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Development and Testing of Single-Parameter Precipitation Distributions

JOHN S. SELKER AND DOUGLAS A. HAITH

Department of Agricultural Engineering, Cornell University, Ithaca, New York

A general procedure was developed for calibrating multiparameter probability distributions of daily precipitation to single-parameter distributions. The approach uses monthly precipitation summaries and data from U.S. Weather Bureau Technical Paper 57 (Miller and Frederick, 1966). The three-parameter beta-*P* model of daily precipitation amount was calibrated for 33 sites east of the Rocky Mountains. The resulting single-parameter Weibull distribution and two other single-parameter precipitation distributions were compared with respect to their fit to Paper 57 summaries and historical daily precipitation records. The Weibull model was shown to yield significant improvement over the other models in reproducing precipitation probability distributions.

Introduction

Mathematical models of wet day precipitation amount are useful in a variety of water resource applications. When available, historical records of daily precipitation may be used to estimate parameters of appropriate probability distributions. However, in many cases, these records are either inaccessible or nonexistent and information is limited to regional summaries of mean monthly precipitation and number of wet days. In such situations, precipitation probability distributions based on a single parameter (mean wet day precipitation) are required [*Haith*, 1986; *Richardson*, 1985; *Steenhuis et al.*, 1984].

The exponential distribution is probably the most widely used single-parameter distribution of daily precipitation amount [*Todorovic and Woolhiser*, 1974; *Richardson*, 1981; *Pickering et al.*, 1988]. Although appealing for its simplicity, the exponential distribution has been recognized to underpredict extreme events, which is undesirable in many engineering applications [*Skees and Shenton*, 1974; *Pickering et al.*, 1988].

Single-parameter probability distributions can be derived by calibrating multiparameter distributions. For example, a special case of the beta-*P* distribution [*Mielke and Johnson*, 1974] was calibrated to a single-parameter model by *Pickering et al.* [1988] based on 25 years of weather data available at three sites. This model was shown to provide consistently better results than the exponential distribution, particularly in the case of extreme event prediction for these sites.

Calibration procedures based on historical records are of limited interest, since these same records would permit the direct use of the presumably more accurate multiparameter models. However, calibration from summarized precipitation data such as those contained in U.S. Weather Bureau Technical Paper 57 [*Miller and Frederick*, 1966] (subsequently referred to as Paper 57) is also possible. This information was used in the present study to calibrate the three-parameter beta-*P* distribution to a one-parameter model which is a member of the Weibull family of distributions.

The research described in this paper had two major objectives: (1) to illustrate the use of Paper 57 information to calibrate wet day precipitation probability distributions, and (2) to compare three single-parameter distributions, the exponential, Pickering et al. calibrated beta-*P*, and calibrated Weibull.

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Derivation of a Weibull Distribution for Daily Precipitation

Generalized Three-Parameter Beta-P Distribution

A beta-*P* model has shown good potential as the basis of simple precipitation models. In addition to the useful properties of having closed form expressions for the cumulative distribution and moments, it is invertible, and performs well in precipitation modeling [*Mielke and Johnson*, 1974; *Pickering et al.*, 1988]. In its general form, the beta-*P* distribution, which is also known as a Burr type XII distribution [*Burr*, 1942; *Rodriguez*, 1977; *Tadikamalla*, 1980], has three parameters, and is given by

$$F_X(x) = 1 - [1 + (x/b)^c]^{-a} \tag{1}$$

for $x \geq 0$. The moments of this distribution are

$$E[X^v] = ab^v\beta[1 + v/c,\ a - v/c] \tag{2}$$

for $-c < v < ac$. Here, β represents the beta function [*Abramowitz and Stegun*, 1965]:

$$\beta(r, s) = \frac{\Gamma(r)\Gamma(s)}{\Gamma(r + s)} \tag{3}$$

and $\Gamma(\cdot)$ is the gamma function. Thus the mean of the distribution is given by

$$E[X] = ab\beta[1 + 1/c,\ a - 1/c] \tag{4}$$

For any values of a and c, and the mean daily precipitation $[E(X)]$, the parameter b is given from (4) as

$$b = E(X)/[a\beta(1 + 1/c,\ a - 1/c)] \tag{5}$$

Model Calibration

Paper 57 provides the expected number of 24-hour precipitation events exceeding 0.5, 1, 2, and 4 inches (1 inch = 2.54 cm) in each month for the continental United States.

The primary source of data for Paper 57 was the records of 648 weather stations, with records of over 18 years at 99% of these stations (most records were for the 30-year period 1931–1960). Supplemental data were used from an additional 556 stations. Thus the record includes one station per 6,500 km^2 or an average distance between sites of about 80 km.

The 24-hour event numbers in any month can be converted to daily numbers using the following regression equation from Paper 57:

$$N_m(x) = M_m(x)/\alpha \qquad (6)$$

in which $N_m(x)$ and $M_m(x)$ are the expected numbers of daily and 24-hour events, respectively, exceeding x in month m and α = 1.2, 1.3, 1.4 for 0.5-, 1-, and 2-inch events, respectively. The 4-inch data were not used in this study, as the number of events of this size is only appreciable for the Gulf States.

This information may be used to calibrate precipitation probability distributions. If X_m is precipitation amount on a wet day in month m, the probability of precipitation in excess of x is

$$Pr\{X_m > x\} = 1 - F_{X_m}(x) \qquad (7)$$

where $F_{X_m}(x)$ is the cumulative probability distribution for wet day precipitation amount. An estimate of the expected number of daily events exceeding x in month m, $N_m(x)'$, is

$$N_m(x)' = n_m[1 - F_{X_m}(x)] \qquad (8)$$

in which n_m is the mean number of wet days in month m. We have assumed that the distribution $F_{X_m}(x)$ is the same for each day in month m since weather summaries such as that of *National Oceanic and Atmospheric Administration* [1985] generally list only mean monthly data for n_m and precipitation. The expected number of events of various sizes can be computed from (8) and compared directly with the values given in Paper 57.

Paper 57 data were used to calibrate the beta-*P* model (equation (1)); 33 sites east of the Rocky Mountains were selected for this evaluation. These sites are listed in the appendix. The expected number of 24-hour events was read from the maps provided in Paper 57 and converted to 0.5-, 1-, and 2-inch daily events using (6).

The parameters a and c in (1) were calibrated by minimizing two measures of fit: the mean squared difference between the model's estimate and the reported number of storms, and a chi-squared measure of the error in fit. Model fit was studied for each storm size independently. The squared error, $S(x)$, was calculated as

$$S(x) = \frac{1}{12(33)} \sum_{i,j} [N_{ij}(x) - N_{ij}(x)']^2 \qquad (9)$$

in which $N_{ij}(x)$ is the reported expected number of events of size x for site i and month j and $N_{ij}(x)'$ is the comparable estimated number of events as given by (8). By dividing each term of this sum by the expected number of storms, we obtain a chi-square measure:

$$\chi^2 = \frac{1}{12(33)} \sum_{i,j} \left[\frac{[N_{ij}(x) - N_{ij}(x)']^2}{N_{ij}(x)}\right] \qquad (10)$$

The squared error criterion was selected to minimize large model errors. The chi-square measure has the advantage of normalizing the errors, so that sites with large expected storm counts do not dominate the sum. Also, for large sample sizes, the variances of minimum chi-square estimates converge to the Cramer-Rao lower bound [*Bickel and Doksum*, 1977]. Hence the chi-square estimator shares the asymptotic efficiency characteristics of maximum likelihood estimators [*Moore*, 1978].

Although not used in this study, a likelihood function can in principle be maximized to estimate the parameters a and c. Letting T_m be the total number of wet days over the period of record in month m at a site, and $t_m(x)$ be the number of wet days in which precipitation exceeded x, the probability of the three observations $t_m(0.5)$, $t_m(1.0)$, and $t_m(2.0)$ is

$$F_{X_m}(0.5)^{[T_m - t_m(0.5)]}[E_{X_m}(1.0) - F_{X_m}(0.5)]^{[t_m(0.5) - t_m(1.0)]}$$

$$\cdot [F_{X_m}(2.0) - F_{X_m}(1.0)]^{[t_m(1.0) - t_m(2.0)]}[1 - F_{X_m}(2.0)]^{t_m(2.0)} \qquad (11)$$

A likelihood function can be formed as the product of similar expressions for each site, month and precipitation threshold. The observation data T_m and $t_m(x)$ are not included in the Paper 57 summaries, but could be inferred from other sources. In general, it appears that maximum likelihood estimates would be most useful in calibrating distributions for specific months and sites using the raw observations of daily precipitation (rather than the Paper 57 summaries).

Calculation of the modeled number of storms was carried out using equations (1), (5), and (8). Figures 1 and 2 show the squared error and chi-squared values with variation in parameter values, for 0.5-inch storms. Plots for other storm sizes have a similar structure, with minima following a well-defined path with increasing values of a. These results suggest that there is some best fitting asymptotic distribution, as a approaches infinity. This limit was given by *Rodriguez* [1977] as

$$F_X(x) = 1 - \exp\{-[\Gamma(1 + 1/c)x/E(x)]^c\} \qquad (12)$$

which is a member of the Weibull family of distributions.

Although the Weibull distribution has often been discussed in relationship to hydrometeorological data [*Wong*, 1977; *Mielke*, 1979; *Wilks*, 1989], we are not aware of its previous use in modeling of daily precipitation.

The parameter c affects the general shape of the distribution as shown in Figure 3 for $E(X)$ = 10 mm. Figure 4 shows the chi-squared and squared error for the Weibull distribution for a range of c values. The two measures of error differ slightly in their minima. Regarding the chi-squared measure, it appears that a value of 0.78 gives a good fit to 1- and 2- inch events, with reasonable fit to the 0.5-inch events. In the case of squared error, a suitable value is c = 0.73. With c = 0.75 the chi-squared and squared error measures are both nearly optimal for all three event sizes; thus this value was selected as the optimized value, resulting in the probability distribution for wet day precipitation in month m given by

$$F_{X_m}(x) = 1 - \exp[-(1.191x/\mu_m)^{0.75}] \qquad (13)$$

in which μ_m is the expected wet day precipitation in month m.

In this calibration exercise a single value of the Weibull

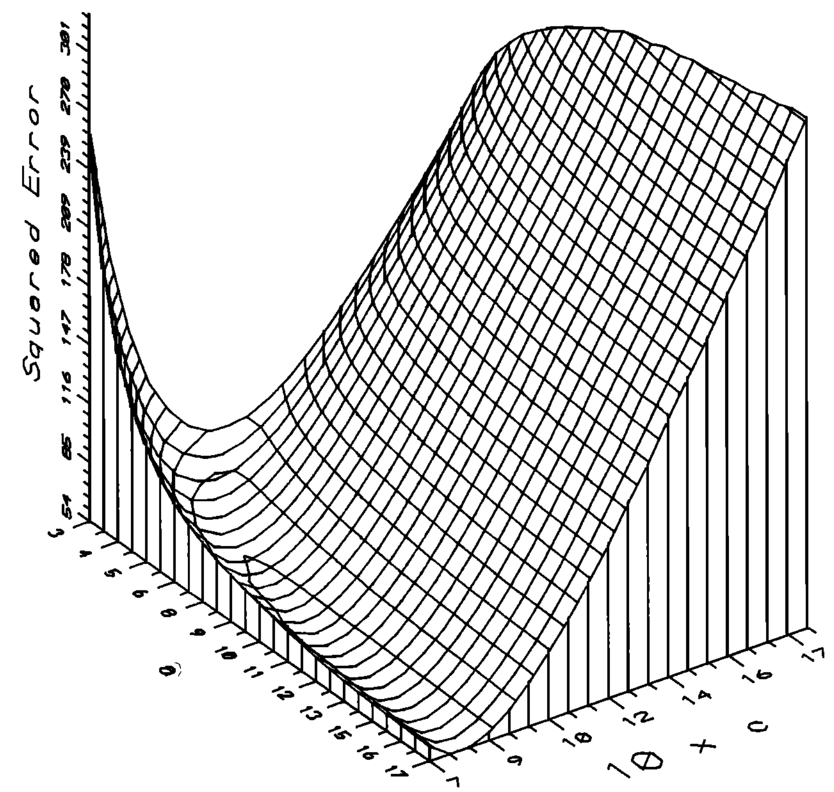

Fig. 1. Squared error for 0.5-inch storms as functions of model coefficients for generalized beta-P distribution (equation (1)).

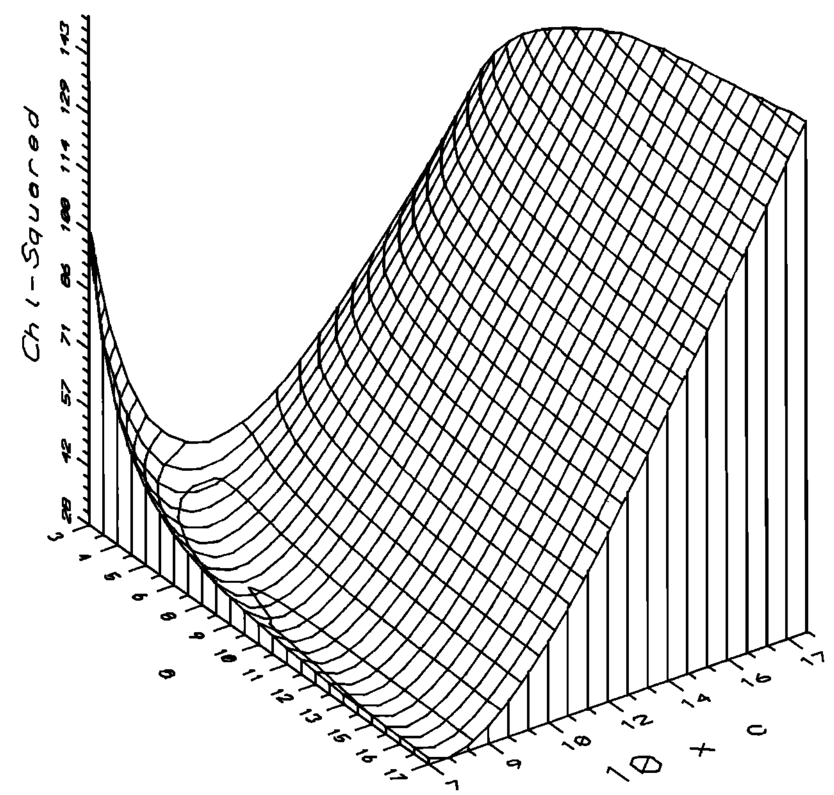

Fig. 2. Chi-squared error for 0.5-inch storms as functions of model coefficients for generalized beta-P distribution (equation (1)).

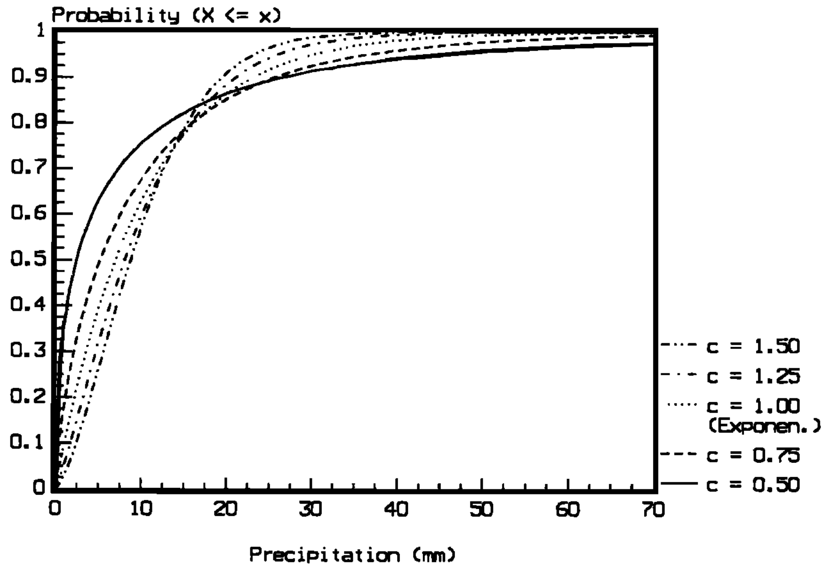

Fig. 3. Weibull distribution as function of c parameter for $E(X) = 10$ mm.

parameter c was optimized for all 12 months and 33 sites. However, the approach is sufficiently general for many other options. Separate values could be obtained for each month, and the sites could be grouped to produce regionalized estimates. Other multiparameter distributions could be similarly calibrated.

Comparison of Single-Parameter Distributions

The calibrated single-parameter Weibull distribution (equation 13)) was compared with two other single-parameter models for daily precipitation: the calibrated beta-P distribution from *Pickering et al.* [1988] and the exponential distribution. The former is given by

$$F_{X_m}(x) = 1 - [1 + x/(9\mu_m)]^{-10} \tag{14}$$

and the exponential distribution is

$$F_{X_m}(x) = 1 - \exp(-x/\mu_m) \tag{15}$$

As indicated in Figure 3, the latter distribution is a special case of the Weibull distribution for $c = 1$.

Comparisons for Selected Storms (Paper 57 Data)

The 33 sites listed in the appendix were used for testing of the three model predictions against the historical values obtained from Paper 57 (as in the calibration procedure). We considered three measures of discrepancies between modeled and reported data. These include the relative error, which is the average of the predicted number of storms divided by the reported number of storms for each site and month. Here a value of one indicates perfect prediction, with

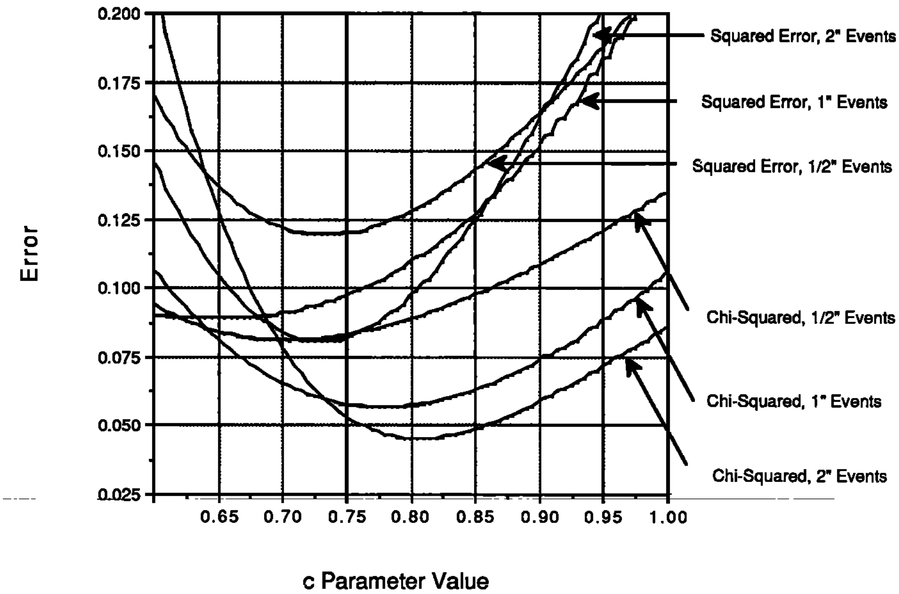

Fig. 4. Chi-square and squared error for Weibull distribution.

TABLE 1. Error Analysis Summary for Alternative Precipitation Models Based on Comparisons to Values Reported in Paper 57

Model	Event Size, in	Relative Error[a] Mean	Relative Error[a] Coefficient of Variation	Average Error,[b] in	Fraction Predicted[c]
Exponential	1/2	0.97	0.47	+0.06	1.03
Beta-*P*	1/2	0.95	0.46	−0.02	0.96
Weibull	1/2	0.97	0.44	−0.08	0.96
Exponential	1	0.64	0.61	−0.19	0.77
Beta-*P*	1	0.79	0.65	−0.16	0.80
Weibull	1	1.02	0.80	−0.03	0.97
Exponential	2	0.27	1.44	−0.12	0.37
Beta-*P*	2	0.59	1.41	−0.09	0.53
Weibull	2	1.27	1.50	−0.02	0.90

1 in = 2.54 cm.

[a]Predicted number of storms divided by reported number of storms for all sites and months.

[b]Mean difference between predicted and reported events.

[c]Mean fraction of total events predicted. *Paper 57* reports an average of 2.086 0.5-inch events, 0.8091 1-inch events, and 0.1908 2-inch storms per year for these 33 sites.

values less than one indicating underprediction of events, and values over one indicating overprediction in the number of events. The mean and coefficient of variation of the relative error are calculated for each event size category. Average error, which is the average of the differences between modeled and reported number of storms over all sites and months (a measure of the bias of the model), is also calculated. Finally, the mean fraction of storms predicted is calculated. This value is the 33-site average number of annually modeled events divided by reported values for each of the three event size categories.

Each of these measures has limitations. Relative error, for instance, is bounded below by zero and thus tends to have a bias toward large values when averaged over many sites. Average error is weighted toward sites with large numbers of

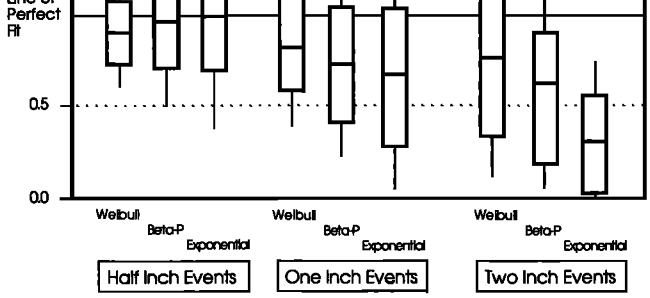

Fig. 5. Relative errors of exponential, beta-*P* and Weibull distributions for three storm sizes.

TABLE 2. Historical Precipitation Records Used for Model Validation

Site	Record Length, yr
Albany, N. Y.	50
Baltimore, Md.	39
Burlington, Vt.	67
Caribou, Me.	48
Charleston, W. Va.	39
East Wareham, Mass.	61
Hartford, Conn.	67
Ithaca, N. Y.	67
Pittsburgh, Pa.	31
Portland, Me.	67
Rochester, N. Y.	61

events, where a small fractional error for a site with a relatively high number of expected events may contribute disproportionally to the overall average. These summary statistics are listed in Table 1.

Table 1 indicates that the three models' performances in predicting 0.5-inch events are very similar. For 1-inch events, the exponential model underpredicts the number of events by about 35%, and the one-parameter beta-*P* by about 20%, with the Weibull predicting close to the reported values. Considering 2-inch events, the exponential model underpredicts event counts by about 70%, the one-parameter beta-*P* underpredicts by about 55%, and the Weibull either overpredicts or underpredicts, depending on the measure employed (that is, the mean relative error indicates overprediction, while the average error and fraction predicted measures show slight underprediction). The models have very similar performance in the distribution of errors relative to the mean error, as indicated by values of the coefficient of variation.

The distributions of relative errors are displayed in Figure 5 as box plots. In these plots, the box shows the interval containing 75% of the data points and having equal number of points above and below the median value, while the whiskers define the region containing 90% of the data from the sample. From the box plots it is again evident that the performance of the models in predicting 0.5-inch storms is roughly equal, and the performance in 1- and 2-inch events shows deterioration in all three models. In these categories, the exponential model substantially underpredicts the number of events. The Weibull appears to provide the most accurate predictions, although it shows a wider spread of error, which is confirmed in Table 1 by a larger coefficient of variation. Note that the median of the Weibull distribution is nearly 2.5 times as large as that for the exponential in storms which yield over 2 inches of precipitation. This difference can be expected to have a significant impact in systems which are dominated by the occurrence of extreme events.

Validation With Historical Precipitation Data

Since the Weibull distribution was calibrated to the 33 sites using Paper 57 data, we might expect a better fit to these data than that provided by the exponential or one-parameter beta-*P* models. To explore the general validity of each distribution, we compared them to empirical frequency distributions determined from historical precipitation records from the 11 sites in the northeast United States listed

TABLE 3. Summary of Chi-Squared Test of Fit of Three Precipitation Models to Historical Records

	Exponential			Beta-P			Weibull		
Site	Chi-Squared[a]	p Value Range[b]	Months Passing 5%[c]	Chi-Squared[a]	p Value Range[b]	Months Passing 5%[c]	Chi-Squared[a]	p Value Range[b]	Months Passing 5%[c]
Albany, N. Y.	1713	<0.000	0	1294	<0.000	0	410	0.095–0.492	6
Baltimore, Md.	1363	<0.000	0	1068	0.000–0.001	0	334	0.023–0.612	10
Burlington, Vt.	2002	<0.000	0	1423	0.000–0.001	0	478	0.000–0.566	6
Caribou, Me.	1558	<0.000	0	1148	<0.000	0	432	0.000–0.379	7
Charleston, W. V.	1074	<0.000	0	822	0.000–0.002	0	376	0.001–0.796	9
East Wareham, Mass.	2053	<0.000	0	1583	<0.000	0	775	0.000–0.008	0
Hartford, Conn.	1996	<0.000	0	1508	<0.000	0	480	0.000–0.217	3
Ithaca, N. Y.	2536	<0.000	0	1810	<0.000	0	592	0.000–0.345	4
Pittsburgh, Pa.	958	0.000–0.001	0	707	0.000–0.003	0	318	0.002–0.881	11
Portland, Me.	2711	<0.000	0	2042	<0.000	0	486	0.000–0.218	5
Rochester, N. Y.	1852	<0.000	0	1286	<0.000	0	488	0.000–0.815	4
Mean	1802		0	1336		0	469		6

[a]Annual χ^2 test values based on sum of 12 monthly (χ^2_{276}).
[b]Calculated from monthly χ^2 p values (level of significance) from χ^2_{23} critical values.
[c]Number of months with χ^2_{23} test value less than the 5% critical value (35.2).

in Table 2. The historical data for this study were obtained from the Northeast Regional Climate Center, Cornell University, Ithaca, New York. The shortest record used was 31 years with an average length of record of 54 years. The parameters required to compute storm numbers for each distribution (mean monthly precipitation, and the mean monthly number of wet days) were calculated from the weather records.

Since the precipitation record is recorded in increments of 0.01 inches, the data are distributed across a discrete set of values. For such data, the chi-squared test can provide an approximate test of fit. The test is somewhat limited in that we must assume that precipitation amounts on different days are statistically independent, but it provides a reasonable means of comparing the three distributions.

In order to obtain comparable data, a set number of observation cells was established for all sites and months. Cells were created by inverting the Weibull distribution to obtain cells with approximately equal expected cell count. All cells were multiples of 0.01 in length to avoid artificially high chi-squared values due to unequal coverage by the distribution functions. Twenty-five cells were used, with expected cell counts of 13 or greater. Table 3 summarizes the chi-squared test results. Here, annual values are presented, calculated from averaged monthly values. The quality of fit of the Weibull distribution is markedly superior to the other models.

The chi-squared test does not give an indication of the type of errors which are occurring. In this regard it is useful to examine graphic evidence of the fit of these distributions. Figure 6 shows the empirical and three analytical distributions for Pittsburgh, Pennsylvania for the month of January.

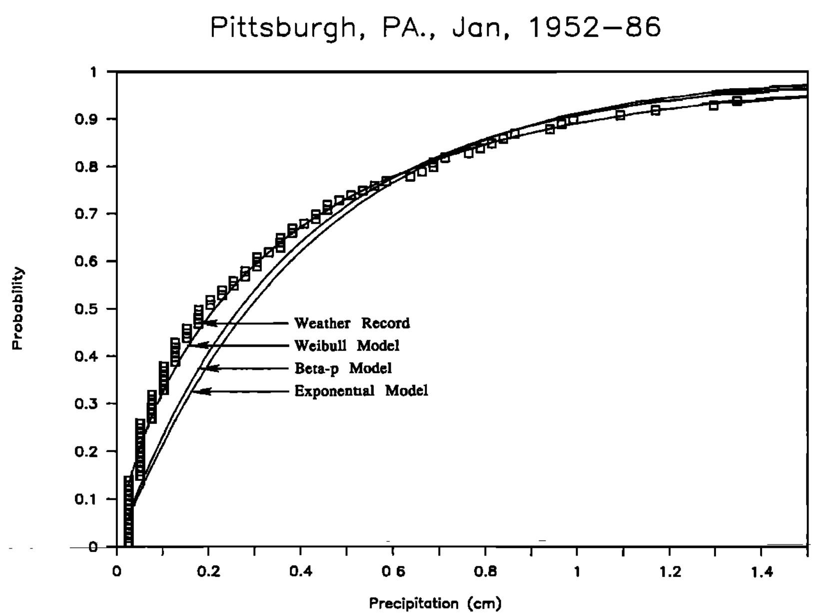

Fig. 6. Comparison of exponential, beta-P and Weibull distributions with empirical distribution for January at Pittsburgh, Pennsylvania.

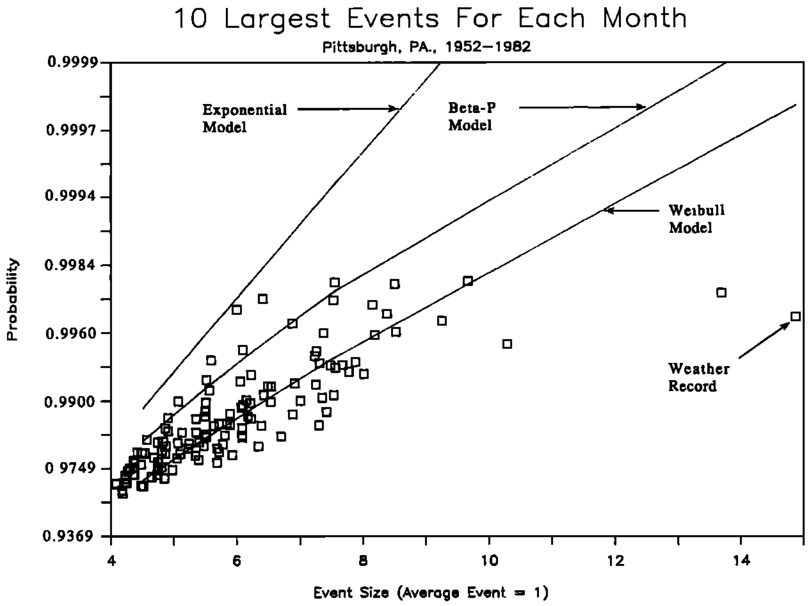

Fig. 7. Comparison of probabilities of large precipitation events for Pittsburgh, Pennsylvania.

The chi-squared values for the distributions are 120.6 for the exponential distribution, 85.9 for the beta-*P*, and 34.2 for the Weibull. Thus the fit is somewhat better than average for the first two, and somewhat worse than average for the Weibull for this month and site, compared to the full 11 sites. Even so, the errors in fit exhibited by the former distributions are quite evident. The beta-*P* and exponential models overpredict the probability of small events, and underpredict extreme events.

In many applications, prediction of extreme large events is of primary importance. Figure 7 gives the plotting positions of the 10 largest precipitation events for each month in Pittsburgh. The y axis gives the probability for the event (logarithmic scale), and the x-axis records the ratio of event size to mean event size. The beta-*P* distribution overpredicts the return period of very large events, while the exponential overpredicts the return periods for the majority of historical events. The Weibull appears to fit the data quite well, without obvious systematic error. Note that for events which yield 8 times the average wet day precipitation the return period of the exponential is 13 times as long as that of the Weibull, while the beta-*P* is 2.5 times as long. Given that this event has a historical return period of about once in 300 wet days, this discrepancy is potentially important for engineering applications.

Conclusions

Single-parameter probability distributions of daily precipitation are useful in a variety of water resource applications. Although such distributions are not generally as accurate as multi-parameter distributions, they can be used in situations where weather data are limited to monthly summaries. This paper has developed a general procedure through which precipitation summaries given in U.S. Weather Bureau Technical Paper 57 can be exploited to calibrate multi-parameter distributions to single-parameter distributions. The approach was used to obtain a single-parameter Weibull distribution for 33 sites in the United States east of the Rocky Mountains.

The summaries presented in Paper 57 are based on analyses of historical precipitation records at more than 1200 locations. The procedures used in the present study demonstrate that this information provides a generally applicable and efficient means of calibrating and testing precipitation models, without requiring reference to daily weather records.

The Weibull distribution was compared with exponential and beta-*P* distributions. Comparisons were based on Paper 57 data and weather records from 11 sites in the northeast United States. The Weibull distribution displayed a significantly improved fit to the historical distribution of events. Of particular interest here, the Weibull model provided large precipitation event probabilities much closer to those found in the weather records.

Appendix

The following *Paper 57* sites were used for calibration of the Weibull precipitation distribution: Abilene, Texas; Albany, New York; Amarillo, Texas; Apalachacola, Florida; Atlanta, Georgia; Austin, Texas; Baltimore, Maryland; Caribou, Maine; Charlotte, North Carolina; Columbia, South Carolina; Dallas, Texas; Dubuque, Iowa; Evansville, Indiana; Fargo, North Dakota; Goodland, Kansas; Grand Rapids, Michigan; Hartford, Connecticut; Houston, Texas; Indianapolis, Indiana; Knoxville, Tennessee; Lexington, Kentucky; Lincoln, Nebraska; Macon, Georgia; Memphis, Tennessee; Mobile, Alabama; Parkersburg, West Virginia; St. Paul, Minnesota; Shreveport, Louisiana; Sioux City, Iowa; Springfield, Missouri; Tulsa, Oklahoma; Wichita, Kansas; Wilmington, North Carolina.

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REFERENCES

Abramowitz, M., and I. A. Stegun, *Handbook of Mathematical Functions*, Dover, New York, 1965.

Bickel, P. J., and K. A. Doksum, *Mathematical Statistics: Basic Ideas and Selected Topics*, 492 pp., Holden-Day, Oakland, Calif., 1977.

Burr, I. W., Cumulative frequency functions, *Ann. Math. Stat.*, *13*, 215–232, 1942.

Haith, D. A., Simulated regional variations in pesticide runoff, *J. Environ. Qual.*, *15*(1), 5–8, 1986.

Mielke, P. W., Some parametric, nonparametric and permutation inference procedures resulting from weather modification experiments, *Commun. Stat. Theor. Methods*, *A8*(11), 1083–1096, 1979.

Mielke, P. W., Jr., and E. S. Johnson, Some generalized beta distributions of the second kind having desirable application features in hydrology and meteorology, *Water Resour. Res.*, *10*(2), 223–226, 1974.

Miller, J. F., and R. H. Frederick, Normal monthly number of days with precipitation of 0.5, 1.0, 2.0 and 4.0 inches or more in the conterminous United States, *Tech. Pap. 57*, U.S. Weather Bur., Washington, D. C., 1966.

Moore, D. S., Chi-square tests, in *Studies in Statistics*, *Stud. in Math.*, vol. 19, edited by R. V. Hogg, pp. 66–106, Mathematical Association of America, Washington, D. C., 1978.

National Oceanic and Atmospheric Administration, *Climate of the States*, 3rd ed., 1572 pp., Gale Research Company, Detroit, Mich., 1985.

Pickering, N. B., J. R. Stedinger, and D. A. Haith, Weather input for nonpoint source pollution models, *J. Irrig. Drain. Eng.*, *114*(4), 674–690, 1988.

Richardson, C. W., Stochastic generation of daily precipitation, temperature, and solar radiation, *Water Resour. Res.*, *17*(1), 182–190, 1981.

Richardson, C. W., Weather simulation for crop management models, *Trans. ASAE*, *28*(5), 1602–1606, 1985.

Rodriguez, R. N., A guide to the Burr type XII distributions, *Biometrika*, *64*, 129–134, 1977.

Skees, P. M., and L. R. Shenton, Comments on the statistical distribution of rainfall per period under various transformations, Proceedings, Symposium on Statistical Hydrology, *Misc. Publ. 1275*, pp. 172–196, U.S. Dep. of Agr., Washington, D. C., 1974.

Steenhuis, T. S., M. Van Der Marel, and S. Pacenka, A pragmatic model for diagnosing and forecasting ground water contamination, in *Proceedings of the Conference on Practical Applications of Groundwater Models*, pp. 515–544, National Water Well Association, Worthington, Ohio, 1984.

Tadikamalla, P. R., A look at the Burr and related distributions, *Int. Stat. Rev.*, *48*, 337–344, 1980.

Todorovic, P., and D. Woolhiser, Stochastic model of daily rainfall, Proceedings, Symposium on Statistical Hydrology, *Misc. Publ. 1275*, pp. 232–246, U.S. Dep. Agr., Washington, D. C., 1974.

Wilks, D. S., Rainfall intensity, the Weibull distribution, and estimation of daily surface runoff, *J. Appl. Meteorol.*, *28*, 52–58, 1989.

Wong, K. W. R., Weibull distribution, iterative likelihood techniques and hydrometeorological data, *J. Appl. Meteorol.*, *16*, 1360–1364, 1977.

D. A. Haith and J. S. Selker, Department of Agricultural Engineering, Riley-Robb Hall, Cornell University, Ithaca, NY 14853.

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