AN ABSTRACT OF THE THESIS OF

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Title: Evaluation of Scheduling Heuristics for Non-Identical Parallel Processors.

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An evaluation of scheduling heuristics for non-identical parallel processors was performed. There has been limited research that has focused on scheduling of parallel processors. This research generalizes the results from prior work in this area and examines complex scheduling rules in terms of flow time, tardiness, and proportion of tardy jobs. Several factors affecting the system were examined and scheduling heuristics were developed. These heuristics combine job allocation and job sequencing functions. A number of system features were considered in developing these heuristics, including setup times and processor utilization spread. The heuristics used different sequencing rules for job sequencing including random, Shortest Process Time (SPT), Earlier Due Date (EDD), and Smaller Slack (SS).

A simulation model was developed and executed to study the system. The results of the study show that the effect

of the number of machines, the number of products, system loading, and setup times were significant for all performance measures. The effect of number of machines was also found to be significant on flow time and tardiness. Several two-factor interactions were identified as significant for flow time and tardiness.

The SPT-based heuristic resulted in minimum job flow times. For tardiness and proportion of tardy jobs, the EDD-based heuristic gave the best results. Based on these conclusions, a "Hybrid" heuristic that combined SPT and EDD considerations was developed to provide tradeoff between flow time and due date based measures.

EVALUATION OF SCHEDULING HEURISTICS FOR NON-IDENTICAL PARALLEL PROCESSORS

by

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EVALUATION OF SCHEDULING HEURISTICS FOR NON-IDENTICAL PARALLEL PROCESSORS

CHAPTER 1. INTRODUCTION

Competition in the marketplace requires production processes to become more economical and efficient. Production organizations have conflicting goals: production costs have to be kept as low as possible while specific customer demands need to be satisfied (Dorn and Foreschl, 1993). Today's production management needs to satisfy several objectives that may conflict with each other, such as maximizing machine utilizations, meeting due dates, minimizing work-in-process inventories and balancing utilization of production resources. As a result, allocation and scheduling of raw materials, jobs, machines, and other resources at the right time to obtain optimal or near optimal solutions play an important role in achieving management objectives. A common situation in manufacturing and service industries is that of assigning jobs to machines or workers (processors) that do not have equal capabilities and capacities. The focus of this study is scheduling of this specific type of system.

This research involves scheduling tasks on multiple parallel, non-identical processors. Each task may consist of a number of jobs. A parallel processor is the situation where a task can be done by more than one processor but only one processor can actually work on the task. Non-identical processors are processors that do not have the same capacities and/or capabilities. The occurrence of parallel, non-identical processors is quite common in both manufacturing and service industries. An example would be a typing pool where any typist could type a document, but only one typist can be assigned the task. Other examples include an airline assigning a type of airplane to service a route and a textile plant assigning jobs to looms.

There has been limited research that has focused on scheduling of parallel processors. An earlier study (Smith, 1993) examined the factors affecting scheduling a system of parallel, non-identical processors using a series of experimental designs. Several factors including loading of jobs on processors, the range and distribution of processor capacities, ranking of jobs for processor assignment, job size distribution, and product demand distribution were examined. The results showed that system loading and job set-up times on processors play a major role in system performance. Furthermore, grouping jobs by product type was also found to minimize set-up times and hence reduce the mean flow time and tardiness but at the expense of controlling individual processor usage. However, Smith's (1993) results are based on only one system, consisting of three machines and ten products.

This research generalizes the results from prior work in this area and examines complex scheduling rules in terms of flow time, tardiness, and proportion of tardy jobs. The results obtained will serve as foundation for further research on dynamic scheduling of non-identical parallel processors.

1.1 Literature Review

Before past research work in this area is reviewed, it will be helpful to identify the relative position of scheduling parallel, non-identical processors problem among other scheduling problems. A scheme for classifying scheduling problems is shown in Figure 1 (Day and Hottenstein, 1970). The framework in Figure 1 classifies scheduling problems into three levels. Based on the nature of arrival of jobs, the first level is divided into two categories: static and dynamic. In the static case, all jobs are available to be scheduled at time zero. In contrast, dynamic problems refer to jobs continuously entering the system over the scheduling period.

The second level is characterized by the number of processors involved: single stage problems and multiple stage problems. Multistage problems can be further classified into three types based on the nature of the job

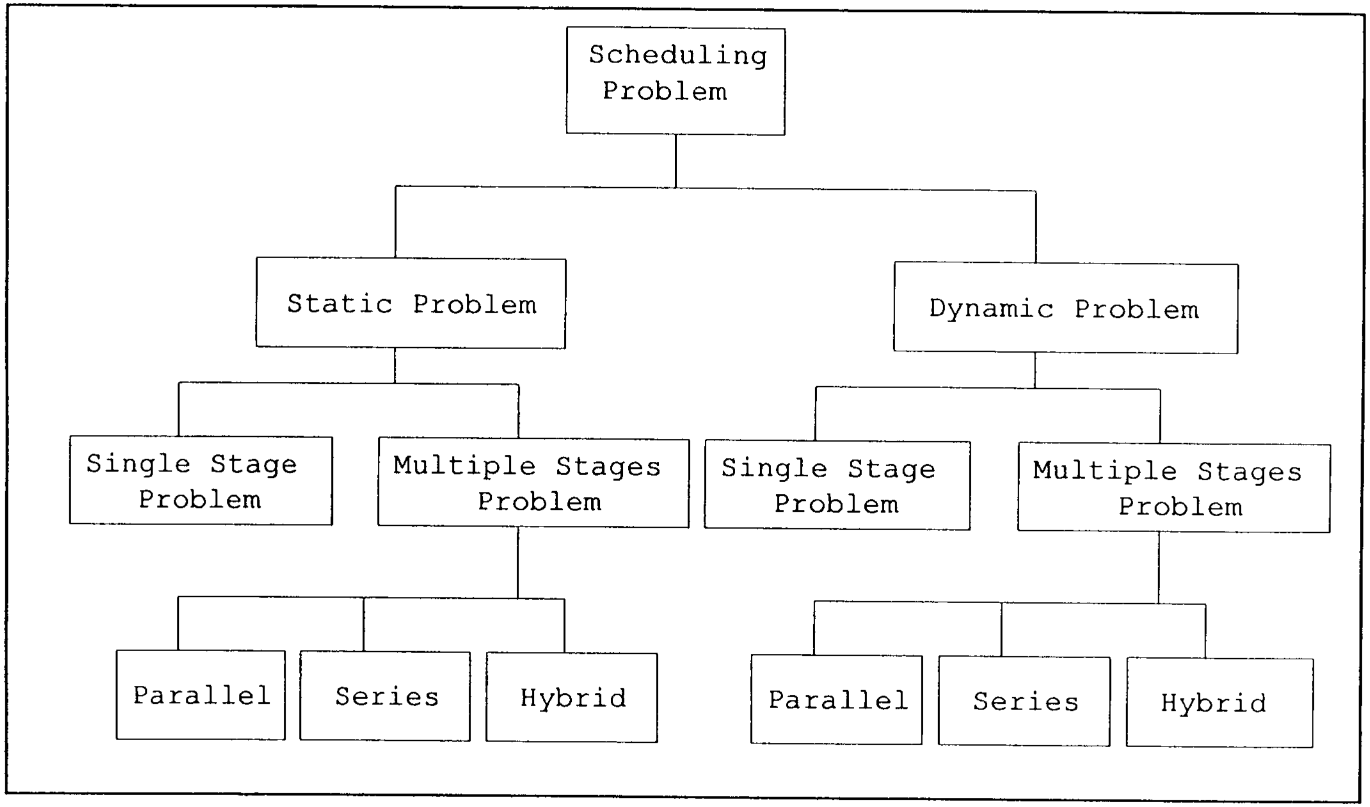


Figure 1: Scheduling problem classification (Day and Hottenstein, 1970)

route. These are parallel processors, processors in series, and a combination of parallel and series processors (or hybrid system). In the parallel processor system, there are more than one processor in the system and each job must be processed exactly once on one of the processors. In the series case, the system consists of several processors performing different operations and jobs are required to be processed on more than one machine in sequence. This is known as the flow shop problems. If there are several identical processors for processing one operation, then the system is called a hybrid system.

Scheduling problems with processors in series have drawn more attention from researchers than those with multiple processors in parallel (Day and Hottenstein, 1970). The system addressed in this research is related to the parallel case with static job arrivals. As mentioned in the previous section, since scheduling jobs on parallel, non-identical processors is very common in both manufacturing and service industries, it is quite surprising to find very little research reported in this area.

There have been three primary studies associated with parallel, non-identical processors. Marsh (1973) was primarily concerned with evaluating optimum solutions to scheduling parallel, non-identical processors to minimize total set-up time. The computation time needed to develop the optimal solution was also investigated. Four programming approaches were studied and only combinatorial programming and heuristic programming were found to be computationally feasible for some problems. However, the findings also showed that the computation time requirements made solving for an optimum solution prohibitive for all but the simplest systems. Several optimization techniques were evaluated, but the focus was on the branch-and-bound programming technique.

In Guinet's (1991) study of scheduling textile production systems, graph theory algorithms were adapted to model the parallel, non-identical processor scheduling problem. An attempt was made to minimize the mean flow time which would in turn minimize the mean tardiness by employing the linear programming approach. Guinet's investigation, like Marsh's, included sequence-dependent set-up times.

As pointed out by Smith (1993), both Marsh (1973) and Guinet (1991) studies showed that an optimum solution for all but the smallest systems was not practical in common, everyday scheduling situations. Furthermore, an understanding of how relationships between the parallel processors, the scheduling system, and product and job distributions affect system performance may lead to decision rules that can aid in developing more effective schedules. The following section provides the background of this study.

1.2 Background

In Smith's (1993) study, "An Experimental Investigation of Scheduling Non-identical Parallel Processors with Sequence-Dependent Set-up times and Due Dates", several factors affecting scheduling of non-identical parallel processors were investigated. The definition of experimental variables used in Smith (1993) is given in Table 1; the experimental settings are summarized in Table 2. The three performance measures used were Mean Flow Time, Proportion of Jobs Tardy, and Processor Utilization Spread. The system consisted of ten product types and three parallel, non-identical machines (processors). There were three main steps in Smith's research. The first step was to screen variables for significance using two statistically designed experiments (experiment one and two). Experiment one was a 24 run, folded Plackett-Burmann design to evaluate main effects only. Experiment two was a 32 run, sixty-fourth fractional factorial design to evaluate whether there were any significant interactions that should be planned for in subsequent experiments. After the first two experiments were run, the next step was to analyze the results obtained and select significant variables for detailed study. In the third step, detailed response surface experiments (experiment three and four) using these variables were

Table 1: Definition of Experimental Factors Used in Smith(1993)

Notation	**Definition**
Processor Spread	The range of capacities of the processor
Processor Distribution	The location of the middle processor of the processor spread described above
Loading	Percent of capacity scheduled
Setup Time	The amount of time needed to change over from one product line to another
Grouping	The situation that all tasks for each product are grouped together and run as one "super" job.
Ranking for Processor Assignment	The rule for ranking jobs for assignment to a processor.
Processor Assignment	Determines the processor to which a task (or product) is assigned to.
Processor Sequencing	Determine how jobs/groups (products) will be sequenced after they have been assigned to a processor.
Job Sequence	Is how tasks are sequenced within a product group.
Product Demand Distribution	The relative demand for individual products
Job Size Distribution	The distribution of jobs quantities
Set-up Considered	Including set-up times when assigning tasks to processors

Table 2: Experimental Factors and Settings Used in Smith (1993)

Effect	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Processor Spread	*	*		
Low (-)	20	20		
High (+)	80	80		
Processor Distribution	*	*		
Low (-)	50	50		
High (+)	75	75		
Loading	*	*	*	*
Low (-)	75	75	75	75
High (+)	90	90	90	90
Setup Time	*	*	*	*
Low (-)	U(0,3)	U(0,3)	0	0
High (+)	U(0,12)	U(0,12)	U(0,5)	U(0,5)
Grouping	*	*		
Low (-)	No	NO		
High (+)	Yes	Yes		
Ranking for Processor Assignment	*	*		Yes, not a variable
Low (-)	LPT	LPT		
High (+)	EDD	EDD		
Processor Assignment	*	*		
Low (-)	Slowest	Slowest		
High (+)	Fastest	Fastest		
Processor Sequencing	*	*		*
Low (-)	Chrono	Chrono		Chrono
High (+)	Optimi	Optimi		Optimi
Job Sequence	*	*		
Low (-)	SPT	SPT		
High (+)	EDD	EDD		
Product Demand Distribution	*	*		
Low (-)	Equal	Equal		
High (+)	Pareto	Pareto		
Job Size Distribution	*	*	*	*
Low (-)	N(1000,50)	N(1000,50)	N(1000,0)	N(1000,0)
High (+)	N(1000,300)	N(1000,300)	N(1000,300)	N(1000,300)
Set-up Considered			*	
Low (-)			No	
High (+)			Yes	

U - uniform distribution Chrono - chronological order
N - normal distribution Optimi - order based on set-up optimization

executed. Both experiments three and four were full 2^4 factorial designs and were identical with the exception that experiment four grouped all tasks by product while experiment three scheduled each job independently.

Based on the experimental investigation, Smith (1993) concluded that:

1.2.1 Mean Flow Time

1. Increased loading and set-up times will increase mean flow time in a static system. To minimize mean flow, tasks should be grouped by product whenever set-up times are required.
2. The method of ranking groups/tasks for assignment to the processors had no effect on the mean flow time. When more than one processor is available to process a job or product group, it does not matter which processor the job or group is assigned to.

1.2.2 Proportion Jobs Tardy

1. Loading significantly effects the proportion of jobs that are tardy, regardless of whether product grouping was used or not.
2. Set-up times were only significant when product grouping was not used.

3. Job size distribution and processor group sequencing were important when tasks were grouped by product.

1.2.3 Processor Utilization Spread

1. Grouping jobs by product will tend to increase the difference in processor utilization.

Based on the results summarized above, system loading and set-up times were identified as the most important factors affecting system performance. Grouping jobs by product will minimize set-up times and hence mean flow time and tardiness at the expense of controlling individual processor usage.

1.3 Research Objectives

The objectives of this research are two-fold:

1. To use the results from Smith (1993) to develop heuristics that focus on multiple objectives.
2. To test the heuristics developed in (1) for a variety of non-identical parallel processor scenarios.

1.4 Research Approach

The heuristics developed in this study were evaluated using simulation. Simulation was used because it allows complex systems to be modeled without being limited by the assumptions inherent in analytical models (Smith, 1993). The simulation model represents the essential features of the system. By loading the model with input data, the system output may be observed. However, to be effective, simulation results need to be carefully analyzed.

Several factors, including process times, job due dates and setup times were considered in developing scheduling heuristics. Since processor utilization spread is a major consideration in many industrial settings, this was also incorporated in developing the heuristics. The three primary performance measures used for evaluating the system are flow time, tardiness, and proportion of tardy jobs.

Through this research, an extensive, systematic analysis of a parallel, non-identical system is carried out. An understanding of the relationship between parallel processors, products and scheduling system lead to the decision rules that can provide feasible and effective production schedules. The schedules may not yield an optimal solution, but solving the problem analytically to obtain an optimal solution is difficult and economically infeasible. Developing feasible schedules using validated heuristics would be beneficial to industry; the results obtained from this study will also serve as a foundation for further work in scheduling of dynamic systems.

CHAPTER 2. EXPERIMENTAL METHODOLOGY

There are three main steps in this study. First, several factors which may affect system performances are identified and their significance is analyzed using the ANOVA method. Second, heuristic decision rules are developed. Third, simulation model was developed and implemented to evaluate the heuristics. The results from these experiments were then statistically analyzed. A detailed description of the methodology follows.

2.1 Terminology

The purpose of this section is to identify key terms used in this thesis and clarify their meaning.

1. Jobs: are individual, distinct, demands for a product or service. Thus, a job means the same as an order.
2. Products: are classifications of jobs. Each product may have one or more than one individual job but a job can only belong to one product type.
3. Tasks: are sets of jobs grouped by product types. Therefore, in the context of this study, task and product can be used interchangeably.
4. Processor: is any resource capable of processing the job. It is synonymous with machine in this study.

5. Quantity: is the size (in units) of a job.
6. Due date: is the deadline or promised delivery date of a job.
7. Set-up Time: is the amount of time needed to change over from one product line to another on a processor. The set-up times in this study include both processor dependent and product dependent setups. Processor dependent setups mean that the setup times only depend on the processors, regardless of the product type. Product dependent setups mean the setup times only depend on the production sequence (product-to-product).

2.2 System Definition and Characteristics

The system consists of several parallel, non-identical processors or machines. Processors may have same or different capacities. The system can produce a number of products but not all product types can be produced on every processor. Each product's machine requirement is determined randomly. Jobs for a product have varying quantities and due dates. Both processor dependent and product dependent set-up times are considered.

The complexity with most systems is due to the number of system variables and their interaction. The system of parallel processors is no exception. To make this study manageable , several assumptions were made:

1. All jobs are available at the start of the scheduling period. This situation is referred to as static situation.
2. All processors are available at time zero.
3. A job can be only scheduled on one processor at a time.
4. Job splitting is not allowed. For example, assume that a job requires processing of 100 units. Once that job is assigned to a processor, all 100 units will be processed on the processor before the next job is scheduled.
5. Product preemption is not allowed. Each product, once started, must be performed to completion.
6. The machines are continuously available without breakdown.
7. Though both processor dependent setups and product dependent setups are considered, the setup time between jobs of same product group is considered negligible.

2.3 Performance Measures

There three basic performance measures considered in this study are flow time, tardiness, and proportion of tardy jobs. Both average values and spread of these variables were examined.

Flow Time is defined as the time a job spends in the system from the time it is available (or ready) to be processed until it is completed. In this study, the flow time for each job is equal to its completion time because all jobs are available for scheduling at time zero (i.e., start of the scheduling period). Smaller flow time is desirable since it indicates that a job flows through the system faster. It also means responding to customers quickly and reducing work-in-process inventories.

Tardiness is defined as the positive difference between completion time of a job and its due date. The Proportion of Tardy Jobs measures the percentage of jobs which are completed after their due dates. Obviously, smaller value of tardiness and tardy jobs are preferred.

Mathematically, the performance measures are defined as follows. Let

F_i = flow time for job i

T_i = tardiness of job i

P_T = proportion of tardy jobs

C_i = completion time of job i

r_i = ready time of job i

d_i = due date of job i

Then,

$$F_i = C_i - r_i$$

$$T_i = \text{Max}\ (0,\ C_i - d_i)$$

$$P_T = \text{Number of tardy jobs / Total jobs}$$

There are two other measures that are important in evaluating production systems. These are processors' utilization and processor utilization spread. Processor or machine utilization depends on the system loading design. This is treated as an independent variable in this study. Processor utilization spread measures the difference in utilization among processors. The objective of many organizations is to minimize this spread. Process utilization spread is explicitly included in developing the scheduling heuristics.

Trying to "optimize" performance measures simultaneously is generally not feasible as some of the measures conflict with others. For example, high processor utilization can only be achieved at the expense of high flow times and more jobs waiting in the system. The aim of scheduling heuristics developed in this research is to provide a balance between some or all of these measures.

2.4 Experiment Variables

2.4.1 Factors

There are three groups of factors defined in this experiment: Product-related, Processor-related, and Others.

- Product related
    1. Number of products: investigates the effect of the number of products on the system performance. The settings used in this experiment are 5, 15, and 25.
    2. Job size distribution: investigates the effect of different job size distributions on system performance. The two distributions used in this study are uniform and normal distributions. The mean of both distributions is 1000 units. The range for the uniform distribution is 800 to 1200 units; the standard deviations for the normal distribution being 300.

- Processor related
    3. Number of processors: investigates how the number of machines affect the system performance. For this study, three levels for the number of machines are considered; these are 3, 5, and 10.
    4. Processor capacities: this variable will identify if the difference in capacity distributions between processors affects the system performance. A uniform distribution was used to model this variable. The range for the low setting is between 80 and 120 units per hour while that for the high setting is between 50 and 150 units per hour.

- <u>Others</u>

5. <u>Loading:</u> is the percent capacity of system scheduled for usage. A 75% loading level is considered as the low setting and 90% as the high setting. The low level is based on the fact that organizations generally consider utilization less than 75% to be unacceptable; utilization higher than 90% would likely result in most jobs being tardy. This factor represents a combination of product and processor characteristics.

6. <u>Set-up Times:</u> is the amount of time needed to change over from one product to another. The set-up times in this study include both processor dependent and product dependent setups. There are three levels of setups considered: 10% (low), 20% (middle), and 30% (high) of total capacity, where the total capacity is the total available machine time in the scheduling period. As an example, in a 480 hours scheduling period and three processors, the high setup time will be approximately 30% of the total capacity [3*480*(total capacity of processors)]. All set-up times were modeled using the uniform distribution.

The experimental variables and level settings are summarized in Table 3.

Table 3: Experimental Factors and Settings

	Factors	Levels	Factor	Values	
	PROCESSOR-related				
1	No. of processors (NMACH)	3	3	5	10
2	Processor Capacity (CAPTY)	2	U(80,120)	U(50,150)	
	PRODUCT-related				
3	No. of products (NPROD)	3	5	15	25
4	Job size distribution (JOB_SIZE)	2	U(800,1200)	N(1000,300)	
	OTHERS				
5	Loading (LOAD)	2	75%	90%	
6	Set up times (hours) (SETUP)	3	Low (10%)	Mid (20%)	High (30%)

U --- Uniform distribution
N --- Normal distribution

2.4.2 Analysis

The Analysis of Variances (ANOVA) technique was used to analyze the results. The Sum of Squares are used to measure deviations from the predicted values obtained using the estimated effects. A 95% confidence level was used for evaluation. Therefore, the P-value of 0.05 or less indicates significant effects. The P-value is the probability of observing data against the null hypothesis (H_0) under the assumption that the hypothesis is correct. In ANOVA, H_0 is defined as the absence of any effects. Since the confidence level was 95%, a P-value of less than 1 minus the confidence level (in this case 1 - 0.95 = 0.05) indicates significant effects.

Statgraphics 5.0 (Statgraphics, 1991), a commercial software package was used to perform the ANOVA analysis. Since there are six variables under study, if all interactions up to sixth order were considered, there will be 63 combinations ($C_1^6 + C_2^6 + C_3^6 + C_4^6 + C_5^6 + C_6^6$). All interactions greater than second order are ignored. There are two reasons for this. First, Statgraphics cannot perform all the interactions at the same time. Second, including all possible interactions requires the use of all 2^n degrees of freedom which eliminates the possibility of correcting for experimental error. Thus, all interactions which are higher than second order are assumed negligible so that their Sum of Squares could be used to estimate the error. Another problem with higher interaction is difficultly in interpreting their meaning.

The first step used to perform the ANOVA was to examine all variables individually without including any interactions. After identifying the significant variables, all interactions between these variables were considered. Use of this methodology compared with examining all possible terms (including interactions) simultaneously simplifies the model and the results. The simpler model is easier to control and interpret. Also, if a certain factor is identified as insignificant, it is meaningless to consider the interaction of this factor with others. To summarize the statistical analysis procedure:

1. Calculate ANOVA table without including any interactions.
2. Eliminate the "most" non-significant factors (terms).
3. Repeat steps 1 and 2 until all terms are significant.
4. Recalculate ANOVA table that considers all second order interactions and main effect factors.

2.5 Scheduling Heuristics

Smith's (1993) results showed that grouping jobs by product type minimized set-up times and hence reduced the mean flow time and tardiness but at the expense of controlling individual processor usage. In order to control the individual processor usage, the Processor Utilization Spread (PUS) is used in developing the scheduling heuristics. The PUS is defined as the difference between the heaviest and least loaded processors and measures how evenly jobs are distributed among the processors. For the study, a processor utilization spread of 10 percent or less of the maximum loading was considered to be "even" loading. If the PUS was greater than 10 percent, the situation was defined as "uneven" and the resulting schedule was not feasible. The schedule would have to be revised to satisfy the 10 percent criterion.

In developing the heuristics, the idea of a two-phase approach (Baker, 1974) was used. The problem of scheduling multiple parallel processors contains both allocation and sequencing dimensions. Allocation means allocating or assigning jobs on processors and sequencing is simply the order in which the jobs are processed through the processors. A sound heuristic procedure should address both the allocation problem and the sequencing problem. Thus, the first step is to allocate (or assign) jobs on processors and the second step is to determine the optimal sequence on each processor separately. Using this two-phase method to schedule jobs on processors may not produce an optimal schedule, but it will tend to provide a very good schedule (Baker, 1974).

The basic heuristic developed in this research consists of four components (Figure 2):

A. Group jobs by product type. These grouped jobs are called TASKs.

B. Assign TASKs to Processors.

C. Evaluate Processor Utilization Spread (PUS).

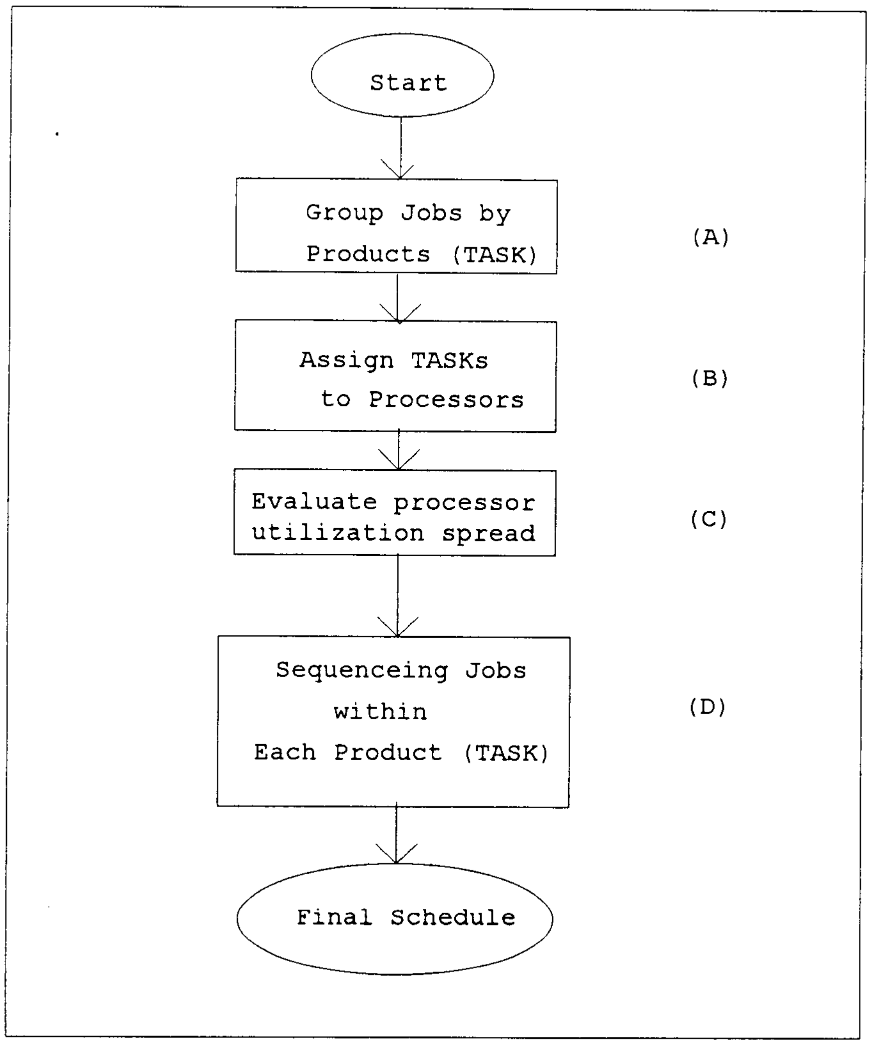


Figure 2: Flowchart of the Basic Heuristic

D. Within each product (TASK), sequence individual jobs using some sequencing rule.

A. <u>Group jobs by product</u>

As mentioned earlier, a job is associated with a product. The first step in the heuristic is to group jobs by products. These grouped jobs are referred to as tasks. Processing similar jobs together would tend to reduce setups between products. A negative result may be violation of due dates or excessive tardiness of some jobs; this concern is addressed by the sequencing component of the heuristic.

B. <u>Assign tasks to processors</u>

The flowchart of phase B is showed in Figure 3.

B.1. Assign products that could only be run on one processor to that processor.

B.2. Identify products that could be run on multiple processors, but not all processors. Order these products by decreasing number of machines that the product can be processed on.

B.3. Identify the processor with the minimum loading.

B.4. Schedule the "minimum-machine" products

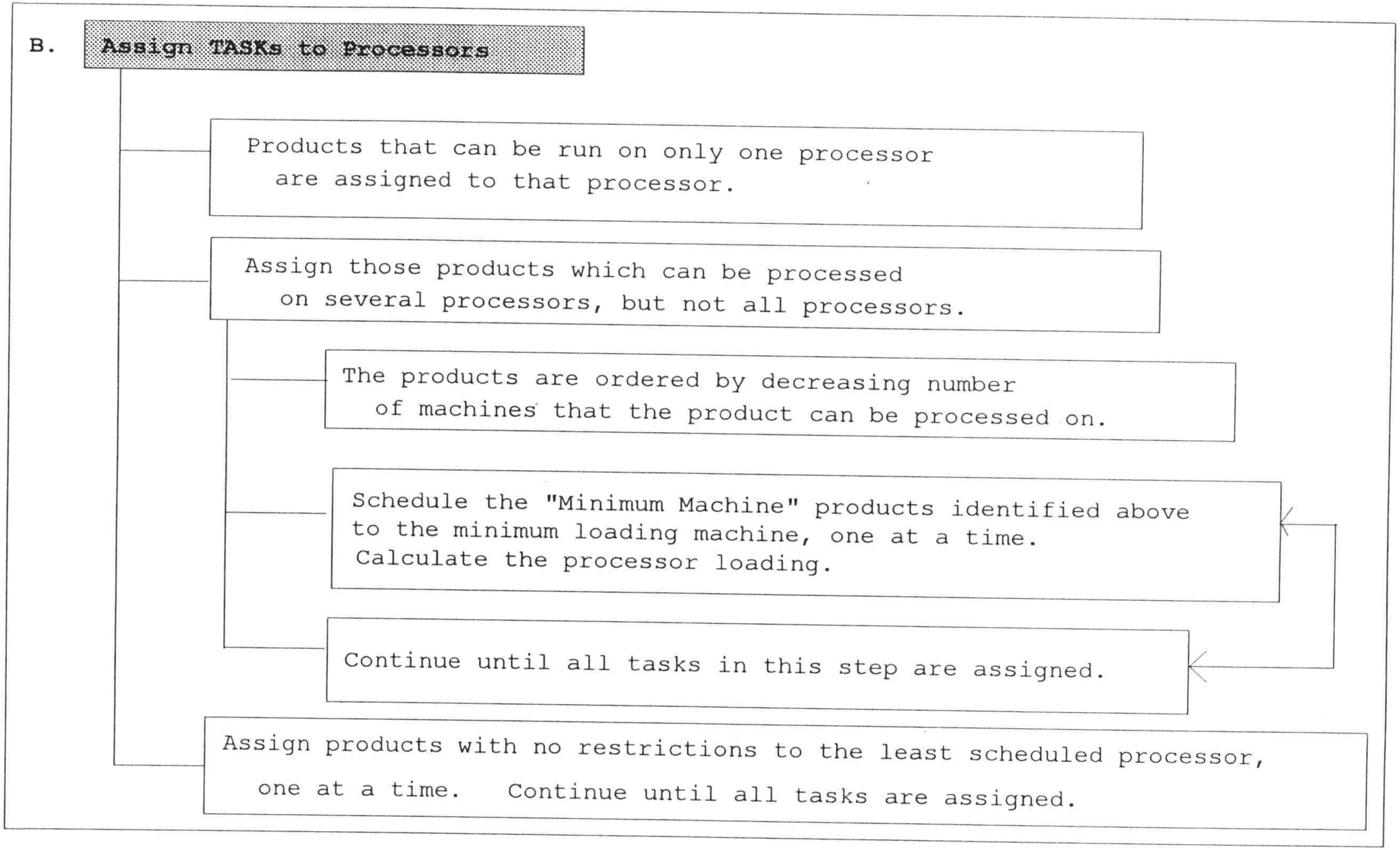


Figure 3: Flowchart of the Basic Heuristic - Phase B

identified in step B.2 to the minimum loading processor, one at a time. Calculate the processor loads.

B.5. Repeat steps B.3 and B.4 until all products identified in B.2 are assigned. If there are still some unassigned products with any processor restrictions, assign them to the processor with the minimum loading.

B.6. Assign products with no restrictions (i.e., products that can be processed on any processor) to the least scheduled processor, one at a time.

B.7. After assigning a product in B.6, calculate the processor loads and identify the minimum loading processor.

B.8. Repeat steps B.6 and B.7 until all products are scheduled.

C. <u>Evaluate Processor Utilization Spread (PUS)</u>

At this stage, all tasks have been assigned to processors. The processor utilization spread criterion (CPUS) is defined as 10 percent of the maximum processor load. The phase C is summarized in Table 9.

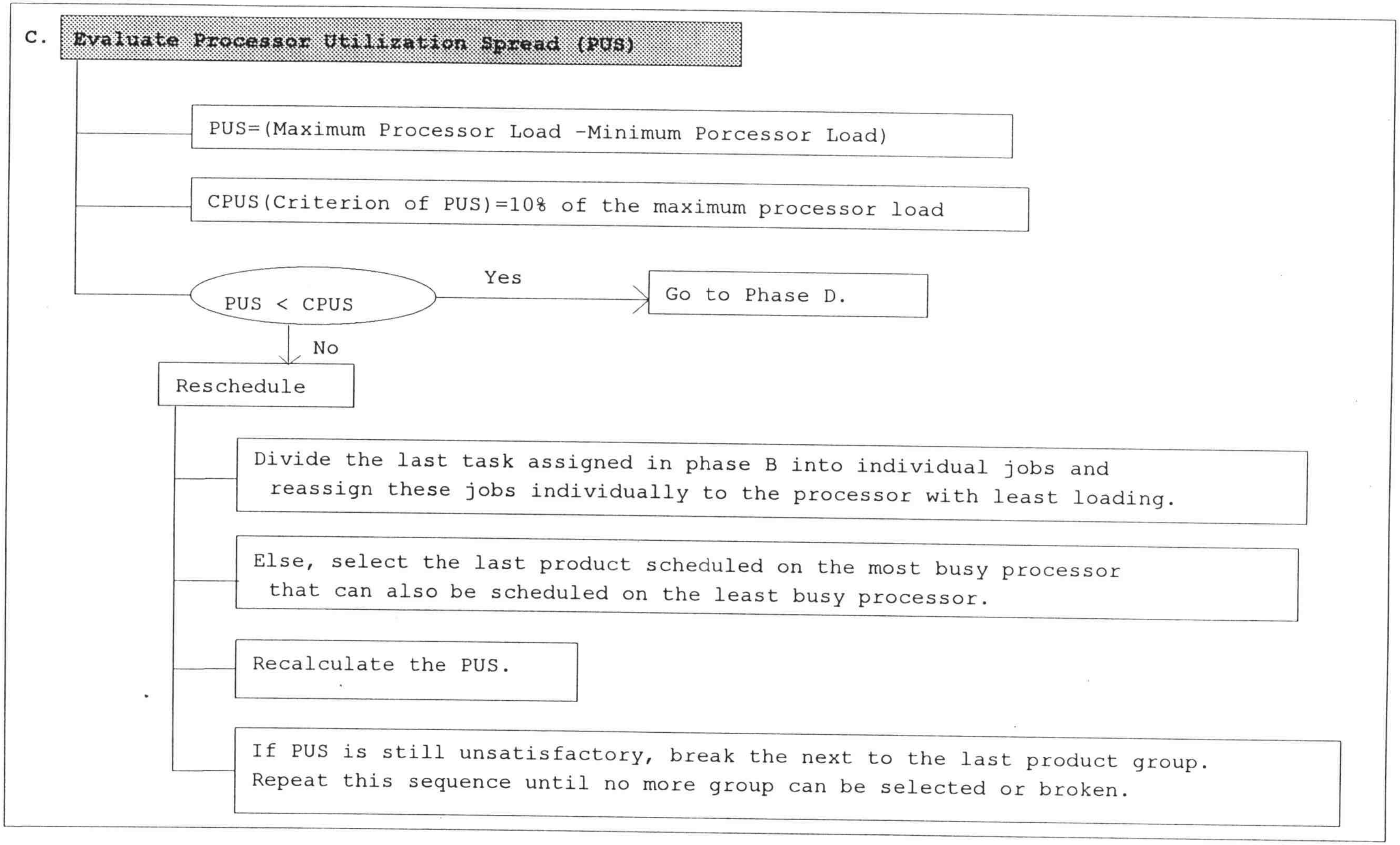


Figure 4: Flowchart of the Basic Heuristic - Phase C

C.1. Calculate the Processor Utilization Spread (PUS), defined as

PUS = (Maximum processor load)
  - (Minimum processor load)

C.2. If PUS is less than CPUS, then go to phase D of the heuristic; otherwise, go to step C.3.

C.3. Divide the last task assigned in phase B into individual jobs and reassign these jobs individually to the processor with the least loading. Else, select the last product scheduled on the most busy processor that can also be scheduled on the least busy processor and recalculate the PUS. If PUS is still unsatisfactory, break up the next to the last product group. Repeat this sequence until no more group can be selected or broken.

D. <u>Sequence jobs on processors</u>

Within a product group, individual jobs are processed in the order generated. Since jobs are generated randomly, this rule represents a random processing order. Alternatives to this are examined in the modification of this basic heuristic.

Some of the performance measures may be improved by considering criterion other than random sequencing order in phase D of the heuristic. There are a number of priority rules that have been developed in scheduling. The shortest Process Time (SPT) rule is one of the most common rules used in production settings since it gives a better solution than other rules in most cases (Bedworth and Bailey, 1987; Conway, 1967; and Day and Hottenstein, 1970). Also, scheduling jobs by Earliest Due Date (EDD) rule is shown to minimize the maximum tardiness in single-stage scheduling problems (Baker, 1974 and Jackson, 1955). A second measure of urgency for a given job is the time until its due date minus the time required to process it, referred to as job's slack time. In particular, among jobs with identical due dates, the shortest slack is the most urgent. Therefore, three extensions of the basic heuristic were developed and evaluated. These differ from the basic heuristic in phase D where job sequence on processors is determined.

- Heuristic Rule 2 (SPT Case)

In phase D, individual jobs within a product group are sequenced by the Shortest Process Time (SPT) rule.

- Heuristic Rule 3 (EDD Case)

In phase D, individual jobs within a product group are sequenced by the Earliest Due Date (EDD) rule.

- Heuristic Rule 4 (SS Case)

In phase D, individual jobs within a product group are sequenced by the Smallest Slack (SS) rule, where slack time is defined as the difference between due date and process time.

2.6 Simulation Model

A simulation model was developed for the parallel, non-identical processor system. There are three main steps in this model. The first step consists of data generation. This includes: number of jobs needed to achieve the desired loading level, job quantities, job product type, machine requirements, and job due dates. The second step was modeling the scheduling heuristics. The last step was calculating necessary statistics and generating the final report. A detail description of the model follows.

2.6.1 Data Generation

Simulation of parallel processor systems requires a large amount of data. This includes number of processors, number of products, machine requirement for each product, and number of jobs needed to reach the specified loading level. The more important of these are discussed below and summarized in Figure 5.

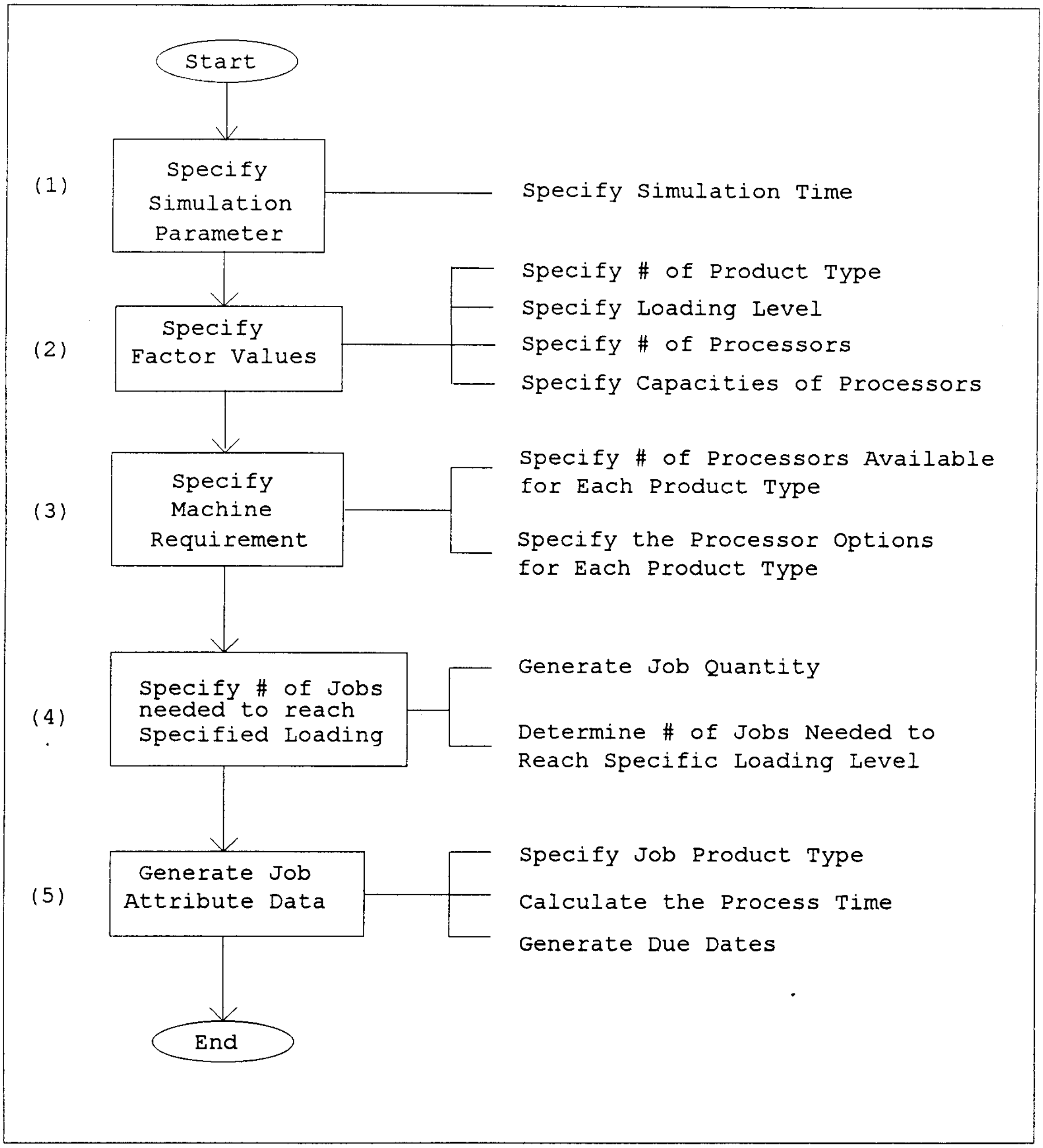


Figure 5: Data Generation

1. Processor related parameters that need to be specified include:
   - Number of processors.
   - Processor capacities using the distributions specified in Table 3. The low setting for the range of machine capacity is 80 to 120; the high setting 50 to 150. Individual processor capacities were determined from the range using a uniform distribution.

2. Product related parameters include:
   - Product type. A number between 1 and number of products is generated randomly and assigned as the product type.
   - Processor requirement for each product type. Each product may be processed on one or more processors. A set of random numbers was used to make this assignment for each product type. The first random number indicates the number of processors available for the product while a second set of random numbers identifies the specific processors. To illustrate, assume that the first random number generated for a product is 2. This means that this product can be processed on two processors. Now, two new random numbers are generated. Let these be 1 and 4, implying that the product can be processed on processors 1 and 4.

- Process time for each job. Since a job may be scheduled on more than one processor, there are several possible process times for each job. The process time used in scheduling depends on the processor to which the job is assigned. A processing time for a job on processor i, Pi, is given by

$$Pi = \text{Quantity} / (\text{Capacity of processor } i)$$

For example, consider a job with a quantity of 1000 units, which can be processed on either processor 1 or processor 4. Processor 1 has product capacity of 100 units per hour while that of processor 4 is 120 units per hour. Then the process time on processor 1 is $P_1 = 1000 / 100 = 10$ hours and the process time on processor 4 is $P_4 = 1000 / 120 = 8.33$ hours.

- Jobs due dates.

Due date assignment for job i (d_i) is specified as a product of two parameters: $d_i = F * U(1,2)$, where the parameter F is a sample from the uniform distribution between the range {maximum processing time, scheduling time span} and U(1,2) is a random uniform variable between 1 and 2.

3. To determine the number of jobs needed to reach a specific loading level, a loading level and job quantities required to meet this level need to be specified. Jobs are accepted for scheduling until the sum of the job quantities is equal to the desired loading level.

2.6.2 Heuristics Modeling

This component is based on the heuristic described in the previous section.

2.6.3 Statistics Generation

Once the final schedule is obtained, the next step is to calculate statistics associated with the system performance measures. The average and variation for each measure were computed and reported.

2.7 Implementation

The simulation model was implemented in FORTRAN 77. The simulation was executed using an 80486-DX computer operating at 33 MHz. Based on the factor settings in Table 3, there were 216 treatments per run. The time needed to run the simulation varied, depending on the job size generated. Generally, for the 3 machine case and an average of 130 jobs for 90 percent loading, it took approximately 3 minutes to obtain a schedule. For the 5 machine case with 220 jobs for 90 percent loading, it took approximately 5 minutes. For 10 machine case, the execution time increased to around 15 minutes to finish 430 jobs. Two runs per simulation were executed. The average of these two runs was used in the statistical analysis. After completing the simulation, the spreadsheet, Quattro Pro 1.0, was used to calculate the average and standard deviation. The data obtained was transferred into Statgraphics 5.0 for generation of necessary statistics.

CHAPTER 3. RESULTS

The simulation model described in the previous chapter was executed to study the performance of the non-identical parallel processors system in terms of job flow times, tardiness, and proportion of tardy jobs. The datasheets showing the input conditions for each run and the performance measures (simulation output) are given in Appendix. The simulation results are summarized in terms of mean value, standard deviation and maximum value which will be discussed later. The standard deviation measures the spread among the observations and the maximum shows the largest value within the groups. Two statistical methods, ANOVA and Pairwise t-test, are used for the analysis. The purpose of using ANOVA is to identify the significant factors in terms of performance measures. The Pairwise t-test was used to test the hypothesis whether the results between two specific cases are significant or not. For the purpose of this research, all results are based on 95% confidence level. A summary of results and discussion follows.

3.1 Results for the Basic Heuristic

In the basic heuristic, within a product group, individual jobs are processed in the order generated. The

ANOVA results for the three performance measures are summarized in Table 4. In ANOVA, the effects of each response variable for all factors are analyzed. All interactions greater than the second order are ignored. Variables and interactions are identified as being significant based on a 95% confidence level with p-value equal to or less than 0.05.

Table 4: Summary of ANOVA Results for the Basic Heuristic

	Basic Heuristic (ORIG)		
FACTORS	**M_FLOW**	**M_TARD**	**PROP_T**
A:NMACH	*	*	
B:CAPTY			
C:NPROD	*	*	*
D:LOAD	*	*	*
E:JOB_SIZE			
F:SETUP	*	*	*
INTERACTIONS			
AC	*	*	
AD	*	*	
AF	*	*	
CD	*	*	

M_FLOW --- Mean Flow Time
M_TARD --- Mean Tardiness
PROP_T --- Proportion of Tardy Jobs
* --- Significant effect

3.1.1 Significant Factors

The three variables, number of products (C:NPROD), loading level (D:LOAD), and set-up time (SETUP), were identified as being significant for all three performance measures.

The primary reason for the number of products being significant is grouping of products. Recall that the first step in the heuristic is grouping of jobs by product. The grouped jobs (TASKs) are then scheduled. As the number of products increases, more TASKs will be scheduled in the first stage. In addition, more products will increase the proportion of setup time because setup time considers both processor and product dependent changeover times. Thus, changing the number of products changes the completion time of the job. Once the completion time has been shifted, all three performance measures, flow time, tardiness, and proportion of tardy jobs, are affected.

The loading level was significant for all performance measures, too. This result is consistent with expectations. As the loading increases, the number of jobs needed to reach the loading level increases, thus the time to complete all jobs increases. Increased loading also translates into more jobs in the system, with subsequent increase in tardiness and proportion of jobs tardy.

The last variable that was identified as being significant for all measures is the set-up time. Increasing the set-up time increases the job completion times and the number of jobs waiting to be completed.

Besides the three factors mentioned above, number of machines (A:NMACH) was significant for flow time and tardiness. As the number of machines increases, the number of jobs needed to reach a specific loading level increases. Since the completion time equals the flow time (ready time for all jobs is zero), the mean flow time will increase and thus the tardiness will be affected. However, it is surprising that this variable was not identified as being significant on proportion of tardy jobs. This may be due to other variables having such a dominant influence that the number of machines effect cannot be detected.

3.1.2 Non-Significant Factors

The other two variables, machine capacity (B:CAPTY), and job size distribution (E:JOB_SIZE) were not significant for any performance measure. For the processor related variable, Capacity (B:CAPTY), the mean of both low and high settings is 100 units per hour (refer to Table 3). Thus, the difference tested for this variable is for the spread of capacity between processors. As processor capacity spread increases, more of the workload is shifted to the faster processors. Therefore, the schedule may not be influenced by the capacity spread distribution. Factors such as set-up time and loading level have a much more significant impact that effectively masks any influence by the processor capacity variable.

The Job-Size variable (E:JOB_SIZE) was also insignificant on all performance measures. This is expected since the mean values of both low and high settings are 1000 units (Table 3). Consequently, this variable was identified as insignificant and the interaction associated with this variable can be ignored.

The simulation results, summarized in terms of the experimental variables excluding the non-significant variables (B:CAPTY and E:JOB_SIZE) for all performance measures, are given in Tables 5 and 6.

3.1.3 Two-Factor Interactions

For the significant variables (A:NMACH, C:NPROD, D:LOAD, and F:SETUP), all the two-factor interactions were examined. The results showed that the four interactions, AC, AD, AF, and CD, were significant for flow time and tardiness (refer to Table 4); and there was no significant interaction for proportion of tardy jobs. The effects of these interactions are discussed below.

For flow time, the two-way interactions of A (number of machines) are significant with the other three significant factors (C, D, and F). First, consider the two-factor

Table 5: Summary of Simulation Results for the Basic Heuristic - Flow Time

NMACH	NPROD	LOAD	SETUP	MEAN	STD-DEV	MAX
3	5	0.75	LOW	194.32	3.10	198.67
			MID	210.34	3.84	216.41
			HIGH	220.68	4.60	228.21
		0.90	LOW	234.82	4.55	242.34
			MID	255.22	7.01	267.18
			HIGH	267.04	8.38	281.19
	15	0.75	LOW	198.64	0.59	199.55
			MID	232.82	4.83	240.52
			HIGH	250.91	2.46	254.31
		0.90	LOW	233.29	0.50	233.86
			MID	263.88	1.53	265.80
			HIGH	283.40	2.30	286.33
	25	0.75	LOW	209.19	2.36	213.26
			MID	261.28	5.85	270.71
			HIGH	296.53	10.86	314.70
		0.90	LOW	243.62	1.60	245.97
			MID	293.20	2.64	296.56
			HIGH	326.20	4.37	331.31
5	5	0.75	LOW	190.24	0.70	190.93
			MID	202.91	2.72	205.70
			HIGH	207.65	2.31	210.78
		0.90	LOW	227.59	2.82	231.24
			MID	242.22	4.77	246.09
			HIGH	247.92	5.48	253.88
	15	0.75	LOW	197.84	2.55	202.20
			MID	213.15	8.96	228.38
			HIGH	224.36	13.38	247.53
		0.90	LOW	228.59	2.57	232.05
			MID	244.29	1.88	246.47
			HIGH	254.86	2.13	257.85
	25	0.75	LOW	195.59	0.64	196.25
			MID	219.13	0.17	219.31
			HIGH	233.72	0.63	234.76
		0.90	LOW	232.35	1.50	234.88
			MID	257.40	2.92	261.76
			HIGH	275.38	6.70	284.83
10	5	0.75	LOW	253.89	9.61	262.09
			MID	297.17	9.40	308.70
			HIGH	330.41	6.78	337.20
		0.90	LOW	314.42	12.37	332.07
			MID	363.77	14.78	378.68
			HIGH	392.34	16.45	405.95
	15	0.75	LOW	193.98	4.98	202.07
			MID	212.18	6.39	221.86
			HIGH	216.76	6.35	227.06
		0.90	LOW	234.76	8.02	244.81
			MID	260.62	18.41	286.85
			HIGH	275.76	20.81	299.45
	25	0.75	LOW	194.04	5.01	202.61
			MID	210.69	6.64	222.09
			HIGH	223.63	10.46	241.75
		0.90	LOW	230.87	3.28	234.93
			MID	243.57	0.93	245.05
			HIGH	253.53	1.12	255.02

Table 6: Summary of Simulation Results for the Basic Heuristic - Tardiness and Proportion of Tardy Jobs

NMACH	NPROD	LOAD	SETUP	MEAN-TARD	STD-DEV	MAX	MEAN-PROP	STD-DEV
3	5	0.75	LOW	36.24	3.62	42.48	0.3077	0.0135
			MID	42.60	4.06	49.45	0.3335	0.0133
			HIGH	47.11	4.27	54.03	0.3552	0.0147
		0.90	LOW	55.30	7.97	62.05	0.3736	0.0169
			MID	67.97	2.66	71.97	0.4119	0.0109
			HIGH	74.36	3.43	78.90	0.4285	0.0138
	15	0.75	LOW	38.63	0.39	39.07	0.3040	0.0122
			MID	51.34	0.81	52.71	0.3514	0.0129
			HIGH	60.24	0.84	61.48	0.3657	0.0082
		0.90	LOW	57.04	1.95	59.72	0.3545	0.0093
			MID	71.37	2.07	74.22	0.3987	0.0111
			HIGH	81.06	3.09	85.27	0.4288	0.0129
	25	0.75	LOW	41.46	2.59	44.94	0.3036	0.0111
			MID	64.85	2.57	68.34	0.3764	0.0166
			HIGH	83.14	7.71	96.25	0.4265	0.0209
		0.90	LOW	60.27	4.67	67.75	0.3657	0.0288
			MID	83.81	6.51	94.88	0.4500	0.0266
			HIGH	103.30	6.42	114.13	0.5086	0.0237
5	5	0.75	LOW	42.42	2.67	45.43	0.3296	0.0098
			MID	46.90	2.00	49.08	0.3476	0.0033
			HIGH	49.89	2.72	52.48	0.3520	0.0111
		0.90	LOW	61.95	3.26	66.73	0.4082	0.0132
			MID	69.46	4.90	74.86	0.4286	0.0157
			HIGH	72.40	5.37	79.13	0.4323	0.0185
	15	0.75	LOW	44.46	2.71	47.68	0.3343	0.0248
			MID	52.29	4.59	56.12	0.3555	0.0140
			HIGH	57.27	5.23	64.53	0.3748	0.0117
		0.90	LOW	57.82	1.52	59.62	0.3783	0.0167
			MID	65.75	1.66	68.07	0.4036	0.0150
			HIGH	71.22	2.22	74.33	0.4266	0.0166
	25	0.75	LOW	44.27	2.38	47.09	0.3247	0.0116
			MID	54.58	3.29	57.71	0.3498	0.0093
			HIGH	61.79	3.13	65.17	0.3673	0.0133
		0.90	LOW	60.57	2.93	65.44	0.3962	0.0245
			MID	73.61	3.49	79.51	0.4352	0.0217
			HIGH	82.70	3.71	87.90	0.4559	0.0117
10	5	0.75	LOW	70.89	7.46	77.23	0.3753	0.0130
			MID	89.82	6.90	97.36	0.4373	0.0108
			HIGH	103.42	7.32	112.34	0.4765	0.0145
		0.90	LOW	112.35	11.68	128.94	0.4454	0.0132
			MID	136.84	13.35	152.53	0.5093	0.0099
			HIGH	153.70	14.40	168.34	0.5418	0.0116
	15	0.75	LOW	37.39	3.10	40.24	0.3288	0.0781
			MID	45.01	4.08	49.46	0.3191	0.0261
			HIGH	47.14	4.80	53.00	0.3237	0.0252
		0.90	LOW	56.18	5.33	62.28	0.3453	0.0177
			MID	68.59	9.15	81.58	0.3961	0.0281
			HIGH	75.10	11.05	87.81	0.4134	0.0296
	25	0.75	LOW	40.77	2.53	44.49	0.3148	0.0069
			MID	46.38	3.31	52.07	0.3325	0.0119
			HIGH	51.98	4.32	58.97	0.3439	0.0159
		0.90	LOW	54.64	1.11	56.23	0.3516	0.0052
			MID	61.30	1.92	63.21	0.3704	0.0075
			HIGH	65.82	2.03	67.91	0.3798	0.0088

interaction between number of machines (A:NMACH) and number of products (C:NPROD). With 3 and 5 machines, as the number of products increases, the mean flow time increases. This is reasonable since more products mean more setup time is needed during processing. Thus, the flow time will increase. However, this relationship does not hold for 10 machines. For 10 machines, the mean flow time decreased as the number of products increased from 5 to 15 and from 15 to 25.

With fewer machines, products need to wait longer to be scheduled, given a constant loading level, resulting in longer completion times. As the number of machines increases in relation to the number of products, more machines are available for processing; thus jobs should flow through the system faster. However, recall that more machines also mean more jobs to reach the same loading level. This results in higher setups. Therefore, with too many machines and too few products, there are a number of factors that affect the system. Jobs are grouped by product, then scheduled. Groups are split in last phase of the heuristic to achieve a balance in processors utilization. However, groups can only be split upto a certain level. Thus too few products compared to the number of machines results in an imbalance in processor utilization as some processor may not be utilized at all. Consequently, the flow time also increases as there are more jobs to be processed in the system and not all processors are utilized. There is a relationship between the number of machines and the number of products that affects job flow time, but this relationship is also dependent on other system variables.

In order to provide more insight into the simulation results, several simulation experiments with seven machines were executed. The results are summarized in Table 7.

Table 7: Summary of Simulation Experiment with Seven Machines

NMACH	NPROD	LOAD	SETUP	M_FLOW	M_TARD	PROP_T
7	5	0.75	LOW	230.40	56.43	0.3796
			MID	276.98	79.11	0.4513
			HIGH	294.10	89.47	0.4980
		0.9	LOW	269.83	71.95	0.4069
			MID	326.44	102.2	0.4637
			HIGH	345.75	115.24	0.4890
	15	0.75	LOW	192.09	43.99	0.3295
			MID	207.77	50.46	0.3333
			HIGH	218.01	54.68	0.3447
		0.9	LOW	228.40	56.37	0.3454
			MID	243.39	62.96	0.3651
			HIGH	252.15	66.49	0.3816
	25	0.75	LOW	196.20	47.32	0.3266
			MID	219.87	58.30	0.3629
			HIGH	233.29	63.48	0.3710
		0.9	LOW	230.71	55.48	0.3443
			MID	253.15	65.05	0.3841
			HIGH	268.35	73.12	0.4007

The results are consistent with the earlier discussions. An increase in the number of products from 15 to 25 for constant loading and setup results in an increase in flow time. The case of NMACH=7, NPROD=5, represents an exception similar to NMACH=10, NPROD=5, in Table 5, and as discussed earlier.

The ANOVA results showed that the F-ratio for the interaction between the number of machines and number of products to be 274.48 was much higher than the other interactions (see Table 8), showing the complex interaction between these two variables.

Table 8: ANOVA Results for Mean Flow Time for Basic Heuristic

Source of variation	Sum of Squares	d.f.	Mean square	F-ratio
MAIN EFFECTS				
A:NMACH	41826.674	2	20913.337	151.449
C:NPROD	21017.875	2	10508.937	76.103
D:LOAD	87824.625	1	87824.625	636.005
F:SETUP	67582.701	2	33791.350	244.709
INTERACTIONS				
AC	151608.56	4	37902.140	274.478
AD	1822.46	2	911.228	6.599
AF	4366.17	4	1091.542	7.905
CD	1940.94	2	970.470	7.028
RESIDUAL	27065.242	196	138.08797	
TOTAL (CORRECTED)	405055.24	215		

The interaction between number of machines and system loading for flow time was also significant. As loading increases from 75% to 90%, the mean flow time increases, independent of number of machines. As mentioned earlier, increased machine utilization can only be achieved at the expense of higher flow time, when all other factors are held constant, as is clear from Table 5.

The relationship between the number of machines and set-up times was also significant. Set-up time has a significant effect on flow time; the flow time increases when set-up time increases. Also, more machines generally means higher flow time, regardless of the setup time level. However, the result showed that the flow time with 3 machines was higher than with 5 machines. This may be explained by the fact that jobs flow through faster in 5 machine system (less waiting time due to more available machines), as compared to the 3 machine case, thus resulting in a more smooth flow that reduces the job completion time.

The same relationship exists in the interaction between the number of products (C:NPROD) and the system loading (D:LOAD). When the number of machines equals 3 or 5, increasing the number of products increases the flow time independent of the system loading. The same is true for combination of 10 machines with 15 or 25 products. The only exception is 10 machines, 5 products case. The jobs mean flow time is abnormally high. The reason again is that there are too few products in the system compared with the number of machines which causes an imbalance in processor utilization.

The four interactions which were significant for flow times were also found to be significant for tardiness (refer to Table 4). Consider the interaction between the number of machines and the number of products. When the number of machines is fixed at 3 or 5, the tardiness increases as the numbers of products increase. With 10 machines, increasing the number of products decreases tardiness. The effect of this interaction seems complicated and hard to interpret. As the definition of tardiness implies, there are two factors which may affect tardiness: jobs' completion times and due dates. Job due dates, though samples from a uniform distribution, are fixed (not a variable in experimental design). The completion time for jobs equals the flow time, since the ready time for all jobs is zero. Thus, as mentioned in the flow time section, when processors utilization balance is achieved, increasing the number of products increases job flow time thus increasing job tardiness while job due dates are held constant.

The interaction between the number of machines and system loading was also significant for tardiness. As loading increased from 75% to 90%, the tardiness increased approximately 40% for all settings of the number of machines (refer to Table 6). This is understandable, since high loading levels mean more jobs waiting in the system and more jobs need to be processed to meet system utilization.

For the interaction between the number of machines and setup times, the tardiness increased when setup time increased independent of the number of machines in the system. The mean tardiness increased approximately 32%, 16%, and 20% for 3, 5, and 10 machine systems, respectively, as setup time was changed from low to middle (refer to Table 6). Increase in setup time from middle to high resulted in increase in tardiness of about 18%, 9%, and 11% for 3, 5, and 10 machine systems, respectively. This is expected since higher set-up times will cause increase in completion time of jobs. With "constant" due dates, this will result in increase in tardiness.

The same reason can be used to explain the change of tardiness for different number of machines. More machines imply more jobs needed for a specified loading level. As the set-up time includes both processor dependent and product dependent setups, more machines imply higher setup time. Therefore, the time jobs spend in the system will be longer. Hence, the tardiness will increase. However, it is also noticed that in both middle and high setup levels, the performance in terms of tardiness was worse for the 3 machine system compared to the 5 machine system with 15 or 25 products. When the number of products is much higher than the number of machines, each job needs to wait longer to be processed. Thus, this effect masks the influence of the number of machines.

The last significant interaction for tardiness was between number of products and system loading. As mentioned before, higher system loading is achieved at the expense of increasing flow time which will increase the tardiness if the jobs due dates are "constant". With regard to the effect of the number of products, no certain relationship seems to exists. When the number of machines equals 3 or 5, increasing the number of products will increase the tardiness. For 10 machines, 25 products results in minimum tardiness and 5 products results the abnormally high tardiness. Again, the reason for this is that there are too few products in the system which cause an imbalance in processor utilization.

3.2 Comparison of Heuristics

The ANOVA results for the basic heuristic and the three extensions are summarized in Table 9.

The results for the EDD heuristic are the same as those for the basic heuristic. In EDD, jobs within TASKs are sequenced based on earlier due date. Since job due dates are fixed (not a variable in experiment design), it is not surprising that the results are similar to the basic heuristic.

With the SPT heuristic, the significant factors and interactions are different from those of the basic heuristic. The SPT heuristic sequenced jobs in phase D of the algorithm by nondecreasing process times. Process time was determined by the ratio of job quantity to a processor's capacity. Therefore, using process time as a criterion to sequence the jobs will be affected by either the machine capacity or the job size. Since the mean for all machine capacities is the same, this effect is not significant. However, the effect of job quantity (E:JOB_SIZE) is

Table 9: Summary Results - Significant Effects for All Heuristics

	MEAN FLOW TIME				**MEAN TARDINESS**				**PROP. OF TARDY JOBS**			
Factors	**ORIG**	**SPT**	**EDD**	**SS**	**ORIG**	**SPT**	**EDD**	**SS**	**ORIG**	**SPT**	**EDD**	**SS**
A:NMACH	*	*	*	*	*	*	*	*			*	*
B:CAPTY												
C:NPROD	*	*	*	*	*	*	*	*	*	*	*	*
D:LOAD	*	*	*	*	*	*	*	*	*	*	*	*
E:JOB SIZE		*		*		*				*		*
F:SETUP	*	*	*	*	*	*	*	*	*	*	*	*
AC:NMACH*NPROD	*	*	*	*	*	*	*	*			*	*
AD:NMACH*LOAD	*	*	*	*	*	*	*	*				
AE:NMACH*JOB SIZE				*								*
AF:NMACH*SETUP	*	*	*	*	*	*	*	*				*
CD:NPROD*LOAD	*	*	*	*	*	*		*			*	
CE:NPROD*JOB SIZE		*		*		*						*
CF:NPROD*SETUP		*		*		*	*					

* indicates significant effect

identified as being significant on all the performance measures.

For the SS heuristic, jobs were sequenced by the slack time which is related to a job due date and processing time. Therefore, the job quantity is significant, as with the SPT heuristic. With both SPT and SS, interactions including those with job size are significant.

3.3 Discussion of Heuristics

In this section, the four heuristics are discussed in terms of the performance measures. The pairwise t-tests are used to compare the results among the four heuristics.

3.3.1 Pairwise t-tests

Pairwise t-tests were used to test the hypothesis whether there is any significant difference in mean values of performance measures obtained from each heuristic based on a 95% confidence level. More specifically, suppose that the mean value (for example, flow time) of performance measure of Heuristic rule 1 and Heuristic rule 2 are μ_1 and μ_2, respectively. The null hypothesis (H_0) is defined as no significant difference between μ_1 and μ_2 (i.e., H_0: $\mu = 0$, where $\mu = \mu_1 - \mu_2$), and the alternative hypothesis (H_1) is defined as $\mu \neq 0$. If there is evidence that the difference between these two values is significant, then H_0 is rejected in favor of H_1. All hypothesis tested below are defined as H_0: $\mu = 0$ and H_1: $\mu \neq 0$, where μ is the difference in mean values of the two heuristics being compared.

The results from the hypothesis tested are summarized in Table 10. The results show that the null hypothesis is rejected for all cases except one: ORIG versus EDD for mean flow time.

Table 10: The Results of Pairwise t-tests

t- statistics	**M_FLOW**	**M_TARD**	**PROP_T**
ORIG - SPT	12.73	8.91	8.79
ORIG - EDD	1.03*	16.44	14.50
ORIG - SS	-14.76	-9.42	3.74
SPT - EDD	-16.02	18.03	9.88
SPT - SS	-16.32	-11.13	-11.80
EDD - SS	-13.71	-17.05	-15.13

M_FLOW --- Mean Flow Time
PROP_T --- Proportion of Tardy Jobs
M_TARD --- Mean Tardiness
* --- indicates accepting H_0: $\mu = 0$

Consequently, applying different sequencing rules to the basic heuristic did affect the system performance measures. Furthermore, from t-statistics in Table 10, the dominating relationships (greater, smaller or equal) can be identified. If the t-statistic is positive, it means the difference between the two cases (eg. case 1 and case 2), is positive. In other words, the mean value of case 1 is greater than in case 2 (assuming that H_0 is defined as $\mu_1 - \mu_2$) in terms of the response variable (or performance measure). For example, for the first value in Table 10, (ORIG - SPT), the t-statistic for mean flow time is 12.73, indicating that the mean flow time for the basic heuristic (ORIG) is greater than that obtained using the SPT heuristic. The relationship among the heuristics based on the results of the t-tests are summarized in Table 11.

Table 11: The Relationship between Cases Based on the Results of Pairwise t-tests

	M FLOW	**M TARD**	**PROP T**
ORIG vs SPT	ORIG > SPT	ORIG > SPT	ORIG > SPT
ORIG vs EDD	ORIG = EDD	ORIG > EDD	ORIG > EDD
ORIG vs SS	ORIG < SS	ORIG < SS	ORIG < SS
SPT vs EDD	SPT < EDD	SPT > EDD	SPT > EDD
SPT vs SS	SPT < SS	SPT < SS	SPT < SS
EDD vs SS	EDD < SS	EDD < SS	EDD < SS

M_FLOW --- Mean Flow Time
M_TARD --- Mean Tardiness
PROP_T --- Proportion of Tardy Jobs

3.3.2 Flow Time

From Table 11, the mean flow time using the SPT heuristic is the smallest. The SS heuristic is the worst case with the largest flow time. The difference between the basic heuristic (ORIG) and EDD is hard to distinguish. The SPT being the best one in terms of flow time is consistent with expectations and past scheduling research. SPT is known as being the best rule in minimizing the mean flow time in the single machine system (Baker, 1974), and in flow shop problems (Baker, 1984; Conway, 1965; and Rowe, 1958). Therefore, it is not surprising that SPT's performance on flow time is superior to other heuristics.

3.3.3 Tardiness

With this performance measure, the results show EDD to be superior to other heuristics and SS to be the worst one with the largest mean tardiness. For the other two cases, SPT was better (smaller) than the ORIG case.

From the theory of scheduling, it is well known that the maximum tardiness is minimized by sequencing jobs in an order of nondecreasing due dates (Jackson, 1955). Thus, EDD performing better on mean tardiness than the other heuristics is consistent with scheduling theory. Sequencing jobs by nondecreasing slack time is also known to maximize the minimum tardiness (Conway, Maxwell, and Miller, 1967). Hence, SS heuristic is not a good choice if the objective is to minimize mean tardiness.

3.3.4 Proportion of Tardy Jobs

The simulation results for proportion of tardy jobs are the same as tardiness, with EDD being the best choice with the smallest proportion of tardy jobs and SS being the worst one.

The number of tardy jobs is known to be minimized by the Hodgson's Algorithm (Baker, 1974). Basically, the algorithm is scheduling jobs using the EDD order. It gives an indication that scheduling jobs by EDD may result in better solution in terms of number of tardy jobs.

3.4 Use of LPT Heuristic

As mentioned in the previous section, sequencing jobs within product group using SPT and EDD will result in a better solution on flow time and tardiness, respectively. A schedule with minimum makespan (time required to complete all jobs) for parallel, identical machine system can also be obtained by applying LPT (Longest Processing Time) rule, but this schedule may not be optimal (Baker, 1974; Bedworth and Bailey, 1987). Therefore, some simulation experiments using the same random numbers were executed. The results are summarized in Table 12.

As shown by the comparison of LPT with other heuristics in Table 12, LPT does not improve performance of any of the measures compared to the other scheduling rules. Therefore, sequencing jobs using LPT is not a good choice for the parallel, non-identical processors system in terms of the performance measures used in this research.

Table 12: Summary of Simulation Experiment using LPT Heuristic

	NMACH	NPROD	ORIG	SPT	EDD	SS	LPT
FLOW TIME	3	5	220.99	211.93	220.75	224.97	224.97
		15	252.15	248.99	252.36	254.52	254.52
		25	290.42	288.99	290.56	291.90	291.90
	5	5	215.45	219.63	236.87	236.49	236.49
		15	214.04	209.93	214.51	218.15	218.15
		25	231.27	228.75	231.02	233.41	233.41
	10	5	328.41	294.20	310.64	321.30	320.21
		15	216.08	213.05	218.43	224.28	224.28
		25	217.30	212.76	217.67	221.40	221.40
TARDINESS	3	5	45.75	40.73	24.64	45.53	45.53
		15	56.10	57.52	47.81	54.49	54.40
		25	73.18	82.40	78.80	83.50	83.50
	5	5	53.74	63.56	26.44	58.51	58.51
		15	56.83	55.55	43.47	56.62	56.65
		25	62.28	60.27	55.57	64.85	64.93
	10	5	103.78	82.18	22.83	101.10	100.65
		15	49.28	49.43	30.55	54.04	54.04
		25	48.33	45.50	34.50	49.58	49.56
PROPORTION OF TARDY JOBS	3	5	0.3118	0.3330	0.1505	0.2688	0.2688
		15	0.3333	0.3504	0.3333	0.3333	0.3333
		25	0.4159	0.4071	0.3894	0.3894	0.3894
	5	5	0.3609	0.3550	0.2249	0.3965	0.3965
		15	0.3587	0.3913	0.3098	0.3696	0.3696
		25	0.3834	0.3990	0.3482	0.3782	0.3782
	10	5	0.4721	0.4441	0.4246	0.4693	0.4721
		15	0.3296	0.3296	0.2290	0.3212	0.3212
		25	0.3288	0.3041	0.2575	0.3096	0.3096

3.5 Sensitivity to PUS Criterion

The criterion for deciding load balance, Processor Utilization Spread (PUS), used in this study was 10 percent of the maximum processor load. In order to test the sensitivity of results to this criterion, the system was evaluated using PUS criteria of 0% (not considering the criterion, just spreading out all the groups into individual jobs) and 20% of the maximum processor load. Some simulation experiments using different criterion were executed in the situation of NMACH=10, NPROD=25, LOAD=0.9, and SETUP=HIGH. The results were summarized in Table 13 and shown graphically in Figures 6 through 8.

Table 13: Summary of Sensitivity to PUS Criterion

Heuristics	CPUS	M_FLOW	M_TARD	PROP_T
ORIG	0%	284.53	77.11	0.4236
	10%	253.53	65.82	0.3798
	20%	253.05	67.00	0.3673
SPT	0%	283.11	76.94	0.4188
	10%	249.10	64.33	0.3676
	20%	247.62	62.40	0.3678
EDD	0%	296.04	66.17	0.4020
	10%	254.71	51.87	0.3151
	20%	253.08	48.92	0.3136
SS	0%	292.58	80.86	0.4498
	10%	259.27	69.56	0.3827
	20%	258.36	68.58	0.3730

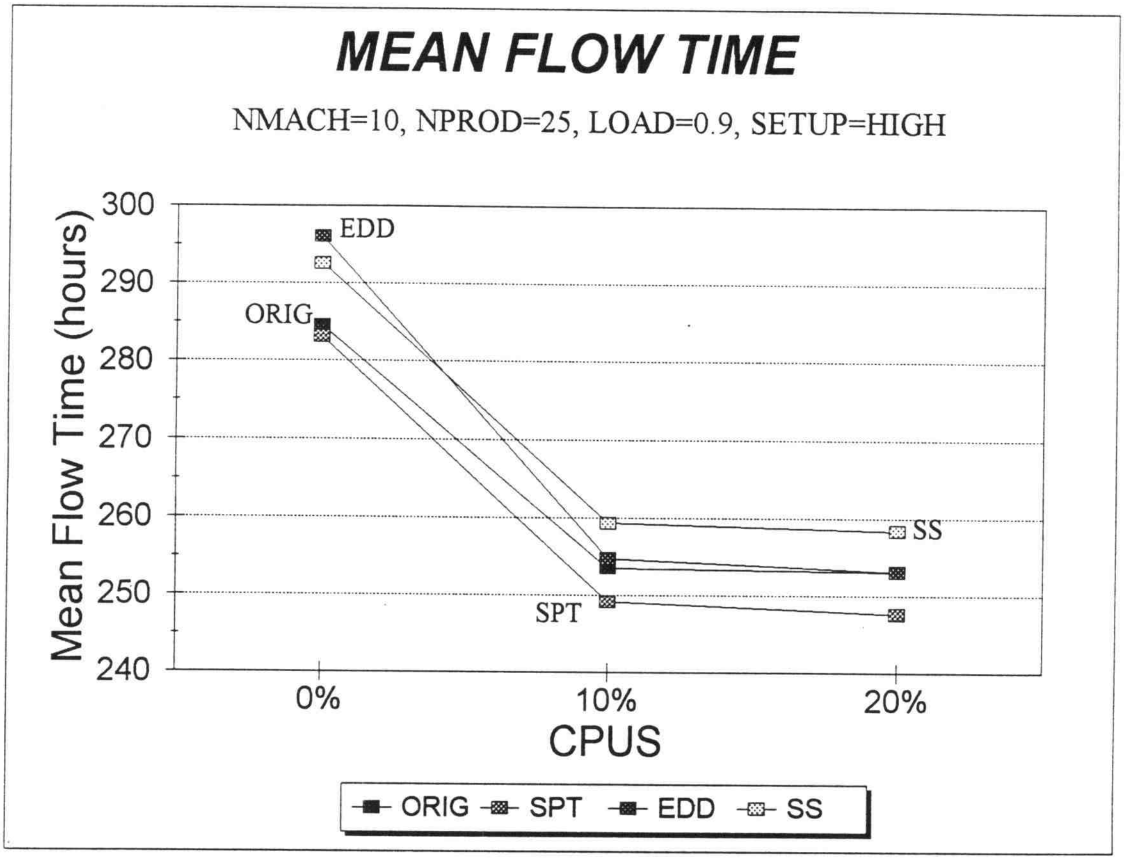


Figure 6: Sensitivity to PUS Criterion - Flow Time

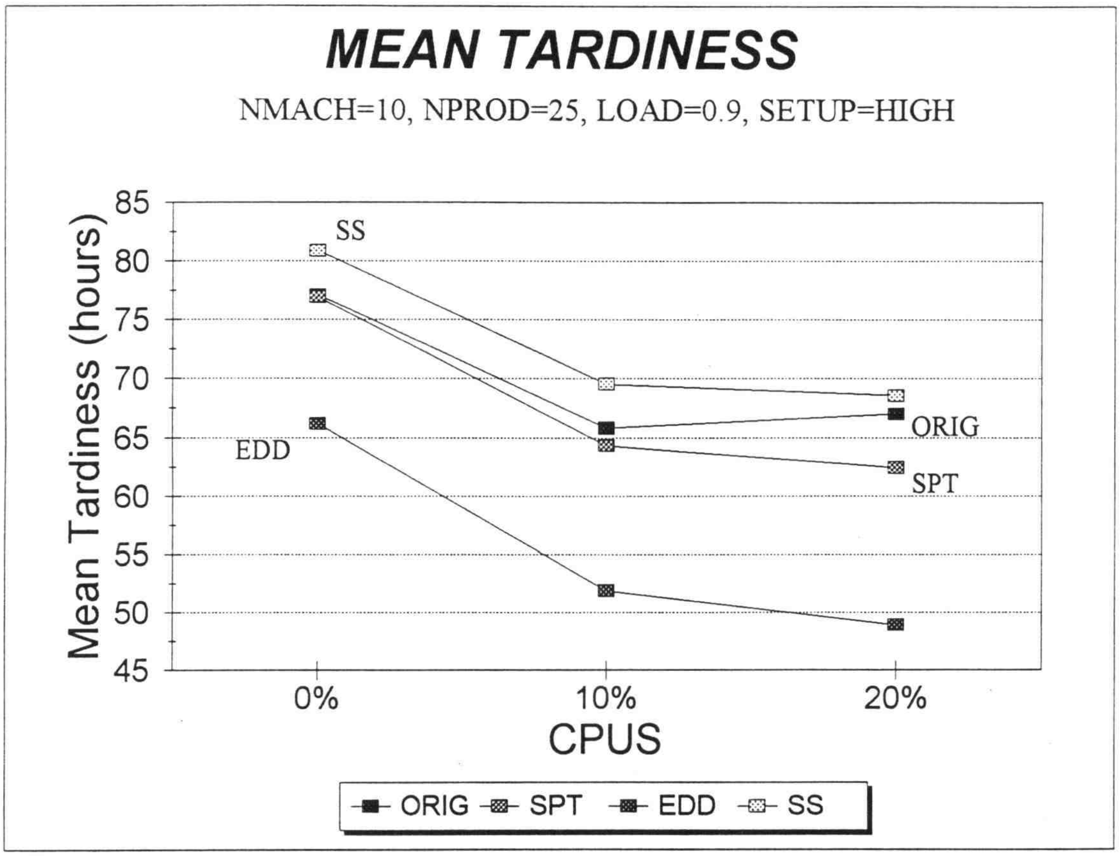


Figure 7: Sensitivity to PUS Criterion - Tardiness

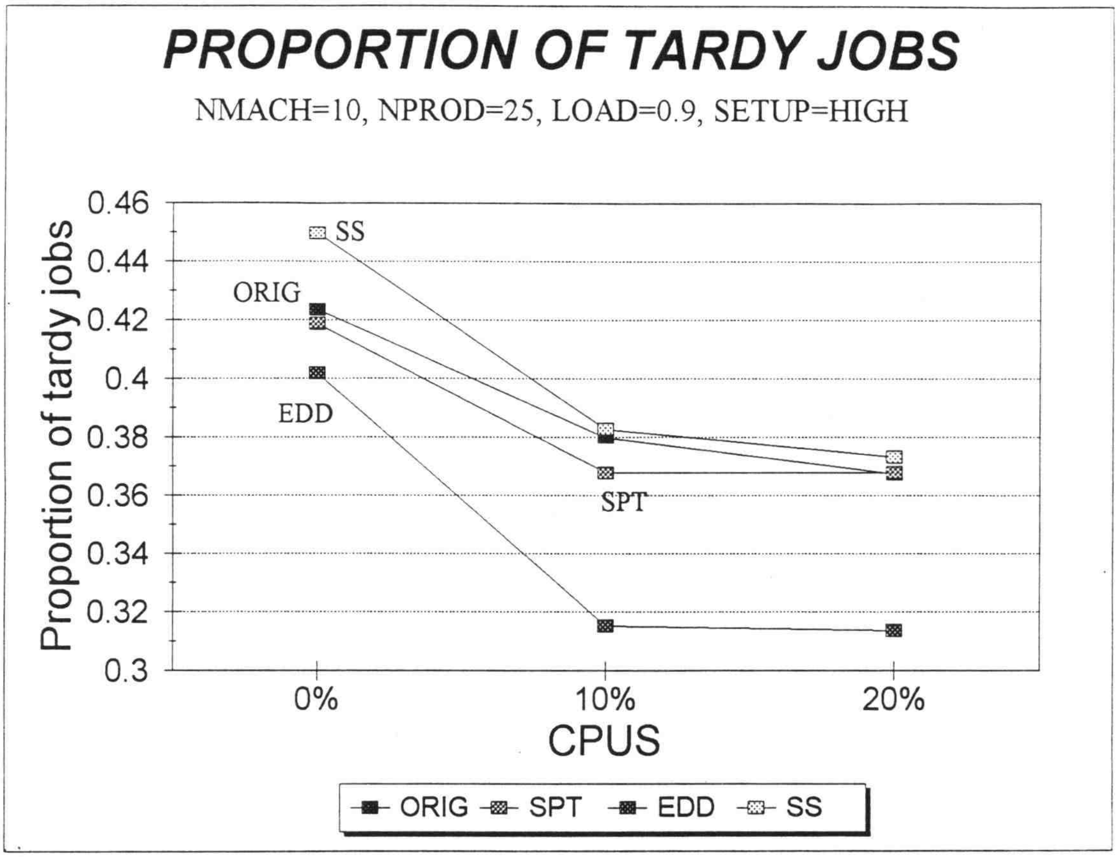


Figure 8: Sensitivity to PUS Criterion - Proportion of Tardy Jobs

The results show that the smaller criterion (0%) did not improve the performance measures. With no criterion (0%), all processors are equally loaded. This is achieved by all groups broken into individual jobs to achieve the criterion. Once all groups are broken into individual jobs then assigned, the proportion of setup time increases; thus flow time, tardiness and proportion of tardy jobs increase. Besides, the computation time needed for scheduling increased by approximately 10 percent compared with the case of using 10 percent criterion. In contrast, the higher criterion resulted in better performance. However, the results using 20 percent criterion improved the performance very slightly over the 10 percent level.

CHAPTER 4. CONCLUSIONS

4.1 Summary of Research

This research focused on scheduling multiple parallel, non-identical processors. Three heuristics that were extensions of the basic heuristic (ORIG) were developed by using three different sequence rules, Shortest Process Time (SPT), Earliest Due Date (EDD), and Smallest Slack (SS). Three performance measures, mean flow time, tardiness, and proportion of tardy jobs, were used to evaluate the system performance.

In general, the three factors, number of products, system loading, and set-up time, were identified as significant factors affecting the system performance in terms of flow time, tardiness and proportion of jobs tardy. For both mean flow time and tardiness, the number of machines was also a significant factor. Perhaps more importantly, a number of two-factor interactions were significant for flow times and tardiness. These included: the interactions between the number of machines and the number of products, system loading, and the set up times, and the interaction between system loading and the set up times.

The experimental results showed that SPT was the best heuristic in terms of job flow time. If the objective is to minimize the mean tardiness or proportion of tardy jobs, EDD would be the best choice. On all three performance measures, SS yielded the worst results. The relationships among the heuristics are summarized in Table 14.

Table 14: Summary of Relationships among Heuristics

Performance Measurements	**Relationships**
Mean Flow Time (M FLOW)	SPT < EDD = ORIG < SS
Mean Tardiness (M TARD)	EDD < SPT < ORIG < SS
Proportion of Tardy Jobs (PROP T)	EDD < SPT < ORIG < SS

4.2 "Hybrid" Heuristic

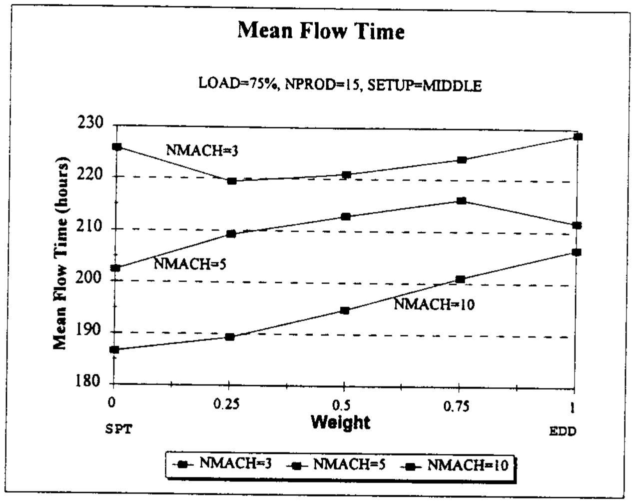
Today's production management needs to satisfy multiple objectives. In order to provide a better balance among the measures, a "Hybrid" heuristic was developed based on above results.

A "Hybrid" heuristic means a mixed heuristic which combines several simple heuristics. For the heuristics developed in this research, SPT and EDD performed better than other heuristics in terms of flow time and tardiness or proportion of tardy jobs, respectively. A hybrid heuristic consisting of these two heuristics is developed. The rank order of processing of jobs is computed from the following relationship:

$$Rank = w_{SPT}*R_{SPT} + w_{EDD}*R_{EDD} ,$$

where w_{SPT} and w_{EDD} represent the weight assigned to minimizing flow times and minimizing due dates, $w_{SPT} + w_{EDD} = 1$, and R_{SPT} and R_{EDD} are the ranks of jobs when they are scheduled using SPT and EDD heuristics, respectively. If SPT heuristic is used exclusively, $w_{SPT}=1$ and $w_{EDD}=0$. Similarly, use of EDD heuristic implies $w_{SPT}=0$ and $w_{EDD}=1$. The Hybrid heuristic schedules the jobs within each product group by the rank determined by the weights, w_{SPT} and w_{EDD}. This heuristic can also be termed as weighted SPT/EDD heuristic. A number of simulation experiments (for different number of machines) with different combinations of weights for SPT and EDD ranking were executed for two situations: (1) LOAD=75%, NPROD=15, and SETUP=MIDDLE (2) LOAD=90%, NPROD=25, and SETUP=HIGH. The results are summarized in Figures 9, 10, and 11.

The results show that higher weight on SPT results in better flow times, as would be expected. The due date results improve with higher weight on EDD. However, depending on management objectives, assigning appropriate weights can provide a compromise. An example situation with NMACH=10, NPROD=25, LOAD=90%, SETUP=HIGH, is shown in Table 15 where the results using SPT, EDD, and the hybrid heuristics with $w_{SPT}=w_{EDD}= 0.5$ are summarized.



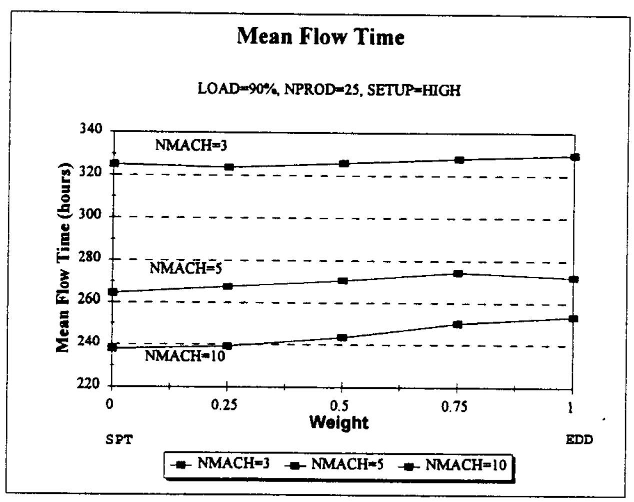
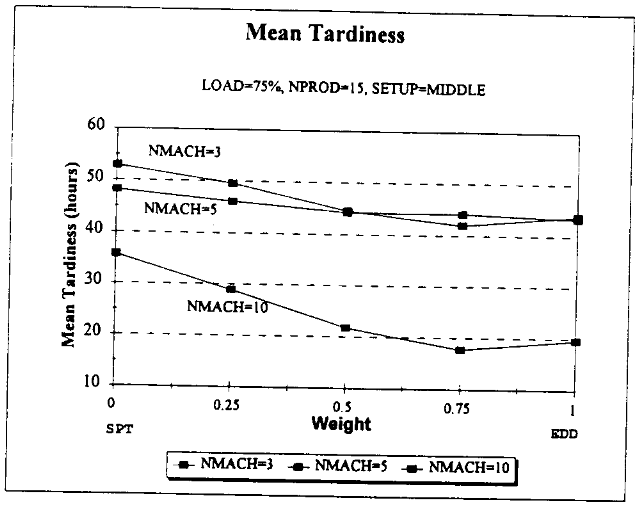


Figure 9: Flow Time for Hybrid Heuristic



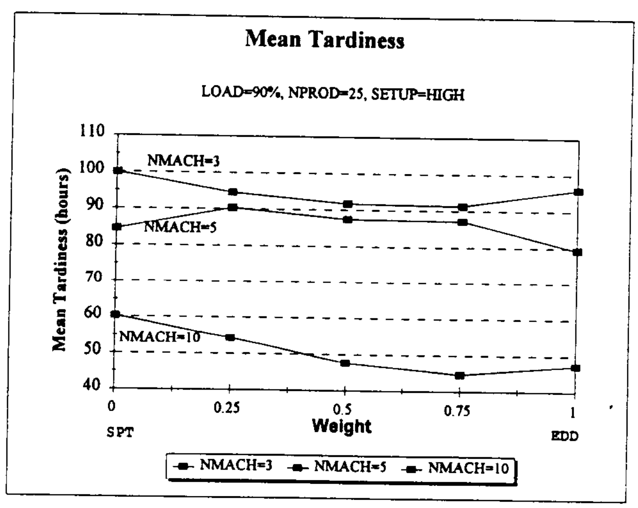
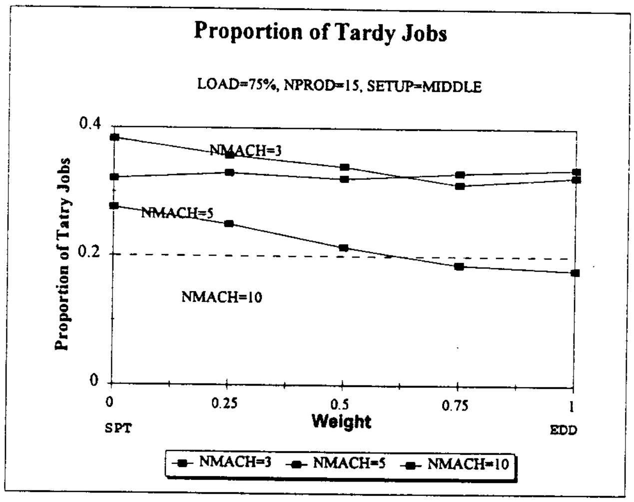


Figure 10: Tardiness for Hybrid Heuristic



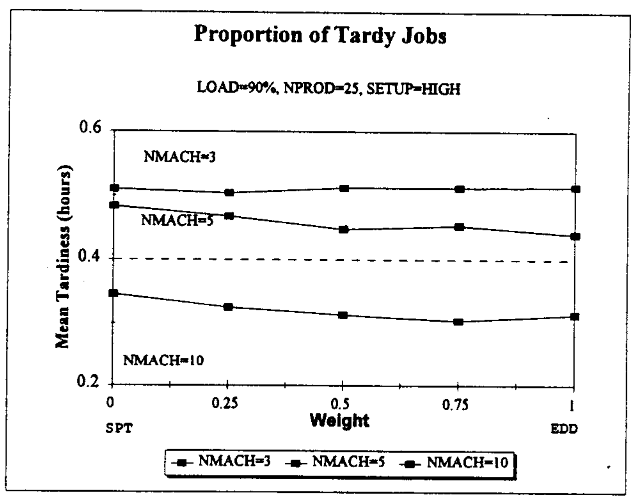


Figure 11: Proportion of Tardy Jobs for Hybrid Heuristic

Table 15: Comparison of Hybrid Heuristic with SPT and EDD Heuristics for NMACH=10, NPROD=25, LOAD=90%, SETUP=HIGH

Heuristic	Mean Flow Time	Mean Tardiness	Proportion of Jobs Tardy
SPT	237.93	60.23	0.3458
EDD	253.35	47.22	0.3140
Hybrid ($w_{SPT}=w_{EDD}=0.5$)	243.70	47.73	0.3136

4.3 Recommendations for Future Study

The results of this study provide guidelines for the design and scheduling of parallel non-identical systems. There are two directions that are identified for future research.

First, the results can be extended by considering additional characteristics not included in this research. These include: product preemption, machine breakdowns, and jobs splitting. Furthermore, an investigation of scheduling the dynamic (jobs arriving in the system continuously) parallel, non-identical system will be valuable.

Second, a natural extension of this study would be to develop and validate the hybrid heuristics which could "optimize" multiple objectives in scheduling. Furthermore, developing a framework for adapting the system developed here for real time control would be valuable.

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APPENDIX

Table 16: Simulation Output

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	3	U(80,120)	5	0.75	U(800,1200)	LOW	97	190.60	-164.81	34.38	0.3002	0.7538	0.0387
2	3		5	0.75	N(1000,300)		115	195.65	-157.58	33.73	0.2925	0.7516	0.0411
3	3		5	0.9	U(800,1200)		129	230.09	-109.88	41.72	0.3482	0.9029	0.0768
4	3		5	0.9	N(1000,300)		125	233.46	-99.58	62.05	0.3937	0.9009	0.0683
5	3		15	0.75	U(800,1200)		116	198.38	-156.72	38.92	0.2889	0.7543	0.0850
6	3		15	0.75	N(1000,300)		107	197.95	-162.24	38.10	0.3073	0.7525	0.0861
7	3		15	0.9	U(800,1200)		132	233.86	-107.66	58.06	0.3429	0.9052	0.0919
8	3		15	0.9	N(1000,300)		132	233.11	-105.31	55.37	0.3489	0.9024	0.0959
9	3		25	0.75	U(800,1200)		115	208.17	-148.58	44.94	0.3054	0.7552	0.1402
10	3		25	0.75	N(1000,300)		106	207.71	-134.19	42.41	0.3113	0.7556	0.1295
11	3		25	0.9	U(800,1200)		131	241.81	-100.33	60.12	0.4041	0.9034	0.1525
12	3		25	0.9	N(1000,300)		123	242.54	-101.57	58.09	0.3303	0.9058	0.1367
13	3	U(50,150)	5	0.75	U(800,1200)		105	192.35	-151.70	42.48	0.3287	0.7559	0.0489
14	3		5	0.75	N(1000,300)		96	198.67	-151.68	34.36	0.3094	0.7546	0.0521
15	3		5	0.9	U(800,1200)		130	233.37	-105.46	58.14	0.3823	0.9328	0.0372
16	3		5	0.9	N(1000,300)		131	242.34	-100.77	59.28	0.3702	0.9271	0.0689
17	3		15	0.75	U(800,1200)		117	199.55	-155.45	39.07	0.2979	0.7539	0.0887
18	3		15	0.75	N(1000,300)		101	198.69	-149.00	38.42	0.3219	0.7536	0.0834
19	3		15	0.9	U(800,1200)		131	233.64	-104.79	59.72	0.3671	0.9021	0.0894
20	3		15	0.9	N(1000,300)		130	232.57	-108.81	55.00	0.3594	0.8999	0.0873
21	3		25	0.75	U(800,1200)		100	213.26	-134.34	37.82	0.2850	0.7781	0.1304
22	3		25	0.75	N(1000,300)		112	207.62	-145.85	40.69	0.3126	0.7558	0.1379
23	3		25	0.9	U(800,1200)		148	244.15	-84.08	67.75	0.3815	0.9060	0.1612
24	3		25	0.9	N(1000,300)		109	245.97	-109.22	55.12	0.3471	0.9066	0.1328
	Average						118	218.98	-127.06	48.15	0.3348	0.8321	0.0942
	STD						14	19.23	25.26	10.51	0.0346	0.0763	0.0369

Table 16: Simulation Output (continued)

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	5	U(80,120)	5	0.75	U(800,1200)	LOW	180	190.17	-135.46	43.60	0.3364	0.7542	0.0390
2	5		5	0.75	N(1000,300)		178	190.76	-139.57	42.46	0.3360	0.7550	0.0393
3	5		5	0.9	U(800,1200)		213	224.99	-90.75	62.47	0.3969	0.9027	0.0387
4	5		5	0.9	N(1000,300)		213	229.42	-83.40	66.73	0.4214	0.9079	0.1713
5	5		15	0.75	U(800,1200)		184	195.69	-129.03	44.33	0.3234	0.7557	0.0715
6	5		15	0.75	N(1000,300)		178	196.96	-127.91	47.68	0.3719	0.7540	0.0722
7	5		15	0.9	U(800,1200)		228	226.90	-98.56	58.05	0.3794	0.9018	0.0859
8	5		15	0.9	N(1000,300)		199	225.47	-94.25	58.20	0.4051	0.9009	0.0726
9	5		25	0.75	U(800,1200)		185	196.06	-131.23	43.73	0.3212	0.7541	0.1065
10	5		25	0.75	N(1000,300)		169	195.43	-129.43	47.09	0.3112	0.7548	0.0976
11	5		25	0.9	U(800,1200)		231	231.82	-89.34	60.08	0.3798	0.9043	0.1292
12	5		25	0.9	N(1000,300)		208	230.93	-85.38	65.44	0.4372	0.9051	0.1212
13	5	U(50,150)	5	0.75	U(800,1200)		213	190.93	-135.57	38.18	0.3127	0.7547	0.0442
14	5		5	0.75	N(1000,300)		209	189.13	-124.71	45.43	0.3332	0.7552	0.0325
15	5		5	0.9	U(800,1200)		214	224.70	-95.89	57.66	0.3933	0.9013	0.0368
16	5		5	0.9	N(1000,300)		234	231.24	-90.18	60.94	0.4215	0.9266	0.0521
17	5		15	0.75	U(800,1200)		176	202.20	-126.77	45.58	0.3041	0.7751	0.0708
18	5		15	0.75	N(1000,300)		176	196.53	-123.21	40.24	0.3381	0.7542	0.0720
19	5		15	0.9	U(800,1200)		201	229.95	-99.16	55.40	0.3624	0.9063	0.0769
20	5		15	0.9	N(1000,300)		219	232.05	-96.15	59.62	0.3664	0.9049	0.0828
21	5		25	0.75	U(800,1200)		164	196.25	-136.30	40.71	0.3235	0.7541	0.0966
22	5		25	0.75	N(1000,300)		169	194.61	-131.06	45.58	0.3431	0.7594	0.1024
23	5		25	0.9	U(800,1200)		232	231.79	-96.98	58.95	0.3925	0.9073	0.1324
24	5		25	0.9	N(1000,300)		215	234.88	-97.51	57.81	0.3754	0.9122	0.1226
	Average						199	212.03	-111.99	51.91	0.3619	0.8317	0.0819
	STD						22	17.80	19.49	8.72	0.0380	0.0753	0.0355

Table 16: Simulation Output (continued)

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	10	U(80,120)	5	0.75	U(800,1200)	LOW	374	257.68	-102.83	71.72	0.3799	0.7469	0.0965
2	10		5	0.75	N(1000,300)		365	262.09	-97.60	77.23	0.3843	0.7561	0.0965
3	10		5	0.9	U(800,1200)		434	332.07	-27.20	128.94	0.4673	0.9109	0.1085
4	10		5	0.9	N(1000,300)		426	304.39	-53.33	103.84	0.4418	0.8999	0.1158
5	10		15	0.75	U(800,1200)		363	191.40	-169.07	37.18	0.2783	0.7506	0.0782
6	10		15	0.75	N(1000,300)		359	188.76	-169.77	32.41	0.2674	0.7516	0.0757
7	10		15	0.9	U(800,1200)		438	227.04	-133.05	51.47	0.3515	0.9013	0.0949
8	10		15	0.9	N(1000,300)		426	226.75	-130.22	50.31	0.3323	0.9019	0.0881
9	10		25	0.75	U(800,1200)		357	191.44	-167.24	38.03	0.3058	0.7501	0.1022
10	10		25	0.75	N(1000,300)		375	189.97	-162.89	38.91	0.3107	0.7517	0.1086
11	10		25	0.9	U(800,1200)		429	227.32	-128.24	54.60	0.3594	0.9008	0.1155
12	10		25	0.9	N(1000,300)		441	233.23	-117.86	54.64	0.3527	0.9037	0.1275
13	10	U(50,150)	5	0.75	U(800,1200)		344	258.28	-89.72	76.15	0.3841	0.7574	0.0845
14	10		5	0.75	N(1000,300)		359	237.51	-118.35	58.47	0.3529	0.7429	0.0865
15	10		5	0.9	U(800,1200)		464	301.37	-55.85	99.22	0.4411	0.8640	0.1019
16	10		5	0.9	N(1000,300)		390	319.87	-26.81	117.39	0.4317	0.9605	0.1033
17	10		15	0.75	U(800,1200)		383	202.07	-150.06	39.75	0.4616	0.7592	0.0792
18	10		15	0.75	N(1000,300)		335	193.69	-152.09	40.24	0.3082	0.7572	0.0682
19	10		15	0.9	U(800,1200)		440	244.81	-105.96	62.28	0.3714	0.9223	0.1009
20	10		15	0.9	N(1000,300)		426	240.46	-112.75	60.65	0.3261	0.9118	0.0880
21	10		25	0.75	U(800,1200)		363	192.13	-159.60	41.66	0.3228	0.7539	0.0988
22	10		25	0.75	N(1000,300)		366	202.61	-149.59	44.49	0.3201	0.7752	0.1062
23	10		25	0.9	U(800,1200)		419	227.98	-125.11	56.23	0.3455	0.9017	0.1137
24	10		25	0.9	N(1000,300)		419	234.93	-122.04	53.10	0.3487	0.9128	0.1230
	Average						395	236.99	-117.80	62.04	0.3602	0.8310	0.0984
	STD						37	41.61	41.54	25.81	0.0543	0.0782	0.0150

Table 16: Simulation Output (continued)

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	3	U(80,120)	5	0.75	U(800,1200)	MID	97	205.93	-149.45	40.32	0.3152	0.7535	0.1132
2	3		5	0.75	N(1000,300)		115	210.34	-142.88	38.99	0.3359	0.7504	0.1245
3	3		5	0.9	U(800,1200)		129	249.38	-90.59	66.14	0.3941	0.9004	0.2318
4	3		5	0.9	N(1000,300)		125	252.87	-80.17	71.97	0.4136	0.8984	0.2305
5	3		15	0.75	U(800,1200)		116	240.52	-124.57	50.68	0.3362	0.7519	0.1925
6	3		15	0.75	N(1000,300)		107	229.33	-130.86	51.19	0.3615	0.7563	0.2283
7	3		15	0.9	U(800,1200)		132	265.80	-75.73	72.39	0.3953	0.9082	0.2090
8	3		15	0.9	N(1000,300)		132	261.60	-76.82	68.93	0.3830	0.9034	0.2358
9	3		25	0.75	U(800,1200)		115	257.57	-99.17	68.34	0.3536	0.7593	0.3077
10	3		25	0.75	N(1000,300)		106	261.39	-80.51	65.29	0.3680	0.7584	0.3073
11	3		25	0.9	U(800,1200)		131	289.38	-51.36	81.25	0.4711	0.9010	0.3421
12	3		25	0.9	N(1000,300)		123	294.41	-49.70	81.01	0.4487	0.9080	0.3104
13	3	U(50,150)	5	0.75	U(800,1200)		105	208.69	-135.37	49.45	0.3525	0.7571	0.1557
14	3		5	0.75	N(1000,300)		96	216.41	-133.94	41.66	0.3303	0.7517	0.1732
15	3		5	0.9	U(800,1200)		130	251.45	-87.39	65.05	0.4167	0.9316	0.1095
16	3		5	0.9	N(1000,300)		131	267.18	-75.94	68.73	0.4235	0.9343	0.2241
17	3		15	0.75	U(800,1200)		117	233.28	-121.73	52.71	0.3415	0.7597	0.2131
18	3		15	0.75	N(1000,300)		101	228.15	-149.54	50.81	0.3666	0.7571	0.2060
19	3		15	0.9	U(800,1200)		131	263.60	-74.83	74.22	0.4132	0.9054	0.1904
20	3		15	0.9	N(1000,300)		130	264.54	-76.82	69.94	0.4035	0.9097	0.2190
21	3		25	0.75	U(800,1200)		100	270.71	-76.88	64.67	0.3959	0.7851	0.3045
22	3		25	0.75	N(1000,300)		112	255.43	-98.04	61.11	0.3880	0.7471	0.3202
23	3		25	0.9	U(800,1200)		148	292.45	-35.77	94.88	0.4731	0.9079	0.3290
24	3		25	0.9	N(1000,300)		109	296.56	-58.64	78.09	0.4070	0.9034	0.2982
	Average						118	252.79	-94.86	63.66	0.3870	0.8333	0.2323
	STD						14	26.35	32.73	13.89	0.0419	0.0767	0.0682

Table 16: Simulation Output (continued)

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	5	U(80,120)	5	0.75	U(800,1200)	MID	180	204.99	-120.64	47.60	0.3483	0.7515	0.1163
2	5		5	0.75	N(1000,300)		178	202.20	-124.39	47.28	0.3527	0.7576	0.1075
3	5		5	0.9	U(800,1200)		213	246.09	-69.65	72.24	0.4329	0.9067	0.1333
4	5		5	0.9	N(1000,300)		213	244.25	-68.67	74.86	0.4424	0.9135	0.1444
5	5		15	0.75	U(800,1200)		184	207.23	-117.48	53.19	0.3506	0.7555	0.1335
6	5		15	0.75	N(1000,300)		178	210.83	-115.55	55.31	0.3776	0.7532	0.1437
7	5		15	0.9	U(800,1200)		228	242.85	-82.61	64.57	0.3968	0.9038	0.1637
8	5		15	0.9	N(1000,300)		199	242.06	-77.66	66.55	0.4178	0.9020	0.1541
9	5		25	0.75	U(800,1200)		185	219.07	-108.22	53.34	0.3593	0.7567	0.2057
10	5		25	0.75	N(1000,300)		169	218.89	-105.97	57.52	0.3375	0.7558	0.2000
11	5		25	0.9	U(800,1200)		231	254.05	-67.11	71.77	0.4231	0.9052	0.2295
12	5		25	0.9	N(1000,300)		208	255.59	-60.72	79.51	0.4710	0.9108	0.2267
13	5	U(50,150)	5	0.75	U(800,1200)		213	205.70	-120.80	43.65	0.3444	0.7620	0.1175
14	5		5	0.75	N(1000,300)		209	198.78	-115.05	49.08	0.3450	0.7555	0.0941
15	5		5	0.9	U(800,1200)		214	234.05	-86.54	61.80	0.4021	0.9045	0.1041
16	5		5	0.9	N(1000,300)		234	244.50	-76.92	68.93	0.4371	0.9188	0.1540
17	5		15	0.75	U(800,1200)		176	228.38	-100.59	56.12	0.3548	0.7818	0.1566
18	5		15	0.75	N(1000,300)		176	206.15	-113.59	44.55	0.3392	0.7529	0.1352
19	5		15	0.9	U(800,1200)		201	246.47	-82.65	63.84	0.3824	0.9130	0.1581
20	5		15	0.9	N(1000,300)		219	245.81	-82.41	68.07	0.4177	0.9077	0.1598
21	5		25	0.75	U(800,1200)		164	219.31	-113.24	49.75	0.3442	0.7570	0.1913
22	5		25	0.75	N(1000,300)		169	219.28	-106.39	57.71	0.3585	0.7651	0.2035
23	5		25	0.9	U(800,1200)		232	258.22	-70.54	72.60	0.4329	0.9260	0.2452
24	5		25	0.9	N(1000,300)		215	261.76	-70.64	70.55	0.4140	0.9174	0.2340
	Average						199	229.85	-94.08	60.43	0.3867	0.8347	0.1630
	STD						22	19.84	20.69	10.34	0.0398	0.0764	0.0433

Table 16: Simulation Output (continued)

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	10	U(80,120)	5	0.75	U(800,1200)	MID	374	300.05	-60.46	90.09	0.4378	0.7452	0.3338
2	10		5	0.75	N(1000,300)		365	308.70	-50.99	97.36	0.4475	0.7545	0.3408
3	10		5	0.9	U(800,1200)		434	378.68	19.41	152.53	0.5204	0.9130	0.3764
4	10		5	0.9	N(1000,300)		426	362.14	3.43	131.59	0.5055	0.9024	0.4163
5	10		15	0.75	U(800,1200)		363	206.06	-154.41	43.13	0.3044	0.7535	0.1737
6	10		15	0.75	N(1000,300)		359	206.81	-151.72	39.26	0.2840	0.7515	0.1627
7	10		15	0.9	U(800,1200)		438	241.54	-118.55	58.20	0.3756	0.9003	0.1951
8	10		15	0.9	N(1000,300)		426	245.24	-111.73	62.06	0.3625	0.9049	0.2169
9	10		25	0.75	U(800,1200)		357	207.79	-150.89	43.87	0.3306	0.7483	0.1863
10	10		25	0.75	N(1000,300)		375	205.50	-147.36	44.51	0.3201	0.7511	0.1849
11	10		25	0.9	U(800,1200)		429	243.39	-112.17	63.19	0.3757	0.8986	0.1962
12	10		25	0.9	N(1000,300)		441	243.35	-107.74	59.77	0.3583	0.9039	0.2132
13	10	U(50,150)	5	0.75	U(800,1200)		344	297.33	-50.67	93.10	0.4444	0.7608	0.2832
14	10		5	0.75	N(1000,300)		359	282.61	-73.24	78.75	0.4195	0.7430	0.3293
15	10		5	0.9	U(800,1200)		464	340.39	-16.84	117.75	0.4949	0.8591	0.3487
16	10		5	0.9	N(1000,300)		390	373.87	27.20	145.50	0.5163	0.9657	0.3810
17	10		15	0.75	U(800,1200)		383	221.86	-130.27	49.46	0.3483	0.7660	0.1701
18	10		15	0.75	N(1000,300)		335	213.99	-131.79	48.18	0.3398	0.7583	0.1468
19	10		15	0.9	U(800,1200)		440	286.85	-63.92	81.58	0.4311	0.9295	0.2168
20	10		15	0.9	N(1000,300)		426	268.84	-84.37	72.54	0.4153	0.9156	0.1878
21	10		25	0.75	U(800,1200)		363	207.39	-144.34	45.07	0.3271	0.7543	0.1675
22	10		25	0.75	N(1000,300)		366	222.09	-130.11	52.07	0.3521	0.7802	0.1871
23	10		25	0.9	U(800,1200)		419	242.49	-110.90	63.21	0.3702	0.8970	0.1869
24	10		25	0.9	N(1000,300)		419	245.05	-111.92	59.03	0.3776	0.9181	0.2059
	Average						395	264.67	-90.18	74.66	0.3941	0.8323	0.2420
	STD						37	54.37	54.24	32.49	0.0671	0.0788	0.0818

Table 16: Simulation Output (continued)

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	5	U(80,120)	5	0.75	U(800,1200)	HIGH	180	210.78	-114.85	52.48	0.3621	0.7533	0.1564
2	5		5	0.75	N(1000,300)		178	208.95	-121.38	50.20	0.3582	0.7587	0.1403
3	5		5	0.9	U(800,1200)		213	245.54	-70.20	71.32	0.4202	0.9007	0.1820
4	5		5	0.9	N(1000,300)		213	252.17	-60.75	79.13	0.4562	0.9150	0.2060
5	5		15	0.75	U(800,1200)		184	216.62	-108.10	57.41	0.3587	0.7566	0.1772
6	5		15	0.75	N(1000,300)		178	215.98	-110.39	57.40	0.3918	0.7532	0.1835
7	5		15	0.9	U(800,1200)		228	253.16	-72.30	69.89	0.4253	0.9036	0.2242
8	5		15	0.9	N(1000,300)		199	252.57	-67.15	72.16	0.4479	0.9021	0.2036
9	5		25	0.75	U(800,1200)		185	233.30	-93.99	60.04	0.3838	0.7573	0.2600
10	5		25	0.75	N(1000,300)		169	233.18	-91.68	64.41	0.3468	0.7545	0.2670
11	5		25	0.9	U(800,1200)		231	267.89	-53.27	78.79	0.4533	0.9047	0.2934
12	5		25	0.9	N(1000,300)		208	270.38	-48.93	84.51	0.4650	0.9087	0.3024
13	5	U(50,150)	5	0.75	U(800,1200)		213	205.74	-120.76	45.39	0.3333	0.7634	0.1429
14	5		5	0.75	N(1000,300)		209	205.16	-108.67	51.49	0.3544	0.7566	0.1379
15	5		5	0.9	U(800,1200)		214	240.11	-80.48	64.44	0.4093	0.9048	0.1474
16	5		5	0.9	N(1000,300)		234	253.88	-67.54	74.71	0.4434	0.9283	0.2180
17	5		15	0.75	U(800,1200)		176	247.53	-81.44	64.53	0.3752	0.7843	0.2364
18	5		15	0.75	N(1000,300)		176	217.33	-102.41	49.73	0.3734	0.7560	0.2053
19	5		15	0.9	U(800,1200)		201	255.86	-73.25	68.50	0.4016	0.9140	0.2002
20	5		15	0.9	N(1000,300)		219	257.85	-70.36	74.33	0.4316	0.9115	0.2082
21	5		25	0.75	U(800,1200)		164	233.63	-98.91	57.56	0.3676	0.7576	0.2510
22	5		25	0.75	N(1000,300)		169	234.76	-90.91	65.17	0.3712	0.7663	0.2628
23	5		25	0.9	U(800,1200)		232	284.83	-43.93	87.90	0.4675	0.9375	0.4114
24	5		25	0.9	N(1000,300)		215	278.43	-53.97	79.62	0.4379	0.9332	0.3024
	Average						199	240.65	-83.57	65.88	0.4015	0.8367	0.2216
	STD						22	22.83	23.07	11.49	0.0410	0.0776	0.0634

Table 16: Simulation Output (continued)

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	3	U(80,120)	5	0.75	U(800,12	HIGH	97	215.75	-139.64	44.28	0.3309	0.7508	0.1597
2	3		5	0.75	N(1000,3		115	219.87	-133.35	42.97	0.3672	0.7509	0.1683
3	3		5	0.9	U(800,12		129	261.77	-78.20	72.52	0.4062	0.8988	0.3404
4	3		5	0.9	N(1000,3		125	265.23	-67.81	78.90	0.4409	0.8984	0.3352
5	3		15	0.75	U(800,12		116	252.16	-102.93	59.51	0.3536	0.7517	0.2788
6	3		15	0.75	N(1000,3		107	248.95	-111.24	59.43	0.3658	0.7565	0.3339
7	3		15	0.9	U(800,12		132	286.33	-55.20	82.76	0.4324	0.9093	0.2624
8	3		15	0.9	N(1000,3		132	279.92	-58.50	77.90	0.4099	0.9026	0.2768
9	3		25	0.75	U(800,12		115	290.19	-66.56	79.44	0.4192	0.7578	0.4490
10	3		25	0.75	N(1000,3		106	294.63	-47.27	80.39	0.3963	0.7451	0.4217
11	3		25	0.9	U(800,12		131	320.32	-20.42	100.13	0.5058	0.9015	0.5089
12	3		25	0.9	N(1000,3		123	329.35	-14.76	101.50	0.5427	0.9100	0.4686
13	3	U(50,150)	5	0.75	U(800,12		105	218.91	-125.14	54.03	0.3667	0.7584	0.2067
14	3		5	0.75	N(1000,3		96	228.21	-122.15	47.18	0.3561	0.7518	0.2679
15	3		5	0.9	U(800,12		130	259.99	-78.85	69.90	0.4280	0.9305	0.1384
16	3		5	0.9	N(1000,3		131	281.19	-61.92	76.12	0.4389	0.9373	0.3160
17	3		15	0.75	U(800,12		117	254.31	-100.70	61.48	0.3667	0.7635	0.2540
18	3		15	0.75	N(1000,3		101	248.21	-99.47	60.56	0.3766	0.7576	0.2940
19	3		15	0.9	U(800,12		131	284.03	-54.39	85.27	0.4459	0.9049	0.2850
20	3		15	0.9	N(1000,3		130	283.33	-58.05	78.31	0.4269	0.9118	0.2956
21	3		25	0.75	U(800,12		100	314.70	-32.90	96.25	0.4514	0.7948	0.4835
22	3		25	0.75	N(1000,3		112	286.62	-66.85	76.48	0.4392	0.7476	0.4065
23	3		25	0.9	U(800,12		148	323.82	-4.44	114.13	0.5102	0.9176	0.4802
24	3		25	0.9	N(1000,3		109	331.31	-23.89	97.45	0.4758	0.9117	0.4166
	Average						118	274.13	-71.86	74.87	0.4189	0.8342	0.3270
	STD						14	34.12	37.60	18.44	0.0530	0.0779	0.1050

Table 16: Simulation Output (continued)

	NMACH	Capacity	NPROD	LOAD	Job-Size	Setup	N_JOB	M_FLOW	M_LATE	M_TARD	PROP_T	PRO_T	SETUP
1	10	U(80,120)	5	0.75	U(800,1200)	HIGH	374	325.13	-35.38	102.20	0.4643	0.7447	0.4759
2	10		5	0.75	N(1000,300)		365	337.20	-22.49	112.34	0.4912	0.7539	0.4885
3	10		5	0.9	U(800,1200)		434	405.95	46.69	168.34	0.5481	0.9142	0.5354
4	10		5	0.9	N(1000,300)		426	395.75	37.04	151.41	0.5471	0.9022	0.5903
5	10		15	0.75	U(800,1200)		363	215.96	-144.51	47.40	0.3279	0.7541	0.2316
6	10		15	0.75	N(1000,300)		359	209.82	-148.71	39.67	0.2855	0.7514	0.2036
7	10		15	0.9	U(800,1200)		438	251.74	-108.35	63.69	0.3881	0.9014	0.2663
8	10		15	0.9	N(1000,300)		426	258.71	-98.26	64.57	0.3804	0.9042	0.2968
9	10		25	0.75	U(800,1200)		357	217.44	-141.24	47.37	0.3292	0.7478	0.2291
10	10		25	0.75	N(1000,300)		375	217.43	-135.43	49.86	0.3387	0.7519	0.2427
11	10		25	0.9	U(800,1200)		429	253.53	-102.03	67.78	0.3828	0.8992	0.2405
12	10		25	0.9	N(1000,300)		441	253.71	-97.38	63.84	0.3674	0.9072	0.2685
13	10	U(50,150)	5	0.75	U(800,1200)		344	322.27	-25.73	106.77	0.4906	0.7623	0.4268
14	10		5	0.75	N(1000,300)		359	337.03	-46.66	92.37	0.4598	0.7410	0.4739
15	10		5	0.9	U(800,1200)		464	364.59	7.36	131.19	0.5218	0.8571	0.4972
16	10		5	0.9	N(1000,300)		390	403.07	56.41	163.87	0.5502	0.9644	0.5384
17	10		15	0.75	U(800,1200)		383	214.19	-137.94	48.48	0.3252	0.7683	0.2132
18	10		15	0.75	N(1000,300)		335	227.06	-118.73	53.00	0.3563	0.7560	0.1961
19	10		15	0.9	U(800,1200)		440	299.45	-51.32	87.81	0.4486	0.9255	0.2539
20	10		15	0.9	N(1000,300)		426	293.15	-60.05	84.35	0.4366	0.9141	0.2490
21	10		25	0.75	U(800,1200)		363	217.90	-133.83	51.71	0.3370	0.7547	0.2147
22	10		25	0.75	N(1000,300)		366	241.75	-110.44	58.97	0.3706	0.7828	0.2488
23	10		25	0.9	U(800,1200)		419	251.86	-101.23	67.91	0.3774	0.8958	0.2280
24	10		25	0.9	N(1000,300)		419	255.02	-101.95	63.75	0.3916	0.9110	0.2563
	Average						395	282.07	-73.92	82.86	0.4132	0.8319	0.3277
	STD						37	63.16	62.22	37.53	0.0782	0.0783	0.1287