ESTIMATING ECONOMICALLY OPTIMUM NITROGEN RATES BY INCORPORATING NITROGEN SOIL TESTS AND MOISTURE FACTORS INTO STATISTICAL WHEAT-FERTILIZER RESPONSE FUNCTIONS ON CONDON SOILS

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CHAPTER I

INTRODUCTION

Fertilizer Use

For more than a century commercial fertilizers have played an increasingly important role in the production of food and fiber in the United States. In the earlier years of their use, fertilizers contributed little to the increase of yields; in part because of limited usage, inferior quality, and lack of understanding of proper application methods. Also, in the earlier years of this country's growth the soils were not greatly depleted from their "original" fertile conditions (22, p. 349-357).

The use of fertilizers in the United States has increased from 6 million tons in 1934 to more than 25 million tons in 1958. This phenomenal increase helps explain the "dynamics of the agricultural supply function". Not only has total fertilizer use increased over this period but also there has been a rapid increase in the percentage of the plant nutrient content. For instance, the percentage of crop yield attributable to the use of fertilizers in 1927 was estimated to be 16%. In 1954 the estimate was 27% (3, p. 125). It seems remarkable that more than one fourth of our nation's food supply comes from commercial fertilizers.

The use of commercial fertilizers has significance beyond the consideration of present food supply. Since fertilizers are now being used to such a great extent, they have essentially provided farmers with means of substituting for other limited resources. Thus farmers can produce the same quantity of food with less labor, less land, less tractor fuel and less of many of the other resources used in agriculture. The resources which are no longer required to produce essential food and fiber can be transferred into other industries, to produce appliances, cars, highways, television sets, and other goods and services deemed necessary in our economy. Thus fertilizers might well be regarded as essential ingredients in our economic growth "mix". It is important that a continuing and adequate supply of them be insured to future generations.

Nitrogen and Plant Growth

Of the various nutrient elements commonly applied to the soil, nitrogen is the most abundant in nature. Although the amounts of nitrogen available in the soil are often small, the atmosphere contains enormous quantities. It is estimated that there are approximately 148,000 tons of atmospheric nitrogen for every acre of land (22, p. 103).

This great abundance, however, does not insure an adequate supply to plants in a form which is usable to them. In fact, nitrogen, in the form in which it commonly exists in nature, is not usable by plants for nutritive purposes. Nature has, however, provided several methods of converting nitrogen into usable forms. One important process involves the action of certain symbiotic and non-symbiotic bacteria which "fix" nitrogen in the soil. Another process is involved with lightning during electric storms. The nitrogen thus converted is usually brought to the soil with rain which may fall during or after the electric storm. A third "natural" process involves the decomposition of manures of various types. Still another source of nitrogen, in a form usable to plants, is the combustion of fuels in our residential and industrial furnaces. This is not exactly a natural source; but it is, none the less, quite significant in some areas (22, p. 120-131, 292-297).

Much of the nitrogen in the soil is part of the soil organic matter, but as such it cannot be utilized by the plant. The organic matter is decomposed by soil organisms which release nitrogen or ammonia. These two forms are then oxidized into the nitrate form of nitrogen. As might be expected, the action of these organisms and the chain of events which yields the nitrate form of nitrogen is determined largely by conditions of moisture, temperature and soil acidity. But nitrate nitrogen is transient in nature, and the amount in the soil varies greatly in comparison to most other elements (1, p. 7). These facts make necessary a greater concern with controlling the amount of nitrogen available to plants than of most other commonly applied elements.

Nitrogen is a constituent of plant protoplasm and is therefore very much involved in plant growth. An adequate supply is necessary for vigorous growth. However, excesses or shortages can have depressive effects on yields. An excess of nitrogen stimulates vegetable growth which may result in delayed maturity and even a decrease in fruitfulness. Shortages, on the other hand, may result in an elongation of root systems but a stunting of total plant growth often occurs (22, p. 103-105).

The Use of Nitrogen Fertilizers

Recognition of the importance of nitrogen to plant growth and the need to supplement the available nitrogen in the soil from an economic and readily available source has caused a tremendous surge in the "consumption" of nitrogen fertilizers in recent years. The use of commercial nitrogen fertilizer in the United States had increased from an average of about 484,000 tons of available nitrogen per year in the 1940-1944 period to 2,672,000 tons of available nitrogen in 1958 (30). A similar increase has been evident in Oregon where, during the same period, the usage of available nitrogen from commercial sources increased from 2,400 tons to 41,689 tons (30). The United States Department of Agriculture estimates that, for the five counties representing the Oregon portion of the Columbia River Basin wheat area, approximately 1,333 tons of dry nitrogen and 9,046 tons of liquid nitrogen were used in 1960. At current prices of these two fertilizer types, the gross value of the fertilizers used in these five counties might well be in excess of one and one half million dollars per year. This represents a rather large expenditure for a single resource to be used in this particular area and might partly justify a concern for an economic utilization of this resource.

CHAPTER II

STATEMENT OF THE PROBLEM

The rapid increase in fertilizer use in recent years is due, in part, to the existence of certain governmental acreage control programs. Economically it has become more and more important for the individual farmer to get large yields on reduced acreages alloted to wheat growers. At the same time farmers have found land values and consequently property taxes increasing. Labor and machinery costs have also advanced rapidly. Because of this "cost-price" squeeze, farmers have had to think twice before allocating resources in their production.

Risk and Uncertainty in Fertilizer Use

Even in the light of the well publicized cost-price squeeze, many farmers would do well to use more fertilizer than they are now using because of the generally favorable ratio between fertilizer costs and product received (3, p. 127). Yet many farmers hesitate to use fertilizers because of the risk and uncertainty /1 involved. Often they do not have a clear understanding of what responses can and should be expected from fertilizer applications. This uncertainty exists partly because

/1 For a recent example of an empirical verification of this commonly believed notion, cf. Myron E. Writh. Production Responses to Agricultural Controls in Four Michigan Farming Areas in 1954. Unpublished M.S. Thesis, Department of Agricultural Economics, Michigan State University, 1956. p. 46.

recommendations are often made in very general terms. When a farmer tries and is apparently unsuccessful in obtaining expected yield increases, his confidence in the recommendations, as well as the value of fertilizers, is diminished. It will be seen in Chapter 7 that an application of 50 pounds of nitrogen under one set of conditions will give sharp yield increases, whereas the same application will give drastic yield reductions under another set of conditions. The reasons for the differences in response are probably numerous. However, the effects of some of the more important factors can be isolated. With the isolation of these factors it is possible to predict, with some accuracy, the most economical application. If these predictions can be made with "sufficient" accuracy, it is conceivable that more fertilizer would be used than is presently being used; and, parenthetically, it might bring a higher level of income to those benefitted by this information. If incomes in general did not rise from further production increases, as is often the case in agriculture, at least there would be certain resources freed to move into other sectors of our economy.

For these and other related reasons it would seem that there is a need for improved information to be given to the farmers upon which their decisions might be based, Some of our research must then be conceived in an effort to estimate more accurately the underlying production parameters. In other words, in the interest of helping farmers to reduce the risk and uncertainty involved with their many fold management decisions it should be the responsibility of reserachers in the agricultural experiment stations to define more clearly the significant variables in the production function and specify as accurately as possible just what the value of these production coefficients are under specified conditions. And even though we may not give the farmer the coefficients of the production function, as such, we should at least present him with recommendations based upon these more closely defined variables.

Much of the work done in the area of soil testing and crop response has been to reveal the underlying physical relationships that exist. Of course, knowledge of these relationships is basic to the extension and correlation of our knowledge in other related areas. But it must also be remembered that the end product of most of our research is in economic applications. Still, much of our research has lacked a basic economic orientation. This is to imply that researchers have sometimes failed to develop the economic implications of their results. Thus we have seen information relayed to farmers couched in terms of "maximum yield" recommendations rather than in terms of an "economic optimum". The importance of this statement need only to be mentioned to be seen, for it is known that maximum yield recommendations are seldom consistent with the goal of maximum profits.

CHAPTER III

OBJECTIVES OF THE STUDY

General Discussion

There has been considerable discussion and work done in an effort to devise methods of analysis in production economics to enhance the predicting ability of researchers by means of statistical yield response functions. Some of the earliest efforts in this regard were made in Iowa using data from the results of single experiments (11). Of course relationships deduced from the results of single experiments under specific and local conditions are of questionable value when extended to other conditions. At least such recommendations must be formulated with extreme care.

It is possible to show, and it has been shown, that the "average" production function generated from a number of experiments over a number of years will give the best possible estimate of the true relationship if no other information is had in advance (5). If such experiments were formulated so as to vary more than one factor at a time it might be that a "best possible" function (surface) could be estimated where these factors were known in advance by the farmers. This is to imply that we might broaden the "base" of our experiments to include several specific conditions thus giving us the advantage of having more applicability to conditions encountered on many farms. Experiments could be conducted over various soil depths while simultaneously varying conditions of soil moisture, rainfall and fertilizer applications. Thus the conditions might be simulated wherein not only the direct effects of such factors as soil depth and fertilizer application rates are observed but also the interacting effects of all of these conditions upon yields. These interacting effects are often as important, or more important, than the so-called direct effects. Because of their significance they have important bearing upon the effectiveness of fertilizer recommendations.

Any one of a number of things can happen to yields as a result of the existence of certain conditions both from direct effects and from indirect effects working through other variables (25). Recommendations which do not take into account the interaction between variables can be very misleading. That is, it might be correct to recommend an increase in fertilizer application based on a certain soil depth. But other factors must be considered at the same time - such as a minimum amount of available moisture or a certain level of available nitrogen already in the soil.

A Basis for Better Fertilizer Recommendations

One of the objectives then of this study is to analyze certain available data with the purpose in mind of developing and testing several types of yield predicting equations whose accuracy would supposedly be enhanced by the incorporation of several quantified factors related to wheat yields. Recommendations relative to fertilizer application rates in Oregon have here to fore been based almost entirely upon a test for soil nitrogen. [1] In some cases, either fall or spring soil moisture has been considered in conjunction with this nitrogen test (20). However, information is woefully lacking as to the effects (especially the magnitude) of these and other important variables on yield responses. While there is much conflict and uncertainty with regard to optimum nitrogen application rates under various soil and moisture conditions, there have been many experiments conducted to determine yield responses in which these factors have been (by design) different but at the same time measured.

It seems appropriate to pursue this study at this time because of the recent development of high speed electronic computing equipment and statistical techniques which lend themselves to the use of such equipment. These techniques and tools while allowing greater detail of analysis also permit predictions with greater precision than would otherwise be possible. Also, some of the recently proposed methods of analysis need testing

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[1] A not unrelated question concerns the nitrogen soil test itself. Nitrogen, as it is (and would be) available to the plant during critical phases of the growing season, is difficult to ascertain. Several methods are presently used but their limitations are generally recognized. However it would be useful (and this is a secondary product of this analysis) to know which of these tests gives the best basis for yield response predictions.

for further clues as to their effectiveness and practicability in future research.

Finally, a development of the economic implications will be undertaken. This is to be in determining "economically optimum" rates of application once the basic physical relationships are explored and determined. Optimum rates will not be computed for each model tested due to the unwieldiness of the task and because it is unnecessary to make these determinations for models which are later eliminated on the basis of other criteria. There would be no justification in relying on the results of one predicting system when it is known that a better one exists.

CHAPTER IV

SOURCE OF DATA

Physical and Economic Conditions in the Area

Two-thirds or more of the wheat grown in Oregon is produced in the Columbia Basin portion of the state. This area is part of a larger wheat producing area, extending from north central Oregon through eastern Washington and northwestern Idaho. Generally favorable physical and economic conditions there have led to a high degree of specialization in wheat production (29, p. 3). For this reason this area has become known as the Oregon wheat area. In the lower rainfall areas of the basin where this study is more particularly centered, it is necessary to store soil moisture by an alternative crop and fallow system. That is, after one wheat crop is harvested in the fall, the ground sits with the stubble and straw until spring. The stubble gives the ground the needed protection from wind and water erosion. The following spring the ground is plowed but left fallow. During the summer months this ground is tilled for weed control and for retaining soil moisture into the following autumn. Usually, the crop for the following year's harvest is planted in August. The wheat grows several inches above the ground and has a fairly firm root system by the time winter snows come. This root system gives the soil considerable protection against erosion from the winter rains and the spring runoff of melting snow. With two seasons' moisture stored in this way, the crop can almost be assured of maturing even if little or no rain comes during May and June. Any rains which do come, however, usually add substantially to the yields.

The Experiment

The data for this project was compiled from experiments conducted in the above described area by the soils department at Oregon State University in cooperation with the United States Department of Agriculture. The original project from which this particular data was extracted was conceived in an effort to provide a basis for accurate fertilizer recommendations under specific conditions. The project entailed conducting experiments in the wheat summer-fallow area of the Columbia Basin country in northeastern Oregon. This area included Umatilla, Morrow, Gilliam, and Sherman counties. This area offered a wide variety of soil, temperature, moisture, and altitude conditions ideal in such an inclusive project.

The experiment was conducted over a period of four years beginning in the fall of 1953 and continuing until after the harvest of 1957. Moisture, rainfall, soil conditions, and other factors were observed and measured through four complete growing cycles. The relationships between these factors and their effects on yields were to be induced. As is suggested by the title of this thesis, this analysis concerned itself only with Condon soils. Condon soils in this area range in depth from two to five feet. Altitude conditions were from 1000 to 3000 feet. Average yearly rainfall in these areas ranged from 10 to 18 inches per year, most of which comes in the winter in the form of snow and rain.

Each year during this experimental period there were from eight to ten farms on Condon soils which cooperated in the study. However, in several cases, important data were lost or not gathered. In these cases it was either rainfall or spring soil moisture readings or both. Consequently, not all data from all of the locations were usable in this analysis. All eight of the locations were usable in 1954, seven out of eight locations were usable in 1955, six out of eight in 1956, and in 1957, data from nine out of ten locations were usable.

In 1954, the procedure, with respect to the nitrogen experiment, was to divide each location (farm) into nine treatment plots, applying fertilizer on four of them in the fall, and on four of them during the spring. The one remaining plot was a check plot; that is, no fertilizer was applied on it. Each plot of land other than the check plot received either 20, 40, 60, or 80 pounds of available nitrogen per acre. The procedure was the same both spring and fall. The nitrogen applied in the fall was applied just before planting. The plots previously untreated were treated in the spring, usually in March. All applications were made by means of a belt type applicator.

The procedure was the same in the 1955-57 period except that the locations were broken into 11 treatment plots. On the ten plots used for spring and fall application, the rates varied in increments of 20 pounds from 20 to 100 pounds per acre instead of 20 to 80 as was done previously. Again one plot was used as a check plot. /1 (See Appendix I for data).

Each fertilizer treatment was replicated four times on plots 8 feet wide and 50 feet long. Farmers who applied fertilizer to their own land were careful not to apply any on plots where the experiment was taking place. Except for fertilization, all plots were handled as any other piece of land on the farm. The farmer used his own tillage practices and tilled the experimental area just as he did the remainder of his land.

Soil samples were taken in the fall and in the spring. The fall samples were gathered over a period of approximately one month, generally just before planting. Spring samples were gathered over a period of about six weeks. The conditions of the access roads and availability of personnel made it impossible to take all samples within a shorter period of time. Generally, however, samples were taken as soon as the experimental area was accessible.

All of the soil samples were taken from unfertilized plots in increments of one foot to a depth of five feet or bedrock, whichever came first. The data used in this analysis are

/1 Check plot means that no nitrogen was applied.

an accumulation of the contents of each (foot) increment of soil, weighted equally. For instance, the amount of nitrogen found in the soil at four feet was weighted just as heavily in the total as was the amount found in the first foot of soil. Thus none of the figures in this data represent a concentration of nitrogen or moisture. Only when each figure is considered in relation to the soil depth is there any indication of concentration.

Laboratory analyses of various types were made. The samples were subjected to three nitrogen soil tests: (1) the nitrate nitrogen test, (2) the nitrifiable nitrogen test, and (3) the ammonia test. /1 Our statistical analysis considered each of these three tests taken alone. It also considered combinations of the nitrate nitrogen plus either the ammonia test or the nitrifiable nitrogen test. Each soil sample was also tested for its moisture content. The procedure again was to measure the moisture in each foot of soil, and to sum the overall moisture content in the entire soil depth. Thus the data include fall and spring moisture readings.

Rainfall gauges were kept at each experimental site. These were attended to by either the cooperating farmer or the local county agent. The precipitation was recorded from the time the spring soil samples were taken up until the time at which the grain began to mature.

In summary then there are 30 different locations upon

/1 Throughout this thesis the term "ammonia test" refers to a test for ammonium nitrogen.

which either 9 or 11 different "treatments" were made. There are a total of 344 yield observations used in this analysis - one from each treatment plot thus described. For each location the analysis considered one fall moisture reading, one spring moisture reading, one summer rainfall reading, and ten "different" measurements of the nitrogen existant in the soil.

It would appear that there is available in the above described experiment, data from an exceptionally detailed study. It seems that it would thus warrant a correspondingly detailed analysis which up to the present time has not been attempted.

CHAPTER V

RESPONSE FUNCTIONS AND FUNDAMENTAL ECONOMIC RELATIONSHIPS

Choosing Appropriate Algebraic Forms

One recurring problem in estimating fertilizer response functions relates to the choice of an appropriate algebraic form. A very large number of mathematical forms could be assumed for the production function. Several general forms which have been used in the past in analysis of fertilizer experimental data are given herewith.

(1) The exponential function. The development and application of this function is due largely to the work of Spillman (27), Mitscherlish (23, p. 413-428), Hartley (9, p. 32-45), Baule (2, p. 363-385) and Stevens (28, p. 247-267). A choice of this function would imply that as fertilizer is added in units of uniform size, yields increase at diminishing rates. This function has the characteristics of approaching, asymptotically, a maximum possible yield. The characteristic of approaching a maximum yield in an asymptotic manner is often a distinct disadvantage when nutrients are applied at very high rates. Particularly is this true in the case of nitrogen applications under certain conditions. Here diminishing yields are often observed at high rates of application.

Another disadvantage of this type of function is that it is very difficult to fit to data when several variables are included in an analysis. However, whenever there is reason to believe that an unknown asymptote is approached as the nutrient is added at very high rates, an exponential function of the Mitscherlich-Spillman type is clearly suitable.

(2) The Power Function of Cobb-Douglas function. This function was first applied to the analysis of production data by Professor Paul H. Douglas (7, p. 139-145). The general form, $y = ax_1^{b_1} x_2^{b_2}$,--------, $x_n^{b_n}$, is linear when appropriate logarithmic transformations are made. In this function, y is the predicted yield, x_1 are the fertilizer inputs, a and b_i are parameters to be estimated. The fact that this function can be transformed to a linear form makes estimation of curvilinear functions (with diminishing returns) possible with a very simple least squares equation. This function can exhibit non-constant elasticity, i.e., increasing, decreasing, or negative marginal returns. It will not, however, exhibit these properties simultaneously with respect to any one input.

A major disadvantage of the Cobb-Douglas function is that, when b_i is greater than zero, the equation implies a continually increasing yield with respect to that input factor. It reaches neither a maximum nor a limiting output. Such an implication is inconsistent with biological phenomena, particularly at higher rates of nutrient application.

Variations of the power function have been proposed and investigated to overcome some of the obvious difficulties associated with the ordinary power function. Carter (6) proposed a modification which, while being more complicated than the Cobb-Douglas function, would make the function somwehat more flexible with respect to its shape. Halter (8) proposes a function of the general form

$$y = cx_1^{a_1} e^{b_1 x_1} x_2^{a_2} e^{b_2 x_2}, \text{------}, x_n^{a_n} e^{b_n x_n},$$

in which y is the total output, x_1, x_2,------, x_n are the quantities of inputs, e is the base of the natural logarithms and c, a_1, a_2,------, a_n, b_1, b_2,------, b_n are parameters. [1] This function closely resembles the classical textbook production function with the possibility of exhibiting increasing, decreasing, and negative marginal returns; it can show this singularly, in pairs, or all three simultaneously. Again estimation of the parameters of this function require extensive calculations especially when two or more inputs are considered. It would ordinarily require programming for automatic calculating machines.

(3) Quadratic Forms. A simple parabolic function of the form $y = b_o + b_i x + b_2 x^2$, where a minus sign before b_2 would denote diminishing marginal returns, does not impose the restrictions common to the power functions or the exponential functions.

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[1] Note that this function is exactly the same as the Cobb-Douglas function when b is 0.

It allows both a declining and a negative marginal productivity. The addition of a cubic term allows the considerations possible in the ordinary parabolic function plus the consideration of increasing marginal productivity. Johnson(18, p. 528-529) contends that the quadratic form is comparatively simple to fit by least squares procedures; and, at the same time, it gives results which in many cases, are equally as good as other, more complicated, functions. Johnson, however, questions the logic, from biological considerations, of implying that yields will decrease from a maximum as fast as they increased toward a maximum.

A variation of the quadratic form, used by Heady, Pesek and Brown (11) in an analysis of fertilizer data, took the form of $y = b_0 + b_1x + b_2\sqrt{x}$. This form also allows diminishing marginal returns (though not constant) just as does the unmodified quadratic. This quadratic square-root equation produces a curve which turns down more slowly than the quadratic parabola. This form might conform somewhat better to the implications of some biological laws; however, the function increases very rapidly at lower rates of nutrient application. This bias may sometimes be inconsequential, since seldom are practical fertilizer recommendations made which would suggest very low application rates. That is, farmers would not bother to make any application unless at least 15 or 20 pounds could be applied. The quadratic parabola may be easier to fit than the square root function in some case.

(4) Form free estimates. A possible approach to the problem

of selecting the appropriate algebraic form is to formulate discrete models which include appropriate qualitative restrictions as suggested by Hildreth (4, p. 62-75). In this approach the investigator may forego the assumption as to form and regard each distinct combination of independent varaibles as a different treatment, related only through appropriate restrictions made upon the variables.

The principal advantage of this procedure is that it avoids some of the biases which may accrue if an inappropriate continuous form is used. In some cases, when several types of functions are fitted to the same data, the economic implications are vastly different even though the conventional statistical criteria, such as multiple correlation coefficients and tests of deviation, differ very little. Hildreth (13) proposes certain procedures for obtaining estimates of points on a production surface under the assumptions that the inputs are subject to diminishing returns. Work in this area is presently being extended.

Drawing Economic Implications from Estimated Response Functions

Once the appropriate algebraic form is chosen and the predicting equation is generated, the economic implications can be analyzed. There are many factors which should be reviewed by a decision maker when confronted by problems of allocating scarce resources among competing alternatives. We will here mention only a few of the factors to be considered which are related to the problem of optimum fertilizer application.

For purposes of exposition it is helpful to assume that an entrepreneur with limited resources will try to allocate his resources in such a way as to maximize his total profit (gross sales minus total costs). In order to do this he must allocate his resource to each possible alternative (whether on the farm or off) in such a way that the profit from each additional unit of input in each enterprise is equal. This means that a farmer with alternatives A, B, C--------,N will put money, or other resources, to work in his most profitable alternative A until the point is reached that, if he were to invest one more dollar in alternative A, the return on that last dollar would be equal to the return he could get by putting the dollar (or other resource) into the second most profitable enterprise B. Thus he would continue to enlarge enterprises A and B to the point at which one more dollar spent in A or B would return the same profit as if he were to invest it in alternative C. In this way he should expand the use of his resources, continuing in the above manner to alternatives D, E,--------,N, until all resources are spent or allocated. If this is done, his profit or net income will in fact be maximized.

It is important to remark while passing that rarely, if ever, do farmers or any other decision makers allocate their resources in precisely this manner. Clearly no one has a certain knowledge of what each existing alternative is, or what the return will be from another dollar. Entrepreneurs often have rules of thumb, habits, "guestimations", and hunches which tend to more or less approximate this classic optimum.

Presumably the farmer would have some kind of production function available which would indicate the returns which could be expected from other available alternatives for his resources. It is not always true that the farmer will have a well defined production function available for each alternative or even one which is formulated in mathematical terms. In fact, farmers seldom have such information available. However, it is probably safe to assume that they think in terms of some "mental approximation" to a classical production function in that they have some idea as to what return they can expect from a unit of resource input in several existing alternatives.

It is also true that for most purposes these vaguely defined production functions are considered to be linear. That is, the farmer might consider that, over the range of resource inputs available for his use, a constant return of $.25 on the dollar can be expected. This is opposed to the concept of a curvilinear production function with diminishing returns which would indicate that, over a given range of investment, the returns from each additional unit of resource would decrease.

It is conceivable that farmers who have at their disposal one or two well defined production functions and several vague, mentally pictured ones, might still use profit maximizing techniques. Let us assume that a farmer was aware of his production function for wheat with respect to the input of nitrogen. If the production function for wheat yields were a quadratic function of nitrogen, such as (5.1) we would have in symbols:

$$\hat{Y} = b_o + b_1N + b_2N^2 + e \tag{5.1}$$

where

$\hat{Y}$ = wheat yield or yield increase over checkplot

N = nitrogen applied

e = an error term, assumed to be normally and independently distributed, with mean zero and variance σ .

From well known principles of economics it can be shown that for a production function such as Equation (5.1) a simple first derivative set equal the input-output price ratio allows an economically optimum solution.

If then, for the above described function, it was our purpose to solve for the value of nitrogen such that the profit with respect to nitrogen was maximized, we would first take the derivative of the function with respect to nitrogen. The next step would be to set the derivative equal to the ratio of the cost of a unit of nitrogen to the value of a unit of wheat. Thus we would have

$$d\hat{Y}/dN = b_1 + 2b_2N = P_n/P_y. \tag{5.2}$$

Solving for N in this simple example we obtain

$$N = \frac{P_n/P_y - b_1}{2b_2} \tag{5.3}$$

Since b_2 must always have a negative sign because of the necessary assumption that any production function has decreasing returns, the denominator has a negative sign. If we multiply the top and bottom of the equation by a minus one, (-1), the equation becomes

$$N = \frac{b_1 - P_n / P_y}{2b_2}$$

where a positive sign is now used for b_2. If no other information about the true production function is available, an estimate of the most profitable rate of nitrogen application can be made, if the "correct" values for P_n and P_y are used. "Correct" values here do not necessarily mean the actual quoted price of elemental nitrogen or the present or future price of wheat. It simply implies that <u>all</u> costs relative to the nitrogen and its application are considered. It also implies that all factors relative to the value of an additional bushel of wheat are considered.

Let us examine first by concept and later by example what this means.

Ownership Status and Capital Position

One important consideration which should be kept in mind when fertilizar recommendations are being considered is the ownership status and capital position of the entrepreneur. If, for instance, a specialized wheat farmer is an owner-operator, does all his own work, has all his own machinery and equipment, has plenty of capital and sees little incentive to invest resources outside his particular farming enterprise, he will presumably make decisions differently than he would if he were a part-owner, a share cropper, or a year-to-year cash renter with similar machinery, capital and investment incentives. In fact, if any one or any combination of the conditions mentioned above were changed significantly, there would be a tendency on the part of the entrepreneur to alter his decision relative to the inputs of factors of production.

To illustrate how a tenant and an owner-operator might make decisions in situations which were identical except for ownership status, consider a certain tenant renting on a 2/3 crop basis while at the same time bearing all the costs of seed, chemicals, fertilizers, and harvesting. That is, the tenant pays the owner 1/3 of the crop for the use of the land. Assuming the price of wheat to be $1.80 per bushel, suppose that the tenant were to make a certain fertilizer application which would increase the yield by 10 bushels per acre. If he were receiving all the crop, the gross value of this yield increase to him would be $18.00. However, as a tenant, receiving only 2/3 of this crop, the value of the same amount of yield increase (and therefore the fertilizer) would be only $12.00. In effect the price of wheat to the tenant in this second situation is only $1.20 per bushel.

If the argument is clear up to this point, it is easy to see that a farmer will judiciously apply less fertilizer in the situation where the effective price is \$1.20, than he would in the situation where the effective price is \$1.80. This can be recognized quickly by examining Equation (5.3) noting that as P_y is decreased from \$1.80 per bushel to \$1.20, the numerator, and thus the value of the whole equation, becomes smaller. Assuming the price of nitrogen to be \$.12 per pound and the values of b_1 and b_2 to be .267 and -.002 respectively, the economic optimum for the owner-operator would be 50 pounds per acre, whereas the economic optimum application rate for the tenant under these conditions, would be about 42 pounds. Thus in general it is safe to conclude that tenants who rent on a share crop basis should apply less fertilizer than owner-operators. /1 In fact, one should reduce the P_y in Equation (5.3) by a factor of (1-s) where s is the share of crop retained by the tenant.

At this point it should be pointed out that not all rental agreements will dictate the same strategy with respect to optimum allocation of resources. In fact if the tenant discussed above were renting on a cash basis he would follow the same economic strategy with respect to fertilizer as an owner-operator with the same capital position.

Often rental agreements include arrangements such that the landlord stands part of the costs involved in such things as chemicals and fertilizers. When this is the case, share crop

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/1 For a more complete analysis of farm management decision theory related to ownership status, see (10. p. 487-638).

tenants will now find that not only is the effective price of wheat decreased but also the effective price (cost) of the inputs is decreased. If a landlord's share of the fertilizer was 1/4, the effective cost to the tenant would be \$.09 instead of \$.12 per pound in the above example. The ratio P_n/P_y would now effectively be .09 / 1.20 where the tenant's share of the crop was 2/3 and the market price of wheat again was \$1.80. If the landlord shared the same percent of the cost of fertilizer as he received in crop, the price ratio P_n/P_y, given the above prices, would be .12 / 1.80 or the same as that ratio used by the owner-operator. This happens simply because the adjustments in the numerator and denominator of the ratio, P_n/P_y, are equal and therefore cancel each other out. /1

Application Costs and Harvesting Costs

For most situations where the farmer makes his own application, it is accurate enough to consider that the labor and machinery costs are no more for an application of 100 pounds of nitrogen per acre than for 20 pounds. If this be the case it is probably not necessary to make any adjustment in the ratio for the cost of applying fertilizer. But suppose that a farmer hires

/1 If a landlord agreed to pay all the costs of fertilizer the effective price of nitrogen to the tenant would be zero. It would therefore, pay him to try to get so-called maximum yields. This is probably the only case where "maximum yield" recommendations are valid.

a custom operator to apply the fertilizer. If the custom rate is \$2.00 per acre plus one cent per pound (the farmer furnishes the fertilizer) the effective cost of the fertilizer is increased by one cent per pound. This would tend to make the optimum allocation solution slightly less. Nevertheless, the predicted optimum rate would be expected to maximize the profit from added nitrogen.

A similar consideration should be mentioned with regard to the harvesting costs. If a farmer can harvest and haul off 50 bushel of wheat at the same total cost as 40 bushel of wheat then no adjustment in the price of wheat, P_y, need be made. But suppose it costs \$12.00 per acre plus \$.20 per bushel to harvest the wheat. Again the judicious farmer will reduce P_y by a factor of \$.20 before solving for an economic optimum.

Some similar situations are reviewed in examples in Chapter 7 using the production function calculated from the data found in Appendix I.

Reliability of an Estimated Production Function

A note should now be made concerning the production function itself. Even with a statistical production function developed from many observations, there still remains a certain amount of "gamble" as to whether estimated function is the "true" production function. Only if it is the "true" function will these so-called optimums be exact. If it is not the true function, in other words, if it does not always show the true relationships, the expected optimum solutions will not, in fact, be truely optimum. It is true that a statistical production function or yield response function is not always the "true" function simply because there is always some discrepancy associated with the predicted value as compared to the actual value. There may be farmers who have experimented with different application rates on their own farms sufficient to know approximately what responses are to be expected. In such a case the farmer might well do better to rely upon his own experience rather than upon recommendations based upon a mathematical equation derived from the results of experiments on several farms other than his own. However, in the situation where little or no information is had relative to the true response function for a particular farm, it would be best to rely upon a least squares estimate of the true response function until more complete information is available for that particular farm.

CHAPTER VI

ANALYSIS OF DATA - PART I

Correlation Analysis

It is very often useful to examine the data to find whether or not the measured observations are correlated with each other. This often gives clues as to which steps to take in any subsequent analysis(26, p. 160). For instance, if two or more variables are highly correlated it is often useful to drop one or more of them from a regression equation so as to avoid a distortion from the effects of co-linearity.

The gross coefficients of correlation, for the data at hand, are listed in Table 6.1. It will be noted that spring moisture is correlated significantly with fall moisture (.69). This is to be expected, in part, since the fall moisture content of the soil usually has a direct bearing upon the moisture content in the spring. An even more significant correlation exists between soil depth and spring moisture (.77). There may be at least two reasons for this phenomenon. First, the measurements represent total moisture readings over the range of the soil depth. Thus if there were any moisture in the soil at all, it would be roughly proportional to the depth. A second reason for the existence of such a high correlation might be that deep soils would not be expected to dry out quite so much as shallow soils under the same conditions of wind, temperature and time periods.

Table 6.1 Correlation Coefficients.

	Spring Moisture	Soil Depth	Spring Rainfall	Yield	Yield Increase /1	Application Rate of Nitrogen
Fall moisture	.69*	.53*	-.45**	.31*	-.09	.02
Spring moisture		.77*	-.46**	.56*	.16	.02
Soil depth			-.23	.65*	.34*	.009
Spring rainfall				.03	.30*	-.03
Yield					.73*	.11
Yield increase /1						.16

* Significant at 1% level or less.

** Significant if complete independence is assumed; not significant if variables are completely dependent.

/1 The increase in yields on the plots fertilized above yields on plots not fertilized.

Also it should not be surprising that soil depth is more highly correlated with spring moisture than with fall moisture. This might be due to the fact that soil depth often becomes a limiting factor in the storage of winter rain and snow. That is, the winter precipitation is often sufficient to give a soil all the moisture it can hold. This would tend to reduce the variation in spring moisture readings between high and low winter precipitations. At the same time the moisture readings would be forced to parallel the soil depth quite closely. On the other hand, precipitation is seldom, if ever, high enough in this area during the summer months, to saturate the soil completely.

The negative correlation between spring rainfall and the other moisture factors seems to be due mainly to normal variations in rainfall patterns which occurred during the four years of this experiment. A high spring rainfall in one year was usually followed by a low spring rainfall the following year. For instance in 1953 /1 the fall moisture readings were generally above average, due, in part, to high rainfall during the previous spring and summer. It happened that the precipitation was below normal the following spring. It must be pointed out, if it is not already clear from the description of the data, that the moisture observations taken during any one year are not independent of each other since weather conditions tend to be somewhat similar from location to location in this comparatively restricted area. This fact will have significant bearing upon any conclusions drawn from subsequent analysis.

The correlation between yield and the fall and spring moisture readings are also to be expected. This correlation is probably because of the fact that high moisture readings were generally found on deep soils. The fact that yields were high

/1 Fall moisture readings taken in 1953 were correlated with moisture readings in 1954 and the moisture readings taken in 1954 were correlated with those taken in 1955, etc.

where moisture factors were high may have been caused by the fact that deep soils can retain soil moisture for longer periods of time. Note, in this connection, the very high correlation between yield and soil depth and the low correlation between yield and spring rainfall.

It is interesting to observe here that neither yield nor yield increase is significantly correlated with the rates of nitrogen application; at least this correlation analysis is not sufficiently effective to detect a statistically significant relationship between nitrogen application and yield. The reasons for this should become apparent from the regression analysis as it is developed.

Regression Analysis - Total Yield

It is interesting and often helpful to know that yield is correlated with one variable or another. However, from an economic decision making point of view, it is much more helpful to know just how much yield can be expected from a given soil depth, moisture reading, fertilizer application, and soil test reading. /1

The method of regression provides a broad and often helpful approach to this problem. It is broad because, within the framework of regression analysis, there are an infinite number of regression equations which could be proposed in an effort to

/1 This term "soil test reading", unless otherwise specified, will have reference to a test for soil nitrogen on plots where no nitrogen applications have been made.

estimate the underlying parameters involved in the production function. Thus there is a problem facing the researcher of determining the equation which is in some way best or better than other existing alternatives. The basic goal in formulating regression equations or predicting models is to formulate an equation which best "characterizes" the data while at the same time conforms to known rules of statistical logic and the implications of proven biological phenomena (15, p. 15).

Model Proposed for the Analysis

One mathematical predicting model recently proposed and currently receiving considerable attention is an extension of the ordinary "least squares" quadratic polynomial equation involving added nitrogen as one of the independent variables. The extension or modification of this rather basic model would be to consider yield to be a function not only of nitrogen added but also of the nitrogen already available in the soil. Thus yield is to be considered a function of the total available nitrogen in the soil. This consideration can be made in several ways, but the unique model presented here is that one proposed by C. G. Hildreth (3, p. 176-186).

Hildreth suggests that if the nutrients measured in the soil by various soil tests were exactly the same as those to be added, and, if the measurements of the nutrients detected were accurate, one should conclude that the total amount of nutrient available in the soil would be simply the amount added plus the amount measured. In order to determine the amount of fertilizer to apply, the farmer or reseracher would first determine the total amount of nutrient necessary for a given yield response. The difference between this total amount and that amount already in the soil would be made up by an artificial application. The applied nitrogen would be assumed to replace the deficiency pound for pound; that is, if a 20 pound deficiency were noted, a 20 pound application would bring the nutrient concentration up to the desired level.

However, there is good reason to believe that there is a chemical and physical difference between the nutrient in the soil and the nutrient applied in artificial form. Furthermore, the quantitative aspects of most soil tests are not well enough developed so that an accurate indication of the amount of nutrient in the soil is given. Especially is this true in the case of nitrogen. Hildreth therefore suggests that for a specific soil type and under certain conditions, a given amount of nutrient in the soil might replace a proportional amount of the nutrient added. If this is the case, then the total amount of nutrient available in the soil can be regarded as the amount of nutrient added plus the product of an unknown constant multiplied by the amount of fertilizer detected in the soil. This written in symbols would be:

$$Y = f(N) + e \qquad (6.1)$$

$$N = n + \lambda st \qquad (6.2)$$

where

Y = total yield

N = total nutrient available in the soil

n = the amount of nutrient added

st = the soil test reading

λ = the unknown factor of proportionality to be determined

e = a random disturbance.

If Equation (6.1) happened to be of a linear form, then it would be a simple matter to estimate, by least squares procedures, the production function parameters and at the same time maintain the conditions expressed in Equation (6.2). However, when the assumption of linearity is dropped, as is necessary in order to determine economically optimum application rates, the procedure is no longer straightforward or simple. Hildreth proposed certain techniques for finding estimates of λ. These are essentially iterative procedures requiring use of highspeed electronic equipment because of the burdensome computing procedures.

One such procedure suggested and used by Hildreth (14) and followed in this project was to assume various values for λ over a range of possible values. For each λ value assumed a regression equation was computed. The purpose of this was to converge on a value of λ which would minimize the sum of squares of the deviations from regression. One would select the range over which the convergence was to be made by weighting such factors as the amount of data, the type of model, the speed and memory capacity of the computer, and the accuracy desired - all of which must be commensurate with the money and time available for the research. One might take extreme values such as three at the upper limit and zero at the lower limit and move by increments of any desired length toward an optimum. It is possible that one could select rather large increments, say .5, for the first "sweep" over the range. After examining the error sum of squares for these lambda values a smaller relevant range can be isolated. The same procedure could then be carried out over the newly isolated range and so on until the desired accuracy is attained.

It would be expected that as λ is changed within the "possible" range of values the error sum of squares would also change. If one were comparing different soil tests, one would note that the error sum of squares approaches the same value as λ approaches zero. This is because, at this value of λ, the predicting equations are exactly the same. N in the model is now equal to n because the effect of any soil test reading, no matter how large or how small, is ignored. However, as λ takes on positive values, the predicting equation now regards the soil test measurement as weighted by the value of λ. Thus if there is any relation between the soil test reading and the true or actual amount of nitrogen available, the predicting equation would supposedly be enhanced in some degree (the error sum of squares being reduced) until such a time as a minimum is reached. This, then, is an iterative least squares estimate of λ. At this point, the accuracy of the predicting model is at a maximum.

It was apparent from the data being used and the known capacity of the computer available for this project that the following model should be fitted for the analysis. This is the familiar quadratic polynomial where, for Condon soils,

$$\hat{Y} = B_0 + B_1N + B_2F + B_3S + B_4D + B_5R + B_6N^2 + B_7N\text{-}F$$

$$+ B_8N \cdot S + B_9N \cdot D + B_{10}N \cdot R \qquad (6.3)$$

subject to restriction

$$N = n + \lambda st \qquad (6.2)$$

where

Y	=	the total yield
N	=	total available nitrogen in the soil after an application
F	=	fall moisture
S	=	spring moisture
D	=	soil depth
R	=	spring rainfall
N•F	=	nitrogen, fall moisture interaction
N.S	=	nitrogen, spring moisture interaction
N•D	=	nitrogen, soil depth interaction
N•R	=	nitrogen, spring rainfall interaction

B_o = an effect common to all variables in this equation

$B_1, B_2, \text{---}, B_{10}$ = coefficients indicating the added effects due to the respective variables

and where

n = amount of nitrogen added

λ = constant of proportionality

st = the nitrogen soil test reading.

Results

This model was fitted to the data in Appendix I for each of the six"different" /1 soil test readings. The value of the error sum of squares for the model using four of the six soil test readings are given in Table 6.2. The values are given at the optimum value of λ for each soil test. Also, values of the regression sum of squares, the variance, and the coefficient of determination are listed. At the bottom of Table 6.2 is listed the corresponding values for $\lambda = 0$. /2

Note that when the spring nitrifiable nitrogen soil test is used as a basis for prediction, the model seems to respond

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/1 There were three tests made in the fall and three tests made in the spring making six different tests. The analytical procedure was the same in the fall and spring. The four readings, which were combinations of the nitrate nitrogen test and one of the other two tests, plus the fall and spring ammonia readings, were not considered extensively when, in the course of finding the optimum value of λ, it became apparent that the results would not be useful.

/2 When $\lambda = 0$ the value of the model is the same for each soil test.

Table 6.2. Comparison of soil test readings.

Soil Test Reading		Optimum λ	Sum of Squares of: Regression	Error	Variance	R^2
(1)		(2)	(3)	(4)	(5)	(6)
Nitrate Nitrogen	F	.3	24204	15966	47.9456	.6025
	S	.1	23904	16266	48.8469	.5951
Nitrifiable Nitrogen	F	2.3	25273	14897	44.7357	.6291
	S	3.0	28229	11941	38.8589	.7027
Ammonia /a	F	1.5	26429	13741	41.2643	.6579
	S	2.0	27450	12720	38.1982	.6833
Soil Test Ignored		0	23897	16273	48.868	.5949

/a Lambda values not determined precisely. (See foot note page 47)

most to changes in the value of λ. Spring nitrate nitrogen, on the other hand, seems to cause the model to respond least when λ is varied. One degree of freedom may be assigned to λ at its maximum value /1 for purposes of testing to see whether it improves the estimate of the yields. It is found that neither the spring nor the fall nitrate nitrogen soil tests add any significant information when used as described in the above model. However the other soil tests do add significantly when added at their optimum λ values. For the model using the spring nitrifiable test there was a decrease in the error sum of squares from 16273 to 11941 when λ was set at 3.0. This decreased the variance from approximately 49 bushels

/1 This assignment is arbitrary since no new information is added, and technically it does not require a degree of freedom.

down to approximately 36 bushels. In terms of the standard deviation this is a decrease of from approximately seven bushels to approximately six bushels.

The values of the error sum of squares are plotted in Figure 6.1 against the corresponding value λ. Note that each of the four functions plotted decreases to a minimum and then begin to rise as λ increases in magnitude. The wide range of optimum λ values may be surprising. They vary all the way from .1 in the case of the spring nitrate nitrogen test to approximately 3.0 in the case of the spring nitrifiable nitrogen test.

On the basis of the smallest error sum of squares it would be concluded that the spring nitrifiable nitrogen test should be used for predicting yields with this model. However, illogical b values tend to throw considerable doubt upon this conclusion. The regression coefficients estimated at the optimum values of λ can be seen in Table 6.3. Also in the same table are listed the coefficients estimated at $\lambda = 0$. This provides a convenient comparison of the five estimates of the b values in the model.

Whenever any of the variables in a regression equation are changed or dropped the corresponding estimates of the parameters change. Thus, in the case of Equation (6.3), as λ is varied from zero to the optimum value of λ for each soil test, the b values change. The extent of the change is observable in Table 6.3 by noting the estimates at $\lambda = 0$ and comparing these

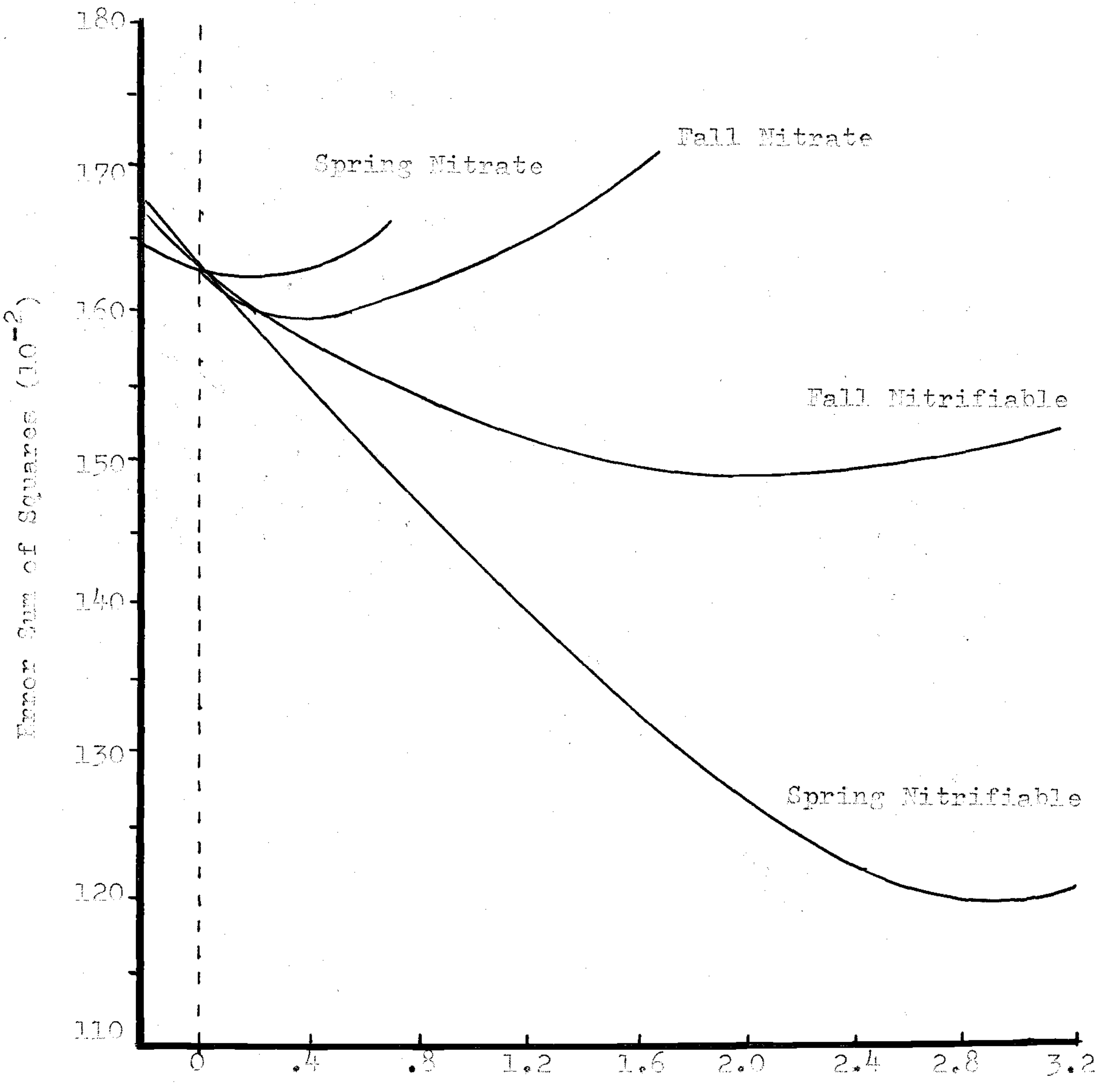

Figure 6.1. Error sum of squares for four soil tests incorporated into Equation(6.1).

at any optimum λ value. In some cases the respective estimates of the B's changed markedly. These changes were of greatest magnitude where the optimum value of λ was in the neighborhood

Table 6.3. Estimates of parameters of Equation (6.1) where four different soil tests were used and where the soil test was ignored.

Soil Test Reading	Fall Nitrate Nitrogen	Spring Nitrate Nitrogen	Fall Nitri- fiable Nitrogen	Spring Nitri- fiable Nitrogen	Soil Test Reading
Optimum λ	(.3)	(.1)	(2.3)	(3.0)	(0)
B_0	8.5998	6.4393	19.0704	23.3290	6.3670
St. Error	4.630	3.774	5.847	5.792	3.755
B_1 (N)	- .11166	-.16426	-.21137	-.29433	-.16545
St. error	.0784	.0741	.0600	.0549	.0739
B_2 (F)	1.4954	1.2154	3.8164	4.6768	1.2154
St. error	.7758	.6545	1.071	.9180	.6524
B_3 (S)	1.7482	1.6951	.05577	2.8498	1.6958
St. error	.8008	.6960	1.131	.9433	.6939
B_4 (D)	1.0548	2.2647	1.5760	-5.5358	2.2952
St. error	1.666	1.411	2.22	1.84	1.405
B_5 (R)	.05240	.77410	-4.6493	-1.5155	.77965
St. error	.8702	.7311	1.253	1.049	.7283
B_6 (N^2) /1	-.1322	-.1349	-.01374	.02096	-.1342
St. error/1	.0353	.0388	.0132	.0130	.0388
B_7 (N.F)	-.03254	-.03400	-.02758	-.04900	-.03415
St. error	.0108	.0109	.0091	.00768	.0109
B_8 (N.S)	.01238	.01648	.01342	-.00190	.01653
St. error	.0115	.0118	.0102	.0083	.0118
B_9 (N.D)	.07753	.07790	.04764	.10739	.07784
St. error	.0381	.0243	.0198	.0165	.0243
B_{10} (N.R)	.04410	.04492	.06010	.03766	.04508
St. error	.0120	.0129	.0105	.00850	.0124

/1 Multiply each number in this row by 10^{-2}.

of from 2.0 to 3.0. This is true in the models incorporating fall and spring nitrifiable nitrogen as well as fall and spring ammonia. /1 Of particular interest is the behavior of the estimate of the quadratic effect of nitrogen. In the case where the model predicted on the basis of a spring nitrifiable test, the sign of the coefficient of N^2 was negative when λ was less than approximately 1.2. At values greater than approximately 1.2 the value of b_6 was positive. In fact in each case the estimates of the quadratic effect became smaller in absolute magnitude as λ increased.

The fact that the estimate of the quadratic effect varied as much as it did, as λ went from zero to its optimum value in each case, tended to throw doubt upon the reliability of the model for purposes of economic analysis of this data. The possibility of such an analysis was almost completely destroyed when the coefficient of N^2 term in the spring nitrifiable nitrogen model was estimated to be a positive value. Also, the fact that the estimates of other equally important parameters were somewhat illogical caused an abandonment of this approach.

Thus only these few brief passing remarks are given to indicate the results of this analysis. However, these results

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/1 An accurate determination of the optimum λ value for the model using fall and spring ammonia was not made when it was decided that the results might not be useful. However, an approximate value of λ was obtained which is probably within .2 or .3 of the optimum. Since the estimate is not exact, the estimates of the λ values are not listed in Table 6.3.

do not mean that this approach is inappropriate in this type of equation. It does mean that this approach did not work very well with this yield and soil test data which was highly variable in many respects.

CHAPTER VII

ANALYSIS OF DATA - PART II

Regression Analysis - Yield Increase

This chapter describes and presents the results of a different appraoch to the problem of predicting yield response functions or surfaces. The alternative approach described in this section incorporates the method of least squares, but predicts the yield increase on a particular location rather than the total yield. /1 Several advantages of such a procedure come immediately to mind. For one thing, total yield is affected by many factors which were not, or cannot be, adequately measured under conditions of these field experiments. Yield is not only a function of the variables measured for this study but it is also a function of such things as present and previous cropping practices, presence or absence of one or many different types of soil nutrients, lengths of growing season, temperature at critical periods of growth (19), hail storms, and many other important factors. In an experiment over an extended area and time, all of the above factors vary from location to location and from year to year. But they remain relatively constant

/1 As mentioned earlier yield increase refers to the difference between yield on a plot where no fertilizer is added and the yield on a plot where a given amount of fertilizer is added. The yield increase data in this analysis are simply the yields on each treatment plot minus the yield on each check plot for each of the 30 locations.

between different plots at the same location.

If the yield increase is considered the dependent variable, the direct effects of such variables as rainfall, soil depth, temperature, soil conditions, etc., are, for the most part, excluded from the analysis. Thus, for the data, the only variable with any direct effect upon yield response would be nitrogen addition.

There are, in addition to the direct effects of nitrogen, certain indirect or interacting effects to be expected. The only independent variable which changes throughout a particular location, is nitrogen applications. The other variables (constant for a location) would affect the yield response through the nitrogen application.

By simply subtracting the check plot yield on each location from the yields at various rates of nitrogen application for each location, the yield data are changed to yield increase data. /1 Subtracting the check plot yield from each treatment yield does not eliminate the possibility of predicting economically optimum nitrogen application rates. In fact, one would

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/1 This procedure automatically gives a zero for each check plot observation. In a regression analysis, the degrees of freedom must be decreased by one for each location. In fact, for the data presented in Appendix I, the above described procedure requires that the degrees of freedom be reduced by two for each check plot yield since there is a fall and a spring application check plot on each location. Thus with 30 locations there are 60 arbitrary zeros in the yield increase data. Therefore the number of degrees of freedom are 344-m-60-1, where m is the number of independent variables in the predicting equation.

obtain essentially the same optimum prediction with "yield increase" data as with total yield data. This can be seen in Figure 7.1 where three arbitrary yield functions are represented by A, B, and C and the corresponding yield increase functions are represented by A', B', and C'. The functions are tangent to the price ratio line (Py/Pn) at a, b, c, and a', b', c' respectively. It will be noted (and it can be shown from theory) that a = a', b = b', and c = c'. Thus it can be expected that if the estimates of these factors are unbiased, the expected value of the estimated economic optimum would be the same for either procedure.

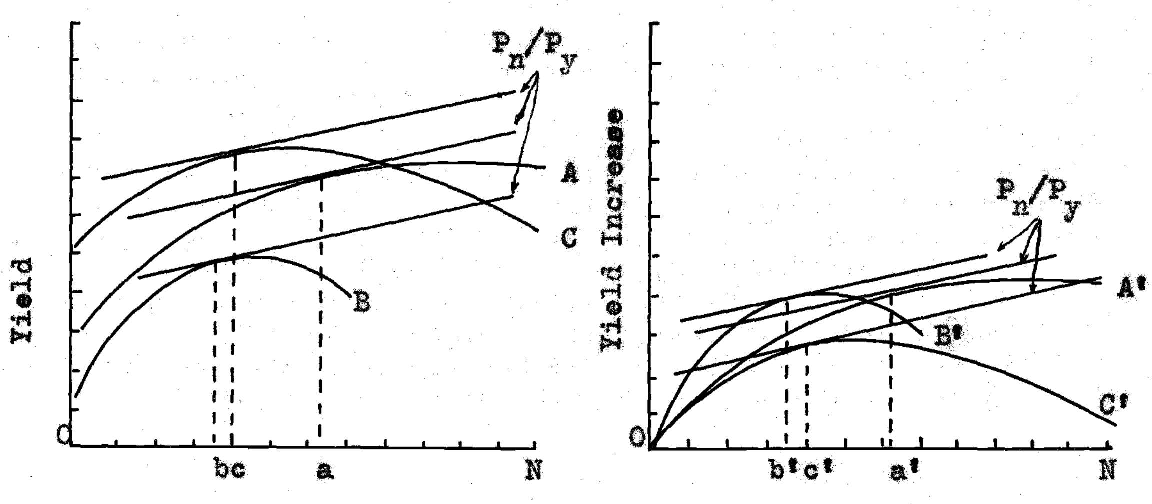

Figure 7.1. A yield function, a yield increase function and the economically optimum solution.

There are at least three distinct ways in which a yield increase predicting model can be constructed. First, the predicting equation can be forced to go through the origin.

This has a certain amount of logical justification inasmuch as it is known that the yield increase will be zero where no nitrogen is applied. This procedure destroys the condition necessary for obtaining an unbiased estimate of the B values. However, the estimates are usually unbiased enough to be useable for all practical purposes. /1

A second feasible procedure would be to let each function go through a particular point (bo) on the yield increase axis. The estimate of the parameters by this procedure are unbiased, but this advantage is offset somewhat by a loss of some logical superiority.

A third procedure which can be used is simply to ignore the zero values on the check plots and to fit a function based on yield increases observed on plots where 20 or more pounds of nitrogen have been added. This procedure gives a lower error sum of squares than either of the two procedures discussed above. /2 At the same time this method can be justified on the grounds that farmers are seldom interested in recommendations below 20 pounds since the cost of application in such instances is often more than the expected returns. However, the most important disadvantage in this procedure from an economic

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/1 Using an unbiased procedure by fitting a yield increase intercept did not change the results significantly with the data of this experiment.

/2 The error sum of squares was not decreased significantly by this procedure in the analysis.

standpoint is that the zero yield observations (check plots) are very important in determining the magnitude of the quadratic effect. If these "observations" are ignored, the estimate of the coefficient of the quadratic term tends to be a smaller absolute magnitude. This can and does have significant implications on any economic interpretations based on these estimated functions.

The Model Proposed for the Analysis

After the various advantages and disadvantages of the above methods were weighed, it was decided that it would be best to follow the procedure of forcing the function through the origin. Furthermore it was decided that the data might best be explained with a continuous parabolic equation. So as to provide for consideration of all the data available for this analysis the following regression equation was proposed. For a given location on Condon soils, and for a specific application time, /1 the predicted yield increase function is

$$\hat{Y}_i = b_1 n_i + b_2 n_i^2 + b_3 n_i \cdot st + b_4 n_i \cdot F + b_5 n_i \cdot S$$

$$+ b_6 n_i \cdot D + b_7 n_i \cdot R + b_8 n_i \cdot A + b_9 n_i \cdot D/st + e_i \qquad (7.1)$$

where

$\hat{Y}_i$ = expected yield increase from the application of i pounds of nitrogen per acre where i = 0, 20, 40, 60, 80, 100.

/1 This refers to either a spring application or a fall application of nitrogen.

n_i = pounds of nitrogen per acre

st = a soil test reading in pounds of nitrogen per acre

S = spring moisture reading in inches

F = fall moisture reading in inches

D = soil depth in feet

A = one (1) when the nitrogen application is made in the spring, 0 when the application is made in the fall

b_1 = the estimated linear effect of one pound of nitrogen

b_2 = the estimated curvilinear effect of one pound of nitrogen

b_3-----b_9 = the estimated interaction effects of the respective variables

e_i = a random disturbance.

The above model allows a test of the hypothesis that yield increase on a given plot, with a given amount of nitrogen applied, is a quadratic function of the nitrogen applied. At the same time it allows a test of the hypotheses that a linear relationship exists between the amount of nitrogen added and (1) the amount of nitrogen detected in the soil before the application, (2) the amount of fall and spring moisture in the soil, (3) the depth of soil alone, (4) the depth of the soil relative to the amount of nitrogen detected, and (5) the time at which the nitrogen is applied.

The hypothesis that yield is a negative, curvilinear function of nitrogen is essential for economic analysis. The quadratic variable (n^2) would be left in the analysis whether or not it was later shown to be statistically significant.

The second hypothesis which can be tested in the above model is that the greater the amount of nitrogen already in the soil, the less will be the yield increasing effect of future additions of commercial nitrogen. Stating the contrapositive of this hypothesis, we would say that the smaller the amounts of nitrogen in the soil, the greater the yield increasing effects of additions of nitrogen. The above model tests this hypothesis in two ways. First, the nitrogen soil test interaction term, the way it is formulated in the model, would take a negative sign if higher soil test readings were followed by smaller yield increases. A second and probably more realistic test is one which makes yield increase a function of the "concentration" of nitrogen already in the soil. A little reflection will indicate why this might be expected to be so. First, the various measurements of growing conditions used in this analysis are all totals. That is, they measure the total amount of moisture or nitrogen from the surface of the soil to a depth of five feet or bed rock. Thus, a deep soil might have a relatively high soil nitrogen reading but at the same time have a relatively low concentration of nitrogen per cubic foot of soil. For this reason it would not be expected that the n-st variable would be able to "detect", as it were, the same relationship as would a variable such as n•D/st in the above model.

It would seem that, if soil depth were divided by the soil test reading, the resulting figure would represent a fairly accurate index of nitrogen "concentration". For instance, if a 3 foot soil had a relatively high soil nitrogen reading, say 90 pounds, the "index" would be relatively small (.0333). A very small yield response would be expected from further application of nitrogen. On the other hand, if a 3 foot soil had a soil test of 10, a large index (.30) would result and, at the same time, a dramatic yield response might be expected. Thus, if this index were to indicate accurately the true concentration of nitrogen in the soil, then the coefficient, b_9, should take a positive sign. /1

A third hypothesis to be examined here stems from a knowledge of the physical relationships between the activities of nitrogen fixing bacteria and the soil moisture during the growing season. Since bacteria require an adequate supply of moisture it is expected that moisture levels would also have to be adequate. If moisture levels were not sufficiently high during initial periods, yields could well be decreased by additions of nitrogen. This relationship may not be true for all levels of moisture, particularly the very high levels; but for the precipitation levels most common in eastern Oregon, it would likely hold true. The hypotheses then to be tested are that (1) spring soil moisture and, (2) spring rainfall have yield increasing effects.

/1 It is conceivable that one of these indicators could completely overshadow the other in its importance in the model. This would presumably happen when one of the two variables was greatly superior to the other in the predicting equation.

In addition to the consideration of soil depth in relation to the amount of nitrogen present, a consideration of the soil depth in relation to the soil moisture as it effects yield responses to nitrogen. Generally, deeper soils hold greater amounts of moisture. This extra moisture gives bacteria longer "productive" periods and gives the plant greater growing potential. Thus, for Condon soils, which are five feet deep or less, in areas where adequate moisture during the growing is critical, yield responses would be expected to be greater on deeper soils and smaller on shallower soils.

As for soil moisture in the fall, it is not well established what its effect would be. Some researchers (4) have indicated that a high moisture level is desirable in the fall so as to insure maximum growth of the plant before winter snows come. Other researchers suspect that this may not be the case. If a high fall moisture reading were to be accompanied by a heavy application of nitrogen, a rank growth would likely result during the fall and ensuing spring. The result in yields, however, may be exactly the opposite. A rank stand of wheat will require greater amounts of moisture to ensure maturity of the grain during the following summer. Therefore, if the available moisture level was not high enough to sustain the rank growth during the following summer, significant yield decreases might well result.

Results

Evaluation of Soil Test Readings

In as much as one of the purposes of this study was to evaluate the nitrogen soil tests presently available and in use, the above described model was fitted separately for each nitrogen test. In fact, since some recommendations are based upon a combination of two of these readings, these "combinations" were also tested. There are presently three types of tests used to detect soil nitrogen. The three tests are for nitrate nitrogen, nitrifiable nitrogen, and ammonia. Recommendations are sometimes made from the result of adding the nitrate nitrogen test to the results of the nitrifiable nitrogen test or to the ammonia test. Thus our analysis covers essentially ten different tests - five fall tests and five spring tests.

In Table 7.1 are listed the error sum of squares, the variance, and the coefficient of determination for the model where the type of nitrogen soil test is varied over the ten different alternatives. By comparing the values of the error sum of squares or the variances, (noting the smallest values in each case) it will be seen that the nitrate nitrogen test seems to give the best results whether one is interested in a fall or a spring application. In other words, where a spring application is to be made, the results of this study indicate that a nitrate nitrogen soil test reading should be used for

Table 7.1. Summary of statistics for comparing the predicting ability of Equation (7.1) using ten different nitrogen soil tests.

Identity	Season	Error SS	Variance	R^2	Sum of Squares of Regression	Standard Deviation
Nitrate Nitrogen	Fall	8686.5	31.5873	.580	12018.2	5.6203
	Spring	5976.9	21.7342	.711	14727.8	4.6620
Nitrifiable Nitrogen	Fall	8794.5	32.0964	.575	11910.2	5.6654
	Spring	8415.4	30.7128	.594	12289.3	5.5419
Ammonia	Fall	9447.2	34.4785	.544	11257.5	5.8718
	Spring	6851.2	25.0041	.669	13853.5	5.0004
Nitrate plus Nitrifiable	Fall	9187.7	33.5679	.556	11517.0	5.7938
	Spring	9214.7	33.6300	.555	11490.0	5.7914
Nitrate plus Ammonia	Fall	9510.1	34.7081	.541	11194.6	5.8914
	Spring	7968.5	29.0818	.615	12736.2	5.3928

purposes of making application rate recommendations. The variance of the model using the spring nitrate reading was 21.7342 as compared to 25.0041 where spring ammonia was used. The poorest indication of nitrogen present, based on its ability to predict yield response through this model, is nitrate nitrogen plus nitrifiable nitrogen with a variance of 33.6300. Thus the spring readings in order of their apparent superiority are nitrate nitrogen, ammonia, nitrate nitrogen plus ammonia, nitrifiable nitrogen, and finally nitrate nitrogen plus nitrifiable nitrogen.

The results of this study would give no justification for the use of a fall soil test reading to predict yield responses from a spring application if a spring reading can be obtained. This is seen by comparing the variance of the model using a soil test taken in the fall with the variance of the model using the corresponding soil tests taken in the spring. In almost every case the variance is higher where a fall reading is used in the model. The only justification for the use of a fall reading would be if it were impossible to obtain a reading in the spring before it was time to make the fertilizer application. Then it would be better to use a fall reading rather than to use no indicator at all.

If a fall application is desired, the results of the fall nitrate nitrogen test should be used. /1 The variance where this reading was used was 31.5874 as compared to 32.0964 when the fall nitrifiable nitrogen test was used and 33.5679 when a combination of a nitrate nitrogen and nitrifiable nitrogen test was used. The poorest indicator of nitrogen present is apparently a nitrate nitrogen reading plus a test for ammonia. The variance in this case was 34.7081. It might be added, however, that as far as fall readings are concerned there is not much basis for favoring any one of these indicators even though, from the

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/1 It would be impossible to use a spring reading to indicate the optimum rate for fall application unless the results of a test made in previous years was relied upon. However, this model has no basis for using the results of readings other than current ones.

evidence posed here, the fall nitrate nitrogen test has a slight superiority.

The Predicting Equation Based upon the Spring Nitrate Nitrogen Soil Test Reading

Since both fall and spring nitrate nitrogen readings seem to perform best through this predicting equation, these two equations may be examined in greater detail. In particular, since the spring reading appeared to be the better of the two, it will be considered first. The equation, with its appropriate coefficients, is

$$\hat{Y}_s = -.27258n - .0011383\,n^2 - .022291n \cdot F + .021837n \cdot S + .055816n \cdot D + .035568n \cdot R + 1.09273n \cdot D/st + .039221n \cdot A \tag{7.2}$$

The nitrogen soil test interaction variable was deleted from the model when an analysis showed that its added effect was not statistically significant. The above equation lists only the coefficients which are significant at the 5% level or less. /1

Fall Moisture

The expected independent effect of fall moisture acting through one pound of nitrogen would be to decrease the yield by .0223 bushel for each inch of fall moisture. If fall moisture were acting through 50 pounds of nitrogen the expected

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/1 The coefficients of Equation (7.2) are listed in Table 7.3 along with their respective standard deviations.

Table 7.2. Estimated regression coefficients and corresponding standard deviations for two predicting equations.

Independent Variable	Equation (7.2)		Equation (7.3)	
	Estimated Regression Coefficient	Standard Deviation	Estimated Regression Coefficient	Standard Deviation
n	-.27258	.02973	-.14458	.03959
n^2 /a	-.11383	.02089	-.13522	.02513
n.st /a	---	---	-.13685	.02408
n.F	-.02229	.00420	-.02820	.00503
n.S	.02184	.00449	.01695	.00539
n.D	.05582	.00964	.07958	.01134
n.R	.03557	.00488	.04763	.00574
n.D/st	1.09273	.08340	---	---
n.A	.03922	.00866	.03922	.01050

/a Multiply each number in this row by 10^{-2} to get the actual value

decrease in yield would be 1.115 bushels for each inch of fall moisture.

The sign of this coefficient adds weight to the suspicions of those researchers who suggest that excessive growth in the fall reduces yields. Excessive fall moisture however, will reduce yields only if soil and moisture conditions are inadequate the following spring. For instance, if we note the coefficient of the n.S variable we see that with 50 pounds of nitrogen, the yield will be increased by 1.092 bushels for each inch of spring rainfall. Thus, if all other things are constant in the equation, and spring moisture is only slightly greater than fall moisture, there will tend to be a net increase in yield from the effects of these two moisture variables.

Soil Depth

Soil depth is by far the most important physical feature in determining yield responses on Condon soils. This is seen by examining the b value related to soil depth alone and the b value related to the soil depth-soil test ratio. Assuming for a moment that farms A and B had exactly the same moisture conditions, the same amount of nitrogen in the soil, say 30 pounds, and soil depth of 3 and 4 feet respectively. The expected yield response from a pound of nitrogen on farm A would be .277 bushels whereas the response on farm B would be .369 bushels. The effect then of this particular foot of soil is to increase the yield by about .108 bushels for each pound of nitrogen added. With a 50 pound application, there would be a difference of approximately 5.4 bushels per acre in favor of the 4 foot soil.

A careful examination of Equation (7.2) will reveal that a difference in soil depth of one (1) foot will not always be expected to induce the same difference in yield response. The variation will be due to the relationship between the soil depth and the soil test reading. For instance, referring again to the above example, if farms A and B both had 100 pounds of nitrogen in the soil, the expected effect of an addition of 50 pounds of nitrogen would be to increase the yield on farm B by 2.3 bushel per acre more than on farm A.

Nitrogen in the Soil

The relationship of the soil test reading and soil depth is shown in Figure 7.2 where yield increase is plotted against soil test readings at various soil depths. The conditions under which these relationships hold true are typical of those found in this experiment - specifically, three inches of fall moisture, four inches of soil moisture and 2.5 inches of spring rainfall. This figure shows the expected response at a 30 pound nitrogen application rate. It will be seen that for deeper soils generally higher yield responses are expected. For instance a five foot soil with a soil test reading of 20 pounds is expected to show approximately a 12 bushel yield increase from a 30 pound application rate. If the soil test indicated 100 pounds rather than 20 pounds, the expected increase would be approximately five bushel.

Shallow soils, on the other hand, tend to show lower responses from nitrogen application, although very insignificant yield increases can be expected if the nitrogen content of the soils is sufficiently low. For instance, a two foot soil with a soil test reading showing 10 pounds of available nitrogen would likely respond as well as a five foot soil with 100 pounds of available nitrogen. However if shallow soils (2 feet or less) have more than 50 pounds of available nitrogen, a decrease in yield is expected from a 30 pound nitrogen application.

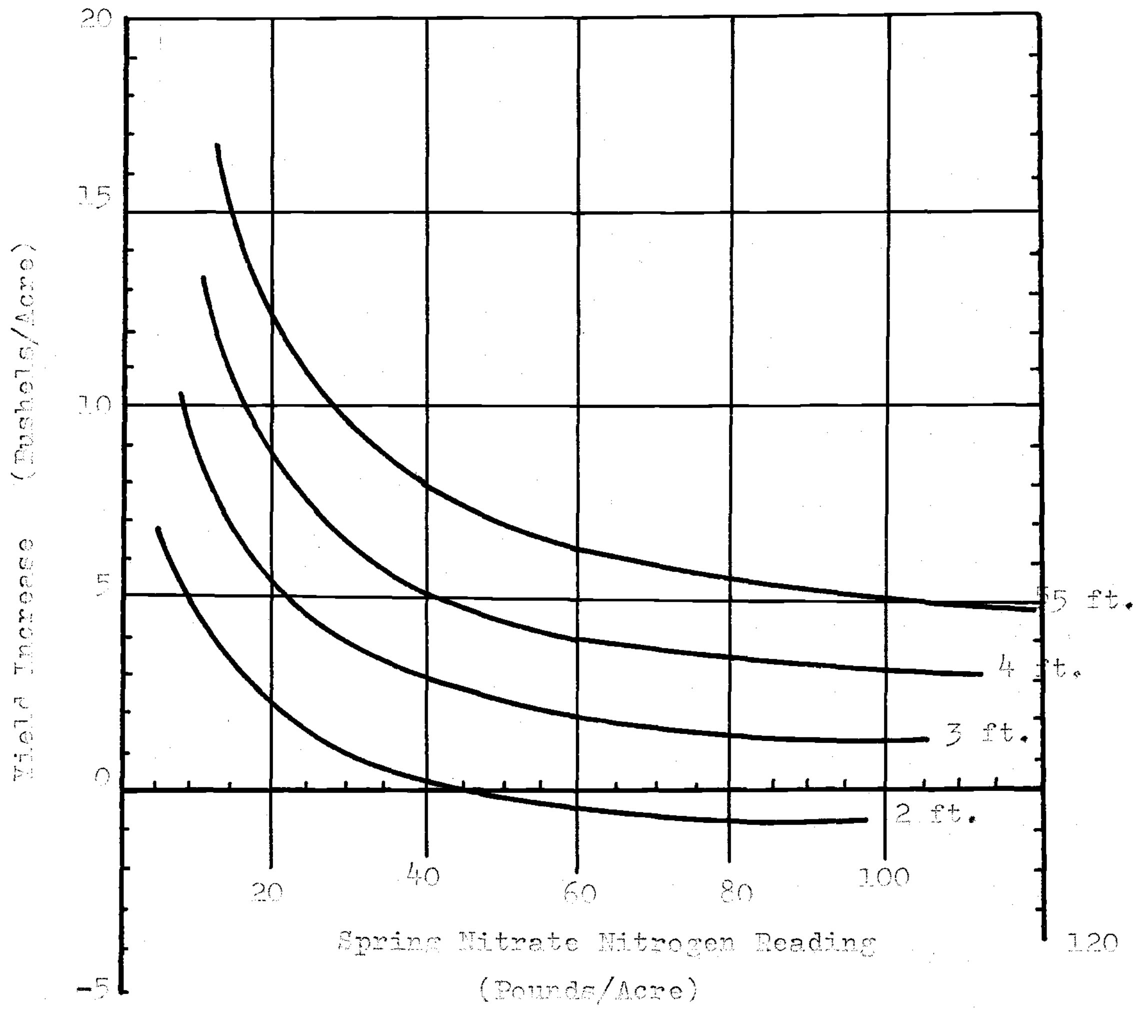

Figure 7.2. Predicted yield increase from a 30 pound application of nitrogen under average moisture conditions on Condon soils.

Application Time

It appears from the analysis of these experiments that a spring application of nitrogen is almost always followed by higher yields than is a fall application where made on Condon soils. Noting the coefficient of the spring application variable in Equation (7.2) and Equation (7.3) we see that the expected yield increase from a spring application over a fall application would be approximately .0392 bushels per pound of nitrogen added. Therefore, for an application rate of 40 pounds, the yield increase due to spring application alone would be anywhere from .7 bushel to 2.4 bushels. /1 If the .0392 represents the true average yield increase per pound of nitrogen because of spring application, the average yield increase at 40 pounds would be about 1.6 bushels. When wheat sells for $1.80 per bushel, a spring application is expected to be worth about $2.88 per acre more than a fall application. At higher levels of nitrogen application, the value of a spring application is greater. At lower application rates a spring application is worth correspondingly less, according to the model.

Spring Moisture and Spring Rainfall

The expected effect of an inch of spring rainfall working through a pound of nitrogen is to increase yields by .0756 bushels per acre. Through 50 pounds of nitrogen the expected effect of each inch of spring rainfall is to increase yields by about 1.7784 bushels. Thus if two inches of rainfall comes after a spring application of 50 pounds of nitrogen it might be expected that 1.778 more bushels per acre would result than if

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/1 This is a statement of the 95% confidence interval. It is assumed that conditions measured in this experiment are unbiased estimates of the real conditions. It must be noted that the procedure of testing several models in order to select the best model induces bias into the estimates of each parameter.

only one inch of rain fell. Spring soil moisture tends to have a yield increasing tendency. Acting upon an application of 50 pounds of nitrogen, each inch of moisture in the soil, at the time when a spring soil test reading is usually made, will tend to increase yields by 1.0918 bushels.

An interesting comparison can be made between the value of various moisture factors and other physical factors. For purposes of comparing the effects of fall moisture, spring moisture, soil depth, soil test, spring rainfall, and spring application of nitrogen, the fact that each of these works through nitrogen can be ignored. This is simply because, for any prediction, the value of n in Equation (7.2) is the same. Thus we see, from a comparison of the b values, that the effect of one inch of fall moisture will be offset only slightly more than an inch of spring moisture. Two thirds of an inch of spring rainfall will offset the yield decreasing effect of an inch of fall moisture. In terms of expected yield increases, an inch and a half of soil moisture in the spring is worth about one inch of rainfall during the growing season. A spring application of nitrogen is worth about one extra inch of spring rainfall or an inch and a half of spring soil moisture. The comparison between soil depth and the other variables is a little more difficult since its effect is shown in two ways. However, it appears from an investigation of several other models in which the soil depth-soil test ratio was excluded, that an extra foot of soil is worth roughly the equivalent of two inches of spring rainfall and three inches of spring moisture.

Estimating Spring Rainfall

When considering yield response predictions based on Equation (7.2), a special consideration must be made with reference to the spring rainfall variable. Spring rainfall is obviously unknown at the time the decision is made to apply fertilizer. It can either be ignored or it can be estimated. In either case, the variance associated with the prediction will be somewhat greater than that variance indicated in Table 7.1. If rainfall can be estimated with some precision, the results, in terms of predicting accuracy, will be enhanced.

It may be that some farms quite consistently get more precipitation in the critical spring months than other farms. This may be because of their favorable locations with reference to established rainfall patterns. The local weather bureau may have information which would be helpful in determining expected spring rainfall in a particular area. Very often there are general long range predictions which would cover the period in question. If no information is available, or if it is not considered worthwhile to obtain this information, the average value for the rainfall measurements taken in this experiment may be used. This value is 3.41 inches.

Predicting Equation Based upon the Fall Nitrate Nitrogen Soil Test Reading

If for some reason it is considered desirable or necessary to make an application in the fall, based upon a fall nitrate nitrogen soil test reading, Equation (7.3) can give some useful guidance with respect to the amounts to apply and the responses to expect. The predicting equation, with the appropriate coefficients, is

$$Y_f = -.144585n - .0013522n^2 - .0013685n \cdot st - .028205n \cdot F + .016950n \cdot S + .079582n \cdot D + .047630n \cdot R + .039224n \cdot A \qquad (7.3)$$

As was mentioned earlier, the results of a fall nitrate nitrogen test can be used to indicate the optimum amount of nitrogen to be applied with a spring application. If this is done, Equation (7.3) should be used. In this case the spring application term, A, takes the value of 1. Generally, however, it is expected that the use of this equation would be limited to the situation where there is either some definite known advantage in making a fall application or where there is some practical necessity involved in the decision. In the latter case the A term takes a zero (0) value. This simply ignores the n•A term in the equation.

From an examination of the coefficients of Equations (7.3) and (7.2) it will be seen that there is a general agreement as to the implied relationships between variables. A spring application of nitrogen is worth roughly the equivalent of an inch of rainfall. At the same time about two-thirds of an inch of spring rainfall offsets the yield decreasing effects of an inch of fall moisture. An additional foot of soil is worth roughly one and one-half inches of spring rain.

The soil depth-soil test ratio did not appear to add any significant information when the fall nitrate reading formed the basis of the ratio. However, the fall nitrate nitrogen soil test did add a significant amount of information when incorporated into the equation in the form of a linear interaction term involving nitrogen application. The implications which can be drawn from both equations are roughly equivalent with respect to nitrogen present in the soil. Both equations indicate that soils which have a high nitrogen content respond less to further nitrogen applications. However, the spring nitrate reading apparently detects something that the fall nitrate test does not; that is, the statistical analysis is able to detect a quite logical relationship between available nitrogen and soil depth when the spring nitrate nitrogen test is used. The fact that the variances of the prediction is lowered further substantiates this conslusion.

Inasmuch as the spring soil moisture reading is not known when the decision to apply fertilizer is made, an estimate of spring moisture must be made. An equation was developed which would predict spring moisture on the basis of the soil depth and the fall moisture readings. The equation for this prediction is, [1]

$$S = .51087\ F + 1.11598\ D \qquad (7.4)$$

where all terms are defined as in the original model. This can be put directly into Equation (7.3) in the place of the spring moisture term. The new equation thus formed is

$$Y_f = -.1445855n - .0013522n^2 - .0013685n \cdot st - .0195453n \cdot F$$

$$+ .0984972n \cdot D + .0476299n \cdot R \qquad (7.5)$$

The spring rainfall must be estimated for Equation (7.5) for the same reasons that it was necessary to estimate rainfall for Equation (7.2). Again historical data relative to the particular location may be useful in determining rainfall expectations. Long term forecasts may not be as accurate where the forecast interval is approximately ten months when a fall application is to be made as compared to three months when a spring application is to be made.

The mean, variance, and standard deviation for the yield increase at each treatment level is given in Table 7.3. Note that there is an increasing variation in the yield response as nitrogen rates are increased. This table indicates that if yield response estimates were based solely upon the mean value of the yield response at the 20 pound treatment level, the estimate would be within approximately 7.34 bushels of the true

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[1] The variance associated with this equation is 1.136 and the correlation coefficient R is 0.84.

Table 7.3. Summary of statistics showing estimated confidence limits of yield increase estimates based on the average yield increases from each treatment rate on Condon soils.

Treatment level	Average yield increase	Variance	Standard deviation	$(t_{.025})$ s	95% Confidence limits	
					Upper	Lower
20	3.565	14.0356	3.7464	7.3429	10.908	-3.778
40	5.328	47.5107	6.8928	13.5099	18.838	-8.182
60	5.417	74.0380	8.6045	16.8649	22.282	-11.448
80	4.690	92.2070	9.6024	18.8207	23.511	-14.131
100	4.261	140.5056	11.8535	23.2329	27.494	-18.972

response 95% of the time. If the yield response estimate were based solely upon the mean value of the responses at 40, 60, 80, and 100 pound treatment levels, the estimates would be within approximately 13.51, 16.86, 18.82, and 23.23 bushels of the true responses respectively, 95% of the time.

Yield response estimates which do not take into consideration the various moisture factors and soil conditions tend to leave a considerable amount of question as to the true application response. In other words, a farmer is taking quite a chance when applying fertilizer based upon recommendations which have not considered soil and moisture conditions. Some farmers obviously would do well to apply 100 pounds of nitrogen per acre, whereas others would do well to apply none at all. At 40 pounds, which is the approximate average economic optimum rate of application, some farmers would lose not only the $4.80 per acre for the cost of nitrogen, but also about \$20 per acre for the decrease in yields. Another farmer, applying 40 pounds, would get an increase of about 25 bushel per acre; but if 100 pounds had been applied, an increase of 35 bushel could have been obtained. Not having applied the extra 60 pounds would have cost him \$10 per acre when compared to the economic optimum. Of course, it is not possible to predict with any model exactly what the true response will be. Thus, there will continue to be a gamble associated with recommendations based on almost any criteria. The point is that the uncertainty can be significantly reduced, when recommendations are based on the results of this study.

The confidence interval of $\hat{Y}$ for Equation (7.2) and Equation (7.5) would be extremely difficult to compute in the usual manner (26, p. 442). The difficulty arises from the fact that each variable in the predicting equation changes for each treatment level. Furthermore, the usual procedure assumes that the variance is equal throughout the treatments. An examination of Table 7.3 will show that the variance is not at all equal. Therefore, a more direct method for obtaining an estimate of the confidence interval was devised. The method followed was to predict first the yield increase from each of the 344 different sets of conditions. From each predicted value was subtracted the actual yield increase value corresponding to each individual set of conditions. The result, of course, was the familiar $(Y_i - \hat{Y}_i)$ or deviation from regression. The deviation squared and summed over each treatment level is an estimate of the error sum of squares. From this procedure the data in Table 7.4 was obtained.

Table 7.4. Confidence interval /1 of the yield increase estimate.

Treatment level (i)	Equation (7.2) Standard deviation (1)	$(t_{.025})s$ (2)	Equation (7.5) Standard deviation (3)	$(t_{.025})s$ (4)
20	2.9447	5.7716	2.9973	5.8747
40	4.4299	8.6826	5.0601	9.9178
60	5.1337	10.0621	6.0514	11.8607
80	5.7367	11.2439	6.8822	13.4891
100	5.3057	10.3992	7.6030	14.9019

/1 Confidence interval is given by $\hat{Y}_i \pm (t_{.025})s$.

$\hat{Y}_i$ plus or minus $(t_{.025})s$ gives the interval within which 95% of the true values fell. Thus, for predictions at 20 pounds of nitrogen based upon Equation (7.2), the true value fell within about 5.8 bushels of the predicted value 95% of the time. At 40 pounds the predicted values were within about 8.7 bushels of the true value 95% of the time. Similarly for Equation (7.5) the true values fell within $\hat{Y}_i$ plus or minus $(t_{.025})s$ (column 4) 95% of the time.

Although there is considerable risk involved with so called optimum nitrogen rate recommendations based upon either Equation (7.2) or (7.5) the risk would be greatly reduced from recommendations based on an average yield response function fitted to nitrogen rate applications alone. This can be seen forcefully by comparing the standatd deviation of Tables 7.3 and 7.4.

It is interesting in this connection to make an estimate of the ability of the model to predict across locations. A sample of nine locations was drawn and the yields on each of the four replication plots on each treatment rate were obtained. From the total sum of squares of this data was subtracted the treatment sum of squares and the replication sum of squares. This left the replication by treatment interaction sum of squares which, after appropriate divisions, gives an estimate of the variance of the mean yields. The estimate of the variance by this method was 6.023 bushels. When this amount is compared to the estimate of the deviations from regression in our best model (21.7342 bushels for the spring nitrate nitrogen reading) an estimate of the ability of the model to predict across different locations is obtained. If the model was a perfect model the two estimates would be the same. Since they are not equal in this case (6.023 versus 21.7342) we conclude that the model is not highly successful in going across locations. At least the greater part of the variation is locational rather than within treatments. This could mean that the variables measured in this analysis are not sufficiently accurate or inclusive enough to eliminate the component of variation due to locational differences. Thus there is considerable model by location variation. This does not indicate that the model is not useful for predictive purposes. It simply means that there is considerable improvement which can be made in both the model and the measurements of factors effecting yields.

Presentation for Practical Application

It is easily recognized that, for the most part, very few farmers or fertilizer dealers are willing or able to solve either Equation (7.2) or Equation (7.5) for optimum solutions. For this reason a simpler method of finding a solution was formulated. The simplified method takes advantage of the fact that most of the relationships in the equation are linear. Equation (7.2) and similarly Equation (7.5) can be expressed symbolically as

$$\hat{Y} = n\ (b_1 + b_3F + b_4S + b_5D + b_6R + b_7D/st + b_8A) + b_2n^2 \quad (7.6)$$

Since the b's in the above equation are constant, the value of that part of the equation inside the parentheses would be constant if each of the variables (F, S, D, R, st) were defined to have a particular value. We could conveniently call the enclosed part of the Equation (7.6) "K_s" (or K_f in the event Equation (7.5) were considered). Thus, for any defined values of soil depth, fall moisture, spring moisture, expected spring rainfall, and soil nitrogen reading, Equations (7.2) and (7.5) can be expressed as

$$\hat{Y}_s = n\ (K_s\ + b_2n) \tag{7.7}$$

and

$$\hat{Y}_f = n\ (K_f\ + b_2n) \tag{7.8}$$

Specifically, K_s represents the solution for the linear portion of Equation (7.2) for specific values of fall moisture, spring moisture, soil depth, expected spring rainfall, and a spring nitrate nitrogen reading. The k_f value likewise represents specific values for the variable factors; however in this case, only the soil depth, the fall moisture, the fall nitrate nitrogen reading, and the expected rainfall need be considered. For convenience, the K values have been computed and tabled. Each variable is tabulated incrementally over a relevant range while the others are held constant. The K_s values are listed in Table 7.5. The K_f values are listed in Table 7.6.

The four sections of Table 7.5 represent 2, 3, 4, and 5 foot soils respectively. Each column in each table represents specified fall and spring moisture readings. Within each column are three tiers representing low, medium, and high spring rainfall expectations. [1] Each line within a tier represents a spring nitrate nitrogen soil test reading. The values of these soil tests range variously from 10 to 100 pounds of nitrogen per acre.

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[1] K values for low, medium, and high rainfall expectations are computed by using 1.5, 2.5 and 3.5 inches respectively in Equations (7.2) and (7.5).

Table 7.5. K_s values

Fall Moisture		1"			2"			3"		
Spring Moisture		2"	3"	4"	3"	4"	5"	4"	5"	6"
Rainfall	Soil Test	(Soil Depth - 2 Feet)								
Low	10	.1716	.1934	.2152	.1711	.1929	.2148	.1706	.1925	.2143
	20	.0623	.0841	.1060	.0618	.0837	.1055	.0614	u.0832	.1050
	30	.0259	.0477	.0695	.0254	.0472	.0691	.0249	.0468	.0686
	40	.0076	.0295	.0513	.0072	.0290	.0509	.0067	.0286	.0504
	50	-.0033	.0186	.0404	-.0037	.0181	.0399	-.0042	.0176	.0395
	70	-.0158	.0061	.0279	-.0162	.0056	.0274	-.0167	.0052	.0270
Medium	10	.2071	.2290	.2508	.2067	.2285	.2503	.2062	.2280	.2499
	20	.0978	.1197	.1415	.0974	.1192	.1411	.0969	.1188	.1406
	30	.0614	.0833	.1051	k.0610	.0828	.1046	.0605	.0823	.1042
	40	.0432	.0650	.0869	.0428	.0646	.0864	.0423	.0641	.0860
	50	.0323	.0541	.0760	.0318	.0537	.0755	.0314	.0532	.0750
	70	.0198	.0416	.0635	.0193	.0412	.0630	.0189	.0407	.0626
High	10	.2427	.2645	.2864	.2422	.2641	.2859	.2418	.2636	.2855
	20	.1334	.1553	.1771	.1330	.1548	.1766	.1325	.1543	.1762
	30	.0970	.1188	.1407	.0965	.1184	.1402	.0961	.1179	.1398
	40	.0788	.1006	.1225	.0783	.1002	.1220	.0779	.0997	.1215
	50	.0679	.0897	.1115	.0674	.0892	.1111	.0669	.0888	.1106
	70	.0554	.0772	.0990	.0549	.0767	.0986	.0545	.0763	.0981

Fall Moisture		2"			3"			4"		
Spring Moisture		3"	4.5"	6"	4"	5.5"	7"	5"	6"	7"
Rainfall	Soil Test	(Soil Depth - 3 feet)								
Low	10	.3362	.3689	.4017	.3357	.3685	.4012	.3353	.3571	.3789
	20	.1723	.2050	.2378	.1718	.2046	.2373	.1714	.1932	.2150
	30	.1176	.1504	.1831	.1172	.1499	.1827	.1167	.1386	.1604
	40	.0903	.1231	.1558	.0899	.1226	.1554	.0894	.1112	.1331
	60	.0630	.0958	.1285	.0625	.0953	.1281	.0621	.0839	.1058
	80	.0493	.0821	.1149	.0489	.0816	.1144	.0484	.0703	.0921
Medium	10	.3718	.4045	.4373	.3713	.4041	.4368	.3708	.3927	.4145
	20	.2078	.2406	.2734	.2074	.2401	.2729	.2069	.2288	.2506
	30	.1532	.1860	.2187	.1528	.1855	.2183	.1523	.1741	.1960
	40	.1259	.1586	.1914	.1254	.1582	.1909	.1250	.1468	.1687
	60	.0986	.1313	.1641	.0981	.1309	.1636	.0977	.1195	.1413
	80	.0849	.1177	.1504	.0845	.1172	.1500	.0840	.1058	.1277
High	10	.4073	.4401	.4728	.4069	.4396	.4724	.4064	.4282	.4501
	20	.2434	.2762	.3089	.2430	.2757	.3085	.2425	.2643	.2862
	30	.1888	.2215	.2543	.1883	.2211	.2538	.1879	.2097	.2315
	40	.1615	.1942	.2270	.1610	.1938	.2265	.1605	.1824	.2042
	60	.1341	.1669	.1996	.1337	.1664	.1992	.1332	.1551	.1769
	80	.1205	.1532	.1860	.1200	.1528	.1855	.1196	.1414	.1632

Fall Moisture		2.5"			3.5"			5"		
Spring Moisture		4.5"	6"	7.5"	5"	6.5"	8"	6"	7.5"	9"
Rainfall	Soil Test	(Soil Depth - 4 feet)								
Low	20	.3043	.3371	.3698	.2930	.3257	.3585	.2814	.3141	.3469
	30	.2315	.2642	.2970	.2201	.2529	.2856	.2085	.2413	.2740
	40	.1951	.2278	.2606	.1837	.2164	.2492	.1721	.2048	.2376
	50	.1732	.2060	.2387	.1618	.1946	.2273	.1502	.1830	.2157
	70	.1482	.1810	.2137	.1369	.1696	.2024	.1253	.1580	.1908
	90	.1343	.1671	.1999	.1230	.1557	.1885	.1114	.1441	.1769
Medium	20	.3399	.3727	.4054	.3285	.3613	.3940	.3169	.3497	.3824
	30	.2671	.2998	.3326	.2557	.2884	.3212	.2441	.2768	.3096
	40	.2306	.2634	.2961	.2193	.2520	.2848	.2077	.2404	.2732
	50	.2088	.2415	.2743	.1974	.2302	.2629	.1858	.2186	.2513
	70	.1838	.2166	.2493	.1724	.2052	.2379	.1608	.1936	.2263
	90	.1699	.2027	.2354	.1586	.1913	.2241	.1470	.1797	.2125
High	20	.3755	.4082	.4410	.3641	.3969	.4296	.3525	.3853	.4180
	30	.3026	.3354	.3681	.2913	.3240	.3568	.2797	.3124	.3452
	40	.2662	.2990	.3317	.2548	.2876	.3203	.2432	.2760	.3087
	50	.2443	.2771	.3099	.2330	.2657	.2985	.2214	.2541	.2869
	70	.2194	.2521	.2849	.2080	.2408	.2735	.1964	.2292	.2619
	90	.2055	.2382	.2710	.1941	.2269	.2596	.1825	.2153	.2480

Fall Moisture		4"			5"			6"		
Spring Moisture		5.5"	7"	8.5"	6"	7.5"	9"	8"	9.5"	11"
Rainfall	Soil Test	(Soil Depth - 5 feet)								
Low	30	.3121	.3449	.3776	.3008	.3335	.3663	.3221	.3549	.3876
	40	.2666	.2994	.3321	.2552	.2880	.3207	.2766	.3094	.3421
	50	.2393	.2720	.3048	.2279	.2607	.2934	.2493	.2820	.3148
	60	.2211	.2538	.2866	.2097	.2424	.2752	.2311	.2638	.2966
	80	.1983	.2311	.2638	.1869	.2197	.2524	.2083	.2411	.2738
	100	.1846	.2174	.2502	.1733	.2060	.2388	.1947	.2274	.2602
Medium	30	.3477	.2805	.4132	.3363	.3691	.4018	.3577	.3905	.4232
	40	.3022	.3349	.3677	.2908	.3235	.3563	.3122	.3449	.3777
	50	.2748	.3076	.3404	.2635	.2962	.3290	.2849	.3176	.3504
	60	.2566	.2894	.3221	.2453	.2780	.3108	.2666	.2994	.3322
	80	.2339	.2666	.2994	.2225	.2553	.2880	.2439	.2766	.3094
	100	.2202	.2530	.2857	.2088	.2416	.2743	.2302	.2630	.2957
High	30	.3833	.4160	.4488	.3719	.4046	.4374	.3933	.4260	.4588
	40	.3377	.3705	.4032	.3264	.3591	.3919	.3477	.3805	.4133
	50	.3104	.3432	.3759	.2990	.3318	.3646	.3204	.3532	.3859
	60	.2922	.3250	.3577	.2808	.3136	.3463	.3022	.3350	.3677
	80	.2694	.3022	.3349	.2581	.2908	.3236	.2794	.3122	.3450
	100	.2558	.2885	.3213	.2444	.2772	.3099	.2658	.2985	.3313

Table 7.6 K_f values.

Soil Depth		2 feet			3 feet			4 feet			5 feet		
Fall Moisture		1"	2"	3"	2"	3"	4"	2.5"	3.5"	5.0"	4"	5"	6"
Expected Rainfall	Soil Test												
	10	.0906	.0711	.0515	.1696	.1500	.1305	.2583	.2388	.2094	.3275	.3079	.2884
	20	.0769	.0574	.0378	.1559	.1363	.1168	.2446	.2251	.1958	.3138	.2942	.2747
	30	.0633	.0437	.0242	.1422	.1227	.1031	.2309	.2114	.1821	.3001	.2806	.2610
	40	.0496	.0300	.0105	.1285	.1090	.0894	.2172	.1977	.1684	.2864	.2669	.2473
Low	50	.0359	.0163	-.0032	.1148	.0953	.0757	.2036	.1840	.1547	.2727	.2532	.2336
	60	.0222	.0027	-.0169	.1012	.0186	.0621	.1899	.1703	.1410	.2591	.2395	.2200
	70	.0085	-.0110	-.0306	.0875	.0679	.0484	.1762	.1566	.1273	.2454	.2258	.2063
	80	-.0052	-.0247	-.0443	.0738	.0542	.0347	.1625	.1430	.1136	.2317	.2121	.1926
	90	-.0189	-.0384	-.0579	.0601	.0406	.0210	.1488	.1293	.1000	.2180	.1985	.1789
	10	.1383	.1187	.0992	.2172	.1977	.1781	.3059	.2864	.2571	.3751	.3556	.3360
	20	.1246	.1050	.0855	.2035	.1840	.1644	.2922	.2727	.2434	.3614	.3419	.3223
	30	.1109	.0913	.0718	.1898	.1703	.1507	.2786	.2590	.2297	.3477	.3282	.3086
	40	.0972	.0777	.0581	.1762	.1566	.1371	.2649	.2453	.2160	.3341	.3145	.2950
Medium	50	.0835	.0640	.0444	.1625	.1429	.1234	.2512	.2316	.2023	.3204	.3008	.2813
	60	.0698	.0503	.0307	.1488	.1292	.1097	.2375	.2180	.1886	.3067	.2871	.2676
	70	.0561	.0366	.0171	.1357	.1156	.0960	.2238	.2043	.1750	.2930	.2735	.2539
	80	.0425	.0229	.0034	.1214	.1019	.0823	.2101	.1906	.1613	.2793	.2598	.2402
	90	.0288	.0092	-.0103	.1077	.0882	.0686	.1964	.1769	.1476	.2656	.2461	.2265
	10	.1859	.1663	.1468	.2648	.2453	.2257	.3536	.3340	.3047	.4227	.4032	.3836
	20	.1722	.1527	.1331	.2512	.2316	.2121	.3399	.3203	.2910	.4091	.3895	.3700
	30	.1585	.1390	.1194	.2375	.2179	.1984	.3262	.3066	.2773	.3954	.3758	.3563
	40	.1448	.1253	.1057	.2238	.2042	.1847	.3125	.2930	.2636	.3817	.3621	.3426
High	50	.1311	.1116	.0921	.2101	.1905	.1710	.2988	.2793	.2500	.3680	.3485	.3289
	60	.1175	.0979	.0784	.1964	.1769	.1573	.2851	.2656	.2363	.3543	.3348	.3152
	70	.1038	.0842	.0647	.1827	.1632	.1436	.2714	.2519	.2226	.3406	.3211	.3015
	80	.0901	.0705	.0510	.1690	.1495	.1299	.2578	.2382	.2089	.3269	.3074	.2879
	90	.0764	.0569	.0373	.1554	.1358	.1163	.2441	.2245	.1952	.3133	.2937	.2742

Low rainfall = 1.5 Medium rainfall = 2.5 High rainfall = 3.5

In Table 7.6 each column represents a specified soil depth and a specified fall moisture reading. The tiers and lines are the same as for Table 7.5.

With Equations (7.2) and (7.5) expressed in the form of (7.7) and (7.8) it is a simple matter to derive equations to determine the economically optimum nitrogen application rates. Taking advantage of each function with respect to n, we get

$$\frac{dy_s}{dn} = K_s + 2b_2n \qquad (7.9)$$

and

$$\frac{dy_f}{dn} = K_f + 2b_2n \qquad (7.10)$$

Setting each equation equal to the nitrogen-wheat price ratio and solving for n gives us the optimizing equations

$$n = \frac{K_s - P_n/P_y}{2b_2} \qquad (7.11)$$

and

$$n = \frac{K_f - P_n/P_y}{2b_2} \qquad (7.12)$$

when signs are changed to account for the fact that b_2 must always be minus. Since the value of $2b_2$ is constant, Equations (7.11) and (7.12) can be further reduced to

$$n = \frac{K_s - P_n / P_y}{.00228} \qquad (7.13)$$

for spring applications and

$$n = \frac{K_f - P_n / P_y}{.0027} \qquad (7.14)$$

for fall applications. With Equations (7.7), (7.8), (7.13), and (7.14) derived, a wide variety of information can be obtained rather easily.

To illustrate the use of Tables 7.5 and 7.6 with Equations (7.7), (7.8), (7.13), and (7.14), one may take a simple example and work out several solutions. Suppose, first, that a farmer with the following information available to him is interested in knowing what the likely yield response would be from a given spring nitrogen application

Soil depth	4 feet
Fall moisture reading	5.2 inches
Spring moisture reading	6.0 inches
Spring soil nitrogen reading	20 pounds/acre
Expected rainfall	2.5 inches

The tabulated K_s value, corresponding to the above data, would be .3169. This value is found in Table 7.5, section 3, column 7, tier 2, line 1 within this tier. The value of the fall moisture reading in the table for this example would be 5 inches since /1 it is closest to 5.2 (the actual value of the reading). With the value of K_s a farmer can now compute the expected yield response for a given range of nitrogen applications by using Equation (7.7). /2 The expected value at 50 pounds in this example would be

$$\hat{Y}_s = 50 \quad [.3169 - (.001138)\ 50]$$
$$= 50 \quad [.3169 - .0569]$$
$$= 50 \quad [.26] = 13 \text{ bushels/acre}$$

The expected yield increase for 60 pounds would be

$$\hat{Y}_s = 60 \quad (.3169 - .0683)$$
$$= 60 \ (.2486)$$
$$= 14.9 \text{ bushels per acre.}$$

Other solutions at other rates of nitrogen application may be made by simply changing the value of n in the equation.

By using Equation (7.13) the farmer can predict the most economical rate of nitrogen application. Assuming that the price ratio of nitrogen to wheat (P_n / P_y) is \$.12 / \$1.80, the optimum solution is simply

$$n = \frac{.3169 - .0667}{.00228}$$
$$= \frac{.2502}{.00228} = 110 \text{ pounds/acre.}$$

/1 If greater accuracy is desired, straight line interpolation may be used.

/2 It must be remembered that Table 7.5 is only to be used in connection with Equations (7.7) and (7.13) while Table 7.6 is to be used with Equations (7.8) and (7.14).

If a farmer had determined to make a fall application and at the same time had the following information available to him, that is,

Soil depth	4 feet
Fall moisture reading	5.2 inches
Expected spring rainfall	2.5 inches
Fall nitrogen soil test reading	20 ppunds

he would use Table 7.6 to find the appropriate K_f value. The appropriate K_f value in the above illustration is .2434. This value is found in column 9, tier 2, line 2 corresponding to a soil test of 20 pounds. The solution in this example, for the expected yield increase from an application of 50 pounds, is 8.8 bushels per acre. The optimum application rate of nitrogen is 65 pounds under the assumption that nitrogen-wheat price ratio is \$.12 / \$1.80. (The reader can verify these solutions in order to make sure he follows the procedure.)

In accordance with the principles set down in Chapter V, what a farmer should do under certain economic conditions can easily be seen. Suppose, for example, that a particular decision maker is a tenant who pays all fertilizer costs but receives only 80% of the crop for his own disposal. Suppose, further, that he has a shortage of operating capital such that he feels he can make a fertilizer investment only if each dollar spent has an expected return no less than \$1.50.

To solve this problem, one would simply adjust the P_n / P_y ratio so that it met the restrictions which are set. The farmer would multiply P_n by 1.50 and would multiply the P_y by .80. The new P_n / P_y would be

$$\frac{P_n}{P_y} = .18 / 1.44 = .125$$

Thus the solution for the optimum application rate under these conditions would be, for a fall application,

$$n = \frac{.2434 - .125}{.0027} = \frac{.1184}{.0027} = 44 \text{ pounds}$$

If the tenant had to hire the wheat harvested and the custom rate was set at a constant rate per acre plus \$.10 per bushel, such a rate would reduce the "effective" price of wheat to the tenant to \$1.34, making the nitrogen-wheat price ratio \$.18 / \$1.34 or .1343. The optimum application rate in this case would now be 40 pounds per acre. A verification of these results will be helpful in making sure that the principles are understood.

It is to be remembered that different economic and physical conditions will effect different economic solutions. Some economic and some physical conditions may be inconsequential and thus warrant no special consideration in an optimum solution. However, such things as ownership status, capital position, the price ratio of nitrogen to wheat, and the type of rental agreements entered into often require special consideration and have a significant effect on any management decision. These considerations should be made in accordance with the principles set down in Chapter V.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

There is much uncertainty with respect to the question of proper nitrogen application recommendations. This uncertainty remains principally because it has not been possible to determine accurately the underlying physical relationships between the various factors which determine yeild responses. Conflict still exists with reference to the effects on wheat yields of moisture at various times during the growing season. The significance of soil depth as well as the nutrient level of the soil needs further study.

A related question deals with the various tests for soil nutrients. The question concerns the ability of any test to measure accurately the amount of existing nutrient in the soil. This question is of particular interest in the case of the various nitrogen tests since the amount of soil nitrogen available for plant use varies widely from one period of time to another. Again, the ability to measure the available nitrogen is particularly important in the context of this analysis since nitrogen is the nutrient most often supplemented by commercial fertilizers in the Oregon wheat area.

Economic considerations are often given inadequate attention in current fertilizer recommendations. Recommendations have gone, and in some cases still go to farmers directed toward the goal of maximum yields. This is seldom consistent with the goal of maximum profits. Profit maximization is likely of more interest to wheat farmers than obtaining maximum yields.

Efforts in this study were directed toward the three questions discussed above. In order to estimate the basic physical relationships between yield responses and the effects of soil and moisture conditions acting through nitrogen applications, two basic regression equations were proposed. These two models (mathematical equations) were fitted to data obtained from experiments conducted jointly by the United States Department of Agriculture and the Soils Department of Oregon State University over the 4 year period between 1953 and 1957. These experiments were primarily designed to estimate the wheat yield responses from nitrogen applications under various soil and moisture conditions.

The first model, examined in Chapter VI, estimated the parameters involved in predicting total yields. This was a quadratic polynomial of the form

$$Y = B_0 + B_1N + B_2F + B_3S + B_4D + B_5R + B_6N^2 + B_7N{\cdot}F + B_8N{\cdot}S + B_9N{\cdot}D + B_{10}N{\cdot}R + e \quad (6.3)$$

subject to the restriction

$$N = n + \lambda st \quad (6.2)$$

where, for a particular location, Y is yield, N is total nitrogen available in the soil after n pounds of commercial nitrogen has been added, S is spring soil moisture, R is spring rainfall, and st is the nitrogen soil test reading. λ , a parameter to be estimated is a proportionality constant which relates the nitrogen measured in the soil to that applied from commercial sources.

The estimates of the parameters using this model were inconclusive, especially when the various soil nitrogen tests were substituted into the model. The extreme variability of the data was seen as the major restriction to the effectiveness of this model.

The second model, discussed in Chapter VII, was designed to estimate the parameters involved in predicting yield increases. /1 This model took the form of the familiar quadratic polynomial. This model was fitted to the data ten different times. A different soil test formed the basis of the estimate of available soil nitrogen. In this way an evaluation of each soil testing method could be made. Each soil test was evaluated on the basis of its ability to reflect (through the model) the true yield response.

The conclusion, based upon this analysis, is that a nitrate nitrogen soil test taken in the spring is superior to any of the other indicators tested. The best fall indicator is the nitrate

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/1 Yield increase for a particular location refers to the difference between the yield where no nitrogen is applied and the yield where a specified application is made. The treatment data for each location was correlated by the check plot yield for each location.

nitrogen test. However, the fall nitrate nitrogen reading is only slightly superior to other fall readings. The spring nitrate nitrogen reading is vastly superior to any of the fall readings and does a considerably better job than any other spring reading.

The predicting equation which best characterized the data when a spring nitrate nitrogen test formed the basis of the estimate of available soil nitrogen was

$$\hat{Y}_s = -.27258n - .0011383n^2 - .022291n \cdot F + .021837n \cdot S + .055816n \cdot D + .035568n \cdot R + 1.09273n \cdot D/st + .039221n \cdot A \tag{7.2}$$

where the symbols are the same as for Equation (6.1) except that A stands for a spring application of nitrogen. This equation would be used to form the basis of predictions made when applications were to be made in the spring. It would, of course, require that a spring nitrate nitrogen reading be used as the appropriate variable.

The predicting equation which best characterizes the data when a fall nitrate nitrogen soil test is used in the model is

$$\hat{Y}_f = -.144588n - .0013522n^2 - .0013685n \cdot st - .028205n \cdot F + .01695n \cdot S + .079582n \cdot D + .04763n \cdot R + .039224n \cdot A \tag{7.3}$$

For use in predicting yield responses based on a fall nitrogen application, two variables would have to be estimated, namely, spring moisture and spring rainfall. Furthermore, A in the equation would take the value of zero. For simplicity a revised form of Equation (7.3) was derived in which spring moisture was estimated from fall moisture and soil depth. This equation is

$$\hat{Y}_f = -.144585n - .001352n^2 - .0013685n \cdot st - .019545n \cdot F + .098497n \cdot D + .047630n \cdot R \qquad (7.5)$$

A spring application of nitrogen will generally give higher yields than a fall application, on Condon soils. The soil depth seems to be the most important factor in determining yield responses from applications of fertilizer. It also appears that excessive fall moisture can reduce the yield response from nitrogen if adequate moisture is not available during the following spring. The estimated magnitude of the effects thus described are given in the coefficients of each of the above equations.

With the basic physical relationships estimated for yield responses on Condon soils, an economic analysis can be made. A general recommendation for all Condon soils under "average" conditions was not made. However, by use of Table 7.5 and 7.6 an estimate of the economically optimum application rate can be made rather easily for any specific set of conditions. The equations to be used in connection with these tables are as follows:

$$n = \frac{K_s - P_n / P_y}{.00228} \qquad (7.13)$$

and

$$n = \frac{K_f - P_n / P_y}{.0027} \qquad (7.14)$$

where Equation (7.13) gives the economically optimum nitrogen rate when a spring application is made based on a spring nitrate nitrogen soil test, and Equation (7.14) is the economically optimum nitrogen application rate where a fall application is made based on a fall nitrate nitrogen test. K_s is the value tabulated in Table 7.5 and K_f is the value tabulated in Table 7.6. The ratio P_n / P_y is the ratio of the cost of one pound of nitrogen to the price of one bushel of wheat.

It appears from an analysis of the several varaince components that there is considerable improvement which could be made in the soil predicting model and also the measurements of physical conditions which effect yields. The greater part of the variance not accounted for by the model is probably due to locational differences. However the model can still be very useful in predicting yield responses and economic optimum nitrogen application rates where no other information is had relative to the true nitrogen-wheat yield response function for a particular location.

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APPENDIX

DATA USED IN THE ANALYSIS

Table 1. Nitrogen application and yield responses on Condon soils, 1953-1957.

Application Time	Application Rate	Yield (Bushels/Acre) Code Numbers /1							
		424	427	431	437	446	454	455	457
Fall	0	31.3	12.6	29.3	28.3	21.8	42.1	25.9	31.6
	20	30.8	16.6	30.0	29.5	27.7	50.4	29.6	41.2
	40	28.9	20.7	30.2	27.4	30.5	52.3	29.3	45.5
	60	23.5	22.1	28.3	24.9	28.5	46.0	29.1	47.8
	80	24.4	25.7	28.7	19.9	31.4	45.0	28.6	48.2
Spring	0	31.3	12.6	29.3	28.3	21.8	42.1	25.9	31.6
	20	34.0	21.0	32.9	29.5	25.4	48.2	30.3	39.7
	40	35.6	26.7	34.4	30.5	30.8	55.1	31.7	39.2
	60	31.8	28.9	31.9	31.5	35.6	47.9	32.7	42.3
	80	33.7	31.6	34.9	30.0	28.5	54.8	31.4	45.8
		Code Numbers							
		525	526	527	535	553	558	5510	623
Fall	0	28.0	18.0	31.4	16.9	29.7	21.6	21.2	27.1
	20	34.8	21.1	40.8	21.6	31.3	22.6	18.4	29.8
	40	35.1	26.2	47.1	23.0	30.1	20.4	14.6	28.0
	60	37.5	28.2	43.8	19.9	32.1	18.5	10.6	29.2
	80	34.2	27.7	46.7	18.3	29.0	17.4	8.4	28.2
	100	31.2	28.2	42.3	17.3	32.5	17.5	6.8	30.6
Spring	0	28.0	18.0	31.4	16.9	29.7	21.6	21.2	27.1
	20	36.2	24.7	35.2	19.2	32.8	21.6	19.5	24.8
	40	39.0	27.4	40.7	20.3	33.2	21.4	16.2	24.3
	60	39.2	28.0	45.7	22.9	32.7	20.7	16.2	26.3
	80	38.7	28.5	42.6	19.3	31.7	19.9	11.5	27.5
	100	33.7	26.1	47.0	19.4	33.5	18.6	9.4	26.9
		Code Numbers							
		632	634	657	658	6510	722	723	
Fall	0	29.4	24.6	33.0	17.8	22.8	40.2	24.6	
	20	39.9	30.9	37.9	24.1	28.0	41.5	21.1	
	40	51.4	33.1	44.9	29.3	30.6	41.2	14.4	
	60	57.5	32.4	54.9	32.9	31.9	39.3	12.9	
	80	62.4	26.7	60.4	29.9	29.9	33.4	10.6	
	100	63.0	32.8	62.9	31.0	32.5	33.8	10.5	
Spring	0	29.4	24.6	33.0	17.8	22.8	40.2	24.6	
	20	40.8	30.2	36.4	24.4	26.9	41.8	19.0	
	40	54.6	31.0	45.2	25.7	33.1	42.5	19.5	
	60	58.8	32.3	51.0	28.6	31.2	42.4	16.1	
	80	59.3	31.9	51.2	25.7	29.6	39.4	14.9	
	100	66.8	34.6	60.2	31.8	30.2	37.4	13.5	
		Code Numbers							
		732	733	751	753	754	756	757	
Fall	0	28.7	30.5	36.8	31.3	35.2	36.6	14.4	
	20	34.8	25.9	42.0	34.0	41.9	37.6	16.2	
	40	35.8	19.2	48.1	36.1	40.4	41.4	13.4	
	60	38.0	18.4	47.1	32.6	40.8	39.2	13.2	
	80	32.7	19.2	42.4	33.8	39.0	35.3	12.6	
	100	39.7	14.3	41.3	29.4	33.9	36.5	9.6	
Spring	0	28.7	30.5	36.8	31.3	35.2	36.6	14.4	
	20	31.2	27.0	43.9	32.0	42.3	40.5	15.9	
	40	37.2	23.5	45.0	34.2	41.6	40.8	16.1	
	60	35.8	21.7	48.2	33.8	40.6	39.8	14.5	
	80	39.0	23.2	45.9	32.0	38.0	41.2	13.4	
	100	36.0	21.0	44.3	31.1	35.7	38.0	14.3	

/1 The first digit in the code represents the year (1953-57) in which the data was gathered. The second digit in the code number represents the county in which the experiment was performed; a 2 represents Gilliam County, a 3 represents Morrow County, a 4 represents Wasco, and a 5 stands for Sherman county. Thus the number 423 would have been performed in Gilliam County on the crop harvested in 1954. It would have been experimental number 3 within that county.

Table 2. Soil and moisture conditions for 30 locations on Condon soils.

Code	Inches of Moisture			Soil Depth (feet)	Soil Test Readings (pounds/acre)					
	Soil Moisture		Spring Rainfall		Nitrate Nitrogen		Nitrifiable Nitrogen		Ammonia	
	Fall	Spring			Fall	Spring	Fall	Spring	Fall	Spring
424	3.2	3.9	2.9	2.75	104	18	51	15	63	94
427	1.5	2.1	2.9	2.50	14	9	26	10	41	61
431	2.3	3.3	4.73	2.25	44	31	27	17	65	67
437	1.3	3.5	3.33	2.75	43	18	35	22	55	60
446	3.5	6.6	2.31	3.50	62	18	17	39	48	69
454	4.6	7.3	2.16	5.00	55	69	19	18	72	130
455	2.5	4.7	2.26	2.50	45	31	19	11	55	32
457	2.6	5.8	2.20	3.25	13	31	12	20	43	77
525	5.4	5.7	3.53	3.50	59	37	27	31	58	56
526	1.8	2.8	2.72	3.00	38	33	17	19	53	48
527	3.9	5.2	1.89	4.00	37	26	10	28	77	57
535	1.2	2.3	3.11	2.75	22	28	20	31	66	56
553	5.0	5.2	3.18	3.25	49	57	17	18	65	57
558	3.9	3.7	2.29	2.75	44	38	20	17	34	49
5510	3.3	3.1	2.67	2.25	50	33	22	13	47	51
623	.9	4.6	3.41	3.00	55	49	42	11	27	54
632	1.1	4.2	3.69	3.00	49	10	61	47	75	180
634	1.3	3.9	3.24	2.50	56	43	85	34	87	133
657	3.2	7.2	3.31	4.50	41	39	36	16	54	88
658	1.3	4.2	3.78	3.50	31	33	27	9	55	119
6510	2.1	4.8	3.61	2.50	20	21	42	7	22	85
722	4.72	5.8	1.77	2.50	48	45	30	10	69	34
723	3.1	4.7	.82	2.25	111	104	39	41	89	53
732	5.69	9.6	1.36	4.00	57	51	28	28	182	168
733	2.53	4.4	.89	3.00	101	55	36	25	113	94
751	4.79	8.0	.98	4.50	58	57	27	14	133	75
753	4.47	5.0	1.12	3.50	34	39	20	11	127	95
754	4.93	7.6	.92	4.00	33	42	21	8	118	78
756	3.54	8.4	.88	4.00	26	61	19	22	84	88
757	1.6	3.5	.92	2.00	27	54	12	26	56	38