AN ABSTRACT OF THE THESIS OF

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Title: DIGITAL SIMULATION OF A SELF-EXCITED CUMULATIVE COMPOUND DIRECT-CURRENT GENERATOR WITH VARIABLE LOAD

Abstract approved Redacted for Privacy
John F. Engle

This thesis develops a simulation model for a cumulative connected compound direct-current generator to varify the operation of this machine under conditions of slowly and rapidly changing loads.

The application of this simulation method will contribute to more exact modeling of power system components. The accuracy gained is in the area of magnitude and time of response of the d-c generator and other systems with which it is a component for simulation.

The simulation uses a table look-up method for the representation of the series field and the armature winding. The first inductance is called subtransient and it represented the effective inductance over a specified length of time called the "time of influence." The second inductance is used after the time of influence has elapsed, and is called the transient inductance.

The shunt field representation is in terms of a subtransient inductance, transient inductance and a time of influence of the subtransient inductance. The mathematical representation of these parameters is expressed as a linear function of the total resistance in the shunt field circuit.

The simulation of the nonlinear generator was programmed in Fortran IV. The program for simulation was based upon an iterative solution to facilitate the integration procedures and the nonlinear component representations.

Test data are compared with the simulation output for eight different conditions.

The comparable results for slowly changing load conditions indicated an extremely close agreement.

The rapidly changing load tests indicated similar agreement except near the beginning of the transient condition where the deviation exceeded 5%.

The use of d-c machines as control components require the increased accuracy of this method for digital control and simulation.

Digital Simulation of a Self-Excited Cumulative Compound Direct-Current Generator with Variable Load

by

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LIST OF NOMENCLATURE

E_i	The internal voltage
E_A	The terminal voltage
E_{A1}	The terminal voltage at the start of dt
E_{A2}	The terminal voltage at the end of dt
I_f	The current in the shunt field
I_{f1}	The current in the shunt field at the start of dt
I_{f2}	The current in the shunt field at the end of dt
I_{f3}	The calculated shunt field current after each iteration
I_ℓ	The load current
$I_{\ell 2}$	The load current at the end of dt
$I_{\ell 3}$	The calculated load current after each iteration
I_s	The current in the series field
I_{s1}	The current in the series field at the start of dt
I_{s2}	The current in the series field at the end of dt
I_c	I_c defined in equation (13)
K_1	The internal voltage due to residual magnetism
K_2	The internal voltage due to one ampere in the shunt field
K_3	The internal voltage due to one ampere in the series field minus the armature reaction
L_{aa}	The self inductance of the armature winding
L_{ff}	The self inductance of the shunt field circuit
L_{ss}	The self inductance of the series field circuit.

L_ℓ	The inductance of the load
M_{fa}	The mutual inductance between the shunt and the series field
M_{fs}	The mutual inductance between the shunt field and the armature windings
M_{sa}	The mutual inductance between the series field and the armature windings
mmf	The magnetomotive force which is equal to the current times the number of turns in a circuit
R_1	The rheostat resistance in the shunt field
R	The total resistance of the shunt field
R_2	Equal to $R \| R_\ell = \frac{R*R_\ell}{R+R_\ell}$
R_a	The resistance of the armature winding
R_f	The resistance of the shunt field windings
R_s	The resistance of the series field windings
R_ℓ	The resistance of the load

DIGITAL SIMULATION OF A SELF-EXCITED CUMULATIVE COMPOUND DIRECT-CURRENT GENERATOR WITH VARIABLE LOAD

I. INTRODUCTION

The cumulative connected direct-current generator is in use in three main areas and operates in both the transient and steady state under variable load conditions. This thesis will develop a mathematical model for both the steady state and the transient operation of the machine.

The application of this simulation method will contribute to more exact modeling of power system components. The accuracy gained is in the area of magnitude and time of response of the d-c generator and other systems with which it is a component for simulation.

The main areas of use for the cumulative-compound generator are:

(i) Generator: (7), (5) The cumulative compound generator is used as low voltage energy source in a d-c system. It can be designed to have a wide variety of voltage characteristics under conditions of changing load. The rise of voltage from no-load to full-load is used to maintain nearly constant voltage at the end of a distribution circuit by compensating for the line drop.

(ii) Control systems: (17) In this area the cumulative direct-current generator has two main applications: First, as a rotating power amplifier. The use is limited to applications where phase lag is unimportant and attenuation is required as frequency is increased. Secondly, it is used in open sequence controls where each motor requires a separate generator for supply and control. The control is accomplished by regulating and reversing the field of the generator. There is a limit to the degree of sluggishness of response that can be tolerated.

The use of d-c machines as control components require the increased accuracy of this method for digital control and simulation.

(iii) Exciter for synchronous motors and generators: The cumulative d-c generator is used as an exciter because of its good steady-state response to slowly fluctuating loads. These generators are slower to respond than separately excited generators.

II. REVIEW OF LITERATURE

The simulation of d-c machines can be divided into two parts, the steady and transient states. It is appropriate to discuss each separately.

Steady State Literature

Most of the papers and books which discuss the response of cumulative direct current machines were printed before 1945, "the age of the computer"; thus most books and papers (7, 4) presented the graphical method which can be summarized by the following steps.

(i) Draw the "no load" magnetization curve and the excitation line with its slope equal to the shunt field resistance.

(ii) At any load, the total mmf is the shunt field mmf (OA) and the series field mmf (AB) minus the mmf due to the armature reaction (BC), Figure 1.

(iii) CF, which is perpendicular to AB from C is equal to the voltage drop in the series field and armature resistance due to the load current.

(iv) Point F should be on the magnetization curve and point A on the line OA, so that the triangle ACF should be moved up and to the right (PA_1K) in order to achieve the desired condition; then OP is the terminal voltage. Steps i-iv are repeated for other values of the load current will produce the curve of terminal voltage as a function of load current, Figure 2.

(v) Steps i-iv are repeated for different excitation lines to produce new load-terminal curves for each resistance value of the shunt field.

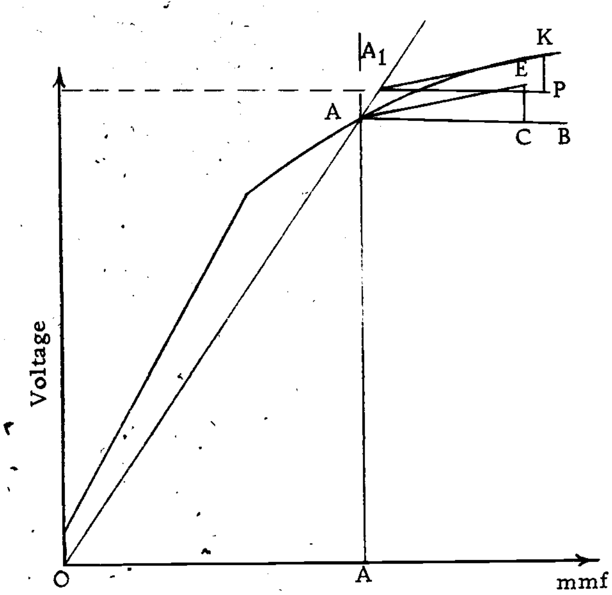


Figure 1: Characteristic Curve

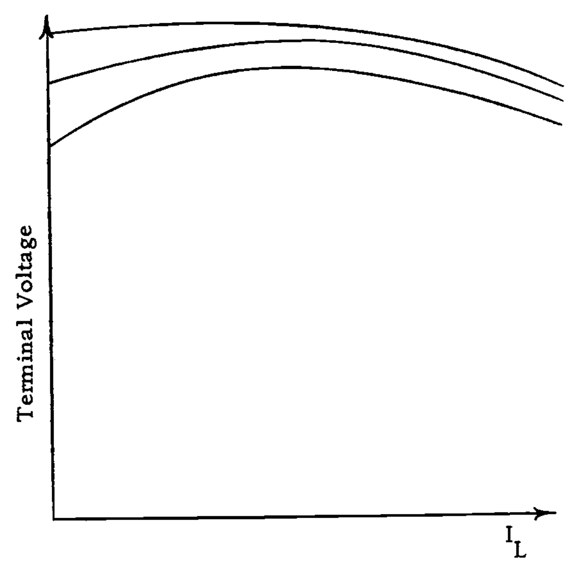


Figure 2. Terminal voltage versus load current

Transient State Literature

The studies of transient state started around 1946 in the form of short circuit characteristics, so that a high degree of reliability and service continuity would be achieved in the d-c system.

The first study was made by G. E. Frost (4), who calculated the short circuit currents by borrowing the concept of the transient and subtransient impedances "as in the a-c machines".[1/] He used the subtransient impedance of the armature alone to calculate the maximum short circuit current, while he used the transient impedance of the armature and the mutual impedances with the field to calculate the decay of the short circuit current. He approximated the magnetization curve by a straight line and disregarded the impedance of the fields. Because of these approximations his calculated curves are not very accurate.

In the same year, T. M. Linville and associates wrote a series of papers ending in 1952 (3, 10, 11). They borrowed the concept of subtransient impedance and the two-reaction theory of Park (by dividing the armature flux into direct and quadrature). They linearized the magnetization curve, used constants for all the parameters, and disregarded the impedance of the series field. They reported better results than Frost.

In 1949, A. T. McClinton and associates (1, 12, 13) used the method of Linville to examine many cases, including motors. The results are incorrect in most cases--more than 30%. Their comment on the method used is

> ...in view of the difference between experimental and computed values of the short-circuit currents using present techniques, it is necessary that the theory of

---

[1/] Impedance means a combination of resistance and inductance. See (5, 6, 7).

> the transient characteristics of d-c machine, be extended to provide the accuracy necessary for attaining the optimum in the shipboard system design (12, p. 1105).

John Cybulskii and John P. O'Conner (2) were the first to suggest the use of the analog computer to solve this problem with the Linville method for different kinds of excitation (separate, shunt and compound). They substituted constant values for the parameters and they linearized the magnetization curve. The results reported were better than those previous, but still not good enough.

H. E. Koening (8) wrote a paper discussing the transient characteristics of the direct current machine. He was the first to report the dependence of the self and the mutual inductances on the shunt field current. He used all the self and the mutual inductances of the machine in his analysis. He used a single value for each inductance and linearized the magnetizing curve so that the system would be linear and solvable without the use of the digital computer. In his second paper (9) he used matrix algebra to analyze the d-c machines. He divided the armature into three parts, the coil under commutation and the other two sections of the armature. He used constants for his matrix of coefficients, and he linearized the magnetization curve. He did not report any comparison of results in either of his articles.

D. H. Schaefer (14) used a unique approach to measure the parameters of the machine by applying an a-c supply with variable frequency. He fitted a transfer function for each curve, and he linearized the magnetization curve so that the equations could be solved analytically. The results he reported were better than any reported previously, and the difference between the computed and the actual results were within plus or minus 10%.

Frederick C. Trutt and Edward A. Erdelyi in their paper (16) compared four methods of representing the magnetization curve in the analysis of the d-c machines; first order, third order, Froelich method and division of the magnetization curve into a number of segments with each segment represented as a straight line. They found that the representation of the magnetization curve by a table lookup procedure was the best of the four methods.

H. B. Hamilton (6) had studied the influence of the iron core on a coil with inductance L, and resistance R. He excited the system by a step voltage V and derived the following equation for the current in coil.

$$i(t) = \frac{v}{r}\left[\epsilon^{-\frac{Rt}{L}} - \frac{4}{\pi^2}\sum_{n,m=1,3,\cdots}\frac{\epsilon^{-\frac{Rt}{L}} - \epsilon^{-\alpha_n xt}}{n^2} + \frac{\epsilon^{-\frac{Rt}{L}} - \epsilon^{-\alpha_m yt}}{m^2}\right]$$

This equation proved to be too complicated to be used in any simulation procedure, especially when this system is made of three electrically coupled circuits.

Most of the electrical books (15) that discuss the transient characteristics of d-c machines used linear constants for the parameters, and substituted for the magnetization curve with a Froelich curve fitting representation.

III. SYSTEM DESCRIPTION AND ANALYSIS

Physical Description

The machine under discussion is a cumulative connected direct-current generator. It is composed of the following parts:

Magnetic Circuit

The field yoke (c), Figure 3, is the principal part. In a small machine like the one under discussion, it is made of rolled steel to attain a uniform magnetic circuit; but for big machines it is made of cast steel. Attached to the yoke are the poles, whose number depend mainly on the dimensions and speed of the machine. The poles are generally made of cast steel (a). At the end of each pole is the pole shoe which supports the shunt and the series field coils and shapes the magnetic field. In big machines the pole shoe is made of laminated iron and it holds the compensating circuit that nullifies the influence of the armature reaction on the main field mmf. The interpoles (b) which are used to stop the sparking of the segment under commutation are made of the same materials as the poles, but their coils and dimensions are less, their number equal to the number of circuits in the armature. The armature core is made of laminated steel (d) which is necessary to reduce the influence of the eddy currents in the core under transient conditions. The armature core is built around the shaft and holds the armature winding.

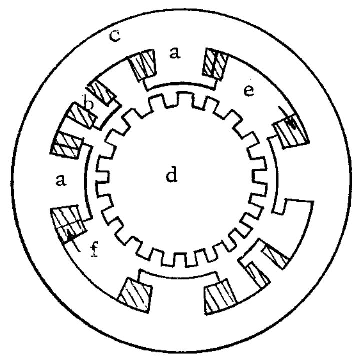


Figure 3. Generator cross section

Field Winding

The flux in the magnetic circuit is generated by the direct current flowing in the field coils minus armature reaction, the field coils conforming to the shape of the poles. They are connected in series so that the adjacent poles are of opposite polarity. In the compound generator there are two coils on each pole. The first is the shunt field coil (e) consisting of a large number of turns with relatively large resistance and requires a relatively small current; the second is the series field coil (f), consisting of few turns with large cross section, small resistance, and is connected in series with the armature.

Commutator and Brushes

The commutator is built at the end of the armature core on the shaft and is made of copper segments insulated from each other and the shaft. The number of these segments equals the number of the slots in the armature core and each end of the armature coil is terminated on a segment. The brushes are made of graphite reinforced by ingredients such as steel powder. They are rectangular in shape, and their number equals the number of poles.

Electrical Analysis

This section contains a study of the components of the generator from the electrical point of view including the interaction of the elements on each other.

Iron Core

In small generators the poles, interpoles, even the armature core in some cases are made of iron. The iron cores influence the parameters of the machine in the transient state. It is not the intention of the author to represent the iron core as an explicit item in the system simulation because it is impractical, as demonstrated in (6).

The influence of the iron core on the steady state response is in the form of the nonlinear behavior of the voltage versus the total mmf.

The iron core makes the self inductance of the shunt, series field, and the armature winding functions of time and not constants. In the case of the shunt field the inductance is also a function of the of the leakage coefficient.

To prove this concept mathematically, let the shunt field coil be discussed alone and approximate the iron core by a coil which has resistance R and inductance L coupled to the shunt field coil of resistance R_f and inductance L_f. The mutual inductance between both coils is M. i_1 is flowing in L_f at t=0 when the switch is closed, as in Figure 4.

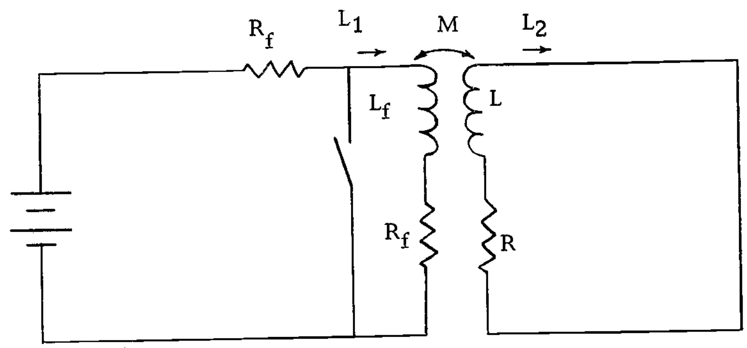


Figure 4. Electrical Representation of Two Coupled Circuits

The equations representing this circuit are:

$$R_f i_1 + L_f \frac{di_1}{dt} - M \frac{di_2}{dt} = 0 \qquad (1)$$

$$R\, i_2 + L \frac{di_2}{dt} - M \frac{di_1}{dt} = 0 \qquad (2)$$

Taking the Laplace Transform of equations (1), (2) and writing them in matrix form:

$$\begin{bmatrix} R_f + SL_f & -SM \\ -SM & R+SL \end{bmatrix} \cdot \begin{bmatrix} i_1(s) \\ i_2(s) \end{bmatrix} = \begin{bmatrix} I_1 L_f \\ 0 \end{bmatrix} \tag{3}$$

Multiplying both sides by the inverse of the coefficient matrix provides the following development.

$$\begin{bmatrix} i_1(s) \\ i_2(s) \end{bmatrix} = \frac{\begin{bmatrix} R+SL & SM \\ SM & R_f + SL_f \end{bmatrix} \cdot \begin{bmatrix} I_1 L_f \\ 0 \end{bmatrix}}{(R_f + SL_f)(R+SL) - S^2 M^2}$$

$$i_1(s) = \frac{i_1 L_f (R + SL)}{(R_f + SL_f)(R+SL) - S^2 M^2}$$

substituting $\frac{L}{R} = t$ $\quad \frac{L_f}{R_f} = t_f$ $\quad \frac{L_f L - M^2}{L L_f} = \sigma$

and dividing by $R_f \cdot R$

$$i_1(s) = i_1 t_f \cdot \frac{1 + st}{(1+st_f)(1+st) - \frac{S^2 M^2}{R_f R}}$$

$$= i_1 t_f \cdot \frac{1 + st}{1 + s(t_f + t) + S^2 \sigma t_f t} \tag{4}$$

The characteristic roots of equation (4) are:

$$S_{1,2} = \frac{-(t_f+t) \pm \sqrt{(t_f+t)^2 - 4\sigma t_f t}}{2\sigma t_f t}$$

expanding the square root into a Taylor series

$$S_{1,2} = \frac{(t_f+t)}{2\sigma t_f t}\left\{-1 \pm \left[1 - \frac{2\sigma t_f t}{(t_f+t)^2} - \frac{2(\sigma t_f t)^2}{(t_f+t)^4} - \frac{1(\sigma t_f t)^3}{(t_f+t)^6} \cdots\right]\right\}$$

$$S_2 = -\frac{t_f+t}{\sigma t_f t} + \frac{1}{t_f+t} + \frac{\sigma t_f t}{(t_f+t)^3}$$

$$S_1 = \frac{1}{t_f+t} - \frac{\sigma t_f t}{(t_f+f)^3} - \frac{\sigma(\sigma t_r t)^2}{(t_f+t)^5}$$

The solution for the above equation in time domain

$$i_1(t) = A\epsilon^{s_1 t} + B\epsilon^{s_2 t} \quad \text{where}$$

$$A = \left.\frac{i_1 t_f(1+st)}{(s+s_2)}\right|_{s = s_1 \simeq -\frac{t_f+t}{\sigma t_f t}}$$

$$A = \frac{i_1 t_f(1 - t\frac{t_f+t}{\sigma t_f t}\ \sigma t_f t}{\sqrt{(t_f+t)^2 - 4\sigma t_f t}} = I_1 \frac{t_f t\,[t_f - \sigma(t_f+t)]}{\sqrt{(t_f+t)^2 - 4\sigma t_f t}}$$

$$I_1 = A + B$$

$$B = I_1 - A = i_1 \frac{[\sqrt{(t_f+t)^2 - 4\sigma t_f t} - t_f t\,(t_t - t_f\sigma - \sigma t)]}{\sqrt{(t_f+t)^2 - 4\sigma t_f t}}$$

The extreme values of σ will give an idea about the behavior of the system. The values of A, B, s_1, s_2, t_1, and t_2 are tabulated in Table 1.

Table 1. Inductances versus leakage coefficient

σ	A	B	S_1	S_2	$\tau_1 = \frac{1}{S_1}$	$\tau_2 = \frac{1}{S_2}$	M
1.0	I_1	0	$\frac{1}{-t_f}$	0	$-t_f$	∞	0
0	$I(1-\frac{t_f t}{t_f+t})$	$I\frac{t_f t}{t_f+t}$	$-\frac{1}{t_f+t}$	∞	$-(t_f+t)$	0	$L_f x L$

From the above mathematical representation the following comments can be made:

(i) When the iron core is not saturated, σ is very small as in the case of the series field acting alone or the armature circuit acting alone. The second term in the equation for S_1 is not significant and can be cancelled so that the reciprocal of S_1 which is equal to $-(t_f+t)$ is a constant. Also the subtransient time constant equals $\frac{1}{S_2} = \frac{t_f t}{t_f+t}$

(ii) If the leakage coefficient is large then the terms in S_1 and S_2 containing σ cannot be disregarded. So $|\tau_1| = |\frac{1}{S_1}|$ is decreased as σ is increased, i.e. the apparent inductance is decreased.

Shunt Field

The copper wire of the shunt field coils is small in diameter so that the resistance is high. This requires the number of turns to be high so that the required mmf can be attained.

Because of the above restrictions, and due to the fact that the poles are made of solid iron, the following experiments were performed for the derivation of the parameters related to the shunt field, since no manufacturers' data were available:

(i) The magnetization curve (characteristic curve).

(ii) Resistance of the shunt field, R_f .

(iii) The self inductance of the shunt field, L_{ff}.

(iv) The mutual inductance between the shunt field and series field, M_{fs} .

The Series Field

It is made of coils wound on the poles. Because of the magnitude of the current the cross section of the wire is large and the number of turns are relatively small. The function of the series field is to supply enough mmf to overcome the voltage drop in the armature, series resistances, and in some cases the voltage drop in the network so that a constant voltage at the terminals of the load is attained. The derived parameters required for the series field are:

(i) Magnetization curve, the ratio of the series field turns to those of the shunt field, and the net mmf supplied by the series field per ampere.

(ii) The series field self inductance L_{ss}, comprised of the transient and subtransient values.

(iii) The resistance of the series field, R_s.

(iv) The mutual inductance between the series field and the armature winding, M_{sa}.

The Armature Winding

The armature wire cross section is relatively large. It is embedded in the slots of the armature core, which are made of laminated sheets of iron. Because of the lamination, the influence of the armature core on the inductance of the armature winding is not severe, although the tips of the slots are driven into saturation:

(i) The armature self inductance influence L_{aa} composed of the transient and subtransient values and the time of the subtransient.

(ii) The armature resistance R_a.

(iii) The mutual inductance with the shunt field M_{fa}.

The armature reaction influence is implicitly included in the series field expression when calculating the net mmf.

The influence of the interpoles is not taken explicitly because their wiring resistance is included with the armature resistance. The mmf of the interpoles is very small and it is included in the armature reaction.

Description of the Generator Used

The direct-current generator used in this study for comparison with the simulated output has the following name-plate data:

Model	48 A 1982 General Electric
Frame	73
Type	CD
Volts	250/250
Winding	compound
Speec	1800 rpm
kw.	10
Amp.	40

IV. DERIVATION AND MEASUREMENT OF THE PARAMETERS

Shunt Field Parameters and Magnetization (No-load) Curve

(i) Magnetization (no-load) curve.

(ii) Resistance of the shunt field.

(iii) Self inductance of the shunt field.

(iv) Mutual inductance with the series field.

The parameters which are associated with the shunt field and tabulated above can be measured or derived by the following experiments.

The first experiment is for the tabulation of the magnetization curve and the resistance of the shunt field.

The circuit configuration to run the above experiment is shown in Figure 5.

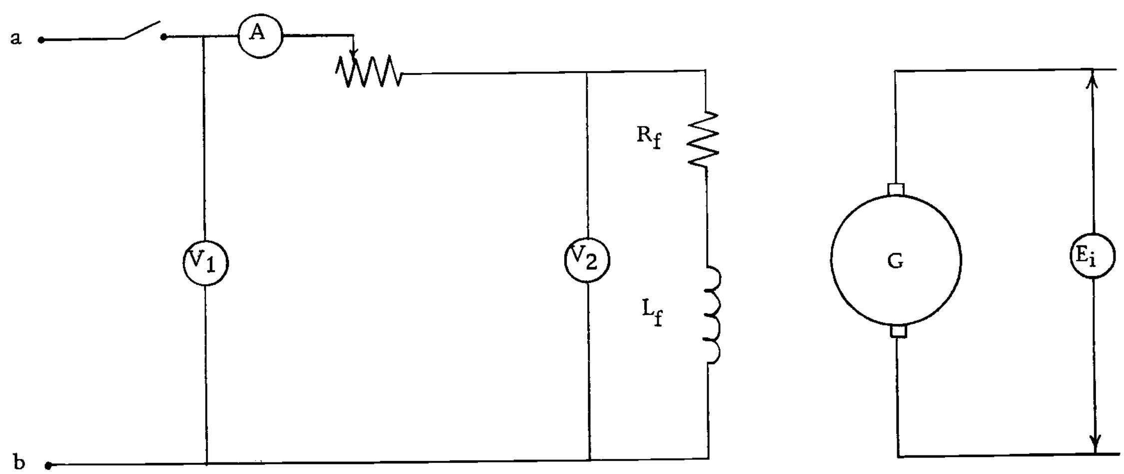


Figure 5. Circuit for evaluation of the magnetization curve (I_f)

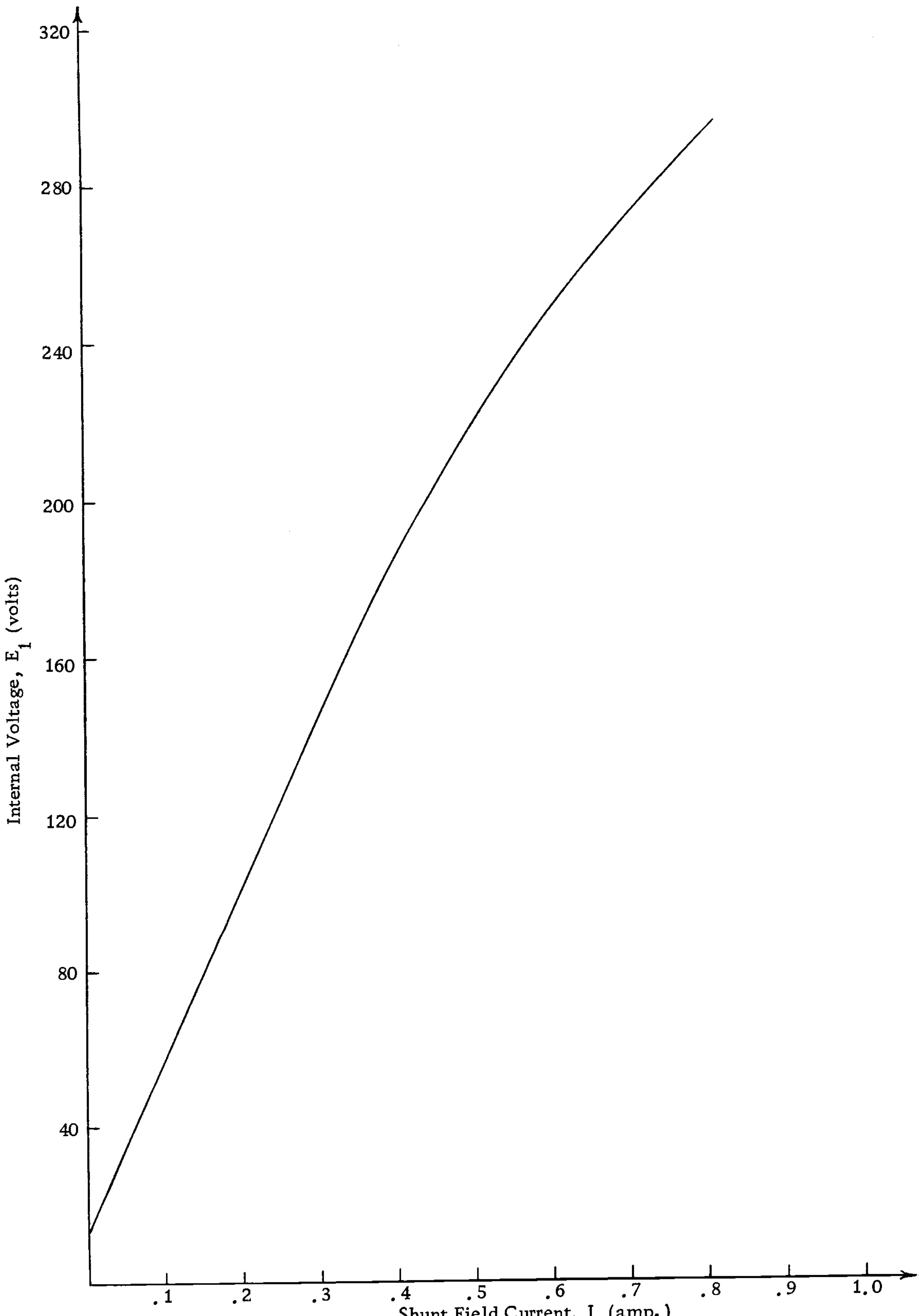


Figure 6. No load magnetization curve due to the shunt field

The prime mover runs at constant speed while a direct voltage is applied at the terminals (a, b). The resistance R_1 is changed to obtain the data which is tabulated in Appendix H.

From Appendix H the curve of the magnetization can be plotted, $E_i \cdots I_f$ as shown in Figure 6.

The resistance of the shunt field is equal to the voltage across the shunt field divided by the current (column 5, Appendix G). The average of this column is equal to the shunt field resistance: 272.1 ohm.

Derivation of Self Inductance

The self inductance of any coil can be calculated from the decay of the current flowing in the coil after it is short circuited. This method is better than applying an a-c supply with variable frequency because of the influence of the iron core. The circuit for the calculation of the self inductance is shown in Figure 7 where the oscilloscope traces the decay of the voltage across the resistance R_1. The series field and armature terminals are kept open. A paper is installed between each brush and the segment under commutation. A camera was mounted on the oscilloscope. The self inductances of the shunt field are derived from Figure 7 for two cases, the first one when the leakage is small and the second when the leakage is large, as determined by I_f.

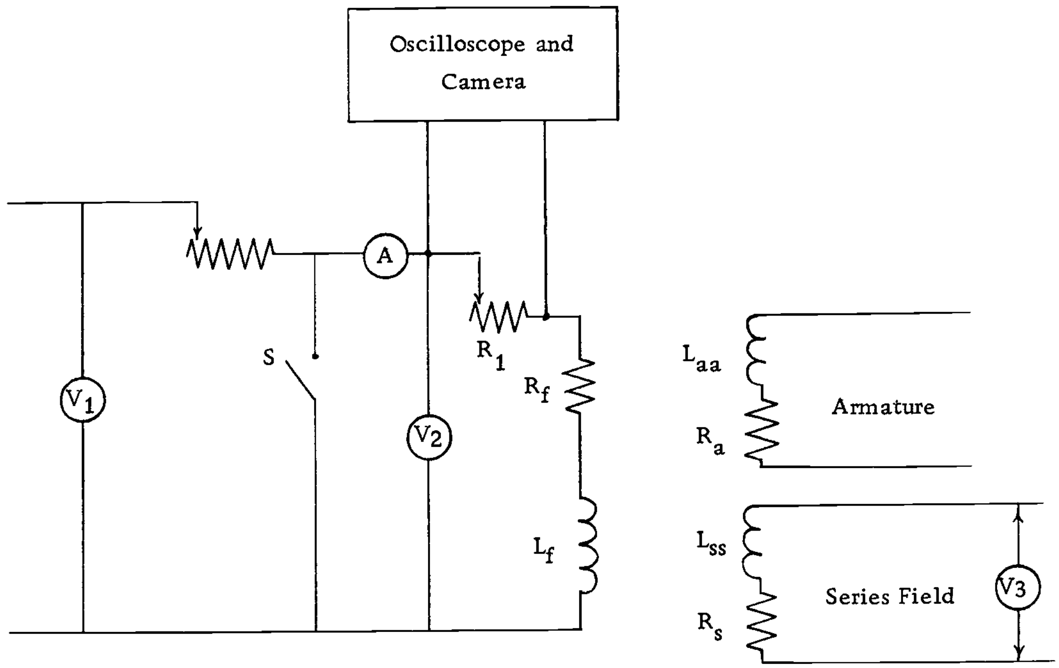


Figure 7. Circuit for measurement of the shunt field inductances

Table 2. Inductances of the shunt field.

I_f (amp.)	R_1 (ohms)	L(tr.) (henry)	L(subtr.) (henry)	Time (subtr.) (sec)
0.36	200	169	79.5	0.14
0.78	99	84	57	0.23

In Figure 8b the pictures are redrawn on a logarithmic scale so that the influence of the subtransient reactance can be seen easily.

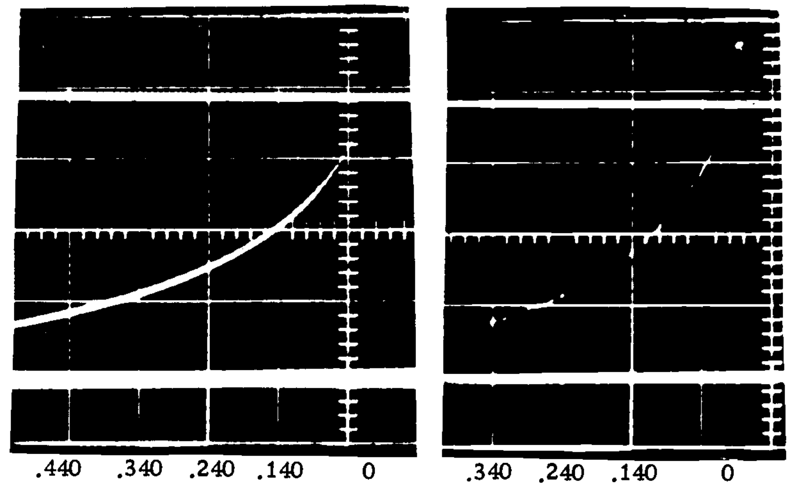


Figure 8a

(2)
$I_f = .78$
Hor. Sc. = 100 m sec/cm
Ver. Sc. = 5 volts/cm

(1)
$I_f = .36$
Hor. Sc. = 100 m sec/cm
Ver. Sc. = 5 volts/cm

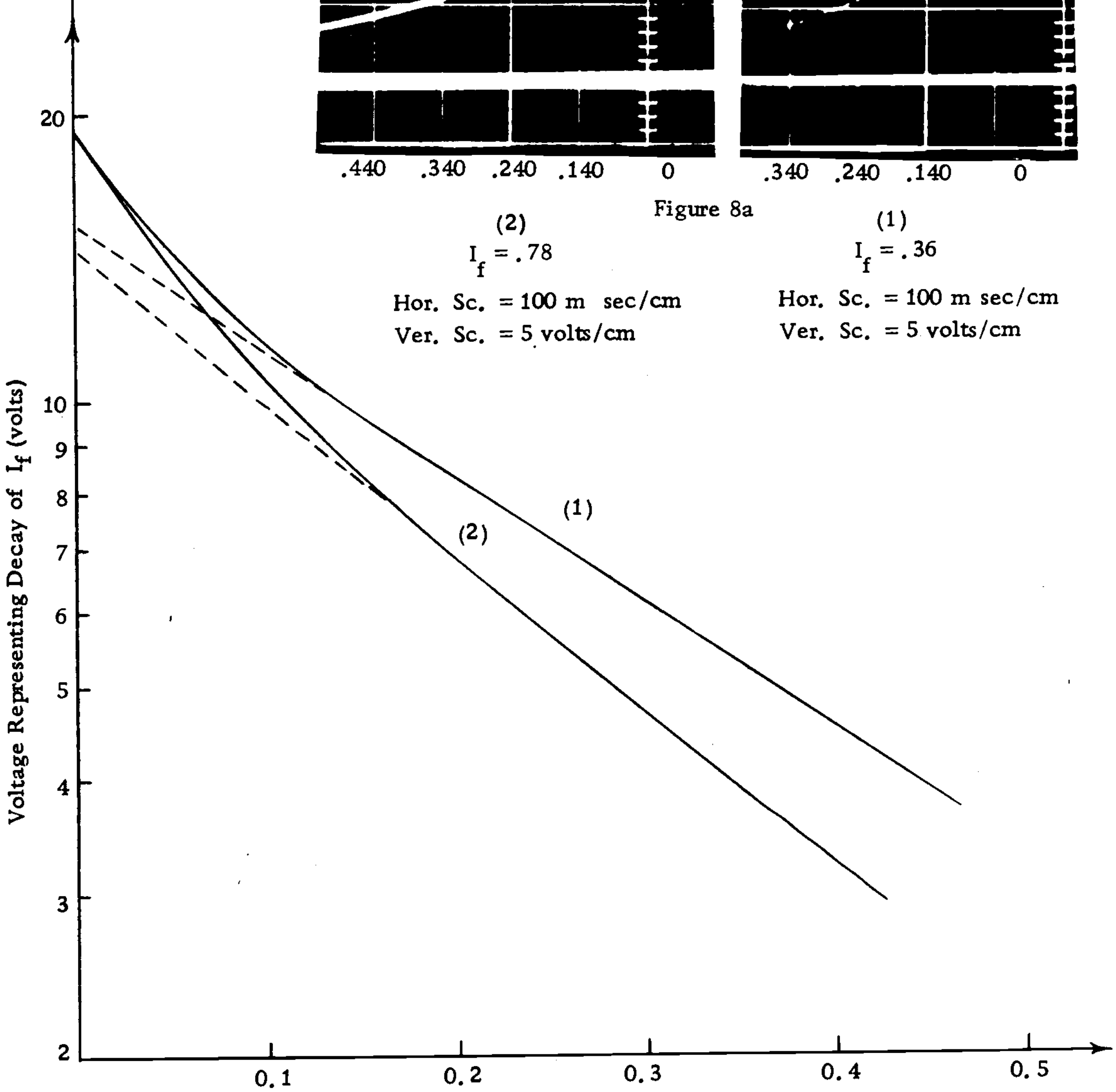


Figure 8b

Figure 8. Decay of the shunt field current

Derivation of the Formulas for Inductances

Since the leakage is unknown at the start of the simulation, the inductances cannot be determined because they are a function of the leakage coefficient, but an indication of leakage is given by the total resistance in the shunt field circuit. This indicated that the inductances might be expressed as function of the expressed resistance in the shunt field. A curve fitted "straight line" is used to represent the inductance. The data points in Table 3 are sufficient to derive the equations for the inductances, "transient and subtransient, and the time of influence of the subtransient." Another point is needed to check the fitness of this representation. This is done for a point where $I_f = 0.41$. It is found that the error is less than 0.2 %.

The equations are:

$$L_{TR} = 0.8415\ R - 228.2 \quad \text{henry}$$

$$L_{sub} = 0.2227R - 35.55 \quad \text{henry}$$

$$T = 0.0009R + .55 \quad \text{second}$$

where

R is the total resistance in the shunt field circuit.

Calculation of the Mutual Inductance Between Shunt and Series Fields (M_{fs})

If the switch is closed in Figure 7 there is a voltage induced in the series circuit though it is open. This voltage equals

$M_{fs} \cdot \frac{dI_f}{dt}$, but $\frac{dI_f}{dt}$ can be found from the previous picture at any time after the closure. The voltage induced in the series field after the closure is traced in Figure 9a and redrawn on a logarithmic scale in Figure 9b.

Table 3 is a summary of the computed value, V3, $\frac{dI_f}{dt}$, t,

$$M_{fs} = \frac{V3}{\frac{dI_f}{dt}} .$$

Table 3. Mutual inductance of the shunt field and the series field

Time (sec)	V3 (volts)	$\frac{dI_f}{dt}$ shunt (ampere/sec)	$M_{fs} = V3/\frac{dI_f}{dt}$ (henry)
0.06	0.75	0.1	0.5
0.16	0.3	0.15/2	0.53
0.25	0.2	0.11/3	0.53

The average value of M_{fs} is equal 0.52 henry.

Series-Field Parameters and Magnetization Curve

(i) Magnetization curve.

(ii) Resistance of the series field.

(iii) Self inductance of the series field.

(iv) Mutual inductance with the armature

(v) Derivation of the series field coefficient for total mmf.

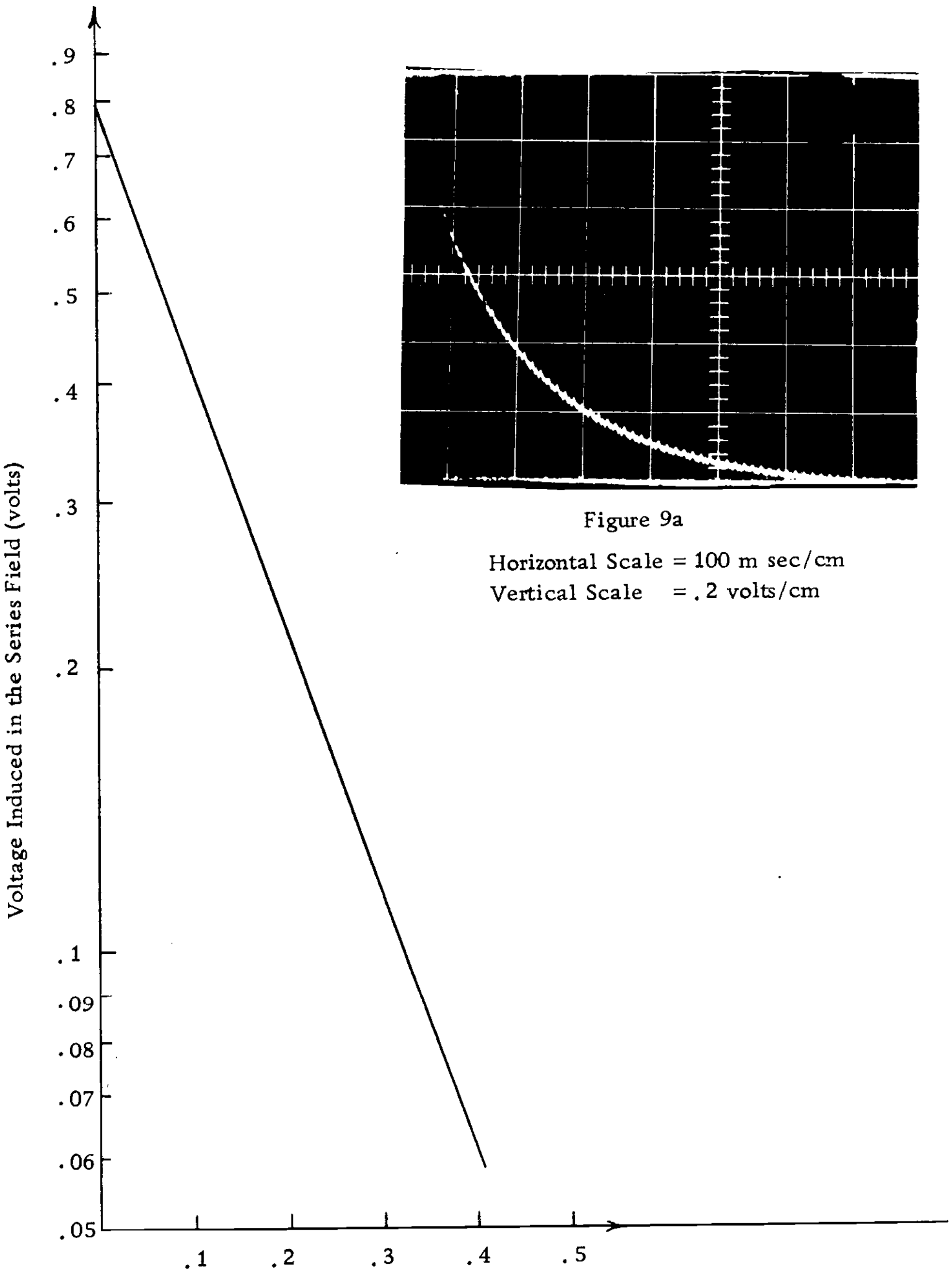


Figure 9a

Horizontal Scale = 100 m sec/cm
Vertical Scale = .2 volts/cm

Time, t (sec)
Figure 9b

Figure 9. Decay of the induced voltage in the series field

The above parameters associated with the series field can be derived or measured by the following experiments.

Magnetization Curve

The first experiment is to derive E_i versus I_s and calculate the resistance of the series field.

The circuit configuration to run the experiment is shown in Figure 10.

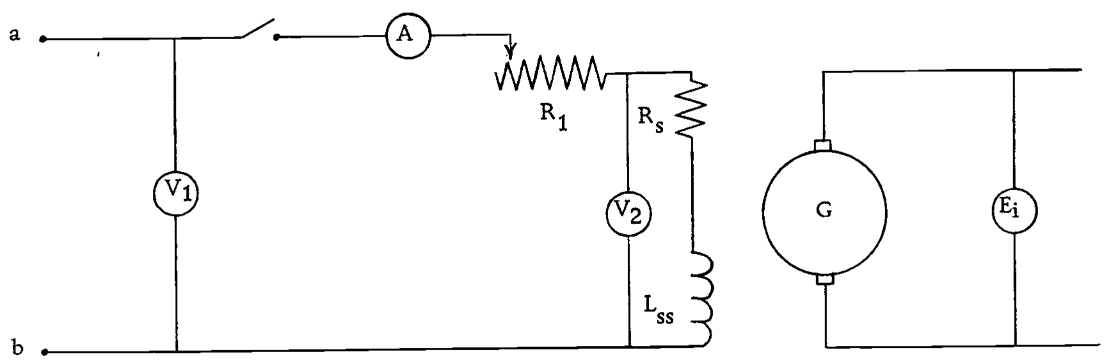


Figure 10. Circuit for magnetization curve ($E_i \rightarrow I_s$)

The prime mover runs at constant speed, while direct voltage is applied to the terminals a-b and the resistance R_1 is changed to obtain the data tabulated in Appendix I. From Appendix I the magnetization curve is plotted in Figure 11.

In Figure 11 the slope of the straight line is equal to 2.6 volts per ampere.

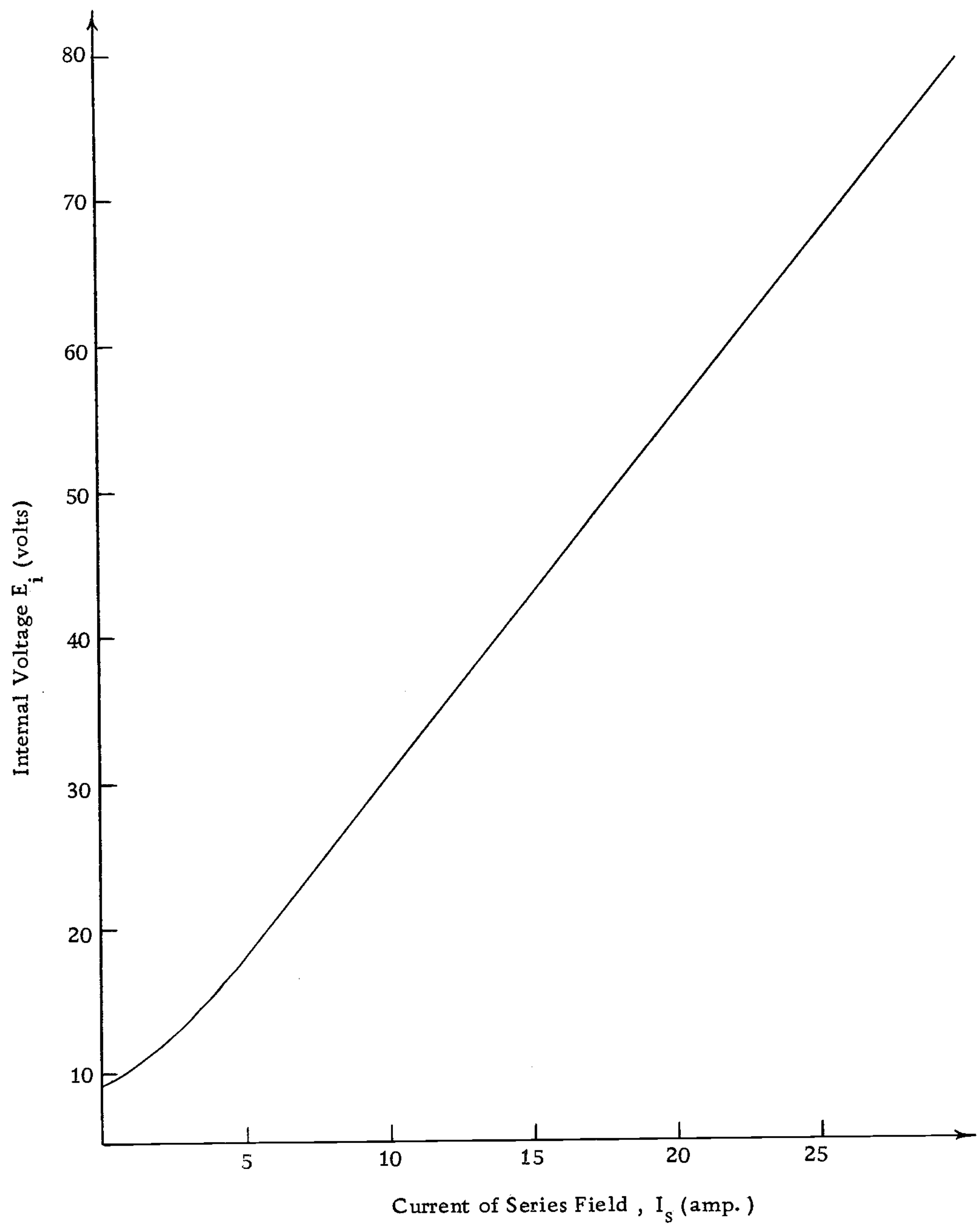


Figure 11. Magnetization curve due to the series field

Resistance of the Series Field

The voltage across the series field is divided by the current of each reading and is tabulated in column 5, Appendix I. The average of the resistance values in column 5 is equal to .034 ohms.

Self Inductance of the Series Field

It is demonstrated in Figure 11 that the flux versus the series field current has a linear relation. This means that the leakage is very small; therefore the transient, subtransient inductances and the time of influence of the subtransient inductances (which is equal to four times its time constant) are all constant.

The circuit for calculation of the self inductances is shown in Figure 12. For this measurement the shunt field and the armature terminals are kept open while paper insulation is installed under each brush, and a camera is installed on the oscilloscope. The picture from that camera is attached in Figure 13a; the decay curve on the picture is drawn on a logarithmic scale in Figure 13b, so that the transient, subtransient inductances and the time of subtransient inductance influence can be easily derived.

Subtransient time constant = 0.032 sec

Subtransient inductance = 0.032 x 0.034 = 0.01088 henry

Transient time constant = 0.078 sec

Transient inductance = 0.078 x 0.034 = 0.026 henry

Time of the subtransient effectiveness 0.04 second

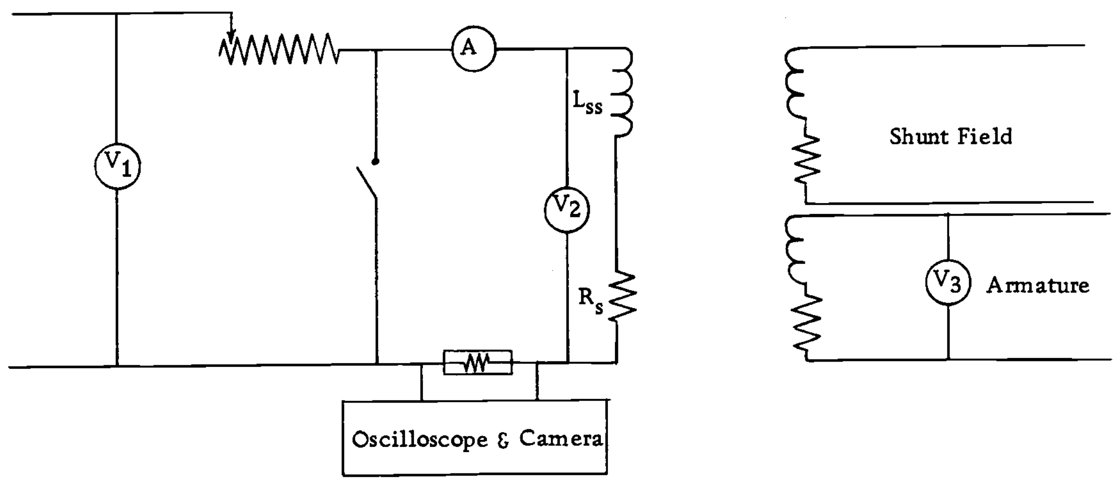


Figure 12. Circuit for measurement of the series field inductances

Calculation of the Mutual Inductance Between the Series Field and the Armature

When the switch is closed in Figure 12 there is a voltage induced in the armature that is equal to $M_{sa} \times \frac{dI_s}{dt}$. $\frac{dI_s}{dt}$ can be found from Figure 13a or 13b. The voltage induced in the armature terminals is shown in Figure 14. The calculated value of M_{sa} taken at t = 1 m. sec. is

$$M_{sa} = \frac{E}{\frac{dI_s}{dt}} = \frac{0.2}{\frac{7.2}{0.064}} = 0.0017 \text{ henry}$$

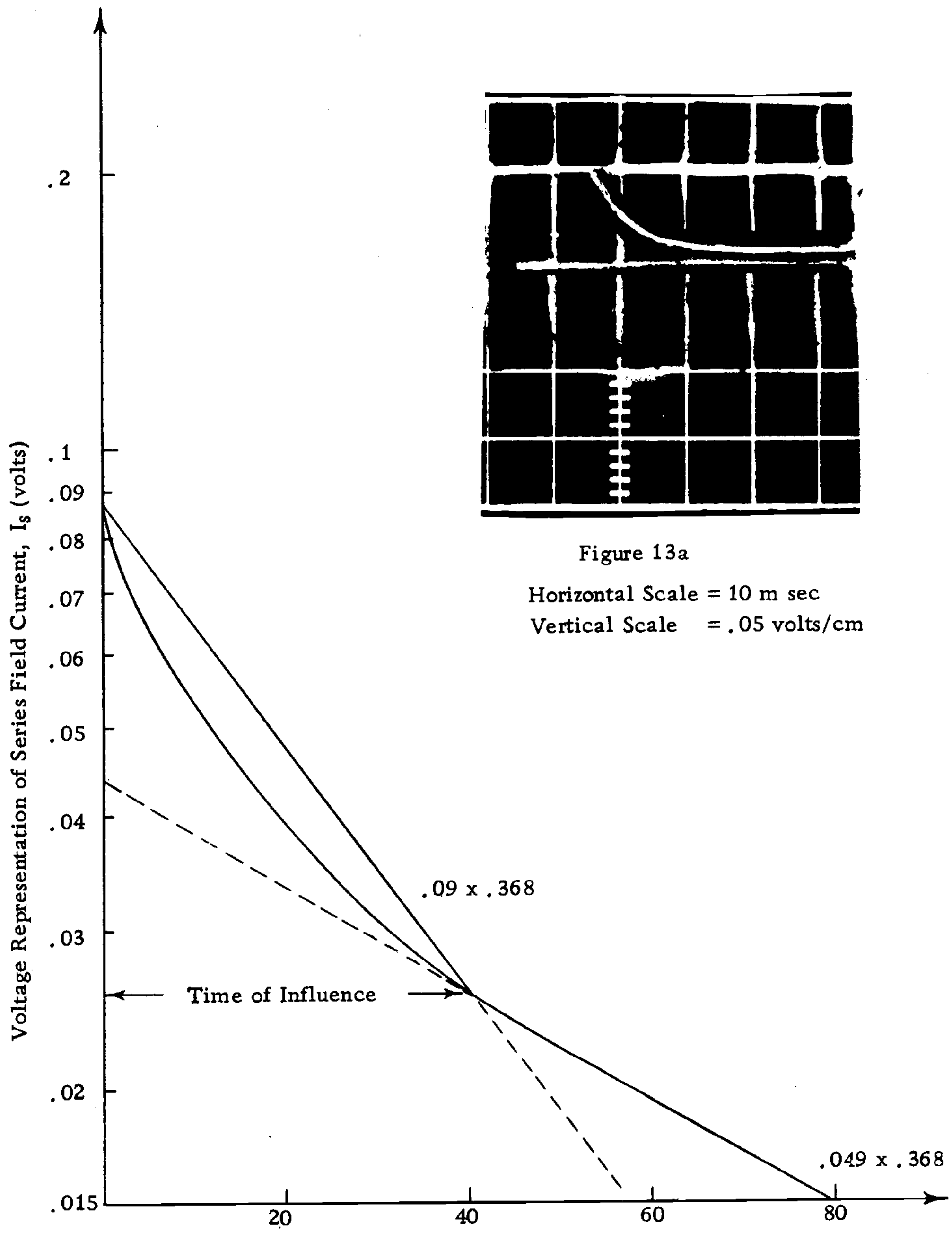


Figure 13a

Horizontal Scale = 10 m sec
Vertical Scale = .05 volts/cm

Figure 13b

Figure 13. Decay of the series field current

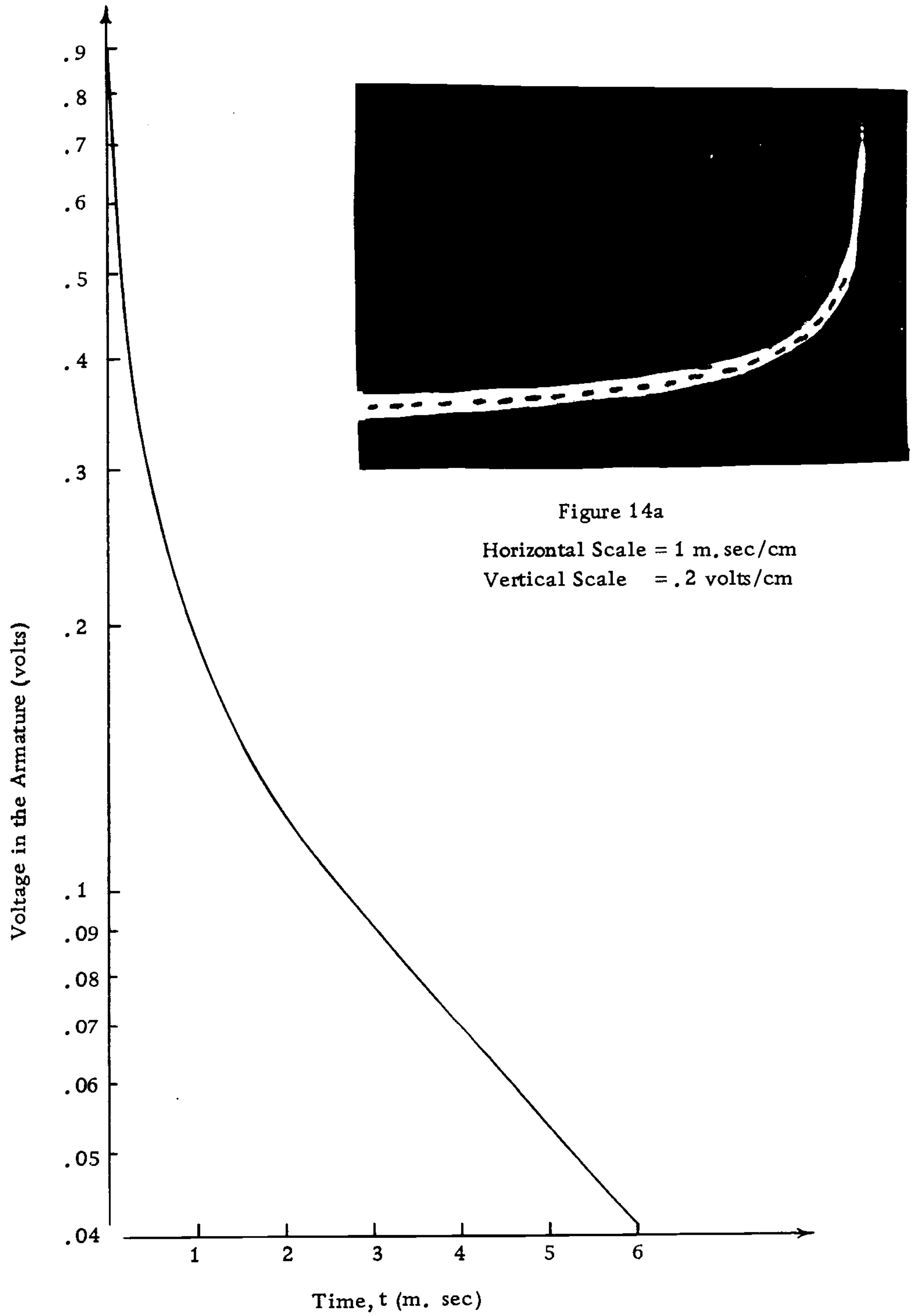


Figure 14a

Horizontal Scale = 1 m. sec/cm
Vertical Scale = . 2 volts/cm

Figure 14b

Figure 14. Decay of the induced voltage in the armature

Calculation of the Net Series mmf

Due to the fact that there is an armature reaction in the generator when it is loaded, the following experiment is performed so that the net results of the series field mmf minus the armature reaction can be found. The generator is loaded with a resistance so that a constant load (30 amp.) is maintained. The rheostat in the shunt field circuit is changed so that a terminal voltage characteristic curve can be produced for that load. The circuit is shown in Figure 15.

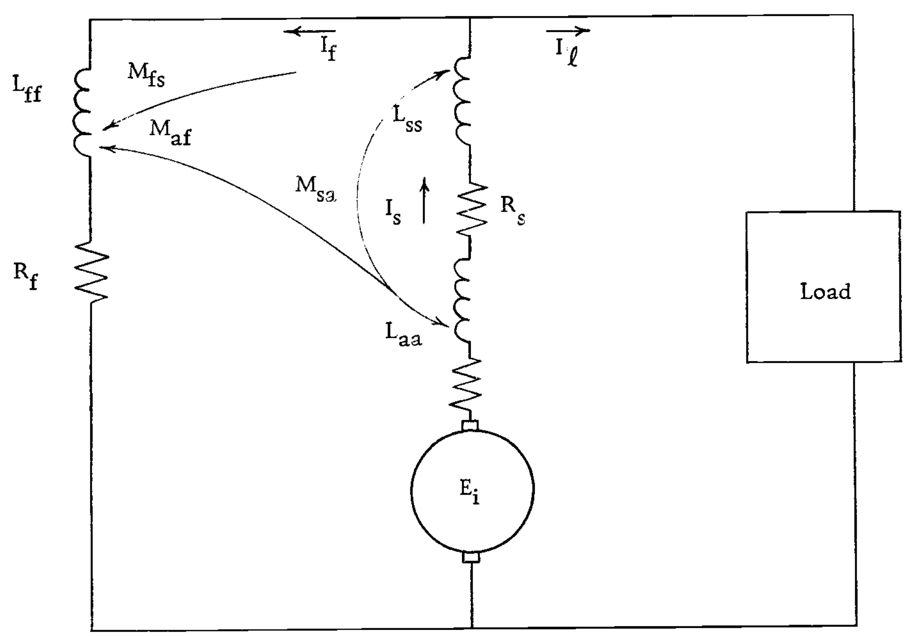


Figure 15. Electrical representation of the cumulative generator

The data derived from this experiment are tabulated in Appendix J. The internal voltage of the machine (column 4) is equal to the terminal voltage plus the voltage drop in the armature resistance and the series field resistance.

The graph for the terminal voltage and the internal voltage versus the shunt field current is shown in Figure 16.

Taking the slope of the magnetization curve at 0.4 ampere from Figure 6, also redrawn in Figure 16, the slope equals 380 volts/amp. The ratio of the slopes of the magnetization curve of the series field to that of the shunt field $= \frac{2.60}{380} = 0.00684$.

mmf for series alone = 0.00684 x 30 = 0.2052 ampere.

mmf of the series field minus the armature reaction from Figure 16 = 0.150.

Therefore the armature reaction 0.2052 - 0.150 = 0.055 which is equal to 25%.

Derivation of the net mmf

It was noticed that on either side of the knee of the magnetization curve the net mmf under load is more than what was previously calculated. This phenomenon had not previously been reported in the literature. An extensive study for different loads proved that this increase is not dependent on the load, but on the magnitude of the shunt field current. It is positive in both directions as tabulated in Table 4.

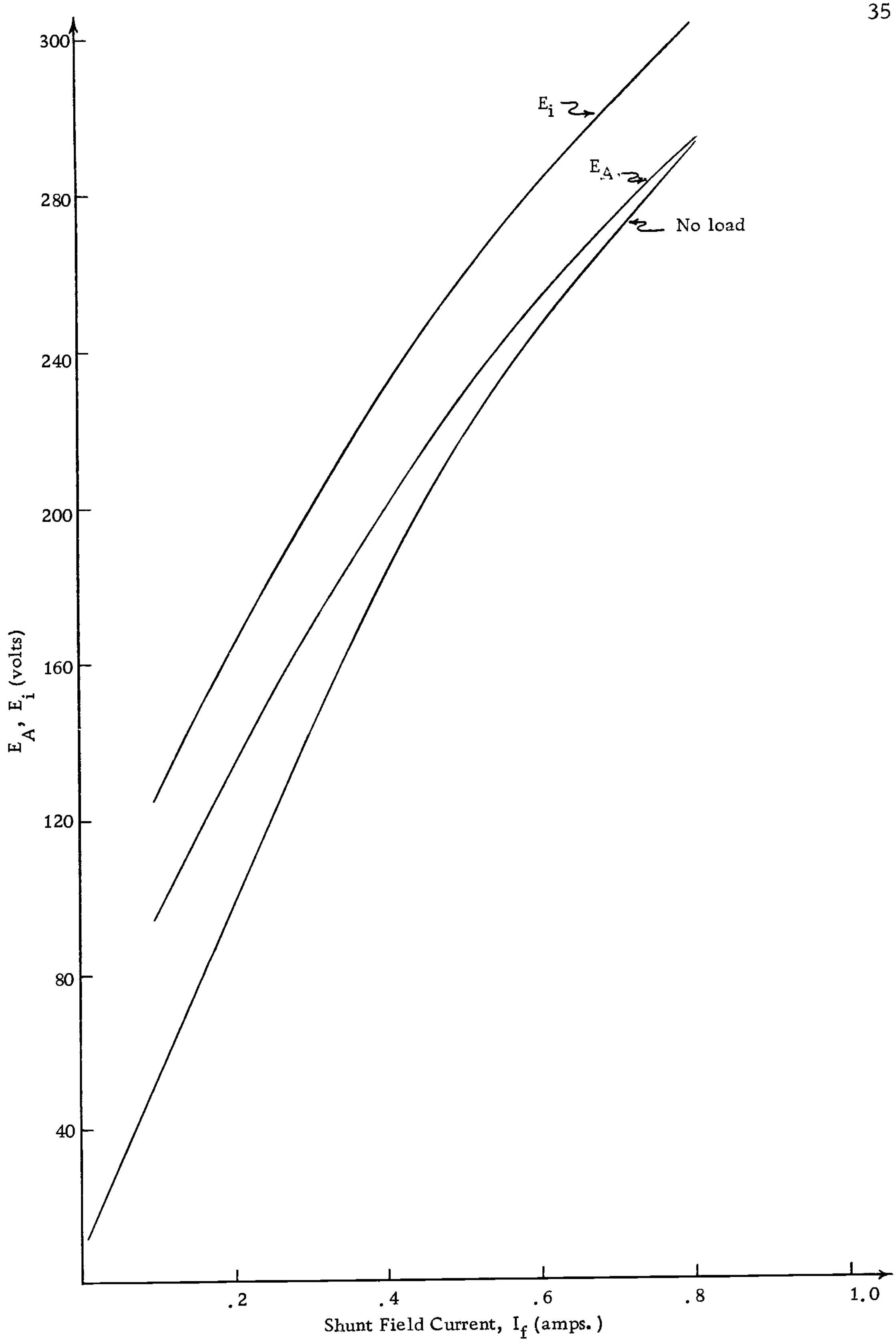


Figure 16. Magnetization curve under load (30 amp.)

Table 4. Influence on armature reaction

I_f amp.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Excess	0.027	0.02	0.11	0.0	0.0	0.13	0.017	0.3

An empirical formula was derived from the data points in Table 4.

The excess mmf under load = 0.08 x ABS (I_f - .45).

The result of the above discussion is:

$$E_i = f(I_c)$$

$$\begin{aligned} I_c &= I_f + .00505\, I_\ell + .08 \times \text{ABS}\,(I_f - .45). & \quad I_\ell > 0.0 \\ &= I_f & \quad I_\ell = 0.0 \end{aligned}$$

Armature Winding Parameters

(i) Self inductance of armature winding.

(ii) Mutual inductance between the armature and the shunt field.

(iii) Armature winding resistance.

Self Inductance of Armature Winding

The first experiment is for measuring the inductances. The circuit is shown in Figure 17 with field circuits left open. A small series resistance is inserted in the armature circuit so that a voltage proportional to current could be traced on the oscilloscope.

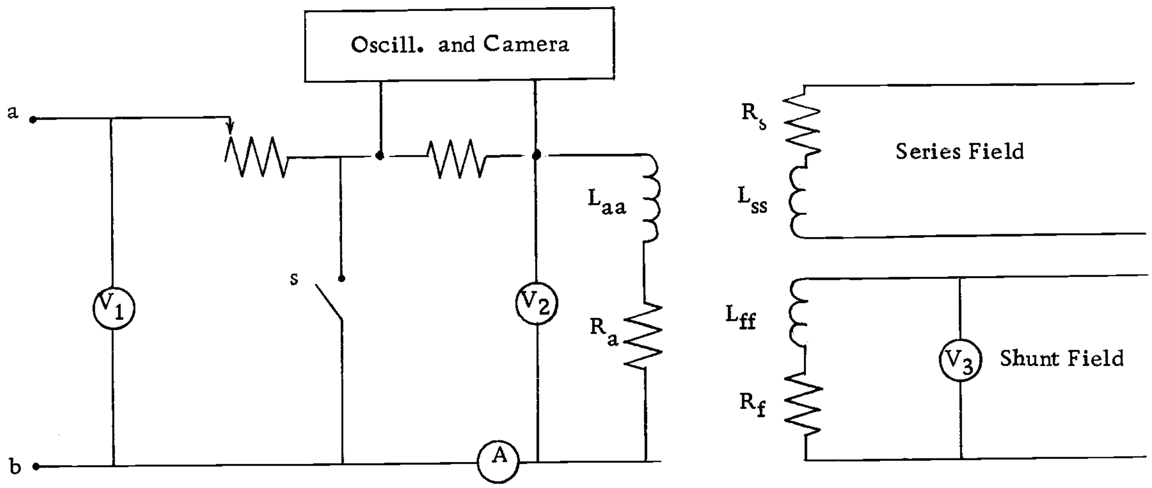


Figure 17. Circuit for measurement of the armature inductances.

The transient decay of the voltage across R is traced in Figure 18a and is also drawn on logarithmic scale in Figure 18b. From Figure 18b the following information was derived:

Subtransient time constant	= 0.075
Subtransient inductance	= 0.075 x 1.61 = 0.12
Transient time constant	= 0.28
Transient inductance	= 0.28 x 1.61 = 0.4508

Time of subtransient effectiveness 0.1 second.

Mutual Inductance with the Shunt Field

The voltage induced in the shunt field due to the decay of current in the armature winding is equal to $M_{fa} \frac{dI_a}{dt}$. The rate of change of the armature current can be calculated from Figure 18b at any specified time. The transient of the voltage induced in the shunt field is shown

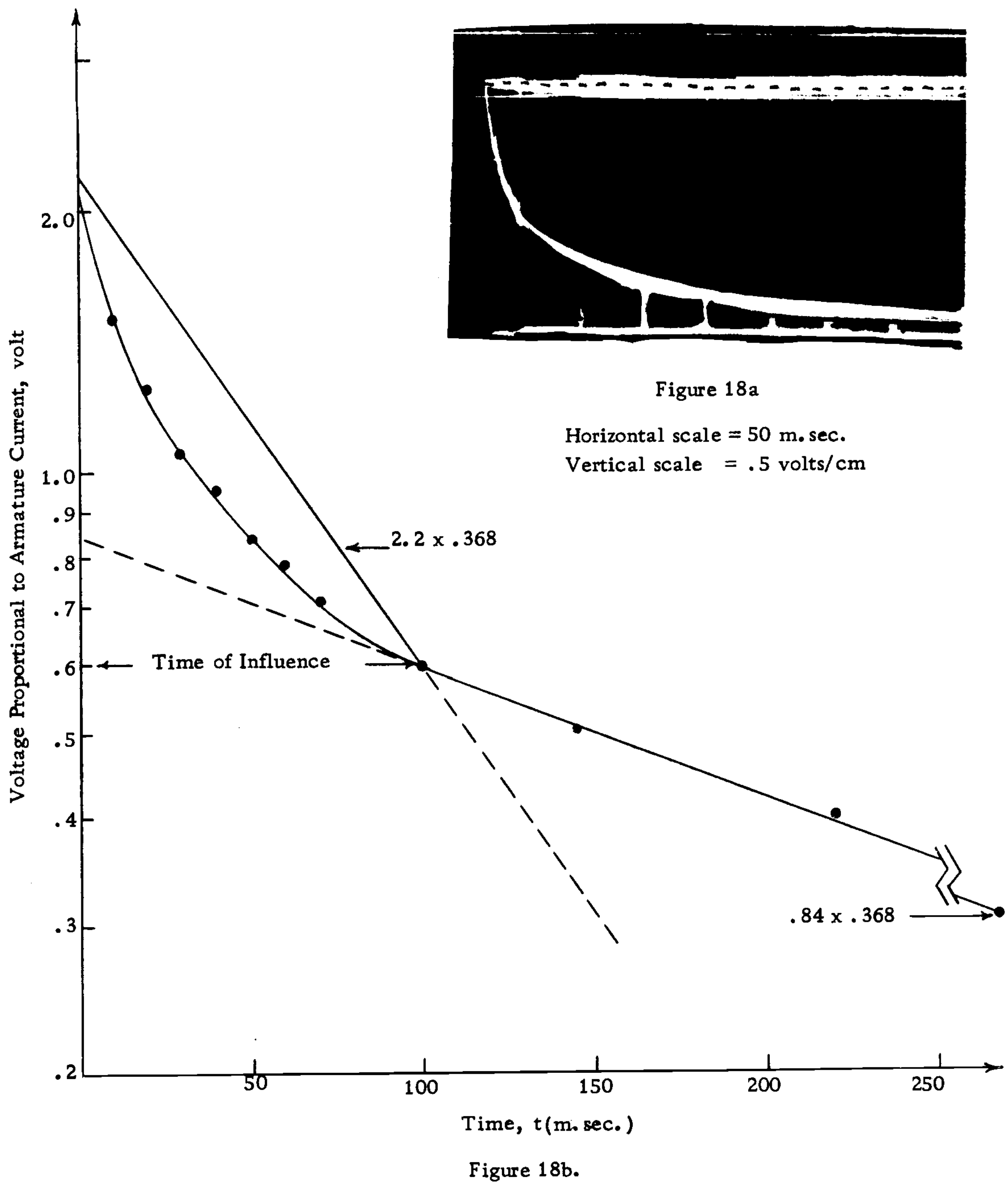


Figure 18. Decay of the current in the armature

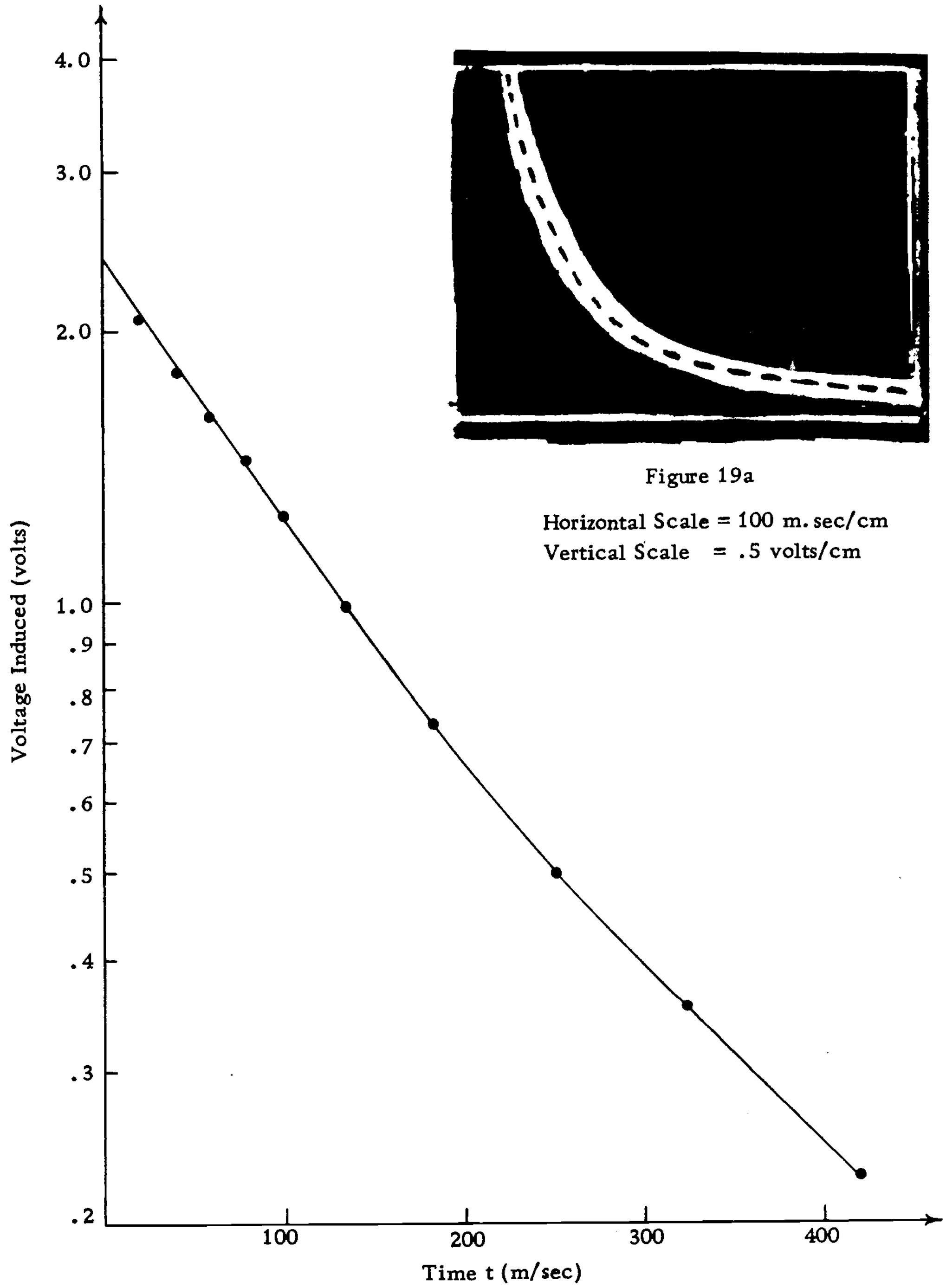


Figure 19a

Horizontal Scale = 100 m. sec/cm
Vertical Scale = .5 volts/cm

Figure 19b

Figure 19. Decay of the induced voltage in the shunt field

in Figure 19a, and also drawn in Figure 19b on a logarithmic scale.

The calculations taken at $t = 50$ milliseconds are:

$M_{fa} = 0.0718$ henry.

Calculation of the Armature Resistance

A voltage source is applied to the terminals of the armature with a variable resistance in series, with normal rotation, so that the effect of the voltage drop in the brushes can be included. The measurement is made two times, first supporting the residual magnetism, and second against it. In Appendix K the readings of the voltages in both cases and the currents are tabulated.

A graph of the voltage versus current is drawn for each case in Figure 20.

The average slope of the two curves is equal to the resistance of the armature and was calculated as 0.94 ohms.

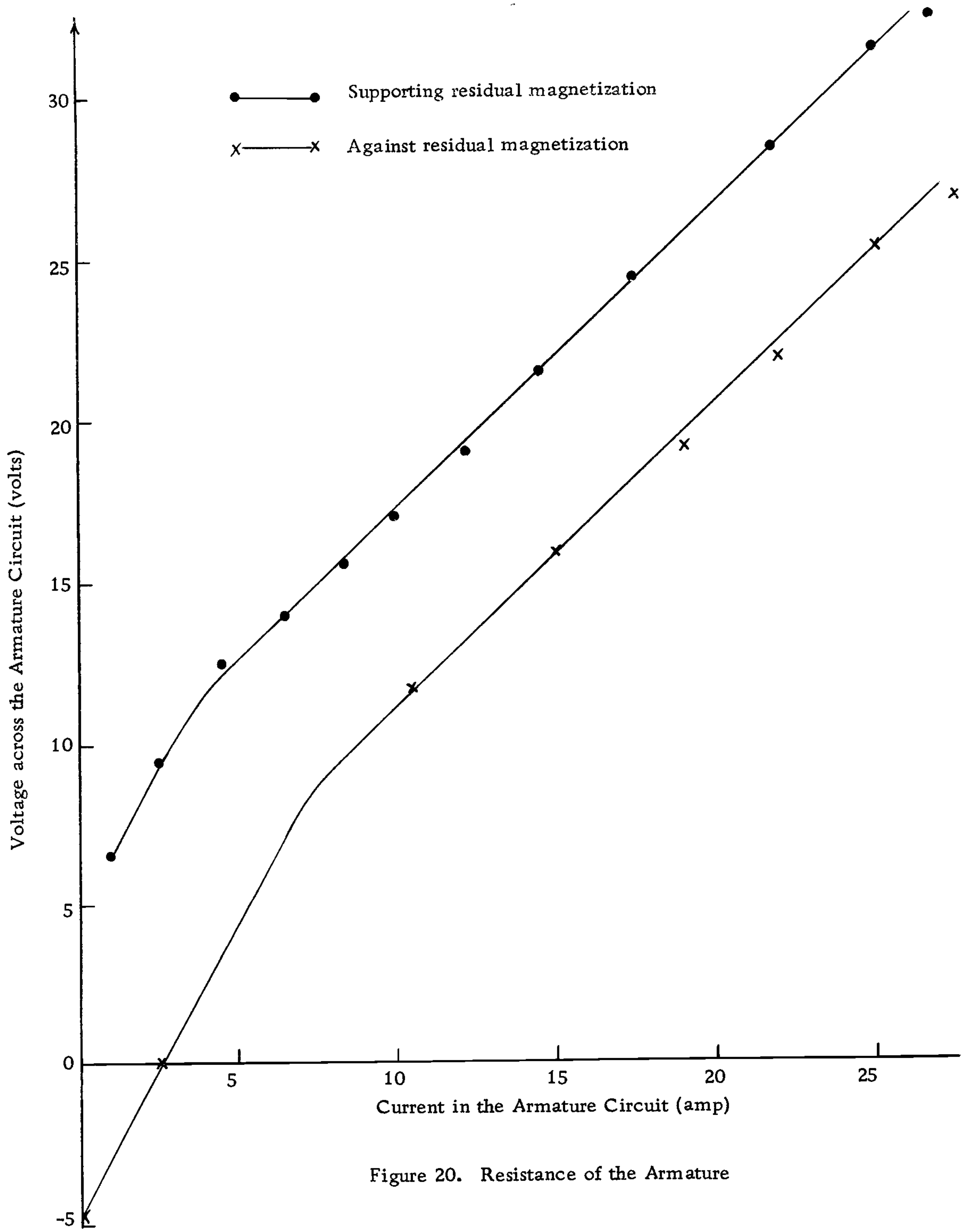


Figure 20. Resistance of the Armature

V. MATHEMATICAL ANALYSIS AND MODELING

Steady State

Since the steady state behavior of the generator under different load and excitation conditions is as essential as the transient behavior for the power engineer, the author felt that a discussion of each separately would shed more light on each operation than a combined discussion.

The steady state representation of the cumulative compound generator is shown in Figure 21.

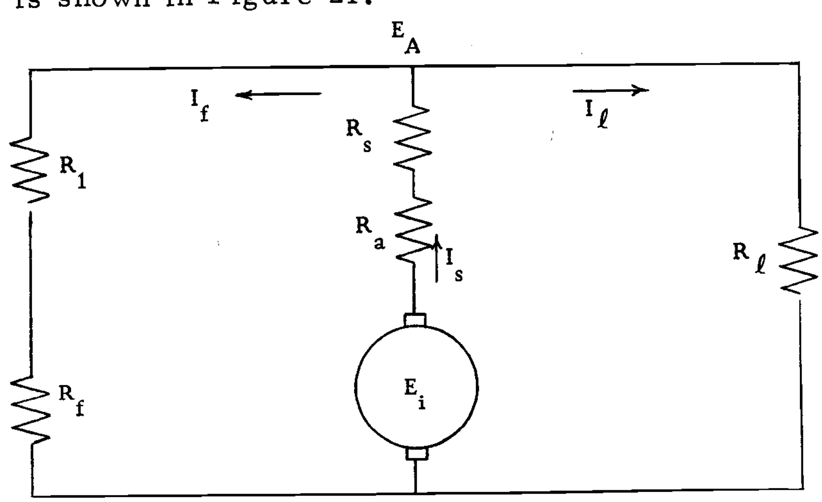


Figure 21. Steady state representation

The resistances R_f, R_a, R_s; K_1; and the representation of E_i as a function of I_f and I_ℓ are each derived in the previous chapter.

The variables are E_i, E_A, I_f, I_ℓ, and I_s.

The relation between the variables can be expressed as:

$$E_i = f(I_f, I_\ell, K_1) = f(I_c) \tag{5}$$

$$E_A = E_i - I_s(R_a + R_s) \tag{6}$$

$$I_f = E_A \div (R_1 + R_f) \tag{7}$$

$$I_\ell = E_A \div R_\ell \tag{8}$$

$$I_s = I_f + I_\ell \tag{9}$$

Before trying to solve the above equations, it is necessary to find the special conditions where there is a zero answer for all the variables, or the solution is indeterminant.

When the iron part of magnetic circuit is unsaturated, $f(I_f, I_\ell)$ can be expressed as

$$E_i = K_2 I_f + K_3 I_\ell + K_1 \tag{10}$$

Equations 5-9 written in matrix form after making proper substitutions are as follows:

$$\begin{bmatrix} 1 & 0 & 0 & -K_2 & -K_3 \\ 1 & -1 & R_s+R_a & 0 & 0 \\ 0 & 1 & 0 & -(R_f+R_1) & 0 \\ 0 & 1 & 0 & 0 & -R_\ell \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} E_i \\ E_A \\ I_s \\ I_f \\ I_\ell \end{bmatrix} = \begin{bmatrix} K_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

Equation (11) expressed in compact form is:

$$\overline{A} \cdot \overline{X} = \overline{b}$$

The value of $\overline{X}$ is indeterminate iff the determinant

$$||A|| = 0 \qquad (12)$$

Determinant $A = (R_f + R_1)(K_3 - R_\ell - R_s - R_a) + R_\ell(K_2 - R_s - R_a)$

With open circuit conditions,

$$R_\ell >> K_3 - R_s - R_a \quad \text{and} \quad K_2 >> R_s + R_a$$

$$\therefore \qquad ||A|| = -R_\ell(R_f + R_1 - K_2) \ .$$

When $R_1 + R_f = K_2$, then the slope of the linear part of the magnetization curve is equal to the resistances in the shunt field circuit, then the value of the variables are undefined. This fact is supported by the behavior of the d-c generators under the above conditions. That is, the system cannot sustain a stable point.

The other case is for vector $\overline{b}$ equal to zero, then the only values the variables can take are zero, i.e., unless there is a residual magnetism the voltage will not build up.

When saturation is considered, K_2 may be equal to $R_1 + R_f$, but not at the point of intersection of the magnetizing curve and the field resistance line because the slope of the characteristic curve is decreasing as mmf increases. Thus in the saturation zone the generator is more stable than in the unsaturated zone.

Mathematical Model

Equations (5-9) rewritten and the values of I_c, R_f, R_s and R_a

substituted:

$$E_i = f(I_c) \tag{13}$$

$$I_c = \left.\begin{array}{ll} I_f + .00505\, I_\ell + ABS(I_f - .45) & I_\ell > 0.0 \\ = I_f & I_\ell - 0.0 \end{array}\right\} \tag{14}$$

$$E_A = E_i - I_s(.94 + .034) = E_i - .974\, I_s \tag{15}$$

$$I_f = E_A \div (R_1 + 272) \tag{16}$$

$$I_\ell = E_A \div R_\ell \tag{17}$$

$$I_s = I_\ell + I_f \tag{18}$$

$f(I_c)$ is a "table look up" where E_i versus I_c is made of 21 small segments of straight lines.

Programming

The set of equations (13-18) cannot be solved directly because of the way E_i is represented, so the only method is by iteration. The method used was Gauss-Seidal which if programmed with great care will lead to the solution.

To nullify the errors, the set of equations (13-18) are simplified so that the iterations will be between two functions, $E_i - I_s(R_s + R_a)$ and $R_2 \cdot I_s$; the other variables can be computed if E_i and E_A are known. Two conditions make selection of E_i and E_A as function of I_s perfect; first the two curves intersect in one point only in the first quadrant, and second the angle between the two curves is large so the round off error is insignificant.

The last step is to arrange the equations so that the iteration direction will lead to the solution and not away from it as seen in Figure 22.

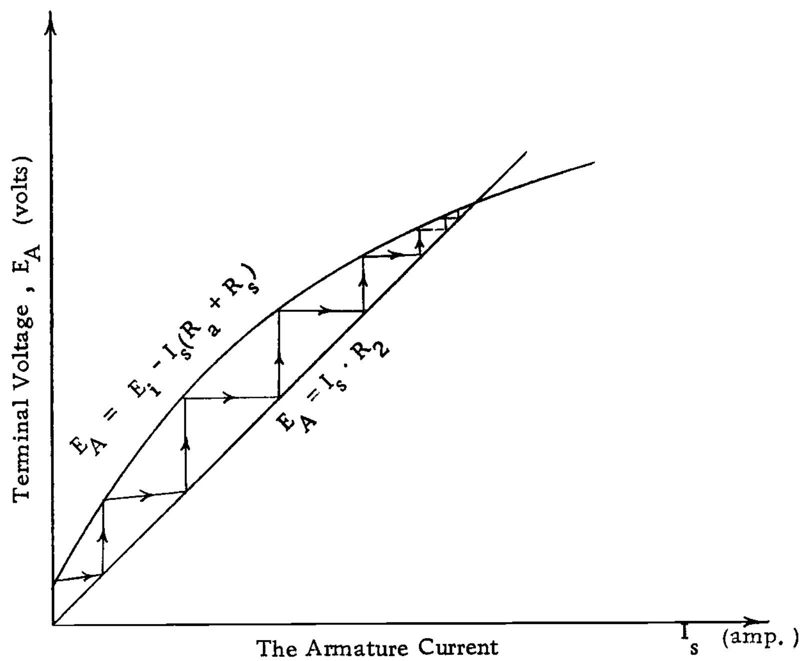


Figure 22. Iteration direction

Equations (13-18) are rearranged to conform to the previously stated conditions.

$$
\left.\begin{array}{ll}
E_i & = f(I_c) \\
I_c & = I_f + .00505\, I_c + \text{ABS}\,(I_f - .45) \quad \left.\begin{array}{l} I_\ell > 0.0 \\ \\ I_\ell = 0.0 \end{array}\right. \\
 & = I_f \\
I_s & = E_i \div (R_2 + R_s + R_a) \\
E_A & = E_i - I_s\,(R_a + R_s) \\
I_f & = E_A \div (R_1 + R_f)
\end{array}\right\} \quad (19)
$$

$$I_\ell = E_A \div R_\ell$$

where

$$R_2 = (R_1 + R_f) \,\|\, R_\ell = \frac{(R_1 + R_f) R_\ell}{R_1 + R_f + R_\ell}$$

The language used for programming was Fortran IV; the flow chart for this program is shown in Appendix A. The program is shown in Appendix B. The results are shown in Appendix C. From these results the relation between terminal voltage and load current is shown in Figure 23. The load resistance changes from 2 to 131 ohms in each graph.

Results and Comments

An experiment is performed to compare the results achieved from the computer simulation with those from actual measurements. No attempt has been made to load the generator beyond 1.25 load,

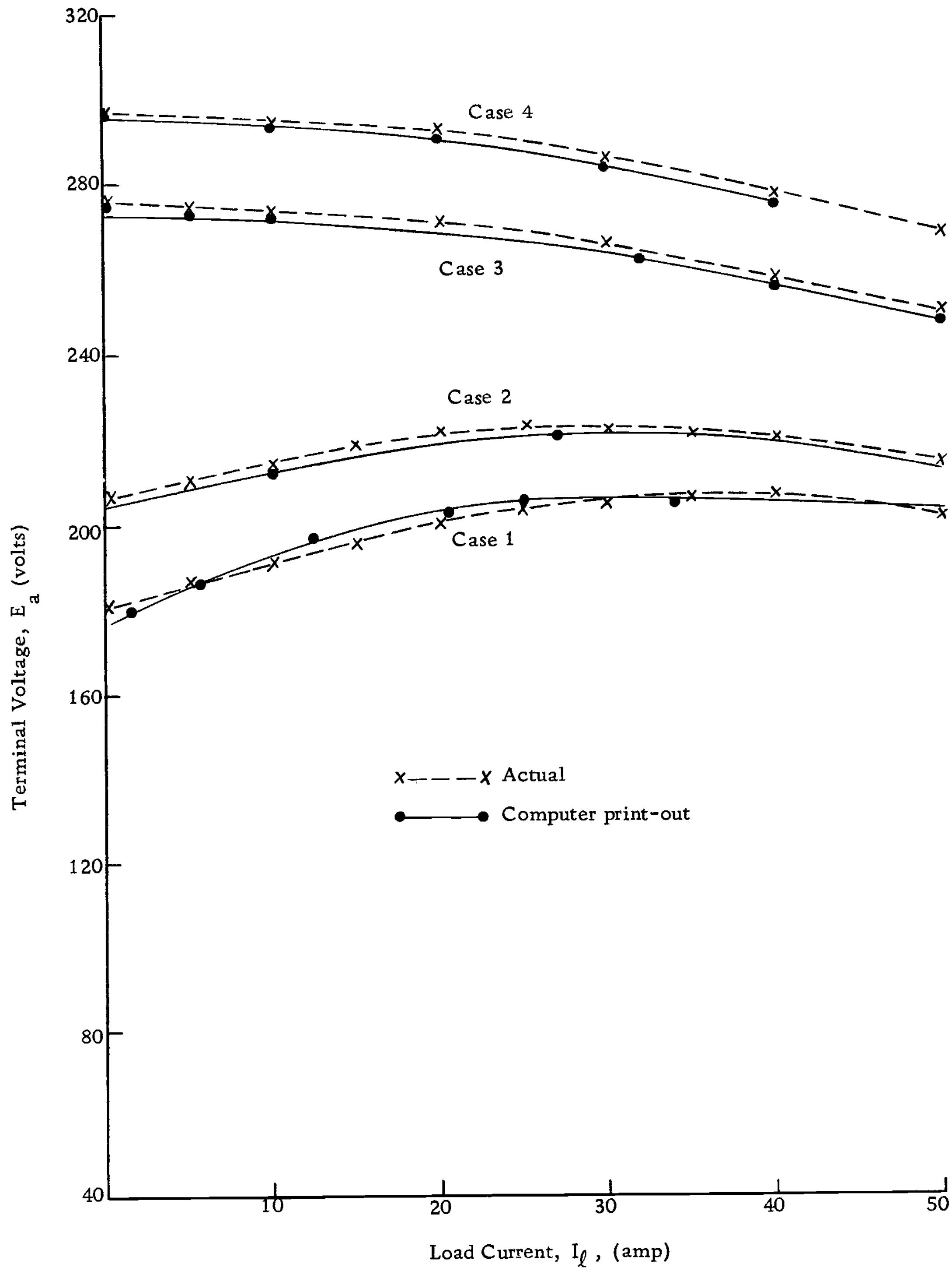


Figure 23. Steady State Response to Variable Load

because of fear of damage. For the same cause the short circuit test has not been tried.

The measured data points for E_A versus I_ℓ for different values of load lines are shown in Appendix D and are drawn as dotted lines in Figure 23.

From Figure 23 it is clearly shown that the agreement between the computer printout and the actual reading is evident and the simulation is reliable.

Transient Response

The electrical representation of the system under transient conditions is shown in Figure 24.

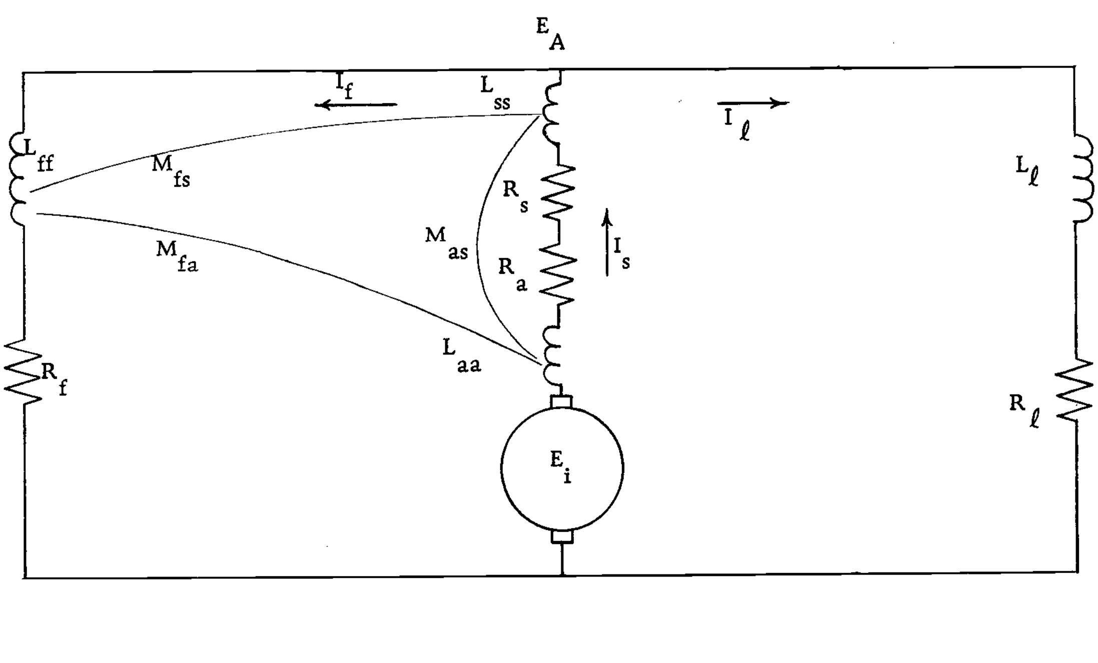


Figure 24. Transient representation

The mathematical equations defining the relation between the variables can be written as:

$$E_i = f(I_c)$$

$$I_c = \begin{cases} I_f + .00505\, I_\ell + .08\, ABS(I_f - .45) & I_c > 0.0 \\ I_f & I_c = 0.0 \end{cases} \quad (13)$$

$$E_A = E_i - I_s(R_a + R_s) - (L_{aa} + L_{ss} + 2M_{as})\frac{dI_s}{dt} - (M_{fs} + M_{fa})\frac{dI_f}{dt} \quad (20)$$

$$I_f = \frac{1}{R_f + R_a}\left[E_A - L_{ff}\frac{dI_f}{dt} + (M_{fs} + M_{fa})\frac{dI_s}{dt}\right] \quad (21)$$

$$I_\ell = \frac{1}{R_\ell}\left[E_A - L_\ell \frac{dI_\ell}{dt}\right] \quad (22)$$

$$I_s = I_f + I_\ell \quad (18)$$

where

$$L_{aa} = \begin{cases} .4508 & t > .1 \\ .12 & t < .1 \end{cases}$$

$$L_{ss} = \begin{cases} .0026 & t > .04 \\ .00109 & t \leq .04 \end{cases}$$

$$M_{as} = .0017$$

$$\left.\begin{aligned} L_{ff} &= .8415\ R - 228.2 && t > T \\ &= .2227\ R - 35.55 && t \leq T \\ T &= -.0009\ R + .55 && \text{second} \end{aligned}\right\}$$

$M_{fs} = .52$

$M_{fa} = .0718$

L_ℓ = inductance of the load

$R = R_1 + R_f$ Resistance in the shunt field circuit.

System Analysis

System analysis is necessary before attempting to solve the system of equations. Difference equations were substituted for the differential equations to facilitate computer solution of the set.

By representing E_A as $I_\ell * R_\ell$, linearizing the internal voltage with the following relation;

$$E_i = K_2 * I_f + K_3 * I_\ell + K_1$$

and substitution for $I_s = I_f + I_\ell$, the Laplace Transform of the set of equations can be made. The set of equations (13, 18, 20, 21 and 22) now has the form

$$\begin{bmatrix} 1 & -K_2 & -K_3 \\ 1 & -(R_a+R_s)-S(L_{ss}+L_{aa}+2M_{as}-M_{fs}-M_{fa}) & -R_a+R_s+R_\ell-(L_{ss}-L_{aa}-2M_{as})s \\ 0 & -(R_f+R_1-S(L_{ff}-M_{fs}-M_{fa}) & R_\ell+S(M_{fs}+M_{fa}) \end{bmatrix} \bullet \begin{bmatrix} E_i \\ I_f \\ I_\ell \end{bmatrix} = \begin{bmatrix} K_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{C} \cdot \overline{x} = \overline{D}$$

Because it is a second order system the necessary and sufficient conditions for the system to be stable are that the coefficients of the characteristic equation of the system should have the same sign and that the middle term is greater than zero.

Expressing the characteristic equation as

$$AS^2 + BS + E = 0$$

where

$$A = -(L_{ss} + L_{aa} + 2M_{as})(L_{ff}) + (M_{fa} + M_{fs})^2$$

$$B = -L_{ff}(R_a + R_s + R_\ell + K_3) - (R_f + R_1)(L_{ss} + L_{aa} + 2M_{as}) + (M_{fs} + M_{fa})(2R_\ell + K_2 + K_3)$$

$$E = (R_a + R_s) R_\ell + (R_a + R_s + R_\ell)(R_f + R_1) - K_2 R_\ell - K_3(R_f + R_1)$$

A is always negative because the first term in A is much greater than the second term, so B and E should be negative and B is nonzero. To fulfill this condition for a value of R_ℓ from $0 \rightarrow \infty$

First condition;

$$R_\ell = \infty$$

$$B = (-L_{ff} + 2M_{fs} + 2M_{fa}) R_\ell \qquad (25)$$

$$E = (R_a + R_s + R_f + R_1 - K_2) \qquad (26)$$

Second condition;

$$B = -L_{ff}(R_a + R_s + K_3) - R(L_{ss} + L_{aa} + 2M_{as}) + (M_{fs} + M_{fa})(K_2 + K_3)$$

$$(M_{fs} + M_{fa})(K_2 + K_3) < L_{ff}(R_a + R_s + K_3) + R(L_{ss} + L_{aa} + 2M_{as}) \qquad (27)$$

$$E = (-K_3 + R_a + R_s)(R) \tag{28}$$

Checking equations (25-28), it is noticed that equation (26) implies the same restriction reported for the steady state. The other restrictions are fulfilled in the system under discussion.

Since $\overline{D}$ must be nonzero so that the variables in (27) have values other than zero, then $K_1 > 0.0$.

Programming

1. Since E_i is represented in a table look up with the other coordinates I_c which is a function of I_f and I_ℓ, then the only way equations (13), (18), (20-22) can be solved is through iteration.
2. It is necessary to transform the set of equations from their differential form into difference form.
3. The set of equations should be arranged in a form suitable for the method of iteration used.
4. It is necessary to apply the relaxation method so that the solution will not be overjumped during the iteration.
5. The restriction and the values of the parameters mentioned in page 50 add a little complication to the programming.

The set of equations will have the following form after all the above modifications:

$$E_i = f(I_c)$$

$$I_c = \left.\begin{array}{l} I_{f2} + .00505\, I_{\ell 2} + .08*ABS(I_{f2} - .45) \\ = I_{f2} \end{array}\right\} \begin{array}{l} I_\ell > 0.0 \\ I_\ell = 0.0 \end{array} \quad (29)$$

$$E_{A2} = E_i - I_{s2}(R_s + R_a) - \frac{(I_{s2} - I_{s1})}{dt} \cdot (L_{ss} + L_{aa} + M_{sa}) - \frac{(I_{f2} - I_{f1})}{dt} (M_{fa} + M_{fa}) \quad (30)$$

$$E_{A1} = E_{A1} + .1\,(E_{A2} - E_{A1})$$

$$I_{f3} = \frac{dt}{L_{ff}} \{E_{A1} - R_f \cdot I_{f2}\} + L_{ff} * I_{f1} - (M_{fa} + M_{fs})(I_{s2} - I_{s1}) \quad (31)$$

$$I_{f2} = I_{f2} + 0.1(I_{f3} - I_{f2})$$

$$I_{\ell 3} = E_{A1} \div R_\ell$$

$$I_{\ell 2} = I_{\ell 2} + 0.1*(L_{\ell 3} - L_{\ell 2}) \quad (32)$$

$$I_{s2} = I_{f2} + I_{\ell 2}$$

The iteration will stop if the deviation between two successive readings of any variable is less than 3×10^{-6}. Then a step in time is programmed.

After each time step (0.01 second) the times of influence are checked to determine if a shift from subtransient to transient inductances should take place.

The flow chart of the program is shown in Appendix E. The program is written in Fortran IV and is shown in Appendix F.

Results and Comments

The simulation program was tested with four different load conditions.

No-load Unsaturated Case. Where the steady state terminal voltage E_A is on the linear part of the magnetization curve, $E_i = 170$, $I_f = .34$. The two curves in Figure 25a are redrawn from the actual results of the terminals of the machine, and from the simulation program. The match is good except at the start, where a deviation is in the range of 5%. It is shown in Figure 25a.

No-load Saturated Case. Where the steady state terminal voltage is 20% above nominal voltage (295 volts) and $I_f = 0.8147$. Because the match is about complete the data points taken from the actual response are drawn x's, the solid curve is from the computer printout, the match is better than case (i). The curves are shown in Figure 25B.

Loaded Unsaturated Case. The terminal voltage for steady state is 183, $I_f = 0.368$ and the load is about 0.3 of rated load, the matching between the actual transient data and the simulation output is very good except in two places, at the start where the actual curve rises slower than the simulated and near the knee of the characteristic curve where the actual rise is sharper than the curve of simulated response. In either case the maximum difference is less than 5%, the two curves are shown in Figure 25c, where the dotted is the actual and the solid is the simulation printout.

Loaded and Saturated Case. The terminal voltage is 20% above the rated voltage (291), and the load current is about 75% of the rated load. The deviation between the two curves occurs during the first half second where the curve of the simulated response rises sharper than the actual curve. The matching between the two curves is perfect with the above exceptions. The curves are shown in Figure 25d.

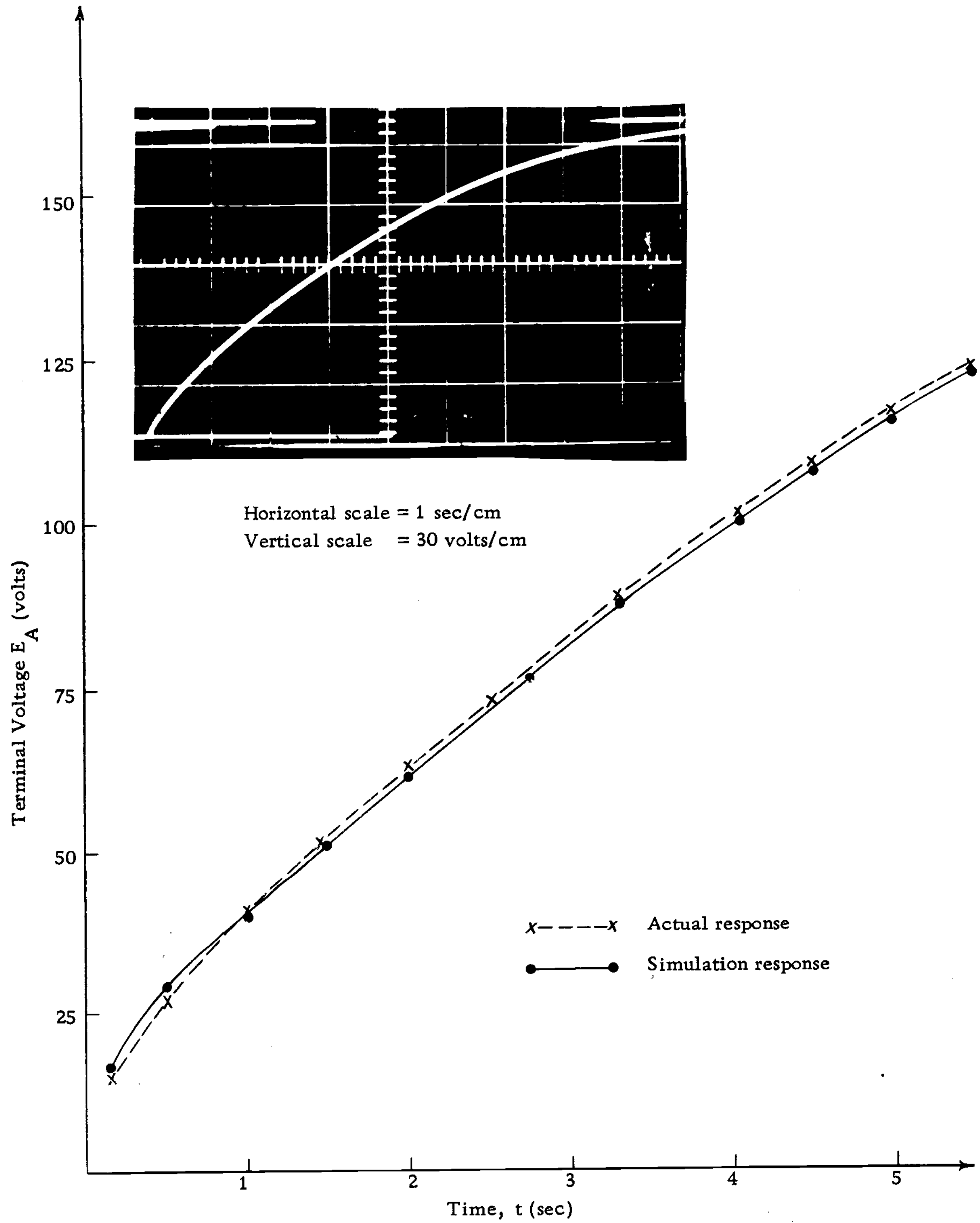


Figure 25a. Case 1 Transient response (I_f = .34 amp.)

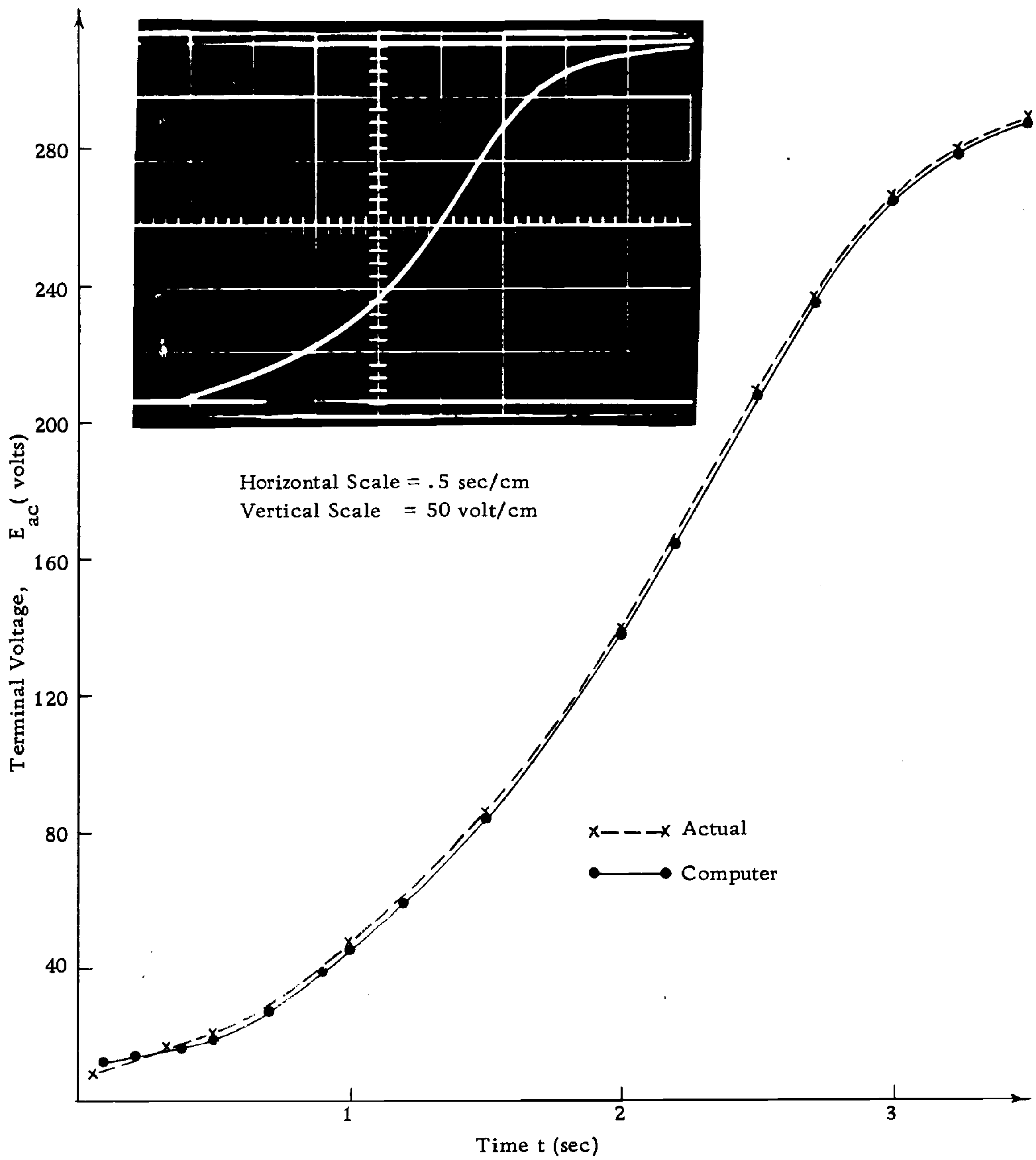


Figure 25b. Case 2 Transient Response (I_f = .78 amp)

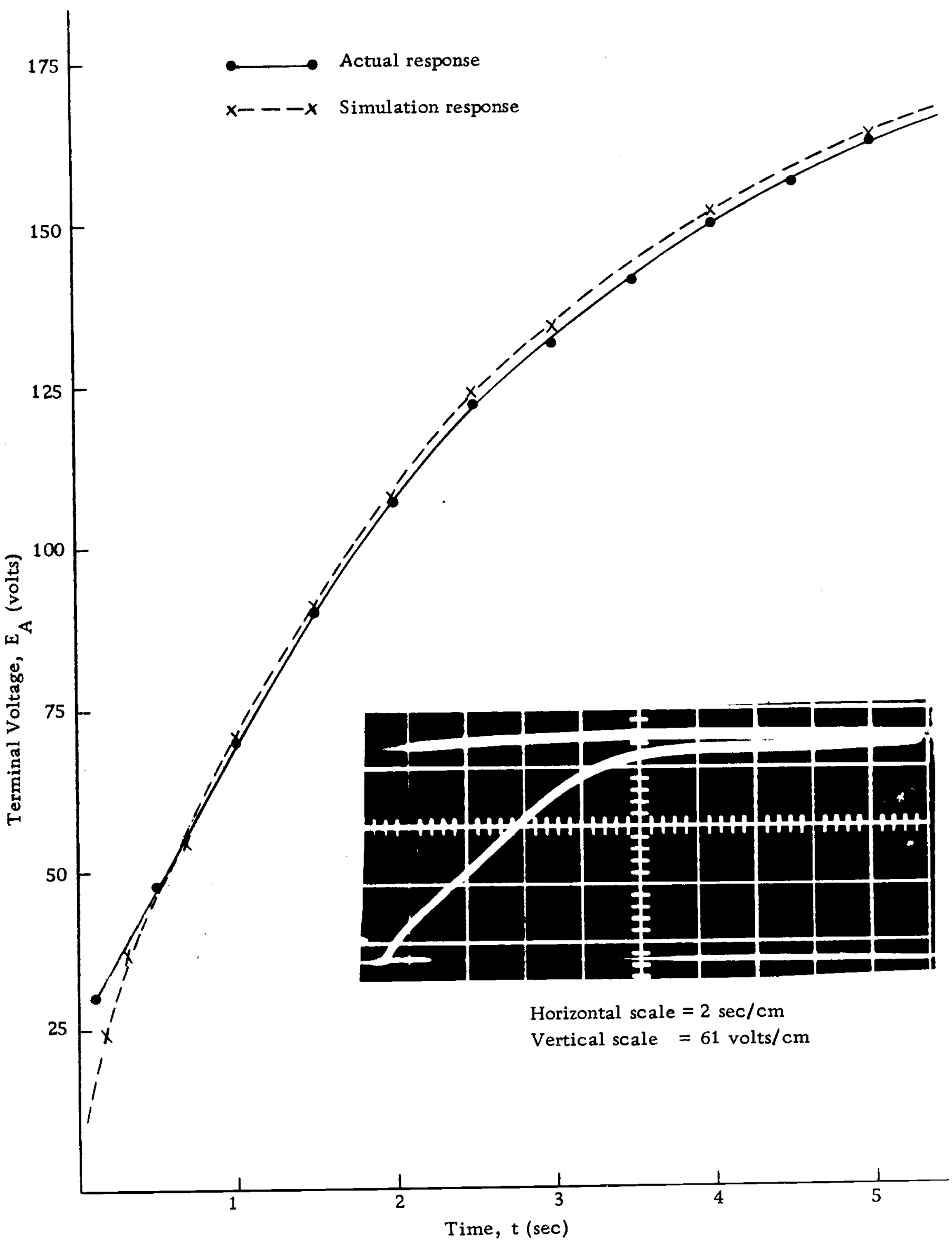


Figure 25c. Case 3 transient response ($I_f = .368$, $I_\ell = 8.19$)

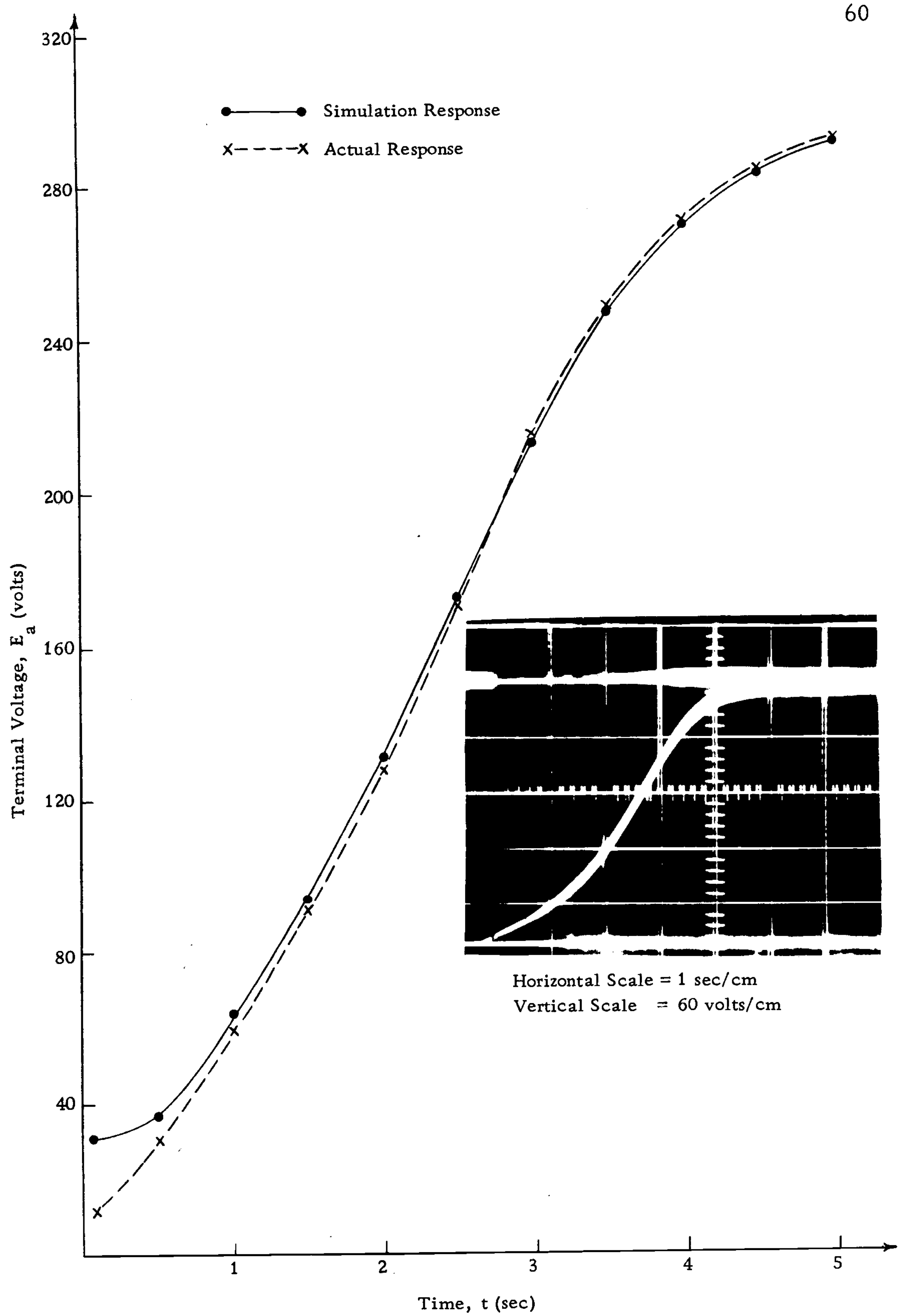


Figure 25d. Case 4 Transient Response ($I_f = .76$, $I_\ell = 29.01$)

VI. CONCLUSION

It has been shown that a fairly simple simulation model is possible for the cumulative connected direct-current generator for both the transient and the steady state response.

It was proved that a good simulation model for a direct-current machine can be achieved if the following items are adopted:

(1) The mmf due to the series field, less the effect of the armature reaction, in the case of a cumulative generator, is not constant for a constant load current. It is a function of the shunt field current.

(2) The magnetization curve is better represented by a table-look-up procedure than by any other curve fitting formula.

(3) The self inductances of the shunt field, the series field and the armature are not constant, but each is a function of time and a leakage coefficient.

(4) The mutual inductances can be approximated by single value constants.

(5) Substitution for the series field inductance can be achieved with less than 10% error by using two constants. The first is called the subtransient inductance and is effective for a limited time, called the time of influence. The second constant is called the transient inductance. Both constants

and the time of influence are derived experimentally.

(6) Substitution for the inductance of the armature can be made with the same procedure as in item (5).

(7) Substitution for the shunt field inductance, due to the leakage coefficient of the shunt field current, can be achieved by two formulas. The first is called the subtransient inductance and used as the value of the inductance for a limited time. This time is a function of total resistance in the shunt field circuit. The second formula is also a function of the total resistance and is called the transient inductance. It is used after the time of influence has elapsed.

(8) An iterative solution is necessary for the simulation solution for both the steady and transient state behavior.

The application of this simulation method will contribute to more exact modeling of power system components. The accuracy gained is in the area of magnitude and time of response of the d-c generator and other systems with which it is a component for simulation.

BIBLIOGRAPHY

1. Cybulski, John, E. L. Brancato and John P. Conner. Transient performance of d-c machinery. Trans. AIEE, February 1953, p. 45-52.

2. Cybulski, John and J. P. O'Connor. Method for short-circuit calculation. Trans. AIEE, April 1952, p. 214-221.

3. Darling, A. G. and T. M. Linville. Rate of rise of short-circuit current of d-c motors and generators. Trans. AIEE, vol. 71, pt. III, January 1952, p. 314-325.

4. Frost, G. E. The short-circuit characteristics of d-c generators. Trans. AIEE, vol. 65, June 1946, p. 394-402.

5. Gandy, Theodore S. and Eimer C. Schacht. Direct-current motor and generator troubles, operation and repair. McGraw-Hill Company, N. Y., 1920. p. 30-32.

6. Hamilton, H. B. Magnetic saturation and eddy current effect in d-c machine voltage buildup. IEEE, June 1964, p. 606-610.

7. Kloeffler, Royce G., Jesse L. Brenneman and Russel M. Kerchner. Direct-current machinery. New York, The Macmillan Company, 1941. p. 1-12, 109-125.

8. Koeing, H. E. Transient response of the direct current dynamo. Trans. AIEE, 1950, Vol. 69, pt. II. p. 139-145.

9. ________________ Application of the network theory to the analysis of rotating machinery, pt. II, computating machines. Trans. AIEE, 1953, pt. II, p. 169-177.

10. Linville, T. M. Current and torque of d-c machines on short circuit. Trans. AIEE. Vol. 65, 1956. p. 956-965.

11. Linville, T. M. and H. C. Ward. Solid short circuit of d-c motors and generators, Trans. AIEE. Vol. 65, pt. 1, 1949. p. 119-122.

12. McClinton, A. T., E. L. Brancato and Robert Panoff. Transient characteristics of d-c motors and generators. Trans. AIEE, Vol. 68, pt. II. 1949. p. 1100-1106.

13. O'Conner, J. P. and J. Cybulski. d-c machines short-circuit calculation and test results. Trans. AIEE, April 1955, p. 222-231.

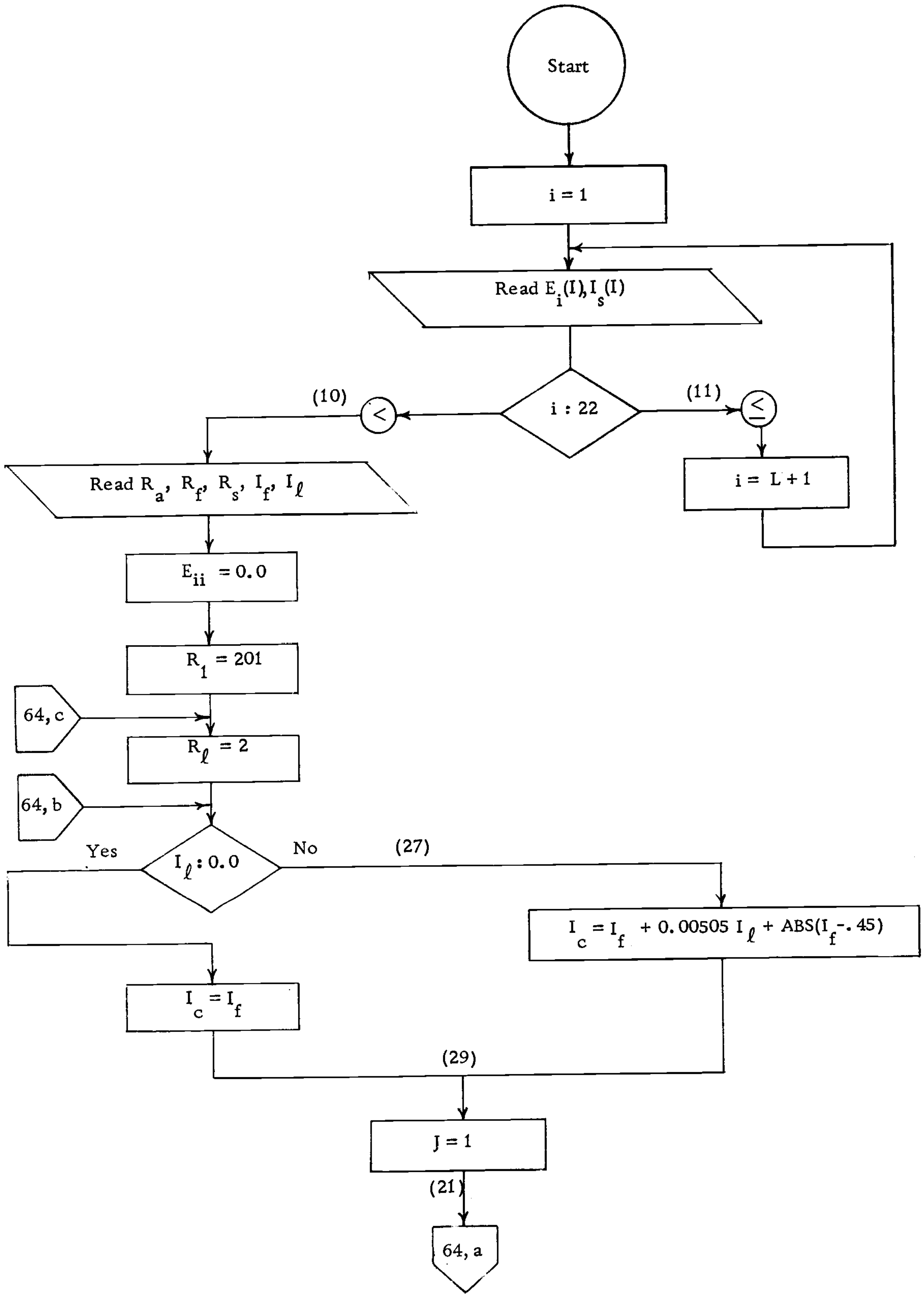
14. Scheafer, D. H. Fault transient and frequency spectra of a d-c generator. Trans. AIEE, April 1953, p. 169-177.

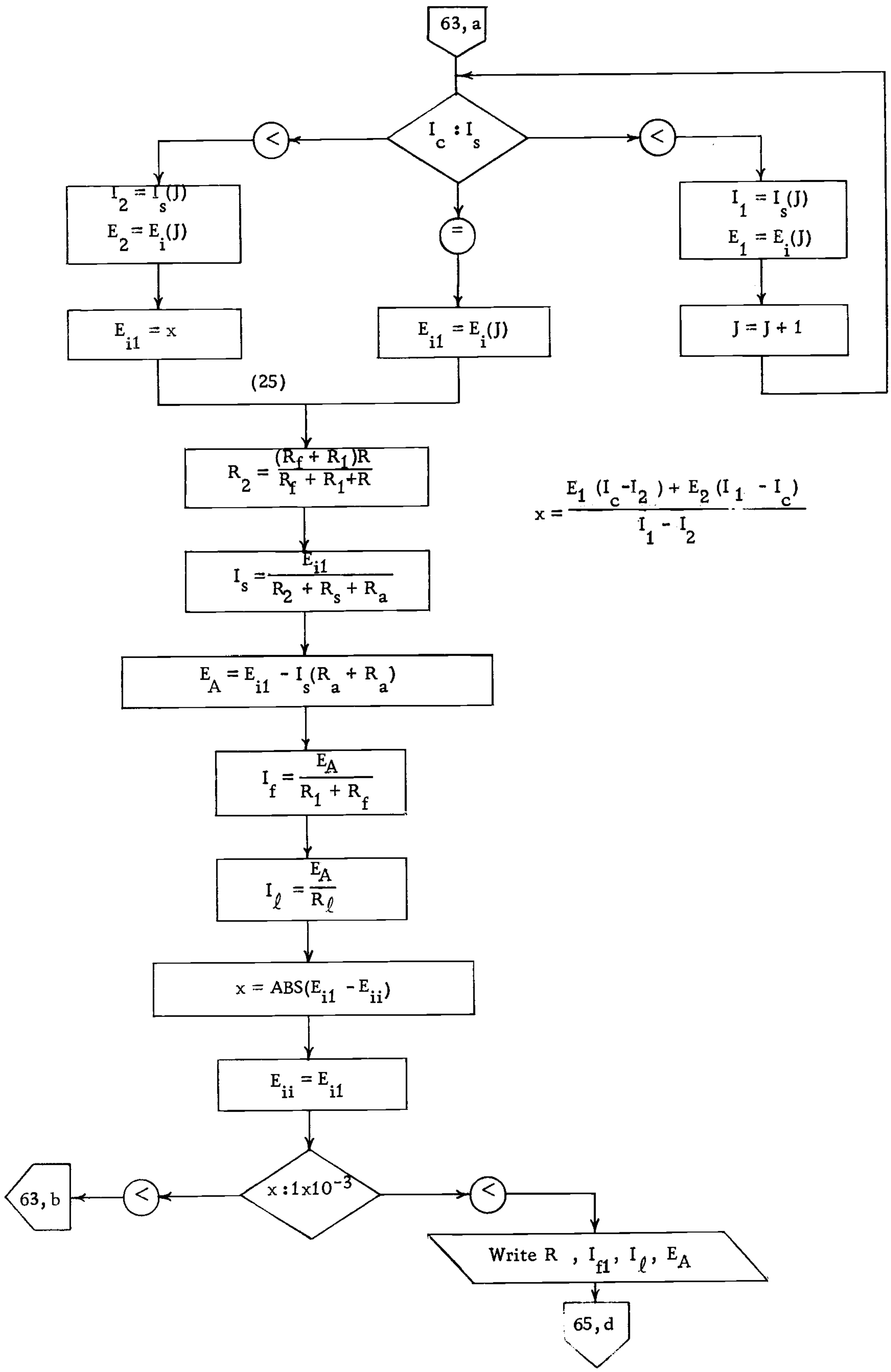
15. Thaler, George J. and Milton L. Wilox. Electric machine dynamics and steady state. John Wiley and Sons, New York. 1966, p. 135-137.

16. Trutt, Frederick C. and Ralph E. Hopkins. Representation of the magnetization characteristics of d-c machines for computer use. IEEE. Vol. Pas-87, No. 3, March 1968, p. 665-669.

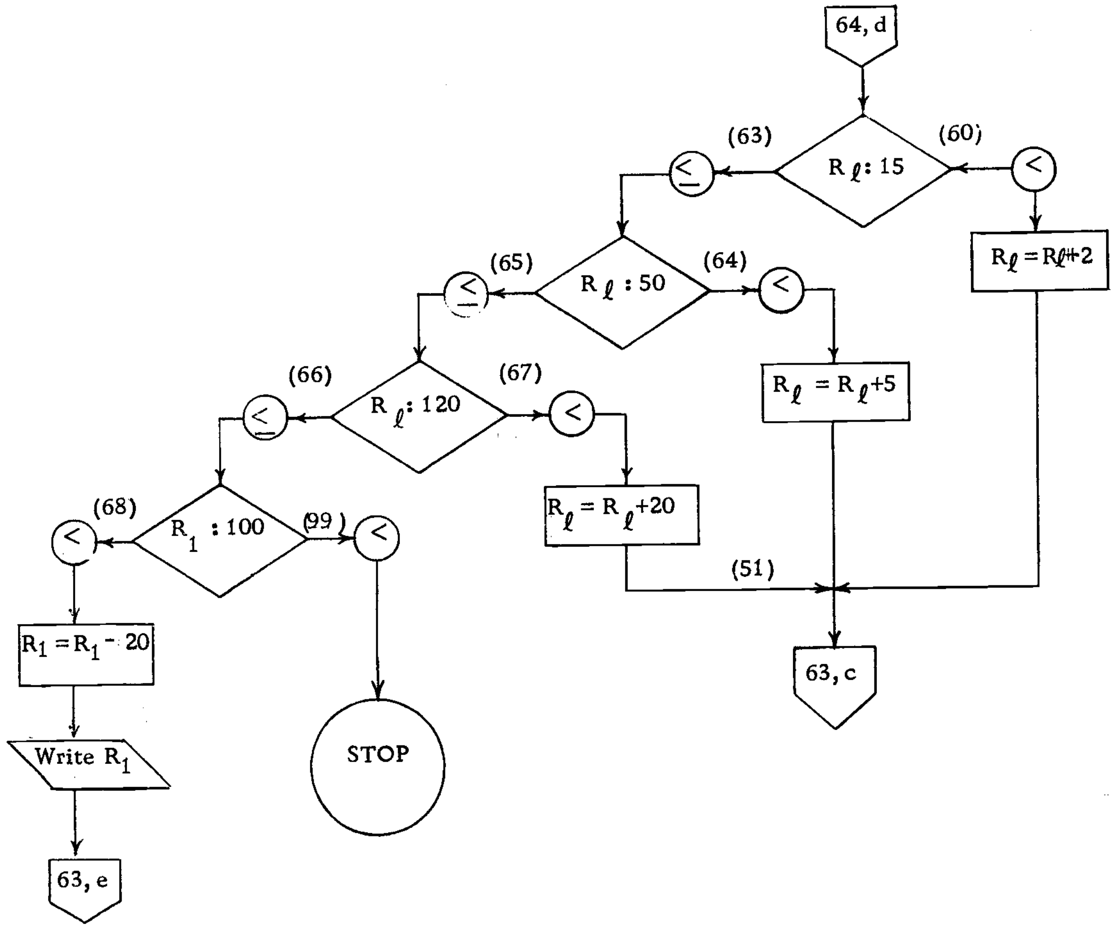
17. Tustin, A. Direct-current machines for control system. F.N. Spon. Ltd, London, 1952, p. 93, 245-246.

APPENDICES


Start
i = 1
Read E_i(I), I_s(I)
i : 22
(10)
<
(11)
≤
i = L + 1
Read R_a, R_f, R_s, I_f, I_ℓ
E_ii = 0.0
R_1 = 201
64, c
R_ℓ = 2
64, b
Yes
I_ℓ : 0.0
No
(27)
I_c = I_f + 0.00505 I_ℓ + ABS(I_f -.45)
I_c = I_f
(29)
J = 1
(21)
64, a



63, a
I_c : I_s
<
=
<
I_2 = I_s(J)
E_2 = E_i(J)
I_1 = I_s(J)
E_1 = E_i(J)
E_i1 = x
E_i1 = E_i(J)
J = J + 1
(25)
R_2 = (R_f + R_1)R / (R_f + R_1 + R)
x = [E_1 (I_c - I_2) + E_2 (I_1 - I_c)] / (I_1 - I_2)
I_s = E_i1 / (R_2 + R_s + R_a)
E_A = E_i1 - I_s(R_a + R_a)
I_f = E_A / (R_1 + R_f)
I_ℓ = E_A / R_ℓ
x = ABS(E_i1 - E_ii)
E_ii = E_i1
x : 1x10^-3
<
<
63, b
Write R , I_f1, I_ℓ, E_A
65, d



64, d
(63)
(60)
R ℓ : 15
≤
<
R ℓ = R ℓ +2
(65)
(64)
R ℓ : 50
≤
<
R ℓ = R ℓ +5
(66)
(67)
R ℓ : 120
≤
<
R ℓ = R ℓ +20
(68)
(99)
R 1 : 100
<
<
(51)
R 1 = R 1 - 20
Write R 1
STOP
63, c
63, e

APPENDIX B

Computer Program for the Steady State Response

```
      PROGRAM    AAA
      DIMENSION EI(30),CS(30)
 2    FORMAT ( 3(F10.4))
 5    FORMAT (5X,4(F10.4,5X))
 1    FORMAT (2(F10.4))
      EII = 0.0
      I = 1
11    READ (60,1)EI(I),CS(I)
      I = I + 1
      IF (I -22)11,11,12
12    READ (60,2) RA,RF,RS
      READ (60,2)CF1,CL1,CS1
      IF (EOF(60)) GO TO 99
      R1 = 201.0
69    WRITE (61,6)
 6    FORMAT (5X,'REHOSTATE RESISITANCE',/)
      WRITE (61,4)R1
 4    FORMAT (10X,F10.4)
      RL = 2.0
51    WRITE (61,7)
 7    FORMAT (5X,'LOAD RES   SHUNT CURRENT   LOADCURRENT   TERMINATVOL',/)
40    IF(CL1-0.005)28,28,27
28    CC = CF1
      GO TO 29
27    CC = CF1 + 0.00505*CS1 + 0.08*ABS(CF1-0.45)
29    J = 1
21    IF(CS(J) - CC) 22,23,24
22    CC1 = CS(J)
      E11 = EI (J)
      J = J + 1
      GO TO 21
24    CC2 = CS (J)
      E12 = EI (J)
      EI1 = E11 + (((CC-CC1)*(E12-E11))/(CC2-CC1))
      GO TO 25
23    EI1 = Ei (J)
25    RR = ((RF+ R1)*RL)/(RF + R1 + RL)
      CS1 = EI1/(RR+RS+RA)
      EA1 = EI1-CS1*(RS+RA)
      CF1 = EA1/(RF+R1)
      CL1 = EA1/RL
      X = ABS(EI1-EII)
      EII = EI1
      IF(X-0.001 )50,50,40
50    EII = 5.0
      WRITE (61,5)RL,CF1,CL1,EA1
      IF(RL-15.0)60,60,63
```

```
60  RL = RL+2.0
    GO TO 51
63  IF(RL-50.)64,64,65
64  RL = RL+5.0
    GO TO 51
65  IF(RL-120.)66,66,67
66  RL = RL+20.
    GO TO 51
67  IF(R1-100.0)99,99,68
68  R1=R1-20.
    GO TO 69
99  STOP
    END

' '
     -10.5        -0.05
      11.5         0.0
      33.5         0.05
      55.5         0.1
      77.5         0.15
      99.          0.2
     120.5         0.25
     142.          0.3
     163.5         0.35
     185.0         0.4
     203.0         0.45
     220.          0.5
     234.0         0.55
     247.          0.6
     259.5         0.65
     272.0         0.7
     283.0         0.75
     293.0         0.8
     303.0         0.85
     313.0         0.9
     323.0         0.95
     325.0         2.5
       0.94      272.0         0.033
       0.0         0.0           0.0
' '
' LOGOFF
```

Note: Whenever C is used in the front of a variable, it means the current.
Example: CF1 = I_{f1}.

APPENDIX C

Results of Computer Simulation for Steady State Response

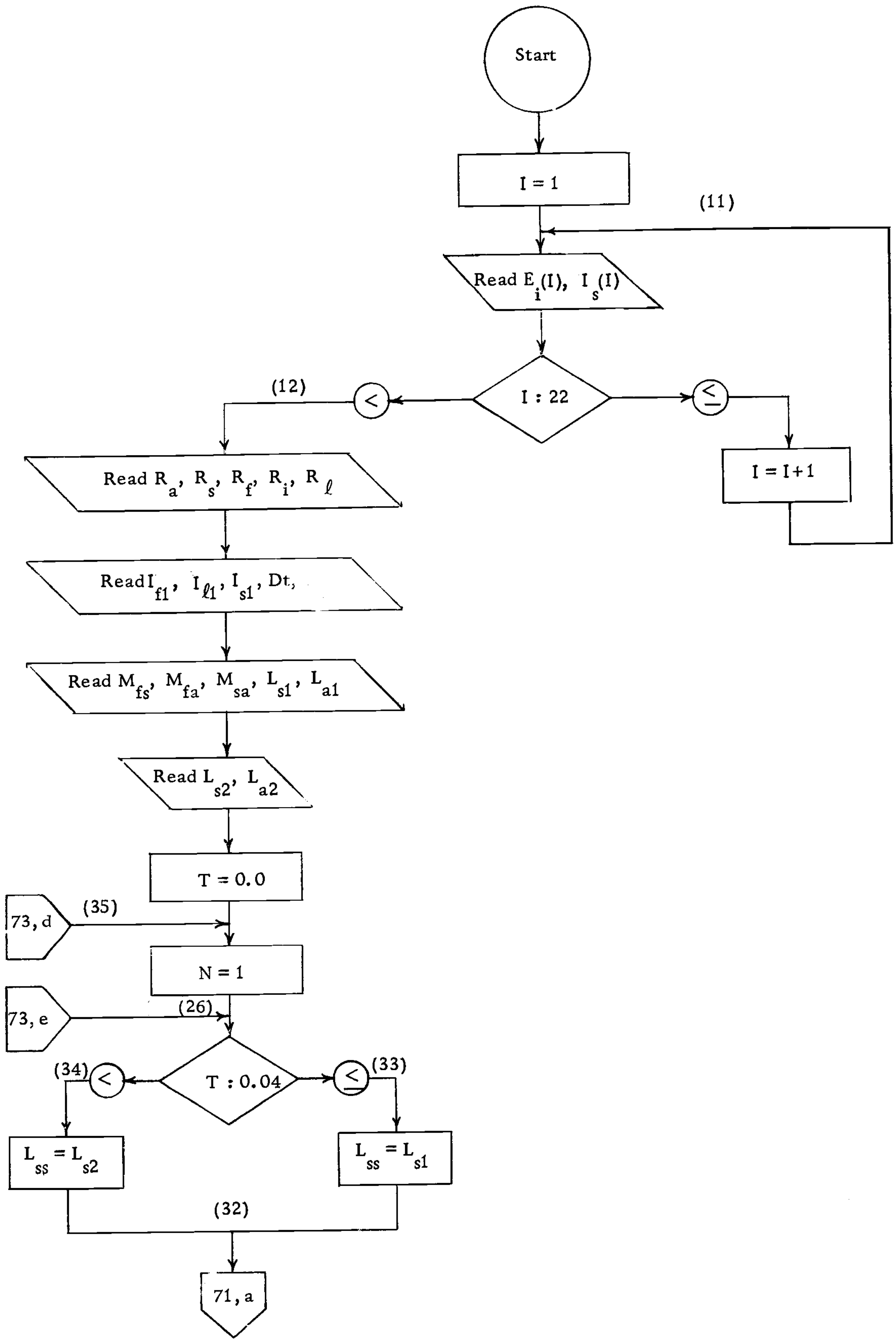
Load Current (amp.)	Terminal Voltage (volt)	Load Current (amp.)	Terminal Voltage (volt)
Case 1		**Case 2**	
1.36	178.59	1.54	201.48
2.00	181.99	2.86	203.36
3.63	184.90	4.03	205.76
5.22	187.79	5.08	208.17
7.37	191.67	8.15	211.40
12.40	198.45	13.39	212.00
16.80	201.80	17.90	215.00
20.30	202.89	21.50	215.40
25.50	204.00	26.95	215.63
34.00	204.56	35.90	214.40
51.05	204.22	53.04	214.18
Case 3		**Case 4**	
2.10	275.27	2.25	295.07
3.02	274.90	4.14	293.90
5.37	273.94	6.36	292.53
7.58	272.93	9.37	290.66
10.43	271.27	13.71	287.97
16.73	267.63	20.28	283.9
22.05	264.54	27.92	279.17
32.32	258.58	34.39	275.15
42.15	252.89		
60.55	292.22	44.29	265.72

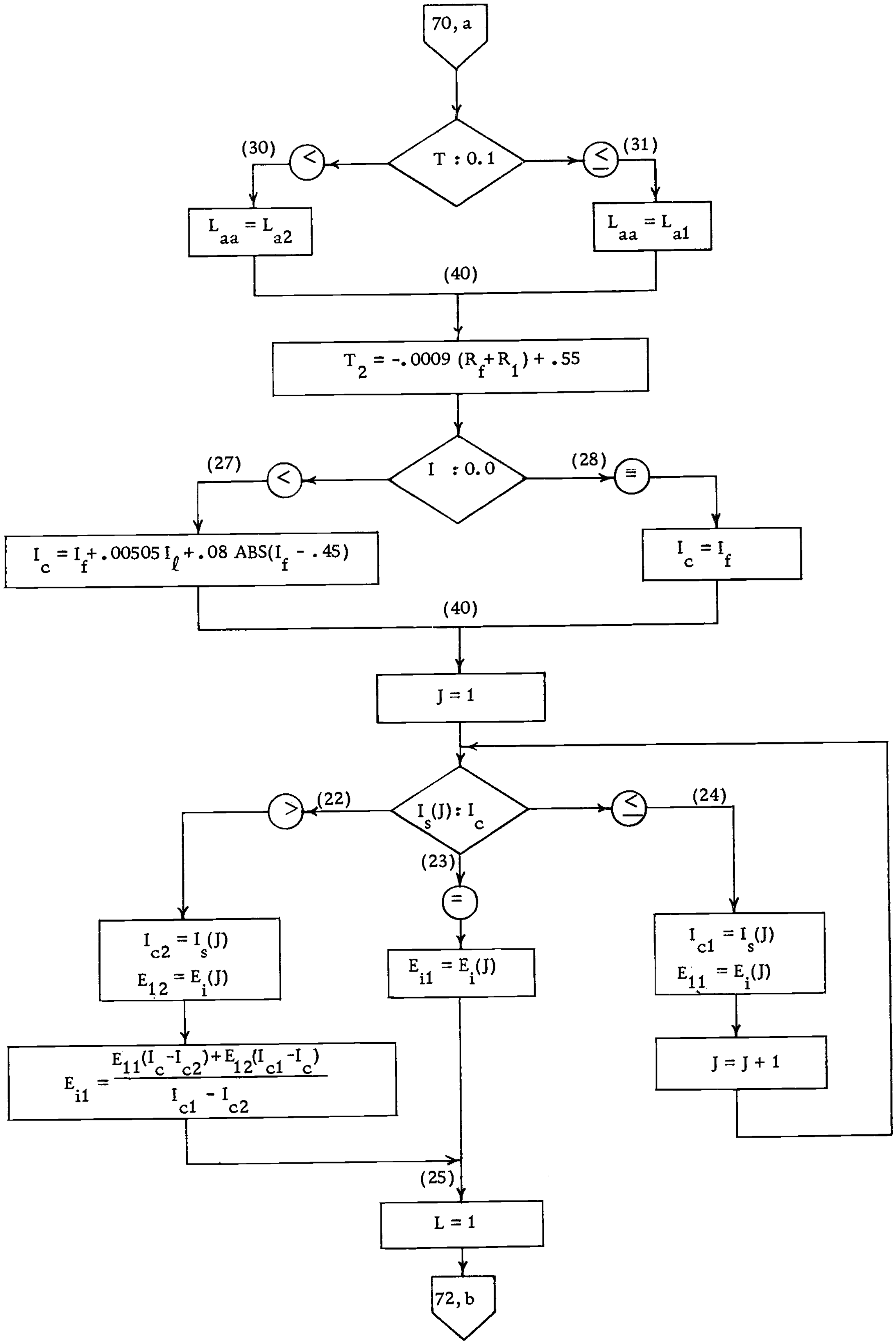
APPENDIX D

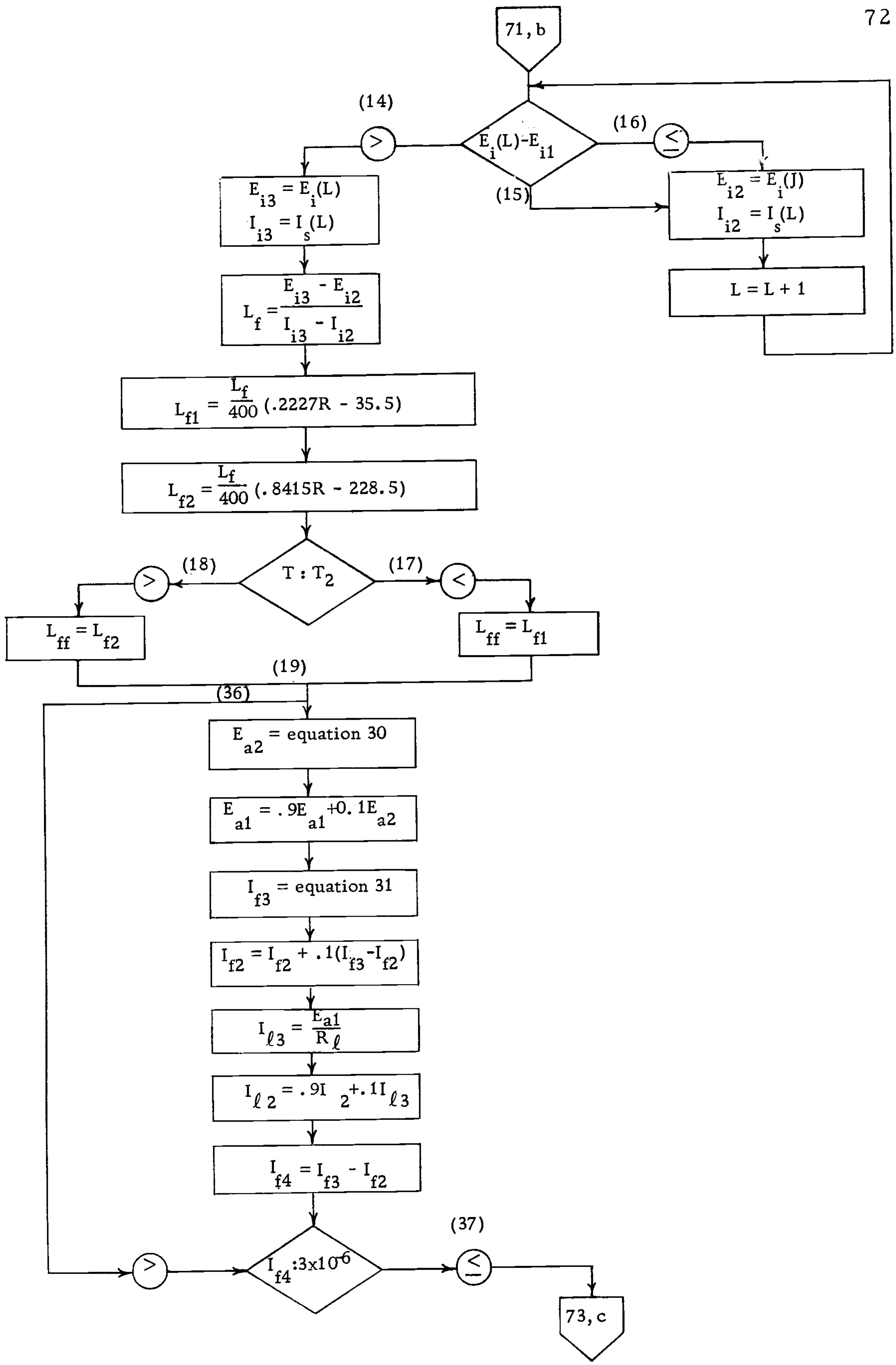
Steady State Voltage Response of the Machine

I_ℓ (amp.)	Case 1 E_A (volts)	Case 2 E_A (volts)	Case 3 E_A (volts)	Case 4 E_A (volts)
0	180	206	276	296
5	186	210	275	295
10	190	214	274	294
15	196	218	272.5	293
20	201	221	271	292
25	204	223	268.5	289
30	205	222	266	286
35	206	221	262	282
40	208	220	258	277
50	201	214	250	268

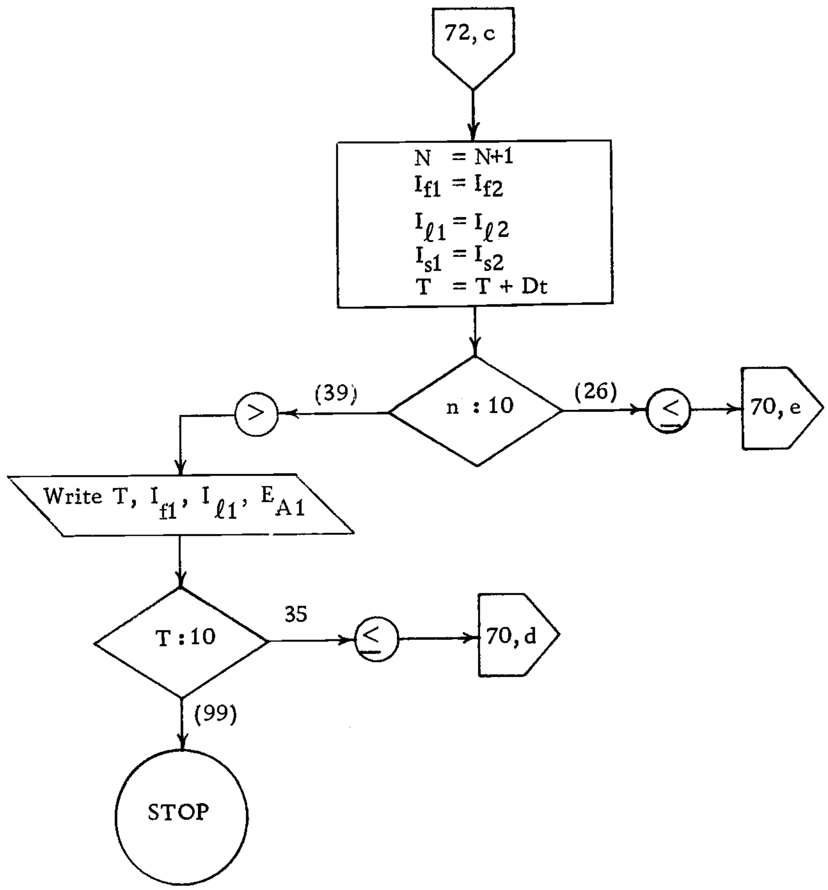
APPENDIX E. Flow Chart for the Transient Response




70, a
T : 0.1
(30)
<
(31)
≤
L_aa = L_a2
L_aa = L_a1
(40)
T_2 = -.0009 (R_f + R_1) + .55
I : 0.0
(27)
<
(28)
=
I_c = I_f + .00505 I_ℓ + .08 ABS(I_f - .45)
I_c = I_f
(40)
J = 1
I_s(J) : I_c
(22)
>
(24)
≤
(23)
=
I_c2 = I_s(J)
E_12 = E_i(J)
E_i1 = E_i(J)
I_c1 = I_s(J)
E_11 = E_i(J)
J = J + 1
E_i1 = [E_11(I_c - I_c2) + E_12(I_c1 - I_c)] / (I_c1 - I_c2)
(25)
L = 1
72, b



71, b
(14)
>
$E_i(L)-E_{i1}$
(16)
≤
(15)
$E_{i3} = E_i(L)$
$I_{i3} = I_s(L)$
$E_{i2} = E_i(J)$
$I_{i2} = I_s(L)$
L = L + 1
$L_f = \frac{E_{i3} - E_{i2}}{I_{i3} - I_{i2}}$
$L_{f1} = \frac{L_f}{400}(.2227R - 35.5)$
$L_{f2} = \frac{L_f}{400}(.8415R - 228.5)$
(18)
>
$T : T_2$
(17)
<
$L_{ff} = L_{f2}$
$L_{ff} = L_{f1}$
(19)
(36)
E_{a2} = equation 30
$E_{a1} = .9E_{a1} + 0.1E_{a2}$
I_{f3} = equation 31
$I_{f2} = I_{f2} + .1(I_{f3} - I_{f2})$
$I_{\ell 3} = \frac{E_{a1}}{R_\ell}$
$I_{\ell 2} = .9I_{\ell 2} + .1I_{\ell 3}$
$I_{f4} = I_{f3} - I_{f2}$
(37)
>
$I_{f4} : 3x10^{-6}$
≤
73, c


72, c
N = N+1
$I_{f1} = I_{f2}$
$I_{\ell 1} = I_{\ell 2}$
$I_{s1} = I_{s2}$
T = T + Dt
n : 10
(39)
>
(26)
≤
70, e
Write T, I_{f1}, $I_{\ell 1}$, E_{A1}
T : 10
35
≤
70, d
(99)
STOP

APPENDIX F

Computer Program for the Transient Response

```
      PROGRAM   AAA
      DIMENSION EI 930),CS(30)
    2 FORMAT (5(F10.4))
    5 FORMAT (5X,4(F10.4,5X))
    1 FORMAT (2(F10.4))
      I = 1
   11 READ (60,1)EI(I),CS(I)
      I = I+1
      IF(I-22)11,11,12
   12 READ (60,2)RA,RS,RF,R1,RL
      READ (60,2)CF1,CL1,ULL,DT,CS1
      READ (60,2)UFS,UFA,USA,US1,UA1
      READ (60,1)UA2,US2
      IF(EOF(60)) GO TO 99
      CF2 = CF1
      CL2 = CL1
      CS1 = CL1+CF1
      CS2 = CS1
      T = 0.0
   35 N = 1
   26 CS1 = CF1+CL1
      IF(T-0.1)30,30,31
   30 UAA = UA1
      GO TO 32
   31 UAA = UA2
   32 IF(T-0.04)33,33,34
   33 USS = US1
      GO TO 40
   34 USS = US2
   40 IF(CL1-0.005)28,28,27
   28 CC = CF1
      GO TO 29
   27 CC = CF1+0.00525*CS1+0.08*ABS(CF1-0.45)
   27 CC = CF1+0.00505*CS1+0.08*ABS(CF1-0.45)
   29 J = 1
   21 IF(CS(J)-CC)22,23,24
   22 CC1 = CS(J)
      E11 = E1(J)
      J = J+1
      GO TO 21
   24 CC2 = CS(J)
      E12 = EI(J)
      EI1 = E11+(((CC-CC1)*(E12-E11))/(CC2-CC1))
      GO TO 25
   23 EI1 = EI(J)
```

```
25  L = 1
13  IF(EI(L)-EI1)14,15,16
14  E12 = EI(L)
    C12 = CS(L)
    L = L+1
    GO TO 13
15  E12 = EI(L)
    C12 = CS(L)
    L = L+1
    GO TO 13
16  E13 = EI(L)
    C13 = CS(L)
    UF = (E13-E12)/(C13-C12)
    R = R1+RF
    T2 = -0.009*R+0.55
    UF1 = UF*(0.227*R-35.55)/400.
    UF2 = UF*(0.8415*R-228.2)/400.
    IF(T-T2)17,17,18
17  UFF = UF1
    GO TO 19
18  UFF = UF2
19  T = T+DT
    TT = 1.0/DT
36  EA2 = EI1-CS2*(RS+RA)-(CS2-CS1)*(USS+UAA+USA)*TT-(UFA+UFS)*(CF2-CF1)
   1*TT
    EA1 = EA1+0.1*(EA2-EA1)
    CF3 = DT*(EA1)-DT*RF*CF2+UFF*CF1-(UFA+UFS)*(CS2-CS1)
    CF3 = CF3/UFF
    CF2 = CF2+0.1*(CF3-CF2)
    CL3 = EA1/RL
    CL2 = CL2+0.1*(CL3-CL2)
    CS2 = CF2+CL2
    CF4 = CF3-CF2
    IF(CF4-0.000003)37,37,36
37  IF(N-10)38,39,39
38  N = N+1
    CS1 = CS2
    CL1 = CL2
    CF1 = CF2
    GO TO 26
39  WRITE (61,5)T,CF1,CL1,EA1
    CS1 = CS2
    CL1 = CL2
    CF1 = CF2
    IF(T-10.0)35,35,99
99  STOP
    END
```

```
''
    -10.5      -0.05
     11.5       0.0
     33.5       0.05
     55.5       0.1
     77.5       0.15
     99.        0.2
    120.5       0.25
    142.        0.3
    163.5       0.35
    185.0       0.4
    203.0       0.45
    220.        0.5
    234.0       0.55
    247.        0.6
    259.5       0.65
    272.0       0.7
    283.0       0.75
    293.0       0.8
    303.0       0.85
    313.0       0.9
    323.0       0.95
    325.0       2.5
       0.94     0.033     465.60     140.      99999.999
       0.0      0.0         0.0        0.01        0.0
       0.53     0.0718      0.0017     0.0106      0.1013
       0.378    0.0257
''
'LOGOFF
```

Note: Whenever U is used in the front of a variable or constant, it means the inductance. Example: U_{fs} is for L_{fs}.

Also whenever C is used in the front of a variable, it means the current. Example: CF1 = I_{f1}.

APPENDIX G

Results of Computer Simulation for Transient Response

Time (second)	Shunt Field Current (amp.)	Load Current (amp.)	Terminal Voltage (volt)
	Case 1		
0.0	0.000	0.0	0.00
0.22	0.022	0.0	20.95
0.44	0.036	0.0	27.61
0.66	0.048	0.0	32.65
0.88	0.060	0.0	37.62
1.1	0.071	0.0	42.50
1.32	0.082	0.0	47.31
1.54	0.093	0.0	52.04
1.76	0.103	0.0	56.70
1.98	0.114	0.0	61.28
2.2	0.124	0.0	65.79
2.42	0.134	0.0	70.22
2.64	0.144	0.0	74.59
2.86	0.154	0.0	78.89
3.08	0.164	0.0	83.07
3.3	0.173	0.0	87.15
3.52	0.182	0.0	91.11
3.74	0.191	0.0	94.97
3.96	0.200	0.0	100.57
4.29	0.218	0.0	104.17
4.51	0.221	0.0	107.68
4.78	0.229	0.0	111.10
4.95	0.237	0.0	114.42
5.17	0.244	0.0	117.65
5.39	0.251	0.0	120.80
5.61	0.259	0.0	123.86
5.83	0.266	0.0	126.84
6.05	0.272	0.0	129.74
6.27	0.279	0.0	132.56
6.49	0.285	0.0	135.31
6.71	0.291	0.0	137.98
6.93	0.298	0.0	140.58
7.04	0.301	0.0	141.86
7.15	0.303	0.0	143.11

APPENDIX G (continued)

Time (second)	Shunt Field Current (amp.)	Load Current (amp.)	Terminal Voltage (volt)
	Case 2		
.1000	.0025	0.0	12.58
.2000	.0054	0.0	13.83
.3000	.0083	0.0	15.10
.4000	.0112	0.0	16.40
.5000	.0143	0.0	17.73
.6000	.0247	0.0	22.21
.7000	.0366	0.0	27.44
.8000	.0494	0.0	33.04
.9000	.0630	0.0	39.02
1.0000	.0776	0.0	45.41
1.1000	.0932	0.0	52.24
1.2000	.1098	0.0	59.53
1.3000	.1276	0.0	67.33
1.4000	.1466	0.0	75.67
1.5000	.1673	0.0	84.54
1.6000	.1893	0.0	93.95
1.7000	.2126	0.0	103.94
1.8000	.2373	0.0	114.54
1.9000	.2636	0.0	125.79
2.0000	.2914	0.0	137.71
2.1000	.3209	0.0	150.36
2.2000	.3523	0.0	163.78
2.3000	.3855	0.0	178.02
2.4000	.4237	0.0	192.67
2.5000	.4658	0.0	207.47
2.6000	.5104	0.0	221.87
2.7000	.5581	0.0	235.05
2.8000	.6026	0.0	246.59
2.9000	.6419	0.0	256.46
3.0000	.6752	0.0	264.80
3.1000	.7036	0.0	271.74
3.2000	.7293	0.0	277.47
3.3000	.7495	0.0	281.94
3.4000	.7662	0.0	285.32
3.5000	.7788	0.0	287.85
3.6000	.7881	0.0	289.73
3.7000	.7950	0.0	291.12
3.8000	.8001	0.0	292.15

APPENDIX G (continued)

Time (second)	Shunt Field Current (amp.)	Load Current (amp.)	Terminal Voltage (volt)
	Case 2 (continued)		
3.9000	.8038	0.0	292.91
4.0000	.8066	0.0	293.47
4.1000	.8087	0.0	293.89
4.2000	.8102	0.0	294.20
4.3000	.8114	0.0	294.43
4.4000	.8122	0.0	294.60
4.5000	.8128	0.0	294.73
4.6000	.8133	0.0	294.82
4.7000	.8136	0.0	294.89
4.8000	.8139	0.0	294.94
4.9000	.8141	0.0	294.98
5.0000	.8142	0.0	295.01
5.1000	.8143	0.0	295.03
5.2000	.8144	0.0	295.04
	Case 3		
.1000	.0046	1.3486	30.5955
.3000	.0224	1.6664	37.7040
.5000	.0458	2.1061	47.6174
.7000	.0678	2.5331	57.1445
.9000	.0885	2.9361	66.1359
1.1000	.1081	3.3162	74.6139
1.3000	.1269	3.6721	82.5501
1.5000	.1444	4.0044	89.9588
1.7000	.1607	4.3145	96.8740
1.9000	.1759	4.6060	103.3282
2.1000	.1902	4.8742	109.3532
2.3000	.2035	5.1263	114.9758
2.5000	.2159	5.3617	120.2245
2.7000	.2274	5.5814	125.1233
2.9000	.2382	5.7865	129.6956
3.1000	.2483	5.9779	133.9640
3.3000	.2577	6.1565	137.9470
3.5000	.2665	6.3233	141.6652
3.7000	.2747	6.4790	145.1358
3.9000	.2824	6.6242	148.3747
4.1000	.2895	6.7598	151.3983
4.3000	.2962	6.8864	154.2204
4.5000	.3024	7.0045	156.8542

APPENDIX G (continued)

Time (second)	Shunt Field Current (amp.)	Load Current (amp.)	Terminal Voltage (volt)
	Case 3 (continued)		
4.7000	.3082	7.1148	159.3128
4.9000	.3136	7.2177	161.6078
5.1000	.3187	7.3138	163.7495
5.3000	.3234	7.4035	165.7489
5.5000	.3278	7.4872	167.6153
5.7000	.3319	7.5653	169.3570
5.9000	.3358	7.6383	170.9831
6.1000	.3393	7.7064	172.5012
6.3000	.3427	7.7699	173.9181
6.5000	.3458	7.8292	175.2406
6.7000	.3487	7.8846	176.4735
6.9000	.3518	7.9335	177.5614
7.1000	.3545	7.9754	178.4911
7.3000	.3567	8.0111	179.2827
7.5000	.3586	8.0415	179.9569
7.7000	.3603	8.0673	180.5310
7.9000	.3617	8.0893	181.0196
8.1000	.3628	8.1081	181.4358
8.3000	.3638	8.1240	181.7900
8.5000	.3647	8.1376	182.0913
8.7000	.3654	8.1491	182.3481
8.9000	.3660	8.1589	182.5663
9.1000	.3666	8.1673	182.7518
9.3000	.3670	8.1744	182.9096
9.5000	.3674	8.1804	183.0439
9.7000	.3677	8.1856	183.1581
9.9000	.3680	8.1899	183.2550
10.0000	.3681	8.1918	183.2977
	Case 4		
0.1000	.0007	3.0931	31.2258
0.2000	.0038	3.2463	32.5904
0.3000	.0068	3.3841	33.9735
0.4000	.0098	3.5248	35.3818
0.5000	.0129	3.6669	36.8037
0.6000	.0237	3.9953	40.4148
0.7000	.0354	4.5134	45.6439
0.8000	.0474	5.0616	51.1441
0.9000	.0597	5.6282	56.8272

APPENDIX G (continued)

Time (second)	Shunt Field Current (amp.)	Load Current (amp.)	Terminal Voltage (volt)
	Case 4 (continued)		
1.0000	.0725	6.2130	62.6925
1.1000	.0856	6.8165	68.7423
1.2000	.0995	7.4354	74.9485
1.3000	.1137	8.0718	81.3293
1.4000	.1284	8.7261	87.8886
1.5000	.1434	9.3986	94.6315
1.6000	.1589	10.0899	101.5629
1.7000	.1749	10.8006	108.6882
1.8000	.1912	11.5311	116.0128
1.9000	.2080	12.2821	1]2.5422
2.0000	.2253	13.0541	131.2823
2.1000	.2431	13.8477	139.2389
2.2000	.2614	14.6645	147.4180
2.3000	.2801	15.5021	155.8259
2.4000	.2995	16.3641	164.4689
2.5000	.3225	17.2176	172.9752
2.6000	.3458	18.0668	181.4564
2.7000	.3702	18.9025	189.8024
2.8000	.3948	19.7224	197.9532
2.9000	.4229	20.4953	205.6546
3.000	.4502	21.2260	212.9125
3.1000	.4768	21.9757	220.4675
3.2000	.5027	22.7176	227.8486
3.3000	.5273	23.4222	234.8585
3.4000	.5506	24.0884	241.4861
3.5000	.5740	24.7099	247.6496
3.6000	.5969	25.2824	253.3294
3.7000	.6190	25.7985	258.4346
3.8000	.6393	26.2619	263.0200
3.9000	.6574	26.6794	267.1061
4.0000	.6734	27.0426	270.7444
4.1000	.6877	27.3701	273.9840
4.2000	.7005	27.6616	276.8684
4.3000	.7118	27.9212	279.4367
4.4000	.7219	28.1523	281.7235
4.5000	.7309	28.3582	283.7597
4.6000	.7389	28.5414	285.5726
4.7000	.7461	28.7046	287.1869
4.8000	.7524	28.8498	288.6242
4.9000	.7581	28.9792	289.9039
5.0000	.7631	29.0944	291.0434

APPENDIX H

Magnetization Curve for the Shunt Field

I_f (amp.)	V_1 (volts)	V_2 (volts)	E_i (volts)	V_2/I_f (ohm)
0.0	0.0	0.0	11.5	0.0
0.05	300	16.0	33.5	320
0.1	300	27.0	55.5	270
0.15	300	40.5	77.5	270
0.2	300	54	99	270
0.27	300	73.7	131	273
0.35	300	94.5	164	270
0.41	300	111	187	272
0.48	300	131	212	272
0.57	300	156	240	273
0.63	300	173	254	275
0.7	300	194	272	277
0.79	300	220	292	277
0.83	300	231	300	272

APPENDIX I

Magnetization Curve for the Series Field

I_s (amp.)	V_1 (volts)	V_2 (volts)	E_i (volts)	V_2/I (ohm)
0.0	0.0	0.0	9.6	0.0
5	120	0.175	19	0.035
7.5	120	0.262	23.5	0.034
10	120	0.342	28	0.0342
15	120	0.52	41	0.0347
20	120	0.65	53.5	0.0325
25	120	0.82	65.5	0.033
30	120	0.98	79.5	0.033

APPENDIX J

Magnetization Curves Under Load (30 amp.)

I_f (amp.)	E_f (volts)	E_A (volts)	E_i (volts)
0.09	29	92	121.5
0.14	40	110	139.5
0.2	56	134	163.5
0.28	76	161	190.5
0.35	97	183	212.5
0.46	127	216	245.5
0.55	153	240	269.5
0.635	176	260	289.5
0.7	194	274	303.5
0.78	218	278	307.5

APPENDIX K

Resistance of the Armature Winding

Supporting		Against	
V (volts)	I (amp.)	V (volts)	I (amp.)
4.7	0.0	4.7	0.0
6.5	1	0.0	2.5
9.4	2.5	5	5.5
12.5	4.5	12	10.5
14	6.5	16	15
15.5	8.5	19.2	19
17	10	22	22
19	12	24.8	25
21.5	14.6	27	27.5
24.5	17.5		
28.6	22		
31.5	25		
32.5	26.5		