AN ABSTRACT OF THE THESIS OF

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Title: Automation and Evaluation of Graduated Dot Maps

Abstract Approved:

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Dot mapping is a traditional method for visualizing quantitative data, but current automated dot mapping techniques are limited. The most common automated method places dots pseudo-randomly within enumeration areas, which can result in overlapping dots and very dense dot clusters for areas with large values. These issues affect users' ability to estimate values. Graduated dot maps use dots with different sizes that represent different values. With graduated dot maps the number of dots on a map is smaller and the likelihood of overlapping dots is smaller. This research introduces an automated method of generating graduated dot maps that arranges dots with blue noise patterns to avoid overlapping dots and uses clustering algorithms to replace densely-packed dots with dots of larger sizes. A user study comparing graduated dot maps, pseudo-random dot maps, blue noise dot maps, and area-proportional circle maps with almost 300 participants was conducted. Results indicate that map-users can interpret graduated dot maps more accurately than the other map types. In addition, map users appear to prefer graduated dot maps to the other map types. These findings suggest that graduated dot maps are more effective and more appealing than conventional dot maps.

Automation and Evaluation of Graduated Dot Maps

by
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Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Nicholas D. Arnold, Author

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1. Introduction

Dot mapping is a method of cartographic symbolization for presenting quantitative information. Dot maps have a distinct advantage over other quantitative symbolization methods because dot maps depict both density and absolute values. Individual dots can be counted or estimated while the distances between dots shows the density of the phenomenon.

The selection of dot size is an important consideration when designing a dot map. Dots that are too small can make a distribution seem sparse and insignificant and dots that are very large can make a distribution seem excessively dense (Robinson et al. 1995, p. 498). The selection of a dot unit value is equally as important. If the unit value is too large, no dots will be placed in areas with lower quantities, and if the unit value is too small, dots may coalesce to form large dark regions (Mackay 1949). Traditionally, there are two divergent schools of thought (Monkhouse and Wilkinson 1978, p. 27): Some posit that dot sizes and values should be chosen such that the dots just begin to coalesce in the area with the highest density of dots (Dent 1985, p.161); others argue that dots should not touch to ensure dots are easily countable (Imhof 1972). Hey (2012) states that dot overlap reduces the legibility of the map and "the overlap of dots interferes with every attempt at counting the dots". This is a relevant argument, but two problems emerge: (1) dot coalescence cannot be avoided in cases where outliers with extremely high values are present; and (2) algorithms in commonly available software do not allow the user to control the coalescence of dots. Furthermore, automated methods rely upon randomly placing dots, which can lead to artificial local clusters of dots. This clustering is often misleading, as the dot patterns suggest a spatial pattern in the data that does not exist.

Graduated dot maps improve upon dot maps by addressing issues of countability of large numbers of dots, and the coalescence of dots. Graduated dot maps use classes of differing dot sizes, each of which corresponds to a proportionally larger unit value. The graduated dot map in Figure 1, for example, uses three dot sizes to visualize a spatial distribution that varies between very dense and very sparse areas. The large dots clearly illustrate pockets of high concentration, while the small dots effectively illustrate the sparse presence of the phenomenon along valley bottoms throughout the mapped area. Conceptually, there are two key advantages to graduated dot maps. First, graduated dot maps reduce the number of dots, so errors in estimation are expected to be smaller. Second, small values can be mapped because multiple dot unit values are used. Despite these advantages, the primary reason this technique is not widely used is due to a lack of both available software and algorithmic methodology described in the literature. Most cartographic textbooks that discuss dot maps do not consider graduated dot maps (for example, Tyner 2010, Slocum et al. 2009, Dent et al. 2009, Robinson et al. 1995) or do so only very scarcely (Kraak and Ormeling 2011, Hake et al. 2002). An exception is Imhof (1972) who discusses design consideration for the combination of graduated dot maps with area features and proportional diagrams.

The goal of this research is to propose a methodology for creating graduated dot maps, evaluate their performance compared to conventional dot maps and area-proportional circle maps, and test whether users prefer graduated dot maps to other visualization techniques.

The proposed algorithmic method for creating graduated dot maps does not pseudo-randomly place dots, but arranges dots in a distribution that exhibits blue noise characteristics, wherein dots have "a large mutual distance and no apparent regularity artifacts" (Balzer et al. 2009). Regions of dense coalescing dots are identified using a clustering algorithm and are replaced with dots of a larger size and unit value.

A user study with almost 300 participants was conducted to evaluate graduated dot maps compared to random dot maps, dot maps with blue noise patterns, and area-proportional circle maps. Results of the user study indicate that graduated dot maps are the preferred method and they do outperform conventional dot maps.

Figure 1: Graduated dot map with three dot sizes for 200, 1000 and 5000 swine (©Atlas of Switzerland, sheet 51, 1977, www.atlasofswitzerland.ch).

2. Literature Review

It is not clear when the first graduated dot map was produced. Robinson (1982) discusses a map by Petermann (1857) that shows the population of Transylvania using area-proportional circles, which were placed to illustrate the population distribution of all towns and farmsteads. While Petermann's map is a hybrid between a dot map and an area-proportional symbol map, various cartographers have used graduated dot maps since then. An early example is a map by Penck (1921, described by Herb 1997) showing the population of Silesia; a later exemplary set of maps is included in the Atlas of Switzerland (1977) showing farm animals (Figure 1).

Various manual and automated techniques have been proposed for conventional dot mapping. In addition, a number of studies have addressed the design considerations and perceptual implications of conventional dot maps. These studies have resulted in a set of design principles and tools to develop dot maps.

2.1. Dot Mapping Techniques

When dots are placed manually, cartographers generally use one of three approaches: uniform, geographically weighted, and geographically based (Slocum et al. 2009, p. 322). When dots are uniformly distributed, one technique consists of placing dots near the center of the enumeration area, with each successive dot being placed in the largest remaining space (Mackay 1949). This is a choropleth mapping technique that is valuable for bivariate or multivariate maps when area color is used to depict different information. This method has the disadvantage that boundaries of enumeration areas are easily detected and the geographic distribution inside enumeration areas is not represented. In the geographically weighted approach, dots are placed such that they are shifted closer to neighboring enumeration areas of higher value, which creates the impression of a continuous phenomena being mapped because enumeration area boundaries are less visible. The geographically based approach places dots based on the use of ancillary data such as land cover information. Although these methods are common in manual cartography, automated techniques that apply these three approaches are not widely available.

The most common automated method of producing dot maps is to pseudo-randomly place dots in an enumeration area. This method uses a random number generator to calculate coordinates of dot locations. Random dot placement is not a common approach in manual cartography because there is no evidence to support random placement and it can lead to unrealistic clusters and gaps in the dot pattern that imply spatial patterns that do not exist (Slocum et al. 2009). Dent et al. (2009) recommend the use of zones of exclusion, which follows the geographically based approach and is available in common GIS software. Zones of exclusion are created with ancillary data to define regions where dots are not to be placed.

Prior to 1949, cartographers selected dot size, unit value, and placement without tools to assist them. In a landmark study, J. Ross Mackay (1949) developed a nomograph to assist in the selection of dot size and value. With the nomograph, a ratio of dot size to unit value is identified that Mackay calls "the zone of coalescence", which enables cartographers to estimate the point at which dots will begin to coalesce, given the size and number of dots per square inch. Although the nomograph has been a widely used tool in cartography, Kimerling (2009) points out that the nomograph has "serious drawbacks in the modern age of computer cartography". He extends Mackay's nomograph to include an automated method to define the amount of dot overlap.

Another method proposed by Lavin (1986) addresses the problem of clusters and gaps in spacing of dots. His dot-density shading technique does not assign a specific unit value to dots; information can only be derived through dot numerousness and spacing (Lavin 1986). Lavin's method is well suited for continuous data, as he suggests, but exact data values cannot be extracted, despite dots being a discrete symbol. He argues that the sheer numerousness of dots in various proximities gives the impression of continually varying tonal values, even though the texture is very coarse.

Based on the suggestion that overlap of dots interferes with countability of dots, Hey (2012) proposed a method to produce dot maps with spiral patterns that do not have overlapping dots. An Archimedean spiral pattern, which is characterized by equidistant change between each spiral revolution, is used to determine the placement of "spiral arms", wherein multiple curves of dots are placed such that dots radiate out from a single point. Although dots do not overlap, the dot clusters still have a very regular appearance. Hey and Bill (2014) refined the spiral-inspired method by introducing a new dot arrangement, addressing the regular appearance. The method is based on calculating potential dot positions for a given area. Larger dots are to reserve the space in which smaller real dots may wander. When calculating the final dot positions on the map, dots are shifted within the reserved space for potential dots to reduce pattern regularity. Dots are allowed to touch but do not overlap.

Berg et al. (2004) studied a problem relating to dot numerousness that has utility in dot mapping. The primary question they study is: Given a point set representing a certain distribution, how can it be automatically simplified, generating a smaller point set? Most relevant to our research, Berg et al. (2004) tested several heuristic algorithms to simplify a point set and generate an approximation of the original dots with the smallest error. Tests included two groups of heuristics: (1) iterative algorithms and (2) clustering algorithms. The most performant of iterative algorithms with the best approximation of the original points were "swapping" heuristics, though they require a good starting configuration (Berg et al. 2004). The second group tested three clustering algorithms: (1) Rows and Columns; (2) Quadtrees; and (3) Dobkin-Tal, each of which require the number of groups as a parameter. Results indicate that the "rows and columns" heuristic is a very fast method of obtaining good quality solutions (Berg et al. 2004). Although these methods could be applied to dot mapping, for our research, the method of finding clusters in dots should not require the user to predefine the number of clusters.

Graphical design principles for generalizing dot maps have been studied by Spiess (1990), and Yan and Weibel (2008) have developed an algorithm for point cluster generalization. They treat four basic types of information including statistical, metric, thematic, and topological information. The primary objective is to ensure that the four types of information are transmitted from the original data to the generalized result. Based on Voronoi diagrams, the method follows three basic procedures: (1) compute a distribution range, which defines the area that dots are potentially placed; (2) delete dots based on their selection probability; and (3) determine the number of dots in the final set. The algorithm presents a potential approach to generalizing clusters in graduated dot mapping applications.

2.2. Dot Map Readability

User studies about dot maps have largely focused on user perception of dots. Provin (1977) indicates that most early work in the area of dot perception was done by psychologists. Freeman (1911) indicated that adults were far better at estimating numbers than children when dots were irregularly spaced. According to Taves (1941), when small numbers of dots were present, users were able to accurately estimate dots however, when seven or more dots were present, user accuracy decreased and dots were underestimated. Kaufman et al. (1949) reaffirmed findings of Taves (1941) and introduced the concept of subitizing to describe how users perceive small numbers of stimuli.

Several cartographic studies dealing specifically with dot maps have been carried out. Olson (1975) tested users ability to estimate dot density, and found that underestimation of dot number is all but universal (Provin 1977). Mashoka et al. (1986) compare readability and preference of dot maps versus proportional circle maps. Their findings further demonstrate that users underestimate numbers of dots and that proportional circle maps were favored over dot maps for their accuracy and simplicity.

3. Method for Creating Graduated Dot Maps

The proposed method for creating graduated dot maps starts by creating a dot map with pseudo-random distribution. The pseudo-randomly placed dots are then rearranged in a blue-noise pattern with the capacity-constrained Voronoi tessellation (CCVT) algorithm, which disperses dense groups of dots such that they have a larger mutual distance while maintaining the density distribution of the original dots. This step does not eliminate coalescing dots in areas where dots are dense. Next, the Density-based spatial clustering of applications with noise (DBSCAN) algorithm is used to identify and subsequently remove dense clusters of dots. The removed dots are then used in another iteration of the CCVT blue-noise algorithm to create the next class of dots of a larger size and unit value. This process is repeated, as many times as there are sizes of dots.

3.1. Creating a Blue-noise Dot Pattern

The term blue noise refers to an even, isotropic, yet unstructured distribution of points (de Goes et al. 2012). This distribution exhibits a spectral density distribution with minimal low frequency components, no spikes in power and translates to dots having "a large mutual distance and no apparent regularity artifacts" (Balzer et al. 2009). Blue noise sampling distributions are ubiquitous in computer graphics (Pharr and Humphreys 2004) and have useful perceptual characteristics that we utilize for creating dot maps (Figure 2). Because blue noise distributions have well-dispersed dots, we are able to avoid local clusters of dots that would imply fictitious spatial patterns.

We use the CCVT algorithm, as proposed by Balzer et al. (2009), to produce blue noise patterns on dot maps. This particular algorithm provides three important functions for the creation of graduated dot maps: (1) it reduces the number of dots; (2) optimizes the distribution of points; and (3) maintains the density distribution of the original dots. The property of reducing the number of dots is important because it is used to develop each dot class. The CCVT partitions space into Voronoi regions and iteratively optimizes the placement of dots. Balzer et al. (2009) note that the number of iterations has a direct effect on the quality of the distribution of dots, pointing out "if the method is not stopped at a suitable iteration step, the resulting point distributions will develop regularity artifacts". In order to avoid these regularities and stop the algorithm, Balzer et al. (2009) introduce a "capacity-constraint", which simultaneously reduces the number of dots for a region while maintaining the original density of the region. The capacity-constraint is a modifiable parameter *k* that we use to reduce the number of dots.

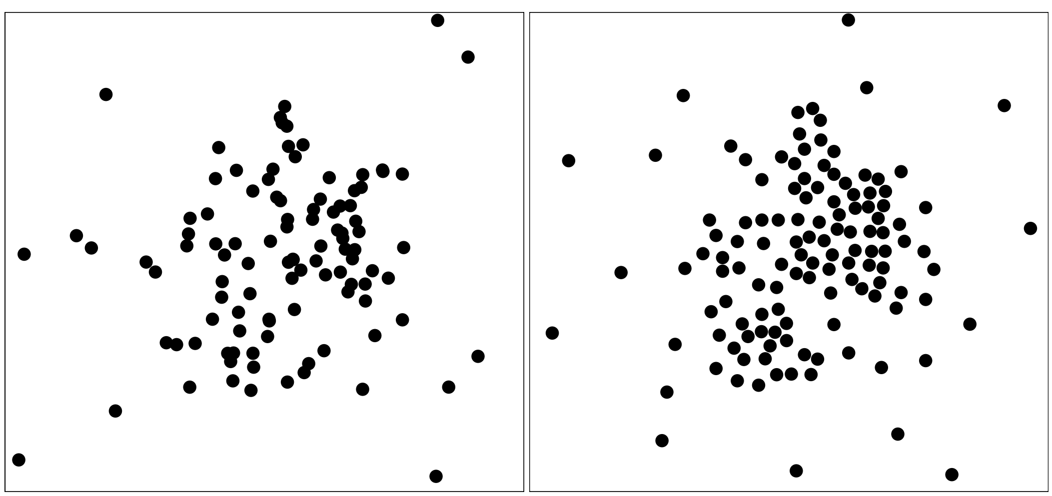

Figure 2: Dots with a pseudo-random distribution (left); dots with blue-noise pattern (right).

3.2. Identifying Dot Clusters

The DBSCAN clustering algorithm was proposed by Ester et al. (1996), and has several advantages over other clustering algorithms for this application. Many clustering algorithms require the number of desired clusters as an input parameter, but in the case of identifying spatial dot clusters, the number of clusters is unknown. Moreover, other techniques require a search distance to identify clusters, which is also an impractical requirement. DBSCAN is a density-based clustering algorithm that requires two parameters. The user selects a density threshold and the minimum number of dots-per-cluster as input parameters. The minimum number of dots parameter is crucial because it determines the minimum number of dots required to constitute a cluster. For example, if the minimum number of dots is four, then three dots that coalesce will not be identified as a cluster and will remain in the distribution. We set the minimum dots parameter to two dots, in an attempt to not allow any dots to coalesce.

The DBSCAN clustering algorithm uses the concept of core and density-reachable points, wherein an arbitrary starting point is selected and its core points are identified using the density threshold. Based upon the minimum number of points chosen, if there is a sufficient number of density-reachable points, a cluster is identified, otherwise the points are considered noise. Although points can be originally classified as noise, they can then be reclassified to a cluster point if a cluster is density-reachable. The algorithm iterates through all points until each point is identified as a cluster point or noise point. The primary advantages of the DBSCAN algorithm are that we only need to specify the density of dots that constitute a cluster and can discover clusters of arbitrary shape (Ester et al. 1996).

3.3. Combining CCVT Blue-noise and DBSCAN Clustering Algorithms

The CCVT blue-noise algorithm and the DBSCAN clustering algorithm are used iteratively to produce a graduated dot map. The input for this algorithm consists of m enumeration areas with values v_m to be mapped, and n dot unit values d_n ordered in increasing order. The output is n sets s_n of dot coordinates.

The initializing step produces a set of pseudo-random dot locations s_0. The following CCVT blue-noise algorithm will reduce the number of dots by a factor k_0, so the dot unit value d_0 for s_0 is $d_0 = d_1 / k_0$. For example, if the unit value of the smallest class of dots is to be 2000 units and k_0 is 2, the dot value d_0 is 1000. The number of pseudo-randomly placed dots per enumeration area is v_m / d_0.

The following procedure is then iteratively applied to create the n output dot sets. The CCVT blue-noise algorithm is run on s_n with the reduction factor k_n. This results in a reduction of the number of dots in s_n by a factor of k_n. Dots in s_n have blue-noise characteristics, and are potentially arranged in dense clusters. The DBSCAN clustering algorithm is run on s_n, which marks each dot as pertaining to a cluster or as a noise dot. Dots that are part of a cluster are removed from s_n and added to s_{n+1}. The final set s_n only contains noise dots. The reduction factor for the next iteration is computed with $k_{n+1} = d_{n+1} / d_n$.

This procedure is executed n times, creating the sets s_n. Note that for the last set the DBSCAN clustering algorithm is not run and no clustered dots are removed. The user selects the number n of dot classes and the dot unit values d_n. Although it is possible to produce many classes of dot sizes, we have found that three classes is a good number in order to avoid confounding users' ability to differentiate between dot sizes. The reduction factor for the initializing step k_0 is the only parameter that users cannot be expected to be specify. In our experiments we used $k_0 = 2$, which resulted in visually satisfactory results.

Figure 3a shows the initialization step, where the set s_0 of pseudo-randomly distributed dots is produced. The following CCVT blue-noise algorithm reduces the number of dots in s_0 by half and stores them in s_1, the unit value is d_1, and the distribution has blue noise characteristics, shown in Figure 3b. The iterative procedure of identifying clusters and replacing clustered dots with fewer, larger dots, begins with using the DBSCAN clustering algorithm to identify clusters from s_n, shown in Figure 4a. Once these dots are identified, they are removed from s_n and added to s_{n+1}. Dots from s_{n+1} are used for the next iteration. Figure 4b shows the result of the next iteration, where the s_n dots from the first iteration were retained and the dots from s_{n+1} were run through the algorithm again. Figure 5 shows the final result of two iterations, which produced three dot size classes.

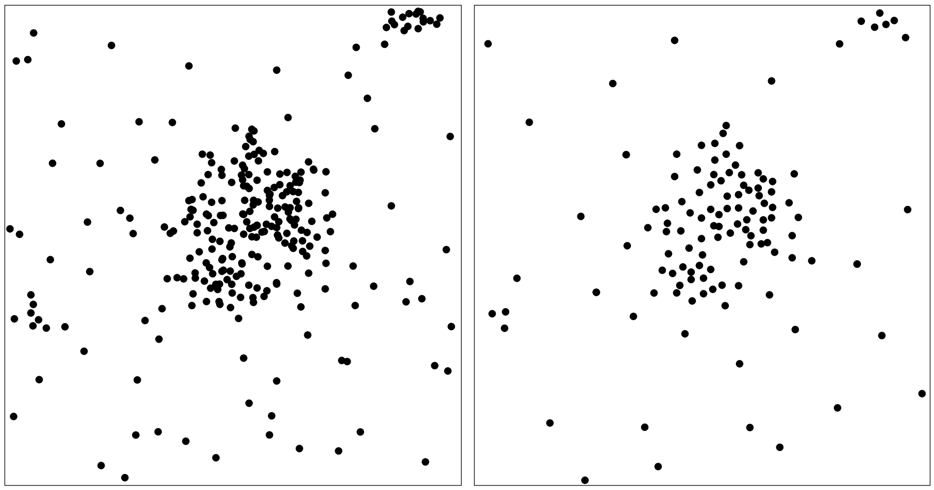

Figure 3a: Pseudo-random dots for the initialization step (left); Figure 3b: CCVT algorithm reduces the number of dots by a factor of two and creates a blue-noise distribution (right)

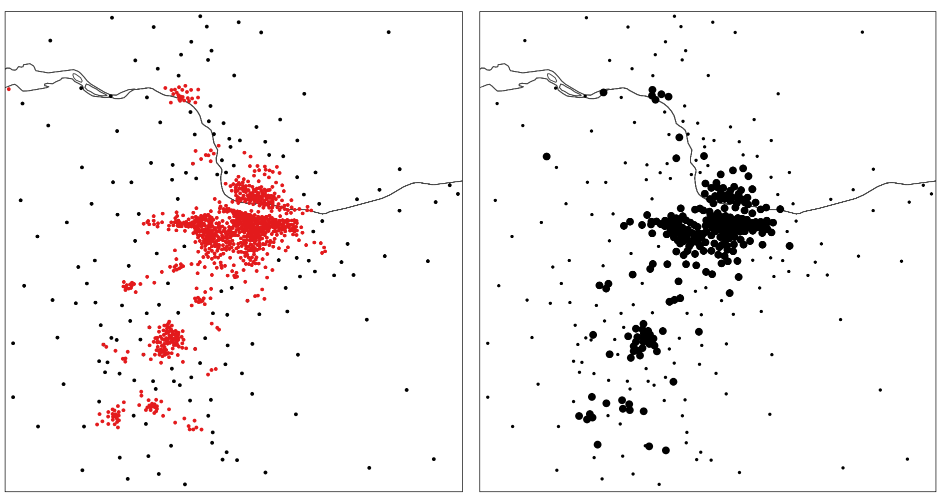

Figure 4a: Clusters identified by the DBSCAN algorithm in red (left); Figure 4b: Dot cluster replacements by the CCVT blue-noise algorithm (right).

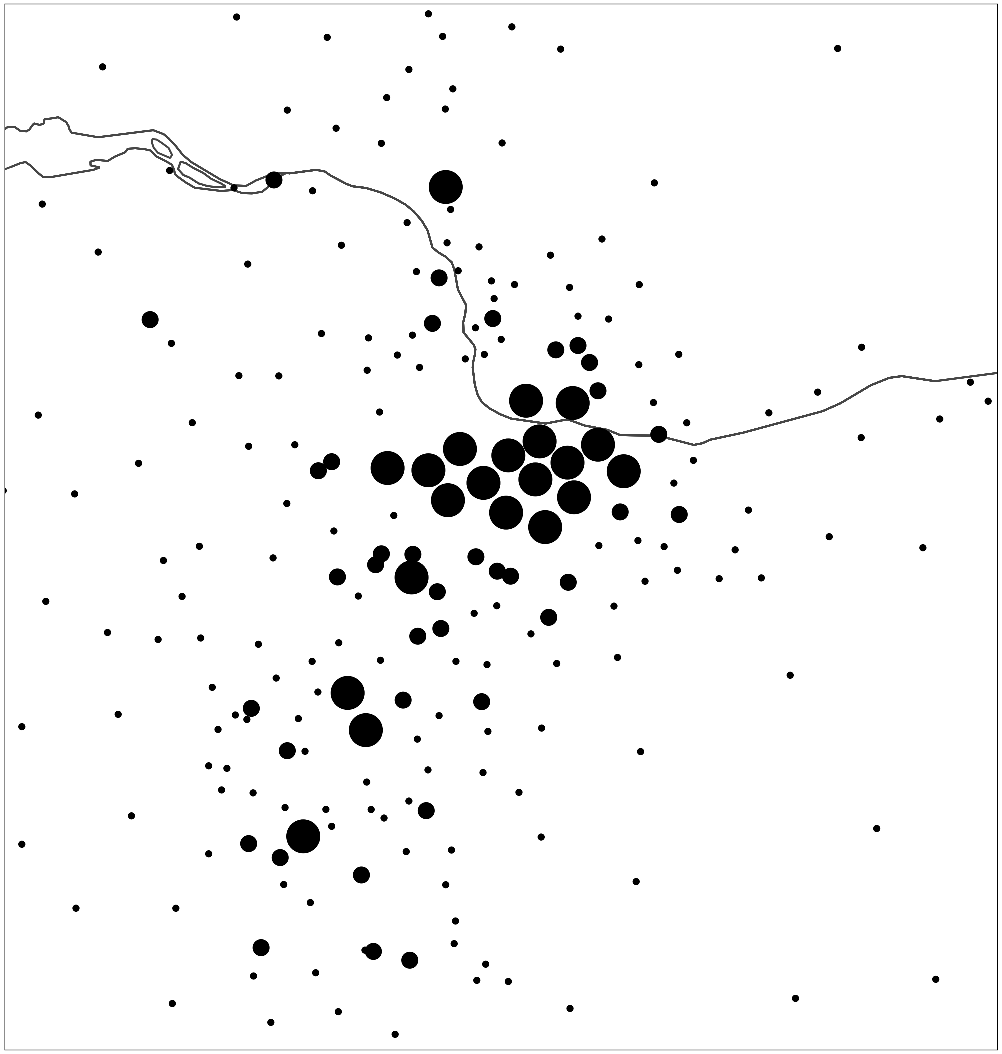

Figure 5: Final graduated dot map with three classes.

3.4. User Evaluation

The objectives of a user study were to evaluate estimation accuracy and user preferences of graduated dot maps compared to other map types. We hypothesize that users will more accurately estimate values for graduated dot maps than the other map types and that users will prefer graduated dot maps over the other map types. The user study compared four map types: dot maps with a pseudo-random dot distribution, dot maps with a blue noise dot distribution, graduated dot maps, and area-proportional circle maps. Perceptual scaling has been applied to the area-proportional circle maps. Proposed by J.J. Flannery, perceptual scaling allows for compensation of the expected value underestimation (Flannery 1971, Slocum et al. 2009, Dent et al. 2009). The graduated dot maps in the user study were created with the described method, and census tract population data were used to place pseudo-random dots for the initialization step.

The user study was built with the Qualtrics survey platform and participants were recruited via Amazon Mechanical Turk, a web-based crowdsourcing service where users complete tasks or surveys and receive micro-payments. Heer and Bostock (2010) found that crowdsourcing is viable for testing graphic perception and provides high-quality responses. Respondents were paid $1.00 for completing the survey. The user study consisted of a demographic survey, a short map reading tutorial, a series of timed map-reading tasks, a map preference survey, and a question whether participants attempted to count or estimate dots. Users were not permitted to go back to any questions once a response was submitted. For the timed map-reading tasks, participants were shown dot maps with pseudo-random distributions, dot maps with blue noise distributions, graduated dot maps, and area proportional circle maps. The preference questions evaluated each of the map types for clarity and preference.

The demographic survey collected information regarding participants' gender, age, country of residence, and education level and was followed by a tutorial covering the map-reading tasks. The tutorial included explanations of how to read conventional dot maps, graduated dot maps, and area-proportional symbol maps and showed a legend for each map type. Two untimed example questions were shown to familiarize users with the questions and then two timed questions were shown. Participants could repeat the tutorial if desired.

The map-reading tasks included 15 maps with pseudo-random dots, 15 maps with blue-noise dots, 15 maps with graduated dots, and 5 area-proportional circle maps. Each dot map had one area highlighted in gray, and users were asked to estimate the value represented by dots for this area. Each area-proportional symbol map had one circle highlighted in gray and users were asked to estimate its value. All pseudo-random and blue-noise dot maps used the same unit value (200). All graduated dot maps used the same three unit values (1000, 10000, and 100000). Participants were shown all maps of one type before being shown the maps of other types. The same enumeration areas were used for the four types of maps.

For each of the three dot map types there were 10 maps with realistic administrative boundaries and 5 maps overlaid with a regular grid, as in Figure 6. To minimize learning effects, the mapped values were changed for each map, the order of maps within each type group was randomized, the enumeration areas of 5 maps were replaced with a regular grid, and maps with administrative boundaries were rotated (Figure 7). For each map, participants were given 10 seconds to view the map and were asked to estimate the value of the gray area. After 10 seconds elapsed, the map disappeared and respondents were required to enter their estimate. The legends for all maps of one group were identical and were shown before the timed maps appeared, to prevent participants from losing time to familiarize themselves with the legend. Each legend for the single-size dot maps showed the individual dot value as well as a sample of three varying densities of dots (Figure 7). Samples of varying densities are common in dot map legends, after Provin (1977) tested the effects of legends on estimating dot values and noted that users showed a marked improvement in average estimates when such legends were present.

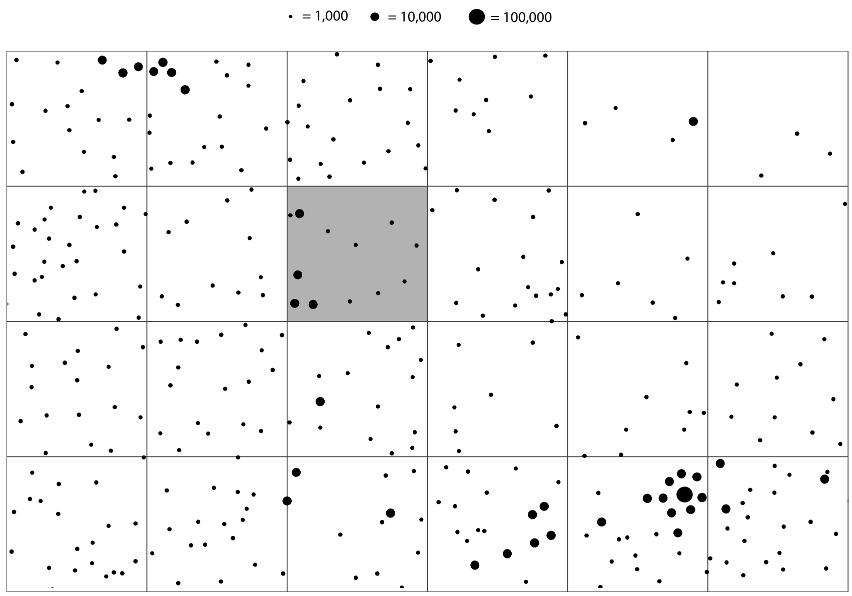

Figure 6: Example graduated dot map with grid overlay for the user study. Subjects are asked to estimate the value for the gray square.

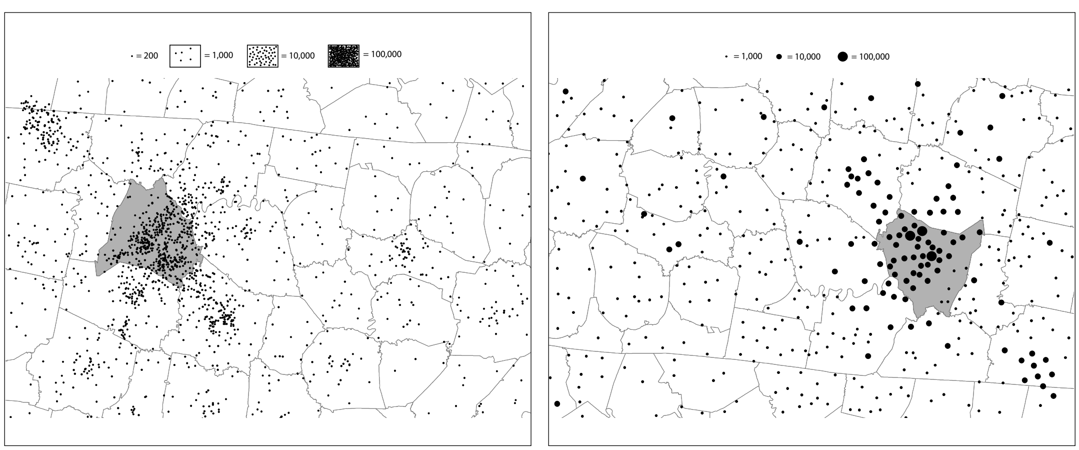

Figure 7: Example maps for the user study. Subjects are asked to estimate the value for the gray areas. Geometry is rotated for the second map.

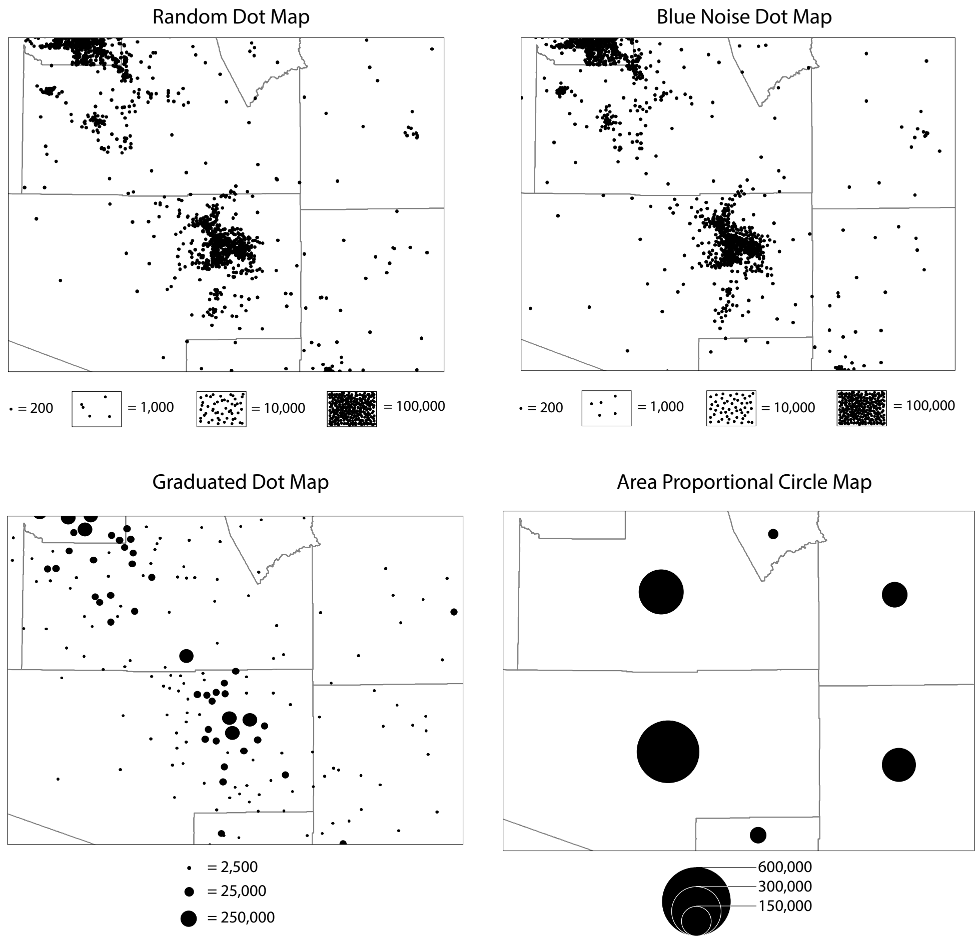

Figure 8: One of two 2 × 2 image matrices for preference map questions.

Five additional test maps with easily readable values were placed throughout the survey. Four test maps were dot maps with five to seven dots in the enumeration area of interest; and one test map showed area-proportional circles with the gray test circle having the same size as one of the circles in the legend. The test maps were used to eliminate responses from participants who entered random values rather than attempting to estimate values.

For the first map preference question, participants were shown two sets of each map type individually and asked to rate each map from 1 to 5 based on 'Clarity & Legibility' and 'Preference & Appeal'. A rating of 5 for both questions indicates that the map is 'Very Clear' and 'Very Appealing', whereas a rating of 1 for both questions indicates 'Not Clear' and 'Not Appealing'. The second type of map preference question showed participants two 2 × 2 image matrices (Figure 8) containing each map type, and participants were asked to rank the maps on each matrix. Responses ranged from 1 to 4, 1 being their 'favorite' and 4 being their 'least favorite'. Appendix 2 shows each of the specific test questions and appendix 3 shows each of the associated test maps.

4. User Evaluation Results

Of the 420 participants in the user study, 123 did not correctly respond to at least 3 of the 5 trivial test maps. The results of these respondents were discarded and only the results from the 297 remaining participants were analyzed. Of the 297 participants, 149 were female, 147 were male, 58 percent were under the age of 35 and 68 percent have completed some level of college education. A total of 264 participants are from the United States, 28 are from India, and five are from other countries.

4.1. Dot Map Estimates

For the dot map estimation tasks, which asked respondents to estimate the value for regions on the map, each of the 15 map sets, containing each type of map, was compared. The Kruskal-Wallis one-way ANOVA test was used to determine if each map group showed a significant difference in the distribution of estimates. P-values for each map set were <0.001 (see appendix 6), indicating that the results are significant. Figure 9 shows the rate of error for each dot map compared to the total number of dots per test area. Appendix 4 shows response statistics and rates of error for each dot map. The scores indicate that user estimations of blue noise dot maps were more accurate than random dot maps, and graduated dot maps are more accurate than blue noise dot maps. In addition, the accuracy of user estimation is highly correlated to the number of dots per estimated value, with increasing numbers of dots resulting in increasing relative error. Error rates for graduated dot maps are low for enumeration areas with small numbers of dots; graduated dot maps with larger numbers of dots do not outperform conventional dot maps. Our results also support previous findings that users all but universally underestimate dot values (Provin 1977).

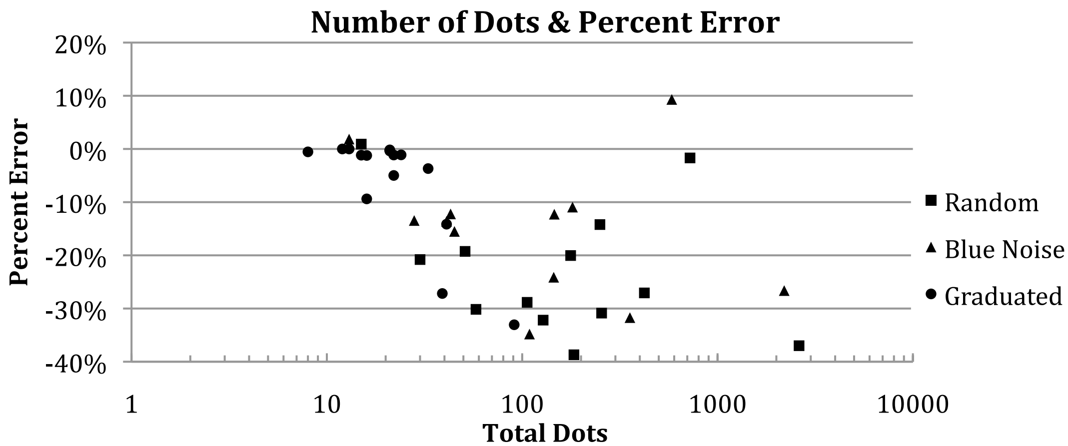

Figure 9: Number of dots per test area versus relative error for the 45 dot maps used in the user survey.

4.2. Graduated Dot Maps vs. Area Proportional Circle Maps

Because graduated dot maps are similar to area-proportional circle maps, the results of the five area-proportional circle map estimations were compared to the results of the 15 graduated dot maps. On average, the area-proportional circles were underestimated by 9.3%. The average underestimation for graduated dot maps was 6.5%. The Kruskal-Wallis one-way ANOVA test was used again to determine if there are significant differences in estimations of area-proportional circle maps and graduated dot maps. All tests returned p-values <0.001 (see appendix 6), demonstrating significant differences in estimations between graduated dot maps and area-proportional circle maps. While area-proportional circle maps are less error prone than single size dot maps (average underestimation of 26.4%), area-proportional circle maps are slightly more prone to errors than graduated dot maps with a moderate number of dots. However, graduated dot maps with large values (and a large number of dots) are more error prone than area-proportional circle maps with similar values. For example, the test area in one graduated dot map has a total of 39 dots (3 dots with a value of 100,000, 31 dots with a value of 10,000, and 5 dots with a unit value of 1,000), a total value of 615,000 and an average error of -26.3% while an area-proportional circle map (with perceptual scaling by

Flannery) with a value of 405,500 has an average error of -5.3%. Appendix 4 shows response statistics and rates of error for each dot map and area-proportional circle map.

4.3. Preference and Clarity

The objective for the preference tests was to compare preferences of each map type. For each map type, users were shown the map and asked to provide a numerical Likert-scale response (see appendix 2) for questions of 'Clarity and Legibility' (1 = Not Clear; 3 = Somewhat Clear; 5 = Very Clear) and 'Aesthetic Preference' (1 = Not Appealing; 3 = Somewhat Appealing; 5 = Very Appealing). The participants were asked to rate two sets of maps; see Figure 8 for the four maps of one set.

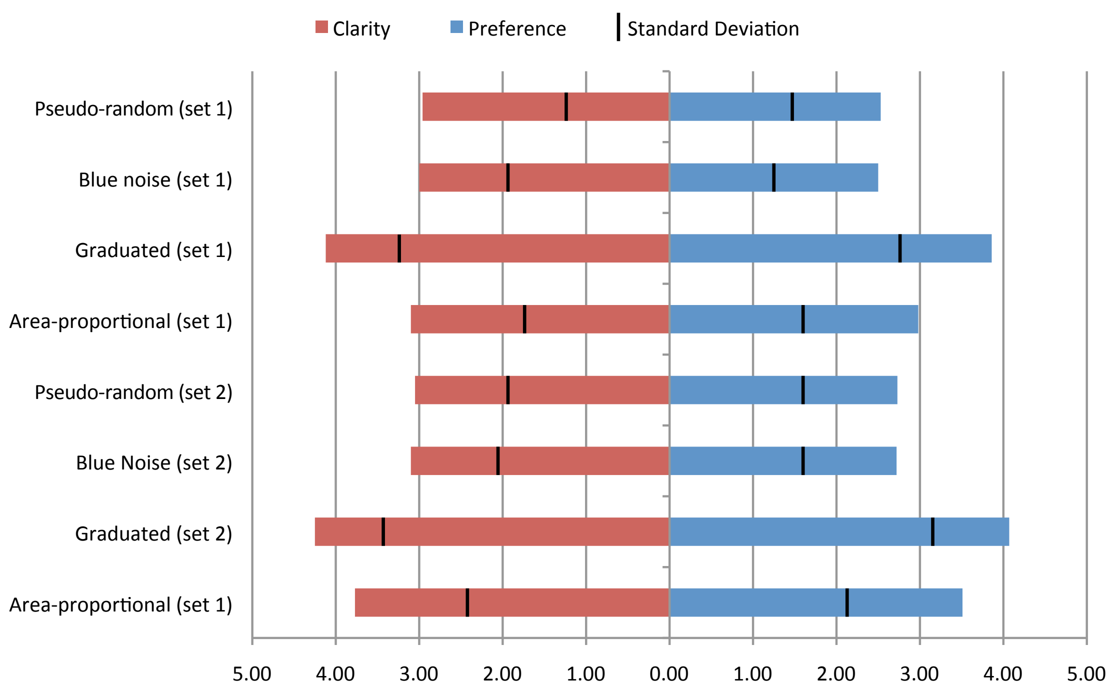

Figure 10: Mean preference and clarity ratings for two groups of four maps of the same area.

Figure 10 shows the average of the preference and clarity responses. Results indicate that users found dot maps with random and blue noise distributions to be least favorable and approximately equal in clarity (mean = 2.5) and preference (mean = 3). Respondents found graduated dot maps to be the most preferred maps with the clearest message. Area proportional circle maps ranked in-between. Results of the Friedman's Test show that there is a significant difference between map types (p-values of <0.001 for all cases) for both sets of test maps (see appendix 6).

4.4. Rank-Order Preference

Subjects were shown the same maps that were used in the preference and clarity question in two 2 × 2 image matrices showing the four types on a single page (Figure 8). Using the matrix, they were asked to rank-order the maps from 1 to 4 (1 = 'favorite' and 4 = 'least favorite').

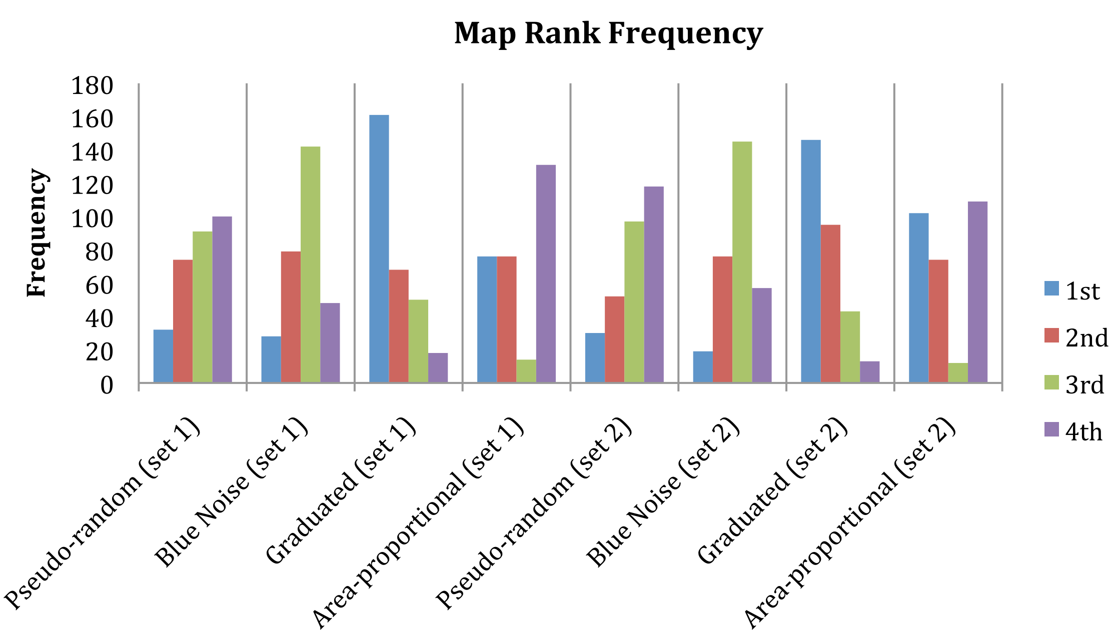

Figure 11: Map rank frequency for two map sets

Figure 11 shows a histogram of responses for each of the map types. The results show a clear pattern of ranks. Users ranked graduated dot maps '1st' more often than any other map type. The results also show that area-proportional circle maps were often ranked '2nd" by users, and blue noise dot maps ‘3rd’. Dot maps with a random distribution were most commonly ranked ‘last’ by respondents. Friedman’s test was used to determine if statistically significant differences were found between each map type in each group. P-values for each test were <0.001, indicating significant differences between each map type.

4.5. Counting vs. Estimating

Participants were asked two final questions: (1) How often users attempted to count the dots; and (2) whether users attempted to count dots more often for graduated dots or single-size dots. For the first question, 78% of respondents indicated that they sometimes tried to count the dots, 19% of respondents indicated that they tried to count the dots every time, and only seven respondents estimated the dots every time. Note that attempting to count the dots for many maps is nearly impossible due to the large number of dots on each map. Responses to the second question were more evenly split. 60% of respondents stated that they counted the graduated dots more often, 39% stated that they counted the single-size dots more often, and only three respondents indicated that they did not count the dots.

5. Conclusion

We present a method for creating graduated dot maps that produces visually pleasing maps with improved estimation accuracy. Our method combines blue-noise dot distributions and a clustering algorithm in an easy to control Java tool, and represents the first automated method for producing graduated dot maps (Appendix 1 shows a screen capture of the graphical user interface of the developed tool, which is available at: http://www.nicknackmaps.com/).

Study participants showed improved accuracy for dot estimation tasks for graduated dot maps compared to conventional dot maps. Our research shows that conventional dot maps result in a high degree of underestimation, which reaffirms the findings of Provin (1977) and Mashoka et al. (1986). Users also underestimate values with graduated dot maps, but to a much lesser extent. We observe that enumeration areas in graduated dot maps with many dots do not have an advantage over conventional dot maps. However, most enumeration areas in graduated dot maps use considerably less dots than enumeration areas on conventional dot maps, which explains the observed advantage. Study participants underestimated the values of area-proportional circles to a slightly higher degree than graduated dot maps, even though Flannery's perceptual scaling was applied to the circles. Study participants preferred graduated dots to conventional dot maps, blue-noise dot maps and area-proportional symbol maps. Respondents indicated that graduated dot maps were clearer, more legible, and more visually appealing than the other maps, and they ranked graduated dot maps as their favorite (ranked 1st) in a rank-order test.

Although the results are favorable for graduated dot maps, there are some limitations to the method and a few open questions. The first limitation is that the notion of dot density can possibly be lost due to the reduction in the number of dots and their placement. Future work is needed to relocate small dots between larger dots to reclaim the density and test the effect on estimation accuracy. Another limitation of the proposed method is that dots are allowed to move outside of their enumeration area when blue-noise dot patterns are created. This is problematic when, for example, terrestrial-related dots are moved over lakes and oceans. Using ancillary data as exclusion areas for placing dots is a potential solution, however future work is necessary to add this additional constraint to the CCVT blue-noise algorithm. Furthermore, the size and displacement of dots in dense regions may create a cartogram effect. Due to large dots being moved outward from the center of small enumeration areas to prevent overlap, it may appear that the data being mapped are for a larger enumeration area, despite underlying geography being undistorted.

Future work is also needed to determine an appropriate number of dot classes for graduated dot maps. We chose three classes of dots in order to prevent confounding users' ability to detect differences between classes, however we do not attempt to evaluate the influence of the number of classes on estimation accuracy. Another open question is related to the concept of subitizing. Coined by Kaufman et al. (1949), subitizing refers to the judgment of small numbers of stimuli, a process that is more accurate, more confident, and more rapid than estimating or counting. Kaufman et al. (1949) indicate that subitizing is done when the number of stimuli is less than six. We hypothesize that graduated dot maps are interpreted with a combination of counting, estimation and subitizing. Future work could evaluate the processes by which users derive values for graduated dot maps. Subitizing seems to also be relevant for the selection of appropriate unit values for graduated dot maps. In order to minimize estimation errors, the unit values could be chosen such that map readers' subitize rather than estimate or count when extracting values for an enumeration area. Optimum unit values could be determined that maximize the number of enumeration areas in a map that only use four or five dots of each class (the numbers suitable for subitizing). The unit values, however, need to be numbers that are simple to sum and multiply and easy to remember, otherwise the advantage of subitizing would be defeated by error-prone calculations necessary to compute enumeration values.

In addition to the number of dot classes, the size of dots represents an area of potential research. Given the purpose of the dot map, future work could determine whether area-proportional dots have an advantage over graduated dots. If the purpose of the map is to allow map readers to count dots, creating area-proportional dot sizes may not be necessary. Conversely, if the purpose is to show density then it could be advantageous to use area-proportional dots.

6. Bibliography

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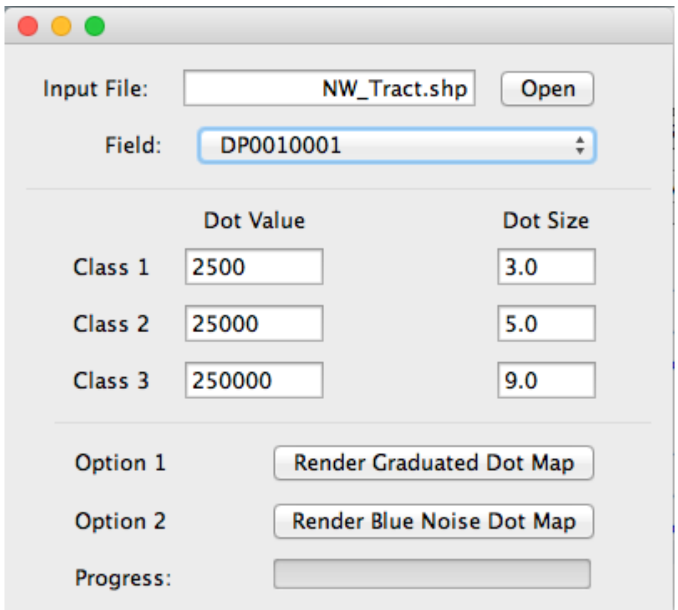
Appendices

1. Figure 12: Graphical User Interface of Java Tool to Create Graduated Dot Maps



The graphical user interface has basic functionality to select the appropriate field for the input shapefile and choose the parameters for the dot map. Users can select the type of dot map they wish to create, which produces a shapefile with the parameters as attributes.

2. Test Questions

The following table includes the four questions posed in the user study. Each question was repeated for each map shown in the survey.

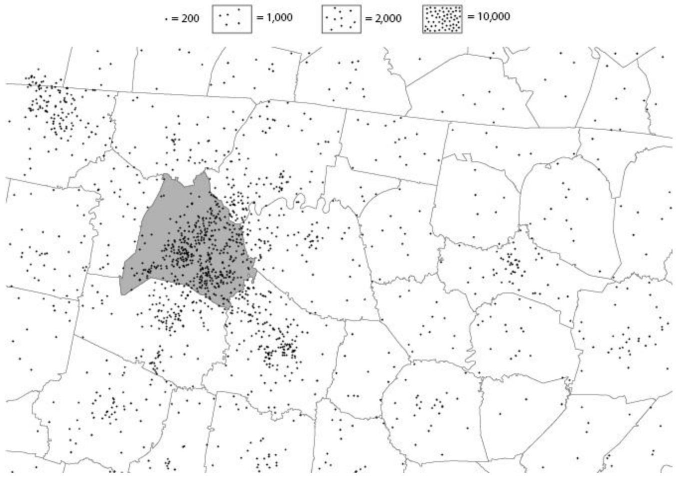
Dot Map Estimates	Estimate the value for the gray area. You have 10 seconds.
Preference & Clarity	In this portion of the survey you will be shown a series of maps and be asked to rate them from 1 to 5 based on two factors: (1) Clarity & Legibility and (2) Aesthetic Preference 1. Clarity & Legibility • 1 = Not Clear • 3 = Somewhat Clear • 5 = Very Clear 2. Aesthetic Preference • 1 = Not Appealing • 3 = Somewhat Appealing • 5 = Very Appealing
Rank-order Preference	Rank the maps from 1 to 4, 1 being your 'favorite' and 4 being your 'least favorite'.
Counting vs. Estimating	1. How often did you count the dots • I never tried to count the dots. I always estimated the values represented by the dots • I sometimes tried to count the dots. • I tried to count the dots every time. 2. Did you count more often with dots with different sizes or dots with a single size? • I counted the dots with different sizes more often. • I counted single-size dots more often. • I did not count the dots for either sizes.

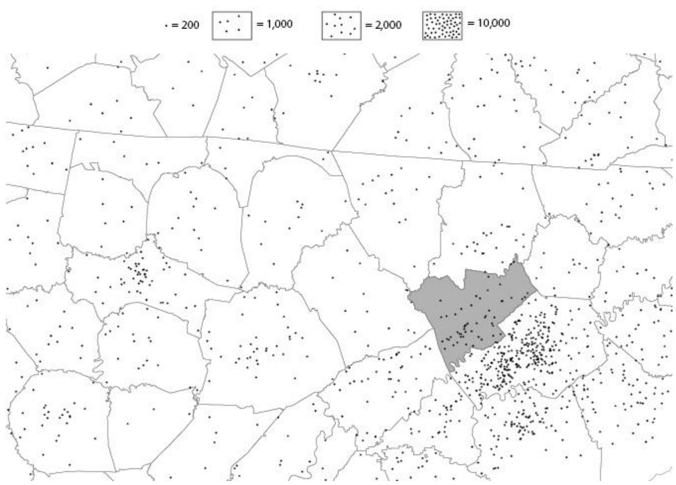
Table 1: Four questions used for the user study.

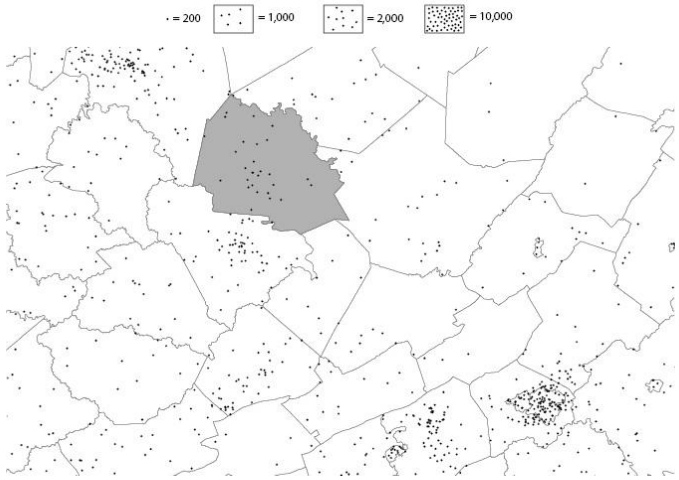
3. Survey Test Maps

The following section contains all test maps used for the user study, including 50 maps used for estimation questions and 8 maps used for preference questions.

Figure 13: Dot Map Estimates: Pseudo-Random Dot Maps



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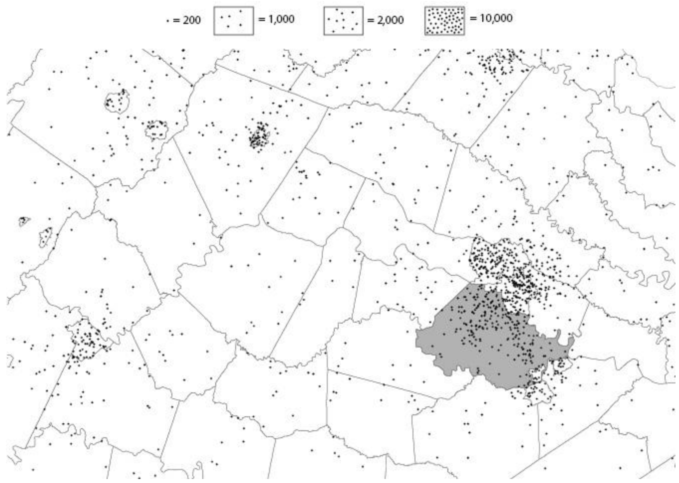
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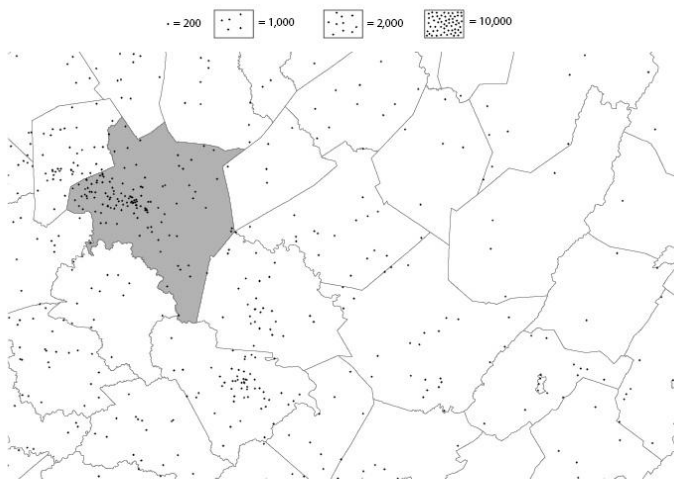
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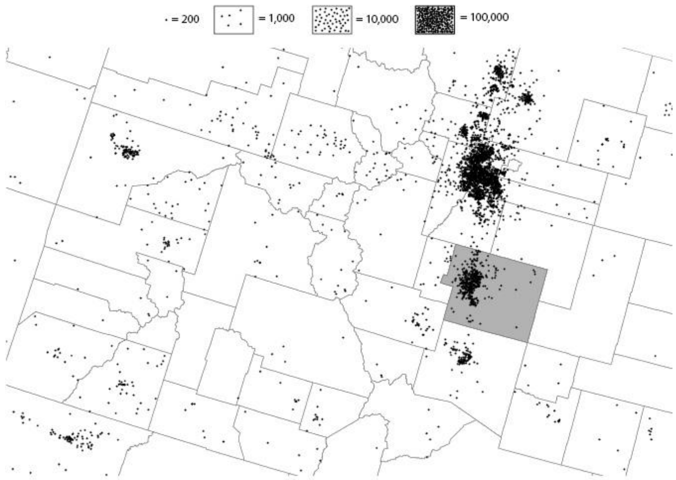
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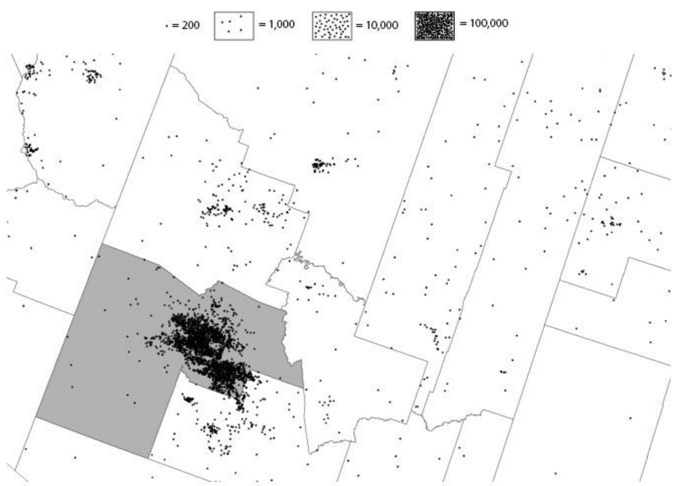
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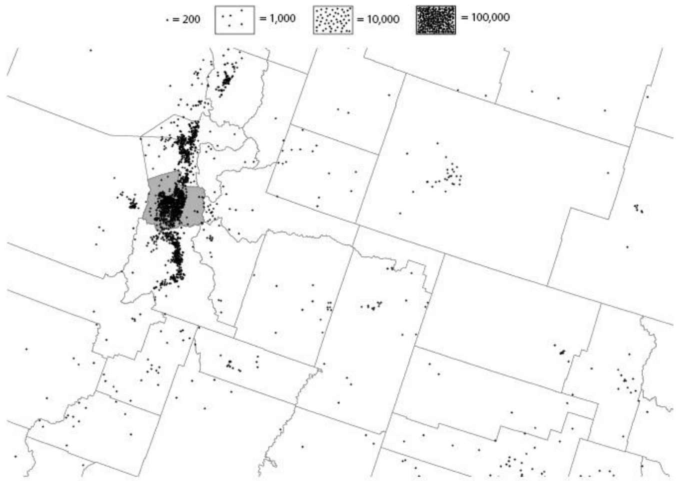
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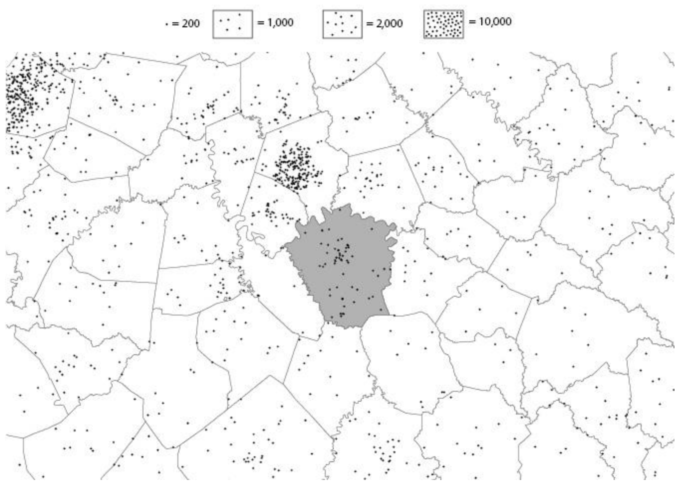
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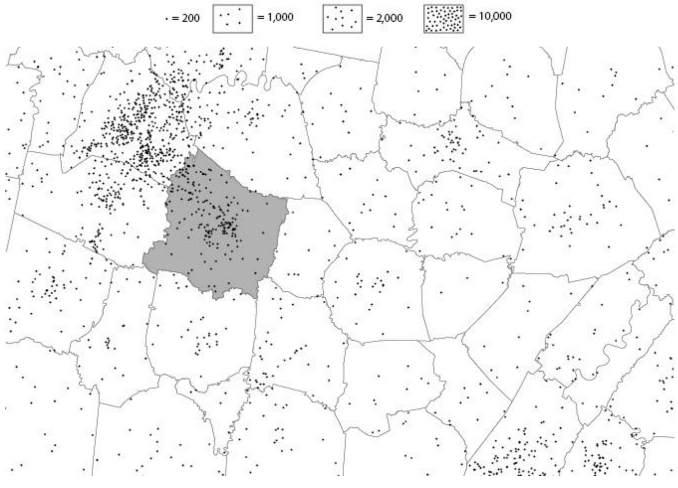
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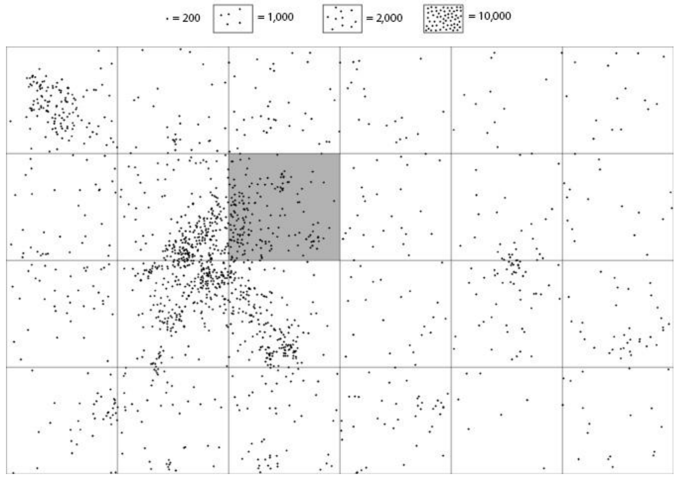
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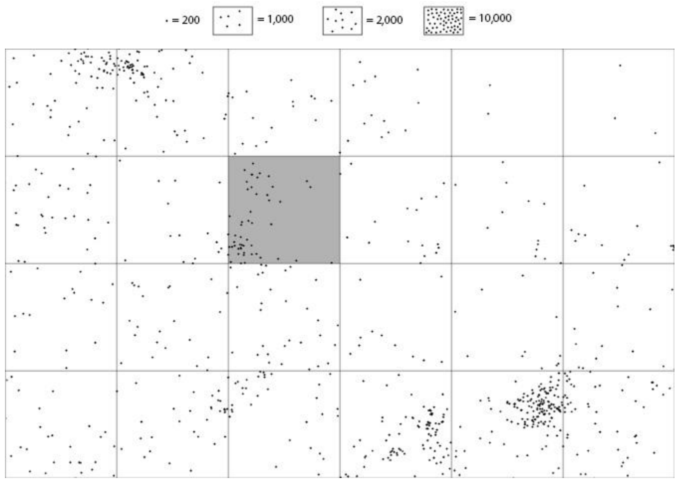
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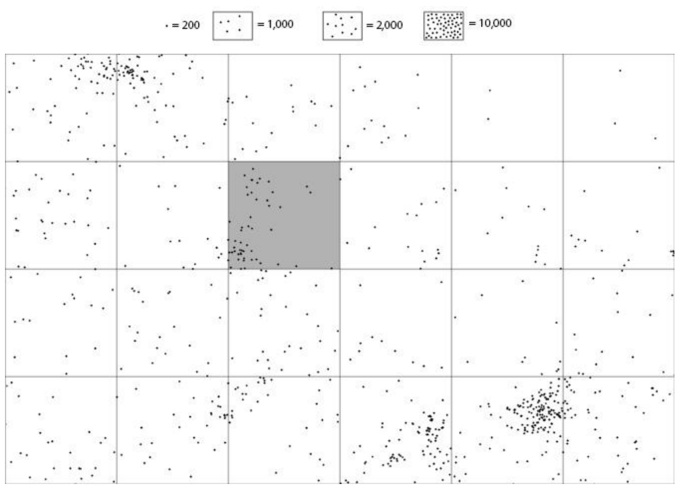
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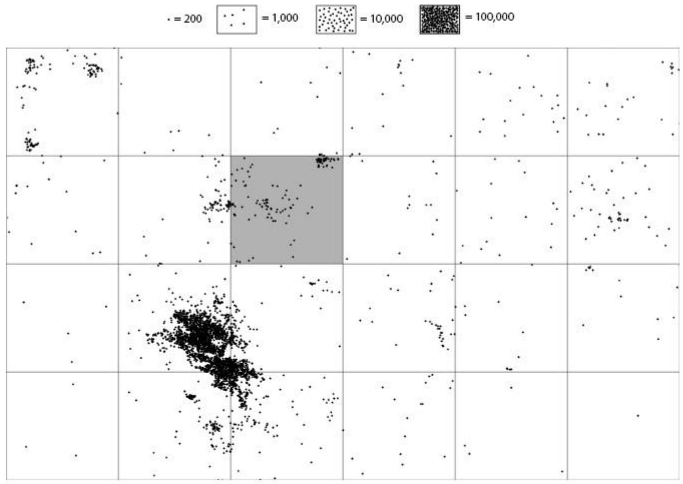
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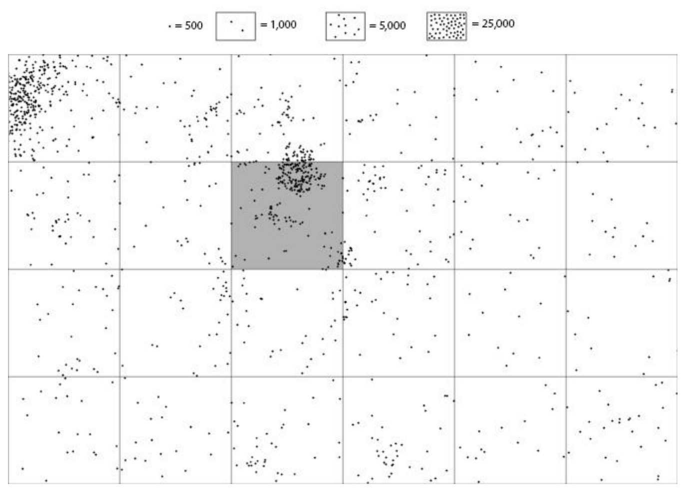
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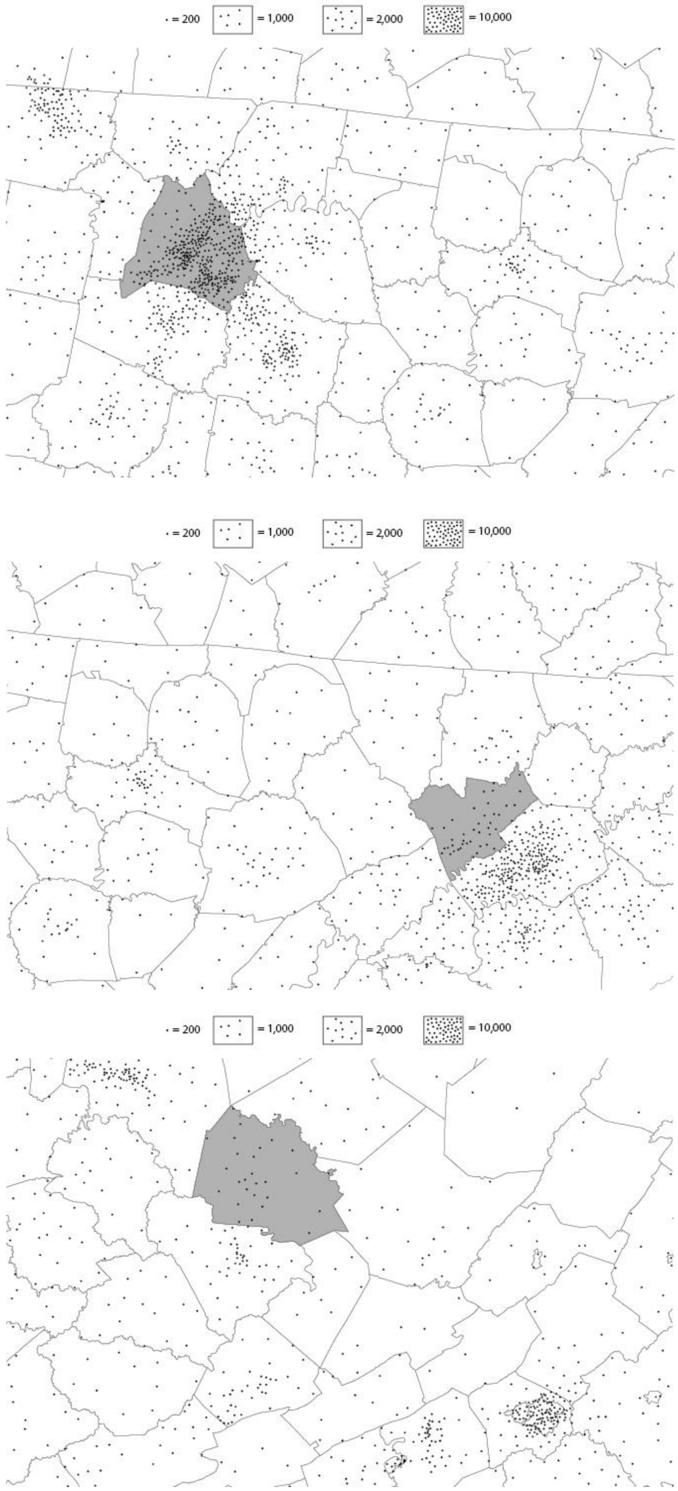
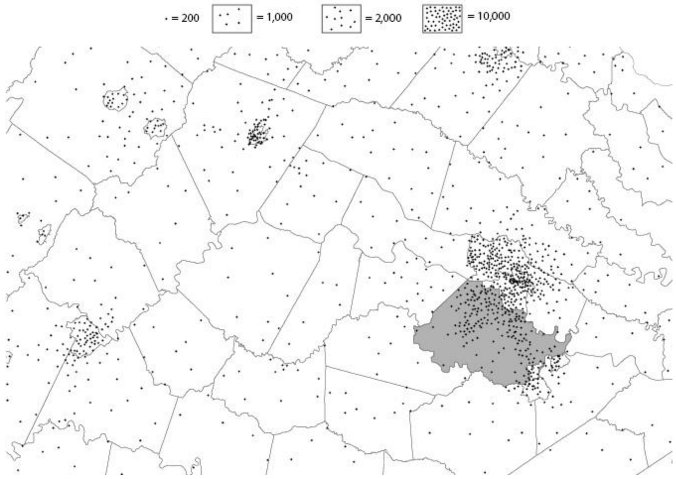
Figure 14: Dot Map Estimates: Blue Noise Dot Maps

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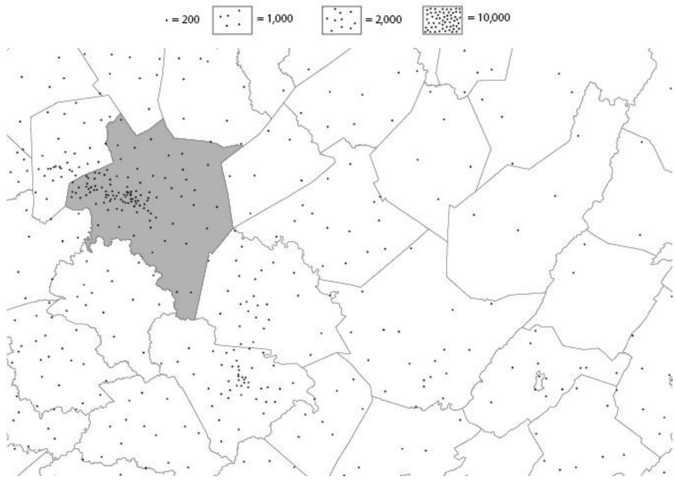
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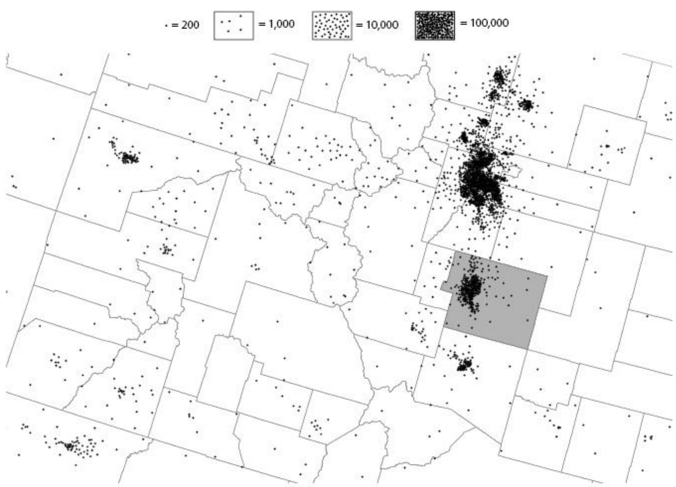
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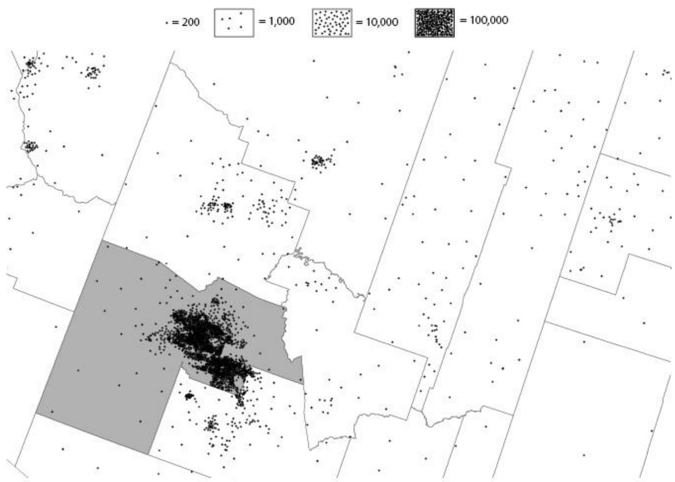
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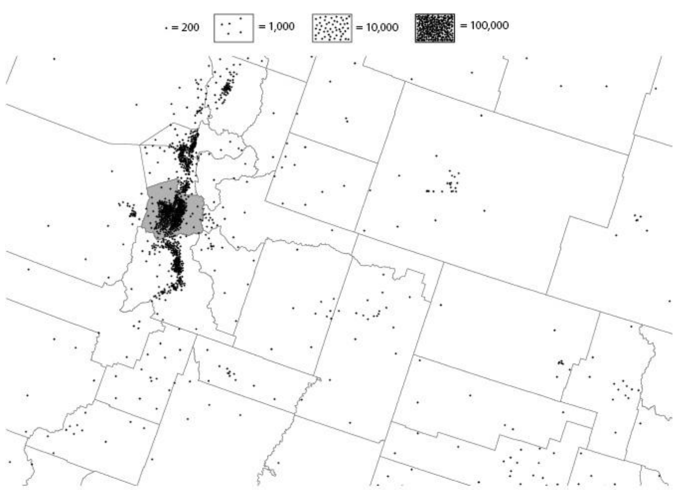
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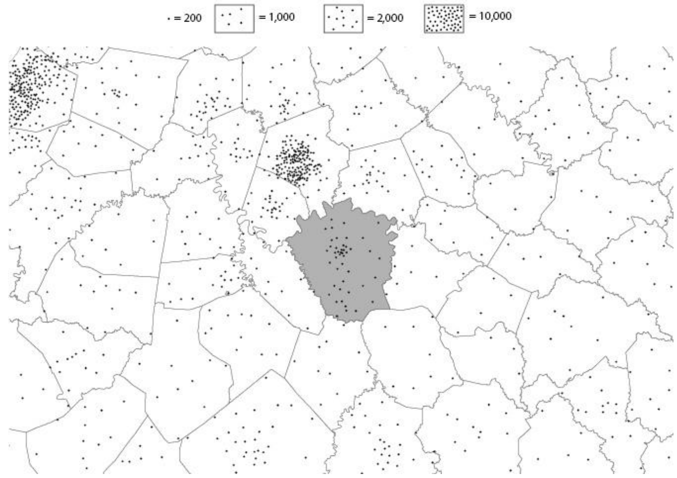
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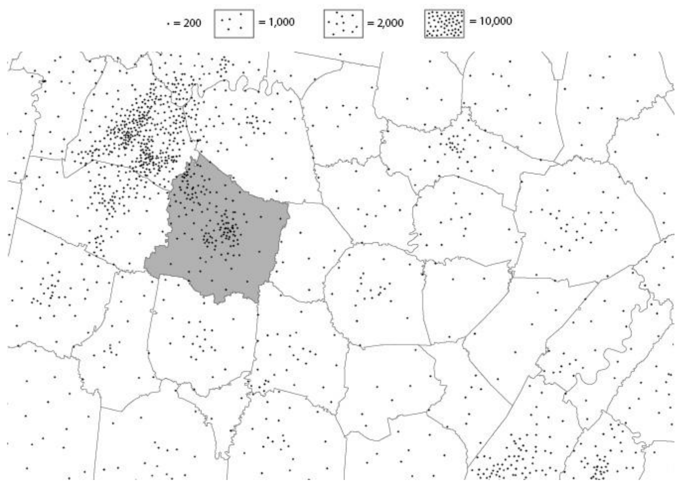
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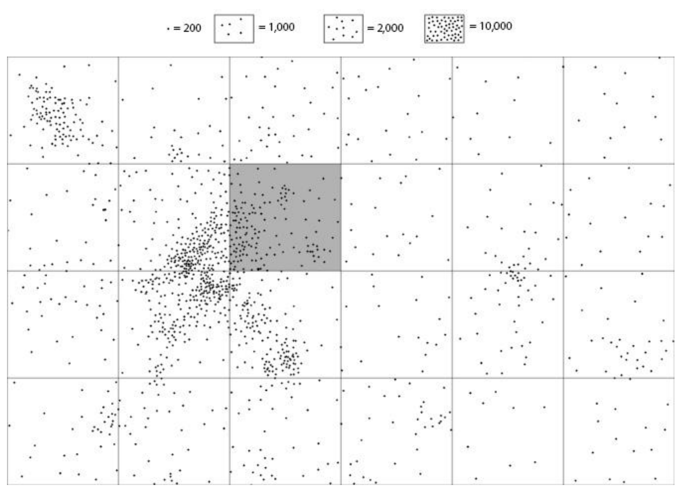
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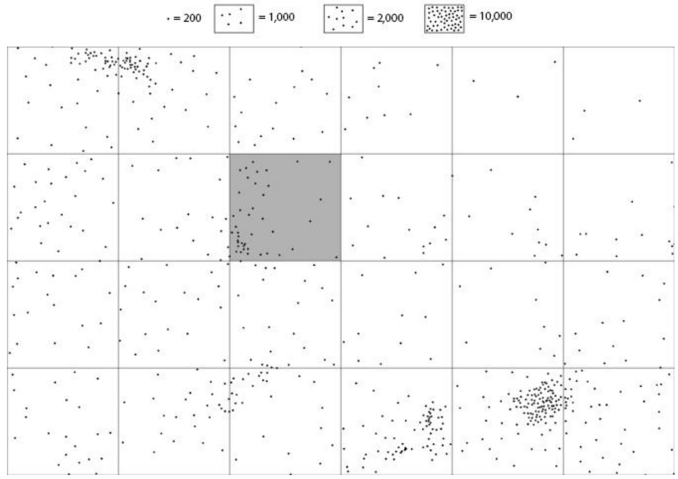
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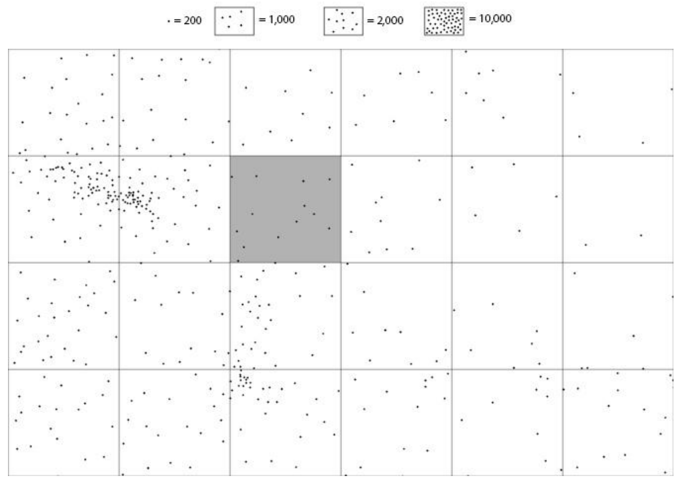
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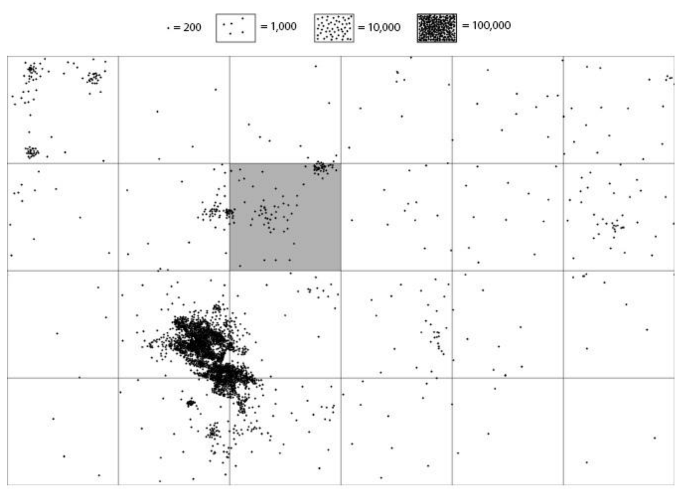
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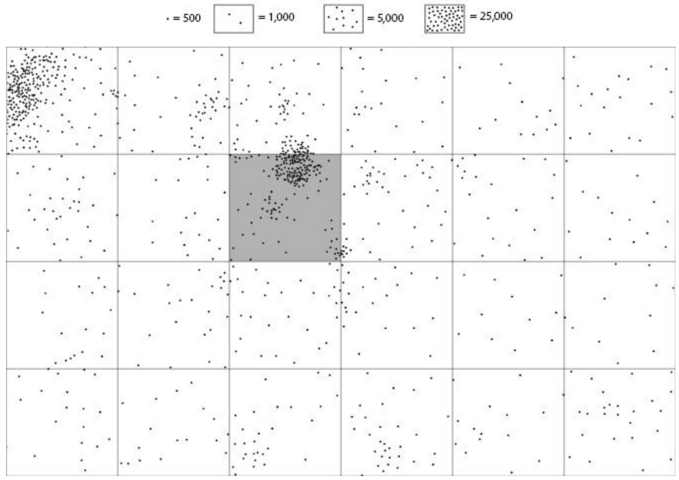
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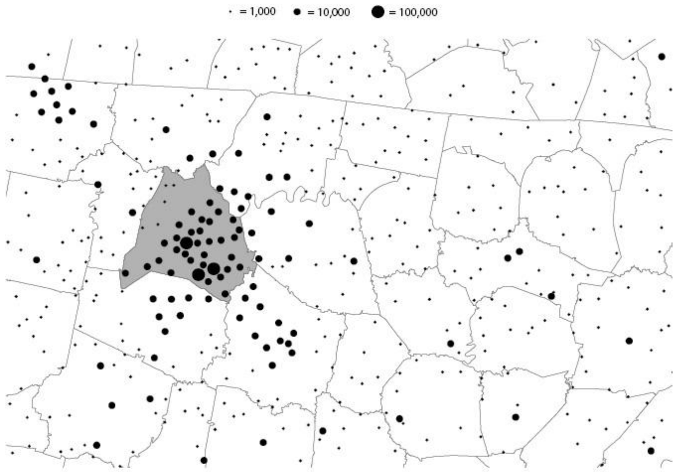
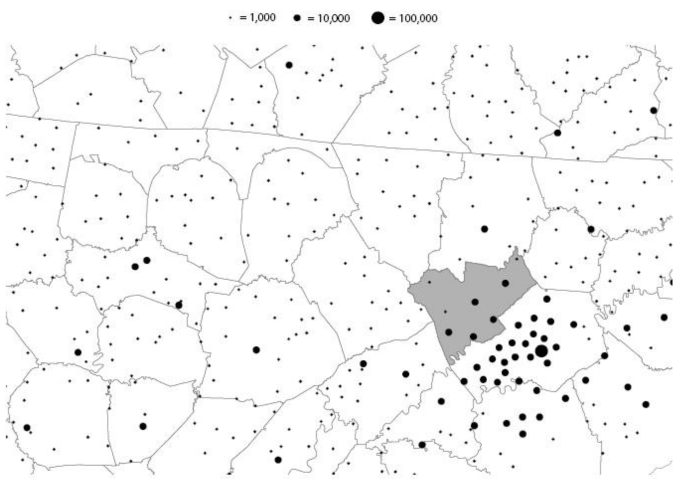
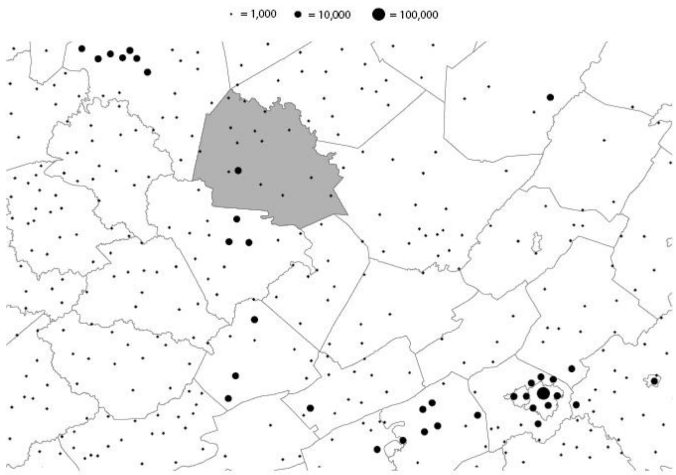
Figure 15: Dot Map Estimates: Graduated Dot Maps



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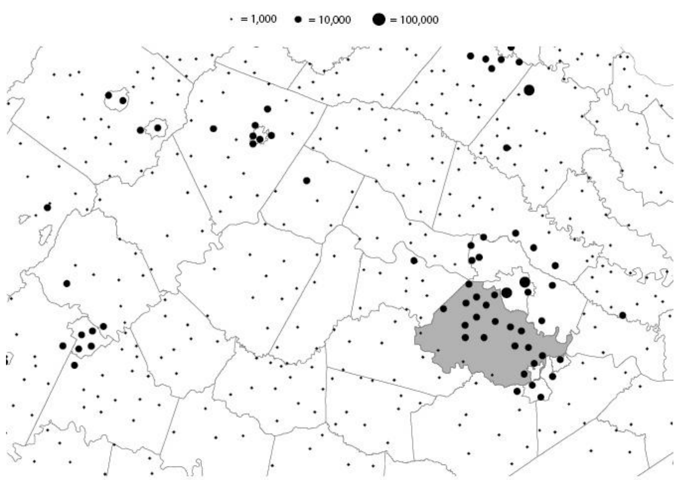
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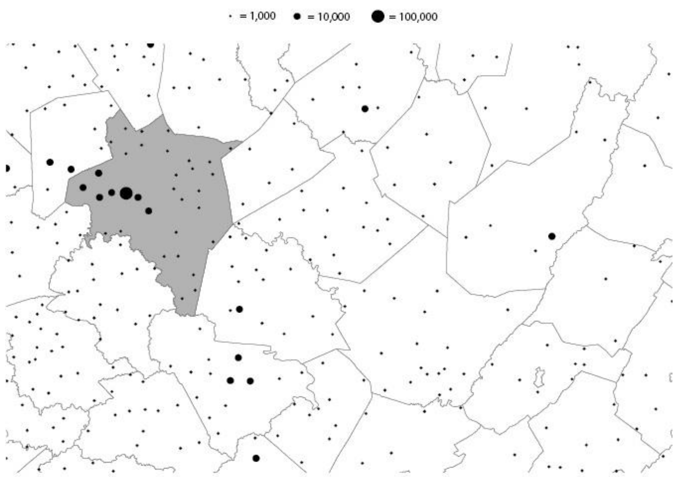
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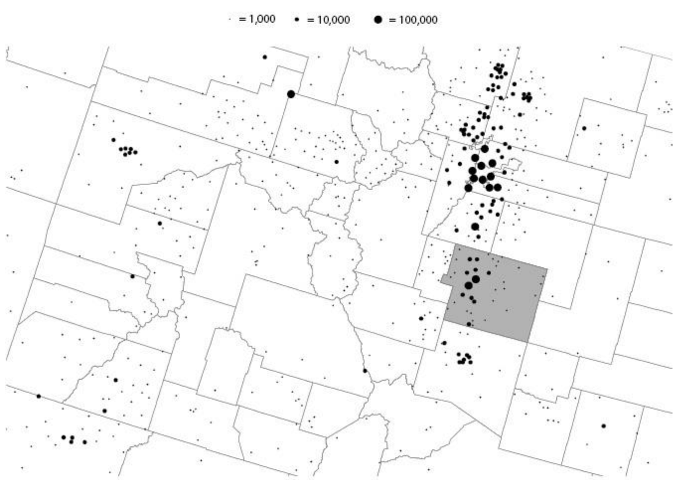
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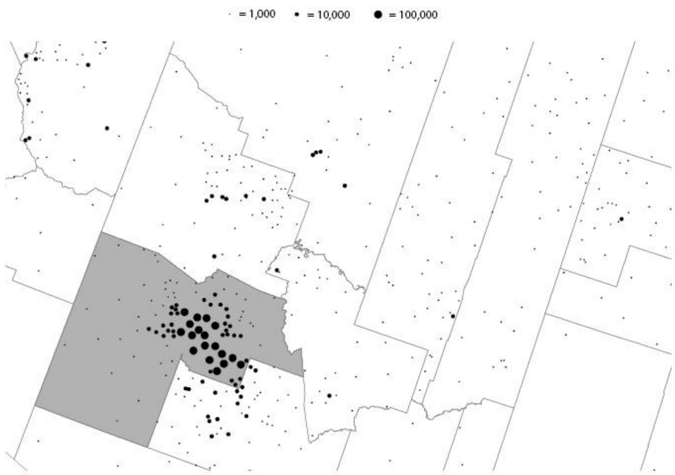
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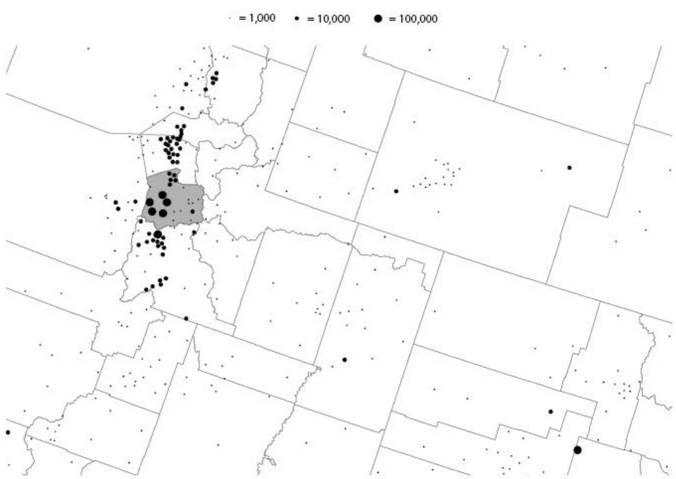
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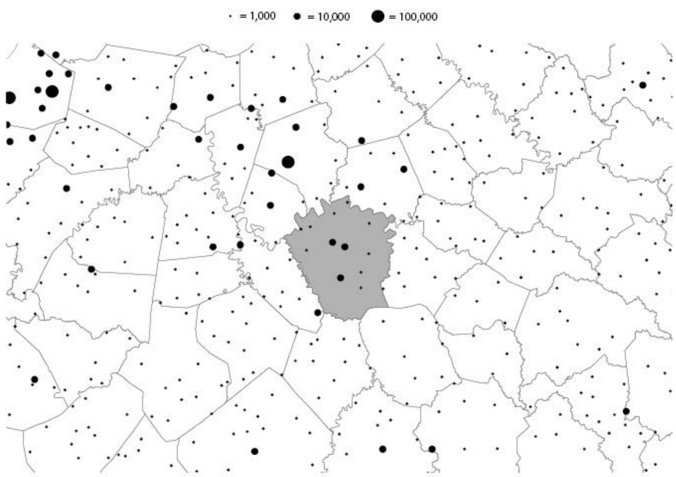
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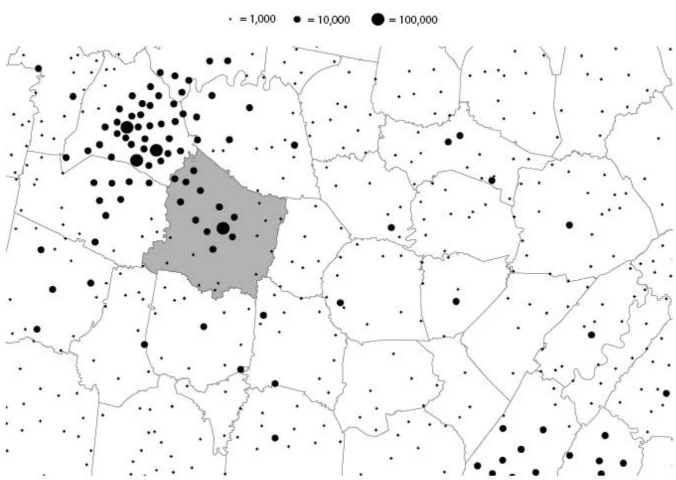
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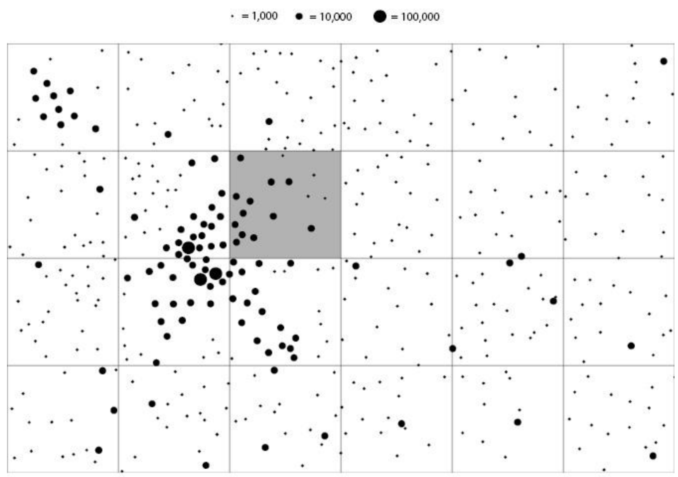

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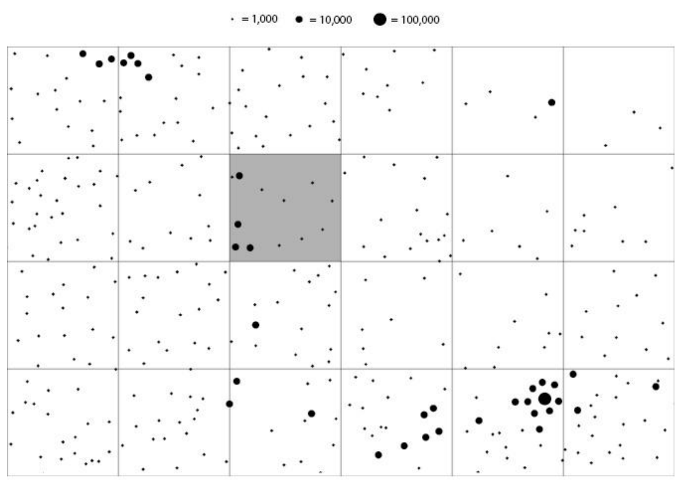

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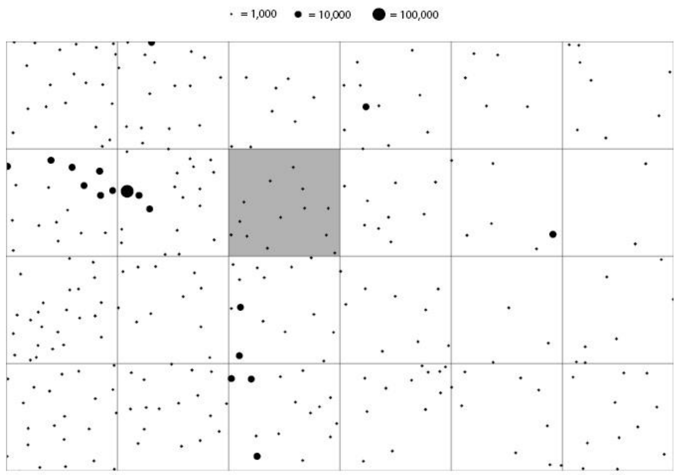
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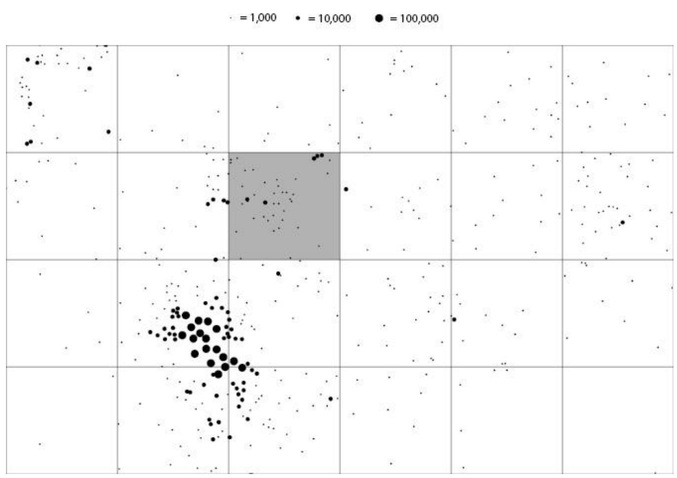
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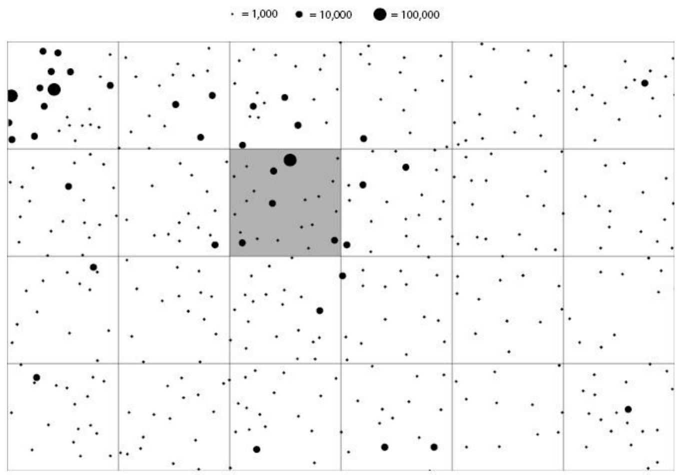
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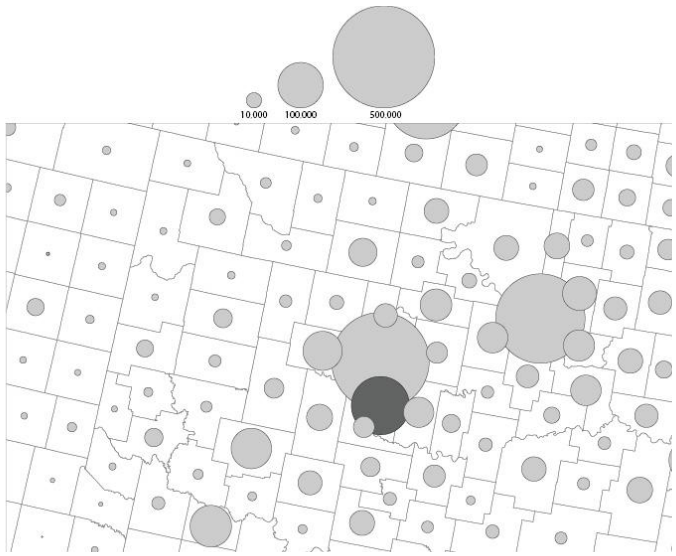
Figure 16: Dot Map Estimates: Area Proportional Circle Maps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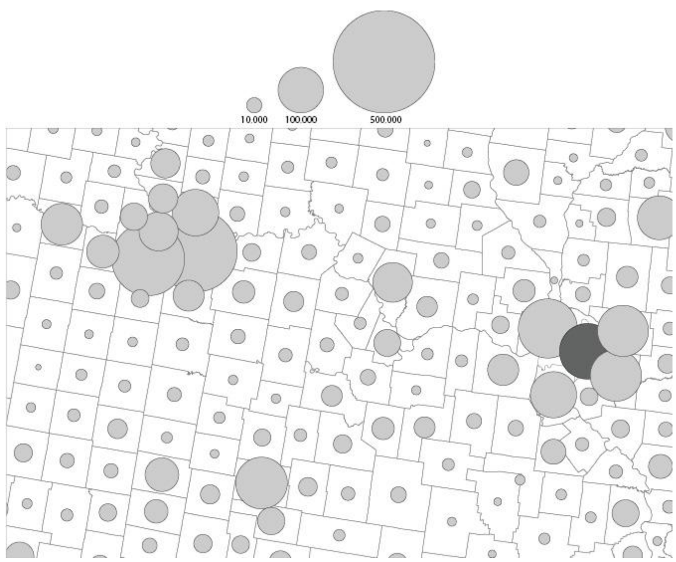
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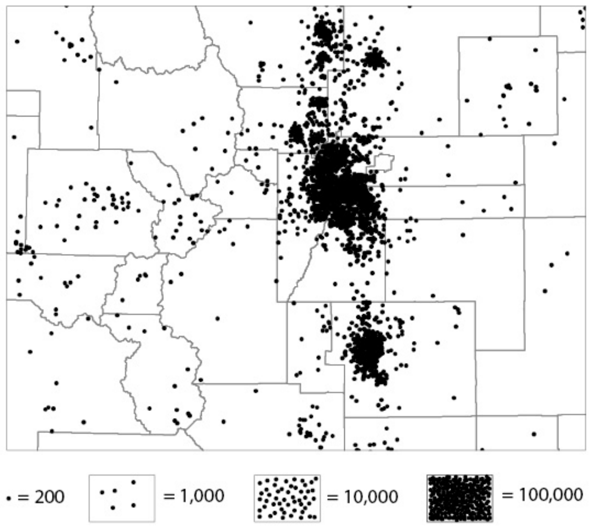
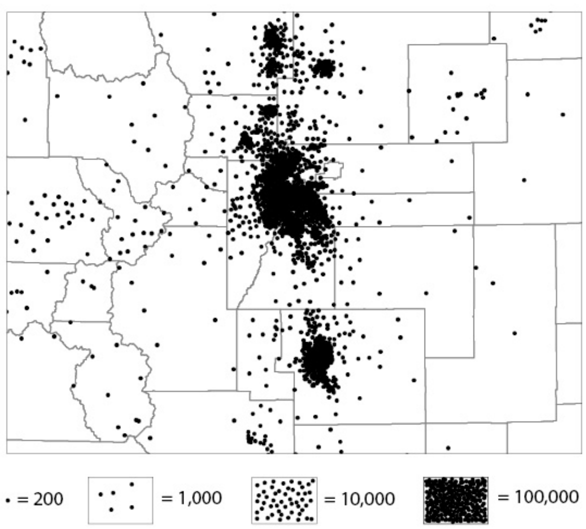
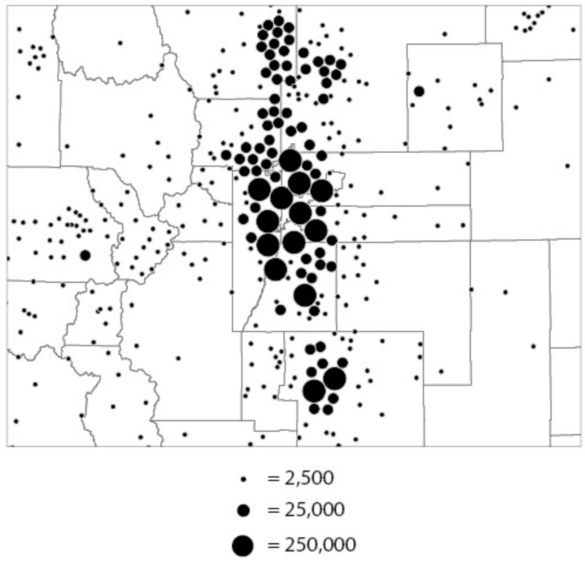
100.000
500.000


10.000
100.000
500.000

Figure 17: Preference and Clarity (set 1):



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= 250,000

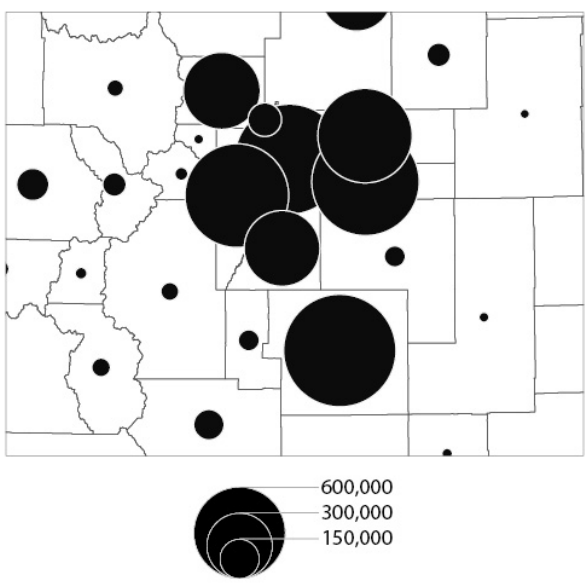
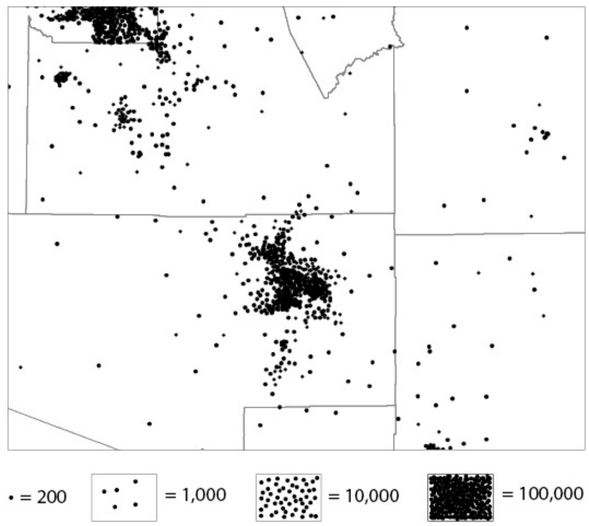
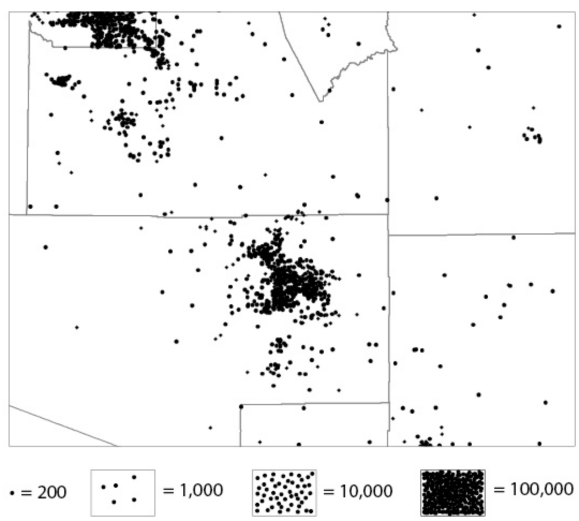
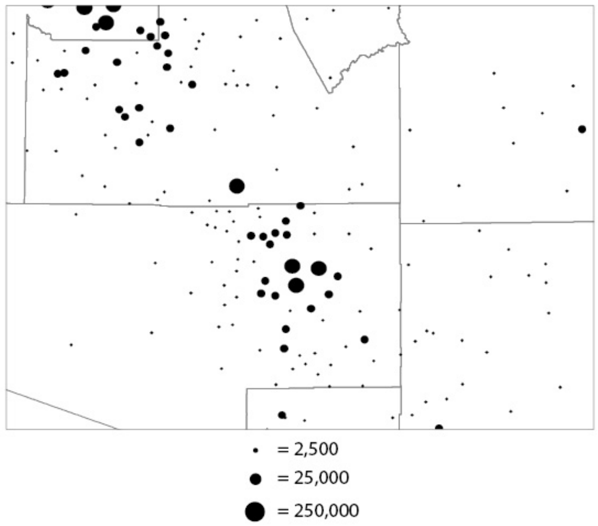

600,000
300,000
150,000

Figure 18: Preference and Clarity (set 2):



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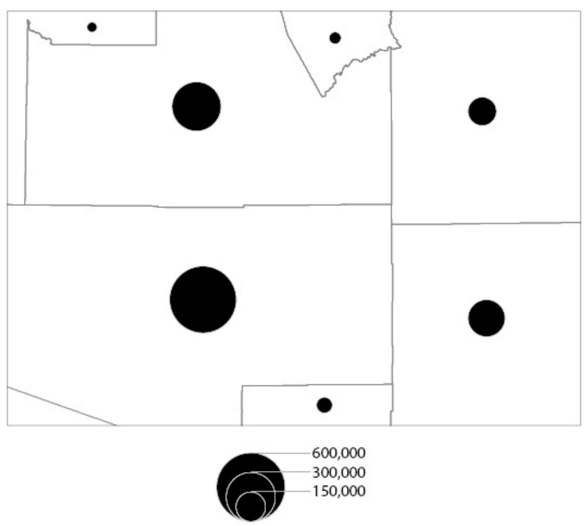

600,000
300,000
150,000

Figure 19: Rank-order Preference (set 1)

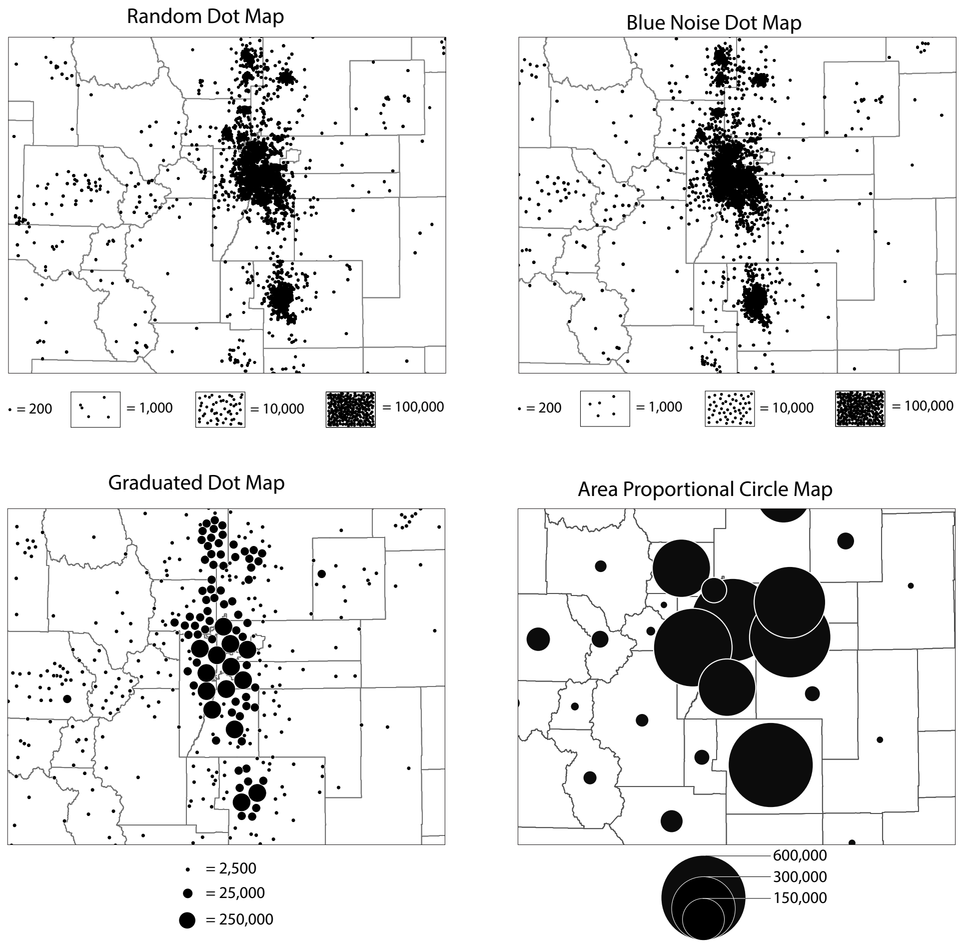
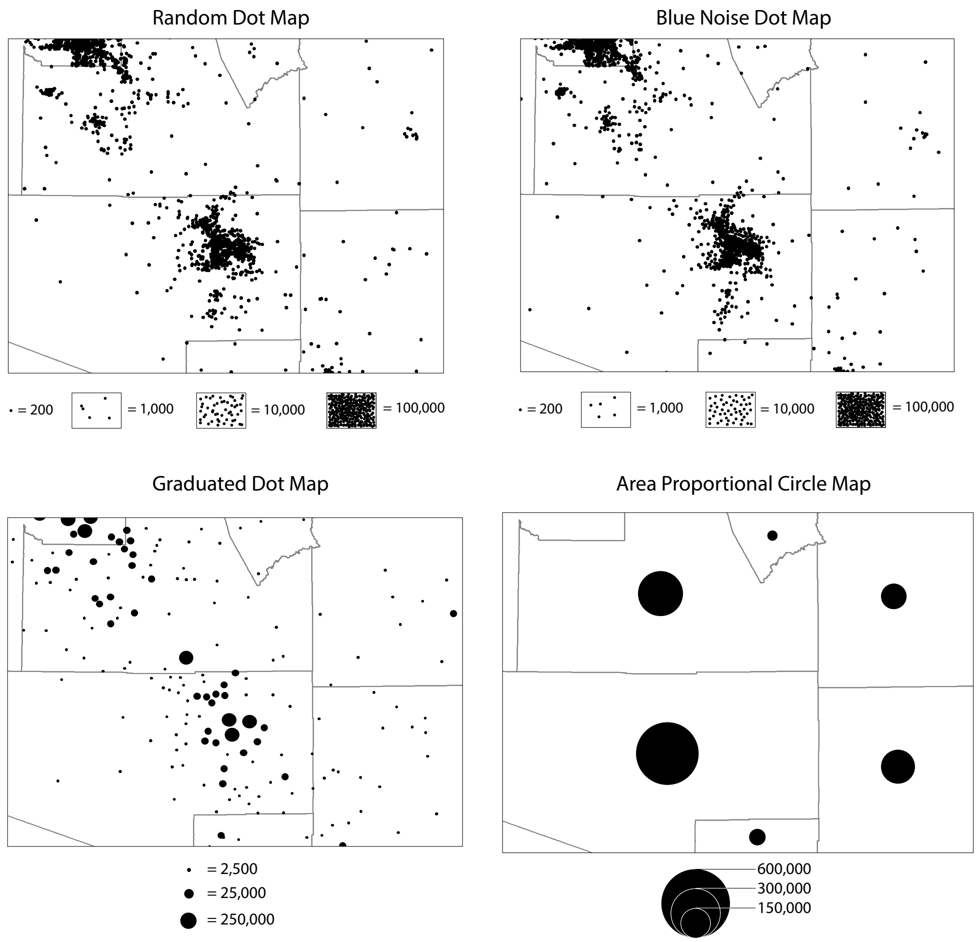


Figure 20: Rank-order Preference (set 2):



4. Map Estimate Results

The values in the first three columns of the table are descriptive statistics of responses generated with the 15 pseudo-random dot maps. The Dot Unit Error is a measure of estimation error expressed relative to the dot unit value. The last column is the total number of dots present for the gray test area on the maps. For the area-proportional circle maps, Error is the total percent error for the test area.

Pseudo-Random Dot Maps

Mean	Median	Mode	Dot Unit Error	No. of Dots
61,557.85	60,000	60,000	-0.27	422
8,235.61	7,900	6,000	-0.19	51
4,752.50	5,000	5,000	-0.21	30
35,260.79	32,000	30,000	-0.31	255
17,361.32	16,000	20,000	-0.32	128
105,998.38	105,000	100,000	0.22	434
329,414.16	305,000	300,000	-0.37	2614
142,160.28	112,000	100,000	-0.02	723
8,101.94	8,000	6,000	-0.30	58
22,560.29	21,000	20,000	-0.39	184
28,306.66	25,000	30,000	-0.20	177
7,258.67	6,800	8,000	-0.29	106
3,027.38	3,000	3,000	0.01	15
42,893.59	25,000	30,000	-0.14	250
47,458.82	42,000	40,000	-0.58	452

Table 2: Pseudo-random dot map statistical results.

Blue Noise Dot Maps

Mean	Median	Mode	Dot Unit Error	No. of Dots
48,622.43	45,000	50,000	-0.32	356
7,549.49	7,500	8,000	-0.12	43
4,846.77	5,000	5,000	-0.13	28
32,241.95	30,000	30,000	-0.11	181
14,213.83	13,000	12,000	-0.35	109
101,805.73	104,000	100,000	0.45	352
321,961.09	300,000	300,000	-0.27	2194
127,453.59	105,700	100,000	0.09	583
7,604.71	7,000	8,000	-0.16	45
22,004.78	22,000	20,000	-0.24	145
25,618.06	23,000	20,000	-0.12	146
7,083.25	7,000	8,000	-0.14	41
2,648.21	2,800	2,800	0.02	13
21,717.40	20,000	20,000	-0.44	194
41,599.45	40,000	40,000	-0.44	372

Table 3: Blue-noise dot map statistical results.

Graduated Dot Maps

Mean	Median	Mode	Dot Unit Error	No. of Dots
447,872.79	450,000	500,000	-0.272	39
52,711.69	53,000	53,000	-0.005	8
23,719.30	24,000	24,000	-0.012	15
174,833.34	180,000	180,000	-0.050	22
179,141.18	180,000	180,000	-0.037	33
286,345.49	300,000	300,000	-0.094	16
1,510,342.75	1,750,000	2,000,000	-0.331	91
569,009.25	570,000	570,000	-0.002	21
154,424.81	155,000	156,000	-0.004	21
219,531.48	220,000	220,000	-0.011	24
122,483.27	124,000	124,000	-0.012	16
48,000.00	48,000	48,000	0.000	12
13,000.00	13,000	13,000	0.000	13
73,837.13	72,000	72,000	-0.141	41
154,224.63	155,000	155,000	-0.011	22

Table 4: Graduated dot map statistical results.

Area Proportional Circle Maps

Mean	Median	Mode	Error
380,814.46	400,000	500,000	-6.09%
349,531.48	400,000	400,000	-7.04%
101,540.72	100,000	100,000	-18.77%
88,146.76	95,000	100,000	-21.30%
197,147.71	200,000	250,000	0.00%

Table 5: Area-proportional circle map statistical results.

5. Map Preference and Clarity Results

Table 6 and 7 show measures of central tendency for the clarity and preference test maps.

Table 8 shows the frequency statistics for each of the rank-order preference maps.

Clarity

	Mean	Median	Mode
Pseudo-random (set 1)	2.96	3	3
Blue Noise (set 1)	3	3	3
Graduated (set 1)	4.12	4	4
Area-proportional (set 1)	3.1	3	3
Pseudo-random (set 2)	3.05	3	3
Blue Noise (set 2)	3.1	3	3
Graduated (set 2)	4.25	4	5
Area-proportional (set 2)	3.77	4	5

Table 6: Clarity statistical results.

Preference

	Mean	Median	Mode
Pseudo-random (set 1)	2.96	3	3
Blue Noise (set 1)	3	3	3
Graduated (set 1)	4.12	4	4
Area-proportional (set 1)	3.1	3	3
Pseudo-random (set 2)	3.05	3	3
Blue Noise (set 2)	3.1	3	3
Graduated (set 2)	4.25	4	5
Area-proportional (set 2)	3.77	4	5

Table 7: Preference statistical results.

Rank Frequencies

	1st	**2nd**	**3rd**	**4th**
Pseudo-random (set 1)	32	74	91	100
Blue Noise (set 1)	28	79	142	48
Graduated (set 1)	161	68	50	18
Area-proportional (set 1)	76	76	14	131
Pseudo-random (set 2)	30	52	97	118
Blue Noise (set 2)	19	76	145	57
Graduated (set 2)	146	95	43	13
Area-proportional (set 2)	102	74	12	109

Table 8: Rank-order statistical results.

Significance Tests

Tables 9-12 show hypothesis test results for map comparisons. The critical values listed were used to reject the null hypothesis. The Sig (p-values) values are the test statistics used to reject the null hypothesis.

Kruskal-Wallis one-way ANOVA		
Map Set	**Critical**	**Sig**
Set 1	0.05	<0.001
Set 2	0.05	<0.001
Set 3	0.05	<0.001
Set 4	0.05	<0.001
Set 5	0.05	<0.001
Set 6	0.05	<0.001
Set 7	0.05	<0.001
Set 8	0.05	<0.001
Set 9	0.05	<0.001
Set 10	0.05	<0.001
Set 11	0.05	<0.001
Set 12	0.05	<0.001
Set 13	0.05	<0.001
Set 14	0.05	<0.001
Set 15	0.05	<0.001

Table 9: Kruskal-Wallis test results for each map set containing three dot map types

Kruskal-Wallis one-way ANOVA		
Map	**Critical**	**Sig**
Map 1	0.05	<0.001
Map 2	0.05	<0.001
Map 3	0.05	<0.001
Map 4	0.05	<0.001
Map 5	0.05	<0.001

Table 10: Kruskal-Wallis test results for the area-proportional circle map set

Friedman's two-way ANOVA		
Map Set	**Critical**	**Sig**
Clarity (set 1)	0.05	<0.001
Preference (set 1)	0.05	<0.001
Clarity (set 2)	0.05	<0.001
Preference (set 2)	0.05	<0.001

Table 11: Friedman's test results for clarity and preference map sets

Friedman's two-way ANOVA		
Map Set	**Critical**	**Sig**
Set 1	0.05	<0.001
Set 2	0.05	<0.001

Table 12: Friedman's test results for rank-order preference map sets