A MATHEMATICAL DESCRIPTION OF LUMBER DRYING

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In former times, lumber and wood products were dried exclusively by air drying: sometimes outside, with no more control than the placement of a cover to keep off the rain; and for more critical cases, by drying in dry-sheds where the lumber could be protected from the direct rays of the sun, and where air velocity could also be controlled. In both cases, much time was needed and production schedules, as we know them today, were non-existent. About a hundred years ago, kiln drying was introduced, and with it came the need for more knowledge of how wood dries, and the need for kiln schedules which would dry lumber fairly quickly with a minimum of degrade.

The development of kiln schedules always was, and still is, a trial-and-error process. When kiln operators are required to develop or modify kiln schedules, they are at a serious disadvantage in several respects: they usually work with kiln charges of about 100 Mfbm, and any disastrous results can be very expensive indeed. Consequently, they tend to be rather conservative in any schedules they develop. Furthermore, they have no way of knowing, apart from intuition and a few crude tests, the results of any schedule until drying is complete. They have an advantage, however, in that they run charges very frequently, and by making many small changes, rather than a few major ones, they can usually arrive at a satisfactory schedule in a reasonable time. Most kiln operators do not make any more improvements than they absolutely have to.

Kiln schedules are also developed by scientists working in research laboratories and universities. Relative to the kiln operator, they have some advantages in this type of work. They work with smaller charges, and are permitted, even required, to dry lumber with schedules that occasionally lead to disaster. This is necessary to establish the safe limits of temperature, humidity or air velocity. Most research kilns are also equipped with scales so that the moisture content of the charge can be determined at any time during drying. Scientists have time, also, to examine the dried lumber in great detail, to observe and measure the effect of the drying schedule on lumber quality. Thus, the advantage of the scientist over the kiln operator in developing schedules is that he has better equipment, and working conditions are more conducive to experimentation.

And yet, in one major area, there is little difference between the scientist and the kiln operator; both approach kiln schedule development by trial-and-error. Many scientists have studied, as part of their education, theories and mathematics related to the drying of wood. Yet, to the best of my knowledge, this theoretical or mathematical approach is seldom, if ever, used in the solution of real-life problems of schedule development or related problems. The approach is almost entirely trial-and-error.

This fact was brought home to me quite forcefully a few years ago at the Western Forest Products Laboratory. After twenty years research in wood preservation and fire-retardant treatments, I was asked to transfer to the Seasoning Section where a lot of research was needed due to the introduction of the new 19% MC drying standards. One of my first assignments was to develop a fast drying schedule for one of our western softwoods. Since it was my first attempt at schedule development, I asked my colleague how I should go about it, and more specifically, I asked what would be the effect on drying rate of increasing the kiln temperature $10^{o}F$, or of changing the wet-bulb temperature a specific number of degrees. I was rather surprised to find that he had no idea. So I searched the literature on the subject, and again I found no information.

Thus, it would appear, in spite of all his education, the university-trained researcher is actually little better off than a good kiln operator in terms of being able to predict drying times with a given change of schedule. I felt that this was an unfortunate situation that should be corrected, and set about to do so.

Drying lumber, from an academic point of view, is the combination of a few processes that are, or should be, describable in terms of the laws of physics. And physics is a branch of science that can be accurately expressed in mathematical terms. Therefore it should be possible to describe drying in terms of mathematics.

Let us consider some other accomplishments of mathematics as an example. When an engineer builds a bridge, he makes a general plan, and calculates the stresses to be sustained by the various components of the bridge. He then calculates the size of the components necessary to withstand the stress. Then he builds the bridge, and it meets the specifications originally set down.

Consider the trips of the astronauts to the moon. Each trip was calculated in detail beforehand, and in each case it came off successfully.

Now, since drying is an exercise in physics in a different area, it should be possible to draw up all the laws and equations, and by a series of calculations predict the moisture content of a charge of lumber throughout the entire process by any schedule we care to choose. Thus, if we want to dry lumber in a shorter time, it should be possible to calculate on paper a schedule that will dry the lumber to the specifications we desire. It should also be possible to calculate the stresses within the lumber at any time, and design the schedule to keep these stresses below a critical level where the wood may be damaged.

It became my ambition, or goal, to describe the drying of lumber in mathematical terms that would apply to a wide range of conditions or schedules. It should be understood that this was not to be the first such attempt. Several scientists have tried to do so in the past, and one such attempt, by Claxton in 1966, was reported to the West Coast Dry Kiln Clubs. However, my attempt was different from previous ones in one critical area: in the equations used to describe the process of diffusion.

The movement of moisture in lumber is primarily by the process of diffusion. Previous investigators have tried to describe this process by using Fick's laws of diffusion, not realizing that Fick's laws have a very limited area of validity. There are several reasons why Fick's laws cannot be used to describe lumber drying. We need consider only one of them: Fick's laws are limited to isothermal conditions. That means that there can be no variation of temperature in the whole system. But, in fact, isothermal conditions are never found in lumber drying. The wood surface is always cooler than the kiln temperature, and even the wood has a distinct temperature gradient within it. There are several other reasons why Fick's laws cannot be used, but this one is enough for illustration.

Before any serious assault could be made on the problem of describing drying mathematically, it was necessary to describe diffusion in terms that are valid for wood. Fortunately, I was given that information during my education: drying takes place in response to a vapor pressure gradient. I have been able to show that this law applies to a wide range of conditions, including that for lumber drying, and have used it to develop my "mathematical model" of lumber drying.

It is because previous investigators were unaware of this fact that their results have been less than successful.

The details of the model are of interest to physicists and mathematicians primarily; articles describing these details are expected to appear in the July 1979 issue of Wood Science.

Let us now consider the principal processes that take place when lumber is dried. These are:

(1) First of all, heat is conducted from the circulating kiln air through the surface boundary layer (the stagnant film of air surrounding the wood), to the wood surface. The rate of heat transfer is determined by the conductance of the boundary layer, and this in turn is affected by the velocity of air passing through the charge, and by the difference in temperature between air and lumber.

$$\text{Rate of heat transfer} = U(T_{air} - T_{wood})$$

where U is the conductivity of the boundary layer, and T is temperature.

(2) After reaching the wood surface, the heat is conducted into the wood. This takes place in conformance with Fourier's laws of heat conduction: the rate depends on the temperature gradient within the wood and the conductance of the wood. In general, wet wood is much more conductive than dry wood, and equations have been worked out showing this relationship.

$$\text{Rate of heat transfer} = K \frac{dT}{dx}$$

where K is the conductivity and dT/dx is the temperature gradient.

(3) Within the wood, the increased temperature "activates" the moisture molecules, permitting them to diffuse. Bound-water is firmly attached to hygroscopic sites in the wood, and as such it cannot diffuse or migrate. In exactly the same way that heat must be added to water to convert it to steam so that it can evaporate, so heat must be added to bound water to convert it to "diffusable" water.

(4) As a result of adding heat and converting the bound water to diffusable water, a "pressure" is generated, a "crowd effect" of the activated molecules with their tickets to somewhere, and anxious to get started. This is the vapor pressure, and can be expressed approximately by the Clausius-Clapeyron equation

$$p = h \exp(a - E/RT)$$

where e is the base of natural logarithms, E is the energy needed to convert bound water to diffusable water, R is the gas constant, and T the absolute temperature of the wood. a is a constant which varies with the units used to measure the pressure. h is the relative humidity in equilibrium with wood at its particular moisture content. Low moisture content wood is in equilibrium with a low humidity, and high moisture content wood with a high humidity. A complex equation is used to relate the relative humidity with the moisture content.
(5) Where a vapor pressure difference occurs within wood, moisture will diffuse from one part of the wood to another, and where the kiln atmosphere has an even lower vapor pressure, moisture will leave the wood, permitting it to dry.

$$\text{Rate of diffusion or evaporation} = D \frac{dP}{dx}$$

These processes and the equations as we have given them make up the skeleton of the mathematical model, which describes the drying process. For the model to be more complete, it requires additional details, such as the heat and moisture conductivities of the surface boundary layer, and the heat and moisture conductivities of the wood itself at various moisture contents. Some of this information is available in the literature. Some had to be determined from experiments we conducted in this investigation.

One interesting process we have not yet mentioned is capillary flow. During the early stages of drying, when the wood still has a lot of free water, threads of moisture move in response to capillary forces toward the wood surface. In permeable woods, this capillary flow is believed to be an important factor in drying. The mathematical model must include equations for this process if it is to be complete. We will describe how these equations were determined in a few minutes.

The Computer Simulation

Now, if we have a complete mathematical model of drying, we should be able to use it. Very simply, we describe the lumber which we start with: its temperature, its moisture content, and its thickness. We assume that, since lumber is packed tight in courses or tiers in the kiln, evaporation can take place only from the upper and lower faces. Now, since our mathematical model is supposed to describe exactly the forces and processes involved in drying, including the rate of drying under any set of conditions, all we need to do is make the appropriate calculations to know the moisture content and several other items of information at any time during drying. Since the calculations are somewhat involved and very tedious, we do not do them manually, but program a computer to do them for us. The computer will then print out all the information we require every hour, or whatever time we choose.

But the fact is, that at the beginning of this project, although we felt we had the skeleton of a mathematical model, we did not know many of the details. Some details we were able to determine rather quickly from experiments, but finding the others turned out to be a major undertaking.

Before we describe some of the details, let us make a comparison. In previous attempts by scientists to develop a mathematical model, the process of diffusion was described by Fick's equations. But these equations are not applicable to wood, and even in systems where they are applicable, they are restricted to isothermal conditions. This meant that any model developed would be usable at only one temperature. The model we were attempting to develop would need to be valid at any temperature, except above the boiling point of water. This restriction is due to the fact that another process is involved above the boiling point, and we felt that the area we were investigating would provide enough problems without taking on any more.

To ensure that the details, the diffusion coefficient and the expressions for capillary flow, would not be restricted to one temperature, we performed two experiments: one at 200/160, and the other at 160/128 (dry bulb/wet bulb). We then plugged numbers and equations into our otherwise complete computer program for the diffusion coefficient and for what we hoped would describe capillary flow, and compared the computer output with our experimental results. After possibly hundreds of changes and computer runs using different values and equations, we finally obtained calculated drying curves that fit both experimental drying curves almost exactly, and our mathematical model was complete in every detail.

But we still had further tests to make. We had to use the computer to predict the drying curves for other schedules not used to develop the model. We chose two schedules that we thought would present quite a challenge. One was a constantly-increasing-temperature schedule, and the other was a step schedule. The agreement between experimental results and computer results was excellent: within about 1% MC all the way.

While it was not necessary for the completion of the project, we felt it would provide an excellent visual presentation of our results to have the computer make a moving picture film for you. My son Paul, who is a computer programmer at Forintek (the successor to the Western Forest Products Lab) prepared another computer program to take the results of my program and graph them on a cathode-ray tube (television screen). We then borrowed a time-lapse motion picture camera, and with some Rube Goldberg equipment, programmed the computer to draw 4000 graphs on the screen and photograph them. The results were shown in a movie.

Future Research

While the studies which resulted in this film are a valuable contribution to our knowledge of lumber drying, they are merely a stepping stone to future research. The model, as it presently stands, will work for any desired lumber thickness, and for a wide range of temperature. The following research is now required to further expand our knowledge of the drying process.

(1) The model is now restricted to a circulating air velocity of 350 fpm. Inconsistent reports in the literature as to the effect of air velocity on heat- and moisture-transfer at the wood surface makes it inadvisable to include equations for the effect of air velocity. Future research is required to establish these effects. When this information is known and expressed in mathematical terms, it can be incorporated into the model, making it of wider application.

(2) Species differ from each other in terms of moisture movement in two important respects: their densities, which affect the value of the moisture diffusion coefficient; and their permeabilities, which affect the ease of liquid flow through the voids. The relationships between density, diffusion coefficient, and capillary flow resistance must be established for the various species, and when this is done their drying rates can be predicted by the model.

(3) One serious consideration limiting the rate of lumber drying is the development of drying stresses within the lumber. At present there is no way to measure or estimate these stresses without destruction of the piece. The model provides a statement of both moisture and temperature distributions within wood during drying. It should be possible to combine these data with plasticity and elasticity data for wood in order to estimate the stresses at any time, and to control the schedule to keep these below a critical level.