FLEXURE OF STRUCTURAL SANDWICH CONSTRUCTION

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FLEXURE OF STRUCTURAL SANDWICH CONSTRUCTION[1]

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SUMMARY

This paper presents some of the basic theories of flexure, particularly as
applied to sandwich constructions. Consideration of various methods of applying
loads and a theoretical analysis of a particular type of loading resulted in
the conclusion that measurements of stiffness and strength may best be obtained
by testing under two-point loading, preferably at two quarter-span points. A
formula is presented for determining the approximate span, so that shear deflec-
tions can be easily measured. Flexural strength, the nonelastic flexure of
sandwich construction, and the clamping of sandwich constructions are discussed
briefly.

Introduction

Flexure testing of a material was perhaps one of the earliest means of evalua-
ting stiffness and strength. Observations on the bending behavior of different
woods probably aided primitive man in selecting the proper stone-ax handle or
bow and arrow. This type of "hand" testing is usually conducted by anyone who
is handed a piece of material to inspect. The tester advances comments con-
cerning the material's amazing stiffness and strength, if appropriate, or per-
haps a comment as to how poorly the material behaves. Uses for the material are
often suggested if it appears to be unusually stiff and strong, or, if the
material is weak, it may be hinted that there is no possible application.

The result of the "hand" testing must be properly interpreted. The ultimate use
of the material and the performance requirements of the finished product must
be carefully considered. A string or wire will fail to meet the lowest flexure
requirements, but both find wide use as tensile structural elements. Often the

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[1] Presented at a meeting of American Society for Testing Materials Committee
   C-19, on structural sandwich construction, in Washington, D. C. October
   22, 1951.

[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

shape of the finished product contributes more to successful performance than the material itself. For example, thin, flexible sheets of steel are now being used for more or less acceptable automobile bodies.

The invention of testing machines has provided means for obtaining more quantitative information concerning materials than can be obtained by "hand" testing. These machines and all of their modern, automatic gadgets, however, must be used with caution in order to arrive at sensible conclusions concerning the behavior of a material.

Possibly, the salient question to be considered before performing a test is "What is to be evaluated?" It certainly would not be helpful to determine the tensile strength of concrete that is to be used in compression. It may be feasible, however, to conduct quality-control tests in tension rather than in compression. Similarly, for most materials, it is not necessary to conduct flexure tests in determining mechanical properties, although it may be feasible to conduct quality-control tests in flexure. Flexure tests may be, in fact, exactly what is needed to evaluate the adequacy of constructions.

Testing, therefore, is somewhat divided into two parts, quality-control tests and evaluation of mechanical properties. The principal concern of this paper is the evaluation of mechanical properties.

The Flexure Test

By means of the flexure test, loads can be applied to the test specimen to produce constant or variable bending moments and constant or variable shears. Conditions at the reactions can be clamped or simply supported. The actual type of loading and support should be considered in view of what information is to be obtained.

Uniformly distributed loads can be experimentally applied by various ways, but these are usually costly and time-consuming.

Clamping of the ends of a beam or the edge of a panel is never accomplished successfully by empirical means. It can be shown, however, that the effective experimental clamping can be empirically determined for use in calculating maximum bending moments and shears.

The method remaining is the use of concentrated loads to produce constant shears and constant or variable bending moments in a specimen simply supported at the reactions.

The simplest load to apply is a single, concentrated load applied at midspan. This center loading produces stress concentrations at the load point of twice the magnitude of corresponding stress concentrations at the reactions. The point of maximum moment is at the load point. Thus, if the material is not homogeneous, a weak spot near the reactions would not be discovered by testing under a single load. In sandwich constructions particularly, the stress at which a facing wrinkle appears is mostly a guess, because the facing is held to the core at the point of maximum moment.

The load may also be applied through two points, preferably symmetrically placed with reference to the midspan point for simplicity. The use of two-point loading produces constant bending moments in the portion of the specimen between the loads and constant shears in the portions outside the loads. Thus, within the central section, more of the material is stressed to the same amount in two-point loading than in center loading, and failure due to moments will begin at the weakest point. Stress concentrations still exist at the load points, but they are one-half the magnitude of those applied with a single load and are equal to concentrations at the reactions.

How are the load points to be spaced? The ultimate use of many constructions is to withstand uniformly distributed loads. If the loads are applied at two quarter-span points, the greatest moments and shears produced will be equal to those applied by uniformly distributed loads. Therefore, the best method of loading in a flexure test is to apply two concentrated loads at the quarter-span points on a beam simply supported at the reactions.

Theoretical Considerations

From the test data of a beam, the stiffness and strength will undoubtedly be computed. For most materials, the beam deflection depends mainly upon the modulus of elasticity of the material. There is always some additional deflection due to shear, but this is usually so small that it is neglected. For sandwich constructions, however, the core material has a relatively low shear modulus, and the beam may deflect a considerable amount because of shear deformations. This low shear modulus of the core will also decrease the buckling loads of plates and shells. Since neglect of proper consideration of the shear modulus of the core may lead to unconservative prediction of deflections or critical loads, it is essential that the modulus be determined in order that its effect may be taken into account.

One way to determine the shear modulus of a core of structural sandwich construction is by means of a flexure test under two-point loading. The two-point loading allows the central portion of the beam to bend under constant moment without shear, and, therefore, the stiffness of the sandwich construction with no shear deflection may also be obtained by proper measurement of deflections. The span of the test specimen should be chosen so that deflections due to shear are sizeable as compared to bending deflections. The following discussion and derivation give the formulas for analyzing the test data for a sandwich beam simply supported at the reactions and loaded at two quarter-span points.

The beam is assumed to be loaded with a total load, $\underline{P}$, equally applied at two quarter-span points, each spaced a distance, $\frac{a}{4}$, from the reactions on a total span, $\underline{a}$.

The general expression for the curvature of a beam, assuming positive deflections downward and the axis origin at the left reaction ($\underline{x}$ positive to the right), is given by[3]

[3]Timoshenko, S. Strength of Materials, Part I. New York. 1940.

$$\frac{d^2y}{dx^2} = -\frac{M_x}{D} + \frac{1}{N}\left(\frac{dV_x}{dx}\right) \tag{1}$$

where y = deflection (measured positive downward)

x = distance from left reaction

M_x = bending moment (positive bending produces positive curvature)

D = bending stiffness of beam

V_x = shear load (shear is positive if the force on the positive side of an elemental beam length is in the positive y direction)

N = shear rigidity of beam

The deflection is determined by solving formula (1). As usual, the beam is considered in its several parts:

Section 1, from $x = 0$ to $x = a/4$,

where $\qquad M_{x1} = \frac{Px}{2} \qquad\qquad V_{x1} = \frac{P}{2}$

and then $\qquad \left(\frac{d^2y}{dx^2}\right)_1 = -\frac{Px}{2D} + \frac{1}{N}\left(\frac{dV_x}{dx}\right)$

$$\left(\frac{dy}{dx}\right)_1 = -\frac{Px^2}{4D} + \frac{P}{2N} + C_1$$

$$y_1 = -\frac{Px^3}{12D} + \frac{Px}{2N} + C_1x + C_2$$

When $x = 0$, $y = 0$. Therefore, $C_2 = 0$

and $\qquad y_1 = -\frac{Px^3}{12D} + \frac{Px}{2N} + C_1x$

Section 2, from $x = a/4$ to $x = 3a/4$,

where $\qquad M_{x2} = \frac{Pa}{8} \qquad\qquad V_{x2} = 0$

and then $\qquad \left(\frac{d^2y}{dx^2}\right)_2 = -\frac{Pa}{8D}$

$$\left(\frac{dy}{dx}\right)_2 = -\frac{Pax}{8D} + C_3$$

When $x = a/2$, $\dfrac{dy}{dx} = 0$. Therefore, $C_3 = \dfrac{Pa^2}{16D}$

and $\left(\dfrac{dy}{dx}\right)_2 = -\dfrac{Pax}{8D} + \dfrac{Pa^2}{16D}$

$$y_2 = -\frac{Pax^2}{16D} + \frac{Pa^2x}{16D} + C_4$$

The constants C_1 and C_4 are found by conditions at $x = a/4$. At this point, the slope due to bending is the same in section 1 as in section 2. Therefore,

$$\left(\frac{dy}{dx} - \frac{V_x}{N}\right)_1 = \left(\frac{dy}{dx} - \frac{V_x}{N}\right)_2$$

from which

$$\left(\frac{dy}{dx}\right)_1 - \frac{P}{2N} = \left(\frac{dy}{dx}\right)_2$$

Then

$$-\frac{Pa^2}{64D} + \frac{P}{2N} + C_1 - \frac{P}{2N} = -\frac{Pa^2}{32D} + \frac{Pa^2}{16D}$$

from which $C_1 = \dfrac{3Pa^2}{64D}$

Therefore, $\left(\dfrac{dy}{dx}\right)_1 = -\dfrac{Px^2}{4D} + \dfrac{P}{2N} + \dfrac{3Pa^2}{64D}$

$$y_1 = -\frac{Px^3}{12D} + \frac{Px}{2N} + \frac{3Pa^2x}{64D}$$

Also, at $x = a/4$, $y_1 = y_2$.

Then $-\dfrac{Pa^3}{768D} + \dfrac{Pa}{8N} + \dfrac{3Pa^3}{256D} = -\dfrac{Pa^3}{256D} + \dfrac{Pa^3}{64D} + C_4$

and $C_4 = -\dfrac{Pa^3}{768D} + \dfrac{Pa}{8N}$

Therefore,
$$y_2 = -\frac{Pax^2}{16D} + \frac{Pa^2x}{16D} - \frac{Pa^3}{768D} + \frac{Pa}{8N}$$

Finally, the deflections of most interest, those at the load points ($x = \frac{a}{4}$) and at midspan ($x = \frac{a}{2}$), are given by

$$y_{a/4} = \frac{Pa^3}{96D} + \frac{Pa}{8N} \tag{2}$$

$$y_{a/2} = \frac{11Pa^3}{768D} + \frac{Pa}{8N} \tag{3}$$

If the deflection at the quarter-span point (formula 2) is subtracted from the midspan deflection (formula 3), it is seen that the deflection of the central portion of the beam is caused by bending only, with no shear deflection. Then

$$y_{a/2} - y_{a/4} = \frac{3Pa^3}{768D} \tag{4}$$

The midspan deflection due to bending alone is therefore 11/3 of the deflection of the central portion of the beam.

From the previous derivation, the deflection due to shear may be obtained by measuring midspan and load-point deflections. Subtraction of the load-point deflection from the midspan deflection yields the bending deflection of the central portion of the beam. Multiplying this by 11/3 and subtracting from the midspan deflection yields the shear deflection. Thus,

$$\frac{Pa}{8N} = y_{a/2} - \frac{11}{3}(y_{a/2} - y_{a/4})$$

which reduces to

$$\frac{Pa}{8N} = \frac{1}{3}(11y_{a/4} - 8y_{a/2})$$

From the last equation, the shear rigidity, $\underline{N}$, is given by

$$N = \frac{3Pa}{8(11y_{a/4} - 8y_{a/2})} \tag{5}$$

If only shear rigidity is to be determined, a mechanical device that automatically measures the bending deflection of the central portion of the beam, multiplies this deflection by 11/3, and subtracts the result from the total midspan deflection can be used to obtain the shear deflection directly during the test.

The bending stiffness may be computed from the bending deflection of the central portion of the beam by

$$D = \frac{3Pa^3}{768(y_{a/2} - y_{a/4})} \tag{6}$$

In order to obtain the shear deflection, it is necessary to subtract the bending deflection from the total deflection. If the specimen is too long, the shear deflection may be reduced to an imperceptible quantity. The span should be chosen so that the shear deflection at the load point is approximately 40 percent of the total deflection, thereby rendering test errors of small consequence. The span may be estimated by the following process: The load-point deflection is given by formula (2):

$$y_{a/4} = \frac{Pa^3}{96D} + \frac{Pa}{8N}$$

The shear deflection, $\frac{Pa}{8N}$, is to be 4/10 of the total deflection; therefore, the total deflection is to be 10/4 of the shear deflection, or $\frac{10Pa}{32N}$. Substituting this value in formula (2) gives:

$$\frac{10Pa}{32N} = \frac{Pa^3}{96D} + \frac{Pa}{8N}$$

or

$$\frac{6Pa}{32N} = \frac{Pa^3}{96D}$$

Therefore,

$$a^2 = \frac{18D}{N}$$

or

$$a = 3\sqrt{\frac{2D}{N}} \tag{7}$$

Formula (7) can be used to obtain the approximate span on which the sandwich beam should be tested. The bending stiffness, D, can usually be computed with sufficient accuracy if the properties of the facings are known. If it is assumed that the core does not contribute to the stiffness of the sandwich construction ($E_F/E_c > 100$, where $\underline{E_F}$ and $\underline{E_c}$ are moduli of elasticity of the facing and core, respectively), the bending stiffness is given by[4]

$$D = \frac{E_F(h^3 - c^3)b}{12\lambda_F} \left[1 - \frac{3(f_2 - f_1)^2}{(1 + \frac{c}{h} + \frac{h}{c})(f_2 + f_1)^2} \right] \tag{8}$$

where E_F = modulus of elasticity of facing

h = total thickness of sandwich construction

c = thickness of core

b = width of sandwich construction

λ_F = 0.91 for isotropic facings and 0.99 for orthotropic facings

$f_{1,2}$ = thicknesses of facings

[4]March, H. W. and Smith, C. B. Flexural Rigidity of a Rectangular Strip of Sandwich Construction. Forest Products Laboratory Report No. 1505. March 1949.

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The appearance of λ_F in formula (8) has been partially verified by test. A wide plate of sandwich construction was tested in bending. Strain gages, which measured strains at right angles to the span and at points across the width of the specimen, showed that the strain was nearly zero at the center of the width and was a sizeable value at the edges. Therefore, the λ_F must be used to describe correctly the deflection of the plate. Subsequent testing of narrower and narrower strips cut from the plate showed no change in stiffness for widths as small as 1 inch; therefore, the λ_F must also be used for narrow strips of sandwich construction.

The shear rigidity, N, of the sandwich construction is given by[2]

$$N = \frac{(h + c)b}{2} G_c \qquad (9)$$

where G_c = shear modulus of the core material.

Values of D and N can be computed and substituted in formula (7) to obtain the span. The span, however, is not to be determined with great accuracy, and it is possible to make some simplifying approximations of the bending-stiffness formula. Formula (8) for the bending stiffness can be approximated by

$$D \approx \frac{chb(f_1 + f_2)E_F}{4} \qquad (10)$$

if

$$1 \gtrsim \frac{f_1}{f_2} > 0.5 \text{ and } c/h > 0.7 \quad .$$

If it is also assumed that $c = h$, formula (7) then becomes:

$$a = 3c \sqrt{\frac{(f_1 + f_2) E_F}{2c \, G_c}} \qquad (11)$$

Values of the shear modulus, G_c, may have to be estimated, but, since the span depends on the square root of the estimate, the error of the estimate may not be great.

After the span is determined by formula (11), the sandwich construction is tested, and the appropriate deflections are measured. The shear modulus of the core material is computed from the test data using the formula

$$G_c = \frac{3Pa}{4(h + c)b \, (11y_{a/4} - 8y_{a/2})} \qquad (12)$$

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[2]Forest Products Laboratory. Sandwich Construction for Aircraft, Part II. ANC-23 Bulletin, p. 39, par. 3.13. May 1951. Published by the Munitions Board.

If the shear deflections are measured directly (by proper instrumentation), shear modulus of the core is given by

$$G_c = \frac{Pa}{4(h + c)by_s}$$ (13)

where y_s = measured shear deflection.

Flexural Strength

The flexure test provides data from which the bending stiffness of the sandwich construction and the shear modulus of its core can be obtained. By continuing the test to failure, the flexural strength can also be measured.

A sandwich construction will fail in the facings by facing compression, tension, or wrinkling, or in the core by shear. Whether the construction fails in the facings or core depends on the relative moments and shears induced by the applied loads.

Experience with materials now available shows that loading at two quarter-span points will usually cause core failures if the sandwich has high-strength facings, such as are used for aircraft constructions. Facing failures will occur if the sandwich has low-strength facings, such as have been used for house constructions. Therefore, it is possible to continue the flexure test under quarter-point loading to produce core failure at a shear stress, τ_c, given by

$$\tau_c = \frac{2V}{(h + c)b}$$ (14)

where V = shear load, or a facing failure at a stress, $\sigma_{F_{1,2}}$, given by

$$\sigma_{F_{1,2}} = \frac{2M}{f_{1,2}(h + c)b}$$ (15)

where M = bending moment.

For large deflections, the bending moment should be increased by the amount

$$\frac{Py}{2} \tan \theta$$

where y = midspan deflection
       θ = slope of beam at reaction

The slope at the reaction can be roughly determined by fastening a protractor to the tilting reaction plate.

The test specimen of sandwich construction should be proportioned so that failures occur where desired. For example, if the shear modulus and shear strength of the core are to be determined, the sandwich construction would be loaded in flexure at two quarter-span points. The facings and core should then be thick enough so that the facings will not fail at a load less than that estimated to cause the core to fail. If the same sandwich construction is to be tested to determine the stress at which the facings fail, the load points can be placed closer together than at quarter-span points, thus increasing the bending moment and also increasing the shear area of the core.

Facings of sandwich constructions having honeycomb cores have been found to fail at stresses that cause dimpling of the facing into the honeycomb-core cells.[6] The stress at which dimpling occurs, σ_{F_1}, is given by[7].

$$\sigma_{F_1} = \frac{E_R}{3}\left(\frac{f_1}{R}\right)^{3/2} \tag{16}$$

where E_R = reduced facing modulus $\dfrac{4EE_T}{(\sqrt{E} + \sqrt{E_T})^2}$

E_T = tangent modulus
E = Young's modulus
f_1 = thickness of compression facing
R = radius of circle inscribed in core cell.

Past experience has shown that rarely does a tension facing fail unless the tension facing is thinner or of a different material than the compression facing. Tension facings fail at the tensile strength of the facing material.

Nonelastic Flexure of Sandwich Construction

Sandwich construction presents a material that is unique in its performance in flexure in that the load-carrying material is sensibly concentrated at its outer surfaces. Therefore, an analysis of behavior above proportional-limit stresses need include the variation of modulus of elasticity as affected only by changing moment along the span of the beam, not by distance from the beam's neutral axis.

This type of analysis was successfully applied to the deflection of aluminum-faced constructions having facings of equal or unequal thickness. The results of the analysis enabled the plotting of a load-deflection curve from the beginning of the test until failure. The curves obtained for the specimens tested agreed fairly well with the load-deflection curve obtained by actual measurements.

[6]Kuenzi, E. W. Edgewise Compressive Strength of Panels and Flatwise Flexural Strengths of Strips of Sandwich Constructions. Forest Products Laboratory Report No. 1827. May 1951.

[7]Norris, C. B. and Kommers, W. J. Short-column Compressive Strength of Sandwich Constructions as Affected by the Size of the Cells of Honeycomb-core Materials. Forest Products Laboratory Report No. 1817. August 1950.

The analysis is outlined briefly as follows: The usual expression for curvature of the beam in terms of the moments is written with the stiffness expressed in terms of the modulus of elasticity of each facing. The tangent modulus of each facing is then written as a function of the stress. This function is determined empirically from stress-strain curves for the facing material. The expressions for the tangent moduli are substituted for the appropriate facing moduli in the expression for curvature and the facing stresses expressed in terms of the moments. The curvature expression is then integrated for the particular type of loading under consideration.

Deflections due to shear deformations in the sandwich core and the effect of exceeding the proportional-limit stress of the core in shear can also be included in the analysis.

The analysis showed that, if a sandwich construction having unequal facings of aluminum is loaded to failure in flexure, the deflection is much larger and the load "hangs on" much longer if the construction is loaded so that the thin facing is in tension rather than in compression.

Clamping of Sandwich Constructions

It may be desired to determine the behavior of a sandwich plate or beam having the edges clamped as they would be in actual use. The theoretical condition of perfect clamping can never be attained. The flexure-test results can be analyzed, however, by determining an effective experimental clamping constant and using this constant to compute the maximum moments and shears.

The procedure is to multiply the theoretical clamped moment by the experimental clamping constant. Using this product of the constant and the theoretical clamped moment, the usual expressions for moments and shears are written, and the deflections determined. All the expressions will be in terms of the constant. The constant can be determined by comparing the experimental deflections with the theoretical deflections for simply supported and clamped end conditions. Thus, the constant is determined experimentally from the deflections, and the maximum moments and shears can then be computed. The constant is independent of shear deflections of the sandwich construction and the shear deflections are the same for both simply supported and clamped end conditions. For some radome constructions that were tested, the effective clamping was 40 to 70 percent. As the stiffness of the sandwich construction increased, the effective clamping decreased.

Conclusion

This discussion of the flexure test, theoretical considerations, flexural strengths, nonelastic flexure, and clamping of sandwich constructions attempts to dispel some of the misconceptions concerning the flexural behavior of sandwich constructions. Undoubtedly, exception can be taken to some of the basic ideas if they are applied blindly to describe the behavior of some extremely special constructions. The discussion, however, was not intended to cover completely the field of sandwich flexure. It should be remembered that testing should only be attempted after careful consideration of what is being evaluated, rather than merely for the sake of testing.