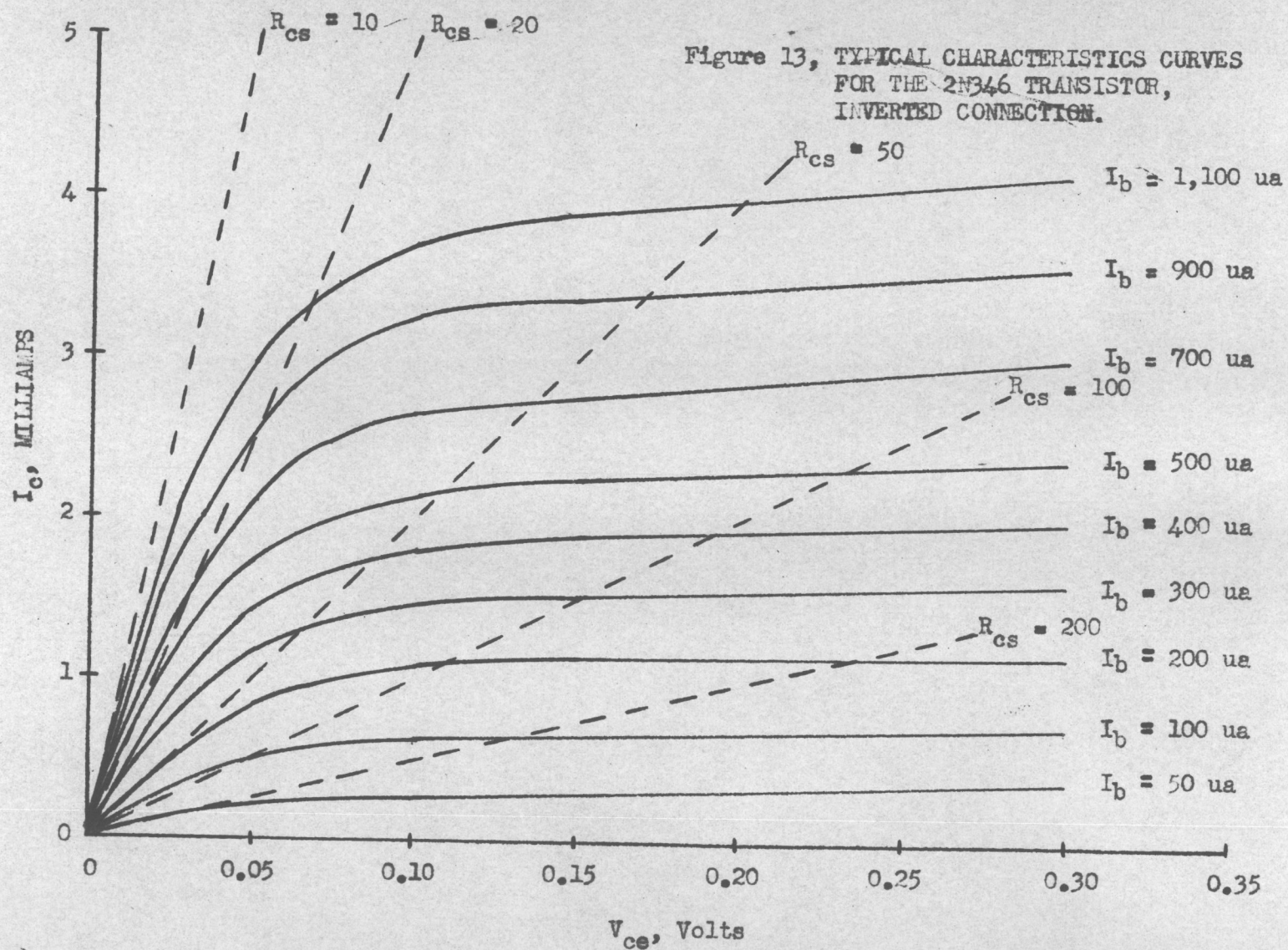


Figure 14, COLLECTOR-TO-EMITTER SATURATION VOLTAGE AND BASE  
CURRENT FOR THE 2N346 TRANSISTOR, INVERTED CONNECTION.

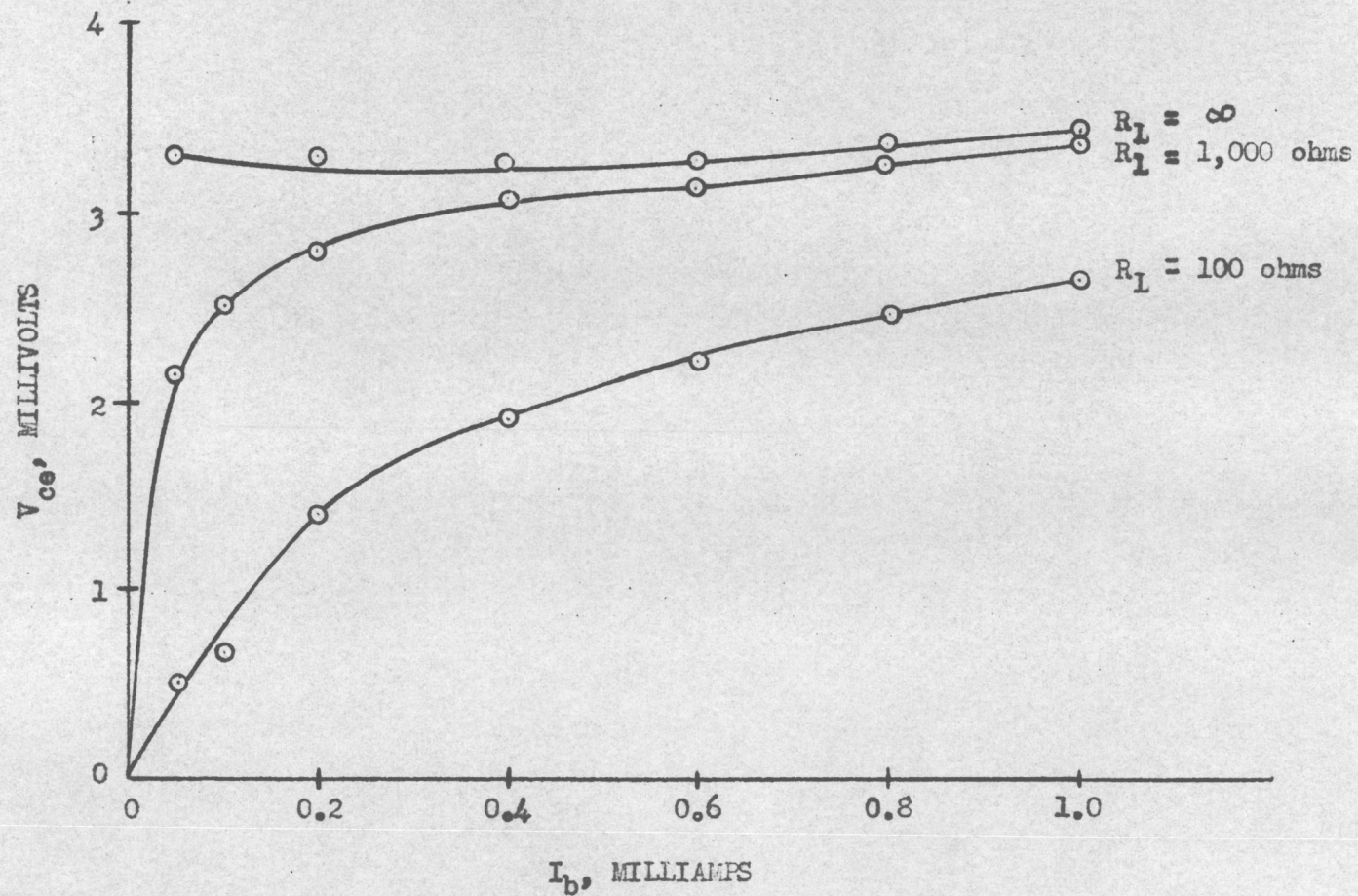
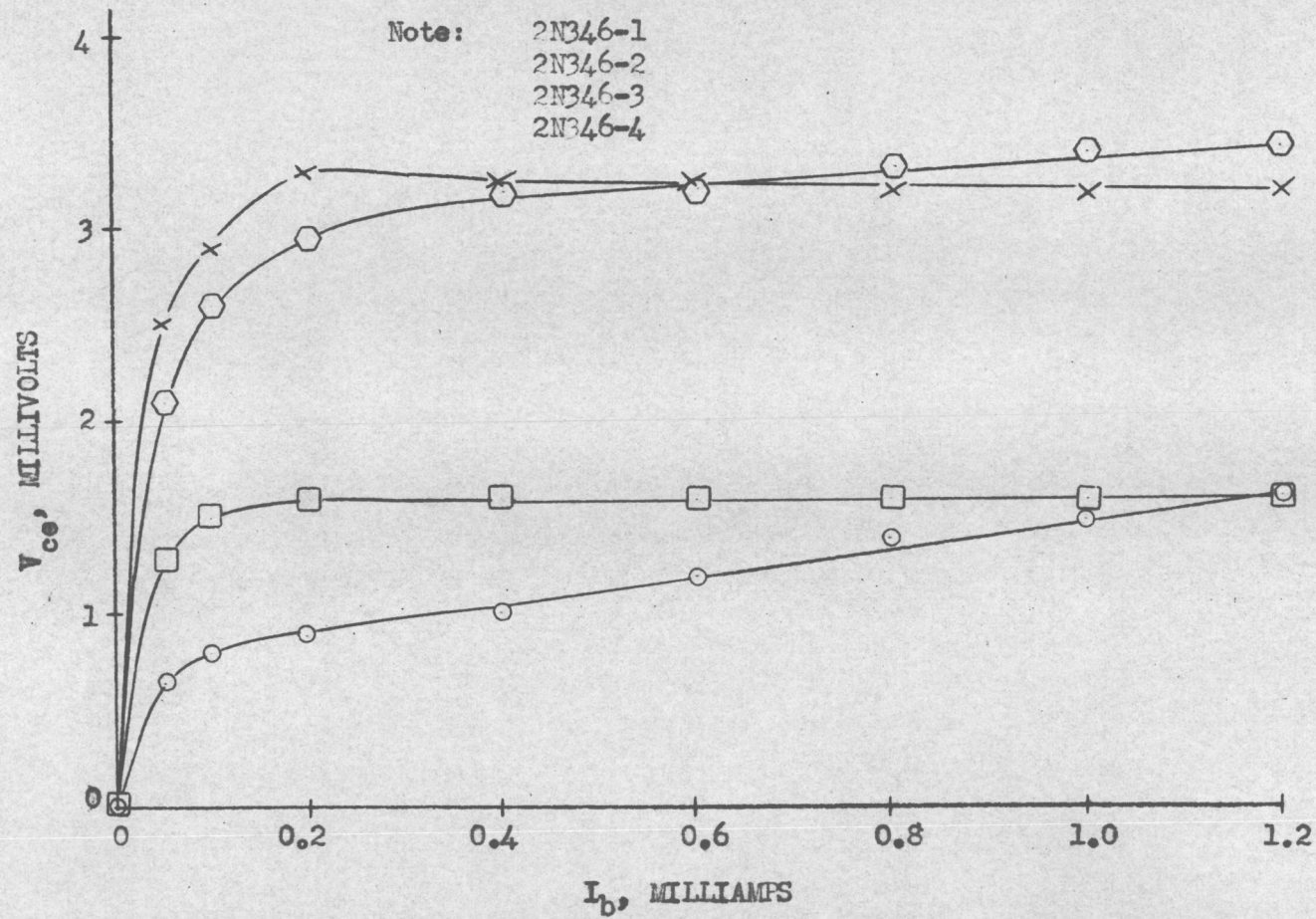




Figure 15, COLLECTOR-TO-EMITTER SATURATION VOLTAGE AND BASE  
CURRENT FOR THE 2N346 TRANSISTOR, INVERTED CONNECTION.  
( Characteristics of four random transistors )



the switch is in the vicinity of 1,000 ohms or greater, the saturation voltage is fairly constant. This is indicative of the source resistance,  $R_{cs}$ , which already has been discussed.

The dependence of the collector saturation voltage upon base current is not very well defined as shown in figure 15. Common to all of these curves, however, is a knee in the vicinity where the acting emitter-base junction becomes saturated. It should also be noted that at zero base-current the collector-to-emitter voltage is not actually zero as indicated by the test instruments. Instead it has a magnitude determined by the relative doping of the P and N regions. This potential, however, cannot be read by a current-drawing device.

The real importance of figure 15, however, lies in the illustration of the difference in the direction of change and rate of change of the saturation voltage with base current. Because none of the transistors tested showed the same rate of change of  $V_{ce}$  with  $I_b$ , it is relatively easy to get an exact match at some particular value of base current, although not at all values. Thus at some specific value of base current two transistors are easily picked which have the same collector saturation voltage. Consequently, when these transistors are inserted in the circuit of figure 12, and driven with the specified amount



of base current, the overall bias error during saturation is zero. This is a very significant result as will be later seen.

The cutoff characteristics of the 2N346 in this low-level application were found to be very good. Despite a predicted value of the cutoff current source, equation 12, of 0.14 ua, the actual magnitude was so small it was difficult to measure --being in the order of a few hundredths of a microamp. Also it was noted that the transistors were quite uniform in this respect which indicates a very good match in the differential type of circuit. Consequently, the cutoff characteristics could very well be considered ideal, for this application.

It was found that the optimum cutoff voltage,  $E_1$ , was not at all critical--a value anywhere from 0.1 to 1.2 volts being satisfactory. However, for other types of transistors this value may be critical and generally speaking the cutoff voltage should not exceed a few tenths of a volt. It is conventional practice to use diodes as the voltage limiting device for this state.

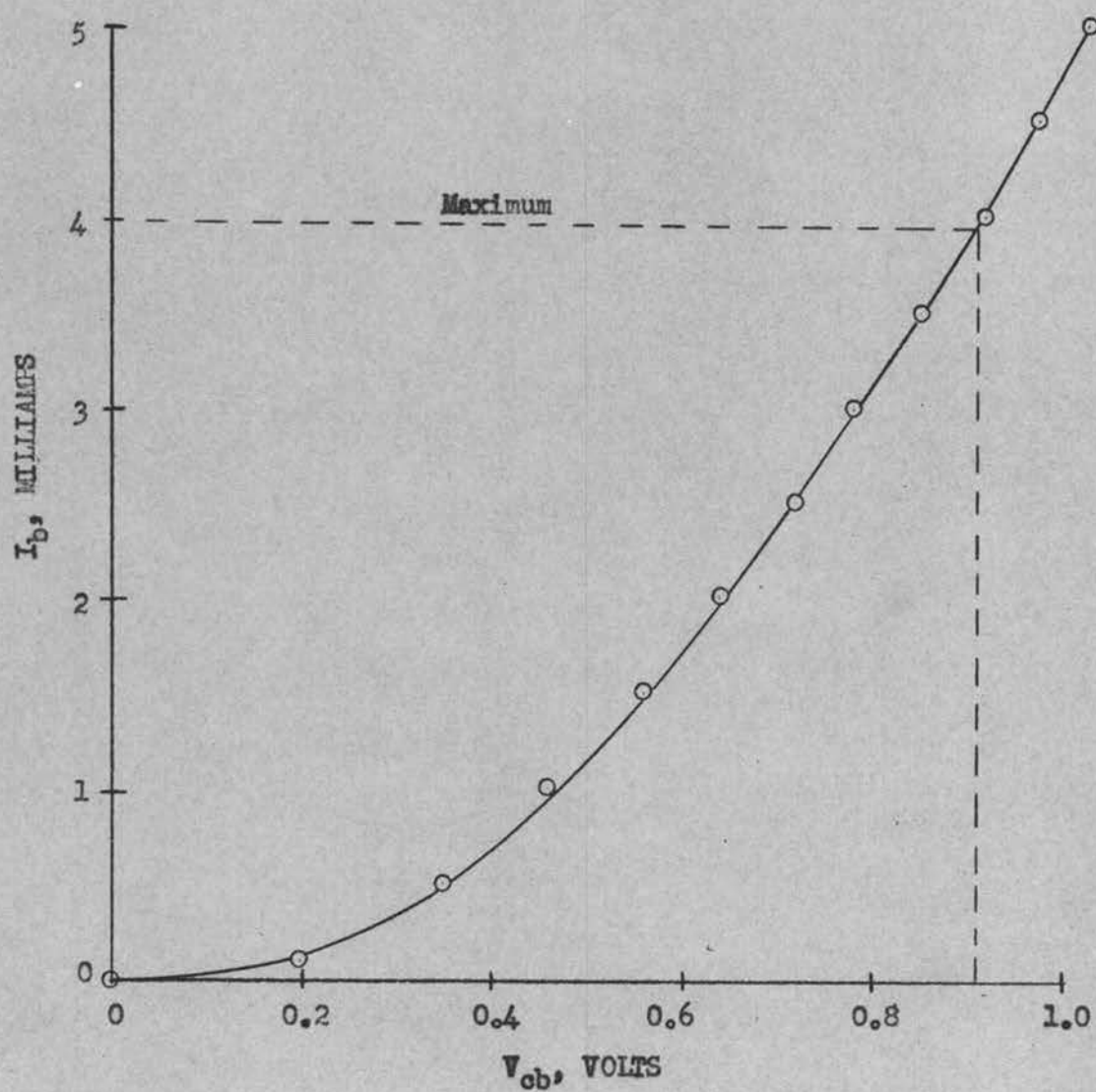
With the cutoff characteristics being very near ideal, it is now very easy to choose a pair of transistors which are also balanced with respect to their individual saturation voltages. First, a circuit is constructed as shown in figure 12, without the external signal source,

and using a square-wave generator as the switching voltage,  $E_1$ , source. Now by monitoring the output waveform with a high sensitivity,  $1 \frac{mV}{cm}$ , oscilloscope one can insert a pair of transistors, adjust the base current drive from zero through maximum, and note if a balance is obtained. If the balance is obtained the output wave form will appear as shown in figure 19. If not, there will always be an offset in the saturation state, as shown in figure 20. Thus it is not necessary to obtain any point by point plots, as shown in figure 15, to obtain a pair of matched transistors.

As far as the switching source, indicated as  $E_1$ , is concerned, either a voltage or current source may be used. However, as has been shown, it is the base current which drives the transistor into saturation, the on state, and the collector-to-emitter voltage which cuts off the transistor. Furthermore, the switching source must be isolated from ground to obtain the best results. For this purpose a pulse transformer was found to be very good.

In the actual chopper constructed for the dc amplifier, a square-wave voltage generator in conjunction with a pulse-transformer was used as the switching source,  $E_1$ . Under these conditions it is necessary to know what base current is produced by virtue of a given voltage. This information is easily obtained and illustrated in figure 16.

Figure 16, BASE CURRENT AND COLLECTOR-TO-BASE VOLTAGE  
FOR THE 2N346 TRANSISTOR, INVERTED CONNECTION.



From this curve, then, a given base current can be imposed by virtue of the applied voltage. The primary reason for obtaining this curve, however, is to find the maximum voltage which can be applied without exceeding the current limitations of the transistor. Since the current is an exponential function of the applied voltage, it is very easy to apply slightly too much voltage, greatly exceed the current rating, and destroy the transistor. This point deserves considerable emphasis.

The transient response of this switch can be seen from an analysis of figures 19 and 20. It is fairly obvious that the primary transient occurs as the switch changes states. This is a result of the instantaneous change of the direction of the current in the base region. The resultant spike or transient is of about 25 to 50 millivolts magnitude and has a duration of about 0.4 microsecond. Aside from this transient the switch is very well behaved. No trouble is encountered with delay time or hole-storage time because of the very thin base region of the surface barrier transistor and the very small magnitude of signals, less than one millivolt potential and about a microamp of current, being switched. Along with this discussion it should be mentioned that given equal quality transistors, the NPN transistor will generally exhibit faster transients than the PNP transistor because



of the higher mobility of electrons as compared to holes.

Based upon this discussion an input chopper was constructed as shown in figure 41, for the proposed dc amplifier. The switch itself exhibited a detectable change in output for less than 10 microvolts input and produced no appreciable bias error in either state, conducting or non-conducting.

### AC Amplifier Design Considerations

The design of the ac amplifier is for the most part a straightforward application of the transistor. Since it was desired to be able to utilize chopping frequencies of from 10 kc to 100 kc, it was necessary that the ac amplifier have a bandwidth of from 10 kc to about 1 Mc. The other primary requirement of the ac amplifier was that it have a voltage amplification of about 20,000 or more in the passband with a minimum of noise, distortion, and temperature variations.

The gain versus bias stability conflict which was described previously does not apply to an ac amplifier because of the possible use of reactive bypass elements. In this connection, it should be noted that the emitter leg bypass capacitors should present a 99% bypass in the applicable frequency range to the resistor which they are to bypass. If the conventional 90% bypass elements are

used, it is very likely that undesired degeneration and phase-shift will occur.

The coupling and composition of the ac section was determined primarily by thermal considerations. It has already been shown that the direct-coupled configuration of the same type, PNP or NPN, has the best thermal stability of any other combination. Consequently this scheme was used.

Of the available transistors the 2N247 and 2N248, both PNP, seemed well adapted to this application since they have a moderate  $h_{fe}$  of about 40 to 80 and a common base alpha cutoff frequency of 30 and 50 Mc, respectively. The noise figures of the transistors were not available, however, they were judged to be acceptable. Also, the linearity of  $h_{fe}$  was considered acceptable from a brief analysis of their characteristic curves.

To obtain the number of stages which are required one must first make an approximation of the voltage amplification which can be effected per stage. Generally speaking, with some current feedback, an amplification of 20 is easily obtained per stage. With this figure as a base, then, four stages would yield a voltage amplification of 160,000. Thus about 20 db of additional feedback could be applied and still maintain an amplification of 20,000. This would generally be considered enough feedback to make

the amplifier quite stable. Consequently four stages were judged to be adequate.

The quiescent operating points and biasing networks were chosen and constructed by conventional procedures (18, p. 82-101). Of the four stages it was found easiest to accomplish a current gain with the first three stages, and to use the fourth stage as a voltage amplifier.

Two different types of feedback were used as indicated on the circuit diagram, figure 41. First, current feedback in the form of an unbypassed emitter resistor of from 50 to 200 ohms was used to obtain some degeneration and stabilization. Additionally this type of feedback offers possibilities of extending the frequency response at the higher frequencies by providing a partial capacitive bypass to lower the input impedance of the stage and hence boost the high frequency amplification.

The second type of feedback is a type of dual voltage feedback, in that a voltage is fed back which is proportional to the output current. This type of feedback was applied from the emitter of one stage to the base of the preceding stage. This type of feedback has the normal advantages attributed to loops which enclose two stages rather than a single stage, and also the fact that the feedback is taken from a low-impedance point and applied to a high-impedance point. Thus it does not load down

the stages enclosed in the loop.

Another point to be noted is that because of the high voltage amplification involved and the use of non-ideal power supplies, decoupling was found to be necessary to prevent random oscillations.

Although in a direct-coupled circuit, each stage is somewhat dependent upon the preceeding stage, transistor interchangeability is not adversely affected. To change transistors, generally only a readjustment of the emitter resistor is required to compensate for the variation in transistor parameters which is encountered.

Utilizing the methods outlined above an acceptable ac amplifier was constructed with an amplification of about 22,000 and a bandwidth of 950 kc. For more results see the chapter entitled "Experimental Results".

#### Demodulator Design Analysis

At this point in the design, the original dc information has theoretically been chopped and amplified. Now it is desired to recover the amplified information, a process referred to as demodulation.

It should be noted, that by necessity the input circuit must contain a series capacitor between the input chopper and the ac amplifier to isolate the chopper from the dc level at the input to the ac amplifier. Since this

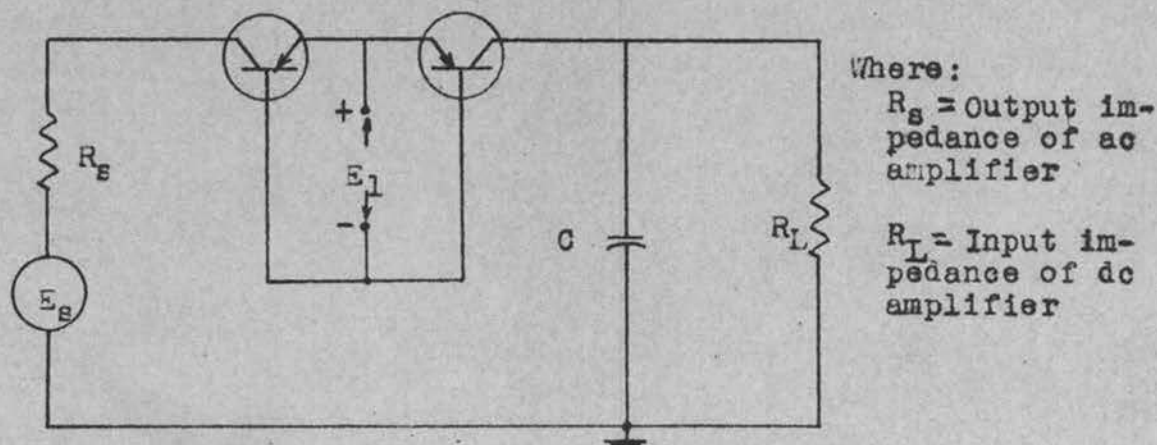


capacitor cannot pass a dc voltage, it distributes the chopped signal about the dc level at the input to the ac amplifier. Thus to recover the chopped and amplified information, only a half of each cycle must be utilized. This is easily accomplished by using a series switch to connect the output of the ac amplifier to a low-pass filter during a half of each chopping cycle.

Since the ac amplifier produces no appreciable phase-shift the positive portion of the chopped wave at the output of the ac amplifier would be in phase with a positive input signal, and the negative portion  $180^\circ$  out of phase with the original signal. Either portion may be recovered depending upon the phase of the recovered signal which is desired. In this specific case the positive in-phase portion was required.

The resultant demodulator then takes the form shown below.

Figure 17. An Equivalent Demodulator Circuit Diagram



For a theoretical minimum noise, the capacitor should charge very fast and discharge very slowly or  $R_s \rightarrow 0$ , and  $R_L \rightarrow \infty$ . Since the common-emitter stage has an output impedance of several thousand ohms, it was decided to use a common-collector output stage on the ac amplifier to feed the demodulator from a low impedance source, about 1,000 ohms. However, it was later found that a source impedance of several thousand ohms was more desirable since the modulated wave contains several undesirable transients. Thus, by making the charging time-constant longer, the demodulated output of the capacitor could not follow the very brief transients and the noise was considerably reduced. Therefore, the common collector ac output stage is not necessary.

The switch itself is similar to the input chopper discussed previously except that the normal series connection is used. This is a better configuration for the high-level application involved here. Also, the 2N588 transistor was chosen over the 2N346 for this application because of its better high-level characteristics.

The parameters of the equivalent circuit, figure 17, which were obtained in the actual demodulator were approximately  $R_s = 6,000$  ohms,  $R_L = 20,000$  ohms, and  $C = 0.02$  uf. Thus the charging time-constant was about 0.12 milliseconds and the discharge time-constant about 0.40 milliseconds,

assuming idealized switching conditions. These parameters were found to be about optimum for a chopping frequency of 80 kc, or a chopping period of 12.5 microseconds.

#### Output DC Amplifier Design Requirements

After constructing and testing the input circuit, input chopper, ac amplifier and demodulator, it was found that the overall amplification from the input to the output of the demodulator was about 5,000. Thus an amplification of at least 4 is required to boost the total amplification to 20,000 or more.

As previously discussed, it was also required that the input impedance to the high-level dc amplifier be in the order of ten or twenty thousand ohms.

To achieve this minimum gain of 4 along with an input impedance of about 20,000 ohms, and acceptable thermal stability, it was decided that silicon transistors would be necessary. This conclusion is apparent after setting up a few theoretical circuits which satisfy the amplification and input impedance requirements, and then calculating the stability factors for the various circuits, allowing a 15°F temperature variation. With germanium transistors it was found that the quiescent collector voltage could easily change a volt or more, while with silicon transistors the change was in the order of

hundredths of a volt.

The NPN type of transistor, in contrast to the PNP transistors used in the ac amplifier, was chosen in order to use the quiescent dc level of -14 volts at the output of the demodulator to the best advantage. In addition to the type, it is easily seen that a moderate  $h_{fe}$  is desired along with an alpha cutoff frequency of several hundred kilocycles.

One more condition must be imposed before a satisfactory transistor can be chosen. This condition is that the transistor must have a maximum collector-to-emitter voltage in the order of 40 volts or more if a  $\pm 15$  volt output swing is to be permitted. This is true because the common-emitter amplification stage must produce a plus or minus 17 or 18 volts which is attenuated to 15 volts by the common-collector output stage and the AZS arrangement.

From these conditions, then, the 2N334 transistor was chosen. Specifically, it is a silicon type NPN transistor with an  $h_{fe}$  of about 40 and an alpha cutoff frequency of 8 Mc. In addition it has a maximum collector-to-emitter voltage of 45 volts.

The output dc amplifier circuit itself is a straightforward conventional type of circuit, which presents no unusual situations. However, the common-collector output stage requires considerably juggling of power supplies



and breakdown diodes until the proper combination of voltages is arrived at whereby the quiescent output level will be zero volts dc.

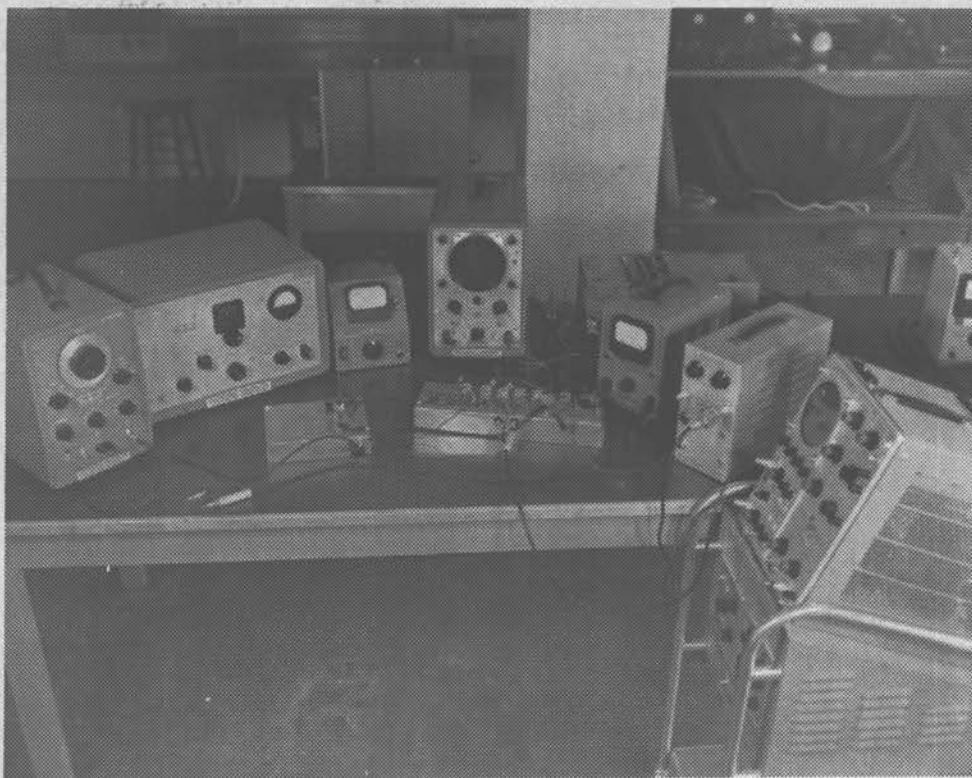
It is naturally understood that the common-collector output stage besides providing an AZS arrangement was asked to provide a low output impedance of about 1,500 ohms. This value can be calculated from circuit equations (16, p. 11-23) or obtained experimentally. The output impedance could easily be lowered to 1,000 ohms or less with a proper choice of power supplies.

Additionally, the low-pass RC filters or shaping networks were found necessary to limit the bandwidth and consequently reduce the output noise.

## EXPERIMENTAL RESULTS

In the manner described previously, a dc chopper amplifier was constructed as indicated on the circuit diagram, figure 41. The completed circuit and test setup appeared as shown below.

Figure 18. Experimental Test Setup



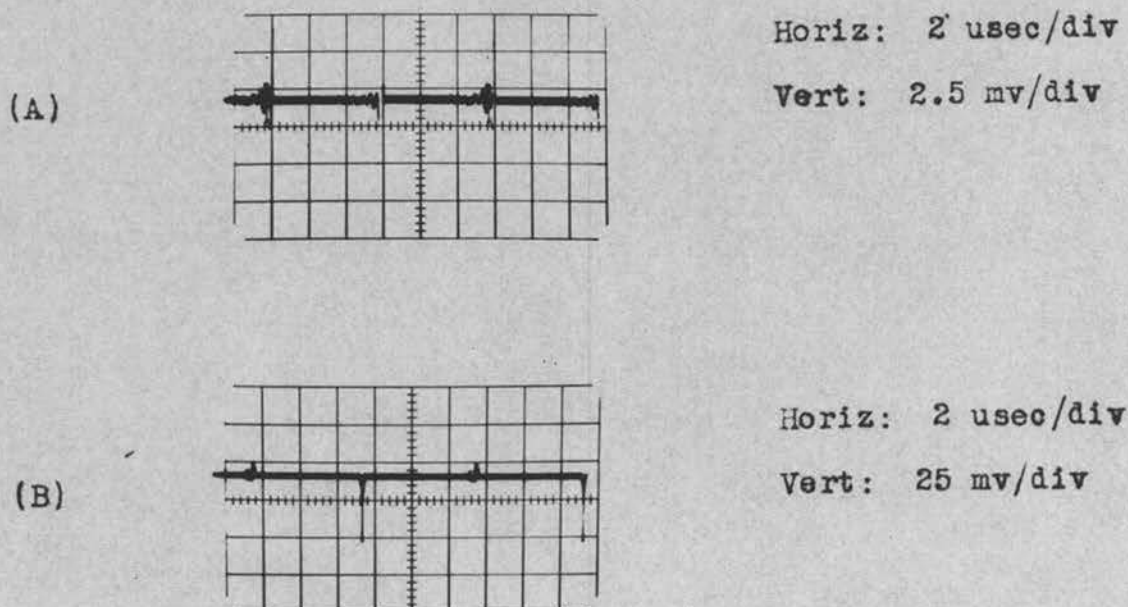
### DC Amplifier Results

Before evaluating the specific results of the total dc amplifier, considerable insight into the operation of the amplifier can be gained by examining the oscillographs recorded at various points in the circuit under various

test conditions. All oscillographs were recorded with an effective oscilloscope bandwidth of about 10 Mc and a chopping frequency of 80 kc unless otherwise noted. Also, the scale factors listed are per major division as shown on the oscillographs.

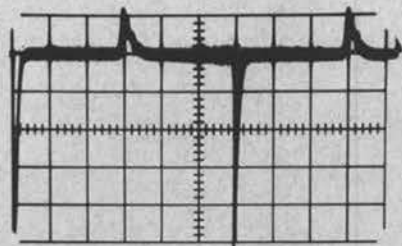
The operating characteristics of the input chopper were difficult to obtain pictorially because of the very small voltages involved, less than one millivolt. However, the output of the input chopper with a zero input signal appeared as indicated below.

Figure 19. Output Voltage of the Input Chopper With A Zero Input Signal





(C)



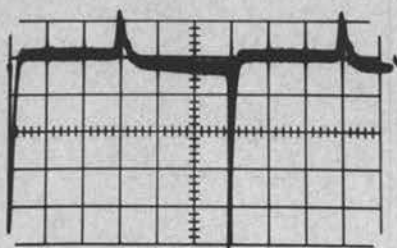
Horiz: 2 usec/div  
(Bandwidth of 1.3 Mc)

Vert: 2 mv/div

The most important characteristic to be noted from these oscillographs are the form and magnitude of the transients which occur while the chopper is changing states. It should also be noted that there is no appreciable bias error produced in either state. Figure 19-C illustrates the waveform approximately as seen by the ac amplifier since the ac amplifier has a bandwidth of 950 kc and the oscillograph was recorded with an effective oscilloscope bandwidth of 1.13 Mc.

The output of the input chopper with an applied external signal is indicated below.

Figure 20. Output Voltage of the Input Chopper With A  
-1 Millivolt DC Input Signal

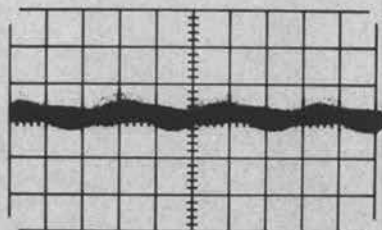


Horiz: 2 usec/div  
(Bandwidth of 1.3 Mc)

Vert: 2 mv/div



Figure 21. Output Voltage of the Input Chopper With An AC Input Signal



Horiz: 2 msec/div

Vert: 2.5 mv/div

The previously shown two oscillographs were recorded on the output side of the capacitor connecting the chopper to the ac amplifier, thus they represent the input to the ac amplifier. The indicated response to a dc signal is quite obvious, in contrast to the response to the ac signal. Figure 21 should contain two traces which have an outline similar to the familiar amplitude-modulated wave. Thus the outlines of the two traces should be  $180^\circ$  out of phase with each other. However, the signal level had to be increased beyond that required for normal operation to obtain a visible trace, and they appear somewhat distorted.

The following figure illustrates the response of the ac amplifier to the chopped waveform with a zero input signal.

Figure 22. Input and Output Voltage Waveforms of the AC Amplifier With Zero Input Signal



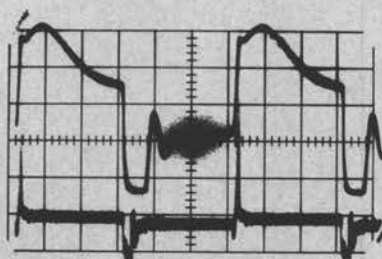
Upper trace:  
Horiz: 2 usec/div  
Vert: 2.5 mv/div

Lower trace:  
Horiz: 2 usec/div  
Vert: 5 v/div

This oscillograph vividly illustrates the hold storage time accumulated by the ac amplifier in response to the large transients caused by the input chopper. This is a basic limit to the speed of chopping which can be used effectively. The portion of the trace immediately to the right of the negative transient is that produced by the chopper being in the conducting or non-transmitting state, and the portion immediately to the right of the positive transient is produced while the chopper is in the non-conducting, or transmitting state.

The response of the ac amplifier to an applied dc input signal is seen in the following figure.

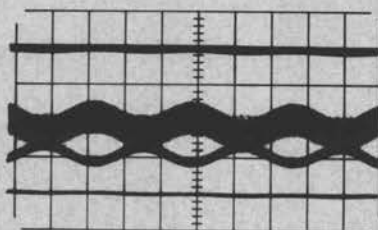
Figure 23. Input and Output Voltage Waveforms of the AC Amplifier With an Applied Input Signal of -0.8 Millivolts



Upper trace:  
 Horiz: 2 usec/div  
 Vert: 5 v/div  
 Lower trace:  
 Horiz: 2 usec/div  
 Vert: 2.5 mv/div

This figure illustrates the change in the waveform at the output of the AC amplifier caused by an input signal of -0.8 millivolt. The lower trace was recorded at the input to the ac amplifier, and the upper trace at the output of the ac amplifier.

Figure 24. Output Voltage Waveform of the AC Amplifier With an AC External Signal



Horiz: 2 msec/div  
 Vert: .5 v/div

This figure illustrates the waveform produced at the output of the ac amplifier by virtue of applying an ac signal at the input to the dc amplifier. It will be noted

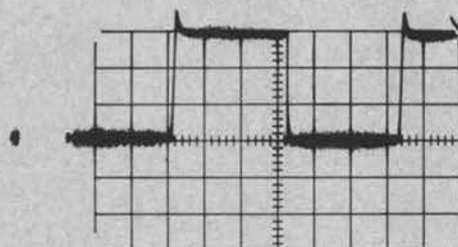


that the two outlines of the ac signal are  $180^\circ$  out of phase as they should be.

The next figure illustrates the ability of the output demodulator switch to control large signals. The illustration was obtained by replacing the input to the switch by a simple dc source and removing the demodulator capacitor. Thus the switch should ideally connect the dc source to the resistive load and disconnect it according to the polarity of the switching voltage.

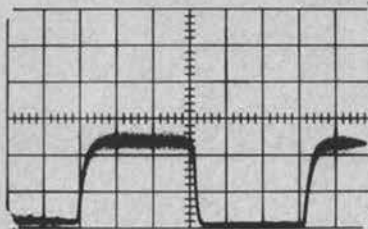
Figure 25. Response of the Demodulator Switch to Large Signals

(A) Applied source of 15 volts



Horiz: 2 usec/div  
Vert: 5 v/div

(B) Applied source of -15 volts



Horiz: 2 usec/div  
Vert: 5 v/div

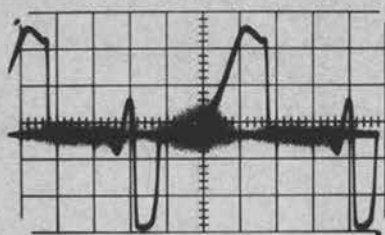


It is seen from this figure that the switch can control positive signals as large as 15 volts very well, however, as is typical of this switch it does not quite completely turn off a negative signal of 15 volts. It was observed, though, to control a negative 10 volts very well. Since the maximum signal to be encountered in application is 5 volts, it was concluded that this series type of switch can control these signal magnitudes very well.

The following illustrations depict the response of the demodulator to an applied dc signal.

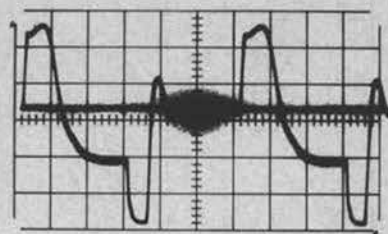
Figure 26. Input and Output Voltage Waveforms of the Demodulator in Response to a Given DC Signal

(A) Zero Input Signal



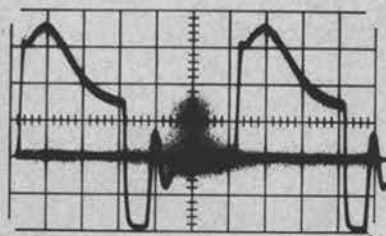
Both traces:  
 Horiz: 2 usec/div  
 Vert: 5 v/div

(B) +0.8 Millivolt DC Input Signal



Both traces:  
 Horiz: 2 usec/div  
 Vert: 5 v/div

(C) -0.8 Millivolt DC Input Signal



Both traces:

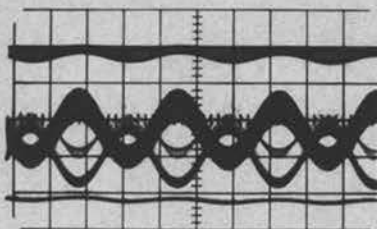
Horiz: 2 usec/div

Vert: 5 v/div

In these oscillographs, both traces were recorded with the same dc orientation, although the dc orientation is not the same for all three oscillographs. The straight line or trace was recorded at the output of the demodulator, while the other waveform was taken from the input to the demodulator, or the output of the ac amplifier common-collector output stage. These figures then show which part of the waveform the demodulator output follows.

Figure 27. Input and Output Voltage Waveforms of the Demodulator in Response to an AC Signal Input

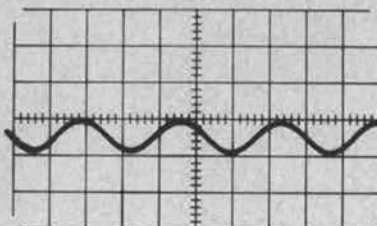
(A)



Horiz: 2 msec/div

Vert: 5 v/div

(B)

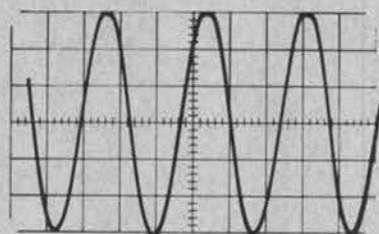


Horiz: 2 msec/div

Vert: 5 v/div

The above two oscillographs were taken with corresponding phase and dc reference. The upper illustration shows the input to the demodulator, and the lower illustration shows the output of the demodulator. From these oscillographs one may easily see that the upper more fuzzy component of figure 27-A is the one which is recovered by the demodulator. The lower one could be recovered by merely changing the polarity of the switching voltage on the demodulator switch, however, it would be  $180^\circ$  out of phase with the original signal, as previously explained.

Figure 28. Output Voltage Waveform of Total DC Amplifier



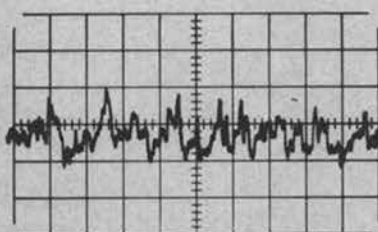
Horiz: 2 msec/div  
Vert: 5 v/div

The above figure illustrates the sinusoidal waveform at the output of the dc amplifier with a full 30 volt swing from peak-to-peak.

The noise at the output of the dc amplifier with a zero input is depicted on the following page.



Figure 29. Noise at Output of DC Amplifier

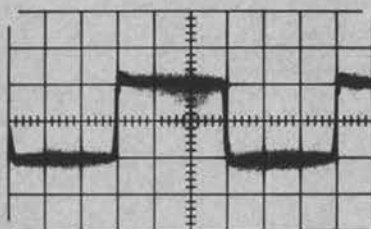


Horiz: 2 msec/div  
Vert: 0.25 v/div

It can be noticed that the noise is primarily composed of low frequency components, and has a maximum peak-to-peak magnitude of about 0.6 volts.

The switching voltage waveform used throughout the tests appeared as indicated below.

Figure 30. Switching Voltage Waveform



Horiz: 2 usec/div  
Vert: 0.25 v/div

A very pertinent characteristic of the overall dc amplifier is the effect of variations in the chopping frequency. This effect is shown in figure 31. (Figures 31 through 41 are contained in Appendix I.)

This figure illustrates that the amplification of



the overall dc amplifier is quite indifferent to changes in the chopping frequency over a fairly large range of frequencies. The overall amplification is within 2 db of maximum with a chopping frequency anywhere from 20 kc to 80 kc, with the 3 db down points at 12 kc and 140 kc. The fall-off at low frequencies is caused partly by the fall-off of the ac amplifier at low frequencies and partly by the inability of the pulse transformers, which supply the switching voltage to the switches, to pass good square waves. At low frequencies their output becomes the derivative of the input square wave. The high frequency fall-off is caused primarily by the fall-off of the ac amplifier and the effects of the transients produced by the input chopper which become more critical as the chopping period is reduced. Both the input chopper and the demodulator switch were found to switch very effectively as high as three or four hundred kilocycles. However, with an ac amplifier response as indicated in figure 32, the ability to pass a good representation of a square wave is limited to about 100 kc. It is generally understood that at least five harmonics are required to produce an approximation of the square wave. This would mean the first, third, fifth, seventh, and ninth harmonic of the fundamental, since the square wave is composed of an infinite number of odd harmonics.

The ac amplifier also causes a time lag from input to output of about 0.3 usec, or a phase lag of about  $9^\circ$  at 80 kc. This characteristic is probably beneficial as it indicates that the demodulator switch changes states slightly before the corresponding part of the chopped signal has been passed. Thus the demodulator retains the potential of the top of the chopped wave rather than a magnitude determined by the transients or round off of the chopped signal.

The frequency response of the overall dc amplifier is shown in figure 33. The amplification is flat from dc to 400 cps with a 3 db down point at 1,100 cps which is just slightly beyond the requirement of 1,000 cps. The phase-shift from input to output is  $117^\circ$  at the 3 db down point.

The frequency response of the overall dc amplifier is primarily determined by the demodulator and output dc amplifier, the remainder being due to the input filter and half-wave chopping technique. This conclusion is easily seen by comparing figure 33 to figure 34 which shows the frequency response for the output demodulator and output dc amplifier. The rather abrupt fall-off in amplification and the consequent rise in phase shift are caused by the demodulating capacitor, and the other low-pass RC filter placed in the output dc amplifier. These

filters were necessary, first, to maintain stability, and second, to reduce the noise of the overall dc amplifier. Since noise is approximately proportional to bandwidth, doubling the bandwidth approximately doubles the noise at the output. Stability could be obtained with a much larger bandwidth, as large as ten or twenty kc, however, the presence of considerable noise required that the bandwidth be reduced to a minimum. This then is the reason for the restricted bandwidth.

The noise at the output of the overall open-loop dc amplifier was found to be about 0.6 volt peak-to-peak, or about 0.3 volts as indicated on an rms voltmeter with a frequency range of 20 cps to 2 mc. Of this total, about 5 millivolts peak-to-peak were contributed by the output dc amplifier alone, and about 0.3 volts peak-to-peak by the ac amplifier alone. The comparatively large noise voltage of the overall dc amplifier, however, was found to be produced by the chopped waveform, entering the ac amplifier, being modulated by the noise of the first couple of ac amplification stages, the noise modulated signal being further amplified in the remainder of the ac amplifier, demodulated in the demodulator, and further amplified by the output dc amplifier. Consequently, to reduce the noise voltage without further limiting the bandwidth, a very low noise ac amplifier must be

constructed. Thus very low noise figure transistors, especially at low frequencies, are indicated.

In this discussion it should also be noted that considerable emphasis must be placed on obtaining power supplies which exhibit very low noise voltages while under load. If the power supplies indicate a noise voltage much in excess of 0.5 millivolts peak-to-peak, the noise figures of the individual components and overall dc amplifier increase very rapidly.

An added feature was also built into the output dc amplifier in that by removing the 10,000 ohms series resistance between the demodulator and output dc amplifier, the amplification of the overall amplifier could be raised from 26,500 to 44,000. This would represent a significant improvement in the dc amplifier although the noise and drift increase proportionally. However, because of the great difficulty involved in obtaining valid open-loop test results at the higher value of gain, the amplifier was left at its lower value for the tests indicated in this discussion.

It will be noted from the previous experimental results, that this system is also wasteful of the amplification built into the different components. With an ac amplification of 22,000 and output dc amplification of 4.3, a total amplification of about 94,000 is possible.



However, out of this possible amplification of 94,000 only 26,500 is realized, the remainder being absorbed by the input circuit to the ac amplifier. In comparison, a very small amount of amplification is lost in the demodulation process. In addition, if the input impedance is increased, the total overall dc amplification is further reduced since the overall dc amplification is approximately inversely proportional to the input impedance.

During the operation of this test circuit it was noted that the long term drift, several hours, at a given room temperature was negligible in comparison to the noise, probably being less than 0.1 volt peak-to-peak.

The non-linear distortion of the overall open-loop dc amplifier was found to be 3.8%. This value represents a minimum value, as the exact settings of the chopping frequency and magnitude were optimized. The distortion is definitely dependent upon the magnitude and frequency of the chopping source, and the magnitude of the switching signal supplied to the demodulator switch; although not in a well defined manner. Consequently these values must be individually adjusted for best results. The non-linear distortion of the ac amplifier and output dc amplifier contribute only a very small part of the total observed non-linear distortion.

The thermal drift of the open-loop overall dc amplifier is indicated in figure 35. It is shown that

the overall dc amplifier has a drift of about 0.25 volt in the temperature range of from 55°F to 85°F, which is the normal operating temperature of laboratory equipment. However, at temperatures much higher than 85°F the thermal drift increases very rapidly. This sudden rise and then fall is caused by the unbalanced temperature tracking qualities of the germanium transistors used in the input chopper. Thus, if one desires a small thermal drift at high temperatures, silicon transistors or temperature matched germanium transistors are indicated.

The contributions to the overall thermal drift by the ac amplifier and output dc amplifier are illustrated in figure 36. The rather non-uniform drift of the ac amplifier is caused by the different temperature tracking abilities of the different stages, diodes, and resistors. The thermal drift of the output dc amplifier is rather uniform although much higher than would be expected. This would lead to the conclusions that either the silicon transistors had higher  $I_{co}$  values than indicated, there were some unusually temperature sensitive resistors in the circuit, or the diodes had large temperature coefficients. It should be noted that the output dc amplifier, particularly, is of an experimental nature. Thus it could be "cleaned up" in a well thought out design and thus made more stable with temperature.

From the preceding discussion, it is seen that the circuit as shown in figure 41 has the following major specifications.

1. Voltage amplification of 26,500.
2. Input impedance equal to 2,000 ohms.
3. Output impedance equal to 1,500 ohms.
4. Maximum output voltage swing of 15 volts.
5. Thermal drift equal to 0.25 volt in the temperature range of 55°F to 85°F.
6. Long term output drift less than 0.1 volt.
7. Bandwidth of dc to 1,100 cps.
8. Phase-shift at 1,000 cps equal to 117°, with reference to the input.
9. Noise at output equal to 0.6 volt peak-to-peak.
10. Non-linear distortion equal to 3.8%
11. The circuit requires matching input chopper transistors which is easily accomplished. Other transistors are interchangeable with only a readjustment of emitter resistor usually necessary. Individual adjustment of chopping frequency and switching signal magnitudes is necessary for optimum results.

As a word of caution, it should be mentioned that the dc chopper amplifier as shown in figure 41 is strictly an experimental circuit and consequently its physical circuitry could be improved.

### Operational Amplifier Results

Since the dc amplifier was designed toward an application to the operational amplifier, its basic characteristics as an operational amplifier network are of interest. (The network, as shown in figure 1, becomes merely an operational dc amplifier if  $Z_1$  and  $Z_2$  are resistors, the ratio of  $R_2$  to  $R_1$  being termed the dc gain of the network. If  $Z_1$  is a resistance, while  $Z_2$  is a capacitor, the network becomes an integrating operational amplifier; and if the resistor and capacitor are interchanged, the network becomes a differentiating operational amplifier.)

The predictability of the operational dc amplifier was found to correspond very well with the theoretical values indicated in figure 44. With  $Z_1$  and  $Z_2$  in the order of 10 ohms to 10 Kohms generally very predictable operations could be performed, the error being in the order of 1%. However, with more incongruous values the error could easily be increased to 100% or more. An exact analysis of the predictability was difficult to obtain because of the required accuracy in components and measuring instruments.

The frequency response of the operational dc amplifier network is indicated in figures 37 and 38. From figure 37 it is noted that the bandwidth is nearly the same regardless of the value of dc gain, thus a trade of



bandwidth for gain is not effected. This characteristic was obtained because the fall-off of amplification with frequency of the overall dc amplifier was not the ideal 6 db per octave, and probably also because the resistors in the operational amplifier network were interacting with frequency sensitive elements within the dc amplifier itself. Consequently, at a dc gain of unity the bandwidth was only about 4,000 cps, at a dc gain of 48 the bandwidth was about 1,700 cps, and at the open-loop dc gain of 25,000 the bandwidth was still 1,100 cps.

The phase-shift response of the operational dc amplifier network at different values of dc gain is illustrated in figure 38. This figure shows that the phase-shift is relatively independent of values of dc gain below 50. This would be expected from the amplification response previously noted.

Also of particular interest is the amount of non-linear distortion which the particular operational amplifier network exhibits. This relationship is shown as a function of dc gain in figure 39. The non-linear distortion in the open-loop configuration was about 3.8% which decreased to less than 0.1% at unity dc gain. In the open-loop configuration, the switching frequency and magnitudes had to be individually adjusted to obtain the optimum results; however, when loaded down with feedback until

the dc gain was in the vicinity of 100 or less, the operational amplifier was quite insensitive to the switching frequency and magnitude.

The noise encountered in the operational dc amplifier was about 0.1 to 0.2 volt peak-to-peak for values of dc gain below 100.

The response of the integrating type of operational amplifier network is depicted in figure 40. With an RC time-constant of 0.005 seconds, the quality of integration would only be expected to be good above 200 cps, as shown. It is seen that the amplification of the network varies inversely as the frequency, and that the phase-shift, with reference to the input, is  $90^\circ$  for all useable frequencies. The phase-shift is not merely the phase-shift relationship of the open-loop dc amplifier plus an additional lag of  $90^\circ$  as might be expected, because the integration relationship is obtained as long as the amplification is high, with no regard to its phase-shift, as long as it is stable.

From a brief investigation of the integrating operational amplifier, it was generally concluded that the network behaved in a predictable manner within its limitations as indicated in Appendix III.

The differentiator type of operational amplifier network is not of very much practical use since its output

is primarily noise. This was easily verified. Consequently no specific data was taken on the differentiator type of network.

A complete analysis of the characteristics of the dc chopper amplifier used in operational amplifier networks is beyond the scope of this report.

## CONCLUSIONS

General Discussion

Of the two possible modes of operation, current or voltage, the operational voltage amplifier network is much better suited to large scale analog computer operations because of the possible use of common power supplies for the operational amplifiers. The specifications for a dc amplifier, to be used in such a network, may deviate considerably from ideal, and still not destroy the usefulness of the amplifier. In particular, the input impedance, output impedance, and phase-shift may vary appreciably from the ideal, and as long as the amplification is high, accurate and predictable operations may be effected.

In selecting a transistor dc amplifier circuit for such an application, considerations of voltage amplification and temperature stability are of paramount importance. In general the direct-coupled type of dc circuitry is unsatisfactory because of the conflict between bias stability and amplification. However, the differential type of circuit is much to be preferred to the single-sided configuration. The conflict between bias stability and amplification in direct-coupled circuits can only be resolved appreciably by using matched or custom tailored



temperature sensitive elements. Thus one usually resorts to a dc chopper-stabilized or dc chopper-amplifier.

Of the chopper-stabilized amplifier or chopper-amplifier, the chopper-amplifier outwardly offers the simplest solution. However, the success of this system is primarily determined by the quality of the chopper. The chopper must also be a high frequency device in order to obtain a suitable bandwidth. Herein, a method has been developed whereby transistors can be easily picked which are matched at a specific value of base current and temperature, and thus exhibit ideal characteristics in a differential type of switch. Thus a very high quality switch can be constructed. This switch utilizes two transistors in the inverted connection, series pair configuration.

The principle disadvantage encountered with this type of chopper, however, is that the transistors are not necessarily balanced at all temperatures, thus considerable mismatch may occur at higher temperatures; and that an appreciable transient of about one-half microsecond duration is produced as the switch changes states. These imperfections are further magnified by the ac amplifier, and as a result the chopper-amplifier designed around this input chopper may have very poor temperature characteristics.

A chopper-type dc amplifier was constructed, as

illustrated in figure 41, which consisted of a series arrangement of input chopper, ac amplifier, demodulator, output dc amplifier, and their associated circuitry. The input chopper was constructed as indicated above, and the overall dc amplifier exhibited a thermal drift of 0.25 volts in a temperature range of from 55°F to 85°F, and an output noise of 0.6 volt peak-to-peak with a bandwidth of from dc to 1,200 cps. Other principle specifications were an input impedance of 2,000 ohms, an output impedance of 1,500 ohms, a maximum output voltage swing of  $\pm 15$  volts, and a voltage amplification of 26,500. A much larger bandwidth could be utilized, however, the noise increases approximately proportional to the bandwidth. The relatively large noise voltage of 0.6 volts peak-to-peak was caused by the noise of the ac amplifier modulating the chopped signal.

Within its limitations the amplifier was found to operate satisfactorily as an operational amplifier in the integrator and dc amplifier operational networks. The dc chopper amplifier as shown in figure 41, however, is an experimental circuit and consequently its physical circuitry could be improved. Its principle value lies in providing the general characteristics of this type of circuit --it is not an end result in itself.

### Possibilities for Future Work

Future work in the field of transistor dc amplifiers could well be directed at improving the characteristics of the dc chopper amplifier which was investigated in this report. The principle disadvantages of this circuit were found to be its excessive thermal drift, noise, and low input impedance.

The use of silicon transistors in the input chopper would definitely improve the temperature characteristics of the circuit and possibly improve the chopping operation. Another possibility of improving the modulation technique lies in eliminating the switching transients. This would allow a much greater chopping frequency.

A delightful possibility of improving the overall dc amplifier is that of using a current-step-up transformer in the input circuit between the chopper and ac amplifier. This could possibly improve the switching characteristics, input impedance, and overall amplification.

An extremely low noise, high gain, ac amplifier is also required to reduce the noise of the overall system.

To further perfect the amplifier, it is quite probable that a phase-lead network could be utilized to correct for some of the undesired phase-shift.

From this investigation, it is definitely seen that the chopper-stabilized amplifier, figure 6-A, deserves

considerable investigation as it may prove to be a better system than the one herein investigated.



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APPENDIX I

ILLUSTRATIONS OF TEST RESULTS



Figure 31, OVERALL DC AMPLIFICATION AND CHOPPING FREQUENCY.

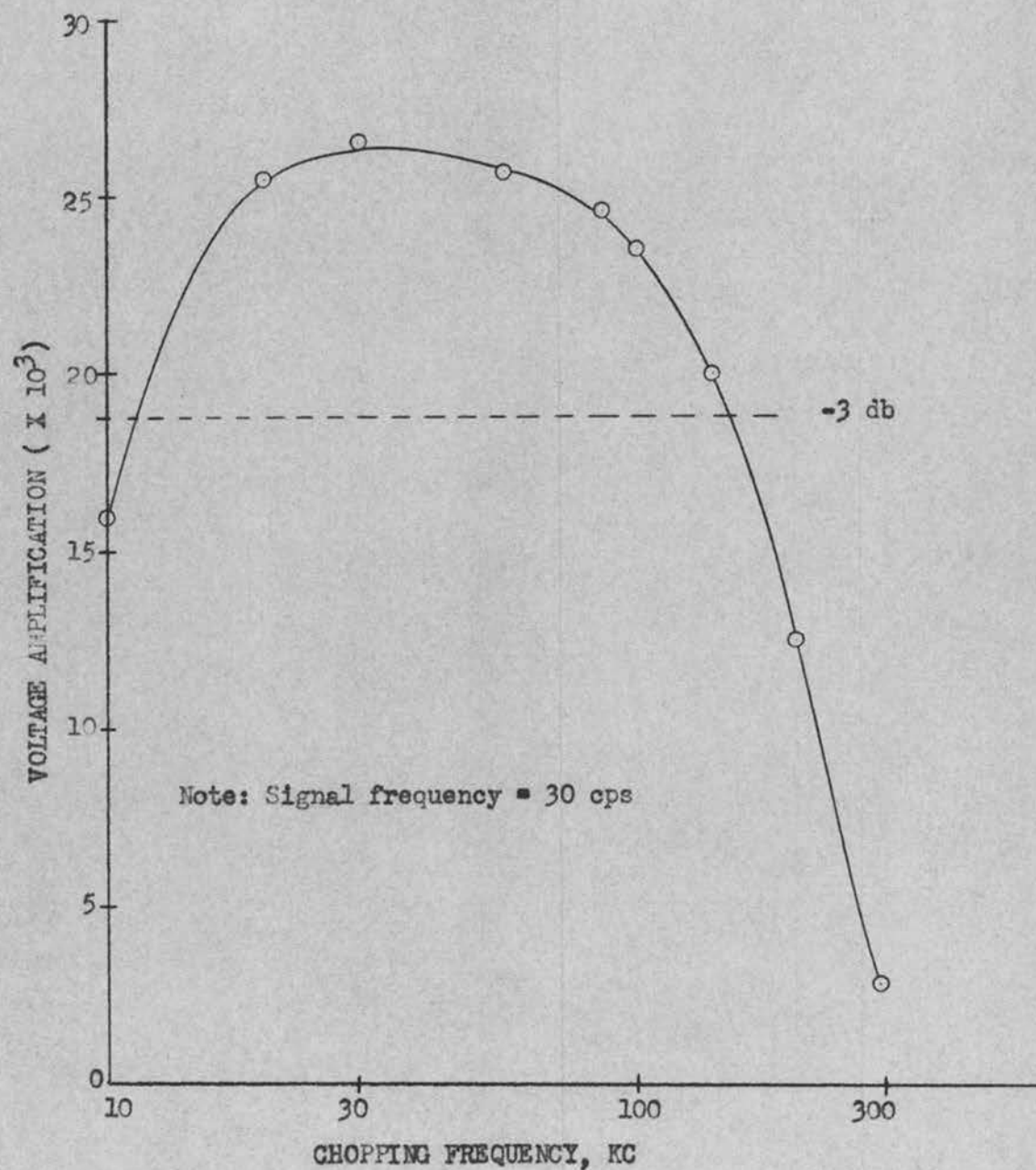
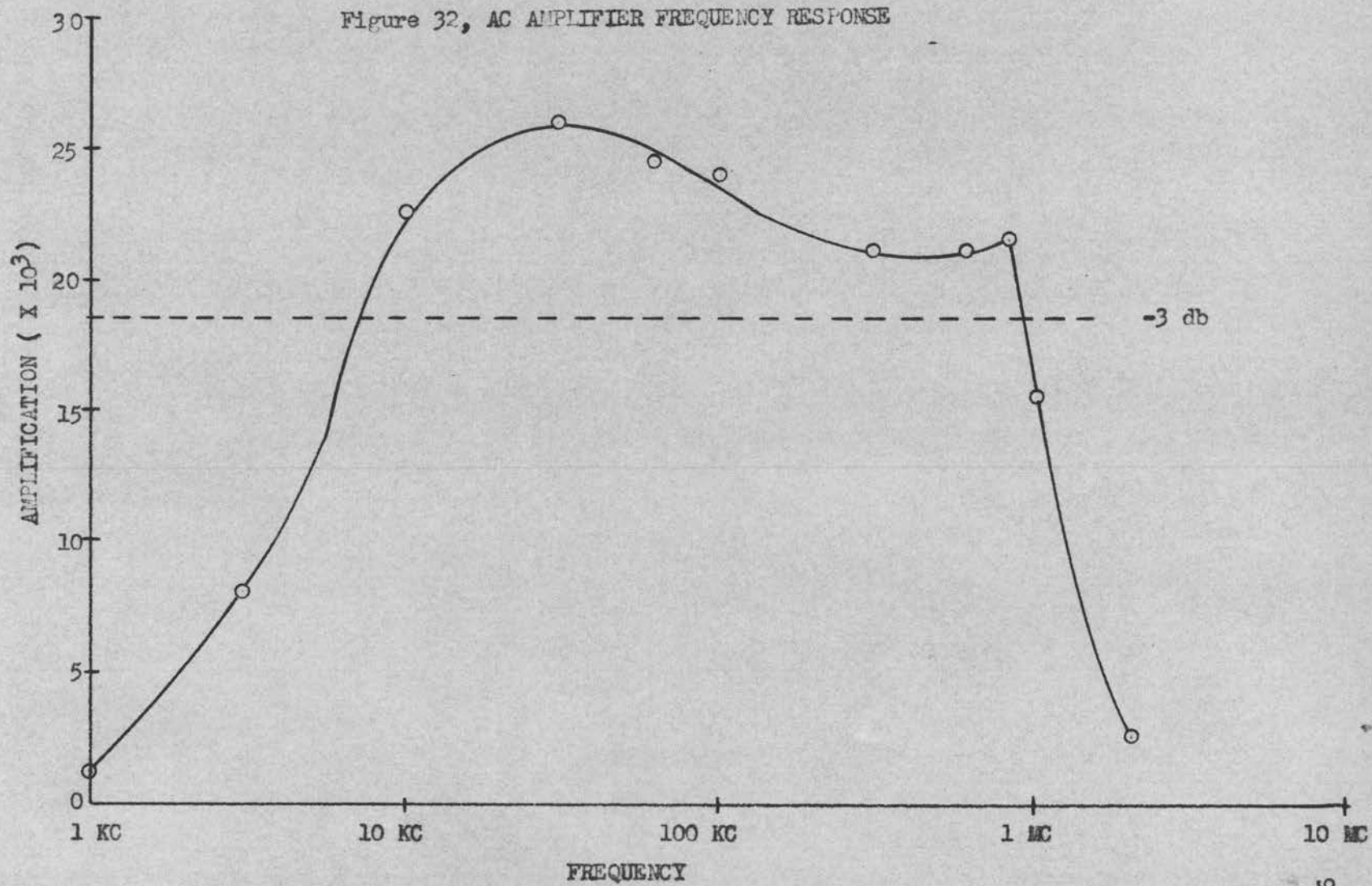


Figure 32, AC AMPLIFIER FREQUENCY RESPONSE



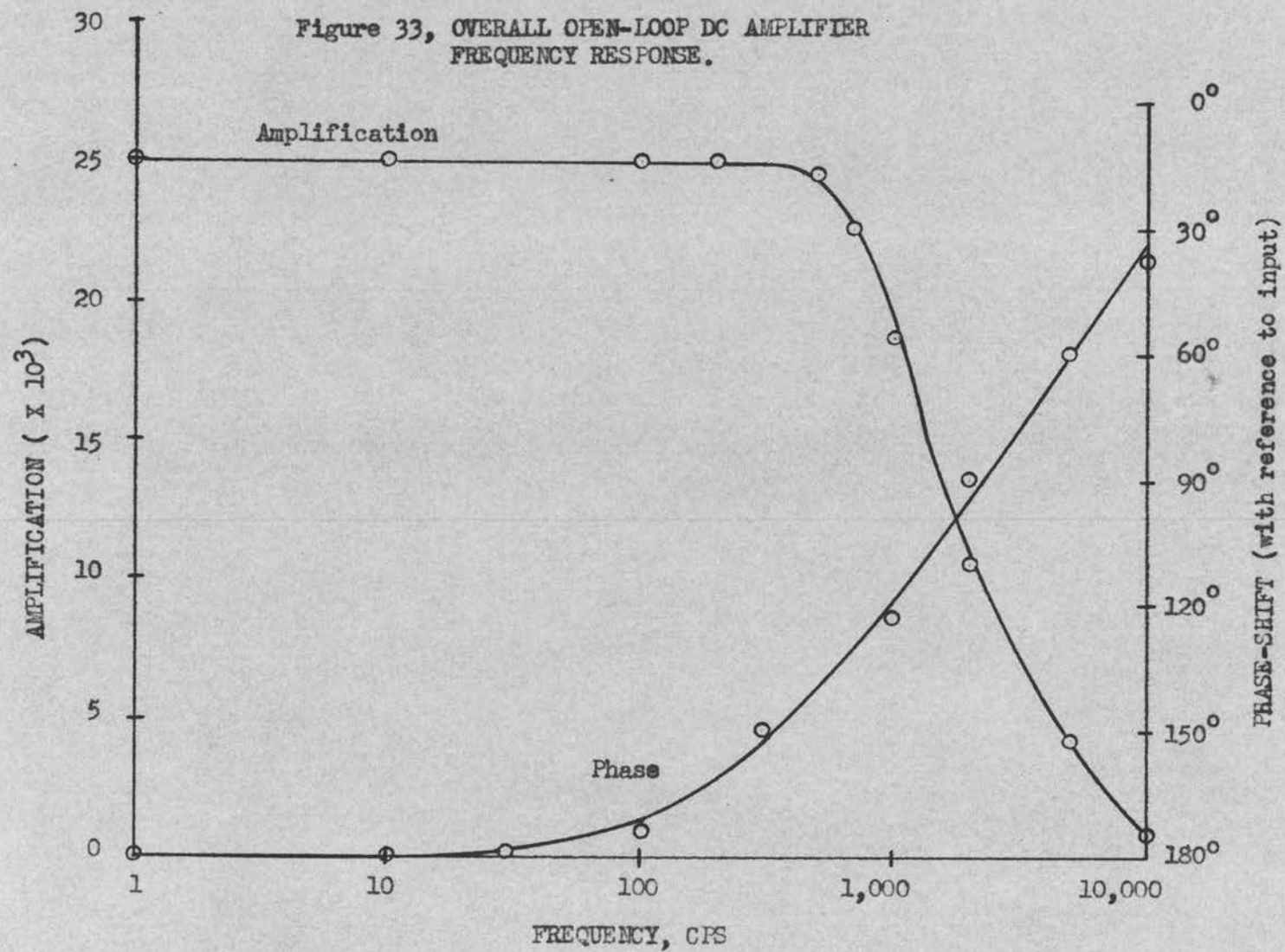




Figure 34, DEMODULATOR AND OUTPUT DC AMPLIFIER  
FREQUENCY RESPONSE.

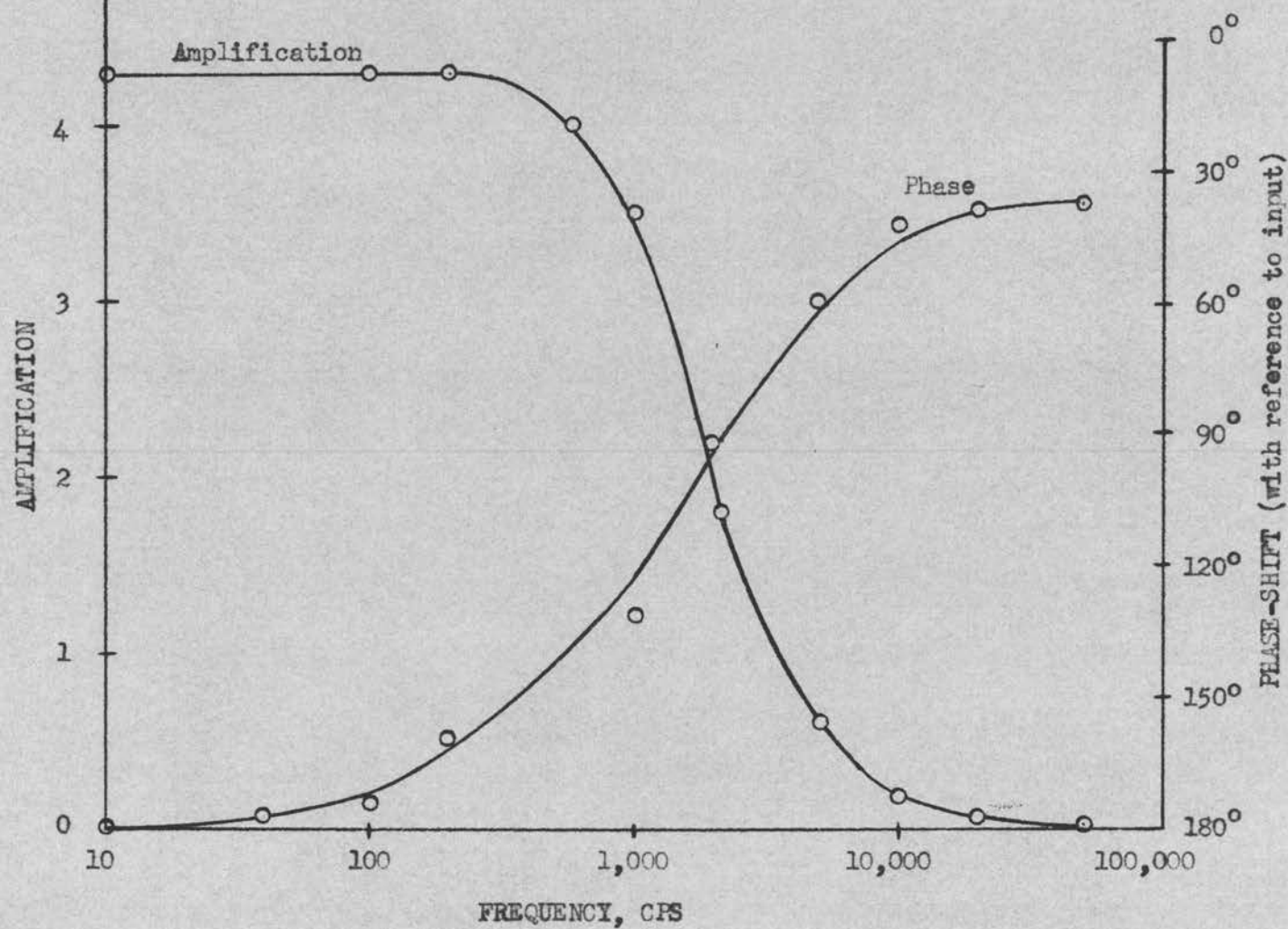




Figure 35, THERMAL DRIFT OF OVERALL OPEN-LOOP  
DC AMPLIFIER.

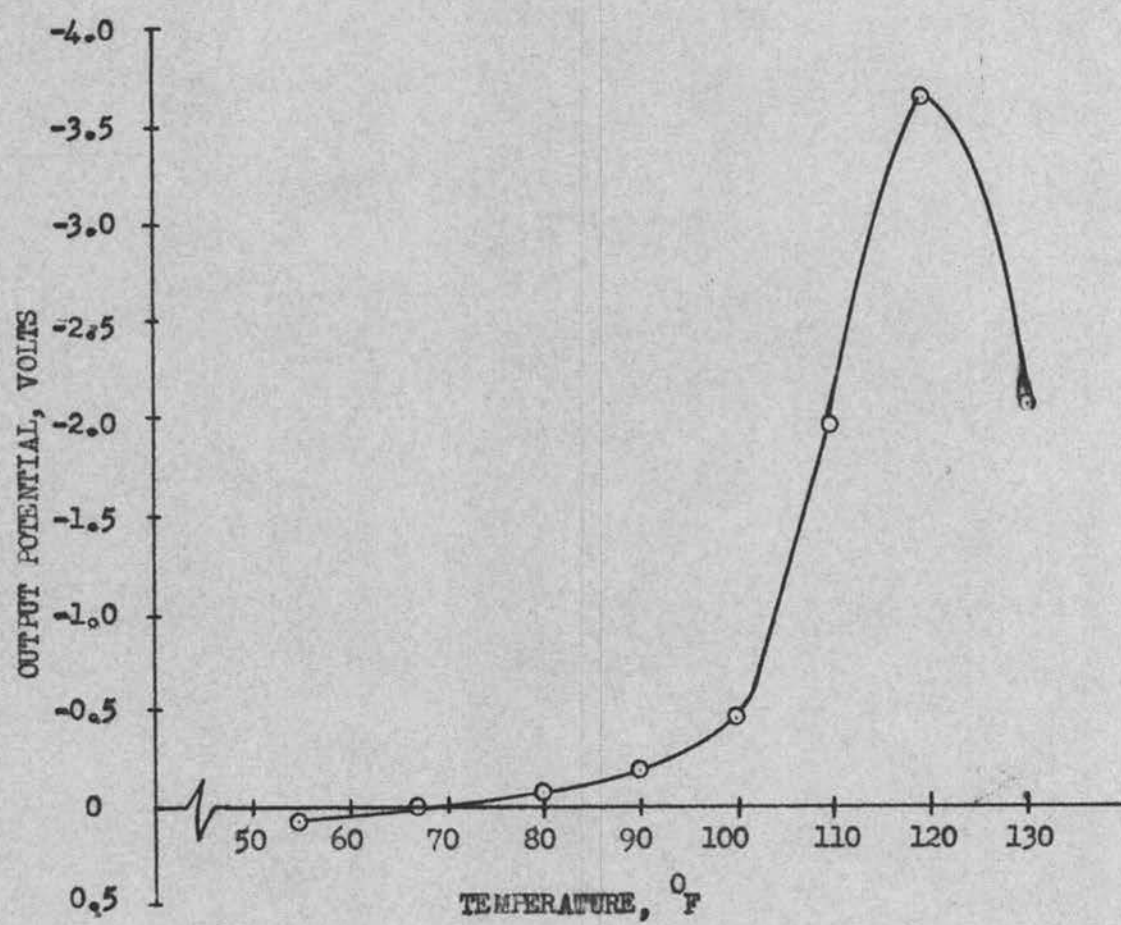


Figure 36, THERMAL DRIFT OF AC AMPLIFIER  
AND OUTPUT DC AMPLIFIER.

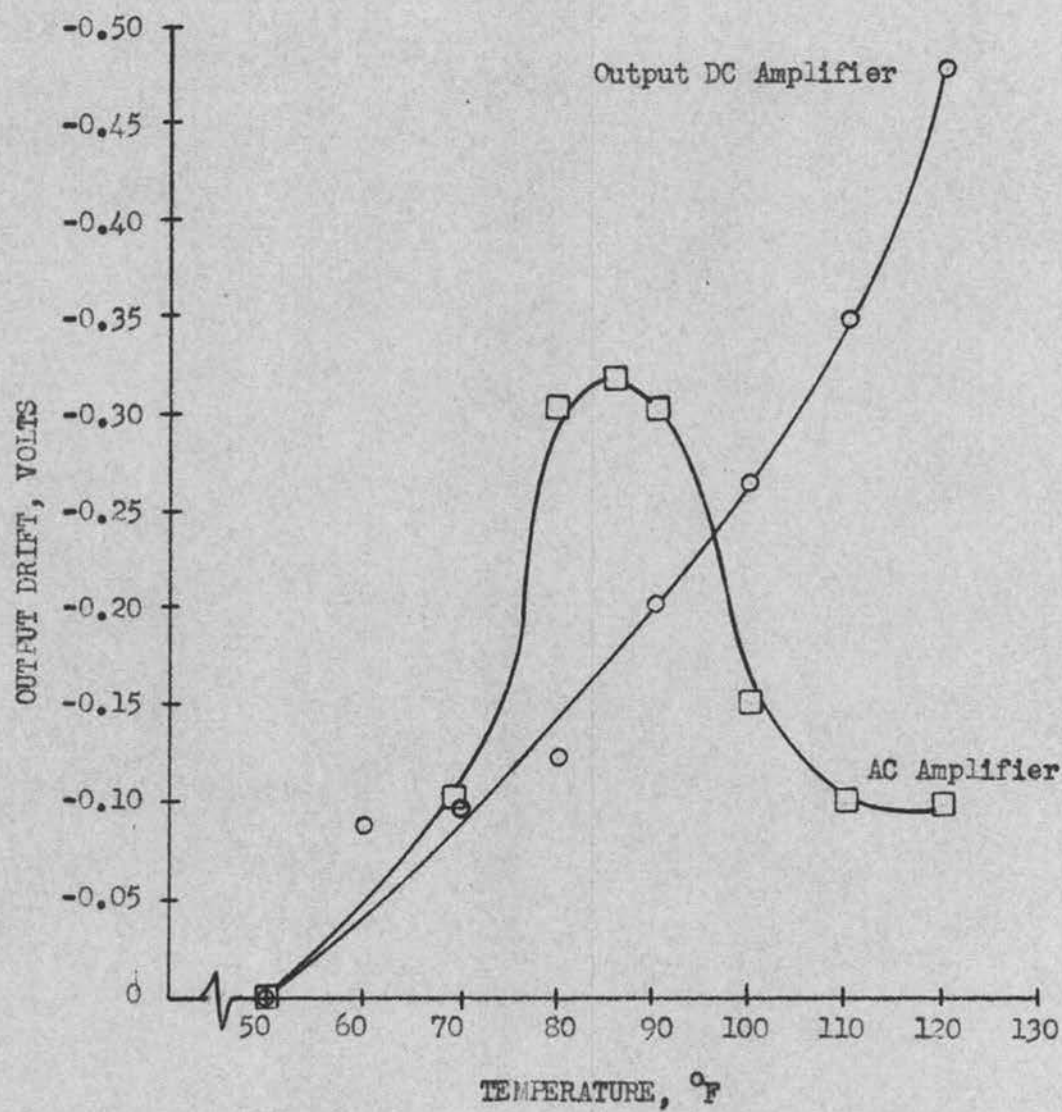


Figure 37, FREQUENCY RESPONSE OF OPERATIONAL  
AMPLIFIER AT DIFFERENT VALUES OF DC GAIN.

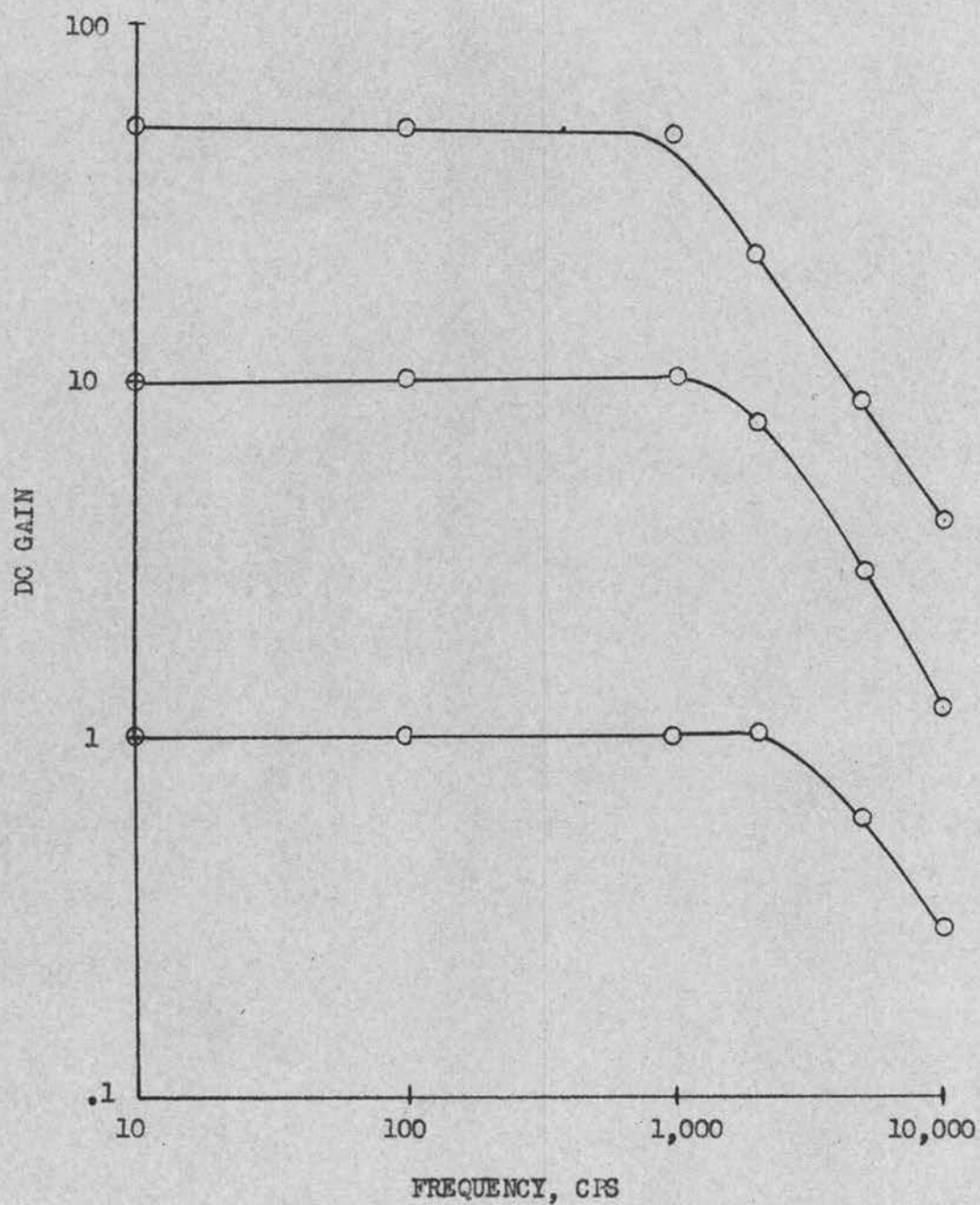




Figure 38, PHASE-SHIFT AND FREQUENCY FOR THE OPERATIONAL AMPLIFIER AT DIFFERENT VALUES OF DC GAIN.

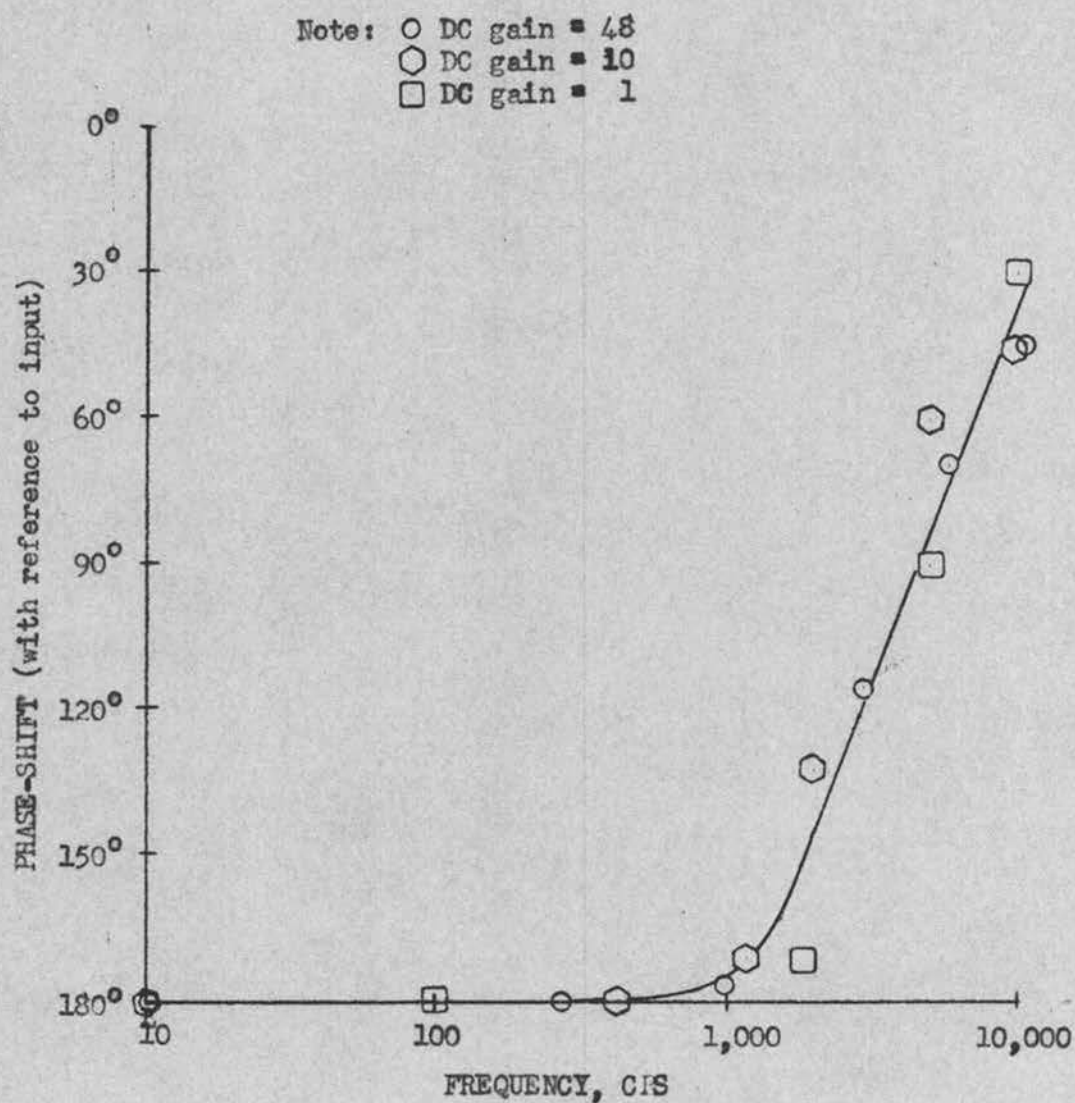




Figure 39, NON-LINEAR DISTORTION AND DC GAIN  
OF THE OPERATIONAL AMPLIFIER.

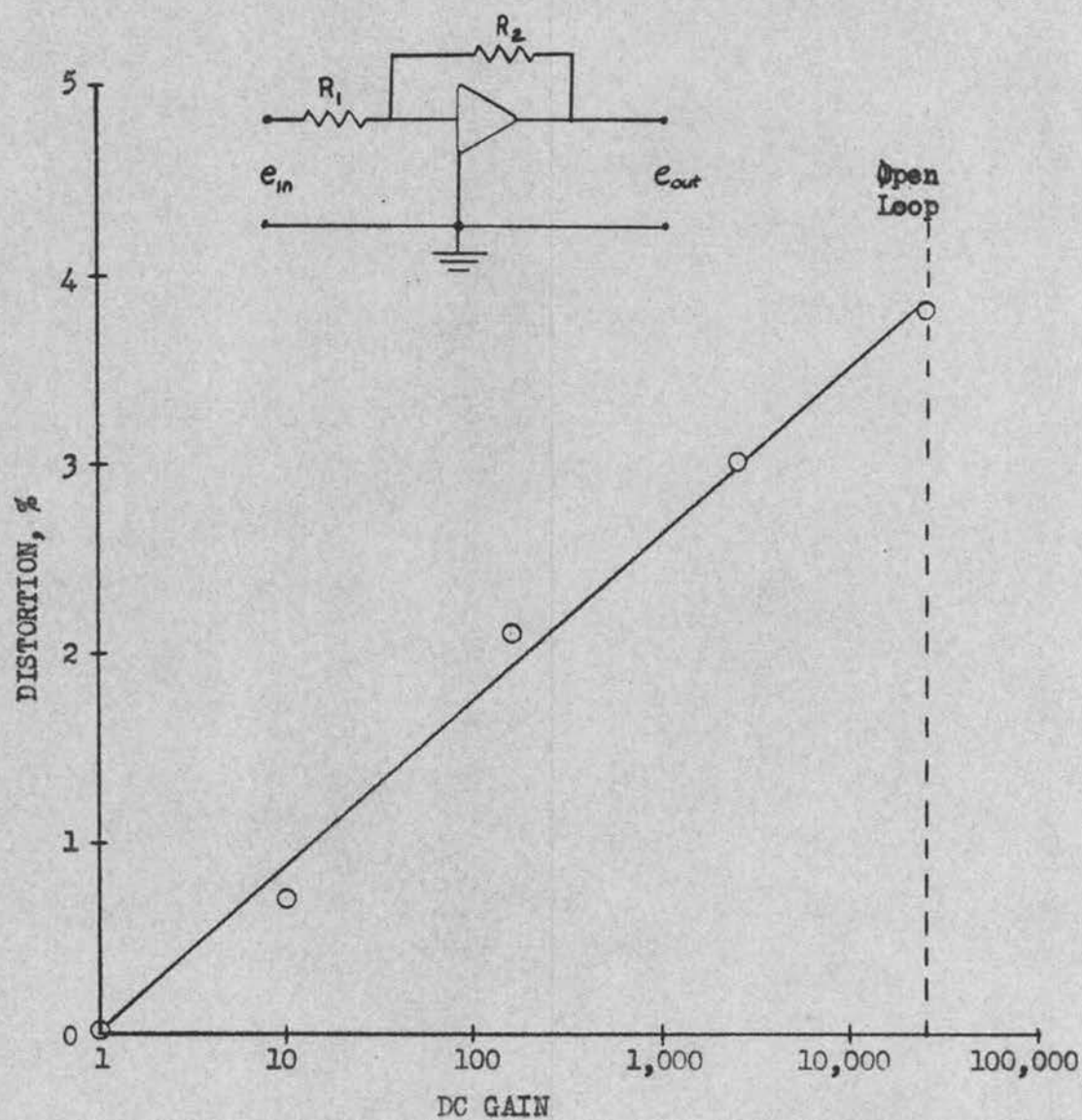
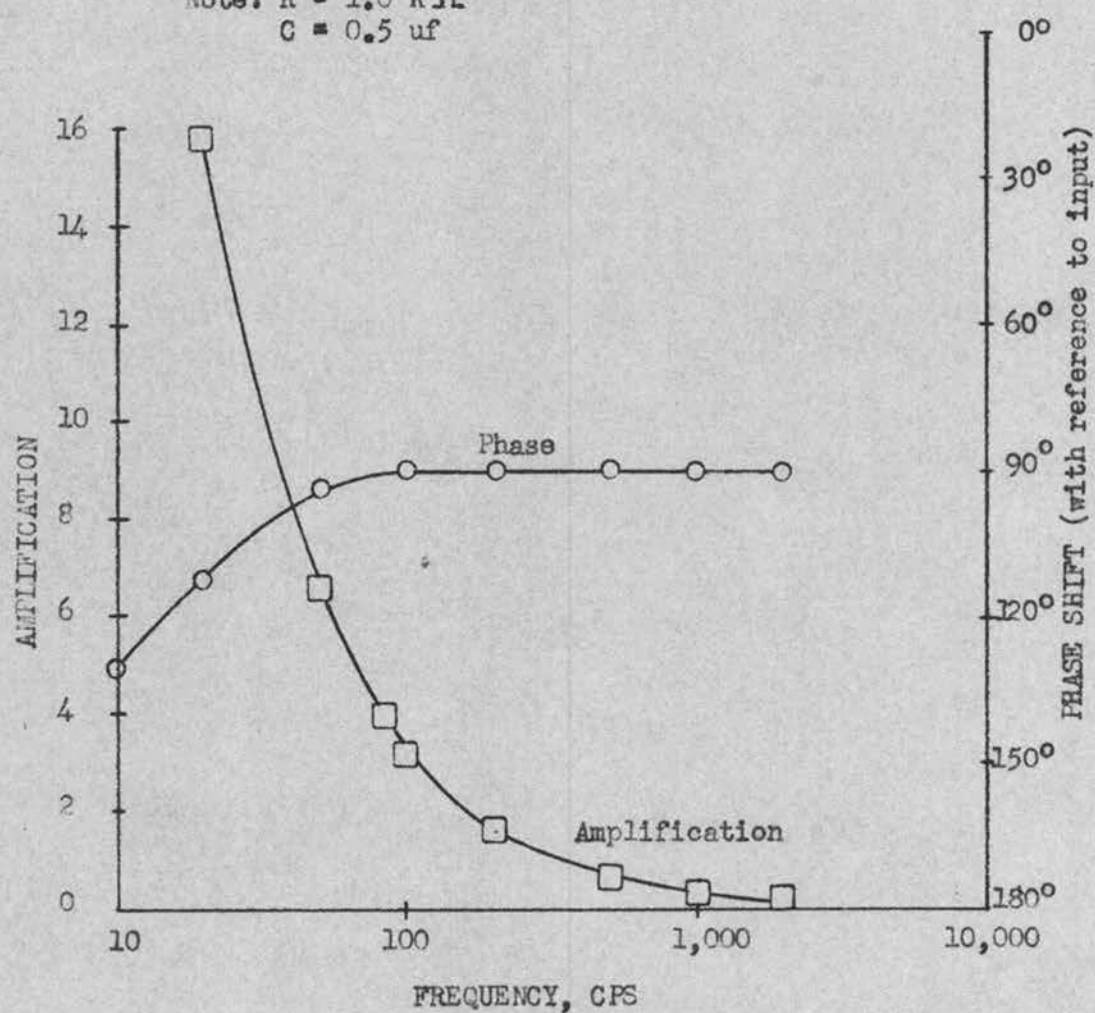
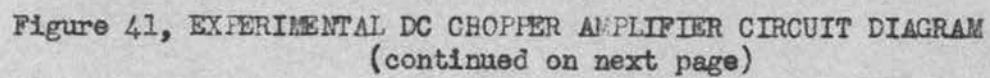


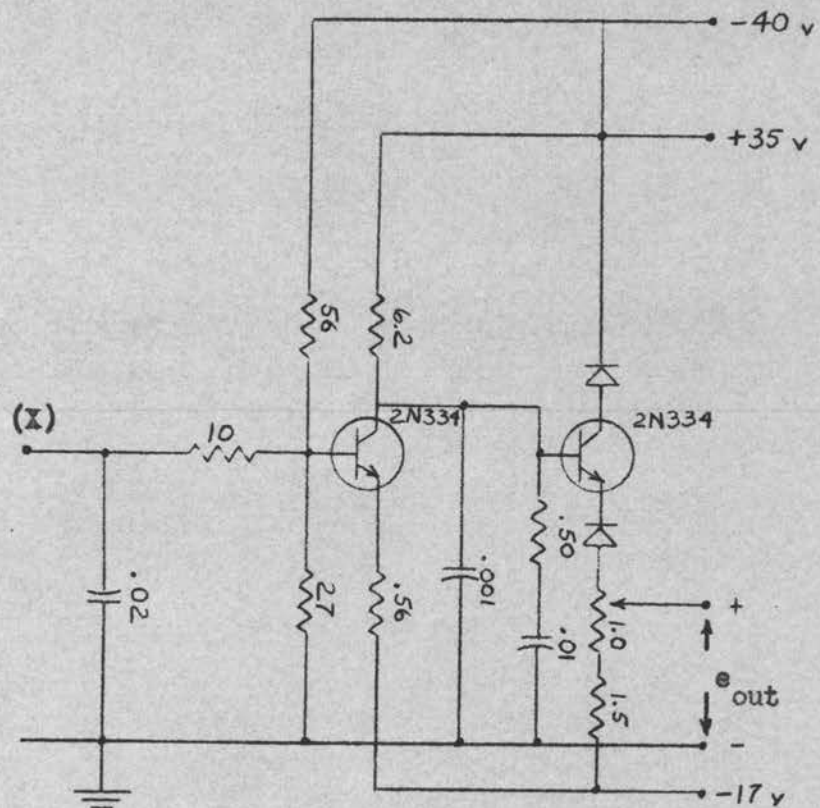
Figure 40, FREQUENCY RESPONSE OF OPERATIONAL  
AMPLIFIER INTEGRATOR NETWORK.

Note:  $R = 1.0 \text{ K}\Omega$   
 $C = 0.5 \text{ }\mu\text{f}$

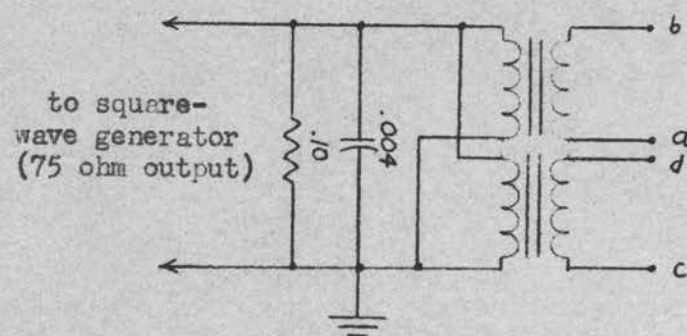








Note: Indicated Units ..  
 All resistors in kilo-ohms  
 All capacitors in micro-farads  
 All inductors in milli-henries



107-A



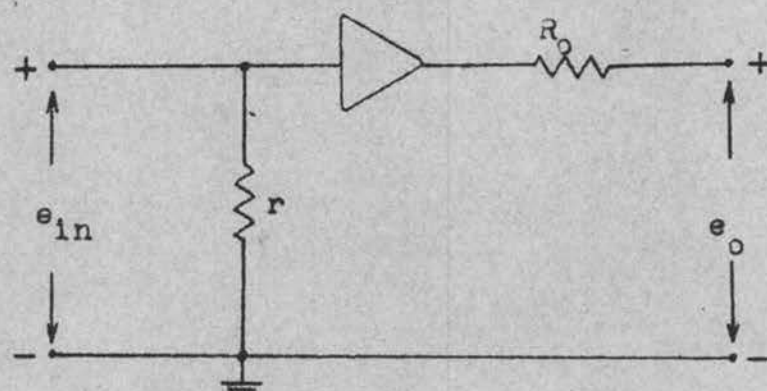
APPENDIX II

DEVIATION AND ILLUSTRATION OF THE ERROR  
CHARACTERISTICS OF A NON-IDEAL  
OPERATIONAL NETWORK

# DEVIATION AND ILLUSTRATION OF THE ERROR CHARACTERISTICS OF A NON-IDEAL OPERATIONAL NETWORK

The non-ideal dc amplifier can be represented in terms of the ideal amplifier as shown below.

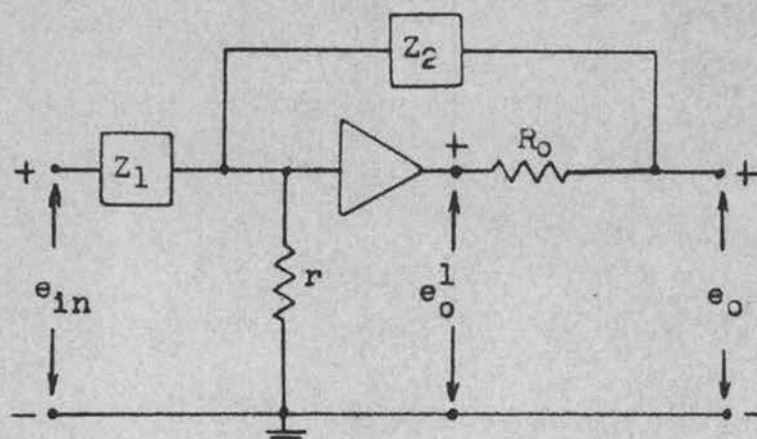
Figure 42. Equivalent Circuit of the Non-Ideal Amplifier



where:  $r$  = input impedance  
 $R_o$  = output impedance

Thus the general operational amplifier network for a non-ideal operational amplifier is represented as shown below.

Figure 43. A Non-Ideal Operational Amplifier Network



Now, if we assume a zero source impedance and infinite load impedance, and equate the sum of currents into the second summing point to zero, we obtain:

$$(1) \quad \left( \frac{e_o^1}{A} - e_o^1 \right) \frac{1}{R_o + Z_2} + \frac{e_o^1}{A} \frac{1}{r} + \left( \frac{e_o^1}{A} - e_{in} \right) \frac{1}{Z_1} = 0$$

A rearrangement yields,

$$(2) \quad \frac{e_o^1}{e_{in}} = \frac{1}{\frac{Z_1}{R_o + Z_2} + \frac{1}{A} \left( \frac{Z_1}{r} + \frac{Z_1}{R_o + Z_2} + 1 \right)}$$

Utilizing the first summing point, we obtain:

$$(3) \quad \left( e_o - e_o^1 \right) \frac{1}{R_o} + \left( e_o - \frac{e_o^1}{A} \right) \frac{1}{Z_2} = 0$$

$$(4) \quad \frac{e_o}{e_o^1} = \frac{Z_2 + \frac{R_o}{A}}{R_o + Z_2}$$

Multiplying expressions (2) and (4) yields,

$$\frac{e_o}{e_{in}} = \frac{Z_2 + \frac{R_o}{A}}{-Z_1 + \frac{1}{A} \left( Z_1 + \frac{Z_1 R_o}{r} + \frac{Z_1 Z_2}{r} + Z_2 + R_o \right)}$$

Now if  $\frac{R_o}{A Z_2} \ll 1$

$$(5) \quad \frac{e_o}{e_{in}}$$

$$= \frac{Z_2}{-Z_1 + \frac{1}{A} \left( Z_1 + \frac{Z_1 R_o}{r} + \frac{Z_1 Z_2}{r} + Z_2 + R_o + \frac{R_o Z_1}{Z_2} \right) - \frac{1}{A^2} \left( \frac{R_o Z_1}{Z_2} + \frac{Z_1 R_o^2}{r Z_2} + \frac{Z_1 R_o}{r_2} + R_o + \frac{R_o^2}{Z_2} \right)}$$

And neglecting the term in  $\frac{1}{A^2}$ ,

$$(6) \quad \frac{e_o}{e_{in}} = \frac{Z_2}{-Z_1 + \frac{1}{A} \left( Z_1 + \frac{Z_1 R_o}{r} + \frac{Z_1 Z_2}{r} + R_o + Z_2 + \frac{R_o Z_1}{Z_2} \right)}$$

Now if  $\frac{1}{A} \left( Z_1 + \frac{Z_1 R_o}{r} + \frac{Z_1 Z_2}{r} + R_o + Z_2 + \frac{R_o Z_1}{Z_2} \right) \ll 1$

$$(7) \quad \frac{e_o}{e_{in}} = - \frac{Z_2}{Z_1} \left[ 1 + \frac{1}{A} \left( 1 + \frac{R_o}{r} + \frac{Z_2}{r} + \frac{R_o}{Z_1} + \frac{Z_2}{Z_1} + \frac{R_o}{Z_2} \right) \right]$$

The error term is easily recognized as

$$(8) \quad \text{error} = \epsilon = \frac{1}{A} \left( 1 + \frac{R_o}{r} + \frac{Z_2}{r} + \frac{R_o}{Z_1} + \frac{Z_2}{Z_1} + \frac{R_o}{Z_2} \right)$$

Solving for  $Z_1$  we obtain

$$(9) \quad Z_1 = \frac{-Z_2 r (Z_2 + R_o)}{Z_2^2 + Z_2 (R + r - A \epsilon r) + Rr}$$



Now if we desire the error limiting values of  $Z_2$  and  $Z_1$ , they can be found from the equation:

$$(10) \quad Z_1 = \frac{-Z_2 r (Z_2 + R_0)}{Z_2^2 + Z_2 (R_0 + r - A \epsilon r) + Rr}$$

For the specific case of:  $A = 25,000$   
 $r = 2,000$   
 $R_0 = 1,000$

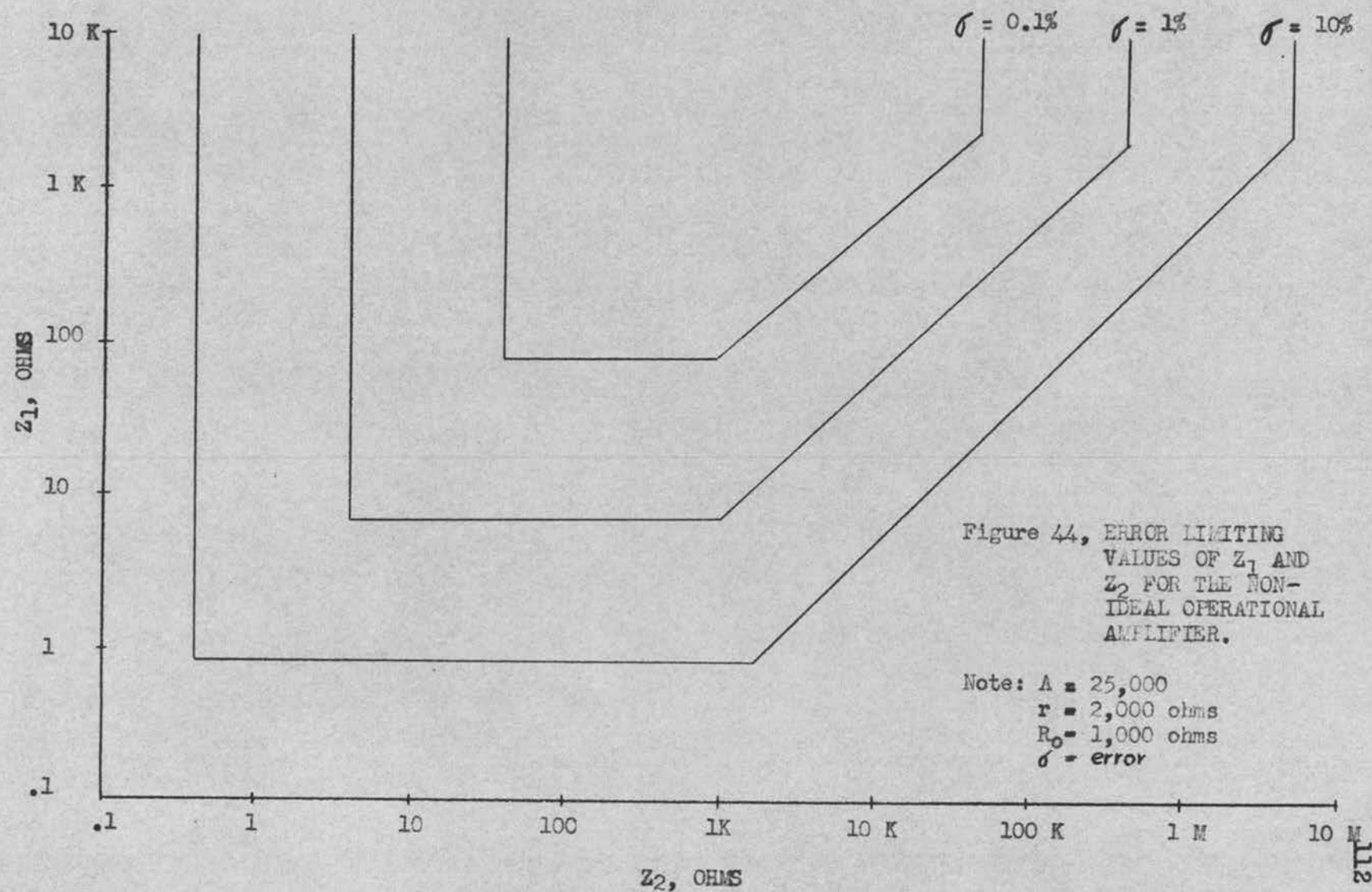
$\epsilon = 10\%$	
$Z_{2,n}$	$Z_{1,n}$
0.4	-
1	1
1 K	0.8
10 K	4.4
100 K	40.8
1 M	500
5 M	-

$\epsilon = 0.1\%$	
$Z_{2,n}$	$Z_{1,n}$
4	-
10	6.67
1 K	8.0
10 K	45.0
100 K	500
500 K	-

$\epsilon = 0.1\%$	
$Z_{2,n}$	$Z_{1,n}$
40	-
100	70
1 K	85
10 K	550
50 K	-

$\epsilon = \text{error}$

This plot is approximated on the following page.



APPENDIX III

DERIVATION OF INTEGRATING OPERATIONAL  
AMPLIFIER NETWORK ERROR  
RELATIONSHIP

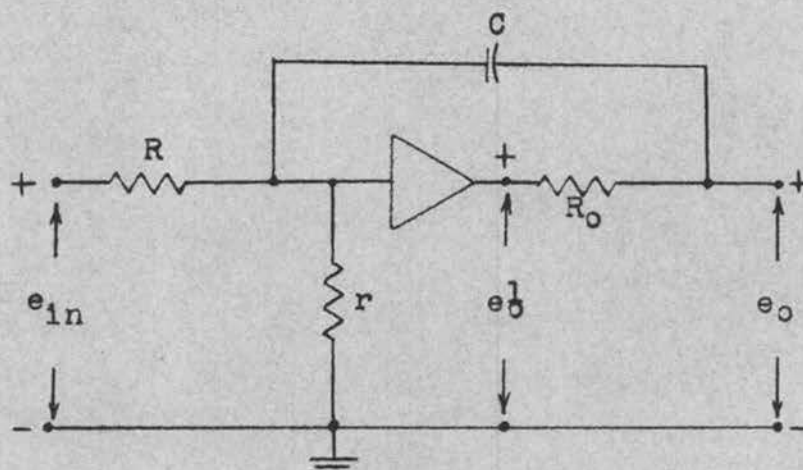
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# DERIVATION OF INTEGRATING OPERATIONAL AMPLIFIER NETWORK ERROR RELATIONSHIP

Proceeding as in Appendix II,

Figure 45. A Non-Ideal Operational Amplifier, Integrator Network



Assumptions: 1. Zero source impedance  
2. Infinite load impedance

In terms of the Laplace operator  $S$ , the current summation around the second node yields,

$$(1) \quad \left( \frac{e_o}{A} - e_1 \right) \frac{1}{R_o + \frac{1}{CS}} + \left( \frac{e_o}{A} - e_{in} \right) \frac{1}{R} + \frac{e_o}{A} \frac{1}{r} = 0$$

Rearrangement yields,

$$(2) \quad \frac{e_o}{e_{in}} = \frac{1}{\frac{-RCS}{R_o CS + 1} + \frac{1}{A} \left( \frac{RCS}{R_o CS + 1} + \frac{R}{r} + 1 \right)}$$



Summing the currents about the first node yields 114

$$(3) \quad (e_o - e_o^1) \frac{1}{R_o} + (e_o - \frac{e_o^1}{A}) C S = 0$$

$$(4) \quad \frac{e_o}{e_o^1} = \frac{1 + \frac{1}{A} (R_o C S)}{R_o C S + 1}$$

Combining the expressions (2) and (4) gives

$$(5) \quad \frac{e_o}{e_{in}} = \frac{1 + \frac{1}{A} (R_o C S)}{-RCS + \frac{1}{A} \left[ RCS + R_o CS + SC \frac{R R_o}{r} + \frac{R}{r} + 1 \right]}$$

If  $\frac{1}{A} (R_o C S) \ll 1$ , and we neglect terms involving  $\frac{1}{A^2}$ ,

$$(6) \quad \frac{e_o}{e_{in}} = \frac{1}{-RCS + \frac{1}{A} \left[ (RCS)(R_o CS) + RCS + R_o CS + \frac{R R_o}{r} CS + \frac{R}{r} + 1 \right]}$$

and, again if  $\frac{1}{A R C S} \left[ (RCS)(R_o CS) + RCS + R_o CS + \frac{R R_o}{r} CS + \frac{R}{r} + 1 \right] \ll 1$

$$(7) \quad \frac{e_o}{e_{in}} = -\frac{1}{RCS} \left[ 1 + \frac{1}{A} \left( R_o CS + 1 + \frac{2 R_o}{r} + \frac{1}{RCS} + \frac{1}{RCS} \right) \right]$$

$$= -\frac{1}{RCS} \left[ 1 + \frac{1}{A} \left( \frac{1}{RCS} + \frac{1}{RCS} + R_o CS + \frac{2 R_o}{r} + 1 \right) \right]$$

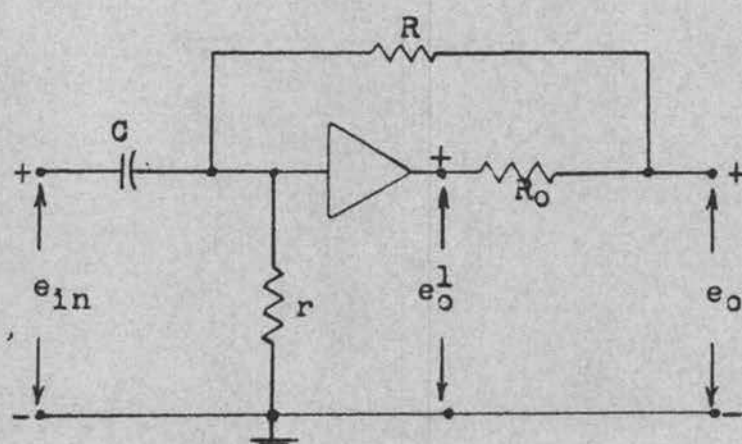
APPENDIX IV

DEVIATION OF DIFFERENTIATING OPERATIONAL  
AMPLIFIER NETWORK ERROR  
RELATIONSHIP

# DEVIATION OF DIFFERENTIATING OPERATIONAL AMPLIFIER NETWORK ERROR RELATIONSHIP

Proceeding as in Appendix I,

Figure 46. A Non-Ideal Operational Amplifier Differentiating Network



- Assumptions: 1. Zero source impedance  
2. Infinite load impedance

In terms of the Laplace operator  $S$ , the current summation around the second node yields,

$$(1) \quad \left( \frac{e_o^1}{A} - e_o^1 \right) \frac{1}{R + R_o} + \frac{e_o^1}{A} \left( \frac{1}{r} \right) + \left( \frac{e_o^1}{A} - e_{in} \right) CS = 0$$

$$(2) \quad \frac{e_o^1}{e_{in}} = \frac{CS}{\frac{1}{R + R_o} + \frac{1}{A} \left( \frac{1}{r} + \frac{1}{R + R_o} \right)}$$

The summation of currents about the first node yields,

$$(3) \quad (e_o - e_o^1) \frac{1}{R_o} + (e_o - \frac{e_o^1}{A}) \frac{1}{R} = 0$$

$$(4) \quad \frac{e_o}{e_o^1} = \frac{R - \frac{R_o}{A}}{R + R_o}$$

Combining expression (2) and (4), yields

$$(5) \quad \frac{e_o}{e_{in}} = \frac{(R - \frac{R_o}{A}) CS}{-1 + \frac{1}{A} \left[ 1 + \frac{R}{r} + \frac{R_o}{r} \right]}$$

Now if  $\frac{1}{A} (1 + \frac{R}{r} + \frac{R_o}{r}) \ll 1$ ,

$$(6) \quad \frac{e_o}{e_{in}} = - \left[ RCS - \frac{R_o}{A} CS \right] \left[ 1 + \frac{1}{A} (1 + \frac{R}{r} + \frac{R_o}{r}) \right]$$

Combining terms and neglecting the term involving  $\frac{1}{A^2}$ ,

$$(7) \quad \frac{e_o}{e_{in}} = - RCS \left[ 1 + \frac{1}{A} (R_o CS + \frac{R}{r} + \frac{R_o}{r} + 1) \right]$$