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Because of increasing competition from non-wood building materials, efforts are being made to improve the design of stud walls. The objective of this study is to develop and verify a theoretical method and computer program that will predict the end fixity in stud walls because of the sheathing being attached to the wall support system. A special computer program was developed from an existing finite element program. The method was verified by comparing analytical results with known theoretical and experimental solutions.

Finite Element Analysis of End Fixity in Stud Wall Panels

by

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FINITE ELEMENT ANALYSIS OF END FIXITY IN STUD WALL PANELS

I. INTRODUCTION

Present light-frame structures, made of wood and wood based materials, may be overbuilt because the existing structural design requirements are based on tradition and simplified analysis. Increased competition from other building materials, such as steel studs and joists, has spurred efforts to improve the analysis and design of light-frame structures. The recent project "Theoretical Analysis and Rational Design Procedure for Stud Wall Systems" at the Forest Research Laboratory, Oregon State University, is aimed at including into the stud wall design factors that contribute to the strength and stiffness of stud walls but are not considered by existing procedures. The application of computers in structural design has enhanced the use of matrix methods in structural analysis and produced the finite element method (5,36). These developments have made it practical to attempt the revision of the existing method for the structural design of wood-stud walls.

The wood-stud wall consists of a series of equally spaced studs with interior and exterior coverings. The exterior wall covering or sheathing may be fiberboard, plywood, particleboard or non-wood materials. The interior wall covering usually is gypsum board. The wall is normally supported by the floor. A typical wood-stud wall support is shown in Figure 1.1. The floor is composed of a series of equally spaced joists and a subfloor with or without an underlayment. The wall and floor components are usually connected with nails but staples or glue may be used instead.

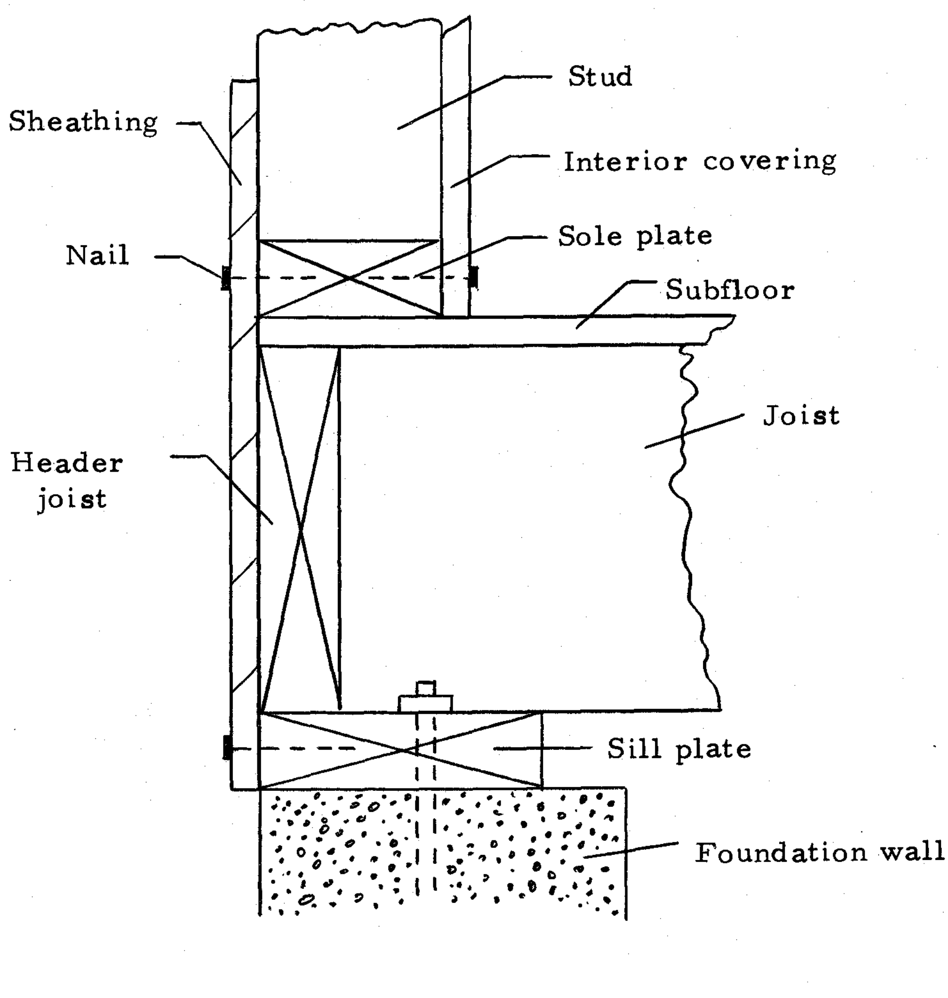

Figure 1.1. Wood-stud wall and wall support.

Lateral wind load, acting on the wall, forces studs to rotate with respect to joists at supports. The attachment of the exterior wall sheathing to the wall supports may restrain this support or end rotation of the stud and produce a partial end fixity, measured by the coefficient of end fixity, defined as

$$\alpha = \frac{M}{\theta} \tag{1.1}$$

where

M - end moment

and θ - angular rotation because of M

Coefficient α is zero for a simply supported beam and infinity for a beam with fixed ends. For wood-stud walls, α should be between these two extremes.

1.1. Previous and Present Outlook

Requirements for stud wall design are stated in standards and building codes, such as the Minimum Property Standards of the United States Department of Housing and Urban Development (29) and the Uniform Building Code of the International Conference of Building Officials (9). Requirements for wall framing in light-frame construction are given in Sec. 2518(f) of the Uniform Building Code. Requirements for stud walls specify design features, such as the size, spacing, bracing and height of studs. Stud spacing, size and grade are based upon wall types. Standards and codes provide requirements for exterior and interior bearing walls and interior nonbearing partitions for one, two and three-story buildings. The grades of lumber are as determined by the rules of an approved lumber grading agency. Some codes restrict the use of Utility grade studs which is the lowest structural grade. The stud nominal size ranges from two in. by three in. to two in. by six in. and three in. by four in. Maximum height, without lateral supports, ranges from 8 ft to 20 ft. Stud spacing is either 12, 16, or 24 in. on center. Various methods are given to brace the ends of all exterior walls and main cross stud partitions against the wall collapse as a diaphragm. Innovative construction or excessive design loads may require that a specific design be furnished.

Conventional analysis of stud walls involves the simple beam-column analysis of a single stud with an uniform lateral load and an axial load. The design loads are specified in the building codes. Stresses and deflections are determined and compared to the allowable values. Allowable stresses are given in the codes (9) and specifications, such as the National Design Specification for Stress-Grade Lumber and Its Fastenings of the National Forest Products Association (19).

Attempts to evaluate the partial end fixity and include it in the analysis and design procedures have been made since the 1930's. The analysis of members with elastically restrained ends has been discussed in textbooks (2, 28). The theoretical or experimental determination of the coefficient of end fixity for steel frame constructions is discussed in a number of articles (3, 11, 14, 18, 22). Early design methods which account for the partial end fixity were either manageable and approximate or very complicated and time-consuming for all but simple structures (10, 11, 13, 26). However, the digital computer and matrix analysis have allowed the use of methods which were previously forbidden because of their complexity (15, 24).

Rodda (23) has developed a theoretical analysis of trusses with semi-rigid joints. His Ph. D. thesis includes an extensive review of the literature on elastic joints in steel structures and nailed timber connections. The ultimate strength design of reinforced timber rigid frames with semi-rigid joints was investigated by Krueger (12). Glenn Schroeder (25) experimentally obtained coefficients of end fixity of five types of wood-stud wall panels.

1. 2. Future Outlook

Improvements can be made in the design procedure for wood-stud walls. Tradition, rigid building codes, inadequate grading methods and over-simplified methods of structural analysis have resulted in walls that are usually overbuilt. Better methods of analysis and more precisely defined properties of wall components and connections can justify less conservative designs.

Stud wall design can be made more economical by reducing the size and grade of the wall components and by increasing the stud spacings. The conventional 16 in. spacing of two in. by four in. studs in walls could be increased to 24 in. spacing. The Uniform Building Code (9) presently allows 24 in. stud spacing for single story dwellings and top stories of multi-story dwellings. Another possible improvement is the use of one in. by four in. studs on 16 in. centers, glued to 3/8 in. plywood sheathing (21). Finally improved design procedures can justify more extensive use of Utility grade studs for structural members.

Improvements in the structural use of wood involve the consideration of the following three factors: establishment of the properties of the structural wood products, precise definition of the actual loads a structure must support, and accurate method of determining the strength and stiffness of wood structures (21).

The establishment of the properties of the structural materials involves experimental determination of strength and stiffness properties for most important types of wood products and connections. As the design of wooden structures becomes more precise, the wide variation of wood properties within a grade will become less acceptable. Improved lumber grading methods should be developed and applied to reduce the in-grade variation of lumber properties. The use of the machine stress-rated lumber is perhaps a practical way to define lumber stiffness and strength more precisely and to segregate the lumber into grades with greater uniformity in their properties.

Assumptions and approximations used for the design loads acting on walls present another problem in structural design. Poorly defined loads often result in overbuilt structures. Therefore, a precise definition of the external loads is also required for an accurate analysis and design procedure for wood-stud walls.

Improved values for material properties and loads are of limited value if accurate design methods are not available. The design method should include as many significant factors of the behavior of the structure as practical. The dynamic, thermal, acoustic, durability and moisture properties of the structure should be considered in addition to the strength and stiffness (27). Improvements in the stud wall design method can be made by including factors that contribute to the strength and stiffness of the wall, but are not included in the current procedure. Provisions could be made to consider the ability of the stronger studs to support the weaker studs due to load distribution through the wall coverings. The stiffness contribution of the composite action between studs and wall coverings should be accounted for. Stiffness and strength increase of walls because of the partial end fixity should also be considered in the improved design method of stud walls.

1.3. Justification and Objectives

Improvements in structural wall design should result in material and perhaps labor savings. These savings should reduce the cost of the structure, diminish the demand for our raw materials and make wood more competitive with building materials made of steel and aluminum.

Maximum deflections and stresses are the criteria for the selection of wall components. Deflections and stresses of a beam decrease with an increase in the coefficient of end fixity. Therefore, a partial end fixity may provide an additional factor in making wood-stud walls more economical.

In his Master of Science thesis, Glenn Schroeder obtained empirical values for the coefficient of end fixity in a few selected types of wood-stud walls (25). Because the end fixity changes with a change in properties of the wall components and connections, it is desirable to have a theoretical model and method to evaluate the partial end fixity for any size and property of wall components and connections. Once the theoretical model, method, and computer program are developed and verified, it is less expensive to theoretically analyze the end fixity than to conduct a series of tests on full-size wall panels. The main aim of this study is to develop such a method.

The specific objectives of this study are:

1. To develop a theoretical method and computer program that will predict the coefficient of the end fixity in wood-stud walls due to the sheathing being connected to the wall supports,
2. To verify the method and computer program, and
3. To apply the method to investigate possible ways of improving the partial end fixity in wood-stud walls.

To meet these objectives, a finite element method based on plane stress elements was applied. The method was verified by analyzing a solid beam with a known theoretical coefficient of the end fixity and by analyzing some of the experimental wood-stud wall panels. Several material properties of experimental wall panels were varied to investigate possible ways of improving the partial end fixity.

II. METHOD OF ANALYSIS

The finite element method was used to evaluate the end fixity of wood-stud walls. The method is ideal for structures with complex boundary conditions and material properties. The results of the analysis, the displacements and stresses within the structure, were used to calculate the coefficient of end fixity which was then applied in the calculation of the midspan deflection of a semi-fixed beam-column. The finite element analysis and the calculation of the coefficient of end fixity and midspan deflection of the beam-column wall panels were done by a computer program.

2.1. Finite Element Method

The finite element method involves separating the continuum of a structure into geometric subdivisions called finite elements. Each element has a discrete number of nodal points. The elements are assumed to be connected only at the nodal points. The nodal displacements are the basic parameters of the finite element analysis and the external loads are applied at the nodal points. The relationship between the nodal displacements and the applied forces is defined by a stiffness matrix derived from the material and geometric properties of the wall components and connections. A stiffness matrix is determined for each element. All the element stiffness matrices are combined to obtain a system stiffness matrix for the entire structure. The effect of the boundary conditions of allowable displacements is included in the system stiffness matrix. External loads and nodal displacements are related by conventional matrix equation (7).

$$[K]\{d\} = \{Q\} \tag{2.1}$$

where

$[K]$ - system stiffness matrix

$\{d\}$ - system nodal displacement vector

and $\{Q\}$ - system external load vector

Equation (2.1) is solved for the unknown displacement vector.

With the nodal displacements known, the element stresses are determined by

$$\{\sigma_e\} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [B]\{d_e\} \tag{2.2}$$

where

$[B]$ - element stress-displacement matrix

$\{\sigma_e\}$ - element stress vector

σ_1, σ_2 - normal stresses

τ_{12} - shear stress

1, 2 - local coordinates denoting the direction of stresses

and $\{d_e\}$ - element nodal displacement vector

2.2. The Finite Elements and Their Stiffness Matrices

Three types of plane stress elements were used to subdivide the stud wall-wall support structure into a finite element model. The mathematical expressions for the stiffness and stress-displacement matrices of these elements are discussed in this section.

The main element is Cook's modified assumed stress hybrid rectangular element (4). The element yields exact displacements and stresses under pure bending. The stresses within the element are given by an assumed field which satisfies the differential equations of equilibrium. The element, shown in Figure 2.1 with its local coordinate system and differential element, has its nodal points i, j, k and ℓ oriented counter-clockwise. The size and location of finite elements are defined in terms of the global Cartesian coordinates x and y. The nodal displacements are u and v in directions of the x and y axis, respectively. The stress in a direction perpendicular to the x-y plane is assumed to be zero in a plane stress element.

Cook (4) defines the element stiffness matrix as

$$[k_e] = [T]^T [H]^{-1} [T] \tag{2.3}$$

where the non-zero elements of the five by eight matrix $[T]$, expressed in terms of the nodal coordinates, are:

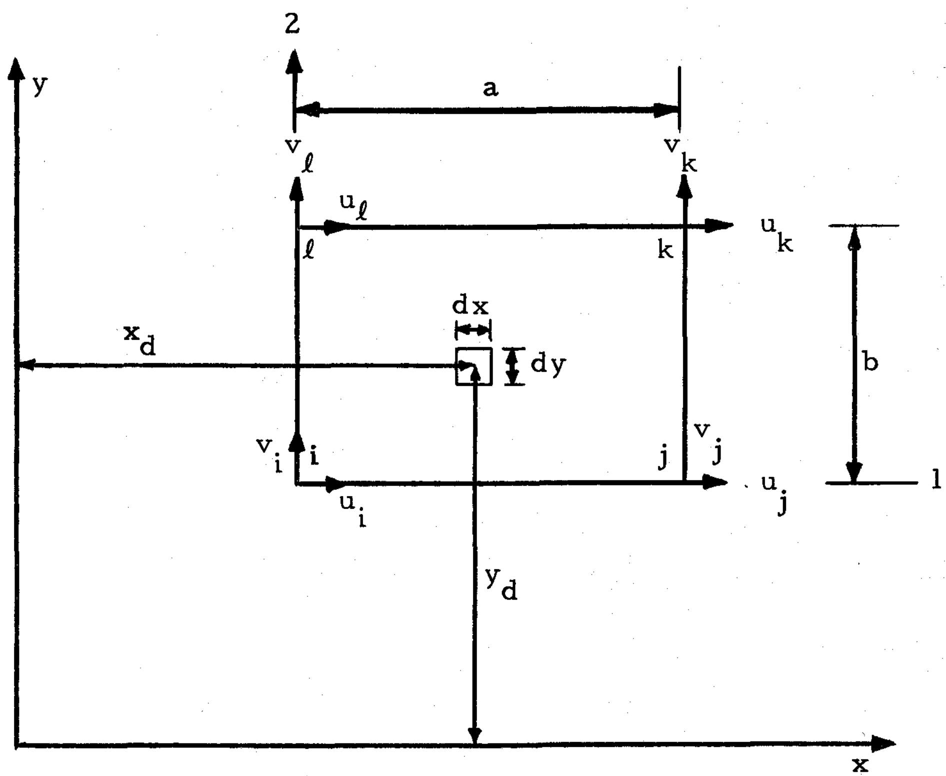

Figure 2.1. Rectangular plane stress element.

$$T_{11} = T_{52} = -T_{15} = -T_{56} = (y_j - y_\ell)/2 \tag{2.4}$$

$$T_{13} = T_{54} = -T_{17} = -T_{58} = (y_k - y_i)/2 \tag{2.5}$$

$$T_{34} = T_{53} = -T_{38} = -T_{57} = (x_i - x_k)/2 \tag{2.6}$$

$$T_{36} = T_{55} = -T_{32} = -T_{51} = (x_j - x_\ell)/2 \tag{2.7}$$

$$T_{21} = (y_j(y_j + y_i) - y_\ell(y_\ell + y_i))/6 \tag{2.8}$$

$$T_{23} = (y_k(y_k + y_j) - y_i(y_i + y_j))/6 \tag{2.9}$$

$$T_{25} = (y_\ell(y_\ell + y_k) - y_j(y_j + y_k))/6 \tag{2.10}$$

$$T_{27} = (y_i(y_i + y_\ell) - y_\ell(y_k + y_\ell))/6 \tag{2.11}$$

$$T_{42} = (x_\ell(x_\ell + x_i) - x_j(x_j + x_i))/6 \tag{2.12}$$

$$T_{44} = (x_i(x_i + x_j) - x_k(x_k + x_j))/6 \tag{2.13}$$

$$T_{46} = (x_j(x_j + x_k) - x_\ell(x_\ell + x_k))/6 \tag{2.14}$$

and

$$T_{48} = (x_k(x_k + x_\ell) - x_i(x_i + x_\ell))/6 \tag{2.15}$$

The five by five matrix $[H]$ is defined as

$$[H] = \iint t[P]^T [C][P]dxdy \tag{2.16}$$

where integration is performed over the entire element area. In Equation (2.16), the symbols are defined as follows:

$$[C] = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{21}/E_2 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \qquad (2.17)$$

where

E_1, E_2 - modulus of elasticity in direction indicated by subscript

ν_{12}, ν_{21} - Poisson's ratio relating the strain in the direction of the second subscript due to the stress in the direction of the first subscript

G_{12} - shear modulus in the 1-2 plane

and t - thickness of the element

Finally,

$$[P] = \begin{bmatrix} 1 & y_d & 0 & 0 & 0 \\ 0 & 0 & 1 & x_d & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2.18)$$

Equations (2.3) through (2.18) are valid for any quadrilateral. Since only rectangles were used in this study, the matrix [H] was simplified and inverted resulting in the matrix [F]. Further simplification was made by assuming the identity

$$E_1 \nu_{21} = E_2 \nu_{12} \qquad (2.19)$$

was true. The non-zeros terms of the five by five matrix [F] for a rectangular element shown in Figure 2.1 are

$$F_{11} = \frac{4E_1^2E_2 - 3E_1(E_2\nu_{12})^2}{(E_1E_2 - (E_2\nu_{12})^2)abt} \tag{2.20}$$

$$F_{21} = F_{12} = \frac{-6E_1}{ab^2t} \tag{2.21}$$

$$F_{22} = \frac{12E_1}{ab^3t} \tag{2.22}$$

$$F_{31} = F_{13} = \frac{E_1E_2(E_1\nu_{21})}{(E_1E_2 - (E_2\nu_{12})^2)abt} \tag{2.23}$$

$$F_{33} = \frac{4E_1E_2^2 - 3E_2(E_2\nu_{12})^2}{(E_1E_2 - (E_2\nu_{12})^2)abt} \tag{2.24}$$

$$F_{34} = F_{43} = \frac{-6E_2}{a^2bt} \tag{2.25}$$

$$F_{44} = \frac{12E_2}{a^3bt} \tag{2.26}$$

and

$$F_{55} = \frac{G_{12}}{abt} \tag{2.27}$$

The matrix multiplication of [F] and [T] to obtain the element stiffness matrix is done by the computer. The stress vector of the differential element is computed by Equation (2.2) where the stress-displacement matrix is

$$[B] = \frac{1}{t}[P][F][T] \tag{2.28}$$

The stress variation within the element in one direction is linear with respect to the perpendicular direction.

A triangular element was added to increase the possible ways of subdividing the structure. The plane stress triangular element is based on a linear displacement function and an anisotropic stress-strain matrix (36, pp. 48-56, 447-450). The element, shown in Figure 2.2, has its nodal points i, j, and k oriented counter-clockwise and a 1-2 local coordinate system. The element stiffness matrix is defined by

$$[k_e] = [SD]^T[SS][SD]\,tA \tag{2.29}$$

where

[SD] - strain-displacement matrix

[SS] - stress-strain matrix

and A - area of triangle

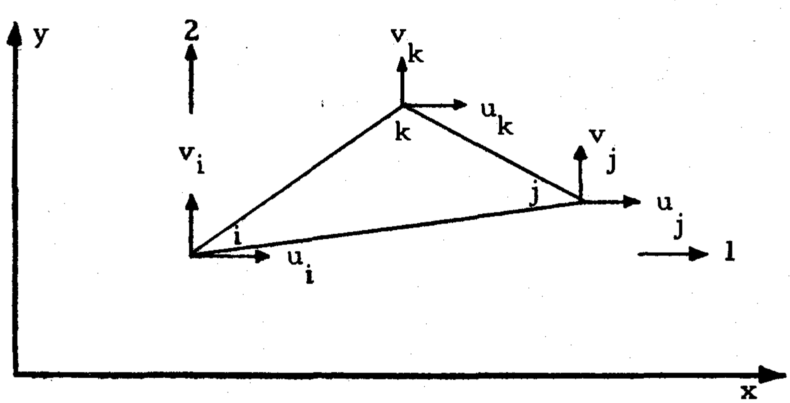

Figure 2.2. Triangular plane stress element.

The strain-displacement matrix of triangular element is defined in terms of the nodal coordinates x and y as

$$[SD] = \frac{1}{2A}\begin{bmatrix} b_j - b_k & 0 & b_k & 0 & -b_j & 0 \\ 0 & a_k - a_j & 0 & -a_k & 0 & a_j \\ a_k - a_j & b_j - b_k & -a_k & b_k & a_j & -b_j \end{bmatrix} \qquad (2.30)$$

where

$$a_j = x_j - x_i \qquad (2.31)$$

$$a_k = x_k - x_i \qquad (2.32)$$

$$b_j = y_j - y_i \qquad (2.33)$$

and

$$b_k = y_k - y_i \qquad (2.34)$$

The stress-strain matrix is

$$[SS] = \frac{1}{(1-\nu_{12}\nu_{21})}\begin{bmatrix} E_1 & E_2\nu_{12} & 0 \\ E_1\nu_{21} & E_2 & 0 \\ 0 & 0 & (1-\nu_{12}\nu_{21})G_{12} \end{bmatrix} \qquad (2.35)$$

The area of triangle is given by

$$A = (a_j b_k - a_k b_j)/2 \qquad (2.36)$$

The stress-displacement matrix for Equation (2.2) is

$$[B] = [SS][SD] \tag{2.37}$$

The stresses are considered constant throughout the element.

The third type of finite element used in this study, the two-dimensional spring element (20), was introduced to represent the connections between the wall and floor components, such as the sheathing and sole plate. The element and local coordinate system 1-2 are shown in Figure 2.3. The element has two nodal point i and j that have the same x and y coordinates, i.e., the length and height of the spring are zero. The location of the two nodal points and springs coincide and can belong to two or more elements. The spring constants in the 1 and 2-directions, denoted as k_1 and k_2, respectively, define the four by four element stiffness matrix as

$$[k_e] = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & 0 & -k_2 \\ -k_1 & 0 & k_1 & 0 \\ 0 & -k_2 & 0 & k_2 \end{bmatrix} \tag{2.38}$$

For a nail, the spring constants are given by the withdrawal and lateral connector moduli.

Equation (2.2) is used to compute the nodal forces of the element. The force-displacement matrix $[B]$ is equivalent to the element stiffness matrix.

The rectangular, triangular, and spring elements are well suited to describe the various material, geometry, and boundary conditions that influence the end fixity of stud wall panels. The finite element analysis gives the element stresses or forces and nodal displacements which are used to compute the coefficient of end fixity.

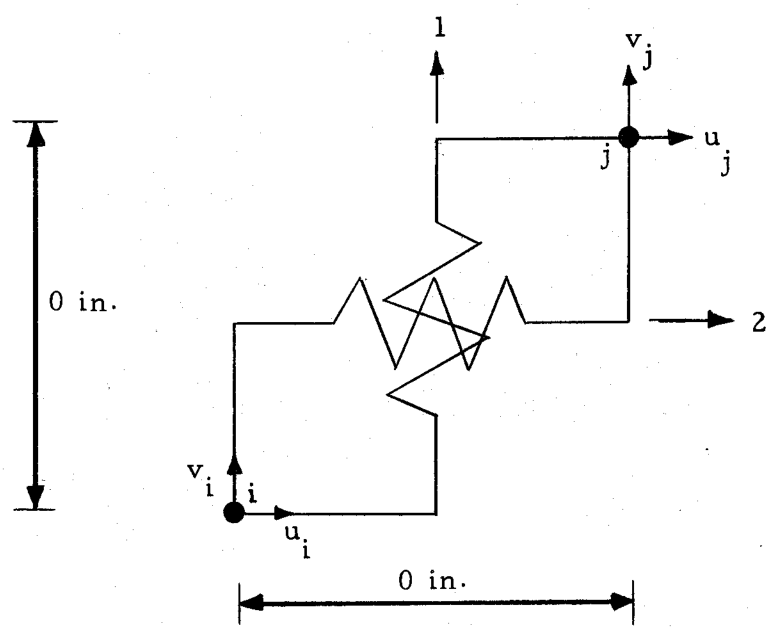

Figure 2.3. Two-dimensional spring element.

2.3. Coefficient of End Fixity

The computation of the end fixity coefficient by Equation (1.1) requires the determination of the moment and corresponding angular rotation at the wall support. To calculate the moment, a series of spring elements is inserted at the cross-section where the end fixity is being determined, as shown in Figure 2.4. A very large spring constant is assigned to the spring element which is part of a continuous member, such as continuum within the wall components or where the spring is in compression, such as on the boundary between wall components that are compressed together. A very small spring constant is used at locations where there is no connection between structural components that are free to separate and the spring is in tension. The connector modulus defines the spring representing a nailed or glued connection that is in tension or shear. The moment caused by the spring forces F_n is

$$M = \sum_{n=1}^{i} F_n h_n \tag{2.39}$$

where

h_n - distance from the spring n to axis A (Figure 2.4)

and i - number of springs in the cross-section

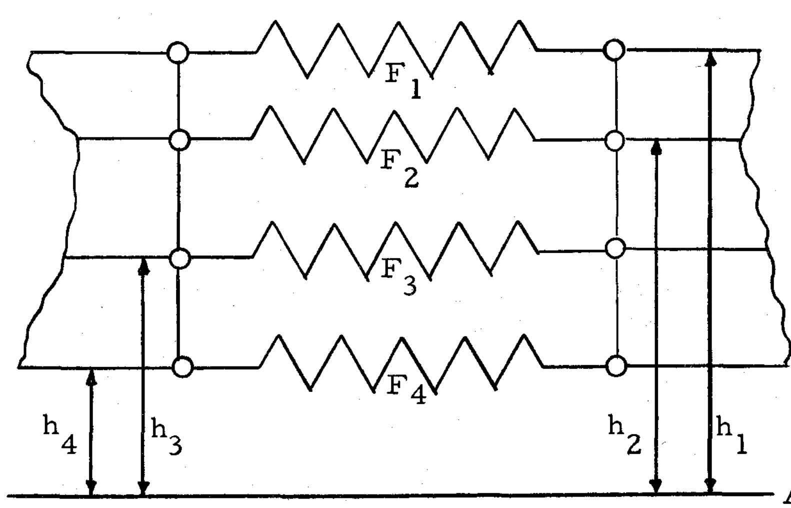

Figure 2.4. Springs for calculation of end moment.

If the forces in the springs represent only a bending moment couple, the moment can be computed with the axis A having any arbitrary coordinate. However, if the springs are all in compression, the axis A must correspond to the neutral axis of the structural member to calculate the moment. The springs are in compression if a compressive axial load acts on the beam-column in addition to the bending load.

The angular rotation of the cross-section at the wall support, shown in Figure 2.5, is assumed to be equal to the slope of the elastic curve. For elastic deformation, the angular rotation can be determined from the relative linear displacement of the top and bottom nodal points at the cross-section. Since the angle is very small, the angular rotation can be approximated by

$$\theta = \operatorname{Tan} \theta = \frac{d_i - d_j}{y_i - y_j} \tag{2.40}$$

where

d_i, d_j - displacements in the x-direction of nodal points i and j of Figure 2.5

and y_i, y_j - y-coordinates of nodal points i and j

Finally, the coefficient of end fixity is computed by substituting Equation (2.39) and (2.40) into Equation (1.1).

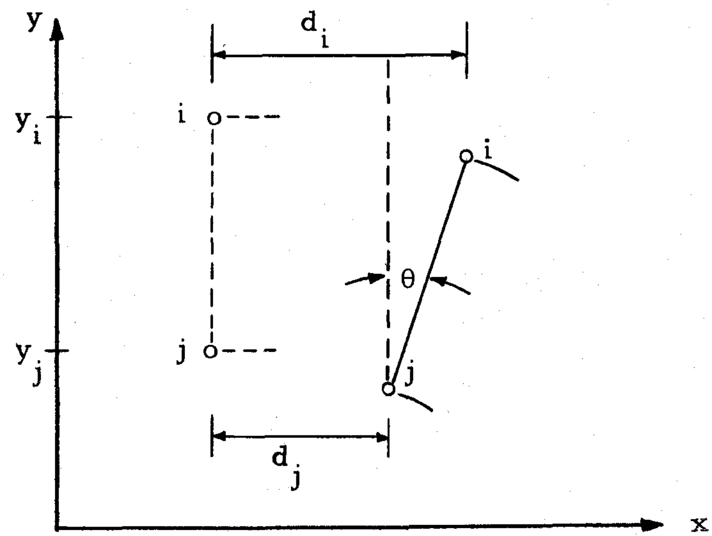

Figure 2.5. Elastic deformation of the cross-section at the stud support.

2.4. Midspan Deflection of Beam-Column

To compare deflections computed on the basis of the theoretical coefficient of end fixity with the corresponding experimental values, the midspan deflections of a beam-column are needed. The beam-column model of the experimental wall panel is shown in Figure 2.6. Because of the symmetry with respect to the midspan, rotation θ and restraining moment M are the same at both supports.

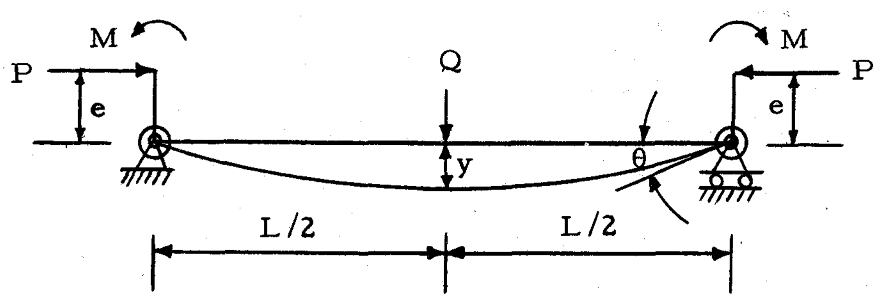

Figure 2.6. Beam-column with partial end fixities and eccentric axial load.

Using the differential equations given by Timoshenko (28, pp. 1-19), the midspan deflection of the beam-column can be shown to be

$$y = \frac{M}{P}(1-\sec(u)) - e(1-\sec(u)) + \frac{Q}{2Pk}(\tan(u)-u) \tag{2.41}$$

where

$\frac{M}{P}(1-\sec(u))$ - deflection due to restraining moment and axial load P

$e(1-\sec(u))$ - deflection due to moment caused by the eccentricity of the axial load

$\frac{Q}{2Pk}(\tan(u)-u)$ - deflection due to lateral load Q and axial load P

$$u = \frac{kL}{2} \tag{2.42}$$

$$k^2 = \frac{P}{EI} \tag{2.43}$$

and EI - flexural rigidity of beam-column

The angle of rotation at the supports is (28)

$$\theta = \theta_o - \frac{ML}{3EI}\psi(u) - \frac{ML}{6EI}\phi(u) \tag{2.44}$$

where

θ_o - angle of rotation at supports of beam-column with hinged ends

$\frac{ML}{3EI}$ - rotation at the support of the hinged beam-column because of support fixity moment M externally applied to the same support

and $\frac{ML}{6EI}$ - rotation at support caused by moment M applied to the opposite support

Expressions $\psi(u)$ and $\phi(u)$, defining the influence of axial load on the angles of rotation, equal

$$\psi(u) = \frac{3}{2u} \left(\frac{1}{2u} - \frac{1}{\tan(2u)} \right) \tag{2.45}$$

and

$$\phi(u) = \frac{3}{u} \left(\frac{1}{\sin(2u)} - \frac{1}{2u} \right) \tag{2.46}$$

The end rotation can be expressed in terms of Equation (1.1) as

$$\theta = \frac{M}{\alpha} \tag{2.47}$$

Equating Equations (2.47) and (2.44) and solving for M gives

$$M = \frac{\theta_o}{\frac{1}{\alpha} + \frac{L}{3EI}\psi(u) + \frac{L}{6EI}\phi(u)} \tag{2.48}$$

The angle of rotation θ_o is

$$\theta_o = \frac{PeL}{2EI}\frac{\tan(u)}{u} + \frac{QL^2}{16EI}\lambda(u) \tag{2.49}$$

where

$$\frac{PeL}{2EI} \text{ - rotation due to moment (Pe)}$$

$$\frac{QL^2}{16EI} \text{ - rotation due to lateral load Q}$$

$$\lambda(u), \frac{\tan(u)}{u} \text{ - influence of axial load P}$$

and

$$\lambda(u) = \frac{2(1-\cos(u))}{u^2 \cos(u)} \tag{2.50}$$

The midspan deflection of the beam-column with partial end fixity is computed by Equations (2.41), (2.48) and (2.49). Values for $\phi(u)$, $\psi(u)$ and $\lambda(u)$ are tabulated in reference 28.

2.5. The Computer Program

Using Equations (2.3) through (2.50) and an existing finite element program, a special FORTRAN IV computer program was prepared for this study. Subroutines were added to Zienkiewicz (36, pp. 435-471) computer program to generate the element stiffness and stress-displacement matrices, to compute the coefficient of end fixity and to compute the midspan deflection of the beam-column. The computer program is listed in Appendix A.

The computer program was designed for the Oregon State University Control Data Corporation 3300 computer system which has a usable memory capacity of 32,000 words. In its present form, the computer program has sufficient storage for 100 nodal points, 200 elements, 25 nodal boundary conditions and 25 different materials. The maximum half-band width of the stiffness matrix is set at 55. About 60 seconds of computer time is required for the example given in Appendices B and C. Additional information on the computer program is given in the commentary within the program and in reference 36.

The first step in the use of the computer program is to create a finite element model of the wall panel consisting of the rectangular, triangular and spring elements. The material and geometric properties of the wall panel components are assigned to the corresponding elements. The nodal points and the elements are numbered for identification purposes. The nodes are defined in terms of the system x-y coordinates. The boundary conditions are inserted by limiting the allowable nodal displacements. The external loads are applied at the appropriate nodal points.

The data on the finite element model is listed in an input file as in Appendix B. The input file in Appendix B is for the problem in Section 3.2. The nodal points and elements for computing the coefficient of end fixity are also listed. The axial load, the eccentricity of the axial load, the midspan lateral load, the length and the factor u of the beam-column are required if the midspan deflection is to be computed. The computer analysis results in the printed output given in Appendix C.

III. VERIFICATION

The verification of the method and computer program consisted of comparing the finite element solutions to known theoretical and experimental solutions. Preliminary verification involved the analysis of solid beams with known theoretical deflections and coefficients of end fixity. Glenn Schroeder's (25) experimental results were used in the final verification.

3.1. Comparisons with Known Theoretical Solutions

A rectangular element based on a linear displacement function (17) was initially used in this study. A simply supported beam with a height of 4.25 in., width of 1.5 in., and a span of 96 in. was evaluated using this initial element. The beam had a modulus of elasticity of 1,990 ksi and a midspan load of 300 lbs. Because of symmetry with respect to midspan, only half of the beam was analyzed. The number of elements was varied from 8 to 48. The deviation between the finite element and classical beam solution for midspan deflection and moment varied from 58 percent to 14 percent with the deviation decreasing with an increase in the number of elements. The substitution of Cook's (4) assumed stress hybrid rectangular element did not improve the results. The accuracy of the results could have been improved by increasing the number of elements, but this number was limited by the memory capacity of the computer.

A semi-fixed beam-column was analyzed to investigate the ability to predict the coefficient of end fixity. The finite element model of half of the beam is shown in Figure 3.1. The partial end fixity was produced by two translational springs at the beam end. The spring constants were 100 kips per in. The beam-column had a modulus of elasticity of 1,990 ksi and a thickness of 1.5 in. The beam-column had an eccentric axial load P and a midspan lateral load Q.

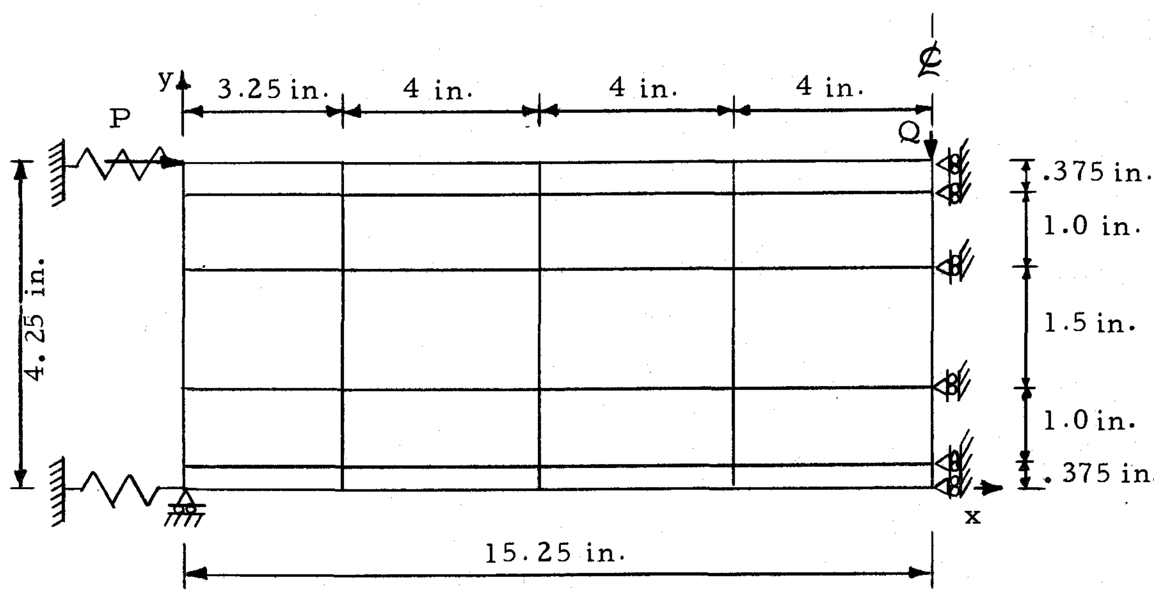

Figure 3.1. Finite element model of a solid semi-fixed beam-column.

A theoretical value for the coefficient of end fixity was determined by equating the work of the two translational springs with the work done by an assumed angular spring. The total work of the two translational springs because of the deformation due to the external loads equals

$$W = 2\left(\frac{Fy}{2}\right) \tag{3.1}$$

where

F - force in the spring due to external loads

and y - translational displacement of the spring

An equivalent angular spring is assumed to replace the set of translational springs at each support. The work of an angular spring because of the deformation due to the external loads equals

$$W = \frac{M\theta}{2} \tag{3.2}$$

where

M - moment or force in spring due to external load

and θ - angular displacement of the spring

The force in a translational spring can be written as

$$F = k_s y \tag{3.3}$$

where k_s - translational spring constant

and the force in an angular spring can be written as

$$M = \alpha\theta \tag{3.4}$$

Inserting Equation (3.3) into Equation (3.1) and Equation (3.4) into Equation (3.2), equating the resulting Equations (3.1) and (3.2) and simplifying gives

$$\alpha = \frac{2k_s y^2}{\theta^2} \tag{3.5}$$

The moment caused by the translational springs equals

$$M = Fh \tag{3.6}$$

or

$$M = k_s yh \tag{3.7}$$

where h - distance between the springs

Because the moment in the assumed angular spring is equal to the moment produced by the translational springs, Equation (3.7) is equated to Equation (3.4) to obtain

$$\frac{y}{\theta} = \frac{\alpha}{k_s h} \tag{3.8}$$

Substituting Equation (3.8) into Equation (3.5) and simplifying gives

$$\alpha = \frac{k_s h^2}{2} \tag{3.9}$$

Substituting the properties of the partially fixed beam-column into

Equation (3.9) results in a coefficient of end fixity of 903 kips-in. per radian.

The finite element analysis was performed for the element distribution shown in Figure 3.1. The loads and the results are given in Table 3.1. The moment was calculated from Equation (2.39) and the nodal forces in the spring elements. The displacements of the top and bottom nodal points at the end of the beam-column and Equation (2.40) were used to compute the angular rotation. The results indicate that the method will give reasonable accurate values for the coefficient of end fixity, even though the element distribution is too coarse to give accurate values for moment and deflection.

Table 3.1. Coefficient of support fixity for partially fixed solid beam-column.

P	Q	Coefficient of Support Fixity: Equation (3.9)	Coefficient of Support Fixity: Finite Element Analysis	Difference
- - (kips)	- -	- - - (kips-in.	per radian) - - -	(percentage)
.442	.15	903	903	0
0	.15	903	876	3

3.2. Comparison with Experimental Wall Panels

The final verification consisted of the finite element analysis of experimental wall panels tested by Glenn Schroeder (25). Three wall panels, identified as wall types no. 1, 2 and 3, with different exterior covering were analyzed and the results compared with the corresponding test results.

3.2.1. Panel Construction and Testing

The test specimens consisted of a wall panel with a 16 in. by 16 in. square floor section at each end as illustrated in Figure 3.2. The floor and wall sections were constructed according to specifications of the Federal Housing Administration of the Department of Housing and Urban Development (29). Wall panels were 16 in. wide and 95.5 in. long. Each wall panel consisted of a clear Douglas-fir nominal two in. by four in. stud, an interior covering of 3/8-in. gypsum board, an exterior covering with three possible choices of sheathing, and a two in. by four in. sole plate at both ends. The exterior sheathing for wall types 1, 2, and 3 were 3/8-in. CD exterior plywood, 5/8-in. CD exterior plywood and 1/2-in. structural particleboard, respectively. The exterior covering was fastened to the stud with 6d box nails spaced 12 in. on center and to the sole plate with three 8d box nails spaced eight in. on center. The gypsum board was fastened to the stud and sole plate with 3d box nails spaced eight in. on center. Two 16d box nails were driven through the sole plate and into the end of the stud.

The wall panels were initially tested without the floor sections as simply supported beams. Each panel was loaded with the axial

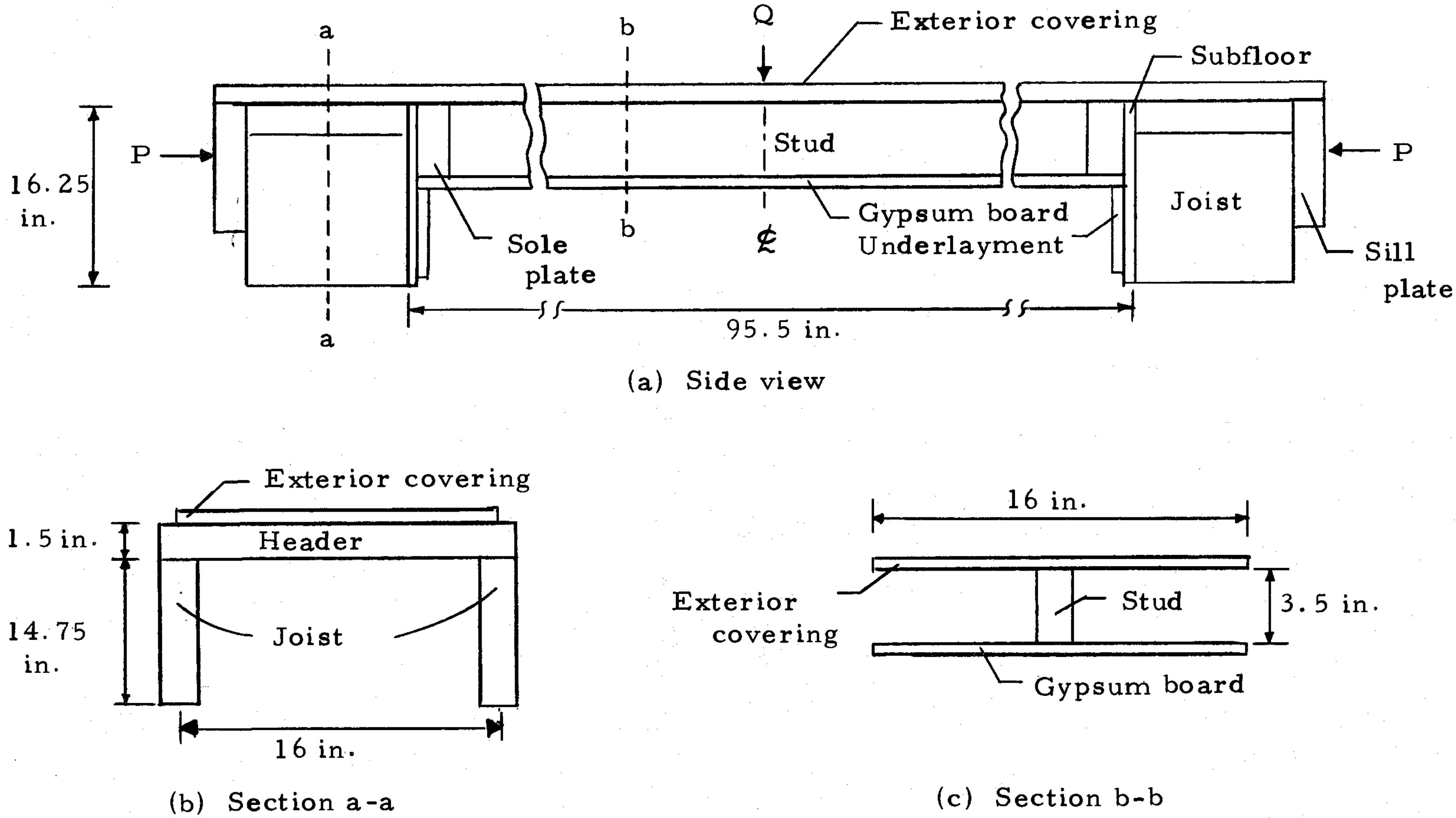

Figure 3.2. Experimental wall panel.

load P of 442 lbs which was kept constant while a lateral load Q was gradually applied at the midspan until it reached 300 lbs. The headspeed was 0.34 in. per minute. The same sequence was then repeated for a 1002 lbs axial load. During the load application, curves relating the lateral load and midspan deflection were recorded on an x-y recorder.

After the free-end test the floor sections were attached to both ends of the wall panel. The floor section consisted of two joists spaced 16 in. on center, a header nailed to each of the joists with two 16d box nails, a two in. by four in. sill plate toe-nailed to each of the joists with a 10d box nail, a 3/8-in. particleboard underlayment and a 1/2-in. plywood subfloor. Joists and headers were made of No. 2 and Better grade Douglas-fir lumber of nominal size two in. by eight in. The floor sections were attached to the wall panels by fastening the sole plate to the subfloor with two 16d box nails spaced 16 in. on center and the exterior sheathing to the sill plate with three 8d box nails spaced eight in. on center.

The experimental wall panels were mounted in a test apparatus that restricted the rotation of the floor joists during testing. The testing procedure was the same as in the free-end tests of wall panels alone.

3.2.2. Evaluation of Load-Deflection Curves

Because Schroeder's experimental results were expressed in a form not suitable for comparison with the theoretical results of this study, his raw experimental data was re-evaluated. The midspan deflections of the wall panel at a lateral load of 300 lbs were scaled from the experimental load-deflection curves. The initial deflection, because of eccentrically applied axial load only, was not recorded by the x-y recorder. To overcome this lack of information, the eccentricity of the axial load was assumed to be zero. The experimental free-end midspan deflection y_{free} and fixed-end midspan deflection y_{fixed} were processed by Schroeder's computer program (25) for no eccentricity to obtain the values of the factor u, defined by Equation (2.42), and the experimental coefficients of end fixity. The results are given in Table 3.2.

3.2.3. Finite Element Model

The first step in a theoretical analysis of end fixity, consisting of a formulation of a finite element model for the wall panel, encountered two major difficulties. The difficulties were the limited capacity of the Oregon State University Control Data Corporation 3300 computer system and the determination of the location of the neutral axis for the wall panel under compression and bending loads. The

Table 3.2. Experimental wall panel results.

Wall Type	P (lbs)	Replication	y_{free} (in.)	y_{fixed} (in.)	u	α (kips-in. /radian)
1	442	1	.435	.387	.279	33
		2	.385	.381	.263	4
		3	.410	.433	.271	-22
1	1002	1	.387	.382	.390	5
		2	.360	.368	.377	-10
		3	.402	.438	.398	-38
2	442	1	.384	.372	.263	12
		2	.388	.332	.264	47
		3	.428	.371	.277	40
2	1002	1	.363	.356	.379	8
		2	.359	.321	.377	39
		3	.381	.380	.388	1
3	442	1	.364	.431	.256	-101
		2	.449	.438	.284	8
		3	.357	.411	.254	-80
3	1002	1	.339	.432	.367	-191
		2	.371	.444	.383	-108
		3	.332	.425	.363	-202

number of nodal points was limited by the computer memory capacity. If the axial load was large in comparison to bending loads, the exact location of the neutral axis could not be determined, because only compressive stresses acted in the cross-section at which the end fixity was evaluated.

The effect of axial load on the end fixity is questionable. Schroeder's experimental results suggest a decrease in the end fixity with an increase in the axial load. However, the results of Table 3.1 indicate that the axial load has little or no influence on the end fixity. Because of this indication and uncertainty in the location of the neutral axis that would have induced an unknown error, the axial load was not included in the finite element analysis. The wall panels were analyzed for a lateral load at the midspan only, which did not require the location of the neutral axis.

Because the experimental wall panels were approximately symmetrical with respect to midspan, only half of the wall panel was included in the model to reduce the size of the elements. Since the end fixity depends on the properties of components and connectors at supports only, the span of the panel had no effect on the end fixity and less than half of the panel needed to be analyzed. The panel was kept long enough to have the effect of shear on the midspan deflection limited to about five percent of the total deflection. Because the influence of the axial load on deflections was related to the panel span, the reduction of the span was another reason for excluding the axial load from the model.

The finite element subdivision of the wall panel is shown in Figure 3.3. Pairs of nodal points were required where the wall components were not rigidly connected. To reduce the size of the finite element system and the number of nodal points, the header, sill plate, joist and subfloor were assumed to be rigidly connected to each other. Because the underlayment as the second layer of the floor covering had no or little effect on the end fixity, it was eliminated from the model. The sole plate was considered to be part of the beam-column and not a part of the supporting system. Therefore, the springs used in calculating the coefficient of end fixity were inserted between the subfloor and the sole plate and in the exterior sheathing. The top and bottom nodal points, connecting the sole plate and subfloor, were used to compute the angular rotation for the evaluation of the coefficient of end fixity.

3.2.4. Properties of Wall Panel Components

Most of the material properties of the beam components were obtained from the literature (16, 25, 30, 31, 32-35). A special test was conducted to determine the connector modulus for the 16d nail in withdrawal. The properties of the finite elements shown in Figure 3.3 and the corresponding wall components and connectors are

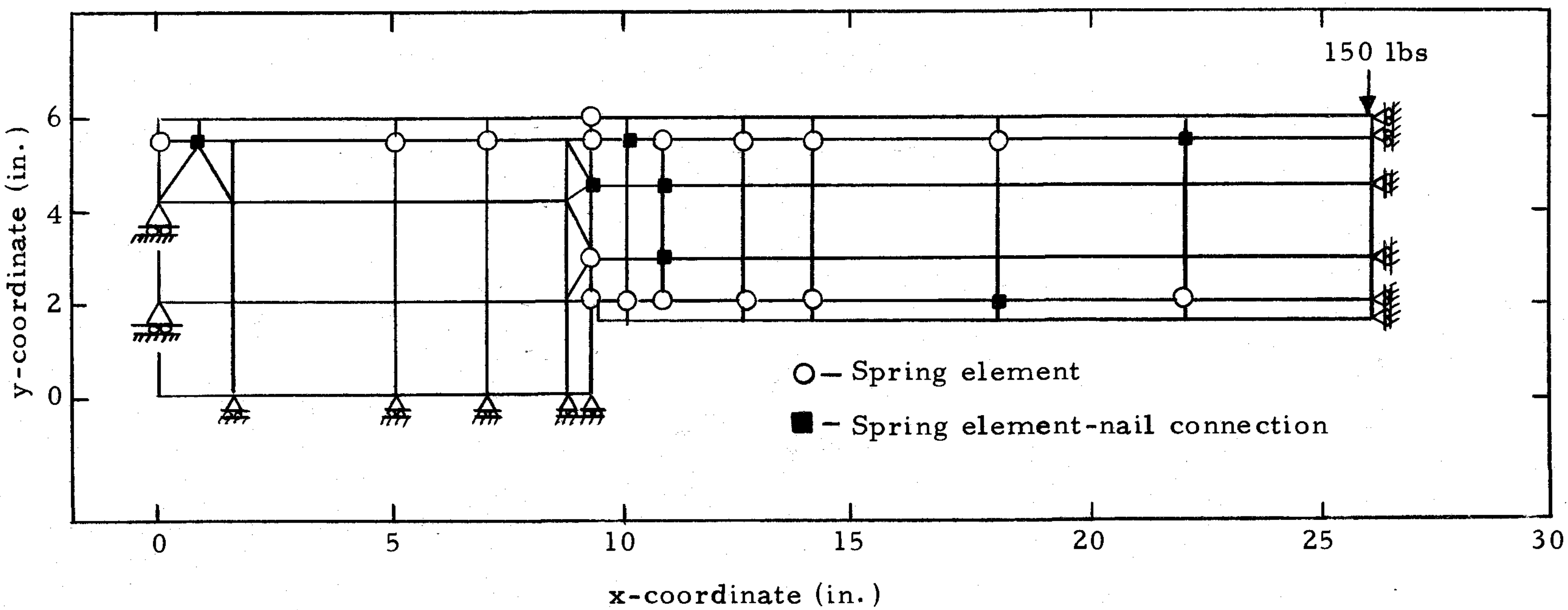

Figure 3.3. Finite element model of wall panel.

listed in Tables 3.3 and 3.4.

Table 3.3. Material properties of wall components.

Property	E_1 (ksi)	E_2 (ksi)	$E\nu$ (ksi)	$1-\nu^2$ --	G_{12} (ksi)	t (in.)
Stud	1990	150	54	.990	141	1.5
Joist	150	1990	54	.990	141	3
Plates and Header	150	150	61	.834	14	16
Subfloor	161	742	44	.985	54	16
Gypsum Board	112	27	17	.938	45	16
3/8-in Plywood	1209	161	47	.989	81	16
5/8-in. Plywood	1092	161	46	.988	74	16
1/2-in. Particleboard	369	32	50	.938	44	16

Table 3.4. Spring constants.

Connection	Nail Size	Number of Nails	k_1 (kips/in.)	k_2 (kips/in.)
Subfloor-Sole Plate	16d	2	3.6	16.8
Stud-Sole Plate	16d	1	1.5	8.4
Stud-Gypsum Board	3d	1	2.6	99,999
Stud-3/8-in. Plywood	6d	1	5.2	99,999
Plates-3/8-in. Plywood	8d	3	16.2	99,999
Stud-5/8-in. Plywood	6d	1	6.3	99,999
Plates-5/8-in. Plywood	8d	3	20.1	99,999
Stud-1/2-in. Particleboard	6d	1	5.7	99,999
Plates-1/2-in. Particleboard	8d	3	16.5	99,999
Sole Plate-Gypsum Board	3d	3	7.7	99,999

Schroeder's thesis (25) and the Wood Handbook (30) provided the properties of the solid wood components. The mean modulus of elasticity in the longitudinal direction E_L, denoted as E_1 in

Table 3.3, of Schroeder's 45 studs was 1,990 ksi with a standard deviation of 84 ksi. The E_L of the studs used in the wall panels of types 1, 2 and 3 range from 1,890 ksi to 2,170 ksi. This difference in E_L has little if any affect on end fixity. The mean moisture content of the studs was 7.3 percent. The Wood Handbook ratios (30, p. 79) were used to obtain the modulus of elasticity in the radial-tangential direction E_2, shear modulus G, and Poisson's ratios ν_{12} and ν_{21}. The values for E_2 were obtained by averaging the modulus of elasticity in the radial R and tangential T directions. The elastic properties for headers, joists and the plates were assumed to be the same as those for the studs.

The plywood properties were derived from the experimental investigation of stud walls by White <u>et</u> <u>al</u>. (31). For C-D 3/8-in. sheathing plywood (PS1.66, 24/0), this investigation gives the value of 1,727 ksi for E_8, the modulus of elasticity in the eight foot direction of the plywood sheet, and 107 ksi for E_4, the modulus of elasticity in the four foot direction. The equations for modulus of elasticity in bending of three ply plywood, in terms of the moduli of elasticity of the plys, are (8)

$$E_4 = (E_L t_1^3 + E_T (t_2^2 - t_1^2))/t_2^3 \tag{3.10}$$

and

$$E_8 = (E_T t_1^3 + E_L (t_2^3 - t_1^3))/t_2^3 \tag{3.11}$$

where

$2t_1$ - thickness of the inner ply

and $2t_2$ - total thickness of the plywood

Solving Equation (3.10) and Equation (3.11) for E_L and E_T, using the experimental E_8 and E_4, gives 1792 ksi for E_L and 42 ksi for E_T.

The elastic properties of the 3/8-in. plywood sheathing acting as a membrane were obtained from the conventional expression relating membrane forces and strains for the cross-section of a ply. This expression, given in a matrix form by Dong et al. (6), is

$$\begin{Bmatrix} N_L \\ N_R \\ N_{LR} \end{Bmatrix}_{ply} = \frac{h}{1-\nu_{LR}\nu_{RL}} \begin{bmatrix} E_L & E_R\nu_{LR} & 0 \\ E_L\nu_{RL} & E_R & 0 \\ 0 & 0 & (1-\nu_{LR}\nu_{RL})G_{LR} \end{bmatrix} \begin{Bmatrix} \epsilon_L \\ \epsilon_R \\ \gamma_{LR} \end{Bmatrix} \tag{3.12}$$

where

N - internal forces per unit length

h - thickness of cross-section

ϵ_L, ϵ_R - normal strains

γ_{LR} - shear strain

and other symbols are defined earlier in the text. Total force vector, acting on the cross-section of the 3/8-in. plywood, is obtained by summing forces acting on individual plies, which results in summing the corresponding moduli of elasticity for individual plies. The sums of the moduli of elasticity are the desired moduli for the plywood cross-section. If all the plies are of equal thickness, the values for 3/8-in. plywood sheathing are

$$E_1 = \frac{2E_L + E_T}{3} \tag{3.13}$$

$$E_2 = E_R \tag{3.14}$$

$$\nu_{21} = \frac{2E_L \nu_{RL} + E_T \nu_{RT}}{2E_L + E_T} \tag{3.15}$$

$$\nu_{12} = \frac{2\nu_{LR} + \nu_{TR}}{3} \tag{3.16}$$

and

$$G_{12} = \frac{2G_{LR} + G_{TR}}{3} \tag{3.17}$$

The values for ν_{RL}, ν_{TR}, ν_{LR}, ν_{RT}, G_{LR}, G_{TR}, and E_R, the elastic constants for Douglas-fir, were taken from the Wood Handbook (30). The elastic properties of the C-D exterior 5/8-in. plywood sheathing and the 1/2-in. plywood subfloor were obtained by determining equations similar to Equations (3.13) through (3.17) and assuming the veneer plies were of the same quality as in the 3/8-in. plywood.

The 1/2-in. particleboard exterior wall covering was Humboldt structural exterior flakeboard. Because no specific values were available, the elastic properties used in this study were obtained by averaging McNatt's (16) experimental results for nine commercial particleboards of various thicknesses and compositions. No data was available for the modulus of elasticity in the y-direction, i.e., perpendicular to the surface. Since the average internal bond strength or tensile strength in the y-direction was nine percent of the tensile strength parallel to the surface, the modulus of elasticity perpendicular to the surface was taken as nine percent of the modulus of elasticity parallel to the surface. Because no values could be found for the Poisson's ratios, the theoretical ratio for isotropic materials of .25 was used.

Experimental results of White et al. (31) were used for the moduli of elasticity of the 3/8-in. gypsum wall board. Like for the particleboard, the Poisson's ratios were taken as .25. The shear modulus was approximated by

$$G = \frac{E}{2(1+\nu)} \tag{3.18}$$

which is a well known relationship for homogeneous isotropic materials.

Theoretical-experimental procedure by Wilkinson (32-35) was used to obtain the lateral connector modulus k_a of nail connections. The modulus k_a depends upon the elastic bearing constants of the connecting components, diameter of the nail and the depth of the nail penetration. The elastic bearing constants, determined from Wilkinson (34, 35), depend upon the type of material, direction of loading with respect to the wood grain, type of nail, prebored lead hole and the specific gravity of the wood. Moduli k_a and the elastic bearing constants for the connecting members 1 and 2, k_{01} and k_{02}, respectively, of specific connections used in this investigation are listed in Table 3.5. The nails were smooth shank box nails driven with no lead holes and the specific gravity of Douglas-fir was taken as 0.5.

The connector moduli of a nail in withdrawal were not available in the literature. For the 6d and 8d box nails, the withdrawal connector moduli were taken as 99,999 kips per in. The use of such a large value was justified since the connections were in compression or negligible tension. A small number of 16d box nails were tested in withdrawal to determine a more precise value for the connector modulus between the sole plate and subfloor which was an influential factor in the end fixity.

Twelve samples, consisting of a nail driven two in. deep into a block of Douglas-fir, were tested in an Instron testing machine. The

Table 3.5. Lateral connector moduli.

Component 1	Component 2	Nail Size	k_{01} (kips/in.^{3})	k_{02} (kips/in.^{3})	k_a (kips/in.)
3/8-in. Plywood	Stud	6d	800	1072	5.2
3/8-in. Plywood	Plates	8d	800	640	5.4
5/8-in. Plywood	Stud	6d	800	1072	6.3
5/8-in. Plywood	Plates	8d	800	640	6.7
1/2-in. Particleboard	Stud	6d	750	1072	5.7
1/2-in. Particleboard	Plates	8d	750	640	5.5
Gypsum Board	Stud	3d	410	1072	2.6
Gypsum Board	Sole Plate	3d	410	640	2.5
Sole Plate	Stud or Subfloor	16d	640	640	8.4

testing apparatus, shown in Figure 3.4, pulled the nail at a rate of .3 cm per minute. The pulling load and extraction slip between the nail and wood were drawn on an x-y recorder. The slope of the recorder traces is the desired connector modulus. The slope was computed from the linear part of the trace which was just before the peak load. The results are given in Table 3.6.

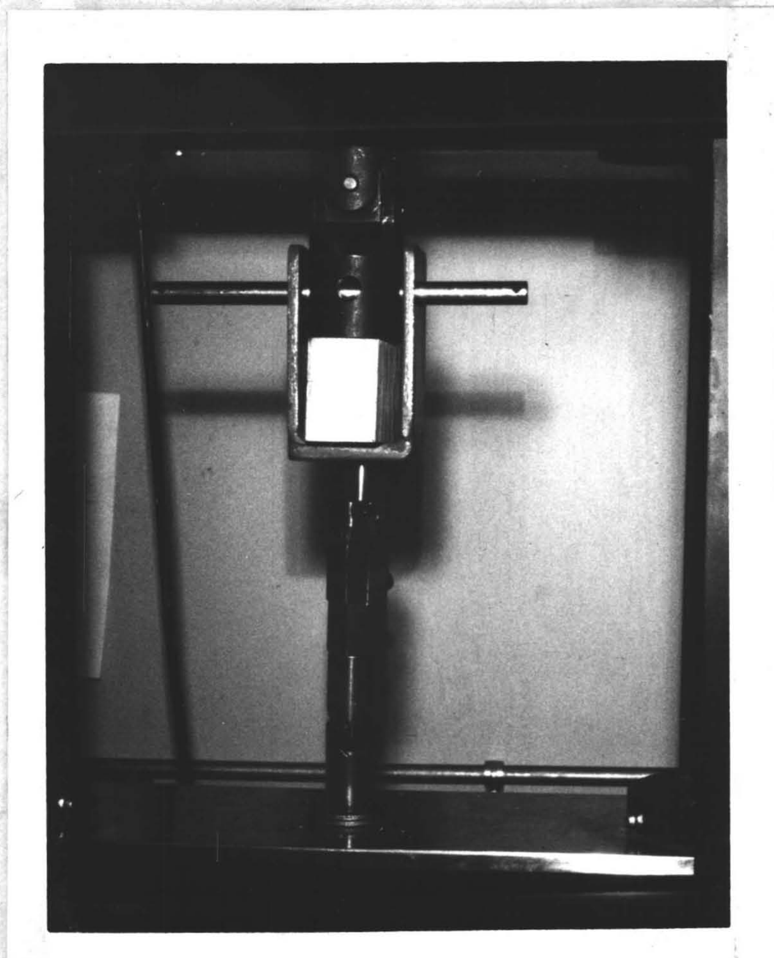

Figure 3.4. Nail withdrawal testing apparatus.

Table 3.6. Connector modulus of 16d box nail in withdrawal.

Sample		Side Grain Connector Modulus (kips per in.)	End Grain Connector Modulus (kips per in.)
1		1.82	1.72
2		1.72	1.17
3		1.68	1.04
4		1.87	1.23
5		--	2.02
6		--	1.76
	Average	1.8	1.5

3.2.5. Computer Analysis

The finite element analysis was performed to obtain a theoretical coefficient of end fixity for each of the three types of wall panels. The theoretical midspan deflections of the wall panels were computed using the beam-column of Section 2.4, the average experimental values for the factor u and the theoretical coefficient of end fixity. The beam-columns had a span of 95.5 in., lateral mid-span load of 300 lbs and a concentric axial load of either 442 lbs or 1002 lbs. In addition, the effective flexural rigidity EI was computed from the average factor u. The input file and printed output for wall type 1 are given in Appendices B and C.

3.2.6. Comparison of Results

The experimental and theoretical results are listed in Table 3.7. The experimental results are the average of three test replications. Three of the six experimental coefficients of end fixity listed in Table 3.7, are negative. Because the coefficient of end fixity is positive by definition, these experimental results must be considered unreliable. The remaining three experimental coefficients are positive and perhaps more reliable.

The percentage differences between the experimental and theoretical coefficient of end fixity are substantial. But the actual differences are reasonable since the coefficient of end fixity can range from zero to infinity. This wide variation is illustrated in the experimental results for each wall panel which are given in Table 3.2. It is best illustrated in the coefficients for wall type 2 and P equal to 1002 lbs which are 1, 8 and 39 kips-in. per radian. The average of these experimental results gives the best agreement with the theoretical coefficient of 19 kips-in. per radian.

The percentage difference is reduced by converting the results to their respective midspan deflections. The percentage difference between the experimental and theoretical midspan deflections are within the 10 percent usually considered reasonable.

Table 3.7. Theoretical and experimental results for wall panels.

	Units	Wall Panel 1		Wall Panel 2		Wall Panel 3	
P	kips	.442	1.002	.442	1.002	.442	1.002
Experimental y_{free}	in.	.410	.383	.400	.368	.390	.347
u	--	.271	.388	.268	.381	.265	.371
EI	kips-in.^{2}	13,700	15,200	14,000	15,700	14,400	16,600
Experimental α	kips-in. per radian	5	-14	33	16	-58	-167
Theoretical α	kips-in. per radian	18	18	19	19	18	18
Difference	percentage	260	*	-42	19	*	*
Experimental y_{fixed}	in.	.400	.396	.358	.352	.427	.434
Theoretical y_{fixed}	in.	.390	.365	.381	.352	.373	.334
Difference	percentage	-2.5	*	6.4	0	*	*

*No comparison made because of unreliable experimental results.

IV. DISCUSSION

Topics included in this discussion are the limitations and applications of the finite element method used in this investigation and the behavior of the semi-fixed supports of the wood-stud walls. Theoretical and experimental results of investigated wall panels are examined in detail. A parameter study is included to demonstrate the method application.

4.1. Method Limitations

The finite element method with plane stress elements used in this study is able to predict the coefficient of end fixity with sufficient accuracy, but there are some limits on how the method should be used. The material properties are assumed to be elastic and constant even though nailed connections and some wall components display inelastic behavior. The number of nodal points, having a pronounced effect on the precision of results, is limited by the memory capacity of the computer being utilized. Finally, compromises have to be made in describing a three-dimensional structure by two-dimensional finite element model.

For most materials, it is reasonable to assume the elastic material properties are similar for tension and compression. The major exception is the behavior of the spring elements connecting components that are not rigidly connected. When the components are being compressed together as a result of the load application, the spring constant or the connector modulus should be infinity. However, the spring constant should be equal to zero or the connector modulus when the components are being pulled apart. Because the method allows only one value of the spring constant for each spring element the finite element analysis has to be repeated several times before the desired conditions in the springs are met. The resulting trial and error procedure is used until all the spring elements, describing the contacts among components, have the proper spring constants. Some materials, such as particleboard, have a different modulus of elasticity for compression and tension. A trial and error procedure similar to that for connections or the average of the two moduli can be used in these cases, depending upon how much the two values differ. For loads resulting in stresses above the elastic limit, a more advanced finite element computer program should be developed to account for the non-linearity of the material properties such as the repeated analysis of the structure by a linear step by step analysis. In this study, the coefficient of end fixity and the material properties were assumed to be independent of the lateral load.

The size of the system stiffness matrix, limited by the memory capacity of the computer, depends on the total number of nodal displacements. The maximum number of nodal displacements can be increased by using a finite element computer program that forms and solves a part of the system stiffness matrix at a time. However the computer program utilized in this study does not have this capacity. A larger number of nodal points than that used in this study would require a larger computer memory capacity than that of the CDC 3300.

Material and geometry variations in the wall panel in the direction perpendicular to the x-y plane are not accounted for and the actual three-dimensional wall panel can not truly be represented by a two-dimensional model. The stress in the wall coverings of the stud wall panel vary from a minimum at the midpoint between two studs to a maximum above the stud. For an I-beam, the actual flange width with a non-uniform stress distribution across the flanges can be represented by an effective width which, if uniformly stressed, would contribute the same amount to the flexural resistance of the beam as the actual flange (1). Several construction features, such as the effect of nail position across the span are not accounted for in the finite element analysis. Two nails connecting the sole plate to the subfloor eight in. from the stud are considered to be located at the stud. Similarly, the nails connecting the wall coverings to the plates, located eight in. away from the stud, are also assumed to be at the stud.

Testing of the experimental wall panels according to the procedures employed by Schroeder suggests that the third dimension can be significant. In the testings, the axial load was applied at the sill plate with an eccentricity. The two joists are eight in. on each side of the stud which is in the center of the wall panel. Because of this separation, more stress was transferred through the header and sheathing than if both the joists and the stud were in the center of the panel. This disproportional stress transfer resulted in an effective eccentricity on the stud that was different than the applied eccentricity.

4.2. Behavior of the Support

The influence of the axial load and the exterior sheathing on end fixity are discussed in this section. The axial load probably does not affect the end fixity substantially. Schroeder's free-end tests illustrate that an increase in the axial load causes a minor increase in the flexural rigidity of the experimental wall panels. Statistical analysis of Schroeder's experimental data resulted in a conclusion that for the plywood coverings an increase in axial loading caused a decrease in the coefficient of end fixity (25). However, the scatter and the large number of negative coefficients of end fixity raises questions about the complete validity of his testing procedures and results. The wall panel model, shown in Figure 3.4, was analyzed with the axial load and the resulting coefficient of end fixity was substantially higher than the values for no axial load. The results of the finite element analysis of the solid wood beam, shown in Table 3.1, suggest a higher end fixity coefficient for a beam with an axial load than for the same beam with no axial load. The acceptable agreement between the results, given in Table 3.7, of the wall panel finite element model analyzed without the axial load and the experimental wall panels tested with the axial load suggests that the axial load does not significantly influence the end fixity. Because neither Schroeder's nor this study gives convincing evidence on the effect of the axial load, there is a need to further investigate the effect of the axial load on end fixity.

Schroeder (25) concluded on the basis of the statistical analysis of the experimental results that the 5/8-in. plywood, 3/8-in. plywood, and 1/2-in. particleboard coverings resulted in the highest, intermediates, and lowest coefficient of end fixity, respectively. The corresponding finite element analysis showed that the difference in exterior sheathing properties does not significantly affect the end fixity (Table 3.7). The stress results from the finite element analysis of the wall panel indicated that the exterior sheathing was subjected to bending instead of the expected tension that would occur if the sheathing was contributing to the end fixity of the wall panel. This suggests that the end fixity occurred because of the connections between the sole plate and subfloor rather than any attachment of the sheathing to the wall support.

4.3. Parameter Study

The number, location, and the size of connectors can be easily implemented into the construction specifications and techniques. Therefore, the connectors may be the key to achieve an end fixity that is worth considering in the wall design. To find out the degree that the nails influence the coefficient of end fixity, a parameter study was performed. The results are given in Table 4.1. The materials were those associated with the wall panel having 3/8-in. sheathing (panel type 1), as listed in Tables 3.3 and 3.4.

Table 4.1. The influence of connection moduli on end fixity.

Modification	Connection	k_1 (kips/in.)	k_2 (kips/in.)	α (kips-in./radian)
1	Sheathing - Sill Plate	32.4	99,000	18
2	Stud - Sole Plate	3.0	16.8	26
3	Subfloor - Sole Plate	7.2	33.6	21
4	Sheathing - Sill Plate Subfloor - Sole Plate	0 5.4	0 25.2	20
5	Sheathing - Header	32.4	99,999	181
	Standard			18

The first four modifications displayed little effect on the coefficient of end fixity. Modification 1 consisted of doubling the connector modulus k_1 of the nails between the sheathing and the sill plate. Modification 2 was to double the connector moduli, k_1 and k_2, of the nails between the stud and sole plate. Modification 3 was obtained by doubling the connector moduli of the nails between the sole plate and subfloor. In modification 4, the connector moduli of the nails connecting the subfloor to the sole plate were increased by 50 percent and the nails between the sheathing and sill plate were eliminated. Results for modifications 2, 3, and 4 in Table 4.1 suggest that an improvement in the connection of the subfloor or stud to the sole plate only slightly improves the end fixity.

To obtain a larger increase in end fixity, two additional sets of three 8d nails, connecting the sheathing to the header, were added at coordinates (5, 5.5) and (7, 5.5) (Figure 3.4) as modification 5. The finite element analysis showed that the additional nails improved the end fixity from 18 kips-in. per radian for the original experimental panel to 181 kips-in. per radian for the modified panel. In the original panel the sheathing was predominately subject to bending stress. Additional nails put the sheathing in tension which increased the coefficient of end fixity.

4.4. Application of Method

The primary application of the method consists of parameter studies, aimed at identifying possible ways of improving the end fixity. Two questions will have to be answered before the end fixity can be included in the structural design of wood-stud walls. The first question deals with the end fixity required to justify its inclusion in the design procedure. The second question is associated with the variability in the end fixity of actual wood-stud walls. The results of this and Schroeder's study suggest that glue or additional nails between the sheathing and the components of the wall support will provide coefficient of end fixity of magnitudes worth including into the design procedure. Schroeder's results indicate that the variability of the end fixity coefficient, because of non-uniformity in the properties of wall components, is considerable.

The experimental and theoretical end fixity coefficients given in Table 3.7 may be slightly higher than the values for an actual wood-stud wall. The wall panels tested and analyzed had the sheathing attached to the plates with three nails along the 16 in. width of the test panels. However, if the nails in the actual wall are spaced eight in. on center, the net number of nails per 16 in. of wall is two.

The application of the method will require additional information on material properties of the components and connections. For the finite element method, the properties of the sheathing need to be defined in terms of the perpendicular axes of the plane of the cross-section. In the literature, most property values for the sheathing were in terms of the perpendicular axes of the plane of the face. Toe-nailing the stud to the sole plate, not included in this investigation, may have an effect on the end fixity. To consider the toe-nailing, proper connector moduli would have to be determined.

The method developed in this study provides an efficient tool to investigate the behavior of the stud wall-wall support and the possible modification of the wall construction to improve the end fixity. Additional experimental work is needed to define the accuracy of the theoretical method more precisely and develop the necessary material properties.

V. CONCLUSIONS

The conclusions of this study are:

1. Using the method and computer program, the coefficient of end fixity of a stud wall can probably be computed with sufficient accuracy.
2. A thorough theoretical-experimental investigation of the end fixity in stud walls is recommended to define the accuracy of the method more precisely.
3. The coefficient of end fixity is 18 kips-in. per radian, which results in a very small reduction of wall deflections, for the wood-stud wall with the plywood or particleboard sheathing nailed to the sill plate with 8d box nails spaced eight in. on center.
4. The end fixity of wood-stud wall can be increased by improving the connection between the exterior covering and the floor system and the connection between the sole plate and subfloor.
5. The method has applications in future investigations of how to improve the end fixity and justify the inclusion of the partial end fixity in stud wall design.

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APPENDICES

APPENDIX A: COMPUTER PROGRAM

```
C         PLANE STRESS FINITE ELEMENT PROGRAM
C           FOR END FIXITY IN STUD WALL PANELS
C
C    TAKEN FROM THE FINITE ELEMENT METHOD IN ENGINEERING
C    SCIENCE (1971) BY ZIENKERWICZ WITH MODIFICATIONS.
C    SUBROUTINES WERE ADDED FOR ADDITIONAL ELEMENTS,
C    END FIXITY, AND MID-SPAN DEFLECTION OF BEAM-COLUMN.
C
      PROGRAM MAIN
      COMMON/DATA/TITLE(12),NP,NE,NB,NMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6
     1),NBC(25),NFIX(25),R1(200),SK(200,55)
C
C   READ INPUT GEOMETRY AND PROP.
C
      CALL GDATA
      NSZF=NP*2
      REWIND 11
C
C    READ LOAD
C
      CALL LOAD
C
C    FORM, THEN SOLVE SIMULTANEOUS EQUATIONS
C
      CALL FORMK
      CALL SOLVE
C
C    CALCULATE FORCES AND STRESSES
C
      CALL STRESS
C
C    CALCULATE END FIXITY
C     IF NF.NE.1
C
      NF=FFIN(5)
      IF(NF.EQ.1)GO TO 200
      CALL FIXITY
 200  CONTINUE
      STOP
      END
C
C
      SUBROUTINE GDATA
      COMMON/DATA/ TITLE(12),NP,NE,NB,NMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6
     1),NBC(25),NFIX(25),R1(200),SK(200,55)
```

```
C
C   READ  TITLE AND CONTROL
C
      READ(5,7)TITLE
      READ(5,1)NP,NE,NEBR,NER,NET,NB,NMAT,I1,X,Y
C   NP= NUMBER OF NODAL POINTS
C   ELEMENTS 1 TO NEBR ARE BENDING RECTANGLES.
C   ELEMENTS NEBR+1 TO NER ARE TENSION-COMPRESSION RECTANGLES.
C   ELEMENTS NER+1 TO NET ARE TRIANGLES.
C   ELEMENTSNET+1 TO NE ARE SPRINGS.
C   NB= NUMBER OF BOUNDARY POINTS.
C   NMAT= NUMBER OF DIFFERENT MATERIALS.
C   I1= Ø IF PRINT INPUT DATA.
C   X,Y = LOCAL COORDINATES OF STRESS RESULTS FOR
C         RECTANGULAR ELEMENTS.
C
      READ(5,7)UNITS
C
C   READ  MATERIAL DATA
C
C    MATERIAL PROPERTIES ARE  E(1), E(2), E(1)*P(2,1),
C     1-P(1,2)*P(2,1), G, AND T.
C       OR   K(1), K(2)
C
      READ(5,8)(N,(ORT(N,I),I=1,6),L=1,NMAT)
C
C   READ NODAL POINT DATA
C
      READ(5,2)(N,(CORD(N,M),M=1,2),L=1,NP)
C
C   READ ELEMENT DATA
C
      READ(5,3)(N,(NOP(N,M),M=1,4),IMAT(N),L=1,NE)
C
C   READ BOUNDARY CONDITIONS
C
C
C     Ø1   =    FIXED IN Y-DIR. ONLY
C     1Ø   =    FIXED IN X-DIR. ONLY
C     11   =    FIXED IN BOTH X&Y-DIR.
C
      READ(5,4)(NBC(I),NFIX(I),I=1,NB)
      IF(I1.NE.Ø)GO TO 5ØØ
C
C   PRINT INPUT DATA
C
      NEBR1=NEBR+1
      NET1=NET+1
      NER1=NER+1
      WRITE(6,1ØØ)TITLE
```

```
C
C   ZERO  LOAD  ARRAY
C
      DO 160 J=1,NSZF
 160  R1(J)=0.
      WRITE(6,100)TITLE
      WRITE(6,101)UNITS
      WRITE(6,109)
C
C   READ ,PRINT AND STORE LOAD CARD
C
 165  CONTINUE
      READ(5,9)NQ,(R(K),K=1,2)
      WRITE(6,9)NQ,(R(K),K=1,2)
      DO 170  K=1,2
      IC=(NQ-1)*2+K
 170  R1(IC)=R(K)+R1(IC)
C
C  IF NODE NUMBER NOT MAX. NODE PT. - GO BACK AND READ MORE
C
      IF(NQ.LT.NP)GO TO 165
   9  FORMAT(I10,3F10.4)
 100  FORMAT('1',12A6//)
 101  FORMAT(//12A6//)
 109  FORMAT('0','LOADS'/5X,'NODE',8X,'X',8X,'Y')
      RETURN
      END

      SUBROUTINE FORMK
C
C  FORMS STIFFNESS MATRIX IN UPPER TRIANGULAR FORM
C
      COMMON/DATA/TITLE(12),NP,NE,NB,NMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6
     1),NBC(25),NFIX(25),R1(200),SK(200,55)
     2,ESTIFM(12,12)
C
C  SET BANDMAX AND NO. OF EQUATIONS
C
C    NBAND=(D+1)F    WHERE   NBAND= HALF-BANDWIDTH
C                           F= NUMBER OF DEGREE OF FREEDOM AT
C                              EACH NODE.
C                           D= MAX. LARGEST DIFFERENCE OF NODAL NUMBERS
C                              OCCURING FOR ALL ELEMENTS.
C
      NBAND=55
C
C  ZERO STIFFNESS MATRIX
C
C
```

```
      DO 300  N=1,NSZF
      DO 300  M=1,NBAND
300   SK(N,M)=0.
C
C  SCAN ELEMENTS
C
      DO 400 N=1,NE
      IF(N.LE.NEBR)CALL STIFT5(N)
      IF(N.GT.NEBR.AND.N.LE.NER)CALL STIFT2(N)
      IF(N.GT.NER.AND.N.LE.NET)CALL STIFT3(N)
      IF(N.GT.NET)CALL STIFT4(N)
      NCN=3
      IF(N.LE.NER)NCN=4
      IF(N.GT.NET)NCN=2
C
C RETURNS ESTIFM AS STIFFNESS MATRIX, STORE ESTIFM IN SK
C         FIRST ROWS
C
      DO 350 JJ=1,NCN
      NROWB=(NOP(N,JJ)-1)*2
      DO 350  J=1,2
      NROWB=NROWB+1
      I=(JJ-1)*2+J
C
C         THEN COLUMNS
C
      DO 330  KK=1,NCN
      NCOLB=(NOP(N,KK)-1)*2
      DO 320 K=1,2
      L=(KK-1)*2+K
      NCOL=NCOLB+K+1-NROWB
C
C  SKIP STORING IF BELOW BAND
C
      IF(NCOL)320,320,310
310   SK(NROWB,NCOL)=SK(NROWB,NCOL)+ESTIFM(I,L)
320   CONTINUE
330   CONTINUE
350   CONTINUE
400   CONTINUE
C
C   INSERT BOUNDARY CONDITIONS
C
      DO 500 N=1,NB
      NX=10
      I=NBC(N)
      NROWB=(I-1)*2
C
C  EXAMINE EACH DEGREE OF FREEDOM
C
```

```
      DO 490 M=1,2
      NROWB=NROWB+1
      ICON=NFIX(N)/NX
      IF(ICON)450,450,420
 420  SK(NROWB,1)=1
      DO 430 J=2,NBAND
      SK(NROWB,J)=0.
      NR=NROWB+1-J
      IF(NR)430,430,425
 425  SK(NR,J)=0.
 430  CONTINUE
      NFIX(N)=NFIX(N)-NX*ICON
 450  NX=NX/10
 490  CONTINUE
 500  CONTINUE
      RETURN
      END

      SUBROUTINE STIFT2(N)
C
C    TENSION-COMPRESSION RECTANGULAR ELEMENTS
C      ELEMENT TAKEN FROM COMPUTERIZED ANALYSIS OF NON-ISOTROPIC
C       STRUCTURES BY P. T. MIKHELSON IN PROCEEDING OF THE
C       SYMPOSIUM ON APPLICATION OF FINITE ELEMENT METHODS
C       IN CIVIL ENGINEERING, NOV. 13-14, 1969.
C
      COMMON/DATA/TITLE(12),NP,NE,NB,MMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6
     1),NBC(25),NFIX(25),R1(200),SK(200,55)
     2,ESTIFM(12,12)
C
C  DETERMINE ELEMENT CONNECTIONS
C
C      LIST  NODES   CLOCKWISE
C
      I=NOP(N,1)
      J=NOP(N,2)
      K=NOP(N,3)
      M=NOP(N,4)
      L=IMAT(N)
C
C  SET UP LOCAL COORDINATE SYSTEM
C
      AJ=ABS(CORD(I,1)-CORD(K,1))
      BJ=ABS(CORD(I,2)-CORD(K,2))
C
C  FORM ELEMENT STIFFNESS MATRIX
C
      A=BJ*ORT(L,1)/(3.*AJ)
      B=ORT(L,3)/4.
```

```
      C=AJ*ORT(L,2)/(3.*BJ)
      D=AJ/(3.*BJ)
      E=BJ/(3.*AJ)
      COM1=ORT(L,6)/ORT(L,4)
      COM2=ORT(L,6)*ORT(L,5)
      ESTIFM(1,1)=COM1*A+COM2*D
      ESTIFM(1,2)=COM1*B+COM2/4.
      ESTIFM(1,3)=COM1*A/2.+COM2*(-1.)*D
      ESTIFM(1,4)=COM1*(-1.)*B+COM2/4.
      ESTIFM(1,5)=COM1*A/(-2.)+COM2*D/(-2.)
      ESTIFM(1,6)=COM1*(-1.)*B+COM2/(-4.)
      ESTIFM(1,7)=COM1*(-1.)*A+COM2*D/2.
      ESTIFM(2,2)=COM1*C+COM2*E
      ESTIFM(2,3)=COM1*B+COM2/(-4.)
      ESTIFM(2,4)=COM1*C*(-1.)+COM2*E/2.
      ESTIFM(2,7)=COM1*(-1.)*B+COM2/4.
      ESTIFM(2,6)=COM1*C/(-2.)+COM2*E/(-2.)
      ESTIFM(2,8)=COM1*C/2.+COM2*E*(-1.)
      ESTIFM(4,6)=COM1*C/2.+COM2*(-1.)*E
      ESTIFM(4,8)=COM1*C/(-2.)+COM2*E/(-2.)
      ESTIFM(5,8)=COM1*B*(-1.)+COM2/4.
      ESTIFM(6,8)=COM1*C*(-1.)+COM2*E/2.
      ESTIFM(7,8)=ESTIFM(3,4)=ESTIFM(2,5)=ESTIFM(1,6)
      ESTIFM(7,7)=ESTIFM(5,5)=ESTIFM(3,3)=ESTIFM(1,1)
      ESTIFM(6,7)=ESTIFM(4,5)=ESTIFM(1,8)=ESTIFM(2,3)
      ESTIFM(5,7)=ESTIFM(1,3)
      ESTIFM(5,6)=ESTIFM(4,7)=ESTIFM(3,8)=ESTIFM(1,2)
      ESTIFM(3,7)=ESTIFM(1,5)
      ESTIFM(3,6)=ESTIFM(2,7)
      ESTIFM(3,5)=ESTIFM(1,7)
      ESTIFM(8,8)=ESTIFM(6,6)=ESTIFM(4,4)=ESTIFM(2,2)
      DO 10  L=1,8
      DO 10 M=L,8
   10 ESTIFM(M,L)=ESTIFM(L,M)
      RETURN
      END

      SUBROUTINE STIFT4(N)
C
C    TWO-DIMENSIONAL SPRING ELEMENT TAKEN FROM
C     FINITE ELEMENT ANALYSIS OF METAL DECK SHEAR
C     DIAPHRAGMS BY NILSON AND AMMAR IN JOUR. OF STRUCT.
C     DIV., ASCE, APRIL 1974.
C
      COMMON/DATA/TITLE(12),NP,NE,NB,NMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6
     1),NBC(25),NFIX(25),R1(200),SK(200,55)
     3,ESTIFM(12,12)
C
C   FORM SPRING ELEMENTS STIFFNESS MATRIX
```

```
C
      DO 10 I=1,4
      DO 10 J=1,4
 10    ESTIFM(I,J)=0.0
      L=IMAT(N)
      ESTIFM(1,1)=ESTIFM(3,3)=ORT(L,1)
      ESTIFM(2,2)=ESTIFM(4,4)=ORT(L,2)
      ESTIFM(1,3)=ESTIFM(3,1)=(-1.)*ORT(L,1)
      ESTIFM(2,4)=ESTIFM(4,2)=(-1.)*ORT(L,2)
      WRITE(11,100)N,((ESTIFM(I,J),J=1,4),I=1,4)
 100  FORMAT(I4,(4E15.4))
      RETURN
      END

      SUBROUTINE STIFT3(N)
C
C         TRIANGULAR ELEMENTS  -   PLANE  STRESS
C
      COMMON/DATA/TITLE(12),NP,NE,NB,NMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6)
     1,NBC(25),NFIX(25),R1(200),SK(200,55)
     2,ESTIFM(12,12),A(3,6),B(3,9)
C
C          DETERMINE ELEMENTS CONNECTIONS
C
      I=NOP(N,1)
      J=NOP(N,2)
      K=NOP(N,3)
      L=IMAT(N)
C                   NUMBER THE NODES COUNTER-CLOCKWISE
C
C         SET UP LOCAL COORDINATE SYSTEM
C
      AJ=CORD(J,1)-CORD(I,1)
      AK=CORD(K,1)-CORD(I,1)
      BJ=CORD(J,2)-CORD(I,2)
      BK=CORD(K,2)-CORD(I,2)
      AREA=(AJ*BK-AK*BJ)/2.
      IF(AREA.LE.0.)GO TO 220
C
C         FORM STRAIN DISP. MATRIX
C
      A(1,1)=A(3,2)=BJ-BK
      A(1,2)=A(1,4)=A(1,6)=A(2,1)=0.
      A(2,3)=A(2,5)=0.
      A(1,3)=A(3,4)=BK
      A(1,5)=A(3,6)=-BJ
      A(2,2)=A(3,1)=AK-AJ
      A(2,4)=A(3,3)=-AK
      A(2,6)=A(3,5)=AJ
```

```
C
C        FORM STRESS STRAIN MATRIX
C
      COMM=1/(ORT(L,4)*AREA)
      ESTIFM(1,1)=ORT(L,1)*COMM
      ESTIFM(1,2)=ESTIFM(2,1)=ORT(L,3)*COMM
      ESTIFM(2,2)=ORT(L,2)*COMM
      ESTIFM(3,3)=ORT(L,5)/AREA
      ESTIFM(1,3)=ESTIFM(2,3)=ESTIFM(3,1)=ESTIFM(3,2)=0.
C
C
C  B IS THE STRESS BACKSUBSTITUTION MATRIX AND IS SAVED ON TAPE
C
      DO 205  I=1,3
      DO 205  J=1,6
      B(I,J)=0.
      DO 205 K=1,3
 205  B(I,J)=B(I,J)+ESTIFM(I,K)/2.*A(K,J)
      WRITE(11,100)N,((B(I,J),J=1,6),I=1,3)
 100  FORMAT(I4,(4E15.4))
C
C        ESTIFM IS STIFFNESS MATRIX
C
      DO 210 I=1,6
      DO 210 J=1,6
      ESTIFM(I,J)=0.
      DO 210 K=1,3
 210  ESTIFM(I,J)=ESTIFM(I,J)+B(K,I)/2*A(K,J)*ORT(L,6)
      RETURN
C        ERROR EXIT FOR BAD CONNECTIONS
C
 220  WRITE(6,101)N
 101  FORMAT('1ZERO OR NEGATIVE AREA ELEMENT NO.',I4/'EXECUTION',
     1'TERMINATED')
      STOP
      END

      SUBROUTINE STIFT5(N)
C
C   BENDING RECTANGULAR ELEMENT TAKEN FROM IMPROVED
C     TWO-DIMENSIONAL FINITE ELEMENT BY R. D. COOK IN
C     JOUR. OF STRUCT. DIV. , SEPT. 1974.
C
      COMMON/DATA/TITLE(12),NP,NE,NB,NMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6)
     1,NBC(25),NFIX(25),R1(200),SK(200,55),ESTIFM(12,12)
     2,T(5,8),P(3,5),H(5,5),A(8,5),B(3,5),C(3,8)
C
C   BENDING RECTANGULAR ELEMENTS STIFFNESS MATRIX
C
```

```
C    DETERMINE ELEMENTS CONNECTIONS (NODES NUMBERED COUNTER-CLOCKWISE)
C     POSITIONS OF NODES:
C
C                      L------------K
C                      !            !
C                      !            !
C                      !            !
C                      !            !
C                      I------------J
C
      I=NOP(N,1)
      J=NOP(N,2)
      K=NOP(N,3)
      L=NOP(N,4)
      M=IMAT(N)
C
C   FORM (T) MATRIX
C
      DO 101 IA=1,8
      DO 101 IB=1,5
 101  T(IB,IA)=0.0
      T(5,2)=T(1,1)=((CORD(J,2)-CORD(L,2))/2.)
      T(5,4)=T(1,3)=((CORD(K,2)-CORD(I,2))/2.)
      T(5,6)=T(1,5)=(-1.)*T(1,1)
      T(5,8)=T(1,7)=(-1.)*T(1,3)
      G=CORD(J,2)+CORD(I,2)
      D=CORD(L,2)+CORD(I,2)
      E=CORD(K,2)+CORD(J,2)
      F=CORD(L,2)+CORD(K,2)
      T(2,1)=((CORD(J,2)*G-CORD(L,2)*D)/6.)
      T(2,3)=((CORD(K,2)*E-CORD(I,2)*G)/6.)
      T(2,5)=((CORD(L,2)*F-CORD(J,2)*E)/6.)
       T(2,7)=((CORD(I,2)*D-CORD(L,2)*F)/6.)
      T(5,1)=T(3,2)=((CORD(L,1)-CORD(J,1))/2.)
      T(5,3)=T(3,4)=((CORD(I,1)-CORD(K,1))/2.)
      T(5,5)=T(3,6)=(-1.)*T(3,2)
      T(5,7)=T(3,8)=(-1.)*T(3,4)
      G=CORD(L,1)+CORD(I,1)
      D=CORD(J,1)+CORD(I,1)
      E=CORD(J,1)+CORD(K,1)
      F=CORD(K,1)+CORD(L,1)
      T(4,2)=((CORD(L,1)*G-CORD(J,1)*D)/6.)
      T(4,4)=((CORD(I,1)*D-CORD(K,1)*E)/6.)
      T(4,6)=((CORD(J,1)*E-CORD(L,1)*F)/6.)
      T(4,8)=((CORD(K,1)*F-CORD(I,1)*G)/6.)
C
C     FORM  (P)  MATRIX
C
      DO 100  IA=1,3
      DO 100  IB=1,5
```

```
 100  P(IA,IB)=0.0
      P(1,1)=P(2,3)=P(3,5)=1.
      P(2,4)=((CORD(J,1)-CORD(I,1))*X)+CORD(I,1)
      P(1,2)=((CORD(K,2)-CORD(J,2))*Y)+CORD(J,2)
C
C  FORM  (H)-INVERSE  MATRIX
C
      AB=ABS(CORD(K,1)-CORD(I,1))
      BA=ABS(CORD(K,2)-CORD(I,2))
      DO 102 IA=1,5
      DO 102 IB=1,5
 102  H(IA,IB)=0.0
      CA=AB**5
      CB=BA**5
      COMM=144./((ORT(M,1)*ORT(M,2)-(ORT(M,3)**2))*(BA*
     1*6)*(AB**6)*ORT(M,6))
      H(1,1)=COMM*(((ORT(M,1)**2)*ORT(M,2)*(CB)*CA/36.)-
     1(ORT(M,1)*(ORT(M,3)**2)*CA*CB/48.))
      H(2,1)=H(1,2)=COMM*(((ORT(M,1)*(ORT(M,3)**2)*CA*(BA**4))-
     2(ORT(M,1)**2)*ORT(M,2)*CA*(BA**4))/24.)
      H(3,1)=H(1,3)=COMM*(ORT(M,1)*ORT(M,2)*ORT(M,3)*CA*CB/144.)
      H(2,2)=COMM*(((ORT(M,1)**2)*ORT(M,2)*CA*(BA**3))-ORT(M,1)*
     1(ORT(M,3)**2)*CA*(BA**3))/12.
      H(3,3)=COMM*((ORT(M,1)*(ORT(M,2)**2)*CA*CB/36.)-
     1(ORT(M,2)*(ORT(M,3)**2)*(CA)*CB)/48.)
      H(4,3)=H(3,4)=((ORT(M,2)*(ORT(M,3)**2)*(AB**4)*CB-ORT(M,1)*
     1(ORT(M,2)**2)*(AB**4)*CB)/24.)*COMM
      H(4,4)=COMM*(ORT(M,1)*(ORT(M,2)**2)*CB*(AB**3)-
     1ORT(M,2)*(ORT(M,3)**2)*(AB**3)*CB)/12.
      H(5,5)=COMM*(ORT(M,5)/144.)*(ORT(M,1)*ORT(M,2)*CA*CB-
     1(ORT(M,3)**2)*CA*CB)
C
C   FORM  ELEMENT STIFFNESS MATRIX
C
      DO 103 NA=1,8
      DO 103 NC=1,5
      A(NA,NC)=0.0
      DO 103 JA=1,5
 103  A(NA,NC)=T(JA,NA)*H(JA,NC)+A(NA,NC)
      DO 104  NA=1,8
      DO 104  NC=1,8
      ESTIFM(NA,NC)=0.0
      DO 104 JA=1,5
 104  ESTIFM(NA,NC)=A(NA,JA)*T(JA,NC)+ESTIFM(NA,NC)
C
C  FORM STRESS-DISPLACEMENT MATRIX AND SAVE ON FILE 11
C
      DO 106  NA=1,3
      DO 106  NC=1,5
      B(NA,NC)=0.0
```

```
      DO 105  JA=1,5
105   B(NA,NC)=P(NA,JA)*H(JA,NC)+B(NA,NC)
106   B(NA,NC)=B(NA,NC)/ORT(M,6)
      DO 107  NA=1,3
      DO 107  NC=1,8
      C(NA,NC)=0.0
      DO 107  JA=1,5
107   C(NA,NC)=B(NA,JA)*T(JA,NC)+C(NA,NC)
      WRITE(11,900)N,((C(I,J),J=1,8),I=1,3)
900   FORMAT(I4,(4E15.4))
      RETURN
      END

      SUBROUTINE SOLVE
C
C     DIRECT SOLUTION BY THE GAUSS ELIMINATION PROCEDURE
C     AND BAND MATRIX TECHNIQUE.
C
      COMMON/DATA/TITLE(12),NP,NE,NB,NMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6)
     1,NBC(25),NFIX(25),R1(200),SK(200,55)
      NBAND=55
C
C   REDUCE MATRIX
C
      DO 300 N=1,NSZF
      I=N
      DO 290 L=2,NBAND
      I=I+1
      IF(SK(N,L))240,290,240
240   C=SK(N,L)/SK(N,1)
      J=0
      DO 270 K=L,NBAND
      J=J+1
      IF(SK(N,K))260,270,260
260   SK(I,J)=SK(I,J)-C*SK(N,K)
270   CONTINUE
      SK(N,L)=C
C
C   AND LOAD VECTOR FOR EACH EQUATION
C
      R1(I)=R1(I)-C*R1(N)
290   CONTINUE
300   R1(N)=R1(N)/SK(N,1)
C
C  BACK-SUBSTITUTION
C
      N=NSZF
350   N=N-1
      IF(N)500,500,360
```

```
 360  L=N
      DO 400  K=2,NBAND
      L=L+1
      IF(SK(N,K))370,400,370
 370  R1(N)=R1(N)-SK(N,K)*R1(L)
 400  CONTINUE
      GO TO 350
 500  RETURN
      END

      SUBROUTINE STRESS
      DIMENSION DIS(2,100),FORCE(200,3)
      COMMON/DATA/TITLE(12),NP,NE,NB,NMAT,NSZF,L1,NER,X,Y,NET,NEBR
      COMMON CORD(100,2),UNITS(12),NOP(200,4),IMAT(200),ORT(25,6
     1),NBC(25),NFIX(25),R1(200),SK(200,55)
     2,B(4,8),R(8)
      EQUIVALENCE(DIS(1),R1(1)),(SK(1),FORCE(1))
C
C     PRINT DISPLACEMENTS
C
      WRITE(6,100)
      WRITE(6,110)(M,(DIS(J,M),J=1,2),M=1,NP)
 100  FORMAT(///,15X,'DISPLACEMENTS'//5X,'NODE',10X,'X',15X,'Y')
 110  FORMAT(I10,2F15.4)
C
C   CALCULATE RECTANGULAR ELEMENTS STRESSES
C
      REWIND 11
      DO 200  NC=1,NER
      IF(NC.LE.NEBR)GO TO 238
      N=NC
C
C   STRESS-DISPLACEMENT MATRIX
C
      L=IMAT(N)
      A=ORT(L,1)/ORT(L,4)
      BA=ORT(L,3)/ORT(L,4)
      C=ORT(L,2)/ORT(L,4)
      D=ORT(L,5)
      E=ORT(L,3)/ORT(L,4)
      I=NOP(N,1)
      K=NOP(N,3)
      AJ=ABS(CORD(I,1)-CORD(K,1))
      BJ=ABS(CORD(I,2)-CORD(K,2))
      B(1,1)=(-(1.-Y)/AJ)*A
      B(2,1)=B(1,1)*E/A
      B(3,1)=(-1.*(1.-X)/BJ)*D
      B(1,2)=B(3,1)*BA/D
      B(2,2)=B(3,1)*C/D
      B(3,2)=B(1,1)*D/A
```

```
      B(1,3)=(-1.*(Y/AJ))*A
      B(2,3)=B(1,3)*E/A
      B(3,3)=(-1.*B(3,1))
      B(1,4)=(-1.*B(1,2))
      B(2,4)=(-1.*B(2,2))
      B(3,4)=(B(2,3)*D/E)
      B(1,5)=(-1.*B(1,3))
      B(2,5)=(-1.*B(2,3))
      B(3,5)=(X/BJ)*D
      B(1,6)=B(3,5)*BA/D
      B(2,6)=B(3,5)*C/D
      B(3,6)=-1.*B(3,4)
      B(1,7)=(-1.)*B(1,1)
      B(2,7)=(-1.)*B(2,1)
      B(3,7)=(-1.)*B(3,5)
      B(1,8)=(-1.)*B(1,6)
      B(2,8)=(-1.)*B(2,6)
      B(3,8)=(-1.)*B(3,2)
      GO TO 239
 238  CONTINUE
      READ(11,401)N,((B(I,J),J=1,8),I=1,3)
 239  CONTINUE
      DO 260 I=1,4
      M=NOP(N,I)
      IF(M.EQ.0)GO TO 260
      K=(I-1)*2
      DO 240 J=1,2
      IJ=J+K
 240  R(IJ)=DIS(J,M)
 260  CONTINUE
      IA=K+2
      DO 300 I=1,3
      FORCE(N,I)=0.0
      DO 300 J=1,IA
 300  FORCE(N,I)=FORCE(N,I)+B(I,J)*R(J)
 200  CONTINUE
C
C   CALCULATE  TRIANGULAR  ELEMENTS  STRESSES
C
      NT=NER+1
      DO 400  NC=NT,NET
      READ(11,401)N,((B(I,J),J=1,6),I=1,3)
 401  FORMAT(I4,(4E15.4))
      DO 460 I=1,3
      M=NOP(N,I)
      IF(M.EQ.0)GO TO 460
      K=(I-1)*2
      DO 440  J=1,2
      IJ=J+K
 440  R(IJ)=DIS(J,M)
```

```
 460   CONTINUE
       IA=K+2
       DO 500 I=1,3
       FORCE(N,I)=0.
       DO 500 J=1,IA
 500   FORCE(N,I)=FORCE(N,I)+B(I,J)*R(J)
 400   CONTINUE
       WRITE(6,105)X,Y
       WRITE(6,101)
C
C  CALCULATE PRINCIPAL STRESSES AND DIRECTIONS
C
       DO 600 N=1,NET
       C=(FORCE(N,1)+FORCE(N,2))/2.
       A=SQRT(((FORCE(N,2)-FORCE(N,1))/2.)**2+FORCE(N,3)**2)
       SMAX=C+A
       SMIN=C-A
       IF(FORCE(N,2).EQ.SMIN)GO TO 700
       ANG=57.29578*ATAN(FORCE(N,3)/(FORCE(N,2)-SMIN))
       GO TO 210
 700   ANG=90.
 210   CONTINUE
C
C  WRITE ALL STRESS COMPONENTS
C
       WRITE(6,111)N,(FORCE(N,I),I=1,3),SMAX,SMIN,ANG
 600   CONTINUE
 105   FORMAT(//' STRESS VALUES FOR RECTANGULAR ELEMENTS ARE',
      1' AT POINT (',F5.2,',',F5.2')')
 101   FORMAT(//6X,'ELEMENT',6X,'X-STRESS',9X,'Y-STRESS',8X,
      1'X-Y-STRESS',7X,'MAX-STRESS',7X,'MIN-STRESS',7X,'ANGLE')
 111   FORMAT(I10,5F17.4,F12.3)
C
C     CALCULATE  FORCES  ON  SPRINGS
C
       NNE=NET+1
       IF(NE.GT.NET)WRITE(6,103)
       DO 201  NC=NNE,NE
       READ(11,401)N,((B(I,J),J=1,4),I=1,4)
       DO 261  I=1,2
       M=NOP(N,I)
       IF(M.EQ.0)GO TO 261
       K=(I-1)*2
       DO 241  J=1,2
       IJ=J+K
 241   R(IJ)=DIS(J,M)
 261   CONTINUE
       IA=K+2
       DO 301 I=1,2
       FORCE(N,I)=0.0
```

```
      DO 3Ø1 J=1,IA
 3Ø1  FORCE(N,I)=FORCE(N,I)+B(I,J)*R(J)
 1Ø3  FORMAT(//1ØX,'ELEMENT        X-FORCE        Y-FORCE        ')
       WRITE(6,1Ø4)N,(FORCE(N,I),I=1,2)
 2Ø1  CONTINUE
 1Ø4  FORMAT(1ØX,I4,5X,(2F1Ø.3))
      RETURN
      END

      SUBROUTINE  FIXITY
C
C    CALCULATION OF THE PARTIAL END FIXITY IN THE
C     STUD WALL PANEL.
C
      REAL NA
      COMMON/DATA/TITLE(12),NP,NE,NB,NMAT,NSZF,X,Y,NET,NEBR
      COMMON CORD(1ØØ,2),UNITS(12),NOP(2ØØ,4),IMAT(2ØØ),ORT(25,6)
     1,NBC(25),NFIX(25),R1(2ØØ),SK(2ØØ,55)
      DIMENSION FORCE(2ØØ,3),DIS(2,1ØØ)
      EQUIVALENCE (SK(1),FORCE(1)),(DIS(1),R1(1))
C
C    CALCULATE ROTATION
C
      NU=FFIN(5)
      NA=1.
      NL=FFIN(5)
      ROT=(DIS(1,NU)-DIS(1,NL))/(CORD(NU,2)-CORD(NL,2))
C
C     CALCULATE MOMENT
C
      EMOM=Ø.Ø
      M=FFIN(5)
      DO 1ØØ NC=1,M
      N=FFIN(5)
      K=IMAT(N)
      I=NOP(N,1)
      J=NOP(N,3)
      ND=M/2
      IF(CORD(J,2).LE.NA)FORCE(N,1)=(-1.)*FORCE(N,1)
      AM=ABS(((CORD(I,2)-NA)+(CORD(J,2)-NA))/2.)
      HT=ABS(CORD(I,2)-CORD(J,2))
 1ØØ  EMOM=FORCE(N,1)*HT*ORT(K,6)*AM+EMOM
      M=FFIN(5)
      DO 1Ø1  NC=1,M
      N=FFIN(5)
```

```
      I=NOP(N,1)
      IF(CORD(I,2).LE.NA)FORCE(N,1)=FORCE(N,1)*(-1.)
      AM=ABS(CORD(I,2)-NA)
  101 EMOM=EMOM+FORCE(N,1)*AM
C
C  CALCULATE END FIXITY
C
      EFIX=ABS(EMOM/ROT)
      WRITE(6,901)
  901 FORMAT(///6X,'DETERMINATION OF PARTIAL END FIXITY')
      WRITE(6,900)ROT,EMOM,EFIX
      CALL DEFLEC(EFIX)
  900 FORMAT(///1X,'      ROTATION AT END = ',F15.7,'  RADIANS'
     1//1X,'      MOMENT AT END = ',F15.5,'  KIPS-INCH'/
     2//1X,'      END FIXITY = ',F15.3,'  KIPS-INCH PER RADIAN')
      RETURN
      END

      SUBROUTINE DEFLEC(EFIX)
C
C  MID-SPAN DEFLECTION OF A BEAM-COLUMN WITH PARTIAL END
C  FIXITY IS CALCULATED. EQUATIONS FROM THEORY OF ELASTIC
C  STABILITY BY TIMOSHENKO.
C
      REAL M,K,L
      N=FFIN(5)
      DO 100 I=1,N
      WRITE(6,900)I
  900 FORMAT(//6X,'ANALYSIS OF BEAM-COLUMN #',I2,' WITH END',
     1' FIXITY')
      READ(5,903)P,E,Q,L,U
  903 FORMAT(6F6.3)
      K=U*2./L
      EI=P*L**2/(4.*U**2)
      U2=2*U
      TANU2=SIN(U2)/COS(U2)
      TANU=SIN(U)/COS(U)
      OU=(3./U)*(1./SIN(U2)-1./U2)
      YU=(3./U2)*(1./U2-1./TANU2)
      AU=2.*(1.-COS(U))/(U**2*COS(U))
      ANG=P*E*L*TANU/(2.*EI*U)+Q*L**2*AU/(16.*EI)
      WRITE(6,902)EFIX,P,E,Q,L,U,EI,OU,YU,AU,ANG
  902 FORMAT(//6X,'EFIX=',F11.4/6X,'P=',F7.3,2X,'ECC=',F7.3
     1,2X,'Q=  ',F6.3/6X,'L=',F6.1,'   U=  ',F5.3,'  EI=',F8.0
     2/6X,'OU=  ',F6.4,'   YU=  ',F6.4,'   AU=  ',F6.4/
```

```
     36X,'ANG=',F8.4)
      TAN=SIN(U)/COS(U)
      A=1./EFIX
      B=L/(3.*EI)
      C=B/2.
      M=ANG/(A+(B*YU)+(C*OU))
      Y=((M/P)-E)*((-1.*COS(U))-(TAN*SIN(U))+1.)
     1+(Q/(2.*P*K))*(TAN-U)
      WRITE(6,901)M,Y
901   FORMAT(//'        FIXITY MOMENT = ',F7.4,' KIPS-INCH',
     1//'        DEFLECTION OF WALL PANEL =',F7.4,' INCHES')
100   CONTINUE
      RETURN
      END
```

APPENDIX B: INPUT FILE

```
3/8 PLYWOOD-NAILED WALL PANEL
  98    80    49    49    57    13    19     0   .5   .50
UNITS FOR INPUT AND OUTPUT ARE KIPS AND INCHES.
   1  1209.    161.      47.       .989     81.     16.
   2  1990.    150.      54.       .990     141.    1.5
   3  112.     27.       17.       .938     45.     16.
   4  150.     1990.     54.       .990     141.    3.0
   5  150.     150.     61.        .834      14.    16.
   6  161.     742.      44.       .985      54.     16.
   7
   8   2.6    99999.
   9 7.7       99999.
  10   5.2     99999.
  11 16.2      99999.
  12   1.5      8.4
  13 .00001    .00001
  14 99999.    99999.
  15   3.6      16.8
  16 99999.    .00001
  17 .00001    99999.
  18 99999.      8.4
  19
          1    26.       1.625
          2    26.       2.
          3    26.       3.
          4    26.       4.5
          5    26.       5.5
          6    26.       5.875
          7    22.       1.625
          8    22.       2.
          9    22.       2.
         10    22.       3.
         11    22.       4.5
         12    22.       5.5
         13    22.       5.5
         14    22.       5.875
         15    18.       1.625
         16    18.       2.
         17    18.       2.
         18    18.       3.
         19    18.       4.5
         20    18.       5.5
         21    18.       5.5
         22    18.       5.875
         23    14.       1.625
         24    14.       2.
         25    14.       2.
         26    14.       3.
         27    14.       4.5
         28    14.       5.5
```

29	14.	5.5
30	14.	5.875
31	12.5	1.625
32	12.5	2.
33	12.5	2.
34	12.5	3.
35	12.5	4.5
36	12.5	5.5
37	12.5	5.5
38	12.5	5.875
39	10.75	2.
40	10.75	3.
41	10.75	4.5
42	10.75	5.5
43	10.75	2.
44	10.75	3.
45	10.75	4.5
46	10.75	5.5
47	10.	1.625
48	10.	2.
49	10.	2.
50	10.	3.
51	10.	4.5
52	10.	5.5
53	10.	5.5
54	10.	5.875
55	9.25	1.625
56	9.25	2.
57	9.25	2.
58	9.25	3.
59	9.25	4.5
60	9.25	5.5
61	9.25	5.5
62	9.25	5.875
63	9.25	0.
64	9.25	2.
65	9.25	3.
66	9.25	4.5
67	9.25	5.5
68	9.25	5.5
69	9.25	5.875
70	8.75	0.
71	8.75	2.
72	8.75	4.
73	8.75	5.5
74	7.	0.
75	7.	2.
76	7.	4.
77	7.	5.5
78	7.	5.5

	79	7.		5.875	
	80	5.		0.	
	81	5.		2.	
	82	5.		4.	
	83	5.		5.5	
	84	5.		5.5	
	85	5.		5.875	
	86	1.5		0.	
	87	1.5		2.	
	88	1.5		4.	
	89	1.5		5.5	
	90	.75		5.5	
	91	.75		5.5	
	92	.75		5.875	
	93	0.		0.	
	94	0.		2.	
	95	0.		4.	
	96	0.		5.5	
	97	0.		5.5	
	98	0.		5.875	
1	7	1	2	8	3
2	9	2	3	10	2
3	10	3	4	11	2
4	11	4	5	12	2
5	13	5	6	14	1
6	15	7	8	16	3
7	17	9	10	18	2
8	18	10	11	19	2
9	19	11	12	20	2
10	21	13	14	22	1
11	23	15	16	24	3
12	25	17	18	26	2
13	26	18	19	27	2
14	27	19	20	28	2
15	29	21	22	30	1
16	31	23	24	32	3
17	33	25	26	34	2
18	34	26	27	35	2
19	35	27	28	36	2
20	37	29	30	38	1
21	47	31	32	48	3
22	39	33	34	40	2
23	40	34	35	41	2
24	41	35	36	42	2
25	53	37	38	54	1
26	49	43	44	50	5
27	50	44	45	51	5
28	51	45	46	52	5
29	55	47	48	56	3
30	57	49	50	58	5
31	58	50	51	59	5

32	59	51	52	60	5
33	61	53	54	62	1
34	70	63	64	71	6
35	74	70	71	75	4
36	75	71	72	76	4
37	76	72	73	77	5
38	78	68	69	79	1
39	80	74	75	81	4
40	81	75	76	82	4
41	82	76	77	83	5
42	84	78	79	85	1
43	86	80	81	87	4
44	87	81	82	88	4
45	88	82	83	89	5
46	91	84	85	92	1
47	93	86	87	94	5
48	94	87	88	95	5
49	97	91	92	98	1
50	95	90	96		5
51	95	88	90		5
52	88	89	90		5
53	66	67	73		6
54	72	66	73		6
55	65	66	72		6
56	71	65	72		6
57	71	64	65		6
58	91	90			11
59	53	52			11
60	13	12			10
61	17	16			8
62	49	48			9
63	42	46			16
64	41	45			12
65	40	44			12
66	39	43			13
67	57	64			16
68	58	65			16
69	59	66			15
70	60	67			13
71	61	68			14
72	62	69			14
73	84	83			13
74	78	77			13
75	68	67			17
76	61	60			13
77	33	32			17
78	25	24			13
79	21	20			13
80	9	8			17

```
      1   1Ø
      2   1Ø
      3   1Ø
      4   1Ø
      5   1Ø
      6   1Ø
     63   Ø1
     7Ø   Ø1
     74   Ø1
     8Ø   Ø1
     86   Ø1
     94   Ø1
     95   Ø1
           6            -.15
          98
 Ø   6Ø     57   Ø   6   67   68   69   7Ø   71   72
2
 .442  Ø.       .3    95.5  .271
1.ØØ2  Ø.       .3    95.5   .388
```

APPENDIX C: COMPUTER OUTPUT

3/8 PLYWOOD-NAILED WALL PANEL

UNITS FOR INPUT AND OUTPUT ARE KIPS AND INCHES.

NUMBER OF NODAL POINTS = 98
TOTAL NUMBER OF ELEMENTS = 80
ELEMENTS 1 TO 49 ARE BENDING RECTANGLES.
ELEMENTS 50 TO 49 ARE T-C RECTANGLES.
ELEMENTS 50 TO 57 ARE TRIANGLES.
ELEMENTS 58 TO 80 ARE SPRINGS.
NUMBER OF RESTRAINED BOUNDARY NODES = 13
NUMBER OF ELEMENT MATERIAL TYPES = 19

MATERIAL PROPERTIES

TYPES	EX	EY	EY*PRXY	1-PRXY*PRYX	G	THICKNESS
1	1209.000	161.000	47.000	.989	81.000	16.000
2	1990.000	150.000	54.000	.990	141.000	1.500
3	112.000	27.000	17.000	.938	45.000	16.000
4	150.000	1990.000	54.000	.990	141.000	3.000
5	150.000	150.000	61.000	.834	14.000	16.000
6	161.000	742.000	44.000	.985	54.000	16.000
7	0	0	0	0	0	0
8	2.600	99999.000	0	0	0	0
9	7.700	99999.000	0	0	0	0
10	5.200	99999.000	0	0	0	0
11	16.200	99999.000	0	0	0	0
12	1.500	8.400	0	0	0	0
13	.000	.000	0	0	0	0
14	99999.000	99999.000	0	0	0	0
15	3.600	16.800	0	0	0	0
16	99999.000	.000	0	0	0	0
17	.000	99999.000	0	0	0	0
18	99999.000	8.400	0	0	0	0
19	0	0	0	0	0	0

NODAL POINTS COORDINATES

NODE NUMBER	X	Y
1	26.000	1.625
2	26.000	2.000
3	26.000	3.000
4	26.000	4.500
5	26.000	5.500
6	26.000	5.875
7	22.000	1.625
8	22.000	2.000
9	22.000	2.000
10	22.000	3.000
11	22.000	4.500
12	22.000	5.500
13	22.000	5.500
14	22.000	5.875
15	18.000	1.625
16	18.000	2.000
17	18.000	2.000
18	18.000	3.000
19	18.000	4.500
20	18.000	5.500
21	18.000	5.500
22	18.000	5.875
23	14.000	1.625
24	14.000	2.000
25	14.000	2.000
26	14.000	3.000
27	14.000	4.500
28	14.000	5.500
29	14.000	5.500
30	14.000	5.875
31	12.500	1.625
32	12.500	2.000
33	12.500	2.000
34	12.500	3.000
35	12.500	4.500
36	12.500	5.500
37	12.500	5.500
38	12.500	5.875
39	10.750	2.000
40	10.750	3.000
41	10.750	4.500
42	10.750	5.500
43	10.750	2.000
44	10.750	3.000
45	10.750	4.500
46	10.750	5.500
47	10.000	1.625

48	10.000	2.000
49	10.000	2.000
50	10.000	3.000
51	10.000	4.500
52	10.000	5.500
53	10.000	5.500
54	10.000	5.875
55	9.250	1.625
56	9.250	2.000
57	9.250	2.000
58	9.250	3.000
59	9.250	4.500
60	9.250	5.500
61	9.250	5.500
62	9.250	5.875
63	9.250	0
64	9.250	2.000
65	9.250	3.000
66	9.250	4.500
67	9.250	5.500
68	9.250	5.500
69	9.250	5.875
70	8.750	0
71	8.750	2.000
72	8.750	4.000
73	8.750	5.500
74	7.000	0
75	7.000	2.000
76	7.000	4.000
77	7.000	5.500
78	7.000	5.500
79	7.000	5.875
80	5.000	0
81	5.000	2.000
82	5.000	4.000
83	5.000	5.500
84	5.000	5.500
85	5.000	5.875
86	1.500	0
87	1.500	2.000
88	1.500	4.000
89	1.500	5.500
90	.750	5.500
91	.750	5.500
92	.750	5.875
93	0	0
94	0	2.000
95	0	4.000
96	0	5.500
97	0	5.500
98	0	5.875

ELEMENTS

ELEMENT	I	J	K	L	MATERIAL
1	7	1	2	8	3
2	9	2	3	10	2
3	10	3	4	11	2
4	11	4	5	12	2
5	13	5	6	14	1
6	15	7	8	16	3
7	17	9	10	18	2
8	18	10	11	19	2
9	19	11	12	20	2
10	21	13	14	22	1
11	23	15	16	24	3
12	25	17	18	26	2
13	26	18	19	27	2
14	27	19	20	28	2
15	29	21	22	30	1
16	31	23	24	32	3
17	33	25	26	34	2
18	34	26	27	35	2
19	35	27	28	36	2
20	37	29	30	38	1
21	47	31	32	48	3
22	39	33	34	40	2
23	40	34	35	41	2
24	41	35	36	42	2
25	53	37	38	54	1
26	49	43	44	50	5
27	50	44	45	51	5
28	51	45	46	52	5
29	55	47	48	56	3
30	57	49	50	58	5
31	58	50	51	59	5
32	59	51	52	60	5
33	61	53	54	62	1
34	70	63	64	71	6
35	74	70	71	75	4
36	75	71	72	76	4
37	76	72	73	77	5
38	78	68	69	79	1
39	80	74	75	81	4
40	81	75	76	82	4
41	82	76	77	83	5
42	84	78	79	85	1
43	86	80	81	87	4
44	87	81	82	88	4
45	88	82	83	89	5
46	91	84	85	92	1
47	93	86	87	94	5
48	94	87	88	95	5
49	97	91	92	98	1

50	95	90	96	0	5
51	95	88	90	0	5
52	88	89	90	0	5
53	66	67	73	0	6
54	72	66	73	0	6
55	65	66	72	0	6
56	71	65	72	0	6
57	71	64	65	0	6
58	91	90	0	0	11
59	53	52	0	0	11
60	13	12	0	0	10
61	17	16	0	0	8
62	49	48	0	0	9
63	42	46	0	0	16
64	41	45	0	0	12
65	40	44	0	0	12
66	39	43	0	0	13
67	57	64	0	0	16
68	58	65	0	0	16
69	59	66	0	0	15
70	60	67	0	0	13
71	61	68	0	0	14
72	62	69	0	0	14
73	84	83	0	0	13
74	78	77	0	0	13
75	68	67	0	0	17
76	61	60	0	0	13
77	33	32	0	0	17
78	25	24	0	0	13
79	21	20	0	0	13
80	9	8	0	0	17

BOUNDARY CONDITIONS

01 = FIXED IN Y-DIR. ONLY
10 = FIXED IN X-DIR. ONLY
11 = FIXED IN BOTH DIRECTIONS

NODE	CONDITION
1	10
2	10
3	10
4	10
5	10
6	10
63	1
70	1
74	1
80	1
86	1
94	1
95	1

3/8 PLYWOOD-NAILED WALL PANEL

UNITS FOR INPUT AND OUTPUT ARE KIPS AND INCHES.

LOADS

NODE	X	Y
6	0	-0.1500
98	0	0

DISPLACEMENTS

NODE	X	Y
1	0	-0.1316
2	0	-0.1356
3	0	-0.1300
4	0	-0.1349
5	0	-0.1323
7	-0.0004	-0.1317
8	.0003	-0.1275
9	-0.0004	-0.1275
10	.0002	-0.1334
11	-0.0002	-0.1282
12	.0002	-0.1322
13	.0000	-0.1322
14	-0.0000	-0.1309
15	.0003	-0.1272
16	-0.0003	-0.1317
17	.0001	-0.1317
18	-0.0002	-0.1253
19	.0001	-0.1310
20	-0.0001	-0.1266
21	-0.0072	-0.0546
22	.0071	-0.0560
23	.0158	-0.2980
24	-0.0147	-0.2931
25	-0.0006	-0.1225
26	.0004	-0.1295
27	-0.0004	-0.1233
28	.0004	-0.1281
29	-0.0034	.0558
30	.0033	.0573
31	-0.0597	-0.1240
32	.0553	-0.1291
33	.0005	-0.1291
34	-0.0006	-0.1219

35	.0004	-0.1283
36	-0.0004	-0.1233
37	.0014	.0649
38	-0.0014	.0634
39	-0.0014	-0.1203
40	.0010	-0.1280
41	-0.0009	-0.1212
42	.0009	-0.1264
43	-0.0263	-0.1048
44	-0.0219	-0.1284
45	-0.0092	-0.1029
46	.0009	-0.1299
47	.0531	-0.0866
48	-0.0492	-0.0810
49	-0.0492	-0.0810
50	-0.0023	-0.0569
51	-0.0260	-0.0830
52	.0166	-0.0554
53	.0166	-0.0554
54	-0.0162	-0.0538
55	.0524	-0.2842
56	-0.0486	-0.2899
57	-0.0241	-0.0112
58	-0.0234	-0.0360
59	-0.0082	-0.0092
60	.0001	-0.0375
61	-0.0436	-0.0002
62	.0426	-0.0019
63	-0.0233	0
64	-0.0241	-0.0001
65	-0.0234	-0.0002
66	-0.0223	-0.0002
67	-0.0217	-0.0002
68	-0.0436	-0.0002
69	.0426	-0.0018
70	-0.0232	0
71	-0.0241	.0001
72	-0.0227	.0001
73	-0.0217	.0001
74	-0.0235	0
75	-0.0231	-0.0001
76	-0.0233	-0.0001
77	-0.0212	-0.0001
78	.0098	.1988
79	-0.0096	.2006
80	-0.0232	0
81	-0.0232	.0001
82	-0.0232	.0001
83	-0.0214	.0001
84	.0237	.0240
85	-0.0232	.0221
86	-0.0231	0

87	-0.0233	-0.0002
88	-0.0230	-0.0001
89	-0.0217	-0.0001
90	-0.0217	-0.0001
91	-0.0217	-0.0001
92	.0212	.0016
93	-0.0232	-0.0001
94	-0.0233	0
95	-0.0230	0
96	-0.0217	.0000
97	-0.0189	.0815
98	.0185	.0807

STRESS VALUES FOR RECTANGULAR ELEMENTS ARE AT POINT (.50, .50)

ELEMENT	X-STRESS	Y-STRESS	X-Y-STRESS	MAX-STRESS	MIN-STRESS	ANGLE
1	-0.0000	-0.0000	-0.0000	.0000	-0.0000	-13.998
2	.4796	-0.0063	-0.0169	.4801	-0.0069	-88.011
3	.1125	-0.0209	-0.0352	.1212	-0.0296	-76.101
4	-0.6470	-0.0426	-0.0302	-0.0411	-0.6485	-2.858
5	-0.0003	-0.0045	-0.0000	-0.0003	-0.0045	-89.755
6	-0.0000	-0.0000	.0000	.0000	-0.0000	14.064
7	.3254	.0047	-0.0186	.3264	.0036	-86.695
8	.1027	.0146	-0.0452	.1217	-0.0044	-67.140
9	-0.4788	.0353	-0.0137	.0357	-0.4791	-1.529
10	-0.0006	.0042	.0000	.0042	-0.0006	.001
11	.0001	.0000	.0000	.0001	.0000	88.604
12	.1740	-0.0049	-0.0158	.1753	-0.0063	-84.999
13	.0832	-0.0077	-0.0355	.0999	-0.0194	-71.743
14	-0.3063	-0.0292	-0.0310	-0.0258	-0.3097	-6.305
15	.0001	-0.0039	-0.0000	.0001	-0.0039	-89.999
16	.0001	-0.0000	.0000	.0001	-0.0000	88.201
17	.0669	.0097	-0.0259	.0769	-0.0003	-68.923
18	.0814	-0.0146	-0.0502	.1028	-0.0361	-66.864
19	-0.1886	.0545	.0012	.0545	-0.1886	.275
20	.0001	.0097	-0.0000	.0097	.0001	-0.001
21	.0001	.0000	-0.0000	.0001	.0000	-85.520
22	.0035	.0032	-0.0055	.0089	-0.0021	-45.768
23	.0806	.0420	-0.0373	.1033	.0193	-58.693
24	-0.1234	-0.0226	-0.0385	-0.0096	-0.1365	-18.696
25	-0.0003	-0.0055	-0.0000	-0.0003	-0.0055	-89.996
26	-0.0018	-0.0026	-0.0021	-0.0001	-0.0043	-50.161
27	.0040	-0.0129	-0.0035	.0047	-0.0136	-78.652
28	-0.0042	.0028	-0.0020	.0033	-0.0047	-14.984
29	.0000	.0000	.0000	.0000	.0000	.007
30	-0.0048	.0026	-0.0020	.0031	-0.0053	-14.203
31	.0041	.0129	-0.0036	.0142	.0028	-19.784
32	-0.0014	-0.0027	-0.0019	-0.0000	-0.0041	-54.166
33	.0005	.0173	-0.0000	.0173	.0005	-0.118
34	-0.0000	-0.0389	-0.0001	-0.0000	-0.0389	-89.797
35	-0.0032	.0449	-0.0010	.0449	-0.0032	-1.161
36	-0.0030	.0097	.0065	.0124	-0.0058	22.886
37	.0015	-0.0000	.0001	.0015	-0.0000	87.715
38	.0016	-0.0054	.0000	.0016	-0.0054	89.991
39	-0.0009	-0.0203	.0007	-0.0009	-0.0203	87.841
40	-0.0047	.0007	-0.0024	.0016	-0.0056	-20.737
41	.0014	.0000	.0001	.0014	.0000	87.175
42	.0005	.0054	.0000	.0054	.0005	.006
43	-0.0001	.0032	.0002	.0032	-0.0001	3.277
44	-0.0046	-0.0020	.0023	-0.0006	-0.0059	30.180
45	.0012	-0.0000	.0001	.0012	-0.0000	87.064
46	-0.0002	-0.0013	.0000	-0.0002	-0.0013	89.970
47	.0000	-0.0000	-0.0000	.0000	-0.0000	-65.972
48	-0.0000	.0000	.0000	.0000	-0.0000	8.544
49	-0.0000	.0007	-0.0000	.0007	-0.0000	-0.000
50	.0029	.0118	.0059	.0147	.0000	26.565
51	-0.0086	-0.0000	.0119	.0083	-0.0170	35.072
52	.0147	-0.0098	.0059	.0161	-0.0111	77.222
53	-0.0001	-0.0014	.0003	-0.0001	-0.0015	77.873
54	.0832	.0003	.0007	.0832	.0003	89.523
55	-0.0244	-0.6081	.0022	-0.0244	-0.6081	89.780
56	.0009	-0.1701	.0049	.0010	-0.1702	88.343
57	-0.0927	-0.6292	.0036	-0.0926	-0.6292	89.612

ELEMENT	X-FORCE	Y-FORCE
58	.000	.000
59	-0.001	.000
60	-0.001	-0.000
61	.001	-0.000
62	-0.001	.000
63	-0.047	.000
64	.012	-0.153
65	.034	.003
66	.000	-0.000
67	-0.048	-0.000
68	-0.003	-0.000
69	.051	-0.150
70	.000	-0.000
71	-0.000	.102
72	.000	-0.102
73	.000	.000
74	.000	.000
75	-0.000	-0.000
76	-0.000	.000
77	-0.000	-0.000
78	.000	.000
79	-0.000	.000
80	-0.000	.000

DETERMINATION OF PARTIAL END FIXITY

ROTATION AT END = .0069293 RADIANS

MOMENT AT END = .12442 KIPS-INCH

END FIXITY = 17.956 KIPS-INCH PER RADIAN

ANALYSIS OF BEAM-COLUMN # 1 WITH END FIXITY

EFIX= 17.9555
P= .442 ECC= 0 Q= .300
L= 95.5 U= .271 EI= 13722
OU= 1.0354 YU= 1.0201 AU= 1.0315
ANG= .0129

FIXITY MOMENT = .2169 KIPS-INCH

DEFLECTION OF WALL PANEL = .3901 INCHES

```
ANALYSIS OF BEAM-COLUMN # 2 WITH END FIXITY

EFIX=     17.9555
P=  1.002  ECC=        0  Q=   .300
L=  95.5  U=  .388  EI=    15176
OU= 1.0750  YU= 1.0426  AU= 1.0668
ANG=    .0120

FIXITY MOMENT =    .2037 KIPS-INCH

DEFLECTION OF WALL PANEL =  .3654 INCHES



END OF FORTRAN EXECUTION
```

APPENDIX D: NOTATION

The following symbols are used in this paper:

a - length of rectangular element, in.

b - height of rectangular element, in.

$[B]$ - element stress-displacement matrix

E, E_1 - modulus of elasticity in direction of subscript, kips per in.^{2}

EI - flexural rigidity, kips-in.^{2}

G, G_{12} - shear modulus in the 1-2 plane, kips per in.^{2}

I - moment of inertia, in.^{4}

i, j, k, ℓ - nodal points, as subscripts they denote the location of displacement

k_a - lateral connector modulus, kips per in.

$[k_e]$ - element stiffness matrix

k_{01}, k_{02} - elastic bearing constants for components 1 and 2, respectively, kips per in.^{3}

k_1, k_2 - spring constants in direction of subscript, kips per in.

L - beam length, in., as subscript it denotes longitudinal direction

M - end moment, kips-in.

P - axial load, kips or lbs

Q - midspan lateral load, kips or lbs

R - radial direction

t - element thickness, in.

T - tangential direction

u, v - displacements in the coordinate directions, in.

u - parameter defined in Equation (2.42)

x, y - global cartesian coordinates

x_d, y_d - global coordinates of differential element and the point where stresses are computed, in.

y - midspan deflection of beam-column, in.

y_{fixed} - midspan deflection of wall panel with partially fixed ends, in.

y_{free} - midspan deflection of simply supported wall panel, in.

α - coefficient of end fixity, kips-in. per radian

θ - angular rotation, radian

ν, ν_{12} - Poisson's ratio relating the strain in the direction of the second subscript due to the stress in the direction of the first subscript

1, 2 - local coordinates, as subscripts they denote direction