AN ABSTRACT OF THE DISSERTATION OF

Pejmon Sadri for the degree of Doctor of Philosophy in Mathematics Education presented on April 29, 2008.

Title: Intentions and Practices of an Expert Middle School Mathematics Teacher in the Context of a Reformed-based Curriculum

Abstract approved:

Margaret L. Niess

The movement to reform mathematics teaching and learning in schools began in the 1980s. After dedicating vast resources to support reform efforts since then, the transition to improve mathematics teaching and learning in schools has not occurred. In the majority of mathematics classrooms teachers still rely on traditional teaching strategies despite the use of reform-based curricula aimed at improving student learning of mathematics. The purpose of this study was to investigate the intended instructional objectives and practices of an expert middle school mathematics teacher in the context of a reform-based mathematics curriculum. Data collected from interviews, audio recording of whole class discussions, classroom observations, and detailed analyses of several lesson segments were used to create a case study for the teacher to describe a model of her practice as an expert mathematics teacher. The theoretical framework used for this

study was Schoenfeld's (1998) model of teaching-in-context that defined key contributories to teachers' practice: beliefs, intentions or goals, and their content and pedagogical content knowledge. The research questions in the study considered the teacher's intentions, her conceptions about mathematics and teaching and learning, the instructional strategies she frequently used, the curricula with which she practiced, and the impact of the classroom and school communities.

The practices of the teacher in this study were reform-based. She used an active, process-oriented, collaborative approach to teaching and learning. She was given the freedom to choose the curricula from which she taught. She chose reform-based curricula for her mathematics classes, and her practices followed these curricula most closely, emphasizing connection making, pattern recognition, and problem solving in collaborative environments. Explaining solutions and consensus building among alternative strategies to solve problems dominated the discourse during whole class and small group activities. The teacher rarely used computing technologies in her teaching. Her own beliefs and corroborating opinions from collaborating middle and high school mathematics teachers were the main reasons for this, although access to the computing technologies at the school was limited. The teacher's collaboration with the members of the school community was strong and this collaboration had a significant impact on her intentions and practice.

This study resulted in a proposed model of expert mathematics teachers' practice in reform-based curriculum situations. This model included the teacher's strong and expanding content knowledge, her career-long commitment to participating in professional development programs and improving her practice, and her enthusiasm about students and the teaching profession as key factors that played an important role with respect to the her decision-making process and the actions she took as she taught in her mathematics classes.

Teaching Mathematics for Understanding: Intentions and Practices of an Expert Middle School Mathematics Teacher in the Context of a Reformed-based Curriculum

by
Pejmon Sadri

A DISSERTATION

Submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

Presented April 29, 2008

Commencement June 2008

Doctor of Philosophy dissertation of Pejmon Sadri presented on April 29, 2008.

APPROVED:

Major Professor, representing Mathematics Education

Chair of the Department of Science and Mathematics Education

Dean of the Graduate School

I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

Pejmon Sadri, Author

ACKNOWLEDGEMENTS

I would like to thank ***Dr. Margaret L. Niess*** for her guidance throughout this important project. I would like to extend her my deepest gratitude for supporting me in all my educational and professional endeavors since I began my doctoral studies at Oregon State University. I thank her for allowing me to try out my ideas about research and for guiding me in the most diligent and thorough approach possible. I would like to thank ***Dr. Lawrence B. Flick*** for his insightful and invaluable comments and input throughout this project, and for working ceaselessly to support the graduate students in the department. Without his guidance and ideas with respect to seeking and acquiring graduate assistantship funding, along those rendered to me by Dr. Niess, financial obstacles would have prevented me from ever attending Oregon State University. I would like to thank ***Dr. Larry Enochs*** for his mentorship and support of all of my educational and professional endeavors. Dr. Enochs, along with Dr. Niess and Dr. Flick, has guided me, mentored me, and has helped me grow and develop my intellectual faculties as an aspiring researcher and academic. I would like to thank ***Dr. Daniel W. Schafer*** and ***Dr. Thomas J. Wolpert*** for accepting to be the other two judges on the committee. Their questions and comments have made insightful and invaluable contributions to the development of this project.

It is my honor to have known and or to have worked with these members of my doctoral program committee.

My wonderful friends at the Educational Opportunities Program (EOP), Dr. ***Lawrence F. Griggs*** and ***Marilyn V. Stewart*** have been two of the kindest and most generous people I have known in my entire life. My admittance to the mathematics education program at Oregon State University would not have been possible without their generous support that gained me a teaching assistantship that reimbursed all my out-of-state tuition. I am once again grateful to Dr. Niess for directing me their way when I asked her guidance to help me find financial support.

The real names of the eight teachers who signed on to be the potential participant in this study, including and in particular the name of the outstanding teacher and a wonderful human being who was selected for this study, and the names of the principals who agreed to listen to what I had to say about the nature and the goals of this study and whose permission to contact the teachers was critical, will remain confidential. However, I am grateful to them beyond words. This project would not have been possible without their help.

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CHAPTER I

Intentions and Practices of an Expert Middle School Mathematics Teacher in the Context of a Reformed-based Curriculum

The Problem

Introduction

Heaton (2000), a nine-year veteran of elementary school teaching and the winner of the outstanding teacher award from the University of Vermont, describes the issues she faced when trying to teach with the Comprehensive School Mathematics Program (CSMP) curriculum - a curriculum that, even though developed prior to the current curriculum standards, promoted a similar reform-based view of mathematics learning and teaching.

> Even though I took lesson plans directly from CSMP, not much of what happened went as planned. Everything seemed "out of sync." I had been following a teacher's guide yet felt lost. I had the urge to move forward but lacked a clear sense of direction. I wanted to engage students in a mathematical discussion but did not know what to talk about. I trusted the teacher's guide but had little confidence in myself and was uncertain whether and what students were learning. I was going through the motions of teaching but doubted that anything I had done in these first 3 weeks of school resembled teaching mathematics for understanding. I had expected changing my practice to be difficult but never imagined it would be this hard. (p. 49)

Heaton's remarks are reminiscent of the difficulties many mathematics teachers face when attempting to adopt other reformed-based and innovative curricula such as those inspired by the National Council of Teachers of Mathematics (NCTM) curriculum standards (1989, 2000). The purpose of this study was to examine the intended instructional objectives and practices of an expert middle school mathematics teacher where a reform-based curriculum was adopted for the mathematics classes, and where the overall classroom and school communities may have played a role in the kind of teaching that took place in the classrooms. The research objective was to analyze a teacher's thinking, instructional practices, and interactions with the professional and school communities as she attempted to implement a curriculum and instruction based on NCTM reform principles.

An increasing number of studies have found that mathematics teachers have continued to have difficulty adopting the recommendations in the reform-based principles (Frykholm, 2004; Grant & Kline, 2000; Haug, 1998; Swarner, 1998) in spite of the availability of new curricula based on the principles. The new curricula emphasize important mathematics with real-world applications that fosters a problem-solving approach to engage and hold students' interest by connecting with what they already know, and demands that students not just provide an answer to problems but justify their solutions through reasoning and sense making. Multiple reform-based curriculum packages have been developed for various mathematics grade level courses based on NCTM's curriculum standards (e.g., *Connected Mathematics, MathScape, Interactive Mathematics Program, SIMMS Integrated Mathematics, and Core Plus*). These curricula packages include instructional guidelines and strategies that rely on recommendations from NCTM's teaching standards (NCTM, 1991) as well as NCTM's curriculum and instruction standards (NCTM, 1989, 2000). These packages present new topics, new organization of topics, and new ways of teaching for teachers as they attempt to implement the new reform-based curricula. Yet, mathematics teachers continue to have difficulty following the recommendations for teaching these curricula in their classrooms.

In light of these on-going difficulties, teacher education programs have assumed the primary responsibility for guiding preservice teachers in incorporating the principles of reform-based teaching (Adamson et al., 2003; Fernandez, 2005; Frykholm, 1996; Frykholm, 2005; McClintock et al., 2005; Wilson & Ball, 1996). However, studies have demonstrated that teacher education programs have not been as successful as expected in changing classroom practices of mathematics teachers, resulting in instruction more in line with the calls in the reform guidelines. Wilson and Ball (1996) assert that "Although many assumptions about teaching and learning are changing, many aspects of classroom practice remain" (p. 128). Manouchehri (1997) indicate that "Many aspiring mathematics teachers have their beliefs about mathematics changed during a methods course, yet the new view does not always carry over to their teaching practice" (p. 198).

Despite calls for improving teaching and learning of mathematics since the 1980s, in the following excerpt, Romberg and Kaput (1999) describe "traditional school mathematics" as not much different from what was described by Welch (1978) over two decades ago:

> The traditional three-segment lesson, which has been observed in many classes, involves an initial segment where the previous day's work is corrected. Next, the teacher presents new material, often working one or two new problems followed by a few students working similar problems at the chalkboard. The final segment involves students working on an assignment for the following day. This mechanistic approach to instruction of basic skills and concepts isolates mathematics from its uses and from other disciplines. (p. 4)

Further, Ball et al. (2001) indicate that "In fact, much about mathematics education [in the schools] has remained the same as it was in 1950 or even 1900" (p. 434).

Statement of the Problem

A large number of mathematics teachers experience serious difficulty teaching with reform-based curricula. Research has shown that teachers need both a strong subject matter knowledge and familiarity with a repertoire of student-centered instructional strategies to be effective in teaching an increasingly diverse pool of students. Teachers are challenged with issues of control and classroom management, issues of producing outcomes that meet or exceed state standards, and issues of working to satisfy the demands of their superiors at schools. Thus, continuously increasing demands of the workplace, whether a condition induced by the classroom environment, the implementation of a new curriculum, or the ecology of the school at large, are putting unprecedented pressure on the shoulders of teachers. Therefore, it is crucial to identify teachers who are able to meet the challenges and to understand how they succeed in such demanding environments.

The idea of reform of the mathematics teaching is no longer novel and yet teachers continue to have difficulty in meeting the challenges of reformed instruction in mathematics. It is inconceivable to observe that mathematics instruction today mirrors itself going back 100 years, as some scholars have indicated. The world outside of the classrooms today appears to be different than the world outside of the classrooms. The nature of discourse and tasks in mathematics classrooms must reflect the type of knowledge and skills needed in today's world.

Research shows that teachers who are highly qualified and experienced have strong content knowledge, foster discourse and performance of tasks that are needed for their students to succeed in the broader society, and are flexible in the instructional repertoire they deploy, enabling them to meet a vast majority of challenges within the classroom environment (Berliner, 2004; Clarke, 1996; Hogan et al., 2003; Shulman, 1987). These same studies call such teachers *expert* teachers.

This research aimed to identify and investigate an expert middle school mathematics teacher, who successfully made the transition in the context of teaching a reform-based mathematics curriculum. The intentions and practices of this teacher in the context of the reform-based mathematics curriculum, and classroom and school communities were studied in order to gain insight into what thinking and strategies she draws from, and how she is able to respond to challenges that arise during the day-to-day task of classroom instruction.

The following research questions aim at disclosing the cognitive aspects of this experienced middle school mathematics teacher's practice who has been able to successfully take on the challenges of transitioning to a reform-based curriculum and instruction program, while remaining equally successful and accountable toward her superiors and other colleagues at the school.

1. What are the intended instructional objectives and practices of an expert secondary-level mathematics teacher when instructing mathematics in a reform-based curriculum program?
2. What aspects of the classroom community and the school community impact the intentions and instructional practices of an expert secondary-level mathematics teacher in a reform-based mathematics curriculum program?

Study Assumption

The key assumption in this study is that a highly qualified and experienced mathematics teacher possesses knowledge and skills to adapt and function in productive ways in the context of any curriculum, reform-based or otherwise.

Definition of Key Terms

The use of several terms mentioned up to this point needs clarification.

An expert mathematics teacher.

For the purpose of this study, an expert mathematics teacher is a teacher who possesses a) principal's support as a highly qualified and experienced teacher, b) a state approved mathematics endorsement, c) a minimum of 7 years of mathematics teaching experience, d) a master's degree or 45 graduate quarter credit hours beyond the bachelor's degree, and e) a recommendation from state mathematics leaders. These criteria are used mainly at the point of first contact with the teachers, and are also a way to discuss qualified potential participants with principals of schools. However, the list is not meant to be a sufficient evaluation method in and of itself. After using the list for the first contact with qualified teachers, the researcher plans to observe and interview each teacher and collect background information before selecting the strongest qualified teacher for this study.

Reform-based mathematics curriculum and instruction.

Many versions of reform-based curriculum and instruction programs exist. Even when the focus is restricted to programs that have been based on the standards (NCTM, 1989, 1991, 2000), there are many versions to be found. Research and review of the reform-based curriculum and instruction programs reveals characteristics that all reform-based curriculum and instruction programs share (Heibert, 2003, p. 15): (1) builds on students' prior knowledge and skills; (2) provides opportunities for both invention and practice through classroom activities that revolve about problem-solving; (3) focuses on discussions about methods of problem solving that are the propriety of the teacher and the students with the intent of unraveling similarities and differences, and advantages and disadvantages; (4) requires students to justify their solutions and provide reasoned explanations of their methods. Examples of reform-based curriculum and instruction packages that have been implemented include *Connected Mathematics*, *MathScape*, *Interactive Mathematics Program*, *SIMMS Integrated Mathematics*, and *Core Plus* (Education Development Center, 2005). Each of these curricula is accompanied by a teachers' guide that describes appropriate instructional strategies for each content area.

School Community.

Sergiovanni (1994, p. xvi) describes a community as a group of individuals who share the same will and are thus bonded by the same goals and ideas. Therefore, for the purpose of this study, the school community includes the school administrators (e.g. the principal, the head of the department) and other teachers at the school with whom the teacher in this study have strong and productive collaboration with respect to mathematics education activities that go on at the school.

Classroom Community.

The term *classroom community* refers to the teacher, the students, and the interpersonal relationship between the two in the classroom (Doveston & Keenaghan, 2006; Ullucci, 2005). Therefore, a classroom environment with little interaction taking place between the teacher and students is not considered to form a classroom community.

Moreover, when present, such interaction must have a purposeful nature with direct connection to mathematics learning of students.

Significance of the Study

The aim of this study is to examine the intentions and instructional practices of an expert middle school mathematics teacher in the context of a reform-based mathematics curriculum within the context of the classroom and school communities to gain insight into the teacher's thinking and strategies, and how she responds to challenges during the day-to-day task of mathematics classroom instruction. This case study is designed to result in a rich description of this expert mathematics teacher to reveal patterns of practice that represent success in incorporating reform-based mathematics curricula for improving student achievement. The study provides basic knowledge with the potential for supporting the development of a theory that is more general in scope. This information also provides important guidelines for teacher professional development designed to prepare teachers for teaching reform-based curriculum as well as guidelines for preservice teacher preparation program. Lastly, although the participant in the study may never ask for any feedback from the researcher regarding the researcher's observations, the researcher will share findings from this study with the teacher.

CHAPTER II

Analysis of Relevant Literature and Research

The overarching purpose of this study was to examine the intended instructional objectives and practices of an expert secondary mathematics teacher in the context of a school community where a reform-based curriculum is adopted for the mathematics classes. The intent was to gain insight into the thinking and strategies of the mathematics teacher, and how the teacher was able to respond to challenges presented in classroom instruction with a reform-based curriculum.

Focus on amelioration of mathematics teaching and learning gained significant momentum after the publication of *A Nation at Risk: The Imperative for Educational Reform* by the National Commission on Excellence in Education (NCEE, 1983). The report cited low and declining student achievement in the areas of language arts and mathematics and used that data to motivate the need for revamping the way teaching was conducted in schools. The following depiction of mathematics teaching and learning is a typical profile of student-teacher interaction that the report identified for criticism:

> First, answers were given for the previous day's assignment, a brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to students working independently on the homework while the teacher moved about the room answering questions. The most noticeable thing about math classes was the repetition of this routine. (Welch, 1978, p. 6)

Research shows, however, that since the publication of NCEE's report, the teaching of mathematics in the majority of classrooms has not changed (Ball et al., 2001; Heibert, 2003), and mathematics achievement for the majority of students has not improved (Batista, 2001; Lee, 2006). Many teacher education programs, professional development programs, and reformed curricula have aimed at raising student achievement by changing the way mathematics teachers present the content. But the results from the literature indicate that key aspects of teaching that lead to student achievement are still missing from classrooms.

Key Instructional Variables

Early research on teaching and learning suggest that, first, human and curricular contexts cannot be ignored in studies aimed at investigating productive teaching practices. Second, each classroom assembly, with its peculiar teacher characteristics, learning habits of the students, context, and individual and prevalent group interactions represents a unique setting and therefore rendering it impossible to propose generic qualities of good teaching (Good et al., 1983, p. 144). Only information on the broadest of teaching qualities such as clarity, flexibility, enthusiasm, task and discourse orientation, and other similar aspects were deemed appropriate key areas of investigation in this research.

With respect to key broad teaching qualities, the literature provides results that are promising and consistent across primary and secondary grade levels. For example, in their empirical study of the impact of various teaching techniques on fourth graders, Good et al. (1983) found that task orientation, clarity, enthusiasm, and frequent teacher talk correlated with improved student learning. Although the work by Good et al. lacked empirical results at the secondary level, an earlier study by McConnell (1977) reported identical results for these same set of teaching behaviors at the secondary level. More recent studies corroborated these earlier findings. Heibert (2003) reviewed the impact of *alternative instructional programs* which were inspired by the standards of the National Council of Teachers of Mathematics (NCTM, 1989, 1991, 2000). In his review of these programs, Heibert (2003, p. 16) found that the bulk of longitudinal studies on elementary school students' learning of arithmetic are rapidly converging on the hypothesis that reform-based "alternative instructional programs" that emphasize a balance between conceptual understanding and mastery of procedural skills produce pupils that possess deeper conceptual understanding of significant mathematics while exhibiting at least the same competency with performing procedural computations as students who are taught traditionally. Moreover, acknowledging that fewer and shorter studies have examined the impact of alternative instructional programs on secondary students, Heibert (2003) confirmed that the results of those studies were similar to the ones conducted on younger learners. Other researchers have underscored the importance of instructional practices that emphasize *both* procedural and conceptual understandings (Desimone et al., 2005; Larson, 2002), and that experienced teachers ignore neither of these two aspects of understanding (Artzt & Armour-Thomas, 1999).

Before delving further into research about the practices of expert teachers in the context of reform-based mathematics curriculum and instruction, the essence of curriculum and instruction standards (NCTM, 1989, 1991, 2000), upon which one or more components of all reformed curriculum and instruction programs are based, warrants description.

The "vision" of school mathematics as portrayed in the standards-based curriculum (NCTM, 1989, 2000), proposes implementation of mathematics curricula that support in-depth and conceptual understanding of mathematical content for all members of the diverse pool of students that occupy schools today. In addition, the curriculum standards require that mathematics curricula emphasize significant content that is coherent across grades, are taught by teachers who are knowledgeable about mathematics, students, and pedagogical strategies, and deploy assessment techniques that help inform and improve instructional decisions, and use appropriate technologies to trigger students' interest and learning. The teaching qualities addressed in this vision are categorized and described in detail in the body of the curriculum standards (NCTM, 1989, 2000) under the equity, curriculum, teaching, learning, assessment, and technology principles. These principles were used to direct the attention of mathematics teachers for shifting from the more traditionally-observed instruction to instruction that would better support all students in meeting the new curriculum standards. *Everyday Mathematics* and *Math Trailblazers* (elementary), *Connected Mathematics* and *Math Thematics* (middle grades), and *Interactive Mathematics Program* and *Math Connections* are all examples of curricula that were created based on the recommendations of the curriculum standards (Educational Development Center, 2005).

Additional teacher standard documents have been added over the past 16 years. In 1989 in response to the calls for the reform and improvement of mathematics teaching, the *Commission on Professional Teaching Standards* began the development of a set of teaching standards published in 1991 as the *Professional Standards for Teaching Mathematics*. While these standards are currently being revisited and are projected to be upgraded in the near future, the original document continues to provide guidelines for the reform of mathematics teaching in schools.

Six standards are proposed in this document for school mathematics teaching (NCTM, 1991). The first standard suggests that teachers' understanding of their students' thinking, knowledge, and how they learn must govern the *tasks* that teachers pose to their students. This standard indicates that such tasks challenge diverse students' intellectual interests engaging them in communicating, reasoning and problem solving, while making connections with ongoing real-world human activity. Thus, rote memorization of procedures and computation of a "correct answer" that ignores the underlying concepts are discouraged while speculation about a solution is encouraged.

The second and the third standards emphasize the importance of *discourse* in the interchange between the teacher and students. They assert that teachers must make discourse a negotiable repertoire in their classroom, (1) knowing when to allow students to discover their solutions during the process of explaining their reasoning and (2) when to provide explicit input into such discussions for the purpose of guiding their direction. In this setting, the teacher ceases to be the only speaker and an environment of mutual respect for the expression of ideas must invite input from everyone present in the classroom. In action, the standards require teachers to begin more of their questions with "why", be selective about following up on ideas that emerge from discussion with and among students, monitor participation by all, and make accurate judgments about initiating whole-group or small-group discussions. By the same token, students must question the teacher as well as one another, engage in evidence-based arguments, and use a variety of reasoning tools to make conjectures about solutions to problems.

The fourth standard requires that the discourse in the classroom be *enhanced* with technological tools, concrete materials for modeling, variety of visual aids, metaphors and analogies, written hypotheses, and oral presentations.

The fifth standard holds teachers responsible for *creating the environment* that is conducive to the practices described in the previous four standards. In action, this standard states that teachers must respect and probe student ideas, while encouraging students to challenge each other's ideas allowing "time for students to respond to questions and must also expect students to give one another time to think, without bursting in, frantically waving hands, or showing impatience" (NCTM, 1991, p. 58). Hence, in this capacity, teachers' classroom management skills are a crucial asset.

Finally, the sixth standard states that teachers must use formative assessment as a means to monitor the effectiveness of their teaching and to adapt their instruction to better respond to the various learning styles of their students.

With their focus on *tasks*, *discourse*, and *the learning environment*, the teaching standards (NCTM, 1991) identify key instructional components in the most general sense. The majority of studies and literature that have addressed effective or "good" teaching, regardless of the specifics of their findings and proposals, have their results proposed along one or more of these same three key dimensions of practice (Batista, 2001; Larson, 2002; Romberg & Kaput, 1999; Rosenshine & Stevens, 1986).

Key Teacher Characteristics

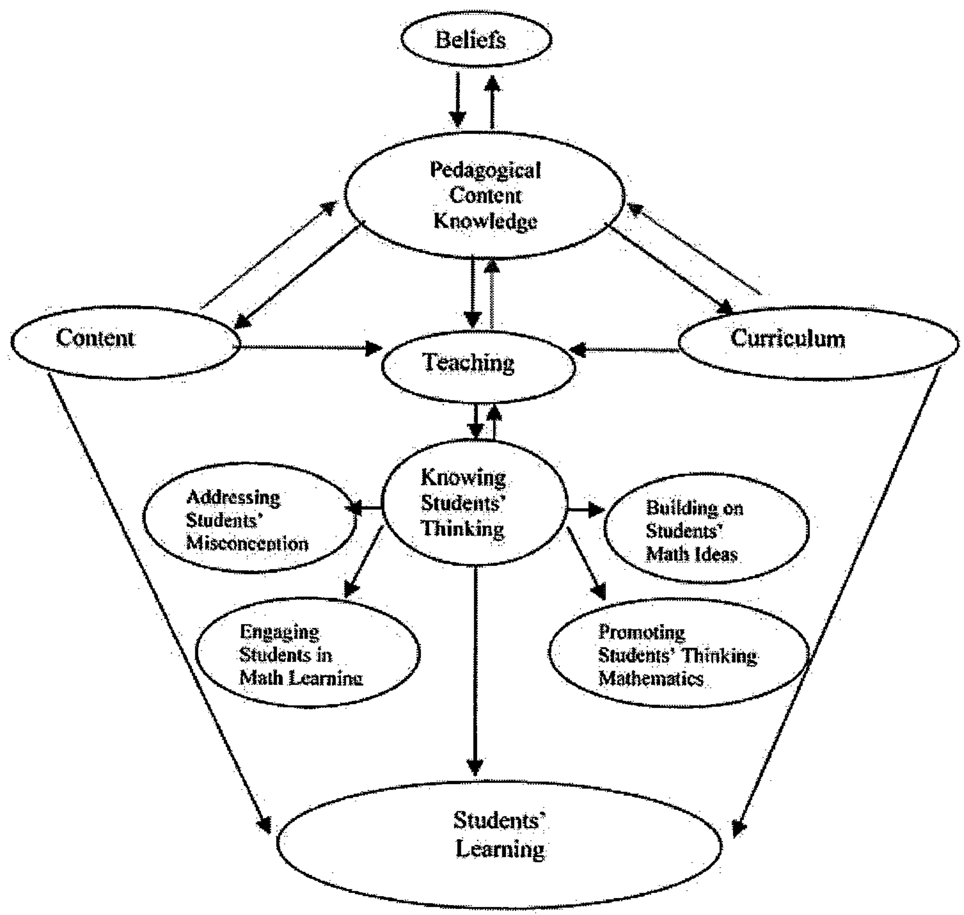
Teachers' Knowledge

Shulman (1987) investigated the nature of the knowledge teachers need to possess in order to be able to do their job properly. From studies of experienced teachers, he formulated the notion of teachers' knowledge base. The knowledge base, functioning as a framework to study teachers' knowledge, consists of content knowledge, general knowledge of instructional strategies (i.e. pedagogical knowledge), knowledge of curriculum, knowledge of how to transform subject matter content knowledge for the purpose of teaching (i.e. pedagogical content knowledge), knowledge of the students, knowledge of educational milieus, and knowledge of the purpose of education (Shulman, 1987, p. 6). Shulman expressed concern that the bulk of studies that had focused on exploration of the work of experienced teachers had placed undue emphasis on what the teachers did to *manage students* in the classroom; that is, teachers' ability to make classrooms a suitable environment for imparting instructional tasks, where pupils can pay attention with minimum distraction, and basically, be able to learn. However, he suggested that the main function of teachers is better captured in the way they are able to *manage ideas* within the discourse in the classroom.

Grossman (1989) used Shulman's conception of teachers' knowledge base to propose the concept of teachers' knowledge. Grossman suggested that teachers' knowledge was composed of both teachers' subject matter knowledge and teachers' pedagogical content knowledge. Moreover, she proposed that teachers' pedagogical content knowledge is comprised of: overarching conceptions of what it means to teach a particular subject, knowledge of curricular materials, knowledge of students' understandings and potential misunderstandings, and knowledge of instructional strategies and representations. Moreover, Grossman asserted that "subject-specific coursework can be a powerful influence on how teachers think about and teach their subjects" (Grossman, 1989, p. 30).

An et al. (2004) borrowed from Shulman's ideas about pedagogical content knowledge, extending them into a network of the knowledge of the content, the curriculum, and students; with the knowledge of the latter given a significant weight (Figure 1).

Figure 1. The network of pedagogical content knowledge



The work by An et al. (2004), with the bulk of its emphasis hinged on teachers' well rounded knowledge of students, reiterates the importance of Shulman's observation as to how flexible "Nancy", the teacher he observed, had been with respect to the strategies deployed in practice. This formulation of the pedagogical content knowledge underscores that an essential characteristic of pedagogical content knowledge is that it pivots about teachers' beliefs, instructional strategies, and types of teacher decision making that *are not fixed* – i.e. it requires continuous adjustment based on the knowledge gained through interaction with students.

In addition to knowledge about the subject, curriculum, students, and pedagogy, more recent research about teachers' knowledge is indicative that teachers' knowledge is also a product of activities, culture, and context within which teachers operate (Calderhead, 1996). This research suggests that teachers constantly reevaluate and reconstruct their understandings of classroom action situations, themselves as teachers, and their assumptions about teaching, based on the norms of interaction in their environment.

Teachers' Intentions/Goals

Jennings (1991) posits that "intentions" or goals may be viewed from three different theoretical perspectives: (1) a cognitive perspective – which focuses on the ability of a person to plan appropriately according to intentions, (2) a self-regulation perspective – which zooms in on the ability of an individual to avoid distractions for the sake of intentions, and (3) a motivational perspective – which hones in on the direction, vigor, and persistence of the intentionally charged behavior rather than the skills necessary to carry out those intentions. The importance of the cognitive perspective can be observed in the work of Thornton (1985). He studied the intentions of three male tenth-grade American history teachers who taught at the same school and had similar ages, years of teaching experience, and educational backgrounds. The study found that not only was there a dissonance between the intensions of two of the teachers and the intended curriculum, but also the teachers could not translate their intentions into corresponding classroom events. Another source of dissonance observed was the contention between the explicit and the implicit curricula that appeared to influence the two teachers' intentions. Because more than 90% of the high school students where Thornton conducted his study were expected to attend the elite colleges in the country after graduation, they were driven by a quest for high test scores. Thornton acknowledged that this student mentality within the high school "subtly undermined the teachers' intentions" (p. 6). Moreover, Thornton believed that these three sources of dissonance between the intentions of the teachers and the intended curriculum were responsible for a phenomenon he called the "intentions' degree of curriculum consonance." He asserted that a "weak consonance does not assure that students will have educational experiences" (p. 2).

Analyzing teachers' intentions from a cognitive perspective, Flick and Dickinson (1997) studied how well four experienced science teachers, three from the middle grades and one from the fifth grade, were able to translate their intentions about inquiry-based learning of science into practice. They found evidence that the intentions and practices of all four science teachers were aligned. Although the students' learning did not always align with the teachers' intentions, inquiry-based learning of science was taking place in the classrooms.

The importance of teachers' intentions and goals and their connection to student learning was also taken into consideration in "a cognitive model for examining teachers' practice in mathematics" by Artzt and Armour-Thomas (1999). In their analysis of knowledge, goals, and beliefs of seven novice and seven experienced mathematics teachers, they observed that the goals of experienced teachers revolved about student learning for understanding, while for novice teachers, the goals usually focused on procedural learning of the content, adequate content coverage, and classroom management. Moreover, Heibert (2003) asserted that when the reform-based "alternative instructional programs" are designed with more ambitious learning goals in mind, it has been shown that these goals can be attained and students can acquire both skills and concepts at levels higher than the students in non reform-based instructional programs.

While these studies considered teachers' intentions and goals from a cognitive perspective, a direct analysis of intentions and goals from the self-regulation or the motivational theoretical perspectives is rare or nonexistent. However, Heibert (2003) expressed that, teachers' beliefs and values do impact their intentions and goals.

Teachers' Beliefs

Distinguishing between knowledge and beliefs, Pajares (1992) predicates that "Belief is based on evaluation and judgment; knowledge is based on objective fact" (p. 313), while Enochs et al. (2000) assert that "Beliefs are part of the foundation upon which behaviors are based" (p. 195). Whatever their nature, however, Pajares (1992) proposed that "beliefs cannot be directly observed or measured but must be inferred from what people say, intend, and do" (p. 314).

In their review of studies that investigated the impact of knowledge and beliefs on what teachers know and do, Borko and Putnam (1996) maintained a discreet, yet, parallel analysis of teachers' knowledge and beliefs as if the two were inseparable. In fact, they proposed that while prospective teachers are immersed in their studies at school, their prior knowledge and beliefs will determine "what and how they learn from their teacher education experiences" (p. 674). Other literature corroborated this finding for prospective teachers (Manouchehri, 1997; McGinnis et al., 2002; Wilcox et al., 1992). With regards to in-service mathematics teachers, it was observed that beliefs have similar powerful impact on teachers' conceptions about students' learning and teaching the subject (Batista, 1994; Manouchehri & Goodman, 1998). Yet, as Pajares (1992) had pointed out, *statement of beliefs* alone can be an illusive measure of exactly what teachers will do in their practice (see e.g. Calderhead, 1996; Cohen, 1990). This phenomenon was also reified in a study by Spillane and Zeuli.

Spillane and Zeuli (1999) studied data from the final stage of a five-year research study that ran from 1992 to 1996, and was analyzed using both quantitative and qualitative methods. A subsample of 25 teachers (17 elementary and 8 middle school) from a sample of 280 mathematics teachers in 6 districts, who had reported teaching mathematics with reform-based strategies and claimed knowledge of the national and state standards for reform, were selected for further observations and interviews. Using the nature of mathematical tasks and discourse that the teachers engaged in with their students, it was determined that the practices of only four of the teachers were truly reform-based.

The review of the literature on the impact of beliefs on practice of teachers, and the nuanced nature of the impact of beliefs on actual practice as discussed by Spillane and Zeuli necessitates the consideration of a set of dimensions along which the impact of beliefs may be studied. Koehler and Prior (1993) proposed a set of dimensions of influence for beliefs that is nearly congruent with Grossman's (1989) conception of the dimensions along which pedagogical content knowledge develops. Koehler and Prior identified four categories of mathematics teachers' beliefs: (1) teachers' beliefs about how students learn, (2) teachers' beliefs about mathematics, (3) teachers' beliefs about student characteristics, and (4) teachers' beliefs about teaching. Some studies of beliefs, particularly in the area of staff development, have concluded that a change in beliefs follows, rather than precede, a change in practice (Calderhead, 1996).

Teachers' Classroom Interactions

Cohen and Ball (2001) discuss the notion in the context of why instructional interventions of the past 18 years may have demonstrated limited success:

> First, no intervention can be completely comprehensive, but most have been very partial. They often take aim at only one element of the complex dynamic of instruction. Many, especially in mathematics and science, have centered on innovative curricula. Some focus on teachers' learning, some restructure school time and space, others aim at incentives – salary programs, merit pay, accountability for outcomes. But few intervene directly on the multiple elements of instruction – the teachers, students, content, and environments – and their interactions. (p. 77)

Cobb et al. (1992) expressed that mathematics teachers, through patterns of interaction established in the classroom, can foster manners of thoughtful discourse among their students if in fact such interactions are "taken-as-shared." Further, mathematical understanding is improved if procedural instructions are presented in an interactional milieu, when making of conjectures and providing mathematical explanations characterize the lesson.

Teachers' Interactions with the School Community

The review of the literature ascertains that school community exerts substantial influence on how teachers practice. In a study that followed four prospective elementary teachers through the first year of their in-service career, Steele (2001) found that reform-based teaching practices fostered by one teacher education program stayed with the program graduates as long as the administration and colleagues at the school where the graduates found work were supportive of the novice teachers' implementation of reform-based instructional strategies. Bay et al. (1999) conducted a review of research on reform concluding that administrative support and collaboration with colleagues were among the top 10 factors necessary for improvement of teaching. Reitzug and O'Hair (2002) stressed that as the size of schools grows and their population of students and teachers balloons consequently, the focus in these schools tends to shift from one that must be geared toward intellectual improvement toward one where the administration attempts to exert control over student *and* teacher behavior. This recognition does not suggest that such influences should be considered as "bad", yet there is evidence that the impact of the school environment on teachers' behavior cannot and should not be ignored. Other scholars have discussed how mathematics teachers simply do not conduct their daily work without interacting intensely with learning from and being inspired by other mathematics teachers in the same school (see e.g. Furman, 2002).

Sergiovanni and Starratt (2007, p. 341) state that communities or groups are important to teachers to the extent that teachers' decisions are influenced by their group membership. Thus, they suggest that often times changing the decisions that individual teachers make is equivalent to changing the thinking in communities or groups to which teachers belong.

Expert Teachers' Practices

Some studies distinguish between expert teachers and experienced teachers *but* claim that the two qualities cannot be separated *in practice* and thus use the terms "expert" and "experienced" interchangeably in their investigation of the practice of teachers (Berliner, 2004). Other studies distinguish between expert and experienced teachers *and* claim that experience does not equate expertise, yet, they view experience as an inseparable component of what expert teachers do (Hogan et al., 2003). Experience and quality in teaching, then, necessitate an in-depth subject mater knowledge combined with the knowledge and experience of how to teach that subject (Clarke, 1996; Shulman, 1987). The literature suggests that studies of both expert and experienced teachers have revealed that these teachers are flexible in their instructional repertoire, operate from rich subject-matter knowledge, and anticipate and are prepared for the demands of virtually any learning situation.

In his comprehensive review of the literature on novice and expert teachers, Calderhead (1996) asserts that the progression of teachers from novice to expert status follows through five stages of development. The first stage, or the novice stage, is marked by the beginning teacher looking for rules to guide her/his actions. In the second stage, or the advanced beginner stage, the teacher begins to bend or even break some formally acquired rules of instruction while seeking contextual and strategic knowledge that might prove useful instead. In the third stage, or the competence stage, the teacher begins to make conscious decisions about what to do and is able to monitor, evaluate, and adapt strategies in case there is need to do so. By the fourth stage, or the proficiency stage, the teacher has developed an intuition about teaching and appropriate strategies and decisions that would be required in response to various learning situations. In the fifth stage, or the expert stage, the teacher and the instructional tasks have become one, and rarely, if at all, anything will cause the teacher to become surprised, because she has fully adapted to and is in control of the didactic environment. In his review of other studies, Calderhead (1996) found that beginning teachers think of good teaching in terms of teaching that incites classroom interaction that is fun and inviting to the involvement of children. Whereas, expert teachers view good teaching in terms of lesson structure and how well the teacher is able to tailor the lesson to the context and the instructional purpose at hand. Yet, in other studies, Calderhead (1996) found that experienced teachers spend a great deal of time considering multiple ways of defining and representing a problem, whereas novice teachers focus on possible solutions to a problem. In other studies, Calderhead (1996) found more evidence that expert teachers not only have a large "domain-specific" knowledge base but also this knowledge base consists of facts and rules that have "become integrated into more holistic patterns of thought and action, situations are perceived in context and can be related to other recent events" (p. 717). These studies once again showed that for novice teachers this knowledge base is smaller and consist of discrete facts and rules.

Ainley and Luntley (2007) pointed out that while research has paid due attention to the knowledge base of experienced and qualified teachers, it has paid little attention to their "attention-dependent knowledge" – a concept they associated with teachers' ability to interact effectively with students during a lesson. It is a spontaneous type of knowledge that cannot be written down as part of a lesson plan, but it helps teachers make the best in-the-moment decisions during classroom events without losing control or get caught by surprise. Berliner (2004) found that teachers claim it takes them between 3 to 5 years before they are fully prepared for what happens inside the classroom and they are no longer "surprised." In both studies, Ainley and Luntley (2007) and Berliner (2004) found that this experience with a wide array of possible learning situations in teaching a subject matter helps teachers achieve automaticity and a level of preparedness in conducting classroom routines that help them accomplish their teaching goals. As the result, experienced teachers are better aware of the demands of the tasks they assign to their students, they anticipate and look for a pattern of learning issues that may crop up among students, they are much quicker than novice teachers to recognize learning issues, and are more flexible with regards to instructional strategies they use to resolve them.

Berliner (1986) asserted that unraveling of the nature of the practice of expert teachers can be used as a starting point for designing teacher education programs, and that studies of knowledgeable and exemplary teachers can provide us with rich and in-depth description of instructional events that may be used to inspire learning of new teachers. This direction is certainly the main long-term aim of this study.

CHAPTER III

Design and Methodology

This chapter describes the methodology and design of a study aimed at answering the following two research questions:

1. What are the intended instructional objectives and practices of an expert secondary-level mathematics teacher for instructing mathematics in a reform-based curriculum program?
2. What aspects of the classroom community and the school community impact the intentions and instructional practices of an expert secondary-level mathematics teacher in a reform-based mathematics curriculum program?

One purpose of the first research question was to identify the dominating goals and intentions that an expert teacher uses in planning and implementing lessons. Another purpose of the first question was to piece together the actions such a teacher undertakes, the instructional strategies this teacher adopts, and the decisions this teacher makes in order to materialize the goals and intentions. In combination, these two agendas formed an overarching purpose for this study that was to construct a model of practice that was predictive of the intentions and classroom actions of the teacher in this study. The purpose of the second question was to determine the impact of the school ecology on the intentions and practices of the teacher. Input from the teacher's science and mathematics colleagues within the school setting, the school principal, and a collaborative connection with mathematics teachers at a nearby school were also explored.

Design of the Study

Patton's (2002) recommendations for designing a research study included clearly articulating the primary purpose of the study, establishing a research perspective, and identifying the focus of the study. The primary purpose of this study was to examine and describe the intentions and practices of an expert secondary level mathematics teacher in the context of a reform-based curriculum. Another main purpose of this study was to explore the classroom and school communities within which the teacher practiced to seek significant sources of influence on the teacher's intentions and practice from these domains. A third purpose of this study was to propose of a model of the expert teacher's practice. Therefore, this research was considered "basic research" (Patton, p. 215) and was conducted using qualitative research methodology to generate a model of the teacher's practice (Auerbach & Silverstein, 2003). Most of what teachers learn about subject-specific teaching comes from experience, but little is known about what instructional strategies mathematics teachers have found to be successful for specific content. The studies that have focused on instructional practices of experienced teachers were in the context of implementing reform-based curricula (Lloyd & Wilson, 1998; Sherin, 2002).

The particular qualitative research methodology used for this study was a case study analysis. Gall et al. (2003, p. 436) described a case study analysis as an in-depth approach to conduct an intensive study of a phenomenon in its natural context. Yin (1994) stated that "case studies, like experiments, are generalizable to theoretical propositions and not to populations and universes" (p. 10), rendering this methodology ideal for constructing a model of the teacher's practice. Yin also concurred that the topic of "decisions" is one major focus of case study analysis.

In this study, Gall et al. (2003) referred to a "phenomenon" as encapsulated in the amalgamation of the intentions and instructional practices of an expert secondary school mathematics teacher in the context of a reform-based curriculum within the classroom and school communities of a particular school.

The nature of this study and the research questions considered prompted the exclusion of quantitative methods. By the same token, within the realm of qualitative data collection and analysis methods, only those methods of collecting data most relevant and appropriate for answering the research questions were selected. For example, videotaping was excluded as a means to collect observational data from the classrooms in favor of the researcher's observational field notes. Videotaping would require the researcher's constant attention to keep track of the teacher's movement in the classroom that would in turn lead to distractions and disturbing the natural flow of the classroom instruction and student interactions. Field notes, on the other hand, in combination with audio recordings of whole class discussions, allowed for an inconspicuous and rich data collection strategy that was minimally intrusive.

Schoenfeld's (1998) teaching-in-context theoretical framework was used as the analytical lens. The central purpose of the framework was to explain, at a fine level of detail, what decisions and actions teachers adopt when teaching, and why and how they do so; that is, to describe at fine level of detail, the ways in which teachers' goals, beliefs, and subject matter knowledge and pedagogical content knowledge interact from moment-to-moment as teachers make decisions and commit themselves to actions in the classroom. Moreover, the framework by Schoenfeld (1998) is accompanied and supported by four other scholarly papers (Schoenfeld, 2000; Schoenfeld, Minstrell, & van Zee, 2000; Zimmerlin, & Nelson, 2000; and Aguirre & Speer, 2000) to form the logic behind the development of the model and its application to various traditional and non-traditional contexts.

Thus, the framework is a comprehensive descriptive/analytical tool that helped to capture the discourse in the classroom and the pedagogical content knowledge of the teacher, while connecting these components of the teacher's practice and beliefs about the subject matter, students, curriculum and approaches to teaching the subject. This framework helped develop descriptive models for select lessons presented by the teacher, where the model depicted the experiences of the teacher in the curricular and classroom contexts as closely as possible.

Analytic Theoretical Framework

The theoretical framework for this study needed to be versatile and flexible to allow the examination of the secondary teacher with respect to the intentions and practices that the teacher identified, as well as incorporating a consideration of social and ecological elements of the teacher's workplace. For this purpose, the analytic framework developed by Schoenfeld (1998) was selected.

The framework is a *descriptive/analytic* tool for characterizing teachers' decisions and actions as they teach. The framework served as a model for describing or analyzing all types of teaching and teaching functions that ranged from cognitive decision making in planning lessons to classroom interactions that required discourse analysis. The model linked the teacher's plans with practice at the most global level of overarching goals and reasoning behind the choice of instructional strategies that crystallized through one-on-one interaction with the students.

Components of the theoretical framework.

Lesson image. The concept of a *lesson image* refers to the teacher's full vision, prior to instruction, of what needed to happen during the lesson implementation and how it needed to be conducted in practice. The concept of lesson image was broader than a lesson plan and included the sequence of all the actions and interactions with students that the teacher had in mind with respect to exercising a lesson. In particular, interviews had the potential for unraveling the teacher's lesson image.

Goals. The framework considered intentions or goals to be *what teachers want to accomplish*. The framework considered intentions or goals to occur at different levels of implementation. That is, intentions included goals for a lesson, a unit, or for the whole year at the school. Moreover, intentions or goals were potentially socially or epistemologically oriented, pre-determined or emergent. A series of goals may have been in operation all at the same time.

Action plans. Action plans were future mechanisms through which the teacher attempted to achieve the goals for the lesson. Action plans were also part of the teacher's lesson image, and like goals, they were rooted either in the teacher's beliefs, knowledge, and prior experiences with teaching the same topic or pieced together impromptu in response to situations for which no familiar effective response existed.

Action sequences. Action sequences were the summation of all that took place in the classroom. In this framework, each lesson was parsed into *episodes* with a particular structural or phenomenological integrity. The process was iterated to parse these episodes into *sub-episodes* when necessary. Each of the episodes was an action sequence. Normally, a close parity existed between the teacher's lesson image and the action sequences that unfold during actual practice of the lesson. Thus, the goals, action plans, and action sequences were linked psychologically, using the information about the teacher's lesson image captured in the lesson interviews and linking it with what actually happened in the classroom. One maxim of the framework was that to every action plan, at least one current goal may be associated.

Routines. Within the category of action sequences, there were routines. These represented repetitive classroom actions whose purpose and nature did not vary from day-to-day activities, and they characterized an entire class period. The teacher might have had more than one routine for accomplishing the same task. For example, during a daily routine review of previous homework problems, the teacher might have decided to quickly give the correct answer, or may have decided to ask individual students to read out loud their answers while the teacher sought consensus about the solutions from the rest of the class.

Scripts. Scripts were the standard way by which the teacher tended to a specific task, or conducted or allowed a certain type of social interaction in the classroom. For example, with respect to problem solving, the teacher may have adopted an approach where it is enough if students had the correct answer, whereas another teacher may have expected students to explain their solutions more thoroughly irrespective of their final answer to a problem. For the former teacher, the marker of the script was to ask students about their final answers, while for the teacher of the latter type, the standard way of proceeding with the task of problem solving was to ask students how they obtained their solutions, putting the emphasis on reasoning and problem solving strategies.

Mini-lectures. Mini-lectures were that special kind of script in which a familiar piece of dialogue was delivered directly, such as a mini-lecture on mathematical induction. Teachers usually gave mini-lectures when they perceived that their students' skills or understandings on a particular activity or topic needed reinforcing. Mini-lectures were usually well-rehearsed through repeated use over time so that the teacher generated them at will. For the most part, mini-lectures were considered to be small and non-interactive, but were usually followed by a "Does that make sense?" type question by the teacher that may have generated some interaction with the students.

Simple talk. Simple talks were small-scale verbal interaction with the students in response to a question or simply just to make a point. Simple talk characterized most short-term conversations that occurred in the classroom.

Beliefs. Beliefs shape the teacher's goals and actions. A variety of interviews strategically conducted were used in this study to gather the teacher's beliefs; that is, the teacher's beliefs about how students learn, student characteristics, mathematics, and teaching.

Knowledge. Schoenfeld's (2000) conception of knowledge rested on Shulman's (1987) conception of the teacher's knowledge base. For the purpose of this study, Grossman's (1989) conception of teacher's pedagogical content knowledge (described in Chapter 2) was used to explore the dimensions of the teacher's knowledge base.

Therefore, the delineation of the teacher's knowledge, intentions/goals, and beliefs through analyses of the action sequences of the practice of the teacher considered the events, beliefs, goals, type of action sequence, kind of knowledge attributed to an action sequence, the part of the lesson image to which the action sequence corresponded, and terminating events that triggered each of the action sequences.

Schoenfeld acknowledges that the framework's focus on teachers, while leaving out students' perspectives, is one of its limitations. By the same token, the teaching-in-context framework ignores the influence of the school community on the practice of teachers, as well. Since this study also considered the impact of the school community on the plans and practices of the teacher, the school community was added onto the framework for this study. Interviews with the teacher in combination with the discussions that were conducted with the principal and three other teachers, two of whom were mathematics teachers and one who was a science teacher, along with the classroom observations, were used to describe the variables and conditions that imparted influence on the thinking and work of the teacher from within the classroom and school communities.

This template was used in this study.

Selection of the School, Classroom, and the Participant

The participant selection strategy for this study engaged purposeful sampling. Gall et al. (2003, p. 165) described purposeful sampling to be appropriate where the goal was to select a case or cases that were likely to be "information rich." In this study, the most outstanding member of a group of eight expert secondary school mathematics teachers was selected. Various aspects of the other seven teachers' practice proved to be less informative to the research questions in this study, thereby excluding them from participation. For example, the teacher with the most number of years of experience (22 years) who was practicing with standards-based curricula at a high school near the researcher's geographical location was excluded from the study despite his strong subject matter knowledge because he did not display reform-based teaching strategies. Little to no classroom interaction was observed between him and his students in the observation that took place during the selection phase of the study. Another teacher who was observed teaching a reform-based lesson during the selection process revealed during the selection interview that the particular lesson with which he practiced was one of "few" interactive lessons that he had developed that provided ample opportunities for collaborative classroom interaction, and technology use. This revelation by the teacher was an indication that the teacher may have been selective with respect to the recommendations expressed in the reform-based curriculum he was practicing with for the past six years. For this reason, the researcher excluded this teacher as the potential primary case in this study. One strong candidate for this study was excluded because she revealed during the selection process that she was going to be on maternity leave for a portion of the course of this investigation. Other teachers who were excluded possessed components of teaching practice that were less adherent to the principles of reform-based teaching than those of the teacher who was ultimately selected for this study; they conducted long lecture sessions, or despite their qualifications as an expert teacher demonstrated little actual teaching in the classroom, spending time on impertinent discussions with students and talking with students about various popular videogames and television programs.

The researcher spent nearly two months in identifying eight teachers who agreed to participate in the study. Several large school districts denied the researcher access to teachers working in their districts for any type of research. In one case, the researcher was informed that it was the policy of the superintendent not to allow any research in his schools because he believed that the research process was a distraction and impeded the normal course of teaching and learning at schools. Many principals politely excused their teachers from participating in this project, providing various or no reasons at all for their excuse. Some principals simply said "No!" to the study. Others asked the researcher to contact the district. Among all the districts contacted, only one district issued a formal letter granting permission to the researcher to conduct the study in the district; several teachers volunteered from this district. Overall, it can be said without exaggeration that *finding* the eight teachers was the most formidable phase of this study. As a whole, though, the school/district communities viewed the research process as potentially harmful in some way. The researcher tried to alleviate some of these concerns by providing a copy of the study abstract to all the teachers, principals, and district officials he contacted. Of those contacted, only a small fraction accepted this offer.

The motivation to select an expert teacher for this study was due to the fact that the experience and highly qualified status of these teachers were known to reflect strong subject matter knowledge (Borko & Putnam, 1996; Hill & Ball, 2005; Shulman, 1987), and research showed that there was a positive correlation between expert teaching and student achievement (Good et al., 1983; Adamson et al., 2003; Desimone et al., 2005). The number of teaching years of experience was an important factor in selecting the teacher because novice teachers tend to focus on issues of pedagogy and classroom management. The criteria to identify a pool of eight expert mathematics teachers from which only one teacher was selected as the participant for this study was established from the outset. These criteria stated that an expert mathematics teacher was stipulated through a) principal's support as a highly qualified and experienced, b) the Oregon Advanced Mathematics Teaching Endorsement, c) a minimum of seven years of mathematics teaching experience, d) a master's degree or 45 graduate quarter credit hours beyond the bachelor's degree, and e) recommendations by state mathematics leaders.

Participant selection began with recommendations from the researcher's major advisor based on these criteria. From this pool of contacts, middle and high school teachers in or near the researcher's geographical location, where reform-based or NCTM-based mathematics curricula were implemented, were identified. The protocol for contacting principals at the identified schools was formulated in the form of a rough conversational script and it has been included in Appendix A. Some principals required that a formal permission from the district to be obtained by the researcher before they agreed to consider the study. Two school districts rejected any research to be carried out in their schools while one district granted the permission to conduct the study after requesting, reviewing, and formally approving an abstract of the study. Two principals did not require permission from their districts and a meeting with the researcher sufficed to acquire their permission to contact the expert mathematics teachers at their school. The researcher then asked each principal to identify potential participants for the study using the selection criteria for qualifications and experience. Five teachers agreed to participate via exchange of a series of email correspondences alone, and four others expressed interest to be part of the study after meeting with the researcher subsequent to an exchange of email messages. All nine teachers received a two-page abstract of the study prior to agreeing to be a participant. All nine teachers signed the IRB's Informed

Consent Form, however, scheduling problems prohibited one teacher to participate in this study.

The remaining eight teachers completed a background questionnaire (Appendix B), and were interviewed immediately after completion of the questionnaire. The background questionnaire provided a profile of the teachers' education and years of experience, and the interviews (using protocol in Appendix C) helped to clarify: (1) the teachers' knowledge of the reform-based curriculum and instruction in general and also with respect to the reform curriculum and instruction standards (NCTM, 1989, 1991, 2000), and (2) the teachers' personal instructional standards. These interviews were audiotaped and transcribed for comparison. To obtain a basic impression of their practice, each teacher was also observed once as they taught a class.

The information obtained from the background questionnaires, the interviews, and the classroom observations were then used to describe and rank the teachers. One teacher from the eight surpassed the others in expertise and was selected to be the participant for the detailed case study.

Ms. Johnson

Ms. Johnson was a middle school mathematics teacher with a bachelor's degree in Business Management, another bachelor's degree in Psychology, and a master's degree in educational leadership. She had begun her teaching career nearly 20 years ago, first, as an elementary school teacher after completing a one year teaching program. Early in her career, Ms. Johnson had been a second grade teacher for two years and then taught sixth grade for two years. Having spent the first four years of her teaching career teaching second and sixth grades, respectively, she had then gone on to teach seventh and eighth grade students before going back to teach fifth and sixth grades for another two years. After this initial teaching of elementary and middle school students, Ms. Johnson transferred to become a middle school mathematics teacher, permanently.

Ms. Johnson was quite enthusiastic about participating in professional development programs throughout her career and, in particular, took advantage of all such opportunities in the area of "reformed teaching of mathematics" – in her own words. The following excerpt from an interview where she explained the history and her commitment to attending reform-based professional development programs is revealing in this respect.

> Well, I'm going to go all the way back to 1989 . . . 1990s. I was in a school district where they put all their extra money in professional development and they would pay teachers to go to workshops and language arts in math. I was an elementary teacher and it was all hands-on, manipulative-based math, and so we worked with our colleagues and they had a person come to us and then we saw how we would use that, you know, it was sort of a model, you use a manipulative, then you use the, you know, concrete-pictorial-abstract and we just worked with that over and over again in different contexts. And the other thing I had during that time is I had a kindergarten teacher who was a math expert, work with me in my class and co-teach with me for math for four years. So, that probably impacted my teaching more than anything. [IMI – Ms. Johnson's career path, Question #1, p. 61 of the field notes]

Furthermore, Ms. Johnson had been a member of the Silicon Valley Math Project in California for nearly five years, and attended "Summer Institutes" that were a collaborative effort between school teachers and the local universities with respect to teaching mathematics. At the time when Ms. Johnson was contacted for this study, she was studying toward a doctorate in educational leadership at a local university, and some of her most recent coursework had involved designing professional development programs for mathematics teachers as well as taking other courses that made training of future mathematics teachers as their focus.

Ms. Johnson had been teaching nine years at her current school, Oxford Middle School. Ms. Johnson taught Math-7, Math-8, and Algebra One courses from *Connected Mathematics Curriculum*. In addition, she taught three Physical Education courses at the school, and served on several school committees that met and made connections with mathematics teachers at a local high school.

During the teacher selection process Ms. Johnson was observed teaching her 50-minute long Algebra One class. The lesson observed followed the NCTM-based reform guidelines closely. She incorporated small group discussions, whole group discussions, and the use of a document camera and overhead projector to teach or have students share their work with other students. During small group discussions, Ms. Johnson constantly asked her students to make conjectures by asking them "why" they had come up with a particular solution. This approach put students in the predicament of having to explain their reasoning. At one point, students were to use "tiles" to build physical and tangible manifestations of algebraic expressions, resulting in students' using open-ended thinking and generating alternative solutions.

Collaboration between team members arranged in groups of two, three, or four students, and also between the teacher and individuals or groups of students dominated throughout the lesson. All students seemed to enjoy themselves while discussing possible solutions to problems. Moreover, real-world settings were used to establish connections between the abstract nature of algebraic expressions and physical and tangible forms, such as the arranging of the tiles. Thus, students were able to involve themselves in worthwhile mental and physical tasks.

The desks in Ms. Johnson's classroom were organized in groups to accommodate teams of up to four students. Eight groups were formed in this classroom, with only one group having two members and all the other groups with three or four members. On this day, the absence of some students from this class of 32 students had caused some groups to have fewer than four students. Only one group, sitting toward the back, seemed to be off task, catching the teacher's attention after about 30 minutes. Ms. Johnson's attention had been occupied through her work with each group, advising and questioning them about what they were doing.

At first, the lesson had begun after Ms. Johnson had asked volunteer students to read short problem statements about which all the students were to write respective algebraic expressions. By the same token, volunteer students were asked to go to the overhead projector and share their answers with the rest of the class. The requirement for any other volunteer student that followed was that he or she was to present an alternative solution that had not been offered by other students. This constituted a warm-up activity for the class that lasted about 10 minutes.

After the warm-up activity, the students were asked to use 8.5x11 "matts" and a box full of color coded tiles already cut to different rectangular sizes to express algebraic expressions using these objects. Each team member was assigned a clear responsibility and the teacher swept through the class, engaging members of each team with questions and asking them to justify their arrangement of the tiles. Correct responses by the students in each team ensued upon Ms. Johnson's demand to see alternative solutions by other students on the team. Ms. Johnson's movement about the class and interacting with members of each team seemed to keep the majority of students engaged and on task. At the same time, students had to justify their answers either in front of their team members or later in front of the whole-class.

Most students at this school were of middle to high socio-economic status. Even so, Ms. Johnson believed that it was a challenge to convince the students' parents as to the merit of the *Connected Mathematics Curriculum* the school had adopted. As far as the effectiveness of the curricula, she indicated during the selection interview:

> OK, I'm going to answer in two ways, because I think the Algebra program that I'm using, that's a high school level Algebra One course, was very effective last year, and having kids learn conceptually algebraic relations and connecting one idea to the other. So, I would say it's been very effective in student achievement. Our middle level program, it's a spiraling curriculum, and it tends to jump around a little bit, and not go in-depth in certain areas that I think that it should, so I sort of question some of the way they've organized the materials. So, I believe it's effective, but I still think you have to do some changing of it to meet student needs.

Based on the NCTM criteria for teaching standards (1991) her lesson possessed all the components for *worthwhile mathematical tasks* and *discourse* among students, within a reform-based *learning environment*, and Ms. Johnson had been consciously *analyzing the effectiveness of the teaching and learning* in order to know what worked and what did not.

Community and School Context for the Selected Participant

Based on the rankings assigned during the selection process, Ms. Johnson was selected to be the participant for this study. Additionally, finalizing this choice were: she was teaching from a reform-based mathematics curriculum, and her school had regularly met mathematics standards as set by the Adequate Yearly Progress (AYP).

Ms. Johnson taught at Oxford Middle School. The school had about 150 students and together with a nearby high school formed the main educational hub in a town of about 5000 people. The majority of the town's male population held jobs in construction, farming, and fishing, and the majority of the town's female population held professions in health care and education. A small portion of the students at the school came from impoverished households. For these few students, the school had a process for providing free and reduced lunch. Ninety eight percent of the student population at the school was Caucasian, with very few African-American and Asian-American students.

Based on the information obtained from the School Accountability Report Card (SARC) for the school year 2006 – 2007, 72.2 percent of the students at the school met or exceeded the standards for "math knowledge and skills", 12.5 percent of the students nearly met the standards, and the remaining 15.3 percent did not meet the standards. According to Ms. Johnson, these percentages were at or slightly above the average statistics for the state. The school had also been able to meet the standard for overall Adequate Yearly Progress (AYP) in mathematics. The overall attendance rate for the students at the school was 95 percent. The small average class size at the school (21 students) may have been one of the reasons for the student performance at the school.

There were three mathematics teachers at the school. Ms. Jenkins, also an expert mathematics teacher, taught 6th graders only. Mr. Hubert, a recently hired mathematics teacher at the school with one year of experience was assigned to teach 7th and 8th grade mathematics to top students at the school, while Ms. Johnson was teaching average to lower achieving students. The sixth graders in Ms. Johnson's Algebra One class, however, were high achieving sixth graders. Ms. Johnson and the one science teacher, the two mathematics teachers, and the principal at the school formed a community that learned from each other. Ms. Buchanan and Ms. Johnson worked together to improve science and mathematics teaching and learning through creating and incorporating the "integrated units." Ms. Johnson coached the new mathematics teacher, Mr. Hubert, to improve his content knowledge and to have better lessons for his students. Moreover, Ms. Johnson provided Ms. Jenkins, the 6th grade teacher with information that she could use to improve teaching in 6th grade. Ms. Johnson and the principal, Ms. Roberts, interacted with the mathematics teachers at a nearby high school to develop a broader vision for the learning of their students at Oxford Middle School. At the same time, Ms. Roberts was generous with respect to spending allocated district funds to reimburse Ms. Johnson for the cost of conferences and professional development programs she wanted to attend. This opportunity helped Ms. Johnson expand her knowledge of the content and teaching methods even more. The learning, coaching, and accommodating cooperation established between Ms. Johnson and these four individuals helped define the community that surrounded Ms. Johnson (Sergiovanni & Starratt, 2007, p. 231).

The school offered four different types of mathematics curricula: Math for sixth graders, Math Seven, Math Eight, and Algebra One. During the study, Ms. Johnson was observed in all mathematics courses she taught; namely, the Algebra One, the Math Seven, and the Math Eight courses. Ms. Johnson taught two sections of the Math Eight course. During the first three weeks of this study, the Math Eight course in the third period was being taught by Ms. Johnson's student teacher. Therefore, for the purposes of this study, the Math Eight course in the fourth period was observed during this time. After the student teacher completed her internship and Ms. Johnson returned to teaching the class, the Math Eight course in the third period was observed until the observations were completed.

The school schedule consisted of six 50 minute class periods per day with the period between 11:39 am. to 12:15 designated for lunch. The first class of the day for all teachers was set to begin at 8:45 am. to allow teachers to have staff meetings prior to start of the classes, as opposed to at the end of the day, when most teachers were either tired or would need to leave the school right away to look after their personal and family obligations.

A written permission was obtained from the principal of the Oxford Middle School, Ms. Roberts, before commencing the study at the school. Mathematics teachers Ms. Jenkins, and Mr. Hubert, and science teacher Ms. Buchanan were members of the school community who were interviewed for this study.

Description of the Courses Observed

The demographics of each of the mathematics courses Ms. Johnson taught at Oxford Middle School are described in Table 1. The seats in the classroom were arranged so that students sat facing each other in groups of four. Only the Algebra One class had enough students for each group to be of size four. In each of the other three classes, groups of three and two students were more usual. Ms. Johnson assigned students to new teams every three weeks. Group memberships were determined semi-randomly and based on the birthday month of students, as the teacher called out the name for each month at random, and students with birthdays in that month could pick their seat where there was room.

Table 1

Demographics of Ms. Johnson's Mathematics Classrooms

	Gender		Ethnicity		
Course	Female	Male	Caucasian	African-American	Asian
Algebra One	14	18	29	0	3
Math Seven	10	7	17	0	0
Math Eight (3rd period)	8	5	13	0	0
Math Eight (4th period)	17	5	20	2	0

In the Algebra One class, the main focus of most of the lessons during the observation period was for students to learn about the connections among patterns in data and writing rules and generating graphs. In the Math Seven class, the overarching goal for many of the lessons was for students to acquire knowledge of operations, and number sense – mainly integers. A major focus in the Math Seven class was to use the guess-and-check strategy to identify a general rule for data with some form of numerical pattern. In the Math Eight courses the main objective for many lessons was to understand proportionality, equivalency between fractions, decimal numbers, and percentages, and constructing and simplifying expressions containing variables of the first and second order. The rules of the order of operations were emphasized in all classes. Students were allowed to use calculators on select problems only.

Ms. Johnson assigned and collected homework only during the week. Her reasons for not assigning homework for over the weekend were twofold. First, she believed that it was important for students to have time to spend with their families during the weekends. Second, she referred to research showing no evidence of the benefits of extra amounts of homework on improving student achievement. However, if the work that was assigned for students to do during class time was not finished, that work added to the homework students received during weekdays and all such work needed to be turned in, promptly.

Method of Data Collection

After Ms. Johnson was selected as the key informant in this study, an "Initial In-depth Interview Protocol" (Appendix D) was used to begin the process of collecting data for the case. This interview took place before field observations of the key informant's classrooms began. Classroom observations began in the following week, and Ms. Johnson was observed in all mathematics classes she taught. The field observation data were collected in the form of the researcher's notes and augmented for clarity of classroom events by transcriptions of audio recordings of the whole-class discussions. This data provided a window for exploring the teacher's intentions and practices in context, thereby illuminating what the teacher knew, and what skills and in what ways the teacher planned for putting that knowledge into practice in accord with the demands of the curriculum, the reaction from students in the classroom, and in response to any influence from the teacher's colleagues and supervisors at the school.

Small-group discussions between students or between the teacher and students were not recorded to avoid interfering with the teacher's intended strategies for implementation of the lessons in the classroom. However, the researcher captured the generality of these conversations in the field notes describing the nature of small group discourse. For example, when the teacher asked one of the groups "How did you get that?" the researcher noted in the field notes as "the teacher is asking students to explain their reasoning." On the other hand, audio recordings of whole-classroom discourse were transcribed daily, the results of which were used to prepare an accurate picture of what transpired in each class on any given day.

Upon the completion of each observed unit in each class, an in-depth post-unit observation interview (Appendix F) provided the teacher with an opportunity to reflect on the impact of instructional strategies over the course of the unit.

Daily, pre and post-observation interviews (Appendix E) with the teacher gathered additional information about more specific, daily intentions in the instruction during the specific unit. This information was used to clarify the teacher's reasons and thinking behind the plans and goals for each class session.

A final interview (Appendix G) was conducted with the teacher after the end of all field observations. The purpose of this interview was to provide the teacher with an opportunity to reflect more globally on her instructional experiences over the course of the study. Also, the interview aimed to probe the teacher's perception about her students' performance, instructional strategies that worked particularly well, and any improvements in her instructional plans for the next time when she will teach the same content.

With respect to studying the impact of the school environment on the teaching behavior of the teacher, the influence of the principal as well as the influence of other mathematics teachers in the school were important for this investigation, because they represented part of the community within which Ms. Johnson functioned and made decisions (Bay et al., 1999; Furman, 2002; Steele, 2001; Sergiovanni & Starratt, 2002). From the outset, Ms. Johnson was asked to identify those individuals with whom she collaborated in regards to her work. Three teachers – two mathematics teachers and one science teacher – were identified by Ms. Johnson as colleagues with whom she coordinated the plans and actions in her classrooms, regularly. Interviews with all of these three teachers and the principal formed the main ecological data for this study. Since Ms. Johnson was a leader in mathematics education in the state, she influenced other teachers' practice more than her own practice was influenced by these colleagues. In addition, because Oxford Middle School was located in a small town, there were close relationship and coordination of efforts between Ms. Johnson and the high school mathematics teachers in the high school nearby. An audio recording of what transpired in one of the meetings between the leading mathematics teachers in both schools, obtained by Ms. Johnson herself, was also collected and considered as data for this study. This audio recording provided valuable information regarding the influence of the high school on the mathematics teaching at Oxford Middle School.

Data Sources

Multiple sources of evidence were used for triangulation to ensure the validity of the case study (Patton, 2002; Yin, 1994). The primary sources of data included the initial (Appendix D) and final in-depth (Appendix G) interviews, the post-unit observation interviews (Appendix F), daily and daylong classroom observations, the audio recordings of the lectures and teacher-directed interactive discourse from inside the classrooms, the daily pre and post-observation interviews (Appendix E) which took place before and after each observation session, and the school community protocol (Appendix H) to guide the interview of the two mathematics teachers, one science teacher, and the school principal. All of these persons had close collaboration with respect to Ms. Johnson's practice. Ms. Johnson was also asked to express her opinion about the influence of the school community (e.g. the principal and other mathematics teachers and colleagues) on her work. These statements by the teacher were compared with the statements of the principal and other mathematics and science teachers about what they saw to be at stake for them in the type of teaching that occurred inside the mathematics classrooms at the school. The intersection between the statements of the teacher's colleagues at the school, and the ones made by the teacher were used in describing the influence imparted by the school community.

The secondary sources of data included handouts of select lessons that the teacher used during observed lessons, and informal and sometimes recorded and transcribed conversations with the teacher that took place throughout the time of observations.

Initial in-depth interview.

To initiate the study of the intentions and practices of the key informant, Ms. Johnson was interviewed using an *initial in-depth interview* protocol (Appendix D). This interview was designed to gather clues and information about: 1) the teacher's overarching intentions for each of the courses she taught as well as those of her general intentions for day-to-day lesson activities; 2) the curricular material she used; 3) the instructional tools and strategies she employed; 4) her knowledge of her students and how they learn; 5) collaboration with other colleagues in regards to what and the way she taught; 6) and whether or not the administrative staff, more importantly the principal, had any impact or influence on what and the way she taught. This interview was audiotaped and transcribed. This interview, in combination with the totality of pre and post observation interviews, helped map Ms. Johnson's cognition about her work (Artzt & Armour-Thomas, 1999).

Classroom observations, audio recordings, and field notes.

A main purpose of this study was to prepare a rich case study description of the practice of an expert mathematics teacher in the context of a reform-based curriculum and instruction program. Thus, classroom observations were a necessary component in this study. Classroom observations began subsequent to the completion of the initial in-depth interview. The researcher observed the teacher on a daily basis in all mathematics classes in the teacher's schedule where the reform-based curriculum was adopted.

A special microphone, Crown's SoundGrabber, with the ability to collect audio signatures from throughout the classroom supported the researcher in each observation. The audio tape recorder and the attached SoundGrabber were placed at a fixed position on a stand near the teacher's desk in the back of the class during all observations. The audio tape recorder was then turned on by the researcher during whole-class discussions and turned off during small group, or the so called "work time", sessions. The overwhelming majority of these audio recordings were transcribed daily. Although the audio taping of the whole-class discussions continued throughout the observation period, transcriptions of the audio recordings discontinued in the latter part of the final week of the observations, since they contained no new information.

The field notes consisted of descriptions of significant mathematical tasks, the teacher's role in classroom discourse, the students' role in classroom discourse, the nature of the interaction between the teacher and the students and the instructional tools and strategies the teacher used to enhance instruction. The dominant characteristics of the learning environment and methods of classroom management were also noted in these field notes. The observations spanned over seven weeks, when it was determined that no new information was gathered.

The combination of the field notes and transcribed audio recordings of whole-class discussions provided an accurate picture of what transpired in each lesson in each class on any day. By comparing the teacher's actual classroom practice and her responses in the interviews about her practice, the researcher developed a more accurate picture of the teacher's intentions and practices.

Whole-class discussions were given close attention, and the consistent nature of small group discussions was also noted in the most general sense of what the teacher wanted students to do in their group "work time" activities. The questions asked in the classroom, and the nature of the classroom discourse along with tasks performed by students were of utmost interest. For example, in the encounters between the teacher and students during small group activity, Ms. Johnson repeatedly asked students to explain how they had arrived at their answers, whether anyone had been able to come up with an alternate solution strategy, and if not, whether there was consensus among members of the groups about the final answer to a problem. Similar observations of classroom events and teacher-directed interactive discourse revealed that the teacher favored a process-oriented teaching strategy that emphasized consensus building and alternative solutions among students.

Pre and post observation interviews.

Pre and post observation interviews (Appendix E) were conducted and transcribed daily for each lesson in each class. During the transcriptions, if a point was made by the teacher that did not seem clear to the researcher, the point was discussed with the teacher for clarification the next day. In general, these interviews, together with the field notes provided information about the teacher's cognition during the pre-active stage, interactive stage, and post-active stage of each lesson (Artzt & Armour-Thomas, 1999). More specifically, the purpose of these brief interviews was to gather the teacher's instructional plans and intentions for student learning and to compare these with the implementation of those objectives and expectations through classroom observation of each lesson. These interviews also attempted to identify if the school community affected the teacher's plans for her classes. This process helped to unravel how the teacher's thinking and intentions about instructional strategies aligned with her practice of teaching from a reform-based curriculum, and also how she was able to overcome or circumvent any obstacles in the way of the stated objectives and expectations.

For most of the study, the pre observation interviews were conducted together for all classes in a day, and all the post observation interviews were conducted together. The researcher aimed at to minimizing intrusion in Ms. Johnson's work while collecting data from these interviews, and hence several adjustments in the timing of the interviews were made.

Post-unit interviews.

Post-unit interviews (Appendix F) for each class probed Ms. Johnson's perceptions about the students' performances, her rationale behind instructional strategies used, and the reasoning for any improvement plans in her instruction in future teaching. These interviews also probed the teacher's plans for the up-coming unit and rationale for use of particular instructional strategies as well as student involvement in the instruction. A post-unit interview was conducted at the end of each unit in each of the classes taught by Ms. Johnson. The teacher conducted a post-unit test in each of her classes, and it was only after the results of these tests were known to the teacher that the post-unit interviews were conducted.

Final interview.

At the end of all the classroom observations the teacher was interviewed using the *In-depth Final Interview Protocol* (Appendix G). The purpose of this interview was to engage her in reflecting on her experiences in teaching over the past units, plans for implementation of future lessons, and the students over the entire course of the study. Connections were made between what the teacher had said in the beginning and what was actually observed in the classrooms, trying to make sense of any discrepancies between those plans and the real-world classroom experiences. This interview provided a more global scan of the nature of her practice and the impact of the classroom community on the instructional strategies. This interview sought to uncover areas of challenge that persisted throughout the observation period, looking into how Ms. Johnson did or intended to resolve those challenges, while creating and maintaining a learning environment aligned with the reform-based curriculum and instruction program. The aspects of the teacher's ability that enabled her to meet these challenges were incorporated in the descriptive model of the intentions and practices of a highly qualified and experienced middle school mathematics teacher in the context of reform-based mathematics curriculum and instruction.

Another benefit of the final in-depth interview was to provide the additional information necessary for triangulating what the teacher believed to be hampering or supporting her practice within a reform-based curriculum and instruction context with the emerging results from the analysis of the field notes and the pre and post observation, and post-unit observation interviews. This type of triangulation of the data was used to minimize potential researcher error or bias.

Impromptu interviews.

In addition to the wealth of information that Ms. Johnson's practice and interviews provided for the benefit of this study, her ceaseless efforts to accommodate the needs of this study allowed the researcher to conduct additional interviews. These additional and impromptu interviews gave further depth and insight to examining Ms. Johnson's practice. All such interviews were recorded and transcribed, and included a wide variety of topics. These interviews focused on: 1) the socio-economic status of the students at the school; 2) the impact of her educational background and the numerous professional development programs she had attended on the formation of her intentions as a teacher and her teaching philosophy and strategy; 3) her approach to the practice of giving a new lesson as opposed to one that was a review; 4) her reasons for refraining from assigning homework for the weekends; 5) her thinking and intentions about the homework she assigned during the week; 6) her thinking behind the type of discourse she intended to generate during her lessons; 7) her general expectations as a teacher; and 8) statement of her beliefs about how students learn, her beliefs about mathematics, her beliefs about student characteristics, and her beliefs about teaching. This latter interview was the only interview of this kind not conducted face-to-face.

School community interviews.

All three teachers with whom Ms. Johnson collaborated: Ms. Jenkins – an expert middle school mathematics teacher herself, Mr. Hubert – a newly hired mathematics teacher with only one year of teaching experience, and Ms. Buchanan – an experienced science teacher, were interviewed (Appendix H) to gain insight into the nature and extent of their collaborations with Ms. Johnson. The arrangements for these interviews were made by Ms. Johnson.

An interview with the principal (Appendix H), Ms. Roberts, also revealed significant contributions made by the principal at Oxford Middle School to the success of Ms. Johnson.

Each of the interviews with the three school community teachers and the principal took place face-to-face. All four interviews were recorded and transcribed on the same day in order to detect and clarify any unclear statements. However, no such clarifications became necessary as each interviewee provided a comprehensive look into the nature of their collaboration with Ms. Johnson.

In addition, Ms. Johnson expressed and acknowledged that regular meetings with mathematics teachers at the nearby high school provided her with input and feedback that she took into account while teaching her mathematics classes, especially since many of the students from Oxford Middle School transitioned to that high school. One such meeting occurred during the study in November and Ms. Johnson suggested that she take the researcher's audiotape recording equipment and record the discussions at the meeting. The audiotape from the discussions with the high school mathematics teachers also revealed important avenues of influence on the practice of Ms. Johnson.

The researcher and researcher's journal.

With the qualitative nature of this study, the researcher was the proverbial filter through which all the data collected were interpreted. Hence, the issue of the impact of the researcher's personal views and biases had to be addressed. At the time of this study, the researcher was involved in the third year of a series of yearlong studies of the impact of technology-based professional development programs on the teaching of secondary mathematics teachers. Prior to this position, the researcher had spent three years teaching developmental algebra and introductory statistics courses to college level students on two different university campuses. The researcher's three years of teaching experience and his work as a researcher for three years shaped his views and biases about teaching mathematics, content important for students to learn, and students' overall view of mathematics and statistics.

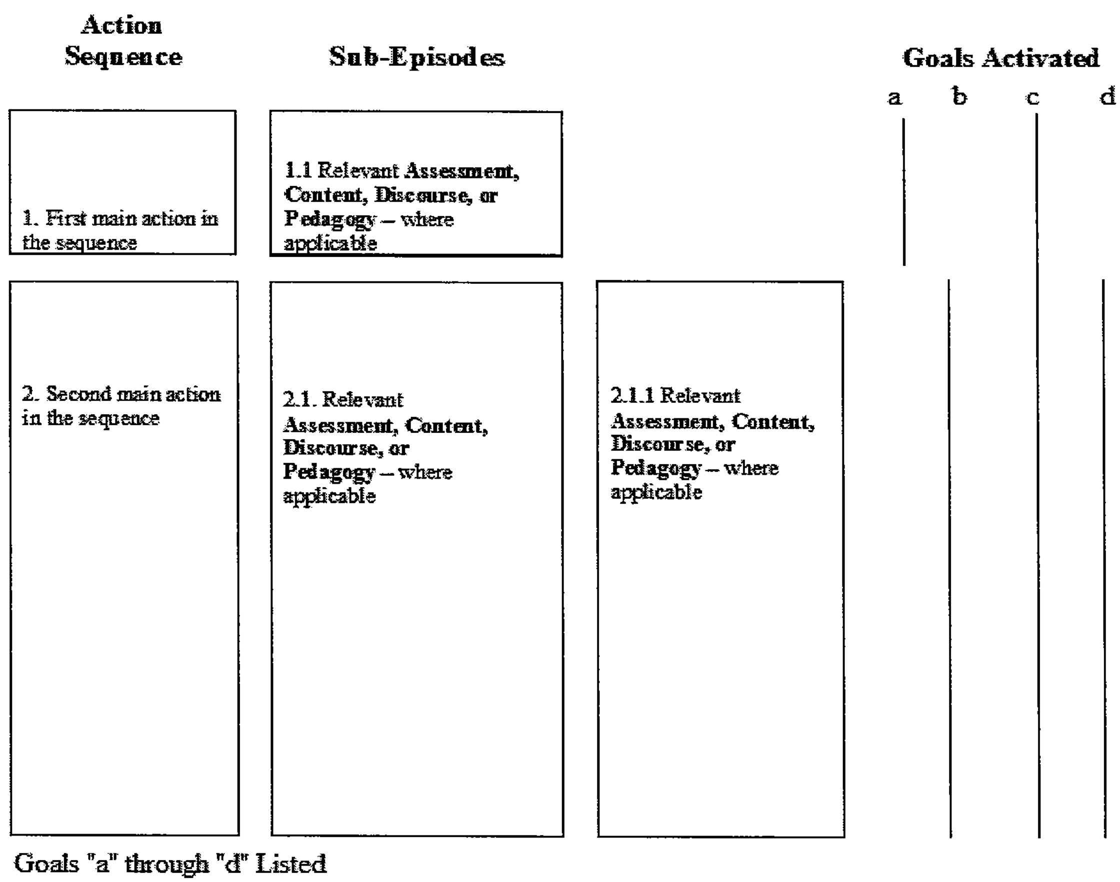
In light of these influences, the researcher recognized the importance of the accuracy of his interpretations of the collected data in representing the essence of the teacher's practice and the role of the class and school environments in shaping that practice. From the beginning, a close partnership was established between the researcher and the teacher that lasted throughout the study. Through this close partnership, the researcher was able to establish a free and unfettered communication between himself and the teacher. Through this communication, the nature of the main emerging themes of the study were brought to the attention of the teacher for discussion to ensure an accurate and unbiased capture of the thinking, intentions, and the nature of Ms. Johnson's practice as well as all the influences imparted by the classroom and school communities. Excerpts of analyzed data were made available to the teacher for her review and feedback. Discussions of the ramifications of the findings of the research with the teacher occurred freely and willingly throughout the study.

From the communication with the teacher, and through his own observations, the researcher was also able to keep a journal of personal opinions, thoughts, and ideas as they evolved over the course of the study. The main emerging themes in the study were challenged through confirmation by triangulation of the collected data and those findings were verified with the teacher to ensure accuracy and unbiasedness. The researcher's reflections recorded in the journal were used to maintain a personal analytical eye on the data as it was collected throughout the course of the study. Focus on the research questions was ensured through these detailed actions.

Data Analysis

The qualitative study involved accumulating and analyzing observational and interview data from the work and statements of the teacher as well as interviews with key members of the school community. In light of the research questions, the researcher examined the teacher's intentions, practice, and the influence of the workplace on her intentions and practice using methods that had emerged from the review of the literature. Schoenfeld's (2000) analytical framework for describing a model of the teaching process was used to prepare and describe typical models of the lessons and teaching process for each of the mathematics courses the teacher taught. The models helped create maps of the connections between the goals and intentions of the teacher and the manner in which she put those goals and intentions to practice. Models of each of the teacher's selected lessons were created to comprise the teacher's goals and the action sequences. The action sequences were then parsed by the type of assessment the teacher used during the lesson, the material content of the lesson, the discourse involved, and the pedagogy used. A generalized model format used for this process is shown in Figure 2.

Figure 2. Generalized depiction of a detailed model of a lesson exhibiting major action sequence and goal trace



Vertical trace lines, as shown in Figure 2, were used to identify the action sequences and sub-episodes during which the pursuit of each stated objective remained active. Not every lesson observed for each class was parsed. From the lessons observed for each class on a daily basis, a series of information-rich lessons that addressed the same topic over a span of two or three days and represented the teacher's typical instructional approaches, were selected and parsed using the framework. The purpose of the components of the model of a lesson was to dissect the entire lesson into segments that were more homogeneous in nature so that, where applicable, the types of assessment, discourse, and pedagogy deployed could then be described. Patterns of assessment, discourse, and pedagogy were then examined in relation to mathematical content. Dominant aspects of the teacher's approach were incorporated in the final descriptive model of the teacher's practice.

In addition, the broad scope of common components of teachers' pedagogical content knowledge that emerged from the review of the literature (An et al., 2004) was used as *an amalgam of themes* (see Figure 1 in Chapter II) to enrich the emerging picture of the model of the teacher's practice with more meaning and resolution.

For example, "knowing students' thinking" formed one of the themes from the model of the pedagogical content knowledge proposed by An et al. (2004). The evidence gathered about this theme indicated that the teacher used multiple methods of formative assessment for exploring her students' thinking. She used classroom assignments that were completed and turned-in by students daily; she conducted daily whole-class discussions that were based on a guided question-response format; and she toured small group discussions of all groups repeatedly on a daily basis, posing a series of questions that ranged from looking at students' conceptual understanding of the content to knowing how close students had come to mastering basic mathematical procedure.

The pre and post observation interviews added to the reliability of the data resources in developing the analysis, since they provided repeated look into how the teacher planned her lessons, gauged student understanding, disclosed the teacher's own thinking about how the implementation of each lesson turned out, and her developing ideas about planning and practicing the upcoming lessons. Thus, at a narrower and more focused level of analysis of the lessons, pre and post observation interview protocols provided information about the teacher's intentions for each lesson. These lesson-based statements of intention and goal were utilized in two main ways. First, they provided initial information at the point of entry into Schoenfeld's model of a lesson. Second, they provided data that was used as a continuing checking mechanism in respect to and triangulating with the teacher's responses to the initial, post-unit, and final interviews. Moreover, daily transcription and study of the pre and post observation interviews, six in all for each day, provided particularly useful method of understanding and making sense of the field observations.

Data from the initial and final interview protocols and from the post-unit observation protocol provided access to the teacher's more global intentions for instruction as well as learning of the students. Data from all interviews were compared in light of the field observations of the lessons throughout the duration of the study to detect any discrepancies between the teacher's developing statements of plans and objectives made during the interviews and the actual practice of the lessons. Any such discrepancies were rare and occurred mainly due to changes made to regular daily schedule of all classes to accommodate for events (fire drill, field trips, and the "snake man" show) at the school.

The discussions with the teacher about the events in the classroom and school communities led to identification of two mathematics teachers, one science teacher, and the school principal as key individuals within the school community. The interviews with these individuals, between the teacher and the principal at the middle school and the mathematics teachers at a nearby high school, were analyzed to determine the influence from these individuals on the teacher's practice and the development of her intentions at work. Statements from the transcriptions of these interviews and the collected audio recording were compared with the statements of the teacher and what was observed in the classrooms to determine the depth and scope of the influence from the school community.

The researcher recorded his personal reflections about the teacher's intentions and practice and the influence of the school community, in the form of entries into a journal. Some of these journal entries were recorded as the researcher was collecting the data and some were recorded outside the premises of the school and while the researcher was reviewing and analyzing the data. Some of these journal entries led to the impromptu interviews with the teacher. Yet, the content of some of these journal entries were simply discussed with the teacher in casual lunch-time conversations. Maintaining a record of the researcher's reflections in a journal was key in keeping the researcher's analysis on track with the research questions, ensuring elimination or minimization of bias or gaps in understanding on the part of the researcher.

Ethical Considerations

As a matter of conducting an ethical study, pseudonyms were used to protect the identity of all teachers and the location of the study. A two page abstract of the study was provided to all prospect key informants as a means of ensuring full disclosure to these teachers before they signed the informed consent form for interviews and observations. The researcher offered each of the eight teachers a full copy of the methodology section for the study. None chose to see the full scope of the study, sufficing to see the abstract only.

CHAPTER IV

Results

The purpose of this study was to investigate the intended objectives and practices of an expert secondary mathematics teacher within the context of a reform-based mathematics curriculum. The investigation focused on how the teacher used the curricular material in her lesson, the level of interaction observed between the teacher and students, the types of tasks that students were engaged with, and the nature of discourse that took place during the course of the lesson.

Schoenfeld's (2000) model of the teaching process was used to parse *typical* lessons given by the teacher in all of the mathematics classes she taught, connecting her stated objectives and the objectives that the researcher gathered by observing the lessons with the actual instructional strategies she used in the classroom (see Appendices I – O). Table 2 provides a summary of the lesson topics parsed.

Table 2

The Range of Topics Represented Among the Parsed Lessons

Course Title	The topics	Appendices
Algebra One	- Solving equations for any term or one variable - The distributive property, and Independent and dependent variables - Generalizing of patterns to abstract rules	I, J, K

Table 2 (continued).

Course Title	The topics	Appendices
Math Seven	- Connecting the guess-and-check strategy to abstract generalization, and order of operations - Operation sense and number sense in operations involving integers	L, M
Math Eight (3^{rd} & 4^{th} periods)	- Operational sense in adding or subtracting terms of algebraic expressions - Extending the guess-and-check strategy to constructing solvable algebraic equations	N, O

The lesson models described the intentions or goals (the why) and practices (the what and how) of what was observed in the teacher's practice. Homogeneous chunks of each selected lesson were described according to the assessment used, content covered, discourse initiated, and pedagogy deployed, and these were *linked* to the teacher's intentions.

The guiding principle for this analysis follows Schoenfeld (1998); he proposed that if one has a good understanding of the teacher's beliefs, intentions, and the main components of knowledge base (subject matter knowledge and pedagogical content knowledge) in a particular context, then one should be able to ascertain with great detail what the teacher does and for what reason. "Asymmetric aspects of teachers' work" was an important element of the theory for this analysis. Schoenfeld called situations of this type "when something happens," and pointed out that circumstances like these could affect the *balance* between teachers' beliefs, intentions, and knowledge, and what they typically did.

This chapter begins with a detailed description of the physical, human, and curricular contexts within which the teacher worked. Next, the teacher's beliefs, intentions, content knowledge, and pedagogical content knowledge are discussed. Finally, these attributes of the teacher's practice will be used to shed light on the research questions:

1) *What are the intended instructional objectives and practices of an expert secondary-level mathematics teacher for instructing mathematics in a reform-based mathematics curriculum program?*
2) *What aspects of the classroom community and the school community impact the intentions and instructional practices of an expert secondary-level mathematics teacher in a reform-based mathematics curriculum program?*

First, it is necessary to describe a coding scheme to facilitate evidential references in support of the researcher's claims, and also give an overview of how this chapter is meant to tie in together.

The Data Source Coding Scheme

The data presented in this section as evidence in support of the teacher's practice has been coded based on the source from which it came from. Table 3 shows each code and its translation in terms of the source of data.

Table 3

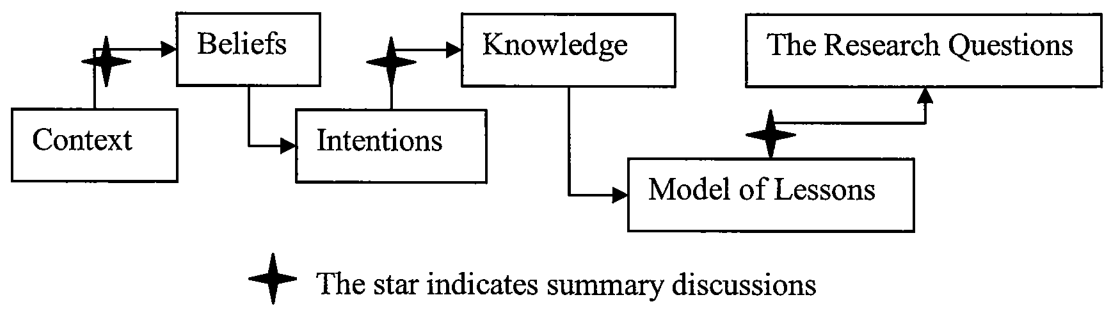
Data Source Codes

Code	Data Source
CA	Classroom audio
FI	Final interview
FN	Field notes (i.e. classroom observations)
II	Initial interview
IMI	Impromptu interview
POI	Post observation interview
PRI	Pre observation interview
PUI	Post unit interview
SCI	School community interview
SI	Selection interview

Intent of the Chapter Organization and a Summary of the Results

In order to answer the research questions in this study, information about several key factors of the teacher's practice were gathered. Following a topical outline of the chapter, the main argument was outlined with the descriptive data. Each factor was dependent on or interacted with at least one of the other factors. For example, *beliefs* have an impact on *intentions*, and both beliefs and intentions are influenced by the *context*. Therefore, a discussion of intentions is more properly put into perspective by a prior coverage of the teacher's beliefs, and a study of the teacher's beliefs and intentions is better illuminated, given a knowledge of the ecological factors interacting with the participant. Moreover, upcoming examples of models of the teacher's lessons (Schoenfeld, 2000) bear more meaning for the reader if a description of the teacher's beliefs, intentions, and knowledge is already described. Given the interdependence among these various factors that imparted significant influence on the work and thinking of the teacher, Figure 3 depicts the topical hierarchy of the results presented in this chapter. Each star indicates a point along the chapter where a succinct discussion of the upcoming results is presented.

Figure 3. Topical hierarchy of the presented results



This chapter shows how Ms. Johnson, as an expert teacher, created part of her own physical, human, and curricular realities in order to accomplish her goals. From the group arrangement of seats and desks and mathematical posters that hung on the walls to facilitate demonstrations, to her role in the collaborative school community, to the curricular materials she chose or the materials she gathered over the years, she was not the subject upon whom various contexts acted, but was the one who shaped them. The presentation in this chapter is designed to show how Ms. Johnson's beliefs about students' various learning abilities led her to design and offer various learning experiences ranging from working with independent problem solving strategies and practice of mathematical procedures, to the use of manipulatives that involved all students in the group, to mathematical games and puzzles that engaged all students in the class. Ms. Johnson's practice possessed a strong social aspect with small group activities forming the greater portion of each lesson. This practice created a classroom environment that appealed to students, but also presented a challenge. Ms. Johnson found it difficult to persuade students to focus on learning problem solving strategies and sharing of their mathematical ideas rather than spending their time socializing.

The presentation in this chapter shows how Ms. Johnson's beliefs and intentions were in alignment with each other with respect to choosing reform-based mathematics curricula that emphasized assessing students' prior knowledge, and both conceptual understanding and procedural fluency. Her belief, that students enjoyed and learned a great deal from each other, prompted her to give lessons that encouraged students working and talking with each other to share their solution ideas. And, her belief that students have different learning modalities prompted her to utilize multiple representations of ideas. In her Algebra One class she emphasized relating patterns in data to graphical representations and from there to derive symbolic formulations. In her Math Seven and Math Eight classes she used manipulatives extensively to teach algebraic operations using integers and fractions. In poster presentations and when solving homework problems, students were required to show their work in multiple ways, such as procedurally as well as with drawings and graphs and in writing.

The reader will find a close link between Ms. Johnson's career-long participation in professional development programs and her knowledge base. Her relentless questions solicited participation by students in the lessons she gave. She aimed at engaging students right from the start, not by lecturing, but by using a computational or whole class activity that made use of problem solving strategies. The reader will also find a link between Ms. Johnson's beliefs and intentions with respect to teaching and learning mathematics and the pattern recognition and connection making strategies she used to teach. Students' misconceptions did not escape her, and she fostered conceptual understanding to eliminate misconceptions (e.g. the misconception of a variable representing a fixed value) as well as achieve procedural proficiency among students.

Ms. Johnson based her teaching on continuous assessment of students' existing knowledge, and her teaching emphasized learning of procedures when she had to re-teach topics taught much earlier during the year or in the previous years (e.g. long division). Samples of models of her lessons described how she was able to link her intentions with various pedagogies (see Appendices I – O).

Furthermore, it became apparent that the classroom community exerted its influence on Ms. Johnson's intentions and practices only when a lesson did not go as planned, and thus Ms. Johnson adopted strategies to adapt to the new situation. When, for example, some students protested a review lesson and expressed that they "know it all," Ms. Johnson conducted a diagnostic test and sent those who scored 100 percent on the test to work on problem solving using computers under the supervision of a student teacher. In another occasion, Ms. Johnson made changes to the content and requirements for getting classroom homework done in order to curb overly social behavior. This strategy was somewhat successful, but overly social behavior and student motivation remained challenges for Ms. Johnson until the end of the study. Other than these special circumstances, Ms. Johnson remained committed to her own beliefs, intentions, and knowledge of what was required for teaching and learning mathematics.

Finally, Ms. Johnson's collaboration with the school community helped her improve her knowledge base by acquiring time and permission to attend professional development programs nationally and sharing that knowledge with her colleagues in seminars. Her collaboration with the school community also helped her establish quality control mechanism by which she and other science and mathematics teachers at the school focused on coherency across grades and sharing and using instructional strategies that worked to students' success.

At this point, the details of the various contexts within which she either worked or helped to create are presented.

Physical, Human, and Curricular Contexts

Physical Context

The Oxford Middle School was located in a heavily wooded and rural area, but only within a few miles of a large urban area. The surrounding forests were often used for students' field trips, one of which took place during the course of this study. Neither Ms. Johnson nor the researcher went on this trip nicknamed "forest field day." The only high school in town was located barely 100 yards away, where a large portion of finishing eighth graders were expected to attend school the following year.

On the school premises, Ms. Johnson taught all of her mathematics classes at Oxford Middle School in the same classroom. The classroom was equipped with an overhead projector, a document camera, a whiteboard, and a large projector screen that were located in front of the classroom. Ms. Johnson's desk, her laser printer, and a computer workstation were in the back of the classroom. When needed, she moved the computer from her desk to the front of the class for the student teacher who was being trained under her supervision. To the left of the entrance to the classroom, and along the wall, a table set up was available for students who wanted to work independently, if and when they could not get work completed in their groups. A sink equipped with a water fountain and cabinets where various supplies, including manipulatives and 10 graphing calculators were being kept were positioned along the wall up to the front of the class. Beneath the ceiling cabinets and on top of the floor cabinets, there were baskets for each class, where students could turn in their homework, spiral notebooks, and other artifacts. The top of the walls contained motivational slogans to brighten the room. On the opposite wall, various reusable math learning posters were hung. One poster depicted a large plastic Cartesian coordinate system, where students could locate ordered pairs and draw graphs, erasing them when they were done. Another poster depicted a large number line with equivalent fractions, decimal, and percent values. The wall ended to a glass door that opened into the school yard and students could use the door as a shortcut for traversing school premises during recess. On the back wall and right behind Ms. Johnson's desk, there were more cabinets, shelves, and racks for students to put their winter clothing, backpacks, and other supplies they did not use during the class.

Human Context

The overall atmosphere prevalent in the classroom and between teachers in the broader school environment was friendly. In the classroom, Ms. Johnson often took time during recess to talk to students about their personal lives. Ms. Johnson also taught physical education classes at the school and often engaged students about their favorite football teams and football players outside of the class time. Recess times were not always spent on social discussions, though, as Ms. Johnson was often observed helping students with their mathematics problems about half-an-hour before the commencement of classes at 8:45 a.m. Unfriendly incidents among students were highly uncommon. Nevertheless, a rare altercation between a female and a male student in the third period eighth grade class was resolved amicably as Ms. Johnson interfered and spoke to both students alone while the other students were gone, ordering the male student to "stop teasing" the female student.

The relationship between the teachers at the school always appeared professional as well as warm and cordial. The teachers' lounge was almost always packed with teachers during the lunch hour, chatting and socializing in a friendly atmosphere. The school principal often joined the rest of the teachers at the lounge. Ms. Johnson was mostly absent from these luncheons during the first few weeks of the study. She was in charge of troubleshooting problems with the computer systems at the school, and ate lunch at locations where she was fixing a computer problem. However, a new person was hired to troubleshoot computer problems at the school, and after that Ms. Johnson attended luncheon meetings regularly and brought considerable life and energy to the discussions between teachers and the school staff.

The teachers and school staff with whom Ms. Johnson collaborated with respect to her work were Ms. Jenkins (an expert sixth grade mathematics teacher with nearly 25 years of teaching experience), Mr. Hubert (a novice mathematics teacher with only one year of teaching experience), Ms. Buchanan (a science teacher with twelve years of teaching experience), and Ms. Roberts (the school principal). This list does not include mathematics teachers at the nearby high school with whom Ms. Johnson and Ms. Roberts had monthly meetings about their vision of mathematics teaching and learning at both schools.

The interview with Ms. Jenkins revealed key aspects of her thinking about Ms. Johnson as a colleague. She said "She is very interested in learning more about how to best teach mathematics…her positive enthusiastic attitude about teaching mathematics [gets] everyone involved with that" [SCI – Ms. Jenkins, Question #1]. She further elaborated that:

> My role in this collaboration has been one of, we're peers with very similar philosophies about how students can learn mathematics and that it's the teacher's responsibility to find ways to make sure kids can learn and understand and remember, and I think it's that collaboration that has worked for us. [SCI – Ms. Jenkins, Question #3]
>
> We tend to meet on a regular basis. If I'm doing a big math project, she's one that comes up and looks at the student material and the same with when she's doing a big project. We share our ideas about what was successful. [SCI – Ms. Jenkins, Question #6]

The collaboration between Ms. Johnson and Mr. Hubert, the new mathematics teacher at the school, was also strong. Mr. Hubert said "We've interacted quite a bit. Every week. When we designed lessons based on the observations based on scores, or impressions of formative and summative assessments" [SCI – Mr. Hubert, Question #6].

The collaboration between Ms. Johnson and the science teacher, Ms. Buchanan, was no less intense than with the mathematics teachers at the school. Ms. Buchanan said the following about her work with Ms. Johnson:

> It's an equal partnership, and we just find ways to connect everyday and talk about what needs to be done and a good example would be just last week when students weren't getting the math in our forest field day unit, and this was an integrated unit and they weren't understanding the pages and I was getting a lot of questions mathematically, you know, and they weren't able to do the computation, and so I just talked to her that morning and they… and I said 'Is there any way you guys could go over this, and she scratched what she was going to do that day and they went over it, and did a practice run. So, you know, we work together and we are flexible that way and we just, you know, want students to succeed. [SCI – Ms. Buchanan, Question #3]

The school principal, Ms. Roberts, reiterated the collaborative atmosphere that existed between Ms. Johnson and the other teachers at the school. She had the following remark to say about Ms. Johnson: "She shares her knowledge, her information, and what she has learned in her instructional practices with other teachers. So, there is a lot of sharing and teamwork, collegiality in our school" [SCI – Ms. Roberts, Question #1]. Perhaps the most insightful comments about the atmosphere at the school came from the principal. She also had the following to say:

> Another challenge we have is because we're such a small staff, we wear many hats, and Ms. Johnson is on a ton of different teams: she is on technology team; she is staff; she is on Billie Pride; she is our tech coordinator. And so it's hard for her to be at so many different places at the same time and to devote that kind of energy to all these different things. So, we struggle with that, and again being a small school. We've overcome some of the challenges, because we are small and we care about each other and we care about our students. We devote the extra time, even though the teachers aren't necessarily reimbursed for it. [SCI – Ms. Roberts, Question #3]

One of the most interesting dimensions of all the interviews conducted with the school community is captured with the following three quotes:

"Our [Ms. Johnson and Ms. Jenkins] goal is that every child can learn mathematics and can be successful to the highest potential" [SCI – Ms. Jenkins, Question #4].

"She [Ms. Johnson] really cares about students first, and wants to make sure that they succeed" [SCI – Ms. Buchanan, Question #1].

"We [Ms. Johnson and Mr. Hubert] are constantly tweaking the curriculum . . . [with] the ultimate goal that I mentioned: the kids are successful in mathematics" [SCI – Mr. Hubert, Question #6].

It was apparent from all the four interviews that Ms. Johnson's name lit up the words "caring" and "students' success" in the minds of all four interviewees.

Curricular Context

Ms. Johnson determined the curricular context for all the courses she taught. She was given the freedom to choose her curriculum for each of the classes she taught, and, taking advantage of this freedom, she chose the *Connected Mathematics Curriculum* for her classes. These were: *Algebra Connections* for her Algebra One class, *Algebra Foundations I* for her Math Seven class, and *Algebra Foundations II* for both of the Math Eight classes she taught. The following excerpt from the selection interview with Ms. Johnson demonstrates her reflective thinking with respect to the curricula she chose and worked with.

> I think the algebra program that I am using, that's a high school level Algebra One course, was very effective last year, and having kids learn conceptually algebraic relations and connecting one idea to the other. So, I would say it's been very effective in student achievement. On middle level program, it's a spiraling curriculum, and it tends to jump around a little bit, and not go in-depth in certain areas that I think that it should, so I

> sort of question some of the way they've organized the materials. So, I believe it's effective, but I still think you have to do some changing of it to meet student needs. [SI – Question #2]

Ms. Johnson was reflective about the impact of the curricula she chose on students' thinking and work.

> I've always used a reform-based curriculum, and so I don't think my expectations for student work has changed. I did see the quality of the work change with this algebra program that I am using, compared to the algebra . . . I mean it's the same people who wrote the first one and the second one. The quality of what students put in their journals increased with better curriculum and better questions. [SI – Questions #3]

Ms. Johnson considered that teaching from reform-based curricula had helped her develop and maintain a sense of consistency and coherency that were not there when she taught her classes without them. The following excerpt elaborates this point.

> [Having] the curriculum has built a consistency with my students so that there is a beginning, a nice common thread and everything connects together to the end. So, it's I think. . . it's brought a consistency to what I teach and connecting everything together. Whereas, when you pick and choose yourself good lessons, sometimes they don't have that continuity. And sometimes you'll have a week where you're like 'Oh, my gosh, what am I going to teach this week?', and having that curriculum has allowed me to have a good math lesson everyday as opposed to great ones for two weeks and then maybe not great for a day or two and then another great one. So, I think they may not all be great but there is a consistency that wasn't there. And how the curriculum is designed to be taught a certain way was, you know, that balance of teamwork and individual and teacher led problems or teacher led discussions. So, really it does define how you structure a lesson. I mean there is some flexibility there, but I try and

> teach it true to the way it was written and that . . . that definitely affects the way I teach. [II – Question #17]

The reform-based curricula were not the only source of material for Ms. Johnson's lessons. Over the years as a mathematics teacher, Ms. Johnson gathered "good" lessons that came from various sources, including scholarly publications. These were lessons with which she had a positive experience with respect to their effectiveness in terms of student learning. Ms. Johnson provided the researcher with access to one of the large binders that held all such lessons. Samples of practice worksheets from some of these lessons that were implemented during this study were collected from the teacher. The lessons meshed with the content discussed in each course's curriculum and reinforced those objectives. One sample practice sheet, for example, emphasized students *constructing* mathematical equations that were representations of a pattern hidden in the shown information. This task required several attempts at speculating the pattern before acquiring the right one, and students were encouraged to work collaboratively on finding the right pattern. Figure 4 shows two such activities Ms. Johnson's Algebra One students worked on. For the table on the left, students were expected to construct the equation $(x*2)+1=y$ based on their exploration and discovery of the pattern that the data in the table represented. For the table on the right, students were expected to recognize the pattern as $\frac{x}{2}+1=y$.

To discover each of the patterns, students were not bound to use any particular procedure, but they were given the freedom to choose any of the four algebraic operations and write them in any combination to write expressions that made sense. Students were given the freedom to express their ideas about the patterns, mathematically

(NCTM, 1991, p. 25). Students were then to use the expressions they found to complete the tables in the figure.

Figure 4. Sample activities from a practice sheet distributed by Ms. Johnson

In (X)	Out (Y)
8	17
-2	
	9
12	25
10	21

In (X)	Out (Y)
100	51
4	
6	4
30	16
	31

One final source of material for Ms. Johnson's lessons came from the *integrated units*. The *integrated units* were the result of the collaboration between Ms. Johnson and the school science teacher, Ms. Buchanan. The units fostered teaching students about problem solving, and they aimed at broadening students' understanding of the applications of the mathematics they learned by mixing it and teaching it with relevant science contexts. The teaching of one such unit entitled "The Forest Field Day" was observed as taught by Ms. Johnson. Students were brought from the science class and mixed with the mathematics students from Ms. Johnson's class. Some of the richest and most engaging classroom teacher-directed interactive discourse observed during the course of this study took place during the teaching of this unit in Ms. Johnson's Math Seven class. The following excerpts from the lesson's pre and post observation interviews depict Ms. Johnson's plans for the lesson and her reflections afterwards.

> The plan is to take all the seventh graders [from both the science and the math classes] that are left after our field-trip and work through a mock of a proposed forest field day so that they could see all the decisions they're going to make and the math and the accounting and positive and negative numbers. So, to just walk them through one process with them, engaging them in the process. [PRI – October 16th, Math Seven, Question #2]

> I didn't have any ideas as to how we would get to data, or vote, or so I was just trying to find the most efficient way for kids to make decisions so that we could get the math on papers so they'd have calculations. And then I was going to do it all calculator, thinking it was a really appropriate calculator problem when I found out it wasn't so talking about how to put the calculator down. So, I made some adjustments in conversation there . . . [the kids] responded with more enthusiasm than I thought. And I think it was because they do realize that they need to know this. It's important for them to know it, because they're all going to do it in their science class. And there were a few kids though that weren't mine [science class students] that I felt might not have been engaged as much as they could've. But I didn't know their names, so that made it harder. [POI – October 16th, Math Seven, Questions #1 and #3]

Summary Discussion One

The aforementioned discussion about contextual aspects of Ms. Johnson's work unraveled the considerable freedoms she enjoyed in her professional life at Oxford Middle School. School atmosphere was friendly and conducive to establishing collaborative relationships with colleagues at professional and personal levels. It became clear that in this context, little obstructed the teacher from acting on her beliefs and pursuing her intentions.

The next few sections discuss Ms. Johnson's beliefs and intentions. Interviews with Ms. Johnson disclosed close connections between Ms. Johnson's beliefs and intentions. She believed mathematics teaching and learning was more than conducting mere lecture sessions and expecting students mastery of mathematical procedures. Her intentions for students to demonstrate their understanding of mathematics in multiple ways and settings reflected that belief. The reader will find more detailed evidence in the upcoming sessions.

Ms. Johnson's Beliefs

Pajares (1992) stated that "beliefs cannot be directly observed or measured but must be inferred from what people say, intend, and do." This perspective was adopted in examining Ms. Johnson's beliefs about how students learn, about mathematics, about student characteristics, and about teaching.

Beliefs about How Students Learn Mathematics

Ms. Johnson asserted that:

> Each student has a modality that dominates their learning. Most students have a strong visual modality. Then a small portion learn by touching or doing. Another small portion of students learn by hearing. Students learn best when the new information can be learned and connected to previous learning using their dominant modality. I also believe that learning can be a social experience where emotions can help learning take place. Students learn best when they are motivated by either intrinsic or by external factors. Peers and perception of what others think is one external motivating factor in middle school. Learning happens most when students start to take charge and ask questions they need to ask to understand. Socrates's philosophy of waiting for students to ask the questions aligns with my philosophy of how students learn. With this in mind we need to realize that times have changed some. Teachers need to teach students how to ask questions and how to be reflective on how they learn. [IMI – Ms. Johnson's beliefs, Question #1]

Ms. Johnson's intentions and practices corroborated her statements. The overwhelming majority of tasks she assigned during her classes were active and incorporated doing, touching, and discussing in a socially charged context. The many mathematical games she implemented, for example, often engaged students in computational tasks while reading, writing, competing with other students or groups, arranging tiles, sharing ideas, and explaining their solutions. She said: "I think [games are] a good way for kids to practice, because it doesn't seem work to them, but it is" [POI – October 18^{th}, Math Eight, Question #2].

This excerpt from a pre observation interview demonstrates Ms. Johnson's thinking and approach for making learning mathematics a fun and social experience for her students.

> I am going to do a chapter review in a game show type format, with a little bit of a competitiveness to it . . . They are going to be in their study teams and solve all different types of math problems together. [PRI – October 18^{th}, Math Seven, Questions #1, #2]

Typical approaches included students completing a matrix, ordering of numbers along the real number line, or winning a race, while committing themselves to complete various computations in the process. These activities often necessitated students to move between their seats and a range of media in front of the class and to communicate their ideas in various written, visual, and abstract or numeric modes, while speaking with their peers. The range of media consisted of a document camera, an overhead projector, large paper or plastic papers hanging from the whiteboard or one of the classroom sidewalls, or the whiteboard itself, where students were expected to present their ideas to the rest of the class. The following excerpt from the transcription of an interview was representative of Ms. Johnson's classroom approach to teaching mathematics followed through with her stated beliefs about how students were different in their learning skills and used different modalities to learn mathematics. In one interview right before a lesson she said: "Our objective is for students to see multiple representations of multiplying fractions" [PRI – October 16^{th}, Math Eight class, Question #1]. And the next excerpt from her response to another question on the same interview demonstrated her thoughts, in more detail, about teaching equivalent fractions to students. "They're going to multiply fractions in a visual model, and then they're going to multiply fractions in a game setting, to think about values, and so the whole focus is multiplying fractions, but in different settings" [PRI – October 16th, Math Eight class, Question #2].

Therefore, multiple representations of ideas in the form of reasoned discussions and making connections, completing information matrices, drawing graphs, arranging math tiles, preparing and presenting posters, completing a series of computational procedures, and active participation of students in a small group or whole-class game-like activities were the norms in each of the courses Ms. Johnson taught, providing various modes of participation and learning for students.

The social aspect of Ms. Johnson's teaching was unwavering. She said:

> I expect students to work as a team, and talk about their solutions, and ask questions so that they know if they didn't come up with the answer, how somebody else came up with it. So, having math conversations. I expect that to happen. And each team has different levels of that kind of engagement, and so there are some teams, if I can get all teams to work like that, it'll be a dream class. [IMI – Ms. Johnson's remarks about her general expectations as a teacher, p. 237 of the field notes]

Inside Ms. Johnson's classes, students sat in groups of two to four while working on classroom assignments or participating in whole-class activities. The teacher-directed interactive classroom discourse, whether part of a whole-class discussion or small group collaboration, revolved about consensus building that imparted and ascribed a sense of value to everyone's ideas and thoughts, as well as their participation. The following excerpt from the transcription of the classroom audio of an Algebra One lesson describes a typical pattern of teacher-directed interactive discourse between Ms. Johnson and her students:

> Did you guys share on figure 1 to see if you agree on the letter A on 1 and 5? Now, talk to each other and see if you agree before you keep going. Did everybody draw it the same? [CA – October 15th, Algebra One class]

Questions rooted in dissonance in students' thinking, while prompting students to build consensus and "agree", were common during whole-class as well as small group discussions.

Ms. Johnson believed that *student motivation* played an important part in their mathematics learning, and she considered student motivation the number one challenge she faced when teaching mathematics. Her assertion in the excerpt that "Peers and perception of what others think is one external motivating factor" [IMI – Ms. Johnson's beliefs, Question #1] was consistent with Festinger's *Theory of Cognitive Dissonance* that proposed "dissonance, that is, the existence of non-fitting relations among cognitions, is a motivating factor in its own right" (Festinger, 1963). The following excerpt from an interview transcription is expressive of some of the thoughts and concerns she had.

> Positive feedback for working hard and having these moments when those kids get to go "I get it!", and if you have enough of those early on, then they may stay motivated. And making sure that I'm introducing new enough material that it doesn't . . . it's not something they already learned in 5th and 6th grade. So, making sure that it's novel, you haven't learned this yet, and that gets motivating; to learn something you've never learned before and not just review, review, review. [II – Question #13]

Observation of her practice showed that Ms. Johnson frequently used words and phrases like "Good!" or "Good work!" [CA] repeatedly to convey positive feedback to her students. At the same time, she used a variety of problem solving strategies to avoid learning of mathematics from becoming monotonous for students.

Other more tangible means of motivating students in Ms. Johnson's mathematics classes were the *Billie Bucks*. *Billie Bucks* were paper money that were of value only within the walls of the school, and could be used by students to take field trips, or attend shows and events at the school. Ms. Johnson distributed *Billie Bucks* only at her own discretion; participating productively in classroom discourse, obtaining a high score on a challenging exam, or implementing creative problem solving strategies and a willingness to share these ideas with the rest of the class were some of the possible ways for students to acquire *Billie Bucks*.

Ms. Johnson's statement of belief about how students learn mathematics was consistent with her response during the initial in-depth interview, when she was asked what instructional strategies she had found to work well. She said that students learn best when they take charge of their own learning by asking questions.

Ms. Johnson also believed that direct lecture could only reach and benefit a small percentage of students and that she would have to do more to engage student thinking effectively. She said the following in response to a question on the in-depth initial interview questioning her about what instructional strategies she thought did not work very well and which strategies she believed were more effective:

> In general, I would say, me just standing up and telling them how to do things. . . it's good for about 25% of the kids, they can learn that way, but there are some kids that don't engage at all. So, when I just stand up and do direct instruction, it's good sometimes for high end kids, but it doesn't get everybody. So, I would say that doesn't work well because it doesn't meet *all* students' needs. And the strategy that works well is that idea of introducing problems and then facilitating their learning and letting students ask the questions as they need them. [II – Question #6]

Beliefs about Mathematics

Ms. Johnson asserted that "All students can learn mathematics," and that:

> Mathematics is a language developed to interpret the world. It can be understood on a continuum from very concrete to completely abstract. Most concepts can be introduced with concrete models with patterns leading to the understanding of the abstract. It is a universal language meant for all humans to learn. [IMI – Ms. Johnson's beliefs, Question #2]

In her classes, Ms. Johnson taught mathematics both to the procedure and to abstract thinking about concepts. She used pencil and paper exercises, and or showing and discussing problem solving strategies from the whiteboard to teach about various mathematical procedures (e.g. algebraic operations, absolute value, long division, etc.). And she used algebra tile models, pattern recognition, and connection making to teach abstract and conceptual thinking when solving mathematical problems. Algebra tiles, for example, were used to instill among her students operational sense and to teach for conceptual understanding when multiplying and dividing integers. The main idea behind all of Ms. Johnson's lessons was making connections and to teach for conceptual understanding of the content she taught, and concrete models provided one tool for this overarching purpose. In many pre and post observation interviews she reiterated this goal with respect to various lessons. The following excerpt from one of the impromptu interviews is expressive of her thoughts and beliefs more comprehensively, that is, her aims for the teacher-directed interactive discourse and tasks implemented in the classroom.

> Ideally, it would be about conceptual understanding . . . [Then] procedurally, I would hope procedure would come from them out of their conceptual understanding in a perfect setting. So that you are not ever teaching procedure, [and] they are telling you what procedure they've figured out. Then, you know they have a concept that they could figure out a procedure that would work. [IMI – Ms. Johnson's remarks about her aims with respect to discourse and other classroom tasks, p. 193 of the field notes]

<u>Beliefs about Students</u>

Ms. Johnson believed that every student had strengths that could help him or her learn mathematics better. She believed that it was part of her job to find strengths in her individual students and use them to help them become better students. She remarked that:

> Students come to school with varying backgrounds, diverse skills in mathematics and varying levels of motivation. It is human nature to learn and feel happiness from success. Most students are social creatures that thrive in settings with activity and interactions. Students have different experiences with having to create and think. [IMI – Ms. Johnson's beliefs, Question #3]

Ms. Johnson was aware of students' differing and individual talents and mental abilities. Her practice and use of multiple representations of ideas in the classroom were consistent with her thinking that aimed at reaching out and helping *all* students understand the content she taught. Her approach to foster connection-making the centerpiece of her instructional strategy was consistent with her belief that students have “diverse skills,” because inherent in the concept of connection making was that, first, various representations of the same concept should be established and discussed; hence, she rendered the making of connections relevant to the function of teaching. Knowledge of the tabular representation of information, for example, was shown to be connected with knowledge of the graphical or visual, or other physical (e.g. algebra tile models) representation of that same information, and that patterns in both of these forms of knowledge could have a symbolic and abstract representation. Recognizing students’ differing intellectual characteristics had played a central role in Gardner’s (1993) seminal work, as well. “It is a pluralistic view of mind,” Gardner wrote “recognizing many different facets of cognition, acknowledging that people have different cognitive strengths and contrasting cognitive styles” (p. 6).

A broader student characteristic that Ms. Johnson took into account while teaching was student’s motivation. She pointed out in her initial in-depth interview that not all of her students in the Algebra One class had the skills necessary to be in that class, yet, she observed that students in that class were highly motivated in general and worked hard to learn the content. On the other hand, she saw a more heterogeneous group of students in her Math Seven and Math Eight classes, with students ranging from not motivated to those who were almost ready to be in her Algebra One class. With respect to those students who were not motivated and where their skills lagged behind the rest of the class, she said “I’m not booting them out, they’re just going to work harder” [II – Question #8].

Beliefs about Teaching

Ms. Johnson believed that:

> Teaching is providing an environment, monitoring the climate of the learning community, choosing and using curriculum materials, knowing and understanding your students so that you can facilitate the maximum amount of learning for each student. The first step is getting to know your students. You need to know who they are, what outside influences or interests do they have, what do they already know and what would help them learn more. [IMI – Ms. Johnson's beliefs, Question #4]

These beliefs meshed with the real-life setting within which Ms. Johnson taught, since the *Connected Mathematics Curriculum* from which she taught were her choice. Acknowledging the complex nature of teaching, though, she also added:

> In teaching mathematics it is important to have curriculum materials that have open entry and are open ended. Students need to have the prerequisite skills to actively engage with the materials while challenging them to learn and connect more knowledge. Teaching is a balancing act. I focus on improving student learning by having positive rapport with all students, open communication with parents and delivering extra help to students. [IMI - Ms. Johnson's beliefs, Question #4]

Ms. Johnson's Intentions

Intentions Induced by Ms. Johnson's Beliefs

Follow Reform-Based Curricula

Ms. Johnson's intentions aligned most closely with the recommendations of the *Connected Mathematics Curriculum* for teaching for conceptual understanding and numerical fluency. When she was asked about her goals for her students for the next nine weeks, she said:

> I want my Algebra [One] students to become comfortable with developing rules for patterns and graphing and seeing the connections as to understanding that where … what slope looks like in a graph, what it looks like in a table. And also having some fluency with solving equations,

> cause that's just going to be a going thread throughout algebra, of course. So, I think that's it for them. And then for Math Seven, my overall goal is that the idea of order of operations and how to use parentheses to regroup and that number fluency again, or building number concept using . . . using that as a vehicle. And for 8th grade, I want them to be able to add, subtract fractions fluently without a calculator, using probability questions as the application level of fractions. [II – Question #2]

Though, Ms. Johnson's statements about teaching the seventh and eighth graders may portray her as a teacher that emphasized learning of mathematical procedures when, in fact, she often used algebra tiles in the lessons she gave in all of her classes, as one of the tools she used to teach for conceptual understanding by connecting ideas in the arrangement of tiles to numerical fluency. She also emphasized pattern recognition and making connections between tables, graphs, and rules in her Algebra One class; she worked to have students in all of her mathematics classes to transition raw information, provided in word problems or guess-and-check tables they developed, to writing solvable equations of one variable; she used manipulatives and tile models and multiple representations of ideas to teach students about writing and simplifying algebraic operations and algebraic expressions, thereby again making connections to physical and visual models; and she asked students to share their problem solving strategies with each other. All of these practices followed the intended objectives of the *Connected Mathematics Curriculum* that Ms. Johnson chose for her classes.

<u>Assess Students' Prior Knowledge</u>

Moreover, she intended to assess and know what students already knew with respect to the content. A large part of that assessment was conducted using daily and casual classroom conversations with students. The following excerpt from a pre observation interview was typical of her approach to assessing what students knew in every class.

> I'm going to try to talk to each group to see how they are thinking and how they are developing their rules and weather one person's thinking and they're [other students in a group] writing it down or they're making connections themselves. [PRI – October 16th, Algebra One, Question #4]

Ms. Johnson also conducted "diagnostic tests" to determine what students knew.

> I'm worried . . . I think I'll do a pre-test [a reference to what she also called a diagnostic test] with kids and then collect that and then maybe even a post test to just get the idea of who knows it and who doesn't and maybe differentiate so that the kids that already know it don't have to go through this lesson. They can do something different. [PRI – October 17th, Math Eight, Question #4].

The post observation interview, a sample of which is shown next, verified that Ms. Johnson followed through with her intention and gave students a pre-test. She used the result of the pre-test to send some of the students who scored 100 percent to the computer lab with a student teacher. The students were to spend time on computer problem solving activity until she was done giving the lesson to the other students.

> I think that the pre-test had its positives and negatives, but I think it's worth doing, cause it allowed me to go at a pace that sort of matched the students'. And if those other kids were in the room [she is referring to the kids who scored 100 percent on the pre-test and went to the computer lab instead of staying for the review], I would have felt that I would need to rush so they wouldn't be bored. [POI – October 17th, Math Eight, Question #2]

Post unit tests were another means by which Ms. Johnson intended to find out what students knew about a previous unit before teaching a new unit. She administered a post unit test in each of her mathematics classes during the course of this study. In all post unit tests she was able to determine exact topics with which students were having difficulty.

> I'm still going to work on how to make it so that the algebra tiles, they have a positive attitude about the algebra tiles and I think the negative attitude comes from lack of conceptual understanding and they don't want to have to think. So, make that connection stronger and more powerful. [PUI – Algebra One, Question #6]

> It's maybe not start this unit until integers were stronger, so maybe put in another unit of building integers, because the order of operations was hard for them if they didn't have an understanding of integers. So, if they have an operational sense with integers, then they couldn't do order of operation problems that involved them. [PUI – Math Seven, Question #6]

> One thing that I would do is do more teacher classroom discourse about the connections between the representations and the rectangular model with probability to ways to solve probability without making long cumbersome charts, cause only the high kids that paid attention to the questions in the book got it and I think there could be some classroom discourse in the connections there. [PUI – Math Eight, Question #6]

Also part of Ms. Johnson's philosophy for assessing students was that she intended for her students to take charge of their own learning. As part of implementing this intention, Ms. Johnson had developed an objective sheet for each of her classes. Figure 5 shows a sample of an objective sheet from the Algebra One class. The objective sheet was distributed to students in each class every Monday, and it was a weekly syllabus that gave students an overview of the activities for the week. For example, in

Figure 5, the objective for Monday, which was also the first day of the final week of the first quarter for the school year, was for students to "identify different types of solutions."

Figure 5. Sample objectives sheet

Name__________________________
Quarter 1 Week 9 Final Week of Q. #1
Monday, October 29
Obj.: Identifying different types of solutions
3-78, 79, 80, 81
82, 83, 84, 85, 86______________

Tuesday, October 30
Obj.: Practice solving equations
3-87, 88, 89, 90, 91
92, 93, 94, 95, 96_______________

Wednesday, October 31
Obj.: More solving equations
3-97, 98, 99
100, 101, 102, 103, 104__________

Thursday, November 1
Obj.: Make connections using vocabulary
Mini quiz
Vocabulary Work_______________

Total points Week 9_____________

Moreover, the numbers below the day's objective indicated the homework problems to be worked out in the class *and* at home, in this case, problems 78 through 86 for the first chapter. Thus, Ms. Johnson's expectation was that any problem not completed in the classroom was to be worked on at home. The blank line following each day's last homework problem provided the space for students to record the points associated with the number of homework problems they had solved, correctly.

At the end of each week, students turned in their "spiral" notebook together with the scored objectives sheet to Ms. Johnson. Ms. Johnson then matched scored objectives sheets against the work of students in their notebooks. Recurring and prevalent errors in the scores with respect to the work shown in the notebooks was evidence that a student was not paying attention in class, prompting Ms. Johnson to talk with that student. In this way, students were put in charge of checking and scoring their own work, while Ms. Johnson entered the process if a student was not following the class work and homework protocols, conscientiously.

Teach and Assess for Conceptual Understanding and Procedural Fluency

Ms. Johnson wanted to teach and assess for conceptual understanding and procedural fluency, yet, she wanted procedural fluency to emerge from students' conceptual understanding. She asserted that:

> [Conceptual understanding is] what kids already know about the topic and then adding on to that [connecting to it], cause that would help your instruction. If you know what they already know, then you know what to add on. Then procedurally, I would hope procedure would come from them out of their conceptual understanding in a perfect setting. [IMI – Ms. Johnson's remarks about her aims with respect to discourse and other classroom tasks, p. 193 of the field notes]

In her written assessments, assigned homework, and poster presentations, Ms. Johnson demanded that students show evidence of their conceptual understanding (e.g. draw pictures of algebra tiles right alongside mathematical procedures; write a few sentences explaining their thinking; or show connection between patterns, graphs, and symbolic representations) as well as procedural proficiency.

> My idea is to make sure that kids have a good understanding of adding, subtracting, and multiplying integers. And they are going to display it in a poster… I have a rubric for the poster for scoring it, that they have and working on it. So, I'll score each poster for correctness, and visual models, and multiple examples. [PRI – November 15th, Math Seven, Questions # 2 and #4]

Ms. Johnson used the result of assessments and students' written work to determine her intentions and points of emphasis for instruction and classroom practice in the upcoming class meetings. For example, on a day when she wanted to give students more classroom practice with the distributive property, a concept with which many students in the Algebra One class and both Math Eight classes had difficulty with, she said the following:

> I think I might have them turn in their double-digit warm up [a reference to a warm up activity that involved students working with distributing numbers and the negative sign into a parentheses containing two terms], so that I can see which kids still really need more work with it. [PRI – November 15th, Math Eight, Question #4]

Make Learning Mathematics Active and Engaging for Students

Following her belief that students thrived on social interaction and enjoyed learning from each other, Ms. Johnson wanted to maximize sharing of ideas among her students, exposing them to as many reasoning and problem solving strategies as possible. Following this intention, a greater portion of each lesson was conducted in small group rather than whole class activities. The groups to which students belonged were not permanent. Ms. Johnson reassigned students to new groups every three weeks.

The initial in-depth interview revealed that Ms. Johnson believed that introducing a new topic was motivating to students. When asked what she planned to do to engage her students, part of her response was:

> [Make] sure that I'm introducing new enough material that it doesn't, it's not something they already learned in 5th and 6th grade. So, making sure that it's novel, 'You haven't learned this yet!', and that gets motivating; to learn something you've never learned before and not just review, review, review. [II – Question #13]

Pre and post observation interviews with Ms. Johnson revealed that another way with which she intended to engage students' interest was by using mathematical games and puzzles and manipulatives.

> They're going to play the Pig Race, which is a game that has them shoot for target numbers. So, they'll have to manipulate the numbers to hit certain targets. . . It's the Pig Race. They're going to have fun. [PRI – November 1st, Math Seven, Question #2 and #3]

> Making it a game made them do a little bit of math, even though they didn't want to. [POI – November 1st, Math Eight, Question #2]

> I think the game . . . the target game . . . is motivating for them. So, they were all doing calculations and their satisfaction when they get to the answer. [POI – November 6th, Math Eight, Question #2]

Despite all her attempts, engaging students to work on mathematical tasks collaboratively and on a continuous basis proved to be an insurmountable intention for Ms. Johnson as nonacademic social discussions often presented an obstacle. Her response to an interview question as to how she was going to improve a lesson was typical of her thinking and intentions in all of the mathematics classes she taught. She said: "I would say deciding how to get more kids involved when we're sharing with the whole class. I saw a lot of kids that did a lot of good thinking and yet they aren't sharing that" [POI – October 15th, Algebra One, Question #5]. Toward the end of the observations, asking Ms. Johnson about how she planned to improve her lessons echoed the same concerns she had expressed in the beginning of the study; she said:

> It's to maybe break it into . . . [break] the instruction into centers and have like a computation center, and then let's do this problem as a group and see. I don't know. But even in a small group, if they're not motivated, they tend to be one step behind, and you tend to repeat yourself and they're not doing the thinking. They're just trying to leach off of other students' answers. [POI – November 29th, Math Eight, Question #5]

Intentions Induced by the Classroom and School Environments

In addition to the influence that moment-to-moment interaction with students exerted on the way Ms. Johnson made decisions during the course of teaching, the social climate in the classroom and the broader school environment also affected what actions Ms. Johnson took inside the classroom.

For example, she asked students to turn in their work for class "work time" activities at the end of each class period and made problems for classroom problem solving strategies more challenging. These two decisions were a direct reaction to excessive socializing among students during small group activities. In one interview, when asked what she would have liked to do to improve her lessons, she said: "[Having] them turn in their work at the end of the period and not give more time for the kids that need more time, but just accepting what they get done" [POI, October 23, Math Eight, Question #5]. When she was asked the same question the next day, she reiterated in her response that: "Get rid of some the [social] drama, and just not let drama come into my classroom; that is, when they bring their social problems into math class and let it bleed into interrupting academics" [POI, October 24, Math Eight, Question #5].

Although, Ms. Johnson was committed to principles of reform-based mathematics teaching and changing mathematics education from rote memorization of facts and procedures to learning mathematics as a human activity, it was apparent that she felt powerless in the face of some of the issues reform-based mathematics teaching had brought her, as in this instance, she felt the criteria to have small group activities had its own drawbacks in terms of promoting unstoppable nonacademic social behavior among students, distracting from her goals for the lessons.

Another major influence of the classroom on Ms. Johnson's decisions and actions came from disruptions in her plans. Schoenfeld (1998) termed this idea as what teachers ended up doing "when something happen[ed]." Field observations indicated that when reform-based approaches to teach for conceptual understanding and making of connections between mathematical ideas proved not to be effective, Ms. Johnson reverted back to focusing on teaching the procedures. That is, whenever students did not ask questions when they should, when they were not able to connect between mathematical ideas and solution strategies, or discuss alternative solutions when working in groups, Ms. Johnson took it upon herself to talk about the main points in the curricula by talking about the strategies and procedures involved. For example, in the work that was turned in to Ms. Johnson in the Algebra One class, students demonstrated that the majority of them did not understand how to distribute a negative sign placed in front of parentheses to the terms inside the parentheses. She said "I did hope that [someone] would ask about parentheses so that we could again talk about what to do when there is a negative sign in front of a parentheses" [POI – October 30th, Algebra One, Question #1]. But because the question was not brought up by students, she used a mini-lecture, a rare aspect of her practice, to discuss the content procedurally.

Teaching the procedure was also observed when Ms. Johnson tried to review or reteach content that was taught weeks earlier, if not in the previous year, though her approach and intentions were to teach new material conceptually through making connections, pattern recognition, use of algebra tiles, or relating the content to other ideas from within or without mathematics; that is, she followed the instructional guidelines recommended in the reform-based curriculum for each class.

Ms. Johnson's collaboration with the mathematics teachers at Oxford Middle School as well as with the mathematics teachers at a nearby high school rendered its own influences as well. These intentions varied greatly in scope and focused on issues that concerned Ms. Johnson and other teachers in the same way. For example, the collaboration between Ms. Johnson and the school science teacher, Ms. Buchanan, produced a series of "integrated units" that were added onto the regular curricula for seventh and eighth grade mathematics. The intention of these *integrated units* was to combine students from the science class with the students from Ms. Johnson's mathematics classes such that Ms. Johnson could teach mathematics content with applications to relevant science areas all at the same time. This action allowed students to study some mathematical content areas in context with their applications in science.

On the other hand, Ms. Johnson's collaboration with the school community mathematics teachers was one of conducting a quality control to investigate teaching strategies that produced in-depth understanding of mathematical content across grades versus those that were not as effective. This collaboration affected what all mathematics teachers intended to do in their classrooms rather than simply Ms. Johnson.

The collaboration with the mathematics teachers at a nearby high school affected Ms. Johnson's intentions to some extent. Although Ms. Johnson believed that the use of calculators in a middle school mathematics curriculum should be kept to a minimum, with the exception of some graphing features, the interaction with the high school mathematics teachers reinforced those beliefs and intentions in Ms. Johnson, as they too discouraged the use of calculators with elementary and middle school mathematics curricula. An audio recording from a meeting between Ms. Johnson, Ms. Roberts, and the high school mathematics teachers provided clear evidence of the high school teachers who were adamant in their pleas that Ms. Johnson and other middle school mathematics teachers not let their students use calculators. The high school teachers discussed their deep concern and experience with entering high school students that lacked the most basic skills in performing algebraic operations, manually. Ms. Johnson was heard defending the rights of every mathematics teacher to teach with or without the technology and according to their own discretion when teaching, even though she herself approved of students using calculator only when the *Connected Mathematics Curriculum* recommended it. The following exchange between Ms. Johnson and one of the students during whole class discussions was insightful with respect to Ms. Johnson's thinking and intentions in the context of the recommendations of the curriculum and using calculators.

Ms. Johnson: OK, one thing about our book, last year's book used to have little bold calculator symbols, and then, you [inaudible] not to use it. In this book, I think the idea should be, unless it says "Use your calculator!", don't grab a calculator.
Student: None of these says "Use your calculator!"
Ms. Johnson: No, that's not true. When they were checking your calculator, they wanted to know whether your calculator was scientific, you guys need to put your calculators away, because if you did those order of operations problems with your calculator, you are not learning what needs to be learned, because it's all memorized. They have preprogrammed your calculator. Just pushing buttons on your calculator won't give you practice. Do your practice! Do your exponents! Do your multiplying and dividing! Then do your adding and subtracting, so that becomes part of your programmed memory. You need to be programmed like your calculator, and you're going to get that by practicing. [CA – October 18th, Algebra One]

It is difficult to ascertain if the high school teachers had a stronger influence on Ms. Johnson's views than her own beliefs with respect to her decision as to when to use and when not to use calculators. Nevertheless, it can be said that the views of the high school mathematics teachers seemed to reinforce Ms. Johnson's own ideas about calculators having potentially harmful effect on students' fluency with mathematical procedures, when used excessively and beyond recommendations of the curricula.

Summary Discussion Two

Ms. Johnson believed that different individuals learned differently and as a rule expressed a propensity to determine what type of a learner were the people she met. On the first day of the observations, she asked the researcher to show her how he worked out an algebra problem similar to the ones shown during instruction in the class. When the researcher explained that he would solve the problem step-by-step until the expression was fully simplified, Ms. Johnson told the researcher he was a linear learner. Ms. Johnson was observed teaching her mathematics classes with the same intention. She wanted to know as much as she could about her students' thinking. This approach impacted Ms. Johnson's intentions for student assessment. She often asked students to express their solutions in more than one way (e.g. procedures, drawings, graphs, or written text), emphasizing that students pick at least two methods from the list.

Ms. Johnson believed that learning mathematics *began* with conceptual understanding, not with procedural proficiency. At times when she had the chance to examine students' work immediately after it was turned in, she stood by the baskets where students placed their papers before leaving class and skimmed through the papers, looking for evidence of students' conceptual understanding. In one occasion, Ms. Johnson sounded disappointed as she performed the inspection. The researcher asked Ms. Johnson what with respect to students' work disappointed her. And she showed the researcher several student papers, where correct procedural solutions were accompanied by incomplete and faulty drawings of rectangles that depicted alternative solutions based on the arrangement of algebra tiles with which students were taught. From this, Ms. Johnson gathered that the majority of her students had not been able to make connections between the mathematical procedures and the visual representations with which they were taught. In other words, in a class where a reform-based mathematics curriculum was being used by a teacher that followed the recommendations of the curriculum with conviction, the teacher's intentions for students' conceptual understanding of the content was still not being met.

Moreover, Ms. Johnson taught based on her own beliefs, intentions, and perceptions about what it meant to be an effective mathematics teacher. The classroom environment influenced her intentions only to the negligible extent that she wanted to adapt to the demands of new situations as they sprung forward during practice.

Ms. Johnson believed that children were used to learning from each other when playing games and interacting with each other as they grew up. She intended to emulate that spirit in her classes by asking students to share their mathematical ideas with each other in small groups. In the coming sections about Ms. Johnson's practice the reader will find that this strategy of having students become engaged with the mathematics while sharing ideas in small groups often did not work well in real classroom settings as students spent most of their times socializing.

The group arrangement of seats and desks helped impart the most significant influence on Ms. Johnson's practice from the classroom community and Ms. Johnson's intentions for students to work together. The impact of the school community on Ms. Johnson's intentions and practice was more significant than the influence of the classroom community. The principal and all the science and mathematics teachers at the school worked with Ms. Johnson with the single intention of improving instructional strategies and providing more coherent and consistent curricula for all students at the school. Table 4 summarizes Ms. Johnson's overarching intentions and examples of her practice that materialized those intentions. The ensuing sections give further detailed evidence about how Ms. Johnson used her knowledge base to put her beliefs and intentions into practice.

Table 4

Ms. Johnson's Overarching Intentions and Supporting Actions

Intention	**Observed Action or Practice**
Create a coherently reformed mathematics education environment at the school that looks to the recommendations of the standards documents as well as her own knowledge of the broader scholarly literature on effective teaching strategies	Chose and relied on reform-based mathematics curricula for her classes; collected and incorporated lessons from published journals into her practice; created a classroom environment conducive to students working independently, collaboratively, and with her; was not selective about using the curricula; sought and collected evidence of student learning to determine the success of her strategies
Wanting to "know what students already know"	Questioned students verbally and continuously; talked with students about their mathematical ideas and pressed for student feedback and participation by all; used brief (10-15 minute long) diagnostic tests; used post-unit tests

Table 4 (continued).

Intention	**Observed Action or Practice**
Show students that mathematics is not only a science of numbers, but it is a science of understanding and modeling patterns, a science of various related problem solving strategies, and a reasoning tool	Asked students to explore patterns in the form of tabular numerical data, geometry of shapes, and trends in graphs; asked students to solve problems in more than one way and show the connection between the different strategies (e.g. connection between manipulatives and algebraic operations); asked students to justify their answers using mathematical definitions and procedures
Make mathematical learning active and engaging such that students complete various lively activities rather than take notes and be docile for the duration of the class	Used manipulatives, poster presentations, mathematical games and puzzles, whole class and small group discussions; asked students with the best of ideas to use the document camera and share their ideas with the class
Collaborate with the school community	Collaborated with the principal and all the science and mathematics teachers at the school; served on several school committees representing the school

Content and Pedagogical Content Knowledge

Content Knowledge

After completing a bachelor's degree in business management and another one in psychology, Ms. Johnson had attended a one-year long program that earned her a teaching license. She obtained a master's degree in education leadership, and at the time of this study she was enrolled at a local university to obtain a doctorate in education leadership. Her most recent coursework in the doctoral program centered about learning to teach teachers how to teach mathematics.

Beginning early in her teaching career, Ms. Johnson had been enthusiastic about teaching mathematics and began attending numerous professional development programs that fostered reform-based teaching of mathematics. Borko and Putnam (1996) agreed that, even for experienced mathematics teachers, "intensive professional experiences can help teachers develop more powerful understandings" (p. 690) of the content they teach. In addition to the professional development program, Ms. Johnson's main experience with teaching mathematics was formed during her membership with the Silicon Valley Math Project in California, where she continued with the program for nearly five years. During this time, she taught mathematics at elementary and middle school while attending "summer institutes" that connected the project members with a local university program that focused on mathematics teaching and learning. As a lifelong learner and highly enthusiastic mathematics teacher she described her philosophy as "I like to go learn more new ways to do things [in mathematics]" [IMI – Ms. Johnson's career path, Question #2, p. 62 of the field notes].

Ms. Johnson had started her teaching career by teaching the second grade for two years, teaching the sixth grade for two years, and then moving to teach mathematics in seventh and eighth grades. She returned to teaching the fifth and sixth grades for another two years, but then moved to teach mathematics in middle school, continuing teaching at this level to the present time. At the time of this study, Ms. Johnson was in her 20th year as a mathematics teacher, 10 years of which had been at the current school.

Pedagogical Content Knowledge

Ms. Johnson used algebra tile manipulatives, and whole-class and small group discussions of problem solving strategies with facility and extensively in all of her classes (see Appendices I through O). At times she used "math games" to make her lessons engaging. Her facility with creating an engaging environment for teaching problem solving strategies seemed connected with her training and experience teaching primary school students, where use of manipulatives and creating an active learning environment were of prime importance. In addition, her career long participation in reform-based professional development programs, and her five year involvement with the Silicon Valley Math Project in California, where through collaboration with a local university, she received additional training with respect to how to teach mathematics. According to Ms. Johnson, the combination of these experiences formed the core knowledge behind her teaching philosophy [IMI – Ms. Johnson's career path, Questions #1 through #3, p. 61 - 63 of the field notes].

Ms. Johnson considered "finding out what kids already know about the topic and then adding on to that" a central goal of classroom discussions:

> ...cause that would help your instruction: if you know what to add on. Then, procedurally, I would hope procedure would come from them out of their conceptual understanding in a perfect setting. So that you are not ever teaching procedure. They are telling you what procedure they've figured out. Then, you know they have a concept that they could figure out a procedure that would work. [IMI – Ms. Johnson's remarks about her aims with respect to discourse and other classroom tasks, p. 193 of the field notes]

Ms. Johnson had been given the freedom to choose the curricula from which she taught. In the interview that was conducted with Ms. Johnson during the selection process, she revealed that she had switched to a "more problem-based book" for her Algebra One class in the year before, while this year was her third year of teaching from *Connected Mathematics Curriculum* for both Math Seven and Math Eight classes, where problem solving strategies, as opposed to rote practice of procedure, was the focus [SI – Question #1].

Teaching conceptual understanding through connection making and pattern recognition, and procedural fluency were central aims of Ms. Johnson's curriculum as she followed the recommendations of the reform-based curricula to that effect. The following excerpts from the initial in-depth interview show what overall instructional strategies she aimed at using within the context of the curricula she used.

> I would say a combination of shorts: introduction to new ideas or vocabulary or a topic, and then having the students use problems or the problems within our book to learn the mathematics, so I let the investigation or math problems that they're asked to solve be the teachers, and I facilitate that learning, and some study teams where students work together, but with individual accountability. [II – Question #5]

> I would say an opening; opening up with an idea where all students have to engage in some kind of thinking and responding either a math warm-up, or a math discussion, or note-taking with questions, but really short 5-minutes introduce, or introducing a problem, launching them, and getting them going, and then introducing a set of problems that they're going to work through that are developing a particular idea for the day and then some work time on review, preview, practice type problems that they get done in class than in a perfect world. In other settings, they might've gotten it done as homework, but they start that in class. [II – Question #14]

The observations of Ms. Johnson's teaching showed that these aims were accomplished by whole class discussions that usually lasted 15 minutes, and followed by small group discussions that continued for the duration of each class (see Appendices I – O). This trend of activities were at times broken by quick five to 10 minute long warm-up algebra exercises in the beginning of the class, and sometimes at the end by wrapping up the class with a whole class discussion of small group activities. However, each day of each class consisted of whole class and small group activities.

Ms. Johnson led whole class and small group discussions with questions. The nature of questions was virtually identical in both whole class and small group discussions. Students were asked to describe their thinking in completing a mathematical procedure or they were prompted to talk about the connections they were able to make between ideas and strategies, and the patterns they had discovered. In essence, each whole class discussion modeled the type of thinking and discussions students were to have during their small group activities. Ms. Johnson solved a few problems during whole class discussions while soliciting answers from students using questions that either looked for specific numeric answers or questions that looked deeper into students' thinking by asking "How did you get that?", "Come up here and show us how!", "Who wants to share a different answer?", "Is there anyone who got a different answer?", "Who wants to share what they got and how they got it?" [CA – October 15th through November 30th, all classes] Yet, these questions rarely led to students explaining their thinking beyond uttering a single number or a few words. The only time that this type of questioning produced the detailed expression of students' thinking Ms. Johnson was looking for was during poster presentations, when each group of students had to stand up in front of the class and defend the patterns they had found, connections they were able to make, and graphs and equations they had produced. Although the students in each of the mathematics courses Ms. Johnson taught prepared posters once during the course of this study, only the Algebra One class students got to present and discuss their posters in front of their classmates. In other classes, students simply turned their posters in for her assessment. Yet again, other times students tried to explain themselves, but their voices were not captured audibly on the audiotape recorder used in this study.

The following excerpts are typical of Ms. Johnson's expectations that she tried to imbue and instill in her students in the type of thinking they did and answers they gave.

> In your learning logs, make sure that you have explained what intersections mean on a graph. So, when two lines intersect, a good thorough explanation of what it means, and if you want to give an example back to this problem or another problem, you can. [CA – November 27th, Algebra One]

> Some of you are drawing the problem and drawing the answer and I really need to see the operation that you started with this and took you this away. Multiplying, you have this many groups of this many. Not just drawing me the answer. So, the operation has to show. On the number line, don't just draw a number line. Make sure you have a sentence to explain it. Whether this one is getting bigger or smaller, or how many groups you have [CA – October 19th, Math Seven]

Although difficult to get students to explain their reasoning, it was apparent throughout the entire observations that that was one of Ms. Johnson's main goals.

Other components of Ms. Johnson's pedagogical content knowledge were examined through the model described by An et al. (2004).

***Knowing students' thinking*.**

Ms. Johnson's main method of assessing students' thinking and what they knew was by speaking and listening to them. The following statements by Ms. Johnson typify her approach during both whole class and small group discussions: "I'm going to walk around and listen as they play the game and I'll see what their notebooks say on Friday; see whether the models they drew work" [PRI – October 16th, Math Eight, Question #4].

> I'm going to try and talk to each group to see how they are thinking and how they are developing their rules and whether one person's thinking and they're [others in the group] writing it down or they're making connections themselves. [PRI – October 16th, Algebra One, Question #4]

> Instead of going over like a whole-class with the group, I went to every table to see what they were thinking so that I didn't give away any secrets and every group had to figure it out a little bit. [POI – November 14th, Algebra One, Question #1]

During talks with small groups, Ms. Johnson used similar questions as during whole group discussions to elicit students' thinking. Questions and assertions included: "How do you know? Show me!" "Have you checked your answer?" "What if . . .?" "What's another way?" "Tell me how you got this!" "Did all of you solve this the same way? Any of you got something different?" All these questions were commonly heard by the researcher during whole group as well as small group discussions.

Two trends emerged with respect to Ms. Johnson's understanding of students' thinking: first, she acknowledged that *students would respond differently and individually* in their understanding of the topics that was laid out before them, and second, for most students, *passage of time* and exposure to more practice with manipulating mathematical concepts and procedures was essential to their understanding of the content. The former aspect of Ms. Johnson's understanding of students' thinking was encapsulated in the following assertion made by her during a pre observation interview:

> I think some kids are going to feel really confident, because they're really good at seeing patterns and other kids are going to be a little frustrated cause they don't know what's going on in other kids' heads to get there, and silent, so they're not talking to each other to give everyone that think time. So, some might be frustrated and not feeling really smart and others are going to feel really smart, cause pattern is . . . it's a hard . . . it's individual. [PRI – October 16th, Algebra One, Question #3]

Ms. Johnson was cognizant about the impact of passage of time on students' thinking and understanding of the content. The response "They're getting there," [POI – November 8th, Math Eight, Question #4] was often heard as the study progressed and Ms. Johnson was asked if her students were acquiring the understandings she hoped they acquire. She repeatedly expressed that some time would have to pass before she could determine the impact of her instruction on students' thinking.

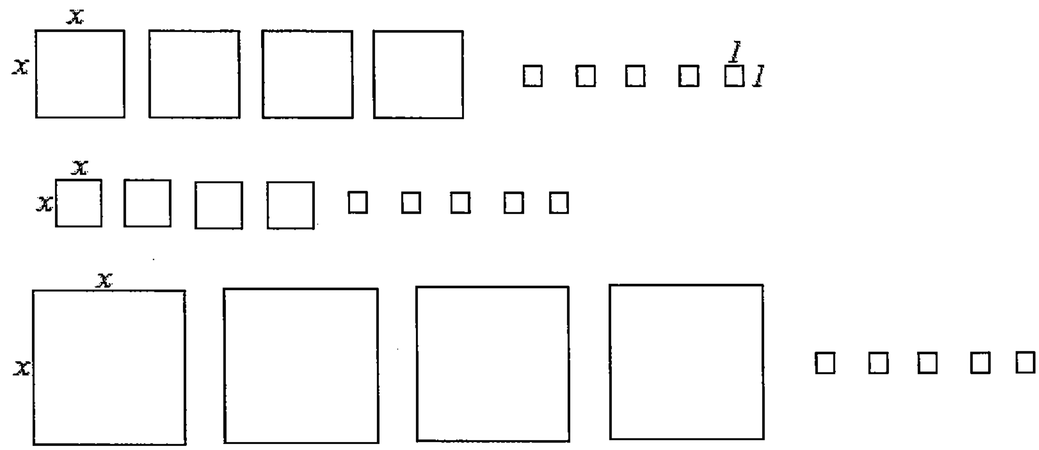
Addressing students' misconceptions.

Ms. Johnson was cognizant of students' potential misconceptions. In her Math Eight class she said "On the tiles, I added that '*x*' equals different values for '*x*' to make sure that they don't think that '*x*' is always 5, cause that's a problem with misconception" [POI – November 13th, Math Eight, Question #1].

Her intention was for students, not her, to recognize and correct their own misconceptions in order to understand the content better. In one interview about a lesson she gave in her Math Seven class she expressed that "I think that whole class worked well, because kids caught errors as they were going and so we caught this misconception before they can practice it over and over, again" [POI – October 25th, Math Seven, Question #2]. And again in another interview she acknowledged that "[I want to] correct misconceptions, but not be explicit about correcting it so that I'm telling them" [POI – November 13th, Algebra One, Question #5].

Moreover, Ms. Johnson used her prior knowledge of students' potential misconceptions to improve her future lessons. For example, she was aware that in her Math Eight classes, some students often perceived a variable as the representation of a fixed value. Therefore, when using math tiles to teach about constructing algebraic expressions, she often asked students to construct the variables in an algebraic expression using different tiles of unknown length for sides while keeping the same tiles for the constant value; that is, a constant number in an expression, say 5, was represented by five identical squares whose sides were the unit length, whereas, to show a variable term, say $4x^2$, students were asked to use *alternative* ways of constructing the term using identical squares of various sizes in each trial. Figure 6, for instance, depicts how students were expected to present three alternative ways of constructing an expression such as $4x^2 + 5$ [FN – November 14th, Math Eight].

Figure 6. Sample arrangement from Ms. Johnson's tile lesson about constructing algebraic expressions to eliminate students' misconceptions about variables



Building on students' mathematical ideas.

An et al. (2004) stated that mathematics teachers can build on students' ideas using any of the four major approaches: 1) making connections to students' prior knowledge, 2) using concepts or definitions, 3) making connections to pictorial or physical models, and 4) using rules and procedures. Asking questions, such as "How many of you have seen this before?", or administering a *quick test,* were two main approaches Ms. Johnson used to explore students' existing knowledge before making connections to it when teaching new content. The following excerpt from an interview is typical of what she had to say in terms of determining students' knowledge: "I'm going to do a pre-test again for fraction/decimal/percent to see where they are, and then I'm going to have students place fractions/decimals/percents to . . . to get that idea of converting and showing equivalences" [PRI – October 19th, Math Eight, Question #2].

Ms. Johnson also used mathematical definitions or properties she had discussed in previous lessons to teach and make sense of new problem solving strategies. The excerpt shown below from a classroom audio demonstrates one such example captured in the classroom audios. In this lesson, her idea was to elicit from students an understanding of how they can use mathematical properties, such as the *identity property of 1,* as a problem solving strategy.

Ms. Johnson: OK, raise your hand if you know what D=r*t stands for, please! At least 3 of you should know. That's my prediction. There's one! I need two more. Think about it, please. Think of trains! Think of walking! Think of jogging! Think of driving!

Students debating amongst themselves.

Ms. Johnson: Think about it please! Yes, Jack?
Student1: D represents distance.
Ms. Johnson: Good! OK, what about *r*? Think about it please! Tyler?
Tyler: Rate?
Ms. Johnson: It is rate. Yes! OK, let me give you an example. If I'm traveling at 3 miles per hour, and I travel for two hours, please tell me how far would I have traveled? Nicole?
Nicole: 6 miles.
Ms. Johnson: 6 miles! Please write this down. Now, if I'm given rate and time, I can usually calculate distance. I want you to use algebra to manipulate this equation so that we, if we want to know rate, but we have the distance and time. So, I want you to change this so it say "r" equals to. What algebra steps are necessary to make it "r" equals to? Right now, it's "r*t".
Student4: $r = \frac{D}{t}$
Ms. Johnson: OK, I want to know the algebra to get there. So, what do I have to do to move it to that? It has to do with *the identity property of 1*. Identity property of 1. I want this "r" all by itself. So, I want it to be worth one "r". How can I change the "t" into a "1"?
Student4: Divide by r.
Ms. Johnson: No! That would make it a "t". I want one "r".
Student4: Divide by "t".
Ms. Johnson: Divide by "t". If I divide by "t" over here, what do I have to do over here?
Student4: Divide by "t".
Ms. Johnson: Divide by "t". So, if I now rewrite this equation, what does it say?
$r = \frac{D}{t}$. [CA – October 29th, Algebra One]

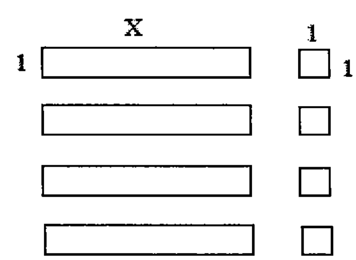
With respect to An et al.'s third method of building on students' mathematical ideas, for example, the following excerpt from the field observation notes and the accompanying classroom audio demonstrates how Ms. Johnson used students' knowledge of algebra tiles or manipulatives as a way to teach her Math Eight students about abstract thinking and reasoning in mathematics.

Ms. Johnson: One of the things that I want you to get is flexibility and how you can say the same thing a whole bunch of different ways. Because when you are solving algebra equations, sometimes you're going to want it in one way and other times a different way, and you need to be able to go back and forth. So, let's take 4X+4. Build the rectangles [the teacher is referring to algebra tiles that students are handling] for this.

Students busily work with their algebra tiles arranging them in different ways to build the rectangles. Ms. Johnson checks with students to see their progress and after a few minutes, she approaches the document camera in front of the class.

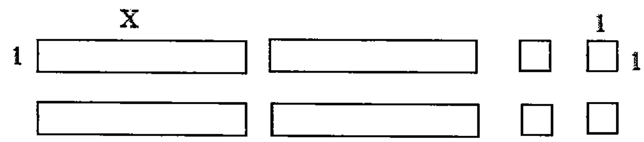
Ms. Johnson: O.K. Most of you should have this:

Figure 7. The tile arrangement for expression 4X+4



Ms. Johnson: This is good. And what's the top row?
Student1: X + 1
Ms. Johnson: So, can we rearrange the rectangles and write this a different way?
Student1: [the student describes a new arrangement but specific wording was inaudible]
Ms. Johnson: Good! So, we can write this as a combination of multiplication and addition problem, like 2(2X+2) and show it like this:

Figure 8. The tile arrangement for expression 2(2X+2)



Ms. Johnson: How about a different way? What's another way?

No response from the students.

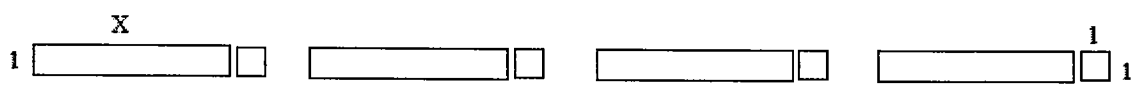
Ms. Johnson: Anyone have a different way of arranging the tiles for the same expression?

Ms. Johnson makes a long pause but still no response from the students.

Ms. Johnson: How about building the tiles for 4(X+1)? Like this?

The document camera screen is not large enough for the required tile arrangement, and Ms. Johnson uses a marker to draw the following diagram on the whiteboard.

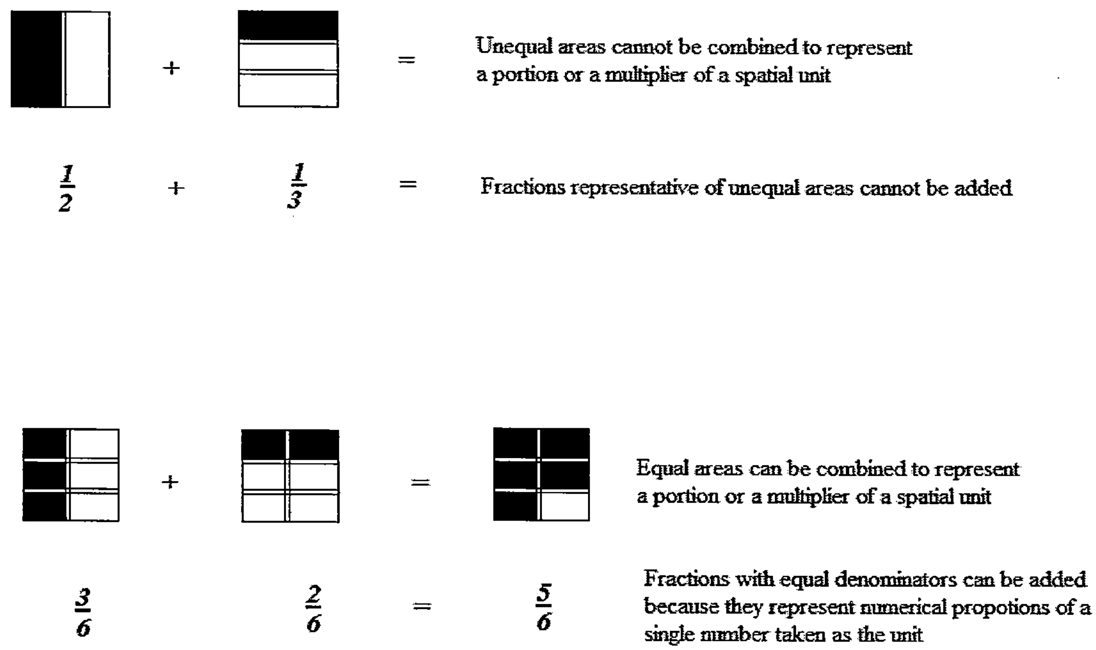
Figure 9. The tile arrangement for expression 4(X+1)



[CA – November 14th, Math Eight] and [FN – November 14th, Math Eight]

Ms. Johnson used connections to manipulatives extensively and in her lesson about adding fractions, as well. Figure 10, for example, depicts how Ms. Johnson used spatial reasoning ability of her students to teach her Math Eight class operational sense when discussing why fractions must possess a common denominator before they were added.

Figure 10. Ms. Johnson's idea for teaching her students about adding fractions, using students' ability for recognizing equal spatial models [FN – October 17th, Math Eight]



In the first set of squares, the tile representing the number 1/2 has a different area than the tile area representing the number 1/3. Ms. Johnson used this disparity in the areas of the two regions to demonstrate to students the difficulty with determining the resultant of an addition operation between two unequal quantities. In the second set of squares on the bottom, the large square containing the 1/2 portion is divided further into three more horizontal segments, which in turn create six smaller areas comprising the entirety of the large square. On the other hand, the large square containing the 1/3 portion is divided into two more vertical segments, which once again create six smaller areas comprising the entirety of the large square. Because both large squares are identical in size, dividing each along the lines of division present in the other, ultimately, leads to creation of identical areas in both squares, therefore, rendering the addition of shaded cells for both squares proportionally meaningful.

Ms. Johnson connected the necessity for creating a common spatial proportion as the unit of division in both squares prior to adding their shaded contents to the necessity for creating a common numerical denominator, as a unit proportion maker, before adding two fractional and unequal quantities. Built into this pedagogy was Ms. Johnson's assumption that representing logical and abstract relationships spatially helped to scaffold students' numerical literacy. In fact, Kilpatrick et al. (2001) asserted that "Students' informal notions of partitioning, sharing, and measuring provide a starting point for *building* the concept of rational number." (p. 7) Other research studies that have focused on the impact of visual imagery on improving cognitive understanding and skills in utilizing strategies to solve mathematical problems bolster this assumption, as well (Dreyfus, 1995).

Moreover, Ms. Johnson believed that multiple representations of ideas helped her to reach out and teach more of her students than she would have been able to otherwise. Dettori & Lemut (1995) wrote: "Introducing the partition meaning of division by means of an image, the teacher has been able to unblock some pupils to solve a problem, stimulating a relationship among the problem data" (p. 27).

Ms. Johnson used students' prior knowledge of procedures to build new knowledge. In the following classroom audio of one of the classroom sessions from her Algebra One class [CA – October 19th, Algebra One], Ms. Johnson uses a student's question to transition into students' understandings and application of a new problem solving strategy; that is, she extended students' prior knowledge of the guess-and-check strategy in solving algebra word problems to the new knowledge and strategy of constructing and solving equations of one variable. This particular approach was observed in her Math Eight class.

Student1: For problem number 26, I got 1036!
Ms. Johnson: 1036.
Student2: I got 2072.
Ms. Johnson: Ahh! Anybody got a different answer?
Student3: 1179.
Ms. Johnson: 1179. Here we go, let's quickly. . . it asks us to do guess-and-check, but my whole point is I like us to use an equation for this. So, turn to 26. We're going to take time to solve it using an equation. It's really efficient. Here we go: "West High's population is 250 students fewer than twice the population of East High. So, I want you to start by defining the variables. So, East High number. . . write this down please! This is 26, using an equation. East side number, we're going to call X. I really need you to transition from guess-and-check to equations. This is a good opportunity. Eon, I'm not going to say it again [Ms. Johnson warns an unruly student]. Did you use an equation?
Eon: Well, it says use a guess-and-check table.
Ms. Johnson: No, no, no. You used a guess-and-check table. So, I really need you to use an equation with a variable. So, please, hear me, and write this equation. When it says "Write a guess-and-check table," you can transition right to this. OK, who could tell me what expression I would use for West High? If the problem says their population is 250 students fewer than twice the population of East High? Alexandria?
Alexandria: X = Y + . . .
Ms. Johnson: I don't want Y in there, though.
Alexandria: Alright! Alright! X*2-250.
Ms. Johnson: X*2 – 250. I want you to write X*2 – 250 = West. One thing about writing equations is that we still have to communicate our thinking clearly. So, we have to define the variable, so when I get the answer to X, I know what it relates to; West is this. Then, what else do we know? They have a total of?
Student6: 2858.
Ms. Johnson: 2858. And the final total will be? That! So, you'll have one expression: X + X*2 – 250 = 2858. And I hope you guys don't mind, I'm going to move the 2 in X*2 in front to make it 2X, just because it's easier for me. So, does everybody have that expression? Any questions as to where this came from? How many people are following along and it makes total sense to them? OK, first step in solving equations?
All Students: Combine!
Ms. Johnson: Combine! 3X – 250 = 2858. Raise your hand if you know the next step, please. OK, Connor!
Connor: Something divided by 3, I'm not sure what.
Ms. Johnson: OK, we are going to divide by 3, but if you want to think about solving equations, one strategy is order of operations backwards. So, . . . Yeah?
Connor: It's 2858 – 250.

Ms. Johnson: OK, why are you not adding instead of subtracting? Because it turns this term into zero and makes our Xs all alone. So, show that on both sides, please. Pay attention to where your equal sign is, cause our equations get longer and longer, it's pretty easy for kids to lose what an equal sign is, and sometimes they start adding everything to the same side; when the equations get longer than this. So, focus on it with simple equations; it won't happen otherwise. So, does everybody realize that we now have just the 3X here: 3X = 2858 + 250? OK, what's the total here?
Connor: 3108.
Ms. Johnson: There you go: 3X = 3108. And our last step, Connor, what were you going to do next?
Connor: 3108 divided by 3.
Ms. Johnson: Good! And in algebra, whatever we do on one side, we better show on the other. It's got to show on both sides. So, what is 3X divided by 3 equal to?
All Students: 1.
Ms. Johnson: X = ? Anybody get the answer that's in the book, 1036?
All Students: Yes!
Ms. Johnson: Are you sure?
All Students: Yeah!
Ms. Johnson: OK, and is this a calculator problem, or not a calculator problem?
Student8: Not a calculator problem.
Ms. Johnson: Not a calculator problem. OK, anytime the book says use a guess-and-check table, you can use a guess-and-check table, but you need to transition to writing an equation. So, the whole idea of the table was, this was your first guess, this was your second guess, this was your third column. So, it would have transitioned into this same equation. [CA – October 19th, Algebra One]

Other instances where Ms. Johnson used rules and procedure to build on students' mathematical ideas were in the context of "math games" or puzzles, problem solving strategies, or rare occasions during whole-class or small group discussions. With respect to the latter, for example, Ms. Johnson used students' prior understanding of the place of *parentheses* in the order of operations rules to discuss the order of operation for the *absolute value* notation. The following classroom audio [CA – October 17th, Math Seven] depicts this approach.

Ms. Johnson: OK, the other word that is going to show up, *absolute value*, is the distance a number is from zero. So, those of you who were at the concert yesterday, make sure that you write this down. What else do we know about absolute value? Jacob!

Jacob: Well, the symbol for absolute value is two lines like this.
Ms. Johnson: Beautiful! The symbol for absolute value looks a lot like parallel lines, almost. And...who else can tell me something else about absolute value? Shelby?
Shelby: Its order of operations is the same as the parentheses.
Ms. Johnson: OK. Good! So, you use them like parentheses in the order of operations. OK, so now, what do we know about distance? Yeah, Holly?
Holly: It's always positive.
Ms. Johnson: Yeah! It's always positive. I think if you haven't written that down, you should write it. Distance is always positive. [CA – October 17th, Math Seven]

Engaging students in mathematics learning.

Ms. Johnson acknowledged that motivating students and engaging them in mathematics learning was a significant challenge for her. Field observations of her practice showed that Ms. Johnson had a much easier time of engaging students' attention in mathematics learning during whole class discussions than when students were involved in their daily small group activities. Mathematical games and puzzles, or Ms. Johnson's rapid and repeated questioning of students during regular (non-game related) whole class activities seemed to focus students' attention for the most part. The following interview excerpts are demonstrative of Ms. Johnson's thinking and actions with respect to engaging students in the learning process. "I think some kids were really motivated by the idea that it was a game and a competition" [POI – October 18th, Math Seven, Question #2]. "I think the game is a good way for kids to practice, because it doesn't seem work to them, but it is" [POI – October 18th, Math Eight, Question #2]. "Making it a game [the Pig Race] made them do a little bit of math, even though they didn't want to. . . [And] the game format, and the cooperation . . . I think working with a partner helped keep them motivated" [POI – November 1st, Math Eight, Questions #2 and #3]. "I think the game, the target game, is motivating for them. So, they were all doing calculations and . . . their satisfaction when they get to that answer" [POI – November 6th, Math Eight, Question #2]. Samples of some of the games Ms. Johnson used in her classes have been included in Appendix P.

Ms. Johnson also believed that in order to attract and maintain students' interest in her instruction and the mathematics she taught, she had better dedicate a portion of her lessons to a new topic. The following interview excerpts describe her thinking.

> And making sure that I'm introducing new enough material that it doesn't, it's not something they already learned in 5^{th} and 6^{th} grade. So, making sure that it's novel, 'You haven't learned this yet!', and that gets motivating; to learn something you've never learned before and not just review, review, review. [II – Question #13]

> I think it was different than last year as they thought of solving equations more new to them. So, they were more engaged, actually, because it was novel and they made more connections to the last year's class and I don't know why it was different. They just have a different attitude toward math. [PUI – Algebra One, Question #3]

Ms. Johnson believed in and gave positive feedback to students as a means to motivate their learning mathematics as well. Warm-up computational activities were another method Ms. Johnson used in order to bring focus to mathematics learning in her classes. The following interview excerpt depicts her vision of a typical day in her classes, including what she pictured herself doing to engage students mathematically.

> I would say an opening; opening up with an idea where all students have to engage in some kind of thinking and responding to either a math warm-up, or a math discussion, or note-taking with questions, but really short 5-minutes introduction, or introducing a problem, launching them, and getting them going, and then introducing a set of problems that they're going to work through that are developing a particular idea for the day. [II – Question #14]

Ms. Johnson was creative in thinking of activities on the spare of the moment for the purpose of engaging students' mathematical thinking. For example, after discussing a lesson on equivalent fractions, decimals, and percents, and with only seconds left to the end of the class, Ms. Johnson stood at the door of the class and announced that students were allowed to leave the room at the sound of the bell, only if they could give her an example of a fraction and its equivalent decimal and percent values. Students unable to complete this action on the first trial were asked to step aside inside the room and think of another example. In another instance, after returning graded tests to students, Ms. Johnson asked students to compute the percentage of the exam they had scored correctly.

Keeping students engaged with mathematical tasks during small group activities was, by her own admission as well as the impression of her lessons gathered during field observations, the most challenging aspect of Ms. Johnson's instruction. During class "work time" or small group activities, Ms. Johnson's relentless visits to each team of students and asking questions when engaging each group kept students active and on task when she was with a group, though, when she moved away, students in the group tended to begin discussions that were off topic. Ms. Johnson often expressed her dismay and frustration in post-observation interviews about students wasting class time discussing their social lives. Another point of disappointment for her was that many students could not finish the class homework assigned for the day due to wasting time socializing with their peers. This prompted her to make two changes that improved students' commitment to on-task behavior. For one, she required students to turn in work they had finished during small group activities at the end of the class period. Another change was that she made the classroom homework problems more challenging. This latter decision had emerged after observing students finish classroom homework early just to be able to socialize with their friends for the rest of the class period. To a lesser extent students still continued with their off topic discussions during small group activities.

A significant portion of the teacher-directed interactive discourse that took place in Ms. Johnson's classes emphasized students' sharing ideas about problem solving strategies. Ms. Johnson downplayed the significance of final answers, and used discrepancies in students' answers to start a discussion about their problem solving strategies. Therefore, she pushed for students' sharing of their problem solving strategies. The following excerpt from the classroom audio of a Math Seven lesson [CA – November 28th, Math Seven], including an accompanying figure obtained from the field notes, represents the type of teacher-directed interactive discussion Ms. Johnson often conducted in her classrooms:

Ms. Johnson: OK, if you have a different way, a diagram that shows a different way, I want you to take over after his speech. OK.

Ms. Johnson signals the student, Jacob, standing at the helm of the document projector to commence.

Jacob: Me and Mike, what we did, our parallelogram, well one side, this side you've got to cross it out and then you put it over onto this side. So, it makes it a perfect rectangle. Then, that will be 48, oh yeah, this right here is worth 8. So, then, altogether, 10 times . . . or 4 times 10 is 40. 40 plus 8 is 48.
Student5: What?
Student6: [making buzzing sound to show disagreement with Jacob's solution].
Ms. Johnson: [talking to the student who had buzzed to display disagreement], Parker, start talking! OK, before though, does everybody agree that this triangle on the right can fit here on the left and make a perfect rectangle?
All Students: Yeah!
Ms. Johnson: Parker, if you don't think it's 48, so come and explain what you think it is?
Parker: So I got 40.
Ms. Johnson: But Jacob said it was 48!
Parker: cause he added an 8 or something.

Jacob: OK, now, just this alone is 40, but you forgot to add that [he is pointing to the triangular slice that was moved to the other side]

Other students unanimously reject his idea – NO!!!!

Ms. Johnson: OK, who wants to tell him how?
Parker: Just count! [the student is pointing to the number of squares on the graph paper.]

A female student gets up, goes to the front and joins the other two boys already in front debating the solution.

Student7: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. If you take this away [she points to the triangular piece that was moved] it's only 8. So, when you put it over there, it's still 10. 10 times 4 is 40.
Ms. Johnson: OK, check your dimensions. See if you can come up with a proof. There is more than one way.

A fourth student gets up and joins the other three students already standing and discussing the solution in front.

Ms. Johnson: Wait! It's Rick's turn. Rick has the floor!
Rick: You just times the height, which is 4, by the base, which is 10. It'll give you 40.
Ms. Johnson: Is this a rectangle?
Student9: No! But it still works.
Ms. Johnson: Janet has the floor!
Janet: It's still a rectangle. If you just move part of it, then it's a rectangle and it's still the same in area, so it works. [CA – November 28th, Math Seven]

Promoting students' thinking mathematics.

Ms. Johnson used several tasks and teacher-directed interactive discourse strategies to promote students' thinking mathematically. Tile or sketch models were used extensively in her practice as part of an overarching goal to use multiple representations of mathematical concepts to promote understanding of content for all students. "I think," she said in a post-observation interview after a session in her Math Seven class, "going over the 'minimal array' helped them and then made them really see the edges to the middle [of the array configuration]. So, they were thinking mathematically" [POI – November 14th, Math Seven, Question #2]. She used the concept of "minimal array" to convey to students the meaning and purpose of simplifying algebraic expressions. In her Math Eight class, she had similar reflections:

> I think using the tiles to make sense of all those algebraic expressions worked well for some kids. They're starting to see it's not a foreign language. I think it was good. Some kids held on to the tiles, kept using them, and some kids just did drawings, and other kids were trying to jump into next thinking. So, it sort of matches all different levels of thinking. So that was good. [POI – November 13th, Math Eight, Questions #2 and #3]

Asking students to exchange and share ideas by talking to each other was another way Ms. Johnson sought to extend students' mathematical thinking by exposing their minds to mathematical ideas of others. In this context, students were asked to share ideas aloud during whole class discussions or small group work, and difference between students' ideas set the stage for classroom discourse which then Ms. Johnson guided toward building of a consensus using her comments and questions. In her Algebra One class, the majority of such classroom discussions revolved about pattern recognition and making connections, though, consensus building was part of the classroom discussion in every mathematics course Ms. Johnson taught. The following sample transcription of classroom audios depicts how Ms. Johnson initiated a process of consensus building.

> Ms. Johnson: Did you guys draw figures 1 and 5?
> Student: Yeah, I have!
> Ms. Johnson: Have you shared? You haven't talked to each other, yet? Yeah, make sure you agree on figure 1.
>
> The teacher walks up to another team.
>
> Ms. Johnson: Did you guys share on figure 1 to see if you agree on the letter A on 1 and 5? Now, talk to each other and see if you agree before you keep going. Did everybody draw it the same?

The teacher walks up to another one of the 8 teams.

Ms. Johnson: So, you guys agree on the first figure?
Student: Yeah!
Ms. Johnson: You did a number to help you figure out the 100? Did you guys agree? One way to check it would be to make your table: 1, 2, 3, 4, 5 . . . see how many tiles there are, and see if the pattern will work for the 100. OK, can you write a rule? If X is your figure number, how many tiles you have?

The teacher walks up to another team.

Ms. Johnson: You guys have your table. Once you have your table for the growth, can you write a rule? If I tell you the figure number, can you tell me how many tiles will you have?

The teacher walks up to another team and repeats this routine. The teacher used consensus building to create a habit of checking work.

Ms. Johnson: Make sure you check with your team to make sure you're all moving in the same direction. [CA – October 15^{th}, Algebra One]

Mini-lectures that lasted only a few minutes and reiterated and reinforced important mathematical ideas were another way Ms. Johnson used to improve mathematical fluency of her students. She used mini-lectures as a review of key mathematical concepts about which students were struggling to develop proficiency; the notion of absolute value and its place in the order of operations and the procedure to calculate long division were examples of two mini-lectures Ms. Johnson gave.

Impact of the Classroom Community and the School Community

Ms. Johnson collaborated with Ms. Jenkins, the expert sixth grade mathematics teacher at the school, to make the mathematics curricula at the school compatible with state standards [SCI – Interview with Ms. Jenkins, Question #2]. She and Ms. Johnson held monthly meetings and were inspired by the same basic goal. "Our goal," Ms.

Jenkins said "is that every child can learn mathematics and can be successful to the highest potential" [SCI – Interview with Ms. Jenkins, Question #4]. When asked about the nature of her collaboration with Ms. Johnson, Ms. Jenkins said:

> We share our ideas about what was successful . . . We'll talk about content areas that I've covered in sixth grade and that she's going to cover in seventh grade, and we'll talk about what's the best way to teach that, the best manipulatives to use, we'll talk about what we did in the sixth grade and how she can support that in seventh grade so that again kids are on some kind of continuum that makes sense to them. You know, making sure that we're using the same vocabulary, the same language, and that's what we do. [SCI – Interview with Ms. Jenkins, Question #6]

Ms. Johnson met with the novice mathematics teacher at the school, Mr. Hubert, on a weekly basis and helped him to design lessons that met the needs of all students in the class, "not just the needs of a few" [SCI – interview with Mr. Hubert, Question #1]. When asked about the nature of his collaboration with Ms. Johnson, Mr. Hubert said:

> We discuss the results of formative and summative assessments in class, and we respond accordingly by designing lessons that will give something to the higher end kids but still allow the kids that are perhaps struggling to get it. So, everyone gets the opportunity to succeed in the classroom. [SCI – interview with Mr. Hubert, Question #2]

Ms. Johnson spoke to Ms. Buchanan, the science teacher at the school, on a daily basis and had worked with her to develop "integrated units" that were already put in place at the time of this study. The purpose of the integrated units was to identify areas of middle school mathematics with applications in middle school science and combine the units from both mathematics and science units and teach the contents together. Thus, when it was time to teach the integrated units, students from Ms. Buchanan's science class and Ms. Johnson's mathematics class merged, and Ms. Johnson taught the mathematics content while making connections to the relevant science content and real world contexts. The teaching of one of these integrated units coincided with the timing of this study, providing some of the most engaging lessons observed. When asked to elaborate about the nature of her collaboration with Ms. Johnson, Ms. Buchanan said:

> We have several in the seventh grade, and so we've developed skills together with science and the math that would take place in that unit, and I think it's been very successful; the units, the integrated units are really fun for students and they enjoy them and I think they get a lot, a lot, out of them and it just makes sense to integrate them together. So, but that's the only aspect of developing a curriculum I've had with her. [SCI – interview with Ms. Buchanan, Question #2]

Principal Roberts worked closely with Ms. Johnson and facilitated her attendance at in-service and summer courses and workshops year round. Her willingness to grant Ms. Johnson permission to attend workshops, hiring substitutes when needed, worked well with Ms. Johnson's enthusiasm to attend as many professional development programs as possible. In return, Ms. Johnson returned with her knowledge of the proceedings gained at workshops and shared them with other interested teachers at the school. Therefore, the principal's generosity with making time and district funds available for the purpose of staff development at the school benefited Ms. Johnson's aspirations to keep current with the latest innovations in mathematics education as it was pointed out by the science teacher, Ms. Buchanan.

> She really cares about students first, and wants to make sure that they succeed and like math, and goes about it by trying to find a lot of different ways and techniques to help them learn and so she's innovative and stays

> up with the current things that are happening in mathematics and ways that students learn it and seems to . . . is always willing to try new things to help them master . . . and also she is very intuitive of what they need to know, you know, she stays in contact with the high school and you know, and where they are coming from. [SCI – interview with Ms. Buchanan, Question #1]

The interview with Principal Roberts disclosed other dimensions along which Ms. Johnson had established collaboration with the school community.

> She has the opportunity to go to many workshops and in-service throughout the year. The district pays for those. She gets release time for those. Also, Ms. Johnson is part of our mathematics curriculum meetings. And so she shares her knowledge and her opinions about curriculum and teaching math in those classes. She also works as a member of a team with other teachers, and so, . . . our K5 – K8 team. So, she shares her knowledge, her information, and what she has learned in her instructional practices with other teachers. So, there is a lot of sharing and teamwork, collegiality in our school. [SCI – interview with Ms. Roberts, Question #1]

The generosity of the principal with the funds that were allocated to the school by the district impacted Ms. Johnson's professional activities and her knowledge of teaching practices.

> The teamwork and collegiality is important. We also have district funds available for substitutes for when Ms. Johnson is out of the classroom, attending workshops. We also have district funds to pay for her attendance at workshops, in-services and summer courses. And we have our curriculum team meetings. And in fact, we're going to have one tomorrow. We have tuition reimbursement funds available for all teachers. And that's the fund we do every year, and I think Ms. Johnson has applied for that several times." [SCI – interview with Ms. Roberts, Question #2]

The principal's mention of "tuition reimbursement" refers to Ms. Johnson's tuition costs for attending a doctoral program in educational leadership at a local university. Ms. Roberts's revelations pointed to Ms. Johnson as a lifelong learner and her enthusiasm in the educational process. Ms. Johnson used her participation in professional development programs to benefit her colleagues at the school as this next interview excerpt demonstrates.

> We have very strong, committed math teachers. We have Ms. Jenkins and Ms. Johnson, and they head up our whole math curriculum program. Ms. Jenkins is sixth grade teacher, and they are passionate and committed to math. Every summer they take several workshops. They come back, they share those practices and those instructional techniques with the rest of the staff. [SCI – Ms. Roberts, Question #4]

In addition, Principal Roberts and Ms. Johnson attended monthly meetings held with key mathematics teachers and administrative staff at the high school next door, sharing ideas. One such meeting took place during the course of this study and Ms. Johnson recorded a 30 minute conversation with the attending mathematics teachers at the high school. District statistics on the Technology Enhanced Student Assessment (TESA) scores were discussed at length. Also discussed was the use of calculators at schools, particularly at K-5 level. The attending high school teachers argued that the quality of school mathematics education was suffering due to spreading use of calculators at schools. Ms. Johnson expressed that each mathematics teacher was to conduct teaching in her/his classes at her/his own discretion.

Models and Descriptions of Ms. Johnson's Lessons

All of Ms. Johnson's lessons in the Algebra One, Math Seven, and three weeks of the Math Eight class (4^{th} period) and four weeks of the Math Eight class (3^{rd} period) were observed during the course of the study. Sequences of lessons typical of Ms. Johnson's practice, irrespective of content topics, were identified from each course category and parsed for further analysis. For the Algebra One class, these included lessons on solving equations for any of several variable terms, solving equations for one variable, re-teaching the distributive property, and constructing rules based on generalized patterns in tables of numerical data. For the Math Seven class, the parsed lessons included the lessons on extending the guess-and-check strategy to constructing solvable equations, tile models representing operational sense and number sense with respect to multiplying and dividing integers, and "minimal array" tile model representations of the idea behind simplification of algebraic expressions. And, for the Math Eight class, the parsed lessons included tile model representations of operational sense with respect to constructing and simplifying algebraic expressions, constructing symbolic representations of tile model representations of the concepts of rectangular area and perimeter, and extending the guess-and-check strategy to constructing solvable equations of one variable.

Analyses of the lesson models, constructed by review of the respective transcripts for pre and post observation interviews, classroom audios, and field observation notes, revealed patterns of practice common to all Ms. Johnson's lessons. Detailed versions of these models have been included in Appendices I, J, and K for the Algebra One class, in Appendices L and M for the Math Seven class, and in Appendices N, and O for the Math Eight class. Each model includes major action sequences of the lesson segments and a maximum of two parsings of sub episodes of each action sequence, where the assessment, content, discourse, and pedagogy observed were noted. The vertical thin lines to the right of sub episodes demonstrate a goal trace during the lesson segment showing when each goal was active. The goals for each lesson segment were listed at the bottom of episodic descriptions, emanating from both the statements made by the teacher during pre observation interviews and those attributed to the lesson segment by the researcher based on his observations of the lesson and casual conversations with Ms. Johnson.

The results reported in this section contain summary descriptions of some of the more detailed versions of lesson models included in the appendices. They were selected as one lesson segment from each class and meant to give the reader a sense of the typical strategies and order of events in Ms. Johnson's classes.

Model of an Algebra One Lesson: Solving Equations of One Variable

This lesson, shown in Figure 11, was observed two weeks into the classroom observation portion of this study. This third day in a sequence of Ms. Johnson's lessons was on solving equations of one variable [FN – October 30th, Algebra One]. No classroom audio was recorded for this lesson because almost the entire lesson was conducted in small groups. During the pre observation interview, Ms. Johnson stated the goals of the lesson as correcting the homework from the previous day, using the "tile problems" to practice solving equations to the point where error in students' work was minimized, and teaching students how to correct and eliminate errors in their own work. The lesson began with a routine to discuss homework assigned in the previous class session. Next, Ms. Johnson followed with a mini-lecture on how to distribute a negative sign placed in front of parentheses to the terms inside the parentheses. Judging by the number of students who were having difficulty with the operation, Ms. Johnson had anticipated this question to come from students. However, in the absence of the question from students, she decided to go ahead and discuss the concept in a brief lecture.

Figure 11. Model of an Algebra One lesson: solving equations of one variable

Action Sequence	**Sub-Episodes**		**Goals Activated** a b c d
1. Whole-class routine (5 min.)	1.1. **Assessment**: Correcting previous homework 1.2. **Content**: A mini-lecture on the distributive property		
2. Small group work on solving equations (45 min.)	2.1. **Assessment**: The teacher interacts with each group continuously throughout the class period **Content**: Textbook problems on solving equations of one variable (e.g. 45=6x+3). **Discourse**: The teacher uses hints and guiding questions **Pedagogy**: Students working independently, combined with small-group collaboration, and the teacher helping with guiding questions	2.1.1. **Content**: Students are assigned to solve 10 problems from the textbook **Discourse**: The teacher emphasizes students' checking their own answers and by building consensus amongst themselves **Pedagogy**: The teacher wants students to abandon the guess-and-check strategy	

Goals:

a. Correct previous homework and provide opportunity for students to ask questions
b. Use a mini-lecture to review a topic with which many students were having difficulty: distribution of a negative sign placed in front of a parentheses
c. Assess students' understanding of the procedure to solve equations of one variable
d. Provide guided practice to students to solve equations independently or with peers, using the symbol-manipulation strategy

Upon finishing the mini-lecture, Ms. Johnson quickly moved to assign classroom homework problems to be worked on by students in small groups. Students had experience solving equations for any of a number of variables, without obtaining a numeric answer. The primary purpose of this lesson was to extend that knowledge to solving equations of one variable for a particular solution. Thus, the lesson focused on operations (multiplying, dividing, subtracting, or adding) needed to rearrange the terms in an equation in order to isolate the unknown term. Because the entire lesson was conducted in small groups from start to finish, it only had one segment. Students were assigned a list of 10 problems from the textbook that they were to try to finish before the end of the class.

Ms. Johnson went from group to group answering questions, monitoring and assessing students' progress and understanding. Each group was visited multiple times by Ms. Johnson. In each of her contacts with the groups, Ms. Johnson wanted students to show her the strategies they were using to work out the problems, whether they had checked their answers by substituting or by comparing their solutions with those of others, and whether they had determined more than one way for solving each problem. Many students reverted to using the guess-and-check strategy, avoiding using the new technique of solving equations. Ms. Johnson made an emphatic request to students that they must use symbolic manipulation, instead. Ms. Johnson's agility in interacting with students and her continuous assessment of students' thinking throughout the class period were key strategies for keeping students on task.

Model of a Math Seven Lesson: Operational Sense and Number Sense

This lesson shown in Figure 12 was observed five weeks into the classroom observation portion of this study. This first lesson of a five-day sequence of lessons on operations involving integers was spent with majority of students struggling to construct tile models of algebraic operations. Students were observed struggling with the same tasks on the second day of the lesson as well. On the third day, students received an assignment to create posters of three integer problems solved using alternative tile or graphical solutions; the graphical solution involved the sketching of as many rectangles to represent physical tile structures, but similar to the tile models, no numbers were used in the expression of solutions [FN – November 13th, Math Seven]. Students were also instructed that the problems they chose for their posters were to be connected to real world situations. A rubric for scoring the content of students' posters listed a guideline for designing the posters and was distributed to students on the third day of the sequence. The fourth day was spent presenting the posters, and on the fifth day, the best posters and strategies were reexamined.

Ms. Johnson began the lesson with a whole-class discussion of how algebra tiles could be used. She explained to students that a black tile represented a "+1" value, and a red tile represented a "-1" value. Further, she described how a correct arrangement of tiles included two sets of "edge tiles" and a single set of "center tiles", and multiplying the edge tiles was to result in the value of the center tiles, while dividing the center tiles by any of the edge tiles was to result in the correct value for the other edge tiles.

After explaining the meaning of the tiles and proper ways of working with them, Ms. Johnson used a document projector to demonstrate examples of different

Figure 12. First action sequence, Ms. Johnson's lesson on teaching students operational sense and number sense when working with integers

Action Sequence	Sub-Episodes		Goals Activated a	b
1. Whole-class discussion of algebra tile problems (25 min.)	1.1. The teacher explains rules for working with the algebra tiles	1.1.1. **Assessment**: A document projector aids the whole-class discussion, while the teacher distributes a "recording sheet" where students will have to sketch and write their thoughts and explanations. The sheet is to be turned in at the end of the class. **Content**: Students struggle with and share ideas about how to read the tiles mathematically as 3x4=12, 4x3=12, 12/4=3, and 12/3=4. **Discourse**: Students discuss and share ideas about the arrangement of the tiles in their groups, and once they agree on an arrangement, they raise their hand and share their configuration for the tiles with the class. **Pedagogy**: Whole-class discussion of the operations and tile arrangements, while students work independently and collaboratively to connect tile arrangements and number operations		

Goals:

a. Use visual models along numerical operations to convey to students an understanding of the operation sense and number sense when working with integers
b. Have students to maintain a log of their work with the tiles on the "recording sheet", turning it in at the end of the class period for the teacher's assessment

arrangements of the tiles. For each arrangement, she asked students to decipher its meaning by giving the numeric representation of it. On the other hand, she displayed numerical representations of a series of algebraic operations and asked students to construct the tile models that represented them. A "recording sheet" was distributed to students, where she asked students to record all their work in response to the classroom questions and exercises and turn it in at the end of the class. Students used this sheet to record their ideas for answering the questions. Some students worked independently and others were collaborating with their teammates. Students were asked to raise their hand and share their ideas with the rest of the class as soon as they jotted down an answer. Nonstop questions and comments by Ms. Johnson kept the whole-class discussion moving at a steady and brisk pace before the small group phase of the lesson. No classroom audio was recorded for this lesson due to momentary equipment failure.

Figure 13 shows the second action sequence for the same lesson as in Figure 12, yet here the events in the classroom during the small group activity are represented. The small group activity mirrored the type of thinking and physical activities and tasks Ms. Johnson modeled for students during the whole-class discussions. Students were assigned a series of problems from the textbook and were to identify either an algebraic expression that matched a tile arrangement or construct a tile arrangement that matched an algebraic expression involving only integers.

Ms. Johnson walked around the room with agility and interacted with all the groups multiple times, while asking them to show their reasoning, progress, and a way that showed they had checked and verified the accuracy of their work. She used the consensus building strategy not only to have students become aware of different thinking

Figure 13. Second action sequence, Ms. Johnson's lesson on teaching students operational sense and number sense when working with integers

Action Sequence	Sub-Episodes		Goals Activated a	b
2. Small group activity (25 min.)	2.1. **Assessment**: The teacher talks to each group during the small group activity and students are to turn in their "recording sheet" **Content**: Students are working on textbook problems (e.g. arranging tiles to get 1x4=4, 4x1=4, 4/1=4, and 4/4=1). **Discourse**: The teacher wants students to build consensus when working on solutions to the problems. **Pedagogy**: Collaboration in small groups combined with the teacher's guidance	2.1.1. **Content**: The teacher uses the algebra tiles to explain to students why the ratio 0/0 is undefined in mathematics.		

Goals:

a. Small groups to use visual models along numerical operations for the purpose of understanding the operation sense and number sense with respect to multiplying and dividing integers
b. Have students maintain a log of their work with the tiles on the "recording sheet", to be submitted at the end of the class period for the teacher's assessment

strategies but also as a way to check on the accuracy of their own work. When a student raised her hand to ask Ms. Johnson a question, she asked her "Have you asked your question from your teammates?", and when the answer came "No!", Ms. Johnson requested that the student ask her peers first and if not satisfied with the answer they gave her, then ask her. The words: "Talk to each other!" in the context of students building consensus about solutions were repeatedly heard as Ms. Johnson interacted with groups. Toward the end of the lesson, Ms. Johnson briefly explained to students why the operation 0/0 was "undefined" in mathematics, since using the tiles, no possible arrangements could be defined for the operation. [FN – November 13th, Math Seven]

Model of a Math Eight Lesson: From Guess-and-Check to Symbolic Manipulation

This lesson was observed in the beginning of the sixth week of the classroom observations. [FN – November 19th, Math Eight] This first lesson in the sequence of three consecutive lessons on the topic focused on transitioning from a guess-and-check strategy of solving word problems to constructing and solving equations of one variable, thus, connecting different problem solving strategies within mathematics. The lesson consisted of three main action sequences. Figure 14 gives a summary of the lesson model for the first two action sequences, and Figure 15 gives a summary description for the third action sequence.

Ms. Johnson began the class with a brief review of school announcements for the day. During the second action sequence, Ms. Johnson began a whole-class discussion, demanding everyone's participation in making guesses and following through with computations. Other students were expected to express their thoughts, agreement or disagreement and their reasons if they disagreed, with respect to the guess and solution of the speaking student. Building consensus was meant as a way to check the accuracy of each solution while exploring alternative solutions in a collaborative environment. Students who avoided volunteering were called upon to express their thoughts.
After a correct solution was found, the "guess-and-check table" was examined in a whole-class discussion to formulate a symbolic representation of a solution in the form of a solvable equation of one variable. Three problems were solved in this way, before the small group activity began. Figure 15 depicts the lesson model for the action sequence

Figure 14. First and second action sequences, Ms. Johnson's lesson on how to transition from guess-and-check strategy of solving problems to one that aims at constructing equations and using symbolic manipulation

Action Sequence	**Sub-Episodes**		**Goals Activated** a b c d
1. The teacher reads school announcements (3 min.)			
2. Whole-class discussion (25 min.)	2.1. **Assessment**: The teacher assesses students' understanding of the content as different students volunteer to provide guesses and follow through with the operations involving the guess **Content**: The subject of discussions are the word problems from the textbook **Discourse**: The teacher asks different students to provide a guess for each step in solving each problem and follow through with it **Pedagogy**: Whole-class discussion of guesses and algebraic operations, and building consensus about solutions	2.1.1. **Assessment**: The teacher demands all students' participation in the activity **Content**: After checking a series of guesses, a generalized equation is formulated. Three problems are solved in this way **Discourse**: All students are expected to participate in making guesses and following through with the operations verbally while the rest of the class is checking and commenting on their thinking and strategy	

Goals:

a. Deliver school announcements for the day
b. Assess students' understanding of the content during whole-class discussions
c. Review the guess-and-check strategy of solving problems
d. Extend the guess-and-check strategy to solve problems and connect it to the symbolic generalization of mathematical ideas in the form of solvable algebraic equations

Figure 15. Third sequence, Ms. Johnson's lesson on how to transition from guess-and-check strategy of solving problems to one that aims at constructing equations and using symbolic manipulation

Action Sequence	Sub-Episodes		Goals Activated a	b
3. Small group discussion (22 min.)	3.1. **Assessment**: Substantial verbal interaction between the teacher and students, and students to turn in their work at the end of the class **Content**: More textbook problems follow the theme from the whole-class discussions **Discourse**: The teacher's questions guide students' thinking and activities. **Pedagogy**: Small group discussion of the guess-and-check strategy and connecting ideas within mathematics. Repeated interaction with students keep them on task	3.1.1. **Discourse**: Continuous and repetitive verbal interaction between the teacher and students. The teacher's questions stem from her expectations and goals for the lesson. **Pedagogy**: Extending ideas from one strategy to another in a problem solving environment. The teacher is student-centered as she interacts with and responds to students' thinking, extensively		

Goals:

a. Assess students' understanding of the content during small group activity
b. Students work independently and collaboratively to extend the guess-and-check strategy to symbolic representation of solutions

representing the small group activity.

Substantial interaction was evidenced between the teacher and groups of students [CA – November 19th, Math Eight; FN – November 19th, Math Eight]. The interaction involved Ms. Johnson asking students to interpret the statements in a word problem into abstract mathematical relationships. Students were clear about Ms. Johnson's expectations. They were expected to work independently or collaboratively, but reach a consensus in their group about a solution or alternative solutions. They were expected to have a clear reason and explanation for their interpretations. At the same time, the continuous and repetitive verbal interaction with students kept them on task as well as giving Ms. Johnson a good assessment of where students were in terms of their understanding of the topic. The researcher did not observe any instance of Ms. Johnson providing answers to students who were having difficulty, but the students' questions were answered by questions and comments that guided students through hints.

Summary Discussion Three

Ms. Johnson's beliefs and intentions provided the main motivation for her practice, and her knowledge base provided the vehicle by which her beliefs and intentions were materialized in the classes she taught. She chose and implemented mathematics curricula that she thought best improved students' conceptual understanding and procedural fluency. She used various assessment strategies to learn what students knew to the extent possible, and presented lessons that built on a common denominator of student knowledge. She wanted to make learning mathematics an active, collaborative, and engaging endeavor for her students, but small group activities presented a particular challenge for her, because students wasted time socializing with each other rather than spending their time on-task and productively. This issue remained unresolved for the most part for the duration of this study.

Ms. Johnson believed that *time* was critical for her students to develop the multidimensional mathematical understanding she wanted them to have. This belief was evident in her practice as she showed patience despite overly social behavior keeping its momentum. In addition, Ms. Johnson expressed dismay at the fact that some students did all the work while others "leach[ed] off of them." Observations of Ms. Johnson's lessons showed that she rarely used a rubric for measuring accountability among students during small group activities. Poster presentations, for example, were one exception, where each group member was expected to complete and present his or her part. Students turned in their *work-time* assignments at the end of each class, yet, without a more cheat-proof rubric to account for each student's participation individually, the majority of students could continue to skip work while sufficing to copy the results from the work of their peers. The severity of this problem was underscored as Ms. Johnson revealed to the researcher that there were also students whose incomplete and mainly faulty track record for completing classroom assignments stood in sharp contrast with respect to impeccable quality work they completed at home. Since small group activities formed the greater portion of each lesson (see Appendices I – O), overly social behavior by students shaped the most insurmountable challenge in this reform-based mathematics education environment.

The following sections scrutinize the study results from the perspectives of each of the two research questions.

Results of the Research Questions

The overarching purpose of this study was to explore how an expert secondary mathematics teacher teaches within a reform-based curriculum program, and what actions from the workplace influenced her teaching.

The First Research Question

The first research question asked: *What are the intended instructional objectives and practices of an expert secondary-level mathematics teacher when instructing mathematics in a reform-based curriculum program?*

Ms. Johnson's intended instructional objectives.

The data suggested that the foundation for Ms. Johnson's overarching intentions about how to teach and what it meant to teach mathematics were formed early in her career. Her participation in numerous reform-based professional development programs, her membership with the Silicon Valley Math Project, her enthusiasm "to go learn more new ways to do things [in mathematics]," and her philosophy that "every student can learn mathematics" appear to have influenced her choice of the mathematics curricula from which she taught.

Examination of her day-to-day practice across the middle grades revealed not only that her intentions aligned with the *Connected Mathematics Curriculum* but also she was able to design and plan lessons based on the intended curricula and successfully implement the lessons. When asked how the curricula from which she taught had affected her practice, she expressed that:

> Having that curriculum has allowed me to have a good math lesson everyday as opposed to great ones for two weeks and then maybe not great

> for a day or two and then another great one. So, I think they may not all be great but there is a consistency that wasn't there. And how the curriculum is designed to be taught a certain way was, you know, that balance of teamwork and individual and teacher led problems or teacher led discussions. So, really it does define how you structure a lesson. I mean there is some flexibility there, but I try and teach it true to the way it was written and that . . . that definitely affects the way I teach. [II – Question #17]

Given the reform-based curricula, Ms. Johnson's intentions were to assess students' prior knowledge, model a process of discussing various problem solving strategies while making use of what students already knew, and then assume the position of a facilitator as students grappled with problems from the book on their own or within a team atmosphere. That Ms. Johnson wanted to play the role of a facilitator occurred repeatedly during discussions and interviews with her. When asked to describe her vision of a typical day in her classes, she stated that:

> I would say [opening up] with an idea where all students have to engage in some kind of thinking and responding, either a math warm-up, or a math discussion, or note-taking with questions, but really short 5-minutes introduce, or introducing a problem, launching them, and getting them going, and then introducing a set of problems that they're going to work through that are developing a particular idea for the day and then some work time on review, preview, practice type problems that they get done in class. [II – Question #14]

Ms. Johnson was able to implement lessons according to her intentions that aligned with the reform-based *Connected Mathematics Curriculum*. This understanding was verified when her statements during pre observation interviews were compared with aspects of her practice captured in the field notes and classroom audios. Yet, post observation interviews revealed that her intentions for students' learning outcomes often did not materialize immediately. Flick and Dickinson (1997) observed the same phenomenon with respect to the intentions of four expert science teachers. The following excerpts from several interviews demonstrate some of Ms. Johnson's reflections to this effect of whether students learned from a lesson as expected:

> Some, they did. But you'll be shocked at the operational sense with "times" [a reference to the multiplication operation] and "equal." It's such a simple idea, yet they have been struggling with that operational sense of where the equal sign is. [POI – October 15th, Math Eight, Question #3]

> I think some kids are going to feel really confident, because they're really good at seeing patterns and other kids are going to be a little frustrated cause they don't know what's going on in other kids' heads to get there. [PRI - October 16th, Algebra One, Question #3]

> Not necessarily. Lots of kids did the tip of the graph backwards, so they don't have dependent, independent variables down, or they don't know how to read. [POI – October 22, Algebra One, Question #4]

During the post-unit interviews and throughout the study, Ms. Johnson acknowledged that a significant time lag existed between her implementation of curricular and teaching intentions and the achievement of her intentions for her students' learning outcomes. A good portion of students had difficulty grasping either conceptual or procedural, or both aspects of the content. The final in-depth interview with Ms. Johnson revealed that this recognition impacted her intensions for teaching each of the classes in the future differently. With respect to her Algebra one class, she expressed that the curriculum was strong and students were just beginning to see the connections between tables, graphs, and equations, the main focus of the course during the observation period. She judged that the curriculum was strong and students needed time to understand the content. On the other hand, with respect to Math Seven and Math Eight classes, she stated that:

> The seventh grade, I still keep trying to tweak it so that the integer operations and integer understanding is more conceptual and stronger. So, I would change how to teach the integer, and then I also . . . I think I would do deeper units with the three major content areas for seventh grade. Integers, proportional reasoning, and the beginning algebraic thinking . . . And eighth grade, I think, I would use the same material, but I might take my lower end kids and do something different so that kids that are ready for all that algebraic thinking, get it. [FI – Question #6]

Other salient examples of Ms. Johnson's intended instructional objectives were observed and are discussed in the section for addressing the second research question, when considering the influences of the classroom and school communities. The tasks, discourse, and learning environment fostered by Ms. Johnson revolved about two main instructional goals: 1) assessing students' prior knowledge, 2) connecting ideas and strategies within mathematics.

Assessing students' prior knowledge.

Ms. Johnson began the majority of her classes with the answers to previous day's homework. Students' questions were discussed in class in detail, giving Ms. Johnson the opportunity to clear misconceptions or difficult points of understanding prior to starting to teach new material.

The most common method Ms. Johnson used to assess students' prior knowledge was by use of questions during whole-class or small-group discussions. Direct questions such as "How many of you have seen this?", or the types of responses students provided while being inundated by Ms. Johnson's questions often was sufficient to give Ms.

Johnson a measure of where her students stood with respect to mathematical concepts and procedures.

Students turned in their "spirals" or notebooks, containing their homework for the week at the end of each week. Several weeks through the observations, Ms. Johnson decided that students had to do the same on a daily basis with respect to work completed inside the classroom. This discussion was intended to keep students on-task during class "work time" or small group activities.

Diagnostic tests were short quizzes targeting students' comprehension of key topics that Ms. Johnson administered to her students prior to making the decision to move forward with discussion of the new content. The outcome of diagnostic tests determined Ms. Johnson's course of action in the immediate future, yet she acknowledged that there was a limit to the time she could spend on concepts and procedures that were not well understood by all students. She said "We don't wait for conceptual understanding to develop. And that's our big fault in our program, cause we only have them for 180 days" [IMI – November 15th, p. 193 of the field notes].

Post-unit tests, on the other hand, targeted measuring students' comprehension of a variety of concepts and procedures, they ran for the entire class period, and provided Ms. Johnson information about students' depth of understanding with respect to an entire unit already completed. The outcome of post-unit tests determined Ms. Johnson's approach to teaching the upcoming unit. For the most part, post unit tests showed that Ms. Johnson was concerned with assessing each student's gaps in understanding more accurately and addressing those needs earlier and before it was too late. "I think it's doing a better job of assessing individual skills so that we can meet individual holes that kids have in their skills. And do something more to [improve] each student's skills so that they are all successful." [PUI – Math Seven, Question #7]

> I think the key is that some kids do all the work, yet are making errors, and so I'm going to do something to make sure that we can have kids focus on their own errors, so that they don't continue to make the same errors over time. So, some accountability piece, because the kids that are making errors about collecting like terms, that shouldn't be happening this late in the year. So, we have to do something about that. [PUI – Algebra One, Question #7]

> There were small group of students, like 5, that at the end of the unit, when it was time to make the fraction poster couldn't pass the quiz to do it. So, maybe having that assessment and accountability piece earlier. So that to raise their level of concern for learning it. So, there was a small group of kids that slipped through the cracks, and here is the time to show everything you know, and they didn't know it. So, something to put that accountability piece earlier. [PUI – Math Eight, Question #7]

Connecting ideas and strategies within mathematics.

In the Algebra One class, for example, the emphasis for many lessons had been to identify slope in a table of data, on a graph, and in an equation. In the Math Seven class, algebra tile models were used to connect properties of tile arrangements, such as the color of tiles and their count, with results from order of operations involving integers. In similar ways and in both Math Eight classes, various algebra tile arrangements were used to connect geometrical properties of rectangles to constructing algebraic expressions. "Minimal array" tile counts and arrangements were used to demonstrate the logic behind simplified algebraic expressions. Naturally, this method of teaching incorporated multiple representations of mathematical ideas at its core.

Teacher-directed interactive classroom discourse.

The fundamental purpose of daily classroom discussions initiated and directed by Ms. Johnson was to engage students in participating in classroom activities. Student responses that were procedural in nature were acceptable to Ms. Johnson's criteria for students' participation as well as those responses that gave an explanation of how students arrived at their solution strategies; albeit the latter type of responses was observed to occur far less frequently than the former type of responses and mainly during poster presentations.

With this main goal of having students to participate in mind, *consensus building* was Ms. Johnson's strategy of choice, which she set forth in whole-class as well as small group interactions. Discrepancies in the final answers to problems were often used to launch classroom discussions toward building consensus.

Whole-class and small group discussions were the daily norm in every class. In addition, students prepared posters and presented them in groups or independently, while other students or Ms. Johnson asked questions or commented on the presentations. The following excerpt from the transcripts was recorded during one poster presentation by a number of teams in the Algebra One class. Each team of presenting students consisted of four students that held a 4-feet-by-3-feet poster, while each student on the team took turns to explain their portion of the contributions to create the poster. This following excerpt from a classroom audio depicts the typical nature of teacher-directed interactive discourse in Ms. Johnson's classrooms.

Ms. Johnson: Now, we want to know how you got it.
Student1[team 1]: So, we put this square over here, so then it would be a square and it would be the figure number [inaudible] to 4 plus 2, and then the figure number this way plus 3 and then plus 3 on the top. And the figure 100 is 10, 686 tiles.
Ms. Johnson: So, you guys did the math and checked the math using this rule? You put the 100 in there and get this, right?

One of the students, Jack, raises his hand to ask a question.

Ms. Johnson: What do you think, Jack?
Jack: Is the graph linear or non-linear?

Some students laugh at the question, while Ms. Johnson, repeats the question for the presenting team.

Ms. Johnson: Does it curve or is it a straight line? Does it go up by the same amount each time?
Students [team 1]: No!
Ms. Johnson: So, from 6 to 13 how much is it going up? Looking at that picture, to get from 0 to 9 it's going up how much?
Students [team 1]: 7.
Ms. Johnson: And from 13 to 22, how much?
Students [team 1]: 9
Ms. Johnson: 9, and from 22 to 33?
Students [team 1]: 11.
Ms. Johnson: 11, and the next one is going to go up by?
Students [team 1]: 15.
Ms. Johnson: 15. So, we know that it's not going up by the same amount every time. Then it shouldn't be a [straight] line. It's not a line. It's growing faster than a [straight] line. So, it should have a curve in the graph. It's hard if you are only doing a small section. It might look like a [straight] line. So, the scaling of your axes……you could trick it into being a [straight] line if you scaled your axes differently. But if you scaled evenly, the X, and then scaled your Y, then it won't be a [straight] line. So, check your scaling on the graph, OK? Because it looked like a [straight] line when you showed it to us.

Ms. Johnson turns toward the class.

Ms. Johnson: Did you guys get how if you scaled the graph, you can make a curve look like a [straight] line by using the wrong scaling? By not doing equal intervals? OK, any more questions? OK, next team.

The next team goes to the front too present their poster.

Ms. Johnson: OK, we're ready.
Students [team 2]: Sure…yeah…our graph is this….Will?

Will: OK, figure zero is 6435.
Ms. Johnson: Connections. How did you get there? You started with the X-Y table?
Will: By using this graph and then the information here.
Ms. Johnson: So, you started with the X-Y table?
Students [team 2]: Yeah!
Ms. Johnson: Could you figure out the pattern by just looking at the table?
Students [team 2]: Yeah!
Ms. Johnson: You did get the rule by just looking at the table?
Students [team 2]: Yeah!
Ms. Johnson: Really?
Students [team 2]: Yeah!
Ms. Johnson: It's not how I remember it, but…

Ms. Johnson is referencing the discussions she had had with the students in the classroom while they were working on their poster. What follows are explanations by the students on the team, but this portion of the students' remarks were inaudible.

Ms. Johnson: I need to know how you solved for the pattern in the figure. Because that's what's the goal of you coming out here.

The students on the team are explaining, but this was inaudible.

Ms. Johnson: Good!

Then the students are explaining their graphs.

Ms. Johnson: So, is it a linear or is it going to be a parabola? Quadratic, or exponential?
Students [team 2]: Sure.
Ms. Johnson: A parabola?
Students [team 2]: Yeah!
Ms. Johnson: A half parabola.
Students [team 2]: OK, and this is the rule: $y = 5x + x^2 + 6$
Ms. Johnson: Well, I have a question. On your graph, does it cross at y axis at 6?
Students [team 2]: Yeah!
Ms. Johnson: That might mean that your rule is right. OK, I have another question, do you think if I didn't have a table, could you have made a graph without a table?
Students [team 2]: Sure.
Ms. Johnson: Eventually the goal in this class is that you don't have to create a table when you want to graph. OK, next team!

The next team of students prepares to go to the front of the class as the presenting team goes back to their seats.

Ms. Johnson: Before you guys start, this is really important. One thing that I am looking for is not what you have but how you got there, OK? That connection…how did you create your table? Or how did you know that a graph would look a certain way? Or how did you come up with your rule? Not just what your rule was.

The Second Research Question

The second research question asked: *What aspects of the classroom community and the school community impact the intentions and instructional practices of an expert secondary-level mathematics teacher in a reform-based mathematics curriculum program?*

Influences of the classroom community and the school community.

Ms. Johnson's knowledge, beliefs, and intentions played a more significant role in determining the instructional strategies she chose and actions she took inside the classroom than her day-to-day interactions with students. The influence imparted by the classroom community was observed only "when something happen[ed]", to put it in terms used by Schoenfeld (1998), or when students' skills and motivation level required an alternative instructional strategy. Otherwise, for the most part and in the normal course of teaching, she made decisions based on her own experience and expertise, beliefs, and the recommendations of the intended curricula. In the following excerpt from the final in-depth interview, Ms. Johnson described in what ways the classroom environment could influence her work.

> For content, if it was a new idea, then I would be more concrete in their using tiles, and if it was a review idea, then I might be more procedural and so that was one thing that I would consider. And my students, their motivation, affects how I would do it, and then their skill level would affect how I'd set it up and considering whether it all had to be learned in class or practice outside of class could help. And then the classroom environment, I think that idea of how to group students so that they're most academic and performing. [FI – Question #7]

Observation of her classes provided one example of how Ms. Johnson changed her intentions and strategies due to challenges that arose from students' lack of motivation to complete their assigned work. During several post observation interviews, Ms. Johnson expressed her frustration with students spending too much time socializing in their groups, discussing off topic subjects, and not completing their *work-time* assignments in class; *work-time* assignments, as Ms. Johnson called them, were assignments for students to complete independently or collaboratively, during class. As the problem worsened, Ms. Johnson required students to turn in their finished work at the end of each class period. This new requirement caused several of the students to rush to finish their classroom assignments in order to have time to socialize with their teammates; these off task activities rarely occurred between groups, however. Noticing the new problem, Ms. Johnson decided to make classroom assignments more challenging. This decision improved the situation, yet, the problem of students socializing excessively in their groups and not having the motivation to work on their classroom assignments steadily, persisted throughout the study.

Ms. Johnson had developed significant collaboration with the school community. This collaboration influenced what Ms. Johnson did in the classroom in direct and indirect ways. First, Ms. Johnson's collaboration with the science teacher, Ms. Buchanan, had produced the "integrated units" which were then added onto the regular mathematics curricula. The *integrated units* required students from the science and mathematics classes to merge and be taught together. The mathematics topics that Ms.

Johnson taught in these units meshed with their applications in science, thereby connecting mathematics and science concepts for the benefit of students.

Ms. Johnson's collaboration with the expert sixth grade mathematics teacher, Ms. Jenkins, and the novice mathematics teacher, Mr. Hubert, developed along a quality control dimension. Ms. Johnson conveyed to Ms. Jenkins the content areas she could emphasize more to better prepare students for the seventh and eighth grades. At the same time, Ms. Johnson took it upon herself to share her approach for giving lessons with the novice teacher, Mr. Hubert. The collaboration between Ms. Johnson and Ms. Jenkins incorporated their efforts to bring the mathematics curricula taught at the school up to the levels that met the requirements set by the state standards.

The impact of the collaboration between principal Roberts and Ms. Johnson on the way Ms. Johnson practiced was more indirect. Ms. Johnson showed generosity with respect to making district funds available to teachers who wanted to attend in-service workshops and various professional development programs throughout the year. As a leader in mathematics education in the state, Ms. Johnson continued to benefit from these funds, attending workshops and conferences nationwide and sharing the information obtained from these various forums with other teachers at the school and even with the district. In this way, Ms. Johnson kept up to date with the latest innovative methods in mathematics education and shared those ideas with her colleagues as well as using them in her own practice. Ms. Johnson expressed her views about collaborating with the school community as follows:

> There is a teacher upstairs, Ms. Jenkins, that we [Ms. Johnson and Mr. Hubert] talk with to make connections to what happens in 6th grade and

> then we do meet with the high school on a regular basis, because trying to make it so that our students' transition to high school with a little better success. So, we are working on that area . . . so, that affects what I do, because they say, they tell me certain things they want their kids to know, and then I have to adjust instruction to make them happy. [II – Question #15]

Key Attributes of Ms. Johnson's Practice

The collected data and the emerging themes were used to piece together several key attributes of Ms. Johnson's practice. These attributes take into consideration interacting aspects of Ms. Johnson's practice as well as a trajectory of her career over a span of nearly 20 years from the date she obtained her license to teach. Table 5 describes these attributes, definitions used to define the attributes, and references to evidence supporting each assertion. The key attributes provide the foundation for a proposed diagram of the teacher's practice discussed in the next chapter.

Table 5

Assertions on Key Attributes of Ms. Johnson's Practice

Assertions about the Teacher Attributes	**Definitions**	**Evidence**
1. Develops strong collaboration with the school community	(Sergiovanni, 1994, p. xvi): School community: a group of individuals who share the same will and are thus bonded by the same goals and ideas	Interview data with the school community indicated that they had: Regular meetings with the teacher to discuss collaborative teaching projects (the science teacher), share ideas and lessons (the mathematics teachers), her cooperation to "wear many hats." (the principal) P. 67 – 70 ; 91 - 92

Table 5 (continued).

Assertions about the Teacher Attributes	Definitions	Evidence
2. Sustained and career-long participation in reform-based professional development programs	Career-long participation in teacher development programs while an in-service teacher	Interview data with the teacher indicated that: She began participating in reform-based professional development (PD) programs since obtaining her license in 1989. She continued with a five-year participation in the Silicon Valley Math Project, and various PD programs nationally and whenever available throughout the years. P. 38, 121
3. Sustained enthusiasm about the profession, students, new ways of teaching mathematics, and the process of schooling	Participates in the life of the school as a whole: Collaborates with and helps other teachers; answers the principal's call and volunteers to serve on various committees; arrives at the school early, meeting with students to help them with mathematics; fixes other teachers' computers; teaches courses in physical education; continues her studies toward a doctorate	Interview data with the teacher and the school community and observations indicated that: the teacher participated in the life of the school as a whole. She fixed the school computers during lunch hours when the school was looking to hire someone for the job; she taught physical education courses; served on various school committees; studied scholarly journals and obtained lessons from them; pursued her doctorate in educational leadership P. 67 – 70 ; 91 – 92; 120 - 121
4. Able to practice teaching mathematics in line with her beliefs, and intentions about effective ways of teaching mathematics	Expression of her stated beliefs and intentions to follow the Connected Mathematics Curriculum for the purpose of teaching conceptual and procedural mathematics	Interview and observation data indicated that: The teacher's beliefs aligned with her intentions and actions in the classrooms (e.g. her belief about students thriving on social interaction and that learning from one another acted as a motivating factor for students, and her practice of having small group activities as an indispensable part of her practice) p.75 – 93 Parsed lessons (Appendices I – O)

Table 5 (continued).

Assertions about the Teacher Attributes	Definitions	Evidence
5. Uses active teaching methods	Brophy & Good (1986): Includes both small and whole group work/discussions; 2)Whole class instruction models the type of tasks and thinking students are expected to do in small groups; 3) The teacher monitors and supervises student work during whole class and small group activities	Field notes indicated that: The teacher held whole class discussions and small group activities every day; her demonstration of tasks and discourse during whole class discussions exemplified for students her expectations for students' small group activities; she monitored and supervised classroom activities vigorously. Parsed lessons (Appendices I – O)
6. Continuously assesses students' knowledge and understanding	Assessment as "coaching activity": On most occasions, the teacher assessed students' understanding through classroom discussions and provided immediate feedback to students – similar to a coach that catches his/her players' mistakes during a game or competition and corrects them as they play	Field observations and interviews indicated that: The teacher did not suffice with written tests and quizzes to assess students' knowledge. Most often she used conversations with students to assess what they knew and their grasp of the new material "I'm going to try to talk to each group to see how they are thinking and how they are developing their rules" [* PRI – October 16th, Algebra One, Question #3]

* PRI = Pre Observation Interview

Table 5 (continued).

Assertions about the Teacher Attributes	Definitions	Evidence
7. Uses process oriented teaching methods to teach both the concepts and mathematical procedures	Yang (2002), and Brophy & Good (1986): 1) incorporates mathematics questioning extensively; 2) accepts and clarifies student ideas; 3) encourages student participation	Classroom audios and field observations indicated that: The teacher's ultimate goal was for students to ask questions about gaps in their understanding, however, often when students failed to ask questions, she supplemented her own questions during classroom activities. Most importantly, she used questions to have students participate and share their ideas with others. She expressed: "I saw a lot of kids that did a lot of good thinking and yet they aren't sharing that. So, my experience would be what can I do to get . . . so not the same five students are raising their hands." (p. 88). P. 75, 77, 80
8. Promotes both conceptual and procedural teaching of mathematics	Kilpatrik et al. (2001): Conceptual teaching of mathematics: Promotes comprehension of mathematical concepts, operations, and relations; makes connections within and among knowledge of mathematics, students, and pedagogy The teacher's own definition: Find out what students know and what ideas they have and connect to that when teaching new material (** FN – p. 193)	Interview data indicated that: The teacher believed "Students have various learning modality. Some are visual learners, some learn by listening, and some by doing and constructing." Her pedagogy emphasized comprehension of ideas by students with various learning skills, making connections between numerical-visual-and-abstract ideas in mathematics; used manipulatives and connected it to abstract equations; Connected one mathematical strategy to another (e.g. guess-and-check to symbolic manipulation) and connects ideas (tabular-graphical-geometrical-symbolic) and emphasized operations. "Making sure that all kids make that connection between the numbers in the pattern and the figure numbers so they're really seeing the growth." [PRI – October 15th, Question #3]

** FN = Field Notes

CHAPTER V

Discussion and Implications

The purpose of this study was to investigate the intended instructional objectives and practices of an expert middle school mathematics teacher where a reform-based curriculum was adopted for the mathematics classes, and where the overall classroom and school communities may have played a role in the kind of teaching that took place in the classrooms. One expert mathematics teacher and a leader in mathematics education in the state, Ms. Johnson, was observed over a span of seven weeks in all the mathematics classes she taught: Algebra One, Math Seven, and Math Eight. Ms. Johnson chose the mathematics curricula from which she taught; the *Connected Mathematics Curriculum* for Algebra One entitled *Algebra Connections, for Math Seven entitled: Foundations to Algebra – Year 1, and for Math Eight entitled: Foundations to Algebra – Year 2.* The teacher was selected for this study based on her experience and educational background, her reputation as a leader in mathematics education in the state, her familiarity with reform-based curricula, her teaching during the selection process, and her success in the classroom. A detailed case study was prepared for this teacher that contained information about her intentions, teaching practices, and the teaching contexts within the classroom and the broader school environment in order to depict a descriptive profile of effective teaching within reform-based curricula. The primary sources of data used to create this case study included initial and final in-depth interviews, post-unit interviews, pre and post observation interviews, observation field notes, detailed analyses of lesson segments that considered typical lessons for each course category Ms. Johnson taught, classroom audios of whole class discussions, and impromptu interviews about the teacher's practice.

Chapter IV presented the results in response to the two research questions about the teacher's intentions, classroom practices, and the impact of the school context on her work. The two research questions were:

1. What are the intended instructional objectives and practices of an expert secondary-level mathematics teacher for instructing mathematics in a reform-based curriculum program?
2. What aspects of the classroom community and the school community impact the intentions and instructional practices of an expert secondary-level mathematics teacher in a reform-based mathematics curriculum program?

This chapter contains an analytical discussion of the observations about the teacher's intentions, classroom practices, and the impact of the classroom and school communities on her work. The first item to be discussed is Ms. Johnson's intentions and the role they played in shaping her practice. This section is then followed by different aspects of the teacher's practice, including her beliefs, subject matter knowledge, and pedagogical content knowledge. A review of the analyses of the impacts of the classroom and school community on the teacher's practice ensues. After addressing the conclusions about the teacher's practice, implications of this study with respect to designing professional development programs and secondary mathematics teacher preparation are discussed. In closing, a discussion of the strengths and limitations of the study as well as recommendations for further research are presented.

Intentions of Ms. Johnson

Schoenfeld's (1998) model of teaching-in-context defined intentions or goals as what teachers *want to accomplish*, and they can be established by what teachers say. In this project, various interviews were used to establish the teacher's intentions. Pre and post observation interviews, for example, in combination with observations of her actions, established Ms. Johnson's intentions for each lesson.

Ms. Johnson often expressed and demonstrated that her intention was to find out what students already knew. She believed that only when she knew what students knew she could begin to build on their knowledge. Knowing what students knew as she taught meant that she could not suffice with occasional paper-pencil tests where time gaps between tests kept her guessing about how much students knew. Her solution to the problem of knowing what students knew was to dedicate the majority of her time and energy in her classes to speaking with them. In an overwhelming majority of the cases, these discussions with students helped Ms. Johnson determine how much of the topic discussed for that day was absorbed by her students, and the discussions also helped increase student on-task behavior. Nevertheless, for the most part, students were not eager to discuss their mathematical thoughts with the teacher. And this reluctance made it difficult for Ms. Johnson to determine exactly how much of the topics discussed for the day was absorbed.

Despite students' lack of enthusiasm to engage in mathematical discussions, the dialogic aspect of Ms. Johnson's approach to teaching mathematics and its continuity during whole class and small group discussions (she was as active during small group activities as she was during whole class activities) characterized her approach in every mathematics class she taught. This energetic approach to finding what students knew was counterbalanced by students' indifference and limited cooperation with the process; after all, blank looks to the desk or stares into the empty space when faced with a question by Ms. Johnson did not reveal much about students' understanding.

Ms. Johnson stated that she followed the recommendations of the reform-based *Connected Mathematics Curriculum* she used, and given the fact that it was she who had chosen the reform-based curricula for all the mathematics classes she taught, it became clear that another one of her intentions was to create a reformed mathematics education environment in her classrooms, one that was accepting and responsive towards the latest findings on effective teaching and learning of mathematics (see Desimone et al., 2005; Larson, 2002; NCTM, 2007).

Interviews with Ms. Johnson and observations of her practice indicated that she intended for her students to gain both conceptual and procedural understanding of mathematics. To foster conceptual understanding (Kilpatrick et al., 2001), she made the point of connecting every mathematical procedure she discussed with at least one visual representation of the procedure. Research has shown the importance of visual representations on improving cognitive understanding (Dreyfus, 1995), and using students' informal notions of partitioning and measuring on conceptual understanding (Kilpatrick et al., 2001). For example, manipulatives were one form of visual representation she used in her classes extensively (see Figures 6, 7, 8, and 9 in Chapter IV). In the Algebra One class, graphs were also commonly used. When students displayed difficulty expressing mathematical procedures using visual representations, she asked them to simply use words to explain their thinking. Every student was required to connect stepwise mathematical procedure with at least in one other form of knowing and reasoning. In other examples commonly observed in her teaching of the Algebra One and Math Eight classes, she connected the guess-and-check problem solving strategy to symbolic manipulation of constants and variables in an equation, demonstrating to students the advantage of symbolic representation as a more efficient and accurate strategy. In the same way, she rehearsed teaching of concepts (e.g. slope, geometrical area, absolute value) and procedures (e.g. algebraic operations between integers and fractions and expressions) to students.

Ms. Johnson *wanted* to make mathematics learning engaging and fun for her students. This intention led to her commitment to implementation of two strategies. First, she believed that students thrived on social interaction, thereby making small group activity a permanent aspect of her pedagogy (see Appendices I – O). Throughout the study, Ms. Johnson was observed and heard asking her students to talk to each other and share ideas when working in their groups. Brophy and Good (1986) considered small group activity an essential aspect of active teaching strategies, and Driver et al. (1994, p. 9) recognized how discussions among students can introduce them to a community of knowledge. On average, two-thirds of the daily activities in each class were conducted in small groups, where two to four students formed a group. Ms. Johnson's intention for students to share their mathematical ideas with each other in this setting was rarely materialized as often times students were observed either working independently in these groups or socializing and wasting their time. Overly social activity was rampant among groups. Ms. Johnson curbed this undesirable byproduct of small group activity to some extent by visiting each group multiple times, bringing curricular focus to what students did through the use of questions and the demand that students finish and turn in small group work at the end of each class period. A combination of having students work on more challenging problems and demanding that they turn in completed work each day increased the time students spent on task. On task behavior, then, was tantamount to students sharing ideas, exploring alternative solutions, and checking the accuracy of their solutions. Activities such as group poster presentations were meant to improve student accountability, although gaining full control on student accountability remained a challenge for Ms. Johnson until the end of the observations.

A second method Ms. Johnson used to make mathematical learning engaging and interesting for her students was by using various mathematical games and puzzles. Researchers and scholars (Olson, 2007; Sriraman & Lesh, 2007; Tzur & Clark, 2006) have also found the use of games a helpful method to create situations where pupils are asked to *make connections* between their knowledge of various mathematical concepts and strategies. The games Ms. Johnson used (see Appendix P for examples) required participation by all students in the classroom. These games often required students to get up from their seats, make their computational contribution, and then go back and have a seat until their turn again. Teammates often cheered each other on during games and gave each other high-fives when a member completed a calculation correctly and won his or her team points. In a casual conversation with the researcher, Ms. Johnson expressed her view that students at this age possessed a great amount of play energy and it was unrealistic to expect to gain their attention during lessons without ample activities that allowed that energy to release itself while they learned mathematics. Nearly a dozen mathematical games and puzzles were observed during the course of this study in all the mathematics classes Ms. Johnson taught, and in other times, students were kept engaged with their mathematical tasks by working with manipulatives and or preparing poster presentations.

Although Ms. Johnson was able to plan and implement lessons according to her intentions for each course she taught, her intentions for students' learning outcomes often did not materialize immediately or upon giving a lesson. This disparity between achieving immediate results after a lesson and her intentions for each lesson was evident in the responses Ms. Johnson provided to the post observation interview questions. Ms. Johnson's efforts to make her lessons more successful and efficient with respect to achieving intended learning outcomes varied from course to course and depended on the type of students she had in each class. According to her, the Algebra One students were motivated and ready for the material taught in the *high school level* class. And when she observed any lag in students' understanding, she was confident that with time and more exposure to and exercise with the curricular material the students could meet her targets for their understanding of the material. In the Math Seven and Math Eight classes, however, the main obstacle against students' grasp of the material was overly social behavior in class and lack of motivation to work on assignments outside of the class. At these two levels of mathematics classes, Ms. Johnson expressed disenchantment with students failing to maintain the pace. The issue remained unresolved for the duration of this study.

Outside of the classroom environment, Ms. Johnson's intention was to collaborate with her colleagues, exchanging ideas with them, and working with them in various professional and personal capacities. She was a major driving force behind creating a work climate at the school in which she, the principal, and all the other science and mathematics teachers could ask for help from each other without the fear of being rejected. On two different occasions, students of Ms. Buchanan, the science teacher, and Mr. Hubert, the novice mathematics teacher, were brought and mixed with Ms. Johnson's students as she taught. These teachers had recognized their own weaknesses teaching or incorporating some topics in mathematics and sought help from Ms. Johnson. Ultimately, this collaboration among these teachers benefited students.

Moreover, Ms. Johnson's collaboration with the school community created a purposeful quality control mechanism among the mathematics teachers at the school that saw as its priority, the creation of a coherent mathematics curriculum and instruction program, one where there was consistency in the strategies the teachers used to teach students, the language they used in the classrooms, and the curricula from which they practiced.

Ms. Johnson's Practice

Ms. Johnson's instructional practices, as a whole, emphasized a multidimensional approach to teaching and learning mathematics; that is, she rarely lectured, but used mini-lectures from time to time to reiterate significant curricular concepts that were missed by a majority of her students (e.g. distributing a negative sign to the terms inside a parentheses); she taught mathematical procedures, but only as one of several solution strategies presented to and expected from students (knowledge of manipulatives, connections between geometrical properties and algebra, graphs, and even explaining and reasoning in words were other ways of knowing she promoted in connection with each procedural strategy). She was responsible for choosing the *Connected Mathematics* curriculum in her classes and followed the recommendations in the curriculum with conviction and without only selecting certain portions of the curriculum. The official curriculum was not Ms. Johnson's only source for teaching material. She had created volumes of binders, stacked with lessons she found helpful over the years when teaching students about a variety of topics.

The evidence collected about Ms. Johnson's practice is discussed herein mainly from the perspective of Schoenfeld's (1998) teaching-in-context framework. Aspects of Ms. Johnson's practice are also juxtaposed and compared with respect to the recommendations of the curriculum and professional standards (NCTM, 2000, 2007).

Ms. Johnson's Practice from the Perspective of Schoenfeld (1998)

Central to Schoenfeld's framework were the teacher's beliefs, intentions, and knowledge. These components were set to work in sync with each other when everything went according to the teacher's plans for instruction. In this sense, instructional plans were considered the teacher's *lesson image*; that is, generally speaking, the teacher's overall vision for how any given lesson was to unfold and turn out. For Ms. Johnson the *lesson image* was comprised of continuous assessment of students' moment-to-moment knowledge about the content, students' continuous engagement with relevant mathematical tasks and discourse, and on-task collaboration between students. She primarily used questions to implement the first two intentions, where the questions and directives for students to talk and share their ideas with each other were incorporated to implement the last intention. Ms. Johnson's overarching intention was for students to gain both conceptual (knowledge comprised of various connected concepts) and procedural fluency in the mathematics she taught, with the procedural fluency stemming from the conceptual one.

Schoenfeld (1998) recognized that when everything proceeded according to instructional plans, the teacher's high priority intentions were in alignment with the teacher's high priority beliefs, and the actions the teacher took were consistent with those beliefs and intentions. Moreover, the actions undertaken by the teacher drew upon the subject matter knowledge and pedagogical content knowledge that defined the teacher's expertise until something happened. Schoenfeld posed the question that when "something happens," and the teacher's vision for a lesson, or the *lesson image*, is disrupted with something unexpected, what would the teacher do? Schoenfeld (1998, p. 3) proposed that in the event that "something happens," the combination of the teacher's beliefs, intentions, and knowledge worked together to create a new high priority intention and subsequent relevant actions. One example was a lesson in the fourth period Math Eight class, where Ms. Johnson's intention was for students to see multiple representations of multiplying fractions [PRI – October 16th, Math Eight]. Ms. Johnson's *lesson image* was for students to multiply fractions in a visual model, and then have them to multiply fractions in a game setting, focusing mainly on multiplying fractions but in different settings. An unexpected event occurred when one of the students, Asia, protested, asking why they were redoing a lesson they already knew. Asia's statement was reaffirmed by other students. At this moment, Ms. Johnson was challenged whether to eliminate the entire lesson and teach something new. At the same time, she knew that not all students in class knew how to multiply fractions very well, and besides, she was uncertain whether the students were simply overconfident about their knowledge of the material. In this case, Ms. Johnson's response was a *delayed action.* This decision led students who had expressed confidence in knowing the material to disengage from the rest of the lesson, distracting other students for the rest of the class period. Although, Ms. Johnson continued with her lesson for the day, the next day she incorporated a diagnostic test to determine which students adequately understood algebraic operations involving fractions.

Ms. Johnson's decision based on her knowledge and beliefs resulted in the creation of a new high priority goal: to recognize the students who knew the material well and provide them with a new and relevant activity they could engage themselves with. She pursued this goal using a *routine* diagnostic test to determine qualified students, and next she used the results of the test to have the students work on a non-routine activity to generate a novel experience for the students who craved it; that is, solving mathematics problems on computers. Ms. Johnson's decision was significant because it revealed that she was flexible in working with students and did not ignore their feelings and emotions as they sat in the classroom; students' feelings and thoughts mattered to her. Also, as expressed earlier in the study, Ms. Johnson indicated that she wanted students to take charge of their own learning and that was what they were doing in this case.

Although a follower of the mathematics content and process standards (NCTM, 2000), the experience highlighted a strategy missing from Ms. Johnson's instructional repertoire - the use of computing and calculator technologies. The standards indicated that "Initially, the teacher must decide if, when, and how technology will be used" (NCTM, 2000, p. 26), and superimposing a portion of her intentions with the intentions of the reform-based mathematics curricula, Ms. Johnson avoided the use of technology whenever the curricula did not recommend it. Perhaps other more technology-focused, reform-based curricula might engage Ms. Johnson in the integration of technology in her mathematics classrooms.

A second *event* that qualified under Schoenfeld's perspective of when "something happens," occurred when, despite Ms. Johnson's expectations, students did not ask for clarification on an important Algebra One topic. According to Ms. Johnson's review of students' homework, the majority of students were having difficulty with the concept of distributing a minus sign placed in front of a parenthesis to the terms inside the parenthesis. In this case, the *event* of interest was students not asking the crucial question they needed to ask as Ms. Johnson provided the answers to the previous day's homework problems. The *event* created a high priority goal: to reteach a procedure rather than focusing on the concept. She taught a mini-lecture, not her preferred instructional strategy. This experience demonstrated that when Ms. Johnson taught new topics, her knowledge, beliefs, and intentions were seamlessly connected, though, when she had to reteach an old topic, one that was taught previously, her intention and actions shifted to a more procedural approach.

When Ms. Johnson's plans proceeded with no unexpected events, the question was whether or not Ms. Johnson's beliefs and intentions were consistent with the actions she took and the decisions she made. She believed that students learned in different modalities, with majority of them being visual learners; her practice emphasized multiple representations of mathematical ideas in the numeric, graphical or visual, and symbolic forms. She believed that students thrived on social interaction; and small group activities where students worked together on problem solving strategies were permanent feature of her classes. She believed pattern recognition and making connections between mathematical concepts were as much part of mathematics as numerical fluency and mastery of procedures; and in following the reform-based curricula from which she taught she used ample pattern recognition and connection making exercises. She believed that an important part of her job was to get to know her students so that she could create for them an environment that was conducive to their learning of mathematics; and, she incorporated games, puzzles, and manipulatives to create a relaxed and cooperative learning environment.

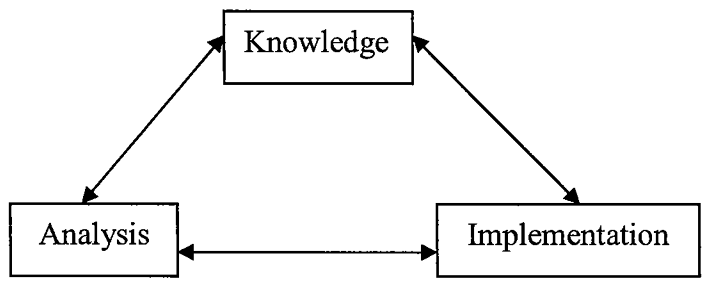
Ms. Johnson's statements indicated that she maintained two overarching intentions for all the mathematics courses she taught. Her first overarching intention was to create a coherent, reformed mathematics education environment at the school (NCTM, 2000, p. 14), and the second was to instill in students that mathematics was not just a science of numbers but a science of modeling patterns, connecting ideas, and a reasoning tool. The ramifications of these two overarching intentions were present in every *lesson image*, only interrupted by rare and unforeseen events such as those discussed earlier. Both intentions were activated at high priority levels from the outset and formed the background against which Ms. Johnson followed her more immediate intentions. The first of these overarching intentions was made possible with the close collaboration and trust that existed between Ms. Johnson and the school principal. The accomplishment of the second overarching intention was achieved through Ms. Johnson's adherence to the instructional recommendations in the curricula. In all, Ms. Johnson was a principled and knowledgeable teacher that was loyal to working with her colleagues and loyal to the curricula she practiced.

More immediate intentions held by Ms. Johnson were pursued using various *routines* and *scripts* (Schoenfeld, 1998, 2000; also see Chapter III). For example, Ms. Johnson's intention to learn about students' existing knowledge and building on that knowledge was pursued using three assessment routines. The first routine involved a homework correction routine she followed at the beginning of most classes (see parsed lessons in Appendices I – O). The second routine Ms. Johnson used to assess students' existing knowledge was by speaking with them. This routine took place throughout whole class and small group discussions and it also aimed at keeping students on task (see Appendices I – O). These two assessment routines were part of Ms. Johnson's every *action plan* (Schoenfeld, 1998, 2000; also see Chapter III). The third assessment routine was comprised of written tests after the end of each unit, or tests conducted at any other time during the instruction on a unit; the latter type were *diagnostic* tests that usually lasted between 10 to 15 minutes, as opposed to post unit tests that lasted the full class period. The written tests emphasized solving problems in more than one way; that is, they were reflective of solution strategies practiced by students during classroom exercises. Students were expected to show the procedures and juxtapose them with drawings or graphs or written explanations of the procedural solution. The purpose was Ms. Johnson's oft-stated intention for students to show that they were making connections between various mathematical ideas and gaining conceptual understanding (Kilpatrick et al., 2001; NCTM, 2000).

Another high priority intention of Ms. Johnson was to make learning mathematics engaging for her students. Ms. Johnson used various *scripts* where students built algebra tile structures based on what was provided to them in the form of symbolic representations. Or students were asked to go in the opposite direction and provided with tile structures and were asked to decipher them into symbolic mathematical representations. Mathematical games and puzzles provided another version of the type of *scripts* that engaged students in highly interactive learning tasks. These games and puzzles required the participation of all students and were always collaborative in nature. Students used their knowledge of mathematics they had gained through curricular exercises to compete with one another in groups.

The discussion so far explained Ms. Johnson's global and immediate classroom intentions and related them to examples of her practice (also see Appendices I – O) that described what she did to pursue the intentions. Another aspect of Ms. Johnson's practice was her knowledge base. Schoenfeld (1998) recounted the works of Shulman (1986) and Borko and Putnam (1996) in his concept of teachers' "The Knowledge Inventory." It was to be the teacher's knowledge about the subject matter, facts, terms, and concepts, as well as her knowledge of the organizing mathematical ideas, connection among mathematical ideas, ways of reasoning, classroom management, learning, teaching, instructional strategies, and creating a learning environment. Information to complement the notion of *Knowledge Inventory* in terms of what teachers do to build knowledge was also required. The underlying description of teachers' knowledge in the *Cycle of Teaching Activity* (NCTM, 2007), and shown in Figure 16, echoes many of the components of teachers' knowledge accounted for by the scholars, while asserting that: "Knowledge is developed in preparation for teaching, as a result of teaching experiences, and through career-long professional development" (NCTM, 2007, p. 8).

Figure 16. The cycle of teaching activity



This aspect of the cycle of teaching activity characterized the trajectory of knowledge development for Ms. Johnson (see Chapter IV). Her first teaching experience began when co-teaching second grade mathematics with an expert primary school mathematics teacher, and was supplemented by attending reform-based professional development programs. Collaboration with her colleagues at the school and attendance to reform-based professional development programs were part of the career life of Ms. Johnson at the time of this study, nearly 20 years after her first teaching experiences took place. The physical environment of the nature of school classrooms gives the impression that teachers work autonomously. This study has shown the opposite. Indeed, with the mutual trust and collaboration between Ms. Johnson, the other teachers in the school and feeder schools and the principal in this school community supported her in pursing her intentions. Ms. Johnson's deep seated and broadly based collaboration were evidence of her commitment to "Reflection on Teaching Practice" (NCTM, 2007, p. 60).

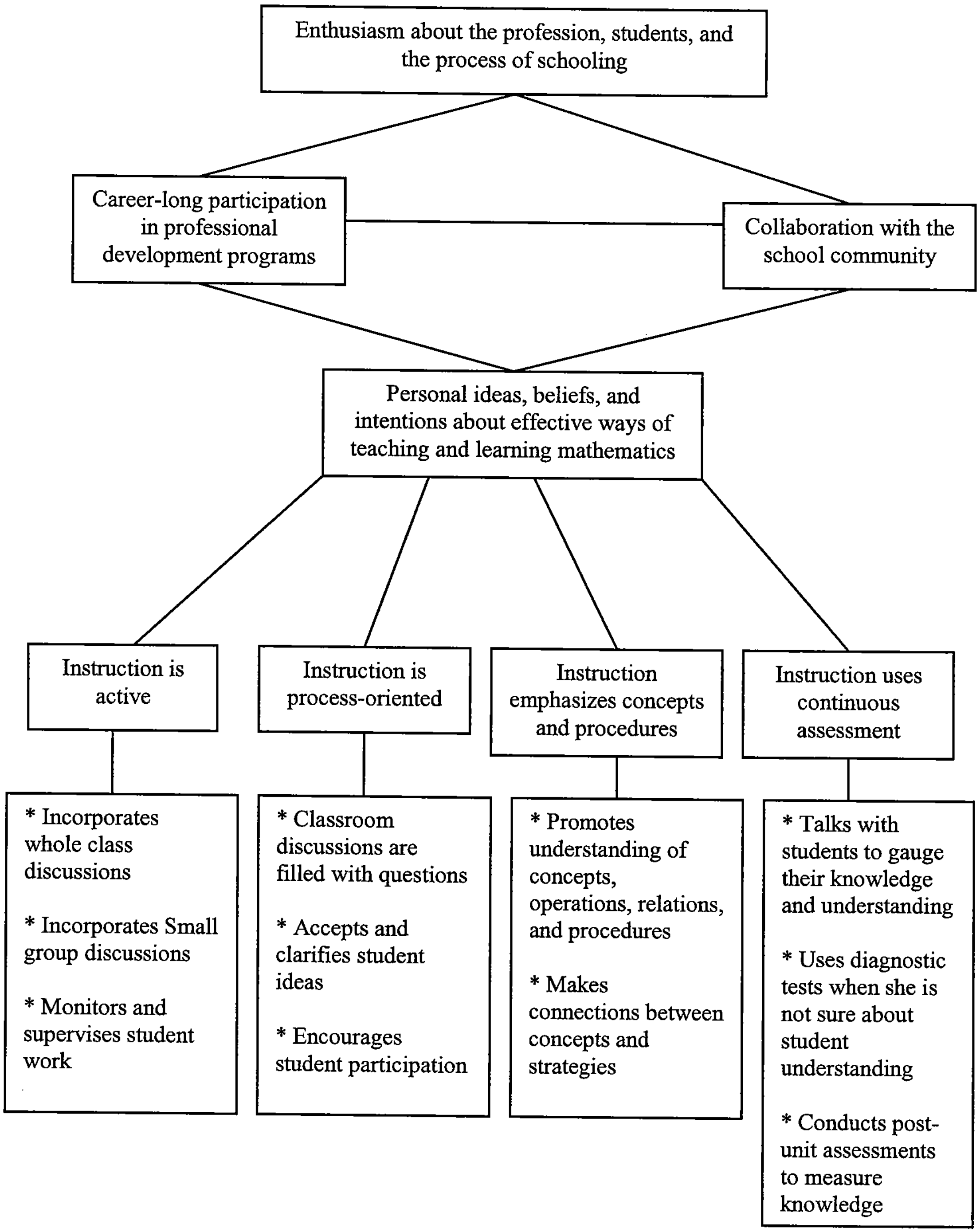
Diagrammatic Depiction of Ms. Johnson's Practice as an Expert Secondary Mathematics Teacher

Research has highlighted the importance of subject matter knowledge in the practice of expert teachers (Borko & Putnam, 1996; Hill & Ball, 2005). Teachers with rich subject matter knowledge are not only more effective in improving their students' achievement but also they are more likely to partake in sustained professional development activity (Desimone et al., 2006).

Observation of Ms. Johnson's practice and consideration of her ideas expressed during the interviews provided a wealth of information for describing her intended objectives and practice as an expert teacher. This information revealed that Ms. Johnson's teaching was charged with enthusiasm and drive to improve her teaching from the beginning of her teaching career, extending to the present. Her strong and continuous enthusiasm and drive to improve as a teacher paved the way for an evolutionary trajectory of her thoughts and practice that began immediately after her successful obtainment of a license to teach. The forces that propelled and shaped this evolution, along with its dominant attributes, were combined to form a diagram of this expert teacher's practice depicted in Figure 17.

The impact of Ms. Johnson's enthusiasm and affection about her work, students, and the teaching and learning process as a whole, on her practice and the way she chose to connect with students and the school community were both evidenced in this study and immeasurable in scope. Her belief in the changing of mathematics teaching to be a necessary ingredient for improving students' learning and achievement in mathematics,

Figure 17. A diagram of Ms. Johnson's practice as an expert secondary mathematics teacher working with reform-based mathematics curricula



her early and continued participation in professional development programs standing as a significant sign of her willingness to change, and her enthusiasm about the subject matter and finding new ways to learn and teach it, were powerful forces that shaped Ms. Johnson's practice throughout her career. In all, these attributes are proposed as key supportive attributes of the practice of an expert mathematics teacher in reform-based curricula.

Researchers have obtained conflicting results about learning outcomes related to reform-based mathematics teaching as opposed to practices that emphasized teacher-centered teaching. McCaffrey et al. (2001), for example, found that when teachers used reform-based instructional strategies to teach reform-based curricula, the improvement in student achievement was marginal. Rosenshine and Stevens (1986, p. 379) found evidence and proposed a "General Model of Effective Teaching" mathematics that included a regiment of review of the old material, guided practice of the new, feedback, and independent practice of procedures, can lead to higher levels of student achievement, depending on *how* each of the lesson components are completed.

The diagrammatic depiction of Ms. Johnson's practice is different from the model proposed by Rosenshine and Stevens (1986) in fundamental ways. For example, a critical aspect of the diagram is its dynamic nature in terms of the teacher reflectiveness about her work and its clear emphasis on what the teacher does throughout her career, both inside and outside the classroom in order to decide and prepare instructional strategies that best suit the students and the content being taught. According to NCTM (2007, p. 60), participating in learning communities (e.g. professional development) and collaboration with colleagues are signs that the teacher is reflective about her practice.

Ms. Johnson's practice was dynamic and superseded a singular and year-to-year strategy of having students *practice* review and new problems in great volume. Also, neither assessing students' prior knowledge nor collaborative nature of classroom discussions and activities were addressed in the model proposed by Rosenshine and Stevens. The model proposed by Rosenshine and Stevens represented a one dimensional perspective about mathematics teaching and learning that reduced it to what was shown on the blackboard and in the thickness of students' notebook for homework.

A discussion of the components of the diagram that encapsulates Ms. Johnson's practice over her entire career is presented next.

Ms. Johnson's Enthusiasm

An interview with Ms. Johnson about her career and involvement with reform-based professional development programs elicited a response that said "Well, I'm going to go all the way back to 1989 . . ." [IMI – Ms. Johnson's career path, Question #1, p. 61 of the field notes]. The rest of her response depicted a trajectory of dedication and commitment to the work of teaching and improving her practice that extended to the school community, both within the school and within the school district. Such indicators as these helped to define Ms. Johnson as an enthusiastic mathematics teacher.

Career-long Participation in Professional Development

Ms. Johnson began participating in reform-based professional development programs soon after obtaining her teaching license and starting her career as a mathematics teacher. She later went on to participate in the Silicon Valley Math Project for five years; a program that brought local university faculty and interested school mathematics teachers together for the purpose of fostering the best ways to teach mathematics. After moving to Oxford Middle School Ms. Johnson continued her participation in professional development programs, while sharing her findings with other interested teachers at the school and the district.

Collaboration with the School Community

Ms. Johnson collaborated with the principal and all the science and mathematics teachers at the school as well as with some of the mathematics teachers at a nearby high school, developing a practice and vision that incorporated knowledge and feedback from other professionals. The *integrated units* were a byproduct of her collaboration with the science teacher, adding a problem solving component to Ms. Johnson's practice. Her collaboration with the mathematics teachers at the school was part of an internal quality control system set up by the three teachers to improve mathematics education at the school across grades. Ms. Johnson's collaboration with the school principal allowed her to collect outside knowledge about effective ways of practicing mathematics teaching and learning and incorporate them in her own work as well as sharing its principles with other teachers at the school.

Personal Ideas, Beliefs, and Intentions

Ms. Johnson's enthusiasm was the force that drove her commitment to participating in professional development programs throughout her career, and the motivation behind her collaboration with the school community wherever she worked. These experiences shaped Ms. Johnson's ideas, beliefs, and intentions when teaching within reform-based mathematics curricula. A good measure of Ms. Johnson's ideas, beliefs, and intentions about the reform of mathematics education in schools was her choice of the *Connected Mathematics* curriculum to which she adhered closely.

Observation of her practice revealed that her practice was active and process-oriented, emphasized conceptual understanding and procedural fluency of the curricular content, and was driven by continuous assessment of students' existing knowledge.

Active Teaching Strategies

Ms. Johnson's teaching was active (Brophy & Good, 1986). She played the role of a supervisor and a facilitator as students worked on tasks during whole class and small group activities. Ms. Johnson began a new lesson by developing a concept during whole class discussions and demonstrations, preparing students for their work in small groups. She monitored students' progress working in groups with agility and frequently during each class session, providing constructive feedback that emphasized conceptual understanding, procedural proficiency and sharing of ideas. She conducted additional review exercises with respect to concepts missed by the majority of students, such as the distributive property of the "minus sign." After a brief 10 to 15 minute long presentation and demonstration of new material, she regularly provided ample opportunities for application and practice of mathematical procedures in small groups. Ms. Johnson rarely lectured, and spent the majority of class time in a two-way conversation with her students, creating an environment where students were asked to interpret the information at their disposal (Borko & Putnam, 1996).

Process-Oriented Approach to Teaching

In their review of studies that focused on process-oriented research, Brophy and Good (1986) examined studies that compared the use of *direct instructional strategies* (product oriented) such as lecturing, giving directions, criticizing, and teacher giving justifications, with those that made use of *indirect instructional strategies* (process-oriented) such as asking questions, accepting and clarifying ideas or feelings, and praising and encouraging. They concluded that higher levels of indirect instruction may be more appropriate for tasks that involved abstract reasoning activity. The data collected in this study revealed that Ms. Johnson primarily used indirect instructionals methods to engage students in the ideas; only rarely did she lecture and then that lecture was for a brief period. Thus, Ms. Johnson's practice was more process-oriented. Yang (2002) described mathematics questioning, often used by Ms. Johnson, as one of key strategies middle school mathematics teachers could use to create a process-oriented learning environment.

Conceptual Understanding and Procedural Proficiency

Ms. Johnson's practice emphasized both conceptual understanding and procedural fluency of the curricular content (see Chapter IV). She wanted procedural fluency to stem from students' conceptual mastery of the content materials. Nevertheless, this intention was hampered by students exhibiting various learning skills. For some students, conceptual understanding presented a formidable challenge, and yet, for others, procedural fluency proved to be difficult to grasp. In dealing with these learning difficulties and in the face of complying with the recommendations of the *Connected Mathematics* curriculum, Ms. Johnson emphasized both conceptual understanding and procedural proficiency during whole class discussions, and emphasized them individually when working with small groups and according to each student's needs. Her re-teaching of the previously-taught content however, primarily focused on procedural proficiency rather than conceptual understanding.

Continuous Assessment Approach

Ms. Johnson continuously assessed students' prior knowledge and developing knowledge. Bransford et al. (2004), recognized the importance of this contemporary view of learning where new knowledge and understanding needs to be based on students' existing knowledge. "There is a good deal of evidence," Bransford et al. stated "that learning is enhanced when teachers pay attention to the knowledge and beliefs that learners bring to a learning task" (p. 11). Using questions to constantly assess students' understanding of the content helped Ms. Johnson determine the best pace for her instruction as to when to teach the new material and when to assign and work on additional review exercises. Brief 10 – 15 minute long diagnostic tests and full class period post-unit tests provided additional formative assessments that gave Ms. Johnson further feedback on students' existing knowledge and helped her select the most effective instructional strategies from one day to the next.

Limitations of the Study

The conclusions of this study can only be attributed to the case. However, this qualitative case study methodology was purposefully chosen for this project. Auerbach and Silverstein (2003) state the hypothesis or model generating function as the primary purpose of a qualitative study. Generating a hypothesis of the intentions and practices of the expert teacher chosen for this study was the main purpose of this study. The case study approach allowed a deep exploration of the teacher's work that was necessary for generating a proposed hypothesis that could be investigated further toward the goal of testing the hypothesis and further identifying typical attributes of the general class of expert mathematics teachers that practice in a reform-based curriculum.

Another limitation of this study reflects the circumstances under which the data were collected. Pre and post observation interviews provided invaluable insight into the teacher's intentions and thinking. Often times in the beginning of the study, the interviews were collected under hurried conditions as the teacher was in the middle of tending to school obligations. This situation prompted the teacher to opt for shorter responses. As a result, the researcher conducted the interviews in chunks during early hours before the beginning of classes and again during lunch. This strategy allowed for accumulation of more comprehensive data under more relaxed conditions for the teacher. The impact of this particular process of collecting the pre and post observation interview data in chunks on the final conclusions of this study is unclear. Yet, the significance of this impact was lessened as the data were collected over a span of seven weeks, providing sufficient time for adequately triangulating the data from which conclusions about the teachers' practice were drawn.

The researcher was the sole interpreter of the collected data. The researcher shared his conclusions and developing perceptions about the teacher's work with the teacher, aiming to minimize the impact of researcher's bias on the final results of this study. This sharing helped the researcher reduce bias in describing the teacher's practice.

Implications

A primary implication of this study is the development of a framework for the development of programs to support mathematics teachers in successfully incorporating reformed-based curricula. The professional development program might guide the teachers in incorporating more active teaching, process-oriented strategies, helping them to incorporate teacher-directed interactive mathematical discourse in their classrooms. Also, the results of this study suggest that such professional development programs focus on the integrating of these strategies along with the more traditional strategies that teachers use in teaching mathematics rather than attempting to replace the teachers' current strategies as a result of one professional development program. This professional development toward more extensive use of the reform-based instructional strategies must be envisioned as a long-term goal, rather than a quick fix for change.

A second implication of this research project was connected with technology use in school mathematics classrooms. During this study the computing and calculator technology played small to no roles in the teaching practice of Ms. Johnson. On the other hand, as an established example of mathematics reform documents, the *Principles and Standards for School Mathematics* (NCTM, 2000) clearly emphasized the *Technology Principle* as an important element in the drive toward the reform of mathematics teaching. What percentage of the expert teachers refrain from using technology in the classroom was unclear. This study provides motivation for an investigation of various uses of technologies in mathematics classrooms that do rely on reform-based curricula. Perhaps more professional development programs might foster the integration of appropriate technologies in learning mathematics.

Another implication of this study was with respect to the type of teaching observed, and emulating the strategies used to determine if any of the strategies used by Ms. Johnson improved teaching and learning in classrooms focused on reform-based curricula. For example, an appropriate mix or balance of whole class and small group discussions and work on problems that provides every student in the class the opportunity to express and defend their ideas based on mathematical evidence can be tried, along with its impact on students' performance measured. Equally important is the consideration of the appropriate mix of traditional and reform-based strategies for enhancing student achievement in reform-based curricula.

A final implication arising from this study was the importance of the school community and its benefits on the readiness of teachers to educate students. Expert teachers can use their experience to mentor novice teachers for incorporating effective instructional strategies, while all teachers can benefit from the cooperation of administrators who are in keeping with the developing ideas about the reform of teaching and learning and the resources that need to be dedicated to that cause for its implementation.

Suggestions for Further Research

The diagram of Ms. Johnson's practice as an expert mathematics teacher in a reform-based curricular setting needs further investigation. The task of confirming the various attributes with other expert school mathematics teachers' practice remains to be examined. If this diagram of expert mathematics teacher attributes validly represents patterns of practice supportive of effectiveness in teaching in a reform-based curriculum, its implications in terms of the design of reform-based mathematics teacher education will be significant.

Ms. Johnson repeatedly discussed the lack of students' motivation to apply themselves to learning of mathematics as the number one challenge she faced on a daily basis. The ubiquitous remark by students as to "Why do I need this?" was also heard from some of Ms. Johnson's students. Students were not the main focus of this study, however, the design of curricula that attract students' attention and interest is of importance. Multiple resources provide potential for improving mathematics learning of motivated students at all levels. Therefore, more studies are needed to determine curricula formats and teaching practices that prove to be highly motivating and engaging to students.

One important evidence of students' motivation is their willingness to engage themselves with meaningful mathematics discourse. In their analysis of Magdalene Lampert's execution of a 10-lesson fifth grade unit, Leinhardt and Steele (2005) pointed out that meaningful instructional dialogue requires the establishment of rules to guide openings and closures, and transition points for changing to a new topic or yielding to another speaker, or clarifying points. On the other hand, in her study of 15 seventh grade mathematics students, Jansen (2008) found that if students felt threatened by mathematical discussions, they avoided engaging in meaningful and conceptual mathematical discourse and sufficed with talking about procedures. Studies like these exemplify teacher and learner centered issues that may impede meaningful discourse from taking place in mathematics classrooms. Unraveling how to confront such issues in teacher education programs can significantly impact the practice of prospective teachers, promoting higher understanding and student achievement.

This study focused specifically on the expert teacher's intentions and instructional plans but did not focus on the student-teacher interactions. Additional research needs to investigate these interactions in more depth to gather a better understanding of the nature of classroom instruction that is successful in implementing the reform-based curricula. What is the nature of the interactions in successful implementations? With this knowledge, another study needs to consider the preparation of teachers to establish the types of interactions that are connected with success in learning mathematics.

Concluding Remarks

This study described how one expert middle school mathematics teacher was able to use her education, training, and experience for the learning benefit and enjoyment of her students. Other mathematics teachers may have found alternative ways of improving student success in mathematics. Looking at the problem of improving student achievement to levels much higher than the 18 percent described in the work by Batista (2001), though, may require consideration beyond the parameters of the educational system alone, since even in the best case scenario, students will not spend any more than three percent of their time with an expert teacher during any given week. The other 97 percent of students' time spent outside of a mathematics classroom each week is important in motivating students to learn about mathematics or depriving their desire to do so. Ms. Johnson, an expert mathematics teacher, identified students' motivation to learn mathematics as the number one challenge in her work. Her assessment pointed toward the problem with respect to lagging student achievement in mathematics. Therefore, the obsession with the question of what can an expert teacher do to improve student achievement within the classroom environment may, in time, transform itself to the question of what can an expert teacher do to support students in gaining success to more mathematics, thereby motivating them to learn and think about mathematics even when not seated in the confines of a mathematics classroom.

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APPENDICES

APPENDIX A

Protocol for Contacting Principals

The purpose of this protocol is to identify middle or high schools in the state of Oregon, where reform-based mathematics curriculum and instruction programs have been implemented and the mathematics teachers whose students' mathematics progress meets the mathematics standards set by the Adequate Yearly Progress (AYP) indicators.

The Researcher: Hi Mr./Ms. (?) I am a graduate student at OSU and Dr. Maggie Niess is my major Professor. I am conducting a dissertation research about classroom teaching of today's middle or high school mathematics teachers in light of the reform of mathematics curriculum and instruction. I am wondering if your school has had the chance to adopt a reform-based mathematics curriculum?

If the principal responds with a "yes", then the researcher will follow with the next question.

The Researcher: Great! Can you please tell me if students' mathematics progress at your school meets the mathematics standards set by the Adequate Yearly Progress (AYP) indicators?

If the principal responds with a "yes", then the researcher will follow with the next question.

The Researcher: May I speak with the mathematics teachers at your school to see if they would be interested in participating in my research for my dissertation?

If the principal responds "no" to any of the above questions, the researcher will politely thank the principal and hang up.

APPENDIX B

Teacher Background Questionnaire

Instruction: Please provide the information or place a check for the appropriate option.

1. Education (please check all that apply)
   ___ Bachelor's Degree
   ___ Bachelor's Degree + 45 hours or more
   ___ Master's Degree
   ___ PhD

2. Undergraduate college major ____________________

3. Teaching endorsement (please check)

   ______ Basic Mathematics
   ______Advanced Mathematics
   ______ Other (please specify)

4. Teaching authorization (please check)
   ______ Middle level
   ______ High school level
   ______ Secondary
   ______ Other (please specify)

5. Total number of years teaching mathematics__________________

6. Total number of years teaching ____________________________

7. Years of teaching at the current school _______________________

8. Mathematics class(es) taught for at least the past three years:

   a. Current curriculum adopted by your district/school for the mathematics class(es)

   b. How many years has this curriculum been used at your school?

   c. How many sections of the class will you be teaching this year?

APPENDIX C

Teacher Selection Interview Protocol

The purpose of this interview is to identify the highly qualified and experienced teachers with the most amount knowledge about reform-based curriculum and instruction programs, and highest of personal instructional standards.

1. How long have you been teaching with this reform-based curriculum and instruction program?

2. What is your opinion of the curriculum and instruction program? Has it been effective in improving student achievement, or have you known other curriculum and instruction programs that were more effective? Please explain.

3. Overall, how did your personal expectations of acceptable student work change after adopting this reform-based curriculum?

4. What were some of the challenges you faced in transitioning to the reform-based curriculum and how were you able to overcome those challenges?

APPENDIX D

Initial In-depth Interview Protocol

The purpose of the initial interview will be to collect data about the participant's overarching goals for the course, preferred instructional strategies, and plans for the students.

1. What are your basic goals and objectives for your students in the next nine weeks of lessons?

2. What is (are) the most important topic(s) you would like your students to have mastered by the end of the next nine weeks?

3. What are some topics in this class that you have found to be especially difficult for your students to understand and how do you address those difficulties in your instructional plans?

4. What curricular materials are you using throughout next nine weeks of lessons?

5. What is your preferred instructional strategy when teaching mathematics?

6. In general, what instructional strategies have you found work well with your students in this particular mathematics class? What instructional strategies have you found do not work well with your students in this class?

7. What instructional tools are you hoping to have access to for teaching this mathematics class?

8. Describe the characteristics of a typical student from this mathematics class.

9. Describe how you connect with your students when you first meet them in class. In the beginning of the year, how do you encourage your students to participate in classroom discussions?

10. What skills would you like your students to have before they begin your class?

11. What skills do you hope your students will have by the end of the class?

12. If you ask your students "What have you learned?" at the end of the year, what do you think they would say?

13. How do you plan to motivate your students to engage in these first units?

14. Describe your vision of a typical day in this class.

15. How has what you do in the class been affected by the other teachers in your school who are teaching the same classes?

16. How has what you do in this class been affected by directions from your principal and others in the school's administrative staff?

17. How has what you do in the class been affected by the particular curriculum that you are using?

APPENDIX E

Pre and Post-Observation Interview Protocols

Pre-Observation Interview

The purpose of the pre-observation questionnaire is to find out about the teacher's objectives, and plans for each class session before observation, and to find out about their expectations from their students.

1. What content objectives or concerns do you have for your students for today's lesson?

2. What's the plan for the day?

3. How do you expect your students will react to your plans for this lesson?

4. What special assessment techniques will you use to assess students' understanding of your instructions for today?

Post-Observation Interview

The purpose of the post observation questionnaire is to find out from the teacher how well expectations about the implementation of instructional plans, and with respect to the students' understanding were met.

1. Describe any changes in your original lesson plan you made while you were teaching. Explain your reasons for making this adjustment.

2. What instructional strategies worked well today and how are you able to make that conclusion?

3. Did the students respond to the lesson as you expected them to? If yes, how? If no, why not and how would you change the lesson to meet their specific needs?

4. Did students learn what you wanted them to learn?

5. What might you do to improve today's lesson?

APPENDIX F

Post-Unit Observation Interview Protocol

The purpose of this interview is to provide the teacher with an opportunity to reflect on the impact of instructional strategies over the course of the unit. This interview will also probe the teacher's perception about students' performance, and the reasoning for any improvements in the instructional plans for future teaching of the same unit. The teacher's plans for the upcoming unit and rationale for use of particular instructional strategies as well as student involvement in the instruction will also be discussed.

1. What content ideas did you want your students to learn from this entire unit?

2. Were those objectives met? If yes, how were you able to assess this? If not, why not?

3. Were the reactions of your students toward the material in this unit typical upon their exposure to it, or was anything different this time if you have previously taught this unit? Please explain.

4. Describe any changes, if at all, you had to make to your lesson plans when teaching this unit? For what reasons did you make these changes?

5. Overall, what instructional strategies would you say work well when you cover the material in this unit, and why do you use these strategies?

6. Are there any changes that you would like to make for when you teach the same unit next year?

7. What have you learned (about your teaching strategies, your students and their preparations, etc.) from teaching this unit that you will use in teaching the coming unit?

8. Are you satisfied with the current level of participation by the students in classroom activities or are there any facets of student participation in coursework that you would like to see change? Please explain.

APPENDIX G

Final Interview Protocol

The purpose of this interview is to provide the teacher with an opportunity to reflect more globally on her instructional experiences over the course of the study. Also, the interview aims to probe the teacher's perception about her students' performance, instructional strategies that worked particularly well, and any improvements in her instructional plans for the next time when she will teach the same content.

1. Now that you have had the chance to work with this cohort of students for some time, describe your perception of how well your students are doing.

2. What content areas did your students particularly performed well on? Why do you think they did so well?

3. What content areas gave your students difficulty? What do you think they had difficulty with understanding these content areas?

4. What instructional strategies worked particularly well? Describe why.

5. What instructional strategies did not prove as successful? Describe why.

6. If you teach this course next year, what changes will you be making in your planning for teaching the same content areas?

7. What aspects of the content, your students, or the classroom environment affected your plans for instruction?

8. How would you envision the ideal teaching situation? Given your vision, what changes or adjustments would you make to your current plans for teaching the same class?

APPENDIX H

School Community Interview Protocols

The purpose of this interview will be to collect data about the impact of the school community on the way the teacher in this study plans her/his lessons, thinks, and conducts the actual practice of teaching.

School Community Protocol – The Collaborating Teachers

1. The teacher in this study is a recognized expert mathematics teacher and is viewed as a leader in her area of teaching. Please explain your views on why she is so successful?

2. Have you worked with this teacher in developing the mathematics curriculum at this school? If so, what is your impression of the outcome of this collaboration?

3. The teacher in this study has identified you as one of three teachers at this school with whom she collaborates with respect to her work. What has been your role in this collaboration?

4. What goal do all mathematics teachers at this school work toward?

5. What is your impression of the student growth in achieving math goals in your school?

6. Have you interacted with this teacher about teaching in the middle school mathematics classes? If so, please describe how this teacher's instructional practices support the students in meeting these goals?

School Community Protocol – The Principal

1. Please describe how you support what the teacher in this study does in her mathematics classrooms.

2. What other support structures, facilities, and or resources in your school are in effect to support this teacher's success with her students?

3. Please explain challenges, if any, against bringing together of all of these support structures, facilities, and resources, and what portion of these challenges have you been able to overcome? And how have you been able to overcome these challenges? *Specifically, the purpose of this question here is to identify challenges that may most specifically impact all mathematics classrooms at this school, if any at all.*

4. What is your vision for maintaining a high standard of mathematics teaching and learning at this school?

APPENDIX I

Detailed Models of Lesson Segments: MS. Johnson's Approach to Solving an Equation (Algebra-1 Class)

Day 1: Action Sequence #1, #2: Distribute objective sheet for the week, review topic form last class on solving an equation for any term in the equation

Action Sequence	Sub-Episodes		Goals Activated a	b	c	d
1. Teacher begins the day by distributing the objectives sheet for the week (1 min.)	1.1. Objective sheets have daily objectives for each day of the week, as well as a list of problems to be done in class					
2. Whole-class warm-up activity (15 min.)	2.1. **Assessment**: Seeking volunteer students to respond to her questions and "sharing" their ideas **Content**: Solving $D = r*t$ for r or for any of the other terms **Discourse**: Teacher begins with what the overall equation represents. She wants at least 3 students to raise their hands that they know. After the volunteer students give their answers, teacher asks the rest of teh class if they agree with what they've heard **Pedagogy**: Leading with guided-inquiry, and using whole-class discussion to build consensus among students to create a mechanism for discovery of correct answers	2.1.1. **Discourse**: Teacher uses the idea of distance=speed*time to give meaning to the abstract relationship. When solving for r, teacher reminds students about the identity property of 1, connecting the content to other areas of mathematics. At the end, teacher uses the "number of problems per minute" to extend the original idea to another real-world example.				

Goals:

a. Distribute objectives sheet for the week, containing goals and a list of in-class homework problems
b. Review topic from last class
c. Provide additional real-world context for application of concepts from last class
d. Involve students in a whole-class discussion of their ideas and reasoning

Day 1: Action Sequence #3, #4: Conduct a "diagnostics test" to assess the computational skills of students, and conduct a "math game" to find and discuss alternative ways students could translate words into mathematical expressions

Action Sequence	**Sub-Episodes**		**Goals Activated** a	b	c	d
3. Teacher hands out "Diagnostics test" (15 min.)	3.1. **Assessment:** Teacher wants to conduct a formative assessment of students' computational skills 3.2. Teacher aks students to spend their extra time checking their solutions					
4. Math game (20 min.)	4.1. **Assessment**: Formative and occurred as different students volunteered their answers to the teacher's questions. **Content**: Translating sentences into algebraic expressions. **Discourse**: Students declare their solutions. The teacher then asks if other students "agree" with the solution. The teacher solicits alternative solutions from others and uses consensus building among students to guide the discourse. **Pedagogy**: The teacher reads out loud a sentence from the textbook, followed by independent work by students, followed by sharing of ideas in small groups, followed by whole-class discussion of solutions and alternative solutions	4.1.1. **Discourse**: In this context, solutions that were simplified to different degrees, or solutions in which the final terms were written in different order were considered alternative solutions (e.g. 3x+5 was considered an alternative solution to x+x+x+5). The discourse was meant to hash out all such alternatives				

Goals:

a. Using a diagnostics test to assess students' computational skills
b. Using students' independent work and thinking to highlight alternative solutions
c. Using whole-class discussions and questioning to guide student understanding and to find out what students already know
d. Promoting consensus building among students to convey to students a means for verifying and checking what they already know

Day 2: Action Sequence #1, #2: Correct previous homework, and provide more guidance to students on solving equations working independently and in small groups

Action Sequence	**Sub-Episodes**		**Goals Activated** a b c d
1. Whole-class routine (5 min.)	1.1. **Assessment**: The teacher starts by quickly giving the final answers to homework problems. Students are expected to interrupt and ask questions if their answers don't match. 1.2. **Content**: The teacher uses whole-class discussion and a mini-lecture to emphasize the procedure for distributing a negative sign to the terms inside a parentheses		
2. Small group work on solving equations (45 min.)	2.1. **Assessment**: The teacher walks to each group multiple times, asking students questions to assess their understanding and progress in solving problems that were assigned. **Content**: Textbook problems on solving equations of one variable (e.g. 45=6x+3). **Discourse**: The teacher uses hints and guiding questions to direct student thinking and learning, working in small groups. **Pedagogy**: Independent work combined with small-group collaboration, and the teacher helping with guiding questions	2.1.1. **Content**: Students are assigned to solve 10 problems similar to the example during the 45 minutes class time. **Discourse**: The teacher emphasizes to students to "check" their answer by substitution and also by discussing and agreeing among themselves. **Pedagogy**: A good portion of students are using the "guess-and-check" strategy to solve the problems. The teacher prompts them to use the “symbol manipulation” strategy instead.	

Goals:

a. Correct previous homework and provide opportunity for students to ask questions
b. Use a mini-lecture to review a topic with which many students were having difficulty: distribution of a negative sign placed in front of a parentheses
c. Assess students’ understanding of the procedure to solve equations of one variable
d. Provide guided practice to students to solve equations independently or with peers, using the symbol-manipulation strategy

Day 3: Action Sequence #1, #2: Correcting previously assigned homework, and providing more opportunities for students to translate textual information into mathematical equations, and improve their understanding of symbolic manipulation method of solving equations of one variable

Action Sequence	Sub-Episodes		Goals Activated a	b	c
1. Whole-class routine (5 min.)	1.1. **Assessment**: The teacher gives the correct answer for yesterday's homework problems very quickly, anticipating students to compare these with their own answers and ask questions when they have any		✓		
2. Whole-class discussion (20 min.)	2.1. **Assessment**: The teacher assesses student understanding by asking students questions in order to generate the solutions she writes on the board. **Content**: Using exercises in the textbook to first translate textual information into mathematical equations of one variable, and then using symbolic manipulation to solve them **Discourse**: The teacher uses guided inquiry to invite student participation and improve student understanding. Each step to solving each equation emerges from the responses students provide to the teacher's questions. **Pedagogy**: Whole-class discussion led entirely by guided inquiry as well as whole-class reflection on each response individual students provide. Once again, emphasis is on abandoning "guess-and-check" strategy in favor of "symbol manipulation"	2.1.1. **Content**: Examples of the content are: 1/5*x=20, and 6w+6=78 that are derived, first, from translating textual information, read out loud by the teacher, into mathematical expressions. **Discourse**: The teacher repeatedly tells students who have not participated to raise their hands to provide a response to her questions. She declines to accept response from students who have already had something to say. **Pedagogy**: The discourse is not only based on guided inquiry but also the teacher demands consensus building by asking "Do you agree?" from the rest of the class		✓	✓

Goals:

a. Correct previously assigned homework
b. Review last class's lesson on solving equations to further students' understanding
c. Assess students' understanding of the content by demanding participation by all students during classroom discourse

Day 3: Action Sequence #3: Students translate textual information into mathematical equations independently, share their ideas with their team members, and get quizzed on their understanding of the whole process by the teacher

Action Sequence	Sub-Episodes		Goals Activated a	b	c
3. Small group activity on solving equations of one variable (25 min.)	3.1. **Assessment**: The teacher walks to each group, questioning group members on their progress in solving the problems that were assigned as in-class homework. **Content**: The teacher has assigned some more problems similar in nature with what was discussed during whole-class discussions. **Discourse**: The teacher asks students in each group to express their thinking by showing or explaining to her "how" they have come up with their answers. **Pedagogy**: The teacher demands that students work independently as well as together on solving the assigned problems, making sure they share ideas and that their independent answers "agree" with one another. Student team work is followed by the teacher verbally quizzing group members on their understanding of the symbolic manipulation technique.	3.1.1. **Content**: Each student reads textual information from the book, translates it into an equation of one variable, and attempts to solve it. **Discourse**: Some students discuss their independent solutions with others in their group. Meanwhile, the teacher cuts in with her own set of questions about whether group members' independent solutions agree, or if alternative solutions exist. The teacher guides students by asking questions, without giving any specific directions.			

Goals:

a. Provide further opportunity for students to work independently and with peers on solving equations of one variable
b. Assess students' understanding or misconceptions about solutions of equations of one variable while applying the symbolic manipulation strategy
c. Prompt students to communicate and share ideas when solving problems

APPENDIX J

Detailed Models of Lesson Segments: Ms. Johnson's Approach to Re-teaching the Distributive Property and Independent and Dependent Variables (Algebra-1 Class)

Day 1: Action Sequence #1, #2: Return graded "diagnostic test" to students, and reteach distributive property by emphasizing generalization of ideas and metacognitive thinking

Action Sequence	**Sub-Episodes**		**Goals Activated** a	b
1. Independent student work (3 min.)	1.1. **Assessment**: The teacher returns graded diagnostic test back to students. **Content**: The teacher asks students to convert their grades into percent form. **Discourse**: The teacher provides guiding hints **Pedagogy**: Students work independently on converting their grade to percent form as the teacher waits for students to finish.	1.1.1.**Content**: The teacher tells students to use long division and no calculators to convert their grade to decimal form, and also convert that into percent form **Pedagogy**: Connecting old material about equivalent fractions, decimals, and percents to a real-world application	|	
2. Whole-class discussion and re-teaching of the distributive property (7 min.)	2.1. **Assessment**: The teacher is highly concerned about one test problem that was solved incorrectly by "almost everyone in class". **Content**: Difficult test question, which is related to the application of the distributive property. **Discourse**: The teacher asks students questions during her mini-lecture about solving the difficult test problem. **Pedagogy**: Mini-lecture combined with questions to engage and assess students' thoughts and understanding	2.1.1. **Assessment**: The teacher asks students what the next step in the procedure should be. **Content**: Solving 3(m-2)=-2(m-7) for the value of m. And generalizing to 2(x-4+y) **Discourse**: Different students volunteer to describe procedure at each step after the teacher asks questions. **Pedagogy**: The teacher emphasizes generalizing ideas and metacognitive thinking		|

Goals:

a. Provide feedback on the previous diagnostic test results and connect previously taught material with a real-world application
b. Reteach the distributive property by emphasizing generalization of ideas and metacognitive thinking

Day 1: Action Sequence #3, #4: Assign students to new teams, connect the ideas or vocabulary of input-output to the relationship between independent and dependent variables and the labeling of the X-Y axes

Action Sequence	Sub-Episodes		Goals Activated: a	b	c	d
3. Students are assigned to new teams (5 min.)	3.1. The teacher assigns students to new teams every 3 weeks. Students line up against one side of the classroom. The teacher calls out the names of the months in random order and those with birthdays in those months are allowed to choose their new spot in the class.					
4. Whole group discussion of independent and dependent variables (10 min.)	4.1. **Content**: Expanding students' vocabulary and understanding of independent and dependent variables. **Pedagogy**: Mini lecture to convey the "idea" of input-output to describe independent-dependent relationship, and connecting the relationship between the independent-dependent variable system to the X-Y coordinate system.	4.1.1. **Content**: The teacher asks students to open up to pages in the textbook that discuss and connect examples of input-output with independent-dependent relationships on X-Y graphs, and the position of each type of variable with respect to the axes. Also, metacognition and generalization of ideas are touched upon again to generalize application of the distributive property from two terms inside parentheses to three or more terms.				

Goals:

a. Assign students to new groups
b. Teach students vocabulary that would help them to understand the upcoming concepts better
c. Connect the vocabulary and idea of input-output relationship to the idea and relationship between independent and dependent variables, and extending the latter to a labeling scheme on the X-Y axes
d. Discuss the use of metacognition and generalizing to help students understand the distributive property

Day 1: Action Sequence #5: Provide small group "work-time" opportunity for students to grapple with the ideas of connection making and metacognition discussed during the whole-class discussion in the context of solving problems from the textbook

Action Sequence	Sub-Episodes	Goals Activated a	Goals Activated b
5. Small group work on textbook examples (25 min.)	**Assessment**: The teacher conducts formative assessment as she walks from group to group, discussing and questioning students on their understanding of the content. **Content**: Connection making between the idea of input-output and independent-dependent relationship and the extending of those ideas to the X-Y axes of a graph. Examples making use of metacognition to generalize the application of the distributive property were also included in this content. **Discourse**: Sharing of ideas about the content in small groups. **Pedagogy**: Small group discussion of the content ideas, with the teacher asking questions to guide student thinking and understanding	|	|

Goals:

a. Have students solve textbook problems in small groups, while sharing ideas, and making connection and extending ideas in respect to independent and dependent variables
b. Have students use metacognitive techniques from the textbook section "How am I thinking?" to generalize their own thinking when using the distributive property

APPENDIX K

Detailed Models of Lesson Segments: Ms. Johnson's Approach to Teaching Pattern Analysis, Connections, and Generalizing to Algebraic Rules and Graphical Representations
(Algebra-1 Class)

Day 1: Action Sequence #1, #2: Use patterns between initial numeric and geometrical data to construct rules, and derive from these rules their graphical representations

Action Sequence	**Sub-Episodes**		**Goals Activated** a	b
1. Whole-class focus on chapter objectives (5 min.)	1.1. **Content**: The textbook chapter called "algebra connections". The teacher asks volunteer students to read out loud segments from the chapter's overall objectives sheet. **Pedagogy**: Direct lecture as the teacher explains and expands on the statements read from the book by students	1.1.1. **Content:** The content is about exploring and discovering patterns, extending and generalizing those patterns to construct algebraic rules, and drawing a graph for the rules.		
2. Small group work on select set of exercises from the chapter (40 min.)	2.1. **Assessment**: The teacher moves from group to group, discussing the chapter activities with students. The teacher challenges students to explain how they got their rules and whether they have tested the accuracy of their rules, and if yes, how. **Content**: Students must make connection between "figure numbers" and "dimensions of each figure", and generalize this connection or pattern into an algebraic rule. **Discourse**: The teacher interacts with the members of each group. **Pedagogy**: Small group discussions led by the teacher's guided inquiry	2.1.1. **Content**: Figure number and dimensions lead to : $(n+2)*(n+3)=5n+n^2+6$, in one case. **Discourse**: The teacher asks members of each group to justify their answers by explaining how they have gotten their rule. Then, she asks if they have alternative solutions or all their solutions agree with each other. Next, she asks if they have tested the accuracy of their rules. **Pedagogy**: Small group discussion, aided and guided by the teacher's inquiry and explanation		

Goals:

a. Use "figure numbers" and associated "geometric figures" from the chapter in the textbook to establish a pattern, and use this pattern to construct an algebraic rule and then graph it.
b. Work in teams to find all possible connections between numbers and figures, rules, and graphs

Day 1: Action Sequence #3: Student teams to collaborate and prepare poster presentation of three problems they solved: discovering patterns, constructing rules, and drawing graphical representations of the rules

Action Sequence	Sub-Episodes		Goals Activated
3. Whole group discussion of poster presentations (5 min.)	3.1. **Assessment**: The teacher wants each team member to explain their understanding of the process of algebraic connection making in the context of a team presentation. **Content**: Members of each team are to collaborate and prepare poster presentation of the three of the problems they solved for the day. **Discourse**: Students in each team are to set the stage to share their ideas and reasoning about making "algebra connections", while other students will be given the opportunity to challenge their ideas. **Pedagogy**: Preparing students to explain their thoughts and to engage in conceptual and divergent thinking strategies, and multi-representation of ideas	3.1.1. **Pedagogy**: Conceptual thinking that entails extending concepts of pattern to abstract algebraic rules; divergent thinking that entails providing of alternative solutions, while being challenged by other students; and multi-representation of ideas such that an association between "figure numbers" and "geometric figures" is embodied in the form of an abstract rule, and the rule is shown connected to a representative graphical representation of itself	a

Goals:

a. Organize solutions of the chapter problems on "algebra connections" into team poster presentations for the purpose of presenting to and sharing with other students during the next class

APPENDIX L

Detailed Models of Lesson Segments: Ms. Johnson's Approach to Connecting Guess-and-Check Strategy to Equations, and Involving Absolute Value in the Order of Operation Problems (Shortened day in Math-7 Class)

Day 1: Action Sequence #1, #2: Correct previous day's homework and involve students in a game of guess-and-check strategy, leading to constructing generalized algebraic rules

Action Sequence	Sub-Episodes		Goals Activated a	b
1. Whole-class discussion of the previous homework problems (5 min.)	1.1. **Assessment**: The teacher reads answers to previous day's homework problems and listens for questions. **Content**: order of operations problems from the textbook. **Discourse**: two students have the same question for the teacher. **Pedagogy**: whole-class discussion of the question	1.1.1. **Discourse**: The questions spark a whole-class discussion. **Pedagogy**: The teacher explains the answer at the whiteboard quickly and makes sure the students are clear about the solution before moving on		
2. Whole-class discussion of the guess-and-check strategy (15 min.)	2.1. **Assessment**: Formative and occurs as the teacher demands and receives participation of students in the guess-and-check game. **Content**: 52 small or large bowls, with the capacity of 3 and 5 fish respectively, should be filled with fish such that the total number of fish in all the bowls adds up to 202. How many small and how many large bowls should there be? **Discourse**: The teacher provides leading questions and demands that different students provide the answers and their reasoning. **Pedagogy**: Whole-class discussion of students' guesses	2.1.1. **Pedagogy**: The teacher demands that different and all students participate in providing guesses. The teacher acts as a facilitator to expose students' reasoning and thinking behind their guesses and writes students' answers on the board for everyone to see. Students must share their thinking behind their guesses. The teacher allows students to make mistake but discover their own mistake along the way to solve the problem		

Goals:

a. Assessing and correcting student understanding of the previous homework problems
b. Teach students how to use the guess-and-check strategy as a tool to develop algebraic rules

Day 1: Action Sequence #3: The teacher implements a review of the procedure to carry out order of operation problems that involve absolute value terms

Action Sequence	Sub-Episodes		Goals Activated
			a
3. Small group activity in applying absolute value in the order of operation problems (12 min.)	3.1. **Assessment**: The teacher walks around to each group, questioning them. **Content**: Order of operation problems involving absolute value terms from the textbook. These problems reflect back on the questions that students asked when the teacher was correcting homework from the previous day in the beginning of the class. **Discourse**: Students discuss the procedure to do the problems with the teacher and with their team members. quite a bit of socializing is also taking place. **Pedagogy**: Small group work on procedures, guided by the teacher's input as she interacts with each group.	3.1.1. **Assessment**: The teacher drops by each group, asking them questions like: "What's -3 times -2? Now, what's the absolute value of -3 times -2?", "What's the absolute value of -12?", "What's -8+3? Now, what's the absolute value of -8+3?". The teacher asks students to turn in their work on the problems at the end of the class period. **Discourse**: Is assessment oriented.	

Goals:

a. Review the role of absolute value in order of operation problems in the context of students working on textbook problems in small groups

APPENDIX M

Detailed Models of Lesson Segments: Ms. Johnson's Approach to Teaching Operational Sense and Number Sense with Respect to Integers (Math-7 Class)

Day 1: Action Sequence #1: The teacher uses algebra tiles to teach operations' sense and number sense when working with integers

Action Sequence	Sub-Episodes		Goals Activated a	b
1. The teacher distributes plastic containers full of algebra tiles to each team and starts a whole-class discussion of making sense of some sample tile arrangements (25 min.)	1.1. Rules for arranging the tiles: black tiles denote a "positive one" value and red tiles denote a "negative one" value. The tiles are to be arranged in "edge" tiles and "center" tiles in a rectangular spread. Multiplying the edge tiles will give the integer value representing the number of tiles in the center, and dividing the center tiles by any one set of edge tiles will give the integer number of tiles on the other edge. Each single black tile in the edge or the center can cancel out another single red tile in the same set and vice versa.	1.1.1. **Assessment**: Classroom discussions aided by a document projector. Also, the teacher distributes a "recording sheet" where students will have to sketch and write their thoughts and explanations. The sheet is to be turned in at the end of the class. **Content**: Students struggle and read the tiles as 3x4=12, 4x3=12, 12/4=3, and 12/3=4. The teacher gives another example to be solved during whole-class discussions. Volunteer students share their arrangement for the tiles with the rest of the class at the request of the teacher. **Discourse**: Students discuss and share ideas about the arrangement of the tiles in their groups, and once they agree on an arrangement, they raise their hand and share their configuration for the tiles with the class. When the teacher has an arrangement for the tiles on the document projector screen, students who have not said anything before are invited to give the operation for the particular arrangement of the tiles. **Pedagogy**: Whole-class discussion of the operations and tile arrangements, while students work independently and collaboratively to connect tile arrangements and number operations		

Goals:

a. Use visual models along numerical operations to convey to students an understanding of the operations' sense and number sense when working with integers
b. Have students to maintain a log of their work with the tiles on the "recording sheet", turning it in at the end of the class period for the teacher's assessment

Day 1: Action Sequence #2: The teacher uses algebra tiles in small groups to teach operations' sense and number sense when working with integers

Action Sequence	Sub-Episodes		Goals Activated a	b
2. The teacher transitions from whole-class discussion to small group activity (25 min.)	2.1. **Assessment**: The teacher walks to each group, verbally quizzing them about what they are doing with the tiles. Also, students are to turn in their "recording sheet", containing the tile arrangements and their reasoning for each set of tiles and algebraic operations. **Content**: Students are working on textbook problems (e.g. arranging tiles to get 1x4=4, 4x1=4, 4/1=4, and 4/4=1). **Discourse**: Students within each group discuss and share ideas about the arrangement of the tiles for each problem. The teacher has insisted that members of each team must reach an agreement about the arrangement of the tiles before working on another problem. She insists that students ask their team members their questions before asking her. **Pedagogy**: Small group discussion of the multiplication and division operations and the associated tile arrangements. The teacher wants students to collaborate and build consensus when solving each problem as she moves to each group, keeping an eye on students' work and providing feedback	2.1.1. **Content**: The teacher uses the operation sense aspect nested in the tile arrangements for the algebraic operations to convey to students why the ratio 0/0 is undefined in mathematics.		

Goals:

a. Small groups to use visual models along numerical operations for the purpose of understanding the operations' sense and number sense when working with integers
b. Have students to maintain a log of their work with the tiles on the "recording sheet", turning it in at the end of the class period for the teacher's assessment

Day 2: Action Sequence #1: Make connections between algebra tile structures and sets of equations involving algebraic operations of integers

Action Sequence	**Sub-Episodes**		**Goals Activated** a	b	c
1. Whole-class discussion of "minimal array" tile structures and their connection with simplification and operations involving integers (15 min.)	1.1. **Assessment**: The teacher uses the document projector to project a tile arrangement on the screen, while questioning students' understanding about the "net value" of tiles in the center and on the edges. **Content**: Each black (+1) tile cancels each red (-1) tile within a single edge or in the center of an array of tiles; hence the concept of net value. Students are to construct tile structures for numerical equations involving integers. **Discourse**: Guided by the teacher's questions with questions like: "What is the net value of the array?", "What is the net value of the array along the top edge?", "What is the net value of the array along the side edge?" The teacher asks different students to answer for each tile structure, but seeks "agreement" or the consensus of the whole-class before moving forward with another question. **Pedagogy**: Whole-class discussion guided by the teacher's leading questions, while answers are accepted based on a consensus throughout the class.	1.1.1. **Content**: Example: for a tiles' array structure with 12 tiles in the center, 4 of which are red and the rest black, the correct response expected from students for the "net value" would be +8. For a tiles' array structure on the side edge, comprising of 3 red tiles and 1 black tile, the correct student response for the "net value" would be -2. Thus, the "net value" for a division operation between the center tiles and the side edge tiles would be -4. At the same time, for a multiplication operation between the side and center tiles arrays, the correct response would be -16. For each case, the numerical set of equations (4 equations in total; two for the division operation and two for the multiplication operation) from the textbook were noted, and a tiles' arrays structure was built to match the set of equations; hence the backward design strategy			

Goals:

a. Make connection between the concepts of "minimal array" and "simplification" of integers in the context of algebraic operations
b. Make students develop operation sense as well as number sense about how algebraic signs interact
c. Implement backward design instructional strategy to have students build tile arrangements that match a set of algebraic equations involving integers

Day 2: Action Sequence #2: Students share ideas about making connections between algebra tile structures and sets of equations involving algebraic operations of integers

Action Sequence	**Sub-Episodes**		**Goals Activated** a b c d
2. Small group work with and discussion of "minimal array" tile structures and their connections to algebraic operations and sign manipulation (35 min.)	2.1. **Assessment**: The teacher walks from group to group asking questions about the main objective of the lesson: making connection between tile array structures and algebraic operations involving integers. The teacher asks "How did you build your array?". Also, students are to prepare and submit a poster of their class work at the end of the class period. **Content**: Algebraic operations involving integers from the textbook and assigned by the teacher for in-class "work time". **Discourse**: The teacher enters each team by either directing "Build your own array!" or by asking "How did you build your array?" This prompts students to take action if they haven't started already, or explain their thoughts and reasoning if they have built an array structure. **Pedagogy**: Small group discussions rooted in the main objective of the lesson, and implemented through the backward design strategy	2.1.1. **Content**: A combination of right colors (red or black) and numbers of tiles was necessary to recreate each set of equations involving only integers in the form of an array structure. **Pedagogy**: The main objective of the lesson was for students to develop operation sense as well as number sense with respect to algebraic operations involving integers	

Goals:

a. Have students collaborate and share ideas about solving problems
b. Make connection between the concepts of "minimal array" and "simplification" of integers in the context of algebraic operations
c. Make students develop operation sense as well as number sense about how algebraic signs interact
d. Implement backward design instructional strategy to have students build tile arrangements that match a set of algebraic equations involving integers

APPENDIX N

Detailed Models of Lesson Segments: Ms. Johnson's Approach to Use Visual Models to Teach Operational Sense to Convey Meaning in Operations Involving Algebraic Expressions (Math-8 Class)

Day 1: Action Sequence #1, #2: Students receive the day's announcements, and use algebra tiles to build visual models of algebraic expressions

Action Sequence	Sub-Episodes		Goals Activated a b c d e
1. Whole-class announcements of the upcoming school events (5 min.)	1.1. The teacher is sitting at her computer and, to save paper, reads from her computer screen a series of announcements for the students		a
2. Warm-up activity: whole-class discussion of "building" tile structures from algebraic expressions (15 min.)	2.1. **Assessment**: Discussions aided by the overhead projector and constructing algebraic tile structures that match the algebraic expressions in the textbook. **Content**: The teacher demonstrates constructing tile structures for the expressions: 4x+4, 5x+10, and 3x+12. **Discourse**: The teacher asks students to tell her the number and the type of tiles she should use in the center and at the edges of the tile arrays. She emphasizes alternative solutions and participation of all students. **Pedagogy**: Whole-class discussion of an active lesson, using manipulatives and the backward design strategy	2.1.1. **Assessment**: Classroom wide discussions with students, emphasizing that all students participate. **Content**: The width of each tile is taken as 1, though, the lengths represent an unknown or the variable X in this exercise. Thus, 3X is shown as three rectangular tiles of width 1 and unknown lengths arranged tip-to-tail one after another. By the same token, to represent 3X+1, a square tile measuring a unit on the sides is added to the array of the three such rectangular tiles. **Discourse**: The teacher emphasize alternative solutions such as tile arrangements for 4X+4=2(2X+2)=4(X+1)=4*1+4*X. The teacher seeks consensus building among students	b c d e

Goals:

a. Read out loud the school announcements of the day
b. Use algebra tiles to build visual models of algebraic expressions, identifying variables and constants
c. Review and reteach the distributive property in the context of alternative solutions
d. Teach students operation sense and symbolic manipulation when simplifying algebraic expressions
e. Assess understanding of all students through conducting whole-class discussions

Day 1: Action Sequence #3, #4: Students collaborate with their team members to build alternative visual tile models of algebraic expressions, and the "quick-check" test

Action Sequence	**Sub-Episodes**		**Goals Activated** a b c d e
3. Small group discussion and work on constructing visual models of algebraic expressions (20 min.)	3.1. **Assessment**: The teacher walks from group to group looking at and discussing the tile structures students are building about algebraic expressions. **Content**: Algebraic expressions are obtained from the textbook. **Discourse**: The teacher asks students to construct each algebraic expression term-by-term, and then find equivalent structures or alternative solutions in the visual models or in symbolic forms. For those who have their first visual model built correctly, the discourse revolves about alternative visual models. And for those who have not built their first visual model, the discourse revolves about the general characteristics of the tiles and what they can represent in a visual model. **Pedagogy**: Small group collaboration and independent work by students, guided by the teacher's questions and expectations.	**Discourse**: The teacher also teaches 3 physical education classes at the school and some students talk to her about that. In general, discourse is not compulsory or forced, and the teacher and students function as a team, discussing mathematical tasks at hand.	
4. Assessment: Conduct a "quick-check" test of the students' understanding of constructing visual models of algebraic expressions (5 min.)	4.1. **Assessment**: The "quick-check test" at the end of the class consists of students drawing on a piece of paper the tile configuration for the expression 4(X+2) in two alternative ways.		

Goals:

a. Have teams of students collaborate and use algebra tiles to build visual models of algebraic expressions, identifying variables and constants
b. Review the important distributive property in the context of students' presentation of alternative solutions
c. Teach students operation sense as well as symbolic manipulation when simplifying or manipulating algebraic expressions
d. Assess understanding of all students through conducting small group discussions
e. Assess understanding of all students using a "quick-check" test

Day 2: Action Sequence #1: Answer and clear students' questions about the previous day's homework

Action Sequence	**Sub-Episodes**		**Goals Activated** a	b
1. Routine: whole-class discussion of previous day's homework (20 min.)	1.1. **Assessment**: The teacher conducts a formative assessment of students' understanding of the content as students ask questions about the homework while she is reading the answers to previous day's homework problems. Many students have questions. **Content**: Students express difficulty with the homework which was about constructing sketches representing visual tile models of algebraic expressions. **Discourse**: The teacher discusses students' questions while asking them to provide answers to her questions, guiding the discussions. Students are engaged and everyone provides input and explanation of their reasoning for constructing the visual models which the teacher is displaying on the overhead projector screen. **Pedagogy**: Whole-class discussion of student questions and ideas about the homework problems, which made use of the backward design strategy	1.1.1. **Pedagogy**: It was rare for students in any class the teacher taught to have this many questions, however, the teacher took the time to clear any questions students had. This is directly related to the teacher's oft stated strategy to teaching: build on students' prior knowledge		

Goals:

a. Assess students' understanding of previous homework and content material
b. Build on students' prior knowledge

Day 2: Action Sequence #2, #3: Students to find symbolic representation of visual tile models in terms of the area and the perimeter of the tile arrangements

Action Sequence	Sub-Episodes		Goals Activated a b c d e
2. Whole-class discussion of visual models of algebraic expressions (10 min.)	2.1. **Assessment**: The teacher gathers information about student understanding as she guides students through the lesson using questions. **Content**: The teacher uses the concept of area and perimeter to discuss visual models and algebraic expressions. **Discourse**: The teacher is displaying an arrangement of tiles on the projector screen, asking students to give her the algebraic expressions for the total area and the total perimeter of the tile arrangement. **Pedagogy**: Whole-class discussion of student ideas and reasoning	2.1.1. **Assessment**: Teams of students express and share their ideas with the teacher and the class **Content**: In one example, the desired expressions are: 4X+6 for the perimeter, and X^2+2X+2 for the area. **Discourse**: Includes alternative solutions in discussions of the distributive property	
3. Small group discussion of visual tile models (10 min.)	3.1. **Assessment**: The teacher discusses with each team their approach to writing the area and perimeter expressions for two separate tiles array structures. **Content**: Students are to come up with the expressions: 8X+6 (perimeter) and $2X^2$+4X+2 (area) for one set of tiles, and 6X+6 (perimeter) and X^4+5X (area) for another arrangement of tiles. **Discourse**: Students share ideas in small groups. **Pedagogy**: Small group discussion of ideas by students and the teacher		

Goals:

a. Assess students' understanding of the content during whole-class discussions
b. Assess students' understanding of the content during small group discussions
c. Use the concepts of area and perimeter to extend the meaning of visual tile models in connection with alternative symbolic representations
d. Emphasize the distributive property as a way of writing alternative algebraic expressions
e. Foster students' understanding of what it means to combine like terms in writing simplified algebraic expressions

APPENDIX O

Detailed Models of Lesson Segments: Ms. Johnson's Approach to Extending the Guess-and-Check Strategy to Develop Symbolic Generalization in the Form of Solvable Algebraic Equations (Math-8 Class)

Day 1: Action Sequence #1, #2: Students hear about the school announcements, and participate in a whole-class discussion of the guess-and-check problem solving strategy, extending and summarizing their ideas into solvable algebraic equations

Action Sequence	Sub-Episodes		Goals Activated: a	b	c	d
1. The teacher reads school announcements out loud to students in the class (3 min.)			✓			
2. Whole-class discussion of the guess-and-check strategy leading to symbolic generalization (25 min.)	2.1. **Assessment**: The teacher assesses students' understanding of the content as different students volunteer to provide a guess and follow through with the operations involving the guess; hence implementing the guess-and-check strategy. **Content**: From the textbook the question is: what is the number of girls and boys in the camp if there are 6 more girls than twice the number of boys, and there being a total of 156 boys and girls at the camp. **Discourse**: The teacher asks for various students to provide a series of guesses for the number of girls and boys. **Pedagogy**: Whole-class discussion of guesses and algebraic operations	2.1.1. **Assessment**: The teacher calls on those who do not volunteer to get their input, as well. **Content**: After a series of guesses and finding the answer, a generalized algebraic expression for the problem is worked out. A total of 3 problems are solved during the whole-class discussions **Discourse**: Sometimes students volunteer to provide a guess and sometimes the teacher calls out on individual students to get their input. The student providing the guess is then expected to carry the guess through a series of operations for one possible answer		✓	✓	✓

Goals:

a. Deliver school announcements for the day
b. Assess students' understanding of the content during whole-class discussions
c. Review the guess-and-check strategy of solving problems
d. Extend the guess-and-check strategy to solve problems and connect it to the symbolic generalization of mathematical ideas in the form of solvable algebraic equations

Day 1: Action Sequence #3: Students work in small groups to practice the guess-and-check problem solving strategy under the guidance of the teacher as they extend and summarize their ideas into solvable algebraic equations independently and collaboratively

Action Sequence	Sub-Episodes		Goals Activated a	b
3. Small group discussion of the guess-and-check strategy leading to symbolic generalization in the form of solvable algebraic equations (22 min.)	3.1. **Assessment**: The teacher enters each group, asking questions about students' guesses and the methods students are using to solve problems. **Content**: Assigned problems from the textbook and follows the content similar to the 3 examples discussed during the whole-class discussions. **Discourse**: Mostly dependent on the teacher's directions as she interacts with each team. Collaborative discussions tend to acquire a social tilt when the teacher walks away from the group. **Pedagogy**: Small group discussion of the guess-and-check strategy and connecting ideas within mathematics	3.1.1. **Discourse**: A key reason for keeping small group discourse on track is the teacher's non-stop walking around and talking with the members of each group about the goals of their activity. At each point of interaction with each group, the teacher asks questions about the content. The questions stem from her expectations and goals for the lesson. **Pedagogy**: Requiring students to turn in their work at the end of the class and making class homework problems more challenging have had a tangible positive impact on reducing socially oriented conversation within small groups and have increased completion rate for classroom homework		

Goals:

a. Assess students' understanding of the content during small group discussions as well as having students turn in their class work at the end of the class period
b. Students work independently and collaboratively to extend the guess-and-check strategy of solving problems, connecting it to the symbolic generalization of mathematical ideas in the form of solvable algebraic equations

APPENDIX P

Sample Games Used by Ms. Johnson

(Game 1: October 16th, Math Eight)

2/9	1/24	2/28	3/4
1/16	21/25	8/30	2/49
9/14	6/14	2/18	8/60
18/30	12/35	16/45	6/10

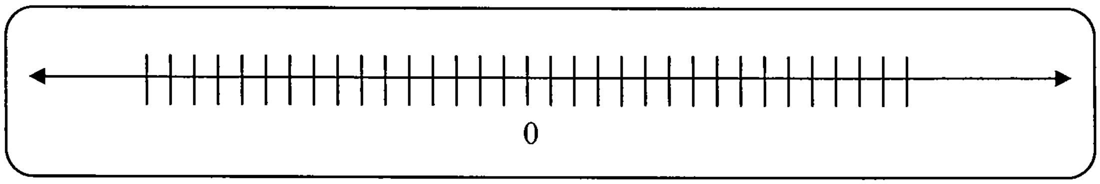
1/3	8/15	1/8	2/3	1/7	1/2	3/2	1/4	1/5	7/5	2/7	6/5	3/4	3/5	2/9	3/7

The game: There were two paperclips that could be moved along the numbers seen in the horizontal table shown below the top table. The student teams were to decide where the paperclips must be placed along the bottom table such that the resultant from multiplying the fractions in the cells, where the paperclips were, was the fraction in a single cell of the top table. When a team got the numbers in the top table correctly, it placed a symbol on top of the number. Any alignment of four symbols of the same kind in the table won the team the contest.

(Game 2: October 18th, Math Seven)

The game: the seats were arranged such that there was a seat for each team at the front of the class, facing the whiteboard. Each team of students still sitting back in their seats was given a color name that represented the team (e.g. team green, team red, etc.). Each team member in a group was assigned a number from 1 to 4 depending on how many students were on the team. From each team, members #1 sat in the seats that lined up in a row in front of the class and the game began as Ms. Johnson slowly uncovered a computational problem to be solved. The students sitting in the active seats got to work and as soon as each person finished, he or she got up and wrote the answer to the problem on the board and went back to his/her team. The first team with the right answer received 1000 points. The second team with the right answer received 500 points, and the third and all the other teams that came in third received 100 points each. Members #2 took their seats at the front as Ms. Johnson uncovered the second problem at the document camera. And the same process was repeated for as many problems as there were. Every computational problem was different than the next and students did not know what the problem was until Ms. Johnson uncovered it. The team that accumulated the most number of points at the end of the class won the game. This particular game was fast paced and lasted for 40 minutes.

(Game 3: October 19th, Math Eight)



The game: A large 7feet-by-2feet poster of the real number line was hung in front of the whiteboard that was located in front of the classroom. The increments on the real number line were 1/10th each. The teacher gave all students small post-its with a fraction written on each. The students were then asked to get up and stick the fractions where they belonged on the real number line. After this, students were asked to determine the accuracy of the fraction placements. Once any error was fixed by students who got up and corrected any problems, the students were randomly given equivalent decimal values for the fractions on the poster and asked to place them very near the fractional equivalent. As this task was finished as before, any errors were corrected once again by students getting up and placing the sticky on the right location. Yet, the students were given a series of mixed sticky notes with equivalent percent values of the numbers already on the poster written on them. The students were to get up and place the equivalent percent values very near or on the bottom of their equivalent fraction and decimal numbers. Any error placements were corrected as before.